ON THE FUNCTIONAL RELATIONSHIP BETWEEN TARIFFS AND WELFARE*

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Received November 1973, revised version received December 1974

This paper uses a model of trade in two commodities between two countries to establish the following proposition. If the foreign offer curve has no points of inflection and if for each home rate of duty the equilibrium most favorable to the home country is selected (or else there is only one equilibrium), then as the rate of duty increases from zero, home welfare first rises then declines while foreign welfare steadily falls.

1. Introduction

Bhagwati and Kemp (1969) have used the standard model of two-country two-commodity trade to argue that for a country which faces an imperfectly elastic foreign offer curve, under certain circumstances it will always be true that successive increases in the tariff from zero will raise home welfare until some level t_w is reached; successive increases in the tariff thereafter will reduce welfare until the tariff becomes prohibitive. They showed that the proposition may break down if the exportable commodity is so inferior in home consumption that for some levels of the home tariff the home offer curve intersects a given terms-of-trade line more than once.

In the figures accompanying their proof of the proposition Bhagwati and Kemp drew the foreign offer curve so that it intersected each terms-of-trade line only once and had no points of inflection although they did not make these assumptions explicit. Subsequently Riley (1970) showed that the proposition may break down also if the foreign offer curve intersects a given terms-of-trade line more than once, which may occur if the foreign country has a tariff and its exportable is inferior in foreign consumption.¹ These authors assumed that each

*Also Simon Fraser University and the U.S. Treasury. The author is indebted to J.N. Bhagwati, T. Borchering, J.S. Chipman, J.W. Dean, D. Graham, H.G. Grubel, H.G. Johnson, M.C. Kemp, C. Lloyd, J.G. Riley, and P.A. Volker for valuable comments on earlier drafts of this paper.

¹Also, Kemp (1969, pp. 305–306) noted that the Bhagwati–Kemp analysis assumes that ‘the foreign offer curve has the conventional shape and passes through the origin, so that the optimal tariff is positive’.

country's aggregate demand is generated by a simple aggregate utility function, an assumption which we maintain throughout the paper.

However, even with Riley's modification the analysis is still incomplete. As this paper shows, the Bhagwati-Kemp proposition may break down as long as the foreign offer curve has any points of inflection, even if it doesn't cross any terms-of-trade line more than once or intersect any home offer curve more than once. Moreover, it necessarily breaks down if any home offer curve exhibits multiple equilibria with the foreign one, but it makes no difference whether the home offer curve has multiple intersections with a given terms-of-trade line. Thus a sufficient set of conditions for the Bhagwati-Kemp proposition to hold is that (i) the foreign offer curve have no points of inflection and (ii) multiple intersections of the two offer curves are ruled out. We also establish a generalization of the Bhagwati-Kemp proposition: *If the foreign offer curve has no points of inflection and if for each home rate of duty the equilibrium most favorable to the home country is selected (or else there is only one equilibrium), then as the rate of duty increases from zero, home welfare first rises then declines while foreign welfare steadily falls.*

Then we demonstrate that the condition for the existence of inflection points in the foreign offer curve is not very stringent, and explore how our italicized proposition is altered when we consider only stable equilibria. Finally, we show that the proposition still holds even when we introduce a non-tradeable good, although the introduction of a non-tradeable along with a tariff implies that even in the absence of inferior goods an offer curve can cross a given terms-of-trade line more than once.

2. The welfare ranking of tariffs

Fig. 1 shows a foreign offer curve superimposed on a set of non-intersecting home trade indifference curves (denoted by I 's) which are fixed in position and have positive first derivatives and non-positive second derivatives² (reflecting

²These are fairly restrictive conditions, but they will hold when we assume away domestic distortions and increasing return, if we further assume that (i) all residents have preference maps which are identical and homothetic, or if (ii) all residents have homothetic preferences and incomes are always distributed in the same proportion between them, or if (iii) they have identical tastes and incomes and are always distributed evenly between them, or if (iv) the government continually redistributes income accruing to the private sector in order to reach the highest possible Bergsonian welfare contour. If these curves are rationalized by route (i), normative significance can be attached to them in that they represent potential welfare. As Chipman (1974, p. 27) notes, if rationalized by route (ii) the curves correctly indicate actual welfare provided the Bergsonian welfare function is of the Cobb-Douglas type for 'then it is optimal for the percentage distribution of income to be held constant (i.e., invariant with respect to changes in prices and aggregate income), the optimal shares being the respective exponents of the welfare function'. If rationalized by route (iii), normative significance is automatically attached to them, for movement to a higher curve implies that every resident is better off. Finally, if rationalized by route (iv), these curves are contours of the Bergsonian welfare function drawn with imports and exports on the axes instead of utilities, so that again

Fig. 1

constant or diminishing rates of transformation in production and diminishing rates of substitution in consumption). For the time being we ignore the dashed line. Let us suppose that each tariff-distorted home offer curve intersects the foreign offer curve only once. Then when the home country is a free trader, equilibrium will be at F , where a home indifference curve is tangent to the terms-of-trade line and the two offer curves intersect. As Johnson (1958, p. 34) notes: 'The reciprocal demand curve traced out by a higher tariff rate will always lie inside the curve traced out by a lower tariff rate . . .' Thus as the home tariff is increased, the equilibrium will move along the foreign offer curve from F to O . If the foreign offer curve has no points of inflection and does not bend back in a southwesterly direction, the foreign offer curve can be tangent to only one home indifference curve, and there can be only one such point of tangency. In this case successive increases in the tariff must cause home welfare to increase monotonically until this point is reached and to decline monotonically thereafter until the tariff becomes prohibitive.

This conclusion continues to hold even when the foreign offer curve curls around as shown by the dashed line in fig. 1. In this case a point of tangency between the foreign offer curve and a home indifference curve may occur at T

normative significance is automatically attached to them. In this case, the Bergsonian indifference curve corresponding to a Bergsonian welfare level w_1 , when drawn with consumption of the two goods on the axes is the envelope of the Scitovsky indifference curves which permit the government to reach the Bergsonian level w_1 .

With regard to alternative (iv), theorem 4 of Chipman and Moore (1972) establishes that if the Bergsonian welfare function is strictly quasi-concave or is of the Leontief fixed-proportions type and if individual preference orderings are continuous, strictly monotone and strictly convex and are represented by continuous concave utility functions, then the social utility function which they define will be strictly quasi-concave. Also when the Bergsonian contours are right-angled, the social indifference curves are a consistent, non-intersecting family of Scitovsky indifference curves. For definitions of terms used in this paragraph, see Takayama (1974, pp. 63, 109), and for further discussion of community indifference curves, see Chipman (1965, sect. 2.3) and Chacholiades (1973, ch. 5A).

in the range where the former is sloping to the southwest, with still other tangency points like T' even further along the foreign offer curve beyond T . However, if there is only one intersection between the foreign offer curve and the home free-trade offer curve, it must occur between T and F^* , where F^* in each of our figures lies on trade indifference curve I^* and is the equilibrium corresponding to the optimum home tariff.³ Moreover, increases in the home tariff will move the equilibrium further away from T toward O via F^* . Thus no equilibrium can occur along the foreign offer curve in the range beyond T , and home welfare continues to be a single-peaked function of the tariff so long as inflection points are ruled out.

Fig. 2

However, this need not be the case if the foreign offer curve has points of inflection. Fig. 2 illustrates such a case. In that figure, the foreign offer curve is shown to have points of inflection, but its slope is positive throughout, and it intersects each ray from the origin only once. Note that a clockwise movement along the offer curve from F to O causes a rise in home welfare from I_3 to I^* , then a decrease to I_1 , an increase to I_2 , and finally a decline to I_0 . Thus we have

³This is easily seen. Given that the home trade indifference curves are convex, the point of tangency between a home trade indifference curve and the terms-of-trade line through O and T must lie between O and T , whereas the terms-of-trade line through O and F^* must be tangent to a home trade indifference curve beyond F^* . Thus, the two offer curves must cross between T and F^* .

proven that the Bhagwati-Kemp proposition does not necessarily hold if the foreign offer curve has points of inflection.

Secondly, if at least two equilibria do exist at a given domestic tariff, then by examining the equilibria corresponding to a slightly higher and a slightly lower tariff, it will always be possible to find one of the equilibria at the intermediate tariff where welfare is less than welfare at one of the low-tariff equilibria and also less than welfare at one of the high-tariff equilibria. Thus, if there are multiple equilibria, welfare can never be a single-peaked function of the tariff.

Fig. 3

It should be emphasized that multiple intersections of the offer curves can occur without points of inflection in either offer curve. Moreover, offer curves can have points of inflection even in the absence of inferior goods and multiple intersections between the two offer curves. Finally, in a two-good world an offer curve can intersect a terms-of-trade line more than once only if it has inflection points, and such multiple intersections require that the country have a tariff and its exportable be inferior in its own consumption.

Now we determine whether the home country's welfare is a single-peaked function of its tariff, assuming that the home country is always able to select the equilibrium most favorable to it and that the foreign offer curve has no inflection points. This situation is illustrated in fig. 3. It is clear from the way in which

offer curves are constructed that a country's welfare must be an increasing function of the distance along any of its tariff-distorted offer curves from the origin, so the last intersection of any home offer curve with the foreign one must be most preferred from the home standpoint. As mentioned above, the offer curve corresponding to a higher tariff must lie inside that corresponding to a lower one. This means that the home offer through F^* , call it H^* , must lie uniformly inside any offer curve like H' which is generated by any tariff which is below the optimum level. Since F^* is the optimum equilibrium from the home standpoint, it must be the last intersection of H^* with the foreign offer. Hence, it is impossible for H' to have the shape indicated by the dashed line. Clearly, then, as long as the tariff is below the optimum level and the foreign offer curve does not bend back to the southwest, the best equilibrium from the home standpoint will be further along the foreign offer curve than F^* and will be the closest one to F^* , where distance is measured along the foreign offer curve, and this equilibrium will necessarily move closer to F^* as the tariff is increased.

The analysis is more complex when the foreign offer curve bends back to the southwest and there is a tangency point at T as shown in fig. 1. However, even if the foreign offer curve has an intersection with the home free-trade offer curve along the foreign offer curve beyond T , it will also have an intersection along the foreign offer curve between T and O , which will lie further along the home offer curve and be preferred from the home standpoint to any intersection along the foreign offer curve beyond T . Since any home tariff-distorted offer curve will lie inside the home free-trade offer curve, this characteristic will be true of all home offer curves. Thus, it is correct for us to restrict our attention to equilibria along the foreign offer curve between T and O , and our conclusion remains intact even when the foreign offer curve curls to the southwest.

Similarly, after the optimum tariff has been reached and exceeded, F^* must lie outside the home offer curve and the best equilibrium from the home standpoint will lie along the foreign offer curve between O and F^* , and it will be the closest one to F^* . Finally, it is easy to see that as the tariff is increased further, this equilibrium will recede along the foreign offer curve closer to the origin. Thus, we have proven that home welfare steadily rises with the home tariff up to some point after which it steadily falls. Moreover, foreign welfare will be a declining function of the home tariff throughout its entire range, for each increase in the home tariff moves the equilibrium along the foreign offer curve closer to the origin, which implies a steady fall in foreign welfare. Thus, we have established the italicized proposition in the introduction.

3. On the difficulty of ruling out inflection points

Now let us find what sort of excess demand function implies an offer curve with inflection points in it. Let p denote the relative price of the foreign import good, and let M be the quantity of foreign imports. Since trade is always balanced

along the foreign offer curve, its slope can be written as

$$\sigma = dM/d(Mp) = 1/\{[1 - (1/e)]p\}, \quad (1)$$

where

$$e = -(dM/M)/(dp/p), \quad (2)$$

and e is the elasticity of foreign excess demand for the import. We restrict our attention to the range in which the excess supply of the export is an increasing function of its relative price, which implies that the offer curve is upward sloping and that the elasticity of excess demand for the import must be greater than 1. Differentiating (1) yields

$$d\sigma/dp = \{-[1 - (1/e)] + (p/e^2) de/dp\}/\{[1 - (1/e)]p\}^2. \quad (3)$$

Thus, in the range of concern the second derivative of the offer curve will be negative if and only if $d\sigma/dp > 0$, and from (3) this inequality holds if and only if

$$(de/dp)/(e/p) > e - 1. \quad (4)$$

Thus, we conclude that multiple tangency points between the home indifference curves and the foreign offer curve may exist if and only if the price elasticity of the elasticity of foreign excess demand for imports exceeds the elasticity of foreign excess demand minus one. In fact, we would expect the elasticity of excess demand to rise with price ($de/dp > 0$) since price increases move the system closer to autarky and cause imports as a fraction of consumption of the import to decline.

To demonstrate that one should not be surprised to encounter inflection points in a simple 2-good world where demand curves are generated by a single aggregative utility function, let us consider the case in which the elasticities of demand for and supply of the foreign import good are constant within a small range. Denoting the demand and supply elasticities by $\eta > 0$ and $\nu > 0$ respectively, and foreign production and consumption of the good by S and C , we have

$$-\eta = (dC/dp)/(C/p), \quad (5)$$

and

$$\nu = (dS/dp)/(S/p). \quad (6)$$

The foreign elasticity of excess demand for the import is

$$e = -[d(C-S)/dp]/[(C-S)/p], \quad (7)$$

where $C-S > 0$ since the good is imported. Upon substitution from (5) and (6),

(7) becomes

$$e = [\eta C + \nu S]/[C - S]. \quad (8)$$

Differentiating (8) totally, and substituting for dC and dS from (5) and (6) yields

$$de/dp = [\eta + \nu]^2 CS/[p(C - S)^2]. \quad (9)$$

Dividing (9) by e/p , substituting into (4), replacing e by the expression in (8) and rearranging indicates that the inequality will be satisfied if and only if

$$S\nu(\nu + 1) > C\eta[\eta - 1]. \quad (10)$$

Thus, if foreign imports are small relative to foreign production and consumption and if both elasticities are constant within a small range where the elasticity of supply exceeds the elasticity of demand, the second derivative of the offer curve will necessarily be negative. Since this configuration of quantities and elasticities is not an unreasonable assumption, it seems reasonable to postulate the existence of inflection points when constructing models of trade.⁴ Therefore, those who calculate optimum tariffs should be wary of multiple optima. Worse still, since two maxima must be separated by a minimum, he who has used first-order conditions alone in his search for an optimum may well have found a pessimum.

4. Dynamic considerations

Our analysis so far has indicated that the single-peakedness result in no way depends on the assumption that the home exportable is a superior good in home consumption. However, when we admit the possibility of inferiority and concern ourselves with stability of equilibrium, our proposition requires modification. It follows from Kemp's (1968) analysis that equilibrium when the terms of trade are given will be stable if and only if the home offer curve intersects the terms-of-trade line from above. Let us postulate that governments are quick to pass on changes in tariff revenues as changed consumption subsidies,⁵ but that some supra-national secretariat buys and sells goods at relative prices fixed at each moment, but variable in the light of inventories. Thus, the secretariat is assumed

⁴Harry G. Johnson has suggested to me yet another way to obtain inflection points in the offer curve upon which the following discussion is based. Suppose the foreign production frontier is curved except for a flat segment in the middle (with a slope of say -1), and suppose that the foreign consumption indifference curves are homothetic and one is tangent to the production frontier where the opportunity cost of the import is higher than that necessary to be on the flat segment of the production frontier. Then even if the foreign offer curve has the normal curvature when the relative price of the import is different from one, for some range it will still coincide with the terms-of-trade line along which the relative price of the import equals one. Ergo, it must have inflection points.

to adjust downward the relative price of whichever good it is accumulating.⁵ Then, even when the terms of trade are variable, for stability the home offer curve must intersect the terms-of-trade line from above.

We also use fig. 3 to illustrate this complication. When H' is the tariff-distorted home offer curve, F' is the equilibrium which maximizes home welfare. With a higher tariff which generates the distorted home offer curve H'' , U_1'' is the equilibrium most preferred by the home country, but both it and U_2'' are unstable, which means that the home country must settle for F'' , where home welfare is lower and foreign welfare is higher than at F' . Thus our proposition breaks down when we admit the possibility of the home exportable being inferior in home consumption and at the same time constrain the home country to choose between stable equilibria. However, it is well known that for every tariff there exists a quota on imports or exports which has equivalent static effects. Thus, if the home country wished to attain point U_1'' , it could impose a quota on imports or exports which caused its offer curve to pass through U_1'' , and there will be either an import quota or an export quota which causes U_1'' to be stable. Thus, we could restate our proposition in terms of tariff equivalents and thereby rehabilitate it. One need only substitute 'tariff or tariff equivalent' for 'rate of duty' in the statement of the proposition in the introduction.

5. Non-traded goods

Until now we have assumed that both countries produce only the two tradeables. However, even when both countries produce a non-tradeable as well as the two tradeables, as Kemp and Tower (1975) have noted, it is still possible to construct trade indifference curves with the two tradeables on the axes. These curves will be non-intersecting, will have the normal curvature and in equilibrium their slopes will equal the price ratio faced by consumers and producers. Moreover, offer curves are constructed from trade indifference curves in exactly the same way as for a two-commodity world. Similarly, welfare will be an increasing function of the distance along any offer curve (whether distorted by a tariff or not). Finally, offer curves which are distorted by higher tariffs will lie inside those which are distorted by lower tariffs. Thus, our italicized proposition continues to hold even in the presence of non-tradeable goods. In this context it is interesting to note that Batra (1973) building on Komiya (1967) has shown that for a country which is too small to influence its terms of trade and produces a non-traded good in addition to an exportable and an importable, an increase in an already positive import duty which causes an infinitesimal change in the equilibrium position may raise welfare even in the absence of domestic distortions and inferior goods. However, since an offer curve which is

⁵This approach is formally equivalent to that employed in section 2 of Jones (1961), although there were no tariffs in his model, so that multiple intersections of one offer curve with a single terms-of-trade line were ruled out.

distorted by a high tariff must be within that which is distorted by a lower tariff, at any point in the range where a tariff-distorted offer curve can lie, the slope of the trade indifference curve must exceed that of the terms-of-trade line through that same point. Thus it is easy to see geometrically that Batra's result is possible only when the home offer curve intersects the given terms-of-trade line from below. Such an equilibrium would be unstable according to our dynamic analysis, although it could be stable under other dynamic hypotheses.⁶ More important though, as Kemp and Tower (1975) have shown, for each equilibrium which affords Batra's result, there must be another equilibrium further along the tariff-distorted home offer curve which intersects the foreign offer curve in the normal way and generates maximum home welfare given the home tariff;⁷ at this equilibrium it is easy to see that home welfare will be a decreasing function of the tariff until it becomes prohibitive. *Thus, for a small country, which faces given terms of trade and produces a non-tradeable, it is still true that maximum attainable welfare is greater with a lower tariff than with a higher one until the tariff becomes prohibitive.*

⁶Kemp (1967 and 1969) has shown that if the foreign offer curve is tangent to a home indifference curve at the point where the former intersects a terms-of-trade line from above, then the optimum tariff corresponding to this equilibrium will be negative. Note that according to our dynamic hypothesis it will also be unstable. However, under an alternative dynamic hypothesis where relative prices adjust much more rapidly than changes in tariff revenues are distributed, the equilibria considered may well be stable, as I have shown in an unpublished paper, available upon request.

⁷Batra (1973, p. 706) writes: 'Despite the claim to the contrary by Murray Kemp, we find that an increase in the rate of tariff may actually lead to a rise in welfare even when distortions are absent.' However, his footnote 1 refers to Kemp's (1969, p. 267) statement: 'The proof could be modified to accommodate nontraded goods,' but Kemp's 'The proof' refers to the proposition that restricted trade is superior to no trade, not that an increase in the rate of tariff will lead to a fall in welfare. Moreover, Kemp (1969, pp. 290-291) writes with world prices given: 'in tariff-ridden situations competitive equilibrium may not be unique; and when multiple equilibria exist it is *not* true that each 5 percent equilibrium is necessarily superior to every 10 percent equilibrium. It can be shown only that for every high-tariff equilibrium there exists at least one superior low-tariff equilibrium . . .'. Thus it is not clear that Kemp has been refuted.

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