Potential Vorticity Dynamics of Boundary Currents in a Quasi-geostrophic Ocean*

M. SUSAN LOZIER AND STEPHEN C. RISER

School of Oceanography, University of Washington, Seattle, Washington

(Manuscript received 20 September 1988, in final form 3 March 1989)

ABSTRACT

Boundary layer potential vorticity dynamics for a quasi-geostrophic, eddy-resolving general circulation ocean model are studied using both Lagrangian and Eulerian analyses. Active western boundary layers are found not only in the upper wind-driven layer but also in the lower layers, despite the lack of a direct vorticity input to the deep ocean. At the western wall dissipative and inertial boundary regimes are exclusively controlled by the time-mean dynamics except for the deepest layer where eddy fluxes drive the mean flow across mean potential vorticity contours. Boundary layers formed at the southern wall in this model are dynamically distinct from the western boundary layers; they are controlled solely by the eddy flux of potential vorticity found in this region of active baroclinic instability. Basin-integrated vorticity balances reveal a strong contribution to the vorticity cycle by the lateral boundaries with such input overshadowed by the vorticity exchange across the midbasin gyre boundary in the surface layer.

1. Introduction

Due to the Lagrangian nature of its formulation, potential vorticity, q , may be viewed as a dynamic tracer of ocean circulation. For this reason the changes in q of a water parcel traveling the circuit of a wind-driven gyre have long been of interest. The potential vorticity supplied by the curl of the wind stress in a Sverdrup interior must be balanced by a corresponding loss if a parcel is to maintain a finite q value along its path. Beginning with the work of Stommel (1948), frictional western boundary layers have been appended to wind-driven gyres to provide this necessary sink. Bottom friction, as was used by Stommel, or lateral friction, introduced by Munk (1950), have commonly been used to create a viscous western boundary layer in order to close the vorticity balance of a gyre. The work of Stommel and Munk provided the foundation for a succession of analytical wind-driven circulation models that have been concisely described by Veronis (1981).

The study of wind-driven circulation has been advanced during the past two decades by explicitly including the effects of mesoscale eddies on ocean dynamics. Eddy-resolving general circulation models (EGCMs), introduced by Holland and Lin (1975a), have been employed to assess eddy-mean flow interactions. Notably, the study of Holland and Rhines

(1980) shows how eddies can induce the mean circulation of the deep ocean through the action of inviscid interfacial eddy stresses. While eddies have been shown to play other roles in the character of the large scale flow, their particular effects on the western boundary regime in an EGCM have been largely ignored. This oversight stems mainly from the use of models whose grid spacing has been insufficient to adequately resolve boundary layers.

This study aims to examine the nature of the western boundary in an eddy-resolving general circulation model and to assess the boundary's contribution to the overall gain/loss cycle of potential vorticity for the wind-driven surface layer and for the deep ocean. The analysis of the western boundary is subsumed by a broader examination of the vorticity features for all lateral boundaries where intense currents exist. The results presented in this paper are from a single numerical experiment, with parameter choices based on past parameter studies of a similar model (Holland 1978; Haidvogel, private communication). This model realization is believed to yield a representative model ocean. The intent here is to give an extensive analysis of the boundary currents and to explore the manifestations of these boundary currents in a Lagrangian frame. Traditionally, the analyses of numerical model results have proceeded from an Eulerian framework in which energy and vorticity budgets have been calculated for distinct regions of the numerical ocean. One problem associated with this approach is the a priori choice of dynamically distinct regions. Another problem, as pointed out by Harrison and Holland (1981), is that although regional budgets improve on pointwise estimates, the determination of horizontal eddy transports is statistically unreliable even for spatial averages

* This is Contribution Number 1806 from the University of Washington.

Corresponding author address: M. Susan Lozier, School of Oceanography WB-10, University of Washington, Seattle, WA 98195.

over eddy length scales. Finally, the use of an Eulerian approach for a vorticity analysis does not take advantage of potential vorticity's capability as a dynamic tracer. Using quasi-geostrophic dynamics, potential vorticity represents the sum of contributions from planetary, stretching, and relative vorticity. Although q is constrained along fluid paths, these individual terms are not. The balance and compensation of these q components creates a Lagrangian signature of the flow that provides a useful perspective on the mesoscale dynamics of the ocean circulation. For these reasons the analysis of the boundary layers in this study will be approached from a Lagrangian viewpoint with complementary Eulerian analyses. In order to gain reliable Eulerian eddy statistics and independent Lagrangian releases this study required 80 hours of Cray XMP computer time. Though sensitivity experiments are desirable they are also costly for an analysis of this scope. Extrapolation to other parameter choices will be discussed in section 8.

2. ECGM and model equations

This study is conducted within the convenient framework of a quasi-geostrophic eddy-resolving model that traces its roots to the two-layer numerical model first used by Holland (1978). This revised model (which was developed at the National Center for Atmospheric Research by D. Haidvogel) has three layers and a stretched grid in the east-west direction. This stretching, which provides the enhanced resolution necessary to examine the details of the western boundary dynamics, is imposed on the numerics through the introduction of a stretched coordinate. The dependence of the stretched coordinate on the zonal coordinate, x , is given in Table 1 with further details of the modified equations and their solution contained in the work of Riser and Haidvogel (1988). Briefly, the x grid spacing begins at 4 km adjacent to the western wall and, with an e -folding scale of 200 km, relaxes to 20 km as the interior is approached. The grid spacing for the meridional coordinate, y , is uniform everywhere at 20 km.

The imposed sinusoidal wind stress produces a double-gyre ocean with an anticyclonic gyre in the southern basin and a cyclonic gyre in the northern basin. Due to the symmetry of the forcing about midlatitude, these gyres are dynamically equivalent. In quasi-geostrophic flow, the potential vorticity equations for each layer may be written as

$$\frac{Dq_1}{Dt} = \nu \nabla^2 \xi_1 + \frac{\mathbf{k} \cdot \nabla x \tau}{H_1}, \quad (1a)$$

$$\frac{Dq_2}{Dt} = \nu \nabla^2 \xi_2, \quad (1b)$$

$$\frac{Dq_3}{Dt} = \nu \nabla^2 \xi_3 - \epsilon \xi_3, \quad (1c)$$

TABLE 1. Model parameters.

Basin dimension	$L \times L = 4000 \text{ km} \times 4000 \text{ km}$
Layer thicknesses	$H_1 = 300 \text{ m}$ $H_2 = 700 \text{ m}$ $H_3 = 4000 \text{ m}$
Total ocean depth	$H = 5000 \text{ m}$
Reduced gravity	$g'_{12} = .03 \text{ m s}^{-2}$ $g'_{23} = .02 \text{ m s}^{-2}$
Coriolis parameters	$f_0 = 9.3 \times 10^{-5} \text{ s}^{-1}$ $\beta = 2 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$
Lateral dissipation coefficient	$\nu = 1 \times 10^2 \text{ m}^2 \text{ s}^{-1}$
Bottom friction coefficient	$\epsilon = 1 \times 10^{-7} \text{ s}^{-1}$
Wind stress	$\tau = (-\tau_0 \cos(2\pi y/L)/\rho_0, 0)$ $\text{cm}^2 \text{ s}^{-2}$
Wind stress amplitude	$\tau_0 = 1 \text{ dyn cm}^{-2}$
Mean state density	$\rho_0 = 1 \text{ g cm}^{-3}$
Rossby radii of deformation:	
1st baroclinic	$l_1 = 47 \text{ km}$
2nd baroclinic	$l_2 = 20 \text{ km}$
Rossby number	$2\pi\tau_0/\rho_0\beta^2HL^3 = 6.5 \times 10^{-4}$
Ekman number	$\nu/\beta L^3 = 6.3 \times 10^{-7}$
Stretched coordinate	$\xi(x) = x + a\lambda(1 - \exp(-x/\lambda))$ $\lambda = 200 \text{ km}$ $a = 4$
Number of gridpoints	201 in y 241 in x

where

$$q_1 = \xi_1 + f + \frac{(\Psi_2 - \Psi_1)f_0^2}{g'_{12}H_1}, \quad (1d)$$

$$q_2 = \xi_2 + f + \left(\frac{-(\Psi_2 - \Psi_1)f_0^2}{g'_{12}H_2} + \frac{(\Psi_3 - \Psi_2)f_0^2}{g'_{23}H_2} \right), \quad (1e)$$

$$q_3 = \xi_3 + f - \frac{(\Psi_3 - \Psi_2)f_0^2}{g'_{23}H_3}, \quad (1f)$$

and

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + J(\Psi_i),$$

with J representing the Jacobian operator, and $\mathbf{k}$ designating a local vertical unit vector. For each layer i ($i = 1, 3$) the quasi-geostrophic streamfunction is represented by Ψ_i with the horizontal velocity field (u_i, v_i) given by $\mathbf{u}_i = (-\Psi_{iy}, \Psi_{ix})$. The potential vorticity is designated as q_i and its three components, relative vorticity, planetary vorticity, and stretching vorticity, are shown as the first, second, and third terms, respectively, on the right side of Eqs. (1d-f). The planetary vorticity, f , is approximated by $f_0 + \beta(y - L/2)$, where L is the basin's north-south and east-west extent, and the

FIG. 1. A schematic diagram for the vertical structure of the three-layer model.

stretching vorticity term will be denoted as η throughout this paper. Table 1 defines all parameters and Fig. 1 is a schematic of the model's vertical structure. A finite difference method is used for solution with a second-order leapfrog scheme in time (time step = 0.5 hour) and a centered second-order representation in space. The solution method for this three-layer model is analogous to the two-layer model solution outlined by Holland (1978).

According to the first term on the right side of Eq. (1), changes in q along the path of a particle in all three layers are brought about by lateral dissipation, which is parameterized as Laplacian friction. Additionally, wind forcing in layer 1 affects the q balance and bottom friction, parameterized as Rayleigh friction, dissipates q in layer 3. The choice of Laplacian friction, rather than biharmonic friction, to model the small scale dissipative effects was made primarily to avoid the introduction of higher order boundary conditions (i.e., $\nabla^2 \xi = 0$), which would complicate the interpretation of the boundary dynamics. Using Laplacian friction the imposed lateral boundary conditions are limited to

$$\xi_i = 0,$$

and

$$\mathbf{u}_i \cdot \mathbf{n} = 0,$$

where $\mathbf{n}$ is the unit vector normal to the boundary. These conditions yield free slip on all walls and stretching vorticity terms that are constant along the boundaries. The combination of the zero relative vorticity boundary condition and the parameterization of lateral dissipation as the Laplacian of relative vorticity transforms the boundary into a source or sink of potential vorticity, if there is significant relative vorticity within a dissipative length scale of the boundary. Additionally, the zero relative vorticity condition can create spatial

gradients of bottom friction at lateral boundaries in layer 3.

Although momentum is transferred from one layer to another, neither flow nor potential vorticity can cross an isopycnal surface under the constraints of quasi-geostrophic dynamics. As seen from an inspection of Eqs. (1d-f), however, the potential vorticity in adjoining layers is coupled via the stretching term. With the use of the thermal wind relations the stretching vorticity terms can be equivalently expressed as

$$\eta_1 = \frac{f_0}{H_1} h_{12}, \quad (2a)$$

$$\eta_2 = \frac{-f_0}{H_2} (h_{12} - h_{23}), \quad (2b)$$

$$\eta_3 = \frac{-f_0}{H_3} h_{23}, \quad (2c)$$

where h_{12} and h_{23} are the positive deviations of the interface heights, shown in Fig. 1. These deviations also yield information on the interfacial vertical velocities w_{12} and w_{23} , via the equations

$$\frac{Dh_{12}}{Dt} = \frac{\partial h_{12}}{\partial t} + J(\Psi_{12}, h_{12}) = w_{12}, \quad (3a)$$

$$\frac{Dh_{23}}{Dt} = \frac{\partial h_{23}}{\partial t} + J(\Psi_{23}, h_{23}) = w_{23}, \quad (3b)$$

where Ψ_{12} and Ψ_{23} are interfacial streamfunctions evaluated from Ψ_i ($i = 1, 3$) via linear interpolation. As the bottom is flat and the surface is rigid, the vertical velocities vanish at $z = 0$ and $z = -H$.

3. Lagrangian paths

From an initial state of rest the quasi-geostrophic layered equations (1a-c) were integrated in time until a statistical steady state was reached. Clusters of 16 particles ("floats") were then released at 26 locations throughout the southern basin. The deployment scheme, shown in Fig. 2, is identical for all three layers.

The subsequent "path" of each particle is determined by the kinematic requirement that

$$\frac{D\mathbf{x}_L}{Dt} = \mathbf{u}(\mathbf{x}_L, t), \quad (4)$$

where $\mathbf{u}$ is the horizontal velocity field and $\mathbf{x}_L$ describes the location of each particle within a layer. Integration of Eq. (4) using a Runge-Kutta scheme yields particle locations at each time step of the Eulerian numerics. Because the particle paths will not necessarily cross Eulerian gridpoints, interpolation is necessary to find the appropriate Eulerian velocity at the instantaneous float locations. A two-dimensional polynomial fit for Ψ is constructed using the 16 nearest Eulerian gridpoints to a given particle. This Ψ field is then analytically

FIG. 2. Initial float locations within all layers. Each letter represents 16 floats spread evenly across the bracketed line. The floats are spaced 44 km apart for clusters M through T; elsewhere the initial spacing is 20 km.

ically differentiated spatially and evaluated at the float's location to find the appropriate velocity (u_L , v_L) for the Lagrangian integration. Details of the interpolation scheme are given by Riser and Haidvogel (1989).

Using a beta-plane spectral model, Haidvogel (1982) has shown that Lagrangian paths constructed in the above manner are subject to tracking errors. These errors are inherent in a numerical scheme with finite horizontal resolution as the exclusion of subgrid scale Eulerian dynamics causes the loss of Lagrangian information. Riser and Haidvogel (1988) tested the validity of the tracks in this finite difference model by launching particles into a frozen circulation pattern and then calculating the rms departure of the particle's streamfunction value from its initial value at launch. After 180 days they found the rms departure for the upper layer to be only 0.1% of the mean, with the errors in the lower two layers an order of magnitude smaller. However, due to the small scale of the relative vorticity field the accuracy of the tracks does not guarantee the accuracy of the particle's interpolated vorticity value.

In order to establish the validity of a q estimate for a float, the q that has been interpolated from the surrounding Eulerian gridpoints is compared to the potential vorticity calculated from the numerical Lagrangian integration of Eqs. (1a–c). Just as the path was established via the time integration of the velocity field, so too may the q value subsequent to launch be tracked in time by integrating the sum of nonconservative forces. Ideally the two q values would be equal but, due to both interpolation and tracking errors, in time

the two estimates diverge from their common initial value. Although the float trajectories were determined for 182 days, the time series was truncated if the difference between the q estimates at a time t_n exceeded 20% of the dynamic signal of the q components. The dynamic signal is measured as the largest change experienced by a q component in the time interval t_{n-1} to t_n . In the boundary layers, where particle velocities are large, the two q values have little time to diverge, and therefore the q balances remain valid. Additionally, the enhanced resolution at the western boundary increases the accuracy of the vorticity estimates. For this realization of the model, only in the upper layer in the vicinity of the midlatitude jet are the q estimates in serious disagreement in less than an eddy time scale. This results from the turbulent evolution of the jet which produces a vorticity field that contains scales small relative to the grid spacing. A small error in a float's location can, therefore, lead to large errors in the estimate of its q value. The results of this study, which deals with float behavior in boundary layers, are not influenced by such tracking errors.

Floats from one release were tracked for 182 days and then reinitialized at their original launch position for the next release. This study includes 10 serial releases, which for 26 clusters of 16 floats each amounts to 75 712 float-days per layer.

If the circulation were steady the Lagrangian paths could be reconstructed from the $\bar{\Psi}$ fields, but with transient motion the Lagrangian information can yield a picture of potential vorticity conservation distinct from that painted by the mean Eulerian fields. When q is divided into time mean and eddy components, $q = \bar{q} + q'$, the conservation statement for potential vorticity may be rewritten as

$$\bar{\mathbf{u}} \cdot \nabla \bar{q} = -\nabla \cdot \bar{\mathbf{u}} q' + \bar{F}, \quad (5)$$

where $\bar{F}$ represents the mean forcing and dissipation collectively. In regions where $\bar{F}$ is weak, mean Eulerian flow may be driven across $\bar{q}$ contours by the divergence of eddy potential vorticity flux, yet under these circumstances an individual fluid particle will retain its initial q value. It is instructive, therefore, to conduct this study with a combination of Lagrangian and Eulerian information.

4. Eulerian field characteristics

The time-averaged streamfunction fields for the 4000 km $\times$ 4000 km numerical ocean are shown in Fig. 3 for each of the three layers. The averaged fields represent means over 3600 days of Eulerian data, subsampled at 2-day intervals. In the text to follow, an overbar will designate such a time-averaged value. Because the $\bar{\Psi}$ contours at the western and southern boundaries are crowded in such a large scale mapping, enlargements of these boundary fields are provided in Fig. 4.

FIG. 3. Mean streamfunction fields for the entire basin. Contour interval scaled by $10^{-3} \text{ m}^2 \text{ s}^{-1}$. Dashed lines designate negative values for this and subsequent contoured fields. (a) $\bar{\Psi}_1$; (b) $\bar{\Psi}_2$; (c) $\bar{\Psi}_3$.

The familiar features of a wind-driven gyre are reproduced in the upper layer. The Sverdrup interior sweeps into a western boundary current, which itself culminates in an intense eastward midlatitude jet. In addition to the strong inertial recirculation directly south of the jet, the westward return flow is also intensified at the southern wall of the basin. This boundary current results from a strong interfacial upwelling, as will be shown in section 6. The $\bar{\Psi}$ structure in layer 2 reveals the weakening of the midlatitude jet and the poleward shift of the wind-driven gyre. Also, a weak cyclonic gyre at the basin's southern edge creates an eastward boundary current along the southern wall in this middle layer. Due to the presence of both a cyclonic and anticyclonic gyre, southward and northward flowing western boundary currents exist in the southern basin of this layer. Finally, in the bottom layer, the southern basin contains multiple gyres, with the northernmost gyre containing the remnants of the wind-driven gyre.

The mean potential vorticity field, $\bar{q}$, for the entire basin is shown in Fig. 5 for each of the layers with the detailed structure for the western and southern boundaries shown in Fig. 6. In the upper layer sharp $\bar{q}$ gradients appear at both the western and southern boundaries. Of particular interest is the southward gradient of $\bar{q}$ from the southern wall until midgyre where the wind stress curl and the mean interface deformation are maximum. In the ocean interior where the relative vorticity may safely be neglected, the mean meridional gradient of q is given by

$$\frac{\partial \bar{q}_1}{\partial y} = \beta + \frac{f_0}{H_1} \frac{\partial \bar{h}_{12}}{\partial y}.$$

A southward height gradient and a sufficiently small upper layer thickness act to reverse the influence of β and create a $\bar{q}$ field that is dominated by stretching vorticity rather than planetary vorticity. This equatorward $\bar{q}$ gradient contrasts the poleward gradient found in the layer below. A reversal of $\bar{q}$, with depth creates conditions favorable for baroclinic instability and, in fact, it is this subsequent instability that leads to the existence of the cyclonic southern gyre in layer 2.

Intensifications of $\bar{q}$ at the western and southern walls and a vast pool of homogenized $\bar{q}$ straddling the border separating the northern and southern basins are also found in the middle layer. This layer, isolated from direct forcing or bottom friction, meets the criteria of the theory proposed by Rhines and Young (1982). Stretching and relative vorticity contributions in layer 3 are generally insufficient to deform the meridional planetary vorticity gradient. Exceptions occur at the western boundary and at midlatitude. There is no enhancement of the $\bar{q}$ gradient at the southern wall in this layer.

The boundary regions of primary interest, as seen in these mean fields to be located at the western and

FIG. 4. Details of the mean streamfunction fields. Contour interval scaled by $10^{-3} \text{ m}^2 \text{ s}^{-1}$. (a)–(c) $\bar{\Psi}_1$, $\bar{\Psi}_2$ and $\bar{\Psi}_3$ at the western boundary; (d)–(f) $\bar{\Psi}_1$, $\bar{\Psi}_2$ and $\bar{\Psi}_3$ at the southern boundary.

FIG. 5. Mean potential vorticity fields for the entire basin. Contour interval scaled by $100/f_0$. (a) $\bar{q}_1$; (b) $\bar{q}_2$; (c) $\bar{q}_3$.

southern walls, will be discussed in sections 5 and 6. Sverdrup dynamics, with characteristically weak flow, extend to the eastern wall. The gentle southward flow along the eastern wall is devoid of any significant relative vorticity making the boundary condition of zero relative vorticity of little consequence. At this wall, neither $\bar{\Psi}$ nor $\bar{q}$ intensification occurs for this model run, although parameter changes in basin dimensions and wind forcing could force the midlatitude jet to reach the eastern boundary and, in turn, force a southward flowing current at the eastern boundary. Holland and Lin (1975b), in a parameter study of an EGM, were able to produce such an eastern boundary current.

The artificiality of the southern boundary in this closed basin simulation of a world ocean results in an artificial southern boundary current. However, since the oceanographic community uses these closed basin models to study ocean dynamics, it is important to understand the effect that the introduction of such a boundary has on the general circulation. The intent here is not to model an observed oceanic boundary current but rather to understand the role the southern boundary plays in this model's vorticity dynamics.

5. Western boundary layer

a. Layer 1

Intensifications in both $\bar{\Psi}$ and $\bar{q}$ are present at the western wall, with the former creating a broader boundary layer (~ 70 km) than the latter (~ 20 km). Boundary layer behavior is also observed from the paths of 60 floats released from Cluster Q in the southern portion of the basin and tracked for 182 days. As seen in Fig. 7 the particles are uniformly swept into the western boundary after launch and then deposited in the midlatitude jet. Two distinct boundary regimes can be differentiated by the Lagrangian signature of these floats. Over 600 floats were released in the western boundary layer and their subsequent vorticity balances were analyzed. Rather than deal with the difficult concept of the Lagrangian mean, individual paths, representative of the group, have been chosen to illustrate Lagrangian behavior. Shown in Fig. 8a is the trajectory of a Cluster A float launched at a position 5 km from the western wall and purposely truncated at the midlatitude jet entrance. The float path is superimposed on the background $\bar{q}$ contours and the plot size is tailored to the trajectory length. To assess the changes in q and its components along a path, the balance of terms

$$\xi - \xi_0 + f - f_0 + \eta - \eta_0 = q - q_0, \quad (6)$$

is computed every two days from launch. All vorticities are referenced to their values at launch as designated by the subscripted term. The right side of Eq. (6) designates not only the sum of the individual vorticities but also represents the sum of the integrated nonconservative forces. Within the boundary layers the wind

FIG. 6. Details of the mean potential vorticity fields. Contour interval scaled by $100/f_0$. Maximum values designated by (H) and minimum values designated by (L). (a)–(c) $\bar{q}_1$, $\bar{q}_2$ and $\bar{q}_3$ at the western boundary; (d)–(f) $\bar{q}_1$, $\bar{q}_2$ and $\bar{q}_3$ at the southern boundary.

FIG. 7. Spaghetti plot of floats released from Cluster Q in the upper layer. Ten releases from each of six float locations were tracked for 182 days.

forcing is negligible so that $q - q_0$ represents the integrated dissipation alone. The balance of terms associated with the trajectory are shown in Fig. 8b. The large gain in q , indicated by an uncompensated gain in planetary vorticity, results from strong dissipative forcing along the path. This behavior sharply contrasts with the behavior of a particle which is launched initially at $x = 65$ km. As seen from its trajectory in Fig. 9a, the particle does not penetrate the $\bar{q}$ boundary layer but stays in the $\bar{\Psi}$ boundary layer in its northward journey. Inspection of its balance of q components (Fig. 9b), shows that while there is the necessary gain in planetary vorticity, it is compensated by a decrease in relative vorticity so that q is approximately conserved.

The similarity of the model boundary layers to frictional and inertial boundary layers proposed by analytical models implies a minimal contribution by eddies to the boundary dynamics. Indeed, the magnitude of the eddy flow is small relative to the intense northward mean boundary current (mean velocities reach a maximum of 220 cm s^{-1} with a standard deviation of only 15 cm s^{-1}) and deviations from the $\bar{q}$ field average only 10%. The importance of eddies for this study, however, is measured by their contribution to the potential vorticity dynamics expressed in Eq. (5). The values of $\bar{u}_1 \cdot \nabla \bar{q}_1$ and $\nabla \cdot \bar{u}_1 \bar{q}_1$, the advection of mean potential vorticity by the mean flow and the eddy flux of eddy potential vorticity, respectively, are contrasted with the value of dissipative forcing, $\nu \nabla^2 \bar{\xi}$, at the western boundary in Fig. 10a. Until the jet entrance is reached ($y = 1800$ km) the dissipative western boundary dynamics are controlled by the mean vorticity and mean flow fields.

The Eulerian mean profiles of $\bar{\xi}$ and $\bar{\eta}$ (Fig. 10c and 10d) are used to further elucidate the differences in the two boundary regimes. The boundary condition ($\xi = 0$) has created a sharp gradient in the $\bar{\xi}$ field leading from the wall to a minimum value a dissipative length scale away. The profile of $\nu \nabla^2 \bar{\xi}$ (Fig. 10b) shows that the $\bar{\xi}$ minimum delineates the seaward extent of the significant dissipative forcing and of the $\bar{q}$ intensification. Despite the manifestation of the boundary conditions in the N-S alignment of $\bar{\xi}$ and $\bar{\eta}$, the meridional planetary vorticity gradient causes the contours to intersect the wall. Particles, however, cross the $\bar{q}$ contours as they travel northward due to the dissi-

FIG. 8. Lagrangian signature for an individual float released from Cluster A, layer 1, with initial position given by ($x = 5$ km, $y = 100$ km). (a) Float trajectory superimposed on $\bar{q}$ contours (origin designated by "o" and "+" marks every 10 days). Contour interval scaled by $100/f_0$; (b) Changes from initial vorticity values.

FIG. 9. As for Fig. 8 except for float's initial position: ($x = 185$ km, $y = 100$ km).

pative sink. The dominant balance of terms from Eq. (1a),

$$\beta v = \nu \nabla^2 \xi,$$

yields a western boundary layer with a characteristic width, λ , scaled as

$$\lambda = \left(\frac{\nu}{\beta} \right)^{1/3}.$$

For the parameter choices of this model realization the boundary layer width equals 17 km, which closely matches the measured layer thickness. Although the scaling and vorticity balance favor the analogy, the model's dissipative boundary layer is not a Munk layer in the strict sense due to the choice of lateral boundary

conditions. Free slip conditions, used by this model, partial slip, or no slip conditions, used by Munk, all create the necessary ξ gradient for the establishment of a dissipative boundary layer, but the strength of the dissipation varies according to the strength of the relative vorticity source provided at the wall relative to that of the boundary current.

The sharp $\bar{q}$ changes within the dissipative boundary layer are due to sharp increases in both vorticity components as $x = 0$ is approached. Past the minimum of $\bar{\xi}$ ($x > 17$ km), however, the changes in $\bar{\xi}$ and $\bar{\eta}$ partially offset each other and, although intensification is still present in the $\bar{\Psi}$, $\bar{\xi}$ and $\bar{\eta}$ fields, there is no longer a crowding of $\bar{q}$ contours. Through this outer inertial boundary layer passes the remainder of the interior flow. In sharp contrast to the mean flow of the dissipative layer, the mean flow and the Lagrangian paths within the inertial boundary layer follow $\bar{q}$ contours. Since planetary vorticity gradients are crossed there must be compensation. Relative vorticity plays this role as observed from the Lagrangian behavior and also seen from the $\bar{\xi}$ profile. The only alternative, stretching vorticity, has contours nearly aligned with $\bar{\Psi}$ within the inertial region; a reflection of the strength of the upper layer flow through this boundary layer. An evaluation of the terms in Eq. (5) for this inertial regime yields a balance of

$$\bar{\mathbf{u}} \cdot \nabla \bar{\xi} + \beta \bar{v} \sim 0,$$

which leads to a characteristic width of

$$\lambda \sim \left(\frac{4\pi\tau_0}{H_1\rho_0\beta^2L} \right)^{1/2}.$$

Substitution of parameters yields a calculated thickness of 51 km, which roughly matches the observed thickness in the model. These results are consistent with Holland's (1967) analysis of frictional-inertial boundary layers. For the parameter choices of this model the solutions to the boundary layer equations are characterized by two decay rates; one applies to a frictional sublayer and the other to the outer inertial layer.

b. Layer 2

The general behavior of the floats in this layer is shown in the spaghetti plot of Fig. 11. The initial launch position (Cluster Q) was situated near where the $\bar{\Psi}$ contours diverge to form the northern and southern branches of the western boundary flow. Those floats in the northern branch exhibit uniform behavior, while those that turn south are characterized by meandering and highly variable paths. This behavior will be shown to be consistent with the dynamics at the southern boundary and will be discussed in the next section.

Both the northward and southward flowing boundary currents in this middle layer display significant dissipative forcing (Fig. 12a), which, despite the increased

FIG. 10. For the western boundary of layer 1: (a) Constituents of Eq. (5) with the residual shown by the dashed line; (b) $\rho \nabla^2 \xi$ with the meridional extent chosen so as to exclude the southern boundary and midlatitude jet dynamics. Contour interval scaled by 10^{-12} s^{-2} ; (c) ξ_1 with contour interval scaled by $100/\rho_0$; (d) η_1 with contour interval scaled by $100/\rho_0$.

eddy strength in this layer, is still balanced by the mean advection of planetary vorticity. Lagrangian signatures in this regime, the relative magnitude of the components of Eq. (5) and the characteristic boundary thickness mimic those of the dissipative layer in the upper layer. A difference arises in the inertial regime of layer 2, where conservation of q results from a balance between the gain in planetary vorticity and a loss of stretching vorticity. The importance of η in compensating f is evident from the Lagrangian signature of a float that was launched from Cluster C and traveled within the $\bar{\Psi}$ boundary layer (Fig. 13). The loss of η results from the northwestward flow in this region running headlong into the $\bar{\eta}$ contours pictured in Fig. 12b. The stretching vorticity, as seen in Eq. (2b) depends upon the deviation in the interfaces sandwiching the

layer, and although both interfaces shoal westward to the wall, the thickness of layer 2 increases (hence η_2 decreases) since the upper interface has the larger slope. In this middle layer the stretching vorticity plays the compensatory role which the relative vorticity fulfilled in the upper layer. The reduced importance of relative vorticity reflects the reduction of flow intensity in a layer isolated from direct forcing.

c. Layer 3

Brief mention will be made of the fact that dissipative boundary layers are also found in layer 3 for each of the northward or southward flowing boundary currents that have significant relative vorticity near the western

FIG. 11. Spaghetti plot of floats released from Cluster Q in the middle layer. Ten releases from each of six locations were tracked for 182 days.

wall. Planetary vorticity completely dominates the $\bar{q}$ field throughout the basin except for a slight concession to relative vorticity at the western wall. The concentration of the total dissipative forces in this layer, lateral friction plus bottom friction (Fig. 14b), coincides with the deviations of the $\bar{q}$ field at the western wall. The width of the dissipative layer is comparable to the boundary thicknesses of layers 1 and 2, thereby suggesting the dominance of lateral dissipation over bottom friction. The presence of bottom friction alone would create a Stommel boundary layer governed by $\beta v = -\epsilon \xi$. Scaling yields a layer thickness of ϵ/β , which for these model parameters equals 5 km, a length scale only marginally resolved in the model. The profiles in Fig. 14a confirm that the small length scales at the boundary favor lateral dissipation and that even in this deepest layer, where the eddy relative vorticity is the same order as the mean value, the vorticity dynamics within the dissipative layer are mainly controlled by the time-mean flow.

Outside the dissipative layer the structure of the $\bar{q}$ field precludes the existence of a mean inertial regime, where the mean flow would have free passage along $\bar{q}$ contours. Therefore, since the mean flow does cross the $\bar{q}$ contours the eddy flux of potential vorticity must compensate. (Bottom friction is also negligible outside the lateral dissipative layer.) Referring again to Eq. (5), the contribution of $\nabla \cdot \bar{u} \bar{q}'$ is now needed to negate the $\bar{u} \cdot \nabla \bar{q}$ term. Specifically, it is the divergence of the eddy relative vorticity which allows for the nonzero $\beta \bar{v}$ values. In this bottom layer, with a large mean layer thickness (4000 m), the stretching vorticity's role is much reduced and it is the relative vorticity (mean and eddy) which compensates for individual particle gains or losses of planetary vorticity.

6. Southern boundary

As mentioned earlier, the reversal with depth of the meridional $\bar{q}$ gradient in the lower half of the southern basin creates a favorable environment for baroclinic instability. As discussed in the work of Gill et al. (1974) the mean available potential energy of the ocean can be converted into eddy energy via this instability. Eddy buoyancy fluxes directed down the mean density gradient tend to flatten the isopycnals, thereby reducing the mean potential energy. Within the context of a layered quasi-geostrophic model, the height field is dynamically analogous to the density field so that the energy conversion term of interest is $\bar{u} \bar{h}' \cdot \nabla \bar{h}'$, where $\bar{u}$ is the velocity field at the interface. A widespread area of eddy production in the southern half of the

FIG. 12. For the western boundary of layer 2: (a) $\bar{u} \nabla^2 \bar{\xi}$ with contour interval scaled by 10^{-12} s^{-2} ; (b) $\bar{\eta}_2$ with contour interval scaled by $100/\rho_0$.

FIG. 13. As for Fig. 8, except for a float launched from Cluster C, layer 2, with initial position given by $(x = 305 \text{ km}, y = 1000 \text{ km})$.

model basin is indicated by ubiquitous downgradient eddy flux vectors, $\bar{u}'_{12}h'_{12}$ (Fig. 15). As there is no flow normal to the boundary, the eddy flux $\bar{v}'_{12}h'_{12}$ must vanish at $y = 0$. The resultant divergence contributes to a positive mean vertical velocity at the southern boundary as seen through an expansion of Eq. (3a) into mean and eddy components:

$$\bar{w}_{12} = \bar{\mathbf{u}}_{12} \cdot \nabla \bar{h}_{12} + \nabla \cdot \bar{\mathbf{u}}'_{12} h'_{12}.$$

The two terms on the right side are evaluated as $J(\bar{\Psi}_{12}, \bar{h}_{12})$ and $J(\bar{\Psi}'_{12}, h'_{12})$, respectively. At the southern boundary the eddy contribution dominates and produces a strong upwelling velocity on the order of $1.5 \times 10^{-3} \text{ cm s}^{-1}$, which extends from $y = 0$ to approximately 150 km from the southern boundary. This compares with a maximum Ekman velocity at midgyre

of $1.7 \times 10^{-4} \text{ cm s}^{-1}$ and a mean vertical velocity at the western wall that is dominated by the $\bar{\mathbf{u}}_{12} \cdot \nabla \bar{h}_{12}$ term.

The divergence of the eddy flux at the southern boundary essentially drives the cyclonic gyre in the middle layer. As described fully in the work of Holland and Rhines (1980) and Rhines and Holland (1979), the pattern for the interfacial eddy stress may be likened to a zonal-wind distribution that establishes the gyre structure. Just as the Ekman velocity imposed at the surface produces a large-scale anticyclonic gyre in the southern basin, so too does the localized interfacial upwelling in the vicinity of the southern boundary produce a cyclonic gyre in the middle layer (with the provision that the vertical velocity at the second interface does not create a negative $\delta w / \delta z$ for the middle layer; a condition which holds in this region). Finally,

FIG. 14. As for Fig. 10a and 10b but for the western boundary of layer 3 and with the inclusion of bottom friction for (a). Contour interval for (b) scaled by 10^{-13} s^{-2} .

FIG. 15. Vector plot of $\overline{u'_{12}h'_{12}}$ superimposed on $h_{12}(m)$ field. Maximum vector length represents value of $3.6 \times 10^{-4} \text{ m s}^{-1}$.

we note that although the presence of the southern boundary in this numerical model creates a divergence of the eddy flux, a divergence could result in the absence of such a boundary. Spatial decay of the eddies away from the region of strong instabilities would produce a similar, although weaker, divergence. An example of this is Holland's (1978) two-layer model where an eddy-induced cyclonic gyre in the southern basin of the middle layer lies far removed from the southern boundary. Similarly, eddy divergences, independent of the boundary, produce the observed multiple gyre pattern in the bottom layer of this model.

a. Layer 1

As at the western wall, the $\bar{\Psi}$ boundary layer at the southern wall is created from the return of the interior flow, yet here the mean boundary current does not encounter a planetary vorticity gradient, which was vital to the western boundary dynamics. The width of this southern boundary current measures approximately 100 km before it turns northward. The $\bar{q}$ boundary layer, however, spans only the width of the first grid space from the southern boundary. Contrary to the fine resolution at the western boundary, the southern boundary has a uniform grid spacing of 20 km which only marginally resolves the $\bar{q}$ structure at the boundary. The boundary profiles of the $\bar{\xi}$ and $\bar{\eta}$ fields (Fig. 16c and 16d) substantiate the narrowness of the $\bar{q}$ boundary layer since past the minimum of $\bar{\xi}$ both vorticity components increase rapidly as the southern wall is approached. The location of the $\bar{\xi}$ minimum at 20 km is responsible for the marginal resolution of the $\bar{q}$ boundary layer. However, based on

the characteristics of the $\bar{\xi}$ field at the western wall, it is believed that further resolution near $y = 0$ would serve only to move the location of the $\bar{\xi}$ minimum, and hence the seaward edge of the $\bar{q}$ boundary layer, slightly southward. It is sufficient that the $\bar{q}$ boundary layer exists at the southern wall and the details of its geographical extent are not a grave concern, because here the $\bar{q}$ layer is not equivalent to the dissipative field. Since there is significant relative vorticity up to 100 km from the southern boundary the reach of the dissipative field (Fig. 16b) extends beyond the confines of the $\bar{q}$ intensification. (Note: It is the $\bar{\xi}$ minimum and not the entire $\bar{\xi}$ field which is marginally resolved.) The resolution of the $\nu \nabla^2 \bar{\xi}$ field, while not optimum, is sufficient to understand the qualitative aspects of the southern boundary dynamics.

Trajectories from 60 floats (10 releases from six locations) launched from the base of the southern boundary (Fig. 17a) indicate that in contrast to the trajectories at the western wall, these paths defy uniformity and cover a broad section of the numerical ocean in 182 days. This contrast holds even for floats that are initially positioned within the dissipative boundary layer (Fig. 17b and 17c). Contrary to the Lagrangian behavior at the western wall, the floats at the southern boundary, due to strong meandering, stray in and out of the dissipative and inertial boundary currents. Initial float location does not dictate travel in one regime alone, therefore the Lagrangian signature of the dissipative and inertial boundaries are interwoven.

As discussed earlier, this region is fraught with eddy production at the expense of mean potential energy. The growth of eddies, however, affects not only the energy distribution but also the balance between the mean and eddy potential vorticity. The strength of the eddy vorticity at the southern wall is evidenced by standard deviations of 80% and 25% for the $\bar{\xi}$ and $\bar{\eta}$ fields, respectively. As shown by Holland and Rhines (1980), the eddy potential vorticity, or enstrophy (q'^2), has as its source the eddy flux down the mean $\bar{q}$ gradient. Given that the stretching vorticity, η , is linearly related to the interfacial height, h_{12} , the down-gradient eddy heat flux in this region corresponds to a downgradient eddy η flux which, along with a down-gradient eddy ξ flux, creates a downgradient q flux. A divergence of this eddy flux at the boundary offsets the dissipative forcing as seen in Fig. 16a. Therefore, contrary to the deep layers at the western wall where eddy vorticity values are also significant, here the eddies manifest their strength in the control of the potential vorticity dynamics. For approximately 70 km from the southern wall the dominant balance of Eq. (5) is

$$\nabla \cdot \overline{u'q'} = \nu \nabla^2 \bar{\xi}.$$

Therefore, the potential vorticity that would be lost from an eddy q flux out of the area is maintained by the vorticity source at the wall. The small scale of the

FIG. 16. As for Fig. 10 but for the southern boundary of layer 1. Contour interval for (b) scaled by 10^{-13} s^{-2} .

variation in the Fig. 16a profiles, as contrasted with that at the western wall, is indicative of the strong eddy activity.

The mechanics of the eddy flux are illustrated by the Lagrangian signature for a float launched at the southern wall (Fig. 18a). The meandering across the $\bar{q}$ contours takes the parcel into and out of the dissipative region so that despite the compensatory efforts by ξ and η a q gain results (Fig. 18b). (Planetary vorticity changes during the meandering are relatively weak). Although the final value of $\bar{q}$ is obviously dependent on where the trajectory is truncated (the choice here was to end the path prior to the float's entry into the western boundary regime) it is evident from the meandering that a parcel's gain in q can not directly correspond to a gain in $\bar{q}$. Just as q and its components are evaluated for any given time step at a Lagrangian lo-

cation, so too may the Eulerian time-averaged vorticity values be interpolated from the surrounding gridpoints. Therefore, changes in ξ and η along a path may be subdivided into time-mean and eddy changes. As seen in Fig. 18c the overall increase in the eddy vorticity components accounts for the gain in potential vorticity along the path. This issue will be further discussed for a float at the southern boundary in layer 2 where the meandering effects are more pronounced.

b. Layer 2

The mean flow field of layer 2's southern gyre is reminiscent of the flow pattern of the midlatitude jet. The southward flowing western boundary current feeds into the southern boundary and a tight recirculation develops. This current, like the midlatitude jet, is also

host to instabilities. The resulting eddy variability causes the disarray of Lagrangian trajectories which were shown in Fig. 11. The positive relative vorticity of the eastward current transforms the southern boundary into a sink ($\nu \nabla^2 \xi < 0$) and a corresponding source is sought. In this region, as mentioned earlier, the upper interface controls the stretching vorticity of layer 2 making η_2 proportional to η_1 (or $-h_{12}$). Therefore, the divergence of the eddy stretching vorticity in the upper layer is accompanied by a convergence of the same flux in the middle layer. Here the mean q that would be gained from an influx of eddy potential vorticity is lost to the vorticity sink at the southern wall.

In this middle layer the eddy relative vorticity is the same order of magnitude as the mean relative vorticity while the eddy stretching vorticity, despite the controlling influence of its divergence, remains approximately 20% of the mean value. The strength of the transient motions is reflected in the float path shown in Fig. 19a. This float strongly meanders as it travels eastward along the southern wall in agreement with the direction of the cyclonic gyre's mean boundary current. From these strong excursions a characteristic signature emerges for a float within an unstable and dissipative regime. The vorticity changes along the path have the expected overall q decrease and the compensatory changes by ξ and η , but the mark of the eddy q flux in this region is borne by the float's v' and q' values (Fig. 19b). Due to the presence of the dissipative gradient, the meandering (even if north and south excursions have the same tilt relative to the mean q contours) results in a phase-shifted negative correlation among the v' and the q' components. Positive q' is carried into the region and negative q' is carried out with the dissipation within the boundary layer absorbing the vorticity gain. In the Eulerian frame this corresponds to a significant convergence of eddy potential vorticity while in the Lagrangian frame particles leaving the boundary have lost potential vorticity. The observed eddy flux and the Lagrangian behavior could result from baroclinic Rossby waves, but in this active region of eddy activity such wave signals are unfortunately obscured. Only in the quiescent eastern part of the basin are Rossby wave dynamics evident in a Lagrangian signature.

c. Layer 3

Due to the slight deviations of the second interface near the southern boundary, the $\bar{\Psi}$ structure of the

FIG. 17. (a) Spaghetti plot of floats released from Cluster Z in the upper layer. Ten releases from each of 6 locations were tracked for 182 days; spaghetti plot of ten releases from same initial position in layer 1. All releases were tracked for 182 days. (b) Floats released at western boundary, Cluster A ($x = 5$ km, $y = 100$ km); (c) Floats released at southern boundary, Cluster Z ($x = 2200$ km, $y = 5$ km).

FIG. 18. (a) and (b) as for Fig. 8 except for float's initial position ($x = 2200$ km, $y = 25$ km). (c) Mean and eddy vorticity values.

FIG. 19. (a) As for Fig. 8(a) except for float with initial position of ($x = 300$ km, $y = 5$ km); (b) Eddy potential vorticity and eddy meridional velocity.

cyclonic gyre in the middle layer is imprinted on the layer below. Although eastward boundary flow creates a positive relative vorticity gradient, it is sufficiently weak to be counteracted by the negative stretching vorticity gradient. Therefore, the $\bar{q}_3$ gradient is dominated by βy and no $\bar{q}$ intensification results. However, a broad dissipative layer is present at the southern wall (Fig. 20b). As in the layers above this dissipative sink balances the vorticity source provided by $\nabla \cdot \bar{u}_3 \bar{q}_3$ (Fig. 20a). In contrast to the western boundary layer, bottom friction now contributes significantly to the total dissipation due to the larger length scale of the Ψ field. Together, lateral dissipation and bottom friction create a broad (150 km) but weak boundary layer relative to the boundary regimes in the upper two layers. Particle

FIG. 20. As for Fig. 14 but for the southern boundary of layer 3. Contour interval for (b) scaled by 10^{-14} s^{-2} .

behavior in this region is not strongly influenced by such weak dissipative effects.

7. Basin integrated potential vorticity balances

Although the natures of the western and southern boundaries were elucidated in the previous sections, a quantification of their contributions to the comprehensive vorticity cycle is also of interest. Though boundaries are the sole sink of vorticity in a single-gyre ocean, for an ocean with gyres of opposite wind forcing an additional sink is created from the complementary vorticity exchange across the gyres' shared border. Within this double-gyre numerical ocean both sink mechanisms exist and, through an integration of the time-averaged vorticity equations, the distribution of the overall sink is determined for each layer. As mentioned in section 1, Eulerian vorticity budgets have

been previously determined for distinct regions of numerical ocean basins (Harrison and Holland 1981; Holland and Rhines 1980), but here, since we are specifically interested in the contribution of the ocean boundaries to the vorticity budget, we choose the area of integration to be the entire southern basin.

With the use of Green's theorem, the surface integral of Eqs. (1a-c) over the area A enclosed by the southern basin can be written as

$$H_1 \frac{\partial}{\partial t} \int_A q_1 dx dy = -H_1 \oint_C q_1 (\mathbf{u}_1 \cdot \hat{\mathbf{n}}) ds + \nu H_1 \oint_C \frac{\partial \xi_1}{\partial n} ds + \int_A \text{curl} \mathbf{r} dx dy, \quad (7a)$$

$$H_2 \frac{\partial}{\partial t} \int_A q_2 dx dy = -H_2 \oint_C q_2 (\mathbf{u}_2 \cdot \hat{\mathbf{n}}) ds + \nu H_2 \oint_C \frac{\partial \xi_2}{\partial n} ds, \quad (7b)$$

$$H_3 \frac{\partial}{\partial t} \int_A q_3 dx dy = -H_3 \oint_C q_3 (\mathbf{u}_3 \cdot \hat{\mathbf{n}}) ds + \nu H_3 \oint_C \frac{\partial \xi_3}{\partial n} ds - \epsilon H_3 \oint_C \frac{\partial \Psi_3}{\partial n} ds, \quad (7c)$$

where C represents the basin's boundary, $\hat{\mathbf{n}}$ is a unit outward normal vector, and s is the arclength of C . With the multiplication by the mean layer thicknesses, the equations express the time rate of change of the volume integrated potential vorticity. For an ocean in statistical steady state the left side of the equations should vanish after sufficient time averaging, thus leaving the terms on the right side to mutually balance. With the choice of the basin as the integration area, all dissipative terms and the advective term were reduced to line integrals in order to emphasize the role of boundaries and to make the computations cleaner. Taking into account that the velocity normal to a solid boundary vanishes, the first two terms on the right side of all three equations are subdivided into contributions from the midlatitude line, the eastern, southern, and western boundaries as follows:

$$-H_i \oint_C q_i (\mathbf{u}_i \cdot \hat{\mathbf{n}}) ds = -H_i \int_0^L v_i(x, L/2) q_i(x, L/2) dx, \quad (8a)$$

$$\begin{aligned} \nu H_i \oint_C \frac{\partial \xi_i}{\partial n} ds &= \nu H_i \left[\int_0^L \frac{\partial \xi_i}{\partial y}(x, L/2) dx - \int_0^L \frac{\partial \xi_i}{\partial y}(x, 0) dx \right. \\ &\quad \left. + \int_0^{L/2} \frac{\partial \xi_i}{\partial x}(L, y) dy - \int_0^{L/2} \frac{\partial \xi_i}{\partial x}(0, y) dy \right] \quad (8b) \end{aligned}$$

for $i = 1, 3$. The term on the right side of Eq. (8a) represents the sole advective flux at $y = L/2$, while the terms on the right side of Eq. (8b) are diffusive fluxes at $y = L/2$, $y = 0$, $x = L$, and $x = 0$. The third term on the right side of Eq. (7c) may be similarly subdivided as

$$\begin{aligned}
 & -\epsilon H_3 \oint_C \frac{\partial \Psi_3}{\partial n} ds \\
 & = -\epsilon H_3 \left[\int_0^L \frac{\partial \Psi_3}{\partial y}(x, L/2) dx - \int_0^L \frac{\partial \Psi_3}{\partial y}(x, 0) dx \right. \\
 & \quad \left. + \int_0^{L/2} \frac{\partial \Psi_3}{\partial x}(L, y) dy - \int_0^{L/2} \frac{\partial \Psi_3}{\partial x}(0, y) dy \right]. \quad (9)
 \end{aligned}$$

Therefore, in addition to the lateral diffusive fluxes at all four boundaries and a centerline advective flux found in the upper two layers, four dissipative fluxes due to bottom friction contribute to the q balance in the deep layer.

At 2-day intervals, for a time period of 12 years, each integral has been evaluated. The time-averaged results, with the inclusion of the northern basin for completeness, are shown in Fig. 21. Error was measured as the difference between the time-averaged left side and right side of Eq. (7). In all layers the error was less than 2% of the source for each basin. For layer 1 the wind forcing is taken to be the source while for layers 2 and 3 the advective flux at the centerline is used as the source value. Using this convention a sink is identified as an influx of vorticity of the opposing sign from the source.

a. Layer 1

Cross-gyre exchange of vorticity with the northern basin is the dominant sink for vorticity in the upper layer's southern basin, as 66% of the negative vorticity induced by the wind is advected north across midlatitude. The western boundary dissipates 27% of the source with the southern boundary responsible for only 6%. The numbers shown in Fig. 21 do not reveal the latitudinal or longitudinal dependence on the q gain or loss which is important in determining the large scale features of the potential vorticity cycle. An intense localized flux could dominate the integrated value and mask the general effect of the boundary on the gyre. From the earlier analysis of the Eulerian boundary layer fields it is expected that this is not the case, and as shown in Fig. 22a, this expectation is confirmed. The diffusive flux along the western boundary outlines the

FIG. 21. Potential vorticity volume fluxes (units of $m^3 s^{-2}$) for northern and southern basins of all three layers. Advective fluxes (broad arrows), lateral diffusive fluxes (solid arrow), and bottom friction fluxes (dashed arrows) are given at boundaries and at midlatitude. Wind source value (curved arrow) is basin integrated.

FIG. 22. Boundary profiles of potential vorticity flux. (a) Diffusive flux at western boundary ($x = 0$) for layer 1; (b) diffusive flux at southern boundary ($y = 0$) for layer 1; (c) eddy advective flux across midlatitude ($y = L/2$) for layer 1. Inset shows eddy flux within $0 \leq x \leq 100 \text{ km}$; (d)–(f) as for (a)–(c) but for layer 2; (g)–(i) as for (a)–(c) but for layer 3.

general features of the q loss at the western boundary. A similar profile at the southern boundary, Fig. 22b, reinforces the validity of the integrated flux values for the boundaries.

An integrated flux value is locally dominated only at midlatitude where the advective flux is heavily concentrated near the western wall. The advective flux, unlike the diffusive fluxes, can be subdivided into mean and eddy components, as

$$-H_i \int_0^L \overline{v_i q_i} dx = -H_i \int_0^L \bar{v}_i \bar{q}_i dx - H_i \int_0^L \overline{v'_i q'_i} dx,$$

(10)

where v_i and q_i are evaluated at $(x, L/2)$ and the overbar designates a 12-year time mean. A determination of the two right hand side terms of Eq. (10) shows the eddy flux to contribute 72% to the integrated total with the mean flux providing the remainder. The mean advective flux across the centerline has a small net contribution, which would diminish upon further averaging, due to the meandering of the jet. The eddy flux is also subject to the midlatitude jet's position and some cancellation occurs (Fig. 22c), but the major area of activity near the western boundary does not have a counterpart. This area is situated at the boundary exit region and the location of the flux maximum approximately coincides with the edge of the dissipative

FIG. 22. (Continued)

boundary layer (see inset of Fig. 22c). Although this local eddy flux dominates the cross-gyre exchange, it is not inconsistent with the dynamics of a jet entrance region. Particles from the northern and southern basins carry vorticity of opposing signs into the jet entrance and, through efficient mixing, the eddies in the jet entrance transfer vorticity between the basins.

b. Layer 2

In the southern basin of the middle layer the positive vorticity imported from its neighboring basin is the source which offsets the sinks at the boundaries. The direction of this advective flux indicates the penetrating effect of the wind, while the weakened magnitude re-

sults from the presence of the pool of homogenized potential vorticity. As in the upper layer, the eddy advective flux across $y = L/2$ is dominant and concentrated at the western limit, yet now this flux essentially vanishes beyond $x = 30$ km, the westernmost extent of the homogenized area (Fig. 22f).

In sharp contrast to the upper layer, the integrated sink at the western boundary is slight; as shown in Fig. 22d, however, this is due to the near self-cancellation of the two opposing gyres in the southern basin. The southern boundary profile (Fig. 22e) outlines a westward shift of significant vorticity exchange with local flux values comparable to those at the western wall. Additionally, the boundary's contribution is in the opposite sense from the upper layer's, as the cyclonic gyre

FIG. 22. (Continued)

in this layer turns the southern boundary into a sink of positive vorticity. With positive vorticity fed into the upper anticyclonic gyre from both the western boundary and centerline flux, and with positive vorticity lost to both the western and southern boundaries of the lower, cyclonic gyre, significant exchange must occur across their shared gyre front as well. As interior diffusive fluxes are small, it is the advective flux which must provide this exchange. In this region of instability the eddy flux of potential vorticity provides the mechanism for the upper gyre of the southern basin to drain its vorticity to the basin's lower gyre.

c. Layer 3

The wind's influence is seen to also extend to this layer, as evidenced by the positive vorticity advective

flux from the northern to the southern basin. This flux, 77% of which is eddy q flux, is not limited to the western corner but enjoys a broader influence (Fig. 22i), signifying not so much the enhanced exchange in the interior but rather the weakened exchange near the boundary. At the centerline the bottom friction diffusive flux outweighs that provided by lateral friction, as the larger length scale there, relative to the length scale at the western boundary layers, favors the former. Also, the relative contribution of bottom friction to the vorticity cycle is larger at the southern boundary than at the western boundary, as anticipated from the prior discussion.

The multiple gyres within the southern basin make a basin integral somewhat unsuitable for analysis of the boundary contributions, but as seen in Fig. 22g

and 22h, the sinks and sources at the western boundary are in accord with the general features of the gyres, and the southern boundary diffusive flux (away from the western wall) is negligible, in agreement with the results of section 6c. As in the middle layer, the exchanges across the individual gyres within a basin are necessary for the vorticity acquired from the northern basin to find its sink.

8. Summary and discussion

Both dissipative and inertial boundary layers exist in this numerical ocean, with the latter arising from the concentrated recirculation of the interior flow and the former a result of the boundary's role as a vorticity sink. The parameterization of the submesoscale activity as a diffusion of relative vorticity provides the mechanism for the boundary to be an active dissipator. The resolution of western boundary layers in this model clarifies the importance of lateral dissipation in shaping boundary dynamics. Two important factors distinguish the western and southern boundary layers. First, the meridional planetary vorticity gradient causes the potential vorticity contours to intersect the western wall, while the q contours remains aligned with the southern wall. The dissipation of relative vorticity at the western wall allows for the resultant cross contour flow and establishes the western boundary as a sink of mean q . Although the mean southern boundary current flows along $\bar{q}$ contours, the particle trajectories at the southern wall cross the $\bar{q}$ field. This discrepancy between Lagrangian and Eulerian mean behavior arises from the second major difference distinguishing these boundary regimes—dominance of eddy activity. At the southern wall the enstrophy cascade allows for the Lagrangian paths to cross $\bar{q}$ contours.

The sensitivity of this study's results to parameter choices is an issue which will be addressed next. For this EGCM the results of this study fall between two extreme cases which are differentiated by their ratio of inertial to viscous forces. At one extreme, when the viscous effects dominate, the solution becomes steady with no motion in the lower layers (Holland and Lin 1975b). All vorticity in the surface wind-driven layer would be lost at the lateral boundaries via strong dissipation. At this limit a frictional boundary only would result as all paths must be thread through a vorticity sink region. With other parameters held the same a three- or fourfold increase in the lateral viscosity coefficient would produce such a result. At the other extreme is when inertial effects dominate and vorticity is exchanged exclusively across a highly unstable mid-latitude jet. Though spatial gradients of relative vorticity would still be present the dissipative field at the boundaries would be relatively weak. Here the inertial boundary layer would accommodate the majority of the return flow. The case here, as measured by the amount of vorticity exchanged at midgyre and the

Rossby/Ekman numbers ratio points to a model run with inertial effects favored. Other parameter choices are believed to lead to characteristics qualitatively similar to this model run where both inertial and dissipative boundary layers exist and vorticity loss is shared between boundaries and cross-gyre exchange. The Lagrangian signatures of potential vorticity for the dissipative boundary layers are expected to remain the same within the limits of the two extremes. For this realization compensation for the change in planetary vorticity within the inertial boundary layers comes from mean relative vorticity, mean stretching vorticity, and eddy relative vorticity in the upper, middle, and bottom layers, respectively. This progression of control reflects the hierarchy of vorticity strength found within the upper layer alone: mean relative vorticity outweighs mean stretching vorticity which outweighs the eddy relative vorticity. With a weakened return flow, hence a weakened mean relative vorticity, the middle layer's inertial regime is controlled by the mean deformation of its overlying interface. Finally, in the bottom layer, stretching vorticity, insignificant in such a deep layer, yields to the eddy relative vorticity. This hierarchy is imposed by the time and space scales resulting from a wind-driven surface layer and eddy-driven interior layers. These vorticity strengths are vulnerable to parameter choices, but for a parameter set which yields a representative model ocean it is believed that these generalizations would hold.

A southern boundary current in a numerical model is not necessary and its desirability is questionable. A southern boundary current may be avoided through a parameter change just as eastern boundary currents are avoided by controlling the strength of the jet penetration. Here it was shown that the presence of a southern boundary current creates a vorticity sink, negligible in the upper layer but of consequence in the lower two layers. A reduction in wind strength and/or a deepening of the upper layer would ease the strong return flow at the southern wall. The price to pay for a broad baroclinically unstable region at the southern flank of the anticyclonic gyre is a vorticity sink at the southern wall.

No-slip boundary conditions are not expected to add a qualitative difference to the boundary layer as this can be viewed as a strong positive source of vorticity (for the anticyclonic gyre) relative to a zero relative vorticity source offered by free-slip conditions. For a time-mean dominated western boundary layer the introduction of biharmonic friction would also not qualitatively change the western boundary dynamics. For an appropriate choice of the frictional coefficient (Holland 1978) boundary layer widths and strengths would be comparable. Changes would occur if the boundary layer were heavily dominated by eddy terms because of the different scale selectivities of the two parameterizations. Significant deviations from these model results could occur due to gross changes in the structure

of the wind forcing and in the nature of the instabilities. Additionally, the introduction of topography, coastal geometry and/or thermohaline forcing could lead to dynamically distinct boundary layers from those studied here. Finally, it should be noted that the major constraint on these results is the assumption of quasigeostrophic dynamics.

Acknowledgments. This work was generously supported by the National Science Foundation under Grants OCE 82-14541, OCE 84-16537 and OCE 85-10848 and by the Office of Naval Research under contract N00014-86-K-0049 to the University of Washington. Computer resources were provided by the National Center for Atmospheric Research and valuable programming assistance was provided by Julianna Chow. The authors wish to thank D. B. Haidvogel and P. B. Rhines for their input to this study.

REFERENCES

- Gill, A. E., J. S. A. Green and A. J. Simmons, 1974: Energy partition in the large-scale ocean circulation and the production of mid-ocean eddies. *Deep-Sea Res.*, **21**, 499-528.
- Haidvogel, D. B., 1982: On the feasibility of particle tracking in Eulerian ocean models. *Ocean Modeling*, **45**, 4-9.
- Harrison, D. E., and W. R. Holland, 1981: Regional eddy vorticity transport and the equilibrium vorticity budgets of a numerical model ocean circulation. *J. Phys. Oceanogr.*, **11**, 190-208.
- Holland, W. R., 1967: On the wind-driven circulation in an ocean with bottom topography. *Tellus*, **19**, 582-599.
- , 1978: The role of mesoscale eddies in the general circulation of the ocean—Numerical experiments using a quasigeostrophic model. *J. Phys. Oceanogr.*, **8**, 363-392.
- , and L. B. Lin, 1975a: On the origin of mesoscale eddies and their contribution to the general circulation of the ocean. Part I: A preliminary numerical experiment. *J. Phys. Oceanogr.*, **5**, 642-657.
- , and —, 1975b: On the origin of mesoscale eddies and their contribution to the general circulation of the ocean. Part II: A parameter study. *J. Phys. Oceanogr.*, **5**, 658-669.
- , and P. B. Rhines, 1980: An example of eddy-induced ocean circulation. *J. Phys. Oceanogr.*, **10**, 1010-1031.
- Munk, W. H., 1950: On the wind-driven ocean circulation. *J. Meteor.*, **7**, 79-93.
- Rhines, P. B., and W. R. Holland, 1979: A theoretical discussion of eddy-driven mean flows. *Dyn. Atmos. Oceans*, **3**, 289-325.
- , and W. R. Young, 1982: Homogenization of potential vorticity in planetary gyres. *J. Fluid Mech.*, **122**, 347-367.
- Riser, S. C., and D. B. Haidvogel, 1989: Particle dispersion in an eddy-resolving model of the wind-driven ocean circulation (in preparation).
- Stommel, H., 1948: The westward intensification of wind-driven ocean currents. *Eos, Trans. Amer. Geophys. Union*, **29**, 202-206.
- Veronis, G., 1981: Dynamics of large-scale and ocean circulation. *Evolution of Physical Oceanography*, B. A. Warren and C. Wunsch, Eds., The MIT Press, 140-183.