Efficient Computation of Electromagnetic Waves in Hydrocarbon Exploration Using the Improved Numerical Mode Matching (NMM) Method

by

Junwen Dai

Department of Electrical and Computer Engineering
Duke University

Date: ______________________

Approved:

Qing Huo Liu, Co-Supervisor

William T. Joines, Co-Supervisor

Jian-Guo Liu

Martin A. Brooke

Stéphane Larouche

Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Electric and Computer Engineering in the Graduate School of Duke University

2016
ABSTRACT

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Abstract

In this study, we developed and improved the numerical mode matching (NMM) method which has previously been shown to be a fast and robust semi-analytical solver to investigate the propagation of electromagnetic (EM) waves in an isotropic layered medium. The applicable models, such as cylindrical waveguide, optical fiber, and borehole with earth geological formation, are generally modeled as an axisymmetric structure which is an orthogonal-plano-cylindrically layered (OPCL) medium consisting of materials stratified planarly and layered concentrically in the orthogonal directions.

In this report, several important improvements have been made to extend applications of this efficient solver to the anisotropic OCPL medium. The formulas for anisotropic media with three different diagonal elements in the cylindrical coordinate system are deduced to expand its application to more general materials. The perfectly matched layer (PML) is incorporated along the radial direction as an absorbing boundary condition (ABC) to make the NMM method more accurate and efficient for wave diffusion problems in unbounded media and applicable to scattering problems with lossless media. We manipulate the weak form of Maxwell’s equations and impose the correct boundary conditions at the cylindrical axis to solve the singularity problem which is ignored by all previous researchers. The spectral element method (SEM) is introduced to more efficiently compute the eigenmodes of higher accuracy with less unknowns, achieving a faster mode matching procedure between different
horizontal layers. We also prove the relationship of the field between opposite mode indices for different types of excitations, which can reduce the computational time by half. The formulas for computing EM fields excited by an electric or magnetic dipole located at any position with an arbitrary orientation are deduced. And the excitation are generalized to line and surface current sources which can extend the application of NMM to the simulations of controlled source electromagnetic techniques. Numerical simulations have demonstrated the efficiency and accuracy of this method.

Finally, the improved numerical mode matching (NMM) method is introduced to efficiently compute the electromagnetic response of the induction tool from orthogonal transverse hydraulic fractures in open or cased boreholes in hydrocarbon exploration. The hydraulic fracture is modeled as a slim circular disk which is symmetric with respect to the borehole axis and filled with electrically conductive or magnetic proppant. The NMM solver is first validated by comparing the normalized secondary field with experimental measurements and a commercial software. Then we analyze quantitatively the induction response sensitivity of the fracture with different parameters, such as length, conductivity and permeability of the filled proppant, to evaluate the effectiveness of the induction logging tool for fracture detection and mapping. Casings with different thicknesses, conductivities and permeabilities are modeled together with the fractures in boreholes to investigate their effects for fracture detection. It reveals that the normalized secondary field will not be weakened at low frequencies, ensuring the induction tool is still applicable for fracture detection, though the attenuation of electromagnetic field through the casing is significant. A hybrid approach combining the NMM method and BCGS-FFT solver based integral equation has been proposed to efficiently simulate the open or cased borehole with tilted fractures which is a non-axisymmetric model.
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Abbreviations

- NMM: Numerical Mode Matching.
- SNMM: Spectral Numerical Mode Matching.
- OPCL: Orthogonal-Plano-Cylindrically Layered.
- PML: Perfectly Matched Layer.
- DoF: Degree of Freedom.
- PPW: Points Per Wavelength.
- FDTD: Finite-Difference Time-Domain.
- FEM: Finite Element Method.
- SEM: Spectral Element Method.
- MoM: Method of Moments.
- PEC: Perfect Electric Conductor.
- GLL: Gauss-Lobatto-Legendre polynomial.
- DBA: Distorted Born Approximation.
- BCGS: Stabilized Bi-Conjugate Gradient.
- FFT: Fast Fourier Transform.
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1.1 Problem Statement

The goal of this study is to develop and improve the efficiency and accuracy of the numerical mode matching (NMM) method for the electromagnetic wave propagation problem in an orthogonal-plano-cylindrically (OPCL) as shown in Fig. 1.1. This structure represents many models in engineering applications, including cylindrical waveguides, integrated optical components, and earth formation in geophysical exploration. In this study, we focus on simulating the electromagnetic (EM) wave propagation in the borehole with invaded zone to investigate the performance of the electromagnetic well-logging tools including induction and resistivity logging tools.

Since the induction logging tool was invented by Henri-Georges Doll in 1947 [1] and first successfully commercialized in 1956, many evolutionary developments have been brought into this technique, resulting in its extensive applications in the oil and gas [2], groundwater [3], mineral [4], and other geophysical explorations [5]. The induction logging tool in general consists of a transmitting coil and multiple receiving coils. The alternating current driven transmitter excites primary electromagnetic
Figure 1.1: The geometry of an orthogonal-plano-cylindrically layered medium: concentrically layered medium with multiple horizontal discontinuities. This model represents typical borehole model in earth formation with invaded zones field which induces eddy currents around the surrounding formation. A secondary electromagnetic field arises from the eddy currents is proportional to the electrical conductivity of the formation. The receiving coils can record the signal produced by the secondary field and further invert the conductivity of the formation.

The resistivity logging tool is an important branch of well logging, which use electrodes to inject a known current on the formation and to measures voltages at different points to determine the formation resistivity. Formation resistivity is the ability of a substance to impede the flow of an electrical current which is a key parameter in determining hydrocarbon saturation. Because the existence of the resistivity difference of formations filled with salty waters (low resistivity) and those filled with hydrocarbons (high resistivity), the resistivity measurements are used to obtain values of water saturation to help evaluate producibility of the formation.
In practice, the EM well-logging tools operate in a complex environment which exhibit inhomogeneities such as layered beds, dipping beds, and invaded zones and anisotropies with materials. It is essential to model and analyze the response of these tools in complex formations to obtain accurate formation evaluation. With the advantage of computing capabilities, computer simulation has become the most cost-effective way to investigate the tool response in complex formations. Hence, this study is devoted to develop an efficient approach to study the response of EM well-logging tools in complex geophysical environments.

1.2 Numerical Simulation for EM Well-Logging Tools

To investigate the performance of the logging tool, several 3D full wave numerical solvers have been adopted to model the well-logging tools and compute the response of electromagnetic waves in the borehole, such as 3D finite element method (FEM) [6–11], 3D finite volume method (FVM) [12], method of moments (MoM) [13], and finite difference time domain method (FDTD) [14].

Although all of the aforementioned full wave solvers have great flexibility to analyze various kinds of borehole, they are suffering from heavy computational complexity when dealing with three dimensional problems. For example, the complexity of 3D MoM solver is about $O(N^2)$, where $N$ is the number of unknowns. The contradiction becomes more acute when dealing with 3D formation parameters inversion (anisotropic permittivity and conductivity), especially in 3D formation profile imaging. It is important to propose an efficient and effective analytic or semi-analytic solution to compute the tool response in the oilfield.

Fortunately, the model of borehole and earth formation to be simulated is axisymmetric about the borehole axis. An orthogonal-plano-cylindrically layered (OPCL) structure which consists of materials stratified planarly and layered concentrically can appropriately present the model of borehole in oilfield. Thus, the electromagnetic
wave propagation in such an axisymmetric structure can also be solved by different kinds of 2D solvers based on the body of revolution (BOR) theory. The BOR theory has been applied to several numerical methods including the FDTD method [15], the MoM [16] and the FEM [17], [18] which greatly reduce the computational complexity and improve the efficiency. Furthermore, compared with these BOR based approaches, a semi-analytical solver, i.e., numerical mode matching method can further reduce the computational complexity by integrating analytical solution about \( z \) [19], [20]. This advantage of high efficiency will become more significant when the dimension in the \( z \) direction increases and when there exists a slim structure, e.g. hydraulic fracture.

1.3 Numerical Mode Matching Method

The propagation of EM wave in such a medium is in general difficult to be solved using conventional methods such as the finite element method (FEM), the method of moment (MoM), and the finite difference time domain (FDTD) method when the model is electrically large. The two-dimensional (2D) or two-and-a-half dimensional (2.5D) numerical mode matching method is a semi-analytical solver which combines a one-dimensional (1D) numerical implementation of the FEM in the radial direction and an analytical solution in the vertical direction to efficiently compute the axisymmetric or non-axisymmetric electromagnetic waves in such an isotropic OPCL medium [19–24]. This method has been extensively exploited in optical and wireless communication component design, for example with optical fiber and cylindrical waveguide, and in geophysical exploration, such as in boreholes drilled for oil, gas and mineral exploration.

Based on the conventional NMM method, several improvements have been made to satisfy new demands. Wang et al. [25] developed the NMM method to simulate the multi-component induction tool and efficiently compute the electromagnetic field
In horizontally stratified inhomogeneous transversely isotropic formations. However, as with all current NMM methods [20, 23, 24], a small radius was used to approach the origin of coordinates to avoid the singularity at \( \rho = 0 \) when implementing a 1D FEM along the radial direction. This approximation will reduce the accuracy of the calculation of EM fields on the \( z \) axis, which is critical in many circumstances: for example, the single well data at the borehole center is frequently required in oilfield exploration. In addition, without an absorbing boundary condition, a sufficiently large computational domain is required to assure accuracy in an unbounded medium, which greatly increases the computational complexity. To truncate the computational domain without reflection, the NMM with the perfectly matched layer for modeling induction tool is considered in [26]. However, so far the PML integrated NMM is only applicable to the axisymmetric case because no formulation on the vector wave equation for the 2.5D non-axisymmetric case has been derived.

In contrast to conventional NMM, Pai et al. [27, 28], Li and Shen [29] investigated the wave propagation in the same structure through obtaining vertical components of eigenmodes using sinusoidal basis functions and matching these modes in the radial direction. Hue [30] and Wang [31] improved the vertical NMM by integrating the PML along the vertical direction to analyze the coupled \( \text{TM}_z \) and \( \text{TE}_z \) modes generated by a tilted coil antenna in 3D cylindrically layered and transversely isotropic earth formations. Although the NMM based on the vertical components of eigenmodes is equivalently important and useful as the conventional NMM, the vertical NMM is underperformed in terms of computational complexity because of more PML elements needed for both sides along the \( z \) direction in the 1D FEM implementation. Moreover, all of the aforementioned NMM methods can only model transversely isotropic media and the source is confined to the \( z \) axis, except for that in [23, 24], which is only for isotropic media. We also note that the NMM method has been developed for 3D direct current problem [32].
1.4 Hydraulic Fracture Technique in Geophysical Exploration

Hydraulic fracturing is a widely used well-stimulation technique [33–37] in wells of low porosity and low permeability, such as shale oil, shale gas, tight oil and other unconventional hydrocarbon reservoirs, to increase the exposure to the producing formation so as to enhance the flow rate of oil and natural gas into the wellbore, therefore, to ultimately promote the productivity.

Knowledge of the information of hydrofracture, including width, length, thickness, location and inclination, will provide two aspects of benefits. First, based on these data, the altered induction data (conductivity or resistivity distribution) in the presence of fracture filled with conductive or resistive proppant (the object filled in the fracture) can be calibrated. On the other hand, the position of borehole perforation into the reservoir can be correctly and readily determined in cemented or cased boreholes to attain high productivity as that in the open hole. By far, the widely employed technique to diagnose the fracture is based on tiltmeter [33, 35], and microseismic mapping [36, 37]. However, these two approaches are relatively expensive and cannot acquire the profile of fractures after they have formed [33, 37].

To overcome the shortcomings mentioned above, several affordable tools based on electromagnetic techniques, for example the induction logging tool and resistivity logging tool [38, 39], can be utilized. In this study, we will discuss the application of the electromagnetic induction tool to study the fracture characteristics. The induction logging techniques have been employed to analyze the formation parameters: resistivity anisotropy, structural profile [40–42]. Recently, different induction tools have been designed to obtain the information of hydraulic fractures [9–11, 43, 44]. However, the operating frequency of these instrument are all above kHz, which restricts the usage of induction tools for large scale fractures. Although the low-frequency induction tool for fracture diagnostics has been investigated [45, 46], it is still uncer-
tain about the applicability of logging tool for fracture detection in practice, because the detectability of the weak scattered field from fractures is not validated in these two papers. Hence, it is important to experimentally and numerically investigate the performance of the induction tool for fracture detection.

Obviously, the aforementioned 3D full-wave numerical solvers can be adopted to investigate the performance of well-logging tools for fracture diagnostic. Although all of such full-wave solvers are flexible to analyze various kinds of borehole models with different kinds of fractures, they all require heavy computational complexity, especially when hydrofractures and casings are considered simultaneously in the model. Hence, it is important to develop an efficient analytical or semi-analytical solution to compute the tool response to fracture in oilfield.

Fortunately, the orthogonal-plano-cylindrically layered (OPCL) structure which comprises materials stratified planarly and layered concentrically [20, 23, 24, 47] can appropriately represent the borehole model with some special hydraulic fractures in oilfield. Thus, electromagnetic wave propagation in such an axisymmetric structure can be simulated by the aforementioned NMM method. Although this approach has been extensively employed to efficiently compute the electromagnetic field in the hydrocarbon exploration, it is the first time to introduce the NMM to determine the feasibility of the induction tool for hydrofracture detection. This study demonstrates that this method attains significantly high efficiency in the borehole simulation, especially when the dimension in the \( z \) direction increases or when there exists thin structures, e.g., hydraulic fracture (usually less than 1 cm thick). Because the NMM method can sophisticatedly treat thin structures without dramatically increasing the number of meshes, the computational efficiency is significantly improved when comparing with conventional 3D full-wave solvers.

In addition, it has been shown that the existence of casing in borehole will significantly affect the propagation of electromagnetic signal [48–52]. And most of the
aforementioned induction techniques [53] (except [45]) are only used to study the models in the absence of casings to avoid high computational complexity. Thus, it is valuable to make use of the inherent advantages of the NMM solver to efficiently determine the electromagnetic signal sensitivity in cased holes.

1.5 Dissertation Overview

This dissertation is organized as follows. In chapter 2 we elaborate the principle of the numerical mode matching method including Maxwell’s equation decomposition, eigenmode calculation and analytical solution solving. Several improvements have been introduced to increase the accuracy of the NMM method and to reduce computational time, and to further extend its applications. In chapter 3 several cases are presented to validate the accuracy and analyze the efficiency of the NMM method in homogeneous or cylindrically layered media. In chapter 4 the theory of numerical mode matching are illustrated from a simple model with only two horizontally stratified layers to a complex model with arbitrary multiple layers. In chapter 5 several more complex models with both cylindrical and horizontal discontinuity are simulated to validate the accuracy and robustness of the NMM solver. Then the NMM solver is employed to study the performance of EM well-logging tools including the induction and resistivity logging tools. In chapter 6 we apply the NMM solver to calculate the response of logging tool in several circumstances in the presence of hydraulic fracture. Then we compute the secondary field and normalized secondary field from the fracture with different thicknesses, lengths, conductivities to evaluate the possibility of using induction logging tool to diagnose the fracture. We also quantitatively analyze the sensitivity for different tool configuration and propose the method to design the induction logging tool. Then we study the effect of casing from the perspective of primary field and secondary field. Finally, a hybrid approach combining the NMM method and the BCGS-FFT solver based integral
equation method is proposed to investigate the scattering problem of the fracture propagating arbitrarily with complicated shape.
Decomposition of Maxwell’s Equation

The 2.5-D NMM method is a semi-analytical solver which combines a one-dimensional (1D) numerical implementation of the FEM/SEM in the radial direction \([19–21, 23–25, 47]\) and the Fourier serial summation in the azimuthal direction, plus an analytical solution in the vertical direction to efficiently calculate the electromagnetic response due to various kinds of sources. This chapter illustrates the application of the FEM/SEM for obtaining the eigenmode and analytical solutions for calculating the excited EM field in a cylindrically layered media.

2.1 Formulation

The model shown in Fig. 2.1 is a cylindrically layered media with infinite length in the vertical direction, which is a simplified model of that in Fig. 1.1. The first step of NMM is to numerically calculate the eigenmode which is completed in this model, preparing the eigenmode of the system for mode matching.
Figure 2.1: The geometry of a cylindrically layered medium: concentrically layered medium without horizontal discontinuities.

2.1.1 Maxwell’s Equations in the Cylindrical Coordinate System

In the media shown in Fig. 1.1 or Fig 2.1, Maxwell’s equations in frequency domain can be written as:

\[ \nabla \times \mathbf{E} = -j\omega \bar{\mu} \mathbf{H} - \mathbf{M} \]  
(2.1a)

\[ \nabla \times \mathbf{H} = j\omega \bar{\varepsilon} \mathbf{E} + \mathbf{J} \]  
(2.1b)

\[ \nabla \cdot \bar{\varepsilon} \mathbf{E} = \rho_c \]  
(2.1c)

\[ \nabla \cdot \bar{\mu} \mathbf{H} = \rho_m \]  
(2.1d)

where the time convention \( e^{j\omega t} \) is implied; the permittivity \( \bar{\varepsilon}(\rho) = \bar{\varepsilon}_r(\rho) + \frac{\bar{\sigma}_r(\rho)}{j\omega} \) and permeability \( \bar{\mu}(\rho) = \bar{\mu}_r(\rho) + \frac{\bar{\sigma}_m(\rho)}{j\omega} \) are both anisotropic and inhomogeneous in the cylindrical coordinate system within each layer. Furthermore, we assume they are both piecewise constant in each radial layer and they could have three different non-zero diagonal elements as shown in the following tensors:
Substituting equation (2.1b) into the curl of $\bar{\mu}^{-1}$ multiplied by equation (2.1a) and making use of Gauss’ law (2.1c), the vector wave equation governing the transverse components of electric field can be written as [20], [23]

$$\bar{\mu}_t' \nabla_t \times \mu_z^{-1} \nabla_t \times \mathbf{E}_t - \nabla_t (\bar{\epsilon}_z^{-1} \nabla_t \cdot \bar{\epsilon}_t \mathbf{E}_t) - \left( \omega^2 \bar{\mu}_t' \bar{\epsilon}_t + \frac{c^2}{\partial z^2} \right) \mathbf{E}_t = -\mathbf{F}_{et} \ (2.2)$$

where $\bar{\epsilon}_t(\rho) = \text{diag} \{ \epsilon_\rho, \epsilon_\phi \}$, $\bar{\mu}_t(\rho) = \text{diag} \{ \mu_\rho, \mu_\phi \}$, $\bar{\mu}_t'(\rho) = \text{diag} \{ \mu_\phi, \mu_\rho \}$, $\nabla_t = \hat{\rho} \frac{\partial}{\partial \rho} + \hat{\phi} \frac{\partial}{\partial \phi}$, and $\mathbf{E}_t$ contains the transverse components of electric field. The source term in equation (2.2) is

$$\mathbf{F}_{et} = \nabla_t \left( \frac{\rho_e}{\bar{\epsilon}_z} \right) + j \omega \bar{\mu}_t' J_t + \bar{\mu}_t' \nabla_t \times \hat{z} \mu_z^{-1} M_z + \bar{\mu}_t' \nabla_z \times \bar{\mu}_t^{-1} M_t \ (2.3)$$

Because the geometry in Fig. 2.1 solved by NMM is periodic in the $\phi$ direction, the EM field can be expanded in terms of a Fourier series, $\mathbf{E}_t(\rho, \phi, z) = \sum_n \mathbf{E}_t(\rho, n, z) e^{jn\phi}$. Thus, the derivative $\partial / \partial \phi$ with respect to $\phi$ can be replaced by $jn$, where $n$ is the azimuthal mode index. In addition, different from literature [20]- [23], the variables $\epsilon_\rho \mathbf{E}_\rho$ and $\mathbf{E}_\phi$ are expanded in terms of basis functions rather than those multiplied by $\rho$ to treat the singularity at $\rho = 0$ more conveniently. Expanding equation (2.2) term by term, we can obtain the following equations in a compact matrix form:

$$\mathbf{L}_s \cdot \mathbf{E}_t + \frac{c^2}{\partial z^2} \mathbf{L}_m \cdot \mathbf{E}_t = \mathbf{F}_{et} \ (2.4)$$
where the operator matrices are as below:

\[
L_s = \begin{bmatrix} w_{\ell\rho} & \ell_{\rho\phi} \\ w_{\ell\phi} & \ell_{\phi\phi} \end{bmatrix}, \quad L_m = \begin{bmatrix} w_{11} & 0 \\ 0 & \ell_{22} \end{bmatrix} \quad (2.5a)
\]

\[
E_t(\rho, z) = \begin{bmatrix} \varepsilon_{\rho r} E_\rho / w \\ E_\phi \end{bmatrix}, \quad F_{et} = \begin{bmatrix} F_{e\rho} \\ F_{e\phi} \end{bmatrix} \quad (2.5b)
\]

The coefficient \( w \) is introduced to ensure the mass matrix \( L_m \), yielding from the procedure of testing to \( L_m \), well-conditioned without affecting the governing wave equation (2.2). When the computational domain contains highly conductive media (large \( \varepsilon_{\rho r} \)) with low relative permeability, the mass matrix becomes ill-conditioned. This situation is often encountered in geophysical exploration, such as a borehole cased by metallic pipe. The scale factor can be carefully chosen to reduce the contrast of the elements in mass matrix. The operators in the matrix can be expressed as

\[
\ell_{\rho\rho}[\cdot] = \left[ \frac{\partial}{\partial \rho} \left( \frac{1}{\rho} \frac{\partial \rho}{\partial \varepsilon_{\rho r}} \frac{\partial \varepsilon_{\rho r}}{\partial \rho} + \frac{k_0^2 \mu_{\phi r}}{\varepsilon_{\rho r}} \right) \right][\cdot] \quad (2.6a)
\]

\[
\ell_{\rho\phi}[\cdot] = \left[ -\frac{\mu_{\phi r}}{\mu_{\rho r}} \frac{j \rho \varepsilon_{\rho r}}{\rho^2 \varepsilon_{\rho r}} \frac{\partial \varepsilon_{\rho r}}{\partial \rho} + j \rho \varepsilon_{\rho r} \frac{\partial \varepsilon_{\rho r}}{\partial \rho} \right][\cdot] \quad (2.6b)
\]

\[
\ell_{\phi\rho}[\cdot] = \left[ \frac{j \rho \mu_{\phi r}}{\rho^2 \mu_{\rho r} \varepsilon_{\rho r}} \frac{\partial \varepsilon_{\rho r}}{\partial \rho} - j \rho \mu_{\phi r} \frac{\partial \varepsilon_{\rho r}}{\partial \rho} \right][\cdot] \quad (2.6c)
\]

\[
\ell_{\phi\phi}[\cdot] = \left[ \frac{\partial}{\partial \phi} \left( \frac{1}{\rho} \frac{\partial \rho}{\partial \mu_{\phi r}} \frac{\partial \mu_{\phi r}}{\partial \rho} - \frac{n^2 \mu_{\phi r}}{\rho^2 \mu_{\phi r} \varepsilon_{\rho r}} + k_0^2 \varepsilon_{\phi r} \right) \right][\cdot] \quad (2.6d)
\]

\[
\ell_{11} = \frac{1}{\varepsilon_{\rho r}}, \quad \ell_{22} = \frac{1}{\mu_{\rho r}} \quad (2.6e)
\]

The \([\cdot]\) denotes dot product. The source terms are as following:

\[
F_{e\rho} = \frac{\partial \varepsilon_{\rho r}^{-1} \rho_e}{\partial \rho} + j \omega \mu_0 J_\rho + \frac{\mu_{\phi r}}{\rho} \frac{\partial (\mu_{\phi r}^{-1} M_z)}{\partial \phi} - \frac{\partial M_\phi}{\partial z} \quad (2.7a)
\]

\[
F_{e\phi} = \frac{1}{\rho \mu_{\rho r}} \frac{\partial \varepsilon_{\rho r}^{-1} \rho_e}{\partial \phi} + j \omega \mu_0 J_\phi - \frac{\partial (\mu_{\phi r}^{-1} M_z)}{\partial \rho} + \frac{1}{\mu_{\rho r}} \frac{\partial M_\rho}{\partial z} \quad (2.7b)
\]

where \( \varepsilon_{\rho r} = \varepsilon_{\rho} / \varepsilon_0 \) and similarly for all other relative permittivity and permeability components. \( J_\rho, J_\phi, J_z \) and \( M_\rho, M_\phi, M_z \) are the components of electric and magnetic
sources. The continuity equation governing the conservation law of electric charge is employed.

\[ \nabla \cdot \mathbf{J} = -j\omega \rho_e \]  

(2.8)

2.1.2 Flowchart of NMM

To efficiently solve the vector wave equation (2.4) containing the transverse electric field components, the numerical mode matching method (NMM) \[20,23,24,47\] is employed in this paper. In this approach, the 2.5D problem is decomposed into three 1D problems which can be easily and efficiently solved using different methods. These three 1D problems can be solved using a numerical computation of eigenmodes along the radial direction, a simple summation of harmonics (Fourier series) in the \(\phi\) direction and a 1D analytical solution along the \(z\) direction. Thus, the EM field generated by a source, for example a point or line source, in the 2D axisymmetric medium can be expressed as the summation of all of the eigenmodes for all azimuthal harmonic modes, as Equation (2.9). And each azimuthal harmonic can be expressed as the summation of the radial eigenmodes with expansion coefficients \[24\], as Equation (2.10).

\[ \mathbf{E}_t(\rho, \phi, z) = \sum_{n=-N}^{N} e^{jn(\phi - \phi_s)} \mathbf{E}_t(\rho, n, z) \]  

(2.9)

\[ \mathbf{E}_t(\rho, n, z) = \sum_{p=1}^{P} c_p(n)a(z)\mathbf{E}_p(\rho) \]  

(2.10)

where \(n\) is the azimuthal harmonic index, which is truncated at \(N\)-th mode to obtain a convergent result from the practical point of view. \(c_p(n)\) is the unknown expansion coefficient of the \(p\)-th eigenmode for the \(n\)-th harmonic which is determined by the excitation, and \(a(z)\) is the analytical solution about \(z\), which is attainable for several
NMM Approach

\[ E_z(\rho, \phi, z) = \sum_{n=-N}^{N} e^{j n \phi} \sum_{p=1}^{P} c_p(n) a(z) E_p(\rho) \]

1. Fourier series
   Axisymmetric about the z axis

2. Analytical solution
   Three kinds of solutions \( a(z) \)

3. Eigen mode
   1D FEM/SEM (PML) \( E_p(\rho) \)

4. Coefficient
   Determined by the source \( q(n) \)

5. Mode Matching
   Calculate the global reflection coefficient

**Figure 2.2:** The flowchart of NMM.

kinds of excitation; both will be derived later in this paper. \( E_p(\rho) \) is the \( p \)-th eigenmode which need to be found first by a 1D numerical solver along the \( \rho \) direction, \( P \) is the number of eigenmodes used in the numerical simulation. Fig. 7.2 shows the steps of the NMM solver.

### 2.2 Calculation of Eigenmodes

In this section, we will illustrate two kinds of numerical solvers, the finite/spectral element method, to calculate the eigenmodes of the model of cylindrical layered media, as shown in Fig. 2.1. The PML is introduced to reduce the size of computational domain, therefore to decrease the computational time. Meanwhile, the accuracy is increased. In addition, a singularity problem along the \( z \) axis is successfully resolved to further improve the results.
2.2.1 Finite Element Method

The eigenmodes of a concentrically layered medium with infinite length in the \( z \) direction as shown in Fig. 2.1 is first considered. The eigenfunctions of the propagating EM field in this medium have \( e^{-jk_{z}z} \) dependence. The \( p \)-th eigenfunction for the electric field satisfies the following equation:

\[
L_{s} \cdot \mathbf{E}_{p} - k_{zp}^{2}L_{m} \cdot \mathbf{E}_{p} = 0 \quad (2.11)
\]

The electric field \( \mathbf{E}_{p} \) can be approximated in a linear vector space spanned by the basis set \( \mathbf{S}(\rho) \). Hence

\[
\mathbf{E}_{p}(\rho) = \begin{bmatrix} \varepsilon_{\rho}E_{\rho} \\ E_{\phi} \end{bmatrix} = \mathbf{S}'(\rho) \cdot \mathbf{e}_{p} = \begin{bmatrix} \mathbf{S}_{1}(\rho) & 0 \\ 0 & \mathbf{S}_{2}(\rho) \end{bmatrix}^{t} \begin{bmatrix} \mathbf{e}_{\rho} \\ \mathbf{e}_{\phi} \end{bmatrix} \quad (2.12)
\]

where \( \mathbf{S}_{1}(\rho), \mathbf{S}_{2}(\rho) \) are column vectors containing \( S_{i1}(\rho), i = 1, \ldots, N_{\rho}, \) and \( S_{j2}(\rho), j = 1, \ldots, N_{\phi}, \) which are used to represent \( \varepsilon_{\rho}E_{\rho} \) and \( E_{\phi} \), respectively. We can choose polynomials, Bessel functions and other basis functions. To balance the accuracy and computational complexity, second order polynomials for each of the \( N_{e} \) elements are employed in the implementation. The three basis functions over each element are shown in Fig. 2.3, the expressions are as following

\[
\phi_{1}(\rho) = \frac{(\rho - \rho_{2i})(\rho - \rho_{2i+1})}{(\rho_{2i-1} - \rho_{2i})(\rho_{2i-1} - \rho_{2i+1})} \quad (2.13a)
\]

\[
\phi_{2}(\rho) = \frac{(\rho - \rho_{2i-1})(\rho - \rho_{2i+1})}{(\rho_{2i} - \rho_{2i-1})(\rho_{2i} - \rho_{2i+1})} \quad (2.13b)
\]

\[
\phi_{3}(\rho) = \frac{(\rho - \rho_{2i-1})(\rho - \rho_{2i})}{(\rho_{2i+1} - \rho_{2i-1})(\rho_{2i+1} - \rho_{2i})} \quad (2.13c)
\]

where \( i \) is the index of elements, starting with one.

Substitute (2.12) into (2.11), and left multiply the equation by a factor \( \rho \mathbf{S}(\rho) \) and integrate with respect to \( \rho \) from \( \rho_{\text{min}} \) to \( \rho_{\text{max}} \). Then the partial differential equation
Figure 2.3: Second order basis functions for $\mathbf{E}_t$: (a) Basis functions for $\epsilon_\rho E_\rho$. (b) Basis functions for $E_\phi$. Each element has three nodes. Basis functions $\phi_1$, $\phi_2$ and $\phi_3$ are over first element.

(2.11) will be converted into a linear equation

$$\mathbf{L}_s \cdot \mathbf{e}_p = k_{zp}^2 \mathbf{L}_m \cdot \mathbf{e}_p$$

(2.14)

where $\mathbf{L}_s = \langle \rho \mathbf{S}(\rho), \mathbf{L}_s \cdot \mathbf{S}'(\rho) \rangle$, $\mathbf{L}_m = \langle \rho \mathbf{S}(\rho), \mathbf{L}_m \cdot \mathbf{S}'(\rho) \rangle$ and the inner product of two functions $g(\rho)$ and $f(\rho)$ is defined as $\langle g(\rho), f(\rho) \rangle = \int_{\rho_{\text{min}}}^{\rho_{\text{max}}} g(\rho) f(\rho) d\rho$. The $N_\rho + N_\phi$ eigenvalues $k_{zp}^2$ and corresponding eigenvectors $\mathbf{e}_p$ can be found by solving equation (2.14).

The eigenmodes of magnetic field can be computed using the transverse components of electrical field. The following equation relates the transverse components of magnetic field $\mathbf{H}_t(\rho)$ and electric field $\mathbf{E}_t(\rho)$:

$$\mathbf{H}_t = \mathbf{M} \cdot \mathbf{E}_t$$

(2.15)

where $\mathbf{H}_t = \begin{bmatrix} H_\phi \\ \mu_\rho H_\rho \end{bmatrix}$ and the operator matrix is as follows

$$\mathbf{M} = \frac{1}{\omega k_z} \begin{bmatrix} \frac{k_z^2}{\mu_0} - \frac{\mu^2}{\mu_\rho \epsilon_\rho \mu_\rho} & \frac{j n}{\epsilon_\rho} \frac{\partial}{\partial \rho} - \frac{\partial^2}{\partial \rho^2} - \frac{k^2}{\mu_\phi} \\ \frac{j n}{\epsilon_\rho} \frac{\partial}{\partial \rho} - \frac{\partial^2}{\partial \rho^2} - \frac{k^2}{\mu_\phi} & \frac{\mu^2}{\mu_\rho \epsilon_\rho \mu_\rho} \end{bmatrix}$$

(2.16)
The magnetic field can also be represented in a linear vector space spanned by the same basis set, i.e. $H_t = S^t \cdot h_p$. By testing equation (2.15), we can obtain the eigenvectors $h_p$ for $H_t$

$$h_p = R^{-1}M \cdot e_p$$  

(2.17)

where $R = \langle \rho S(\rho), R \cdot S^t(\rho) \rangle$, $M = \langle \rho S(\rho), M \cdot S^t(\rho) \rangle$, and $R = \text{diag}\{1, 1/\mu_{pr}\}$.

By using the same method, we can obtain the eigenvectors of vertical components of electric and magnetic fields $e_z$ and $h_z$ for $E_z = S^t_3 \cdot e_z$ and $H_z = S^t_3 \cdot h_z$

$$e_{zp} = Q^{-1}N \cdot e_p$$  \hspace{1cm} (2.18a)

$$h_{zp} = Q^{-1}W \cdot e_p$$  \hspace{1cm} (2.18b)

where $Q = \langle \rho S_3(\rho), S_3^t(\rho) \rangle$, $N = \langle \rho S_3(\rho), N \cdot S^t_3(\rho) \rangle$, $W = \langle \rho S_3(\rho), W \cdot S^t_3(\rho) \rangle$, $S_3(\rho)$ is column vector containing $S_{k3}(\rho)$, $k = 1, ..., N_z$. And from equations (2.1a) and (2.1c), the operators $N$, $W$ are given by:

$$N = \frac{1}{j k_z \varepsilon_z} \left[ \frac{\hat{\varphi}}{\rho} \begin{array}{c} j n e_\phi \\ \rho \end{array} \right]$$  \hspace{1cm} (2.19a)

$$W = \frac{1}{j \omega \mu_z} \left[ \frac{j n}{\rho^2 e_{pr}} - \frac{1}{\rho} \frac{\partial}{\partial \rho} \right]$$  \hspace{1cm} (2.19b)

2.2.2 **Spectral Element Method**

Alternatively, the eigenmodes can be obtained by employing the spectral element method based on Galerkin formulation. Similarly, equation (2.11) is converted into a discretized linear system of equations (2.14) by using the Galerkin’s method. The eigenvalues $k_{zp}^2$ and corresponding eigenvectors $e_p$ can be found by solving equation (2.14). And $\bar{L}_s = \langle \rho S(\rho), L_s \cdot S^t(\rho) \rangle$, $\bar{L}_m = \langle \rho S(\rho), L_m \cdot S^t(\rho) \rangle$. However, the Lagrange polynomials are chosen as the basis functions in $S(\rho)$, whose expression are as following:

$$S_i(\xi) = \prod_{j=0, j \neq i}^{N} \frac{\xi - \xi_j}{\xi_i - \xi_j}$$  \hspace{1cm} (2.20)
where the $\xi_i$, $\xi_j$ are the permutations of the Gauss-Lobatto-Legendre points.

In the SEM method, the Gauss-Lobatto-Legendre (GLL) points and the corresponding weights are used for the quadrature to numerically evaluate the integration. The details of the implementation of the integration can be referred in literature [54, 55]. It has been demonstrated that the computational error of the SEM decreases exponentially as the order of polynomial, resulting in high order accuracy [54–57]. The resultant block-diagonal mass matrix from the application of orthogonal basis functions can be easily inverted which ensures the efficiency of this approach.

In addition, the spectral-element method can be considered as a special class of the general finite-element method with a different choice of nodal points and quadrature integration points. However, a fast convergence of solution to the exact solution is realized with fewer degrees of freedom of the structure in comparison with FEM. Therefore, the SEM-based NMM is the extension of conventional FEM-based NMM, which can achieve higher accuracy and computational efficiency. The extended numerical mode matching method can be abbreviated as spectral NMM or SNMM. The eigenmodes of other four field components, i.e. the transverse magnetic filed $H_t(\rho)$ and vertical electric and magnetic fields $E_z$ and $H_z$, can be derived from the transverse components of the electric field using the Galerkin’s formulation based SEM as those obtained in section 2.2.1.

2.2.3 Boundary Conditions

At the $z$ axis, where $\rho = 0$, the EM field values should retain continuity along any $\phi = \text{constant}$ line. Therefore, there are three kinds of conditions for different harmonics [18], [58], [59], [60] in the implementation of FEM or SEM. At the outermost point $\rho_{\text{max}}$, a perfectly electric conductor is imposed, for both bounded and unbounded problems with perfectly matched layers discussed later in section 2.2.5. It
is reasonable to set the PEC boundary condition at $\rho_{\text{max}}$, because the amplitude of the field at this position is negligible due to the absorption of PML. These boundary conditions restrict the choice of basis functions:

(a) For $n = 0$, the transverse components of EM field at the axis are equal to zero and the vertical components are in general nonzero:

\begin{align}
E_{\rho,0} &= (\nabla \times \mathbf{E})_{\rho,0} = 0 \\
E_{\phi,0} &= (\nabla \times \mathbf{E})_{\phi,0} = 0 \\
E_{z,0} &\neq 0, (\nabla \times \mathbf{E})_{z,0} \neq 0 
\end{align}

and $E_\phi(\rho_{\text{max}}) = 0$. Thus, the eigenvectors for the transverse components of electric field satisfy $e_\rho(0) = e_\phi(0) = 0$, and $e_\phi(\rho_{\text{max}}) = 0$. Therefore, the first basis functions of $\mathbf{S}_{1}(\rho)$ and $\mathbf{S}_{2}(\rho)$ satisfy that $S_{11}(\rho) = 0$ and $S_{12}(\rho) = 0$. The number of unknowns or non-zero basis functions for $\varepsilon_\rho E_\rho$ and $E_\phi$ are $N_\rho = qN_e$ and $N_\phi = qN_e - 1$, respectively, where $q$ is the order of basis functions and $N_e$ is the number of elements. The basis functions for each element for $\mathbf{E}_t$ are shown in Fig. 2.3, except the first basis function $\phi_1$ in first element for $\varepsilon_\rho E_\rho$ is unnecessary.

(b) For $n = \pm 1$, the EM field at the axis $\rho = 0$ satisfies:

\begin{align}
E_{\rho,\pm1} &= \mp j E_{\phi,\pm1} \\
(\nabla \times \mathbf{E})_{\rho,\pm1} &= \mp j (\nabla \times \mathbf{E})_{\phi,\pm1} \\
E_{z,\pm1} &= (\nabla \times \mathbf{E})_{z,\pm1} = 0
\end{align}

According to the boundary condition, i.e. $e_\rho(0) \neq 0$ and $e_\phi(0) \neq 0$, the first basis function should be nonzero, and $e_\phi(0)$ is dependent on $e_\rho(0)$. Therefore, $e_\phi(0)$ can be calculated directly by using equation (2.22a) without testing. The basic functions satisfy that $S_{11}(\rho) \neq 0$, $i = 1, \cdots, qN_e + 1$; and $S_{12}(\rho)$ is unnecessary; $S_{qN_e+1,2}(\rho)$ is also unnecessary as $E_\phi = 0$ at $\rho_{\text{max}}$; and $S_{j2} \neq 0$, $j = 2, \cdots, qN_e$. The number of unknowns or non-zero basis functions are $N_\rho = qN_e + 1$ and $N_\phi = qN_e - 1$, respectively. The basis functions for each element for $\mathbf{E}_t$ are shown in Fig. 2.3.
(c) For $|n| > 1$, the EM field at the axis satisfies:

$$E_{\rho,n} = E_{\phi,n} = E_{z,n} = 0$$  \hspace{1cm} (2.23a)

$$\nabla \times \mathbf{E}_{\rho,n} = (\nabla \times \mathbf{E})_{\phi,n} = (\nabla \times \mathbf{E})_{z,n} = 0$$  \hspace{1cm} (2.23b)

The outer boundary conditions at $\rho_{\text{max}}$ for $e_{\rho}$ and $e_{\phi}$ is exactly the same as case (a). Hence, the choice of basis functions follows the same way. Similarly, the basis functions used for the calculation of $h_{\rho}$, $h_{\phi}$, $h_{z}$ and $e_{z}$ are determined by the above boundary conditions.

### 2.2.4 Singularity Elimination at the Axis

When employing the 1D FEM/SEM to compute the eigenmodes for the harmonics $(n = \pm 1)$, two elements, i.e. $\langle \rho S_{11}(\rho), \ell_{\rho\rho} S_{11}(\rho) \rangle_{11}$ and $\langle \rho S_{11}(\rho), \ell_{\rho\phi} S_{12}(\rho) \rangle_{11}$, in the stiffness matrix are singular if operators (2.6a) and (2.6b) are directly used for calculation due to $\rho = 0$ in the first element. A sufficiently small value of $\rho_{\text{min}} \neq 0$ is commonly adopted to avoid this problem [20], [23], [24], [61]. However, this kind of approximation is not accurate enough, especially when the background medium is lossless. To solve this singularity problem, the vector wave equation with considering the boundary condition (2.22a) is applied here. Then the vector wave equation consisting of operators (2.6a) and (2.6b) together with (2.22a) for harmonic $n = \pm 1$ will reduce to:

$$\ell_{\rho\rho}[\epsilon_{\rho\rho} E_{\rho}] + \ell_{\rho\phi}[\epsilon_{\phi\rho} E_{\phi}] = 1 \frac{\partial}{\partial \rho} \frac{\partial}{\partial \rho} E_{\rho} + \frac{\mu_{\phi\rho}}{\mu_{\rho\rho}} \frac{1}{\rho} \frac{\partial}{\partial \rho} E_{\rho} + k_{\text{0}}^2 \mu_{\phi\rho} \epsilon_{\rho\rho} E_{\rho}$$  \hspace{1cm} (2.24)

We do not need to handle the singularity in the operator (2.6c) and (2.6d), because $e_{\phi}(0)$ can be computed from equation (2.22a) without testing. Similarly, the two elements in the first row of the operators matrix $M$ in equation (2.16) used for calculating $\mathbf{H}_t$ for $n = \pm 1$ will also reduce to:

$$m_{\rho\rho}[\epsilon_{\rho\rho} E_{\rho}] + m_{\rho\phi}[\epsilon_{\phi\rho} E_{\phi}] = \frac{1}{\omega k_z} \left( \frac{1}{\mu_{\rho\rho}} \frac{\partial}{\partial \rho} E_{\rho} + \frac{k_{\text{0}}^2}{\mu_{\rho\rho}} \epsilon_{\rho\rho} E_{\rho} \right)$$  \hspace{1cm} (2.25)
It is also unnecessary to consider the two elements in the second row of \( \mathbf{M} \), because \( H_\rho(0) \) can also be computed from equation (2.22b). The above procedure completely removes the singularity problem in the NMM method at \( \rho = 0 \).

2.2.5 PML Incorporation

For an unbounded medium, by incorporating the FEM/SEM with PML in the radial direction, the reflected wave from the outer boundary can be made negligible due to fast attenuation in the PML absorber. Thus, the accuracy of the yielded field can be greatly improved. It provides a robust way to calculate the field in lossless or low-loss media, and greatly reduce the number of elements for lossy media. By using the stretched-coordinate system [62]- [63], the permittivity and permeability in the PML can be easily chosen as:

\[
\varepsilon_{\text{PML}} = \varepsilon_{\tilde{\Lambda}} \quad (2.26a)
\]
\[
\mu_{\text{PML}} = \mu_{\tilde{\Lambda}} \quad (2.26b)
\]

where the diagonal tensor \( \tilde{\Lambda} = \tilde{\rho}\tilde{\rho}\Lambda_\rho + \tilde{\phi}\tilde{\phi}\Lambda_\phi + \tilde{\zeta}\tilde{\zeta}\Lambda_z \), and

\[
\Lambda_\rho = \frac{\tilde{\rho}e_z}{\rho e_\rho} ; \quad \Lambda_\phi = \frac{\rho e_\rho e_z}{\rho e_z} ; \quad \Lambda_z = \frac{\tilde{\rho}e_\rho}{\rho e_z} \quad (2.27)
\]

where \( e_\rho = a_\rho - jb_\rho \) and \( e_z = a_z - jb_z \) are the PML complex stretching variables [62] along the \( \rho \) and \( z \) directions; and \( \tilde{\rho} = \rho_0 + \int_{\rho_0}^{\rho} e_\rho d\rho \). The real and imaginary parts of \( e_\rho \) and \( e_z \) should be sophisticatedly chosen to optimize the computational time and accuracy. Second order polynomial profile is employed in our simulation as references [18, 63–66]. For the \( \rho \) direction, \( a_\rho = 1 + \alpha \left( \frac{\rho - \rho_m}{t_\rho} \right)^2 \), \( b_\rho = \beta \left( \frac{\rho - \rho_m}{t_\rho} \right)^2 \), where \( \rho_m \) and \( t_\rho \) are the starting point and thickness of the PML, \( \alpha \) and \( \beta \) should be chosen based on the frequency and thickness of PML. For the \( z \) direction, \( a_z = 1 \) and \( b_z = 0 \), because there is no need for complex stretching due to the infinite length in the \( z \) direction for the eigenvalue problem.

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2.3 EM Field Excited by Different Excitations

Once the eigenmodes in concentrically layered medium without any vertical discontinuity are found, the field excited can be calculated by summing up all of the eigenmodes with coefficients which are determined by the excitation. We will separately deduce the formulas for different types of source, including dipole, bipole (also named line current), and surface current.

2.3.1 EM Field Excited by a Magnetic Dipole

A magnetic dipole located at \( (\rho_s, \phi_s, z_s) \) can be expressed in terms of a Fourier series \[23\]

\[
M_{\rho s}, M_{\phi s} \text{ and } M_{z s} \text{ are the magnitudes of the magnetic dipole in three directions; and } M_{\rho n} = \frac{M_{\rho s}}{2\pi}, M_{\phi n} = \frac{M_{\phi s}}{2\pi}, M_{z n} = \frac{M_{z s}}{2\pi}. \]

Substitute equations (2.9), (2.12) and (2.28) into (2.4). The \( e^{jn(\phi - \phi_s)} \) dependence can be canceled out. Integrating the yielded equation left multiplied by a factor \( \rho S_p \rho q \) with respect to \( \rho \) from \( \rho_{\text{min}} \) to \( \rho_{\text{max}} \) and making use of equation (2.14), we can obtain the equation in a matrix form

\[
\left( \frac{\partial^2 a_p(z)}{\partial z^2} + k^2_{zp} a_p(z) \right) \tilde{L}_m \cdot V_n \cdot C_n = \bar{F}_{et} \tag{2.29}
\]

where \( V_n \) denotes the matrix containing \( P \) eigenvectors \( e_p \) of \( n \)-th harmonic; \( C_n \) is a column vector containing \( P \) expansion coefficients for corresponding eigenvectors; The excitation vector \( \bar{F}_{et} = \langle \rho S(\rho), F_{et} \cdot S'(\rho) \rangle = \left[ F'_{ep}; F'_{et} \right] \).

Consider the transverse components of a magnetic dipole, i.e. \( \textbf{M}(r) = \hat{\rho}M_{\rho s} + \)
The right hand side of (2.29) can be expressed as
\[
\vec{F}_{et} = \left[ -\frac{M_{\phi n}}{\mu_{n\rho}} S_1(\rho_s) \right] \delta'(z - z_s) = S_{et} \delta'(z - z_s) \tag{2.30}
\]

Separating the variables in equation (2.29), we can obtain
\[
\frac{\partial^2 a_p(z)}{\partial z^2} + k_{zp}^2 a_p(z) = \delta'(z - z_s) \tag{2.31}
\]

Solving equation (2.31) yields [61]
\[
a_p(z) = \frac{1}{2} e^{-j k_{zp}|z - z_s|} \text{sgn}(z - z_s) \tag{2.32}
\]

Combining all of the results yields the following semi-analytical solution of the transverse components of electric field
\[
E_t(\rho, n, z) = S(t)(\rho) \cdot V_n \cdot e^{-j K_{nz}|z - z_s|} \cdot B_n \tag{2.33}
\]
where \( K_{nz} \) is a diagonal matrix containing \( P \) eigenvalues of \( n \)-th harmonic, \( k_{zp} \), whose imaginary parts are nonpositive; and \( B_n \) contains the excitation coefficient, which is determined by the source. For transverse component of magnetic dipole, \( B_n = \frac{1}{2} C_n \text{sgn}(z - z_s) \), and \( C_n \) can be obtained by simply inversion of matrix as following
\[
C_n = (\vec{L}_m \cdot V_n)^{-1} S_{et} \tag{2.34}
\]

Similarly, we can obtain the following equations for the vertical component of a magnetic dipole, i.e. \( \mathbf{M}(r) = \hat{z} M_{zs} \):
\[
\vec{F}_{et} = \left[ \begin{array}{c} j n \mu_{\omega, \rho} M_{x n} \\ \mu_{x r} \rho_s \end{array} S_1(\rho_s) \right] \delta(z - z_s) = S_{et} \delta(z - z_s) \tag{2.35}
\]
\[
\frac{\partial^2 a_p(z)}{\partial z^2} + k_{zp}^2 a_p(z) = \delta(z - z_s) \tag{2.36}
\]

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The solution to equation (2.36) is

\[ a_p(z) = \frac{e^{-jk_{zp}|z-z_s|}}{-2j k_{zp}} \tag{2.37} \]

Then the transverse components of electric field can be calculated using equation (2.33) with \( B_n = \frac{1}{2} j K_n^{-1} \cdot C_n \).

Equation (2.30) and (2.35) are used for the cases whose source locates at \( \rho_s > 0 \).

As for \( \rho_s = 0 \) case, the formulas reduce to those in reference [61].

### 2.3.2 EM Field Excited by an Electric Dipole

An electric dipole located at \( (\rho_s, \phi_s, z_s) \) can be expressed in terms of a Fourier series

\[
\mathbf{J}(r) = \left( \hat{\rho} J_{\rho s} + \hat{\phi} J_{\phi s} + \hat{z} J_{z s} \right) \frac{\delta(\rho - \rho_s)}{\rho} \delta(\phi - \phi_s) \delta(z - z_s)
\]

\[
= \left( \hat{\rho} J_{\rho n} + \hat{\phi} J_{\phi n} + \hat{z} J_{z n} \right) \frac{\delta(\rho - \rho_s)}{\rho} \delta(z - z_s) \sum_{n=-\infty}^{\infty} e^{jn(\phi - \phi_s)} \tag{2.38}
\]

where \( J_{\rho s}, J_{\phi s} \) and \( J_{z s} \) are the magnitudes of the electric dipole in three directions; and \( J_{\rho n} = J_{\rho s}/2\pi, J_{\phi n} = J_{\phi s}/2\pi, J_{zn} = J_{zs}/2\pi \).

Similar to the magnetic dipole, the semi-analytical solution (2.33) can be used to compute the EM field excited by an electric dipole with different excitation vectors.

For a transverse electric dipole \( \mathbf{J}(r) = \hat{\rho} J_{\rho s}, \mathbf{B}_n = \frac{1}{2} j K_n^{-1} \cdot C_n \); the excitation vector \( \mathbf{F}_{et} = \langle \rho \mathbf{S}(\rho) \rangle = \langle \mathbf{F}'_{\rho \rho}, \mathbf{F}'_{\rho \phi} \rangle \) whose elements can be expressed as

\[
\mathbf{F}'_{\rho \rho} = \frac{j J_{\rho n}}{\omega \varepsilon_z} \left[ -\frac{S_1(\rho_s)}{\rho_s^2} + \frac{S_1'(\rho_s)}{\rho_s} + S_1''(\rho_s) \right] + j \mu_0 \varepsilon \mathbf{J}_{\rho n} \mathbf{S}_1(\rho_s) \tag{2.39a}
\]

\[
\mathbf{F}'_{\rho \phi} = \frac{J_{\rho n}}{\omega \varepsilon_z} \left[ -\frac{S_2(\rho_s)}{\rho_s^2} + \frac{S_2'(\rho_s)}{\rho_s} \right] \tag{2.39b}
\]
Similarly, we can obtain the following equations for a transverse electric dipole $\mathbf{J}(\mathbf{r}) = \hat{\phi} J_{\phi s}$:

$$F'_{e\rho} = \frac{nJ_{\phi n}}{\omega \varepsilon_z} \left[ \frac{S_1(\rho_s)}{\rho_s^2} + \frac{S_1'(\rho_s)}{\rho_s} \right]$$

(2.40a)

$$F'_{e\phi} = \frac{-jn^2 J_{\phi n} S_2(\rho_s)}{\rho_s^2 \omega \mu_\rho \varepsilon_z} + j\omega \mu_0 J_{\phi n} S_2(\rho_s)$$

(2.40b)

For a vertical electric dipole, i.e. $\mathbf{J}(\mathbf{r}) = \hat{z} J_{zs}$, $\mathbf{B}_n = \frac{1}{2} C_n \text{sgn}(z - z_s)$. And the expression of the excitation vector elements are as following:

$$F'_{e\rho} = \frac{jJ_{nz}}{\omega \varepsilon_z} \left[ \frac{S_1(\rho_s)}{\rho_s} + S_1'(\rho_s) \right]$$

(4.41a)

$$F'_{e\phi} = \frac{nJ_{nz} S_2(\rho_s)}{\omega \mu_\rho \varepsilon_z}$$

(4.41b)

### 2.3.3 Field Excited by a Bipole

In this section, the EM field excited by an electric or magnetic current line source are considered. The semi-analytic solution is the same as equation (2.33) for electric bipole and transverse components of magnetic bipole. And $\mathbf{B}_n = \frac{1}{2} j K_{zn}^{-1} \cdot \mathbf{C}_n$ for these five kinds of bipoles. Thus, we only need to derive the excitation vectors. In addition, we will derive the semi-analytical solution for the vertical component of magnetic current line source.

Consider the electric current line source along the $\rho$ direction, the bipole expression is $\mathbf{J}(\mathbf{r}) = \hat{\rho} J_{\rho s} [u(\rho - \rho_a) - u(\rho - \rho_b)]\delta(\phi - \phi_s)\delta(z - z_s)$. Substituting it into equation (2.7) and doing testing, we can obtain the excitation vector
\[
F'_{e\rho} = \frac{jJ_{\rho n}S_1(\rho)[\delta(\rho - \rho_a) - \delta(\rho - \rho_b)]}{\omega_z} |^N _0
\]

\[
-\frac{jJ_{\rho n}}{\omega_z} \left[ \frac{S_1(\rho_a)}{\rho_a} + S'_1(\rho_a) - \frac{S_1(\rho_b)}{\rho_b} - S'_1(\rho_b) \right]
\]

\[
+j\omega\mu_0 J_{\rho n} \int_0^{\rho_N} S_1(\rho)[u(\rho - \rho_a) - u(\rho - \rho_b)]d\rho
\]

\[
F'_{e\phi} = \int_0^{\rho_N} \rho S_2(\rho)F_{e\phi}(\rho)d\rho = \frac{nJ_{\rho n}}{\omega\mu_0\varepsilon_z} \left[ \frac{S_2(\rho_b)}{\rho_b} - \frac{S_2(\rho_a)}{\rho_a} \right]
\]

(2.42a)

(2.42b)

where \( J_{\rho n} = J_{\rho s}/2\pi \), and \( u(\rho - \rho_a) \) and \( u(\rho - \rho_b) \) are both Heaviside step function.

Consider the electric current line source along the \( \phi \) direction, the bipole expression is \( J(r) = \hat{\phi} \frac{J_{\phi s}}{\rho} [u(\phi - \phi_a) - u(\phi - \phi_b)]\delta(\rho - \rho_s)\delta(z - z_s) \). This expression can be expanded in terms of a Fourier series,

\[
J(r) = \hat{\phi} \frac{\delta(\rho - \rho_s)\delta(z - z_s)}{\rho} \sum_{n=-\infty}^{\infty} J_{\rho n} e^{jn(\phi - \phi_a + \phi_b)}
\]

(2.43)

where \( J_{\rho n} = J_{\rho s}\frac{\phi_b - \phi_a}{2\pi}\text{sinc}\left(n\frac{\phi_b - \phi_a}{2\pi}\right) \). Substituting it into equation [2.7], we can obtain the excitation vector

\[
F'_{e\rho} = \frac{nJ_{\rho n}}{\omega_z} \left[ \frac{S_1(\rho_s)}{\rho_s^2} + S'_1(\rho_s) \right]
\]

(2.44a)

\[
F'_{e\phi} = \frac{-jn^2J_{\rho n}S_2(\rho_s)}{\rho_s^2\omega_z\mu_\rho \varepsilon_z} + jnJ_{\phi n}\mu_0 S_2(\rho_s)
\]

(2.44b)

Consider the electric current line source along the \( z \) direction, the bipole expression is \( J(r) = \hat{z} \frac{J_{z s}}{\rho} [u(z - z_a) - u(z - z_b)]\delta(\rho - \rho_s)\delta(\phi - \phi_s) \). Substituting it into equation [2.7] and doing testing, we can obtain the excitation vector

\[
F'_{e\rho} = -\frac{jJ_{zn}}{\omega_z} \left[ \frac{S_1(\rho_s)}{\rho_s} + S'_1(\rho_s) \right]
\]

(2.45a)

\[
F'_{e\phi} = -\frac{\frac{nJ_{zn}}{\omega}\mu_\rho \varepsilon_z}{\rho_s\mu_\rho \varepsilon_z}
\]

(2.45b)

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where $J_{zn} = J_{zs}/2\pi$.

Consider the magnetic current line source along the $\rho$ direction, the bipole expression is $M(r) = \hat{\rho} \frac{M_{\rho s}}{\rho} [u(\rho - \rho_a) - u(\rho - \rho_b)] \delta(\phi - \phi_a) \delta(z - z_s)$. Substituting it into equation [2.7] and doing testing, we can obtain

$$F'_{\phi \rho} = \frac{M_{\rho n}}{\mu_0} \int_0^{\rho_N} S_2(\rho) [u(\rho - \rho_a) - u(\rho - \rho_b)] d\rho$$

(2.46)

where $M_{\rho n} = M_{\rho s}/2\pi$.

Consider the magnetic current line source along the $\phi$ direction, the bipole expression is $M(r) = \hat{\phi} \frac{M_{\phi s}}{\rho} [u(\phi - \phi_a) - u(\phi - \phi_b)] \delta(\rho - \rho_s) \delta(z - z_s)$. Substituting it into equation [2.7] and doing testing, we can obtain

$$F'_{\rho \phi} = - \int_0^{\rho_N} M_{\phi n} \delta(\rho - \rho_s) S_1(\rho) d\rho = -M_{\phi n} S_1(\rho_s)$$

(2.47)

where $M_{\phi n} = \frac{M_{\phi s}}{2\pi} \text{sinc} \left( \frac{\phi_b - \phi_a}{2\pi} \right)$.

Consider the magnetic current line source along the $z$ direction, the bipole expression is $M(r) = \hat{z} \frac{M_{zs}}{\rho} [u(z - z_a) - u(z - z_b)] \delta(\rho - \rho_s) \delta(\phi - \phi_s)$. Substituting it into equation [2.7], doing testing, and using the variable separation method, we can obtain the following equation

$$\frac{\partial^2 a_p(z)}{\partial z^2} + k_{zp}^2 a_p(z) = [u(z - z_a) - u(z - z_b)]$$

(2.48)

The solution to equation (2.48) is the integral of equation (2.37), which is the solution of equation (2.36).

The semi-analytic solution is

$$E_t(\rho, \phi, z) = \sum_{n=-\infty}^{\infty} e^{j n (\phi - \phi_s)} S_t^{n}(\rho) \cdot V_n \cdot K_n(z) \cdot B_n$$

(2.49)
where

$$K_n(z) = \begin{cases} 
  e^{-jK_n|z-z_b|} - e^{-jK_n|z-z_a|} & z > z_b \\
  e^{-jK_n|z-z_a|} - e^{-jK_n|z-z_b|} & z < z_a \\
  2I - e^{-jK_n|z-z_b|} - e^{-jK_n|z-z_a|} & z_a < z < z_b \end{cases} \quad (2.50)$$

And $B_n = \frac{1}{2}K_{zn}^{-2} \cdot C_n$.

Similarly, we can obtain the excitation vector for the vertical magnetic dipole:

$$F'_{e\rho} = \frac{jm\mu_\phi M_{zn}S_1(\rho_s)}{\rho_s\mu_{\rho z}}$$
$$F'_{e\phi} = \frac{M_{zn}}{\mu_{\rho z}} \left[ \frac{S_2(\rho_s)}{\rho_s} + S'_2(\rho_s) \right] \quad (2.51a, b)$$

### 2.3.4 Field Excited by a Surface Current Source

In this section, the EM field excited by an electric surface current source is considered and magnetic surface current source can be easily extended.

Consider the electric surface current source along the $\rho$ direction in the $\rho\phi$ plane, the source expression is $J = \hat{\rho} \frac{\hat{\rho}}{\rho} \left[ u(\rho-\rho_a) - u(\rho-\rho_b) \right] \left[ u(\phi-\phi_a) - u(\phi-\phi_b) \right] \delta(z-z_a)$. Substituting it into equation [2.7] and doing testing, we can obtain the excitation vector

$$F'_{e\rho} = \int_0^{\rho_N} \rho S_1(\rho) F_{e\rho}(\rho) d\rho = \frac{\hat{J}_m S_1(\rho) \left[ \delta(\rho - \rho_a) - \delta(\rho - \rho_b) \right]}{\omega_\rho |_{\rho_a}^{\rho_b}}$$
$$- \frac{j\hat{J}_m}{\omega_\rho} \left[ S_1(\rho_a) - S_1(\rho_b) \right] + S'_1(\rho_b) - S'_1(\rho_a) \quad (2.52a)$$

$$F'_{e\phi} = \int_0^{\rho_N} \rho S_2(\rho) F_{e\phi}(\rho) d\rho = \frac{n\hat{J}_m}{\omega_\rho} \left[ S_2(\rho_b) - S_2(\rho_a) \right]$$

where $J_{\rho m} = J_{\rho m} \frac{\phi_b - \phi_a}{2\pi} \operatorname{sinc} \left( \frac{n \phi_b - \phi_a}{2\pi} \right)$. And the semi-analytical solution is (2.37).

Consider the electric surface current source along the $\rho$ direction in the $\rho z$ plane, the source expression is $J = \hat{\rho} \frac{\hat{\rho}}{\rho} \left[ u(\rho-\rho_a) - u(\rho-\rho_b) \right] \left[ u(z-\rho_a) - u(z-\rho_b) \right] \delta(\phi-\phi_a)$.
Substituting it into equation [2.7] and doing testing, we can obtain $\mathbf{F}_{e\rho}'$ and $\mathbf{F}_{e\phi}'$ are the same as equation (2.42); and the semi-analytical solution is equation (2.50).

Consider the electric surface current source along the $\phi$ direction on the $\phi z$ plane, the source expression is $\mathbf{J}(r) = \hat{\phi} J_{0\phi} \delta(\rho - \rho_s)[u(\phi - \phi_a) - u(\phi - \phi_b)] [u(z - z_a) - u(z - z_b)]$. Substituting it into equation [2.7] and doing testing, we can obtain the excitation vector, $\mathbf{F}_{e\rho}'$ and $\mathbf{F}_{e\phi}'$ are the same as equations (2.44); and the analytical solution is (2.50).

Consider the electric surface current source along the $\phi$ direction in the $\rho \phi$ plane, the source expression is $\mathbf{J}(r) = \hat{\phi} J_{0\phi} [u(\rho - \rho_a) - u(\rho - \rho_b)] [u(\phi - \phi_a) - u(\phi - \phi_b)] \delta(z - z_s)$. Substituting it into equation [2.7] and doing testing, we can obtain the excitation vectors

$$\mathbf{F}_{e\rho}' = \frac{-n J_{\phi n} \mathbf{S}_1[u(\rho - \rho_a) - u(\rho - \rho_b)]}{\omega \varepsilon \rho} \bigg|_{0}^{\rho_N} + \frac{n J_{\phi n}}{\omega \varepsilon z} \int_{0}^{\rho_N} [u(\rho - \rho_a) - u(\rho - \rho_b)] \left( \frac{\mathbf{S}_1(\rho)}{\rho^2} + \frac{\mathbf{S}_1'(\rho)}{\rho} \right) d\rho \quad (2.53a)$$

$$\mathbf{F}_{e\phi}' = \frac{-j n^2 J_{\phi n}}{\omega \varepsilon \mu \varepsilon r} \int_{0}^{\rho_N} [u(\rho - \rho_a) - u(\rho - \rho_b)] \frac{\mathbf{S}_2(\rho)}{\rho^2} d\rho + j \omega \mu_0 J_{\phi n} \int_{0}^{\rho_N} [u(\rho - \rho_a) - u(\rho - \rho_b)] \mathbf{S}_2(\rho) d\rho \quad (2.53b)$$

and the analytical solution is (2.37)

Consider the electric surface current source along the $z$ direction in the $\rho z$ plane, the source expression is $\mathbf{J}(r) = \hat{z} J_{0z} [u(\rho - \rho_a) - u(\rho - \rho_b)] \delta(\phi - \phi_s) [u(z - z_a) - u(z - z_b)]$. 
Table 2.1: Summary of the analytical solutions and corresponding excitations

<table>
<thead>
<tr>
<th>Solution</th>
<th>Equation</th>
<th>Dipole</th>
<th>Bipole</th>
<th>Surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(2.32)</td>
<td>$M_{\rho}, M_{\phi}, J_z$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>(2.37)</td>
<td>$M_{\rho}, M_{\phi}, J_{\rho}, J_{\phi}, J_z$</td>
<td>$J_{\rho}(\rho\phi), J_{\phi}(\rho\phi), J_z(\rho z), J_z(\phi z)$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(2.50)</td>
<td>-</td>
<td>$M_z$</td>
<td>$J_{\rho}(\rho z), J_{\phi}(\phi z)$</td>
</tr>
</tbody>
</table>

Substituting it into equation [2.7], we can obtain the excitation vector

$$ F'_{\epsilon \rho} = \left. \frac{jJ_{zn}S_1(\rho)[u(\rho - \rho_a) - u(\rho - \rho_b)]}{\omega \varepsilon_z} \right|_{\rho = 0}^{\rho = N} $$

$$ F'_{\epsilon \phi} = -\frac{jJ_{zn}}{\omega \mu \varepsilon_z} \int_0^{\rho = N} \frac{[u(\rho - \rho_a) - u(\rho - \rho_b)]}{\rho} \left[ \frac{S_1(\rho_s)}{\rho_s} + S'_1(\rho_s) \right] d\rho $$

the analytical solution is (2.37).

Consider the electric surface current source along the $z$ direction in the $\phi z$ plane, the source expression is $J(\mathbf{r}) = \hat{z} \frac{J_{zn}}{\rho} \delta(\rho - \rho_a)[u(\phi - \phi_a) - u(\phi - \phi_b)][u(z - z_a) - u(z - z_b)]$.

Substituting it into equation [2.7] and doing testing, we can obtain the excitation vector $F'_{\epsilon \rho}$ and $F'_{\epsilon \phi}$

$$ F'_{\epsilon \rho} = \left. -\frac{jJ_{zn}}{\omega \varepsilon_z} \left[ \frac{S_1(\rho_s)}{\rho_s} + S'_1(\rho_s) \right] \right|_{\rho = 0}^{\rho = N} $$

$$ F'_{\epsilon \phi} = -\frac{nJ_{zn}S_2(\rho_s)}{\omega \rho_s \mu \varepsilon_z} $$

the analytical solution is (2.37).

The three analytical solutions and the corresponding excitation are summarized in Table 2.1.

2.4 Relationship between Opposite Harmonic Mode

The EM fields due to different types of source for the harmonics with opposite mode index in this section, we will determine the relationship between the harmonics with
opposite mode indices due to an excitation. Once the relationship is established, we only need to calculate one harmonic.

For positive mode, the eigenmode can be denoted as $V_n = \begin{bmatrix} E_{\rho 1} & E_{\rho 2} \\ E_{\phi 1} & E_{\phi 2} \end{bmatrix}$. Then the coefficient can be obtained by substituting these variables into equation (2.34)

$$C = \begin{bmatrix} C_{\rho} \\ C_{\phi} \end{bmatrix} = \begin{bmatrix} \bar{l}_{11} E_{\rho 1} & \bar{l}_{11} E_{\rho 2} \\ \bar{l}_{22} E_{\phi 1} & \bar{l}_{22} E_{\phi 2} \end{bmatrix}^{-1} \begin{bmatrix} S_{e\rho} \\ S_{e\phi} \end{bmatrix}$$ (2.56)

The inverse of the matrix is

$$\begin{bmatrix} X & Y \\ Z & T \end{bmatrix}^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \begin{bmatrix} a^{-1} + a^{-1}b k c a^{-1} & -a b k \\ - k c a^{-1} & k \end{bmatrix}$$ (2.57)

where $a, b, c, d$ denote four entries in equation (2.56), and $k = (d - c a^{-1} b)^{-1}$.

Then the EM field can be obtained by equation, without considering the $n$ independent variables $K_{zn}$ and basis function $S^t(\rho)$,

$$\begin{bmatrix} E_{\rho t} \\ E_{\phi t} \end{bmatrix} = \begin{bmatrix} E_{\rho 1} & E_{\rho 2} \\ E_{\phi 1} & E_{\phi 2} \end{bmatrix} \begin{bmatrix} X & Y \\ Z & T \end{bmatrix} \begin{bmatrix} S_{e\rho} \\ S_{e\phi} \end{bmatrix} = \begin{bmatrix} (E_{\rho 1} X + E_{\rho 2} Z) S_{e\rho} + (E_{\rho 1} Y + E_{\rho 2} T) S_{e\phi} \\ (E_{\phi 1} X + E_{\phi 2} Z) S_{e\rho} + (E_{\phi 1} Y + E_{\phi 2} T) S_{e\phi} \end{bmatrix}$$ (2.58)

Similarly, for negative mode, the eigenmode can be written as $V' = \begin{bmatrix} E_{\rho 1} & -E_{\rho 2} \\ -E_{\phi 1} & E_{\phi 2} \end{bmatrix}$, since the $\bar{l}_{\rho\phi}$ and $\bar{l}_{\phi\rho}$ are $n$-dependent. And the EM field can be expressed

$$\begin{bmatrix} E_{\rho t} \\ E_{\phi t} \end{bmatrix} = \begin{bmatrix} E_{\rho 1} & -E_{\rho 2} \\ -E_{\phi 1} & E_{\phi 2} \end{bmatrix} \begin{bmatrix} X & -Y \\ -Z & T \end{bmatrix} \begin{bmatrix} S'_{e\rho} \\ S'_{e\phi} \end{bmatrix} = \begin{bmatrix} (E_{\rho 1} X + E_{\rho 2} Z) S'_{e\rho} - (E_{\rho 1} Y + E_{\rho 2} T) S'_{e\phi} \\ - (E_{\phi 1} X + E_{\phi 2} Z) S'_{e\rho} + (E_{\phi 1} Y + E_{\phi 2} T) S'_{e\phi} \end{bmatrix}$$ (2.59)

$S'_{e\rho}, S'_{e\phi}, S_{e\rho}$ and $S_{e\phi}$ are determined by the source term in equation (2.7), the signs of different types of sources for opposite harmonics. The relationship of harmonic
Table 2.2: Relationship between harmonics with opposite indices for different types of excitation

<table>
<thead>
<tr>
<th>Field</th>
<th>$J_\rho$</th>
<th>$J_\phi$</th>
<th>$J_z$</th>
<th>$M_\rho$</th>
<th>$M_\phi$</th>
<th>$M_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_\rho$</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$E_\phi$</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$E_z$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$H_\rho$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$H_\phi$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$H_z$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

due to different types of excitation between opposite mode indices are summarized as Table 2.2. The computational time can be reduced by half, if this relationship is applied in the calculation.
3

Application

In this section, several examples are studied to demonstrate the accuracy and efficiency of the improved FEM or SEM based NMM solver. First, a two-layer circular waveguide is investigated to study the accuracy by comparing the eigenvalues with analytical solution. Then, the EM field due to a magnetic dipole in a homogeneous media is computed to illustrate the accuracy of the improved NMM solver. The importance of singularity elimination at the \( z \) axis is also demonstrated.

3.1 Two-layer Circular Waveguide

It is critical to first exam the eigenmodes which determine the accuracy that the NMM solver can achieve. The propagation constants \( k_z \) of a two-layer circular waveguide are computed to investigate the accuracy of the numerical solver, either the FEM or SEM solver. It is filled with two concentric layers of lossless materials. The inner and outer radii are 0.5 and 1.0 m, respectively. The relative permittivity and permeability combination of the inner and outer layers are (2,2) and (4,1). In the 1D FEM implementation, the number of radial elements is 40 and second order basis functions are adopted. And the number of radial elements is 20 in the 1D SEM
implementation and fourth order GLL basis functions are used. At the operating frequency of 300 MHz, there are seven supported propagating transverse modes and seven hybrid modes. Their propagation constants are listed in Table 3.1. And the analytical solution of eigenvalues of a two-layer cylindrical waveguide can be calculated by applying the boundary conditions. We can find that the SEM solver yields more accurate results than the FEM solver, though they have the same degree of freedom (DoF). Furthermore, Fig. 3.1 shows the relative error of the first four propagation constants of the first order harmonic versus the order of basis functions adopted in the SEM implementation. It can be observed that the relative error decreases exponentially as the order of basis functions.

### 3.2 EM Field in an Isotropic Homogeneous Media

#### 3.2.1 High Frequency Wave

To further check the performance of the NMM, the EM field excited by a magnetic dipole embedded in a lossless homogeneous media is computed. The relative per-

<table>
<thead>
<tr>
<th>Mode</th>
<th>Analytical Solution</th>
<th>SEM (4\textsuperscript{th}, 20)</th>
<th>SEM (2\textsuperscript{nd}, 40)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM\textsubscript{01}</td>
<td>4.90671636485579</td>
<td>4.90671674572941</td>
<td>4.90663091671909</td>
</tr>
<tr>
<td>TE\textsubscript{01}</td>
<td>7.73560728365156</td>
<td>7.73560732382138</td>
<td>7.73558130584931</td>
</tr>
<tr>
<td>TM\textsubscript{02}</td>
<td>8.82578904557653</td>
<td>8.82578905726824</td>
<td>8.82577881592224</td>
</tr>
<tr>
<td>TE\textsubscript{02}</td>
<td>10.2936838650846</td>
<td>10.2936838656807</td>
<td>10.2936817439107</td>
</tr>
<tr>
<td>TM\textsubscript{03}</td>
<td>11.3877959519884</td>
<td>11.3877959519851</td>
<td>11.3877958062899</td>
</tr>
<tr>
<td>TE\textsubscript{03}</td>
<td>12.0701093462126</td>
<td>12.0701093462101</td>
<td>12.0701093478188</td>
</tr>
<tr>
<td>TM\textsubscript{04}</td>
<td>12.2507889015481</td>
<td>12.2507889015445</td>
<td>12.2507889835687</td>
</tr>
<tr>
<td>HE\textsubscript{11}</td>
<td>5.28156079928254</td>
<td>5.2816491148633</td>
<td>5.28139987192794</td>
</tr>
<tr>
<td>EH\textsubscript{11}</td>
<td>7.57671988061177</td>
<td>7.57670522022346</td>
<td>7.57671349225912</td>
</tr>
<tr>
<td>HE\textsubscript{12}</td>
<td>8.9903125556716</td>
<td>8.99031694132314</td>
<td>8.99029257731675</td>
</tr>
<tr>
<td>EH\textsubscript{12}</td>
<td>10.2816456497761</td>
<td>10.2816435699151</td>
<td>10.2816488207815</td>
</tr>
<tr>
<td>HE\textsubscript{13}</td>
<td>11.5604328715301</td>
<td>11.5604329931874</td>
<td>11.5604321263447</td>
</tr>
<tr>
<td>EH\textsubscript{13}</td>
<td>11.957224626282</td>
<td>11.9572246520652</td>
<td>11.9572248369942</td>
</tr>
<tr>
<td>HE\textsubscript{14}</td>
<td>12.3950829367627</td>
<td>12.395082937022</td>
<td>12.3950829590577</td>
</tr>
</tbody>
</table>
mittivity and permeability of the homogeneous media is 8 and 2. The magnetic
dipole operating at 300 MHz locates at \((\rho_s, \phi_s, z_s) = (0.2m, 30^\circ, 0m)\). The magnetic
dipole has three components, i.e. \(\mathbf{M(r)} = (\hat{\rho} + \hat{\phi} + \hat{z}) \cdot \delta(\mathbf{r} - \mathbf{r}_s)\). The receivers
distribute along the radial direction, i.e., \(\mathbf{r}_r = (\rho_r, \phi_r, z_r) = (0.0 \sim 0.5m, 0^\circ, 0.2m)\).
In the NMM simulation, the computational domain along the \(\rho\) direction is 1 m
including a 0.25 m thickness PML . The number of elements, \(N_e\), in the 1D FEM
solver is 80 and first order basis functions are used. The number of elements, \(N_e\), in
the 1D SEM solver is 20 and fourth order basis functions are used. The sampling
density (SD) for these two simulations are both 20 points per wavelength (PPW).
Table 3.2 shows the comparison of the degree of freedom (DoF), the number of harmonics required to obtain converged results, the CPU time, and the relative error of EM field. The number of harmonics required to obtain converged results with required accuracy depends on the operating frequency and the location of excitation and observation points. Both simulations are performed in a desktop with an Intel(R) Core(TM) i5-2320 CPU@ 3.00 GHz. In addition, all of the simulations in this study are run in this computer. It can be seen that the NMM solvers based on both
Table 3.2: Comparison between FEM and SEM for Circular Waveguide

<table>
<thead>
<tr>
<th>Method</th>
<th>DoF</th>
<th>Harmonics</th>
<th>CPU time (s)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM (1\textsuperscript{st})</td>
<td>160</td>
<td>17</td>
<td>3.0</td>
<td>2.1</td>
</tr>
<tr>
<td>SEM (4\textsuperscript{th})</td>
<td>160</td>
<td>17</td>
<td>1.79</td>
<td>0.092</td>
</tr>
</tbody>
</table>

FEM and SEM are rather efficient. Furthermore, the SEM based NMM can give more accurate numerical results with the same sampling density and it can reduce the CPU time.

3.2.2 Extreme Low Frequency Wave

Because the extreme low frequency EM wave has important applications in the controlled source electromagnetic, through-casing induction tool etc, it is meaningful to investigate the performance of the NMM solver at this frequency range. The EM field excited by a magnetic dipole embedded in a lossy homogeneous media is computed. The relative permittivity and permeability of the homogeneous media are both 1, and the electrical conductivity is 0.01 S/m. The magnetic dipole operating at 100 Hz locates at \( \mathbf{r}_s = (\rho_s, \phi_s, z_s) = (20\text{m}, 0^\circ, 0\text{m}) \). The magnetic dipole has three components, i.e. \( \mathbf{M}(\mathbf{r}) = (1\hat{\rho} + 1\hat{\phi} + 1\hat{z}) \cdot \delta(\mathbf{r} - \mathbf{r}_s) \). In the NMM simulation, the computational domain along the \( \rho \) direction is 1000 m including the PML with a thickness of 500 m (around one skin depth at this operating frequency). The number of elements \( N_e \) in the 1D FEM solver is 35 and second order basis functions are used. The number of elements in the 1D SEM solver is 7 and sixth order basis functions are used. The receiver array are along the radial direction, i.e., \( \mathbf{r}_r = (\rho_r, \phi_r, z_r) = (0.0 \sim 80\text{m}, 0^\circ, 10\text{m}) \). Table 3.3 shows the comparison of the degree of freedom (DoF), the number of harmonics required to obtain converged results, the CPU time, and the relative error of EM field. It can be seen that the NMM solvers are quite robust and stable without low-frequency breakdown phenomenon.
Table 3.3: Comparison between FEM and SEM for Homogeneous Media

<table>
<thead>
<tr>
<th>Method</th>
<th>DoF</th>
<th>Harmonics</th>
<th>CPU time (s)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM (2nd)</td>
<td>139</td>
<td>30</td>
<td>2.27</td>
<td>0.46</td>
</tr>
<tr>
<td>SEM (6th)</td>
<td>83</td>
<td>30</td>
<td>0.85</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Moreover, the SEM based NMM are more accurate and efficient than that based on FEM.

3.3 EM Field in a Transverse Isotropic Homogeneous Media

To further study the performance of the improved NMM with singularity elimination, the EM field excited by a magnetic dipole embedded in a lossless homogeneous infinite space is computed. The homogeneous background has transversely isotropic relative permittivity and permeability, i.e. \( \varepsilon_{\rho r} = \varepsilon_{\phi r} = 2, \varepsilon_{zr} = 3 \) and \( \mu_{\rho r} = \mu_{\phi r} = 3, \mu_{zr} = 2 \). The magnetic dipole operating at 200 MHz locates at \( \mathbf{r}_s = (\rho_s, \phi_s, z_s) = (1 \text{ m}, 30^\circ, 0 \text{ m}) \). The receivers locate along the \( \rho \) direction, specifically at \( \mathbf{r}_r = (\rho_r, \phi_r, z_r) \), where \( \rho_r = 0 \sim 1.6 \text{ m}, \phi_r = 0^\circ \) and \( 180^\circ \), and \( z_r = 0.4 \text{ m} \). In the NMM simulation, the computational domain along the \( \rho \) direction is 2 m and the thickness of the PML is 0.4 m. The number of elements \( N_e \) in the 1D FEM solver is 40 and second order basis function is used. In this simulation, thirty-one harmonics are sufficient for convergence to one percent of the magnitude of electric field. And the total CPU times are 2.0 seconds. Figs. 3.2 and 3.3 show the comparison of electric and magnetic fields which are obtained by (a) NMM with boundary conditions applied (solid lines); (b) NMM without boundary conditions (dashed and dotted lines); (c) COMSOL (crosses and pluses). In addition, each figure shows the real part of field components (solid line with pluses and dots) and imaginary part of field components (solid line with crosses and dash). The positive and negative values of \( \rho \) stand for two directions corresponding to \( \phi_r = 0^\circ \) and \( \phi_r = 38^\circ \).
Figure 3.2: Electric field excited by a magnetic dipole, $E_{\rho}, E_{\phi}$ and $E_z$ versus $\rho$.

Figure 3.3: Magnetic field excited by a magnetic dipole, $H_{\rho}, H_{\phi}$ and $H_z$ versus $\rho$

$180^\circ$, and they are along a line. The real and imaginary parts of the excited electric and magnetic fields from NMM with boundary conditions applied and singularity removed at $\rho = 0$ in Figs. 3.2 and 3.3 show very good agreement with the results obtained from COMSOL. We also observe a significant jump which deviates from the correct curves if an approximation is made at $\rho = 0$ when performing the NMM solver without removing the singularity.
In chapter 2, we have elaborated the approach to numerically obtain the eigenmode in a cylindrically layered media, which is the first step of the NMM method. In this chapter, we will illustrate the principle of mode matching to compute the coupled non-axisymmetric EM fields excited by an imposed source in a planarly stratified and cylindrically layered media, as shown in Fig. 1.1. It represents many models in engineering applications, including cylindrical waveguides, integrated optical components, and borehole in an invaded formation.

4.1 EM Field in a Two-Layer Medium

For a two-layer model, as shown in Fig. 4.2, reflection and transmission occur at the junction discontinuity, \( z = d \). For example, if the source is in region 1 at \( z = z_s \), by the theory of mode conversion [20], the field in region 1 can be written as

\[
\begin{align*}
E_{t1}(\rho, z) &= E_{t1}(\rho) \cdot \left[ e^{-jK_{z1}|z-z_s|} \cdot B_1 + e^{-jK_{z1}(z-d)} \cdot R_{12} \cdot e^{-jK_{z1}(z_s-d)} \cdot B_1 \right] \quad (4.1a) \\
H_{t1}(\rho, z) &= H_{t1}(\rho) \cdot \left[ e^{-jK_{z1}|z-z_s|} \cdot B_1 - e^{-jK_{z1}(z-d)} \cdot R_{12} \cdot e^{-jK_{z1}(z_s-d)} \cdot B_1 \right] \quad (4.1b)
\end{align*}
\]
Figure 4.1: The geometry of a concentrically layered medium with a horizontal discontinuity.

The additional term in equation (4.1) is due to the reflection from the discontinuity. The meaning of different variables are explained in section 2.3. However, the notation $n$, which denotes the index of harmonics, is dropped for brevity. In addition, the subscript 1 and 2 in following equations denote the EM field associated with region 1 and 2 respectively.

In region 2, the transmitted wave can be written as

$$E_{t2}(\rho, z) = E_{t2}(\rho) \cdot e^{-jK_{z2}(d-z)} \cdot T_{12} \cdot e^{-jK_{z1}(d-z)} \cdot B_1$$

(4.2a)

$$H_{t2}(\rho, z) = H_{t2}(\rho) \cdot e^{-jK_{z2}(d-z)} \cdot T_{12} \cdot e^{-jK_{z1}(d-z)} \cdot B_1$$

(4.2b)

$R_{12}$ and $T_{12}$ are reflection and transmission matrices which are unknown. The boundary condition for the continuity at $z = d$ of tangential electrical and magnetic fields between two media will be applied to seek the reflection and transmission matrices.

$$\bar{\epsilon}_1^{-1}E_1^t(\rho, d) = \bar{\epsilon}_2^{-1}E_2^t(\rho, d)$$

(4.3a)

$$\bar{\mu}_1^{-1}H_1^t(\rho, d) = \bar{\mu}_2^{-1}H_2^t(\rho, d)$$

(4.3b)
Figure 4.2: The geometry of a concentrically layered medium with two horizontal discontinuities.

Solving equations (4.3) yields the reflection and transmission coefficient

\[
R_{12} = \frac{\tilde{\epsilon}_2^{-1}E_{t2}(\rho)\tilde{\mu}_1^{-1}H_{t1}(\rho) - \tilde{\epsilon}_1^{-1}E_{t1}(\rho)\tilde{\mu}_2^{-1}H_{t2}(\rho)}{\tilde{\epsilon}_2^{-1}E_{t2}(\rho)\tilde{\mu}_1^{-1}H_{t1}(\rho) + \tilde{\epsilon}_1^{-1}E_{t1}(\rho)\tilde{\mu}_2^{-1}H_{t2}(\rho)} \quad (4.4a)
\]

\[
T_{12} = \frac{2\tilde{\epsilon}_1^{-1}E_{t1}(\rho)\tilde{\mu}_1^{-1}H_{t1}(\rho)}{\tilde{\epsilon}_2^{-1}E_{t2}(\rho)\tilde{\mu}_1^{-1}H_{t1}(\rho) + \tilde{\epsilon}_1^{-1}E_{t1}(\rho)\tilde{\mu}_2^{-1}H_{t2}(\rho)} \quad (4.4b)
\]

where \(\tilde{\epsilon}_i^{-1} = \text{diag}\{\epsilon_{pri}^{-1}(\rho), 1\}\) and \(\tilde{\mu}_i^{-1} = \text{diag}\{\mu_{pri}^{-1}(\rho), 1\}\), where \(\epsilon_{pri}(\rho)\) and \(\mu_{pri}(\rho)\) are the radial component of the relative permittivity and permeability of \(i\)-th region.

Therefore, the EM fields for all regions are derived.

4.2 EM Field in a Three-Layer Medium

For a three-layer problem as shown in Fig. 4.2, the source located at region 1 will excite the EM field over the entire domain. And the EM field can be expressed in terms of eigenmode and reflection matrices. The field in region 1 is
\[ E_{t1}(\rho, z) = E_{t1}(\rho) \cdot \left[ e^{-jK_{z1}z} \cdot B_1 + e^{-jK_{z1}(z-d_1)} \cdot \tilde{R}_{12} \cdot e^{-jK_{z1}(z_s-d_1)} \cdot B_1 \right] \quad (4.5) \]

where \( \tilde{R}_{12} \) is a generalized reflection matrices that incorporates subsurface reflections. And the field in region 2 is

\[ E_{t2}(\rho, z) = E_{t2}(\rho) \cdot \left[ e^{jK_{z2}(z-d_2)} + e^{-jK_{z2}(z-d_2)} \cdot R_{23} \right] \cdot A_2 \quad (4.6) \]

The field in region 3 is

\[ E_{t3}(\rho, z) = E_{t3}(\rho) e^{jK_{z3}(z-d_2)} \cdot A_3 \quad (4.7) \]

The unknowns \( A_2 \) and \( \tilde{R}_{12} \) need to be determined by imposing constraint condition at boundary \( z = d_1 \) [20]. At this boundary, the reflected wave is the sum of the reflected wave of the downward wave in region 1 and the transmitted wave of the upward wave from region 2.

\[ \tilde{R}_{12} \cdot e^{-jK_{z1}(z_s-d_1)} \cdot B_1 = \tilde{R}_{12} \cdot e^{-jK_{z1}(z_s-d_1)} \cdot B_1 + T_{21} \cdot e^{-jK_{z2}(d_1-d_2)} \cdot R_{23} \cdot A_2 \quad (4.8) \]

In addition, the downward wave at boundary \( z = d_1 \) is the sum of the transmitted wave of the downward wave from region 1 and the reflected wave of the upward wave from region 2.

\[ e^{jK_{z2}(d_1-d_2)} \cdot A_2 = T_{12} \cdot e^{-jK_{z1}(z_s-d_1)} \cdot B_1 + \tilde{R}_{21} \cdot e^{-jK_{z2}(d_1-d_2)} \cdot R_{23} \cdot A_2 \quad (4.9) \]

Solving equation (4.8) and (4.9), one can obtain the generalized reflection matrix, \( \tilde{R}_{12} \), and field amplitude in region 2, \( A_2 \).

\[ \tilde{R}_{12} = R_{12} + T_{21} \cdot e^{-jK_{z2}(d_1-d_2)} \cdot R_{23} \cdot e^{-jK_{z2}(d_1-d_2)} \cdot P_{22} \cdot T_{12} \quad (4.10) \]

\[ A_2 = e^{-jK_{z2}(d_1-d_2)} \cdot P_{22} \cdot T_{12} \cdot e^{-jK_{z2}(z_s-d_1)} \cdot B(\rho_s) \quad (4.11) \]
where
\[ P_2 = \left[ I - R_{21} \cdot e^{-jK_{2z}(d_1-d_2)} \cdot R_{23} \cdot e^{-jK_{2z}(d_1-d_2)} \right]^{-1} \] (4.12)

And the amplitude of field in region 3, \( A_3 \), is the transmission of \( A_2 \), which is
\[ A_3 = T_{23} \cdot A_2 \] (4.13)

### 4.3 EM Field in a Multiple-Layer Medium

If we consider an \( N \)-layer model, the excited EM field can be expressed as eigenmode and generalized reflection matrices. By generalizing equation (4.10), the generalized reflection matrices can be written as
\[ \tilde{R}_{m,m+1} = R_{m,m+1} + T_{m+1,m} \cdot e^{-jK_{z,m+1}(d_{m}-d_{m+1})} \cdot \tilde{R}_{m+1,m+2} \]
\[ \cdot e^{-jK_{z,m+1}(d_{m}-d_{m+1})} \cdot P_{m+1,-} \cdot T_{m,m+1} \] (4.14)

And the generalized \( P_{m+1,-} \) is given as
\[ P_{m+1,-} = \left[ I - R_{m+1,m} \cdot e^{-jK_{z,m+1}(d_{m}-d_{m+1})} \cdot \tilde{R}_{m+1,m+2} \cdot e^{-jK_{z,m+1}(d_{m}-d_{m+1})} \right]^{-1} \] (4.15)

The generalized reflection matrices \( \tilde{R}_{m,m+1} \) can be recursively found by starting with \( R_{N,N+1} = 0 \). A similar recursive formula can be derived to obtain the generalized reflection matrices from region of higher index to that of low index, as following
\[ \tilde{R}_{m,m-1} = R_{m,m-1} + T_{m-1,m} \cdot e^{-jK_{z,m-1}(d_{m-2}-d_{m-1})} \cdot \tilde{R}_{m-1,m-2} \]
\[ \cdot e^{-jK_{z,m-1}(d_{m-2}-d_{m-1})} \cdot P_{m-1,+} \cdot T_{m,m-1} \] (4.16)

And the generalized \( P_{m-1,+} \) is given as
\[ P_{m-1,+} = \left[ I - R_{m-1,m} \cdot e^{-jK_{z,m-1}(d_{m-2}-d_{m-1})} \cdot \tilde{R}_{m-1,m-2} \cdot e^{-jK_{z,m-1}(d_{m-2}-d_{m-1})} \right]^{-1} \] (4.17)
The generalized reflection matrices $\tilde{R}_{m,m-1}$ can be recursively found by starting with $\tilde{R}_{10} = 0$.

The fields in the $m$-th region where source lies in are the superposition of upward and downward fields, hence, it is given as:

$$E_{tm}(\rho, z) = E_{tm}(\rho) \cdot \left[ e^{-jK_{zm}|z-z_s|} \cdot B_m + e^{-jK_{zm}z} \cdot U_m + e^{jK_{zm}z} \cdot D_m \right] \quad (4.18)$$

Two constraint conditions at boundaries $z = d_m$ and $z = d_{m-1}$ are imposed to derive the unknowns $U_m$ and $D_m$. At boundary $z = d_m$, the upward wave is the reflected wave of the sum field of primary field and downward wave. Thus,

$$e^{-jK_{zm}d_m} \cdot U_m = \tilde{R}_{m,m+1} \cdot \left[ e^{-jK_{zm}(z_s-d_m)} \cdot B_m + e^{jK_{zm}d_m} \cdot D_m \right] \quad (4.19)$$

At boundary $z = d_{m-1}$, the downward wave is the reflected wave of the sum field of primary field and upward wave. Thus,

$$e^{jK_{zm}d_{m-1}} \cdot D_m = \tilde{R}_{m,m-1} \cdot \left[ e^{-jK_{zm}(d_{m-1}-z_s)} \cdot B_m + e^{-jK_{zm}d_{m-1}} \cdot U_m \right] \quad (4.20)$$

Solving equations (4.19) and (4.20), one can obtain the expression for $U_m$ and $D_m$ as following

$$U'_m = e^{-jK_{zm}d_m} \cdot U_m = \tilde{M}_{m+} \cdot \tilde{R}_{m,m+1} \cdot \left[ e^{-jK_{zm}|d_m-z_s|} \cdot B_m \
+ e^{-jK_{zm}(d_{m-1}-d_m)} \cdot \tilde{R}_{m,m-1} \cdot e^{-jK_{zm}|d_{m-1}-z_s|} \cdot B_m \right] \quad (4.21)$$

$$D'_m = e^{jK_{zm}d_m} \cdot D_m = \tilde{M}_{m-} \cdot \tilde{R}_{m,m-1} \cdot \left[ e^{-jK_{zm}|d_{m-1}-z_s|} \cdot B_m \
+ e^{-jK_{zm}(d_{m-1}-d_m)} \cdot \tilde{R}_{m,m+1} \cdot e^{-jK_{zm}|d_{m-1}-z_s|} \cdot B_m \right] \quad (4.22)$$

where

$$\tilde{M}_{m+} = \left[ I - \tilde{R}_{m,m+1} \cdot e^{-jK_{zm}(d_{m-1}-d_m)} \cdot \tilde{R}_{m,m-1} \cdot e^{-jK_{zm}|d_{m-1}-z_s|} \right]^{-1} \quad (4.23)$$
\[
\tilde{M}_{m-} = \left[ I - \tilde{R}_{m,m-1} \cdot e^{-jK_{m,z}(d_{m-1}-d_m)} \cdot \tilde{R}_{m,m+1} \cdot e^{-jK_{m,z}(d_{m-1}-d_m)} \right]^{-1}
\] (4.24)

And for \( z \geq z_s \) in \( m \)-th region, the other field components can also be written as

\[
E_{zm}(\rho, z) = E_{zm}(\rho) \cdot \left[ -e^{-jK_{zm}|z-z_s|} \cdot B_m - e^{-jK_{zm}z} \cdot U_m + e^{jK_{zm}z} \cdot D_m \right]
\] (4.25a)

\[
H_{tm}(\rho, z) = H_{tm}(\rho) \cdot \left[ -e^{-jK_{zm}|z-z_s|} \cdot B_m - e^{-jK_{zm}z} \cdot U_m + e^{jK_{zm}z} \cdot D_m \right]
\] (4.25b)

\[
H_{zm}(\rho, z) = H_{zm}(\rho) \cdot \left[ e^{-jK_{zm}|z-z_s|} \cdot B_m + e^{-jK_{zm}z} \cdot U_m + e^{jK_{zm}z} \cdot D_m \right]
\] (4.25c)

And for \( z < z_s \) in \( m \)-th region, the EM field components can also be written as

\[
E_{tm}(\rho, z) = E_{tm}(\rho) \cdot \left[ e^{-jK_{zm}|z-z_s|} \cdot B_m + e^{-jK_{zm}z} \cdot U_m + e^{jK_{zm}z} \cdot D_m \right]
\] (4.26a)

\[
E_{zm}(\rho, z) = E_{zm}(\rho) \cdot \left[ e^{-jK_{zm}|z-z_s|} \cdot B_m - e^{-jK_{zm}z} \cdot U_m + e^{jK_{zm}z} \cdot D_m \right]
\] (4.26b)

\[
H_{tm}(\rho, z) = H_{tm}(\rho) \cdot \left[ e^{-jK_{zm}|z-z_s|} \cdot B_m - e^{-jK_{zm}z} \cdot U_m + e^{jK_{zm}z} \cdot D_m \right]
\] (4.26c)

\[
H_{zm}(\rho, z) = H_{zm}(\rho) \cdot \left[ e^{-jK_{zm}|z-z_s|} \cdot B_m + e^{-jK_{zm}z} \cdot U_m + e^{jK_{zm}z} \cdot D_m \right]
\] (4.26d)

To calculate the field in region \( n > m \), we should first obtain the amplitude of the downward wave at boundary \( z = d_m \), which is

\[
A_{Dm} = e^{-jK_{zm}|d_{m-1}-d_m|} \cdot B + D_m'
\] (4.27)

And the amplitude of the field in \( n \)-th region can be derived by using the following recursive relation

\[
A_n = e^{-jK_{zn}|d_{n-1}-d_n|} \cdot A'_n
\] (4.28)

with \( A_m = A_{Dm} \) as the starting variable. Where

\[
A'_n = P_{n,-} \cdot T_{n-1,n} \cdot A_{n-1}
\] (4.29)
Hence, the field in $n$-th region can be written as
\[
E_{tn}(\rho, z) = E_{tn}(\rho) \cdot \left[e^{-jK_{zn}(d_{n-1}-z)} \cdot A_n + e^{-jK_{zn}(z-d_n)} \cdot \tilde{R}_{n,n+1} \cdot A_n \right] \quad (4.30a)
\]
\[
E_{zn}(\rho, z) = E_{zn}(\rho) \cdot \left[e^{-jK_{zn}(d_{n-1}-z)} \cdot A_n - e^{-jK_{zn}(z-d_n)} \cdot \tilde{R}_{n,n+1} \cdot A_n \right] \quad (4.30b)
\]
\[
H_{tn}(\rho, z) = H_{tn}(\rho) \cdot \left[e^{-jK_{zn}(d_{n-1}-z)} \cdot A_n - e^{-jK_{zn}(z-d_n)} \cdot \tilde{R}_{n,n+1} \cdot A_n \right] \quad (4.30c)
\]
\[
H_{zn}(\rho, z) = H_{zn}(\rho) \cdot \left[e^{-jK_{zn}(d_{n-1}-z)} \cdot A_n + e^{-jK_{zn}(z-d_n)} \cdot \tilde{R}_{n,n+1} \cdot A_n \right] \quad (4.30d)
\]

Similarly, to calculate the field in region $n < m$, we should first obtain the amplitude of the upward wave at boundary $z = d_{m-1}$, which is
\[
A_{Um} = e^{-jK_{zm}(d_{m-1}-z_m)} \cdot B + U_m' \quad (4.31)
\]

And the amplitude of the field in $n$-th region can be derived by using the following recursive relation
\[
A_n = e^{-jK_{zn}(d_{n-1}-d_n)} \cdot A_n' \quad (4.32)
\]
with $A_m = A_{Um}$ as the starting variable. Where
\[
A_n' = P_{n,+} \cdot T_{n+1,n} \cdot A_{n+1} \quad (4.33)
\]

Hence, the field in $n$-th region can be written as
\[
E_{tn}(\rho, z) = E_{tn}(\rho) \cdot \left[e^{-jK_{zn}(d_{n-1}-z)} \cdot \tilde{R}_{n,n-1} \cdot A_n + e^{-jK_{zn}(z-d_n)} \cdot A_n' \right] \quad (4.34a)
\]
\[
E_{zn}(\rho, z) = E_{zn}(\rho) \cdot \left[e^{-jK_{zn}(d_{n-1}-z)} \cdot \tilde{R}_{n,n-1} \cdot A_n - e^{-jK_{zn}(z-d_n)} \cdot A_n' \right] \quad (4.34b)
\]
\[
H_{tn}(\rho, z) = H_{tn}(\rho) \cdot \left[e^{-jK_{zn}(d_{n-1}-z)} \cdot \tilde{R}_{n,n-1} \cdot A_n - e^{-jK_{zn}(z-d_n)} \cdot A_n' \right] \quad (4.34c)
\]
\[
H_{zn}(\rho, z) = H_{zn}(\rho) \cdot \left[e^{-jK_{zn}(d_{n-1}-z)} \cdot \tilde{R}_{n,n-1} \cdot A_n + e^{-jK_{zn}(z-d_n)} \cdot A_n' \right] \quad (4.34d)
\]

Therefore, the EM field in all regions excited by the source embodied in $m$-th layer can be calculated. Three variables $U'_n$, $D'_n$, and $A'_n$ are introduced for formulating to prevent overflow in computation. We can also refer literature [20] for better understanding of the physical interpretation.
Application in Layered Media

In this section, we will study several cases to validate the improved NMM method. Firstly, the EM field scattered by several kinds of objects with different electric properties are computed to illustrate the flexibility of the NMM solver in anisotropic problems. Secondly, the EM field due to a magnetic dipole in the Oklahoma formation is computed to illustrate the efficiency and accuracy of the improved NMM solver. Then the apparent conductivity of the Oklahoma formation with a borehole or invaded zones are computed to investigate the performance of induction tool. Thirdly, the EM field due to a line current in a five-layer oilfield model is calculated to simulate the current line source used in resistivity logging tool. Finally, the EM field due to a surface current source is calculated for its potential application in antenna design. The NMM-yielded results are validated by comparing them with the results from RF (radio frequency) module of COMSOL, which is based on the finite element method.
5.1 Scattering Field by Cylindrical Blocks

As one of the important features of the improved NMM solver, the incorporated PML renders this solver applicable to lightly lossy or even lossless background media. In this section, the EM field scattered by several kinds of stacked cylinders are computed.

5.1.1 One Isotropic Cylindrical Block

As an extended application of the NMM method, we will calculate the EM field scattered by a cylindrical block with a finite length in free space due to a magnetic dipole, as shown in Fig. 5.1(a). The radius and height of the cylinder are 0.3 m and 0.4 m, respectively. The background is free space. The center of the cylinder is at the origin. The cylinder has isotropic relative permittivity and permeability, \( \varepsilon_r = 4 \) and \( \mu_r = 2 \). The magnetic dipole operating at 1.0 GHz locates at \( \mathbf{r}_s = (\rho_s, \phi_s, z_s) = (0.4 \text{ m}, 30^\circ, 0 \text{ m}) \). The magnetic dipole has three components, i.e. \( \mathbf{M}(\mathbf{r}) = (1\hat{x} + 1\hat{y} + 1\hat{z}) \cdot \delta(\mathbf{r} - \mathbf{r}_s) \). The receivers locate along the \( \rho \) direction, specifically at \( \mathbf{r}_r = (\rho_r, \phi_r, z_r) \), where \( \rho_r = 0 \sim 0.5 \text{ m} \), \( \phi_r = 0^\circ \) and \( 180^\circ \), and \( z_r = 0.1 \text{ m} \). The positive and negative values of \( \rho \) in Fig. 5.2 stand for these two directions.
corresponding to \( \phi_r = 0^\circ \) and \( \phi_r = 180^\circ \), and they are along a line. To implement the simulation in the NMM solver, this model should be interpreted as a three-layer model which has two free-space layers on the top and at the bottom and one middle layer with one discontinuity in the \( \rho \) direction. The number of elements \( N_e \) in the 1D FEM solver is 60. The CPU time for each step is 1.2 seconds. A total of thirty-five harmonics are used in this simulation. Therefore, the total CPU time is 21.6 seconds. Whereas it takes approximate 30 minutes to complete this simulation in COMSOL 3D FEM solver which consumes nearly 24 GB memory. Fig. 5.2 shows good agreement between the results from the NMM solver (solid line and dashed lines) and COMSOL (crosses and stars). Each figure shows the real part of field components (solid line and crosses) and the imaginary part of field components (dash and stars).

The EM field scattered by a larger scatterer with 1 m height (9.5 times of the wavelength in the object) has also been investigated. The CPU time of the NMM remains the same. However, COMSOL fails to do the simulation on a computer with 24 GB RAM. We present the result of the NMM in Fig. 5.3. This comparison reveals the advantage of the NMM method whose computational complexity depends on the number of layers rather than the dimensions of the object in the \( z \) direction.
Figure 5.3: Electric and magnetic field scattered by one cylindrical block with 1.0 m height and three-layer model with an infinite length as in Fig. 5.1 (b) due to a magnetic dipole in a lossless medium. (a) Electric field $E_\rho$, $E_\phi$ and $E_z$ versus $\rho$. (b) Magnetic field $H_\rho$, $H_\phi$ and $H_z$ versus $\rho$.

Furthermore, the height of this model is extended to infinite as shown in Fig. 5.1 (b). The height and radius of the cylinder in region 2 is 1.0 and 0.3 meter; and the cylinders in region 1 and 3 are semi-infinite. The relative permittivity and permeability of each region are shown in pairs. The fields obtained from NMM solver are shown in Fig. 5.3. However, existing numerical or analytical methods can hardly solve such a problem.

This geometrically symmetrical problem can also be solved using BOR based FEM method which has been implemented in COMSOL 2D axisymmetric solver. However, due to the limitation of this feature, only the EM field excited by a magnetic dipole oriented the $z$ direction can be calculated. As for computational efficiency comparison, we did an example with magnetic dipole placed at $z$ axis which only requires one harmonic for the NMM. The total CPU time for NMM method and COMSOL are 1.2 s and 5.3 s for the smaller object, i.e. the 0.4 m height cylinder, as shown in Fig. 5.1(a), while it takes the NMM solver and COMSOL 1.2 s and 10.1 s to simulate the larger model, i.e. a 1.0 m height cylinder. We can notice the high computational performance of the NMM solver especially when the structure
5.1.2 Five Layers Anisotropic Inhomogeneous Media

We will next investigate a case with more complicated geometry and electric properties. As shown in Fig. 5.4, the model has five layers and four stacked cylinders. The geometrical dimensions and arrangement of the four cylinders and the boundaries of the horizontal layers can be referred in Fig. 5.4. The relative permittivity, permeability and conductivity (unit: S/m) of the cylinders and all layers are either transversely isotropic or diagonally anisotropic, which are given in the curly brackets. The magnetic dipole operating at 100 kHz is located at $r_s = (\rho_s, \phi_s, z_s) = (2.0 \text{ m}, 45^\circ, 0 \text{ m})$. The receivers are located along the $\rho$ direction, specifically at $r_r = (\rho_r, \phi_r, z_r)$, where $\rho_r = 0 \sim 14.0 \text{ m}$, $\phi_r = 0^\circ$ and $180^\circ$, and $z_r = 3.0 \text{ m}$. To validate the results from NMM solver, the wave propagation in the planarly layered anisotropic background with four stacked cylinders are simulated. In the NMM simulation, the computa-
tional domain along the $\rho$ direction is 14.0 m and the thickness of the PML is 2.0 m. The number of harmonics used in the NMM simulation is twenty-seven. Good agreement between the results of the EM fields from the NMM solver and COMSOL is observed in Figure 5.5.

5.2 EM Well-Logging Tools

One of the most important applications of the NMM is to simulate the induction tool, which is widely used in well logging to survey the electrical properties of the earth formation.

5.2.1 Oklahoma Formation without Borehole or Invaded Zone

In this section, the EM field excited by a magnetic dipole embedded in the Oklahoma formation is computed. This formation consists of twenty-eight planarly stratified layers whose bed thicknesses and conductivities range broadly, which is considered as one of the most challenging model for electromagnetic simulation. The formation parameters, such as the boundary and electric conductivity are shown in first two columns of Table 9.1. All of the layers are non-magnetic media. The magnetic dipole
operating at 30 kHz is located at \( r_s = (\rho_s, \phi_s, z_s) = (1.0 \text{ m}, 0^\circ, 26 \text{ m}) \). The magnetic dipole has three components, i.e. \( M(r) = \left(1\hat{\rho} + 1\hat{\phi} + 1\hat{z}\right) \cdot \delta(r - r_s) \). And the receiver array locates along the \( \rho \) direction, i.e. \( r_r = (\rho_r, \phi_r, z_r) = (0 \sim 10 \text{ m}, 45^\circ, 25 \text{ m}) \). In the NMM simulation, the computational domain along the \( \rho \) direction is 30 m and the thickness of the PML is 30 m (around one skin depth at this operating frequency). The number of elements \( N_e \) in the 1D FEM solver is 70 and first order basis functions are used. The number of elements \( N_e \) in the 1D SEM solver is ten and fifth order basis functions are used. Figs. 5.6 and 5.7 shows the comparison of electric and magnetic fields between NMM and FEKO. An excellent agreement between them can be observed. Furthermore, Table 5.1 shows the comparison of the degree of freedom (DoF), the number of harmonics required to obtain converged results, the CPU time, and the relative error of EM field. We can find that the SNMM can give more accurate numerical results with less computational complexity and reduce the CPU time. The benchmark is from the sommerfeld integral based method. The relationship of the EM field due to different types of excitation between opposite harmonics are employed to save half computational time.

### 5.2.2 Oklahoma Formation with Borehole and Invaded Zone

In this section, the apparent conductivity acquired by the induction tool will be calculated in three kinds of model, (a) Oklahoma formation, (b) Oklahoma formation with borehole, (c) invaded zone is further considered. The apparent conductivity is
Figure 5.6: Comparison of electric field between NMM and FEKO.

Figure 5.7: Comparison of magnetic field between NMM and FEKO.

transformed from the magnetic field using the formula [67]:

\[
\sigma_a(z) = \frac{4\pi L}{\omega \mu} \text{Im}(H_{zz})
\]  

(5.1)
where \( L \) is the transmitter-receiver spacing (TR spacing) of the induction logging tool.

Because this formula is deduced in homogenous media based on low frequency asymptotic approximations, the use of this equation in a heterogeneous media will cause the yielded value deviate the true conductivity. In practice, the apparent conductivity recorded by a induction tool differs from the true conductivity of the formation due to the presence of borehole mud, invaded zones, and the influence of adjacent beds. Therefore, we will use the NMM solver to simulate the induction tool in these three circumstances.

In the simulation, the radius of the borehole is 0.1 meter. And we investigate two kinds of borehole conditions, filled with conductive water-base (\( \sigma_f = 1.0 \text{ S/m} \)) and resistive oil-base (\( \sigma_f = 0.001 \text{ S/m} \)) mud. The radii and the conductivity of all invaded zones are listed as column four and five in Table 9.1. The operating frequency of the tool is 30 kHz.
Fig. 5.8 and 5.9 show the apparent conductivity of the Oklahoma formation with borehole filled with oil-base mud, acquired by centered and eccentered tool, respectively. Fig. 5.10 and 5.11 show the apparent conductivity of the Oklahoma formation with borehole filled with water-base mud, acquired by centered and eccentered tool, respectively. In addition, the tool spacings corresponding to the left and right panels of all of the figures are 0.1 and 1.0 meter, respectively.

We will analyze the effects of borehole and invaded zone, and the influence from adjacent beds as well as the tool eccentricity effect based on these simulations.

Comparing Figs (a) and (b) for all four cases, it can be found that the apparent conductivity acquired by shorter spacing tool is much closer to the true formation conductivity. Without borehole nor invaded zone, the red line curves in Figs. (a) are almost equal to the true conductivity. That can be explained by the low influence from adjacent beds when short spacing tool is used. In contrast, the apparent conductivity obtained by long spacing tool can not delineate the formation con-
ductivity, which need further post-processing to remove the shoulder-beds effects, especially when the model consists of thin beds. However, the induction tool with short spacing is sensitive to the influence of borehole and invaded zone, as show in Figs. (a), the altered dashed black and dash-dotted magenta lines. The apparent conductivity is the comprehensive effect of borehole mud and invaded zone with the formation.

Comparing Figs 5.11 and 5.10, it can be seen that the apparent conductivity acquired by the eccentered induction tool with short spacing is closer to the true formation conductivity, because the tool can sense the formation due to shorter distance. However, we can not see obvious eccentricity effect for the case with low conductivity contrast between borehole mud and formation [68]. For example, as shown in Figs. 5.9 and 5.8, the apparent conductivities are quite similar. In addition, the eccentricity effect is not appreciable for long spacing tool. We can conclude that the eccentricity effect is affected by the tool spacing and the conductivity contrast

Figure 5.10: Apparent conductivity of Oklahoma formation with water-base mud borehole, acquired by centered tool.
In summary, an effective approach is necessary to retrieve the true of the earth formation by eliminating the effect of the borehole and invaded zone, and the eccentricity effect.

5.3 Current Line Source

In this section, the electromagnetic field excited by an electric or magnetic line current are calculated and validated against COMSOL. The oilfield model is comprised of five layers with air as the top layer, as in Fig. 5.12. The permittivity and conductivity of the borehole, the invaded zones, and each bed are shown in pairs in this figure. The air in this model is slightly lossy for the sake of convergence in the COMSOL simulation. The current line source consists of two electrodes, which can be also placed either in the borehole or on the ground surface, forming a surface-to-surface, borehole-to-borehole, or borehole-to-surface pair. For example,
the borehole-to-surface pair as shown in Fig. 5.12 injects the current from the ground to the borehole. The cable carrying the current source (indicated as the red line) can be modeled as two segments of line current, i.e., the radial (segment AB, from point A(0,0,24.9) m to point B(0,10,24.9) m) and the vertical (segment OA, from point O(0,0,0) m to point A(0,0,24.9) m) current line. We will use this model to validate the electrical and magnetic line current source. The operating frequency for both types of current line source is 100 Hz. In the electromagnetic well logging, one typical configuration of the electrode-type resistivity logging tool injects electrical current from electrode B to electrode O, which can be decomposed into two segments of electrical current, i.e., $J_\rho$ (AB) and $J_z$ (OA). A pair of electrodes is commonly used to measure the voltage which can be obtained simply by integrating the electric field along the path between these two electrodes. In order to validate the result, the electrical field along line $(x_r, y_r, z_r)$ (-20:1:20, 5, 24.7) m are computed and compared with the results from COMSOL as shown in Fig. 5.13. A good agreement between these two results can be observed. Furthermore, the EM field due to a line current located from (-5, 8.66, 24.9) m to (5, 8.66, 24.9) m is computed to validate the field.
Figure 5.13: Electric field $E_{\rho}$, $E_{\phi}$ and $E_z$ versus $\rho$ excited by an electric line current in a five layer medium.

Figure 5.14: Electric field $E_{\rho}$, $E_{\phi}$ and $E_z$ versus $\rho$ excited by an electric bipole in a five layer medium.

excited by the $J_\phi$ type line current whose curve is an arc with 10 meter radius and azimuthal angle from $-120^\circ$ to $-60^\circ$. The electric field is shown in 5.14.

5.4 Surface Current Source in a Layered Media

In this section, the electromagnetic field excited by a surface current source are calculated and validated against COMSOL. This model consists of five layers with
different permittivities and permeabilities, as shown in Fig. 5.15, along with the boundaries. The operating frequency is 1 GHz. The receivers are located along line \((0\sim0.3, 0, 0.2) \text{ m}\). The transmitter is in the area of \((0.1\sim0.15 \text{ m}, 0\sim30^\circ, 0 \text{ m})\) in cylindrical coordinate. Two types of electrical surface current, \(J_\rho\) and \(J_\phi\), are considered in this subsection. The excited electric fields due to the \(J_\rho\) and \(J_\phi\) sources are computed and shown in Figs. 5.16 and 5.17, respectively. The results are validated by comparing them with the results from COMSOL, showing very good agreement.

**Figure 5.15:** A five layer model with surface current source
Figure 5.16: Electric Field excited by a surface current, $J_\rho$, in a five layer model as shown in Fig. 5.15.

Figure 5.17: Electric Field excited by a surface current, $J_\phi$, in a five layer model as shown in Fig. 5.15.
In this chapter, the improved NMM method is used to rapidly compute the induction logging response due to a magnetic dipole embedded in an open or cased well with hydraulic fractures. Because the NMM method can sophisticatedly treat thin structures without dramatically increasing the number of meshes, the computational efficiency is significantly improved when comparing with conventional 3D full-wave solvers. In addition, we need to point out that this chapter only focuses on orthogonal transverse fracture (OTF) which is perpendicular to the borehole axis, although the naturally or hydraulically induced fractures could propagate in arbitrary directions. The rudimentary research of this model can provide guidance for applications in more complicated circumstances.

6.1 Validation against Experiment

In this section, the numerical results obtained from the NMM solver are validated against the experimental data, showing very good agreement.
6.1.1 Experimental Model Description

A scaled down model of borehole with a fracture was designed to fit our laboratory size for experiments. Fig. 6.1 shows the 2D vertical cross section of this experiment model, since the model is axisymmetric about the \( z \) axis. A 125 \( \mu \)m thick carbon steel foil was attached to a plastic pipe with a 21 mm outer radius and a 3 mm thickness, forming a cased borehole. The electrical conductivity and relative magnetic permeability of the carbon steel casing are 6.99 MS/m and 100. A donut-shaped thin nickel sheet was used to represent the conductive proppant filled in the hydraulic fracture. The sheet was centered at the origin of coordinate system and its conductivity is 10,546 S/m corresponding to a conductivity of 264 S/m in field. The inner and outer radii of the sheet are 25.5 and 82.0 mm respectively, and the thickness is 0.33 mm. The induction logging tool, operating at 100 kHz, consisted of a pair of transmitter and receiver. A circular electric current loop (solenoid) was applied to excite \( \text{TE}_z \) wave which was equivalent to that produced by a vertical magnetic dipole \( M_z \).
6.1.2 Simulation and Experiment Measurement

In the carbon steel cased borehole, the transmitter and receiver were aligned along the \( z \) axis as an induction logging unit working in fixed-spacing mode. They were separated by different spacings ranging from 80.9 mm to 140.9 mm with a 20 mm step. The transmitter started from \(-281.5\) mm and ended at \(118.5\) mm with a 10 mm step. In the NMM simulation, the model is interpreted as three stratified layers with two discontinuities in the first and third layers and four discontinuities in the second layer along the \( \rho \) direction as shown in Fig. 6.1. The computational domain in the \( \rho \) direction is 0.7 m with a 0.3 m long PML. A total of 140 radial elements in the \( \rho \) direction are used in the simulation, and second order polynomial basis functions are adopted. The total computational time was 10.2 seconds for 41 logging points.

In the experiment, the incident field without the effect of fracture, also defined as primary field, was measured first. The scattered field from the fracture, also defined as the secondary field, was accurately measured using the bucking technique. Fig. 6.2 shows the normalized secondary field, which is the ratio of the secondary field to the primary field, versus the logging depth defined as the middle point of transmitter and receiver.
receiver. One main reason to use the normalized value is that the amplitude of the secondary field should be more than one percent of that of the primary field to ensure the detectability in practice. A very good agreement between experimental data and the results from the NMM solver is observed. We can find that the normalized secondary field due to the fracture remains at detectable level (above 1%) even the electromagnetic field was significantly weakened by the highly conductive casing.

6.2 Sensitivity Analysis for Open Borehole

In this section, the induction response of hydraulic fractures with different geometrical and electrical properties is computed, such as the radius of fracture, and the electric conductivity and magnetic permeability of the filled proppant. We evaluate the possibility of using induction logging tool to diagnose hydraulic fractures and analyze the sensitivity of the secondary field from fractures with different properties.

6.2.1 Field Model Description

Fig. 6.3 shows the 2D vertical cross section of an axisymmetric borehole model in oilfield including a borehole, a 1 cm thick carbon steel casing, a 2 cm thick cement layer, and a hydraulic fracture. The conductivity, relative permittivity and permeability of the background, borehole fluid, cement are shown in Fig. 6.3. The widely used casing in oilfield is made of carbon steel, which has a high relative magnetic permeability (typical value is 100) and a high conductivity (6.99 MS/m). The fracture is hydraulically induced by injecting a high-presure fluid to allow petroleum flow more freely into the borehole. The fracture is thin with a thickness of a few millimeters. This research focuses on the orthogonal transverse fracture (OTF) for both horizontal and vertical wells, which is perpendicular to the borehole and axisymmetric with respect to the borehole axis. The radius of a hydrofracture may extend to 100 meters. The induction logging tool is equipped with transmitter and
Figure 6.3: 2D vertical cross section of the cased borehole model including a fracture in oilfield.

receiver array consisting of coil sondes. In this paper, two kinds of induction logging configurations are set to measure the electromagnetic field. The first one consists of one transmitter and multiple receivers with fixed spacings, as shown in Fig. 6.3. The transmitter radiates electromagnetic wave and the receivers record the returned signal as the tool moves along the borehole, which is called the fixed-spacing mode. The second one contains one transmitter located statically at a fixed position and one receiver moving along the borehole axis, which is called the fixed-transmitter mode. A vertical unitary magnetic dipole $M_z$ (solenoid) is employed for both configurations to excite the $TE_z$ wave and induce the response from fractures. Unless indicated otherwise, the operating frequency of the induction logging tool is 100 Hz. In this section, the open borehole model, i.e., the casing and cement are absent, is simulated to validate our results against the results calculated from COMSOL. Note that although fractures in the field are more complicated, the sensitivity study for OTFs is valuable.
6.2.2 Effects of the Conductivity of the Proppant

In this section, the NMM solver is validated furthermore by comparing the normalized secondary field with that obtained from COMSOL. The induction tool works in fixed-transmitter mode. The position of the transmitter is \((r_t, z_t) = (0, 0)\) m and the receiver is located at \((r_r, z_r) = (0, 1\sim100)\) m. In this case, the thickness and radius of the fracture are 5 mm and 80 m, respectively. The relative permittivity and permeability of the filled proppant are 80 and 1. And the conductivity of the proppant is sampled at 1, 10, 10\(^2\), 10\(^3\), 10\(^4\), 10\(^5\), 10\(^6\) S/m. In the NMM simulation, the computational domain along the \(\rho\) direction is 120 m and the thickness of the PML is 20 m. The number of elements in the 1D FEM solver is 125. The total CPU time was 8.6 seconds for each simulation. In contrast, it took 302 seconds to complete one simulation in COMSOL 2.5-D FEM solver which consumed nearly 24 GB memory. Fig. 6.4 shows the normalized secondary field \((H_z)\) recorded by the receiver moving along \(z\) axis and the transmitter is located statically at origin. A very good agreement can be observed between the results from NMM and COMSOL solvers. We can also find that the normalized secondary field is small when the TR spacing (the distance between transmitter and receiver, denoted by \(z\) for all figures in this paper) is small, and it increases as the receiver moves away from the transmitter. The reason is that the primary field attenuates much faster than the secondary field with the increase of TR spacing. The primary and secondary fields are validated simultaneously through this comparison, while they are both justified respectively in our study. In addition, we will only present the NMM results and omit COMSOL simulation for convenience in later study.

From Fig. 6.4, we can observe that the secondary field increases commensurately with the conductivity of the proppant. The reason is that the induced current in the filled proppant is enhanced proportionally as the conductivity grows. The secondary
field is linearly proportional to the conductivity of the proppant ranged between the conductivity of background and $10^4$ S/m defined as upper limit, while its increase rate becomes small when the conductivity is above $10^4$ S/m. As can be seen in Fig. 6.4 that the secondary fields from the proppant with conductivity $10^5$ and $10^6$ S/m are overlapped when the TR spacing is longer than 20 meters. That means the conductive proppant has its maximum capacity of enhancing the scattering field. Further investigation reveals that the upper limit gets bigger as the length of the fracture becomes smaller and this effect is also related to the frequency of the EM wave. In addition, the proppant with conductivity higher than $10^3$ S/m will result in big and sensitive enough secondary field that satisfies the requirement of detection. However, a highly conductive proppant is not easy to fabricate. Therefore, we will mainly focus on the proppant with $10^3$ or $10^4$ S/m conductivity in this paper.

6.2.3 Effects of the Permeability of the Proppant

From Fig. 6.5, we can observe that the secondary field from the fracture can be enhanced by using a magnetic proppant even with slightly magnetic permeability.
Figure 6.5: Normalized secondary field along the z axis from a 80 m long fracture filled with proppant with different permeabilities, $\sigma = 1$ S/m.

The secondary field increases with the increase of proppant magnetic permeability. Fig. 6.6 shows the normalized secondary field along the z axis from fractures with different lengths filled with magnetic proppant ($\mu_r = 2$) with a 1 or 1000 S/m conductivity, respectively. The secondary field is not sensitive to the length of the fracture if the conductivity of the filled proppant is 1 S/m, though it is strong enough for detection. The sensitivity of the secondary field increases to a certain extent, if the conductivity of the proppant increases to 1000 S/m. The sensitivity of the response cannot be improved using magnetic proppant though it can enhance the secondary field, especially when the conductivity of the proppant is small. Combining with the foregoing analysis in Section 6.2.2, it can be concluded that the sensitivity of the secondary field is determined by the conductivity of proppant. Thus, it is unnecessary to use a magnetic material as proppant, since non-magnetic proppant with high conductivity will satisfy the requirement of both signal amplitude and sensitivity, if the magnetic dipole is applied in the induction tool. In summary, the above results indicate that the use of magnetic proppant does not substantially increase the sensitivity of the electromagnetic response from fractures.
Figure 6.6: Normalized secondary field along the $z$ axis from fractures with different lengths filled with proppant with a 1.0 (upper) or 1000 (lower) S/m conductivity, $\mu_r = 2.0$.

6.2.4 Effects of the Length of Hydraulic Fracture

The length of a fracture will also affect the secondary field. The hydraulic fracture with a larger length will result in stronger scattering field. However, it is not linearly proportional to the dimension of the fracture. We can observe this phenomenon from Fig. 6.7. This figure shows the ratio of the secondary field from fractures with different radius, i.e., 5, 10, 20, 50, and 80 m, to that from the fracture with a 1 m length. The increase rate of the secondary field becomes small as the length of the fracture elongates. We also notice that the growth rate becomes small as the proppant conductivity increases, which means the sensitivity decreases. Because the incident field decays fast along the radial direction, the secondary field is mainly contributed by the proppant near the borehole. And the contribution from near to far part of the proppant becomes less and less. Thus, the secondary field will become indistinguishable when the extension of the fracture is out of the range that the radiated EM wave can approach due to loss in the formation. The maximum length of fracture can be distinguished is related to the operating frequency, conductivity
**Figure 6.7:** Ratio of the secondary field from the fracture with different lengths, i.e., 5, 10, 20, 50, 80 m to that from the fracture with a 1 m length for different conductivities.

**Figure 6.8:** The normalized secondary field $H_z$ from the fracture with different lengths. Proppant: $\sigma = 10^3$ S/m and $\mu_r = 1$.

of the background and the proppant.

Fig. 6.8 shows the normalized secondary field $H_z$ along the $z$ axis when the transmitter is located at the origin. We can find that the normalized secondary field is not sensitive to the length of the fracture when the TR spacing is small. The sensitivity is enhanced as the TR spacing increases while the amplitude of the
Figure 6.9: The normalized secondary field $H_z$ from fractures with different lengths (proppant: $\mu_r=1, \sigma = 1000 \text{ S/m}$), acquired by the induction tool working in fixed-spacing mode. (a) TR spacing is 40 m; (b) TR spacing is 70 m.

Figure 6.10: The normalized secondary field $H_z$ from fractures with different lengths (proppant: $\mu_r=1, \sigma = 1000 \text{ S/m}$), acquired by the induction tool working in fixed-spacing mode. (a) TR spacing is 40 m; (b) TR spacing is 70 m.

secondary field decreases. Thus, it is critical to choose the optimal TR spacing of the induction tool for best performance in sensitivity and signal strength.

Fig. ?? shows the normalized secondary field $H_z$ from fractures with different lengths, and the conductivity of the filled proppant is 1000 S/m. The induction tool
works in fixed-spacing mode with 40 and 70 m TR spacings, respectively. We can see the difference of the responses from fractures with different lengths ranging from 1 to 50 m when the transmitter and receiver are separated by a 40 meter as shown in Fig. 6.9, while the responses from 50 and 80 m long fractures are nearly overlapped. This can be improved by increasing the TR spacing, as the result shown in Fig. 6.10 with a 70 m TR spacing, while both the sensitivity and strength for short fractures decrease. Therefore, the TR spacing is generally chosen in the range of the length of fracture.

6.3 Sensitivity Analysis for Cased Borehole

It is necessary to study the primary field induced by a magnetic dipole source in the borehole in the presence of various kinds of casings before studying the effect of casing on fracture detection, which is commonly neglected to reduce computational complexity. However, the effect of casing is significant, because the electromagnetic field normally decreases dramatically due to fast attenuation in highly conductive media. Furthermore, it is meaningful to study the effect of the casing on the secondary field for the detection of the hydraulic fracture using the induction logging technique. Thus, we will calculate the primary and secondary fields in the presence of casings with different thicknesses, conductivities, permeabilities for investigation of casing effect.

6.3.1 Effects of the Thickness of the Casing

Fig. 6.11 shows the primary field \( (H_z) \) component distributions along the \( z \) axis from both NMM and COMSOL solvers due a unitary magnetic dipole located at the origin operating at 10 Hz and 100 Hz in a borehole enclosed by casings with different thicknesses, i.e., 0.5, 1.0, 1.5 and 2.0 cm. The conductivity and relative permeability are \( 6.99 \times 10^6 \) S/m and 100. For the purpose of comparison, we also
present the primary field due to the same dipole antenna in an open borehole (the magenta line) for all the three parameters analysis hereafter. We should point out the model difference used in these two simulations, i.e., the NMM simulation is with infinitely long casing while the COMSOL simulation is with a 60 meters long casing. We can find that the primary field is weakened by the casings due to high loss in the casing, and it becomes smaller with the increase of the casing induction factor. The amplitude of the primary field of the 100 Hz EM wave is much smaller than that of the 10 Hz EM wave as the result of higher loss. Especially, it may be difficult to measure the primary field of 100 Hz EM wave with a weak amplitude, when the thickness of the casing is 1.5 or 2.0 cm.

For 10 and 100 Hz EM wave, the normalized secondary field along the $z$ axis due to the fracture with a 50 m length filled with proppant with a 1000 S/m conductivity for an open or cased borehole with different thicknesses are shown in Fig. 6.12. We can find that the normalized secondary fields in various cased boreholes remain at the same level of that in open borehole as shown in Fig. 6.12, although the primary and secondary field are both weakened by the casing. This is also validated by the scaled experiment in section 6.1.2. That allows the application of induction logging tool for fracture detection in the presence of casing. Although the amplitude of the primary field of 10 Hz EM wave is stronger than that of 100 Hz EM wave, its normalized secondary field is around one order less than that of the later one from the proppant with same conductivity. It reveals that the EM wave with lower frequency suffers from less attenuation through high conductive media, while it results in weaker secondary filed from the conductive scatterer. Therefore, it is important to determine the optimal operating frequency to minimize the attenuation and obtain appropriately strong echo from the proppant for better detection.

In the NMM simulation, the number of segment elements in the 1D FEM implementation of the NMM solver is 144 and the second order polynomial basis functions
Figure 6.11: The primary field for the casings with different thicknesses for 10 Hz (upper) and 100 Hz (lower). The conductivity is $6.99 \times 10^6$ S/m, the permeability is 100, and the thickness is variable.

Figure 6.12: The normalized secondary field for the casings with different thicknesses for 10 Hz (upper) and 100 Hz (lower). The proppant: $\sigma = 1000$ S/m, $\mu_r=1$ are adopted. Because the discretization is only implemented in the radius direction with linearly increased steps, the number of elements in the NMM solver can be greatly reduced. Furthermore, the avoidance of discretization in the vertical direction and combination with analytical solution can save computational complexity. The total computational time is 14.8 seconds for each simulation.
As shown in Fig. 6.13, triangular elements are used in the 2D surface discretization. To obtain a high-performance mesh, elements with smoothly increased size from thin to thick structures should be generated, leading to considerable computational complexity. The coexistence of casing and thin fracture further increases the computational complexity, compared with the open borehole model simulated in Section 6.2. The number of elements is 2.04 millions. It took 90 minutes to discretize the model and 310 seconds to complete one simulation in COMSOL 2D FEM solver which consumed 23.2 GB memory. It can be concluded that the inherent advantage of the NMM solver improves the computational efficiency significantly.

Although these two models are different, the primary fields inside the casing \((z=1\sim30\text{ m})\) are close. This phenomena is termed as localization [49], because the impact of the casing on the field strength is localized and the effect of casing length change (in a reasonable range) is insignificant. However, the primary field raises at the end of the casing, i.e. \(z=30\text{ m}\). The end effect due to electric charge at the end of conductive media can enhance the amplitude of the EM field nearby.
6.3.2 Effects of the Conductivity of the Casing

Fig. 6.14 demonstrates the primary field ($H_z$ component) distributions along the $z$ axis due a unitary magnetic dipole located at the origin operating at 10 Hz and 100 Hz in a borehole enclosed by casings with different conductivities, i.e., $6.99 \times 10^4$, $6.99 \times 10^5$, and $6.99 \times 10^6$ S/m. The thickness of casing is 1 cm and the relative permeability is 100. It appears that the magnitude of the primary field decreases as the casing induction factor increases due to the increase of conductivity. This can be explained by the fact that the conductive casing acts as a long electrode antenna [51, 52] whose induced current is excited by the magnetic dipole, radiating electromagnetic wave and leaking energy. Hence, the field strength inside the casing reduces. And higher operation frequency and casing conductivity will cause more radiation, resulting in the field strength inside the casing become even smaller.

The normalized secondary fields along the $z$ axis (omitted in this paper) due to proppant with a 1000 S/m conductivity for cased borehole with different conductivities retain the same as those in Fig. 6.12. This result confirms the feasibility of the method [50] applied to recover the formation information in cased borehole by removing the effect of casing once the paramters of casing are known.

6.3.3 Effects of the Permeability of the Casing

Fig. 6.15 shows the primary field ($H_z$ component) distributions along the $z$ axis due a unitary magnetic dipole operating at 10 Hz and 100 Hz located at the origin in a borehole enclosed by casings with different relative permeabilities, i.e., 1, 10, 100, and 200. It indicates that the magnitude of the primary field strength decreases with the increase of the casing induction factor due to the increase of relative permeability. However, the behavior of decay attributed to the casings with conductivity and magnetic permeability are different, even they have same casing induction factor. A closer inspection of results of different casings in Fig. 6.14 reveals that the primary
<table>
<thead>
<tr>
<th>$H_z$ (A/m)</th>
<th>Primary Field $H_z$ v.s. $z$</th>
</tr>
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<tbody>
<tr>
<td>$10^{-4}$</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>$10^{0}$</td>
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<tr>
<td>$10^{2}$</td>
<td>$10^{4}$</td>
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Figure 6.14: The primary field for the casings with different conductivities for 10 Hz (upper) and 100 Hz (lower). The thickness is 1 cm, the permeability is 100, and the conductivity is variable.

field is amplified when the magnetic casing is adopted. The amplitude of the primary field in the borehole enclosed by a casing with $6.99 \times 10^4$ S/m conductivity and 100 permeability is stronger than that of a model with $6.99 \times 10^6$ S/m conductivity and 1 permeability, provided they have the same casing induction factor (0.53). The phenomenon is also testified in our experiment, as the primary field due to a magnetic dipole placed inside the carbon steel casing is larger than that in the copper casing, even the former model has bigger casing induction factor. It can be concluded that the magnetic casing acting as a transformer can magnify the electromagnetic field. Once again, the normalized secondary fields along the $z$ axis (omitted in this paper) due to proppant with a 1000 S/m conductivity for cased borehole with different magnetic permeabilities retain the same as Fig. 6.12.

6.4 Induction Response of Multiple Hydraulic Fractures

It is important to investigate the induction response in a borehole model consisting of a group of hydraulic fractures, which is more realistic in the oilfield. In order to
Figure 6.15: The primary field for the casings with different relative permeabilities for 10 Hz (upper) and 100 Hz (lower). The thickness is 1 cm, the conductivity is $6.99 \times 10^6$ S/m, and the relative permeabilities is variable.

study the electromagnetic cluster, a carbon steel cased borehole with three hydraulic fractures is simulated. The configuration of this model is the same as that in Fig. 6.3 except the number of fractures. The radii of these fractures are all 50 meter, which are located at -5, 0 and 10 meter in the vertical direction. The conductivity of the filled non-magnetic proppant is 1000 S/m. The operating frequency is 100 Hz. It took 64.5 seconds for the NMM solver to complete the simulation while COMSOL failed to run the simulation due to huge computational complexity in a 24 GB memory equipped computer.

Fig. 6.16 shows the induction response of these three hydraulic fractures acquired by the logging tool working in fixed-spacing mode with different transmitter-receiver separations. With the increase of TR spacing, the peak of normalized secondary field increases. It can be seen that the normalized secondary field is not symmetric anymore, because the distribution of fracture is asymmetric. In addition, we can roughly estimate the locations of fractures from the local maximum points.
Figure 6.16: The normalized secondary field $H_z$ from three fractures all with a 50 m length (proppant: $\mu_r=1, \sigma = 1000 \text{ S/m}$), acquired by the induction tool working in fixed-spacing mode for different TR spacings.
Hybrid Method

Because the naturally or hydraulically induced fractures usually propagate in arbitrary directions, the 2.5-D NMM is not applicable due to asymmetry of model. Although the 3D full-wave solver, such as FEM, integral equations method or finite difference are still applicable. However, they are suffering from extremely heavy computational burden, taking very long implementation time, especially when the hydraulic fracture is considered.

In this section, we demonstrate a hybrid approach which combines the NMM method and BCGS-FFT [69–72] which is an efficient method to solve integral equation. The proposed method can efficiently calculate the scattering field from the hydraulic fracture with minor sacrifice of accuracy. In addition, the response of triaxial induction tool due to a multi-component dipole provide more comprehensive information for fracture diagnostic. Two examples are studied to demonstrate the accuracy and efficiency of the hybrid approach.
Figure 7.1: A 3D borehole model with tilted fracture.

7.1 Electric Field Volume Integral Equation

In the problem of calculating the scattering field from the fracture as show in Fig. 7.1, we assume all of the material (including the background formation, borehole, and fractures) in this model are non-magnetic, which is true for most of the borehole model in the oilfield. Then the total field due to the dipole can be computed using the electric field volume integral equation

\[ E(r, r_T) = E^p(r, r_T) + j\omega\epsilon_b \int_V G^{EJ}(r, r') \chi_{\epsilon}(r') E(r', r_T) dV \tag{7.1} \]

where \( E(r, r_T) \) is the total field, \( E^p(r, r_T) \) is the primary field due to the dipole without considering the fracture; \( r \) and \( r_T \) are the locations of observation and source points, and \( r' \in V \) which is occupied by the fracture. \( k_b \) is the wavenumber of the background; \( G^{EJ} \) is the dyadic Green’s function for electric field due to electric
source; $\chi_\epsilon(r)$ is the permittivity contrast defined as

$$\chi_\epsilon(r) = \frac{\epsilon(r) - \epsilon_b}{\epsilon_b} \tag{7.2}$$

Once the total electric field $\mathbf{E}(r', r_T)$ inside the fracture is obtained by solving the equation (7.1), the scattering EM field, i.e. secondary field, can be calculated using

$$\mathbf{E}^s(r, r_T) = j\omega\epsilon_b \int_V G^{EJ}(r, r') \chi_\epsilon(r') \mathbf{E}(r', r_T) dr' \quad (7.3a)$$

$$\mathbf{H}^s(r, r_T) = j\omega\epsilon_b \int_V G^{HJ}(r, r') \chi_\epsilon(r') \mathbf{E}(r', r_T) dr' \quad (7.3b)$$

where $G^{HJ}(r, r')$ is the dyadic Green’s function for magnetic field.

### 7.2 BCGS-FFT solver

When the background is homogeneous, the electric field volume integral equation can be rewritten as

$$\mathbf{E}(r, r_T) = \mathbf{E}^p(r, r_T) + j\omega \left[ 1 + \frac{1}{k_b^2} \nabla \cdot \nabla \right] \mathbf{A}(r) \quad (7.4)$$

where $\mathbf{A}(r)$ is electric vector potential

$$\mathbf{A}(r) = j\omega\mu_b\epsilon_b \int_V g(r, r') \chi_\epsilon(r') \mathbf{E}(r', r_T) dr' \quad (7.5)$$

To solve this equation numerically, the EFVIE is tested by a set of testing functions. The weak form EFVIE is given by

$$\int_V w_t(r) \cdot \mathbf{E}(r, r_T) dr = \int_V w_t(r) \cdot \mathbf{E}^p(r, r_T) dr + j\omega \int_V w_t(r) \cdot \left[ 1 + \frac{1}{k_b^2} \nabla \cdot \nabla \right] \mathbf{A}(r) dr \quad (7.6)$$
The unknown electric field and vector potential can be expanded in terms of basis functions as described in previous work [69, 70].

\[ E^{(q)}(\mathbf{r}) = \sum_i e_i^{(q)} \Phi_i^{(q)}(\mathbf{r}) \]  
\[ A^{(q)}(\mathbf{r}) = \sum_i \sum_m a_i^{(q)} \Psi_{i,m}^{(q)}(\mathbf{r}) \]

The testing function \( \mathbf{w}_i(\mathbf{r}) \) and basis function \( \Phi_i^{(q)}(\mathbf{r}) \) is the rooftop basis function which is defined in [69], and \( \Psi_{i,m}^{(q)}(\mathbf{r}) \) is the second order curl basis functions, which is defined in [69, 71, 72]. \( a_i^{(q)} \) is the expansion coefficients of \( A^{(q)} \), which can be fast calculated using the following equation [71]

\[ a_i^{(q)} = j \omega \epsilon_b \mu_b \Delta V \mathcal{F}^{-1} \left\{ \mathcal{F} \left\{ G_{i,m}^{(q)} \right\} \times \mathcal{F} \left\{ \chi_\epsilon \left( \mathbf{r}_K^{(q)} \right) e_\mathbf{K}^{(q)} \right\} \right\} \]  

where the discrete values of \( G_{i,m}^{(q)} \) is defined in [22]. Substituting equations (7.7), (7.8) into (7.4), the discretized weak form electric field equation can be written in terms of matrix

\[ Ze = E^p \]  

where \( Z \) is a dense matrix given by the weak form of EFVIE, \( E^p \) is the primary electric field; the expansion coefficients \( e \) can be solved using iterative stabilized bi-conjugate gradient (BCGS). The CPU time of this iterative solver is \( O(N \log N) \).

### 7.3 Hybrid Method

The electric field equation (7.1) in the borehole model with fracture can be solved by directly using the conventional MoM whose computational complexity is \( O(N^2) \) and CPU time is \( O(N^3) \). The computational burden is quite heavy when the number of unknowns \( N \) is large. Unfortunately, the aforementioned BCGS-FFT method can not be employed directly to solve the scattering problem of model in Fig. 7.1,
because the presence of borehole and casing causes the Green’s function no longer translationally invariant. The procedure of FFT in the BCGS-FFT method requires translational symmetry of the Green’s function, resulting in this method can be only applied to the model with homogeneous background. In order to solve this problem within reasonable time, a hybrid approach combining the NMM, DBA (distorted Born approximation) and BCGS-FFT is introduced. It is named NMM-DBA-BCGS-FFT. To implement this hybrid approach, one should follow the steps as below:

- **Step1:** Utilize the NMM solver to compute the primary field $E^p$ in the area of fracture (as the yellow region in Fig. 7.2b) due to a dipole source in the earth formation in the presence of borehole or casing and in the absence of fracture.

- **Step2:** Calculate the total field in the region occupied by the fracture using the BCGS-FFT method. The weak form of electric field equation (7.6) or (7.9) is used to find the total field. To efficiently evaluate the matrix $Z$, the

Figure 7.2: Flowchart of Hybrid NMM-DBA-BCGS-FFT Method.
FFT is adopted in equation (7.8) to calculate the expansion coefficients \( a_{i,m}^{(q)} \). One should note that the Green’s function in homogeneous media is used to approximate the inhomogeneous background.

- **Step 3:** Calculate the scattering field from the fracture using equation (7.3). The dyadic Green’s function \( G^{EJ} \) and \( G^{HJ} \) are provided by the NMM for the model without the fracture. The reciprocity theorem can be utilized to reduce the computation.

In step 2, an extended Born approximation is made by imposing the Green’s function in homogeneous media to the inhomogeneous background, which can be named distorted Born approximation (DBA) [73–76]. This approximation only stands for the problem with weak coupling between the background media and the scatter. In summary, the proposed hybrid approach is more accurate than Born approximation which ignores the electromagnetic coupling, guaranteeing the numerical accuracy.
7.4 Numerical Simulation

In this section, we will use two cases to demonstrate the validity and efficiency of this hybrid approach.
Figure 7.6: Comparison of secondary fields ($H_z$ component) from the fracture. The source is magnetic dipole $M_z$.

7.4.1 Thick Fracture

In this example, we will calculate the secondary field from the donut-shaped circular disk whose dip and azimuthal angle are $\theta$ and zero respectively, as shown in Fig. 7.3. The inner and outer radii of the disk are 0.15 and 5.0 meter. The thickness of disk is carefully chosen as 0.5m to ease the computational complexity for COMSOL which provides the reference results. The radius of the borehole is 0.1 meter. The conductivity of the earth formation, borehole fluid and fracture are 0.05, 0.5 and 100 S/m. The relative permittivity of these three objects are 50, 80, and 100. And all of the material in this model are non-magnetic. A magnetic dipole with three kinds of different polarization operates at 1 kHz. And the receiver records the scattering field along the borehole axis from 0 to 30 meter.

Fig. 7.4-7.6 shown the secondary field ($H_x, H_y$ and $H_z$ components) from the disk with dip angle $\theta = 0$ due to a magnetic dipole orienting $x$-, $y$-, $z$-direction, respectively. A good agreement of the results between the hybrid approach and
Figure 7.7: Comparison of secondary fields ($H_x$ components) from the fracture. The source is magnetic dipole $M_x$.

Figure 7.8: Comparison of secondary fields ($H_z$ components) from the fracture. The source is magnetic dipole $M_x$.

COMSOL can be observed. Fig. 7.7-7.11 show the secondary field from the same disk with dip angle $\theta = 45^\circ$. It can be seen that these two results agree with each other overall except a slight jump at $z = 3$ m which need further investigation.
Figure 7.9: Comparison of secondary fields ($H_y$ component) from the fracture. The source is magnetic dipole $M_y$.

Figure 7.10: Comparison of secondary fields ($H_x$ components) from the fracture. The source is magnetic dipole $M_z$.

7.4.2 Cased Borehole with Fracture

In this example, we will calculate the secondary field from a thin fracture whose dip and azimuthal angle are $45^\circ$ and zero respectively, as shown in Fig. 7.12. The
Figure 7.11: Comparison of secondary fields ($H_z$ components) from the fracture. The source is magnetic dipole $M_z$.

inner and outer radii of the disk are 0.15 and 100.0 meter. The thickness of fracture is 5 mm. The radius of the borehole is 0.1 meter. The thickness of the casing adhered to the borehole is 1 cm. The relative permeability of the casing is 100, and its conductivity is $6.99 \times 10^6$ S/m. The conductivity of the earth formation, borehole fluid and fracture are 0.05, 0.5 and 1000 S/m. The relative permittivity of these three objects are 50, 80, and 100. The transmitter and receiver are working in induction mode, the operating frequency is 100 Hz. The normalized secondary field due to three different magnetic dipoles are shown in Fig. 7.13-7.15. It can be observed that the secondary field is strong enough for fracture detection even the signal is greatly weakened by the conductive casing.
**Figure 7.12:** A 3D model of a carbon steel cased borehole with a thin tilted hydraulic fracture.

**Figure 7.13:** Comparison of normalized secondary fields from the fracture. The source is magnetic dipole $M_x$. 

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Figure 7.14: Comparison of normalized secondary fields from the fracture. The source is magnetic dipole $M_y$.

Figure 7.15: Comparison of normalized secondary fields from the fracture. The source is magnetic dipole $M_z$. 
Conclusions

We have formulated the vector wave equation for investigating the wave propagation in an anisotropic orthogonal-plano-cylindrically layered medium due to a source located at arbitrary position using the numerical mode matching method. Furthermore, the NMM method is successfully employed to investigate the performance of the induction logging tool for the detection of hydraulic fracture in open and cased borehole.

8.1 Conclusion on the NMM method

In this study, several improvements have been introduced to increase the accuracy of the NMM method and to reduce computational time, and to further extend its applications. The contribution of this work are as follows:

(1) We formulate Maxwell’s equations for anisotropic media with permittivity and permeability having three different diagonal elements in the cylindrical coordinate system. This expands the NMM method to the media with more general properties.

(2) With the deduced formulation, it is then possible to incorporate the PML in the radial direction, thus extending the application to lossless and low-loss me-
dia. And the NMM method incorporated with the PML can greatly reduce the computational complexity and improve the accuracy.

(3) The spectral element method (SEM) is implemented to seek for eigenmodes for each horizontal layers instead of FEM. In the SEM method, the Gauss-Lobatto-Legendre (GLL) points and the corresponding weights are used for the quadrature to numerically evaluate the integration. The resultant block-diagonal mass matrix from the application of orthogonal basis functions can be easily inverted which ensures the efficiency of this approach. It has been demonstrated that the computational error of the SEM decreases exponentially as the order of polynomial, resulting in high order accuracy. Therefore, the spectral NMM is the extension of conventional FEM-based NMM, which can achieve higher accuracy and computational efficiency.

(4) When dealing with the 1D FEM solver along the radial direction, a new mathematical manipulation and the boundary conditions are employed to remove the singularity at $\rho = 0$ and to obtain more accurate field responses along the $z$ axis. The singularity problem at $\rho = 0$ has been completely resolved for the first time in the NMM method.

(5) A scale factor is introduced to ensure the mass matrix well-conditioned without affecting the equation, especially when the computational domain contains highly conductive media with relative low permeability, causing the mass matrix ill-conditioned. This situation is often encountered in geophysical exploration such as cased borehole circumstance at low frequency. The scale factor can be carefully chosen to reduce the contrast of the matrix in mass matrix to obtain more accurate field responses along the borehole axis.

(6) The field relationship between opposite mode indices for different kinds of sources are proved. By applying these relationship, the computational time can be further reduced by half.

(7) The EM fields excited by an electric and magnetic dipole oriented in an
arbitrary direction at any position are simulated, thus allowing for comprehensive applications in the electromagnetic well logging where the sources may be off the z axis. And the formulas to compute the excited EM fields due to a current line source located are deduced, which enables the NMM method to be applied to simulate the resistivity logging tool. The excitation is even extended to surface current source which enables the NMM method to simulate the cylindrical patch antenna.

8.2 Conclusion on the Hydraulic Fracture Detection

One important contribution of this work is to apply the NMM to investigate the performance of induction logging tool in hydraulic fracture detection. It is rather efficient to use the NMM solver to simulate the electromagnetic wave propagation in an open or cased well together with orthogonal transverse fractures, which is axisymmetric with respect to the borehole axis. The comparisons between the results obtained from the improved NMM solver and that from experiments or COMSOL reveal the accuracy and efficiency of the NMM solver. From the simulation, we find that the response from the fracture is linearly proportional to the electric conductivity of the filled proppant in certain range. We recommend to use the proppant with conductivity in the range of $10^3$-$10^4$ S/m to achieve strong and sensitive enough scattering field. The study of the primary and secondary field reveals that the induction logging tool is suitable to diagnose the hydraulic fracture in an open or cased borehole. The performance of induction logging tool in the cased borehole is similar to that for open borehole, although the electromagnetic field is weakened by the highly conductive casing. We conclude that the TR spacing for logging tool is proportional to the length of the fracture, which is longer than traditional induction logging tools. However, the induction logging tool has its maximum detectable length of fracture, which is determined by the operating frequency and the electrical conductivity of the background. In addition, some phenomena including localization,
end effect and casing effects have been systematically studied and can be applied to modeling simplification for the cased oil and gas borehole.

8.3 Conclusion on Hybrid Approach

The hybrid approach is proposed to compute the multi-component induction response of the hydraulic fractures orienting arbitrarily without axis-symmetry. The dyadic Green’s function without considering the fracture can be acquired numerically using the NMM method. Then the distorted Born approximation and the BCGS-FFT based integral equation method are introduced to compute the response of the model containing the hydraulic fracture. The hybrid approach can efficiently and accurately compute the response of fracture in open or cased wells for simulating the multi-component induction logging tool.
Appendix One
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Biography

Junwen Dai was born in Zhangzhou, Fujian, China. He received the B.S. degree in electronic information science and technology from Xiamen University, Xiamen, China, in 2008, and the M.S. degree in radio science from Fudan University, Shanghai, China, in 2011. He is currently working toward his Ph.D. degree in electrical engineering at Duke University.

Since 2011, he has been a Research Assistant with the Department of Electrical and Computer Engineering at Duke University, Durham, North Carolina, US. He is the recipient of Chambers Fellowship from Duke University in 2011. He is also a recipient of travel fellowship grant awards from the U.S. National Committee for the International Union for Radio Science in 2015. His research interests include theory of electromagnetic field, computational electromagnetics, fast numerical method, well logging, microwave imaging and target reconstruction.