Beach and Sea Cliff Dynamics as a Driver of Rocky Coastline Evolution

by

Patrick Wayland Limber

Earth and Ocean Sciences
Duke University

Date: _____

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Lincoln Pratson

Peter N. Adams

Dissertation submitted in partial fulfillment of
the requirements for the degree of Doctor of Philosophy in
Earth and Ocean Sciences in the Graduate School
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ABSTRACT

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Abstract

Rocky coastlines, with wave-battered headlands interspersed with calm sandy beaches, stir imaginations and aesthetic sensibilities the way few other landscapes do. Despite their prevalence (sea cliffs or bluffs are present along nearly 75% of the world's oceanic coastlines), we know very little about how rocky coastlines evolve. Quantitative studies of large-scale (>1 km) rocky coastline evolution are just beginning, and this work asks several unresolved and fundamental questions. For example, what determines the planform morphology of a rocky coastline? Can it reach an equilibrium configuration and cross-shore amplitude? What rocky coastline processes and characteristics scale the formation time and size of sea stacks?

The overarching theme of the following four chapters is the dynamics between beaches and sea-cliffs. Sea-cliff erosion and retreat is a primary source of beach sediment on rocky coastlines. As cliffs contribute sediment to the beach, it is distributed by alongshore sediment transport, and the beach can control future rates of sea-cliff retreat in two main ways: in small amounts, sediment can accelerate cliff retreat by acting as an abrasive tool, and in larger amounts, the beach acts as a protective cover by dissipating wave energy seaward of the sea-cliff. These feedbacks have been observed on rocky coastlines and in laboratory experiments, but have not been explored in terms of their control on large-scale and long-term (i.e., millennia) rocky coastline evolution. The aim of this dissertation is to explore the range of ways that beach and sea-cliff dynamics can

drive rocky coastline evolution with simple analytical and numerical models, and to generate testable predictions.

In Chapter 1 (Limber and Murray, 2011), we build a simple model where wave-driven sea-cliff retreat is a nonlinear function of beach width, as discussed above, and cliff retreat is maximized by sediment abrasion and minimized by either a lack of beach sediment or too much sediment. As sea cliffs retreat, beach sediment is produced and distributed alongshore by wave-driven sediment transport, and local beach widths determine future cliff retreat rates. Numerical experiments indicate that through such interactions, rocky coastlines can reach an equilibrium configuration where headlands and embayments remain stable through time, even in the absence of alongshore variations in sea-cliff lithology. Furthermore, the equilibrium coastline configuration, or the alongshore proportion of rocky headland to cliff-backed pocket beach, can be predicted analytically. Initial tests suggest that predictions match well qualitatively with actual landscapes.

Chapter 2 builds on Chapter 1. It is an analytical exploration of the feedbacks between beaches and sea cliffs using linear stability analyses. In the context of large-scale plan-view rocky coastline evolution, results suggest that there is no autogenic alongshore headland spacing, and that headland spacing (and alongshore embayment length) is dependent on initial conditions. The growth and decay rates of headlands are a function of rock strength, susceptibility to abrasion, and beach properties (e.g., the protective capacity of the beach), and are largely independent of alongshore sediment transport

rates. In addition, relative rates of beach sediment gains from sea cliff retreat and beach sediment losses can be used to explain rocky coastline morphology, or the abundance of headlands and pocket beaches. The strength of coupling between beaches and sea cliffs is controlled by sea cliff height and composition, where beaches backed by tall and/or sand-rich cliffs tend to respond faster to a change in width. This has implications for human impacts to rocky coastlines in that beaches backed by low, sand-poor sea cliffs may be more vulnerable to impacts such as river damming than those backed by tall, sand-rich cliffs. Finally, we can relate the infinitesimal-scale stability analysis to finite-scale rocky coastline morphology using a landscape Peclet number. The Peclet number describes the competition between sandy shoreline diffusivity (determined by the wave climate) and the coastal sediment budget, or total beach sediment inputs over a given alongshore length scale.

Chapter 3 continues exploring abrasion in terms of sea stack formation. Sea stacks are common and often spectacular coastal landforms, but few details are known about how, how quickly, and under what conditions they form. We use a numerical model of sea stack formation as a function of beach and sea cliff dynamics to address these geomorphic questions. Along the flanks of rocky headlands where pocket beaches are often curved and narrow due to wave field variability, abrasion can drive alongshore-directed sea cliff erosion. Eventually, a channel is cut through the headland, separating headlands from the mainland to become sea stacks. Under a symmetrical wave climate (i.e., equal numbers of waves approaching the coastline from the right and from the left),

analytical and numerical model results suggest that sea stacks form faster and are larger in plan-view area when abrasion intensity and initial headland area are large, respectively. Sea stack formation is discouraged when the sediment input from sea cliff retreat is too high (i.e., sea cliffs are tall and/or sand-rich) or the wave climate is highly asymmetrical. Therefore we propose that sea stacks are largest in plan-view area and most common on rocky coastlines where 1) the wave climate is symmetrical, 2) beach sediment input is below a threshold that is a function of sea cliff height, composition, and retreat rate, as well as any sediment losses from the nearshore system, 3) abrasion intensity is high, and 4) initial headland size is large. Initial comparisons to actual coastlines suggest that the number of sea stacks per kilometer of coastline may be inversely proportional to coastline sediment supply, as proposed by model results.

Finally, in Chapter 4, we move away from abrasion. On rocky coastlines where abrasion is not a significant factor, and hydraulic wave impacts, for example, drive sea-cliff retreat, the beach acts only as a protective cover. Therefore, as beach width increases, the rate of sea cliff retreat monotonically decreases. In the absence of abrasion, plan-view morphology is controlled by alongshore differences in rock type. More resistant rock extends seaward to form headlands, and less resistant rock retreats landward to form embayments. It remains unclear, though, if headlands and bays can reach equilibrium cross-shore amplitude, what physical characteristics might scale that amplitude, and over what time scales the coastline morphology evolves. To address these questions, we use an exploratory analytical model of rocky coastline evolution along

lithologically diverse rocky coastlines. The length of coastline that produces beach sediment changes as the coastline evolves, with higher-amplitude coastlines having more exposed and erodible sea cliffs and therefore yielding more sediment. In embayments, beach sediment collects due to alongshore sediment transport and the beach acts as a protective cover, slowing sea cliff retreat there. As a result, the headland and embayment retreat rates equalize and coastline amplitude can reach a persistent, stable equilibrium. In addition, these internal dynamics can produce headlands of different shapes (defined as an aspect ratio) ranging from needle-like (alongshore length is much smaller than cross-shore length), to stubby headlands (alongshore length is greater than cross-shore length). Equilibrium coastline amplitude depends on physical coastline and oceanographic characteristics, and it increases 1) as beach sediment input (a function of sea cliff height and composition) decreases; 2) the bare-rock sea cliff retreat rates increase; 3) beach sediment losses increase; 4) the time-averaged incoming wave angle increases; 5) headland spacing increases; and 6) alongshore headland width decreases. However, when sediment input is too large, headlands become transient, and the coastline amplitude and headland aspect ratio decrease to zero. The characteristic time scale over which lithologically diverse rocky coastlines evolve is longer for slower rates of sea cliff retreat, high-amplitude coastlines, and higher-angle wave climates that shadow embayments on the leeward side of headlands. Using simple coastline characteristics, our model gives insight into how and why cross-shore amplitude and headland shape vary on rocky coastlines.

Dedication

To Mom, Dad, and Noah.

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1. Beach and sea-cliff dynamics as a driver of rocky coastline evolution and stability

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1.1 Introduction

On cliff-backed rocky coastlines (Figure 1), the evolution of planform features such as headlands and embayments results largely from the interplay between incoming wave energy (Adams et al., 2002), alongshore lithologic or structural variations (Trenhaile, 1987), and the amount of beach sediment present (Sunamura, 1976, 1982; Sallenger et al., 2002). Previous observations suggest that rocks that can better resist wave attack extend seaward as headlands, and weaker rocks retreat landward as bays (e.g., Trenhaile, 1987, and references therein). While rock resistance is an important control on large-scale (i.e., >1 km) coastline shape, beaches also play a role by regulating the intensity and frequency of wave-induced sea-cliff retreat. Beach sediment can act as an abrasive tool to enhance sea-cliff retreat, or as a protective cover to hamper it (Sunamura, 1976, 1982; Robinson, 1977; Walkden and Hall, 2005). Accordingly, cliff retreat is minimized in two ways: when a lack of beach sediment reduces abrasion intensity, and when too much sediment hampers waves from reaching the sea-cliff. Because of this nonlinearity, sections of the coastline that are not fronted by a beach (i.e., headlands) may retreat more slowly than, or at the same rate as, neighboring sections

where a beach is present (i.e., embayments), suggesting a simple mechanism of headland persistence that is independent of alongshore lithologic variations. Many previous models of rocky coastline evolution have focused on the evolution of cross-shore profiles (e.g.,

Figure 1. Cluffed rocky coastline examples. A: Few or no beaches. B: Pocket beaches interspersed with bare headlands. C: Continuous beach. (Photos from Google Earth, 1 November 2010).

Sunamura, 1982; Trenhaile, 2000), but recently the evolution of plan-view coastline features has been considered (Adams, 2004; Walkden and Hall, 2005). Building on both bodies of work, we present analytical and numerical modeling to explore the range of

behaviors produced by simple interactions between cliff erosion, sediment production, sediment redistribution, and coastline evolution. We address here the following questions relevant to plan-view coastlines on millennial time scales: (1) what factors control the stability of headlands and embayments on rocky coastlines; (2) is it possible for a wave-dominated rocky coastline to reach planform equilibrium; and (3) how can observed rocky coastline types (Figure 1) be characterized using simple physical parameters?

1.2 Model development

We use an exploratory approach that involves purposefully simple parameterizations of large-scale and long-term emergent geomorphic interactions (Murray, 2007). For simplicity, alongshore variations in lithology are neglected, and terrestrial cliff erosion is assumed to be insignificant compared to mechanical wave erosion. The millennial time scales considered here (10^3 to 10^4 yr) are much longer than those associated with storm-driven, seasonal, or decadal variations in sea-cliff retreat rates, and the long-term cliff retreat rates and constant wave heights represent implicit time averages. Changes in relative sea level that could affect coastline behavior (e.g., Walkden and Hall, 2005) are not considered, representing a coastline that evolves over a sea-level highstand.

On our hypothetical cliff-backed rocky coastline (Figure 2), cross-shore sea-cliff retreat is a nonlinear function of beach width (Figure 2, inset), derived from the work of Sunamura (1976, 1982). Maximum sea-cliff erosion (E_{max}) occurs when the beach height (measured from the shore platform-cliff junction) is approximately equal to the still-water

depth, which is time-averaged over surface wave and tidal fluctuations (Sunamura, 1982).

Assuming a uniform beach slope, such that sediment is distributed or removed evenly over the entire height of the beach, we convert the height threshold to a width threshold,

Figure 2: Schematic of rocky coastline model setup. Inset shows equation 1, and steady-state beach widths when beach sediment loss rate equals beach sediment supply rate from cliff retreat (i.e., sea-cliff retreat times cliff height). Physical constraints and variables are explained in text.

referred to as the critical width (w_{crit}). Using these physical constraints, a nonlinear relationship between sea-cliff retreat and beach width is established:

$$\frac{d\eta_c}{dt} = \begin{cases} E_0 + mw(t), & w(t) \leq w_{crit} \\ NE_0 e^{-\frac{(w(t)-w_{crit})}{w_{scale}}}, & w(t) > w_{crit} \end{cases}$$

Equation 1

where E_0 is the bare-rock cliff retreat rate, when beach width equals zero; η_c is the

seaward cliff edge; t is time; w is the time-varying beach width measured at the junction of the shore platform and overlying beach wedge; and w_{scale} is a beach-width scale over which the sea-cliff retreat rate decreases to approximately zero. The variable w_{scale} is determined knowing E_{max} and the maximum beach width where sea-cliff retreat becomes negligible (i.e., $<0.001 \text{ m yr}^{-1}$). Equation 1 is similar to that derived by Anderson (2002) to represent bedrock weathering as a function of regolith thickness, and is discontinuous to be tractable in numerical and analytical models. The value of m is proportional to abrasion strength, which can affect the rate at which headlands grow or decay in cross-shore amplitude. It can be written as

$$m = \frac{E_0(N - 1)}{w_{crit}}$$

Equation 2

where N is the nondimensional ratio E_{max}/E_0 , describing rock abrasion intensity ($N \geq 1$; Sunamura, 1976). The magnitude of N can be estimated from laboratory experiments (Sunamura, 1976, 1982) or from field observations (Robinson, 1977). E_0 is set by using long-term observations of sea-cliff retreat (e.g., Hapke and Reid, 2007), and implicit in the rate are other mechanisms of cliff retreat such as water hammer or air compression (Trenhaile, 1987). The maximum beach width where sea-cliff retreat becomes negligible can be estimated from field observations, or from a wave run-up equation using deep-water wave height and period collected from wave buoys (Sallenger et al., 2002). During retreat, the modeled sea-cliff face maintains a constant, time-averaged vertical profile. Implicit in the average are short-term processes such as undercutting due to basal cliff

erosion, vertical distribution of mechanical wave erosion along the cliff face due to daily tidal fluctuations, and episodic mass wasting events. Many of the studies cited here describe coastlines with weakly lithified or soft-rock cliffs, but we expect that the same processes also occur, but more slowly, along coastlines with more strongly lithified cliffs, with abrasive tools ranging from sand to boulders. Beach geometry is represented simplistically as a wedge at the base of the sea-cliff (Figure 2), with a slope β , and width that represent implicit averages over seasonal, storm, and tidal adjustments. The beach wedge is on top of a shore platform that widens as the sea-cliff retreats. Our model does not address detailed shore platform evolution, which has been modeled extensively (e.g. Trenhaile, 2000). Instead, we assume that the shore platform is an approximately flat surface that can support beach formation.

As sea-cliffs retreat, weathered and eroded cliff material becomes a source of sediment for the beach (e.g., Perg et al., 2003; Walkden and Hall, 2005) and beach width then controls future cliff retreat rates. The change in beach width (w) with time is a function of cliff retreat rate, height, and composition, as well as alongshore sediment transport and a sediment loss rate (note that landward-directed rates of change are implicitly negative):

$$\frac{dw}{dt} = -(\gamma HC) \frac{d\eta_c}{dt} + D_0 \frac{d^2\eta_b}{dx^2} + S$$

Equation 3

where γ is a constant that scales the volume of sediment eroded from the sea-cliff to a change in beach width; H is a time-averaged nondimensional ratio between cliff height

and a characteristic beach height; C is the proportion of weathered sea-cliff sediment that is large enough to remain in the nearshore zone ($0 \leq C \leq 1$; Limber et al., 2008); D_0 is a diffusion coefficient; η_b is beach position; x is alongshore position; and S is a globally imposed constant sediment loss rate. The first term on the right side is the contribution to beach width from sea-cliff retreat, which varies in magnitude depending on what geometry is assumed for the beach. In our case, because the beach is triangular in profile, $\gamma = 2$ (see Appendix A). The variables H and C are hereafter discussed as a single variable, HC , which is the effective nondimensional sea-cliff height. The diffusion term in Equation 3 implies a wave climate dominated by low-angle waves (Ashton and Murray, 2006b), where alongshore sediment transport smoothes planform shoreline shape. The variable D_0 is a time-averaged effective diffusivity describing the wave climate, which depends on the distributions of wave heights, wave periods, and wave approach angles (Ashton and Murray, 2006b). Sediment loss rate S can be interpreted in a number of ways, such as offshore transport due to beach sediment fining in the surf zone (e.g., Perg et al., 2003).

1.3 Stable and unstable beach widths

Figure 2 (inset) shows that in this modeling framework, rocky coastline evolution depends on relative rates of sediment losses, S , and gains from sea-cliff retreat. The intersections of the rates correspond to steady-state beach widths, or fixed points, which are stable and unstable (Figure 2, inset). If beach width is perturbed around the unstable fixed point, positive feedbacks between abrasion, cliff retreat, and beach sediment

availability will push the cliff beach system to either a bare headland or a cliff fronted by the stable steady-state beach width (i.e., a pocket beach). For example, increasing beach width from the unstable fixed point promotes sea-cliff retreat because of abrasion. The increase in the sea-cliff retreat rate adds more sediment to the beach, which increases beach width, which in turn increases the cliff retreat rate further. This process continues until beach width grows to the stable fixed point. Alternatively, decreasing beach width from the unstable fixed point discourages sea-cliff retreat due to a lack of abrasive tools, sediment inputs from cliff retreat become less than sediment losses (S), and beach width decreases to zero. Extending these dynamics alongshore, and including alongshore sediment transport, a coastline could possibly develop alternating, and persistent, headlands and pocket beaches. For further investigation, a numerical analysis is required.

1.4 Numerical modeling

We use a model that discretizes Equations 1 and 3 over a domain with a cell size of 100 m, an alongshore length of 20 km, and periodic boundary conditions. Random initial coastline shapes are generated by sine wave superposition with random amplitudes and wavenumbers. Initial beach width is zero, and all sediment is supplied by sea-cliff retreat. Cliff-top elevation is assumed to be constant alongshore. The sediment loss rate (S) and bare-rock sea-cliff retreat rate (E_0) are represented together in the model as the nondimensional variable $S^* = S/E_0$. Because it is the relative rates of S and E_0 that drive rocky coastline behavior (Figure 2, inset), S^* determines coastline behavior rather than the individual values of E_0 and S . Field observations (e.g., Hapke and Reid, 2007) can be

used to estimate either E_0 or S , although such observations are over a much smaller time scale than the millennial time scales considered here. The beach erosion rate operates on a domain-wide basis so that wherever a beach is present, a constant beach width is removed each time step. In the model experiment reported here, the abrasion-maximized sea-cliff retreat rate, $E_{\max}$, is twice as fast as bare-rock sea-cliff retreat (i.e., $N = 2$), and we use $D_0 = 0.5 \text{ m}^2 \text{ s}^{-1}$, $HC = 2$, and $S^* = 5$ ($E_0 = 0.1 \text{ m yr}^{-1}$). Rock abrasion intensity N does not affect rocky coastline dynamics (assuming $N > 1$), but does determine the value of m , which controls the rate at which the coastline evolves (Equation 2). The diffusion coefficient represents an average wave climate, which was determined using wave statistics compiled for the coast of California (Wingfield and Storlazzi, 2007). A one-line method is used to track shoreline location (Ashton and Murray, 2006a). Finally, beach height is held constant in order to be compatible with our analytical model, where, for simplicity (see Equation 3), beach height is assumed to be constant in the variable H . Using a time-varying beach height does not affect the rocky coastline dynamics, but does change the final coastline configuration.

1.4.1 Numerical modeling results

Figure 3A shows a time series of plan-view sea-cliff and beach position from a representative model run over 15 k.y. As cliffs erode, sediment transport causes sediment to collect in embayments, forming pocket beaches. Initially, because of abrasion, the cliff retreat rate behind the pocket beach is higher than the retreat rate of adjacent sediment-free headlands, and headland amplitude grows (Figure 3C). If a pocket beach continues to

widen, the cliff retreat rate there eventually becomes lower than the headland retreat rate, and headlands can reduce in amplitude, or in some cases disappear. Persistent headlands are larger in cross-shore extent relative to the steady-state beach width. Pocket beaches ultimately reach a stable steady-state width of ~ 45 m. By the end of the model run, there are two beach widths, one equaling zero (headlands) and one at a steady state. The standard deviation of alongshore variations in cross-shore coastline position (Figure 3C) asymptotes toward a steady-state value. The coastline configuration is characterized using a measure of rockiness, R , which is the proportion of the coastline that is bare headland (Figure 3B). The coastline evolves fastest during the first few thousand years,

Figure 3: Representative model run results. A: Plan-view time series of coastline evolution showing initial conditions and coastline at 760 yr and 15 k.y. as it reaches equilibrium configuration. B: Rockiness through time. C: Standard deviation (SD) of cross-shore sea cliff position through time.

approaching its steady-state configuration by ~ 4.5 k.y. Between 4.5 and 15 k.y., the coastline changes little, and the coastline configuration is in equilibrium: headlands and pocket beaches are retreating landward at the same rate, and planform features persist.

1.5 Discussion

The nonlinear relationship between beach width and sea-cliff retreat has terrestrial geomorphic analogs. A similar (but weathering driven) instability between regolith thickness and bedrock weathering can explain the formation and persistence of tors in high alpine and desert environments (Anderson, 2002; Strudley et al., 2006). Also, river channel incision is limited by both a scarcity of abrasive tools and an abundance of tools, which effectively armors the channel (Sklar and Dietrich, 2001). Here, coastal headlands are analogous to tors, but the instability is abrasion driven (like in rivers) rather than weathering driven. On rocky coastlines where other mechanical erosion processes, such as quarrying, are more effective than abrasion, the abrasion-driven feedbacks required for the development and persistence of headlands may not be applicable. In this case, alongshore-heterogeneous lithology could support persistent headlands. However, how, and if, a lithologically variable coastline can reach an equilibrium configuration is unclear (Trenhaile, 1987), and is a topic for future model experiments. Analytical and numerical model experiments (not presented here) show that the time scale for the coastline to approach steady state is inversely proportional to m (Equation 2) and HC . In the model run presented here (Figure 3), the coastline approaches its equilibrium configuration by ~ 4.5 k.y., which is on the order of a sea level highstand. If HC or m is smaller, however, the coastline might evolve too slowly to fully reach equilibrium during a highstand, and consideration of sea-level changes is necessary. The rate of beach position diffusion, D_0 , does not affect millennial coastline evolution because the time

scale for coastline changes from gradients in alongshore sediment transport is relatively small. Rockiness is independent of the initial coastline shape. However, the location and equilibrium cross-shore amplitude of headlands and bays are dependent on the initial coastline shape. Initial coastline shapes could be the result of a newly flooded, fluvially dissected landscape, or a coastline that has been modified by a previous sea-level highstand (Trenhaile, 1987). To understand how equilibrium is achieved, we first define an effective coastal-average net sediment balance, S_{eff} , that is the difference between the sediment loss rate (S) and the contribution of beach sediment from sea-cliff retreat as the coastline configuration (R) changes:

$$S_{\text{eff}} = S - \frac{\gamma HC \frac{d\eta_c}{dt}}{1 - R} + E_0.$$

Equation 4

Beach erosion and accretion occur when $S_{\text{eff}} < 0$ or $S_{\text{eff}} > 0$, respectively. If R decreases over time, beach sediment is spread out by alongshore sediment transport over larger areas as pocket beaches become more prevalent. Then, S_{eff} decreases because each unit length of coastline receives progressively less sediment input. Conversely, if R increases, sediment inputs are distributed over fewer pocket beaches and smaller areas; S_{eff} then increases because cliff sediment inputs more effectively offset sediment losses. If $R = 1.0$, there is no beach present, and S_{eff} approaches infinity and lacks a physical interpretation. In the numerical model run (Figure 3), as cliff erosion produces sediment, R decreases from the initial condition of $R = 1.0$. As R decreases, S_{eff} also decreases, as described

here, and asymptotically approaches E_0 . Eventually the difference between S_{eff} and E_0 becomes negligible, so that pocket beaches and adjacent headlands retreat at the same rate (i.e., $S_{\text{eff}} \approx E_0$). The width of pocket beaches adjusts to the stable steady-state value, the rocky coastline can no longer evolve, and headlands and embayments become permanent features of the coastal landscape. S_{eff} can define the erosion rate term in Figure 2 (inset, dashed line) that dictates the steady-state beach widths and coastline dynamics. As a result, coastline behavior can change through time as R changes, and when the dashed line equals E_0 , the coastline is in equilibrium.

Equilibrium rockiness can be predicted analytically. Using Equation 4, we substitute the equilibrium condition, $S_{\text{eff}} = d\eta c/dt = E_0$, nondimensionalize, and find R :

$$R = \frac{S^* - \gamma HC}{S^*}$$

Equation 5

where $S^* = S/E_0$. The value of R predicted by Equation 5 agrees well with R in the numerical model (root mean square error is 0.03) for 16 model runs with varying values of S^* and HC . The variable R is contoured in Figure 4 as a function of nondimensional variables HC and S^* . Although it is not physically observable, R can be negative (Figure 4, shaded gray area), which is morphologically the same as $R = 0$ where the coastline consists of a continuous cliff-backed beach. When the sediment loss rate (relative to E_0) is high and HC is small, a rocky coastline with few, or no, pocket beaches develops (e.g., Figure 1A). Alternatively, when the sediment loss rate is low and HC is large, the coastline evolves to a configuration of a continuous beach backed by sea-cliffs with no

headlands (e.g., Figure 1C). Finally, when HC and S^* are similar in magnitude, a coastline with alternating headlands and pocket beaches develops (e.g., Figure 1B).

Figure 4: Contours of predicted rockiness (R) as function of effective sea-cliff height (HC) and sediment loss rate relative to bare-rock sea-cliff retreat rate (S^*). A, B, and C correspond to Figure 1.

With measurements of sea-cliff height and retreat rate, net beach erosion, and sea-cliff composition, we made qualitative comparisons of predicted analytical rockiness (R , Equation 5) to rockiness observed from aerial photography (R_0) for three study sites in California: Santa Cruz, Oceanside (Figure 1C), and San Francisco (Limber and Murray, 2011b). The variables R and R_0 were evaluated within three broad coastline types (continuous cliff-backed beach, $R \leq 0$; alternating headlands and pocket beaches, $0 < R < 1$; and no beach, $R = 1$; see Figure 4). R_0 and R for the Santa Cruz coastline were both, on average, alternating headlands and pocket beaches, although there was wide uncertainty

in observed bare-rock cliff and beach retreat rates. Taking a mid-range estimate of S^* , R_0 and R were 0.55–0.75 and 0.58, respectively. For Oceanside, R_0 ranged from continuous beach to beaches with few headlands (0–0.1). R was negative, which is morphologically the same as zero rockiness and represents a coastline with a continuous, cliff-backed beach. Similarly, R_0 for San Francisco ranged from continuous beach to beaches with few headlands (0–0.14), while R was negative. The differences between study sites are largely due to sea-cliff composition and beach sediment supply. The mudstone Santa Cruz cliffs are sand poor and yield little beach sediment ($C \approx 0.1$; Perg et al., 2003), while the Oceanside and San Francisco cliffs are sand rich ($C \approx 0.8$ for both sites; Young et al., 2010; Limber et al., 2008), resulting in an overall rockier coastline configuration in Santa Cruz reflected by higher values of R .

In this initial work, the numerical model intentionally includes a limited set of processes, and we consider only the simplest terrestrial landscapes. Our intent is not to accurately reproduce a particular coastline, but to clarify ways in which the possible interactions between cliff erosion, sediment production, and sediment transport can affect long-term rocky coastline evolution, and to generate testable predictions to assess the generality of the results. Model additions could include variable cliff lithology (and topography) and sediment production rates, nearshore submarine canyons and open boundary conditions, and fluvial sediment inputs.

2. A linear stability analysis of beach and sea-cliff dynamics yields insights into rocky coastline evolution

2.1 Introduction

On rocky coastlines, the frequency and intensity of wave-induced sea-cliff erosion can be controlled by beaches, which reduce or encourage wave erosion depending on their width and height (Sumamura, 1982; Komar and Shih, 1993; Walkden and Hall, 2005; Lee, 2008). In Oregon, for example, sea-cliffs fronted by low beach elevations were subjected to wave attack for longer periods of time than cliffs fronted by higher beach elevations, and experienced more severe erosion (Ruggiero et al., 2001). Similarly, winter storms caused maximum cliff retreat within a Northern California pocket beach where beach width was at a minimum (Sallenger et al., 2002). If beach widths are inversely proportional to cliff retreat rates and eroding sea-cliffs produce a proportion of the sediment found on the beach (e.g. Perg et al., 2003), a negative feedback exists: sea-cliff retreat produces beach sediment, which provides protection from wave attack and reduces future cliff retreat, which then reduces the cliff sediment supply to the beach. Effectively, beach behavior can control cliff behavior, and sea-cliff behavior can control beach behavior (Sunamura, 1976).

In addition to the negative feedback, a positive feedback mechanism between beach width and cliff retreat has been observed. Sunamura (1976, 1982) demonstrated that beach sediment, in small amounts, acts as an abrasive tool to enhance sea-cliff retreat. Abrasion is widely noted as an important erosive process (e.g. Emery and Kuhn, 1982; Trenhaile, 1987; Carter and Guy, 1988; Sunamura, 1992; Bird, 2000), and has also

been observed in experiments by Kamphuis (1990) and in the field by Robinson (1977), and used by Dick and Zeman (1983) and Trenhaile (2005) in profile-based models of sea-cliff and shore platform behavior, respectively.

Considering the positive and negative feedbacks together results in a nonlinearity: in small amounts, beach sediment can encourage sea-cliff retreat, but as the beach gets larger, it can also act as a protective cover to discourage sea-cliff retreat. Accordingly, there is a sea-cliff retreat rate maximum that occurs when beach width is greater than zero, and it is possible for the sea-cliff retreat rate to be the same for two different beach widths (Figure 5).

Figure 5: Schematic of the analytical model set-up. Inset shows equation (6), with constraints that are explained in the text.

Extended alongshore, if sediment-free headlands can retreat landward at the same rate (or a slower rate) as neighboring pocket beaches, this suggests a mechanism for headland and embayment formation and persistence with alongshore-homogeneous cliff lithology. Using a simple numerical model, Limber and Murray (2011) explored how this

mechanism can affect large-scale rocky coastline dynamics and planform morphology. Results suggested that it could lead to an equilibrium, and analytically predictable coastline morphology (i.e., the relative abundance of rocky headlands and pocket beaches) that depends on rates of beach sediment loss and wave-driven headland retreat, as well as sea-cliff height and composition. A linear stability analysis can provide further insight into rocky coastline dynamics. Specifically, we will analytically examine the feedbacks between beaches and sea-cliffs in the context of large-scale rocky coastline evolution, and address four main questions: 1) is there an autogenic alongshore headland spacing and pocket beach length scales on rocky coastlines, 2) what controls the characteristic time scales for headland growth and decay, 3) what controls the strength of coupling between beaches and sea-cliffs, and 4) how do human impacts (i.e., river damming, sand mining) affect rocky coastline behavior?

2.2 Analytical model

We are purposely treating emergent geomorphic processes and interactions on broad spatial (> 1 km) and time scales (millennial), rather than explicitly representing much smaller and faster scale processes. Our modeling approach is exploratory; our goal is to generate insight into overarching rocky coastline dynamics that can be driven by interactions between beaches and sea-cliffs rather than the dynamics of a particular coastline, beach, or sea-cliff. Accordingly, we make several simplifying assumptions. Sea level rise is not considered here, and it is assumed that the model operates during a sea level highstand. We take implicit time averages of sea-cliff and beach behavior such that short-term dynamics, such as tidal, storm, and seasonal changes, are not explicitly

included. The cliff face retains a constant profile, and the beach, which is triangular, rests on a flat shore platform. Shore platform evolution, which has been modeled in detail elsewhere (e.g., Trenhaile, 2000; Trenhaile, 2005), is not included here. Finally, rock composition is constant alongshore.

As described in Limber and Murray (2011), we can explore rocky coastline dynamics using coupled nonlinear differential equations representing sea-cliff retreat, beach width change, and beach position change such that waves erode the cliff, produce beach sediment that is distributed alongshore (implicitly by wave-driven alongshore sediment transport), and beach width then controls future sea-cliff retreat rates. Sea-cliff retreat is a nonlinear function of beach width:

$$\frac{d\eta_c}{dt} = \begin{cases} E_0 + mw(t), w(t) \leq w_{crit} \\ NE_0 e^{-\frac{(w(t)-w_{crit})}{w_{scale}}}, w(t) > w_{crit} \end{cases}$$

Equation 6

where η_c is sea-cliff position; E_0 is the bare-rock cliff retreat rate; w is beach width; w_{crit} is the beach width where abrasion maximizes the sea-cliff retreat rate; N is abrasion intensity relative to E_0 ($N > 1$); and w_{scale} is a length scale constant that depends on N as well as the beach width that provides near complete cover from wave attack so that sea-cliff retreat becomes negligible (i.e. $\sim 1\%$ of the maximum value). m is the slope of the line in Figure 5 between $w = 0$ and w_{crit} :

$$m = \frac{E_0(N-1)}{w_{crit}}$$

Equation 7

Beach width changes through as a function of sediment inputs from sea-cliff retreat (first term), alongshore sediment transport (second term), and sediment losses (third term):

$$\frac{dw}{dt} = -(\gamma HC) \frac{d\eta_c}{dt} + D_0 \frac{d^2\eta_b}{dx^2} + S$$

Equation 8

where γ is a beach geometry constant that converts the volume of sediment released from sea-cliff retreat to a beach width (see Appendix A); H is sea-cliff height divided by a characteristic beach height; C is the proportion of weathered sea-cliff material that is coarse enough to contribute to beach width (Limber et al., 2008); D_0 is a diffusion coefficient that represents the time-averaged wave climate that is acting on the coastline to transport sediment (Ashton and Murray, 2006b); η_b is beach position; x is alongshore position; and S is a sediment sink that can be attributed to, for example, sediment attrition in the surf zone and consequential offshore transport (Hicks et al., 2011; Perg et al., 2003) or human impacts (Thornton et al., 2006).

The change in beach position (the seaward toe of the beach, see Figure 7 for definition) through time is the same as equation 8, except it includes an extra cliff retreat term:

$$\frac{d\eta_b}{dt} = -(\gamma HC - 1) \frac{d\eta_c}{dt} + D_0 \frac{d^2\eta_b}{dx^2} + S$$

Equation 9

Alongshore sediment transport is represented as a diffusional process in a low-angle wave climate. As a result, gradients in alongshore sediment transport will cause bumps in

beach position to decay. Using a conservation equation for beach position evolution combined with a deep-water formulation of the common CERC or Komar (1971) equation for alongshore sediment transport (Ashton and Murray, 2006a), the diffusion coefficient can be written as

$$D_0 = \frac{K_2}{D} T^{1/5} H_0^{12/5} \left\{ \cos^{1/5}(\phi_0 - \theta) \left[\left(\frac{6}{5} \right) \sin^2(\phi_0 - \theta) - \cos^2(\phi_0 - \theta) \right] \right\}$$

Equation 10

where K_2 is a constant that relates volumetric sediment flux to the wave driving forces; D is the depth to the shoreface; H_0 is deep-water wave height; T is deep-water wave period; ϕ_0 is the wave angle; and θ is the orientation of the coastline. The long-term effective diffusivity is a temporal average of equation 10 (Ashton and Murray, 2006b).

2.2.1 A saddle node bifurcation

A steady-state beach width, or fixed point, exists when beach sediment gains from sea-cliff retreat equal sediment losses. Fixed points can be stable or unstable. Figure 6 is a bifurcation diagram (explained below) with beach width trajectories (arrows) that are driven by the key dynamics in our analytical model (circled numbers). Each feedback or process numbered in Figure 6 is explained below:

1. If beach width is increased from a stable fixed point, negative feedbacks will push the beach width back to the fixed point. This happens because the beach acts more effectively as a protective cover, slows down sea-cliff retreat, and slows down the rate that sediment is contributed to the beach from sea-cliff retreat relative to beach sediment losses. So, beach width decreases.

2. For a beach width decrease from a stable fixed point, the opposite happens: a narrower beach allows the sea-cliff to retreat faster, which produces sediment faster relative to beach sediment losses, and accordingly beach width increases.

Figure 6: Bifurcation diagram and beach width trajectories driven by beach and sea cliff dynamics (circled numbers; explained in text).

3. If beach width is increased from the unstable fixed point, positive feedbacks between beach width, abrasion, and sediment losses will increase the beach width to the stable fixed point. A beach width increase also increases the availability of abrasive tools, which in turn increases the rate of sea-cliff retreat. Then, beach sediment production increases, which further increases abrasion, and the feedback continues until beach width reaches the stable fixed point.

4. If beach width is decreased from the unstable fixed point, positive feedbacks will decrease beach width to zero (a bare sea-cliff or headland). Here, by reducing beach width, the availability of abrasive tools is also reduced, so sea-cliff retreat decreases. As retreat decreases, less beach width is produced, which reduces abrasion more, and the feedback continues until $w = 0$. Considering that the dynamics described by numbers 3 (beach width decrease) and 4 (beach width increase) occur in the same phase space, extended alongshore these dynamics can result in the persistence of headlands and sediment-filled embayments (i.e. pocket beaches).
5. When $S > HCNE_0$, no beach can exist because sediment losses are always greater than sediment production, so beach width decreases to zero.

Fixed points can be created and destroyed by varying S relative to $HCd\eta_c/dt$. For example, when $S < HCE_0$, there is a single stable fixed point; when $HCE_0 < S < HCNE_0$, two fixed points exist, one stable and the other unstable; and when $S > HCNE_0$, the fixed points are destroyed. As fixed points are created and destroyed (i.e., the system bifurcates), the beach and sea-cliff system undergoes changes in behavior in which different steady-states and planform coastline morphologies are possible. Figure 6 shows steady-state beach widths (w) as a function of S , and, if these dynamics are extended alongshore, the rocky coastline morphology that results (discussed in Section 2.3.4).

2.3 Linear stability analyses: results and discussion

The variables considered in the analyses are cliff height and composition (HC), plan-view shoreline-shape diffusivity (D_0), m (equation 7), and alongshore wavenumber

of the perturbations, k . In each case, the initial condition is a straight coastline a beach backed by sea-cliffs. The configuration is at either a stable or unstable equilibrium, and the feedbacks that will be evaluated are numbered and explained in Figure 6. Figure 7 is a schematic of the analyses, how the perturbations are applied, and how each analysis relates to one another. As discussed in the Introduction, the stability analyses are designed to address preferred headland spacing and controls on the time scales of rocky coastline evolution, and also elucidate what controls the strength of coupling between beaches and sea-cliffs.

Figure 7: Flow diagram of the linear stability analyses showing how the perturbations are applied to the coastline, and the feedback loops that result.

2.3.1 Headland growth and the unstable fixed point

Here we will activate the feedbacks shown in Figure 6, circled numbers 3 and 4, which can lead to the development of headlands and bays. Starting with a straight planform shoreline (i.e., constant alongshore beach width), the addition to beach width from cliff retreat equals the subtraction from beach width by the sediment sink. We begin the stability analysis by perturbing sea-cliff position, η_c , where the perturbation, η_c' , is sinusoidal:

$$\eta_c' = \varepsilon_1 e^{\sigma t} \sin kx$$

Equation 11

where ε_1 is the amplitude of the sinusoidal perturbation; σ is the perturbation growth rate; and x is alongshore position.

Imposing η_c' causes beach width to vary alongshore (Figure 7). Because cliff retreat is a function of beach width, sea-cliff retreat rates also vary alongshore. Incipient embayments, where the beach is slightly wider, are now retreating faster than incipient headlands because of the greater availability of abrasive tools. As a result, the sediment yield from cliff retreat per unit time is greater in embayments than on headlands, and the beach protrudes seaward in front of embayments. Beach position, η_b , and beach width are consequently perturbed from their steady-state position (Figure 7). Like the sea-cliff position perturbation, η_b' is imposed on the steady-state beach position:

$$\eta_b' = \varepsilon_2 e^{\sigma t} \sin kx$$

Equation 12

where ε_2 is the amplitude of the beach width perturbation with constant alongshore curvature. Because beach width is the difference between η_c and η_b , the cliff position perturbation and the responding beach position perturbation can be combined so that the beach width perturbation is

$$w' = e^{\sigma} \sin kx[\varepsilon_1 + \varepsilon_2].$$

Equation 13

Note that the beach position perturbation, η_b' , is phase shifted by π relative to η_c' (Figure 7) because the sea-cliff sediment yield is highest where the beach is widest. In those locations, η_b' bulges seaward at a distance of ε_2 . The size and curvature of ε_2 depends on a balance between the rate that diffusion is smoothing the bumps and the rate that cliff retreat is supplying sediment to them. Considering that small planform beach shapes respond to wave-induced alongshore sediment transport on time scales of days to months (Ashton and Murray, 2006b), and planform sea-cliff shapes evolve over century or millennial time scales, we can assume a quasi-steady state exists between the beach and sea-cliff perturbations, where the smoothing of ε_2 by diffusion and the rate of sediment supply to ε_2 are balanced. If the rate of sediment diffusion is high relative to the rate at which cliff retreat perturbations is supplying sediment to the seaward bulges, the quasi-steady state bulges can be maintained by diffusion at low curvatures and will therefore be small. The alongshore beach width variation, w' , and cliff perturbation growth rate, σ , will then be minimized. The opposite also applies: if diffusion is weak relative to sediment supply rate, the quasi-steady state beach bulges will be larger (with greater curvature) and will maximize w' and σ .

To derive an equation representing the quasi-steady state assumption, we consider how the cliff position and beach width perturbations affect beach position over time. By inserting the beach width perturbation, w' , equation 9 reduces to a balance between the rate of diffusion and the rate of beach sediment supply from sea-cliff retreat:

$$D_0 \frac{\partial^2 \eta_b}{\partial x^2} = (\gamma HC - 1) m w'.$$

Equation 14

Now, we can solve for the beach position perturbation amplitude, ε_2 , by substituting the beach position and width perturbations (equations 12 and 13) into equation 14, yielding

$$\varepsilon_2 = \varepsilon_1 \left[\frac{m(\gamma HC - 1)}{k^2 D_0 - m(\gamma HC - 1)} \right].$$

Equation 15

Next, using quasi-steady-state assumptions and substituting equations 11, 13, and 15 into equation 6 for $w(t) \leq w_{crit}$, we get an expression for the change in the cliff position perturbation with time,

$$\frac{d}{dt}(\varepsilon_1 e^{\sigma t} \sin kx) = m w' = m \left[e^{\sigma t} \sin kx (\varepsilon_1 + \varepsilon_2) \right].$$

Equation 16

Finally, equation 16 can be solved for the growth rate constant, σ , which is the perturbation's tendency to grow as a function of k , m , D_0 , and effective sea-cliff height (HC):

$$\sigma = m \left[\frac{k^2 D_0}{k^2 D_0 - m(\gamma HC - 1)} \right].$$

Equation 17

The magnitude of effective nondimensional cliff height, HC , is typically less than 10^2 . But what are viable values of D_0 , m , and k ? Restricting the analysis to low-angle waves, taking 23-year (1980-2002) winter averages of wave characteristics from six deep-water California wave buoys (Wingfield and Storlazzi, 2007), and using the methods in Ashton and Murray (2006b), our range of values for D_0 is 0.01 to $\sim 1 \text{ m}^2 \text{ s}^{-1}$. The variable m (equation 7) depends on the sea-cliff retreat rate, abrasion intensity (N), beach slope, and still water level relative to the shore platform. Assuming N is between 2 and 4 (Sunamura, 1976), still water level is 2-3 m (Sunamura, 1982), beach slope is $\sim 3^\circ$ (Trenhaile, 2005), and sea-cliff retreat rates are on the order of $10^{-1} \text{ m yr}^{-1}$ (Hapke et al., 2007; Moore and Griggs, 2002), values of m are on the order of 10^{-3} . Tenable values of headland spacing reach a feasible maximum of about 20 km (see below) so that k is approximately 10^{-5} . Using these physical constraints, and converting D_0 to years, typical values for D_0 are well above 10^3 , or roughly an order of magnitude larger than HC . Given that D_0 is large relative to effective sea-cliff height, equation 17 reduces approximately to one. Therefore, on most coastlines, the perturbation grows at a rate determined largely by m rather than by alongshore wavelength because gradients in alongshore sediment transport keep the beach nearly flat for all wavelengths. The rates m and σ are inversely proportional to the abrasion-maximized beach width (w_{crit}), and proportional to abrasion strength (N) and the bare-rock sea-cliff retreat rate (E_0).

In order for the unstable perturbation growth rate to vary significantly with headland spacing, the rate of beach position diffusion must be very small (i.e., $\sim 0.001 \text{ m}^2 \text{ s}^{-1}$), combined with large alongshore wavelengths and effective cliff heights (i.e., > 50).

As spacing increases for a given value of D_0 , the amplitude and curvature needed to maintain ε_2 in the quasi-steady state becomes higher (as discussed above). At alongshore scales generally greater than 10-20 km, the size and shape of ε_2 is physically unrealistic. In addition, our initial condition of a straight, featureless rocky coastline is not likely for such large wavelengths. So, we restrict our analysis to wavelengths less than about 10 km. There are wavelength ranges that maximize (i.e., $\sigma > 1$) the headland growth rate; however, they occur at large wavelengths (~ 100 km) that are beyond the limits of our assumptions.

The stability analysis indicates that there is no tenable autogenic alongshore headland spacing, given homogeneous alongshore lithology. Alongshore headland spacing is likely a product of initial conditions (Limber and Murray, 2011) and could be a result of, for example, heterogeneous alongshore lithology (Trenhaile, 2000), or regularly spaced coastal-draining rivers and streams (e.g. Perron et al., 2010). One of the defining characteristics of rocky coastlines is the coexistence of contrasting time scales, specifically those of alongshore sediment transport, or beach position diffusion, and sea-cliff retreat. Sediment transport can operate on time scales from days to months ($\sim 10^{-3}$ to 10^{-2} years), while sea-cliff retreat and rocky coastline evolution occurs over decadal to millennial time scales (10 to 10^3 years). This is apparent in our stability analyses where beach position diffusion (D_0) keeps the beach nearly flat on the long time scales relevant to rocky coastline evolution (equation 17). Using a linear stability analysis, Murray and Ashton (2003) found a growth rate for small-amplitude perturbations to sandy coastlines, σ_{sandy} :

$$\sigma_{sandy} = D_0 k^2$$

Equation 18

It is interesting to note that the perturbation growth rate for rocky coastlines (equation 17) involves the single term in equation 18; however, in a low-angle wave climate, because σ_{sandy} is so large, the rocky coastline growth rate reduces to m (equation 7) and is largely independent of σ_{sandy} (and alongshore wavenumber). The rocky coastline growth rate depends instead on the properties of the rock that composes the sea-cliff, and the presence of sediment (i.e. beach width), but not the rate that it is distributed alongshore.

Without abrasion, the positive feedbacks necessary for the formation of headlands and bays in homogeneous lithology would not exist. If we assume instead that the beach acts solely as a protective cover and an increase in beach width leads to a monotonic decrease in sea-cliff retreat rates, then headlands will be transient features because they will always retreat faster than beach-fronted cliffs. Along coastlines where abrasion is not a dominant mechanical erosion process, persistent planform coastline morphology could be the product of variations in alongshore rock strength, where headlands are more erosion-resistant relative to surrounding sea-cliffs. The role that beach and sea-cliff dynamics play in this scenario, and how they control headland amplitude, is the subject of ongoing research (Limber and Murray, *in prep*).

The stability analysis suggests characteristic time scales for headland growth. Using equation 17, the e-folding times for headland growth and decay are shown in Figure 8 as a function of N , E_0 , w_{crit} , and w_{max} . The variables w_{crit} and w_{max} are normalized by w_{max} , which we assume to be on the order of 100 m, and N and E_0 are considered

together as a single variable. Large values of NE_0 could mean either that abrasion is intense, or that E_0 is large and NE_0 ranges from a minimum of 0.01 to a maximum of 1 m yr^{-1} . Starting with the unstable fixed point (Figure 8, left panel) the characteristic time scale for headland growth is, as expected, inversely proportional to abrasion intensity, N , and proportional to w_{crit} . The time scales range from approximately one year, when NE_0 is large and w_{crit} is small, to 10^6 years when NE_0 is small and w_{crit} is large.

Figure 8: e-folding times (years) for headland growth and decay as a function of abrasion-maximized sea cliff retreat rate, w_{crit} , and w_{max} . Note that the scale on both axes is logarithmic.

2.3.2 Headland decay and the stable fixed point

As we have already discussed, negative feedbacks between sea-cliff retreat, beach width, and beach sediment losses control the stable steady-state dynamics so that perturbations to cliff position or beach width will decay to the original steady-state (Figure 6). Applying the methodology used for the unstable fixed point to the stable fixed point, perturbations to alongshore sea-cliff position will simply decay to a straight plan-view coastline configuration at a rate determined largely by the slope of equation 6 for $w > w_{crit}$ (m_1), or

$$m_1 = \frac{-NE_0 e^{-\frac{w-w_{crit}}{w_{scale}}}}{w_{scale}} = \sigma_{stable}.$$

Equation 19

Because the slope around the stable fixed point is negative, headlands decay. Alongshore sediment transport is likely to overcome any beach sediment contributions from sea-cliff retreat and maintain a flat beach so that ε_2 is negligible (as explained above) and the headland decay rate is simply m_1 . Similar to the headland growth rate, the headland decay rate, σ_{stable} , is inversely proportional to the length scale $w_{crit} - w_{max}$, and proportional to N and E_0 . However, a key difference between the stability analysis results for the stable and unstable fixed points is that, given the same cliff position perturbation, the responding beach position perturbation is phase-shifted by π (i.e. the seaward beach position bumps correspond with seaward cliff bumps; Figure 7).

For the stable fixed point, the time scale for headland decay is inversely proportional to both w_{crit} and NE_0 (Figure 8, bottom). Therefore, coastlines with high

values of E_0 and N will evolve fastest. The rate of E_0 is largely dependent on rock properties (Benumof et al., 2000), as well as wave energy (Adams et al., 2005), so coastlines that receive more wave energy and are composed of weaker rocks are more likely to evolve further over a single sea level highstand. When headlands decay, the value of w_{crit} relative to w_{max} becomes important: coastline with larger values of w_{max} will generally evolve more slowly.

2.3.3 Perturbing the beach to provoke a sea-cliff response

We can gain additional insight from the stable fixed point by perturbing beach width instead of sea-cliff position. By doing this, the negative feedbacks between beach width and sea-cliff retreat, as shown in Figure 6 (see circled number 1), are activated, and we can determine what coastline properties determine how quickly the beach width returns to the stable steady-state after a temporary perturbation. This could lead to insights about the strength of coupling between beaches and sea-cliffs and how a rocky coastline will respond to human impacts, such as fluvial sediment reductions (Willis and Griggs, 2003) or sand mining (Thornton et al., 2006).

We begin by perturbing alongshore steady-state beach width, such that the perturbation w' is

$$w' = \varepsilon_3 e^{\sigma t}$$

Equation 20

where ε_3 is the perturbation amplitude. Equation 20 lacks a spatial modulation (unlike equation 12, for example) and is instead applied uniformly alongshore so that the beach remains straight. This simplifies the analysis by preventing alongshore gradients in both

sediment transport and sea-cliff retreat, so we can neglect beach position diffusion and focus on sea-cliff and beach coupling. Substituting equation 20 into equation 8 and eliminating steady-state conditions, we find that

$$\frac{dw'}{dt} = -\sigma_1 w$$

Equation 21

and the perturbation decays at a rate σ_1 :

$$\sigma_1 = \gamma H C m_1.$$

Equation 22

This suggests that the response rate of the coastline to a stable beach width perturbation is proportional to sea-cliff height and abrasion strength, and inversely proportional to the difference between w_{max} and w_{crit} . For a landward perturbation (i.e., decrease in beach width), this is because 1) tall and/or sand-rich cliffs yield more beach sediment per unit length of landward retreat, and therefore must retreat less than small or sand-poor cliffs to respond to the perturbation, and 2) a larger difference between w_{max} and w_{crit} means that sea-cliff retreat rates will be changed less for a given beach width perturbation. The beach and sea-cliff system is therefore more tightly coupled for rocky coastlines with tall and/or sand-rich cliffs, or where abrasion is a more effective mechanical wave erosion process.

The characteristic, or e-folding, time scales for a perturbation to a flat and stable beach width are shown in Figure 9. As discussed above, the beach returns to its original state faster when tall and/or sand-rich cliffs are present. This has been observed along the coast of California, where Hapke et al. (2009) showed that beaches tended to experience

less variability along high-relief sections of coastline relative to medium- or low-relief sections, because sea-cliff failure along high-relief coasts adds more sediment to the beach. This has implications for alterations to the coastline, such as those caused by, for example, river damming (Willis and Griggs, 2003) or sand mining (Thornton et al., 2006) in that beaches on rocky coastlines with low and/or sand-poor cliffs may be more vulnerable relative to those that are backed by tall, sand-rich cliffs.

Figure 9: Results from the stable beach width uniformly alongshore. Left panel shows the e-folding times for perturbation decay as a function of effective sea cliff height (HC) and E_0 . The right panel shows changes in equilibrium beach width as a function of beach sediment loss rate. When it is backed by tall and/or sand-rich sea cliffs (i.e. when HC is large), the beach changes less for a given change in S. C.I. is the contour interval.

Along these lines, we can ask how much the stable equilibrium beach width might change, instead of returning to its original width, as a result of a permanent change in sediment supply. Using equation 8 and neglecting beach position diffusion so that we are just looking at beach and sea-cliff dynamics, the equilibrium beach width is

$$w_{eq} = (w_{crit} - w_{scale}) \ln\left(\frac{S}{\gamma HC N E_0}\right)$$

Equation 23

Any human-induced changes to beach width, such as sand mining, river damming or beach nourishment, can be represented by increases (or decreases) to the beach sediment loss term, S (Limber and Murray, 2011c). Figure 9 shows equilibrium beach width as a function of the beach sediment loss rate (S) and nondimensional sea-cliff height (HC ; contours). When the value of HC is high, beach width changes less for a given change in S because the beach and cliff system are more tightly coupled, as discussed above, so that the abundant sediment input from sea-cliffs effectively masks any alterations to beach width. Similarly, when the rate of S is high, beach width changes less for a given change in HC . However, along populated rocky coastlines that have been heavily altered by human activities, such as sea wall construction, the beach and sea-cliff system can become completely decoupled so that beaches and sea-cliffs behave independently (Hapke et al., 2009).

2.3.4 General discussion

The specific growth and decay time scales, because they are determined by m , are dependent on the choice of the functions in equation 6. For example, if we had chosen a linear function to define equation 6 for $w > w_{crit}$, the e-folding times would be larger. The shape of equation 6 (Figure 5) is an approximation and, in addition, likely varies from place to place. Changing the shape of the curve (i.e., changing w_{crit} , w_{max} , or defining the peaked curve differently) would affect the e-folding times shown here, but would not

affect the dynamics involved. Numerical modeling by Limber and Murray (2011) showed that the values of w_{crit} and w_{max} do not affect planform coastline morphology.

The dynamics and results presented here also do not depend on the specific values of w_{crit} , w_{max} , and N . The variable w_{crit} , which is the threshold where beach width changes from encouraging to discouraging sea-cliff retreat, is difficult to assign value to, as there are few observations. Through wave tank experiments, Sunamura (1982) provided a physically intuitive threshold that occurs when beach height (measured at the beach/cliff junction) reaches the still water level; below that level, sediment encourages cliff retreat, and above that level the beach begins to impede waves from reaching the sea-cliff. Dick and Zeman (1982) also used this threshold. To convert this to a beach width, one must at least estimate a beach slope and a still water depth. Accordingly, w_{crit} would tend to be larger on dissipative coastlines with low-sloping beaches, and smaller on reflective beaches. Although w_{crit} is not well constrained, several researchers have made observations on the protective capacity of the beach on rocky coastlines that are useful in defining w_{max} . Different metrics such as beach volume (Lee, 2008; Komar and Shih, 1993), beach height (Ruggiero et al., 2001; Sunamura, 1982, 1976; Sallenger et al., 2002), and beach width (Everts, 1991; Sallenger et al., 2002) have all been shown to be inversely proportional to cliff retreat rate. For example, Everts (1991) observed that a subaerial beach width of about 60 m was enough to protect sea-cliffs from wave attack, and Sallenger et al. (2002) made estimations of a protective beach width threshold to scale observed cliff retreat using an empirical wave runup model by Ruggiero et al. (2001) that includes deep-water wave height and period, as well as beach slope. The

variable N has been measured along the Yorkshire, UK coast to be between 15 and 18.5 (Robinson, 1977), and Sunamura (1982; 1976) estimated it to be between 4 and 20 through laboratory experiments and an analytical model.

The stability analyses only address infinite perturbations, but we can connect them to emergent behavior and finite rocky coastline morphology. The stability and number of fixed points, as shown in Figure 6, control coastline morphology. If $S < HCE_0$, the sea-cliff retreat rate where there is a steady-state beach is slower than the bare-rock cliff retreat rate (E_0) because the beach acts as protective cover. Bare headlands in this case are transient landscape features, and the coastline will evolve to a configuration fronted by a continuous stable beach. If $HCE_0 < S < HCNE_0$, there is an unstable beach width in addition to the stable beach width. Here, the sea-cliff retreat rate where there is a steady-state beach is greater than or equal to the headland cliff retreat rate (E_0), and therefore headlands can persist or grow in cross-shore amplitude. The coastline can then develop into a planform configuration consisting of alternating bare headlands and pocket beaches. Finally, when $S > HCNE_0$, the fixed points disappear. In this case, a steady-state beach cannot exist because sediment is being removed faster than it can be supplied, and the planform coastline configuration consists of sea-cliffs with no beach present (i.e. plunging cliffs). So, the value of S relative to E_0 , as well as HC , control coastline morphology, which was shown using a numerical model by Limber and Murray (2011). E_0 , N , w_{crit} , and w_{max} , then, control how quickly the coastline will evolve to its eventual configuration.

The coexistence of different time scales (sediment transport and sea-cliff retreat) that is evident in the stability analyses offers additional insights into finite-amplitude rocky coastline morphology. Alongshore sediment transport distributes sediment according to coastline curvature, which includes both beach and sea-cliff position. So, in low-angle wave climates, sediment is swept alongshore from headlands, which generally produce sediment at a slower rate than waves can transport it (a detachment-limited system), into embayments to form pocket beaches (Limber and Murray, 2011). But, what if the rocky coastline system is dominated instead by a higher-angle wave climate, which is not considered in the stability analyses, so that alongshore sediment transport does not smooth the sandy coastline as effectively? This could increase the beach position diffusion time scale so that it is closer to the time scale for rocky coastline evolution. Alternatively, sea-cliff retreat, or other sediment sources, could produce beach sediment faster than waves could distribute it alongshore (a transport-limited system). In either case, headlands may remain covered by a veneer of beach sediment, and seaward bumps in beach position could persist.

We can evaluate the competition between shoreline diffusivity and sediment inputs from sea-cliff retreat with the nondimensional ratio

$$\frac{-\gamma HC \frac{d\eta_c}{dt} + S}{D_0 \frac{\partial^2 \eta_b}{\partial x^2}}$$

Equation 24

which is similar to the quasi-equilibrium between beach position and sea-cliff position in equation 14. Noting also that $\delta^2 \eta_b / \delta x^2$ is in units of inverse length, where the length is an alongshore length scale (L), we have

$$P = L \left(\frac{-\gamma HC \frac{d\eta_c}{dt} + S}{D_0} \right)$$

Equation 25

where P is analogous to a landscape Peclet number (Haff, 2007). The numerator is essentially a coastal sediment budget (e.g., Best and Griggs, 1991; Patsch and Griggs, 2008) that balances sediment inputs with outputs, and the denominator governs how effectively the sediment inputs are spread out over an alongshore length scale, L , which could be the size of a littoral cell ($10^1 - 10^2$ km; Patsch and Griggs, 2008), or smaller (10^0 km). The numerator can be generalized further to include the rates of any other rocky coastline sediment inputs, such as fluvial sediment sources, balanced with sediment outputs:

$$P = L \left(\frac{S_B}{D_0} \right)$$

Equation 26

where S_B is the net rate of all littoral sediment inputs. A time-averaged D_0 can be calculated using wave buoy data and methods presented by Ashton and Murray (2006b), and the numerator can be estimated using existing sediment budgets or other field data. Small values of P mean that diffusivity is dominant or sediment inputs are small, so that planform coastline morphology will be rockier (i.e., more headlands, with pocket beaches; Limber and Murray, 2011). Larger values of P mean that diffusivity is weak or

sediment inputs are large, so that the large-scale planform morphology will be less rocky (i.e., fewer headlands, where beaches are more likely to be continuous).

2.4 Conclusions

Although simple, our analytical model of sea-cliff and beach dynamics reveals several insights into large-scale and long-term rocky coastline evolution. The abrasion-driven nonlinearity in equation 6 (Figure 5), when coupled with an equation for beach width change (equation 8), results in stable and unstable beach widths, or fixed points, that can cause headlands to grow or decay in cross-shore amplitude through the geomorphic feedbacks shown in Figure 6. Analyses of the fixed points provide the following insights:

- 1) Starting from the unstable fixed point, headlands can grow in amplitude at a rate determined by abrasion intensity, the bare-rock retreat rate of the headland, and the beach width that maximizes the sea-cliff retreat rate due to abrasion. Because there is such a large difference in the time scale between the growth rate of beach and sea-cliff perturbations, the rate that sediment is distributed alongshore by waves does not strongly affect the rate of rocky coastline evolution. So, the speed of large-scale coastline evolution depends on the physical properties of the sea-cliffs (i.e., rock type) as well as how the beach controls the frequency and intensity of wave impacts on the sea-cliff.
- 2) There is no autogenic alongshore headland spacing. Headland spacing and pocket beach length is therefore likely controlled by initial conditions and/or external forcing, such as heterogeneous alongshore lithology and/or coastal-draining rivers and streams.

- 3) Starting from the stable fixed point, headlands decay at a rate proportional to the bare-rock retreat rate and abrasion intensity, and inversely proportional to w_{max} , the beach width that prohibits waves from reaching the sea-cliff.
- 4) Starting again from the stable fixed point, but perturbing an alongshore-uniform beach instead of sea-cliff position (i.e., so that there are no headlands or embayments), the beach returns, as expected, to its initial position. But, the rate at which it returns is a function of sea-cliff height and composition. This has implications for human-induced alterations to beach width, and suggests that beaches on rocky coastlines backed by low and/or sand-poor sea-cliffs are less resilient to human impacts than those with tall and/or sand-rich cliffs, where in the latter case the beach and sea-cliff are more tightly coupled.
- 5) As the relative values of S and E_0 change, a saddle node bifurcation can occur that destroys or creates the fixed points, leading to changes in planform coastline morphology (Figure 6; discussed above). This analytical framework could provide a widely applicable explanation for different global rocky coastline morphologies, such as why some rocky coastlines are largely sediment-free, while others consist of pocket beach and headland sequences or a continuous cliff-backed beach.

Drawing on the stability analyses, we introduce here a simple testable prediction that involves measuring the relative efficacy of shoreline diffusivity and inputs to the coastal sediment budget (P , equation 25). When P is small, littoral sediment inputs are small and/or easily distributed alongshore by a diffusive wave climate; when P is big, sediment

inputs are large and/or diffusion is less effective, so that the coastline will be more likely to be covered by a veneer of sediment.

3. A new model of sea stack formation

3.1 *Introduction*

Sea stacks (Figure 10) are among the world's most recognizable landscape features. Because of their rugged beauty, sea stacks are popular tourist destinations; well-known examples include the Twelve Apostles in Victoria, Australia (Figure 10), Old Harry Rocks on the Dorset Coast, England, and Haystack Rock on the northern coast of Oregon, USA. Despite their popularity, little is known about the processes that control sea stack formation, the time scales over which they develop, or why some rocky coastlines are dotted with stacks while others are not. Generally, sea stacks form when sections of a cliffed and actively eroding rocky coastline become cut off from the mainland, forming small islands. The textbook explanation of how this happens involves pre-existing rock joints, fractures, or other structural weaknesses that are excavated by mechanical or hydraulic wave erosion, often along the flanks of a protruding rocky headland (Shepard and Kuhn, 1983; Trenhaile, 1987; Clark and Johnson, 1995; Trenhaile et al., 1998). As the weaker rock preferentially erodes, more resistant sea-cliff remnants are left behind in the surf zone, sometimes progressing morphologically from a cave to an arch, and finally to a stack after the arch collapses.

Using a series of photographs, Johnson (1925) and Shepard and Kuhn (1983) described this process happening over a 23-year period on the coast of Cape Breton Island, Canada, where sea caves were excavated laterally (i.e., in the alongshore direction) by wave erosion along both sides of the base of a headland. An arch formed when the caves coalesced in 1908, and by 1921, the arch had collapsed, leaving behind

Figure 10: Sea stack examples. The top picture shows two sea stacks backed by a beach on the coast of California, USA near Big Sur. The middle picture is of the Twelve Apostles sea stack field in Victoria, Australia (from Google Earth). Finally, the bottom picture depicts a narrowing, beach-flanked headland and proto-sea stack on the California coast.

two sea stacks. Sunamura (1992) observed a similar series of events on the coast of Japan, where a ~30m-wide mudstone headland was eroded to form a small sea stack within 6 years. Other notable areas where sea stack formation has been studied include Hopewell Rocks in the Bay of Fundy, Canada (Trenhaile et al., 1998) and the northern Pacific coast of Baja California, Mexico (Clark and Johnson, 1995). In both of these locations, sea stacks were attributed to the preferential excavation of joint sets by wave impacts.

The role of beach sediment in sea-cliff erosion has been widely discussed (e.g., Sunamura, 1976, 1982; Sallenger et al., 2002; Walkden and Hall, 2005; Lee, 2008; Limber and Murray, 2011), but its role in sea stack formation has not. Wood (1968), however, made several key observations on stack-sediment interactions on the southeast coast of England. He noted that abrasion, the mechanical scraping of the sea-cliff surface by sand grains entrained in wave swash, accelerates sea-cliff erosion rates. Abrasion has been observed along many other rocky coastlines (e.g., Sunamura, 1976, 1982; Robinson, 1977; Kamphuis, 1990; Limber and Murray, 2011) and in other settings such as river channels (Sklar and Dietrich, 2001). Accordingly, when a joint, for example, is dissected along the side of a rocky headland and a cave begins to form, it provides a small area for sediment to accumulate. As sediment accumulates, it becomes a tool to enhance lateral headland erosion, eventually leading to a sea stack through the process described by Johnson (1925).

Recently, Limber and Murray (2011) showed that abrasion-driven interactions between beaches and sea-cliffs can lead to the development of kilometer-scale planform

rocky coastline morphology, from interspersed headlands and bays to continuous cliff-backed beaches. Building on that work, we consider the role of abrasion in sea stack formation and suggest a new model that does not depend exclusively on structural or lithologic heterogeneities, but instead on the geomorphic interplay between rock erosion, sediment transport, and the regional wave climate. For the first time, sea stack formation can be assessed quantitatively, and we can address how quickly sea stacks evolve, what controls sea stack size, and finally, what rocky coastline and wave climate characteristics (e.g. sea-cliff height and composition, wave incidence) provide the most conducive environment for sea stack formation.

3.2 Numerical model

As in Limber and Murray (2011), our modeling approach is exploratory (Murray, 2007). The goal of this study is to provide general insight to the dynamics of sea stack formation, and make basic scaling arguments that can be compared to nature. Our sea stack model is a hybrid of the coastline evolution models presented by Ashton et al. (2001), Valvo et al. (2006), and Limber and Murray (2011). In the model, wave-driven sea-cliff erosion produces beach sediment that is distributed by alongshore sediment transport. The direction and magnitude of alongshore sediment transport depends on the angle of the incoming wave relative to the orientation of the coastline (Ashton and Murray, 2006a). A new wave angle and direction is chosen each model day from a probability distribution function. If the wave is approaching obliquely to the coastline from, say, the left (looking offshore) and a seaward protrusion is present, like a rocky headland, a shadow zone can exist on the headland's leeward side (right-hand side, in this

case; Figure 11). In the shadow, there is little sediment transport in the direction of the incoming wave; however sediment can enter this zone during model days when the waves approach from the opposite direction. This can allow beach sediment to accumulate through time on the sides of headlands (and less so in the middle of embayments where there is little shadowing), giving pocket beaches a curved, scalloped shape where beach width progressively narrows along headland flanks.

As sea-cliffs retreat, they produce beach sediment that controls future sea-cliff retreat rates. Depending on beach width, sediment can encourage or discourage sea-cliff erosion (and cause further changes to beach width) by acting either as an abrasive tool or a protective cover that dissipates wave energy seaward of the cliff, respectively (Sunamura, 1982; Limber and Murray, 2011; Figure 12, inset). Thus, along the flanks of headlands where the beach is curved and narrow, abrasion can drive lateral (i.e., alongshore-directed) sea-cliff erosion. It is through this process that rocky headlands can separate from the mainland and become sea stacks. Figure 11 is a plan-view schematic of the model approach, showing wave shadowing and defining directions of sea cliff retreat mentioned in the text (i.e., lateral and landward). Also shown are several length scales that are of importance: initial headland width and amplitude, ℓ_H and ℓ_A , and the cross-shore distance that the zone of lateral abrasion-driven headland erosion is spread, y_C .

Because of abrasion, sea-cliff retreat in our model is a nonlinear function of beach width:

$$\frac{d\eta_c}{dt} = \begin{cases} E_0 + mw(t), & w(t) \leq w_{crit} \\ NE_0 e^{-\frac{(w(t)-w_{crit})}{w_{scale}}}, & w(t) > w_{crit} \end{cases}$$

Equation 27

where η_c is sea-cliff position; E_0 is the bare-rock cliff retreat rate; w is beach width; w_{crit} is the beach width where abrasion maximizes the sea-cliff retreat rate; N is abrasion intensity relative to E_0 ($N > 1$); and w_{scale} is a length scale constant that depends on N as well as the beach width that provides near complete cover from wave attack so that sea-cliff retreat becomes negligible (i.e. $\sim 1\%$ of the maximum value). The variable m is the slope of the line in Figure 12, inset, between $w = 0$ and w_{crit} :

$$m = \frac{E_0(N-1)}{w_{crit}}$$

Equation 28

Figure 11: Plan-view schematic of sea stack formation on a rocky headland. ℓ_H is headland width; ℓ_A is cross-shore headland amplitude; y_c is the cross-shore length of the lateral zone of abrasion-driven sea cliff retreat; and LBR is lateral bare-rock sea cliff retreat.

Equation 27 is derived largely from the work of Sunamura (1976; 1982), Anderson (2002), and Strudley et al., (2006). Bare-rock sea-cliff retreat rate can be estimated using historical observed retreat rates (Hapke and Reid, 2007), and although few observations exist, abrasion intensity has been estimated to be between 15 and 18.5 along the Yorkshire, UK coast (Robinson, 1977), and between 4 and 20 in laboratory experiments (Sunamura, 1982; 1976). Sunamura (1982) observed that the abrasion maximized sea-cliff retreat rate occurred when beach height approximately equaled the time-averaged nearshore water level. Accordingly, the variable w_{crit} can be estimated using the mean beach slope, shoreface depth, and beach geometry (Figure 12). The beach width that provides near complete protection to sea-cliffs from wave attack can be approximated using field observations (Everts, 1991; Lee, 2008) or using wave run-up models (e.g., Sallenger et al., 2002). Everts (1991) estimated that this beach width is roughly 80 m, and we take similar values of between 75 and 100 m. It is important to note that changing the protective beach width, the value of w_{crit} , or the value of N does not change the coastline dynamics presented here, only the speed at which they occur (Limber and Murray, *in prep*).

Figure 12 is a cross-section schematic of the model. Sea-cliff retreat occurs evenly over the entire profile, from the shoreface depth to the top of the subaerial cliff. The subaerial sea-cliff profile is assumed to remain constant in the long-term, and is averaged over short-term changes like storms and the vertical distribution of wave energy across

Figure 12: Schematic of model set-up. Inset shows the nonlinear relationship between sea cliff retreat and beach width. E_0 is the bare-rock sea cliff retreat rate, N is abrasion intensity, and w_{crit} is the beach width that maximizes sea cliff retreat.

the cliff face from tides (Walkden and Hall, 2005) and cliff failure events. Similarly, the rate of sea-cliff retreat is time-averaged and implicitly includes such shorter-term changes. For simplicity, topography on top of the sea-cliff is flat, as on a marine terrace.

Numerically, the change in sandy coastline position (η_b) through time is expressed as

$$\frac{\partial \eta_b}{\partial t} = -(1 - \gamma HC) \frac{d\eta_c}{dt} + S - \left(\frac{1}{D} \frac{\partial Q_s}{\partial x} \right)$$

Equation 29

where H is sea-cliff height divided by the depth to which the beach extends; C is the proportion of sea-cliff material that is coarse enough to contribute to beach width

(Limber et al., 2008); Q_s is alongshore sediment transport (Ashton and Murray, 2006a; Komar, 1971); x is alongshore position; S a beach sediment loss rate; and η_b is beach position. The first term on the right-hand side represents sediment inputs to the coastline system as sea-cliffs retreat. The taller and more sand-rich the sea-cliffs are, the more sediment will be produced per unit length of sea-cliff retreat. There is an additional $d\eta_c/dt$ term because the beach stays pinned to the sea-cliff, so beach position must move landward as the sea-cliff retreats. The variables H and C are considered together as an effective nondimensional sea-cliff height. The second term, beach sediment losses, can be physically interpreted in many ways, such as the fining and resultant offshore transport and loss of sediment in the surf zone (Perg et al., 2003; Hicks, 2011).

The third term is the change in beach position due to gradients in alongshore sediment transport, Q_s . The term Q_s is defined by the common CERC sediment transport equation, discussed in detail elsewhere (Ashton et al., 2001; Valvo et al., 2006; Ashton and Murray, 2006a, 2006b). Essentially, the magnitude of sediment transport is a sensitive function of incoming wave angle relative to the orientation of the coastline, and tends to increase when wave height and period increase. Although a new wave angle is chosen each model day, the cumulative effects of wave field can be thought of as a time-averaged wave climate that has a characteristic diffusivity and wave asymmetry. Diffusivity is the ability of waves to smooth beach position that varies with incoming wave angle, and asymmetry is the proportion of wave influences on alongshore sediment transport coming from the right and left directions (Ashton and Murray, 2006a). When waves encounter shore-parallel depth contours at less than approximately 45 degrees,

diffusion will smooth out bumps in shoreline position. When waves approach at greater than 45 degrees, however, the shoreline becomes unstable and bumps can grow into capes or flying spits (Ashton et al., 2001). In this initial work, we consider only low-angle (< 45 degrees), diffusive wave climates, but vary the asymmetry. Wave climate asymmetry, A , equals zero or one, all waves are traveling right to left, or left to right, along the coastline. A symmetrical wave climate is $A = 0.5$, when an equal influence on alongshore transport comes from the right and the left. As the sandy shoreline changes position, beach sediment is spread evenly over the entire beach profile (Figure 12).

Equation 29 is discretized over a model domain. In each 60-m model cell, rock and sand are tracked as a proportion of cell volume. For example, a cell can be 0.5 rock, and 0.5 sand. Knowing beach and sea-cliff geometries, this can be interpreted as beach and sea-cliff position. A sea stack forms in the model when rock cells become separated from the mainland by a row of cells that are completely empty of rock. Just before a sea stack forms, however, the proto-stack can sometimes be separated from the mainland by a row of cells that are partially full of rock (this row can only be up to 2 cells long in the alongshore direction). Because of how the model works, sediment is allowed to pass through this corridor between the proto-stack and the mainland. So, the corridor can be interpreted as a sea arch, or a collapsed arch leaving behind a permeable transport pathway.

3.3 Results

A time series of frames from a representative numerical model run is shown in Figure 13, left. Values of key variables are: $E_0 = 0.2 \text{ m yr}^{-1}$; $S = 0.5 \text{ m yr}^{-1}$; $H = 3$; $C = 0.2$;

Figure 13: Time series of numerical model runs. Time series A shows sea stacks forming from initial headlands. In time series B, the coastline is too sediment-rich, and sea stacks do not form.

$N = 4$; and $A = 0.5$. The initial conditions include a sediment-free coastline, with two rocky headlands. As the model runs, sea-cliffs erode and produce beach sediment that accumulates in embayments as pocket beaches. Wave field variability pushes sediment back and forth in the embayments, and wave shadowing in the lee of headlands causes an

intermittent veneer of sediment to accumulate on headland flanks. Lateral sea-cliff erosion begins on the headland flanks (e.g., Figure 10, bottom), and because of abrasion, it outpaces the bare-rock cliff erosion occurring on the headland seaward of the beach. Eventually, after about 550 years, the zones of lateral erosion on both sides of the headland coalesce into a continuous sediment transport pathway, and a sea stack is formed.

If the beach sediment supply is too high (e.g., cliffs are tall and/or sand-rich), embayments fill up with beach sediment and the wide pocket beach acts as a protective cover to inhibit embayment sea-cliff erosion. The greater rates of retreat on bare-rock headlands cause them to disappear, and prevent sea stacks from forming. A numerical model time series illustrating this process along a sediment-rich coastline is shown in Figure 13, right. By the end of the model run, headlands are decaying without having split into sea stacks, as opposed to the model run on the left, where the sediment supply is lower and stacks have formed. A rocky coastline will produce more sediment when sea-cliffs are tall and/or sand-rich, the beach sediment loss rate (S) is low, and/or the bare-rock sea-cliff retreat rate (E_0) is high. In addition, a greater sediment supply causes the average pocket beach width to increase and shift the zone of lateral abrasion-driven cliff retreat further seaward along the headland. This increases the length scale y_C (Figure 11) and decreases plan-view sea stack size, if a stack forms.

Figure 14 illustrates the relationship between sea stack size, the length scale y_C , and beach sediment supply. Eleven model runs are shown where the effective nondimensional sea-cliff height, HC , is progressively increased. All other variables

remain constant and are the same as listed above for Figure 13. An increase in HC increases the beach sediment supply, and the size and color of the data point corresponds to the magnitude of HC . In general, as HC increases, the mean beach width increases, and

Figure 14: Numerical model results showing the relationship between effective nondimensional sea cliff height (HC), the distance the sea stack forms offshore (y_c), and sea stack area. The size and color of the data points correspond to the magnitude of HC .

the sea stack forms at a more seaward location along the headland. Accordingly, y_c also increases and the sea stack includes less of the initial headland area. Sea stack area therefore decreases as y_c and HC (i.e., sediment supply) increase. It is interesting to note that a sea stack does not form (stack area is zero) when HC equals zero and when HC is too large (in this case, $HC = 7$). A nonlinearity results: a stack cannot form without beach sediment and abrasion, and cannot form when the amount of beach sediment is too high.

The results of an additional thirty-two model runs are shown in Figure 15. In each, the initial conditions are the same as in Figure 13, with two headlands protruding from a sediment-free cliffed rocky coastline. Except for abrasion intensity (N) and wave climate

Figure 15: Numerical model results. Solid lines indicate a symmetrical wave climate; dashed lines indicate an asymmetrical wave climate. The top plot shows the time scale results for different values of abrasion intensity and headland width, and the bottom plot shows the sea stack size results for different values of abrasion intensity and plan-view headland area.

asymmetry (A), which are varied, the values for key physical parameters that are listed for Figure 13 remain the same. The top plot addresses the effects of initial headland width, abrasion intensity, and wave climate asymmetry on the time scales of sea stack formation. We can see for a wave asymmetry of 0.5 that the time scales for the 240 m headland width (circles, Figure 15) are larger than those for the 180 m headland width (squares) because there is simply a bigger alongshore length for lateral abrasion-driven cliff retreat to propagate through. Abrasion intensity (N) accelerates the rate at which lateral sea cliff retreat propagates through the headland, so as abrasion intensity increases the time scales decrease.

When the wave climate asymmetry is 0.1, meaning that 90% of the wave influence is approaching the coastline from the right and 10% from the left, the results are very different: overall, it takes longer for sea stacks to form, if they form at all. A data point located at zero on the y-axis in Figure 15 means that a stack did not form. Figure 16 is a time series of the development of a sea stack in an asymmetrical wave climate. It takes longer for a sea stack to form in an asymmetrical wave climate because the beach sediment piles up along the right side of the headland and is relatively scarce on the left (downdrift) side, causing the zones of abrasion-driven lateral sea-cliff erosion on either side of the headland to be offset (i.e., it is further seaward on the right, and further landward on the left). In a symmetrical wave climate, the zones are located at the same position on both sides of the headland, and can work in tandem to erode through the headland base. In the cases where a sea stack does not form, it is because the sediment on the updrift side overtakes the headland, and the headland rapidly decays. This happens

more often for higher values of N because 1) wave asymmetry continuously sweeps beach sediment to the left side of embayments, leaving a thin veneer of sediment on the right side of the bay (Figure 16), and 2) the thin veneer of sediment on the right-hand side combined with a large value of N causes sediment to be produced rapidly and swept to the left, until the beach grows far enough seaward to overtake the headland. For both

Figure 16: Time series of a numerical model run with a highly asymmetric wave climate ($A = 0.1$). The net sediment transport direction is to the left. In an asymmetric wave climate, the zone of lateral abrasion-driven cliff retreat is shifted seaward by length y_c , and the eventual sea stack is smaller than it would be in a symmetrical wave climate.

headland widths, the time scales for an asymmetrical wave climate are also more irregular than for a symmetrical wave climate. Collectively, these results suggest that sea stacks are more common in a nearly symmetrical wave climates than in asymmetrical climates.

Figure 15, bottom, shows the effects of abrasion intensity (N) and initial headland size on eventual sea stack area under symmetrical and asymmetrical wave climates. For the symmetrical wave climate, as abrasion intensity and initial headland size increase, sea stack size increases. Size increases with abrasion intensity because higher abrasion intensity decreases the formation time scale, and the faster the sea stack forms, the less time lateral bare-rock sea cliff retreat (see Figure 11) has to reduce sea stack size. However, when the wave climate is asymmetrical ($A = 0.1$), sea stacks are smaller and do not clearly vary with abrasion intensity or headland size. Because beach sediment accumulates on the updrift side of the headlands and the zone of lateral abrasion-driven cliff erosion is shifted seaward, the length scale y_C (Figures 11 and 16) increases, and the sea stack develops further offshore. Consequently, if the sea stack forms, it will be smaller. In addition, because the zones of lateral sea-cliff retreat are offset, the headland is more dissected by abrasion and the shape of the eventual sea stack is more uneven than for a symmetrical wave climate (e.g., Figure 16). Combined with the time scale results above, results suggest that sea stacks will be larger, less dissected, and more common in symmetrical wave climates.

3.3.1 Analytical predictions

From the numerical model results, we can make basic analytical predictions of sea stack formation time and size. Considering that lateral abrasion-driven sea-cliff retreat occurs at a maximum rate NE_0 , and assuming a constant time-averaged maximum rate of erosion on both sides of a headland (which is not always the case, see the Discussion Section below), the *minimum* time (t_0) it takes for erosion to cut through a headland of alongshore width ℓ_H (Figure 11) can be expressed as

$$t_0 \propto \frac{\ell_H}{2NE_0}.$$

Equation 30

So, equation 30 predicts that sea stacks will form faster when abrasion intensity is high, or when the alongshore headland width is small. This is consistent with the numerical model results in Figure 14, discussed above, under a symmetrical wave climate.

Knowing an approximate time scale, we can now predict plan-view sea stack size. Size is a result of the competition between *lateral* bare-rock and *lateral* abrasion-driven cliff retreat rates on headlands (Figure 11). Under a symmetrical wave climate, the faster the abrasion-driven rate is (or the higher abrasion intensity is), the faster a sea stack will form, and the less time lateral bare-rock retreat will have to decrease sea stack size. So, we can estimate alongshore sea stack length (ℓ_{Hstack}) by subtracting the amount of alongshore bare-rock headland lost during the time it takes a sea stack to form (t_0 , equation 30), from the initial headland width, ℓ_H ,

$$\ell_{Hstack} = \ell_H - 2E_0t_0 = \ell_H \left(1 - \frac{1}{N}\right).$$

Equation 31

To get sea stack area, we also need to know the sea stack's eventual cross-shore length, ℓ_{Astack} . The length scale ℓ_{Astack} can be estimated by subtracting the cross-shore length lost to landward-directed cliff erosion from the initial cross-shore headland amplitude, ℓ_A , during the time it takes the sea stack to form (t_0). An additional consideration, however, is the size and location of the zone of lateral abrasion-driven headland erosion (i.e., the cross-shore length of the beach-veneered space between the stack and the mainland, y_C). If the zone is wide, the stack will be smaller; if the beach is wide and the zone is located near the seaward end of the headland, rather than at the base, the sea stack will also be smaller. Numerical model results show that y_C is proportional to sediment supply (Figure 14) and wave asymmetry (Figures 15 and 16). The length y_C must be subtracted from cross-shore sea stack length, resulting in

$$\ell_{Astack} = \ell_A - E_0t_0 - y_C = \ell_A - \frac{1}{2N} - y_C.$$

Equation 32

Now, multiplying the alongshore stack length by cross-shore length to find rectangular plan-view sea stack area, $Size_{stack}$, we have

$$Size_{stack} \propto \frac{-\ell_H(N-1)(2N[\ell_A - y_C] + \ell_A)}{2N^2}$$

Equation 33

where y_C is proportional to sediment supply and wave asymmetry. Equation 33 is a maximum estimation of sea stack area because we use the minimum time scale for sea

stack formation (t_0). Even though we do not expect equation 33 to exactly predict sea stack area in the field (thus the proportional sign), it is consistent with numerical model results and captures the qualitative relationships between abrasion intensity, initial headland size, and resultant sea stack size: size increases with headland size (ℓ_A and ℓ_H) and abrasion intensity, and decreases when the zone of abrasion-driven cliff erosion (y_C) gets larger or is located further seaward along the headland due to higher sediment supply or an asymmetrical wave climate.

3.3.2 Initial comparisons to nature

Limber and Murray (2011) also developed a testable prediction for plan-view coastline morphology. If the beach sediment input from sea cliff retreat is too high, headlands will be transient features and the coastline will evolve to a configuration featuring a continuous cliff-backed beach. If the beach sediment input is below a certain threshold, however, headlands can persist. The lower the sediment input gets, the more common rocky headlands will be. This prediction is expressed as

$$R = \frac{S^* - \gamma HC}{S^*}$$

Equation 34

where S^* is S/E_0 and R is rockiness, or the proportion of coastline that consists of bare-rock headland (as opposed to sediment-filled embayment). When $R \leq 0$, the coastline will feature a continuous beach and no headlands. When $R > 0$, headlands, interspersed with pocket beaches, will persist. This matches our numerical model results in that sea stacks are discouraged when the sediment supply is too high (Figure 14) and suggests that sea stacks will be more common along coastlines where R is above zero.

To compare our results to real rocky coastlines, two sites were selected in California for which the variables in equation 34 are known (alongshore lengths in parentheses): Santa Cruz (32 km) and Oceanside (37 km). Santa Cruz has a relatively small littoral sediment input from sea-cliff retreat and rivers (Perg et al., 2003; Best and Griggs, 1991). On average, $R > 0$ in Santa Cruz (see Limber and Murray, 2011 for details), making it a place where sea stacks are likely to form. Oceanside, with tall sand-rich sea-cliffs (Young et al., 2010), has a larger sediment input and more continuous beaches. Here, $R < 0$ (Limber and Murray, 2011), and compared to Santa Cruz, may therefore have fewer sea stacks. Oceanside does, however, have a lower wave asymmetry, where $A = 0.7$ (National Data Buoy Center, Station 46086), compared to Santa Cruz where $A = 0.9$ (National Data Buoy Center, Station 46042).

Using a continuous series of low-altitude oblique aerial photography made available by the California Coastal Records Project (online at www.californiacoastline.org), we counted sea stacks along each coastline. For these purposes, a sea stack is defined as a small, isolated nearshore island that is raised above the ocean surface and is not regularly submerged by waves or tides. We also counted “sea stumps”, which are small rocky islands at or near sea level that are regularly submerged. Sea stumps may be sea stack remnants (Shepard and Kuhn, 1983), or pieces of dissected shore platform. Because it is difficult to distinguish between them, only true sea stacks are included in the tallies presented here.

We counted twenty-eight sea stacks along the Santa Cruz coastline (not including 65 sea stumps and 4 arches), making the average alongshore stack density 0.88 stacks

km⁻¹. In Oceanside, we found no sea stacks, but 9 sea stumps were observed. Even though the wave climate at Santa Cruz is highly asymmetrical, the sediment input is apparently small enough to allow rocky headlands to be exposed and sea stacks to form. Oceanside, however, is too sediment-rich, and the continuous beach acts as a protective cover to discourage sea stack formation.

At the southern end of the Oceanside study area is the La Jolla submarine canyon. The canyon extends into the nearshore and intercepts sediment transport (Covault et al., 2007), marking the end of the Oceanside littoral cell and reducing the net southward sediment flux. Immediately south of the canyon, the rocky La Jolla peninsula juts seaward. The beaches along the peninsula are fewer and less continuous than those to the north of the canyon, suggesting a lower sediment input, and the coastline morphology is much rockier. Interestingly, the sea stack density along this 7 km stretch of coastline is 0.71 stacks km⁻¹.

Although sea stack density is inversely proportional to sediment supply in these initial comparisons, there are other physical factors at play here (see Discussion below). It is difficult to tease out the separate effects of wave climate, sediment supply, and other geological factors such as rock joint density on sea stack formation. In order to better understand their relationship and relevance to model results, a more robust survey of how sea stack density and size relate to rocky coastline morphology and wave climate is required.

3.4 Discussion

Our model is consistent with the traditional explanation of sea stack formation. Pre-existing joints, fractures, or faults can facilitate the process by acting as a zone of sea stack initiation (Wood, 1968), or by effectively increasing N or E_0 through local weakening of the headland cliffs. As a result, the presence of structural weaknesses may increase sea stack incidence, and obscure or augment the effects of wave asymmetry or sediment supply as discussed for the Oceanside, Santa Cruz, and La Jolla study sites. However, we argue that structural weaknesses are not essential for sea stack formation. Stacks can form from lateral abrasion-driven sea-cliff retreat resulting simply from the presence of a beach along the headland flanks. Shepard and Kuhn (1983) observed that many stacks may form from the erosion of a headland on both sides, as observed by Johnson (1925), and we provide the mechanism (abrasion) for that observation. Along coastlines where abrasion is not a significant factor and hydraulic impacts or subaerial weathering, for example, are more dominant, then other mechanisms will cause sea stacks to form. In the numerical model, when abrasion is switched off and the sea-cliff retreat rate monotonically decreases with increasing beach width (the beach acts only as a protective cover), headlands decay and sea stacks do not form. Other mechanisms of stack formation include rock solution in limestone sea-cliffs, or surge channels cut by the convergence of wave energy on one side of a headland (Shepard and Kuhn, 1983). In addition, some rock types, because of bedding patterns or composition, may be more conducive to sea stack formation than others (e.g., soft rock).

For simplicity and to provide clarity to these initial insights, alongshore lithology is homogeneous in the numerical model and our analyses. We could easily make the headland lithology more resistant than the surrounding lithology, and sea stacks will still form, albeit more slowly. But, the dynamics and mechanism of sea stack formation would remain the same. Recently, Limber and Murray (2011) showed how headlands could persist in homogeneous cliff lithology from abrasion-driven sediment dynamics. This study differs from Limber and Murray (2011) in that wave field variability is considered; in the prior study, waves were assumed to approach the coastline directly, and there was no wave shadowing on the leeward side of headlands. The beach accordingly remained nearly straight (instead of scalloped, Figure 11), and lateral headland retreat was negligible. In the numerical model presented here, if lateral cliff retreat is negligible, headlands persist in homogeneous lithology and sea stacks do not form.

Although our analytical scaling arguments for sea stack formation time and plan-view size in equations 30 and 33 generally reflect the numerical model results, they could be adjusted in many ways. First, as mentioned above, the time scale (equation 30) is defined using a constant maximum sea-cliff retreat rate. This would require the beach on the headland flanks to be of width w_{crit} constantly for an extended period of time. From the numerical model results, this is clearly not the case. Beach sediment is pushed back and forth in embayments as the wave approach angle and direction change daily, causing local beach width to vary. On average, the lateral abrasion-driven sea-cliff rate is therefore less than NE_θ , and the time scale for stack formation is longer. In addition, when abrasion carves a notch along the headland flanks prior to stack formation (see

Figure 13, left middle panel), sediment can get “stuck” in the indentation if it becomes a wave shadow zone. If enough sediment accumulates, the lateral sea-cliff retreat rate can approach zero until either the wave approach direction changes or the notch reduces in alongshore depth (i.e., lateral bare-rock sea-cliff retreat outpaces lateral abrasion-driven sea-cliff retreat, Figure 11) so that the trapped sediment can be transported away. This also causes the sea stack formation time scale to be longer than is estimated by equation 30.

The scaling of sea stack plan-view area, (equation 33) as mentioned above, is a maximum estimate because the minimum time scale is used to derive it. Also, we assumed that the stack is rectangular when in reality it may be more circular or oval-shaped. The sea stacks generated in the numerical model are accordingly smaller than those predicted by our sea stack size scaling argument. Also, in the model, waves can push sediment along the entire length of the headland, so that a thin veneer of sediment sometimes covers parts of the headland that are assumed to be bare (and retreat at the bare-rock rate) in the derivation of equation 33. Retreat rates on these parts of the headland are hastened by abrasion, making the eventual sea stack smaller. We could straightforwardly add constants so that the analytical scaling arguments agree better with the model results. But, they match qualitatively well without them, and our intent is to make basic scaling arguments that can be easily interpreted. Therefore constants are not added to prevent our arguments from becoming overly complicated.

Sea stack formation is a process by which a rocky coastline can straighten and maintain more continuous, efficient sediment transport pathways. As pathways are

established, sea stacks are left stranded in the nearshore. It is conceptually similar to a meandering river, where river bends are cut off and a straighter channel is established, leaving an oxbow lake behind. Sea stacks, though, can still exert control over the coastline after they form. By shadowing the coastline behind them from approaching waves, they can create a local convergence in alongshore sediment transport. As a result, tombolos may develop and reconnect stacks to the mainland, and in the process protect the sea-cliffs behind them from wave impacts. Eventually, the sea stack will erode and the wave shadow zone will disappear, causing coastline behavior to change again. Future work will involve modeling the evolution of sea stacks and their effect on large-scale rocky coastline evolution as stacks interact with alongshore sediment transport and with each other.

3.5 Conclusions

We present the first numerical model and detailed quantitative assessment of sea stack formation. Numerical model results suggest that sea stacks form faster and are larger in plan-view area when abrasion intensity and initial headland size are large, and vice versa. Sea stack formation is discouraged when the sediment input from sea-cliff retreat is too high (i.e., sea-cliffs are tall and/or sand-rich), and when the wave climate is highly asymmetrical. Based on these observations, we present basic analytical predictions of sea stack formation time and plan-view size. Overall, we suggest that sea stacks are most common and largest in plan-view area on rocky coastlines where 1) the wave climate is symmetrical, 2) beach sediment input is below a threshold that is a function of sea-cliff height, composition, and retreat rate, as well as any sediment losses from the

nearshore system (equation 34), 3) abrasion intensity is high, and 4) initial plan-view headland area is large. Initial comparisons to the Santa Cruz, Oceanside, and La Jolla study sites suggest that sea stack density may be inversely proportional to coastline sediment supply. However, additional work is needed to understand the many geological and oceanographic effects that drive sea stack formation.

4. Controls on the equilibrium cross-shore amplitude of lithologically diverse rocky coastlines

4.1 Introduction

4.1.1 Overview

The diverse plan-view shapes of rocky coastlines have long been a topic of scientific and general interest. A quick look at Google Earth will show large-scale (roughly defined here as greater than or equal to one kilometer) rocky coastline shapes ranging from crenulated to straight, and rocky headlands interspersed with smooth sediment-filled embayments. One will also readily notice that the cross-shore relief, or amplitude, of rocky coastlines varies from nearly zero to hundreds of meters (Figure 18). A dynamic balance between the differing rock strength of headlands and bays has been offered to explain the persistence of coastline amplitude, assuming that headlands are composed of relatively more resistant rock (Trenhaile, 1987). Wave energy focuses on headlands (and disperses in bays) as the coastline amplitude increases, equalizing the erosion rates of the softer and harder rocks and preserving the plan-view coastline shape through time (Muir-Wood, 1971; Trenhaile, 1987; Trenhaile, 2002). However, no process-based quantitative model has been proposed and it remains unclear (1) how, and if, lithologically diverse coastlines reach equilibrium cross-shore amplitude, (2) what physical characteristics scale that amplitude, as well as headland shape, and (3) over what time scales the coastline morphology adjusts.

In addition to lithologic variations, the production and distribution of beach sediment is another key factor in the evolution of headlands and bays. Beaches can

Figure 17: Examples of headlands and embayments on rocky coastlines. The top panel is a series of needle-shaped headlands from northern coast of Wales, UK. The bottom panel shows a series of headlands and bays, including Montara Beach, California, USA, which is sandwiched between two erosion resistant headlands. Montara Beach will be discussed further in the text.

decrease or increase the rate of sea-cliff erosion by acting either as a protective cover or as an abrasive tool, respectively (e.g., Sallenger et al., 2002; Robinson, 1977; Sunamura, 1976, 1982; Lee, 2008; Kamphuis, 1990; Ruggiero et al., 2001; Limber and Murray, 2011). Sea-cliff height, composition, and retreat rate, as well as any rivers that drain to the coastline, control the production of beach sediment, while the wave climate (i.e. wave height, angle, etc.) and shape of the coastline control the alongshore distribution of sediment (Ashton and Murray, 2006b). Focusing on abrasion, recent research by Limber and Murray (2011) suggests that the feedback process, where beach sediment is produced

by sea-cliff erosion, distributed by alongshore sediment transport, and controls future sea-cliff erosion rates, can create persistent headlands and bays even in the absence of a geologic heterogeneity and wave refraction. However, on coastlines where abrasion is not a significant factor, beach width will act solely as a protective cover and sea-cliff retreat will decay monotonically as beach width increases. In this case, headlands will not persist in homogeneous lithology, and alongshore lithologic variations are key in supporting plan-view coastline morphology.

Accordingly, we present a simple model where internal sediment dynamics coupled with alongshore variations in rock strength and wave climate can control the cross-shore amplitude and aspect ratio of headlands and bays on lithologically diverse rocky coastlines. As sea-cliffs erode and retreat, beach sediment is produced. Beach sediment then controls the frequency and efficacy of wave attack on the sea-cliff, and as beach width grows, it acts as a protective cover and slows sea-cliff retreat rates. If we assume that alongshore sediment transport sweeps beach sediment into embayments to form pocket beaches (May and Tanner, 1973; Komar, 1985; Carter et al., 1990; Valvo et al., 2006) and erosion-resistant headlands remain sediment-free, then pocket beach width can grow and reduce sea-cliff retreat rates in bays that consist of weaker and faster-eroding lithology. As beach width grows and armors the sea-cliff, retreat rates in bays eventually become equal to headland retreat rates (similar to the suggestion that wave refraction can equalize headland and bay retreat rates) and the coastline can reach equilibrium amplitude. But, what scales the equilibrium amplitude?

If rocky coastline amplitude changes through time, so does the total alongshore length of the coastline (Figure 19). Then, if the coastline is cliff-backed, the amount of erodible cliff face will change, too. So, larger amplitude headlands will produce more beach sediment than smaller amplitude headlands, and will likewise shed more sediment to surrounding bays. If amplitude is too large, for example, sediment production from cliff retreat will be high, and pocket beaches will grow wide enough to stop cliff retreat in bays. Because headlands are now retreating faster than the sediment-filled bays, amplitude will decrease. The opposite happens when coastline amplitude is too small (Figure 19): sediment production is too low to sufficiently slow cliff retreat in the softer-rock bays, and amplitude increases. Amplitude growth spurs sediment production, which becomes high enough to increase pocket beach width and equalize sea-cliff retreat between the headlands and bays. Essentially, without relying wave-energy redistribution from refraction, headlands can control their own size (and shape) through internal feedbacks between coastline length, amplitude, and sediment production/distribution.

Using this framework, we can make basic arguments on how different length scales (sea-cliff height, headland spacing, and headland width), physical properties (rock composition and retreat rate), and wave climates scale the cross-shore amplitude of lithologically diverse rocky coastlines. Focusing on kilometer-scale headland and bay systems backed by sea-cliffs, we will present exploratory, process-based analytical and numerical modeling to address how and how quickly a lithologically diverse rocky coastline can reach equilibrium, and how beach and sea-cliff dynamics can produce different headland shapes.

Figure 18: Schematic of dynamics and feedbacks in the analytical coastline model. Alongshore sediment transport distributes beach sediment into bays, and coastline sediment production changes with amplitude (and coastline length). When the retreat rate of the bay equals the retreat rate of the headland, the coastline is in equilibrium.

4.1.2 Background

Early observers of rocky coastlines suggested that exposed headlands, if rock strength was constant alongshore, would be continually reduced in cross-shore amplitude by wave impacts until the coastline reached a straight, featureless equilibrium configuration (Gilbert, 1908; Johnson, 1919). It was also recognized early on that alongshore variations in rock strength could support more persistent coastline shapes (Gulliver, 1899; Johnson, 1919; Trenhaile, 1987), and this view is still predominant (Trenhaile, 1987; Benumof et al., 2000; Trenhaile, 2002; Dickson et al., 2004). Most

recently, though, Limber and Murray (2011) showed the range of large-scale coastline morphologies, from smooth and sandy to crenulated and rocky, that could result from simple nonlinear beach and sea-cliff dynamics. In their model, due to internal sediment dynamics, headlands and bays could persist without variations in rock type.

There has been a relatively small body of work addressing large-scale and long-term planform rocky coastline evolution. In Nova Scotia, Canada, Carter et al. (1990) observed the coexistence of narrow “needle”-shaped headlands (i.e., high aspect ratio) and “stubby” headlands with wider bases (low aspect ratio), and found that the stubby types tended to be located in exposed higher-energy wave zones, while needle types were found in more sheltered areas. In addition, along with May and Tanner (1973) and Komar (1985), they noted the importance of headlands as an alongshore sediment transport boundary. Beach sediment is distributed away from the headland to adjacent embayments, and headland spacing thus can affect coastline evolution. Similarly, Trenhaile (2002) related bay amplitude to headland spacing, as did Short (1999) and Bowman et al. (2009). The idea is that headland spacing can affect the wave energy distribution across the embayment so that more energy reaches the bay when headlands are spaced further apart. Bowman et al. (2009) also found a scaling relationship between coastline amplitude and beach width along the Catalan coast, Spain, where the widest pocket beaches were found in small amplitude embayments.

The equilibrium shape of log-spiral or headland-bay beaches on kilometer scales has received considerable attention (e.g., Sylvester, 1960; Yasso, 1965; Littlewood et al., 2007; Weesakul et al., 2010; Jackson and Cooper, 2010; Hsu et al., 2010). These studies

are mostly concerned with the curved, crenulate shape of the sandy bay that results from wave refraction and/or diffraction around a stationary headland rather than the how the headland amplitude evolves. The equilibrium amplitude of these coasts has been addressed (Hsu et al., 1989), but it, and the log-spiral shape, is found empirically (Hsu et al., 2010) rather than through a dynamical process-based approach as is presented here. They also do not include the geomorphic feedbacks between beaches and sea-cliffs, or between coastline length and sediment production.

Only a few authors have modeled the plan-view evolution of rocky coastlines. Prior models of rocky coastline evolution have focused on the cross-shore development of features such as shore platforms (Trenhaile, 2000, 2005), sea-cliffs (Sunamura, 1982), marine terraces (Anderson et al., 1999; Trenhaile, 2002), and erosional continental shelves (Sunamura, 1978; Trenhaile, 2001). Adams (2004) built a numerical model where shoaling waves drive sea-cliffs of different rock types landward to form an embayment. Valvo et al. (2006) modeled the evolution of sandy coasts underlain by a geological framework, where beach sediment was derived from shoreface weathering. If shoreface lithology varied alongshore, the coastline developed equilibrium undulations similar to headlands and bays. The highest amplitude undulations were found where the soft lithology was eroding much faster than the hard lithology and/or the soft lithology yielded little coarse beach sediment.

Sea-cliff erosion and retreat are complex and rely on many intertwined terrestrial and marine processes (Emery and Kuhn, 1982). On high-energy, wave-dominated coastlines, wave attack is primarily responsible for long-term cliff retreat (Trenhaile,

1987; Sunamura, 1992). But, beaches can mitigate the frequency that cliffs are subjected to wave impacts, depending on beach width and height (Sumamura, 1982; Komar and Shih, 1993; Lee, 2008). Ruggiero et al. (2001) and Sallenger et al. (2002), for example, showed that sea-cliff erosion in Oregon and California was more intense when beaches were narrow, allowing waves to regularly impact the cliff. Subaerial processes, such as groundwater percolation and surface runoff also affect sea-cliff stability (Emery and Kuhn, 1982; Hampton, 2002), along with human impacts (Runyan and Griggs, 2003) and bioerosion (Naylor et al., 2012).

4.2 Analytical and numerical modeling background

4.2.1 Analytical model set-up

As in Limber and Murray (2011), our modeling approach is purposefully simple to maximize the clarity of insights into large-scale dynamics (Murray, 2007). Our goal is to make basic scaling arguments based on rocky coastlines processes and characteristics that can be compared to nature to assess their generality. The initial analytical model set-up assumes a low-angle wave climate (Ashton and Murray, 2006a), so that headlands are divergent sediment transport boundaries (May and Tanner, 1973; Carter et al., 1990) and all sediment produced along their perimeter is swept into bays to form pocket beaches. The entire coastline is cliff-backed, and produces beach sediment as the sea-cliffs retreat at a rate that is a function of cliff height, composition, and retreat rate. Sea-cliff retreat is driven by wave impacts (hydraulic and mechanical), and we do not consider subaerial or other rock weathering mechanisms. We assume constant cliff profiles, beach geometries, and long-term sea-cliff retreat rates that are averaged over seasonal changes, storms, and

tides. In addition, beach sediment cannot be transported out of bays once it is there, which is not always the case in nature as sediment can be shared between bays during storm events (Storlazzi and Field, 2000). (This assumption can be relaxed in the numerical modeling discussed below.) Sea level rise is not included in this initial work, although it could be added (e.g., Ashton et al., 2011; Wolinsky and Murray, 2008). Sea-cliff lithology variations are assumed to extend landward to infinity (or to the bottom edge of the numerical model domain).

Dimensional and nondimensional model variables are listed in Table 1. H and B subscripts refer to headlands and bays, respectively, and landward rates of change are negative. In the model (Figure 20), the change in coastline amplitude through time is simply the alongshore difference between the headland and embayment cliff retreat rates, so that when headlands are retreating faster than bays, amplitude decreases and vice versa (as in Figure 19):

$$\frac{dA}{dt} = \frac{\partial}{\partial x} \left(\frac{d\eta_c}{dt} \right) = E_{0H} - \frac{d\eta_{cB}}{dt}$$

Equation 35

where A is coastline amplitude; t is time; x is in the alongshore direction; η_c is sea-cliff position; η_{cB} is embayment sea-cliff position; and E_{0H} is the bare-rock sea-cliff retreat rate of the headlands. Amplitude is measured relative to beach width, so that when amplitude equals zero the cross-shore headland and beach position are equal. The first and second terms on the right-hand side of equation 35 represents headlands and embayments respectively. Because cliff-derived sediment is transported away from the headlands and collects in neighboring embayments, the headland retreat rate is the bare-

Table 1: List of variable definitions

Variable	Description	Units
A	Cross-shore coastline amplitude	m
ℓ_H	Alongshore headland basal width	m
ℓ_B	Alongshore embayment width	m
w	Beach width	m
w_{max}	Beach width where cliff retreat rate equals zero	m
$E_{0(H,B)}$	Bare-rock headland (H) or bay (B) retreat rate	m yr ⁻¹
η_{cB}	Embayment sea cliff position	m
S	Beach sediment loss rate	m yr ⁻¹
t_0	Characteristic time scale (e-folding time)	yr
w_{scale}	Length scale used in equation (36) to relate embayment sea cliff retreat to beach width	m
$H_{H,B}$	Bay or headland sea cliff height divided by shoreface depth	
$C_{H,B}$	Proportion of coarse sediment in bay or headland sea cliffs	
γ	Beach geometry constant	
ψ	Wave shadowing parameter	
A^*	Headland aspect ratio, A/ℓ_H	
S^*	Nondimensional sediment loss rate, S/E_{0H}	
w^*	Beach width normalized by w_{max}	
ℓ^*	Ratio of bay and headland widths, ℓ_B/ℓ_H

Figure 19: Schematic of the analytical model set-up, showing length scales of interest and variable definitions.

rock rate and the bay retreat rate is a function of beach width. As discussed above, the beach acts as a protective cover and reduces wave impacts at the cliff toe. Accordingly embayment sea-cliff retreat is inversely proportional to beach width (Sallenger et al., 2002; Valvo et al., 2006; Lee, 2008):

$$\frac{d\eta_{cB}}{dt} = E_{0B} e^{-\frac{w(t)}{w_{scale}}}$$

Equation 36

where E_{0B} is the bare-rock embayment retreat rate; $w(t)$ is beach width as a function of time; and w_{scale} is a characteristic length scale determined by E_{0B} and the beach width that causes the sea-cliff retreat rate to decrease to near zero, w_{max} . The variable w_{max} can be estimated using field measurements (e.g., Everts, 1991) or wave run-up models (e.g., Sallenger et al., 2002), and its chosen value does not affect qualitative rocky coastline dynamics but rather how quickly they occur (Limber and Murray, *in prep*).

Beach width change is simply a balance of sediment gains from sea-cliff retreat, and any sediment losses (Limber and Murray, 2011):

$$\frac{dw}{dt} = S - \left(\gamma H C_B \frac{d\eta_{cB}}{dt} + \frac{\gamma H C_H E_{0H} (2A + \ell_H)}{\ell_B} \right)$$

Equation 37

where S is a beach sediment loss rate; ℓ_B is the alongshore embayment length; γ is a beach geometry constant that converts the volume of material eroded from the sea-cliff into a beach width; H is sea-cliff height divided by the depth of the beach; C reflects sea-cliff composition and is the proportion of cliff-derived sediment that is coarse enough to contribute to beach width (Limber et al., 2008); and ℓ_H is the alongshore length of the

headland measured at its base (Figure 20). The variable S can be interpreted as an offshore loss when sand grains are abraded in the surf zone (Perg et al., 2003; Hicks, 2011; Limber and Murray, 2011). The second term on the right-hand side of equation 37 represents sediment production from sea-cliff retreat as determined by the height and composition of the cliffs. H and C can be considered together as an effective nondimensional cliff height. For example, a small value of HC does not necessarily mean that the cliff is small; it may also mean that the cliff supplies little sediment to the beach, or the effective shoreface is relatively deep. The sediment yield from each cliff type is multiplied by its respective alongshore length, then summed. The cumulative sediment production for the entire domain is then distributed across the embayment length, implicitly simulating alongshore sediment transport from the headlands to the bays (Limber and Murray, 2011). So, for a given input of beach sediment, beach width changes more for smaller embayment lengths than for larger ones because sediment is not as spread out.

4.2.2 Dynamics and equilibrium

Equations 35-37 are coupled as a dynamical system. If a beach width exists that can drive sea-cliff retreat in the embayments at the same rate as on headlands, then the system will reach a stable equilibrium and coastline amplitude can persist through time (Figure 21). The equilibrium is stable because of the negative feedbacks between coastline amplitude (or length) and sediment production, as depicted in Figure 19. For example, if beach width is increased from the equilibrium, embayment cliff retreat will

slow, and amplitude will decrease. Sea-cliff sediment production will then decrease as well, and beach width will return to its original width.

Figure 20: Sea cliff retreat decreases monotonically with increasing beach width. When the embayment retreat rate (a function of beach width) equals the headland bare-rock retreat rate, the coastline amplitude is in a stable equilibrium. The variable w' is a change in the equilibrium beach width. When beach width decreases, 1) embayment retreat gets faster, 2) the coastline amplitude increases and produces more beach sediment, and 3) beach width increases and decreases embayment retreat back to equilibrium. When beach width increases, 1) embayment retreat slows, 2) amplitude decreases and produces less sediment, and 3) beach width decreases and increases embayment retreat back to equilibrium.

Tall, sand-rich, and/or quickly retreating sea-cliffs, small embayment lengths, or low sediment loss rates can overwhelm the feedbacks between amplitude and sediment production and cause headlands to decrease to zero amplitude. In this case, the large equilibrium beach width drives embayment retreat at a slower rate than headlands. An amplitude decrease to zero is not enough to slow sediment production and decrease beach width, so headlands disappear and plan-view coastline relief is transient. Thus there is a geomorphic window of amplitude persistence determined by the coastline's physical properties (sea-cliff height, composition, embayment length, etc.) that can explain why

some lithologically variable rocky coasts exhibit frequent headland and embayment sequences, while others evolve to a relatively flat and smooth configuration.

4.2.3 Numerical model

We adapt a coastline evolution model originally developed by Ashton et al. (2001), Ashton and Murray (2006a), and Valvo et al. (2006) for use on cliffed, rocky coastlines. The change in sandy coastline position (η_b) through time is expressed as

$$\frac{\partial \eta_b}{\partial t} = -(1 - \gamma HC) \frac{d\eta_c}{dt} + S - \left(\frac{1}{D} \frac{\partial Q_s}{\partial x} \right)$$

Equation 38

where Q_s is alongshore sediment transport (Ashton and Murray, 2006a); and x is alongshore position. The first term on the right-hand side is sediment inputs to the coastline as sea-cliffs retreat. There is an additional $d\eta_c/dt$ term because the beach stays pinned to the sea-cliff and retreats landward. The second term, beach sediment losses, is the same as in equation 37. A key difference between the numerical and analytical model is the dependence of sea-cliff retreat on daily incoming wave angle. The wave energy flux reaching the coastline is maximized when waves approach a model cell orthogonally, and decreases as wave angle relative to shoreline orientation increases:

$$\frac{d\eta_{c(B.H)}}{dt} = \cos(\phi - \theta) E_{0B} e^{-\frac{w(t)}{w_{scale}}}$$

Equation 39

where ϕ is the angle of the incoming deep-water wave, and θ is the orientation of a particular model cell. When a model cell containing sea-cliff is in a wave shadow zone, cliff retreat is zero during that time period.

The term Q_s is defined by a deep-water formulation of the common CERC sediment transport equation that is discussed in detail elsewhere (Ashton et al., 2001; Valvo et al., 2006; Ashton and Murray, 2006a, 2006b). The direction and magnitude of Q_s is a sensitive function of incoming wave angle relative to the orientation of the coastline, and sediment transport occurs faster when wave height and period increase. Here, we consider low-angle (< 45 degree incidence where the waves first encounter shore-parallel depth contours), diffusive wave climates, and slightly asymmetric wave climates. Asymmetry, A , is the proportion of wave influences on alongshore transport from the left (looking offshore). A symmetrical wave climate is $A = 0.5$, when net sediment transport is driven by waves coming from both directions. Equation 38 is discretized over a grid of 100 m cells, and as the sandy shoreline changes position, beach sediment is spread evenly over the entire depth of the beach deposit (Figure 22), which is assumed to have shore parallel contours. The rock behind the beach retreats along with the sea-cliff and is of the same rock type.

Figure 21: Cross-section of the numerical model set-up, showing sea cliff retreat and shoreface geometry.

4.3 Modeling results

4.3.1 Coastline amplitude and headland shape

To simplify the analysis and reduce the number of variables, we introduce several nondimensional terms (Table 1). Coastline amplitude, A , is divided by headland width, ℓ_H . The resulting variable, A^* , is headland aspect ratio, which can be used to address headland shape as well as headland and coastline amplitude, with the assumption that headland width remains constant as the coastline retreats landward. Similarly, beach width, w , is normalized by w_{max} to form the variable w^* , and ℓ_B and ℓ_H are combined so that $\ell_B/\ell_H = \ell^*$. We will begin with the analytical modeling results exploring general dynamics, and following that we will see how each variable affects coastline amplitude and headland aspect ratio.

Figure 23 contains three example phase space diagrams showing the co-evolution of A^* and w^* . Variable values for the top panel (A) are $HC_B = 2$, $HC_H = 4$, $E\theta_B = 0.2 \text{ m yr}^{-1}$, $E\theta_H = 0.05 \text{ m yr}^{-1}$, $S = 0.3 \text{ m yr}^{-1}$, $w_{max} = 100 \text{ m}$, and $\ell^* = 2$. Panels B and C are variations on Panel A, showing how steady-state headland aspect ratio changes by altering the values of some of the variables listed above. In Panel A, equilibrium A^* is about 0.5, meaning that headland width is about twice as large as headland amplitude. We can pick two trajectories and follow them to equilibrium to illustrate rocky coastline system dynamics (shadowed, light-colored paths and associated numbers, Panel A). Figure 7 shows the evolution in time of the first trajectory after solving the dynamical system numerically. Starting in the upper left corner of the phase space, where aspect ratio is about 0.8 and beach width is zero, pocket beach width increases rapidly over

Figure 22: Phase planes of analytical model runs. Each panel is discussed in the text.

~750 years as sea-cliffs retreat and produce beach sediment (1). As the trajectory moves to the right, amplitude and aspect ratio increase slightly as well because the more resistant headlands are retreating more slowly than the bays, and the beach is not yet wide enough to slow bay retreat and equalize their rates. By ~300 years, pocket beach width becomes large enough to decrease the bay retreat rate below that of the headland, and the A^* trajectory begins moving downward (2). Beach width, however, has grown larger than

its steady-state size because amplitude (and coastline length) is too large, and as a result headlands are shedding too much sediment to the pocket beaches. Now, coastline length becomes smaller, producing less sediment, and pocket beach width decreases through time in sync with the headland aspect ratio decrease (3). Finally, by ~20,000 years, aspect ratio decreases enough that the pocket beach reaches a width that drives embayment retreat at nearly the same rate as headland retreat, and the coastline amplitude is in equilibrium. (This process is shown schematically in Figure 2.)

Figure 23: Headland aspect ratio and nondimensional beach width as a function of time. This corresponds to the top-most numbered trajectory in Figure 22, Panel A.

Now, we will start from the opposite side of the phase plane, in the bottom right quadrant. Here, beach width is 100 m and aspect ratio is about 0.25. First, the trajectory moves to the left, and beach width decreases, because the small coastline length is not producing enough sediment to support the wide beach and outpace sediment losses (1). Next, the trajectory turns upwards, and aspect ratio begins to increase as the pocket beach narrows enough to allow the embayment to retreat faster than the headlands (2). Aspect ratio continues to increase, as do cliff sediment production and beach width (3), until the

beach reaches an equilibrium width that drives embayment and headland retreat at equal rates.

Panel B in Figure 6 is the same as Panel A, except that E_{0H} is reduced to 0.02 m yr^{-1} . The coastline has evolved to quite a different outcome, with steady-state aspect ratio around 2.8 and beach width wider than in Panel A. This aspect ratio corresponds to a “needle-like” headland (Carter et al., 1990) rather than the low aspect ratio “stubby” type in Panel A. In order to equalize the greater difference between bay and headland retreat rates, the pocket beach must evolve to a greater width. So, headland size increases until it can produce enough beach sediment to sufficiently widen the beach. In Panel C, we find a steady-state aspect ratio of zero. Here, $HC_B = 10$, and all other variable values are the same as in Panel A. Sea-cliff retreat is producing too much sediment (i.e., cliffs are tall and/or sand-rich) and overwhelms the negative feedbacks between coastline amplitude and sediment production so that changes to coastline amplitude do not significantly affect sediment production. Headlands are therefore transient features.

In Figure 6, steady-state headland aspect ratio is inversely proportional to the headland retreat rate, beach sediment production capacity (i.e., sea-cliff height and proportion of coarse beach sediment), and incoming wave angle. To see exactly how these variables relate to each other, equations 35 and 37 are set equal to zero and solved for aspect ratio. The result is our main scaling argument:

$$A^* \propto \frac{\ell^*}{2\gamma HC_H} (S^* - \gamma HC_B) - \frac{1}{2}$$

Equation 40

where S^* is a nondimensional beach sediment loss rate (Table 1) defined as S/E_{0H} . A proportional sign is used rather than an equal sign to acknowledge the model's simplicity, and that we would not expect real-world coastline aspect ratio or amplitude to correspond exactly to equation 40. But, we can argue, based on equation 40, that headland amplitude and aspect ratio are proportional to the spacing between headlands (ℓ_B) and the beach sediment loss rate (S). It is inversely proportional to headland width (ℓ_H), the height of and proportion of beach sediment in the sea-cliffs (HC_H and HC_B), and the headland retreat rate (E_{0H}). When A^* is greater than zero, headlands can be persistent features of the coastal landscape; but when it is less than or equal zero, headlands are transient features.

To better understand equilibrium headland size and shape described by equation 40, the variables (except for wave angle) can be divided into two categories: length scale (ℓ^*) and sediment production (HC_H and HC_B , and S^*). Using these groupings, contour plots of equilibrium headland aspect ratio are plotted in Figure 8. Proceeding right along the x-axes is equivalent to decreasing net sediment production by increasing the beach sediment loss rate (S), decreasing the headland retreat rate (E_{0H}), and/or decreasing sea-cliff height and the proportion of coarse sediment in the cliffs (HC_H and HC_B). Proceeding upwards along the y-axes is equivalent to increasing the space between headlands (ℓ_B), or decreasing headland width (ℓ_H). Aspect ratios have been partitioned into four groups to correspond with a particular headland shape: stubby (0 - 1), elongate (1-2), needle (2-3), and super needle (>3). Examples of stubby and needle types are shown in Figure 1. A range of different headland morphologies can result simply through

internal sediment dynamics as well as alongshore headland position. In general, when sediment production is low or headland spacing is large, coastline aspect ratio is high and the coastline will tend to develop high-amplitude needle or super needle headlands; when sediment production is large or headland spacing is low, aspect ratio and amplitude are lower and headlands tend to be elongate or stubby. If sediment production is too large, amplitude goes to zero and headlands will be transient features.

Figure 24: Contour plot of equilibrium headland aspect ratio showing the different headland types that are possible just by changing the coastline variable along the x and y axes.

4.3.2 Numerical model

In Figures 27 and 28 are the results of two model runs over 20,500 years. The initial conditions consist of two headlands with an aspect ratio of 1.25 (amplitude = 500

m, headland width = 400 m). Headland lithology is more resistant than surrounding embayment lithology, and retreat slower. The headlands are separated by two embayments that are 4,900 and 2,300 m in length (note that due to periodic boundary conditions, the shorter embayment is continuous from the right edge of the domain to the left edge of the domain). Other key parameter values are $HC_B = 2$, $HC_H = 4$, $E\theta_B = 0.2 \text{ m yr}^{-1}$, $E\theta_H = 0.01 \text{ m yr}^{-1}$, $S = 0.5 \text{ m yr}^{-1}$, and $w_{max} = 100 \text{ m}$. The wave climate for both runs is entirely low-angle and slightly asymmetrical ($A = 0.6$; 60% of wave influence is from the left).

Figure 25: Numerical model results. Initial conditions are in the top panel. The subsequent two panels show the end result of model runs discussed in the text.

Figure 26: Numerical model results showing maximum aspect ratio through time.

In Figure 27, Panel A, the coastline has reached equilibrium and developed two scalloped and sediment-filled embayments. The maximum aspect ratio of the headlands on each bay is plotted in Figure 28 (gray lines). Because cross-shore amplitude and aspect ratio are measured relative to the beach, the aspect ratio is variable over short time scales due to stochasticity in the wave climate and beach position. Aspect ratio (and amplitude) increases in the larger bay from 1.25 to about 1.75 over ~3,000 years, but remains approximately the same in the smaller bay. This result is consistent with the analytical model, where headland aspect ratio and coastline amplitude increase as headlands are spaced farther apart. Panel B, Figure 27, shows a model run where effective nondimensional sea-cliff height (HC) and the bare-rock headlands retreat rate are increased so that sediment inputs are greater than in Panel A. The domain fills with

beach sediment, causing embayments to retreat slower than headlands. As a result, headlands disappear. The aspect ratio plotted in Figure 28 (gray line) decreases to zero by ~2,000 years, occasionally dipping below zero when beach width is greater than headland amplitude. This is also consistent with the analytical model: when sediment input is too high, beach sediment protects much of the embayment sea-cliffs from wave impacts, and headlands become transient coastline features.

The scalloped-shaped bays are the result of wave shadowing on the leeward side of headlands. Net sediment transport is from left to right across the domain, which leaves the left side of embayments sediment-free and susceptible to retreat. At around 16,000 years, the aspect ratio of the large bay in Panel A suddenly decreases. This is because of a temporary reversal of alongshore sediment transport that causes sediment to accumulate in the left side of the bay. Beach width increases relative to headland amplitude, and aspect ratio decreases.

4.4 Discussion

4.4.1 Runaway headlands and additional negative feedbacks

In some cases, particularly when E_{0H} is very small ($<0.01 \text{ m yr}^{-1}$), headland aspect ratio predicted by equation 40 must become unrealistically large (i.e., tens to hundreds of kilometers) in order to reach equilibrium. Adams and Anderson (2002) also observed this “runaway” headland scenario in modeling rocky coastline amplitude response to wave energy delivery. It suggests that we are missing a key feedback or process in our analytical model and resulting scaling argument. In the numerical model, coastline amplitude does not run away and reach an unrealistically large length, even though

headland retreat is very slow (0.01 m yr^{-1}). This is because of the dependence of sea-cliff retreat on incoming wave angle: as the embayment deepens and amplitude grows, more of the bay is within a wave shadow zone, so sea-cliff retreat slows down there, tending to prevent runaway amplitude growth. So, in addition to sediment dynamics, there is an additional negative feedback between headland amplitude, incoming wave angle, and sea-cliff retreat that controls coastline amplitude.

Wave shadowing on the leeward side of headlands can be added to the analytical model using a dynamic scaling parameter. As the amplitude of the coastline changes through time, and depending on the wave angle relative to the coastline, a length of the embayment on the leeward side of the embayment will be sheltered from incoming wave energy (Figure 20). The shadow length increases with increasing headland amplitude and wave angle, and within the shadow, sea-cliff retreat and beach sediment losses cease (during the time waves approach from that angle). So, by reducing wave impacts in the embayments relative to the headlands, the wave shadow effectively concentrates wave energy on the headlands as they become larger, similar to how the way wave refraction focuses wave energy on headlands. We can determine the mean proportion of embayment that is likely to be shadowed as a function of the mean wave angle and headland amplitude as follows:

$$\psi = 1 - \frac{A(t)}{\ell_B} \tan(\bar{\varphi})$$

Equation 41

where ψ is the proportion of bay that is shadowed and $\bar{\varphi}$ is a representative wave angle.

If the wave angle is zero (i.e., the wave is approaching directly onshore) or the amplitude

is zero, then there will be no wave shadow effect and $\psi = I$. Note that equation 41 is only valid for $\psi \geq 0$ and becomes negative when $A/\ell_B \tan \bar{\varphi} > 1$. To avoid this we must restrict our analysis to low wave angles ($< 45^\circ$), and/or coastline amplitudes less than about 5 km.

Including ψ in equations 35 and 37 reduces headland aspect ratio and coastline amplitude. Equation 40, our main scaling argument, becomes

$$A^* \propto \frac{\ell^*}{2\gamma HC_H + S^* \bar{\varphi}} (S^* - \gamma HC_B) - \frac{1}{2}.$$

Equation 42

Figure 29 demonstrates the effect on headland aspect ratio of adding wave shadowing in embayments. It is the same as Figure 26, except mean wave angle is 45° . In Figure 26, when the mean wave angle is zero, the maximum aspect ratio is 4.5, corresponding to a super needle headland type. In Figure 29, mean wave angle is 45° and maximum aspect ratio is about 1.5 because wave impacts are concentrated on headlands rather than in bays. So, for higher angle wave climates, we expect to find lower amplitude rocky coastlines, with all else being equal.

A factor that we do not consider here is wave energy. As coastline amplitude grows, more wave energy is concentrated on headlands (and less in bays) due to wave refraction (Trenhaile, 1987; Adams and Anderson, 2002; Adams, 2004). Headland retreat therefore becomes faster with higher amplitude. This is an additional negative feedback

Figure 27: Contour plot of headland aspect ratio. This plot is the same as Figure 26, except the mean incoming wave angle is 45 degrees instead of zero degrees.

that can control coastline morphology and hasten the rate that it evolves. The bare-rock sea-cliff retreat rates could be defined in terms of coastal wave energy delivery, following Adams et al. (2002). Because wave energy is a function of wave height, we could then address additional questions such as how wave climate affects rocky coastline evolution and morphology. Another factor we do not include is sediment input from coastal-draining rivers and streams. In California, for example, much of the littoral sediment budget is derived from fluvial sediment, and rivers are common features in coastal embayments (Willis and Griggs, 2003; Patsch and Griggs, 2008). Fluvial sediment can be added to our model as a constant offset the sediment loss rate, S (Limber and Murray, 2011b). The additional sediment input tends to reduce coastline amplitude and headland aspect ratio much in the same way that increasing sea-cliff height (or C) does.

4.4.2 Applications to Montara Beach, California, USA

Sallenger et al. (2002) documented the response of sea-cliff retreat to variations in beach width at Montara Beach in California, USA (Figure 18). Montara is a pocket beach contained between two headlands that are lithologically different and more erosion resistant than the cliffs that comprise the embayment (Limber, 2005). During a series of storms, sections of sea-cliff that were fronted by a subaerial beach experienced less retreat than those that were not. Montara provides an ideal site to examine the relevance of the rocky coastline dynamics and the scaling relationship for coastline aspect ratio presented here.

We can start by estimating the terms in parentheses in equation 40: S^* and γHC . If $S^* - \gamma HC$ is negative, then aspect ratio is negative, which is equivalent to zero. If it is positive, then aspect ratio is likely to be greater than zero. The sediment loss rate, S , can be estimated using measurements of sandy shoreline change by Hapke et al. (2006). Although these measurements represent net shoreline change, and S is gross sediment loss, they provide at least a minimum estimate of S . Over the long-term (132 years), the beach has accreted at an average rate of 0.17 m yr^{-1} . This suggests that the sediment loss rate is small. However, between 1956 and 1998, the beach retreated at an average rate of 0.43 m yr^{-1} . Given these observations, we pick minimum and maximum rates of S of 0 and 0.5 m yr^{-1} . The average bare-rock headland retreat rate, E_{0H} , is 0.73 m yr^{-1} measured over 69 years (Hapke and Reid, 2007). Thus the range of S^* values is 0 - 0.7. Embayment sea-cliff height is, on average, 15 m (Limber, 2005). If we assume that the depth to the underlying shore platform is between 2 and 5 meters, then H is between 3 and 7.5. The

embayment sea-cliffs are composed of soft sandy alluvial deposits. Limber (2005) collected sea-cliff and beach samples and determined through grain size analysis that the proportion of cliff sediment that is coarse enough to remain in the nearshore is approximately 0.7. Depending on beach geometry, γ equals one if the beach is rectangular (as in Figure 22) or two if the beach is wedge-shaped or triangular (Limber and Murray, 2011). Combining all of these observations yields a range for γHC of 2.1 – 10.5. Therefore, $S^* - \gamma HC < 0$, and equilibrium headland aspect ratio and amplitude is likely to be near zero. A quick look at Montara Beach (Figure 18) shows that headland amplitude is greater than zero, and indeed headlands extend seaward up to 200 m beyond the subaerial beach. But the cliff retreat rates show that headlands are retreating faster than the embayment, which is retreating at an average rate of 0.45 m yr^{-1} (Hapke and Reid, 2007). If this is the case, then the coastline is still decreasing in amplitude, and perhaps has not yet reached a steady-state.

4.4.3 Characteristic time scales

Our model is driven by negative feedbacks between sea-cliff retreat and beaches, as well as internal sediment dynamics. These simple interactions can produce a range of observable headland and embayment amplitudes and aspect ratios on lithologically diverse rocky coastlines, from stubby to needle-shaped (e.g., Carter et al., 1990). But, what factors control how quickly a rocky coastline will evolve and reach equilibrium? Using a linear stability analysis, Limber and Murray (*in prep*) derived characteristic time scales for rocky coastline evolution based on beach and sea-cliff dynamics. Results suggested that the local derivative of equation 36 around a steady-state beach width

(slope of the curve around the fixed point in Figure 21), representing the sensitivity of cliff retreat rate to beach width, controlled the growth or decay rate of headlands. Taking the derivative of equation 36 with respect to beach width, and inserting the equilibrium beach width that causes embayments to retreat at the same rate as headlands (meaning coastline amplitude is in equilibrium), we can derive an amplitude growth or decay rate. The inverse of the growth rate yields a characteristic time scale, or e-folding time, t_0 :

$$t_0 = -\frac{w_{scale}}{E_{0H}} = -\frac{w_{max}}{E_{0H} \ln \left[\frac{E_{min}}{\psi E_{0B}} \right]}.$$

Equation 43

The time scale depends largely on the headland retreat rate, as well as the beach width that causes embayment sea-cliff retreat to decrease to near zero, and the wave shadow scaling parameter (equation 41). The e-folding times as a function of the headland retreat rate and ψE_{0B} are shown in Figure 30. The times are inversely proportional to both bare-rock retreat rates. Remember that ψ increases (representing shadowing of more of the embayment) as coastline amplitude and the time-averaged incoming wave angle increase, and as headland spacing decreases. So, the characteristic time scale is longer for 1) large equilibrium headland amplitudes and mean wave approach angles, 2) headlands that are spaced closer together, 3) headlands and embayments that retreat more slowly, and 4) large values of w_{max} . As can be seen from the e-folding times and the model trajectory time scale (Figure 24), it can take tens of thousands of years for a rocky coastline to evolve and reach equilibrium. Unless sea-cliff retreat rates are high, a single sea level rise

highstand may not be long enough. Therefore, in some cases, the coastline will not reach equilibrium unless it is inherited from a previous highstand (Trenhaile, 1987).

Figure 28: e-folding times for rocky coastline amplitude and headland aspect ratio as a function of wave shadowing and headland retreat rate.

4.5 Conclusions

Along lithologically diverse rocky coastlines, equilibrium coastline amplitude and headland aspect ratio increase 1) as beach sediment input (a function of sea-cliff height and composition) decreases; 2) the bare-rock sea-cliff retreat rates increase; 3) beach sediment losses increase; 4) the time-averaged incoming wave angle increases; 5) headland spacing increases; and 6) alongshore headland width decreases. Using simple coastline characteristics, our model gives insights into how and why cross-shore amplitude and headland shape vary on rocky coastlines. Future work will involve additional numerical modeling; adding rivers, topography, and wave energy to the model; and using spectral analysis (wavelets) to analyze coastline shape and compare the model to nature.

Appendix A

The volume of beach sediment released during sea cliff erosion and retreat, determined by cliff height and composition, is scaled to a beach width by the beach geometry constant, γ . For simplicity and tractability in the analytical model, we assume that the beach wedge at the base of the cliff is triangular with a constant height. So, for example, if the volume of sediment eroded from the sea cliff through time is determined by the product of effective cliff height (HC), cell size, and cliff retreat distance, that sediment volume must be deposited cumulatively at the base of the cliff while maintaining the shape of a triangle with a time-varying beach width, w (Figure 19, main text). Therefore, in our case, the cumulative sediment released from cliff retreat must be multiplied by a factor of two (because the volume of the triangular beach wedge is one-half the product of beach height, width and cell size) to determine the beach width. For different beach geometries the constant γ must be adjusted accordingly.

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Biography

Patrick Limber was born on September 9, 1978 in Waynesboro, Virginia and grew up in Asheboro, North Carolina. He graduated from the University of North Carolina School of the Arts in 1997 with a high school diploma in Music and Fine Arts (classical guitar), and went on to receive a B.A. (*summa cum laude*) in Environmental Science from SUNY-Purchase College in 2002. After briefly working as a disgruntled hotel housekeeper in Cairns, Australia, Patrick happily enrolled at the University of California, Santa Cruz where he earned an M.S. in Earth Science (coastal geology) and subsequently received a two-year NOAA Coastal Management Fellowship in Raleigh, N.C. In 2007, Patrick began his doctoral work at Duke University under the guidance of A. Brad Murray.