Evaluation of flow direction methods against field observations of overland flow dispersion

Stefano Orlandini,1 Giovanni Moretti,1 Mauro A. Corticelli,1 Paolo E. Santangelo,1 Alessandro Capra,1 Riccardo Rivola,1 and John D. Albertson2

Received 1 March 2012; revised 2 September 2012; accepted 5 September 2012; published 10 October 2012.

[1] The D8, D8-LTD, D∞-LTD, D∞, MD∞, and MD8 flow direction methods are evaluated against field observations of overland flow dispersion obtained from novel experimental methods. Thin flows of cold water were released at selected points on a warmer slope and individual overland flow patterns originating from each of these points were observed using a terrestrial laser scanner and a thermal imaging camera. Land microtopography was determined by using laser returns from the dry land surface, whereas overland flow patterns were determined by using either laser returns or infrared emissions from the wetted portions of the land surface. Planar overland flow dispersion is found to play an important role in the region lying immediately downslope of the point source, but attenuates rapidly as flow propagates downslope. In contrast, existing dispersive flow direction methods are found to provide a continued dispersion with distance downslope. Predicted propagation patterns, for all methods considered here, depend critically on the size of grid cells involved. All methods are found to be poorly sensitive in extremely fine grids (h ≤ 2 cm), and to be poorly specific in coarse grids (h = 2 m). Satisfactory results are, however, obtained in grids having resolutions h that approach the average flow width (50 cm), with the best performances displayed by the MD8 method in the finest grids (5 ≤ h ≤ 20 cm), and by the MD∞, D∞, and D∞-LTD methods in the coarsest grids (20 cm < h < 1 m).


1. Introduction


“...the rate of rainfall or snowmelt exceeds the interception requirements and the rate of infiltration, water starts to accumulate on the surface. At first the excess water collects into the small depressions and hollows, until the surface detention requirements are satisfied. After that water begins to move down the slope as a thin film and tiny streams. This early stage of overland flow is greatly influenced by surface tension and friction forces. With continuing rainfall the depth of surface detention and the rate of overland flow increase, but the paths of the small streams on the surface of the catchment are still tortuous and full of obstructions. Every small obstruction causes a delay until the upstream level has risen to overflow the obstacle or to wash it away. On release a small wave speeds downstream and merges with another little rivulet. The merging of more and more of these little streams culminates in the river which drains the whole catchment in question.”

[3] More detailed descriptions of atmospheric forcing, land surface topography, and surface-subsurface flow interaction than available at present are, however, required to model explicitly these overland flow processes. In fact, overland flow is often described as a sheet flow on a regular surface characterized by its topography and some resistance coefficient [e.g., Ivanov et al., 2004; Kollet and Maxwell, 2006]. Alternatively, the concept of hydraulic geometry introduced by Leopold and Maddock [1953] is used to describe the complex network of rivulets occurring on the land surface during storm events [e.g., Orlandini and Rosso, 1998; Camporese et al., 2010].

[4] Sheet and rivulet flow models are commonly based on the assumption that overland flow is essentially driven by gravity. Under this assumption, flow lines are coincident with slope lines, or lines of slope, defined by Cayley [1859] and Maxwell [1870] as those lines that intersect contour lines at right angles. However, if the flow is also affected by surface forces implying for instance significant pressure or stress gradients, then flow lines are not necessarily the same as slope lines and flow dispersion becomes a relevant process. The distinction between slope and flow lines is outlined by Tarboton [1997] and Orlandini et al. [2003],
and is explicitly made by Gallant and Hutchinson [2011]. In light of this distinction, the term “flow direction method” used in terrain analysis, hydrology, and fluvial geomorphology may not be entirely appropriate. Nevertheless, no alternative terms are introduced in this paper, and the term flow direction method is used to denote either single flow direction methods (nondispersive methods designed to represent slope lines and flow lines in gravity-driven overland flows) or multiple flow direction methods (dispersive methods designed to represent flow lines in flows driven by gravity and other forces). The attention is focused in this paper on dominant classes of terrain analysis methods designed to process grid digital elevation models. Reviews of these methods can be found by Tarboton [1997], Orlandini and Moretti [2009], and Gallant and Hutchinson [2011]. Methods designed to process contour digital elevation models [e.g., Moretti and Orlandini, 2008] or based on the aspect driven method of Lea [1992] [e.g., Costa-Cabral and Burges, 1994] are not evaluated in this paper.

Flow direction methods presented in the literature are grouped here in three dominant classes. The first class is composed of single flow direction, nondispersive methods. The D8 method by O’Callaghan and Mark [1984] and the D8-LTD method by Orlandini et al. [2003] belong to this class. These methods were designed to approximate slope lines and to provide a basis for the description of nondispersive flows essentially driven by gravity. The second class is composed of multiple flow direction, moderately dispersive methods. The D∞ method by Tarboton [1997] and the D∞-LTD method by Orlandini and Moretti [2009] belong to this class. Although these methods determine for each grid cell a single steepest direction, they actually disperse flow along two (grid-referenced) directions approximating this steepest direction. Hence, these methods are classed here as multiple flow direction, dispersive methods. The D∞ and D∞-LTD methods were designed to mitigate the artifacts inherent in the use of single flow direction methods. However, as explicitly reported by Tarboton [1997], they are based on the idea that planar dispersion in terrain analysis should be minimized and physical dispersion inherent in the overland flow process, if necessary, should be modeled separately. The third class is composed of multiple flow direction, fully dispersive methods. The MD8 method by Freeman [1991] and Quinn et al. [1991] as well as the MD∞ method by Seibert and McGlynn [2007] belong to this class. The MD8 method was introduced to overcome the limitations of the D8 method in the computation of (specific) drainage area, but it was also considered to describe the planar dispersion inherent in overland flow processes [e.g., Huggel et al., 2003]. The MD∞ method was explicitly designed to provide a realistic description of overland flow dispersion [Seibert and McGlynn, 2007].

Despite this broad effort to account for flow dispersion in models, actual planar dispersion experienced by overland flow along a natural slope has not been measured so far, and the ability of terrain analysis methods to reproduce this dispersion has not been evaluated. The aims of this paper are to introduce novel experimental methods for the observation of planar overland flow dispersion occurring along natural slopes, and to evaluate the dominant classes of terrain-based flow direction methods against the field data collected. Experimental methods are based on the combined use of a terrestrial laser scanner and a thermal imaging camera as described in section 2. The full set of terrain-based flow direction methods reviewed in the above paragraph are evaluated by comparing visually and numerically predicted propagation patterns and observed overland flow patterns as described and discussed in sections 3, 4, and 5. Conclusions and directions into future research are reported in section 6.

2. Field Experiments

The experimental methods presented in this paper were tested during the field work shown in Figure 1. This effort was carried out in June 2010 along a natural slope located in the Italian Apennines (Figure 1, inset a). Latitude and longitude of the selected site are 44°24′18.16″ N and 10°55′48.86″ E, respectively. The slope was selected on the basis of displaying (1) divergent, or at least not markedly convergent, morphology, (2) sparse vegetation, and (3) accessibility by equipment and instruments. The average terrain slope along the selected field site is approximately 50%. The soil is composed of clay and small stones. The vegetation cover is sparse grass.

During the field campaign, a water tank (WT in Figure 1) equipped with a small tap (Figure 1, inset b) was supplied with cold (2–10°C) water carried in the field using a refrigerator. From the tank, a thin water flow was released on the slope as detailed in inset b of Figure 1. Two thin layers of metal were driven vertically into the ground to produce a small convergent channel in which the water flow released from the tap was entirely collected and conveyed toward a 1 cm wide exit (b). This exit is the source of the overland flow. Three flow experiment replicates, denoted in this paper as F1, F2, and F3, were carried out by selecting three different locations at which flows were released and point sources were formed. Flow experiment F1 is that illustrated in Figure 1. Flow experiments F2 and F3 were carried out using the same instruments and methods. Overland flow originated at the three selected point sources and propagated downslope over a dry land surface forming three noninteracting patterns. During the experiments F1, F2, and F3, average flow discharges equal to 8.9, 9.6, and 18.9 cm$^3$ s$^{-1}$ were released for time intervals of 29.1, 32.2, and 15.5 min, respectively. At the end of experiments F1, F2, and F3, the downstream lengths of flow patterns were 4.55, 6.65, and 5.20 m, whereas the average flow widths were 0.63, 0.47, and 0.66 m, respectively. The average terrain slope along the flow patterns observed in experiments F1, F2, and F3 were 48%, 54%, and 56%, respectively. The average velocities with which the flow patterns F1, F2, and F3 extended downslope were 2.6 × 10$^{-3}$, 3.4 × 10$^{-3}$, and 5.6 × 10$^{-3}$ m s$^{-1}$, respectively. Inertial forces are likely to play a minor role under these circumstances. No soil erosion was observed during the experiments. Flow experiments from each individual point source were not repeated in the field campaign reported in this paper. It is, however, acknowledged that estimates of the precision with which overland flow patterns can be observed may be provided in future, more extensive, field campaigns by repeating flow experiments from fixed point sources.
Prior to each experimental water release (F1, F2, and F3), a terrestrial laser scanner (TLS in Figure 1) was used to determine the microtopography of the dry land surface. The terrestrial laser scanner used in this investigation is the ScanStation C10 platform by Leica Geosystems. This platform includes a pulsed laser scanning system that produces a 3R class laser beam having a wavelength of 532 nm. ScanStation C10 is a panorama scanner that ensures a full $360^\circ$ field-of-view, with a measurement range of 300 m at 90% reflectivity and a scan rate up to 50,000 points per second. To ensure the automatic correction of the vertical axis error, the instrument is also equipped with a dual-axis compensator having accuracy equal to $7.2722 \mu$rad (0.0004°). The study area was surveyed by combining the data acquired from four scanning positions to ensure that each flow experiment replicate is observed from at least two positions and that a homogeneous point cloud is obtained. For each flow experiment replicate (F1, F2, and F3) laser scans were performed from a tripod having height of about 1.80 m, which was positioned downslope of the tank releasing water such that the line of sight was as close to nadir as possible in the center of the area involved in the flow experiment. This implies that the center of the terrain portion involved in the flow was in the center of the field of view. The distance between the TLS and the points of the surveyed area was everywhere less than 20 m. Both the horizontal and vertical angular resolution were set in the scanning system to 0.5 mrad (0.0286°). The resulting mean point cloud densities across the areas surveyed prior to each flow experiment were found to be 26, 28, and 21 points cm$^{-2}$, respectively.

The positions of eight Leica Geosystems high definition surveying (HDS) retroreflecting targets placed along each of the three terrain portions involved in the flow were acquired to provide an accurate alignment of point clouds captured from the four scanning positions. The alignment process was performed by using the Leica Cyclone 7.0 software. The documentation of this software can be found on the Leica Geosystems website. The topographic map of the terrain involved in flow experiment F1 is represented, for instance, in Figure 2a by setting the contour interval to 10 cm. The inset of Figure 2a illustrates, for demonstration, the point cloud obtained by scanning a tuft of grass.
The overland flow patterns generated during the experiments F1, F2, and F3 were observed using both the terrestrial laser scanner and a thermal imaging camera. The terrestrial laser scanner was found to observe overland flow patterns by detecting the contrasted reflectance of the dry and wet land surface portions. Reflectance, also known as total hemispherical reflectivity or reflectivity factor, is the average fraction of incident laser radiation reflected by the land surface. Averages are made over the reflected directions of the hemisphere and over all the wavelengths of the incident laser radiation. Wetted portions of the land surface absorb a significantly larger fraction of the laser radiation than dry portions and display, therefore, a drastic reduction in reflectance with respect to the dry land surface. A second point cloud (in addition to that used for the topographic survey) was therefore captured by the terrestrial laser scanner immediately following the flow experiments, using the same scanning positions and the same scanning system settings used for the topographic survey. Since the laser and reflectance signals are acquired and processed internally by the same instrument, reflectance returns were readily projected on the land topography. Using the projected reflectance data, the overland flow patterns were clearly identified as shown, for instance, in the 1 cm resolution false color raster map reported in Figure 2b. The inset of Figure 2b illustrates, for demonstration, the reflectance of a tuft of grass.

During the flow experiment replicates F1, F2, and F3, flows of cold water on the warmer slope were monitored by means of a thermal imaging camera (TIC in Figure 1). Thermal imaging cameras detect radiation emitted from target surfaces in the infrared range of the electromagnetic spectrum and produce images of that radiation called thermograms. The higher the surface temperature, the more the atoms and molecules move and the more infrared radiation they emit. Since the amount of radiation emitted by an object increases with temperature (to the fourth power), thermography allows one to see even small variations in temperature. The inversion of the received radiation is captured here with consideration of the surface emissivity. The thermal imaging camera used in this investigation is the Avio Advanced Thermo TVS-500EX camera by Nippon Avionics. It is provided with a microbolometric focal plane array (FPA) sensor, which operates in the longwave spectrum range 8–12 μm. The camera features an IEEE-1394 interface, allowing for a maximum acquisition frequency of 60 frames per second, a field-of-view of 19.4° (H) 14.6° (V), a spatial resolution of 1.07 mrad, and a measurement distance ranging from 30 cm to ∞. Effective pixels are 320 (H) × 240 (V). The infrared sensor ensures a temperature resolution better than 0.05°C. Calibration is automatically performed by the instrument itself. The same Leica Geosystems HDS retroreflecting targets used for the registration of the point clouds captured by the laser scanner

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**Figure 2.** Determination of the overland flow pattern observed during the flow experiment F1. The microtopography of the slope, determined by the laser scanner prior to experimental water release, is represented in the topographic map (Figure 2a), where the contour interval is set to 10 cm. The inset of Figure 2a illustrates, for demonstration, the point cloud obtained by scanning a tuft of grass. Laser reflectance of the land surface, determined by the laser scanner immediately following the flow experiment, is shown in false color map Figure 2b. Figure 2b illustrates, for demonstration, the reflectance of a tuft of grass. The land surface temperature, determined by the thermal imaging camera at the end of the flow experiment, is shown in the false color map (Figure 2c.) The boundary of the observed flow pattern, shown in Figures 2a–2c, is identified by integrating reflectance and temperature data sources as explained in section 2.
were used for the projection of thermal images on the surveyed terrain. A photogrammetric procedure based on the recognition of homologous points in the thermal images and in the land surface model was applied to perform this projection. More than 11 homologous points were used to correctly adequately the image deformation. Land surface temperature immediately following flow experiment F1 (Figure 1) is represented, for instance, in the 1 cm resolution raster maps reported in Figure 2c. The temperature of the dry land surface shown in Figure 2c ranges approximately from 15 to 30°C. The spatial variability displayed is primarily due to the shadowing effects produced by grass and land microtopography.

In the present study flow patterns observed in the field at the end of flow experiments are delineated manually by integrating reflectance and infrared data sources (Figure 2). Where the outlines of the wetted area provided by reflectance and temperature images differ, photographs acquired during the experiments and physical reasoning are used to help identify the most plausible flow pattern. It is, for instance, considered that land surface reflectance images may be less sharp in the upper part of the flow pattern, where flow depth starts to decrease immediately after water release from the tank stops (Figure 2b). On the other hand, land surface temperature images may be less sharp in the lower part of the flow patterns because of the warming of released water as it propagates downslope (Figure 2c). In addition, it must be considered that the extension of the cold area above the point source of water displayed in land surface temperature images is due to the presence of the water tank along the line of sight between the thermal imaging camera and the ground surface, and is produced when the thermogram pixels are projected on the land surface (Figure 2c). This artifact is not displayed by land surface reflectance images as they are obtained using data acquired from different scanning positions (Figure 2b). The resulting experimental uncertainty in overland flow pattern delineation is of the order of a few centimeters.

3. Numerical Experiments

Topographic data points obtained from the laser scanner survey (of the dry land surface prior to flow experiments) were used to generate alternative grid-based digital elevation models having resolution \( h = (1, 2, 5, 10, 20, 50, 100, 200) \) cm. The natural neighbor interpolation strategy was used for this task [Sibson, 1981]. The grids used to generate the digital elevation models were constrained to have one cell centered on the point in which the water flow was released and one of the sets of parallel lines forming an angle \( \gamma \) of \( 0^\circ, 22.5^\circ, \) or \( 45^\circ \) with the north direction (to support analysis of the effect of grid orientation on the results). The D8, D8-LTD, D\( \infty \)-LTD, D\( \infty \), MD\( \infty \), and MD8 flow direction methods are applied to the generated digital elevation models to evaluate the ability of these methods to reproduce the observed flow patterns. The physical quantities considered to evaluate methods are the fractions of flow released at the point source that propagates through grid cells. The value 1 is assigned to the cell centered on the point source. Values less than or equal to 1 are obtained for the downslope cells by applying the flow direction methods and the related flow accumulation algorithms [O’Callaghan and Mark, 1984; Orlandini et al., 2003; Orlandini and Moretti, 2009; Tarboton, 1997; Seibert and McGlynn, 2007; Freeman, 1991; Quinn et al., 1991].

Results obtained for flow experiment F1, grid orientation \( \gamma = 0^\circ \), and grid cell sizes \( h = (1, 2, 5, 10, 20, 50, 100, 200) \) cm are reported in Figure 3. Maps are arranged to form an \( 8 \times 6 \) matrix, in which each row reports results obtained for a given grid cell size, and each column reports results obtained from a given method. Dispersion in the predicted propagation patterns is generally observed to increase within the matrix from the upper left map (1A) to the lower right map (8F). Results obtained for the flow experiment F1, with \( \gamma = 22.5^\circ \) and \( h = 2 \) cm, are reported in Figure 4 to further illustrate how methods perform in extremely detailed digital elevation models. Results obtained for the flow experiment F1, with \( \gamma = 22.5^\circ, 45^\circ \) and \( h = 50 \) cm, as well as for the flow experiments F2 and F3, with \( \gamma = 0^\circ, 22.5^\circ, 45^\circ \) and \( h = 50 \) cm, are reported in Figure 5 to further illustrate how methods perform in digital elevation models having grid cells comparable in size with the average width of the observed flows.

4. Evaluation Methods

The ability of flow direction methods and related flow accumulation algorithms to reproduce the observed flow patterns is evaluated numerically for all the flow experiments, grid orientations, and the grid cell sizes, by determining suitable computational domains and by analyzing data sets representing the predictions and the data available for the comparison. For each flow experiment, a rectangular computational domain \( D \) is defined to include the predicted and the observed flow patterns as sketched in Figure 6. The rectangle \( D \) has two opposite sides oriented along the direction in which the observed flow pattern extends \( \eta \), and the other two opposite sides oriented along the transverse direction \( \xi \). Since the flow may not be fully developed around the tongues of the observed flow patterns, only the upper \( 80\% \) of length of the flow patterns is included in the computational domain. The resulting computational domain \( D \) is therefore a rectangle formed by the cross section segment passing through the point source (0% of the flow pattern length along \( \eta \)), the cross section segment located at the 80% of the flow pattern length along \( \eta \), and two segments having direction \( \eta \) and such that cross sections inside \( D \) are sufficiently long. Only the cell portions lying inside \( D \) are considered.

Predictions of the fraction of flow released at the point source that propagates through the grid cells forming the computational domain \( D \) are provided directly by flow direction methods and related flow accumulation algorithms. The data used for the comparison, however, are not provided directly from observations but rather are derived from the observed flow patterns by overcoming two problems. The first problem is that the experimental methods developed in this study do not provide direct information on the distribution of flow along the cross sections of the observed flow patterns but just the extent of the flow patterns. A distribution of flow along the cross sections must therefore be assumed in order to provide suitable data for the comparison. In this investigation the data available for the comparison are determined by considering a uniform distribution of flow along the cross sections of the observed
Figure 3
flow patterns, with flow density along each cross section equal to the reciprocal of the total flow width. This is the simplest and most objective assumption that can be made. The total width $w$ of the observed flow pattern is sampled at about 50 cross sections and the values of $w$ in the other cross sections are determined by linear interpolation.

[18] The second problem is that the observed flow patterns have irregular shapes and a procedure is needed to match the data predicted over the square cells forming the computational domain $D$ and the data for the comparison derived from the observed flow patterns $F$. This problem is especially important when the grid cell size is comparable to or larger than the width of observed flow patterns. In this investigation this problem is addressed by considering that the observed flow pattern $F$ partitions the computational domain $D$ in two sets, either inside or outside $F$, and by defining three sets of grid cells, namely the internal, external, and boundary cells (Figure 6). Cells of $D$ lying entirely inside $F$ are called internal cells. Cells of $D$ lying entirely outside $F$ are called external cells. Cells of $D$ lying both inside and outside $F$ are called boundary cells. Boundary cells contain the boundary of $F$. While single values representing the predictions and the data available for the comparison could be assigned to each internal and external cell, pairs of values must be assigned to each boundary cell to represent predictions and data available for the comparison over the two cell portions lying inside and outside $F$. It is then convenient to treat the $N$ cells lying, at least in part, inside the computational domain $D$ using a unique, general formalism.

[19] By denoting with $a_i$ the fraction of the $i$th cell of $D$ $(i = 1, \ldots, N)$ lying inside $F$, $a_i = 1$ for internal cells, $0 < a_i < 1$ for boundary cells, and $a_i = 0$ for external cells. Predictions are represented by the set $(m'_1, \ldots, m'_N, m''_1, \ldots, m''_N)$, where $m'_i (i = 1, \ldots, N)$ is the quantity associated with the cell portion lying inside $F$ (even when this portion is equal to zero), and $m''_i (i = 1, \ldots, N)$ is the quantity associated with the cell portion lying outside $F$ (even when this portion is equal to zero). Similarly, the data available for the comparison are represented by the set $(d'_1, \ldots, d'_N, d''_1, \ldots, d''_N)$. For the $i$th cell of the computational domain $(i = 1, \ldots, N)$, it can be written that

$$m'_i = m_i$$

and

$$m''_i = m_i$$

where $m_i (i = 1, \ldots, N)$ is the predicted fraction of flow released at the point source that propagates through that cell. In addition, it can be written that

$$d'_i = \frac{h}{\cos \Theta} \frac{1}{w_i}.$$

where $h$ is the grid cell size, $\Theta$ is the angle formed by cross sections having direction $\xi$ and grid lines having direction $x$ (which are selected in such a way that $\Theta \leq 45^\circ$), and $w_i$ is the total width of the observed flow pattern along to the cross section passing through the center of the $i$th cell, and that

$$d''_i = 0. \quad (4)$$

It is stressed that the quantity considered here is the fraction of flow released at the point source that propagates through the cells. A single value of this quantity $m_i$ is predicted at the $i$th grid cell (as represented by a uniform green shade inside the cell) and this value is assigned to both $m'_i$ and $m''_i$ as expressed by equations (1) and (2), respectively. Different values of the quantity considered must, however, be assigned to $d'_i$ and $d''_i$ as expressed by equations (3) and (4), respectively. The values provided by equations (3) and (4) are valid approximations of the observed flow passing through internal and external cells, respectively, and they can be used to provide weighted averages $d'_i a_i + d''_i (1 - a_i)$ representing reasonable approximations of the observed flow passing through either internal, external, or boundary cells. Equations (1)–(4) provide, therefore, a suitable basis to perform a meaningful comparison between predicted and observed flow patterns.

[20] The ability of flow direction methods and related flow accumulation algorithms to reproduce the observed flow patterns is evaluated numerically by using the weighted Pearson coefficient of correlation

$$R = \sum_{i=1}^{N} \frac{[d'_i - \bar{d}](m'_i - \bar{m}) a_i + [d''_i - \bar{d}](m''_i - \bar{m})(1 - a_i)] b_i}{\delta_{d} \delta_{m}}$$

where $b_i (0 < b_i \leq 1)$ is the fraction of the $i$th cell lying inside $D$,

$$\bar{d} = \sum_{i=1}^{N} \frac{[d'_i a_i + d''_i (1 - a_i)] b_i}{\sum_{i=1}^{N} b_i}, \quad (6)$$

$$\bar{m} = \sum_{i=1}^{N} \frac{[m'_i a_i + m''_i (1 - a_i)] b_i}{\sum_{i=1}^{N} b_i}, \quad (7)$$

$$\delta_{d} = \sqrt{\sum_{i=1}^{N} [(d'_i - \bar{d})^2 a_i + (d''_i - \bar{d})^2 (1 - a_i)] b_i},$$

and

$$\delta_{m} = \sqrt{\sum_{i=1}^{N} [(m'_i - \bar{m})^2 a_i + (m''_i - \bar{m})^2(1 - a_i)] b_i}, \quad (9)$$

Figure 3. Comparison between the flow pattern observed during the experiment F1 and propagation patterns predicted from different methods (columns: D8, D8-LTD, D∞-LTD, D∞, MD∞, and MD8) using grid digital elevation models having fixed grid orientation ($\gamma = 0^\circ$) and variable resolution (rows: $h = 1, 2, 5, 10, 20, 50, 100, 200$ cm). The point source is indicated by the empty circle, the boundary of the observed flow pattern is indicated by the blue dotted line, and the predicted propagation pattern is indicated by green shaded cells. Cell shades indicate the fraction of released water passing through the cell. Pearson correlation coefficient $R$, computed over the domain bounded by the two cross sections in cyan, is reported in the top right corner of each map.
Figure 4. Comparison between the flow pattern observed during experiment F1 and propagation patterns predicted from different methods (D8, D8-LTD, D∞-LTD, D∞, MD∞, and MD8) using grid digital elevation models having variable grid orientation (γ = 22.5°, 45°) and fixed resolution (h = 2 cm). The point source is indicated by the empty circle, the boundary of the observed flow pattern is indicated by the blue dotted line, and the predicted propagation pattern is indicated by green shaded cells. Cell shades indicate the fraction of released water passing through the cell. Pearson correlation coefficient $R$, computed over the domain bounded by the two cross sections in cyan, is reported in the top right corner of each map.
In equations (5)–(9) the terms related to the cell portions lying inside and outside $F$ are weighted by $a_i$ and $(1 - a_i)$, respectively. For internal cells only the terms related to the cell portion lying inside $F$ contribute to the summations, whereas for external cells only the terms related to the cell portion lying outside $F$ contribute to the summations. Weighted averages performed inside the $i$th ($i = 1, \ldots, N$) cell are further weighted by considering the fraction $b_i$ of that $i$th cell lying inside $D$. It can be readily obtained that

$$R = \frac{\sum_{i=1}^{N} (d_i^i a_i - \bar{d})(m_i - \bar{m}) b_i}{\delta_d \delta_m}, \quad \bar{d} = \frac{\sum_{i=1}^{N} d_i^i a_i b_i}{\sum_{i=1}^{N} b_i},$$

$$\bar{m} = \frac{\sum_{i=1}^{N} m_i b_i}{\sum_{i=1}^{N} b_i}, \quad \delta_d = \left[ \frac{\sum_{i=1}^{N} (d_i^2 a_i - 2 d_i^i a_i + \bar{d}^2) b_i}{\sum_{i=1}^{N} b_i} \right]^{1/2},$$

$$\delta_m = \left[ \frac{\sum_{i=1}^{N} (m_i - \bar{m})^2 b_i}{\sum_{i=1}^{N} b_i} \right]^{1/2}.$$  

The Pearson coefficient of correlation given by (5)–(9) is the most suitable measure for evaluations involving nonbinary data [Baldi et al., 2000]. Relative entropy is not directly applicable in this study because zero values are present in the data sets [Baldi et al., 2000].

[21] It can be noted that the Pearson correlation coefficient $R$ given by (5)–(9) is not affected by the value of $\cos \Theta$ in (3). In addition, it can be verified that the value of $R$ does not depend significantly on the extension of the computational domain $D$. Correlation coefficient results for the various flow direction methods (D8, D8-LTD, D∞-LTD, D∞, MD∞, and MD8), when used to reproduce the flow patterns F1, F2, and F3, are computed over the domain lying between the two cross sections at 0% and 80%, respectively, of the flow pattern length (Figure 6). These cross sections are reported in cyan in the maps of Figures 3–5. For each of these maps, the value of $R$ is reported in the upper right corner. These values of $R$ are found to depend strongly on the size $h$ of the grid cells involved. The results obtained for different values of $h$ are therefore considered separately. For each grid cell size and for each flow direction method, the mean (M), standard deviation (SD), and coefficient of variation (CV) of the values of $R$ obtained over all flow experiment replicates and grid orientations are computed. The obtained results are reported in Figure 7.

5. Discussion

[22] In all the flow experiment replicates F1, F2, and F3, planar dispersion is found to play an important role in the upper parts of the flow patterns, immediately downslope of the point source. The role of dispersion is, however, found to decrease in the lower part of the observed flow patterns, where flows display a virtually constant width of about 1 cm to 2 m, Figure 3, maps 1A–8B, where the notation 1A–8B indicates the maps lying in the rectangle having corner maps 1A and 8B), and are not meant to describe overland flow dispersion. When fine resolution digital elevation models are used (e.g., $h = 2$ cm, Figure 3, maps 2A and 2B, and Figures 4a, 4b, 4g, and 4h), the predicted flow patterns describe surface flow paths that may lie within the observed flow patterns, but that only cover a minor part of them. In these cases, the methods may be specific, but they cannot be sensitive. When coarse resolution digital elevation models are used (e.g., $h = 2$ m, Figure 3, maps 8A and 8B), the predicted flow patterns describe surface flow paths that may cover most of the observed flow patterns, but grid cells are clearly too large to provide an accurate reproduction of observed flow patterns. In these cases, the methods may be sensitive, but they cannot be specific. As the size of the grid cell involved increases from 1 cm to 2 m, the D8 and D8-LTD methods generally display increasing sensitivity and decreasing specificity, with the best results obtained when the grid cell size $h = 50$ cm, which is comparable with the width of flow, is used (e.g., Figure 3, maps 6A and 6B, Figure 5, maps 1A–8B).

[23] Multiple flow direction, moderately dispersive methods (D∞-LTD and D∞) are found to provide predicted propagation patterns with variable degree of dispersion depending on grid cell size (e.g., Figure 3, maps 1C–8D). When fine resolution digital elevation models are used
Figure 5
(e.g., $h = 2$ cm, Figure 3, maps 2C and 2-D, and Figures 4c, 4d, 4i, and 4j), the predicted propagation patterns display a low degree of dispersion and reproduce results similar to those obtained from nondispersive methods (e.g., Figure 3, maps 1A and 1B, and Figures 4a, 4b, 4g, and 4h). In these cases, the methods may be specific, but they cannot be sensitive. When coarse resolution digital elevation models are used (e.g., $h = 2$ m, Figure 3, maps 8C and 8D), the predicted propagation patterns display again a low degree of dispersion and reproduce essentially the results obtained from nondispersive methods (e.g., Figure 3, maps 8A and 8B). In these cases, the methods may be sensitive, but they cannot be specific. Dispersion is, however, produced by the $D_{1}$-LTD and $D_{1}$ methods when intermediate grid cell sizes ($h = 5; 10; 20; 50; 100$ cm) are used (e.g., Figure 3, maps 3C–7D, and Figure 5, maps 1C–8D). Satisfactory results are obtained for grid cell sizes $h = 10; 20; 50; 100$ cm (Figure 3, maps 4C–7D, Figure 5, maps 4C–7D). It is, however, noted that the methods provide a continued dispersion with distance downslope, whereas this continued dispersion is not observed in the field.

Multiple flow direction, fully dispersive methods (MD$_{\infty}$ and MD8) display different results. The MD$_{\infty}$ method produces more dispersion than the moderately dispersive methods (D$_{\infty}$-LTD and D$_{\infty}$) (e.g., Figure 3, maps 1C–8E; Figures 4c, 4d, 4e, 4i, 4j, and 4k; and Figure 5, maps 1C–8E), allowing for slightly improved predictions in terms of sensitivity, specificity, and correlation for $h < 20$ cm (Figures 7a–7c). Results comparable with those obtained from other methods are obtained for $h > 20$ cm (Figure 7a). The MD8 method produces even more dispersion than the MD$_{\infty}$ method (e.g., Figure 3, maps 1E–8F; Figures 4e, 4f, 4k, and 4l; and Figure 5, maps 1E–8F), allowing for significantly improved predictions over all the other methods examined in this paper in terms of sensitivity, specificity, and correlation for $h < 20$ cm (Figures 7a–7c). However, slightly worse results than those obtained from the MD$_{\infty}$, D$_{\infty}$, and D$_{\infty}$-LTD methods are obtained for $h > 20$ cm (Figure 7a).

The consistency of replicated flow direction method applications over different experiments (F1, F2, and F3) and grid orientations ($\gamma = 0^\circ, 22.5^\circ, 45^\circ$) is discussed here.

Figure 5. Comparison between the flow patterns observed during experiments F1, F2, and F3 and propagation patterns predicted from different methods (D8, D8-LTD, D$_{\infty}$-LTD, D$_{\infty}$, MD$_{\infty}$, and MD8) using grid digital elevation models having variable grid orientation ($\gamma = 0^\circ, 22.5^\circ, 45^\circ$) and fixed resolution ($h = 50$ cm). The point source is indicated by the empty circle, the boundary of the observed flow pattern is indicated by the blue dotted line, and the predicted propagation pattern is indicated by green shaded cells. Cell shades indicate the fraction of released water passing through the cell. Pearson correlation coefficient $R$, computed over the domain bounded by the two cross sections in cyan, is reported in the top right corner of each map.

Figure 6. Sketch of the computational domain (D) defined to evaluate flow direction methods against observed flow patterns (F). For the $i$th cell of the computational domain $D$ ($i = 1, \ldots, N$), $w_i$ is the total width of the observed flow pattern along the cross section passing through the center of that cell, $a_i$ is the cell fraction lying inside $F$, $b_i$ is the cell fraction lying inside $D$, $m_i$ and $m_i'$ are the predictions associated with the cell portions lying respectively inside and outside $F$, $d_i$ and $d_i'$ are the data for the comparison associated with the cell portions lying respectively inside and outside $F$ (section 4).
D for the various flow direction methods (D8, D8-LTD, persive methods (MD...tions (Figures 4g–4j). The multiple flow direction, fully dis-

4, 7b, and 7c), displaying in some cases inaccurate solu-

correlation coefficient $R$ (equations (5)–(9)) obtained

MD∞, D∞, and D∞-LTD methods (Figure 7a). The means, standard deviations, and coefficient of variation of the Pearson correlation coefficient reported in Figure 7 reveal essentially that the highest figures of merit and the lowest degrees of inconsistency are offered by the MD8 method for $h \leq 20$ cm, and by the MD∞, D∞, and D∞-LTD methods for $h > 20$ cm.

6. Conclusions

The experimental methods for the observation of overland flow dispersion developed in this study (sections 2, Figures 1 and 2) made it possible to perform a first evaluation of the dominant classes of flow direction methods currently in use (sections 3 and 4, Figures 3–7). In all the flow experiment replicates examined in this study, overland flow dispersion was observed to play an important role immediately downslope of the point source, but attenuates rapidly as flow propagates downslope (Figures 1–5). None of the methods were able to reproduce the observed overland flow patterns in a consistently sensitive and specific manner (Figures 3–5). In fact, predicted propagation patterns were found to depend critically on the size $h$ of grid cells involved (Figures 3 and 7). All methods were found to be poorly sensitive when extremely fine grids ($h \leq 2$ cm) were used (Figures 3 and 4), and to be poorly specific when coarse grids ($h = 2$ m) were used (Figure 3). Satisfactory results were, however, obtained in grids having resolutions $h$ that approach the average flow width (that is in this application approximately equal to 50 cm), with the best performances displayed by the MD8 method in the finest grids ($5 \leq h \leq 20$ cm), and by the MD∞, D∞, and D∞-LTD methods in the coarsest grids (20 cm < $h \leq 1$ m) (Figures 3, 5, and 7).

Further work is needed to provide consistently accurate descriptions of overland flow dispersion in grid-based digital elevation models. The results obtained in this study suggest further testing of terrain analysis methods over sufficiently extended flow patterns in order to evaluate the capabilities of these methods in distributed catchment models having limited grid resolution as a result of computational constraints. Scale issues affecting the relation between land surface microtopography, dispersion, and size of grid cells involved need then to be addressed to provide a hydrologic model of flow partitioning along the slope directions identified by terrain analysis methods. The relative roles of forces acting on overland flows have to be determined in this phase. In addition to individual overland flow patterns originated at single point sources, more complex flows may be generated from multiple point sources, nonpoint sources extending over single cells, or nonpoint sources extending entirely over hillslopes to study mixing in overland flows. This task may be facilitated by exploiting the ability of the thermal imaging camera to monitor flow propagation and by using innovative methods for tracing the flow elements within the mixing flows [e.g., Tauro et al., 2012a, 2012b]. In addition to the terrain analysis methods considered in the present study, methods based on the aspect driven concept [e.g., Costa-Cabral and Burges, 1994] and methods designed to process contour digital elevation models [e.g., Moretti and Orlandini, 2008] may be considered in combination with modest smoothing of
topographic data to investigate the relation between characteristic scales in topographic data and in terrain analysis modeling.

[29] Acknowledgments. This study was carried out under the research program PRIN 2008 (grant 2008A7EBA3) funded by the Italian Ministry of Education, University, and Research. The authors are grateful to Giovanni Sebastiano Barozzi (Università degli Studi di Modena e Reggio Emilia, Italy), and three anonymous reviewers for comments that led to improvements in the manuscript.

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