Causality across rainfall time scales revealed by continuous wavelet transforms

Annalisa Molini,^{1,2} Gabriel G. Katul,^{1,2} and Amilcare Porporato^{1,2,3}

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[1] Rainfall variability occurs over a wide range of time scales owing to processes initiated by cloud microphysics and sustained by atmospheric circulation. A central topic in rainfall research is to determine whether rainfall variability at a given scale is caused by dynamics acting at some other scales. Random multiplicative cascades (RMCs) are standard approaches for describing rainfall variability across such a wide range of time scales. Their popularity stems from their ability to reproduce rainfall self-similarity and long-range correlations as well as intermittency buildup at finer scales. However, standard RMCs only predict instantaneous flow of variance (energy or activity) from large to fine scales and cannot account for scale-wise causal relationships. Such relationships reveal themselves through noninstantaneous cascade mechanisms, namely, large-scale events influencing finer-scale events at later times (i.e., forward causal cascade) or conversely (inverse causal cascade). The presence of causal cascade signatures within the rainfall process is explored here using both continuous wavelet decomposition (CWT) and scale-by-scale causality measures such as cross-scale correlation and linearized transfer entropy. The causality hypothesis is further tested against results from toy models, surrogate data, and a scalar turbulence time series (water vapor) to ensure that rainfall causality is not an artifact of the estimation method or resulting from the redundancy in CWT. The analysis demonstrates the presence of causal cascades (mainly forward) in rainfall series when sampled at fine temporal resolutions (seconds). These causal relationships tend to vanish when rainfall is aggregated at coarser time scales (hours and longer).

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Physics has stopped looking for causes for 'there are no such things'. The law of causality, I believe, like much that passes muster among philosophers, is a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm.

Russell [1913, p. 1]

1. Introduction

[2] Intermittency and long-range correlations are known to characterize the rainfall process over a wide range of space and time scales [Molini *et al.*, 2002, 2006; Koutsoyiannis, 2002; Roux *et al.*, 2009; Waymire, 2006; Zawadzki, 1973]. Random multiplicative cascades (RMCs), with their theoretical underpinning in the multifractal formalism, are now classical tools used to represent such long-range power law correlations and intermittent patterns [Sornette, 1998].

Hence, their application in describing rainfall downscaling from large (meteo-climatic) to fine (hydrological) space and time scales has received significant attention in hydrology [see Deidda, 2000; Gupta and Waymire, 1993; Lovejoy and Schertzer, 1995; Menabde *et al.*, 1997] (and references therein).

[3] The genealogy of RMCs may well trace its mathematical formalism to turbulence studies, where it proved successful in explaining how intermittency buildup occurs in the turbulent kinetic energy cascade [Yaglom, 1962; Frisch, 1995; Kraichnan, 1974; Meneveau and Sreenivasan, 1991; Monin and Yaglom, 1975; Pope, 2000]. These early RMCs (and later refinements) found experimental support when instantaneous turbulent kinetic energy dissipation rate measurements (or their estimates from high-frequency sampled velocity fluctuations) turned out to be well approximated by a lognormal distribution with a long-range power law correlation structure. Such cascade models often rely on phenomenological arguments that are kinematic in nature, and actual connections to the dynamics (as given by the Navier-Stokes equations) have resisted rigorous derivation [Benzi *et al.*, 1984; Jiménez, 2000]. In the case of rainfall, all the governing laws are not entirely known

¹Nicholas School of the Environment, Duke University, Durham, North Carolina, USA.

²Department of Civil and Environmental Engineering, Pratt School of Engineering, Duke University, Durham, North Carolina, USA.

³Visiting professor at Laboratory of Environmental Fluid Mechanics and Hydrology, École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland.

thereby making the link between kinematic arguments and the actual dynamics elusive [Marsan *et al.*, 1996; Venugopal *et al.*, 1999, 2006a, 2006b]. This is one main reason why the kinematic approach remains an attractive modeling framework in rainfall research. One possible shortcoming of a kinematic approach in representing the true evolution of the process is the presence of causal relationships across scales. These causal relationships are not reproduced by classical RMCs approaches [Schmitt and Chainais, 2007] because RMCs generally assume an instantaneous cascade [Jiménez, 2000].

[4] The overall goal of this work is to develop schemes that can assess causality signatures in rainfall time series measurements across different time scales. This assessment has both theoretical and practical implications. On the theoretical side, such assessments are necessary first steps to build a comprehensive theory for the dynamics of rainfall. Also, these schemes may offer blueprints on how to proceed with possible revisions to RMCs that can then improve modeling performances or statistical downscaling skills. The proposed causality detection schemes are based on scale-by-scale decomposition of rainfall time series using continuous wavelet transform (CWT) and on extensions to the scale-time domain of causality statistics originally developed in the time or frequency domains. The schemes are then subjected to a battery of tests to ensure that any causality in rainfall is not an artifact of the proposed methodology. These tests include the use of toy models such as a simple downscaling scheme based on the β model (BM) [Gupta and Waymire, 1993], the binomial cascade (BC) [Riedi, 2002; Meneveau and Sreenivasan, 1991], and both shuffled and iterative amplitude adjusted Fourier transform (IAAFT) surrogate data [Schreiber and Schmitz, 2000; Roux *et al.*, 2009]. Also, the proposed scheme is applied to scalar turbulence (water vapor) [Katul *et al.*, 2006], and dissimilarities in causal properties between turbulence (sampled in time) and rainfall were emphasized. We should note here that the adopted toy models are neither intended to reproduce a particular rainfall data set nor to provide an exhaustive summary of all plausible rainfall representations. Rather, they are intended to explore the fidelity of the proposed causality measures when applied to cascades whose causal scale-wise structure can be extrapolated a priori. Moreover, these toy models do offer robust reference cases to assess causality strength in experimental data sets. The measured water vapor concentration fluctuations are also used here as another toy model because of connections between rainfall intensity, precipitable water, and water vapor concentration fluctuations. These connections can be inferred from simplified models such as the cylindric column representation of a steady state convective storm cell circulation [Wiesner, 1970].

2. Causality, Time Asymmetry, and Cascades

[5] As earlier mentioned, RMC models applied to rainfall data do not account for possible directionality in the energy (here interpreted as the local variance in rainfall intensity) flow from large to fine scales, and are unable to reproduce causality across time scales. This implies that RMCs cannot capture how the dynamics at a given scale influence the

evolution of the process at finer (forward cascade) or larger (inverse cascade) scales at later times.

[6] From a modeling perspective, proposed solutions for introducing causal effects in cascade models incorporate a time dependence in the cumulant generating function of the cascade [Marsan *et al.*, 1996] and resort to more complex cascading schemes such as continuous cascades [Bacry *et al.*, 2008; Schmitt and Marsan, 2001; Schmitt and Chainais, 2007]. Nevertheless, all these solutions require a priori knowledge of the underlying causal relationships. Hence, a method that is able to detect causality in the measured rainfall time series has the decisive advantage of informing whether RMCs can be applied without modifications or whether more complex rainfall models are warranted.

[7] In the time domain, causality generally implies that a response occurs at a later time when compared to a perturbation. This implies that causality measures such as Granger causality [Granger, 1969] are traditionally employed to explore causal relationships in multivariate time series (i.e., whether variable X is causing variable Y) as earlier done, for example, in rainfall-soil moisture feedback studies [Salvucci *et al.*, 2002; Tessier *et al.*, 1996; Wei *et al.*, 2008]. As a consequence, they cannot be readily implemented for the univariate case.

[8] Here, we are concerned with how rainfall variability at a given scale a is influencing variability at a larger-scale $a + \Delta a$ and vice versa in a univariate time series of rainfall intensity $R(t)$. We propose to overcome the limitation in assessing causality from univariate time series by using continuous wavelet transform (CWT), which are reviewed elsewhere [see, e.g., Kumar and Foufoula-Georgiou, 1997, and references therein]. Because of the time-scale decomposition and the interest in cross-scale information flows, CWTs do offer the possibility of applying multivariate causality measures to wavelet decomposed univariate series. Statistical tools adopted to analyze cross-scale causality flows in rainfall range from delayed cross-scale correlation functions [Arnéodo *et al.*, 1998a] to metrics developed in the context of information theory such as transfer entropy [Schreiber and Schmitz, 2000].

[9] Since the interest here is in variability across time scales, we introduce a local rainfall variance measure $\sigma_a^2(t)$ that depends on both scale a and time t . The $\sigma_a^2(t)$ will be formally defined in section 4.1 using continuous wavelet transform (CWT), but roughly speaking, it describes the variability of the rainfall process $R(t)$ observed with a window of typical width a centered at time t . This definition is analogous to the usage of the local kinetic energy in turbulence cascades. Like rainfall intensity, the turbulent kinetic energy varies across scales and hence is not a conserved quantity throughout the cascade (though the mean turbulent kinetic energy dissipation rate is actually conserved across scales).

[10] Using RMCs, it is possible to decompose the local variance of $R(t)$ in a multiplicative manner. In particular, $\sigma_{a_n}^2$ at the finest-scale a_n can be assumed to be linked to the variability at some large (integral) time scale a_0 by a random cascade

$$\sigma_{a_n}^2 = \prod_{i=0}^{n-1} M_{a_{i+1}, a_i} \cdot \sigma_{a_0}^2 \quad (1)$$

over any discrete decreasing sequence of scales $\{a_i\}_{i=0,\dots,n}$, being the M_{a_{i+1},a_i} independent realizations of a random variable M subject to the following constraints [Amblard and Brossier, 1999; Arnéodo et al., 1998a; Frisch, 1995]:

$$M \geq 0, \langle M \rangle = 1, \langle M^q \rangle < \infty \forall q > 0. \quad (2)$$

Note that such a multiplicative cascade mechanism guarantees scale similarity but does not include any explicit dependence on time and simply shows how energy at one scale produces instantaneously energy at another scale (labeled as Markovian causality in scale). Moreover, rainfall self-similarity is assured only over a limited subrange of time [Fraedrich and Larnder, 1993; Malamud and Turcotte, 1999; Menabde et al., 1997; Veneziano et al., 1996; Venugopal et al., 1999, 2006a, 2006b] and space scales [Deidda et al., 2006; Gupta and Waymire, 1990, 1993; Kumar and Foufoula-Georgiou, 1993; Lovejoy and Schertzer, 1985; Lovejoy et al., 2008], and different scaling regimes do arise when analyzing climatological or single storms patterns [Fraedrich and Larnder, 1993]. Such deviations from self-similarity can still be accounted for in RMCs by providing scaling rules adjusted to different rainfall regimes (i.e., the scaling regimes can be shifted across some scales). These simple schemes do not encode any information about causal properties in time arising from the dynamics of the rainfall process.

3. Rainfall Data Sets

[11] We apply scale-wise causality detection schemes to rainfall time series sampled at different frequencies and across various hydroclimatic regimes. The scheme performance is tested on both toy models (where causal properties are a priori known) and measured scalar turbulent series (i.e., an atmospheric water vapor time series). Scalar turbulence usually resembles a lognormal bounded cascade [Katul et al., 2006] and can also be used as an example of a simple cascade toy model. The analysis is performed on both high-resolution rainfall data spanning typical duration of rain events and historic time series sampled at coarser resolution but covering much longer periods (from 8 to 118 years). The high-resolution data set is composed of seven rainfall events, each of different duration (ranging from 3 and 24 h), recorded between May 1990 and April 1991 at the Iowa Institute of Hydraulic Research (IIHR), Iowa City (IA) and publicly available at <http://www.iihr.uiowa.edu/>. The series were sampled at a frequency of 0.1 to 0.2 Hz using a ORG 705 optical rain gauge manufactured and calibrated by Scientific Technology Incorporated, USA [Georgakakos et al., 1994; Venugopal and Foufoula-Georgiou, 1996]. In Figure 1, four of the seven studied events are presented, together with their power spectra $S(f)$ estimated via an orthonormal wavelet transform (OWT). The OWT spectra presented are normalized by the variances. Average power spectra computed with standard Fourier techniques are known to suffer from energy leakage at high frequencies given the on-off nature of the data sets. For this reason, scaling laws were determined using OWT spectra computed by a Symmlet wavelet [see, e.g., Mallat, 1989]. It was demonstrated elsewhere [Katul and Parlange, 1994] that OWT spectra provide robust estimation of the power law

exponent irrespective of the choice of the wavelet basis function.

[12] Spectral slopes here are presented over the typical range of subevent scales (1 h down to 5–10 s) where scaling is more evident. These high-resolution events present sharply different scaling regimes, with events characterized by higher intensities (i.e., event 1) displaying stronger memory (and stronger long-range correlation) when compared with low-intensity events (event 4).

[13] Recently, it was shown that the nature of the multiplicative cascading mechanisms for this data set is predominantly local and bounded within single pulses defining a typical storm event time scale [Venugopal et al., 2006a, 2006b]. This finding suggests that any causality, if it exists, must at minimum be explored at these fine scales. To assess whether causality is eventually preserved from fine to large (climatological) time scales, an 8 year long time series collected at Duke Forest, near Durham, North Carolina (hereafter referred as DF) with a 30 min resolution [Katul et al., 2007] and a 118 year historic daily time series recorded at the Meteorological Observatory A. Bianchi in Chiavari, Italy (CHV) [Molini et al., 2005] are analyzed. The two rainfall depth time series are shown in Figure 2 together with their $S(f)$. Spectral exponents for the DF series are obtained in the range 1–12 h, while the CHV exponent is estimated for time scales spanning between 1 day and 6 months. As expected, scaling laws of rainfall spectra at climatological scales possess a stronger memory when compared with their subdaily counterparts due to the vanishing of intermittent features with increasing time aggregation.

4. Methods of Analysis

4.1. Wavelet Decomposition and Local Log Variance

[14] We already pointed out (section 1) how the scale-by-scale decomposition of univariate time series represents the starting point in analyzing the energy (or variance) flow among scales. In the present section, the basic tools adopted in such a time-scale decomposition, namely the continuous wavelet transform CWT, and its connection with the multiplicative scheme in equation (1) are presented.

[15] For a rainfall intensity time series $R(t)$, the CWT at scale a and time location t is defined as the set of scalar products $W_\psi[R]$ between R and the t -translated, a -dilated and normalized version of an analyzing function $\psi(t)$,

$$W_\psi[R](t, a) = g(a) \int_{-\infty}^{+\infty} R(t') \psi\left(\frac{t' - t}{a}\right) dt', \quad (3)$$

where $g(a)$ is a scale-dependent normalization factor and the $W_\psi[R]$ are also called wavelet coefficients (WCs). To detect causal relationships through asymmetry in time, the analyzing wavelet ψ (also called “mother wavelet”) should be symmetric.

[16] The “Mexican hat” (the second derivative of a Gaussian mother wavelet) and higher-order even derivatives of Gaussian wavelets are symmetric and were previously adopted in the analysis of rainfall series [Venugopal et al., 2006a]. The causality analysis presented throughout is based on the mother wavelet Gauss4, i.e., the fourth-order derivative of a Gaussian function. However, we did repeat all the causality analysis using different mother wavelets and

Figure 1. (left) Rainfall intensity $R(t)$ and (right) normalized OWT power spectra $S(f)$ for four events extracted from the Iowa City data set. From top to bottom: event 1 (originally sampled at 10 s) and events 4, 5, and 6 (sampled at 5 s). Note that the ordinates are different in the rainfall intensity plots. These events span a wide range of intensities with event 1 sampling a maximum of about 120 mm/h while event 4 sampling a maximum of about 9 mm/h. The spectral exponents at subevent scales (from a few seconds to 1 h) oscillate between the 1.6 for event 4 to 0.77 for event 1, revealing the tendency of higher-intensity events to possess longer memory.

found that the effects of different ψ on causality statistics remain minor (not shown).

[17] The local variance $\sigma_a^2(t)$ at scale a and time t can be estimated from the wavelet coefficients $W_\psi[R]$, as

$$\sigma_a^2(t) \equiv \frac{1}{a^2} \int \chi\left(\frac{b-t}{a}\right) |W_\psi(b, a)|^2 db \quad (4)$$

where χ is a bump function (or a simple box function) guaranteeing the energy conservation at each scale. Such a filtering is applied to avoid edge effects related to the wavelet basis [Moret-Bailly et al., 1991]. The analysis is thus conducted on the magnitudes (or log variances) of $\sigma_a^2(t)$ given by

$$\omega_a(t) = \frac{1}{2} \ln \sigma_a^2(t). \quad (5)$$

A similar approach was employed in economic time series analysis, where marked directionality in information flow of this quantity across different time scales was documented [Arnéodo et al., 1998b].

[18] The log transform in equation (5) has a Gaussianizing effect on the local variance at different scales, allowing us to restrict attention to the case of linear causal relationships (see section 4.3). Also, it is preferred here because our causality analysis is essentially confined to fine scales Venugopal et al. [2006a] known to be energetically small in the rainfall series. Finally, it is worth noting that redundancy effects due to the usage of CWT are minor at fine scales [Farge, 1992].

4.2. Scale-Wise Cross Correlation

[19] Once a rainfall time series is decomposed on a scale-by-scale basis, cross-scale linear correlation coefficients can

Figure 2. Rainfall depths $h(t)$ for (top left) DF and (bottom left) CHV time series. For these long time series, sampled at coarse resolution (30 min for (top right) DF and 1 day for (bottom right) CHV) the wavelet spectrum is shown. As expected, the CHV time series has longer memory over a wide range of scales (from about 3 months to 2 days).

be estimated to assess eventual asymmetries in the local variance cascade. These time asymmetries signify that a local excursion at a given scale impacts the energy or activity of events at another scale and at later times, thus suggesting causality in the cascade. Correlation coefficients are computed from

$$C_{a+\Delta a, a}(\Delta t) = \frac{\langle \tilde{\omega}_{a+\Delta a}(t) \cdot \tilde{\omega}_a(t + \Delta t) \rangle}{(\langle \omega_{a+\Delta a}^2 \rangle \cdot \langle \omega_a^2 \rangle)^{1/2}}, \quad (6)$$

with $\tilde{\omega}$ being the centered magnitudes.

[20] Assuming $\Delta a > 0$, we have from equation (6) that $C_{a+\Delta a, a}(\Delta t)$ represents the correlation between large and small scales at later times, while $C_{a+\Delta a, a}(-\Delta t)$ can be thought of as a measure of how fine scales are correlated with large scales at later time. As a consequence, if $C_{a+\Delta a, a}(\Delta t) > C_{a+\Delta a, a}(-\Delta t)$, larger scales are said to cause finer ones (forward causal cascade) and if $C_{a+\Delta a, a}(\Delta t) < C_{a+\Delta a, a}(-\Delta t)$, small scales are said to influence large scales (inverse causal cascade). Finally, if $C_{a+\Delta a, a}(\Delta t) = C_{a+\Delta a, a}(-\Delta t)$, we obtain

the instantaneous cascade with no causality. A schematic representation of the different forms of causal cascade is reported in Figure 3. Note that the decomposition of the signal in n subscales results in a $n \times n$ correlation matrix. For visual presentation, we report $C_{a+\Delta a, a}$ referenced to the original aggregation scale a_n (or generally the finest sampling scale). This is a choice consistent with our main interest on small-scale dynamics. For $\Delta a = 0$, the single-scale autocorrelation $C_a(\Delta t)$ already discussed elsewhere for the Iowa series [Roux et al., 2009; Venugopal et al., 2006a] is recovered and will not be discussed here.

[21] While the scale-wise cross-correlation analysis indicates the potential for net causal relationships among scales, it provides no information about the monodirectionality or bidirectionality of causality and the different components contributing to it. In section 4.3, how different components of causality in the time-scale half-plane can be estimated via formal application of classical causality measures such as Granger causality and more recent interpretations such as the linearized transfer entropy are reviewed.

Figure 3. Diagram representing basic concepts in causal cascades and their signatures in cross-scale correlation analysis. (top) Schematic representation of a wavelet decomposed variable (e.g., local variance) at smaller (a_0) and larger ($a_0 + \Delta a$) scales exhibiting (a) forward, (b) symmetric, and (c) inverse cascade schemes. (bottom) The corresponding cross-scale correlation functions. On average, forward causal cascades suggest that oscillations at larger-scale $a_0 + \Delta a$ precede in time oscillations at smaller-scale a_0 , resulting in an asymmetric cross-scale correlation function similar to the one shown in Figure 3a (bottom). In the inverse cascade (Figure 3c), small-scale oscillations precede oscillations at larger scales leading to a scale-by-scale correlation $C_{a+\Delta a,a}(\Delta t) < C_{a+\Delta a,a}(-\Delta t)$.

4.3. Causality in the Wavelet Domain

[22] Popular causality measures in the time domain include Granger causality and its extended nonlinear forms, such as transfer entropy [Hlavackova-Schindler et al., 2007; Schreiber and Schmitz, 2000], predictability improvement and similarity index [Lungarella et al., 2007] among others. Extensions to the frequency domain are concerned with Fourier homologue of the temporal measures such as the spectral Granger causality described by Geweke [1982] in terms of coherence. It is not the intent here to compare all these measures, and we restrict our attention to the most elementary representation of causality measures (i.e., linear) as a starting point for our analysis in the wavelet domain.

[23] Consider two scales $a + \Delta a$ and a , where Δa is positive. The local magnitudes $\omega_{a+\Delta a}(t)$ and $\omega_a(t)$ can be regarded as realizations of a stochastic processes that depend on both time and scale. If this stochastic process can be assumed jointly stationary at fixed scales $a + \Delta a$ and a , one

way to proceed to quantify causality is to assume an autoregressive (AR) structure in the time domain for each scale given by [Ding et al., 2006; Granger, 1980]

$$\begin{aligned} \omega_a(t) &= \sum_{j=1}^{\infty} \alpha_{1j} \omega_a(t - j\Delta t) + \epsilon_{1t}, & \text{var}(\epsilon_{1t}) &= \Sigma_1 \\ \omega_{a+\Delta a}(t) &= \sum_{j=1}^{\infty} \delta_{1j} \omega_{a+\Delta a}(t - j\Delta t) + \eta_{1t}, & \text{var}(\eta_{1t}) &= \Gamma_1 \end{aligned}, \quad (7)$$

while jointly they can be represented as

$$\begin{aligned} \omega_a(t) &= \sum_{j=1}^{\infty} \alpha_{2j} \omega_a(t - j\Delta t) + \sum_{j=1}^{\infty} \beta_{2j} \omega_{a+\Delta a}(t - j\Delta t) + \epsilon_{2t} \\ \omega_{a+\Delta a}(t) &= \sum_{j=1}^{\infty} \kappa_{2j} \omega_a(t - j\Delta t) + \sum_{j=1}^{\infty} \delta_{2j} \omega_{a+\Delta a}(t - j\Delta t) + \eta_{2t} \end{aligned}, \quad (8)$$

with noise contributions ϵ_{it} and η_{it} uncorrelated in time and the covariance matrix of the process in (8) given by

$$\Xi = \begin{pmatrix} \Sigma_2 & \Upsilon_2 \\ \Upsilon_2 & \Gamma_2 \end{pmatrix}, \quad (9)$$

where $\Sigma_2 = \langle \epsilon_{2t}^2 \rangle$, $\Gamma_2 = \langle \eta_{2t}^2 \rangle$ and $\Upsilon_2 = \langle \epsilon_{2t}, \eta_{2t} \rangle$. Independence among scales implies $\Upsilon_2 = 0$, $\Sigma_1 = \Sigma_2$ and $\Gamma_1 = \Gamma_2$, so that the global interdependence among magnitudes at scales a and $a + \Delta a$ can be defined as

$$F_{a,a+\Delta a} = \ln \frac{\Sigma_1 \Gamma_1}{\det(\Xi)}. \quad (10)$$

The influence of large scales on finer ones (i.e., forward causal cascade) can be quantified via

$$F_{a+\Delta a \rightarrow a} = \ln \frac{\Sigma_1}{\Sigma_2}, \quad (11)$$

while feedbacks from small scales to larger ones (inverse causal cascade) are given as

$$F_{a \rightarrow a+\Delta a} = \ln \frac{\Gamma_1}{\Gamma_2}, \quad (12)$$

and instantaneous causal effects due to factors exogenous to the two scales considered such as common drivers are obtained by

$$F_{a-a+\Delta a} = \ln \frac{\Sigma_2 \Gamma_2}{\det(\Xi)}, \quad (13)$$

so that the total interdependence between two scales a and $a + \Delta a$ described in equation (10) can be written as

$$F_{a,a+\Delta a} = F_{a \rightarrow a+\Delta a} + F_{a+\Delta a \rightarrow a} + F_{a-a+\Delta a}, \quad (14)$$

which again denotes a “net” or “global” measure of correlation among scales (similar to the $C_{a+\Delta a, a}$ described in section 4.2), now expressed in terms of variance of the autoregressive prediction error, i.e., more in terms of predictability rather than correlation [Dhamala et al., 2008; Ding et al., 2006].

[24] Nevertheless, the application of such a classical form of Granger causality based on an underlying AR process to the estimation of causal relationships across rainfall scales is limited. This limitation stems from nonstationary effects in the scale-wise variability and contingent drawbacks in the proper determination of the AR model order. When we applied this analysis to rainfall data by fitting simple monovariate and bivariate AR models to the local variance series at different scales, high AR model order (>20) were obtained through the use of both Schwarz’s Bayesian and Akaike’s Final Prediction Error (FPE) criteria. Despite the use of these higher-order AR models, large and highly correlated residuals remained (results not shown). This is an effect of the strongly intermittent nature of rainfall in time, which cannot be eliminated even after local filtering and detrending via CWTs.

[25] Because of these AR modeling limitations, a preferable approach to testing causality is given by the so-called transfer entropy $T_{a+\Delta a \rightarrow a}$, i.e., a measure of the information

flow from scale $a + \Delta a$ to scale a . This can be interpreted as a deviation from the generalized Markov property by the Kullback statistic

$$p\{\omega_a(t + \Delta t) | \omega_a^{(k)}(t)\} = p\{\omega_a(t + \Delta t) | \omega_a^{(k)}(t), \omega_{a+\Delta a}^{(l)}(t)\}, \quad (15)$$

where p is a mass probability, $\omega_a^{(k)}(t) = \{\omega_a(t), \dots, \omega_a[t - (k + 1)\Delta t]\}$ and $\omega_{a+\Delta a}^{(l)}(t) = \{\omega_{a+\Delta a}(t), \dots, \omega_{a+\Delta a}[t - (l + 1)\Delta t]\}$ are state vectors. Namely, the absence of information transfer from $a + \Delta a$ to a is equated to a lack of influence of scale $a + \Delta a$ on the transition probabilities of the system at scale a [Schreiber and Schmitz, 2000]. Then, from equation (15) we have

$$T_{a+\Delta a \rightarrow a} = \sum p\{\omega_a(t + \Delta t), \omega_a^{(k)}(t), \omega_{a+\Delta a}^{(l)}(t)\} \cdot \log \frac{p\{\omega_a(t + \Delta t) | \omega_a^{(k)}(t), \omega_{a+\Delta a}^{(l)}(t)\}}{p\{\omega_a(t + \Delta t) | \omega_a^{(k)}(t)\}}. \quad (16)$$

Due to computational reasons, we assumed $k = l = 1$. Hence, the superscripts (k) and (l) are dropped hereafter for notational simplicity. Note that $T_{a+\Delta a \rightarrow a}$ is explicitly nonsymmetric (contrary to other information transfer statistics such as mutual information).

[26] When the causal structure is linear, a linearized form of $T_{a+\Delta a \rightarrow a}$ can be used. If we assume that the whole causal structure between the process at scale $a + \Delta a$ and the one at scale a is linear, then a plausible choice for their joint probability structure is the multivariate Gaussian distribution. We emphasize that the Gaussian distribution assumption here is for the joint and not individual distributions and is primarily intended to separate the nonlinear from linear causality components. It has the benefit of allowing robust estimation of joint properties from small samples (as is the case here). In the Gaussian case, the correlation structure of the stochastic process is entirely described by its linear correlation matrix. Under such an assumption, we can obtain a linearized version of $T_{a+\Delta a \rightarrow a}$,

$$T_{a+\Delta a \rightarrow a}^{\text{lin}} = \frac{1}{2} \log \frac{|\Gamma[\omega_a(t) \otimes \omega_{a+\Delta a}(t)]| \cdot |\Gamma[\omega_a(t + \Delta t) \otimes \omega_a(t)]|}{|\Gamma[\omega_a(t + \Delta t) \otimes \omega_a(t) \otimes \omega_{a+\Delta a}(t)]| \cdot |\Gamma[\omega_a(t)]|}, \quad (17)$$

where $\Gamma(X \otimes Y)$ is the covariance matrix of the joint process $X \otimes Y$ and $|\cdot|$ is the determinant operator [Kaiser and Schreiber, 2002; Paluš, 2007]. Equation (17) is based on the assumption of a Gaussian joint probability structure of the log variances at different scales with the benefit of allowing a robust estimation for small samples. Unlike the traditional Granger causality, the linear transfer entropy allows the assessment of linear causal relationships without requiring a parametric estimation (e.g., the order of a AR process).

5. Testing Causality Against Surrogates and Toy Models

5.1. Surrogate Data

[27] Causal relationships in rainfall, if any, could be an artifact of the estimation method or the redundancy in the CWT [Yamada and Ohkitani, 1991; Katul et al., 1994]. For

this reason, multiscale causality must be tested against surrogate data [Schreiber and Schmitz, 1996] whose correlation properties are known a priori. We adopted both simple shuffled surrogates (randomized in the time domain) and iterative amplitude adjusted fourier transform (IAAFT) data, recently used in testing rainfall nonlinearities in time [Roux *et al.*, 2009]. The former can be derived by randomizing the amplitudes of the original series in time. Surrogates obtained by this method preserve the distribution of the original data, while the original correlation structure is lost. Hence, shuffled data resemble the properties of a δ -correlated random process and their spectral densities $S(f)$ are independent of the frequency f . IAAFT surrogates are computed using an iterative algorithm only able to preserve the distributional and spectral properties of the original rainfall series. However, the nonlinear correlations, which are encoded by correlations in the phase angle in Fourier space are destroyed through the use of IAAFT.

5.2. Random Cascades From Toy Models

[28] We also compare cross-scale causality features of rainfall with ones from two simple cascade models, namely β model (BM) and the binomial cascade (BC) [see Meneveau and Sreenivasan, 1991, and references therein]. BM has been largely adopted in space-time rainfall downscaling (see references from Molnar and Burlando [2005]) and is one of the first intermittent models of cascades used in turbulence. In this model, the cascade generators M_{a_{i+1},a_i} are nonzero and equal to a fraction β of the new offspring, but zero on the other fraction $(1 - \beta)$ of the offspring. Simple scaling properties reveal themselves when β is independent from the scale a_i . Moving from global to local scale invariance, one of the simplest cascading scheme is the binomial cascade whose weight M has a bimodal distribution with only two possible values, say $M_1 = p_1$ and $M_2 = p_2$. Note that such a cascade scheme is not suitable for rainfall modeling purposes and is presented here only as an illustration of time symmetry across scales. We run 100 realizations of both BM and BC (with the probability parameter of the binomial distribution $p_1 = 0.6$) and analyzed average causality. The BM series were obtained by the downscaling of the DF series, previously aggregated to 24 h, to the final aggregation of 30 min. Each BM and BC realization is given by ensembles of 50,000 data points.

5.3. Scalar Turbulence Series

[29] The presence of causal relationships in the rainfall process is also contrasted with an analysis conducted on water vapor (WV) molar concentration fluctuation time series collected at the Blackwood Division of the Duke Forest, near Durham, North Carolina [see Katul *et al.*, 2006, and references therein]. Measurements were obtained above a 17 m tall managed temperate Loblolly Pine plantation using an open path infrared gas analyzer (LI-7500, Licor, Lincoln, Nebraska) at a sampling rate of 10 Hz. At such fine scales, fluctuations in H_2O concentration resemble passive scalar turbulence displaying a bounded lognormal turbulent cascade [Katul *et al.*, 2006]. In Figure 4, a 7 day sample extracted from the WV time series is shown (Figure 4a) together with the wavelet power spectrum (Figure 4b) calculated for the entire series. The obtained spectral exponent at fine scales is 1.64 in agreement with the theoretical 5/3

value predicted by Kolmogorov's (1941) theory within the inertial subrange. For computational reasons and consistency with the length of the examined rainfall records, we analyzed a series of subsamples extracted from the original time series each 150,000 points (or about 4 h). One of these subsamples is presented in the inset in Figure 4a. This subsample preserves the power law properties of the full series (see Figure 4b inset), which is expected given that energetic scalar eddies have characteristic time scales of tens to 100 s [Cava *et al.*, 2008]. Analogous results were obtained for all the other subsamples (not shown).

6. Analysis and Results

[30] Results and discussion are structured along the two main themes earlier presented in sections 4.2 and 4.3, namely the estimation of net causality flows across scales via the delayed correlation function $C_{a+\Delta a,a}(\Delta t)$ and the untangling of different directionality and their strength using the transfer entropy $T_{a+\Delta a \rightarrow a}^{lin}$. The causality measures are applied to both the measured rainfall time series and the battery of tests earlier discussed.

6.1. Correlation Among Scales and Time Asymmetry

[31] To assess global causality effects across fine scales, we first evaluate the cross-scale correlation function $C_{a+\Delta a,a}(\Delta t)$ of the Iowa city high-resolution rainfall events. Sample results are reported in Figure 5, where the single-scale autocorrelation functions $C_a(\Delta t)$ over a selected subrange of scales (from 10 s to about 26 min with time step of 5 min) are also reported. From top to bottom, the $C_a(\Delta t)$ (Figures 5a, 5c and 5e) and $C_{a+\Delta a,a}(\Delta t)$ (equation (6)) in the $(\Delta t, \Delta a)$ half-plane (Figures 5b, 5d and 5f) are shown for event 1 (2 December 1990), 5 (1 November 1990 A) and 6 (3 May 1990) though the calculations are conducted for all seven events. The two-dimensional section of the time-scale correlation space for $C_{a+\Delta a,a}(\Delta t)$ is computed by fixing the reference scale a_n to the original rainfall sampling rate (smaller scale) so that Δa on the ordinate scale represents the scale shift from such a reference scale. The three events presented here strongly differ in causal signatures (i.e., $C_{a+\Delta a,a}(\Delta t) \neq C_{a+\Delta a,a}(-\Delta t)$), although they all confirm the predominantly local nature of the causal relationships for rainfall at finer time scales.

[32] Event 1, for example, shows the strongest asymmetric correlation structure. Note that for this event, correlation values >0.5 are persistent for high (>30 min) Δa , with higher-correlation values concentrated in the negative Δt half-plane. This fact suggests a causal influence of fine scales on the larger ones (i.e., what we termed as inverse causal cascade) over scales ranging between few seconds to about 30 min, while the other two events display forward causal cascades. Analogous forward causal cascades were obtained for all the other four events in weaker or stronger forms but are not shown here. Also, single-scale correlations present a variety of different decaying behavior, ranging from a slow and substantially homogeneous decay for event 1, strong correlations with strength increasing along scales for event 5 and a weaker memory in event 6. In short, considering that event 1 is also the only one with such high intensities for the longest duration, a tendency of extreme events to be driven by fine-scale dynamics more than the large-scale ones seems to be a

Figure 4. (a) High-resolution (10 Hz) WV time series recorded above a pine forest [Katul *et al.*, 2006] and (b) its normalized wavelet power spectrum. Insets represent the subsample series adopted in our analysis (inset in Figure 4a) and its $S(f)$ (inset in Figure 4b). The estimated spectral slopes are both close to the K41 value of $5/3$ in the inertial subrange.

plausible explanation. This fact has also relevance to modeling extreme events like event 1 that develop in different dynamical context, with stronger correlations in time (see Figure 1), a marked asymmetry in cross-scale couplings, and a predominant role of clustering of small-scale events to produce an intense event over an extended period.

[33] To further explore the robustness of these conclusions for rainfall, the causal correlation structure of event 4 (Figure 6b) with its shuffled (Figures 6c.1 and 6c.2) and IAAFT (Figure 6d) surrogates are compared. In Figure 6a a comparison is shown between the wavelet spectra of event 4 (solid circles) and its shuffled and IAAFT surrogates. As expected, IAAFT transform is able to preserve the power spectrum of the original series, while the shuffled data destroy correlations in time leading to a white noise spectrum. Moreover, randomization in time destroys both single-scale and cross-scale correlations, resulting in total absence of causal patterns in the shuffled data structure. The wavelet decomposition here does not introduce any asymmetry in the cross-scale correlation structure and the CWT based scale-wise cross-correlation analysis appears capable of

“fingerprinting” the existence of causality in rainfall. On the other hand, when considering results from IAAFT surrogates, weak residual cross-scale correlations do reveal themselves and are possibly connected to the remnant linear correlation structure in the original series.

[34] In addition, a test for symmetry in scale-wise correlations is provided in Figure 7 where causal correlation results for the BC and BM simulation are provided. The correlation structure of the BC is symmetric along scales while the one obtained for the BM, though substantially symmetric in time, exhibits more complex patterns (as expected for these classes of cascade schemes).

[35] Such a symmetry is preserved when the analysis is extended to the Duke forest water vapor concentration time series subsample, representative of a turbulent passive scalar (Figures 7c, 7f and 7i). Causal patterns in such a passive turbulent scalar are in fact predominantly symmetric, and cross-scale correlations $C_{a+\Delta a,a}(\Delta t)$ turns out to be minor when compared with single-scale correlations, displaying strong memory in time. At first glance, the fact that the passive scalar turbulent cascade is “instantaneous” or

Figure 5. (a, c, and e) Autocorrelation coefficients $C_a(\Delta t)$ and (b, d, and f) cross-scale correlation coefficients $C_{a+\Delta a,a}(\Delta t)$ (equation (6)) in the $(\Delta t, \Delta a)$ half-plane for event 1 (2 December 1990, Figures 5a and 5b), 5 (1 November 1990 A, Figures 5c and 5d) and 6 (3 May 1990, Figures 5e and 5f). The bidimensional section of the time-scale correlation space for $C_{a+\Delta a,a}(\Delta t)$ is computed by fixing the reference scale a_n to the rainfall sampling scale (finest scale) so that Δa on the ordinate represents the scale shift from such a reference scale.

Figure 6. (b, c, and d) (top) Autocorrelation coefficients $C_a(\Delta t)$ and (bottom) cross-scale correlation coefficients $C_{a+\Delta a,a}(\Delta t)$ in the $(\Delta t, \Delta a)$ half-plane for event 4 (Figure 6b), the same event after shuffling (Figure 6c), and its IAAFT surrogates (Figure 6d). (a) A comparison between the wavelet spectra of the original event (solid circles) and its shuffled (solid squares) and IAAFT (solid stars) surrogates. As expected, IAAFT transform is able to preserve the mean spectral characteristics of the series. The shuffled data wipe out all linear and nonlinear correlations and essentially resembles a white noise spectrum. In Figure 6a the gray shadow represents the range of scales on which the scale-by-scale autocorrelations $C_a(\Delta t)$ are estimated.

“noncausal” may appear counterintuitive to what is known about turbulent cascades. There are a number of plausible reasons for this finding. The turbulent cascade is generally three-dimensional in space while the analysis here is temporal, and at best, may reflect one-dimensional cuts along the mean wind direction (if *Taylor’s* [1938] frozen turbulence hypothesis is adopted). For such a one-dimensional spatial cut, large and small eddies do simultaneously coexist thereby weakening our abilities to detect the time lags arising during the breakup of large eddies into smaller ones.

[36] Symmetries (or lack of causality) in rainfall series are restored when dealing with long time series like the DF and CHV, whose single-scale and cross-scale correlation structures are shown in Figure 8. Nevertheless, a weak asym-

metric pattern (revealing a weak forward causal cascade) seems to emerge again for the long historic time series of Chiavari at scales between 1 and 3 months (Figure 8d). The same analysis was repeated on the log amplitudes of the wavelet coefficients rather than the variances, and the results were qualitatively the same as in Figures 5–8 (figures not shown).

[37] Summarizing, net causality across rainfall time scales seems to manifest itself on a restricted range of fine scales (between few seconds and 30 min) and tends to disappear at large interevent and climatological scales. However, the directionality of such causal relationships appear mixed. Based on the limited data set analyzed here, low- to average-intensity events seem to favor a forward causal cascade and

Figure 7. Scale-by-scale correlation analysis for (a, d, and g) binomial cascade (BC), (b, e, and h) β model (BM) simulated time series, and (c, f, and i) the water vapor series. A sample of the simulated time series (Figures 7a, 7b, and 7c), its scale-by-scale autocorrelation (Figures 7d, 7e, and 7f), and scale-wise correlation (Figures 7g, 7h, and 7i) are shown. Note that BM simulated events appear more clustered if compared with the original DF ones (Figure 2, top left). This is a well-known drawback of BM-based disaggregation schemes [Molnar and Burlando, 2005], though not influencing the causal nature of the cascade. Also, for the water vapor series, resembling a lognormal bounded cascade, cross-scale correlations are much weaker when compared with rainfall events. Here, symmetry is present in $C_{a+\Delta a,a}(\Delta t)$, suggesting an instantaneous cascade of variance across scales.

the single high-intensity event favors an inverse causal cascade, where small scales appear to impact the dynamics of larger ones. The usual cautionary notes must be emphasized whenever conclusions from such a restricted data set are to be generalized. In the following, we analyze the directionality of the linear components of causality through the linearized form of transfer entropy $T_{a+\Delta a \rightarrow a}^{lin}$ discussed in section 4.3.

6.2. Linearized Transfer Entropy Across Scales

[38] Through the linearized transfer entropy $T_{a+\Delta a \rightarrow a}^{lin}$, it is possible to quantitatively characterize component-wise cross-scale causality. Namely, the detected asymmetry in cross-scale correlation functions can be decomposed in its causal (directional) and simple coupling components. Before proceeding, it is worth noting that $T_{a+\Delta a \rightarrow a}^{lin}$, as well

as traditional Granger causality, cannot be objectively normalized and can only provide a (relative) measure of strength of causality when compared to a suitable surrogate data [Lungarella et al., 2007]. Hence, the assessment of causality strength in rainfall series is again achieved by using surrogate data and toy models, which due to their known lack of causality allow evaluation of the significance of $T_{a+\Delta a \rightarrow a}^{lin}$ for the experimental data.

[39] Selected results from the high-resolution rainfall data set are shown in Figures 9a–9d, where $T_{a+\Delta a \rightarrow a}^{lin}$ for subevent scales (up to 1 h) are presented. Here, T^{lin} is represented as a function of scales a_1 and a_2 , the causing and caused scale, respectively, with $a_i = 1$ being the smallest scale. $T_{a+\Delta a \rightarrow a}^{lin}$ is then represented in a (a_1, a_2) plane, allowing us to investigate the whole scale-to-scale connections for a given time delay Δt between causing and caused scales. Results are

Figure 8. (a and c) Scale-wise autocorrelations and (b and d) cross-scale correlations for the DF (Figures 8a and 8b) and CHV (Figures 8c and 8d) time series. Weak asymmetric features reveal themselves at CHV for scales between 1 and 3 months.

shown for a time delay corresponding to the maximum Δt at which a significant ($C_{a+\Delta a,a}(\Delta t) > 0.5$) asymmetry is still detected in the cross-scale correlation functions.

[40] The displayed events are event 1 (Figure 9a) and event 4 (Figure 9b), together with the shuffled and IAAFT versions (Figures 9c and 9d) of event 4. The maximum asymmetry Δt is about 20 min for event 1 and 10 min for event 4. Recall from section 6.1 that events 1 and 4 are examples of inverse and forward net causal cascade across scales, respectively. Again, from Figure 9, it is evident that a stronger directional component of causality is from a discrete range of large scales to the finest ones (bottom part of Figure 9b) for event 4, while event 1 displays high values of T^{lin} in the top part of Figure 9b, with small scales influencing the larger ones. In fact, the peak of causality for event 1 is above the diagonal $a_1 = a_2$. This means that small scales are influencing large scales more frequently than large scales are influencing small scales. Similarly, in Figure 9b, $T^{lin}_{a+\Delta a \rightarrow a}$ has its peak and its overall higher values for $a_1 < a_2$ so that a forward causal cascade mechanism is favored. Still, the shuffled version of event 4 does not reveal any significant directionality in the causality flow, while IAAFT data preserve a weak causal structure that may indicate the main role of the nonlinear components of the series generating linear causal relationships across scales. Similar results were obtained for the shuffled and IAAFT surrogates of all the other events and time series.

[41] When coarser resolution rainfall time series are considered (Figures 9e and 9f) directionality and causality tend to disappear, confirming that causal relationships are essentially localized (and confined) within small scales. Only weak forward causal patterns can be observed for the DF series around the 4 h scale and for the CHV series around 45 days. This pattern corresponds to an almost indistinguishable asymmetry of cross-scale correlation functions in Figure 8, highlighting the better performance of $T^{lin}_{a+\Delta a \rightarrow a}$ in distinguishing between different causality flows across scales. Also, the directionality is entirely lost in BC, while a spurious but weak effect of causation of large scales on the smaller ones is preserved by the BM (represented in

Figures 10a and 10b, respectively). This is probably due to the fact that BM simulated series are obtained from the disaggregation of the Duke rainfall time series, thereby preserving part of its weak causality at small contiguous scales. The weak causal structure of rainfall at coarse aggregation scales may have been retained in the BM. Finally, for the water vapor series (Figure 10c), directionality is negligible with weak influence from large scales to fine ones restricted to a range of up to 15 s. As mentioned earlier, the lack of strong directionality here can be attributed to the nature of the sampling of the phenomenon, not the phenomenon itself. The turbulence cascade is three-dimensional and one-dimensional cuts indirectly inferred from time series may weaken the directional causality inferences because eddies of all sizes coexist simultaneously for those cuts. We should note that this argument does not preclude the fact that one-dimensional cuts can still capture the multifractal spectrum (i.e., statistical properties) of scalar turbulence as already demonstrated by Prasad *et al.* [1988].

[42] A criticism of this analysis is the Gaussian assumption for the scale-wise joint distributions, which was adopted given the interest in the linear components of causality and the need for robust estimation of the joint distributional properties from limited data. We have conducted an analysis on these joint distributions for all the high-frequency rainfall series and found that while these distributions are not precisely Gaussian, they do not diverge appreciably from Gaussian (i.e., the body of the joint distribution is near-Gaussian though minor asymmetry can be detected; figure not shown). Hence, we may infer from this analysis that the linear components of causality are the primary terms in the overall causal statistics here, even if the nonlinear terms are still present.

7. Discussion and Conclusions

[43] The correlation and causal structure of the rainfall local variance across different time scales were investigated using CWT, cross-scale correlations, and linearized transfer entropies in the wavelet domain. The causality hypothesis

Figure 9. Linearized transfer entropy $T_{a_1 \rightarrow a_2}^{lin}$ as a function of causing scale a_1 and caused scale a_2 for (a) event 1, (b) event 4, and its (c) shuffled and (d) IAAFT versions, (e) the DF rainfall time series, and (f) the CHV time series. The $a_i = 1$ scales here represent the smallest scale. Results are shown for a time delay corresponding to the maximum Δt for which significant ($C_{a+\Delta a, a}(\Delta t) > 0.5$) asymmetry is detected in the cross-scale correlations of Figures 5, 6, and 8. The inset in Figure 9e represents a zoom over fine scales for DF.

Figure 10. Average $T_{a_1 \rightarrow a_2}^{lin}$ for the (a) BC and (b) BM ensemble of simulations and the (c) Duke Forest WV subsample represented in the inset in Figure 4a. The inset in Figure 10b reports a zoom over fine scales for the BM. As in Figure 9, $a_i = 1$ represents the smallest scale.

was tested against synthetic data sets including surrogate data, realizations of diverse cascade models and scalar turbulence records. Causality patterns emerging from the two analyses were highly synergetic, suggesting that asymmetry in the scale-wise cross-correlation functions with time lags (i.e., positive and negative) may be used to fingerprint causality in rainfall. This scale-wise correlation analysis demonstrated that causality in rainfall cascades was revealed at time scales ranging from few seconds to tens of minutes (i.e., the storm scale). Different storm events resulted in different causality strengths, further confirming the local nature of the cascade. In particular, extreme events such as event 1 (presenting peaks of intensities of 100–120 mm/h), seemed to be essentially driven by microscale features in which fine-scale events appeared to coalesce together to form intense events at larger scales and later times. The precise mechanism leading to such coalescence and subsequent formation of intense storms are difficult to discern from a single realization in time.

[44] The next logical steps building on this work include: (1) expanding this analysis to include nonlinear causal relationships beyond what was reported here for the linear causality case and assessing whether such nonlinear causality persists over significantly longer time scales and (2) establishing whether there is a relationship between the time scales at which significant causality in the cascade exists and microclimatological conditions triggering the rainfall processes (e.g., strong convective rainfall events triggered by previous soil moisture conditions [Juang *et al.*, 2007]). Also, given that fine subevent scales are the ones at which intermittency and clustering effects are mostly developed, a connection between causality and anomalous scaling found in rainfall [Roux *et al.*, 2009; Venugopal *et al.*, 2006a] may exist and is the subject of an ongoing investigation.

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G. G. Katul, A. Molini, and A. Porporato, Nicholas School of the Environment, Duke University, PO Box 90328, Durham, NC 27708-0328, USA. (annalisa.molini@duke.edu)