Measuring Tail Risks at High Frequency

Brian Weller*
Northwestern Kellogg
April 26, 2016

Abstract

I use the cross section of bid-ask spreads to develop a new measure of extreme event risk. Equilibrium spreads embed tail risk information because (1) sharp changes in underlying asset values harm liquidity providers and (2) price movements and potential costs are linear in factor loadings. Using this insight, simple regressions relating spreads and trading volume to factor betas recover tail risks for common factors in stock returns. My methodology disentangles financial and aggregate market risks during the 2007–2008 Financial Crisis; quantifies jump risks associated with FOMC announcements; and anticipates extreme market risks before the 2010 Flash Crash.

JEL: C58, G01, G12, G14, G17
Keywords: Tail Risks, High-Frequency Market Making, Bid-Ask Spreads

I. Introduction

Fear of extreme market movements lies at the heart of rare disasters explanations for the equity premium. Although crashes are readily identified after the fact, anticipating extreme events remains a pressing econometric challenge. The rarity of extreme market events complicates the assessment of disaster risks using standard dynamic forecasting techniques on historical data. For this reason, existing approaches to risk assessment instead typically use options data to extract market forecasts of extreme events. However, the availability of liquid, deep out-of-the-money options limits these procedures’ estimation frequency and potential scope. The objective of this paper is to introduce a complementary methodology to overcome these limitations.

In this paper, I exploit the cross section of bid-ask spreads to develop a new, real-time measure of extreme event risk. By drawing on high-frequency quote data for thousands of U.S. stocks, I advance conditioning frequencies from months to minutes and the set of potential factors from those with liquid traded options to any factors that explain the cross section of realized stock returns. More generally, I demonstrate that information recovered from the behavior of market intermediaries offers a rich new resource for understanding aggregate economic shocks and potential systemic threats. Rather than impeding the recovery of information about factor risks, trading frictions themselves embed a wealth of unexploited information about future price movements.

Extreme market events take many forms, and the tail risks detected by my approach consist primarily of sharp and sudden factor crashes or jumps. This set of anticipated risks encompasses sharp factor price movements of several basis points, on the order of the median half-spread, to extreme price jumps, realized, for example, during the Black Monday crash of October 19, 1987, or during any of several market crashes at the height of the 2007–2008 Financial Crisis. These extreme market movements jeopardize years of investment returns in minutes or hours, and they are endemic to equity, currency, and commodity markets alike. Such tail realizations are particularly damaging to market makers because they are difficult to hedge and translate into trading losses with high probability, but they are also of broader concern to market participants and policymakers because they occur too quickly to be managed well.
Methodology Intuition

The market-making sector provides a natural setting for recovering conditional estimates of anticipated risks.\footnote{I use the term “market maker” to encompass all market liquidity providers rather than designated market makers (DMMs) alone. Equity market makers differ from traditional securities dealers in operating competitively in limit-order markets, but the name continues to be used to describe modern liquidity providers. For example, KCG Holdings, one of the world’s largest liquidity providers in stocks by share volume, refers to itself as a market maker even for its non-DMM roles (https://www.kcg.com/market-making).} Liquidity providers set quotes to balance expected gains from intermediation against potential losses from trading against better-informed market participants. One important source of adverse selection is “picking-off” risk, or failing to adjust quotes in response to sudden changes in the latent value of an asset before another party trades against them. Large factor innovations are especially damaging to market makers because picking-off losses are realized in many securities simultaneously. High-frequency liquidity providers harness extensive market data and advanced algorithms to manage this source of risk, and they continually revise their quotes in response to anticipated sudden price changes. Prevailing quotes thus reflect market-maker risk assessments on a near-instantaneous basis for every exchange-traded security.\footnote{Partly as a result of continual market-maker quote revisions, the ratio of order volume to trading volume typically exceeds 30 for U.S. stocks and 500 for U.S. exchange-traded products (Market Information Data Analytics System, http://www.sec.gov/marketstructure/datavis/ma_overview.html). Exchange-traded products are defined as CRSP securities with share code 73 and primarily consist of exchange-traded funds.}

My approach extracts tail risk estimates using this quote information for a large cross section of stocks. The technique consists of a simple empirical system in the style of Fama and MacBeth (1973) regressions for estimating factor prices. The econometrician first recovers rolling factor exposures $\beta_{ik}$ for each asset $i$ and return factor $k$ via time-series regressions:

$$r_{it} = \alpha_i + \sum_k \beta_{ik}^{(t)} f_{kt} + \epsilon_{it}, \forall i.$$  \hspace{1cm} (1)  

These estimated betas enter as independent variables in a cross-sectional regression for the subsequent time interval $t$. I adapt the standard second-stage regression by replacing returns with the
average effective half spread $h_{it}$ multiplied by traded volume $V_{it}$ during the interval:

$$h_{it}V_{it} = \gamma_{it} + \sum_k \xi_{kt} \beta_{ik} + \delta_{it}, \forall t. \tag{2}$$

Equation (2) relates average intermediation revenues to factor exposures. $\xi_{kt}$ loses its interpretation as a price of factor risk in this context, but in its place, $\xi_{kt}$ gains the interpretation of anticipated factor tail risks. To see why, consider the optimization problem of competitive liquidity suppliers in the presence of picking-off risk. If market makers are sufficiently fast in updating their quotes in response to information, picking-off risk arises exclusively from latent factor or idiosyncratic jumps rather than from continuous variation in underlying values. In equilibrium, half-spreads are set such that the expected gains per unit time from intermediating to liquidity consumers, $E[h_{it}V_{it}]$, exactly offset the expected cost per unit time from exposure to jump risks. Selecting a spread lower than this level does not recoup expected losses from picking off of quotes at stale prices, whereas selecting a spread higher than this level results in undercutting by other market makers.

Larger factor exposures $\beta_{ik}$ generate greater picking-off risks for a given factor $k$. All else equal, bid-ask spreads must be larger for high-beta securities to offset greater expected losses from being adversely selected on rapid price changes. Moreover, the size of the difference in spreads across securities with different factor loadings scales with the anticipated jump arrival intensity and magnitude, but only for jumps large enough to move latent security prices outside the quoted best bid and offer and induce picking-off events. It follows that the cross-sectional slope of spreads with respect to factor loadings $\xi_{kt}$ represents the tail expectation of the distribution of potential jumps for factor $k$ at date $t$. To the best of my knowledge, this insight represents the first use of liquidity

\footnote{Notably, the cross section of spreads alone is insufficient to extract factor risk information. The Internet Appendix considers spreads as the dependent variable and shows that recovered quantities are often negative, i.e., higher beta stocks have lower spreads.}

\footnote{In the main text, I divide the left-hand side of Equation (2) by quoted depth to deliver a composite quantity $\frac{h}{d} V$ (expressed in percent). Quoted depth enters theoretically in response to binding minimum tick sizes. When the minimum tick size exceeds the expected cost of intermediation, liquidity providers respond by quoting greater depth, and equilibrium is restored when expected costs equal expected benefits for the marginal quoted share. Empirically, spreads and volume dominate depth in contributing to variation in $\xi$, and I drop depth here to simplify exposition.}

\footnote{Such high-frequency factor structures have strong empirical support and are the basis for an active literature on high-frequency covariance estimation and jump regressions (e.g., Aït-Sahalia and Xiu (2015), Li et al. (2014), and Bollerslev et al. (2016)). To the extent that sampled price movements do not instantaneously reflect jumps in fundamentals because of residual stale limit orders, market makers suffer picking off on these events when trade occurs (Foucault et al. (2003), Christensen et al. (2014)). I discuss timing and synchronicity in Section III.}
measures to recover information about standard return factors.

An important difference between Equation (2) and Fama-MacBeth cross-sectional regressions is that the researcher observes and estimates expectational variables for tail risks rather than stochastic tail realizations. Like options prices, which represent a risk-neutral expected payoff, spreads incorporate expectations of potential tail risks, and the cross section of spreads delivers an expected cost of extreme realizations for each time interval rather than a realized cost. Consequently, coefficient estimates for anticipated tail risks $\xi_{kt}$ are much more precise than analogous risk prices in a Fama-MacBeth framework, and cross-sectional slopes are of independent interest as conditional expectations rather than as inputs for a single average jump tail risk estimate.

To facilitate exposition, I have assumed in this example that picking-off risk is the sole source of the bid-ask spread. The primary empirical threat to my analysis is the inventory and non-jump adverse selection risks that belie this assumption. In confronting this threat, my two-stage methodology confers important robustness advantages over analyzing the liquidity of individual factor-mimicking ETFs. In general, in addition to picking-off risk, equilibrium spreads must account for market maker compensation for carrying positions or trading against informed counterparties. Manipulative quoting behaviors (e.g., quote stuffing and spoofing) or idiosyncratic liquidity demand surprises also may contribute variation to the liquidity composite $h_{it}v_{it}$ for reasons other than differential exposure to tail risks. For individual assets, including factor-mimicking ETFs, distinguishing among these sources of variation is difficult or impossible. By contrast, in my two-stage methodology, these alternative sources of variation must align with factor betas to contaminate tail risk estimates.\textsuperscript{6} Likewise, noisy quoting behavior or liquidity demands wash out from the cross section unless they correlate with factor exposures across a large number of securities.

\textsuperscript{6}Section V.B verifies empirically that such contamination does not occur. To rule out contamination by inventory risk, I rerun my primarily analysis using the adverse selection component of the bid-ask spread in place of the effective spread under the identifying assumption that this component of the spread is unrelated to inventory risk. To rule out contamination by non-jump adverse selection, I include controls for a commonly used measure of “slow” adverse selection, the probability of informed trading, or PIN (Easley and O’Hara (1992); Easley, Kiefer, O’Hara and Paperman (1996)). Implied market tail risks are nearly identical to the baseline specification in both analyses.
Applications

Figure I illustrates the output of my methodology using a market-factor model of returns. Figure I plots the recovered market tail risk $\xi_{mt}$ for each trading day in 2004–2013 with hourly estimates of the cross-sectional slopes. For each hour, I recover slopes from a filtered sample consisting of approximately 2,800 stocks to identify potential tail variation, resulting in narrow confidence intervals for anticipated tails (dashed blue). This precise tail risk estimation at high frequency distinguishes my approach from time-series methods that rely on high-frequency data series on the order of weeks or months.

The recovered market tail risk series aligns well with measures of anticipated and realized jump tails. The correlation with weekly left jump tail estimates from options data (Bollerslev and Todorov (2014)) exceeds 75%, and a one standard deviation increase in the jump tail measure is associated with 5.47 more jumps per hour exceeding 10 basis points (t-statistic of 18.54). In addition, the extracted tail-risk measure correlates strongly with the CBOE S&P 500 implied volatility index (VIX), a 30-day forward volatility measure, and realized return variation on the SPDR S&P 500 ETF (SPY), in part because both series feature significant comovement with realized jump intensities.

In addition to serving as a real-time barometer of market-factor risks, the measure performs well across diverse and challenging economic environments. I apply the methodology to the May 6, 2010 Flash Crash as a prototypical large and plausibly unexpected systematic jump. Existing tail estimation techniques do not have sufficient resolution to anticipate the Flash Crash or to reliably distinguish changes in tail risk ex post. Consequently, regulators possessed little ability to intervene until long after the event concluded because they lacked reliable tools for evaluating and responding to near-term catastrophe risks.

My measure helps to fill this gap by providing intraday assessments of tail risks. My tail risk measure is a natural leading indicator for liquidity crashes because it draws directly from liquidity providers’ revealed expectations for jump tail events. For this event, the implied market tail risk increases several standard deviations relative to the prior day’s value an hour before the Flash Crash begins, 17 standard deviations in the quarter hour before the Flash Crash begins, and a
remarkable 104 standard deviations at the height of the event. By contrast, realized volatility and implied idiosyncratic tail risk (corresponding with level changes in spreads) meaningfully increases only as the crash develops, suggesting that market makers correctly anticipated a liquidity crisis in the market factor and only later adjusted spreads to accommodate liquidity spillovers uncorrelated with the SPY market index.

The Flash Crash also serves as an example of the dual uses of my measure; in addition to utilizing it as a forward-looking indicator, I apply the tail risk measure retrospectively to assess whether market makers register persistently elevated crash fears after the event. Both market and idiosyncratic anticipated jump risks quickly revert to pre-Crash levels and are statistically indistinguishable from the pre-Crash period after the ensuing weekend. This analysis serves as a first step toward an assessment of permanent impacts of liquidity-driven market meltdowns.

I next exploit the methodology’s new intraday resolution on conditional tails to document the evolution of tail risks around major scheduled macroeconomic news. I show that anticipated jumps vary throughout Federal Open Market Committee (FOMC) announcement days in regular patterns of decreased tail risk (relative to non-announcement days) prior to the announcement, heightened tail risk in the quarter hours before and containing the announcement, and slightly elevated tail risk after the announcement. This finding suggests that the pre-FOMC announcement drift documented by Lucca and Moench (2015) and the anomalous performance of the CAPM documented by Savor and Wilson (2013, 2014) cannot be rationalized by unobserved market jump risk without concurrent time variation in risk premia.

Finally, I demonstrate that the methodology separately identifies tail risks in a multifactor setting, even when candidate factors are very highly correlated. For this purpose, I study the coevolution of aggregate market and financial sector risks during the 2007–2008 Financial Crisis. Despite the Financial Select Sector SPDR ETF (XLF) having an annual average daily correlation of 89% with the SPY over this period, differences in factor loadings in the cross section nonetheless

---

7 Incidentally, the only other occasions that register at least a 16 standard deviation increase in tail risk relative to the previous day correspond with: the largest stock market decline in four years and the rollout of the NYSE’s Phase IV Hybrid Market (February 27, 2007); the market plunge at the height of the “Quant Quake” of August 2007 (August 9, 2007); the U.S. House’s rejection of Paulson’s financial stabilization plan (September 29, 2008); and the S&P downgrade of the U.S. federal government credit rating (August 5, 2011).

8 The Financial Select Sector SPDR (XLF) is one of nine partitioning sector-specific ETFs associated with the
enable precise estimates for anticipated shocks specific to the financial sector. The most extreme changes in the time series of financial sector tail risks often differ from those of the market jump series and correspond with major uncertainty innovations specific to financial firms, e.g., bank nationalization rumors and congressional votes on Fannie Mae and Freddie Mac rescue packages. The methodology thus offers a unique and useful tool for understanding the 2007–2008 Financial Crisis and assessing ongoing sector risks. Incidentally, this application provides a novel link between Brunnermeier and Pedersen (2009)’s concepts of market and funding liquidity by using market intermediaries’ behavior to inform on fears of extreme disruptions to banks and the financial sector.

II. Related Literature

A. Tail-Risk Measurement

The primary objective of this study is to develop a forward-looking measure of instantaneous tail risk for a variety of return factors. The two prevailing alternatives for tail risk measurement take advantage of options panels or of high-frequency time series for individual securities. The most closely related work in this literature is Bollerslev and Todorov (2014), which uses a cross section of S&P 500 index options to recover time-varying jump tails. In so doing, the authors exploit the fact that differential exposure to jump risk is the key source of variation in prices of close-to-maturity deep out-of-the-money options. Likewise, this paper makes use of the insight that differential exposures to jump risk drive variation in the size of (volume-adjusted) bid-ask spreads—themselves interpretable as prices of very short-dated options (Copeland and Galai (1983))—across stocks with different factor betas.

Todorov (2011a,b) supplement options analyses with high-frequency data and extreme value in-fill arguments to estimate jump tails for the aggregate market.

Similarly to Kelly and Jiang (2014), this paper takes a cross-sectional approach to obtain conditional tail risk estimates. Kelly and Jiang (2014) show that the aggregate market tail inherits individual asset tail dynamics if asset return tails follow a power law. If tail realizations are not too infrequent, this cross-sectional approach can detect physical market-factor tail shapes with short panels on the order of one month. My approach differs in two key respects. First, my estimation strategy relies on spreads rather than on tail return realizations. Because every bid-ask spread is informative at all times rather than only in the “rare event” states associated with extreme returns, I significantly increase the conditioning frequency at which tail risk estimates can be constructed. Secondly, my measure recovers tail expectations, which jointly summarizes factors’ ex ante tail position and shape, rather than the realized tail shape beyond a time-varying threshold value. As an example of this distinction, Kelly and Jiang (2014)’s time-varying tail threshold increases sharply during the 2007–2008 Financial Crisis, and the implied tail shape looks no more extreme than during the preceding years as a result.

My methodology complements these approaches. This paper adds the ability to estimate tail risks (1) in the very near term, (2) for a broad set of factors, (3) with high-frequency conditioning, and (4) under alternative sets of assumptions. Options-based approaches have difficulty assessing near-term risks because option maturities are long relative to intraday or daily events, and many options on individual names are too illiquid to be used for recovering non-market factor information. Likewise, combining realized jumps with extreme value theory can only recover very slow-moving variation in jump tails, and it is not yet applicable to candidate factors not directly traded in liquid factor-mimicking securities (e.g., size, value, and momentum). Conversely, my approach is limited in not being able to describe the full distribution of potential jump events or to gauge the persistence of negative shocks in a forward-looking way.

---

9Even Carr and Wu (2003) filters out options with time to maturity less than one week, and their important study explicitly focuses on option price dynamics as time to maturity goes to zero.
B. Market Microstructure

The key relation between spreads and tail risks emerges from Budish, Cramton and Shim (2015)’s model of high-frequency market making. Budish et al. (2015) represent high-frequency market making as a story of two speed races: a race to be first in the order book to provide liquidity to uninformed traders, and a race to be first to modify orders in response to discontinuous changes or “jumps” in fundamental asset values. If the marginal liquidity provider(s) loses the second race, she offers intermediation services at stale prices and suffers losses when trades are executed at these old prices by other fast traders. These potential losses are “picking-off risk,” an important source of the bid-ask spread. Forerunners in developing this source of risk include Copeland and Galai (1983), Harris and Schultz (1997), and Foucault, Röell and Sandås (2003), among others. Indeed, the equilibrium condition of Budish et al. (2015) and this study can also be motivated using the quotes-as-options framework of Copeland and Galai (1983). Viewed from this perspective, bid-ask spreads naturally extend short-dated options to the near-instantaneous expiration horizons that are especially well-suited for isolating jump risks (Bollerslev and Todorov (2011b)).

Budish et al. (2015) embed a compound jump process as the source of time variation in prices. I augment their model by imposing a factor structure on the jump process and by considering the resulting cross section of spreads across multiple assets. In so doing, I convert Budish et al. (2015)’s statement about spreads in continuous-time markets into a useful empirical relation between spreads and asset-pricing risks. Section III develops this relation in depth and considers several additional issues that arise when inverting the model to recover underlying tail risks.

Many other works investigate the information content of the limit order book. Of this set, Foucault, Moinas and Theissen (2007) is closest to this paper in showing that the pre-HFT limit order book contains volatility information in addition to directional information, albeit at the individual asset level. Nagel (2012) shares the spirit of this paper in relating returns to intermediation to forward-looking market volatility. Specifically, Nagel (2012) shows that short-term reversal returns are very highly correlated with the VIX, and he interprets this relation as evidence that financially constrained intermediaries are less able to provide liquidity in times of financial uncertainty. I take a different approach and instead derive a similar relationship as a robust consequence of
picking-off risk associated with movements in the market factor. Jump risks in the market factor must be compensated in equilibrium by higher returns to market making for assets with greater market exposure. When anticipated jump risks—a component of the VIX—are greater, the level of equilibrium returns is higher and the slope of spreads with respect to factor loadings is steeper.

A key contribution of this work is to extend this intuition to a broad set of factor risks and show that bid-ask spreads embed rich information about the underlying structure of asset returns. In this sense, my paper also relates to the broad literature on common factors in liquidity and trading volume (e.g., Chordia, Roll and Subrahmanyam (2000), Lo and Wang (2000), Hasbrouck and Seppi (2001), and Korajczyk and Sadka (2008)). Prior work such as Lo and Wang (2000) and Hasbrouck and Seppi (2001) also recover common factors in volume and liquidity measures, but these factors differ from realized or excess return factors typically studied in the asset pricing literature.

III. Spreads and Asset-Pricing Risks

I now turn to developing the relation between the cross section of bid-ask spreads and instantaneous factor tail risks. I first build intuition for the link between bid-ask spreads and factor exposures for individual assets in a multi-asset variation of a simple model of high-frequency liquidity provision. Competitive market makers set spreads to balance anticipated gains from liquidity supply against expected losses from picking off of stale quotes in response to extreme factor realizations. This model motivates a two-stage, cross-sectional approach for inverting the distribution of realized liquidity provision to recover factor tail risks.

A. Picking-Off Risks and Return Tails

The equilibrium bid-ask spread of a competitive market-making sector is set by balancing expected gains per unit time from liquidity provision against expected costs per unit time from displaying potentially stale quotes. I adapt the picking-off risk model of Budish et al. (2015) as formal motivation for this key condition. This condition also can be derived in alternative setups under different assumptions, e.g., those of Copeland and Galai (1983), so empirical results should not be viewed as requiring the Budish et al. (2015) framework in order to hold. In either case, I assume for
now that picking-off risk is the sole source of cross-sectional variation in volume-weighted spreads. I postpone in-depth discussion of this assumption until Section V, where I show empirically that inventory risk and non-jump adverse selection risk have minimal effect on tail risk estimates.

Jump Risks and Equilibrium Spreads

The investible universe consists of $N$ securities indexed by $i$. The fundamental value of each asset evolves as a compound Poisson process with arrival-rate parameter $\lambda_{i, \text{jump}}$ and jump distribution $F(J_i)$. By contrast with continuous variation in prices, jumps represent large, discontinuous changes in asset value over an infinitesimal interval. All market participants observe a public news source on each asset’s value. Many important intraday news events share these properties. For example, Federal Open Market Committee announcements affect discount rates for all assets and are embargoed until a rigidly enforced dissemination time,\(^{10}\) and Thomson Reuters and RavenPack package and distribute algorithmically readable company-specific or “macro” news to subscribers within milliseconds of release.\(^{11}\)

The economy consists of two types of traders: uninformed liquidity consumers with unit demands for a specific asset and a continuum of potential market makers, each capable of offering one unit of liquidity in a single asset; I institute the unit liquidity demand and supply assumption only to build intuition, and I relax it shortly. Market makers “race” to enter the limit-order book to intermediate for the liquidity consumer at a price $h_i$, the half-spread. $h_i$ arises from price competition among the competitive market makers. The order book respects price-time priority, although queuing order is random from the perspective of market makers. Potential market makers that lose the speed race to intermediate at $h_i$ instead participate as stale-quote “snipers”—if the underlying value of the asset jumps outside of the quoted prices given by the National Best Bid and Offer (NBBO), the fast non-market makers consume liquidity for a profit of $J_i - h_i$ at the offering market maker’s expense.

---

\(^{10}\) The sudden arrival of previously embargoed news should not be conflated with known release times. Actual news release times vary around scheduled dissemination times by several minutes until 2013. I treat the case of fixed arrival times (and violations of this Poisson assumption) in Section VI.D.

\(^{11}\) The RavenPack News Analytics service provides “analytics on more than 170,000 entities in over 100 countries and covers over 98% of the investable global market.” As of this writing, associated documentation claims that the average overall latency from story publication to dissemination of machine-readable sentiment analysis is 250 milliseconds. By comparison, an average human blink takes 300–400 milliseconds.
The timeline of events is as follows. At time 0, the first “race” commences among potential market makers of equal speed. The winners in each asset are selected at random, e.g., by noise in the exchange clock. Time then advances until either of two events occurs to force an update to the limit order book. One possibility is that an information event occurs that induces a jump in an underlying asset value. The fringe of stale quote snipers trade with the resting market maker if the jump exceeds the half-spread for the affected assets.\textsuperscript{12,13} Subsequent orders conditioning on the new information set establish a new bid and offer. Alternatively, a fundamental trader arrives and takes available liquidity from the order book. In this scenario the market maker earns the half-spread $h_i$, and the set of potential market makers again race to refill the order book centered at the same price as before. Because refreshing the book occurs in both event categories, the market making sector’s optimal policy for liquidity provision entails providing exactly one unit of liquidity and replenishing it if and only if liquidity is taken.

Figure II illustrates the sources of market-maker profits and losses per unit of offered liquidity. The rate of gains from intermediation equals the half-spread $h$ multiplied by the arrival rate of liquidity consumers $\lambda_{FT}$ (I suppress subscripts for visual clarity). The probability of this event for a particular unit of liquidity is $\frac{\lambda_{FT}}{(\lambda_{FT} + \lambda_{jump})}$. The expected costs from being picked off equal the arrival rate of jumps in the fundamental value of the asset $\lambda_{jump}$ multiplied by the tail expectation of jump sizes $J$ above the spread $h$, as taking liquidity is desirable only when the price movement exceeds the half-spread. Equating these quantities obtains the equilibrium condition of Budish et al. (2015):

$$\frac{\lambda_{FT} \times h}{E[\text{benefit/time}]} = \frac{\lambda_{jump} \times \Pr(J > h) \times E[J - h | J > h]}{E[\text{cost/time}]}.$$  \hspace{1cm} (3)

Equation (3) relates anticipated price movements $J$ to the half-spread $h$ at each date $t$. Higher arrival rates of liquidity consumers drive spreads toward zero, whereas faster information arrivals

\textsuperscript{12}The assumption of picking off with certainty is a simplifying assumption only. The Budish et al. (2015) model obtains an identical equilibrium condition absent this assumption by making the participation of HFTs endogenous.

\textsuperscript{13}The salient feature for picking off to occur is for quotes to reflect the asset’s pre-jump valuation, and for liquidity takers to take advantage of post-jump information. Non-instantaneous, but still rapid price responses accompanied by trade are perfectly consistent with picking off in that liquidity providers lose to arbitrageurs on interim trades before observable prices reflect assets’ updated valuations (as in Christensen et al. (2014)).
or larger jumps conditional on information arrivals increase $h$.\textsuperscript{14} Focusing on costs, only underlying price movements larger than $h$ result in costs to the market maker in trading at unfavorable prices.

The baseline model thus far supposes that desired trading volume is fixed at one unit. However, even traders engaged in order splitting frequently trade larger quantities in a single order, and picking-off strategies routinely execute against many or all of the orders available at a given price. Moreover, non-unit liquidity supply and demand do not have symmetric effects on the market maker’s equilibrium optimal liquidity provision policy. It is therefore essential to incorporate non-unit liquidity demands and quoted depths, or quantities available to buy or sell at given prices as indicated by the order book, into the model before proceeding to relate the factor structure of expected jump returns to market maker liquidity provision.

In the baseline model, the expected benefit of intermediating a (unit) order exactly offsets the expected cost of providing the option to trade to stale-quote snipers. If more depth is added at the NBBO, the potential cost to market makers of stale-quote sniping increases by a factor of $d$ because snipers (in the aggregate) pick off all supplied liquidity at the stale bid or offer. Conversely, providing depth also increases the potential benefits of offering liquidity in that intended liquidity consumers can trade a larger number of shares before they themselves become liquidity suppliers.

Adjusting Equation (3) for variable quantities of liquidity demanded and supplied, market makers’ expected benefit per unit time of facilitating trade for a stochastic quantity $q$ and an offered depth $d$ becomes

\[
\lambda_{FT} \times h_i \times (d + (E[q|q<d] - d))P_r(q<d).
\]

The new parenthetical term encapsulates the expected amount of liquidity consumed given a bid or offer size $d$. Market makers only intermediate for desired trade sizes up to $q = d$—beyond that point, larger liquidity demands instead convert into resting limit orders or “walk the book” to consume liquidity at higher prices, for which a different equilibrium condition applies.

By contrast, the expected cost of offering $d$ shares increases linearly in $d$ without bound, as

\textsuperscript{14}This equilibrium condition applies asset by asset and date by date. Any factor structures in trader arrival rates should be offset by spreads unless the factor structures arise in response to differential jump risk exposures. For this reason, I can link standard realized return factors to covariation in my composite liquidity measure regardless of the structure of underlying order flow commonality.
the mass of stale-quote snipers picks off the entire offered depth across all venues when given the opportunity.\textsuperscript{15} The net benefit per share of depth quoted is thus weakly decreasing in $d$, because fundamental traders do not always consume up to $q = d$ units of liquidity, whereas arbitrageurs do.

If queue positions are known, market makers earn positive rents on inframarginal units, and market makers (or orders) toward the back of the book are closer to break-even in expectation. However, in high-frequency settings, continual churn in the limit order book makes position order uncertain (Yueshen (2014))—recall that the order-to-trade volume ratio exceeds 30 for stocks and 500 for exchange-traded products. In the random-sequencing limit, all units of liquidity offered satisfy a depth-adjusted equilibrium condition of Equation (3), because each unit of depth quoted experiences the NBBO-average liquidity costs and benefits:

$$
\lambda_{FT} \times h \times \{E[q|q < d] \times \Pr(q < d) + d \times \Pr(q \geq d)\} \times \frac{1}{d} = \lambda_{jump} \times \Pr(J > h) \times E[J - h|J > h],
$$

where $q^*$ is the expected quantity intermediated for fundamental traders.

Equation (5) holds for all marginal market makers, i.e., those with the lowest spreads. Allowing for non-unit liquidity demands gives a second margin over which potential market makers can control their liquidity provision because optimal depth is not fixed at one unit. Other market makers may play a strategy of offering more depth at higher costs of liquidity, but these traders satisfy a separate first-order condition.\textsuperscript{16}

This adaptation for non-unit quantities also addresses distortions wrought by minimum tick sizes and discrete price increments. Forcing the half-spread to exceed the expected cost of providing a single unit of depth attracts additional market makers because the price of liquidity provision is artificially high. In turn, competitive liquidity providers respond by adding quotes to the order book until the total depth at the NBBO equates expected intermediation gains and picking-off costs, as in Equation (5).

\textsuperscript{15}Intermarket sweep orders or ISOs execute differently from Reg NMS protected orders and are popular among high-frequency traders. Sending ISOs simultaneously to several exchanges can clear all liquidity at the NBBO without incurring the delays associated with checking for potential price improvement at other market centers.

\textsuperscript{16}These market makers balance expected intermediation gains (after depth at the NBBO is exhausted) against the expected jump cost conditional on the price jump exceeding their price of liquidity $h' > h$. In principle, exploiting the shape of the order book thus provides additional information on the distribution of factor jump risks.
Relating Picking-Off Risks to Return Tails

I now specialize this framework by overlaying a factor structure on the jump processes underlying the bid-ask spread (see, e.g., Todorov and Bollerslev (2010)). Jump or discontinuous innovations in each asset’s fundamental value are decomposed as

\[ r^d_i = \sum_k \beta_{ik} r^d_k + \tilde{r}_i, \tag{6} \]

for a set of return factors \( k \) and idiosyncratic jump return \( \tilde{r}^d \). Recent empirical work suggests that high-frequency traders quickly embed factor news into prices of individual stocks to enforce the factor structure of Equation (6). Brogaard et al. (2014) show that HFT order flow in individual stocks reflects market-factor information, and Ito and Yamada (2015) go further to claim that “HFTs are equipped to aggregate market-wide information and reflect [it] in prices [...] This makes their order flows informative for market-wide information” (my emphasis).

Because the factor structure relates underlying returns in percentage terms rather than in dollar terms, I rewrite Equation (5) in return form as

\[ \hat{h} \lambda_{i,FT} \times q^* = \lambda_{i,jump} \times \int_{\hat{h}}^{\infty} \left( r^d_i - \hat{h}_i \right) f \left( r^d_i \right) dr^d_i \times d, \tag{7} \]

where Equation (7) converts all dollar quantities into returns by replacing dollar half-spreads \( h \) by percentage half-spreads \( \hat{h} \) and dollar jump sizes by discontinuous returns \( r^d \). To economize on notation, I suppress asset subscripts, jump superscripts, and decorative marks from now on unless meanings are otherwise ambiguous.

I make two simplifying assumptions to facilitate taking the adapted model to the data:

**Assumption 1.** Jump arrivals are independent both among factors and between factors and idiosyncratic discontinuous returns.

**Assumption 2.** Idiosyncratic jumps are distributed i.i.d. across assets.

Both assumptions streamline the estimation procedure but are otherwise inessential. Assumption 1 excludes co-jumps and more complex jump dependencies among factors. Whether excluding
co-jumps is reasonable depends on whether the considered return factors are plausibly orthogonal to one another. Relaxing this assumption to allow for co-jumps is readily accommodated by adding cross terms to each cross-sectional estimation, as described in Appendix C. Assumption 2 excludes heterogeneity in the rate of idiosyncratic information arrival among assets. This assumption, too, may be relaxed, for example by estimating security loadings on the common factor in idiosyncratic volatility—"CIV" of Herskovic, Kelly, Lustig and Nieuwerburgh (2016)—under the assumption that such factor loadings extend also to idiosyncratic jumps.

By assumptions 1–2, the jump intensity for stock \(i\) is given by

\[
\lambda_{\text{jump}} = \sum_{k} \lambda_k + \tilde{\lambda},
\]

where \(\lambda_k\) is the jump intensity for factor \(k\) and \(\tilde{\lambda}\) is the jump intensity for the stock’s idiosyncratic component. This representation decomposes short-lived adverse selection risks into factor- and idiosyncratic-news components.

Picking-off costs to the market maker integrate across the joint distribution of potential jumps larger than the half spread. By excluding co-jumps, when a factor \(k\) jumps, discontinuous returns have a simple form, \(r_{i}^{d} = \beta_{ik} r_{k}^{d}\), as coincident jump returns from other sources are exactly zero. Consequently, we can sum over costs associated with each factor independently rather than integrating over the region associated with all potential combinations of jump returns. Substituting Equations (6) and (8) into Equation (7) delivers:

\[
h \lambda_{FT} \frac{q^{*}}{d} = \sum_{k} \lambda_{k} \int_{h/\beta_{k}}^{\infty} (\beta_{ik} r_{k} - h) f (r_{k}) d r_{k} + \tilde{\lambda} \int_{h}^{\infty} (\tilde{r}_{i} - h) f (\tilde{r}_{i}) d \tilde{r}_{i}.
\]

The salient region of the jump distribution for each asset-factor combination is determined by \(h/\beta_{k}\). For each factor \(k\), jump risks can be decomposed into two regions: jumps larger than the half-spread for all assets, i.e., \(r_{k} \geq \bar{h}_{k} \equiv \max_{i} (h_{i}/\beta_{ik})\), and jumps larger than the half-spread for some assets but not for others:

\[
\int_{h_{i}/\beta_{ik}}^{\infty} (\beta_{ik} r_{k} - h_{i}) f (r_{k}) d r_{k} = \int_{h_{i}/\beta_{ik}}^{\bar{h}_{k}} (\beta_{ik} r_{k} - h_{i}) f (r_{k}) d r_{k} + \int_{\bar{h}_{k}}^{\infty} (\beta_{ik} r_{k} - h_{i}) f (r_{k}) d r_{k}
\]
where $\bar{F}$ denotes the counter-cumulative distribution function for $r_k$.

I assume that all jumps are large relative to half-spreads for my set of assets. In support of this assumption, Hendershott et al. (2011) show that by 2004, the first year in my sample, the effective half-spread for smaller-than-average (fourth size quartile) stocks is less than 6 basis points; using a $\beta$ cutoff of 0.5 implies the smallest “large” jump can be less than a 12 basis point change in price, or equivalently, less than an 8–22 cent change in the price of the SPY market proxy during the time period considered. By contrast, the smallest (median) realized jumps detected in the SPY by the Lee and Mykland (2008) and Bollerslev et al. (2013) methodologies are 26.5 (46.6) basis points and 13.5 (32.7) basis points, respectively.\(^{17}\) In estimation, I also impose loose restrictions on the set of assets considered to exclude stocks with extreme spreads and beta loadings to ensure that this assumption holds.\(^{18}\)

Excluding “small” jumps delivers the following simplifying relation:

$$
\lim_{h_k \to 0} \int_{h_k / \beta_k}^{\infty} (\beta_{ik} r_k - h_i) f (r_k) \, dr_k = -h_i + \beta_{ik} \int_{0}^{\infty} r_k f (r_k) \, dr_k. \quad (11)
$$

Equation (9) then reduces to a linear relation between the liquidity consumer arrival rate for each asset and the distribution of jump risks for each factor multiplied by the asset’s factor exposure:

$$
h \lambda T q^s \frac{\sigma}{d} \overset{(6),(8)}{=} \sum_k \lambda_k \int_{h_k / \beta_k}^{\infty} (\beta_k r_k - h) f (r_k) \, dr_k + \tilde{\lambda} \int_{h_k}^{\infty} (\tilde{r} - h) f (\tilde{r}) \, d\tilde{r} \\
\overset{(11)}{=} -\left(\tilde{\lambda} + \sum_k \lambda_k\right) h + \sum_k \beta_k \lambda_k \int_{h_k}^{\infty} r_k f (r_k) \, dr_k + \tilde{\lambda} \int_{h_k}^{\infty} \tilde{r} f (\tilde{r}) \, d\tilde{r},
$$

---

\(^{17}\)Jump detection tests compare five-minute returns against estimates of local volatility. I use a 1% significance threshold in the Lee and Mykland (2008) methodology and $\tau = 4$ standard deviations in the Bollerslev et al. (2013) methodology. Corresponding values for the Financial Select Sector SPDR ETF (XLF) are 18.2 (60.7) and 12.5 (37.4) basis points.

\(^{18}\)Empirically, I find that parameter estimates vary little with the choice of threshold $h_k$ for market (SPY) and financial sector (XLF) test factors. The Internet Appendix describes an alternative approach to estimating likelihoods of factor moves without imposing this “large jump” assumption.
which implies

\[
\left( \lambda_{FT} q^* + \lambda_{jump} d \right) \frac{h}{d} = \sum_{k} \lambda_{k} E \left[ r_{k}^{d} | r_{k}^{d} > \bar{h}_{k} \right] \beta_{k} + \lambda E \left[ r^{d} | r^{d} > \bar{h} \right]. \tag{12}
\]

The left-hand side of Equation (12) represents a liquidity composite of normalized half-spreads \( h/d \) and the expected volume-weighted arrival rate of both trader types, \( \bar{V} = \lambda_{FT} q^* + \lambda_{jump} d \). The coefficient on each \( \beta_{k}, \lambda_{k} E \left[ r_{k}^{d} | r_{k}^{d} > \bar{h}_{k} \right] \) or \( \xi_{k} \), represents the tail risk for factor \( k \). Faster event arrival rates or more damaging potential events must be compensated in equilibrium by larger half-spreads \( h \) or higher anticipated fundamental trading volume \( \bar{V} \).

Notably, very small jumps or continuous price movements are excluded from this coefficient and do not affect the equilibrium spread. The brief quote lifetimes of high-frequency market makers—as short as microseconds\(^{19}\)—ensure that continuous variation of the underlying asset value is highly unlikely to breach the bid-ask spread before quote revisions occur. It is this feature that strips out volatility contributions to my extracted tail risk measure.

**B. Cross-Sectional Recovery of Tail Risks**

Estimating tail risks from Equation (12) takes place in two stages. First, the econometrician computes betas with respect to candidate realized return factors. I estimate backward-looking, rolling annual betas using daily returns \( r_{it} \) on candidate factor realizations \( f_{kt} \) for each stock in the filtered sample \( i \):

\[
r_{it} = \alpha_{i} + \sum_{k} \beta_{ik}^{(t)} f_{kt} + \epsilon_{it}, \forall i. \tag{13}
\]

\(^{19}\)See, for example, quote lifetime distributions from the SEC Market Information Data and Analytics System available at [http://www.sec.gov/marketstructure/datavis/quotelife_etp_lg.html](http://www.sec.gov/marketstructure/datavis/quotelife_etp_lg.html).
Armed with these betas, I estimate (symmetric) tail risks cross-sectionally across stock-level observations via least absolute deviations regression:

\[
\left( \frac{Vh}{d} \right)_{it} = \tilde{\xi}_t + \xi_{t, MKT} \beta_{i, MKT} + \sum_{k \neq MKT} \xi_{tk} |\beta_{ik}| + \delta_{it}. \tag{14}
\]

For much of the ensuing analysis, all variables (and products of variables, where appropriate) are hourly averages over the respective time interval: \(d\) is the bid and offer depth summed across exchanges in 100-share round lots, \(V\) is realized volume in 500-share units, \(h\) is the effective half-spread, and \(\delta_{is}\) is a stock-specific error term for date \(t\). \(\xi_{tk}\) represents the average anticipated jump risk over the interval for factor \(k\). The time fixed effect \(\tilde{\xi}_t\) controls for common movements in asset-level tail risk not associated with the market factor or other return factors.

Equations (13) and (14) resemble Fama-MacBeth regressions for determining prices of factor risk. They differ in that all \(\xi_t\) estimates are of independent interest rather than only inputs into a single time-series average value. Unlike the realized factor premium at each date, the ex ante expected jumps are not (conditionally) stochastic, and their estimation relies on large \(N\) rather than on large \(T\) asymptotics. The key assumption needed for inference on each \(\xi_t\) term is that factor loadings \(\beta\) estimated over the prior year are valid for the subsequent trading day. The choice

---

\(^{20}\)The market factor model is readily estimated in part because reliable negative betas are quite rare among common stocks. Consequently, excluding stocks with betas below a small, positive threshold excludes few stocks so little data is lost. However, imposing this non-negativity restriction with other candidate asset pricing factors is undesirable, as \(\beta\)’s need not be centered around one, and they are frequently negative for several common factors, e.g., size (SMB) and value (HML). Appendix C shows that taking absolute values correctly accounts for negative betas with respect to non-market factors.

\(^{21}\)Equation (12) suggests the use of expected volume rather than realized volume in Equation (14). Realized volume offers a noisy proxy for unobserved market maker expectations because active intermediaries should form unbiased expectations about near-term arrival rates. The measurement error of \(\bar{V}\) has no (asymptotic) effect on recovered tail risk estimates under standard unbiasedness and independence assumptions on the error term. Appendix B discusses implicit assumptions on market makers’ information sets.

\(^{22}\)I deflate realized volume to account for intermediation and reintermediation by market makers. Specifically, I deflate total volume by an estimated intermediation multiplier of \(\kappa = 2.5\), as calibrated in a high-frequency setting for metals futures markets (Weller (2015)), to obtain an estimate for the correct scale of tail risk. However, because the intermediation multiplier may differ across asset classes, I refrain from interpreting the scale of implied risk and instead assume only that the intermediation multiplier is roughly constant across stocks within the 2005–2013 period. I further divide realized volume by two to account for half of volume being buys or sells, on average. Using signed volume techniques such as Lee and Ready (1991) has minimal effect on results in most instances, as detailed in the Internet Appendix.

\(^{23}\)The quoted spread sometimes overstates the true cost of consuming liquidity, as trades may execute inside the (displayed) spread. Replacing effective spreads with quoted spreads gives nearly identical time series of implied risks up to a scaling factor.
of annual betas trades off accuracy for precision in factor-loading estimates, but differences between betas for different horizons are likely to be small.\textsuperscript{24}

Equation (14) also clarifies the conditions under which other sources of the bid-ask spread are potentially problematic. To the extent that inventory risk and non-jump adverse selection risk influence effective spreads, Equation (14) implies that these drivers distort tail risk estimates $\xi_k$ if and only if they are cross-sectionally correlated with factor loadings $\beta_k$. Otherwise, these terms act as measurement error on the left or a level shift on the “idiosyncratic” tail risk estimate $\tilde{\xi}$. I directly test whether tail risk estimates are contaminated by these other sources of the spread in Section V.B.

More positively, Equation (14) illustrates the robustness of my methodology to unexpected liquidity demands or manipulative liquidity supply at the single-security level. Consider, for example, a large portfolio liquidation that surprises market makers with unexpected trading volume. The composite liquidity measure $V_{h/d}$ increases for the affected securities, but factor tail risk estimates $\xi_{tk}$ remain constant unless the unexpected liquidation impact to $V_{h/d}$ aligns with factor exposures, e.g., a stock with $\beta_{i,MKT} = 2$ is sold twice as aggressively as a stock with $\beta_{i,MKT} = 1$. Even allowing for such trading behavior, distortion of $\xi_{tk}$ estimates requires liquidation of a large number of stocks simultaneously in precisely this manner. Similarly, quote stuffing, spoofing, or other order book manipulations may affect $V_{h/d}$ idiosyncratically, but influencing implied factor tail risks is much more difficult. Manipulation must influence a large set of stocks’ liquidity composites in the same direction to contaminate any tail risk estimate, and moreover, it must line up with factor exposures to distort implied factor risks $\xi_{tk}$.

For estimation, I use median regression rather than OLS in the second-stage regressions because the dependent variable features occasional extreme observations. These observations otherwise result in too much weight being placed on fitting a small number of influential points with extreme

\textsuperscript{24}In principle, this procedure can be improved by estimating jump betas with respect to each set of factors (e.g., Todorov and Bollerslev (2010), Li, Todorov and Tauchen (2014)). I take this rather basic approach for three reasons. First, existing methods for computing jump betas are not well suited for multifactor models, especially in the presence of co-jumps. Second, high-frequency beta estimation is challenging across disparate liquidity environments, both over time and across stocks. Third, to a first approximation, jump betas are very similar to continuous betas at the stock level with only rare exceptions: for example, Todorov and Bollerslev (2010) find a cross-sectional correlation of 96.4\% between jump and continuous betas for their sample of forty large stocks. Differences between betas associated with high- and low-frequency variation consequently should be a relatively minor source of measurement error.
volumes or spreads. However, employing this robust regression approach complicates the problem of using estimated rather than known betas from the first stage. Direct corrections to standard errors (as in Shanken (1992)) are unavailable for this hybrid approach, and the discontinuous moment conditions associated with quantile regression make GMM approaches numerically formidable.

Instead, I compute standard errors via pairs bootstrap with \( R = 1,000 \) resamples to resolve the generated regressors problem and account for (heteroskedastic) median regression errors (Koenker (2005)). This procedure first uses the full sample of stocks and 252 rolling trading days to obtain point estimates for \( \xi_{tk} \). Then, for each replication \( r = 1, \ldots, R \), I construct \( \xi_{tk}^{(r)} \) as follows:

1. For each stock \( i = 1, \ldots, N \), draw with replacement 12 strings of 21 trading days during the 252 trading days preceding date \( t \);
2. Estimate \( \beta_{tk}^{(r)} \) for all stocks \( i \) using Equation (13);
3. For each time interval on date \( t \), draw \( N \) stocks with replacement from the set of \( N \) stocks;
4. Estimate \( \xi_{tk}^{(r)} \) using Equation (14) on the \( N \) randomly chosen stocks.

Confidence intervals for \( \xi_{tk} \) are constructed directly from quantiles of \( \xi_{tk}^{(1)}, \ldots, \xi_{tk}^{(R)} \). I use strings of 21 trading days to account for potential serial correlation in deviations from Equation (13). The bootstrap hence accounts for uncertainty in coefficients in both stages by incorporating draws for elements of Equations (13) and (14) simultaneously and at modest computational cost.

**IV. Data Description**

The primary data sources for this study are the Center for Research in Security Prices (CRSP) U.S. Stock Database and the New York Stock Exchange Trade and Quote (TAQ) data. The TAQ data aggregate orders from all Consolidated Tape Association exchanges and are timestamped to the second. I follow Holden and Jacobsen (2014) to recover cleaned effective spreads and market depths from the underlying TAQ data. Traded volume over each interval is directed observed. CRSP provides security attribute data (e.g., share codes), unique ticker-entity mappings, and daily shares outstanding for each security. In addition to CRSP and TAQ, I obtain intraday historical Chicago Board Options Exchange Volatility Index (VIX) data from Pi Trading.
The data sample consists of all common stocks (CRSP share code = 10 or 11) in the TAQ database from January 2004 to December 2013. Although TAQ starts in 1993, the spread may be too coarse prior to decimalization to provide a good guide for market-maker risks, and the large-jumps assumption cannot hold for bid-ask spreads on the order of several percent. In addition, the assumptions of continually updated spreads and minimal order processing costs are not plausible until algorithmic trading improvements in the mid-2000s (Foucault et al. (2003)). Slow market-maker responses to order flow through 2005 (Lyle and Naughton (2015)) suggests that even 2004 may be too early a start date; I focus on 2005–2013 for empirical tests for this reason and because of the model’s reliance on HFT market making.

I restrict the sample to exclude the 15 minutes after market open and before market close. These periods are characterized by unusual trader composition and informational events, such as elevated informed trading activity at market open in response to overnight events. For much of my analysis, I split the remainder of the trading day into six hourly bins running 9:45–10:45am through 2:45–3:45pm. The filtered sample has consists of roughly 2,800 stocks for each hour of each trading day from 2004 to 2013. Additional data cleaning and filtering details are provided in Appendix A.

V. Results

My empirical analysis proceeds in two main steps. In the first step, I recover hourly tail risk estimates for a one-factor market model and compare these estimates to market tail realizations and to alternative near-term forecasts such as the VIX. In the second step (Section VI), I apply the tail risk extraction methodology to verify the performance of the jump tail measure for the 2010 Flash Crash, major macroeconomic news events, and the 2007–2008 Financial Crisis. This last example demonstrates the power of the two-stage procedure for separately identifying factor tail risks in a multifactor setting.

Figure III plots recovered market and idiosyncratic tail risks by hour over the 2004 to 2013 sample period. Total spread-implied market and idiosyncratic tail risks are clearly distinct.\(^{25}\) Idiosyncratic tail shapes contrast with the market and idiosyncratic tail shapes of Kelly and Jiang (2014), for example. The Kelly-Jiang measure exploits power law relations to show that market tail shapes inherit properties of asset-level tail shapes. However, average tail locations do not share this aggregation property.

\(^{25}\)This feature contrasts with the market and idiosyncratic tail shapes of Kelly and Jiang (2014), for example. The Kelly-Jiang measure exploits power law relations to show that market tail shapes inherit properties of asset-level tail shapes. However, average tail locations do not share this aggregation property.
jump risk, plotted in red, is dominated by market risk in several crisis periods. This result is reassuring as the period considered includes extreme market events such as the recent financial crisis and global recession.

Figure III illustrates the jump tail measure’s ability to capture market news in real time. I mark the fifteen largest changes in implied market risks over the preceding 24 hours in green. I compute changes by differencing jump tails for the same hour at date $t$ and $t - 1$ to account for intraday patterns and news that spans multiple trading hours, and I separate increases by a minimum distance of 10 trading days to isolate distinct events. The extracted set of events consists primarily of scheduled Fed-affiliated and macroeconomic announcements. Scheduled and surprise events are captured “in progress”—for example, the 1:45–2:45pm window captures the typical timing of FOMC announcements, and the 2:45–3:45pm window on May 6, 2010 captures the Flash Crash.26

Figure IV indicates that the tail risk measure also captures well-known intraday patterns in volatility and jump risks (e.g., Andersen and Bollerslev (1997), Bollerslev and Todorov (2011b)). Trimming the first and last 15 minutes of trading eliminates the most pronounced jump patterns at the start and end of normal market hours, but the pronounced skewed U-shape pattern nonetheless manifests for each year in the sample.27 Moreover, both plots rank 2007 to 2011 as among the most extreme years in the sample for expected and realized jumps and 2005–2006 and 2012–2013 as the least extreme years in the sample for these measures.

As the figures suggest, my study focuses on the time-series properties of the tail risk measure, particularly with respect to contemporaneous and predictive assessment of factor jumps. However, as the second stage of Equation (14) is new, a brief digression into the cross-sectional regressions is informative about the mechanics of my approach as well as potential sources of error.

The cross-sectional regressions’ $R^2$ has a mean (median) value of 0.0175 (0.0135) across all hour-

26Several of these “peak news” days saw several large jumps realize in rapid succession. Notably, the implied tail risk measure cannot distinguish between market-maker expectations for one large jump or for a high arrival intensity of smaller jumps—the spread embeds minimal information about serial dependence of large price movements if all market-making risks are truly instantaneous. The success of the quarter-hour resolution applications in Section VI suggests that the tail measure may be of use in studying high-frequency tail dependence.

27As suggested by Figures I and III, the tail risk measures achieve extreme values in 2008 for every hour of the trading day, even relative to other crisis years. I omit 2008 from the diagram to maintain resolution on the other years of the 2004–2013 sample.
days using the one-factor market model for jump returns. The low goodness of fit arises from extracting information from a large universe of individual stocks rather than from stock portfolios for which other sources of spreads, volume, or depth unrelated to factor risk may wash out. First, I focus on one source of the bid-ask spread—picking-off risk associated with factor exposures—and market makers may be exposed to inventory risks or adverse selection, which in turn drive variation the composite liquidity measure. Critically for my analysis, these unmodeled sources of the spread should not align cross sectionally with betas, and Section V.B addresses this concern in depth. Second, idiosyncratic jump risks may differ across assets, for example, in line with idiosyncratic volatilities, and this source of market maker costs instead is assumed to be fixed across assets for parsimony. This omission only contaminates estimated factor risks if idiosyncratic jump risks are cross sectionally correlated with factor betas. Third, realized volume is a coarse proxy for expected volume at the single-security level, and the dependent variable suffers from measurement error on account of imperfect market maker expectations.

Relation to Other Tail Risk and Volatility Measures

Table I reports correlations among spread-implied tail risks, volatility measures, and options-implied tail risks from Bollerslev and Todorov (2014). All measures are aggregated as weekly averages because weekly resolution is the highest frequency available for options-based alternatives. Tail measures are highly positively correlated. Relative to the options-implied tails, high-frequency spread-implied tails are more similar to realized volatility and less similar to the VIX. This relationship is expected in that options-implied tails are “low frequency” (on the order of a week) and do not condition on intraweek information relevant to near-term volatility and jump tail risks. Likewise, the spread-based measure does not span the “long horizon” information captured in the monthly VIX that is embedded in options with more than a week to expiration.

The market tail risk measure comoves especially strongly with realized market volatility at the

\[ R^1 = \text{the quantile regression analogue of } R^2 \text{ (Koenker and Machado (1999)).} \]

Median regression selects the coefficients \( b \) that minimize the value \( V = \sum \delta_i (0.5 - 1 (\delta_i < 0)) \) for \( \delta_i = y_i - x_i b \). Koenker and Machado (1999) suggest a goodness of fit criterion \( R^1 = 1 - V_U / V_R \) for an unrestricted coefficient vector \( b_U \) and a restricted coefficient vector \( b_R \) with only an intercept term. This criterion parallels \( R^2 = 1 - S_U / S_R \) for \( S_U \) and \( S_R \) equal to the sum of squared residuals in unrestricted and restricted least-squares models.

28
weekly frequency, which suggests that the crash risks relevant for market makers align well with longer-horizon volatility events. This high correlation validates rather than indicts the tail risk measure for three reasons. First, Bollerslev and Todorov (2011b) show that we cannot reject the hypothesis that realized continuous and jump variation have a one-factor structure at the weekly frequency. If the tail risk measure perfectly forecasts near-term tail realizations, we should expect this correlation to be indistinguishable from one at this level of granularity. Second, the tight link between anticipated jumps and anticipated volatility parallels the relationship between options-implied tails and the VIX. Lower-frequency options-implied tails have an 88% correlation with the forward-looking volatility measure of a similar horizon. Third, I show in the following section that the tail risk measure captures the component of market return variation orthogonal to continuous price movements, that is, to the extent that the one-factor structure does not hold at higher frequencies, the tail risk measure is associated with the residual jump variation in prices.

A. Empirical Tests

Regression Specifications

Taken together, these figures provide suggestive evidence that the spread-based tail risk measure captures high-frequency fluctuations in market risks. I now formally test whether implied tail risk coincides with jump realizations and whether the tail risk measure indeed captures jump risk rather than volatility.\textsuperscript{30}

The key verification regression takes the following form for the market factor (and is later repeated for a financial sector proxy in Section VI):

\[
\text{tail}_\text{realization}_t = \alpha + \beta \xi_{t-\Delta, MKT} + \gamma VIX_t + \delta CV_t + \epsilon_t. \tag{15}
\]

I measure tail realizations in units of spreads and basis points and in event counts and event sums

\textsuperscript{29}By contrast, the dynamics of implied market tail risks at higher frequencies often differ from those of realized volatility. For example, these series diverge dramatically prior to the 2010 Flash Crash, as I discuss in Section VI.A.

\textsuperscript{30}Equation (3) operates under the physical measure if high-frequency market makers are risk neutral. Under this assumption comparing implied jump risks to jump realizations properly accounts for $\Xi$s and $\Omega$s. The tight empirical relationship between implied market tail risks and physical jump realizations suggests that risk neutrality is a good approximation.
(weighting by event size). Given the tail factor’s close coevolution with other forward-looking variation measures, e.g., the VIX, I include the VIX as a control to ensure that the tail measure indeed has additional explanatory power for tail events. I also include total continuous variation $CV$ to isolate the contribution of the tail risk measure to explaining jumps rather than continuous variation. $CV$ is defined as the sum of squared minutely price movements smaller than 2.5 standard deviations of minutely price movements (following the continuous and jump variation decomposition of Mancini (2009), among others). Realized continuous variation comoves very strongly with jump variation, so including it as a control presents a particularly strong test of the interpretation of the recovered coefficients as an estimate of anticipated jump tails.

Table II presents summary statistics for tail realizations. The tail realization measures used in the regression are as follows. Basis-point jumps count the number of events in which the minutely midpoint return exceeds 10 basis points, my implicit “large jump” threshold. The jump sum is a weighted count of the number of events in which the minutely return exceeds 5, 10, 25, and 100 basis points, with respective weights of 5, 10, 25, and 100. Spread jumps count the number of occasions in which the minutely return exceeds 5 quoted half-spreads. The corresponding jump sum measure is a weighted count of the number of events in which the minutely return exceeds 1, 5, 10, and 25 half-spreads, with concomitant weights of 1, 5, 10, and 25. Illiquidity-driven noise in jump counts is not of great concern for the SPY, because it trades at least once during every minute for every market hour from 2005 to 2013.

Large basis point and spread movements typically include the infrequent jumps captured by formal jump detection techniques (e.g., Lee and Mykland (2008) and Bollerslev et al. (2013)), but they also include more frequent medium-scale price movements that may not register as jumps in high volatility environments. I use relatively simple measures of realized tail events for two reasons. First, large price movements generate picking-off opportunities regardless of the underlying volatility environment so long as they move the “fundamental value” outside of the spread within a very short time period. The model does not distinguish between rare, truly discontinuous price movements and extremely rapid continuous ones associated with high local volatility. Second, extreme market movements are quite rare, and including moderately large market movements dramatically increases
the number of non-zero observations for the dependent variable. The weighted count measures in particular strike a compromise between including moderately extreme events and upweighting truly extreme factor realizations.

I run the regression of Equation (15) for three values of $\Delta$. First, I run the contemporaneous regression with $\Delta = 0$. This regression should have high explanatory power if the model is true because market makers adjust their spreads nearly every instant to reflect anticipated risks. In this specification, the recovered tail risk measure $\xi_{t,MKT}$ has the interpretation of the within-hour average anticipated jump risk. Maintaining $\Delta = 0$, I also analyze averages of all variables by hour within each year to assess my measure’s ability to pick up intraday patterns. Second, I assess within-period predictive ability with $\Delta \in (0,1)$ periods ahead by using the average spread and depth for the first minute of the hour for predicting the jump activity in the remainder of the period. This measure represents the start-of-hour anticipated jump risk under the assumption that the market maker accurately anticipate’s near-term trading volume. This variant on the jump measure uses less forward-looking information (realized volume remains), but it also handicaps the jump measure because hourly average spreads capture within-hour variation in perceived jump intensities and magnitudes, whereas the first-minute bid-ask spread and quoted depth do not. Finally, I implement a true forecasting regression with $\Delta = 1$. This specification tests whether the lagged tail risk measure predicts future tail realizations over the next hour. Recognizing that jump intensities, VIX, and continuous variation are persistent, I add lagged tail realizations and explanatory variables in both forecasting regressions. These additions challenge the predictive ability of the tail risk measure because the sizable persistence in volatility and jump risks is differenced out.

Regression Results

Table III presents results from the baseline test of contemporaneous forecast jump tails on realized jumps. I normalize $\xi_{t,MKT}$ by dividing by its standard deviation to facilitate interpretation of coefficients (for comparison, the standard deviation of the VIX in this period is 9.90). For all tail realization measures, an elevated market tail measure coincides with an increase in the number of realized jumps within the hour, and the coefficients are statistically significant and economically
large. For example, a one standard deviation increase in the market jump tail risk is associated with 5.47 additional realized basis-point jumps per trading hour and 69.7 additional weighted jumps (the measure predicts both intensity and size). The jump tail measure is only partly subsumed by the VIX, and they have roughly equal impact on the dependent variable for a one standard deviation change. The coefficient on continuous variation is inconsistent or driven out by the jump tail risk measure; by contrast, the jump tail estimates perform well in explaining the residual variation in realized jumps, which supports its interpretation as a measure of extreme event risk rather than of contemporaneous or anticipated volatility.

Intraday results are slightly smaller in coefficient magnitudes but are comparably significant statistically with and without the VIX control. Including continuous variation results in a serious multicollinearity problem because continuous and total variation are so strongly related in coarsened hour-year averages. This low granularity destroys too much information to reliably distinguish between intraday patterns in jump intensities and continuous variation.

Table IV presents analogous results for forecasting using beginning-of-period (top panel) and lagged tail risk measures (bottom panel). The number of observations falls relative to the Table III regressions because I drop observations with lags using information from the last hour of the preceding trading day. Controlling for lagged quantities removes the artificially high explanatory power of the forecasting variable that derives from persistent tail risks. Coefficients decrease slightly with the use of beginning-of-period tail estimates and the inclusion of lagged tail risk and explanatory variables. However, the estimated predictive ability of the tail risk measure remains economically and statistically large. A one standard deviation increase in $\xi_{t-\Delta, MKT}$ is associated with an increase of 2.4 basis-point jumps per hour and 33.0 weighted basis-point jumps. As before, these estimates survive the inclusion of measures of both future total volatility and contemporaneous continuous variation.

The bottom panel presents a more serious challenge to the forecasting power of the model.

---

31 Standard errors are robust to heteroskedasticity (both panels) and serial correlation of up to 126 trading hours (top panel). Adding hour-year fixed effects has minimal impact on my estimates or their statistical significance. Results are available upon request.

32 I confirm that results are not driven by extreme observations by recomputing both tables with log counts and log sums. All hourly coefficients remain statistically significant at the 99% level. Tables are available upon request.
Coefficients continue to attenuate with the exclusion of all contemporaneous information, but all specifications nonetheless indicate moderately strong predictability of market jump realizations for the hour ahead. The reduced explanatory power for hour-ahead jumps is expected because HFT market makers adapt their spreads and quoted depth almost instantaneously to reflect near-term tail risks.

These tests confirm the suggestive evidence of Figures I, III, and IV. The market tail risk measure is associated with both low- and high-frequency realized jump risks. It contains some of the same information as the forward-looking VIX, but its dynamics are intermediate between those of the VIX and of near-term realized volatility. Importantly, the tail risk measure is not spanned by measures of continuous variation, and in fact it drives out these measures for realized spread jumps. These features accord with the design of the measure as a tool for assessing instantaneous extreme event risks. In addition, the tail risk measure enjoys moderate forecasting power for near-term market jump events. Section VI supplements these forecasting regressions with corroborating evidence for the tail risk measure’s near-term predictive ability in the contexts of the 2010 Flash Crash and FOMC announcements.

B. Other Sources of the Spread

Other sources of the spread constitute the primary empirical threat to interpretation of recovered coefficients as tail risks. An extensive market microstructure literature describes three main sources of the bid-ask spread: order processing costs; inventory risk; and adverse selection, of which picking-off risk is a particular type. In this section, I consider inventory risks and adverse selection costs other than picking-off risks and their respective effects on quantities estimated in the model. In the algorithmic market-making era, order-processing costs are indistinguishable from zero and are ignored.33

---

33Investigating these alternative sources of the spread takes on increased importance in light of the dramatic reductions in trading costs throughout the early 2000s. For example, Hendershott et al. (2011) reports that the average effective half-spread for stocks in the middle market cap quintile declines from roughly 14 basis points in 2001 to about 3 basis points in 2006. The reduction in effective spreads primarily derives from declines in adverse selection associated with better-controlled picking-off risk for idiosyncratic price movements (Lyle and Naughton (2015)), which calls into question the assumption of a constant picking-off probability given a jump. However, Lyle and Naughton (2015) also show that market makers closely monitor and anticipate non-idiosyncratic news events as early as 2002. For this reason, variation in picking-off probabilities for factor jumps is likely to be small and not a
Inventory Risk

Risk-averse market makers must be compensated for exposure to price variation of assets in inventory. If market makers do not hedge inventory risks, e.g., using liquid factor-mimicking indexes, market-maker inventories may contaminate tail risks recovered from bid-ask spreads. To address potential contamination by inventory risk, I isolate the component of spreads contributed by adverse selection risks—including picking-off risk—rather than by inventory risks.

To isolate the contribution of adverse selection to cross-sectional variation in bid-ask spreads, I follow Glosten (1987) and decompose the effective (bid-ask) spread into “realized spread” and “adverse selection” components. The realized spread component captures revenues to liquidity providers net of permanent changes to the security price. This realized spread incorporates required compensation for bearing inventory risk as well as rents extracted from other traders. Conversely, the adverse selection or price impact component of spreads captures market maker losses due to unfavorable permanent price impact. Conceptually, the market maker suffers from adverse selection when she sells at \( t \) and the price subsequently increases or buys at \( t \) and the price subsequently decreases. Glosten (1987)’s now-standard proxy for the adverse selection component of spreads is given by the scaled difference between quote midpoints at time \( t \) and \( t + 5 \) minutes,

\[
adv\_sel_{it} = q_{it} \left( \frac{m_{i,t+5m} - m_{it}}{m_{it}} \right),
\]

where \( m_{it} \) is the prevailing quote midpoint in security \( i \) at time \( t \), \( q_{it} = +1 \) for market-maker sells, and \( q_{it} = -1 \) for market-maker buys. Buys and sells are determined by the Lee and Ready (1991) algorithm. \( m_{i,t+5m} - m_{it} \) represents the “permanent” price impact of transactions as the difference between the 5-minute ahead price and the current price.

I average this adverse selection value by stock and minute to obtain a continuous proxy for adverse selection costs. Replacing effective spreads with the adverse selection component of spreads is a major driver of my time series results.
in Equation (14) delivers
\[
\left( V^{adv\_sel} \right)_{it} = \xi_t + \xi_{t,MKT} \beta_{i,MKT} + \sum_{k \neq MKT} \xi_{t,k} |\beta_{ik}| + \delta_t.
\]

Table V replicates Table III using tail risk estimates recovered from Equation (17). All coefficients retain comparable levels of economic and statistical significance. The close similarity of results with and without compensation for inventory risks in the liquidity composite indicates that the tail risk measure is not contaminated by (omitted) inventory risk throughout the sample.

Non-Jump Adverse Selection

Adverse selection imposes costs on market markets through two qualitatively different modes:

1. Intermediation against informed traders with long-lived information (slow);

2. Picking off by stale-quote snipers (fast).

This contrast has a theoretical basis in models of informed trading by insiders. In insider trading models such as Kyle (1985) or Back (1992), insiders generate losses to the market maker by smoothly trading at an \( O(dt) \) rate. Foucault, Hombert and Rosu (2015) introduce a model of informed trading with Brownian (\( O(dz) \)) trading innovations (and market maker losses) resulting from “news trading” on a flow of signals. In both sets of models, as the time interval approaches zero, the likelihood of a price movement larger than a fixed \( \epsilon > 0 \) threshold converges to zero. By contrast, picking-off costs as a response to jump realizations do not scale even as the time interval becomes small. Hence long-lived information leads to slow erosion of market-maker profits, whereas price jumps contribute to a rapid deterioration of profits (conditional on an event).

The potential confound posed by slow adverse selection is that cross-sectional variation in spreads reflects differences in exposures to long-lived informational risk in addition to picking-off risk. Economically, estimation bias arises when factor exposures align with the risk imposed by long-lived informed traders. For example, for market tail estimates to be biased, assets with high market betas must be more exposed to non-jump adverse selection risk. Formally, slow adverse selection biases tail risk estimates if and only if the volume- and depth-scaled component of half-spreads
not associated with tail risk exposures $h^-$ is cross-sectionally correlated with factor loadings, i.e.,

$$\text{cov}(h^- \frac{V}{\tau}, \beta_k) \neq 0.$$ \footnote{To be precise, the $L^1$ analogue of cross-sectional correlation must be nonzero. \textit{Angrist, Chernozhukov and Fernández-Val} (2006) provide a weighted covariance interpretation of this analogue in discussing omitted variable bias in quantile regression models more generally.}

Such alignment is tantamount to market participants having private information on the underlying factor. \textit{Gorton and Pennacchi} (1993) and others suggest that private informational advantages are unlikely for systematic factors. \textit{Gorton and Pennacchi} (1993) argue that composite products, and by extension, corresponding factors that are mimicked, are exposed to minimal risk of non-public adverse selection because insider information is typically known at the security level rather than at the aggregate level.

I support this argument by showing that controlling for stock-level slow adverse selection does not meaningfully affect recovered tail risks for the market factor. Specifically, I include the probability of informed trading (PIN) measure of \textit{Easley and O’Hara} (1992) and \textit{Easley, Kiefer, O’Hara and Paperman} (1996) to control for the arrival rate of informed traders. The PIN measure is constructed under the assumption that order flow tilts in the direction of information that persists throughout the trading day unbeknownst to the market maker. Such information is long-lived with respect to the horizon of HFT market makers, and as such, PIN should primarily encapsulate costs of slow rather than fast adverse selection. I compute stock-level PIN estimates quarterly for the 2005–2013 sample period using the methodology of \textit{Yan and Zhang} (2014).\footnote{An active literature describes issues encountered in estimating PIN in fast-moving equity markets. \textit{Boehmer, Grammig and Theissen} (2007) discuss downward biases resulting from buy-sell misclassification errors. \textit{Lin and Ke} (2011) describe a potential bias arising from maximum likelihood estimation with large numbers of trades serving as exponential terms. \textit{Yan and Zhang} (2012) detail optimization failures on the boundary of the parameter space of the PIN model, which in turn translate into errors in the implied PIN measure. I find that results are robust across alternate estimation methodologies.}

Table VI reports the results of the contemporaneous regression of realized jumps on implied market tail risk, where the implied tail risk estimation equation adds the stock-date PIN control to the right-hand side of Equation (14). Like the tail risk measure net of potential inventory risk, the tail risk measure net of slow adverse selection risk performs very similarly to the baseline specification (Tables III) in matching time variation in market tail realizations; indeed, no tail risk coefficient in Table VI is statistically distinguishable at the 5% level from its counterpart in Table
III.

Summary

The tail risk series closely resemble each other. Pairwise correlations with the baseline specification are 92.0% controlling for inventory risk and 98.7% controlling for slow adverse selection. Figure V depicts the recovered time series of market tail risks in the baseline, inventory-risk robust, and slow adverse-selection robust specifications. These strong correlations are borne out almost point by point in the normalized series. In sum, neither inventory risk nor slow adverse selection appear to meaningfully contaminate implied market tail risks. While I do not rule out other sources of the bid-ask spread as a contaminant for arbitrary implied factor risks, a similar methodology can be applied for any candidate set of factors.

VI. Applications

The model of Section V develops an equilibrium relation among bid-ask spreads, volumes, and arrival rates. This relation motivates a new empirical methodology for recovering tail risks at high frequency and in (near-)real time. I supplement formal tests of this model with comparisons to known jumps associated with macroeconomic events. These applications serve both as qualitative verifications that the measure picks up and anticipates tail realizations in a variety of challenging macroeconomic settings and as a demonstration of the broad range of settings to which this technique can be applied.

I first demonstrate that the market tail risk measure is elevated shortly before and during the hours of the 2010 Flash Crash and Federal Open Market Committee (FOMC) announcements. These settings offer evidence that the risk measure is anticipatory rather than a reflection of concurrent jumps and that it assesses tail risks rather than volatility. I then show that the tail risk extraction methodology disentangles aggregate market from financial risks. Using a simple two-factor return model, I find that extremes for the financial jump risk factor correspond with major banking events often distinct from those associated with broader stock market movements; importantly, changes in implied factor risks indeed correspond with factor-specific innovations.
These macroeconomic events pose forecasting challenges difficult to meet with other existing methodologies. Although my findings qualitatively match intuition, there are few existing benchmarks against which I can assess results quantitatively—no other methodology allows for measurement of conditional tail risks intraday. Indeed, a principal contribution of this paper is to develop a new tool for quantifying extreme event risks in precisely these types of applications.

A. The 2010 Flash Crash

In a spectacular market episode, the May 6, 2010 Flash Crash saw equity indices decline by 5–6% and revert almost completely within a 30-minute period. Assessing welfare consequences associated with the 2010 Flash Crash has proved even more challenging than explaining the event’s causes. Kirilenko, Kyle, Mehrdad and Tuzun (2011) tabulate buyers and sellers in S&P 500 E-Mini futures (“E-Mini”) during the Flash Crash, but no corresponding data exists to evaluate the extensive knock-on redistributive effects associated with extreme turnover in equities and index products. Moreover, much popular discussion following the 2010 Flash Crash centers on distrust of the market mechanism and fears of future crashes, yet fears of future crashes are inherently difficult to quantify.

My measure of instantaneous jump risks is well-suited to evaluating the costs of rapid jump events. I require only that such events exceed the market makers’ typical holding period and thus contribute to picking-off risk. Market makers fear picking off on both the initial price decline (or rise) and on the return because extended price disruptions of several minutes affect a security’s “terminal value” with respect to the market maker’s trading horizon. Kirilenko et al. (2011) find support for market makers not holding through long crashes: rather than maintaining inventory during the 2010 Flash Crash, high-frequency market makers engaged in extreme turnover, or “hot potato” activity.

36 Explanations for the Flash Crash abound. Among these are that a single large trader’s faulty algorithm caused a severe order flow imbalance (CFTC and SEC (2010)); extreme order flow toxicity drove away market makers and collapsed liquidity (Easley, López de Prado and O’Hara (2012)); and a breakdown in cross-market arbitrage brought about an extreme price of immediacy (Menkveld and Yueshen (2015)).

To demonstrate its utility in assessing the costs of flash crashes, I construct contemporaneous tail risk measures around and during May 6, 2010 using the one-factor market model. A one-factor market model is particularly apt in this instance because the 2010 Flash Crash originated in S&P 500 E-Mini futures, a key price discovery market for the S&P 500. I estimate tail risks every 15 minutes to achieve high resolution on the crash interval (2:30–3:00pm) and surrounding trading hours.

Figure VI plots market and idiosyncratic tail measures from April 28, 2010 through May 14, 2010 for every quarter hour from 9:45am to 3:45pm. To capture innovations and place risk changes in context of normal intraday and slow-moving macroeconomic variation, I difference the previous day’s value at the same quarter hour and divide by the rolling standard deviation of differences for the same quarter hour over the preceding 63 trading days (a calendar quarter). Several features are readily apparent. First, the Flash Crash itself is associated with extreme contemporaneous elevations of both the market (104 standard deviations) and idiosyncratic (71 standard deviations) tail risk measures. Second, jump risks remain elevated for the remainder of the trading day and throughout May 7, 2010. Both risks return to near-normal levels after the May 8–9 weekend. Third, market tail risks increase a quarter hour before idiosyncratic tail risks, likely because the Flash Crash started in the E-Mini, a nearly ideal S&P 500 index proxy. Importantly, market factor risks are identified using the cross section of spreads, depth, and volume rather than the characteristics of any particular security—market tail risk only reflects abnormal liquidity demands in the E-Mini to the extent that they spill over into all stocks in proportion to each security’s market beta.

Intriguingly, the market tail risk measure achieves 17 standard deviations above its quarter-hour norm in the 2:15–2:30pm interval. Market makers anticipate distress conditions even before Waddell & Reed initialized its trading algorithm at 2:32pm (Menkveld and Yueshen (2015)). All told, the preceding relations align with several existing explanations of the Flash Crash and reassure that the proposed risk measure effectively anticipates near-term tail risks.\(^{39}\)

\(^{38}\)Moreover, as detailed in the Internet Appendix, market buy depth systematically declines relative to market sell depth in line with market betas, suggesting that liquidity providers are especially concerned about the risk of market down jumps prior to the Flash Crash. The joint CFTC and SEC report on the 2010 Flash Crash documents a similar, lopsided deterioration of market depth for the E-Mini in the hour preceding the Flash Crash.

\(^{39}\)As noted in the introduction, the only other occasions during which tail risk increases by at least 16 standard deviations are the intraday crashes of February 27, 2007, August 9, 2007, September 29, 2008, and August 5, 2011.
By contrast with my measure, options data used for constructing the VIX and other forward-looking risk measures incorporate volatility and jump information days or weeks beyond the duration of fleeting, mean-reverting flash crashes, and correspondingly are much less affected by such events. Although the VIX is somewhat elevated during the Flash Crash, the Flash Crash is not an extreme event for the VIX except in the rapidity of its increase intraday. An equally large and comparably sharp increase in implied volatility occurs in the same month: the normalized change-in-VIX measure achieves the same level on May 20, a day coinciding with a local maximum for the VIX (Figure VII).

The spread-implied measure also provides a longer-term view of changes in tail risk around the 2010 Flash Crash. The tail risk measure should remain elevated if the 2010 Flash Crash truly increases stability fears among market participants. Evidence for this effect is unambiguously negative. From Figure VII, we observe that all three risk measures return to roughly their pre-Flash Crash levels only days later—these differences are neither statistically significant nor economically large. Although longer-term average tail risks (and spreads) increase slightly in post-Crash weeks, these increases occur after May 10, 2010, several days after the crash. Subsequent tail risk elevations are inconsistent with a story of heightened perceived Flash Crash risk and likely arise from macroeconomic sources. In light of these results, it is difficult to argue that the 2010 Flash Crash had a persistent effect on market fears: high-frequency market makers should be among the most attuned to potential flash crash risk, yet their pricing of crash risks quickly reverts.

B. Federal Open Market Committee Announcements

The Federal Open Market Committee (FOMC) holds eight scheduled meetings per year to discuss salient economic and financial issues and policy responses. At the conclusion of each meeting, the FOMC releases a statement summarizing its views and actions. The release of these statements—typically scheduled to within minutes—is among the most important scheduled macroeconomic news announcements. Several recent papers have documented empirical regularities associ-
ated with these announcements. Savor and Wilson (2013) and Lucca and Moench (2015) find that announcement-day average stock returns comprise a large fraction of the annual equity premium, and Savor and Wilson (2014) find that the CAPM works well for cross-sectional pricing during FOMC days.

Rational explanations for these phenomena require that risk be highly time-varying as measured in FOMC event time. Although realized market volatility is lower than average during the FOMC pre-announcement period, elevated and difficult-to-observe jump risk may offer a partial, rational explanation. High market jump risk requires a higher equity premium, and the increased importance of market jump risk can enforce the CAPM if the CAPM works for discontinuous returns (as suggested by Bollerslev, Li and Todorov (2016)). Moreover, the sample of FOMC announcements may be too short for these jump risks to have been realized.

Existing alternatives cannot provide resolution on such high-frequency tail variation. Cross-sectional methods such as Kelly and Jiang (2014)’s cannot identify daily or intraday variation in jump tails because they rely on realized jumps within the estimation window. Options-based approaches are also inapplicable for short-term jump tail analysis because they require a panel of securities that mature in trading hours surrounding FOMC announcements.

I apply my tail risk extraction methodology to analyze jump risks around FOMC announcements and find evidence against this hypothesis of elevated market tail risk during the high-return period. For each quarter-hour interval and calendar year, I compute the average FOMC announcement date tail risk, subtract the average non-FOMC announcement tail risk, and normalize by dividing by the respective standard deviation of tail risks across all days for each quarter hour and year. I use full-period average spreads to measure jump risks because announcements are typically centered on the start of the quarter hour, so the previous quarter hour’s measure is a better assessment of anticipated risks. Importantly, the scheduling of announcements historically has not been precise enough to violate the assumption of Poisson arrivals of fundamental news: although market participants know the planned FOMC announcement time, there is significant uncertainty about precisely when the news comes out. This uncertainty is on the order of several minutes, as Figure VIII illustrates, and translates in the context of the model into a sharply elevated jump risk for the interval containing
the announcement.

Figure VIII plots deviations in perceived tail risks around FOMC announcements. For every year in the sample, FOMC announcements indeed coincide with sharply elevated perceived tail risk relative to the non-FOMC dates in the same year. Relative to the preceding quarter hour, most years also see a marked, anticipatory increase in implied tails in the quarter hour before the FOMC announcement (typically 2:00–2:14pm). These anticipatory movements in tail risk can be explained by (1) uncertainty in the exact timing of the information release, as suggested by within-year dispersion of the announcement minute around the year’s modal quarter hour, and (2) fear of early information leakage and attendant price jumps, as suggested by the empirical investigations of Bernile, Hu and Tang (2015) and Kurov, Sancetta, Strasser and Wolfe (2015). Notably the measure does not simply reflect contemporaneous realized volatility around the FOMC announcement: Lucca and Moench (2015) instead find that volatility is monotone decreasing in the hours prior to the FOMC announcement (Figure 3 of their work).

Although the tail risk measure registers increased risk in the quarter hours around FOMC news, implied tail risk is typically lower than average prior to the FOMC announcement, in parallel with the period’s reduced volatility. There is little evidence that the high average returns the morning of FOMC announcements can be attributed to market jump fears. The pre-FOMC announcement drift and announcement-day success of the CAPM therefore cannot be attributed to an increase in the (physical) probability or magnitude of market jumps.

C. The 2007–2008 Financial Crisis

I study the 2007–2008 Financial Crisis to demonstrate the potential of my approach to identify factor- or sector-specific extreme event risks. Specifically, I apply the jump extraction technique to discern the magnitude of perceived jump risks to a “financials” factor independent from market risks. The choice of the financial sector is driven by its economic importance during the 2000s as well as the

---

41 Crisis years have a much larger unnormalized FOMC announcement effect, particularly in 2008. However, the large fluctuations in tail risk during 2008–2009 counterbalance the increased differences between FOMC and non-FOMC day means.

42 These early-response results accord with the findings of Jiang, Lo and Verdelhan (2011) in the U.S. Treasury bond market. The authors find increased spreads, decreased depth, and stagnant trading volume in the five minutes before major market news announcements.
difficulty of disentangling financial sector risks from market risks using alternate methods; during
this period, the daily correlation of XLF, my financial sector factor-mimicking portfolio, and the
SPY often exceeds 90%. At the same time, Aït-Sahalia and Xiu (2015) demonstrate that the first
two principal components of high-frequency returns correspond well with market and financial sector
innovations, respectively, suggesting that innovations in these factor risks should be detectable at
high frequency.

The central regression in this analysis modifies Equation (14) to accommodate a financials factor:

\[
\left( \frac{Vh}{d} \right)_{it} = \tilde{\xi}_t + \xi_{i,MKT} \beta_{i,MKT} + \xi_{i,FIN} |\beta_{i,FIN}| + \epsilon_{it}, \forall t. \tag{18}
\]

I exclude co-jump terms for the market and financial factors because (1) the joint risk of tail events
in the market and financial factors is not of independent interest and (2) rank tests around market
jumps find evidence against factor co-jumps with the aggregate market (Li, Lin, Tauchen and
Todorov (2016)). Appendix C discusses this point in additional detail.

Figure IX plots the time series of implied financial sector tail risks. The recovered series of
financial tail risks is visually similar to the one-factor market risks of Figure III, but it differs
somewhat in the events corresponding with the largest changes in financial sector risks. Several
events associated with large market risk increases in the one-factor model are in fact specific to
the financial sector. Large-scale asset purchases, bank bailout legislation, and bank nationalization
news feature prominently for financials, but not for the aggregate market in the two-factor model.
Conversely, the FOMC interest rate target announcements of 2007–2008 and the S&P U.S. credit
rating downgrade have pronounced effects for the aggregate market but not for the financial sector
independently.

I now test formally whether the recovered financials tail risks indeed correspond with jumps in
the financials factor. Table VII presents analogous results with the market-factor analysis in Section
V. As before, I split specifications based on (1) the number of minutely differences of more than 10
basis points (“jump count”) and the weighted sum of jumps of 5, 10, 25, and 100 basis points and
(2) the number of minutely differences of more than 5 half-spreads (“jump count”) and the weighted
sum of jumps of 1, 5, 10, and 25 half-spreads. Rather than using the VIX, I include market jump
tails as a control to quantify the degree to which jump types are successfully disentangled.

Throughout, results are economically and statistically significant. A one standard deviation increase in the jump tail measure corresponds with approximately 8.2 more basis-point jumps and 1.6 spread jumps on a baseline standard deviation of 10.7 and 2.5 jumps per hour, respectively. Market tails have no incremental explanatory ability beyond that captured by $\xi_{t,FIN}$. The financial factor’s explanatory ability is replicated in the averages by hour-year, suggesting that intraday patterns in tail risks are also captured by the jump tail measure. This brief analysis suggests that the jump tail extraction technique successfully and separately identifies market and financial risks (and changes in risks) at high frequency.

D. Limitations

I conclude this discussion of applications with consideration of limitations to my empirical methodology. Although my approach permits assessment of tail risks in a wide variety of economic settings, it is equally important to highlight situations in which my methodology operates differently from advertised.

**Completely Predictable and Unpredictable Events**

The tail risk measurement technique assumes that the timing of jump events is imperfectly predictable using information available to market makers. Although the two-stage regression technique mechanically delivers estimates in other instances, the resulting $\xi$ coefficients should be interpreted with care.

First, market makers cannot adjust their spreads in advance of events such as natural disasters that are not anticipated by any agents in the economy and accessible to market makers in signals such as order flows. This methodology recovers the market’s perception of tail risks, which may differ sharply from true latent tail risks in such settings. However, after the initial shock, these quantities may again coincide if the source of risk persists beyond the initial event.

At the opposite extreme, the tail risk extraction technique may fail if jumps occur at precise and prescheduled times. In the limit as the event’s arrival time becomes predetermined, the key
equilibrium condition of Equation (3) breaks down. Indeed, several important macroeconomic news disclosures—including FOMC announcements from 2013-onward—share this property.

The key failure as local jump arrival rates go to infinity is in the assumption that the jump always exceeds the half-spread. In the case of scheduled arrivals, the zero-profit condition of Equation (7) can only be satisfied when the spread weakly exceeds the entirely of the anticipated jump distribution. It follows immediately from this equation that as the density of the waiting time distribution collapses to zero except at a single point (which can be approximated informally with $\lambda_k \to \infty$), half-spreads $h$ converge to a multiple of the conditional maximum of $r_k^d$. Because uninformed volume in a tight enough neighborhood around the announcement is effectively zero and jump sizes scale with factor betas, the cross section of half-spreads without a volume adjustment reveals this conditional maximum factor jump size.

In the case of FOMC announcements previously considered, there is enough residual uncertainty about the precise announcement time to remain in the Poisson arrival regime, as Figure VIII suggests. Although spreads are elevated and quoted depth is reduced around these events, they do not achieve the extreme values that would suggest a “worst case” and a scheduled-arrival regime. In practice, even when announcement dates are known, uncertainty is not immediately resolved as information continues to be digested after the initial announcement event, and the latent asset value may continue to jump to the market maker’s detriment. For this reason, the “echo” of the initial announcement may contribute to the empirical success of this methodology even when the model assumptions are not satisfied at each instant.

**Long-Horizon Forecasting**

Section III exposits a model of calculated market maker liquidity provision in response to potential factor jumps in the very near term. By contrast with options-implied jumps, my measure cannot anticipate tail realizations beyond a short forecasting horizon, nor does it provide direct information on the persistence or serial correlation of jump events. For these reasons, my measure is best interpreted as market expectations of tail realizations over short horizons, and its use restricted to intraday tail risk assessment or retrospective analysis of crash fears and determinants thereof.
Notwithstanding market makers’ limited planning horizon, empirical linkages between near-term factor risks and disasters of the Rietz (1988) and Barro (2006) variety are surprisingly strong. First, seminal events of the Great Recession and 2007–2008 Financial Crisis also correspond with large changes in implied tail risks in the immediate term. As Figures III and IX indicate, the demise of Bear Stearns, proposals and implementations of large-scale asset purchases, and discussions of bank nationalizations all feature prominently as extreme tail risk events on multiple timescales. Of course, high-frequency market makers and market participants at the time likely could not gauge the persistence of these shocks, but they nonetheless identify key movers of market perceptions of impending macroeconomic upheavals. Second, recent work by Andersen et al. (2015a) suggests that market crashes have their genesis in sequences of “small” jumps, such as those captured by my methodology. From this perspective, we can view elevated tail risks as a necessary but not sufficient forewarning of macroeconomic fluctuations or rare disasters.

In support of this view, Figure X plots the autocorrelation function of market tail risk estimates by hour from 2004–2013. The top subfigure plots raw autocorrelations, and the bottom subfigure illustrates autocorrelations net of intraday patterns, where I adjust for intraday variation in jump risk by dividing each value by the corresponding average market tail risk for the same hour across all dates in the sample. The raw series reveals substantial persistence in tail risks, but the persistence is masked by the pronounced intraday patterns in when the market is likely to see and respond to major news events. The pattern-adjusted series indicates that tail risk levels are highly persistent: the half-life of abnormal tail risks is six weeks. The tail risk measure thus typically informs on longer-run risks far exceeding the market maker’s holding horizon. Persistent elevations in market tail risk agree with the Andersen et al. (2015a) perspective of serially correlated sequences of small crashes contributing to more dramatic economic downturns.

VII. Conclusion

High-frequency market makers continually extract signals from order flow to optimize their provision of liquidity. Intermediaries must pay special attention to signals on potential discontinuous price movements, because such movements can generate losses from “picking off” by other fast
traders. Securities with larger factor loadings are more exposed to discontinuous factor movements than are securities with smaller loadings. As a consequence, liquidity prices are higher for these securities, and the cross section of liquidity costs embeds significant information about near-term return factor risks. The key contribution of this paper is the development of a straightforward methodology for extracting some of this factor risk information in real time.

Bid-ask spreads are unique among existing data sources in their ability to reveal intraday changes in tail fears for return factors. This paper thus takes a first step toward bringing microstructure data to bear on measurement challenges in related fields. My methodology is distinguished from existing methods in its ability to obtain information about (1) a wide array of return factor risks (2) at an intraday frequency (3) for short look-ahead horizons.

This cross-sectional approach offers a valuable tool for researchers to evaluate extreme event risks. High-frequency tail risk estimates provides a viable alternative to the VIX and other leading indicators of market turmoil. Regulators, too, might benefit: the extreme rise in tail risk before the onset of the 2010 Flash Crash suggests that the measure may have predictive power for severe market disruptions. These properties are left for future investigation.


Figure I: Hourly Market Tail Risks, 2004–2013

This figure plots rolling one-month means of hourly cross-sectional slope estimates $\xi_{mt}$ (blue) for an order book depth-adjusted version of Equation (2) for each trading date in 2004–2013. Dashed blue bands depict corresponding 95% confidence intervals. Realized volatility (red) is estimated using minutely squared returns on the SPY and scaled to the hourly frequency. The VIX is plotted using the right axis (gold) for comparison.

Figure II: Potential Intermediation Outcomes

This figure presents potential outcomes of offering liquidity with a half-spread $h$. Values at terminal nodes represent market-maker payoffs, and branch labels represent conditional probabilities. The green line indicates liquidity supply to a fundamental trader, whereas the red line indicates picking off by other potential market makers.
Figure III: Anticipated Jumps by Hour, 2004–2013
This figure plots hourly estimated tail risks for market and idiosyncratic risks in a one-factor market model. The fifteen largest increases in tail risks within a one-month window are overlaid with a green X. Changes are measured as the tail risk at date \( t \) and hour \( h \) less the tail risk at date \( t - 1 \) and hour \( h \). The table below offers a brief description of coincident events of tail risk news days. Standardized values divide by the full time-series standard deviation. Blue events coincide with the most extreme increases in the VIX within 24 hours. Implied market tails on October 10, 2008 are truncated for visual clarity.

<table>
<thead>
<tr>
<th>Hour</th>
<th>Std. Value</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>16-Aug-07 14:15</td>
<td>6.54</td>
<td>Fed approves changes to its primary credit discount window facility</td>
</tr>
<tr>
<td>18-Sep-07 14:15</td>
<td>6.08</td>
<td>FOMC lowers Fed funds target 50 basis points to 4.75%</td>
</tr>
<tr>
<td>31-Oct-07 14:15</td>
<td>5.29</td>
<td>FOMC lowers Fed funds target 25 basis points to 4.5%</td>
</tr>
<tr>
<td>11-Dec-07 14:15</td>
<td>4.84</td>
<td>FOMC lowers Fed funds target 25 basis points to 4.25%</td>
</tr>
<tr>
<td>22-Jan-08 10:15</td>
<td>9.71</td>
<td>FOMC lowers Fed funds target 75 basis points to 3.5%</td>
</tr>
<tr>
<td>14-Mar-08 10:15</td>
<td>5.88</td>
<td>New York Fed drops bailout deal to save Bear Stearns</td>
</tr>
<tr>
<td>19-Sep-08 15:15</td>
<td>13.27</td>
<td>TARP proposed; short-selling ban; global campaign by central banks</td>
</tr>
<tr>
<td>10-Oct-08 15:15</td>
<td>23.71</td>
<td>Stock market crashes in Asia, Europe, and the United States</td>
</tr>
<tr>
<td>28-Oct-08 15:15</td>
<td>9.95</td>
<td>First round of TARP bank bailouts ($115 billion)</td>
</tr>
<tr>
<td>13-Nov-08 14:15</td>
<td>11.31</td>
<td>Large negative jobless claims surprise; most new claims since 9/11</td>
</tr>
<tr>
<td>16-Dec-08 15:15</td>
<td>7.65</td>
<td>FOMC lowers Fed funds target to 0-0.25%</td>
</tr>
<tr>
<td>15-Jan-09 14:15</td>
<td>4.93</td>
<td>Senate approves release of $350 billion of TARP funds</td>
</tr>
<tr>
<td>18-Mar-09 14:15</td>
<td>4.04</td>
<td>FOMC announces $1 trillion in new Treasury bond and MBS purchases</td>
</tr>
<tr>
<td>06-May-10 15:15</td>
<td>16.83</td>
<td>2010 Flash Crash</td>
</tr>
<tr>
<td>05-Aug-11 14:15</td>
<td>4.68</td>
<td>S&amp;P downgrades U.S. government debt to AA+</td>
</tr>
</tbody>
</table>
Figure IV: Intraday Jumps by Hour, 2004–2013

This figure plots hourly means of market tail risks by hour and year (left) and of weighted realized spread jumps by hour and year (right). Realized spread jumps are a weighted average of the number of events in which the minutely return exceeds 1, 5, 10, and 25 half-spreads, with concomitant weights of 1, 5, 10, and 25. The underlying economic model is a one-factor market model. 2008 is a positive outlier and is omitted from both diagrams to preserve resolution on the other years in the sample.

(a) Anticipated Jump Risks (ex 2008) (b) Weighted Realized Spread Jumps (ex 2008)

Figure V: Comparison of Implied Market Jump Risks Net of Alternative Sources of the Spread

Figures plot rolling ten-day means of hourly estimated market tail risks for each trading date in 2004–2013. The blue line is the baseline estimation of Equation (14). The red line replaces the effective half-spread with realized adverse selection. The gold line adds a stock-quarter control for the probability of informed trading. The right plot aligns the series by subtracting series means and dividing by series standard deviations.

(a) Forecast Market Tail Risks (b) Forecast Market Tail Risks (Normalized)
Figure VI: Standardized Deviations in Jump Expectations around the 2010 Flash Crash

This figure plots standardized deviations in jump expectations around the May 6, 2010 Flash Crash. Tail risks are assessed with a market model with 15-minute increments. For each quarter hour, I normalize each value by subtracting the previous day’s value during the same quarter hour and dividing by the 15-minute specific standard deviation of this value across all dates in the preceding 63 trading days. The top figure plots the normalized value for the market factor before (green), during (red), and after (orange) May 6, 2010. The dotted purple line is the normalized 15-minute estimate for realized volatility. Black circles denote the 2:30–3:00pm interval during which the crash and reversion occur. The middle and bottom plots provide the corresponding information for the idiosyncratic jump factor and the VIX.
Figure VII: Long-Term Effects of the 2010 Flash Crash on Implied Jump Risk

This figure plots five-day backward-looking moving averages of the quarter-hour jump measure around the 2010 Flash Crash. Blue and red lines correspond to implied market and idiosyncratic jumps, with their associated scale on the left axis. The orange line corresponds with the VIX, with its associated scale on the right axis. The dashed line marks May 6, 2010, and the dotted line marks two business days after the event, May 10, 2010. The associated table regresses the inferred tail risk measure against a constant and a post-Flash Crash indicator for quarter hours from April 28, 2010 through May 14, 2010. Both May 6, 2010 and May 7, 2010 are excluded. Standard errors are clustered by day.

\[
\xi_t = \alpha + \beta I_{\text{After FC}} + \epsilon_t
\]

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Tail Risk</td>
<td>0.025***</td>
<td>0.027***</td>
</tr>
<tr>
<td></td>
<td>(10.122)</td>
<td>(25.218)</td>
</tr>
<tr>
<td>\text{1(after 5/6)}</td>
<td>0.004</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(1.125)</td>
<td>(0.302)</td>
</tr>
<tr>
<td>Obs.</td>
<td>264</td>
<td>264</td>
</tr>
<tr>
<td>\text{R}^2</td>
<td>0.015</td>
<td>0.001</td>
</tr>
</tbody>
</table>

t-statistics are given in parentheses with stars indicating *** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \).

Figure VIII: Intraday Tail Risks around FOMC Announcements

This figure plots intraday tail risks for the market factor for FOMC and non-FOMC announcement dates from 2005–2013. For each quarter-hour interval and calendar year, I compute the average FOMC announcement date tail risk and subtract the average non-FOMC announcement tail risk. I then normalize this quantity by the standard deviation of tail risks for all days in the same quarter hour and year. Stars indicate FOMC announcement times retrieved by minute from the first post-statement news article on Bloomberg or Dow Jones newswires following Fleming and Piazzesi (2005). The right plot zooms in on the 1:00-3:00pm interval.
Figure IX: Anticipated XLF Jumps by Hour, 2004–2013

This figure plots hourly estimated tail risks for financial and market risks in a two-factor market and financials model. The fifteen largest increases in tail risks within a one-month window are overlaid with a green X. Changes are measured as the tail risk at date $t$ and hour $h$ less the tail risk at date $t-1$ and hour $h$. The table below offers a brief description of coincident events of tail risk news days. Standardized values divide by the full time-series standard deviation. Red events correspond with extreme changes in both factors (using the two-factor model) within 24 hours. Implied tails on October 10, 2008 are truncated for visual clarity.

<table>
<thead>
<tr>
<th>Hour</th>
<th>Std. Value</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>22-Jan-08 10:15</td>
<td>5.88</td>
<td>Worldwide “Black Monday”</td>
</tr>
<tr>
<td>14-Mar-08 10:15</td>
<td>3.48</td>
<td>New York Fed drops bailout deal to save Bear Stearns</td>
</tr>
<tr>
<td>23-Jul-08 10:15</td>
<td>4.70</td>
<td>House passes Fannie and Freddie rescue bill after Bush drops opposition</td>
</tr>
<tr>
<td>19-Sep-08 10:15</td>
<td>19.89</td>
<td>TARP proposed; short-selling ban; global campaign by central banks</td>
</tr>
<tr>
<td>10-Oct-08 10:15</td>
<td>24.13</td>
<td>Stock market crashes in Asia, Europe, and the United States</td>
</tr>
<tr>
<td>28-Oct-08 15:15</td>
<td>6.53</td>
<td>First round of TARP bank bailouts ($115 billion)</td>
</tr>
<tr>
<td>13-Nov-08 14:15</td>
<td>9.18</td>
<td>Large negative jobless claims surprise; most new claims since 9/11</td>
</tr>
<tr>
<td>16-Dec-08 15:15</td>
<td>4.62</td>
<td>FOMC lowers Fed funds target to 0-0.25%</td>
</tr>
<tr>
<td>20-Jan-09 10:15</td>
<td>4.27</td>
<td>Discussion of bank recapitalization, bank stocks down 20–25%</td>
</tr>
<tr>
<td>20-Feb-09 15:15</td>
<td>5.98</td>
<td>Dodd suggests bank nationalization may be necessary</td>
</tr>
<tr>
<td>19-Mar-09 15:15</td>
<td>6.23</td>
<td>FOMC announces $1 trillion in new Treasury bond and MBS purchases</td>
</tr>
<tr>
<td>21-Apr-09 11:15</td>
<td>4.19</td>
<td>BofA reports sharp rise in bad loans, financial stocks drop more than 10%</td>
</tr>
<tr>
<td>05-Jun-09 10:15</td>
<td>4.84</td>
<td>Rumors circulate on FDIC push to gain greater control over Citigroup</td>
</tr>
<tr>
<td>27-Oct-09 10:15</td>
<td>3.92</td>
<td>House Financial Services Committee presents draft “Too Big to Fail” law</td>
</tr>
<tr>
<td>06-May-10 15:15</td>
<td>11.17</td>
<td>2010 Flash Crash</td>
</tr>
</tbody>
</table>
Figure X: Sample Autocorrelation Function for Market Tail Risks, 2004–2013

This figure plots the sample autocorrelation function for implied market tail risks. The market tail risk series consists of hourly estimates extending from 2004–2013. The top subfigure plots the unadjusted autocorrelation function of market tail risks for 0 through 180 hourly lags, or equivalently, 0 through 30 trading days. The bottom subfigure plots the same autocorrelation function for a normalized market tail risk series, where I divide each tail risk estimate by the sample average of tail risk estimates for the same hour across all dates in the sample. The blue line depicts two standard errors from zero serial correlation.

(a) Autocorrelation Function of Raw Market Tail Risk Series

(b) Autocorrelation Function of Intraday Pattern-Adjusted Market Tail Risk Series
Table I: Weekly Correlation of Tail Measure with Other Volatility and Tail Measures

This table reports weekly correlations of tail and volatility measures over the 2004–2011 sample period for which all measures are available. The spread-implied measure uses Equation (14) to compute hourly market tail risk estimates. VIX is the (30-day) CBOE Volatility Index. Realized volatility is the square root of the average squared one-minute SPY returns within each hour. Options-implied tails are the weekly parametric left-tail risk estimates from Figure 7 of Bollerslev and Todorov (2014). All values are centered, equal-weighted averages by week to align with the weekly options-implied estimates.

<table>
<thead>
<tr>
<th></th>
<th>Spread-Implied Tail</th>
<th>Options-Implied Tail</th>
<th>VIX</th>
<th>Realized Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spread-Implied Tail</td>
<td>–</td>
<td>0.75</td>
<td>0.85</td>
<td>0.94</td>
</tr>
<tr>
<td>Options-Implied Tail</td>
<td>0.75</td>
<td>–</td>
<td>0.88</td>
<td>0.80</td>
</tr>
<tr>
<td>VIX</td>
<td>0.85</td>
<td>0.88</td>
<td>–</td>
<td>0.92</td>
</tr>
<tr>
<td>Realized Volatility</td>
<td>0.94</td>
<td>0.80</td>
<td>0.92</td>
<td>–</td>
</tr>
</tbody>
</table>
Table II: Summary Statistics for Tail Realizations

This table presents summary statistics on the distribution of basis-point jumps, spread jumps, and volatility by hour and day for the SPY and XLF ETFs. The count variable sums jumps exceeding 10 basis points or 5 half-spreads, and the sum variables are a weighted average of jump sizes exceeding 5, 10, 25, and 100 basis points or 1, 5, 10, or 25 half-spreads. Realized volatility is the square root of the average squared one-minute SPY returns within each hour multiplied by $\sqrt{390}$ to obtain a daily measure (for market hours).

<table>
<thead>
<tr>
<th>SPY Hourly</th>
<th>Basis-Point Jumps</th>
<th>Spread Jumps</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jump Count</td>
<td>Jump Sum</td>
<td>Jump Count</td>
</tr>
<tr>
<td>Mean</td>
<td>2.947</td>
<td>32.649</td>
<td>12.168</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>6.546</td>
<td>80.745</td>
<td>10.443</td>
</tr>
<tr>
<td>Median</td>
<td>0</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>Max</td>
<td>53</td>
<td>1310</td>
<td>55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SPY Daily</th>
<th>Basis-Point Jumps</th>
<th>Spread Jumps</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jump Count</td>
<td>Jump Sum</td>
<td>Jump Count</td>
</tr>
<tr>
<td>Mean</td>
<td>17.627</td>
<td>195.314</td>
<td>72.793</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>36.111</td>
<td>444.137</td>
<td>55.160</td>
</tr>
<tr>
<td>Median</td>
<td>4</td>
<td>40</td>
<td>55</td>
</tr>
<tr>
<td>Max</td>
<td>273</td>
<td>4960</td>
<td>286</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>XLF Hourly</th>
<th>Basis-Point Jumps</th>
<th>Spread Jumps</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jump Count</td>
<td>Jump Sum</td>
<td>Jump Count</td>
</tr>
<tr>
<td>Mean</td>
<td>6.394</td>
<td>76.610</td>
<td>0.690</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>10.735</td>
<td>145.440</td>
<td>2.472</td>
</tr>
<tr>
<td>Median</td>
<td>1</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Max</td>
<td>59</td>
<td>1670</td>
<td>36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>XLF Daily</th>
<th>Basis-Point Jumps</th>
<th>Spread Jumps</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jump Count</td>
<td>Jump Sum</td>
<td>Jump Count</td>
</tr>
<tr>
<td>Mean</td>
<td>38.366</td>
<td>459.660</td>
<td>4.142</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>60.654</td>
<td>819.032</td>
<td>12.357</td>
</tr>
<tr>
<td>Median</td>
<td>12</td>
<td>120</td>
<td>0</td>
</tr>
<tr>
<td>Max</td>
<td>308</td>
<td>6610</td>
<td>144</td>
</tr>
</tbody>
</table>
This table presents results from a regression of realized jumps against contemporaneous tail risks,

\[ \text{tail}_{\text{realization}} = \alpha + \beta \xi_{t,MKT} + \gamma VIX_t + \delta CV_t + \epsilon_t. \]

Tail realizations are measured in counts of minutely returns exceeding basis point or spread thresholds. The count variable sums jumps exceeding 10 basis points or 5 half-spreads, and the sum variables are a weighted average of jump sizes exceeding 5, 10, 25, and 100 basis points or 1, 5, 10, or 25 half-spreads. Continuous variation is estimated by hour with a 2.5 standard deviation threshold on minutely price movements. Regressions in the top panel consist of the 2005–2013 sample by trading hour, with one-month rolling HAC standard errors (126 observation bandwidth). Regressions in the bottom panel average all variables within each year and hour of the trading day and use White standard errors. \( \xi_{t,MKT} \) is normalized by its standard deviation in both panels.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Jump Count</th>
<th>Jump Sum</th>
<th>Jump Count</th>
<th>Jump Sum</th>
<th>Jump Count</th>
<th>Jump Sum</th>
<th>Jump Count</th>
<th>Jump Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_{MKT} )</td>
<td>5.474***</td>
<td>3.487***</td>
<td>2.520***</td>
<td>69.766***</td>
<td>50.390***</td>
<td>27.123***</td>
<td>7.388***</td>
<td>4.641***</td>
</tr>
<tr>
<td>( VIX )</td>
<td>0.269***</td>
<td>0.261***</td>
<td>2.620***</td>
<td>2.428***</td>
<td>0.371***</td>
<td>0.380***</td>
<td>2.320***</td>
<td>2.286***</td>
</tr>
<tr>
<td>( CV )</td>
<td>4.144</td>
<td>99.655***</td>
<td>17.482</td>
<td>17.482</td>
<td>17.482</td>
<td>17.482</td>
<td>17.482</td>
<td>17.482</td>
</tr>
<tr>
<td></td>
<td>(1.510)</td>
<td>(3.015)</td>
<td>(1.977)</td>
<td>(0.710)</td>
<td>(0.710)</td>
<td>(0.710)</td>
<td>(0.710)</td>
<td>(0.710)</td>
</tr>
</tbody>
</table>

|Obs. | 13538 | 13538 | 13538 | 13538 | 13538 | 13538 | 13538 | 13538 | 13538 | 13538 | 13538 | 13538 | 13538 | 13538 | 13538 | 13538 |
|R² | 0.699 | 0.786 | 0.799 | 0.747 | 0.801 | 0.851 | 0.501 | 0.566 | 0.572 | 0.629 | 0.670 | 0.670 | 0.670 | 0.670 | 0.670 | 0.670 |

<table>
<thead>
<tr>
<th>Variable</th>
<th>Jump Count</th>
<th>Jump Sum</th>
<th>Jump Count</th>
<th>Jump Sum</th>
<th>Jump Count</th>
<th>Jump Sum</th>
<th>Jump Count</th>
<th>Jump Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_{MKT} )</td>
<td>3.404***</td>
<td>2.443***</td>
<td>1.005***</td>
<td>40.061***</td>
<td>30.297***</td>
<td>9.113***</td>
<td>5.903***</td>
<td>5.182***</td>
</tr>
<tr>
<td>( VIX )</td>
<td>0.178***</td>
<td>0.130***</td>
<td>1.810**</td>
<td>1.110***</td>
<td>0.134***</td>
<td>0.105**</td>
<td>0.847**</td>
<td>0.305</td>
</tr>
<tr>
<td></td>
<td>(3.217)</td>
<td>(3.727)</td>
<td>(2.632)</td>
<td>(3.261)</td>
<td>(3.652)</td>
<td>(2.651)</td>
<td>(2.131)</td>
<td>(1.933)</td>
</tr>
<tr>
<td>( CV )</td>
<td>14.811***</td>
<td>218.154***</td>
<td>8.890*</td>
<td>168.962***</td>
<td>(6.701)</td>
<td>(9.814)</td>
<td>(1.684)</td>
<td>(5.005)</td>
</tr>
</tbody>
</table>

|Obs. | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 |
|R² | 0.884 | 0.941 | 0.975 | 0.887 | 0.930 | 0.984 | 0.830 | 0.840 | 0.844 | 0.851 | 0.858 | 0.858 | 0.858 | 0.858 | 0.858 | 0.858 |

t-statistics are given in parentheses with stars indicating *** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \).
This table presents results from regressions of realized jumps against beginning-of-period and lagged estimated tail risks with lagged copies of each variable,

\[
\begin{align*}
tail_{realization_t} &= \alpha + \beta \xi_{t-\Delta,MKT} + \gamma \text{VIX}_t + \delta CV_t + \\
&\quad + \alpha_1 tail_{realization_{t-1}} + \beta_1 \xi_{t-\Delta-1,MKT} + \gamma_1 \text{VIX}_{t-1} + \delta_1 CV_{t-1} + \epsilon_t, \\
tail_{realization_t} &= \alpha + \beta \xi_{t-1,MKT} + \gamma \text{VIX}_t + \delta CV_t + \\
&\quad + \alpha_1 tail_{realization_{t-1}} + \beta_1 \xi_{t-1,MKT} + \gamma_1 \text{VIX}_{t-1} + \delta_1 CV_{t-2} + \epsilon_t,
\end{align*}
\]

corresponding to the upper and lower panels, respectively. Tail realizations are measured in counts of minutely returns exceeding basis point or spread thresholds. The count variable sums jumps exceeding 10 basis points or 5 half-spreads, and the sum variables are a weighted average of jump sizes exceeding 5, 10, 25, and 100 basis points or 1, 5, 10, or 25 half-spreads. Continuous variation is estimated by hour with a 2.5 standard deviation threshold on minutely price movements. Regressions consist of the 2005–2013 sample by trading hour, with one-month rolling HAC standard errors (126 observation bandwidth). \( \xi_{t,MKT} \) is normalized by its standard deviation in both panels.

<table>
<thead>
<tr>
<th>Variable</th>
<th>SPY Basis-Point Jumps</th>
<th>SPY Spread Jumps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jump Count</td>
<td>Jump Sum</td>
</tr>
<tr>
<td>( \xi_{MKT} )</td>
<td>2.392*** (11.213)</td>
<td>2.040*** (11.722)</td>
</tr>
<tr>
<td></td>
<td>32.984*** (11.050)</td>
<td>34.979*** (11.050)</td>
</tr>
<tr>
<td></td>
<td>16.614*** (6.347)</td>
<td>17.149*** (5.935)</td>
</tr>
<tr>
<td></td>
<td>4.077*** (6.328)</td>
<td>4.257*** (5.384)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>SPY Basis-Point Jumps</th>
<th>SPY Spread Jumps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jump Count</td>
<td>Jump Sum</td>
</tr>
<tr>
<td>( \xi_{MKT} )</td>
<td>1.881*** (4.953)</td>
<td>1.543*** (4.950)</td>
</tr>
<tr>
<td></td>
<td>20.849*** (3.834)</td>
<td>20.297*** (3.834)</td>
</tr>
<tr>
<td></td>
<td>3.474*** (5.162)</td>
<td>3.539*** (4.545)</td>
</tr>
</tbody>
</table>

| Obs. | 11275 | 11275 | 11275 | 11275 | 11275 | 11275 | 11275 | 11275 | 11275 | 11275 | 11275 | 11275 |
| \( R^2 \) | 0.808 | 0.834 | 0.812 | 0.837 | 0.882 | 0.722 | 0.739 | 0.740 | 0.767 | 0.787 | 0.798 | 0.860 |

| Obs. | 9012 | 9012 | 9012 | 9012 | 9012 | 9012 | 9012 | 9012 | 9012 | 9012 | 9012 | 9012 |
| \( R^2 \) | 0.802 | 0.820 | 0.824 | 0.800 | 0.816 | 0.824 | 0.737 | 0.755 | 0.765 | 0.772 | 0.780 | 0.782 |

\( t \)-statistics are given in parentheses with stars indicating *** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \).
Table V: Jump Tails and Contemporaneous Market Jumps with an Adverse Selection Proxy

This table presents results from a regression of realized jumps against contemporaneous tail risks,

\[ \text{tail}_t = \alpha + \beta \xi_{t, MKT} + \gamma VIX_t + \delta CV_t + \epsilon_t. \]

Right-hand side tail estimates are constructed using realized adverse selection rather than half-spreads as in Equation (17). Tail realizations are measured in counts of minutely returns exceeding basis point or spread thresholds. The count variable sums jumps exceeding 10 basis points or 5 half-spreads, and the sum variables are a weighted average of jump sizes exceeding 5, 10, 25, and 100 basis points or 1, 5, 10, or 25 half-spreads. Continuous variation is estimated by hour with a 2.5 standard deviation threshold on minutely price movements. Regressions in the top panel consist of the 2005–2013 sample by trading hour, with one-month rolling HAC standard errors (126 observation bandwidth). Regressions in the bottom panel average all variables within each year and hour of the trading day and use White standard errors. \( \xi_{t, MKT} \) is normalized by its standard deviation in both panels.

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_{MK} )</td>
<td>4.907***</td>
<td>2.743***</td>
<td>1.723***</td>
<td>63.132***</td>
<td>39.556***</td>
<td>17.850***</td>
<td>6.669***</td>
<td>3.733***</td>
<td>4.162***</td>
<td>60.032***</td>
<td>39.162***</td>
<td>32.891***</td>
</tr>
<tr>
<td>( VIX )</td>
<td>0.354***</td>
<td>0.315***</td>
<td>3.853***</td>
<td>3.023***</td>
<td>0.480***</td>
<td>0.469***</td>
<td>3.411***</td>
<td>3.171***</td>
<td>3.367***</td>
<td>3.367***</td>
<td>3.171***</td>
<td>3.367***</td>
</tr>
<tr>
<td>( CV )</td>
<td>5.372**</td>
<td>1.14304***</td>
<td>1.14304***</td>
<td>-2.259</td>
<td>33.022</td>
<td>(2.009)</td>
<td>(3.650)</td>
<td>(1.476)</td>
<td>33.022</td>
<td>33.022</td>
<td>(2.009)</td>
<td>(3.650)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Obs.</th>
<th>13538</th>
<th>13538</th>
<th>13538</th>
<th>13538</th>
<th>13538</th>
<th>13538</th>
<th>13538</th>
<th>13538</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
<td></td>
<td>0.562</td>
<td>0.762</td>
<td>0.786</td>
<td>0.612</td>
<td>0.768</td>
<td>0.840</td>
<td>0.408</td>
<td>0.553</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_{MK} )</td>
<td>3.322***</td>
<td>2.250***</td>
<td>0.948***</td>
<td>39.491***</td>
<td>28.273***</td>
<td>8.489***</td>
<td>5.739***</td>
<td>4.536***</td>
<td>4.194***</td>
<td>45.720***</td>
<td>37.734***</td>
<td>25.590***</td>
</tr>
<tr>
<td>( VIX )</td>
<td>0.222***</td>
<td>0.160***</td>
<td>2.325***</td>
<td>1.373***</td>
<td>0.249***</td>
<td>0.233***</td>
<td>1.655***</td>
<td>1.071***</td>
<td>1.655***</td>
<td>1.071***</td>
<td>1.655***</td>
<td>1.071***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Obs.</th>
<th>54</th>
<th>54</th>
<th>54</th>
<th>54</th>
<th>54</th>
<th>54</th>
<th>54</th>
<th>54</th>
<th>54</th>
<th>54</th>
<th>54</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
<td></td>
<td>0.843</td>
<td>0.953</td>
<td>0.972</td>
<td>0.862</td>
<td>0.949</td>
<td>0.982</td>
<td>0.785</td>
<td>0.828</td>
<td>0.829</td>
<td>0.835</td>
<td>0.866</td>
</tr>
</tbody>
</table>

\( t \)-statistics are given in parentheses with stars indicating *** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \).
Table VI: Jump Tails and Contemporaneous Market Jumps Controlling for PIN

This table presents results from a regression of realized jumps against contemporaneous tail risks,

\[ \text{tail}_t = \alpha + \beta \xi_{t,MKT} + \gamma VIX_t + \delta CV_t + \epsilon_t. \]

Tail estimates include a stock-quarter control for the probability of informed trading. Tail realizations are measured in counts of minutely returns exceeding basis point or spread thresholds. The count variable sums jumps exceeding 10 basis points or 5 half-spreads, and the sum variables are a weighted average of jump sizes exceeding 5, 10, 25, and 100 basis points or 1, 5, 10, or 25 half-spreads. Continuous variation is estimated by hour with a 2.5 standard deviation threshold on minutely price movements. Regressions in the top panel consist of the 2005–2013 sample by trading hour, with one-month rolling HAC standard errors (126 observation bandwidth). Regressions in the bottom panel average all variables within each year and hour of the trading day and use White standard errors. \( \xi_{t,MKT} \) is normalized by its standard deviation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_{MKT} )</td>
<td>5.365***</td>
<td>3.238***</td>
<td>2.064***</td>
<td>67.610***</td>
<td>46.109***</td>
<td>22.030***</td>
<td>7.201***</td>
<td>4.210***</td>
<td>4.301***</td>
<td>62.659***</td>
<td>42.993***</td>
<td>32.808***</td>
</tr>
<tr>
<td>( VIX )</td>
<td>0.279***</td>
<td>0.260***</td>
<td>2.824***</td>
<td>2.432***</td>
<td>0.393***</td>
<td>0.394***</td>
<td>2.583***</td>
<td>2.417***</td>
<td>2.583***</td>
<td>2.417***</td>
<td>2.583***</td>
<td>2.417***</td>
</tr>
<tr>
<td>( CV )</td>
<td>5.678**</td>
<td>116.460***</td>
<td>116.460***</td>
<td>116.460***</td>
<td>-0.441</td>
<td>-0.441</td>
<td>-0.441</td>
<td>-0.441</td>
<td>-0.441</td>
<td>-0.441</td>
<td>-0.441</td>
<td>-0.441</td>
</tr>
<tr>
<td></td>
<td>(1.976)</td>
<td>(3.477)</td>
<td>(3.477)</td>
<td>(3.477)</td>
<td>(2.810)</td>
<td>(2.810)</td>
<td>(2.810)</td>
<td>(2.810)</td>
<td>(2.810)</td>
<td>(2.810)</td>
<td>(2.810)</td>
<td>(2.810)</td>
</tr>
<tr>
<td>Obs.</td>
<td>13538</td>
<td>13538</td>
<td>13538</td>
<td>13538</td>
<td>13538</td>
<td>13538</td>
<td>13538</td>
<td>13538</td>
<td>13538</td>
<td>13538</td>
<td>13538</td>
<td>13538</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.672</td>
<td>0.759</td>
<td>0.789</td>
<td>0.701</td>
<td>0.760</td>
<td>0.844</td>
<td>0.476</td>
<td>0.544</td>
<td>0.544</td>
<td>0.581</td>
<td>0.629</td>
<td>0.643</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_{MKT} )</td>
<td>3.383***</td>
<td>2.413***</td>
<td>0.918***</td>
<td>39.702***</td>
<td>29.779***</td>
<td>8.357***</td>
<td>5.793***</td>
<td>5.019***</td>
<td>3.436***</td>
<td>45.151***</td>
<td>40.031***</td>
<td>19.392***</td>
</tr>
<tr>
<td>( VIX )</td>
<td>0.176***</td>
<td>0.123***</td>
<td>1.806**</td>
<td>1.043***</td>
<td>0.141***</td>
<td>0.084*</td>
<td>0.141***</td>
<td>0.084*</td>
<td>0.141***</td>
<td>0.084*</td>
<td>0.141***</td>
<td>0.084*</td>
</tr>
<tr>
<td></td>
<td>(3.077)</td>
<td>(3.662)</td>
<td>(2.535)</td>
<td>(3.190)</td>
<td>(3.219)</td>
<td>(1.837)</td>
<td>(2.100)</td>
<td>(1.837)</td>
<td>(2.100)</td>
<td>(1.837)</td>
<td>(2.100)</td>
<td>(1.837)</td>
</tr>
<tr>
<td>Obs.</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.874</td>
<td>0.926</td>
<td>0.975</td>
<td>0.871</td>
<td>0.912</td>
<td>0.984</td>
<td>0.800</td>
<td>0.810</td>
<td>0.827</td>
<td>0.814</td>
<td>0.822</td>
<td>0.870</td>
</tr>
</tbody>
</table>

t-statistics are given in parentheses with stars indicating *** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \).
Table VII: Jump Tails and Contemporaneous XLF Jumps

This table presents results from a regression of realized jumps against contemporaneous tail risks,

\[ t_{\text{realization}} = \alpha + \beta \xi_{t,FIN} + \gamma \xi_{t,MKT} + \epsilon_t. \]

Tail realizations are measured in counts of minutely returns exceeding basis point or spread thresholds. The count variable sums jumps exceeding 10 basis points or 5 half-spreads, and the sum variables are a weighted average of jump sizes exceeding 5, 10, 25, and 100 basis points or 1, 5, 10, or 25 half-spreads. Regressions in the top panel consist of the 2005–2013 sample by trading hour, with one-month rolling HAC standard errors (126 observations). Regressions in the bottom panel average all variables within each year and hour of the trading day and use White standard errors. \( \xi_{t,MKT} \) and \( \xi_{t,FIN} \) are normalized by their standard deviations in both panels.

<table>
<thead>
<tr>
<th>Variable</th>
<th>XLF Basis-Point Jumps</th>
<th>XLF Spread Jumps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jump Count</td>
<td>Jump Sum</td>
</tr>
<tr>
<td>( \xi_{FIN} )</td>
<td>8.190***</td>
<td>7.566***</td>
</tr>
<tr>
<td>( \xi_{MKT} )</td>
<td>2.664</td>
<td>48.680</td>
</tr>
<tr>
<td></td>
<td>(0.496)</td>
<td>(0.850)</td>
</tr>
<tr>
<td>Obs.</td>
<td>13541</td>
<td>13541</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.581</td>
<td>0.583</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>XLF Basis-Point Jumps</th>
<th>XLF Spread Jumps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jump Count</td>
<td>Jump Sum</td>
</tr>
<tr>
<td>( \xi_{FIN} )</td>
<td>6.983***</td>
<td>7.540***</td>
</tr>
<tr>
<td>( \xi_{MKT} )</td>
<td>-3.650</td>
<td>-76.565***</td>
</tr>
<tr>
<td></td>
<td>(-1.649)</td>
<td>(-7.63)</td>
</tr>
<tr>
<td>Obs.</td>
<td>54</td>
<td>54</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.902</td>
<td>0.903</td>
</tr>
</tbody>
</table>

t-statistics are given in parentheses with stars indicating *** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \).
A. Data Filters

The TAQ database occasionally records erroneous trade price and quantity information. I take three precautions to avoid contamination of the sample and mistaken detection of price jumps in Section V. First, I filter the trade price series following the trade data methodology of Barndorff-Nielsen, Hansen, Lunde and Shephard (2009). This methodology eliminates most obvious data errors. Second, I adapt the outlier removal procedures of Brownlees and Gallo (2006) (similar to Rule Q4 of Barndorff-Nielsen et al. (2009)) to exclude price observations exceeding the centered median price (excluding the current observation) on $[t-10m, t+10m]$ by 2.5 mean absolute deviations plus a 15-basis point granularity parameter. This filter removes most data errors in the form of rapidly mean-reverting jumps in recorded prices. Finally, I use volume-weighted averages of prices within each minute as raw inputs rather than individual trades or quotes. This procedure smooths microstructure noise not of interest for my analysis.

The data is lightly filtered to exclude stocks with poorly estimated betas or extreme illiquidity. To be included in the sample, a stock must have:

1. Traded on at least half of the days in which the market has normal trading hours in the observation year;
2. Quoted spreads less than 5% of the price of the stock;
3. One-sided volume exceeding 200 shares in the trading interval, but less than the 95th percentile of one-sided volume; and
4. A market beta in $[0.1, 2.5]$.

Securities not satisfying all of these conditions are excluded from cross-sectional regressions. Table A.1 breaks down sample attrition by each filter.\footnote{For conciseness, the table reports sample attrition with $\hat{\beta}_s$ assumed to be known, although I apply the market beta filter separately for each bootstrap sample $\hat{\beta}_s^{(r)}$.} The average sample size is approximately 2,800 distinct stocks for each trading hour and 3,200 distinct stocks for each trading day.

The rationale for these filters is as follows. Filters 1 and 2 and the lower bound of filter 3 ensure that the stock is not too thinly traded to be reliable for risk estimation, either for computing betas...
in the time series or for estimating tail risks in the cross section. The upper bound on volume in filter 3 ensures that results are not driven by “influential” assets with extremely high volume. The distribution of volume is roughly lognormal, but the underlying model cannot accommodate a log transform of the data to eliminate potential right skew in the dependent variable. Filter 4 accounts for estimation error in the betas; especially large or small betas are likely to be the result of estimation error. In addition, $\beta$ close to zero makes less tenable the assumption of factor jumps being greater than $h/\beta$.

B. Market-Maker Rationality and Informational Requirements

A. λ Known to Market Makers

Equation (12) relies on the rationality and attentiveness of the collective active market maker. Importantly, no single high-frequency market maker needs to enforce Equation (12). Individual market makers may perceive alternative statistical relationships among securities or no relationships at all. Provided that the product of expected factor tails and risk loadings is unbiased across market makers, the linearity of Equation (12) maintains the conjectured liquidity-provision–risk-pricing relationship by the law of large numbers as the number of market makers and assets grows large. In short, for spreads to reflect asset-pricing risks, market makers must be fast and rational, but they need not be omniscient.

Although all market makers need not perceive the same risks, it is useful—but not essential—for market makers to have intimate knowledge of the arrival rate of traders and trading opportunities. Specifically, I assume the following:
Assumption 3. *Market makers know arrival rates $\lambda_{FT}$ and $\lambda_k$ for all factors $k$.*

Assumption 3 simplifies the equilibrium condition of Equation (12) by enabling the market maker to condition on expected volume in setting $h$. High-frequency market makers specialize in estimating near-term liquidity demands from the order book. By virtue of their fast connections and algorithmic processing, they have the opportunity to exploit a large set of potential signals in real time. The competitiveness of market making combined with the potential costs of misestimating demand contributes to detailed estimates of near-term demand outcomes.

*Market Maker Information and Exogeneity*

Equations (12) and (14) relate spreads and depth to expected jump arrivals. Under Assumption 3, the parameters $\lambda_{FT}$ and $\lambda_{jump}$ that govern expected volume and its composition are known to market makers. Because the conditional average volume $\bar{V}(\cdot; h, d)$ is in the market maker’s information set, the competitive market-making sector sets spreads and depth such that jump risk costs exactly offset intermediation gains. From Equation (12), the market making sector then solves

$$\begin{align*}
\left( \frac{h}{d} \right)^* &= \frac{\sum_k \xi_k \beta_k + \bar{\xi}}{\lambda_{FT} q^* + \lambda_{jump} d} = \frac{\sum_k \xi_k \beta_k + \bar{\xi}}{\bar{V}(\cdot; h, d)} \quad \text{(B.1)}
\end{align*}$$

in establishing half-spreads and quoted depth. Assumption 3 is useful because taking expectations over $1/\bar{V}(\cdot; h, d)$ otherwise introduces a concavity correction term for uncertainty in traders’ arrival rate. The next section relaxes this assumption and discusses appropriate modifications to the estimation technique.

Market-maker liquidity prices $h$ and quantities $d$ are a function only of primitive economic risks and known expected volume. Multiplying both sides of the expression by $\bar{V}(\cdot; h, d)$ recovers expected intermediation revenues per unit of depth and maintains the left-hand side’s dependence only on exogenous jump risks:

$$\begin{align*}
\left( \frac{h}{d} \right)^* \bar{V} &= \sum_k \xi_k \beta_k + \bar{\xi}. \quad \text{(B.2)}
\end{align*}$$
B. λ Unknown to Market Makers

I now relax the assumption that market makers know near-term trader arrival rates with certainty (Assumption 3). Suppose instead that liquidity providers do not know trader arrival rates but observe with error the average potential realization of near-term volume. In particular, let the true average volume be conditionally lognormally distributed given a common signal $s$ observed by market makers, where the disturbance term $\nu$ is orthogonal to perceived jump tail risks:

$$\log \bar{V} = s + \nu, \; \nu \sim N\left(0, \sigma_\nu^2\right). \tag{B.3}$$

This signal structure ensures a positive arrival rate and facilitates an empirically plausible conditionally lognormal distribution for volume.

The competitive market making sector sets expected intermediation profits equal to zero. Solving for the equilibrium spread obtains

$$\left(\frac{h}{d}\right)^* = \left(\sum_k \xi_k \beta_k + \bar{\xi}\right) \frac{1}{\bar{V}} = \left(\sum_k \xi_k \beta_k + \bar{\xi}\right) \exp\left(-s - \frac{1}{2} \sigma_\nu^2\right), \tag{B.4}$$

where the second equality follows from the properties of the lognormal distribution. Multiplying both sides by the true average $\bar{V}$ obtains

$$\left(\frac{h}{d}\right)^* \bar{V} = \left(\sum_k \xi_k \beta_k + \bar{\xi}\right) \exp\left(s + \frac{1}{2} \sigma_\nu^2\right) = \left(\sum_k \xi_k \beta_k + \bar{\xi}\right) \exp\left(\nu - \frac{1}{2} \sigma_\nu^2\right). \tag{B.5}$$

As in the known $\bar{V}$ case, using realized volume in place of expected volume contributes to measurement error on the left-hand side and does not affect $\xi_k$ coefficient estimates asymptotically. However, the multiplicative scaling terms are potentially problematic for two reasons. First, uncertainty in average volume forecasts may vary over time. Implied tail risk coefficients are meaningful if $\sigma_\nu^2$ is constant, but time variation in the precision of volume forecasting distorts estimates of relative levels of tail risk. Second, the precision of volume forecasts may differ across assets.

Perhaps surprisingly, obtaining correct tail risk coefficients up to a scaling term is still achievable for each cross-section under plausible independence assumptions. The image processing literature
offers several solutions to the multiplicative noise or “speckle filtering” problem. For example, one such method, median filtering, is readily applied in the cross-sectional regression context under a suitable distance metric on asset betas. Left-hand side values not similar to those of “near” neighbors are excluded under this filter, as abnormally large local deviations are attributed to multiplicative noise from \( \exp(\nu) \).

With this filter in mind, Assumption 4 is a sufficient auxiliary condition for recovery of tail risks with market maker uncertainty about the true trader arrival rate:

**Assumption 4.** The average uncertainty \( \bar{\sigma}^2 \) is constant over time, and \( \sigma^2_\nu \) is locally independent of factor loadings within each cross section.

The first component of Assumption 4 is tenable if HFT market makers’ ability to predict future volume has been roughly constant over the sample period. The second component of the assumption holds if factor loadings do not have first-order effects on market makers’ ability to forecast order flow.

### C. Additional Empirical Considerations

**Negative Betas and Return Symmetry**

This methodology can accommodate negative asset betas. For illustration, suppose that the additional candidate factor is \( FIN \) or “financials,” and that financial betas can be of either sign. If jump tails are symmetric, recovering expected jumps entails minimal modification of our previous expressions,

\[
\left( \frac{V_h}{d} \right)_{it} = \tilde{\xi} + \xi_{MKT} \beta_{im} + \xi_{FIN} \beta_{i,FIN} 1(\beta_{i,FIN} > 0) - \xi_{FIN} \beta_{i,FIN} 1(\beta_{i,FIN} < 0) + \epsilon_i \\
= \tilde{\xi} + \xi_{MKT} \beta_{im} + \xi_{FIN} |\beta_{i,FIN}| + \epsilon_i. 
\]

(C.1)

The first line acknowledges that negative betas only increase half-spreads for buying the asset when the factor’s jump return is negative. Collapsing the expression in the first line suggests that taking the absolute value of beta suffices for estimation.
I implicitly assume symmetry of jumps under $P$ because the distribution of realized jumps for individual stocks and the SPY and XLF ETFs is very close to symmetric.\footnote{Jumps detected using Lee and Mykland (2008) and Bollerslev et al. (2013) methodologies share this symmetry property.} This condition can be weakened further: the density need not be symmetric around zero for every jump size. It is sufficient that the tail expectation of jump sizes above $\tilde{h}_k$ is equal for positive and negative factor moves. Alternatively, Equation (C.1) can be generalized to allow for asymmetric tail expectations:

$$\left( \frac{Vh}{d} \right)^R = \xi^R + \xi^R_{MKT} \beta_{i,MKT} + \xi^R_{FIN} \beta_{i,FIN} \mathbb{1}_{\beta_{i,FIN} > 0} - \xi^L_{FIN} \beta_{i,FIN} \mathbb{1}_{\beta_{i,FIN} < 0} + \epsilon_1, \quad (C.2)$$

$$\left( \frac{Vh}{d} \right)^L = \xi^L + \xi^L_{MKT} \beta_{i,MKT} - \xi^L_{FIN} \beta_{i,FIN} \mathbb{1}_{\beta_{i,FIN} < 0} + \xi^R_{FIN} \beta_{i,FIN} \mathbb{1}_{\beta_{i,FIN} > 0} + \epsilon_2, \quad (C.3)$$

where $R$ and $L$ denote right- and left-jump tails and the corresponding volume and spread proxies. These expressions are readily stacked to obtain a single regression equation in the unknown left and right tail coefficients.\footnote{I estimate left- and right-jump tails separately in the Internet Appendix and confirm that jump tails are typically (nearly) symmetric, but symmetry breaks down around extreme events such as the 2010 Flash Crash.}

\textbf{Co-jumps}

I now revisit assumption 2 in the context of Equations (9) and (12). Excluding co-jumps eliminates terms in Equation (9) associated with factors moving jointly. As an example, suppose that the econometrician considers only market and financial return factors and allows for co-jumps between them. Again denoting the financial return factor as $FIN$, the additional picking-off risk term associated with co-jumps is

$$\lambda_{\{MKT,FIN\}} \int_{\beta_{MKT} + \beta_{FIN} \geq h} (\beta_{MKT} + \beta_{FIN} - h) d (\tau_{MKT}, \tau_{FIN}). \quad (C.4)$$

This additional term is readily converted into linear terms under the large jumps assumption of Equation (12) if jumps are of the same sign. Under these conditions, the additional term in Equation...
(12) is decomposed as

\[-\lambda_{\{MKT,FIN\}} + \lambda_{\{MKT,FIN\}} E[r_{MKT} | r_{MKT}, r_{FIN} > 0] \beta_{MKT}\]

\[+ \lambda_{\{MKT,FIN\}} E[r_{FIN} | r_{MKT}, r_{FIN} > 0] \beta_{FIN}. \tag{C.5}\]

If jump signs differ, additional terms arise resulting from different combinations of the signs of the co-jump returns.

In my multivariate analysis of market and financial jumps, I omit these additional terms because (1) the correlation between SPY and XLF returns is positive and extremely strong, on the order of 89% in my sample, and (2) the absorption of the co-jump terms \(\lambda_{\{MKT,FIN\}} \times E[r_{MKT} | r_{MKT}, r_{FIN} > 0]\) and \(\lambda_{\{MKT,FIN\}} \times E[r_{FIN} | r_{MKT}, r_{FIN} > 0]\) has a clear associated economic intuition. The total coefficient on \(\beta_{MKT}\) is the tail risk of the market with or without financial co-jumps, which arguably is of greater interest than either component of market tail risk independently.

**Liquidity Maker and Taker Fees**

I add liquidity rebates to effective half-spreads to obtain the gross of fees benefit of liquidity provision that accrues to market makers. I assume that rebates are roughly constant across stocks (i.e., that Tape A vs. B vs. C differences are small) and equal to 22 cents per 100 shares. This average rebate size is found in the present-day NYSE price list for the most active liquidity providers (Tier 1; https://www.nyse.com/markets/nyse/trading-info), in Table 1 of the maker-taker analysis of Foucault, Kadan and Kandel (2013) (who in turn reference a 2009 publication), as well as in a recent comprehensive study of maker-taker fees as the average value for active liquidity providers from January 2008 through December 2010 (Cardella, Hao and Kalcheva (2015)). I omit consideration of taker fees and small-market-share “inverted” fee exchanges. Liquidity taker fees drop out in the linear model of Equation (12) because they are subsumed into the lower bound of integration for each factor.

Because rebates are typically small relative to the spread, reasonable alternatives for the level of
the rebate have minimal effect on results. For example, reducing the rebate from $0.0022 per share to zero maintains the same shape in the time series of jumps while shifting the recovered idiosyncratic tail risk down slightly. In addition, rebate measurement errors contaminate the dependent variable and are likely to be uncorrelated with asset betas, so rebate mismeasurement should have little effect on factor tail risk estimates.