Physical Insights, Steady Aerodynamic Effects, and a Design Tool for Low-Pressure Turbine Flutter

by

Joshua Joseph Waite

Department of Mechanical Engineering and Materials Science
Duke University

Date: __________________________

Approved:

______________________________
Robert Kielb, Supervisor

______________________________
Earl Dowell

______________________________
Kenneth Hall

______________________________
Damian Vogt

______________________________
Thomas Witelski

Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Mechanical Engineering and Materials Science in the Graduate School of Duke University 2016
Abstract

Physical Insights, Steady Aerodynamic Effects, and a Design Tool for Low-Pressure Turbine Flutter

by

Joshua Joseph Waite

Department of Mechanical Engineering and Materials Science
Duke University

Date: __________________

Approved:

______________________
Robert Kielb, Supervisor

______________________
Earl Dowell

______________________
Kenneth Hall

______________________
Damian Vogt

______________________
Thomas Witelski

An abstract of a dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Mechanical Engineering and Materials Science in the Graduate School of Duke University 2016
Abstract

The successful, efficient, and safe turbine design requires a thorough understanding of the underlying physical phenomena. This research investigates the physical understanding and parameters highly correlated to flutter, an aeroelastic instability prevalent among low pressure turbine (LPT) blades in both aircraft engines and power turbines. The modern way of determining whether a certain cascade of LPT blades is susceptible to flutter is through time-expensive computational fluid dynamics (CFD) codes. These codes converge to solution satisfying the Eulerian conservation equations subject to the boundary conditions of a nodal domain consisting fluid and solid wall particles. Most detailed CFD codes are accompanied by cryptic turbulence models, meticulous grid constructions, and elegant boundary condition enforcements all with one goal in mind: determine the sign (and therefore stability) of the aerodynamic damping. The main question being asked by the aeroelastician, “is it positive or negative?” This type of thought-process eventually gives rise to a black-box effect, leaving physical understanding behind. Therefore, the first part of this research aims to understand and reveal the physics behind LPT flutter in addition to several related topics including acoustic resonance effects. A percentage of this initial numerical investigation is completed using an influence coefficient approach to study the variation the work-per-cycle contributions of neighboring cascade blades to a reference airfoil. The second part of this research introduces new discoveries regarding the relationship between steady aerodynamic loading and negative aerodynamic damping. Using validated CFD codes as computational wind tunnels, a multitude of low-pressure turbine flutter parameters, such as reduced frequency, mode shape, and interblade phase angle, will be scrutinized across various airfoil geometries and steady operating conditions to reach new design guidelines regarding the influence of steady aerodynamic loading and LPT flutter. Many pressing topics influencing LPT flutter including shocks, their nonlinearity, and three-dimensionality are also addressed along the way. The work is concluded by introducing a useful preliminary design tool that can estimate within seconds the entire aerodynamic damping versus nodal diameter curve for a given three-dimensional cascade.
To my loving mother, wise father, amazing sister, and the best nieces an uncle could ask for: Kimberly, Donald, Ericka, Abbie, and Stella.
Contents

Abstract iv
List of Tables ix
List of Figures x
List of Abbreviations and Symbols xv
Acknowledgements xviii

1 Overview 1
  1.1 Introduction to Turbomachinery ........................................... 1
  1.2 Motivation for Research ...................................................... 4
  1.3 Chapter Outlines ............................................................... 6

2 Review of Turbomachinery Flutter 7
  2.1 How LPT Cascades Flutter ..................................................... 7
  2.2 Key Control Parameters for LPT Flutter ................................... 10
    2.2.1 Reduced Frequency and Mode Shape .................................. 11
    2.2.2 Steady Loading Parameters ............................................. 13
  2.3 Aeroacoustics ................................................................. 18
  2.4 Three-Dimensional Effects .................................................. 19
  2.5 Nonlinearity ........................................................................ 22
    2.5.1 Linear vs. Nonlinear CFD ............................................... 22
    2.5.2 Friction Damping ............................................................ 23
  2.6 Multi-Row Analysis .............................................................. 25
  2.7 Mistuning ............................................................................ 26

3 LPT Flutter Analysis and Physical Understanding 28
  3.1 Computational Fluid Dynamics Solver ....................................... 28
  3.2 LPT Geometries Considered .................................................... 30
    3.2.1 Standard Configuration 4 .................................................. 30
  3.3 Key Control Parameters for LPT Flutter .................................... 35
    3.3.1 Non-Dimensional Damping Coefficient ............................... 35
4 Influence of Steady Aerodynamics on LPT Flutter

4.1 Loading Study: Variable Reduced Frequency

4.2 Loading Study: Constant Reduced Frequency

4.2.1 Unsteady Pressure Bifurcations

4.2.2 3D Results

4.2.3 Recap of Initial Loading Studies

4.3 Mode Dependent Shock Structure Effects: SC4

4.3.1 Bending Mode Results

4.3.2 Pitching Mode Results

4.3.3 Recap of Mode Dependency Study

4.4 Geometry Dependent Shock Structure Effects

4.4.1 Various Quasi-2D LPT Geometries

4.4.2 Critical Reduced Frequency as a Function of Mode Shape

4.4.3 Bending Mode Results

4.4.4 Pitching Mode Results

4.4.5 Local Work Contributions

4.5 Design Guidelines and Conclusions

5 Preliminary Design Method

5.1 CYCLOPS3D

5.2 FECLOPS3D

6 Summary and Future Work

6.1 Summary of Work

6.2 Future Work Recommendations

A Instruction Manual for FECLOPS3D/CYCLOPS3D

A.1 FECLOPS3D

A.1.1 Input 1: Control File

A.1.2 Input 2: Characteristic File

A.1.3 Input 3: Grid and Mode Files

A.1.4 Running the FECLOPS3D Executable

A.1.5 Output 1: Summary File

A.1.6 Output 2: Sections File

A.1.7 Output 3: Main CYCLOPS Input File

A.2 CYCLOPS3D

A.2.1 Input 1: Main Input File
List of Tables

3.1 Key Geometric Parameters: STCF 4 .............................. 31
3.2 Experimental flow conditions for SC4 Test Cases ................. 33
3.3 Converged steady CFD results for quasi-2D SC4 ................. 34
3.4 Test Case 627: Subsonic Acoustic Resonance IBPA Predictions . 38
3.5 Test Case 628: Supersonic Acoustic Resonance IBPA Predictions . 47
3.6 Test Case 624: Transonic Acoustic Resonance IBPA Predictions . 50

4.1 Bending Mode Sensitivity Regions for SC4 ......................... 76
4.2 Pitching Mode Sensitivity Regions ................................. 80
4.3 Numerical Flow Conditions for LPT Test Cases .................... 88
4.4 Critical Reduced Frequency Sensitivity Regions for Bending Modes:
   SC4/KTH .................................................................. 95

6.1 Categories of Investigation in Chapter 4 Loading Study ............. 118
6.2 Critical Reduced Frequency Sensitivity Regions for Bending Modes:
   SC4/KTH .................................................................. 121
List of Figures

1.1 View inside of the GE90 turboshaft jet engine. . . . . . . . . . . . . . 2
1.2 Schematic view inside a typical jet engine. Arrows indicate axial flow
direction. Original image modified from Wheeler [76]. . . . . . . . . . 3
1.3 Pressure-Volume Cycle for the series of thermodynamic processes
in the Brayton Cycle used in all gas turbines. Source: NASA Glenn
Research Center [46] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3

2.1 Sample 2D airfoil geometry and terminology . . . . . . . . . . . . . 8
2.2 Sketch of three traveling wave modes for a linear cascade of flat plates
(N=12) undergoing pitching motion from Meinzer [41]. . . . . . . . . 9
2.3 Experimentally measured damping vs. IBPA and S-curve stability
parameter fit from Panovsky and Kielb [49]. . . . . . . . . . . . . . 10
2.4 Tie-Dye plot of Standard Configuration 4: \( M_2 = 0.90 \). Neighboring
blades shown for cascade orientation. \( \xi \) AND \( \eta \) normalized by the
chord. Used with permission [40]. . . . . . . . . . . . . . . . . . . . 13
2.5 Investigation of torsion instability boundaries as a function of torque
by Cardinale [8]. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 15
2.6 LPT blade profile and geometry of 3D Vogt experiments [70]. . . . 20
2.7 Comparison of harmonics needed for convergence of small (top) and
large (bottom) amplitude oscillations by Hall, Thomas, and Clark [28]. 23
2.8 Steady-state vibrational amplitudes for a 3-bladed disk with blade-to-
ground dampers by Sinha, Griffin, and Kielb [57]. . . . . . . . . . . . 25
2.9 Isolated-row vs. multi-row flutter analysis by Hall and Ekici [26]. . 26

3.1 Photo of Standard Configuration 4. . . . . . . . . . . . . . . . . . . . 31
3.2 Nonlinear study by Jing Li of unsteady surface pressure coefficient as
a function of vibration amplitude. \( \sigma = -90^\circ \). Scale factors = 0.1, 1.0,
2.0, and 2.9. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 32
3.3 Coefficient of pressure distribution for SC4 Test Case 628. . . . . . 34
3.4 Absolute Mach number distributions. . . . . . . . . . . . . . . . . . . 35
3.5 Fundamental mode shapes considered. Displacements enlarged for
depiction. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 36
3.6 Minimum non-dimensional damping coefficient over all IBPAs as a function of reduced frequency. \( p_2/p^* = 1.21 \) .................................................. 37
3.7 Damping vs. IBPA for subsonic exit flow (Case 627) at three different reduced frequencies ................................................................. 38
3.8 Numbering convention used for a reference blade and cascade. ........ 40
3.9 Magnitude of the GF influence coefficients for subsonic exit flow (Case 627) at three different reduced frequencies. .......................... 41
3.10 Magnitude of the generalized force influence coefficients for subsonic exit flow at four different reduced frequencies. ......................... 42
3.11 Work-per-cycle contributions from the reference blade and three nearest neighbors on both the pressure and suction sides at \( \sigma = -72^\circ \) as a function of reduced frequency. .................................................. 44
3.12 Amplitude of unsteady pressures due to vibrating only the reference blade. Test Case 627: Subsonic flow, \( k = 0.1 \). ................................. 45
3.13 Damping vs. IBPA for supersonic exit flow (Case 628) at three different reduced frequencies. .......................................................... 46
3.14 Magnitude of the GF influence coefficients for supersonic exit flow (Case 628) at three different reduced frequencies. ......................... 46
3.15 2D acoustic resonance conditions for subsonic, supersonic, and transonic SC4 test cases. ............................................................ 49
3.16 Schematic of a concentric cylinder intended to model a turbomachinery core geometry. ................................................................. 50

4.1 Damping vs. IBPA for transonic exit flow (case 624) at three different reduced frequencies ................................................................. 53
4.2 Inlet Mach (\( M_1 \)), exit Mach (\( M_2 \)), and normalized torque as a function of back pressure for SC4 Test Case 624 baseline conditions. ....... 55
4.3 Damping vs. IBPA for transonic exit flow (Case 624) at various normalized back pressures. Green triangles mark the baseline case seen in Fig. 4.1. \( p_2 \) in the legend is the back pressure normalized by \( p_{ref} = 14.7 \) PSI. .................................................. 56
4.4 Reduced frequency as a function of back pressure normalized by 14.7 PSI for SC4 Test Case 624. .......................................................... 57
4.5 Minimum damping vs. steady blade loading for baseline Test Case 624: \( k = 0.1 \) flex mode, \( k = 0.3 \) LE pitch mode, and \( k = 0.3 \) mid-chord pitch mode. \( p^* = 15.24 \) psi, \( (p_2/p^*)_{baseline} = 1.04 \) ............... 57
4.6 Minimum damping vs. steady blade loading for baseline Test Case 627: \( k = 0.1 \) flex mode, \( k = 0.3 \) LE pitch mode, and \( k = 0.3 \) mid-chord pitch mode. \( p^* = 12.33 \) psi, \( (p_2/p^*)_{baseline} = 1.18 \) ............... 58
4.7 Minimum damping vs. steady blade loading for baseline Test Case 628: \( k = 0.1 \) flex mode, \( k = 0.3 \) LE pitch mode, and \( k = 0.3 \) mid-chord pitch mode. \( p^* = 16.63 \) psi, \( (p_2/p^*)_{baseline} = 0.60 \) ............... 58
4.8 Mode-specific comparison of least stable damping vs. loading relations across three different SC4 test case baseline boundary conditions. Back pressures are normalized by case-specific $p^*$ values. Variable reduced frequency. ................................................................. 59
4.9 Variation of inlet/exit Mach number, enforced blade vibrational frequency, exit flow angle, and torque as a function of normalized back pressure for a constant reduced frequency of $k = 0.1$. Baseline conditions are SC4 Test Case 627. ................................................................. 61
4.10 Work-per-cycle contributions of the reference and neighbor blades as a function of steady loading. ................................................................. 62
4.11 Minimum damping over all IBPAs as a function of normalized back pressure for $k = 0.1$ flex, $k = 0.3$ leading edge pitch, $k = 0.3$ mid-chord pitch, and $k = 0.1$ edgewise bending. ................................................................. 64
4.12 Least stable IBPA as a function of mode shape, reduced frequency, and critical choking pressure ratio (refinement of $\sigma = 18^\circ$). ................................................................. 64
4.13 Density gradient contours in the O-block for back pressures on opposite sides of the critical choking pressure. ................................................................. 65
4.14 Coefficient of pressure distribution across airfoil as a function of load. Increasing load (decreasing back pressure) in the downward-right direction. Highest (lowest) $p_2/p^*$ value shown is 1.31 (0.48). ................................................................. 66
4.15 Suction side unsteady pressure bifurcations as a function of increased loading: $k = 0.1$, FLEX MODE, $\sigma = -54^\circ$. ................................................................. 67
4.16 Pressure side unsteady pressure bifurcations as a function of increased loading: $k = 0.1$, FLEX MODE, $\sigma = -54^\circ$. ................................................................. 68
4.17 Minimum aerodynamic damping vs. back pressure for the 3D Hannover airfoil. ................................................................. 69
4.18 Variation of inlet/exit Mach, blade vibrational frequency, and exit flow angle as a function of normalized back pressure for constant reduced frequency, $k = 0.1$. ................................................................. 73
4.19 Depiction of the bending mode sign convention with $+\theta$ shown to move counter-clockwise originating from the fore-looking-aft engine-axis. The “flex” and “edgewise” bending modes are shown as examples. ................................................................. 75
4.20 Critical reduced frequency as a function of every 6° bending direction for select loading conditions. ................................................................. 76
4.21 Minimum non-dimensional damping coefficient and least stable IBPA for 20 IBPAs as a function of bend-direction and critical choking pressure ratio. Reduced frequency is held constant: $k = 0.1$. ................................................................. 77
4.22 Sample sketch of the $a = 0$ “leading edge pitch” mode. Pitching-axis locations are also shown for mid- & trailing-edge pitch. ................................................................. 79
4.23 Critical reduced frequency as a function of chord-line pitching axis location (every 5%) for select loading conditions. Note: markers are for visualization and do not indicate refinement. ................................................................. 79
4.24 Minimum non-dimensional damping coefficient and least stable IBPA for 20 IBPAs as a function of chord-line pitching axis and critical choking pressure ratio. Reduced frequency is held constant: \( k = 0.3 \)  
4.25 Minimum non-dimensional damping coefficient over all IBPAs as a function of choking pressure ratio for select pitching axes. Reduced frequency is held constant: \( k = 0.3 \)  
4.26 Evolution of shock location on the airfoil suction side as a function of pressure ratio.  
4.27 SC4 Near-Field Tie-Dye plot: \( \pm 2.5 \) chords  
4.28 SC4 Near-Field Tie-Dye plot: \( \pm 2.5 \) chords  
4.29 SC4 Near-Field Tie-Dye plot: \( \pm 2.5 \) chords  
4.30 SC4 Near-Field Tie-Dye plot: \( \pm 2.5 \) chords  
4.31 Chord-normalized blade profile comparisons.  
4.32 Inlet/Exit relative Mach numbers and flow turning as a function of critical choking pressure ratio (\( \Pi_c = p_2/p^* \)) for four LPT test cases.  
4.33 Isoentropic Mach number distributions: \( p_2/p^* = 0.83 \).  
4.34 Sketch of bending mode sign convention. Sample stagger-angle dependent flex and edgewise directions are shown.  
4.35 FLAT PLATE: Critical reduced frequency as a function of BENDING angle and load.  
4.36 SC4: Critical reduced frequency as a function of BENDING angle and load.  
4.37 KTH: Critical reduced frequency as a function of BENDING angle and load.  
4.38 BP2: Critical reduced frequency as a function of BENDING angle and load.  
4.39 Critical reduced frequency as a function of chordline PITCHING axis and load.  
4.40 Critical reduced frequency as a function of chordline PITCHING axis and load.  
4.41 Tie-Dye plot showing \( k_{crit} \) as a function of near-field pitching axes. Blade orientation shown for reference: \( \Pi_c = 1.18 \).  
4.42 BP2 airfoil pressure coefficient variation with load.  
4.43 Least stable damping and IBPA for three BP2 loading conditions as a function of pitching mode: \( k = 0.3 \).  
4.44 Local BP2 suction side work contributions for various combinations of pitching mode and IBPA: \( \Pi_c = 0.83, k = 0.3 \).  
4.45 Local work around BP2 as a function of loading: \( \theta = 90^\circ, k = 0.1, \sigma = 75^\circ \).  
5.1 Linear superposition of the three fundamental mode shapes for a rigid body mode with pitching axis \((\xi,\eta)-(1,1)\).
5.2 Coefficient of pressure profiles of two LPT airfoils at various exit Mach numbers from Meingast [40]. Left: EPFL ; Right: LM. 113
5.3 Damping vs. IBPA CYCLOPS3D results for an industrial LPT blade compared with full scale URANS CFD results for two mode shapes (primarily bending and primarily torsion). 114
5.4 Depiction of a 2D airfoil section being discretized into $N_{ax} - 1$ sections for the mass and mass-moment calculations. $N_{ax} = 17$. 116
6.1 Critical reduced frequency vs. bend angle for the BP2 blade. Actual design conditions at 82.5% span: $\Pi_c = 0.525$, $k = 0.19$, $\theta = 84^\circ$. Interpolation between $\Pi_c = 0.48$ and $\Pi_c = 0.55$ yields $k_{crit} = 0.242$. 123
6.2 Visualization of peak-to-peak variation of minimum throat distance normalized by mean throat distance as a function of IBPA and mode shape. 126
A.1 Sample FECLOPS3D control.txt file with line-by-line description below. 130
A.2 Image of undeformed airfoil section (UIF) superimposed with its displaced cosine and sine mode shapes (UOF) that all have a “flipped” positive circumferential sign convention. 131
A.3 Diagram explaining how to calculate the Line 8 input of the FE-CLOPS3D control.txt file. 132
A.4 Sample characteristic file input for FECLOPS3D. 133
A.5 Sample beginnings of a UIF (grid data) and UOF (eigenvector data) file, which are inputs to FECLOPS3D. 134
A.6 Sample portion of feclops3d.out file output and a corresponding check of spanwise modal smoothness. User-specified number of spanwise sections = 20. 136
A.7 Sample beginning of a sections.txt output file listing the axial and min/max circumferential coordinates FECLOPS3D finds to use calculate and integrate mass and mass-moment. One spanwise section with 40 axial sections shown. 137
A.8 Sample format for the beginning of the primary CYCLOPS3D input file, which is the output from FECLOPS3D. 139
A.9 Sample format for the beginning of Baseline Aero files. 140
A.10 Sample damping_vs_nd.txt file. 145

xiv
List of Abbreviations and Symbols

Symbols

\( a \) Chord-normalized pitching axis location along chordline
\( b \) Semi-chord (half the chord distance)
\( A_{i,j} \) Influence coefficient matrix: row i, column j
\( c \) Blade chord
\( f \) Blade excitation frequency (Hz)
\( h \) Bending displacement amplitude
\( i \) Square root
\( I \) Mass-moment of inertia
\( k \) Reduced frequency, \( \frac{\omega c}{2V} \)
\( n \) Nodal diameter, or normal vector
\( N \) Number of blades, or normal vector
\( M \) Mach number
\( p \) Pressure
\( q \) Dynamic pressure
\( R \) Radius
\( s \) Distance between blades
\( t \) Time
\( T \) Temperature
\( V \) Relative flow velocity magnitude
\( W_{cyc} \) Work-per-cycle
\( x,y,z \) Axial, Circumferential, Radial coordinate
\( X \) Influence coefficient (physical blade) domain
\( Y \) Traveling wave domain
\( \alpha \) Leading edge pitch mode or pitching amplitude (rad)
Relative flow angle (axial/circumferential plane)
Critical damping ratio
Chord-normalized axis perpendicular to chord
Stagger angle (or specific heat ratio, $= C_p/C_v$)
Thickness of quasi-2D grid
Mass ratio
Bend angle (or Solidity, $= c/s$)
Chord-normalized axis parallel to chord
Critical choking pressure ratio, $= p_2/p^*$
Density
Interblade phase angle
Modal displacement
Modal velocity
Angular frequency, $= 2\pi f$
Non-dimensional aerodynamic damping
Zweifel coefficient

Subscripts/Superscripts

- $c$ Cosine mode
- $cm$ Center of mass
- $cyc$ Per-cycle
- $i$ Imaginary part of a complex number
- $is$ Isentropic
- $r$ Real part of a complex number
- $s$ Static or Sine mode
- $std$ Steady
- $t$ Total
- $X$ Physical blade displacement
- $Y$ Traveling wave displacement
- $0$ Stagnation
- $*$ Critical choking condition
- $1$ Inlet
Exit
(˘) Differentiated with respect to time
(¯) Complex conjugate
(ˆ) Indication of vector quantity

Abbreviations/Acronyms

BC  Boundary Condition
BP2  Industrial low-pressure power turbine blade descriptor
FP  Flat Plate
GF  Generalized Force
HCF  High-Cycle Fatigue
IBPA  Inter-Blade Phase Angle
Im  Imaginary part of complex number
KE  Kinetic Energy
KTH  Academic low-pressure turbine airfoil descriptor
LCO  Limit Cycle Oscillation
LE  Leading Edge
LPT  Low-Pressure Turbine
MC  Mid-Chord
MUSTANG  Frequency domain RANS solver used for CFD
ND  Nodal Diameter
PA  Pitching Axis
PS  Pressure Side of airfoil
SS  Suction Side of airfoil
SC4  Standard Configuration
TD  Tie-Dye
TE  Trailing Edge
Acknowledgements

First and foremost, an enormous “thank-you” goes out to my committee: Dr. Earl Dowell, Dr. Kenneth Hall, Dr. Damian Vogt, Dr. Thomas Witelski, and my adviser Dr. Robert Kielb. I am truly lucky to have had such great teachers and mentors during my time at Duke. Your kind support, quality teaching, friendly interactions, and great suggestions for the direction of my Ph.D. education at Duke is greatly appreciated. Thank you for all the memories.

Next, a very special thank you goes out to Stephen Clark, who was my mentor upon joining Dr. Kielb’s Turbomachinery Aeroelasticity group in 2012, and quickly brought me up to speed in this brand new research field. Stephen is a good friend, and I will always be grateful for his facilitation of my transition into this group in a selfless and efficient manner.

I also need to recognize and thank Simon Bittner and Christopher Meinzer of the Institute of Turbomachinery and Fluid Dynamics located at Leibniz University Hannover for their help, resources, and collaboration on the Hannover airfoil analyses. Additionally, I would like to acknowledge Paul Petrie-Repar, Professor at the KTH Royal Institute of Technology for his collaboration, support, and providing details vital for an airfoil test case used in this research.

Thank you to the entire Duke Aeroelasticity Group, including but not limited to: Ivan Wang, Chad Gibbs, Eli Giovanetti, Dean Culver, Fanny Besem, and Jing Li. This was a special support group of intelligent colleagues and close friends that were always there to bounce ideas off, help work through problems, or sometimes provide a good laugh. Thanks to a great bunch of Hudson Hall friends.

Lastly, I was fortunate enough to have my LPT flutter research funded by the American Society for Engineering Education. I would like to acknowledge their immense support and generous financial aid through the National Defense Science and Engineering Graduate (NDSEG) Fellowship.
1
Overview

1.1 Introduction to Turbomachinery

Whether it is an aircraft engine providing thrust for a BOEING 777 at an altitude of 35,000 feet, or a land-based gas turbine capable of powering cities, a turbomachine is a complex mechanical system composed of rotating and stationary airfoil-shaped blades whose sole purpose is to extract energy from a working fluid and convert it to useful work. These marvels of engineering are so pervasive, and their importance so critical to society’s function, that a projected 84% of the world’s electricity consumption will be generated from turbomachinery sources by 2040 despite the momentum-gaining trend toward renewable energy[16].

Turbomachines can be classified into three different variations according to the direction of working fluid flow relative to the engine’s axis of rotation: axial, radial, and mixed flow turbomachines. The primary focus herein is on axial turbomachines, such as the GE90 turboshaft jet engine depicted in Figure 1.1, where the fluid flows parallel to the axis of rotation.

Within the axial turbomachine category, there are several variations of gas turbine engines that serve as the power-generating propulsion systems for airplanes or helicopters. For example, turboshaft engines are lightweight units used to generate the shaft power required to spin the large rotor blades seen on helicopters. On the other hand, turbofan engines, commonly seen on commercial aircraft, use the energy extracted by a turbine to power the fore ducted fan that contributes to the majority of thrust in modern high bypass ratio designs. The components of a typical turbofan jet engine are shown in Fig. 1.2. Of course, the overall goal of a jet engine is to provide forward thrust\(^1\). This is accomplished by the team contributions of several rotating and non-rotating components performing their specific tasks. Simplistically

\(^1\) The overall goal of the much heavier, ground-based power turbine is to generate electricity by driving a turbo-generator shaft.
explained, these component tasks are the following:

- **Fan**: Suck-in outside air to supply the core
- **Compressor** (low-pressure (LPC) followed by high-pressure (HPC)): Compress the air for higher energy potential
- **Burner**: Add fuel to air and continuously ignite for powerful explosions
- **Turbine** (high-pressure (HPT) followed by low-pressure (LPT)): Extract the energy from the high-velocity air to drive the fore fan\(^2\).
- **Nozzle** (typically co-annular in turbofans): Expand hot exhaust to ambient atmosphere, control the mass flow (in the case of variable geometry convergent-divergent nozzles), and provide a percentage of the thrust.

Note the stators (vanes) are not pictured in Fig. 1.2. Stators, located between rotor stages, are non-rotating cascades that remove swirl and direct the flow between stages. This effectively provides a better angle of attack for the spinning rotors.

Regardless of engine variety, all turbomachines follow a similar series of thermodynamic processes referred to as the Brayton Cycle\[46\]. Figure 1.3 is a pressure-volume (P-V) diagram which schematically represents the set of idealized thermodynamic processes in a Brayton Cycle. Beginning with the inlet, atmospheric stagnation conditions are indicated by the number 0. This marks the cycle’s starting point. From there, compression stages impart work on the gas and (ideally) increase pressure and temperature isentropically from point 2 to 3. The next process from 3-4 involves a constant-pressure combustion due to the addition and ignition of fuel in the burner. This extremely hot, high-velocity gas exits the burner and useful work is extracted by the turbine stages (point 5) prior to nozzle further isentropically expanding the flow back to free stream conditions, which closes the cycle.

\(^2\) and/or compressor depending on the engine type
**Figure 1.2**: Schematic view inside a typical jet engine. Arrows indicate axial flow direction. Original image modified from Wheeler [76].

**Figure 1.3**: Pressure-Volume Diagram for the series of thermodynamic processes in the Brayton Cycle used in all gas turbines. Source: NASA Glenn Research Center [46]
1.2 Motivation for Research

In engineering, aeroelasticity is the special branch that studies the interactions between aerodynamic, elastic, and inertial forces of a fluid flowing past a solid body. The three major aeroelastic issues found in the turbomachinery blades of jet engines and power turbines are forced response, non-synchronous vibrations, and flutter. Flutter is an unstable, self-excited blade vibration due to a positive feedback loop that is created from unsteady aerodynamic forces coupling with the structural vibration of the rotor blades. The blade amplitude exponentially grows or reaches limit cycle oscillation (LCO), which can lead to high-cycle fatigue (HCF) and eventual failure. In the case of commercial aircraft, HCF blade fragmentation can lead to engine loss. Much worse, in the case of single engine military fighters, the same phenomenon could ultimately lead to loss of life and/or aircraft. Clearly, this is an important dynamic instability and deserves careful attention.

Fans, front compressor stages, and aft turbine stages are the most likely locations flutter will occur in aircraft engines or power turbines. This is partly due to high loading conditions in combination with their relatively high aspect ratios creating an unfavorable aeroelastic ratio of aerodynamic force to structural stiffness. Low-pressure turbine (LPT) blades, which are long and slender in nature, are subject to high flow velocities as the goal of the LPT is to extract flow energy to drive the fan and/or low-pressure compressor. Exacerbating this is a modern trend to reduce the overall engine weight for fuel efficiency purposes. One particular way to achieve this is to redesign the rotor blade by shortening the chord length. For a constant flow path, this further increases the blade aspect ratio which effectively lowers the eigenfrequencies. These conditions make rear LPT stages extremely flutter prone and of great interest to the modern aeroelastic community.

Other modern turbomachinery trends are pressuring designers to create lightweight, ultra-high bypass ratio aircraft engines, and highly supersonic relative tip exit Mach numbers in power turbines. Not only do these design goals decrease the highly-correlated LPT flutter parameter known as reduced frequency, but they also introduce higher per-stage loading conditions and shock structures never before seen in many legacy engine designs, rendering empirical knowledge ineffective. Consequently, the loading and shock structures seen on aft stages of low-pressure turbines are evolving at a fast pace. Since flutter is a high risk event in these low reduced frequency blades, physically understanding and predicting how new operating conditions affect aeroelastic behavior proves crucially important.

Low-pressure turbines have relatively simple unsteady aerodynamics when compared to fans and compressors. That is, the mean flow is clean, linear, and behaves rather two-dimensionally. It is no wonder why we have a relatively greater understanding of turbine aeromechanics compared to compressors and can computationally predict LPT flutter with decent accuracy in most cases. However, unexpected flutter incidents in industry still occur and therefore confirm limitations of current predictive tools. Commercial Computational Fluid Dynamics (CFD) codes still de-
pend on many empirical correlations and simplified CFD models and assumptions. When introduced to atypical blade redesigns or unanticipated dynamic phenomena, these codes may inaccurately assess overall stability. Conceptually understanding the physics of turbomachinery flutter aids engineers in minimizing design delays, meeting project deadlines, and eliminating high-cycle fatigue failures in active jet engines or power turbines. Therefore, a deeper physical understanding of the mechanisms driving flutter is necessary to devise more reliable predictions.

Most detailed CFD codes are accompanied by cryptic turbulence models, meticulous grid constructions, and elegant boundary condition enforcements with mostly one goal in mind: determine the sign of the aerodynamic damping. For the most part, blade aerelasticians ask themselves, “is it positive or negative aero damping?” Of course, this is a watered-down explanation of the complete set of engineering analyses required to eradicate flutter possibilities. In reality, accurate calculations of how negative the value of aerodynamic damping is proves critical in some applications, especially when nonlinear structural damping mechanisms, like friction dampers, are implemented. The reason is that these added friction dampers can be tuned to counteract specific levels of aerodynamic instabilities. In the case of limit cycle oscillation (LCO), the overall level of all damping sources is computed such that the engineer can guarantee the maximum vibration amplitude is well within fatigue limits. Virtually every passenger that has flown commercially on a turbofan-powered aircraft has unknowingly experienced some form of non-catastrophic, intermittent flutter or LCO. This is possible because engineers properly combined the linear aerodynamic flutter analysis with a nonlinear structural damping analysis to ensure low enough vibration amplitudes that meet infinite fatigue life requirements. However, when the engineer blindly uses pre-existing CFD codes with the sole purpose of determining the sign of aerodynamic damping, and completely disregards the complex physics occurring behind the scenes, this gives rise to a black-box effect, which leaves physical understanding behind. Having said that, this work aims to achieve three main objectives:

1. Understand and reveal the physics behind LPT flutter
2. Introduce new discoveries regarding the relationship between steady loading effects and LPT flutter
3. Present a preliminary flutter design tool and propose final design guidelines

Using CFD as computational wind tunnels, a multitude of known low-pressure turbine flutter parameters will be scrutinized across various airfoil geometries and operating conditions. In addition, new steady loading parameters effecting LPT flutter boundaries are introduced. The physics of LPT flutter is numerically investigated using an influence coefficient approach to study the variation of unsteady pressure amplitude and phase as a function of flow condition and other parameters. Many pressing topics encompassing LPT flutter, including acoustic resonance,
three-dimensionality, and nonlinearities, are also addressed along the way prior to concluding the work with a useful preliminary design method and final design guidelines.

1.3 Chapter Outlines

The remainder of this document is organized as follows:

- **Chapter 2**: This chapter begins with describing the physical causation of cascade flutter in turbomachinery and how that differs from conventional coupled mode flutter seen on aircraft wings. Directly following, a review of the major research issues regarding flutter in low-pressure turbines is presented. Specifically, these topics will cover correlated unsteady and steady flutter parameters, aeroacoustics, three-dimensionality, nonlinearity, multi-row effects, and mistuning.

- **Chapter 3**: CFD analyses are conducted by sweeping key unsteady and steady LPT flutter control parameters on the publicly available Standard Configuration 4 LPT test case. Insights are drawn revealing a deeper understanding of how these parameters cause flutter.

- **Chapter 4**: The relationship between LPT flutter and blade loading is fully investigated. A complex, non-monotonic correlation between blade loading and aerodynamic damping is identified to be a function of mode shape and cascade/airfoil geometry. The underlying factors controlling the flutter-load link is identified to be choke-related which manifests itself as variations in shock strength, location, and unsteady pressure bifurcations primarily located on the suction side of the blade.

- **Chapter 5**: An advanced preliminary design method that provides quick estimates of damping vs. interblade phase angle is presented. This design tool uses existing baseline aerodynamic work matrices to radially integrate damping contributions from a series of quasi-2D rigid body mode shapes out to span. Using this tool, engineers can assess how hypothetical changes in airfoil shape, mode, stacking sequence, frequency, or loading may change aeroelastic stability for all traveling wave modes.

- **Chapter 6**: The final chapter concisely summarizes key points from the body of work and remarks on relevant future work remaining in the field.

- **Appendix A**: This appendix includes the instruction manual for the preliminary design tool code suite defined in Chapter 5. A procedure that outlines the generation of Baseline Aero files is also included.
This chapter presents a literature review surrounding several important topics in the realm of low-pressure turbine flutter analysis in turbomachinery. While most of the following literature specifically investigates turbines, some references investigate relevant phenomena found in fans or compressors. A special note is included in the case when different turbomachinery components are being discussed.

2.1 How LPT Cascades Flutter

The geometric terminology associated with airfoils and a sample two-dimensional profile are shown in Figure 2.1. The upper and lower surfaces are called the suction (SS) and pressure side (PS), respectively, and earn their names due to the relative pressure levels of air flowing past the airfoil from left to right in this example. Separating the leading edge (LE) and trailing edge (TE) is the imaginary dashed line called the chord line, which equals the (mean) camber line in the case of symmetric airfoils. However, this particular airfoil is asymmetric, and the degree of asymmetry is indicated by the max camber distribution.

Aircraft wings and turbomachinery blades are 3D objects whose spanwise 2D cross-sections are various airfoil shapes. Both are subject to the coupling effects of aerodynamic and structural forces that sometimes lead to the self-exciting, aeroelastic instability called flutter. However, the physical mechanism initiating this oft-catastrophic dynamical event is quite different for aircraft wings and turbomachinery cascades. Due to their relatively high mass ratios, turbomachinery blades with adequate frequency margin between modes will flutter in a single mode as opposed to aircraft wings which are subject to mode coalescence [10]. The mass ratio is physically defined as linear density ratio of the airfoil cross-section to the fluid circumscribing
it, or mathematically defined as

$$
\mu = \frac{4m}{\pi \rho_{\text{air}} c^2},
$$

(2.1)

where $m$ is the mass per unit span of the blade, $\rho_{\text{air}}$ is the fluid density, and $c$ is the blade chord.

Tuned annular cascades consist of a certain number of identically machined blades sitting equally spaced around the annulus. Disregarding manufacturing defects and tolerances, each blade will practically have the same eigenvectors (mode shapes) and corresponding eigenvalues (natural frequencies). Most flutter events occur at the low frequency range modes such as first bending (1F) and first torsion (1T). For an interblade phase angle (IBPA) of zero degrees, all blades in the excited cascade will oscillate in unison with the same mode shape, vibration amplitude, and phase. However, the minimum potential energy principle may also permit the cascade of blades to vibrate in a traveling wave mode (TWM) with a non-zero IBPA. That is, all blades in the cascade still oscillate with the same vibratory mode shape, frequency, and amplitude, but now with a constant phase shift, or time lag, between neighboring blades called the interblade phase angle. The IBPA (in radians) is mathematically presented as [12]

$$
\sigma = \frac{2\pi N}{N_b},
$$

(2.2)

where $n$ is the number of nodal diameters and $N$ is the total number of blades in the cascade. Note that $n$ can be positive or negative and range from 0 to $N/2$ (for an annulus with an even number of blades). The IBPA is positive or negative depending on whether the TWM is forward or backward traveling, respectively. For illustration purposes, Meinzer provided a simple sketch of how a traveling wave mode may be
visualized for a linear cascade of 12 flat plate airfoils subject to a pitching mode in Fig. 2.2 [41].

![Figure 2.2: Sketch of three traveling wave modes for a linear cascade of flat plates (N=12) undergoing pitching motion from Meinzer [41].](image)

To determine if a cascade is fully stable, negative (unstable) aerodynamic damping must not exceed inherent levels of structural damping or applied mechanical damping mechanisms such as platform dampers or shrouds. This must be true for the entire traveling wave domain, i.e., all IBPAs. One of the most widely used flutter diagnostic tools is the damping vs. IBPA plot. Fig. 2.3 is experimental damping vs. IBPA data for an EPFL test rig study examined by Panovsky and Kielb [49]. In this example, the cascade is unstable for an IBPA range roughly between $\sigma = 80\degree - 115\degree$.

The solid, red curve is a close approximation to the actual data that superimposes the unsteady pressure influences generated only by the reference blade itself (blade 0), and its two nearest neighbor blades on the reference blade’s immediate pressure (blade +1) and suction (blade -1) side. Interestingly, and true of all realistically defined turbine blades, the self-damping contribution, indicated by the mean value of the sine curve, $S_0$, on Fig. 2.3, is always positive (stabilizing). Thus, for turbomachinery blades, unlike aircraft wings with low mass ratios, flutter only occurs when the resulting aerodynamic forces from the entire system of blades is integrated. Simply put, neighboring blades cause cascade flutter. It is not until the unsteady pressure contributions from the nearest neighboring blades ($\pm 1$) are linearly superimposed that certain IBPAs may go unstable (if $S_1$ exceeds $S_0$).

These aerodynamic influence coefficients can be thought of as a Fourier series decomposition, with $S_0$ being the constant value contribution from a reference blade, the immediate neighboring blades providing the first harmonic content, the next closest airfoil neighbors ($\pm 2$) providing the second harmonic content (twice the fre-
Figure 2.3: Experimentally measured damping vs. IBPA and S-curve stability parameter fit from Panovsky and Kielb [49].

quency, half the wavelength), and so on. In most cases, the relative magnitude of the higher harmonic influence coefficients diminish with increasing distance away from the reference blade; therefore, although all blades on the annulus contribute to the unsteady response of a reference blade via linear superposition (for small perturbations), the nearest three blades, i.e., 0 & ±1, are the primary contributors. The small deviations from a pure sine wave in the experimental data of Fig. 2.3 represent the higher-order influences present in the cascade\(^1\) that vary as a function of reduced frequency and exit Mach number [73].

2.2 Key Control Parameters for LPT Flutter

Flutter occurs in turbomachinery blades when positive aerodynamic work (exerted by the fluid on the blade) exceeds inherent structural or designed mechanical damping mechanisms. Therefore, blade designers strive to have the blade doing the “good” work on the surrounding air to create stability. However, when the phase of the aerodynamic excitation lags the blade response, the resulting \(W_{cyc}\) is positive (unstable). Over the years, academia and industry have thoroughly studied and discovered some parameters very closely correlated to aerodynamic stability. First, Section 2.2.1 will discuss certain unsteady parameters (associated with the blade vibration) deemed

\(^1\) These are sometimes the manifestation of acoustic resonance.
to have significant impact on LPT flutter. Then, Section 2.2.2 will briefly discuss a lesser studied area which is the effect of steady parameters (associated with the mean flow and blade loading) on LPT flutter.

2.2.1 Reduced Frequency and Mode Shape

The effects of the reduced frequency parameter, $k$, on aerodynamic damping are well documented [49, 70, 4, 47]. The reduced frequency is defined as

$$k = \frac{\omega (c/2)}{V_2}. \quad (2.3)$$

Herein, the reduced frequency is defined using the semi-chord definition, where $c/2$ is the semi-chord, $\omega = 2\pi f$ is the blade’s angular oscillatory frequency in rad/s, and $V_2$ is the exit velocity relative to the rotating rotor frame of reference. Normalizing by the exit relative velocity is common when working with turbine blades, whereas inlet relative velocity correlates better with fan and compressor flutter. There are two physical interpretations of $k$:

1. The time a fluid particle takes to travel a distance equal to semi-chord ($c/2$) divided by the vibrating blade’s oscillation period.

2. The semi-chord length ($c/2$) multiplied by the wavenumber ($\frac{\omega}{V_2}$).

Generally speaking, airfoils oscillating at lower reduced frequencies have a higher tendency to flutter. Aft stages of low pressure turbines in aircraft engines and power turbines consist of particularly high aspect-ratio blades subject to high subsonic or supersonic relative velocities, which tend to yield airfoil oscillations in the low reduced frequency regime. Adding to this problem is the modern material and weight-saving trend for aircraft engine manufacturers focused on increasing fuel efficiency by maximizing power-to-weight ratio. By reducing the blade chord or thickness, a lighter, lower-reduced frequency blade results. Consequently, this compounds the already low reduced frequency aft LPT stages which deservedly receive more flutter concern than any other turbomachinery component. In summary, the aeroelastic engineer strives to design blades with higher reduced frequencies to avoid unwanted vibrational instabilities.

Another essential unsteady parameter which can greatly affect LPT flutter stability is the blade’s vibratory mode shape [36, 64, 73]. There are two main types of turbine mode shapes: blade-dominated (e.g. bending, torsion, corner, and stripe modes) and disk-dominated (i.e. blades are essentially passive masses attached to the rotor). Kirschner was among the first to verify the importance of mode shape on aerodynamic damping through a series of simplified experiments testing various torsion modes, bending modes, and a combined modes for four elastically suspended blades [36].
Furthermore, since flutter normally occurs in one of the first few natural modes, these three-dimensional modes can be approximated as rigid body translations or rotations with varying spanwise amplitude. Combining this with the fact that LPT flow characteristics are remarkably linear [32], the stability of two-dimensional mode can be approximated by applying the principle of superposition to the 2D CFD of three fundamental rigid body mode shapes: two orthogonal translations and a rotation [49]. To illustrate this, Panovsky and Kielb created a preliminary design tool, the colorfully named “Tie-Dye” (TD) plot, which conveniently maps the critical reduced frequency, $k_{\text{crit}}$, which corresponds to neutral aerodynamic damping for all possible interblade phase angles (IBPAs), as a function of mode. That is, a reduced frequency lower than the critical value is aerodynamically unstable and could lead to flutter. Figure 2.4 is a sample TD plot of Standard Configuration 4 (an LPT described in [6]) for a fixed exit Mach number of $M_2 = 0.90$. There are two orthogonal, chord-normalized axes: 1) $\xi$ - parallel to the chord-line (positive in the direction leading to trailing edge) and 2) $\eta$ - perpendicular to the chord-line (positive in direction of pressure to suction side of airfoil). These axes define the a particular mode’s center of twist or “pitching axis”. For example, here are four points and their corresponding mode shape:

1. $(\xi, \eta) = (0, 0) \rightarrow$ Pitching about the leading edge
2. $(\xi, \eta) = (1, 0) \rightarrow$ Pitching about the trailing edge
3. $(\xi, \eta) = (\infty, 0) \rightarrow$ Translating/bending perpendicular to chord-line or “flex”
4. $(\xi, \eta) = (0, \infty) \rightarrow$ Translating/bending parallel to chord-line or “edgewise”

Note that the $(\xi, \eta)$ location for a pure bending mode shape would be out at infinity because the “bending arm” is infinitely long. As a quick example of reading TD plots, suppose the mode shape of interest is defined as pitching about the leading edge. This mode shape is located at $(\xi, \eta) = (0, 0)$, which has a color code corresponding to $k_{\text{crit}} = 0.25$.

As the pitching axis varies throughout the four-chord window of Fig. 2.4, $k_{\text{crit}}$ changes by an order of magnitude. This simple fact underscores the strong correlation between mode shape and LPT flutter stability for a fixed reduced frequency, and sheds light on how harsh gradient regions should be intentionally avoided by LPT design engineers. Furthermore, it has been shown that standard Tie-Dye map contours compare closely at the qualitative level across a wide variety of airfoil shapes for fixed steady loading conditions.

Peng and Vahdati did a similar study to the TD plot method only they used the ratio of the leading edge to trailing edge displacements as their varying parameter[50]. Their study showcased a critical region where stability sharply dropped for a specific mode shape displacement ratio. This high-gradient region of the aerodynamic damping as a function of their mode shape parameter agrees with the notion of mode shape playing a dominant role in flutter stability.
There are some studies that have peered into the combined effects of reduced frequency and mode shape. Panovsky and Kielb [49] and Vega and Corral [69] both investigated subsonic aerodynamic damping at various reduced frequencies for bending and torsion modes. Both plotted the influence coefficients of these modes for low and high reduced frequencies. Their main conclusions agree that for low reduced frequency bending modes, the blade-on-itself effects and neighboring pressure side (PS) have the largest contribution to the work-per-cycle. Furthermore, for the torsion mode, it was found that influence coefficients far away from the reference blade decay much less than for a bending mode indicating the presence of higher-order pressure wave propagation.

2.2.2 Steady Loading Parameters

While LPT flutter sensitivities are well-understood from an unsteady parameter perspective, much less research has concentrated on the influence of steady parameters on aeroelastic flutter. Recently, evidence observed in engine tests and via numerical methods have indicated steady loading combined with shock location are also key flutter drivers [75, 11]. This hot topic is more important than ever. The future of turbomachinery design demands lighter jet engines and higher energy-output from power turbines. Direct consequences of these trends are higher loading per stage and the present-day push into higher Mach number regimes rife with shock effects. With this trend, physically understanding the link between loading and flutter will soon be paramount. It is imperative to quickly learn how reduced frequency, mode shape,
and loading conditions are all fundamentally interrelated in determining LPT flutter stability.

The few steady parameter studies of the past focused on correlations tied to incidence angle, loading, or flow velocity.

Carta showed the flow incidence angle, which consequently affects the mean loading and blade torque, was a key stability parameter for a mid-chord pitching mode of blades in a linear, subsonic research compressor [9]. Specifically, higher angle of attacks reduced stability by decreasing the critical reduced frequency. Széchenyi showed that at certain higher angles of attack, the aeroelastic stability is dominated by the separated flow behavior near the leading edge [63]. Verifying this experimentally using a low-pressure turbine cascade was the work of He [29]. He investigated various incidence angles and concluded large destabilizing separations inhabited the pressure side of the turbine blade. In addition, the downstream reattachment point was found to be stabilizing which counteracted the detrimental effects of the PS separation bubble.

Industry has recently discovered that a re-designed turbine blade destabilized an aft LPT stage. To be specific, the re-design ended up pushing more load toward the tip (resulting in higher torque) which indicates steady loading variations may be tied with flutter stability. Additionally, an attractive weight-saving trend in gas turbine engines is to decrease the overall number turbomachinery stages. Less stages means more loading per stage which may result in a similar instability as the aforementioned re-design. However, there are very few studies that detail the effect of steady loading on LPT flutter boundaries. A brief mention of this was presented by Cardinale in the early 1980s [8]. Cardinale explored turbine flutter sensitivities in off-design conditions and found an unusual function of loading shown in the power turbine speed sweep seen in Fig. 2.5. By sweeping the power turbine speed while maintaining constant core speed, approximate flutter boundaries were mapped as a function of bucket torque and rotational speed. A strange observation is that, for higher power turbine speeds, increasing the loading incited transitions into and back out of flutter as evidenced by the composite instability.

Another collection of steady parameters were experimentally tested for their correlation to LPT flutter by Nowinski and Panovsky [47]. This work studied three different LPT torsion modes subject to various flow conditions. Tested steady parameters include the static pressure ratio, incidence angle, and the Zweifel coefficient. Of these three steady parameters, the strongest correlation was in the static pressure ratio which approximately varied linearly with the minimum blade-on-itself damping coefficient. The Zweifel coefficient was discussed as a primary loading guideline and is a measure of the pressure differential across the blade normalized by the exit dynamic pressure\(^2\). If we assume \(M^2 \ll 1\) and \(p_{0,1} = p_{0,2}\), the Zweifel coefficient

---

\(^2\) Somewhere along the history of turbomachinery design, it became standard procedure to use exit normalized parameters instead of using inlet parameters like fans and compressors.
Figure 2.5: Investigation of torsion instability boundaries as a function of torque by Cardinale [8].

The equation can be simplified to

\[ \Psi = \frac{2}{\theta} \sin(2\beta_2) \left| \frac{V_1}{V_2} - 1 \right| \]

where \( \theta \) is the cascade solidity, \( \beta_2 \) is the trailing edge relative flow angle, and \( V_1 \) and \( V_2 \) are the inlet and exit relative flow velocities, respectively [82]. Since the Zweifel coefficient had very low correlation to damping, it was concluded to have only secondary effects on the blade-on-itself damping and no effect on the neighboring blades. However, as later discussed in Chapter 3, other steady loading parameters can have a significant effect on LPT flutter stability.

High axial velocity, or supersonic exit Mach numbers, have also been linked to LPT flutter. This is not completely surprising as the exit axial velocity is incorporated into the reduced frequency parameter, and raising the exit velocity serves to decrease \( k \). While most aircraft engines rarely operate with exit Mach numbers exceeding the high subsonic regime, we do see power turbine LPTs with exit Mach numbers approaching \( M_2 = 2 \) in some cases. Previous research by Meingast, Kielb,
and Thomas quantified the effect on LPT flutter ranges due to the variation of the exit Mach number, $M_2$ [40]. It was shown that for subsonic flows, most mode shapes were stabilized as the exit Mach number was increased. Although exit Mach number did affect the flutter boundary, its role was identified as secondary compared to reduced frequency and mode shape. Additionally, a recent study by Vega and Corral concluded that while the phase of the force influence coefficients were relatively independent of exit Mach number, their modulus and therefore work contributions demonstrated correlation [69].

Investigating a small range of pressure ratios, Shibata and Kaji found that increasing the back pressure ratio of a fan rotor stabilized plunging modes [55]. This study was conducted by carefully adjusting the inlet Mach number with back pressure as to not change the overall shock structure. They also asserted that trailing edge shocks destabilized bending oscillations, but also witnessed the cascade instability vanishing as higher back pressure ratios pushed the shock upstream into the passage. Also investigating transonic rotor flutter, Isomura and Giles showed the primary driver of stability fluctuations near the stall line was the large unsteady pressures due to shock oscillations, as opposed to blade stall [30]. Recently, Corral and Vega studied lightly and highly loaded airfoils in the low reduced frequency range ($k < 0.05$) [11]. Introducing a wall-normal velocity gradient parameter, they provided a mathematical formulation showing that the unsteady pressure phase, which determines cascade stability, varies linearly with $k$ for highly loaded airfoils, whereas it departs from $90^\circ$ for lightly loaded airfoils.

In the realm of aircraft wing flutter, Ashley brilliantly investigated the effect of shocks in the “sub-transonic” regime, which is defined as flow conditions having effective subsonic inlet Mach numbers that also induce part-chord shocks [2]. The application included arbitrarily superimposing shock loads (modeled by force doublets) on top of airfoil-distributed unsteady pressure loads which were computed using classical subcritical linearized theory. The shock-induced lift and the nose-up moment contributions on the airfoil pressure side of the two-degree-of-freedom typical section model (mid-chord pitch and plunge) were given, respectively, as:

\[
L_s = 2b(p_{s2} - p_{s1}) (\frac{x_{\text{foot}}}{2b}) e^{i(\omega t - \phi_s)}, \tag{2.5}
\]

\[
M_{ys} = -4b^2 \left( \frac{x_s}{2b} \right) (p_{s2} - p_{s1}) \left( \frac{x_{\text{foot}}}{2b} \right) e^{i(\omega t - \phi_s)}, \tag{2.6}
\]

where $p_{s1}$ and $p_{s2}$ are the pressures directly fore and aft of the shock, respectively, $x_{\text{foot}}$ is the amplitude of the shock foot motion, $x_s$ is the mean shock position, and $\phi_s$ is the shock phase with respect to the airfoil motion. Combining these with subcritical linearized theory, mathematically verbose equations describing the total lift and moment were presented. These expressions included the Theodorsen function and three first-kind Bessel functions with an argument that was a function of free-stream Mach number and reduced frequency only. In addition to the two aforementioned non-dimensional parameters, other important dimensionless quantities appearing in
the final two-degree-of-freedom equations of motion include the ratio of plunge-to-pitch frequency \( \omega_w/\omega_\theta \), the mass ratio \( \mu \), and the non-dimensional location of the center of mass and elastic axis. According to Ashley, tens of thousands of parametric sweeps of these non-dimensional parameters were investigated to determine the overall contribution of shock effects to the critical velocity (inverse of critical reduced frequency) of several different airfoil sections. Of course, all results were heavily dependent on mass ratio; interestingly, the qualitative stability trends somewhat simplified toward high values of \( \mu \) where turbomachinery blades are typically designed. Since Ashley’s research models an isolated wing, and not a cascade of airfoils, some proposed conclusions are not immediately applicable to LPT blades. However, Ashley’s results at high mass ratio and single-mode-dominated frequency ratios (i.e. mostly pure pitch or pure plunge) can be thought of as the effect to a LPT blade’s self-damping, i.e., term 0 in the influence coefficients. One interesting result was that for high mass ratios, the pure pitching mode saw destabilizations due to a shock, whereas pure plunging modes saw markedly favorable increases in critical airspeed. Kielb and Kaza conducted a mistuning study using the same two-degree-of-freedom typical section model [33], only this time investigating a full cascade of airfoils undergoing pitch and plunge subject to the subcritical unsteady aerodynamic loads described by Smith [58]. This study verified the assertion that unless the frequencies of the first-bending and first-torsion modes are extremely close for real, high-mass ratio turbine blades, multi-mode effects are considered negligible to overall stability contributions.

In 2002, Srivastava studied the relationship between shock characteristics and 3D fan flutter after seeing shocks sharply influence local work contributions [62]. Srivastava contended that by raising the fan’s back pressure, therefore increasing the load, the shock foot moves upstream while simultaneously increasing strength and the area of positive (destabilizing) aerodynamic work, especially at higher spanwise locations. Continuing this research in 2004, Srivastava devised an algebraic model showing how interblade phase angle (IBPA) and shock strength/location affected flutter stability [60]. These mathematical expressions, which model a flat plate, show how work-per-cycle contributions due to the shock changes for both the airfoil suction side (SS) and pressure side (PS), and are given as:

\[
W_{SS} = -\bar{\alpha}\bar{a}\pi D(\Delta p_{SS})\sin(\sigma + \epsilon) \tag{2.7}
\]

\[
W_{PS} = +\bar{\alpha}\bar{b}\pi F(\Delta p_{PS})\sin(\sigma + \lambda). \tag{2.8}
\]

In Eqs. 2.7 & 2.8, \( \bar{\alpha} \) is the blade pitching amplitude, \( \bar{a} \) & \( \bar{b} \) are shock motion amplitudes, \( D \) & \( F \) are the distances between the pitching axis and shocks, \( \Delta p_{SS} \) & \( \Delta p_{PS} \) are the pressure differentials across the shocks, \( \sigma \) is the IBPA, and \( \epsilon \) & \( \lambda \) are the shock phases with respect to the blade motion. These expressions imply a strong relation between shock structure, aerodynamic damping, and mode shape. In fact, it suggests there is a linear relationship between work and the gap between the shock and pitching axis. Regardless of operating condition, the IBPA range will be split
in two. Some traveling wave modes will be adversely affected by the shock, whereas other IBPAs will show signs of stabilization. Foreshadowing later discussions, it can be logically inferred from these expressions there exists a strong relation between shock structure, aerodynamic damping, and mode shape.

2.3 Aeroacoustics

Under certain conditions, acoustic waves can propagate through a turbomachinery duct as mechanical waves of pressure and displacement through a fluid medium such as air. A directional propagation condition requires a purely real wavenumber, which is the spatial eigenvalue associated with each mode [23]. This means that for low excitation frequencies, only the fundamental acoustic mode \((\sigma = 0^\circ)\) will propagate. However, as the frequency is gradually increased, additional modes begin to propagate once each mode’s particular “cut-off” (or “acoustic resonance”) frequency has been reached. Once a mode’s cut-off frequency is surpassed, the mode is said to be “cut-on”, whereas the mode is “cut-off” at all lower frequencies. Considering this, flutter may be segmented into three categories:

1. Sub-critical flutter: acoustic waves cannot propagate through the duct
2. Acoustic resonance flutter: a pair of waves are at the brink of propagation
3. Super-critical flutter: at least one pair of waves can propagate

Acoustic resonance consequences can be detrimental or beneficial to the overall aerodynamic damping depending on the phase of resulting acoustic waves. It can only occur when the traveling wave mode (i.e. IBPA) exactly matches the resonance pattern. This phenomenon essentially adds one extra degree of complexity to the aeroelastic system.

According to Whitehead, neighboring blade rows have a suppressive effect to acoustic resonant behavior [78]. Additionally, Vega and Corral investigated the physics of LPT flutter and found acoustic perturbations do not play a strong role in the low reduced frequency regime [69]. Although, for higher reduced frequencies and especially torsion modes, acoustic perturbations were quite important to the overall unsteady effects.

Wu et al. highlighted the relevance of aeroacoustic flutter for fans in 2003 for a high bypass-ratio jet engine [80]. The aeroacoustic characteristics of the intake duct caused the fan blade to flutter. For a narrow range of engine speed, flutter removed a “bite” from the stable operating condition. This sharp, local drop in stability is commonly referred to as the “flutter bite”. Additionally, Vahdati agreed that the intake geometry had a critical effect on upstream pressure wave reflections and fan blade flutter [68]. Specifically, fan stability is dominated by the upstream geometry and acoustics and increasing the intake length moves the flutter bite to a lower operating speed. Vahdati used a bump feature in the CFD grid to artificially
reflect upstream pressure waves and found it was most unstable when there was a 90° phase between the upstream and reflected wave. The worst case scenario is when flow-driven flutter and acoustic flutter coincide at the same speed [67]. Lastly, it was concluded fan flutter can occur when the acoustic pressure field is simultaneously cut-on upstream and cut-off downstream of the rotor because energy cannot transport downstream and is forced to propagate back through the passage [66].

The significant unsteady aerodynamic contributions for a low-pressure turbine blade can mostly be captured by considering only three blades in the entire cascade: the reference blade and its immediate neighbors on the pressure side (PS) and suction side (SS). If the contributions of all other blades are assumed to be identically zero, it can be shown the dependence of aerodynamic damping on inter-blade phase angle (IBPA) takes on the exact shape of a sine wave [49]. The Discrete Fourier Transform (DFT) of this sine wave consists of three components. The first component is the mean sinusoidal value which is actually the blade-on-itself damping [49]. For practical turbine blade designs, this contribution almost always has positive damping indicating flutter cannot occur without neighboring blades. The 2nd and 3rd components of the DFT are the next two adjacent blades (on both sides of the reference blade) which complete the pure sinusoidal shape. If the effects of acoustic resonance and higher harmonic terms (blades farther away) did not exist, damping vs. IBPA plots would be perfectly sinusoidal for tuned cascades. Therefore, a damping curve exhibiting acoustic resonance behavior should appear sinusoidal except near predicted acoustic resonance IBPAs where the curve may encounter positive or negative spikes in damping giving an overall richer harmonic content.

The propagation of subsonic and supersonic acoustic waves has been studied extensively [65, 58, 45]. The equation governing the decay or propagation of acoustic perturbations for a 2D cascade of flat plates was given by Kielb and Kaza [33] as

\[
\left( \frac{\sigma_r + 2\pi n}{s/c} \right)^2 + \frac{4M^2}{M^2 - 1} k \left[ k + \left( \frac{\sigma_r + 2\pi n}{s/c} \right) \sin \gamma \right] = 0, \tag{2.9}
\]

where \( \sigma \) is the IBPA (in radians), \( s \) is the blade spacing, \( c \) is the chord, \( M \) is the inlet or exit Mach number, \( k \) is the inlet or exit reduced frequency, \( \gamma \) is the stagger angle, and \( n \) is an integer multiplier. To find the acoustic resonance IBPAs, solve for \( \sigma_r \) for values of \( n \) permitting \(-\pi \leq \sigma_{r=1,2} \leq \pi\). Take note that for a given set of parameters this equation has two solutions. Thus, a total of four unique solutions are available: two associated with the inlet and two associated with the exit flow (which are later referred to as inlet/exit A and B in Chapter 3).

### 2.4 Three-Dimensional Effects

Not only are LPT aerodynamics remarkably linear, but there three-dimensional behavior is subdued when compared with compressors blades, i.e. LPT flow dynamics
are quite two-dimensional. The experimental Standard Configurations (SC) constructed in EPFL afford us the most extensive, publicly-available source of 2D flutter data [6]. McBean et al. and Witteck et al. looked at the three-dimensional LPT blade effects of Standard Configuration 4 (SC4). They found significant three-dimensionality that disturbed the amplitude and phase of the blade surface’s unsteady pressures [39, 79]. However, these effects mainly operated near the blade tip and therefore the midspan aerodynamics still behaved in a two-dimensional manner. Furthermore, Witteck stated previous tip clearance studies by Micallef were shown to carry negligible changes in flutter predictions [43].

Vogt was one of the first to experimentally investigate three-dimensional flutter mechanisms of low-pressure turbines [70]. He developed a new test facility for a controlled investigation of flutter in 3D mean flow. In the facility, an annual sector of LPT blades featured one free-to-oscillate blade. The blade profile and geometric parameters are given in Figure 2.6.

The rigid body mode shapes investigated were axial bending, circumferential bending, and torsion. By using an influence coefficient approach, the flow data could be recorded at various blades in the cascade and be converted back to traveling wave coordinates. The three-dimensionality of the aeroelastic response was tested over a multitude of combinations of mode shapes, reduced frequencies, subsonic exit flow velocities, and inlet angle of attacks. These tests were validated with good agreement using a non-commercial unsteady numerical prediction tool (VOLSOL).

Vogt segmented his experiments into three degrees of accuracy: two-dimensional (50% span), three-dimensional, and three-dimensional with tip clearance. Some interesting 2D flutter conclusions Vogt made are:

---

**Figure 2.6**: LPT blade profile and geometry of 3D Vogt experiments [70].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real chord (midspan)</td>
<td>c</td>
<td>mm</td>
<td>50</td>
</tr>
<tr>
<td>Axial chord (midspan)</td>
<td>cₜₐₓ</td>
<td>mm</td>
<td>45</td>
</tr>
<tr>
<td>Span</td>
<td>s</td>
<td>mm</td>
<td>97</td>
</tr>
<tr>
<td>Pitch (midspan)</td>
<td>pₚ</td>
<td>deg</td>
<td>4.5</td>
</tr>
<tr>
<td>Solidity (midspan)</td>
<td>βₚ,cₜₐₓ</td>
<td>-</td>
<td>0.68</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>s/c</td>
<td>-</td>
<td>1.94</td>
</tr>
<tr>
<td>Radius ratio</td>
<td>γ</td>
<td>-</td>
<td>1.25</td>
</tr>
<tr>
<td>Hub radius</td>
<td>rₜₜ</td>
<td>mm</td>
<td>383</td>
</tr>
<tr>
<td>Shroud radius</td>
<td>rₜₜ,hₜ</td>
<td>mm</td>
<td>480</td>
</tr>
</tbody>
</table>
1. The aeroelastic response was the highest on the oscillating blade and its two immediate neighbors.

2. The pressure magnitudes about the airfoil surface at midspan were similar for all three modes.

3. Instantaneous passage throat size was of critical importance.

4. The circumferential bending mode pressure amplitude was lowest due having the smallest influence on passage throat.

5. Of the reduced frequencies tested (ranging from k=0.2-1.0), the higher values of k led to higher pressure amplitudes and phase veering on the fore part of blade (toward the LE).

6. Increasing flow velocity had moderate influence on the unsteady pressures.

7. Separated flow due to incidence angle variation leads to considerable changes; however, the effect is negligible after downstream reattachment point.

8. Incidence angle has a high effect on unsteady pressure magnitudes due to higher blade loading and secondary flow effects.

Two types of three-dimensionality were found experimentally. First, the increasing amplitude from hub-to-tip of the mode shape played a role. Second, there were local variations in the unsteady pressure magnitude around the airfoil due to radially prominent mean flow field variations. Some of Vogt’s compelling conclusions regarding 3D flutter effects are:

1. The most pronounced effect is spanwise variations due to respective blade mode shape (increasing response amplitude out to span).

2. Locally varying pressure magnitudes were due to spanwise blade loading variations and viscous hub/endwall effects, especially for the torsion mode.

3. Local variations lead to distinct changes in local stability.

4. For the axial bending mode, the aft suction side was the largest stability contributor due to passage throat influence.

5. For the circumferential bending mode, the fore suction side was the largest stability contributor.

6. For the torsion mode, the aft part dominates stability contributions, but there is a very large variation out to span.

7. Prediction accuracy was greatly improved when including a tip gap in the 3D model.
8. The consequence of not modeling a tip clearance is generally over-predicting the pressure magnitudes. These differences are mainly driven by the pressure differential rather than viscosity.

9. To fully conclude flutter predictions on blade 0, a full three-dimensional analysis is recommended.

10. The spanwise aerodynamic damping contribution tends to decrease from hub-to-tip. Most 2D flutter analyses are performed at the tip because this is typically the point of maximum displacement for low order modes and unshrouded blades.

2.5 Nonlinearity

Low-pressure turbine aerodynamics are remarkably linear [49, 71]. Linear systems permit the application of the principles of superposition. That is, the total system response to a combination of independent forces is exactly equal to the sum of the individual responses to these forces. The response of a one-dimensional linear system has one unique solution. Only the rate at which transients decay to this solution may vary due to the initial conditions. Nonlinear systems, on the other hand, are characterized by their inherent sensitivity to initial conditions. Most unsteady flows are periodic in time. A periodically forced nonlinear system’s sensitivity to starting conditions will change by varying parameters (e.g. forcing frequency, forcing amplitude, aerodynamic/mechanical damping, or stiffness). This means that for certain aerodynamic states or even thermal stresses, variations may alter the nonlinearity of a system via bifurcations which may detrimentally (or beneficially) affect the global predictability [72].

2.5.1 Linear vs. Nonlinear CFD

The onset of flutter is small amplitude and therefore linear. To accurately predict the limit cycle oscillation (LCO) amplitude, however, a nonlinear solver is necessary [17]. Nonlinear studies have infrequently focused on LPT blades due to their nearly linear aerodynamics, but there have been numerous nonlinear CFD analyses on fans and compressors. Hall, Thomas, and Clark proposed a harmonic balance technique for modeling nonlinear flows in turbomachinery [28]. This idea transforms the unsteady conservative variables using a Fourier series in time with spatially varying coefficients. By examining the front row of an unsteady, transonic, viscous high-pressure compressor, they illustrated quite clearly the difference between small and large amplitude CFD analyses. Figure 2.7 shows an example of their analysis. For larger amplitudes, nonlinear effects are evident because a higher number of harmonics are needed to converge to engineering accuracy; however, the overall unsteady pressures do not significantly change when comparing the $\alpha = 0.01^\circ$ and $\alpha = 1.0^\circ$ pitching
cases. As visible, only using one harmonic for the $\alpha = 1.0^\circ$ case yields a spurious result. This shows adequate accuracy can be achieved by using a small number of harmonics for even highly nonlinear flows and that nonlinear fluid dynamic effects can drastically alter the aeroelastic stability for even a single blade row.

![Comparison of harmonics needed for convergence of small (top) and large (bottom) amplitude oscillations by Hall, Thomas, and Clark [28].](image1.png)

**Figure 2.7:** Comparison of harmonics needed for convergence of small (top) and large (bottom) amplitude oscillations by Hall, Thomas, and Clark [28].

### 2.5.2 Friction Damping

Since LPT flow characteristics are predominately considered linear, most nonlinearities that may induce limit cycle oscillations come from the friction forces of platform dampers or shrouds. Design engineers implement nonlinear friction dampers to attenuate resonant amplitudes. This is accomplished by constraining large blade motions along a contact surface in addition to stiffening the high-aspect ratio LPT blades. By increasing the eigenfrequencies and energy dissipation, aeroelastic stability can be improved and the blade service life lengthened.

Typically, the contact normal force is considered a function of engine RPM and damper mass. Therefore, the dynamics have been predominately modeled assuming a constant normal force with a straightforward friction model [24, 15]. Menq et al.
were the first to investigate the influence of variable normal force on friction damped blade vibrations [42]. André numerically investigated the resonant response of tip-shrouded LPT blades using a Coulomb friction model [1]. He studied the optimal normal forces and the effect on resonant amplitudes of the damper and blade mass ratio. Friction damping has received less focus than other fields and currently not many codes exist that properly address the dynamics involving shifts in inter-blade phase angle and mode shapes. An exception to this was the work of Yang and Menq who developed the BDAMPER code [81] to predict the forced response of friction-damped turbine blades.

Sinha, Griffin, and Kielb studied the effect of friction dampers on torsional blade flutter [57]. Since higher engine speeds are more prone to flutter due to an increased fluid velocity relative to the blade, they looked at the feasibility of higher reduced velocities\(^3\) maintaining vibratory stability. One interesting result referred to as the “nose plot” is shown in Figure 2.8. For low reduced velocities (i.e. high reduced frequencies), the vibratory amplitude is exactly 0 and LCO does not exist. When the reduced velocity is increased to approximately 2.25, the solution jumps up to a small amplitude, stable limit cycle oscillation. If the stable branch is followed by further increasing the reduced velocity, the LCO response amplitude does not change significantly until a critical value of \(U_{1, \text{crit}} \approx 3.4\). Beyond this critical value of reduced velocity, the solution is unbounded as flutter causes exponential growth in amplitude. The second (top) branch in the \(2.25 \leq U_1 \leq 3.4\) region is the unstable branch. Intermediate perturbations to the amplitude lying between the stable and unstable branches will decay to the stable branch. However, if the steady state solution has settled on the unstable branch and the amplitude is increased slightly (for a fixed reduced velocity), the steady state response will again grow without bound.

The computation of the final, friction-saturated LCO amplitude of an aerodynamically unstable bladed disk (a.k.a. blisk) is a numerically difficult achievement. Because the aerodynamic flow conditions and nonlinear finite element equations must be solved simultaneously, it is numerically expensive to solve for the nonlinear vibration dynamics. Martel et al. derived a reduced order model that applied asymptotic techniques and the multiple scales method to accurately model this effect at low numerical cost [38]. They implemented a complex friction coefficient to the Olofsson microslip model [48] to create a faster reduced order model of blade vibrations subject to nonlinear friction dampers. Essentially, this complex coefficient allowed linear implementation for small amplitudes of oscillation, and nonlinear implementation for larger amplitudes. In the end, nonlinear friction effects were able to be condensed into one complex coefficient that was a function of amplitude.

\(^3\) Reduced velocity is the inverse of reduced frequency.
2.6 Multi-Row Analysis

The discussion up until this point has focused on LPT flutter from a single blade row perspective. For an isolated cascade analysis, pressure waves created by the vibrating airfoil propagate outward and usually non-reflecting boundary conditions are applied. Realistically, fractions of this wave energy are reflected back by the neighboring rows onto the cascade which can impart additional unsteady forces and largely influence actual flutter stability [7]. Multi-row influences may manifest themselves in the form of induced flow disturbances that can mix the effects of forced response and flutter. A dual gust-flutter response was investigated by Frey and Fleeter and its detrimental or beneficial effect on flutter was found to be a function of the phase between the two mechanisms [21]. Therefore, a fully coupled multi-row analysis should yield more accurate flutter predictions than the simplified isolated blade row case.

Hall and Ekici modeled multi-row effects by implementing a set of vorticity waves that propagate between blade rows called “spinning modes” [26]. Not properly modeling the spinning mode interaction for a stator-rotor stage could lead to large errors in the predicted aerodynamic damping [27, 52]. The reason is the acoustic upstream mode from the rotor onto the stator is scattered and reflected back as a cut-on mode onto the rotor with the same frequency as the stator wake. Hall and Ekici found that multi-row analysis had a much greater impact on 2D flows, but 3D flutter predictions will also benefit from the added accuracy of multi-row coupling. Figure 2.9
shows a comparison of single and coupled row analysis by Hall and Ekici. Clearly, the effects of multi-stage modeling can significantly impact flutter stability especially near $\sigma = 60^\circ$ in this instance. Furthermore, the plot seems to agree that neighboring rows tend to suppress acoustic resonance effects which was Whitehead’s assertion earlier in Section 2.3.

![Figure 2.9: Isolated-row vs. multi-row flutter analysis by Hall and Ekici [26].](image)

Of particular interest to aft LPT stages is the additional modeling of the turbine rear frame struts. There have been some speculations whether this alters aeroelastic stability and may warrant future research.

Lastly, some studies have looked at the combined effect of multistage coupling on mistuned blisks[5, 53]. Mistuning, the next topic of discussion, was found to increase the importance of multistage coupling compared to a tuned system.

2.7 Mistuning

The majority of LPT flutter analyses are conducted under the assumption of a completely tuned cascade of blades. That is, each blade is identical to the next (structurally and aerodynamically). Generally speaking, mistuning a cascade improves flutter stability, yet it exacerbates forced response vibrations. Therefore, a tuned
flutter analysis is considered conservative as manufacturing tolerances and operating wear create unanticipated mistuning which can aerodynamically stabilize [70]. Széchényi found mistuning can effectively condense the imaginary part of aerodynamic damping root locus loop [63]. However, there was a limitation in that if a single-blade was prone to flutter, mistuning alone could not stabilize the cascade.

In 2004, Kielb et al. extended the Fundamental Mistuning Model developed by Feiner and Griffin [19] to include aerodynamic coupling terms from CFD output. The resulting application was compiled into a mistuning code called MISER. Their specific study found the benefits of mistuning on flutter using aerodynamic coupling was largely suppressed when also including a structural coupling model [34].

Crawley and Hall found an alternating mistuning pattern optimally prevents flutter and is insensitive to implementation errors [13]. Adding to this, Silkowski et al. saw the destabilizing influence of the neighboring blades can be analyzed by circumferentially arranging blades with alternating natural frequencies [56]. Nowinski and Panovsky verified this experimentally by vibrating every other blade in a traveling wave mode and determining this effectively diminished the destabilizing aerodynamic influence of the neighboring blades [47].
LPT Flutter Analysis and Physical Understanding

This chapter begins by briefly describing the computational fluid dynamics (CFD) solver used for all results to follow.

3.1 Computational Fluid Dynamics Solver

The last couple of decades bore witness to significant advancements in computational methods used to solve complex fluid dynamics problems in turbomachinery. CFD has matured to levels where this research focus is privileged enough to move away from developing robust codes, and leverage existing state-of-the-art, validated solvers to use as computational wind tunnels.

Steady and unsteady aerodynamic results were obtained using a well-validated, linear, frequency domain Reynolds-averaged Navier-Stokes (RANS) solver that is named MUSTANG [18, 28]. Applying flow periodicity in space and time, this code is faster than traditional time-marching algorithms, and is first and second-order accurate in time and space, respectively. Furthermore, it is time linearized, capable of 3D Navier-Stokes or Euler solutions, and has the ability to model multi-row simulations.

Eulerian CFD solutions assume that viscous friction forces are zero and therefore velocities even at the airfoil surface can be non-zero. However, for viscous CFD simulations, the Navier-Stokes equations must be solved, and the main difference is that now fluid flow “sticks” to the airfoil surface due to friction between the solid wall and fluid. At the onset of the resulting boundary layer, the flow velocity exponentially diminishes to zero at the surface, which is an imposed boundary condition often referred to as the “No-Slip Boundary Condition”. All numerical results shown herein assume viscous flow, and implement the widely used Spalart-Allmaras turbulence model [59]. For the purpose of this research, choice of turbulence model used is assumed to negligibly effect the flutter boundaries.
In conservative form, the Navier-Stokes equations MUSTANG iterates on are

\[ \frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + \frac{\partial H}{\partial z} - S = 0, \]  

(3.1)

where \( U \) is the vector of conservative variables

\[
U = \begin{cases} 
\rho \\
\rho u \\
\rho v \\
\rho w \\
\rho E 
\end{cases}. 
\]  

(3.2)

The flux vectors, \( F \), \( G \), and \( H \), and the source vector, \( S \), are given by

\[
F = \begin{cases} 
\rho u - \rho \dot{f} \\
\rho u^2 + p - \tau_{xx} - \rho u \dot{f} \\
\rho u v - \tau_{xy} - \rho \dot{f} \\
\rho u w - \tau_{xz} - \rho v \dot{f} \\
\rho u I - \tau_{xh} - \rho E \dot{f} 
\end{cases},
\]

\[
G = \begin{cases} 
\rho v - \rho \dot{g} \\
\rho v^2 + p - \tau_{yy} - \rho u \dot{g} \\
\rho u v - \tau_{yv} - \rho v \dot{g} \\
\rho v w - \tau_{yz} - \rho \dot{g} \\
\rho v I - \tau_{yh} - \rho E \dot{g} 
\end{cases},
\]

\[
H = \begin{cases} 
\rho w - \rho \dot{h} \\
\rho u w - \tau_{xx} - \rho \dot{h} \\
\rho v w - \tau_{zy} - \rho \dot{h} \\
\rho w^2 + p - \tau_{zz} - \rho w \dot{h} \\
\rho w I - \tau_{zh} - \rho E \dot{h} 
\end{cases},
\]

\[
S = \begin{cases} 
0 \\
0 \\
\rho(\Omega^2 y + 2\Omega w) \\
\rho(\Omega^2 z - 2\Omega v) \\
0 
\end{cases},
\]  

(3.3)

where \( p \) is the static pressure, \( I \) is the rothalpy, \( \{\dot{f}, \dot{g}, \dot{h}\} \) are the control surface velocities, and the shear stresses, \( \tau \), with their various subscripts are defined by Ekici and Hall [17].

Three important boundary conditions (BCs) are applied for proper flow convergence. First, a solid surface boundary condition is applied to enforce zero flux through a surface, i.e.,

\[
V \cdot \hat{n} = \frac{\partial R}{\partial t} \cdot \hat{n},
\]

(3.4)

where \( V \) is the velocity vector and \( R \) is the vector of a point’s coordinates on the airfoil surface. Second, since a single blade passage is being modeled for efficiency purposes, periodic boundary conditions are applied such that the mean flow, \( U \), is circumferentially periodic by

\[
U(x, \theta, r) = U(x, \theta + \theta_G, r),
\]

(3.5)

where \( \theta_G \) is the circumferential distance between passages. For single traveling wave modes, the unsteady flow, \( u \), is phase shifted by an interblade phase angle, \( \sigma \), by prescribing the following condition:

\[
u(x, \theta + \theta_G, r) = u(x, \theta, r)e^{i\sigma}.
\]

(3.6)
Third, quasi-3D non-reflecting boundary conditions (NRBC) are applied to eliminate artificial reflections of outgoing pressure, entropy, and vortical waves. The implementation and formulation of NRBCs are described by Hall [25] and also Giles [22]. At the far-field boundaries, only waves deemed outgoing are retained in the solution. This is applied two-dimensionally at various spanwise locations making this a “quasi-3D” application. In a recent industrial case, it was discovered that applying 1D NRBCs resulted in the wrong aerodynamic damping prediction of a low-pressure turbine blade. Therefore, it is recommended to always apply at least 2D NRBCs for 3D flutter calculations.

3.2 LPT Geometries Considered

This research will begin focusing on a specific quasi-2D LPT grid of Standard Configuration 4 (STCF 4). By initially testing a thin, quasi-2D grid slice, the groundwork will be laid for identifying how steady loading parameters can drastically affect LPT flutter stability. Once a general understanding is reached, the analysis will transition to studying the same phenomena for the Hannover airfoil - a full 3D LPT blade, among other airfoil shapes and cascade characteristics. The following section briefly describes this LPT configurations for the interested reader. Otherwise, the reader may wish to advance to Section 3.3.

3.2.1 Standard Configuration 4

In the 1980s, the École Polytechnique Fédérale De Lausanne (EPFL) created a collection of “Standard Configurations” (STCF) in an effort to standardize research for unsteady aerodynamics of turbomachinery. Only two STCF cases (4 and 11) were LPT blades, but STCF 4 was the only blade tested for subsonic, transonic, and supersonic exit flows. Due to interest in testing exit Mach effects and the diversity of publicly available experimental flow conditions, STCF 4 was selected as the LPT geometry to analyze. A picture of the STCF 4 experimental apparatus is shown in Fig. 3.1. The axial flow direction in this picture is from bottom to top and STCF 4 is a non-rotating, annular LPT cascade consisting of 20 identically machined blades. Detailed information regarding the STCF 4 geometry can be found in publications by Bölcs or Fransson [6, 20], but the key parameters are listed in Table 3.1.

Steady CFD Results

STCF 4 is modeled in a viscous, quasi-2D sense. To create a 2D cut, the original 3D blade geometry is “chopped” at midspan decreasing the radial dimension, $\Delta z$, to 20% of the original span over four radial cells ($\Delta z = 8.057 \text{ mm} = 0.3172 \text{ inches}$). The H-O-H grid topology (upstream/downstream H-blocks cap the O-block which is comprised of the nodes surrounding the blade) was created using Navier-Stokes (i.e.,
Figure 3.1: Photo of Standard Configuration 4.

Table 3.1: Key Geometric Parameters: STCF 4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{hub}$</td>
<td>160 mm (6.299 inches)</td>
</tr>
<tr>
<td>$R_{tip}$</td>
<td>200 mm (7.874 inches)</td>
</tr>
<tr>
<td>Chord, $c$</td>
<td>74.4 mm (2.929 inches)</td>
</tr>
<tr>
<td>Solidity, $\theta = c/s$</td>
<td>1.28 (midspan)</td>
</tr>
<tr>
<td>Stagger (from axial), $\gamma$</td>
<td>56.6°</td>
</tr>
</tbody>
</table>

viscous) meshing near the airfoil surface. In all, 24,375 nodes comprise the grid over five span-wise sections for the purpose of fast convergence.

All quasi-2D CFD simulations for Standard Configuration 4 were conducted using a linear version of MUSTANG. Implementing nonlinear CFD negligibly changes results at small, reasonable vibration amplitudes. In 2013, Jing Li conducted nonlinear CFD research at Duke for the quasi-2D, supersonic STCF 4 case. Li used a harmonic balance code and retained three harmonics over seven sub-time levels. Li’s results showed that increasing vibrational amplitude had negligible effect until abnormally high amplitudes were reached. This is a common and expected finding for low-pressure turbine aerodynamics. A sample plot from Li’s work is shown in Fig. 3.2.

The completed research will focus on three different experimental test cases recorded for STCF 4: Test Case 624 (transonic exit flow), Test Case 627 (subsonic exit flow), and Test Case 628 (supersonic exit flow). The measured parameters describing these flow profiles are given in Table 3.2. It should be noted that experimental documentation of the inlet total temperature, $T_{i,1}$, could not be located. However, Witteck et al. used a temperature of 330 K (594 R) for Test Case 628 [79].
Their calculation of $T_{t,1}$ assumed isentropic compression from standard day air. Witteck later reported that $T_{t,1} = 330$ K was found in error, and the actual value was closer to 360 K. However, the conclusion was the discrepancy in the inlet temperature boundary conditions produced negligible changes in converged inlet Mach number and flow angle. For the purpose of simplicity, this study will use the same inlet total temperature boundary condition specified in Table 3.2. Also listed in Table 3.2 are values for $k$ and $f$. These are the experimentally reported reduced frequencies and blade excitation frequencies, respectively. Using these values, it is only an algebraic exercise to determine the correct CFD excitation frequencies which correspond to $k = 0.1, 0.3, \text{ and } 0.5$, which are the reduced frequency values investigated later.

Note that for Test Case 628, the experimentally determined inlet flow angle was reported as $\beta_1 = 12.0^\circ$. Subsequent computational studies in literature showed poor agreement using this value. Better agreement was found using $\beta_1 = 28.0^\circ$. The popular consensus with those familiar with Test Case 628 is that this experimental value was reported in error due to the close proximity of the upstream pressure probe and blades (5 mm) [79]. Therefore, $\beta_1 = 28.0^\circ$ was used for all CFD analyses for

\[^1\] The experimentally measured value here is actually $\beta_1 = 12.0^\circ$. Witteck later reported this number was in error and $\beta_1 = 28.0^\circ$ is a better approximation [79].
Table 3.2: Experimental flow conditions for SC4 Test Cases

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 624 (Transonic)</th>
<th>Case 627 (Subsonic)</th>
<th>Case 628 (Supersonic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{1,is}$</td>
<td>0.21</td>
<td>0.19</td>
<td>0.2</td>
</tr>
<tr>
<td>$M_{2,is}$</td>
<td>1.04</td>
<td>0.85</td>
<td>1.39</td>
</tr>
<tr>
<td>$p_{0,1}$</td>
<td>28.86 psi</td>
<td>23.34 psi</td>
<td>31.49 psi</td>
</tr>
<tr>
<td>$p_2$</td>
<td>14.62 psi</td>
<td>14.61 psi</td>
<td>10.04 psi</td>
</tr>
<tr>
<td>$T_{0,1}$</td>
<td>594 R</td>
<td>594 R</td>
<td>594 R</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>21.4°</td>
<td>15.2°</td>
<td>28.0°</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>71.3°</td>
<td>71.5°</td>
<td>68.3°</td>
</tr>
<tr>
<td>$k$</td>
<td>0.0999</td>
<td>0.1187</td>
<td>0.0799</td>
</tr>
<tr>
<td>$f$</td>
<td>149.0 Hz</td>
<td>149.0 Hz</td>
<td>152.0 Hz</td>
</tr>
</tbody>
</table>

For quick validation purposes, Figure 3.3 is plotted to demonstrate the quasi-2D CFD steady state convergence of pressure along the blade surface for Test Case 628 agrees well with measured experimental data. Further, the converged steady CFD results for both test cases 627 and 628 are displayed in Table 3.3. When comparing directly with Table 3.2, there is overall agreement with the flow angles and pressure with a slight drop in $M_{2,is}$. One potential reason for the lower Mach number was thought to be the consequence of the aforementioned $T_{t,1}$ BC discrepancy. However, upon further investigation, a new steady analysis using $T_{t,1} = 358$ K did not affect the converged steady isentropic Mach numbers past the 4th significant figure. More plausible reasons for this minor difference could be the quasi-2D nature of the analysis or that the reported CFD Mach numbers are mass-averaged at the exit. The Mach number distributions for the subsonic and supersonic flow conditions are shown in Figure 3.4.

Section 3.3 will investigate several important control parameters of LPT flutter including steady loading, exit Mach number, reduced frequency, and unsteady mode shape. Before continuing, two of the fundamental rigid body mode shapes used for the following SC4 analysis are introduced.

**Mode Shapes**

Flutter in LPT blades typically occurs at low frequencies and at either the 1st or 2nd modes, which are most typically first-bending and first-torsion. These first-order modes have little to no chordwise bending and therefore are adequately approximated as rigid body modes with variable displacement magnitudes as a function of span. Furthermore, all rigid body mode shapes can be represented by the superposition of one rotational and two translational components. The three fundamental rigid body
Figure 3.3: Coefficient of pressure distribution for SC4 Test Case 628.

Table 3.3: Converged steady CFD results for quasi-2D SC4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 627 (Subsonic)</th>
<th>Case 628 (Supersonic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{1,is}$</td>
<td>0.177</td>
<td>0.202</td>
</tr>
<tr>
<td>$M_{2,is}$</td>
<td>0.829</td>
<td>1.351</td>
</tr>
<tr>
<td>$p_{t,1}$</td>
<td>160.9 kPa (23.34 psi)</td>
<td>217.1 kPa (31.49 psi)</td>
</tr>
<tr>
<td>$p_{s,2}$</td>
<td>102.5 kPa (14.87 psi)</td>
<td>69.4 kPa (10.07 psi)</td>
</tr>
<tr>
<td>$T_{t,1}$</td>
<td>330 K (594 R)</td>
<td>330 K (594 R)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>15.1°</td>
<td>27.9°</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>71.5°</td>
<td>69.3°</td>
</tr>
</tbody>
</table>

modes used by Panovsky and Kielb to create the Tie-Dye plots (refer to Fig. 2.4) are the following:

1. Edgewise bending mode (motion parallel to the chord-line)
2. Flex bending mode (motion perpendicular to the chord-line)
3. Pitching mode (center of rotation at the leading edge)

Due to the ease of numerically generating these mode shapes and to maintain consistency with the Tie-Dye method, the two rigid body mode shapes the majority of this chapter considers are flex bending and leading edge pitching. The edgewise bending mode is not investigated at this point since it does not closely represent a realistic LPT flutter mode due to its inherently high natural frequency. Fig. 3.5 shows the orientation and maximum displacement of the airfoil (exaggerated 250-fold) in relation to the original O-block at midspan for both the flex and pitching modes.
The flex mode unsteady grid was generated as follows. The Standard Configuration 4 test rig was designed such that each blade will rotate (bend) about an axis 88.5 mm (3.48 inches) below the blade root. This center of rotation results in an average bending arm of approximately 91.7 mm (3.61 inches) due to the radial thickness. Defining the bend angle as 0.01 radians in the direction perpendicular to the chord, the average bending displacement is \( h = 0.917 \text{ mm} (0.0361 \text{ inches}) \). As for the pitching mode, \( \alpha = 0.0123 \text{ radians} \) was enforced such that the maximum displacement (at the trailing edge) was also equal to 0.917 mm for consistency. Lastly, the field displacements in the O-block were linearly decayed out to the final circumferential station to ensure zero displacement at the grid boundary.

### 3.3 Key Control Parameters for LPT Flutter

#### 3.3.1 Non-Dimensional Damping Coefficient

A non-dimensional aerodynamic damping coefficient used to assess flutter stability for SC4 is now defined. The generalized force (GF) is the surface integral around the airfoil of the dot product between the pressure and the local modal velocity. In general, it is a complex number due to the variation in unsteady pressure phases over the airfoil surface. The sign of the imaginary part of the generalized force (Im(GF)), i.e. positive or negative, completely determines the stability of the airfoil with the convention of negative being stable. However, it is convenient to use the standard practice of a positive damping coefficient indicating a stable response. To this end, the non-dimensional aerodynamic damping coefficient [49], \( \Xi \), is defined as

\[
\Xi_{flex} = \frac{-W_{cyc}/\Delta z}{\pi q_2 h^2},
\]  

(3.7)
for the flex mode, and
\[ \Xi_{\text{pitch}} = \frac{-W_{\text{cyc}}/\Delta z}{\pi q_2 c^2 \alpha^2}, \quad (3.8) \]
for the pitching mode. Here, \( W_{\text{cyc}} = \pi \Im(GF) \), \( \Delta z \) is the radial grid thickness, \( q_2 \) is the exit dynamic pressure, \( h \) is the bending amplitude of the flex mode, \( c \) is the chord length, and \( \alpha \) is the pitching angle in radians.

### 3.3.2 Sensitivity to Mode Shape and Reduced Frequency

The well-known influence of reduced frequency on aerelastic stability is now investigated in detail for the Standard Configuration 4 LPT blade. It will be shown that as reduced frequency increases, more acoustic modes are cut-on and acoustic resonances may drastically alter stability and cause flutter. To forecast where these acoustic perturbations should occur, exact 2D acoustic resonance conditions are computed later in Section 3.4 and compared directly to the damping vs. IBPA plots for three reduced frequencies: \( k = 0.1, 0.3, \) and \( 0.5 \).

Figure 3.6 shows the minimum damping results of varying the reduced frequency for both aforementioned modes. These CFD results use the baseline boundary conditions shown in Table 3.2. The sole control variable is the frequency of blade vibration. For each mode, all 20 nodal diameters were analyzed with CFD and the least stable data point was saved to create the plot. The critical reduced frequencies are approximately \( k_{\text{flex}} = 0.15 \) and \( k_{\text{pitch}} = 0.22 \) for the flex and LE pitch mode, respectively, which roughly agrees with the TD plot of Figure 2.4. The curves are nearly linear which sheds light on the well-established connection between aerodynamic damping and reduced frequency. A nonlinear veering region occurs near \( k = 0.4 \) and
$k = 0.6$ for the flex and pitching modes, respectively. This can be explained quite simply as a shift in acoustic resonance conditions. That is, for a fixed set of steady flow parameters (e.g. exit Mach, flow angle), the exact 2D acoustic resonance IBPA varies linearly with reduced frequency [73]. However, it is often true that acoustic resonance serves to stabilize the aerodynamic damping quantity, which if true in this case would not impact the minimum non-dimensional damping coefficients shown in Fig. 3.6. Additionally, since SC4 has 20 blades in the cascade, the CFD calculations only calculated every $18^\circ$ of IBPA. Therefore, the reader should not expect purely linear variations in Fig. 3.6. For reference, the least stable IBPAs are roughly $\sigma_{flex} = -54^\circ$ and $\sigma_{pitch} = -72^\circ$ (with small variation) for values of reduced frequency below the veering region, and $\sigma_{flex} = 0^\circ$ and $\sigma_{pitch} = -18^\circ$ for the higher values of $k$.

The non-dimensional damping coefficient vs. IBPA plots for Test Case 627 are shown in Figs. 3.7. They are divided into two plots (one for each mode shape) and include data for three different reduced frequencies: $k = 0.1$, 0.3, and 0.5. To represent all possible forward- and backward-traveling nodal diameters, CFD analysis was completed at every $18^\circ$ (360°/20 blades) of IBPA.

Turning our attention first to Fig. 3.7, we immediately see that SC4 is unstable at $k = 0.1$ for both mode shapes. Each mode shape is most unstable at approximately $\sigma = -54^\circ$, with $\Xi_{flex} = -0.08$ and $\Xi_{pitch} = -0.35$. On the other hand, the system is stable at all tested IBPAs for $k = 0.3$ and $k = 0.5$. This indicates the neutral stability limit lies somewhere between $0.1 < k_{crit} < 0.3$, which agrees quite well with the Meingast results seen back in Fig. 2.4. Also note that Fig. 2.4 suggests if we were to run the same simulations for pitching about the mid-chord, the critical reduced frequency would be approximately 0.45 in a high-gradient region. That would make two of our three tested reduced frequencies unstable. This strongly supports the argument for mode shape being an important factor in LPT flutter.
Figure 3.7: Damping vs. IBPA for subsonic exit flow (Case 627) at three different reduced frequencies.

Table 3.4: Test Case 627: Subsonic Acoustic Resonance IBPA Predictions

<table>
<thead>
<tr>
<th></th>
<th>$k = 0.1$</th>
<th>$k = 0.3$</th>
<th>$k = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet A</td>
<td>8.4°</td>
<td>25°</td>
<td>42°</td>
</tr>
<tr>
<td>Inlet B</td>
<td>−6.2°</td>
<td>−19°</td>
<td>−31°</td>
</tr>
<tr>
<td>Exit A</td>
<td>30°</td>
<td>90°</td>
<td>149°</td>
</tr>
<tr>
<td>Exit B</td>
<td>−4.5°</td>
<td>−14°</td>
<td>−23°</td>
</tr>
</tbody>
</table>

sensitivity analysis.

Acoustic resonance “spikes” are highly visible for both mode shapes and all three reduced frequencies. Refer to Table 3.4 for the predicted acoustic resonance IBPAs which were computed using Eq. 2.9. Despite the IBPA resolution, the reader will see the damping vs. IBPA plots agree quite well with these predictions. If more IBPAs were analyzed, the higher refinement would push the visible acoustic resonance points even closer to these 2D predictions. The seemingly random spikes in damping do not occur at the same IBPA for each value of $k$. This will be made clear in Section 3.4 when it is proven there is a linear relationship between $k$ and the acoustic resonance inter-blade phase angle. Furthermore, acoustic resonance IBPAs are independent of mode shape; however, the severity of their effect on flutter is certainly mode shape dependent as Fig. 3.7 makes evident.
MUSTANG implements phase shifted boundary conditions in its solver. This allows the cascade to be accurately modeled using a single passage. In other words, whatever happens to the reference blade (blade 0) also occurs for all other blades, only phase shifted by the IBPA. For instance, some physical quantity, $Q$, at blade $n$ can be characterized by

$$Q_n = Q_0 e^{i \sigma_n},$$

(3.9)

where $\sigma_n = 2\pi n/N$. MUSTANG solves for the generalized force quantities in the traveling wave domain. The physical blade coordinates, $X$, can be written in terms of the traveling wave coordinates, $Y$, by the summation

$$X_m = \sum_n Y_{\sigma_n} e^{i m \sigma_n}.$$

(3.10)

Equation 3.10 can also be expressed in matrix form as

$$X = [E]Y,$$

(3.11)

where $[E]$ is a Fourier transformation matrix defined by Whitehead [77] to be

$$E_{(m,n)} = e^{2\pi i mn/N},$$

(3.12)

where $i = \sqrt{-1}$, $m$ is the row index, $n$ is the column index, and $N$ is the number of blades.

Using this knowledge, a useful transformation can be applied to convert the aerodynamic force matrix in the traveling wave domain to the physical blade domain. The physical blade aerodynamic forces, or the influence coefficients, are obtained via the transformation

$$[A_X] = [E][A_Y][E]^{-1}.$$

(3.13)

where

$$[A_Y] = \begin{pmatrix}
A_{\sigma_1} & 0 & 0 & \cdots & 0 \\
0 & A_{\sigma_2} & 0 & \cdots & 0 \\
0 & 0 & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & A_{\sigma_{N-1}} & 0 \\
0 & 0 & \cdots & 0 & A_{\sigma_N}
\end{pmatrix}$$

(3.14)

is a diagonal, $N \times N$ matrix with the GF corresponding to the 0th ND as the first entry, then the following order of GFs should correspond to: ND = +1, +2, ..., +N/2, 1-N/2, ..., -2, -1. For symmetric cascades, $[A_X]$ is a full matrix with only $N$ independent terms and is required to be circulant. The structure of $[A_X]$ is shown in Eq. 3.15 and the entries of $[A_X(i,j)]$ can be interpreted as the aerodynamic force
on the $i^{th}$ blade due to only the motion of the $j^{th}$ blade.

$$[A_{X(i,j)}] = \begin{pmatrix}
A_0 & A_1 & A_2 & \cdots & A_{-1} \\
A_{-1} & A_0 & A_1 & \cdots & A_{-2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_2 & \vdots & \vdots & \cdots & A_1 \\
A_1 & A_2 & \cdots & A_{-1} & A_0
\end{pmatrix}$$

(3.15)

In general, the influence coefficient matrix is noted to be tri-diagonally dominated by the $A_0$, $A_{-1}$, and $A_{+1}$ terms [35]. These three terms are the influence coefficients of the reference blade, its neighboring suction side blade, and neighboring pressure side blade, respectively. The blade numbering convention used away from a reference blade is shown in Fig 3.8.

If $[A_X]$ is truly tri-diagonal, then every damping vs. IBPA curve should exactly resemble a sinusoid. In the upcoming sections, however, deviations from sinusoidal shapes arise and are case-specific to each combination of mode shape, reduced frequency, and exit Mach number.

The influence coefficients of the generalized force (GF) magnitude for subsonic Test Case 627 are shown in Figure 3.9.

The influence coefficients are the Discrete Fourier Transform of the traveling wave coefficients. The reference blade (blade 0) is the first term in the DFT which is simply the mean value of the function and represents the blade-on-itself influence, which is
almost always stabilizing for practically designed turbine blades. The ±1 terms would represent the first harmonics, ±2 the second harmonics, and so on. Therefore, if the only non-zero influence coefficients were the 0 and ±1 terms, the resulting damping vs. IBPA plot would be purely sinusoidal. The reader can determine this is approximately the case for the flex mode damping curve at \( k = 0.1 \) in Fig. 3.7a. Observing the corresponding 0 and -1 influence coefficients of Fig. 3.9a, we can verify that these two terms dominate.

Upon inspection of Fig. 3.7, there is recognizably more harmonic content in the pitching mode compared to the flex mode. This richer content leads us to believe the influence coefficients farther away from reference blade 0 will dissipate less for the pitching mode than for the flex mode. This is exactly what is shown in Fig. 3.9, especially for \( k = 0.1 \). Also at \( k = 0.1 \), the reader will notice that the -1 (PS) blade has a higher influence on blade 0 than does the other immediate neighbor at +1 (SS). However, the inverse is true for both mode shapes at \( k = 0.5 \), where the SS blade begins to dominate the PS influence.

Overall, these results share good qualitative agreement with Panovsky and Kielb [49] and Vega and Corral [69] for the subsonic exit flow LPT influence coefficients at these specific reduced frequencies. As each of these independent investigations looked at different LPT geometries, this suggests broad generalizations can be made about LPT influence coefficients at each reduced frequency. One caution to the reader is that Vega and Corral use a full-chord definition of reduced frequency, whereas Panovsky and Kielb and this research use the semi-chord definition.

It is often convenient to examine the influence coefficients in order to determine where exactly the aerodynamic damping contributions originate. The magnitudes of the GF influence coefficients are shown in Fig. 3.10 for select reduced frequencies.
One notable observation is the blade-on-itself contributions dominate, and the relative dominance grows with increased reduced frequency for both modes. However, the size of this relative blade 0 dominance appears to be greater for the flex mode. This phenomenon indicates the flex mode enjoys a greater stability benefit from high reduced frequencies compared to the LE pitch mode since blade-on-itself damping is almost always positive for normal low-pressure turbine assemblies.

While the relative magnitude of GF contributions from various blades in the cascade provides a decent amount of information, it paints an incomplete picture of whether the overall effect improves or impairs LPT flutter stability since the GF is a complex number. A revealing, and more interesting approach is to plot the work-per-cycle contributions from various airfoils around the cascade. The influence coefficients can be thought of in two related ways: 1) the aerodynamic force experienced by neighboring airfoils due to vibrating a central airfoil, or 2) the aerodynamic forces experienced by a central airfoil by vibrating all other airfoils. If one collects the resulting unsteady pressures in the latter case, in addition to the blade’s self-influence, integrating the dot product between each complex pressure contribution and the central airfoil’s modal velocity will yield the individual $W_{cyc}$ for all blades in the LPT cascade. Lastly, to generate these contributions for any IBPA, the simple transformation given by Crawley [12] as

$$W_{cyc,Y} = \sum_{X=-N/2}^{N/2} W_X e^{iX\sigma_Y}$$  \hspace{1cm} (3.16)$$

is applied. In Eq. 3.16, $W_{cyc,Y}$ is the traveling wave work, $W_X$ is the influence coefficient domain work, and $\sigma_Y$ is the IBPA of interest. After applying this transformation for $\sigma_Y = -72^\circ$, which is one of the most unstable IBPAs for this specific
case at low reduced frequencies, Fig. 3.11 was created which illustrates the work-per-cycle contributions as a function of reduced frequency. Now, it is evident the blade’s self-influence (blade 0) has a stabilizing effect for all reduced frequencies since its $W_{cyc}$ contribution is never positive. This is true in general, and outside of pathological scenarios, it is impossible for LPT flutter to initiate without neighboring blades due to high mass ratios ruling out coupled mode behavior. Other observations from Fig. 3.11 include:

1. The blade 0 $W_{cyc}$ contribution forms a nearly linear relationship to $k$, which is a trend we aim to replicate for steady loading later on to prove flutter correlation.

2. The first three neighboring blades on either side of blade 0 tend to destabilize the central airfoil for $k < 0.8$ and $k < 0.6$ for the flex and LE pitch modes, respectively.

3. For higher values of $k$, all neighboring blades within close proximity of the central airfoil tend to stabilize.

4. The neighboring blade contributions vary harmonically with reduced frequency. This is expected, as it is well known that higher influence coefficients have higher harmonic content [49, 73].

3.3.4 Physical Insights of Unsteady Pressure Propagation

We now take a closer examination of the unsteady pressures. The same process of converting the traveling wave aerodynamic force matrix to physical blade space can be applied to the unsteady pressures (by applying a transformation similar to Eq. 3.13). Due to the symmetric nature of the cascade, the resulting transformed matrix will also be circulant with $N$ independent complex values.

These calculations were executed by collecting the unsteady pressures from all 20 IBPA CFD runs for each of 24,375 grid points and executing a piece-wise transformation to the physical domain. In the end, the unsteady pressure influence coefficients were created for each grid point. This allowed generation of some interesting plots in which we can visualize the instantaneous unsteady cascade field pressures created from vibrating only the central, reference blade 0. Figures 3.12a & 3.12b show this visualization for the flex and pitch mode, respectively, for subsonic exit flow at $k = 0.1$.

Recall the influence coefficients in Fig. 3.9 showed the 0 and -1 blades dominated for subsonic exit flow at $k = 0.1$. This information is now visually verified by Fig. 3.12. Clearly the unsteady pressure generated by the reference blade on the reference should be a sizable contribution. Recall the definition of the $-1^{th}$ influence coefficient is the force on the $0^{th}$ blade due to only vibrating the $-1^{th}$ blade. Since this graphic has only the $0^{th}$ blade oscillating, the reader must vertically shift the numbering scheme up one unit to visualize the -1 blade vibrating for the same set of
unsteady pressures. Further, the higher harmonic content seen in the pitching mode of Figure 3.12b is also apparent as the unsteady pressures affect blades \(-1\) and \(-2\) much more so than for the flex mode. According to Vega and Corral, the highest
unsteady pressure is located in the region of the highest Mach numbers [69]. This explains the smaller unsteady pressure magnitudes seen on the blade pressure sides.

3.3.5 Sensitivity to Exit Mach Number

Earlier, we witnessed how the mode shape and reduced frequency affects flutter. To test the effect of a different exit Mach condition, the same analysis conducted for subsonic flow is now applied to the supersonic exit flow conditions (Test Case 628) for SC4. The damping vs. IBPA and influence coefficients are presented in Figs. 3.13 & 3.14, and the predicted acoustic resonance conditions are given in Table 3.5.

Not surprisingly, the least stable reduced frequency is again $k = 0.1$. However, the flex mode is now completely stable while the pitching mode is unstable at only $\sigma = -36^\circ$ which aligns quite well with the predicted Exit-B acoustic resonance prediction. Furthermore, the pitching mode is now unstable for a narrower window of traveling wave space. It is interesting to see the co-existing adverse and beneficial
nature of acoustic resonance. In some cases, it can stabilize the LPT blade, and in others it destabilizes. These changes do show an undeniable LPT flutter sensitivity to exit Mach number; however, the obvious and more pronounced effect manifests itself in the form of acoustic resonance variations.

The influence coefficients of Fig. 3.14 share similar trends to their subsonic counterparts, with a slightly higher GF magnitude here. Perhaps the most notable difference is seen in the pitching mode at $k = 0.3$ and $k = 0.5$. In each of these cases, the $\pm 1$ blades do not have a relatively dominant influence compared to blades farther
Table 3.5: Test Case 628: Supersonic Acoustic Resonance IBPA Predictions

<table>
<thead>
<tr>
<th></th>
<th>$k = 0.1$</th>
<th>$k = 0.3$</th>
<th>$k = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet A</td>
<td>14°</td>
<td>42°</td>
<td>70°</td>
</tr>
<tr>
<td>Inlet B</td>
<td>−10°</td>
<td>−30°</td>
<td>−49°</td>
</tr>
<tr>
<td>Exit A</td>
<td>−6.5°</td>
<td>−19°</td>
<td>−32°</td>
</tr>
<tr>
<td>Exit B</td>
<td>−32°</td>
<td>−95°</td>
<td>−158°</td>
</tr>
</tbody>
</table>

away. This is nearly counter-intuitive, but for the $k = 0.3$ pitching case, the -2/-3 blades have greater than or equal to contributions to the unsteady aerodynamic forces on blade 0 than the immediate ±1 neighbors. A similar result is seen for $k = 0.5$, where the -2 blade clearly dominates all other neighbors. However, the mean blade-on-itself damping increases with reduced frequency resulting in greater stability and making these higher harmonic influences less meaningful.

Conclusions regarding LPT Flutter Parameters

1. Reduced frequency and mode shape are critical LPT flutter design parameters. In general, raising the reduced frequency has a stabilizing effect. Also, changing the mode shape can drastically (beneficially or detrimentally) change the aerodynamic damping and hence flutter boundary.

2. For subsonic flow, 0° IBPA is always cut-on but this is not the case for supersonic flow.

3. For low reduced frequencies, the neighboring suction side blade dominates the unsteady aerodynamic forces seen by the reference blade. This was shown to be true regardless of mode shape or exit Mach number and was supported by other authors. Therefore, this result may be generalized for other LPT airfoil shapes.

4. For higher reduced frequencies, the neighboring pressure side blade has a relatively dominant influence on the unsteady aerodynamic forces seen by the reference blade. Acoustic perturbations were more prevalent for the higher reduced frequencies as the unsteady pressures propagated farther away from the reference blade suction side. However, higher reduced frequency means higher self-influence stabilization which diminishes neighboring unsteady effects.

5. When the exit Mach number was raised from subsonic to supersonic, the pitching mode lost stability. Therefore, variation of the exit Mach number does affect flutter conditions, although the obvious and most noticeable effect manifests itself as acoustic resonance shifts.
3.4 Aeroacoustics

It may not be obvious by visual inspection of Eq. 2.9, but the exact 2D acoustic resonance interblade phase angles are linearly proportional to the reduced frequency. Further, for very high Mach numbers, the slope relating these two quantities is dictated only by stagger angle and solidity. For low Mach numbers, a steep slope exists indicating acoustic resonance behavior will only occur near 0° IBPA across a broad range of reduced frequencies. To prove this result, the following derivation will solve for the slope corresponding to the four acoustic resonance lines. First, simplifying Eq. 2.9, let

\[
F = \frac{4M^2}{M^2 - 1}, \quad G = \sin \gamma, \quad \theta = c/s, \quad \text{and} \quad H = \frac{\sigma_r + 2\pi n}{\theta - 1}.
\]  

(3.17)

The equation now reduces to \(H^2 + Fk^2 + FGHk = 0\). Dividing by \(k^2\) and solving for \((H/k)\) using the quadratic formula yields

\[
\frac{H}{k} = \frac{\sigma_r + 2\pi n}{k\theta - 1} = \frac{-FG \pm \sqrt{(FG)^2 - 4F}}{2}.
\]  

(3.18)

Lastly, solving for \(k\) we obtain the linear relationship between \(k\) and \(\sigma_r\):

\[
k = \frac{2\theta}{-FG \pm \sqrt{(FG)^2 - 4F}}(\sigma_r + 2\pi n).
\]  

(3.19)

Here we can plainly see that the slope of each pair of lines is solely mandated by

\[
m = \frac{2\theta}{-FG \pm \sqrt{(FG)^2 - 4F}}.
\]  

(3.20)

Using the geometric parameters of Table 3.1 and the converged CFD values of Table 3.3, the exact 2D acoustic resonance conditions are predicted from \(k_2 = 0\) to \(k_2 = 0.5\) for the subsonic (Fig. 3.15a), supersonic (Fig. 3.15b), and transonic (Fig. 3.15c) test cases. The tabulated values for the subsonic and supersonic case have already been presented in Tables 3.4 & 3.5, respectively, and now the predictions for the transonic case are given in Table 3.6. Note that the horizontal dashed-lines in Fig. 3.15c are meant to represent the solution instantaneously jumping from \(\sigma = +180^\circ\) to \(\sigma = -180^\circ\). Only the sloped lines of the Exit A solution correspond to actual solutions.

For each case, all acoustic resonance conditions converge to \(\sigma = 0^\circ\) at low reduced frequencies. This, combined with both inlet acoustic resonance predictions (labeled A and B) steeply sloping near \(0^\circ\) for all \(k\) values, offers some explanation why damping vs. IBPA curves often stray from perfectly sinusoidal shapes near \(\sigma = 0^\circ\). Therefore, it is true that for extremely low reduced frequencies the acoustic resonance region of influence is confined to a narrow region of the traveling wave space. This agrees with
The assertion of Vega and Corral that the physics of low reduced frequency airfoils do not involve acoustic perturbations [69].

The inlet cut-on regions can be defined as the area between the two inlet lines, and similarly for the exit cut-on regions of IBPA. Consequently for the subsonic case, Fig. 3.15a shows that $\sigma = 0^\circ$ is always cut-on. However, things get interesting supersonically and this is no longer the case as seen in Fig. 3.15b, which shows the cut-on region associated with the exit conditions does not include $\sigma = 0^\circ$. Upon comparing these predicted cut-on regions to Figures 3.7-3.13 (and Fig. 4.1 which will be introduced later), the damping vs. IBPA curves only resemble a sinusoidal-like shape in the cut-off region (i.e., outside the area included between these lines).

To end this section, we briefly discuss the attempt to predict 3D acoustic resonance conditions by analytically solving the convective wave equation subject to a
Table 3.6: Test Case 624: Transonic Acoustic Resonance IBPA Predictions

<table>
<thead>
<tr>
<th></th>
<th>k = 0.1</th>
<th>k = 0.3</th>
<th>k = 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet A</td>
<td>10°</td>
<td>31°</td>
<td>52°</td>
</tr>
<tr>
<td>Inlet B</td>
<td>−7.5°</td>
<td>−23°</td>
<td>−38°</td>
</tr>
<tr>
<td>Exit A</td>
<td>−125°</td>
<td>−14°</td>
<td>96°</td>
</tr>
<tr>
<td>Exit B</td>
<td>−5.2°</td>
<td>−16°</td>
<td>−26°</td>
</tr>
</tbody>
</table>

Figure 3.16: Schematic of a concentric cylinder intended to model a turbomachinery core geometry.

A concentric cylindrical domain such as the one shown in Fig. 3.16. The convective wave equation is given by Eq. 3.21 as

$$\frac{D^2 p}{Dt^2} - c^2 \nabla^2 p = 0,$$

where

$$\frac{D^2}{Dt^2} = \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2,$$

and the pressure, \( p \), is of the form

$$p(x, \theta, r, t) = f(r) \exp^{i(kx+m\theta-\omega t)},$$

and \( c \) and \( U \) are the speed of sound and incoming free stream velocity, respectively. Again, the sole purpose of this investigation was to improve the prediction of acoustic resonance interblade phase angles in a more physically realistic three dimensional environment. However, after applying the boundary conditions of a concentric cylinder, and solving the resulting system of 1\textsuperscript{st} and 2\textsuperscript{nd}-kind Bessel ordinary differential
equations, the analytical predictions of the first modes were inferior to the highly accurate 2D predictions. This has been attributed to several poor assumptions made in the three-dimensional domain including irrotationality (no swirl), which led to symmetric predictions about $\sigma = 0^\circ$. Higher order modes of the 3D analytical model were only cut-on at higher levels of reduced frequency ($k > 1.2$) and therefore are of less concern for LPT flutter. Future work should involve building a 3D computational or analytical model incorporating swirl and three-dimensional variations of flow field characteristics like the blade chord, flow velocity, and Mach number.
Influence of Steady Aerodynamics on LPT Flutter

As modern turbomachinery trends evolve toward higher Mach number and loading regimes, so grows the importance of physically understanding the onset of flutter as engineers race to keep pace with safe design of future generations of blades. This fact, along with recent experimental and numerical supporting evidence, has motivated the study of investigating the impact of load variations on the aerodynamic damping of turbomachinery blades. Recently, an industrial turbine manufacturer re-designed the blade loading of an aft low-pressure turbine stage. In addition to increasing the hub-to-tip delta-twist magnitude, the blade root was unloaded resulting in more load impacting the thinner tip (outer diameter) profile. It was then discovered that this once stable LPT configuration now fluttered at 100% Goodman in tests as the inlet guide vane angle increased creating a higher load. While aeroelastic flutter has been the focus of many research endeavors, most investigations have concentrated on identifying its key correlations to unsteady parameters (e.g. reduced frequency or mode shape). Much less consideration has been given to influence of steady blade loading on stability boundaries. However, the flutter stability characteristics due to loading and various shock structures will prove critical in reaching future performance goals, as not all flutter events can be avoided using reduced frequency and mode shape design criteria. Deeper understanding of how certain operating conditions inhibit or cause flutter is necessary, and has recently received renewed attention from academia and industry. While previous research has shown secondary correlations to steady parameters (e.g. exit Mach number or angle of attack), this chapter reveals data identifying a strong relationship between steady blade loading and aerodynamic damping. As a consequence, these findings suggest it is no longer adequate to only consider reduced frequency and mode shape when designing LPT blades. Anecdotal evidence suggests most engineers have been aware that increasing the loading at a fixed reduced frequency would decrease stability, but the effect has not been measured. It will be shown that the effect of increased loading does not
monotonically decrease stability. Furthermore, for certain torsion modes, loading causes the LPT cascade to jump into and out of flutter as shown experimentally by Cardinale in Fig. 2.5. Additionally, this connection between loading and flutter will be proven to be a strong function of mode shape, interblade phase angle, and cascade/airfoil geometry. Many of these effects are attributed to the proximity of the static back pressure to the critical choking pressure resulting in drastic changes to the passage shock structure that mutate the unsteady pressure amplitudes and phase [70]. Therefore, steady blade loading is studied as the third parameter to have significant impact on LPT flutter boundaries.

4.1 Loading Study: Variable Reduced Frequency

This initial study serves to demonstrate that increasing blade loading can greatly influence the minimum aerodynamic damping over the entire traveling wave mode range (i.e., over all inter-blade phase angles (IBPAs)). Torque can be increased for a single passage LPT blade by reducing exit static pressure, also known as the back pressure, which is denoted as $p_2$. To begin the study, quasi-2D SC4 CFD results from the baseline Test Case 624 (see Table 3.2) are examined. For reference, the damping vs. IBPA results for this transonic baseline case at three distinct reduced frequencies ($k = 0.1, 0.3, 0.5$) and two previously introduced fundamental modes are shown in Figure 4.1, and the non-dimensional aerodynamic damping coefficients defined in Eqs. 3.7 & 3.8 are retained. Interblade phase angle is varied in 6° increments, which provides high enough resolution to detect the majority of acoustic resonance spikes throughout the curves. Again, the data reveals the well-established LPT flutter guideline that, generally speaking, lower reduced frequencies are less stable.

To analyze the influence of varying the steady loading, the back pressure, $p_2$
will be varied in 5% increments of standard atmospheric pressure \( p_{atm} = 101.3 \) kPa = 14.7 psi). In total, 31 different back pressures are investigated. Figure 4.2 shows the resulting inlet Mach, exit Mach, and torque normalized by \( \tau_{ref} \) plotted as a function of the back pressure normalized by the critical choking pressure, \( p^* \). Reducing the back pressure increases the loading, flow velocity, and torque until fully choked conditions occur\(^1\). Choking conditions, which are quite normal in turbine design [14], indicate the presence of a shock wave. The equation for the critical downstream pressure when choking originates is [54]

\[
p^* = p_{0.1} \left( \frac{2}{\gamma + 1} \right)^{\gamma/(\gamma-1)},
\]

where \( \gamma = \frac{C_p}{C_v} \) is the ratio of specific heats. When \( p_2 \leq p^* \), it is impossible to change the mass flow rate, and therefore \( M_1 \), by further reduction in \( p_2 \) alone. For dry standard atmospheric air, \( \gamma = 1.4 \) and Eq. 4.1 reduces to \( p^* = 0.5283 p_0 \). Therefore, the flow should be choked for \( p_2/p_{0.1} \leq 0.5283 \). Since the back pressures have been normalized by \( p^* \), this effect can be seen in Fig. 4.2 for \( p_2/p^* \leq 1 \) where the \( M_1 \) no longer changes for further increases in loading. However, the exit Mach number and torque magnitude certainly increase with further reduction in the back pressure until below \( p_2/p^* \approx 0.25 \), where the steady CFD results are virtually uniform.

Two nominal combinations of reduced frequency and mode shape are initially selected for investigation. These \( k \) values were specifically chosen to represent values of reduced frequencies that are considered critical for bending and torsion modes. Thus, these combinations are \( k = 0.1 \) for the flex mode and \( k = 0.3 \) for the leading edge pitch mode. Figure 4.3 shows the damping vs. IBPA curves for select values of \( p_2/p_{ref} \). The baseline 624 case of Fig. 4.1 is shown in green where \( p_2/p_{ref} \approx 1 \). The reader will note that the curves of \( p_2/p_{ref} = 0.1 \) and 0.3 are right on top of each other\(^2\). This is an expected result as the steady loading does not change at these low back pressures.

It is only by chance we find the baseline case for the \( k = 0.1 \) flex mode of Fig. 4.3a has the most unstable aerodynamic damping and therefore unfavorable steady loading conditions. In this unique case, either increasing or decreasing the back pressure by 25% would both restore neutral stability. This is not true for the \( k = 0.3 \) pitching mode where the baseline case is fully stable. Raising the back pressure here seems to have little effect whereas dropping the back pressure gives rise to a sharp instability at \( p_2/p_{ref} = 0.55 \). Therefore, Figure 4.3 shows the drastic importance steady loading can have on overall flutter stability which is also heavily dependent on the mode shape.

The effects of acoustic resonance are more evident for the \( k = 0.3 \) pitching case of Fig. 4.3b. This comes as no surprise as the influence coefficients for the leading edge

---

\(^1\) \( \tau_{ref} \) is the steady torque from the baseline 624 CFD results for this case

\(^2\) Note: In the Fig. 4.3 legend, \( p_2 \) values have been normalized by 14.7 psi).
pitch mode do not decay as quickly away from the reference airfoil when compared to the flex mode. Additionally, as the back pressure is varied, the least stable traveling wave mode (i.e. IBPA) shifts because the acoustic resonance conditions change as a function of several steady parameters as can be inferred from Eq. 2.9.

Since the back pressure is the only control parameter manually varied in this study, it is important to note that Eq. 2.3 states the reduced frequency of the blade will decrease as the exit velocity increases (i.e., reduced frequency does not remain constant). In a later section, the blade vibration frequency is enforced to be constant by increasing the physical blade frequency proportionally with the exit Mach number as it rises with load. For the current results, we must look at how $k$ varies with $p_2$ to ensure these changes in the minimum aerodynamic damping curves are related to steady loading and not the reduced frequency. Assuming a constant speed of sound, Fig. 4.4 plots the change in reduced frequency as a function of back pressure. The nominal case values for $k = 0.1$ flex and $k = 0.3$ pitching correspond to $p_2/p_{ref} = 1$ as they should. However, as the back pressure decreases so does the reduced frequency. From this data alone, one should expect a destabilization of the blade due to this effect. However, the cascade stability characteristics do not shift gradually, monotonically, or predictably with reduced frequency. Therefore, the sharp destabilizations
soon to be displayed are attributed to the unique effect of blade loading.

After tabulating the minimum damping value for all 31 tested back pressures, Figures 4.5, 4.6, & 4.7 were created. The reader will notice the results from the subsonic case (627) and supersonic case (628) are now reported in addition to the inclusion of a $k = 0.3$ mid-chord pitching mode. The additional mode was included to further test the dependence of steady loading on mode shape for a constant reduced frequency. Since torque is a function of back pressure as seen in Fig. 4.2, Figures 4.5a & 4.5b correlate closely to one another and share most features.

Six general observations from these plots are presented:

1. Contradicting the notion that loading increases flutter risk, increasing blade loading does not monotonically decrease stability, and in fact is a function of

Figure 4.3: Damping vs. IBPA for transonic exit flow (Case 624) at various normalized back pressures. Green triangles mark the baseline case seen in Fig. 4.1. $p_2$ in the legend is the back pressure normalized by $p_{ref} = 14.7$ PSI.
Figure 4.4: Reduced frequency as a function of back pressure normalized by 14.7 PSI for SC4 Test Case 624.

Figure 4.5: Minimum damping vs. steady blade loading for baseline Test Case 624: $k = 0.1$ flex mode, $k = 0.3$ LE pitch mode, and $k = 0.3$ mid-chord pitch mode. $p^* = 15.24$ psi, $(p_2/p^*)_{baseline} = 1.04$

2. The mid-chord pitch mode nearly decreased stability monotonically with load for this variable reduced frequency study. Thus, this mode and frequency case is in fact least stable at the highest load.

mode shape. For example, the $k = 0.3$ LE pitching mode is completely stable at the highest loading condition investigated, whereas the $k = 0.1$ flex mode is approximately neutrally stable.
Figure 4.6: Minimum damping vs. steady blade loading for baseline Test Case 627: $k = 0.1$ flex mode, $k = 0.3$ LE pitch mode, and $k = 0.3$ mid-chord pitch mode. $p^* = 12.33$ psi, $(p_2/p^*)_{\text{baseline}} = 1.18$

Figure 4.7: Minimum damping vs. steady blade loading for baseline Test Case 628: $k = 0.1$ flex mode, $k = 0.3$ LE pitch mode, and $k = 0.3$ mid-chord pitch mode. $p^* = 16.63$ psi, $(p_2/p^*)_{\text{baseline}} = 0.60$

3. By normalizing the back pressures by their respective critical choking pressure, $p^*$, the curves for all three SC4 baselines (Test Cases 624, 627, & 628) collapse to qualitatively similar shapes as further demonstrated in Fig. 4.8.

4. Providing supporting evidence of modal dependency, the $k = 0.3$ LE pitch
mode features a unique and extremely sharp gradient with loading changes in the choked regime near the $p_2/p^* = 0.6$. Although the MC pitching mode is on average less stable, the LE pitch mode stability demonstrated the largest sensitivity to loading variations among the three modes initially investigated.

5. The most stable back pressure for the LE pitch mode is approximately the most unstable loading condition for the flex mode for all three cases with the exception of the instability spike near $p_2/p^* \approx 0.35$ in Fig. 4.5. This is the region where $M_2$ in Fig. 4.2 is abruptly changing before it reaches its maximum value as the loading is further increased.

6. For Test Case 628, the baseline pressure ratio is $(p_2/p^*)_{\text{baseline}} = 0.60$. At this loading condition, the LE pitch mode of Figure 4.7a sits within a local minimum indicates the loading is at the worst case scenario for this mode. The implication for engineers is that some combinations of loading and mode shape
will inherently have greater stabilization potential. In this specific baseline case, any fluctuations in design or ambient pressure conditions will stabilize the entire cascade.

Therefore, this initial study that constrained only the physical blade frequency, and not the value of $k$, showed the loading condition can drastically change the minimum aerodynamic damping of low-pressure turbine cascades. Furthermore, this phenomenon is mode shape dependent much like the critical reduced frequency. In the next section, this study is extended to enforce constant values of reduced frequency to isolate the underlying reason behind cascade stability modifications.

4.2 Loading Study: Constant Reduced Frequency

For this study, the blade loading will be varied in the same manner by altering the exit static pressure, $p_2$, while enforcing constant reduced frequency through physical blade frequency adjustments. To do this, consider that as $p_2$ is changed, the converged exit velocity also adjusts accordingly due to the changing downstream flow resistance. However, any change in relative exit velocity, $V_2$, also affects the reduced frequency parameter, $k$. This is evident as $V_2$ is the denominator of Eq. 2.3. Thus, to honor the scientific method of maintaining a single control variable, the blade’s vibrational frequency will vary proportionally with $V_2$ in all following CFD analyses in this section such that a constant reduced frequency is enforced.

Since the qualitative results of the influence of loading on flutter stability was similar across all three test cases of SC4 (see Fig. 4.8), this study will focus only on one set of baseline conditions: Test Case 627. Refer to Table 3.2 for the angle of attack, inlet stagnation pressure, and total temperature boundary conditions which are held fixed for this study.

Figure 4.9 displays the converged steady results for the inlet/exit Mach number, required blade frequency to enforce constant $k$, exit flow angle, $\beta_2$, and torque all as a function of the back pressure, $p_2$, which is normalized by the critical choking pressure, $p^*$. As Fig 4.9 plots $p_2/p^*$, the choking restriction on $M_1$ can be seen for $p_2/p^* \leq 1$. However, the exit Mach number and torque magnitude certainly increase with further reduction in the back pressure until below $p_2/p^* \approx 0.25$, where the steady CFD results are virtually unchanged as the flow fully expands. For values of $p_2/p^* < 0.40$, the exit flow angle, $\beta_2$, drops significantly. The amount of flow turning in this high loading region is greatly reduced, hence a larger deviation. It would be unwise to ignore this additional effect at lower back pressures; therefore, to exhibit caution, all following discussions of results will be limited to values of $p_2/p^* > 0.48$ where $\beta_2$ is approximately constant.

Figure 4.10 shows work-per-cycle as a function of $p_2/p^*$ (or loading) in the same influence coefficient perspective as the previous section. The flex and leading edge pitching modes were tested at constant reduced frequencies of $k = 0.1$ and $k = 0.3$, respectively. As expected, the blade 0 $W_{cyc}$ contribution is always stabilizing.
Additionally, it is clearly a function of load much like reduced frequency; however, the relationship between increased loading is noticeably less linear when compared to the reduced frequency case of Fig. 3.11. The correlation seems more complex than before. Although, there are certain pressure ratio regions exhibiting high levels of proportionality. For instance, in Fig. 4.10b, both ranges \(0.4 < \frac{p_2}{p^*} < 0.8\) and \(1.2 < \frac{p_2}{p^*} < 1.8\) are remarkably linear. Perhaps not coincidentally, the zone intermediate to these regions is where the critical choking pressure resides, and thus there is a high likelihood of abrupt changes to the passage shock structure.

Since blades 0, ±1, ±2, and ±3 only account for 7 of 20 blades in the cascade, the sum of the four vertically separated data points in Fig. 4.10 might not appear to add up to the total work line (shown with star symbols). Since the influence coefficient matrix is tri-diagonally dominated, the summation should still be relatively close. Most often, disparities will occur near acoustic resonance points like the one near \(\frac{p_2}{p^*} \approx 0.80\) in Fig. 4.10a. This is because the higher-order influence coefficients
must grow in magnitude to account for the higher-order content in the damping vs. IBPA curves due to acoustic resonance spikes.

Interestingly, the preliminary results in Fig. 4.10 indicate signs of a potential mode shape dependency when quantifying how loading variations affect LPT flutter...
boundaries. As the loading is initially increased, the total work-per-cycle tends to grow slightly more unstable for the flex mode, yet it does the exact opposite for the pitching mode. Confirming this mode shape dependency has important implications to the LPT blade designer. That is, not only would future blade designs require consideration of the critical reduced frequency as a function of mode, but consideration of the loading as well.

To confirm this suspicion, the minimum damping coefficient was calculated over all IBPAs for several combinations of reduced frequency and mode shape. In addition to the previously seen $k = 0.1$ flex and $k = 0.3$ leading edge pitching modes, two new modes are considered in this test: 1) $k = 0.3$ mid-chord pitching 2) $k = 0.1$ edgewise (parallel to chord-line) bending. The tabulated results are plotted in Fig. 4.11, and the following observations are made:

1. While holding reduced frequency constant, the steady loading may still drastically alter the flutter boundary.

2. The significance of steady loading on LPT flutter is very mode dependent which is exactly the same phenomena the critical reduced frequency exhibits as seen with the TD plot of Fig. 2.4.

3. In Fig. 4.11, the flex and edgewise bending modes are nearly anti-symmetric in the unchoked regime of $p_2/p^* > 1$ indicating opposite stability gradients with loading for these orthogonal modes.

4. As soon as $p_2/p^*$ drops below 1, the slope of least stable aerodynamic damping vs. loading changes sign. That is, if loading the blade was increasing stability prior to choke, further increases in load into the choked regime would now decrease stability. This is an early indication that shock waves in the passage may significantly affect LPT flutter stability characteristics, and warrants further consideration.

5. There are a total of six stability inflections for mid-chord pitch mode (i.e., torsion). This is similar to the phenomenon caught by Cardinale’s analysis in which variations in torque for a constant power turbine speed could cause intermittent jumps into and out of flutter (see the right-hand-side of Fig. 2.5).

So far, there have been several mentions of how acoustic resonance shifts affect the least stable IBPA. Figure 4.12 quantifies this data for the same four combinations of reduced frequency and mode shape seen in Fig. 4.11. An excellent piece of supporting evidence that loading conditions are directly correlated to flutter comes from this plot. Whenever the least stable IBPA is relatively constant over a large range of back pressures, the corresponding change in the minimum damping coefficient of Fig. 4.11 is quite linear. This is especially true for higher values of $p_2/p^*$.
Figure 4.11: Minimum damping over all IBPAs as a function of normalized back pressure for $k = 0.1$ flex, $k = 0.3$ leading edge pitch, $k = 0.3$ mid-chord pitch, and $k = 0.1$ edgewise bending.

Figure 4.12: Least stable IBPA as a function of mode shape, reduced frequency, and critical choking pressure ratio (refinement of $\sigma = 18^\circ$).
4.2.1 Unsteady Pressure Bifurcations

To determine the root cause of LPT flutter’s sensitivity to steady loading variations, we now inspect the passage shock structure of two pressure ratios above and below the critical value \( p_2/p^* = 1 \). One of the best experimental methods to visualize fluid flow with varying density is Schlieren photography. Since the data collected was not experimental, the magnitude of the 2D field density gradients is computed, which is given as

\[
|\nabla \rho| = \sqrt{\left( \frac{\partial \rho}{\partial x} \right)^2 + \left( \frac{\partial \rho}{\partial y} \right)^2},
\]

(4.2)
to illustrate the passage shock formations. Figure 4.13 shows the resulting 2D density gradients for back pressures of \( p_2/p^* = 1.01 \) and \( p_2/p^* = 0.95 \). When \( p_2/p^* = 1.01 \) in Fig. 4.13a, the flow is not quite choked; therefore, the suction side (SS) trailing edge is shock-free despite having a back pressure 1% higher than the critical choking pressure. However, when the ratio slightly dips below this critical value to \( p_2/p^* = 0.95 \), the upstream mass flow can no longer increase, choking the flow and forming a visible shock on the aft SS. On the other hand, there is no visible difference on the pressure side (PS), indicating this phenomenon solely affects the suction side. Also of note, there is a smaller shock also being formed at approximately 50% chord on the suction side. As the next plot will show, this shock moves toward the trailing edge with increased loading (decreased back pressure).

![Figure 4.13](image)

**Figure 4.13:** Density gradient contours in the O-block for back pressures on opposite sides of the critical choking pressure.
To provide more insight as to how loading is modified as a function of the critical pressure ratio, the coefficient of pressure at any point \((x, y, z)\) around the airfoil’s surface, which is defined as

\[
C_p(x, y, z) = \frac{p(x, y, z) - p_1}{p_0 - p_1},
\]

is plotted in Fig. 4.14 for various loading conditions ranging from \(0.48 \leq \frac{p_2}{p^*} \leq 1.31\). In this plot, normalized chord is defined as the straight-line distance from the leading edge divided by the chord length. The direction of increasing loading in Fig. 4.14 is indicated by the arrow, with larger values of \(\frac{p_2}{p^*}\) (lower loading) has the least change in \(C_p\) from suction to pressure side. While there is not much variation on the airfoil PS across these loading conditions, the SS experiences significant transitions. Not only does the total loading change, but more importantly the loading distribution. The two pressure ratios seen in Fig. 4.13 are pointed out with arrows here. One can verify the formation of a shock toward the aft SS just when \(\frac{p_2}{p^*}\) decreases below 1. For the curves corresponding to higher values of back pressure, the SS has relatively smooth, shock-free distributions.

\[\text{FIGURE 4.14: Coefficient of pressure distribution across airfoil as a function of load. Increasing load (decreasing back pressure) in the downward-right direction. Highest (lowest) } p_2/p^* \text{ value shown is } 1.31 \text{ (0.48).}\]

This shock formation is now studied to determine its correlation to LPT flutter. To quantify the effect, it is convenient to visually inspect the bifurcations of local unsteady pressure. The phase and amplitude of the unsteady pressure distribution
are shown as functions of critical pressure ratio and normalized chord in Figures 4.15 & 4.16 for both the suction and pressure sides, respectively. The specific case these contour maps describe is the $k = 0.1$ flex mode’s least stable IBPA of $\sigma = -54^\circ$. Note that for the suction side, negative phase angles are destabilizing, with $\phi = -90^\circ$ being the worst case scenario since the unsteady pressure would be in phase with the modal velocity. On the other hand, the opposite is true for the pressure side as the force vector points in the opposite direction. Furthermore, for small unsteady pressure amplitude magnitudes, the phase of the unsteady pressure is less significant as the overall contribution to work-per-cycle diminishes.

First examining the suction side plots of Fig. 4.15, the reader will notice there is a relatively small gradient across all pressure ratios for the fore suction side (first 30%). Now observing the pressure side data in Fig. 4.16, it is noteworthy that the airfoil PS experiences negligible changes in both amplitude and phase despite the increased loading. Therefore, in terms of physically where this phenomenon manifests itself, the answer is the aft suction side. At this key location, there is a sharp bifurcation in the unsteady pressure distribution directly after increasing the load below the critical value of $p_2/p^* = 1$. This solidifies the notion that the primary culprit inducing stability changes as a function of loading is the aft suction side shock formations, which qualitatively change local unsteady pressure magnitudes and phases.

![Figure 4.15: Suction side unsteady pressure bifurcations as a function of increased loading: $k = 0.1$, FLEX MODE, $\sigma = -54^\circ$.](image)

### 4.2.2 3D Results

The Hannover airfoil, which is now analyzed three-dimensionally, was designed by the Institute of Turbomachinery and Fluid Dynamics at the Leibniz Universität Hannover. Its blade design is proprietary and therefore images and sensitive details will remain undisclosed. However, it can be mentioned that the geometry closely
Figure 4.16: Pressure side unsteady pressure bifurcations as a function of increased loading: $k = 0.1$, FLEX MODE, $\sigma = -54^\circ$.

resembles a typical subsonic LPT at approximately 90% span. More specifically, the Hannover airfoil is roughly based on the 11th Standard Configuration (STCF 11) with the main difference being reduced camber. It is a freestanding blade with a spanwise length of 105 mm ($\approx 4.13$ inches) and the cascade contains 33 blades in circumferential direction.

For the 3D investigation of the Hannover airfoil, the first two real mode shapes (first bending and first torsion) are examined. Images of the mode shapes are withheld, however, they nearly resemble the earlier flex mode and pitching about mid-chord only now with increased amplitude from hub to tip. A mesh convergence study is completed for 100% back pressure (i.e. baseline) and the results are mesh independent. The final mesh has an H-O-H topology with a grid point distribution of (17/33/49:161/33/49:17/33/49). To change the loading, the exit boundary conditions (static pressure) are manipulated yet again, while the inlet boundary conditions (e.g., total pressure, total temperature, flow angles) are held constant. Typical shifts in ambient back pressure are of the order of 10% nominal operating condition. Thus, in this 3D analysis, the exit conditions are varied within a smaller range from 85% to 105% back pressure in 2.5% increments. In this range of back pressure, the midspan values of torque and exit Mach vary linearly and the inlet Mach number varies quadratically. This is in general agreement with the quasi-2D corollary of Fig. 4.2. As a result of a tighter investigated window of back pressures, no shock structures can be identified in the surface pressure distributions.

The reduced frequency is held constant for this 3D analysis to once again guarantee that changes in aerodynamic stability are due to the steady loading variation alone without any influence from the reduced frequency parameter shift. Since the exit flow velocity varies with back pressure, the reduced frequency is held constant by using Eq. 2.3 to solve for the required physical blade vibration, $\omega$, and assign
that value in corresponding unsteady CFD simulations. The flutter stability for the Hannover blade is calculated using the familiar non-dimensional damping value, $\zeta$, which can be defined by

$$\zeta = \frac{-W_{cyc}}{4\pi KE},$$

where $W_{cyc}$ is again the work-per-cycle of the fluid on the blade and KE is the kinetic energy during blade vibration.

Figure 4.17 shows the 3D results of the least stable damping value over all nodal diameters as a function of normalized back pressure for the bending and the torsional modes. The actual aerodynamic damping values were stripped from the plot to conceal sensitive information; however, the total range of $\zeta$ is listed to the left of the ordinate axes to provide a gauge that directly measures how sensitive the minimum aerodynamic damping is to back pressure variations (i.e. steady loading fluctuations). The back pressure is once again normalized by this specific rig’s nominal critical choking pressure, $p^*$.

Starting with the bending mode results of Fig. 4.17a, there exists a complex relationship between the loading and least stable aerodynamic damping in this region.
of back pressure. As the loading is first increased (i.e., back pressure decreased), the blade destabilizes until $p_2/p^* = 1.36$ when further increases in loading begin to stabilize the blade. However, there is a relatively small change in aerodynamic damping overall as the scale indicates $\zeta$ varies only 0.014% in this region. This is favorable news in terms of a steady blade loading design sensitivity for the first bending mode. On the other hand, the torsion mode results of Fig. 4.17b indicate a sensitivity nearly five times stronger as the change in $\zeta$ equals 0.065%. Again, this demonstrates the intricate mode shape dependency found in the relation between flutter and steady blade loading. Theoretically, a change of ambient pressure may now realistically cause a once stable configuration to flutter in marginal stability scenarios. The near-linear relation between minimum aerodynamic damping and back pressure for this 3D torsional mode validates the main assertion of this work. That is, the steady aerodynamic loading is indeed an important, strongly correlated parameter that controls LPT flutter. This result is especially trusted now that the reduced frequency was held constant in the 3D investigation of the Hannover airfoil.

4.2.3 Recap of Initial Loading Studies

For years, two design parameters have been well-known to control flutter in low-pressure turbine blades: the reduced frequency and vibratory mode shape. This initial study primarily focused on a quasi-2D slice of the Standard Configuration 4 LPT blade has shed light on a third controlling parameter, the steady loading, which can drastically stabilize or destabilize the least stable aerodynamic damping of an LPT cascade. Results showed that as the blade load increased (by way of decreasing the back pressure), flutter stability visibly changed in a peculiar, mode-dependent manner. The initial quasi-2D results did not fix the reduced frequency, which actually mimics reality for gas turbine engines. Therefore, to maintain consistency with the scientific method, and to prove the hypothesis that steady loading is a parameter strongly correlated with LPT flutter behavior, a control was added. The control was fixing the reduced frequency via adjusting physical vibration frequency proportionally with the exit velocity for two test cases: a quasi-2D slice of SC4 using Test Case 627 baseline conditions and the fully three-dimensional Hannover airfoil. The 3D findings validated the assertion that steady blade loading is indeed a strongly correlated LPT flutter control parameter that is a function of mode shape.

Some main conclusions are now reiterated

1. The steady blade loading can significantly alleviate or worsen the aerodynamic damping of LPT blades. Therefore, the steady loading is a crucial flutter control parameter and should be considered in all LPT blade designs, especially when anticipated ambient pressure fluctuations are high.

2. Based on a limited glimpse of four different quasi-2D and two different 3D mode shapes, a preliminary conclusion can be made that the sensitivity of the least stable aerodynamic damping condition is a function of mode shape. Strong
supporting evidence is presented in Fig. 4.5 where the response of the flex and pitching modes behave distinctively.

3. From an alternative perspective of the LPT airfoil designer, the steady blade loading dictates allowable mode shapes that are aerodynamically stable.

4. By varying the back pressure, the inlet and exit Mach numbers shift which ultimately changes the acoustic resonance conditions of the entire cascade.

5. The influence of steady blade loading on low-pressure turbine flutter also manifests itself in the form of shock structure on the blade surface. The blade shock structure is a function of the back pressure and it is hypothesized to be the cause of the large stability gradient, for example, in the $k = 0.3$ leading edge pitching mode results of Fig. 4.5a.

6. When the back pressure equals the critical choking pressure, $p_2/p^* = 1$, there is a qualitative change in how steady loading affects flutter. This is believed to be a direct result of new shock formations or departures when the mass flow goes into and out of choked conditions.

7. A key driver controlling LPT flutter’s mode-dependent loading sensitivity is the passage shock formation changing the unsteady pressures on the suction side of the blade.

8. In general, loading the blade stabilizes the flex mode and destabilizes the edge-wise mode for back pressures greater than the critical choking pressure, i.e., $p_2 > p^*$.

9. The unsteady pressure characteristics sharply change on the blade’s suction side below the critical value of $p_2/p^* = 1$. These bifurcations create sharp jumps up or down in the minimum aerodynamic damping coefficient.

10. Unlike reduced frequency, the relationship between LPT flutter and blade loading is non-monotonic. For example, the mid-chord pitching mode (i.e., torsion) jumped into and out of negative aerodynamic damping regions multiple times as the load is increased. This suggests an intricate relationship between loading and LPT flutter. If a power turbine blade designer plans to increase blade torque by pushing load out toward the tip or removing stages, the sensitivity of the mode shape to steady loading variations should be carefully considered, especially when ambient pressure fluctuations are high.

4.3 Mode Dependent Shock Structure Effects: SC4

Looking beyond the most common parameters used in flutter-free design, the reduced frequency and mode shape, this section looks into complex, non-monotonic
relationship between flutter and loading. A quasi-2D grid of Standard Configuration 4 Test Case 627 serves as a computational wind tunnel to examine the impact of both rigid body translations and rotations through various levels of load (e.g. subsonic/supersonic, passage-shock/shock-free). Critical reduced frequency is plotted as a function of mode shape, and several distinct modal regions are identified with vital qualitative distinctions in terms of their sensitivity to load and mode. It is shown that for a constant reduced frequency, certain mode shape regions experience either increasing, decreasing, or non-monotonically changing stability as a function of advancing loading condition.

The question must be answered why certain pressure ratios and loading conditions are detrimental for some modes, and beneficial for others’ stability margins. An answer may be in the relationship between pitching axis and shock location. It is known that the proximity of the static back pressure to the critical choking pressure drastically changes the passage shock structure which in turn mutants the unsteady pressure amplitudes and phase [75]. While the pressure side of the airfoil is relatively unaffected, the suction side experiences significant changes in unsteady pressure amplitude and phase drastically altering flutter stability. In other words, it is no longer sufficient to consider only reduced frequency and mode shape. Loading and shock position must be considered simultaneously. An outline of the analyses to test these relations is now presented.

The current analysis considers only the subsonic ($M_2 = 0.85$) quasi-2D SC4 Test Case 627 baseline conditions of Table 3.2 as the reference point. From this baseline, one of the following three control parameters is independently varied: reduced frequency, mode, or back pressure. Investigating numerous combinations of reduced frequency, mode shape, loading, and inter-blade phase angle, this simplified quasi-2D grid is necessary to decrease computational expense for the 24,245 CFD simulations required to generate the results that follow.

All CFD results were collected using MUSTANG, the validated frequency domain Reynolds-averaged Navier-Stokes (RANS) solver presented in Chapter 3. Since the shock motion’s influence on work-per-cycle is linear for small blade oscillations [61], all unsteady CFD enforces small blade vibrational amplitudes below the linear limit. Experience working with LPT unsteady aerodynamics empirically supports this linearity assertion by Srivastava. In CFD tests, as the blade amplitude steadily increases low to high values, linearity holds damping nearly constant as a function of amplitude until unreasonably high vibration magnitudes (e.g., greater than HCF limits) are reached.

To control the loading, the exit static pressure is once again varied. Reducing $p_2$ increases loading, mass flow, and flow velocity until fully choked conditions occur as shown in the steady CFD results of Fig. 4.18. The flow chokes in Fig. 4.18 for the following values of critical pressure ratio, $\Pi_c$:

$$\Pi_c = \frac{p_2}{p^*} \leq 1.$$  \hspace{1cm} (4.5)

When $\Pi_c = 1.01$, the suction side trailing edge is shock-free despite having a back
Figure 4.18: Variation of inlet/exit Mach, blade vibrational frequency, and exit flow angle as a function of normalized back pressure for constant reduced frequency, $k = 0.1$.

For values of $\Pi_c < 0.40$, the exit flow angle, $\beta_2$ drops significantly in Fig. 4.18. Flow turning abilities in this high loading region are greatly reduced. To isolate this potential effect from the current study, all results following are limited to values of $\Pi_c > 0.40$ where $\beta_2$ is relatively constant.

All other boundary conditions (i.e. angle of attack, inlet stagnation pressure, and total temperature) are held constant for this work. By adjusting $p_2$, the converged exit velocity, $V_2$, calibrates accordingly due to the change in downstream flow resistance. This change in exit velocity also alters the reduced frequency, $k$. However, to maintain consistency with the scientific method, the blade vibrational frequency is regulated proportionally to $V_2$ to guarantee constant reduced frequency is prescribed.

To vary the mode shape, three types of rigid body modes are tested: 1) Pure translational modes with various “bend” angles/directions 2) Pure pitching modes
with pitching axes located on the chord-line (connecting the LE & TE) 3) Linear combinations of translations and rotations when constructing sample Tie-Dye plots.

Finally, to quantify aerodynamic damping, a slightly different non-dimensional damping coefficient is used for bending and pitching modes, respectively, by [49]

\[ \Xi_h = -\frac{W_{cyc}}{\pi q_2 h^2} \] \& \[ \Xi_\alpha = -\frac{W_{cyc}}{\pi q_2 c^2 \alpha^2}. \]

(4.6)

Here, \( W_{cyc} = \pi \text{Im}(GF) \) is the work-per-cycle of the air on the blade (i.e. negative \( W_{cyc} \) is stable), \( \Delta z \) is the grid thickness, \( q_2 \) is the exit dynamic pressure, \( h \) is the translational/bending amplitude, \( \alpha \) is the pitching angle in radians, and \( c \) is the chord length. Note that the complex generalized force (GF) is the surface integral around the airfoil of the dot product between the pressure and modal velocity.

### 4.3.1 Bending Mode Results

This section exhibits the relationship between bending modes and loading by investigating 31 equally-spaced translational rigid body modes. These “bend” angles vary in increments of \( \theta = 6^\circ \) and range from \( \theta = 0^\circ \rightarrow 180^\circ \). Thanks to symmetry, the other half of the bend angles need not be considered. Figure 4.19 shows the sign and naming convention of the investigated bending modes. As a reminder, \( \xi \) is the chord-wise coordinate positive going from LE to TE, and is orthogonal to \( \eta \), which is defined positive toward the airfoil suction side. These specific directions are functions of the airfoil’s stagger angle. Specifically, the flex mode is located at \( \theta_\eta = 90^\circ - \gamma \) and the edgewise direction is at \( \theta_\xi = 180^\circ - \gamma \). The stagger angle, \( \gamma \), separates the chordline and axial directions. For this study, the positive \( \theta \) direction moves counter-clockwise originating from the positive engine axis (fore looking aft) as illustrated in Fig. 4.19.

Figure 4.20 shows the critical reduced frequency as a function of bend angle for eight selected critical pressure ratios. Each value of \( \Pi_c \) represents a fixed steady loading condition and shock structure. Note that three values of \( \Pi_c \) exceed the critical ratio of 1, and are therefore shock-free. This figure contains a wealth of information. First and foremost, each loading condition has a unique function of mode shape, and vice versa. For example, at \( \theta = 90^\circ \), loading is a stabilizing factor, whereas at \( \theta = 165^\circ \) loading destabilizes. Second, some bending modes are highly sensitive to the loading condition, whereas other regions maintain fixed \( k_{crit} \). In an attempt to simplify the analysis of Fig. 4.20, the effect of loading on LPT flutter is divided into four regions of bending modes as indicated by Roman numerals I-IV and the vertical dashed lines.

**Bending Region I** covers approximately \( \theta = 0^\circ \rightarrow 30^\circ \). These are nearly “flex” bending modes, or plunge roughly perpendicular to the chord-line with a slight tendency going from SS TE to PS LE. This region is most stable and least likely to see flutter. While the bend angles of the local minima vary as a function of load,
Region I offers the aeroelastic designer the most forgiving sensitivity to load and mode shape, as $k_{crit} < 0.18$ regardless of load or mode.

**Bending Region II** covers approximately $\theta = 30^\circ \rightarrow 100^\circ$. This wide range of modes includes translations ranging from the flex mode to just beyond pure circumferential bending modes. In this region, higher loads, i.e. lower $\Pi_c$ values, benefit from lower critical reduced frequencies, and are therefore considered more stable. Therefore, loading monotonically stabilizes SC4 in Region II. Something interesting occurs between $\theta = 30^\circ - 60^\circ$. In this range, there is a noticeable loading insensitivity for values of $\Pi_c > 0.60$. Furthermore, $k_{crit}$ roughly reduces to a linear function of bend angle throughout this region.

**Bending Region III** covers approximately $\theta = 100^\circ \rightarrow 135^\circ$. These modes are close to edgewise translations. In this region, the effect of load variations can be quite high; however, the trend with load is non-monotonic meaning it is difficult to predict if specific levels of loading are beneficial or detrimental to stability. The instantaneous rate or change of $k_{crit}$ with respect to mode shape is the highest in Region III, indicating a high sensitivity to bend angle. Although, $k_{crit} < 0.4$ for the majority of the region for all loads which signifies a favorable stability situation when compared to zones II & IV. Note that low reduced frequency is unfavorable, but low critical reduced frequency is quite desirable, as more stable operating conditions are in play.

**Bending Region IV** covers approximately $\theta = 135^\circ \rightarrow 180^\circ$. Regions II and IV
Figure 4.20: Critical reduced frequency as a function of every 6° bending direction for select loading conditions.

appear to behave in opposite manners, as the result of increasing the load in Region IV tends to destabilize ($k_{\text{crit}}$ rises). This is especially true for values of $\Pi_c < 1$ where shocks are present.

Table 4.1 briefly summarizes the regional differences in terms of sensitivity to load variations, mode shape changes, and the range of $k_{\text{crit}}$ values.

Table 4.1: Bending Mode Sensitivity Regions for SC4

<table>
<thead>
<tr>
<th>Region</th>
<th>Sensitivity to Loading Changes</th>
<th>Reaction to Bend Angle</th>
<th>$k_{\text{crit}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>low, non-monotonic</td>
<td>low: inflection</td>
<td>$&lt; 0.18$</td>
</tr>
<tr>
<td>II</td>
<td>high, stabilizing</td>
<td>low: $\approx$ linear</td>
<td>$&lt; 0.6$</td>
</tr>
<tr>
<td>III</td>
<td>high, non-monotonic</td>
<td>high: $\uparrow$ gradient</td>
<td>$&lt; 0.4$</td>
</tr>
<tr>
<td>IV</td>
<td>high, destabilizing</td>
<td>low: $\approx$ linear</td>
<td>$&lt; 0.55$</td>
</tr>
</tbody>
</table>

Fig. 4.21 plots the least stable non-dimensional damping coefficient and least stable IBPA as a function of bend angle. The reduced frequency is held constant at $k = 0.1$ which is a typical unstable or critical reduced frequency for real, three-dimensional first-bending modes. The minimum damping is defined as the least
positive or most negative damping value obtained testing 20 different IBPAs ranging from $\sigma = -162^\circ$ to $\sigma = +180^\circ$ in increments of $18^\circ$. Each intersection with the neutral stability dashed-line indicates $k_{\text{crit}} = 0.1$ for that specific mode/load combination, and can be verified in Fig. 4.20.

![Graph showing minimum non-dimensional damping coefficient and least stable IBPA for 20 IBPAs as a function of bend-direction and critical choking pressure ratio. Reduced frequency is held constant: $k = 0.1$.](image)

**Figure 4.21:** Minimum non-dimensional damping coefficient and least stable IBPA for 20 IBPAs as a function of bend-direction and critical choking pressure ratio. Reduced frequency is held constant: $k = 0.1$.

Again, the observation holds that increasing the loading in the beginning of Re-
region II near $\theta = 40^\circ$ tends to stabilize, whereas the effect is reversed in Region IV near $\theta = 165^\circ$ with the exception of the least loaded case of $\Pi_c = 1.31$. One interesting result, seen in Fig. 4.21, is the general trend defining the least stable IBPA varies proportionally with bend angle. There are of course small variations between loading conditions; however, relative to the macro trend, a conclusion is drawn that the least stable IBPA is a stronger function of bending mode direction and less of a function of load, with the exception of edgewise-like modes ($\theta \approx 0^\circ$) where there is discontinuity in the trend-line\textsuperscript{3}. Lastly, as these curves are quite similar in shape and grow in amplitude with larger pressure ratios, future work should investigate a proper scaling factor to collapse these curves for use in preliminary design.

### 4.3.2 Pitching Mode Results

This section displays the relationship between loading and rigid body torsion modes, or “pitching” modes. In all, 21 pitching axes (PAs) that span linearly from LE to TE in 5\% increments of chord are tested. Figure 4.22 illustrates an example of the mean and max rigid body deformation of the LE pitch mode, or pitching about 0\% chord. To simplify matters when referencing certain pitching mode locations, the variable “$a$” is introduced as a chord normalized pitching axis location along the $\xi$-axis. Two other pitching axis locations, mid-chord ($a = 0.5$) and TE pitch ($a = 1$), are indicated for reference. Note the sample mode has exaggerated rotation. In the CFD, all modes rotate 2\% of one radian, or roughly $\alpha = 1.15^\circ$.

Figure 4.23 is much like its bending corollary of Fig. 4.20, but this time, $k_{crit}$ is plotted as a function of pitching axis. For some combinations of mode shape and loading, the critical reduced frequency jumps off the plot above $k_{crit} \approx 1$. When this happens, it indicates only unstable conditions exist for that mode/load pair for $k \leq 1$. Once again attempting to simplify the interpretation of Fig. 4.23, the pitching mode range is divided into four qualitatively distinct regions.

**Pitching Region I** is defined by $0 \leq a < 0.46$. Generally speaking, this zone has the lowest average $k_{crit}$, and is therefore a good design region. The mode sensitivity is fairly low here, as $\left| \frac{dk_{crit}}{d(a)} \right| \ll 1$ for most loads and pitching axes. The most stable pitching axis for each mode in this region is a function of load. For instance, $\Pi_c = 0.95$ is least likely to flutter with a local minimum at $a = 0.45$, whereas the $\Pi_c = 1.31$ max stability point is near $a = 0.39$, and loading conditions $\Pi_c = 0.48, 0.60, 0.78, 0.83$ all have their lowest $k_{crit}$ value with a leading edge pitch mode ($a = 0$). Therefore, while the mode sensitivity is low, the loading sensitivity is high as a small change in loading from $\Pi_c = 0.48$ to $\Pi_c = 0.60$ results in a 5-6x rise in $k_{crit}$ for $a = 0 - 0.25$.

**Pitching Region II** is defined by $0.46 \leq a < 0.55$. This zone is characterized by a very sharp destabilizing gradient in critical reduced frequency with respect to mode shape, i.e. $\frac{dk_{crit}}{d(a)} \gg 1$. Furthermore, $k_{crit}$ is virtually independent of loading condition in Region II, as the mode shape solely dictates stability.

\textsuperscript{3} Perhaps, more accurately, the trendline shifts by reflecting itself across the $\sigma = 0^\circ$-axis.
Figure 4.22: Sample sketch of the $a = 0$ “leading edge pitch” mode. Pitching-axis locations are also shown for mid- & trailing-edge pitch.

Figure 4.23: Critical reduced frequency as a function of chord-line pitching axis location (every 5%) for select loading conditions. Note: markers are for visualization and do not indicate refinement.

Pitching Region III is defined by $0.55 \leq a < 0.85$. This pitching region is
the most unstable with very high $k_{\text{crit}}$ values regardless of mode shape. Increased loading is favorable in Region III which is clearly visible near $a = 0.80$.

**Pitching Region IV** is defined by $0.85 \leq a \leq 1$. Moving the pitching axis closer to the TE in this region improves stability. The critical reduced frequency ranges anywhere from $k_{\text{crit}} = 0.4 - 0.7$. For $\Pi_c < 1$, there is a somewhat monotonic relationship in that increased loading improves stability.

High-level qualitative regional differences in terms of sensitivity to load variations, pitching mode changes, and the range of $k_{\text{crit}}$ values are summarized in Table 4.2.

<table>
<thead>
<tr>
<th>Region</th>
<th>Sensitivity to Loading Changes</th>
<th>Reaction to Pitching Axis</th>
<th>$k_{\text{crit}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>high, non-monotonic</td>
<td>very low</td>
<td>$&lt; 0.3$</td>
</tr>
<tr>
<td>II</td>
<td>very low</td>
<td>high: $\approx$ linear</td>
<td>$0.1 - 0.6$</td>
</tr>
<tr>
<td>III</td>
<td>high, ↑ load stabilizes</td>
<td>high</td>
<td>$&gt; 0.5$</td>
</tr>
<tr>
<td>IV</td>
<td>medium</td>
<td>low: $\approx$ linear</td>
<td>$0.4 - 0.7$</td>
</tr>
</tbody>
</table>

Figure 4.24 plots the least stable non-dimensional aerodynamic damping, $\Xi_\alpha$, and the least stable IBPA as a function of chord-line pitching axis location for a constant reduced frequency of $k = 0.3$. The same 20 IBPAs tested in the bending mode results are retained for this pitching analysis. The foremost pronounced feature of Fig. 4.24a is the generalization that moving the pitching axis aft is destabilizing. At first, this seemed to contradict the Region IV conclusions of Fig. 4.23. However, after generating data similar to Fig. 4.24a this time using $k = 0.6$, it was witnessed that the tail end ($0.9 < a < 1.0$) of all loading curves rise above the neutral stability line (i.e. $\Xi_\alpha = 0$) much like the $k = 0.3$, $\Pi_c = 0.60$ case does near $a = 0.6$. Therefore, one should be careful when making these types of generalizations when inspecting only one reduced frequency value. It is also worth noting, with the exception of the lowest loading condition $\Pi_c = 1.31$, the curves all seem to collapse between $a = 0.6 - 0.7$.

Figure 4.25 now plots the minimum damping as a function of variable pressure ratio at a constant reduced frequency of $k = 0.3$. This alternative type of plot was created to provide an easier visualization tool of how specific modes change in aeroelastic stability as a function of load. For simple reference, the seven values of $\Pi_c$ shown in Fig. 4.24a are indicated by gray, vertical dashed-lines. To analyze Fig. 4.25, the Region definitions of Fig. 4.23 and Table 4.2 are recycled.

Region I modes appearing in Fig. 4.25 correspond to pitching axes of $a = 0.25$, 0.35, & 0.45. According to the conclusions of Table 4.2, there should be a low sensitivity to mode shape and a high sensitivity to load. As for the low sensitivity to mode, this is verified by noting how qualitatively similar the minimum damping curves are for the entire loading region. On the other hand, these three modes are all easily affected by $\Pi_c$ variations. This assertion is especially accurate in the range
0.60 < \Pi_c < 0.90 where there is a steep loss of stability with increasing loads. It is hypothesized that this is attributed to the passage shock moving farther aft as the back pressure drops, creating a larger distance, and therefore bigger moment-arm, between the shock and elastic axis thus amplifying the effect. The path of the suction side shock location (projected onto the chord-line) moving as a function of \Pi_c for
Figure 4.25: Minimum non-dimensional damping coefficient over all IBPAs as a function of choking pressure ratio for select pitching axes. Reduced frequency is held constant: \( k = 0.3 \).

Figure 4.26: Evolution of shock location on the airfoil suction side as a function of pressure ratio.

SC4 is illustrated in Fig. 4.26. It can be seen that the shock moves almost linearly \( \Pi_c \), as the position moves roughly 6%-chord aft for each pressure decrement of \( 0.1p^* \).
There is one Region II mode in Fig. 4.25 located at \( a = 0.55 \) and is an interesting case. By now, there may be a misinterpretation of what is meant by “low” load effect in Table 4.2. While it may be true the absolute range of minimum damping values is relatively small for \( a = 0.55 \) in Fig. 4.25, the impact of increasing the load initiates five distinct jumps across either side of the neutral stability line. This is the same type of intermittent flutter pattern seen by Cardinale in Figure 2.5. So, in Region II, while the impact of loading is “low” in terms of change in damping magnitude, the impact of loading is quite high and truly meaningful from the perspective of stability inflections.

The two Region III modes seen in Fig. 4.25 correspond to pitching axes of \( a = 0.65 \) & 0.75. As expected, these pitching axis locations are most unstable. At least for \( k = 0.3 \), increasing loads is only helpful for either very low or very high values of \( \Pi_c \). Between \( 0.60 < \Pi_c < 1.10 \), the effect of pressure ratio and shocks carry minimal weight in changing stability.

Thus far, the analyses have been limited to rotational modes whose pitching axes are located on the chord-line. To offer an expanded look into how loading changes as a function of other pitching modes, seven Tie-Dye (TD) featuring distinct values of \( \Pi_c \) are shown in Figures 4.27a-4.30b. The layout and contour scale is identical between each TD plot. A black rectangle is placed in the center to indicate the LE/TE positions and provide the chord-length scale. Each plot has a pitching axis resolution of 200x200 spanning \( \pm 2.5 \) chords in both the \( \xi \)- (parallel to chord from LE to TE) and \( \eta \)- (perpendicular to chord going from PS (down) to SS (up)) directions.

Certainly, the sheer amount of data embedded within these eight TD figures is vast. However, there are some high-level observations worth noting. Regardless of load, several areas in the TD plot visibly maintain their shape and relative stability level. A region that first comes to mind is the high \( k_{\text{crit}} \) region located approximately one chord below the LE (in the negative \( \eta \) direction). Pitching axes located in this region are quite unstable for normal levels of \( k < 0.8 \). Of course, an exception arises for \( \Pi_c = 0.48 \), as this high loading condition demonstrates a greater stability tolerance.

The second mainstay of the TD plots, regardless of load, is the high gradient region cutting through the airfoil. This undesirable region of low stability is essentially comprised of Regions II & III as defined above in Fig. 4.23. Region II can be interpreted as the quick transition from blue to green to red at approximately \( a = 0.50 \) for each value of \( \Pi_c \). Again, an exception exists for \( \Pi_c = 0.48 \), as this high loading condition demonstrates a greater stability tolerance.

The third and final region that is pointed out is the fairly stable (blue) region located in the upper left-hand corner of each plot. Although the exact area and value of \( k_{\text{crit}} \) slightly shifts as a function of load, this is without question the largest, consistently stable pitching axis zone. However, for once, the higher loads are not the most stable here as some intermediate (green) values of \( k_{\text{crit}} \) come into play.

In closing this section, we will briefly discuss what is called the miracle of the Tie-Dye plot. When holding Mach conditions constant, the miracle of the Tie-Dye plot
(a) \( \Pi_c = 0.48, \ M_2 = 1.42 \)
(b) \( \Pi_c = 0.60, \ M_2 = 1.30 \)

**Figure 4.27**: SC4 Near-Field Tie-Dye plot: ±2.5 chords

(a) \( \Pi_c = 0.78, \ M_2 = 1.18 \)
(b) \( \Pi_c = 0.83, \ M_2 = 1.07 \)

**Figure 4.28**: SC4 Near-Field Tie-Dye plot: ±2.5 chords

84
(a) $\Pi_c = 0.95$, $M_2 = 0.97$
(b) $\Pi_c = 1.01$, $M_2 = 0.93$

Figure 4.29: SC4 Near-Field Tie-Dye plot: ±2.5 chords

(a) $\Pi_c = 1.19$, $M_2 = 0.79$
(b) $\Pi_c = 1.31$, $M_2 = 0.70$

Figure 4.30: SC4 Near-Field Tie-Dye plot: ±2.5 chords

85
is that $k_{\text{crit}}$ contours are relatively constant even for different blade geometries [40], especially for subsonic conditions. Therefore, despite investigating only one LPT geometry here, it is a reasonable hypothesis that many qualitative trends observed for the SC4 pitching and bending modes may also be readily applied to other LPT geometries.

4.3.3 Recap of Mode Dependency Study

1. The effect of loading on aerodynamic damping is a function of mode shape.

2. For Standard Configuration 4, $k_{\text{crit}}$ variations as a function of bending mode and load can be divided into four regions:
   - **Bend Region I**: ($0^\circ < \theta < 30^\circ$) → Most stable region. Low sensitivity to load (non-monotonic) and mode shape. $k_{\text{crit}} < 0.2$
   - **Bend Region II**: ($30^\circ < \theta < 100^\circ$) → High sensitivity that stabilizes with increased load and low, roughly linear sensitivity to mode. $k_{\text{crit}} < 0.6$
   - **Bend Region III**: ($100^\circ < \theta < 135^\circ$) → High, non-monotonic load sensitivity and high sensitivity to mode shape. $k_{\text{crit}} < 0.4$
   - **Bend Region IV**: ($135^\circ < \theta < 180^\circ$) → High sensitivity that destabilizing with increased load and low, roughly linear sensitivity to mode. $k_{\text{crit}} < 0.55$

3. For non-edgewise bending modes, the least stable IBPA is a strong function of bend direction, and less load dependent.

4. For Standard Configuration 4, $k_{\text{crit}}$ variations as a function of pitching mode and load can be divided into four regions:
   - **Pitch Region I**: ($0\% \leq \text{P.A.} < 46\%$) → Most stable region. High, non-monotonic load sensitivity and high sensitivity to mode shape. $k_{\text{crit}} < 0.3$
   - **Pitch Region II**: ($46\% \leq \text{P.A.} < 55\%$) → Very low sensitivity to load and very high, linear sensitivity to mode. $0.1 < k_{\text{crit}} < 0.6$
   - **Pitch Region III**: ($55\% \leq \text{P.A.} < 85\%$) → Least stable region. High sensitivities to both load and mode. Higher loads stabilize. $k_{\text{crit}} > 0.5$
   - **Pitch Region IV**: ($85\% \leq \text{P.A.} \leq 100\%$) → Medium load sensitivity and low, linear sensitivity to mode. $0.4 < k_{\text{crit}} < 0.7$

5. The least stable IBPA for bending modes is approximately a function of mode with a minor dependence on loading condition. The least stable IBPA for pitching modes is erratic for pitching axes nearest the shock location indicating a mathematical expression exists defining the work-per-cycle contributions as a function of mode, shock location/strength, and IBPA.

4.4 Geometry Dependent Shock Structure Effects

Previous numerical studies on the Standard Configuration 4 LPT concluded loading, and in particular the shock structure, strongly impacted the aerodynamic dämp-
ing as a function of mode shape. This section presents a deeper investigation into the physical explanation behind the interdependency of loading, mode shape, reduced frequency, and LPT geometry. Serving as computational test rigs, four unique quasi-2D LPT configurations are exhaustively analyzed. Three primary contributors to this mode-dependent flutter phenomenon are shown to be the passage shocks, blade geometry, and interblade phase angle. Further, the sensitivity of critical reduced frequency as functions of bending mode and loading are divided into four zonal regions exhibiting qualitatively similar behavior. It is shown in some zones, depending on airfoil geometry and shock strength, that loading will always either increase or decrease stability monotonically. For pitching modes, the shock strength, phase, and location with respect to the rigid body center of torsion prove to play a key part in determining stability. Understanding these shock effects generalized to all LPT geometries yield new design considerations useful for suppressing flutter.

4.4.1 Various Quasi-2D LPT Geometries

This section introduces the four LPT airfoils under investigation. Each airfoil was carefully selected to provide contrasting shape and flow features to attempt generalization of all findings to arbitrary airfoil designs. The chord-normalized blade profiles are plotted in Fig. 4.31, and important flow characteristics are listed in Table 4.3.

![Figure 4.31: Chord-normalized blade profile comparisons.](image)

**Airfoil 1 - Flat Plate (FP):** The FP geometry is the simplest by far. It is a 2D linear cascade representative of a typical last stage steam turbine blade near
Airfoil 2 - Standard Configuration 4 (SC4): EPFL’s Standard Configuration 4 is an annular, 20-bladed, tuned, non-rotating LPT configuration. The subsonic baseline conditions (Test Case 627) are adopted. Detailed information regarding the SC4 geometry can be found in [20].

Airfoil 3 - Academic LPT blade (KTH): The KTH blade’s geometry and flow conditions were obtained from Vogt’s Ph.D. dissertation [70]. In terms of blade thickness and camber, it provides an excellent intermediate option between SC4 and the industrial blade, BP2, described next.

Airfoil 4 - Industrial LPT Blade (BP2): The BP2 blade is a last stage industrial LPT power turbine blade. Since many relevant details are proprietary, baseline flow conditions and geometric profile have either been omitted or skewed.

**CFD Model**

All CFD and airfoils are modeled as linear, viscous, and quasi-2D using a validated in-house frequency domain RANS CFD code called MUSTANG [28] (refer to Section 3.1). By using small, quasi-2D radial grid dimensions, computational time is minimized. Convergence studies on each H-O-H grid were conducted to minimize the number of cells required to achieve quality unsteady pressure results, which is necessary to reduce the enormous computational expense required to investigate all combinations of airfoil shape, mode, IBPA, loading condition, and frequency. Lastly, despite investigating numerous loading conditions that undergo strong shock behavior, according to many independent studies, shock-induced nonlinearities negligibly contribute to local work contributions for reasonable vibration amplitudes [37, 55].

However, the walls at the hub and tip of the quasi-2D domain are considered inviscid to eliminate spurious end wall effects.
By now, we know a turbine blade’s loading and exit velocity increase as the back pressure decreases for a constant inlet total pressure value until the flow fully expands. In addition to this knowledge is the fact that the inlet Mach number chokes at $\Pi_c = \frac{p_2}{p^*} \leq 1$ according to Eq. 4.1. When $p_2 \leq p^*$, the mass flow rate cannot increase by further reducing $p_2$ alone, and a normal shock forms at the passage throat at $p_2/p^* = 1$. For the purpose of directly comparing the four quasi-2D LPT configurations and steady results, each of which possessing unique values of specific heat ratio and inlet stagnation pressure, and thus $p^*$ values, all results as a function of back pressure have been nondimensionalized by $p^*$, which nicely collapses results to provide one-to-one comparisons.

Figure 4.32 directly compares converged steady CFD results for the inlet/exit relative Mach numbers ($M_1$&$M_2$) and the amount of flow turning, which is defined as the difference between the relative exit and inlet flow angles ($\beta_2 - \beta_1$). There is a nice spread in levels of $M_1$ between test cases, which provides this study varying levels of shock strength to investigate. Note that while changes in $M_1$ are small with back pressure, the shock structure changes rapidly for $p_2/p^* < 1$, which yields a plethora of unique loading conditions ranging from normal shocks, oblique shocks, double shocks, and shocks fixed at the trailing edge.

With the exception of BP2, the $M_2$ curves collapse nicely when normalizing by the critical choking pressure ratio, $\Pi_c = \frac{p_2}{p^*}$. There is also a good variety in flow turning as a function of load. The sole rotating case (BP2) has the greatest change.
in turning with load as the incidence angle decreases while approaching the baseline design condition in the high loading regime. Lastly, to demonstrate the aerodynamic uniqueness of the selected airfoils, the isentropic Mach number distribution, \( M_{is} \), across the airfoil surface is shown in Fig. 4.33. The isentropic Mach number is given by Eq. 4.7 as

\[
M_{is} = \sqrt{\left(\frac{p_0}{p}\right)^{\frac{\gamma - 1}{\gamma}} - 1} \star \frac{2}{\gamma - 1}. \tag{4.7}
\]

Despite these four steady distributions being evaluated at the same subcritical pressure ratio of \( p_2/p^* = 0.83 \), the suction side shock location (in terms of percent chord) and shock strength greatly vary, indicating the profile selections are diverse enough to justify the generalization of later assertions across diverse designs of LPT blades.

![Figure 4.33: Isentropic Mach number distributions: \( p_2/p^* = 0.83 \).](image)

From these baseline steady CFD solutions at various loads, unsteady parameters, such as the reduced frequency, vibratory mode shape, and IBPA, will be parametrically swept to systematically investigate their influence on flutter boundaries.

### 4.4.2 Critical Reduced Frequency as a Function of Mode Shape

The critical reduced frequency, \( k_{crit} \), is defined as the reduced frequency, \( k \), in which there is neutral aerodynamic stability, i.e., zero aerodynamic damping, when considering all nodal diameters. This section investigates \( k_{crit} \) as a function of airfoil shape, loading \( (\Pi_c) \), and mode shape. All non-aerodynamic damping sources are ignored.

Since flutter normally occurs in one of the first few natural modes, 3D flutter modes can be accurately approximated as rigid body translations or rotations with varying spanwise amplitudes. As previously mentioned, LPT flow characteristics are quite linear. Therefore, the \( k_{crit} \) of any two-dimensional rigid body mode can be
constructed using the superposition of three fundamental rigid body mode shapes: two orthogonal translations and one rotation [49]. The translational modes chosen are “bending” in the chord-line direction (defined as the $\xi$-coordinate, positive going from LE to TE), and the “flex” mode which translates perpendicular to the chord (defined as the $\eta$-coordinate, positive going from PS to SS).

To calculate the aerodynamic work for each reduced frequency, the unsteady forces and moments due to the blade motion for each fundamental mode shape are calculated using the procedure outlined by Meingast [40]. Then, for any single mode with chord-normalized bending amplitudes of $h_\xi$ and $h_\eta$, and $\alpha$ radians of rotation about the LE, the work-per-cycle that the fluid exerts on the blade can be expressed as:

$$ W_{cyc} = [h_\xi h_\eta \alpha] \begin{bmatrix} W_{\xi\xi} & W_{\xi\eta} & W_{\xi\alpha} \\ W_{\eta\xi} & W_{\eta\eta} & W_{\eta\alpha} \\ W_{\alpha\xi} & W_{\alpha\eta} & W_{\alpha\alpha} \end{bmatrix} \begin{bmatrix} h_\xi \\ h_\eta \\ \alpha \end{bmatrix}, $$

(4.8)

where, for instance, $W_{\alpha\eta}$ is the work computed by dotting the LE pitch mode shape with the unsteady pressures generated by the flex mode.

In all, eight values of reduced frequency ranging from $k = 0 \rightarrow 0.7$ in increments of $k = 0.1$ were investigated over 24 interblade phase angles ranging from $\sigma = -165^\circ \rightarrow 180^\circ$, which is a refinement of $\sigma = 15^\circ$. Critical reduced frequencies of the least stable IBPA for any mode shape can now be computed using a linear interpolation scheme.

### 4.4.3 Bending Mode Results

Bending mode values and the sign convention used are depicted in Figure 4.34. The positive axial direction serves as the datum at $\theta = 0^\circ$, with $+\theta$ rotating counterclockwise toward the positive circumferential (tangential) axis. As a reference, the edgewise and flex modes for this specific airfoil (SC4) are indicated on Fig. 4.34 as $\theta_\xi$ and $\theta_\eta$, respectively.

Critical reduced frequencies as a function of bend angle and loading are plotted for each blade in Figures 4.35-4.38. Due to symmetry, only $180^\circ$ of bending modes need to be considered. All plots have been color-coded proportionally to their $\Pi_c$ value, with higher loads bearing darker shades. Since stagger angles differ between these blades, their unique flex and edgewise bending modes are indicated on each plot.

The general shapes of these curves resemble each other quite nicely, indicating the presence of a macro-trend that might possibly be generalized across all LPT geometries. Note how $k_{crit}$ reaches a local maximum at $\theta = 100^\circ$, corresponding to $10^\circ$ beyond circumferential bending (toward the chordline direction). One interesting finding is that independent of airfoil shape or loading condition, the flex mode is always more stable than the tangential bending mode. This result, which holds for four starkly contrasting geometries and flow conditions, further corroborates a past single-airfoil study conducted by Panovsky & Kielb in 2000 [49]. Another noticeable trend is identified in the region separating the flex and tangential bending mode. As
θ decreases away from the local θ = 100° maximum (i.e., the translational direction shifts toward axial bending), $k_{cr}$ linearly decreases until reaching a local minimum in the neighborhood of the modal region bracketed by θ = 0° and θ = 30°. After witnessing the same effect on all four airfoils, it is concluded this phenomenon is
independent of LPT geometry. Therefore, this local minimum region represents a safe design region for the majority of loading conditions. The one exception to this occurs for BP2 where the optimal bending modes are shifted roughly 20° in the chord-line direction between $\theta = -20°$ and $\theta = 10°$.

Upon inspecting the SC4 and KTH results of Figs. 4.36 & 4.37, their sensitivity to both increased loading and bend angles follow nearly identical qualitative patterns.
These trends have been divided into four distinct zonal mode regions and are listed in Table 4.4. Zone I is the design “sweet spot” region located between $\theta = 0 - 30^\circ$. Although $k_{\text{crit}}$ as a function of loading is non-monotonic and difficult to predict in this zone, the $k_{\text{crit}}$ values are relatively small. In Zone II ($\theta = 30 - 100^\circ$), there is both a monotonic relationship with loading and bend angle. That is, increased loading stabilizes the cascade (reduces $k_{\text{crit}}$), while increasing $\theta$ linearly increases $k_{\text{crit}}$. Interestingly, the opposite phenomenon occurs in Zone IV, which is defined as bending modes located between $\theta = \theta_\xi + 10^\circ$ and $\theta = 180^\circ$, where $\theta_\xi + 10^\circ$ refers to a translation direction $10^\circ$ beyond the chordline direction. In this zone, increased loading results in destabilization, and increasing the bend angle linearly decreases $k_{\text{crit}}$. This load destabilization effect also holds true for FP and BP2 in Figs. 4.35 & 4.38. Although, the reader should note here that Zones III & IV appear to coalesce for BP2, which obfuscates any clear trend in the normal Zone IV region. Visual inspection suggests the BP2 Zone IV lies between $\theta = -10^\circ$ and $\theta = 15^\circ$.

Zone III ($\theta = 100^\circ - \theta_\xi + 10^\circ$) is characterized by $k_{\text{crit}}$ values that are nearly load independent; however, modal sensitivity is high, as there is a sharp increase in stability as the translation direction approaches edgewise bending. The minimum near $\theta = 123^\circ$ for the BP2 configuration in Fig. 4.38 corresponds to a plunge direction parallel to the aft portion of the suction side, which is approximately shaped like a flat plate. In fact, this minimum approximately corresponds to the aft SS portion of the blade for all geometries investigated, and is especially evident for the FP in Fig. 4.35 where the aft SS metal angle equals the edgewise direction. While the FP Zone III appears very unstable, note that a purely 2D flat plate experiences zero force undergoing an edgewise motion since there is no area for the pressure to
act on. Further, all modal velocity components normal to the chord are virtually zero in Zone III, leading to small local work contributions when integrating the dot products with unsteady pressures around the blade surface. Therefore, although the extremely high gradient regions near $\theta = \theta_\xi$ are technically unstable for the FP, the level of positive aerodynamic work, or negative aerodynamic damping, is negligibly small compared with the flex mode.

Table 4.4: Critical Reduced Frequency Sensitivity Regions for Bending Modes: SC4/KTH

<table>
<thead>
<tr>
<th>Zone: $(\theta$ range)</th>
<th>Sensitivity to Increased Load</th>
<th>Sensitivity to Bend Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: $\theta = 0^\circ - 30^\circ$</td>
<td>Non-monotonic</td>
<td>Low: possible design region</td>
</tr>
<tr>
<td>II: $\theta = 30^\circ - 100^\circ$</td>
<td>Stabilizes</td>
<td>High: $\approx$ linear</td>
</tr>
<tr>
<td>III: $\theta = 100^\circ - \theta_\xi + 10^\circ$</td>
<td>Low, $\approx$ load independent</td>
<td>High gradient surrounds $\theta_\xi$</td>
</tr>
<tr>
<td>IV: $\theta = \theta_\xi + 10^\circ - 180^\circ$</td>
<td>Destabilizes</td>
<td>High: $\approx$ linear</td>
</tr>
</tbody>
</table>

Qualitatively speaking, the results of the SC4 and KTH configurations compare better with another, whereas the same can be said when comparing FP and BP2. This may be attributed to the similar shapes of these pairs. For flat-plate-like blades with relatively thin, straight aft portions like the FP and BP2 geometries, increasing the load tends to monotonically destabilize Zone II modes, which is exactly opposite to the effect on SC4 and KTH. Understanding how loading will effect Zone II modes is crucial, as preventing circumferential bending modes is paramount. While geometric differences may be the primary factor differentiating the sensitivities to loading in the various bending zones, the effect may also be partially attributed to the shock strength magnitude of each configuration, as SC4 & KTH have lower values of $M_1$ compared with FP & BP2.

4.4.4 Pitching Mode Results

The same pitching mode definitions used in Section 4.3 and Fig. 4.22 are retained for this analysis. Once again, the variable “a” is used to denote a chord-normalized pitching axis location along the straight line between leading and trailing edges.

Critical reduced frequencies as functions of pitching axes along the chord-line and loading are plotted for each blade in Figures 4.40a-4.40b. It is noted here that an attempt was made to apply a corrected k-factor of $k_{corrected} = (k/\sqrt{(M^2 - 1)})$ on the ordinate axis for all $k_{crit}$ vs. mode results; however, this correction factor did
not collapse the curves as intended, so all plots retain the actual value of reduced frequency for the ordinate coordinate.

Qualitative similarities exist once again for the SC4 and KTH configurations as one pair, and the FP and BP2 as another. The zonal pitching mode tendencies defined by Waite and Kielb in Table 4.2 hold true at a qualitative level for the KTH blade. For example, Zone I is defined as pitching axes between $a = 0 - 0.45$ for SC4, where $a$ is the chord-normalized pitching axis location along the chordline. Here, there is a high, nonmonotonic sensitivity to load change and little sensitivity to pitching axis. The exception to this is the highest loading condition of each configuration, where the aft shock increasingly stabilizes the cascade as the distance between the shock and pitching axis grows. For zone II, which has shifted approximately 5% upstream for the KTH blade ($a = 0.4 - 0.5$), Fig. 4.39b exhibits a strikingly similar high gradient region to Fig. 4.39a for all pressure ratios with the exception of the two extreme loading conditions. Due to Zone II’s inherent high gradient region, it is recommended that airfoil designers do not locate the torsion axis here.

The SC4 and KTH airfoils, which are characterized by thicker fore sections and higher aft turn, display remarkably similar tendencies of $k_{crit}$ as a function of mode and load. Further verification of this is shown in the Tie-Dye plots of Fig. 4.41. In this plot, chord-normalized pitching axes indicate the corresponding value of critical reduced frequency. Clearly, for the same loading level of $\Pi_c = 1.18$, the contours follow correlative behavior, despite the absolute value of $k_{crit}$ not always aligning as SC4 shows predominately less stable conditions at this $\Pi_c = 1.18$.

On the other hand, the different airfoil geometries and higher loading conditions of FP and BP2 exhibit new pitching zone trends that are more difficult to generalize. In particular, when $\Pi_c < 0.69$, $k_{crit}$, while relatively high, is rather insensitive to chordwise pitching axis location. As $\Pi_c$ rises into the transcritical or supercritical ranges in Figs. 4.40a & 4.40b, the shock grows weaker and moves closer to the leading edge. This shifts the $k_{crit}$ local minima which represents the optimal design pitching axis for a given loading scenario. These distinct local minima should be thought as target design regions for each load setting, as the pitching axis range in which $k_{crit}$ remains relatively low provides a design margin of 10-20% of chord. Interestingly, however, for the highest values of $\Pi_c$, where loading is weakest for both BP2 and FP, the $k_{crit}$ vs. pitching axis curves behave precisely as the SC4 and KTH configurations do, and may be classified exactly into the zonal regions of Table 4.2.

Some loading conditions, for example, $\Pi_c = 0.62$ for BP2, make stability impossible at low reduced frequencies regardless of chordline pitching mode. In this very special case, two shocks exist on the airfoil surface. These loads are inherently unstable due to the passage shock structure and severity which are a strong function of IBPA. This double-shock feature can be seen by plotting the coefficient of pressure, as defined in Eq. 4.3, around the airfoil surface. Figure 4.42 shows $C_p$ for three selected BP2 loading conditions. As the back pressure decreases, the load increases, and the shock moves aft prior to forming a double shock at $\Pi_c = 0.62$. The local $k_{crit}$ minima for subcritical loads of BP2 move northeast on Fig. 4.40b, indicating
Figure 4.39: Critical reduced frequency as a function of chordline PITCHING axis and load.

The most stable pitching axis follows the shock aft while also losing stability.

Figure 4.43 shows the least stable aerodynamic damping and IBPA as a function of mode shape for these three BP2 loading conditions. Reduced frequency is held constant at $k = 0.3$. For all following data, actual rigid body mode shapes were used in the unsteady CFD; thus, superposition was not applied. The local maxima of
$\Pi_c = 0.83$ and $\Pi_c = 0.93$ correspond closely to the shock position. Once the pitching axis changes side of the shock foot, the stability characteristics flip.

According to Figure 4.43b, the critical IBPA shifts roughly $180^\circ$ when the pitching axis crosses the normal shock location. Additionally, the local work contribution of the shock is minimized as the distance between the shock and pitching axis is...
Figure 4.41: Tie-Dye plot showing $k_{crit}$ as a function of near-field pitching axes. Blade orientation shown for reference: $\Pi_c = 1.18$

Figure 4.42: BP2 airfoil pressure coefficient variation with load.

zero; therefore, the unsteady work contribution is mitigated due to the large unsteady pressures inherent to the region. This suggests that there are two benefits for the blade designer to locate the pitching axis near (or just aft of) the shock location at design conditions. The first, and most obvious benefit, is the mitigation of harmful local work contributions due to the shock. The second benefit is subtle. It is hypothesized that slightly different pitching axes between neighboring blades (due to intentional design or unintentional manufacturing variations) will create slight differences between exact pitching axis locations. This effect may create a form of aeroelastic mistuning in which certain blades respond to their respective least stable
IBPA, which very well may be in anti-phase with the neighbor. Further computational (and experimental) tests are warranted to further validate this hypothesis on the secondary benefits to placing the pitching axis in close proximity to the fore suction side shock.

4.4.5 Local Work Contributions

Waite and Kielb showed the pressure side unsteady pressure distributions changed negligibly with increased load, and that the controlling stability factor was the unsteady pressure bifurcations on the suction side [74]. Fig. 4.44 illustrates how the suction side local work of the BP2 airfoil changes as a function of pitching axis (Fig. 4.44a) and IBPA (Fig. 4.44b). Reduced frequency is enforced constant at
\( k = 0.3 \) and the data corresponds to \( \Pi_c = 0.83 \).

Clearly, the lone shock at \( a = 0.3 \) causes a large spike in local work, with the amplitude of local work magnifying as the distance between the shock and pitching axis elongates. Further, there is a stability inflection at roughly \( a = 0.15 \) when \( \sigma = -45^\circ \). These results agree well with Srivastava’s flat plate algebraic model given in Eq. 2.7.

Figure 4.44b holds the pitching mode constant \( (a = 0.5) \) for various IBPA values. The reader may note the node in local work contribution at \( a = 0.61 \) which simply corresponds to a spot on the blade SS with zero modal velocity for this particular mode shape. According to Equation 2.7, the suction side work contribution from the shock will be stabilizing for \( 0^\circ < (\sigma + \epsilon) < 180^\circ \) and destabilizing for \( -180^\circ < (\sigma + \epsilon) < 0^\circ \). Therefore, the ultimate phase that dictates stability contributions is the sum of the interblade phase angle and phase of the shock motion with respect to blade motion. Using this information, we know some IBPAs will benefit from shock-induced stabilization, whereas others will destabilize. Figure 4.44b shows this is indeed the case, which again verifies Srivastava’s earlier work. Additionally, since it can be interpolated exactly which IBPA is neutrally stable at the shock foot, the suction side shock phase can be estimated after the fact. Assuming the shock motion phase is constant, it appears that the local work contribution from the shock at \( a = 0.30 \) will equal zero at approximately \( \sigma = 60^\circ \), since the local work is negative and positive at \( \sigma = 30^\circ \) and \( \sigma = 90^\circ \), respectively. Therefore, the shock foot phase would have to be \( \epsilon = -60^\circ \) after solving \( \sigma + \epsilon = 0^\circ \).

Figure 4.45 shows the local work contributions (positive=unstable) as a function of loading for BP2’s tangential bending mode at \( k = 0.1 \) and \( \sigma = 75^\circ \). The PS (left) and SS (right) positions are indicated, with the LE corresponding to zero. Note how certain blade portions react to increasing load. For example, the fore 40% of the pressure side is virtually unchanged with loading. In fact, below \( \Pi_c = 0.83 \), the entire PS local work contributions are roughly constant. However, for \( \Pi_c > 0.83 \), the unstable local work contributions linearly increase which in part explains why the BP2 Zone II bending modes destabilize with increased loading.

Although the fore 30% of the SS is virtually unchanged, the suction side portion aft of the shock at \( a = 0.3 \) demonstrates rich bifurcations in local work as loading increases. Below the critical choking pressure ratio of \( \Pi_c = 1 \), a very narrow, high magnitude strip of helpful (stabilizing) work appears. However, aft of the shock there is an intense region of destabilizing work not present in the unchoked region. As loading further increases, the local work at the \( a = 0.3 \) shock diminishes, and a new shock forms farther aft indicated here by the stabilizing work bubbles in the lower-right hand corner of Fig. 4.45. In this high-load region, cascade stability is a more complicated function of shock structure, IBPA, and solidity; therefore, for levels of \( \Pi_c < 0.83 \), loading does not monotonically destabilize BP2, which is easily verified when inspecting higher loads of Figure 4.38.
Figure 4.44: Local BP2 suction side work contributions for various combinations of pitching mode and IBPA: $\Pi_c = 0.83$, $k = 0.3$

4.5 Design Guidelines and Conclusions

From this work, several design rules follow. Regardless of pitching or bending mode, the LPT blade designer wishing to push into higher loading regimes should avoid pitching and bending mode zones where loading monotonically decreases stability for a given cascade. Additionally, modal regions exhibiting high sensitivity, i.e., unpredictability, to critical reduced frequency as a function of mode or load, such as the Zone II pitching modes near mid-chord in Figs. 4.39a & 4.39b, should also be
avoided where possible in design. Practically speaking, the engineer should compare the blade’s actual $k$ value to $k_{\text{crit}}$ after calculating the approximate rigid body pitching axis near 80% span, where the spanwise damping contributions are roughly largest. Then, if $k < k_{\text{crit}}$, depending on the modal zone, modify the airfoil geometry to move the mode shape in the direction of decreasing $k_{\text{crit}}$ while maintaining roughly the same steady aerodynamics and shock position along the chordline. Oppositely, the designer may just as easily opt to maintain mode shape while altering the airfoil steady loading distribution to push the shock aft/fore if the modal zone shows loading monotonically increases/decreases stability. Finally, several conclusions are now summarized:

1. The sensitivity of critical reduced frequency to loading and bending mode variations can be classified into four unique modal zones defined in Table 4.4.

2. Zone I bending modes ($\theta = 0^\circ - 30^\circ$), represent a safe design region with relatively low values of $k_{\text{crit}}$ regardless of mode shape and loading condition.

3. For Zone II modes ($\theta = 30^\circ - 100^\circ$), which include the flex and circumferential bending modes, stability is a function of loading as $k_{\text{crit}}$ changes monotonically with pressure ratio. However, loading may act as a stabilizing or destabilizing agent depending on airfoil geometry. For blades like SC4 & KTH featuring thicker fore sections with higher aft turn, loading was shown to monotonically increases/decreases stability.
stabilize the cascade. On the other hand, loading destabilized cascades with flat-plate-like blades with relatively thin, straight aft portions like the flat plate and BP2 configurations.

4. Independent of loading or airfoil geometry, the flex mode is more stable than the tangential bending mode. Critical reduced frequency varies linearly between these modes.

5. In bending Zone III \( (\theta = 100^\circ - \theta_{\xi+10^\circ}) \), there is a load-independent linear stabilization as the bend angle transitions from tangential to edgewise bending.

6. Zone IV bending modes \( (\theta = \theta_{\xi+10^\circ} - 180^\circ) \) generally grow less stable with increased loading and are highly sensitive to changes in bend angle.

7. The critical reduced frequency as a function of bend angle reaches a local maximum at approximately \( \theta = 100^\circ \), which corresponds to a translation slightly beyond a pure tangential bending mode. Conversely, \( k_{\text{crit}} \) reaches a local minimum near the axial bending mode, but the exact bend angle of this minimum varies with loading and geometry.

8. For LPT blades with weak shocks, the value of critical reduced frequency as a function of pitching mode falls into four qualitatively distinct modal regions, as defined in Table 4.2.

9. The critical IBPA shifts approximately 180 degrees as the pitching axis moves fore and aft of a single, normal shock.

10. Locating the pitching axis near the shock location for design conditions minimizes destabilizing local work contributions from the shock.
Turbomachinery engineers use preliminary design tools and special guideline parameters to bridge the gap between initial conception and the detailed design stage. In the realm of LPT flutter, these tools are intended to expedite early decision making processes, such as how airfoil shape or three-dimensional stacking configuration might change overall cascade stability. As an example, industrial blade designers historically relied on the use of critical reduced frequency values of \( k = 0.1 \) and \( k = 0.3 \) for first-bending and first-torsion modes, respectively. However, with the introduction of the Tie-Dye plot, a preliminary design tool introduced in Section 2.2.1 from Panovsky and Kielb, it is now well-known that the relationship between critical reduced frequency and mode shape is more complicated than assigning a definitive design limit for a given mode. For instance, there are some pitching axis regions on the Tie-Dye plot, especially near mid-chord torsion modes, that exhibit extremely high \( k_{\text{crit}} \) gradients. An LPT designer would use this information gathered from a preliminary design tool to determine the optimal direction in which to relocate the mode shape.

This chapter introduces an advanced preliminary design method that decreases computational costs associated with the beginning phases of LPT flutter analysis and design. The primary objective of this design technique is to quickly and accurately estimate aerodynamic damping vs. IBPA curves for 3D turbine blades. Required inputs include the airfoil geometry, cyclic symmetric (or real) mode shapes, a control text file, and a flow field (characteristic) file specifying exit relative Mach number, exit relative velocity, and pressure ratio distributions out to span. With this tool, engineers can rapidly assess how hypothetical changes in airfoil geometry, stacking, frequency, mode shape, or even loading will impact overall cascade stability. As an added benefit, key flutter drivers are isolated and potential stability solutions can be inferred by segmenting the local work contributions from a user-specified number of quasi-2D radial sections. For instance, if 75% of the negative aerodynamic comes
directly from the 80-90% span range, an engineer can mitigate this aerodynamically unstable region by modifying the blade profile there as to change either the pitching axis (mode), chord (reduced frequency), or perhaps the loading condition. This preliminary design method is comprised of two Fortran codes named FECLOPS3D & CYCLOPS3D. A complete instruction manual on how to generate input files and run this preliminary design tool package is given in Appendix A.

As a brief introduction, this preliminary design method begins by equating a 3D mode shape into the sum of numerous 2D rigid body modes at each radial section from hub to tip. The rigid body mode shape assumption is plausible because flutter in free-standing LPT blades typically occurs in one of the first three modes. Since these lowest frequency modes (typically first bending, first torsion, or second bending) exhibit negligible chordwise bending motion, the total 3D aerodynamic work can be accurately approximated by integrating a series of local spanwise work contributions due to the various quasi-2D rigid body modes at each radial cut. Further, each quasi-2D mode shape can be represented by its unique pitching axis coordinate as described by Panovsky and Kielb [49]. Additionally, since Tie-Dye plots of different LPT profiles (subject to similar loading conditions) are relatively invariant (see Section 2.2.1), CYCLOPS3D leverages previously existing “Baseline Aero” files from legacy airfoils/engines to eliminate the need to run CFD simulations in this preliminary design tool, and that is where CYCLOPS3D gets its speed advantage over the alternative of running full 3D CFD at every interblade phase angle. These Baseline Aero files allow for the interpolation, scaling, and computation of spanwise work and kinetic energy contributions of the radially varying rigid body mode shapes as a function of reduced frequency \((k)\), interblade phase angle \((\sigma)\), and loading condition \((\Pi_c)\). The format and an in-depth description of Baseline Aero files is included in Appendix A.2.2. Essentially, each Baseline Aero file is composed of numerous 3x3 fundamental work matrices given by Eq. 5.1

\[
W(k; \sigma, \Pi_c) = \begin{bmatrix}
W_{\xi \xi} & W_{\xi \eta} & W_{\xi \alpha} \\
W_{\eta \xi} & W_{\eta \eta} & W_{\eta \alpha} \\
W_{\alpha \xi} & W_{\alpha \eta} & W_{\alpha \alpha}
\end{bmatrix}.
\] (5.1)

In general, these work values are complex numbers and functions of reduced frequency, IBPA, and loading. Although these Baseline Aero files can contain any number of combinations of reduced frequency and interblade phase angles, historically they are created using 8 \(k\)-values and 24 \(\sigma\)-values. Thus, since each Baseline Aero file corresponds to a constant relative exit Mach number, there are typically 192 different fundamental work matrices corresponding to all combinations of \(k\) and \(IBPA\). The fundamental modes are:

1. \(\xi\): 1-chord translation parallel to the chordline
2. \(\eta\): 1-chord translation perpendicular to chordline
3. \(\alpha\): 1-radian rotation about the leading edge.
Each fundamental mode has an unsteady pressure distribution due to their unique vibration at a given steady/unsteady aerodynamic solution. The 9 work entries per combination of \(k\), \(\sigma\), and \(\Pi\) correspond to the surface integral over the airfoil of the dot product between the first subscript’s modal velocity and the second subscript’s unsteady pressure distribution. For example, \(W_{\eta\alpha}\) is the work due to the flex (\(\eta\)) modal velocity dotted with the unsteady pressures generated from pitching about the leading edge (\(\alpha\)). Therefore, the main diagonal consists of the complex work values of the three fundamental modes.

General outlines for the two Fortran subroutines, CYCLOPS3D and its preprocessor FECLOPS3D, are now introduced.

5.1 CYCLOPS3D

CYCLOPS3D is the primary code in this preliminary design tool package. A key assumption that CYCLOPS3D leverages is that LPT blade aerodynamics are similar across many different geometries if exposed to similar loading conditions. This is done by using preexisting baseline aerodynamic work matrices that have already been formed by running numerous CFD runs for three orthogonal rigid body mode shapes at different inter-blade phase angles and exit Mach conditions. The symmetry of annular cascades can be taken advantage of by reducing the full annulus computational model to a smaller portion about any axis of symmetry in order to reduce computational cost. Since most turbine cascades exhibit structural cyclic symmetry, each natural frequency possesses a complex mode shape composed of two real modes 90° out of phase, which are commonly referred to as the cosine and sine modes. Therefore, every cyclic symmetric rigid body mode shape can be represented by two pitching axes. The underlying theory involves converting the 3D LE/TE cyclic symmetric rigid body displacement data into a series of 2D fundamental mode components described at the beginning of this chapter. Then, CYCLOPS3D interpolates between \(k/\sigma\) combinations from fundamental work matrices of existing Baseline Aero files, integrates/scales the quasi-2D spanwise work quantities to obtain total 3D aerodynamic work. Lastly, the total work, \(W_{\text{cyc}}\), is normalized to kinetic energy factor to obtain the non-dimensional aerodynamic damping term given by Eq. 5.2

\[
\zeta_{\text{aero}} = \frac{-W_{\text{cyc}}}{4\pi KE},
\]

where KE is the total (radially integrated) 3D kinetic energy. The end product is the aerodynamic damping as a function of nodal diameter. A side benefit to CYCLOPS3D is the ability to locate the least stable mode and nodal diameter. Because CYCLOPS3D can instantly narrow the search area of the least stable IBPA range, LPT designers will save time when performing mandatory, high-cost CFD simulations to identify regions and quantify levels of instability.

The general outline of the CYCLOPS3D code is now presented.
1. First, CYCLOPS3D reads in six Baseline Aero files at monotonically increasing levels of loading. The user should select a baseline case demonstrating the strongest similarity in steady loading distribution (with an extra emphasis on shock characteristics) to the test case being investigated. Baseline Aero files also contain information about the test case including the reference chord length (for scaling purposes), $M_2$, and $\Pi_c$.

2. Next, CYCLOPS3D begins to loop through each nodal diameter case and reads all spanwise information from the primary CYCLOPS3D input file, which includes LE/TE locations, LE/TE axial/circumferential cosine/sine mode rigid body displacements, and spanwise mass/span, mass-moment/span, $V_2$, $M_2$, and $\Pi_c$.

3. Using the set of LE/TE coordinates at each span, the chord length and stagger angle are computed by applying the distance and arctangent formulas between the set of LE/TE coordinates, respectively. This spanwise chord data is used in conjunction with the relative exit velocity profile to compute the reduced frequency at each radial section.

4. Any rigid body mode shape can be represented by the superposition of two orthogonal translations and one rotation. With CYCLOPS3D, the two fundamental translations, $\xi$ and $\eta$, move parallel with and perpendicular to the chordline with a one-chord magnitude, and the rotational term is a one-radian pitch with a center of rotation at the blade’s leading edge. Figure 5.1 presents a sample breakdown of how a rigid body mode shape can be decomposed into the three chosen fundamental modes. In the undeformed position, the 0 blade’s LE and TE rest at $(\xi, \eta) = (0,0)$ and $(\xi, \eta) = (1,0)$, respectively. After following the sequence of prescribed $\eta$, $\xi$, and $\alpha$ displacements, the blade’s final maximum displacement mode shape is shown by position 3. To locate the rigid body pitching axis, take any two points on the displaced blade, draw two straight lines to those points’ original positions, and then draw two more “infinitely” long lines that are perpendicular to the last lines drawn. The intersection of these two infinitely lines is the rigid body pitching axis; however, note for pure bending modes that these lines will never intersect. Bending modes have pitching axes out toward infinity in the direction perpendicular to their actual motion.

Using the section-specific chord, $c$, and stagger angle, $\gamma$, the fundamental mode shape displacements can be calculated using

$$h_\xi = \frac{dx_{LE} \cos (\gamma) - dy_{LE} \sin (\gamma)}{c},\quad (5.3)$$

$$h_\eta = \frac{dx_{LE} \sin (\gamma) + dy_{LE} \cos (\gamma)}{c},\quad (5.4)$$
Figure 5.1: Linear superposition of the three fundamental mode shapes for a rigid body mode with pitching axis $(\xi, \eta) = (1, 1)$.

\[
\alpha = \frac{(dx_{TE} - dx_{LE}) \sin(\gamma) + (dy_{TE} - dy_{LE}) \cos(\gamma)}{c}. \tag{5.5}
\]

The rigid body pitching axis can be mathematically derived as

\[
[\xi, \eta] = \left[ -\frac{h_\eta}{\alpha}, \frac{h_\xi}{\alpha} \right]. \tag{5.6}
\]

In Equations 5.3-5.6, $dx$ and $dy$ refer to the axial and circumferential displacements and the subscripts $LE/TE$ refer to the leading/trailing edge, all respectively. These calculations are performed separately for the cosine and the sine mode.

5. Next, the kinetic energy is calculated at each span and for both cosine/sine modes as the sum of the translational energy from the $\xi \& \eta$ modes with the rotational energy from the $\alpha$ mode.
6. Using the spanwise \( k/\sigma/M_2 \) combination, CYCLOPS3D locates the two Baseline Aero files having the closest \( M_2 \) values (one higher, one lower) and performs a three-dimensional linear interpolation on the reduced frequency, interblade phase angle, and exit Mach number to obtain the 3x3 fundamental work matrix associated with each radial cut and mode.

7. Furthermore, CYCLOPS3D scales this fundamental work matrix from the Baseline Aero file to more accurately portray the geometric (size) and loading differences between the baseline airfoil and the test airfoil. The scaling factor CYCLOPS3D uses to multiply the interpolated fundamental work matrix is given in Eq. 5.7 as

\[
\text{Scale Factor} = \left( \frac{c_{\text{test}}}{c_{\text{baseline}}} \right)^2 \left( \frac{\Pi_{c, \text{baseline}}}{\Pi_{c, \text{test}}} \right).
\]

8. Now having a properly scaled and interpolated fundamental work matrix for both the cosine and sine modes at each span, CYCLOPS3D calculates a 2x2 cyclic symmetry work matrix at each span, which contains four work-per-cycle terms that correspond to all combinations of the cosine/sine unsteady pressures acting on the cosine/sine mode shapes. The formulation of these 2x2 work matrices is now presented.

The work per cycle done by a fluid onto the blade, \( W_{\text{cyc}} \), is the surface integral over one vibration cycle (having period \( T \) and frequency \( \omega \)) of the local dot product of local modal velocity vector and force vector and is given in Eq. 5.8 by

\[
W_{\text{cyc}} = \oint \int_A \left( \Phi \cdot P \cdot N \right) dt \, dA,
\]

where \( \Phi \) is the modal velocity, \( P \) is the surface pressure, and \( N \) is the outward pointing surface normal vector. Assuming harmonic motion, these quantities can be expressed as

\[
\Phi = \frac{d}{dt} \left[ \text{Re}(\vec{\phi} e^{-i\omega t}) \right] = \frac{d}{dt} \left[ \frac{1}{2} (\vec{\phi} e^{-i\omega t} + \vec{\phi} e^{+i\omega t}) \right] = \frac{i\omega}{2} \left( -\vec{\phi} e^{-i\omega t} + \vec{\phi} e^{+i\omega t} \right),
\]

\[
\vec{N} = \vec{N}_{\text{std}} + \frac{1}{2} (\vec{n} e^{-i\omega t} + \vec{n} e^{+i\omega t}),
\]

\[
P = P_{\text{std}} + \frac{1}{2} (p e^{-i\omega t} + p e^{+i\omega t}),
\]

where \( \vec{\phi} = \vec{\phi}_c - i\vec{\phi}_s \) and \( \vec{n} = \vec{n}_c - i\vec{n}_s \) for a forward traveling wave, \( \vec{N}_{\text{std}} \) and \( P_{\text{std}} \) are the steady normal vector and surface pressure, and \( p \) is the complex unsteady pressure \( p = p_r + ip_i \). Regarding the notation, the subscripts \( c \) and \( s \) refer to the cosine and sine modes, the subscripts \( r \) and \( i \) refer to the real
and imaginary components of a complex number, and the bar over a variable
represents the complex conjugate has been applied. After substituting Eqs. 5.9-
5.11 into Eq. 5.8 and applying a few mathematical simplifications, the work-
per-cycle reduces to
\[ W_{cyc} = \frac{i\pi}{2} \iint \left[ P_{std} \left( \phi \cdot \bar{n} - \bar{\phi} \cdot \bar{n} \right) + \bar{N}_{std} \left( \bar{\phi} p - \bar{\phi} p \right) \right] dA, \] (5.12)

It is interesting to note that Equation 5.12 shows the work-per-cycle has two
components. The first term in the integral is associated with the steady pres-
sure, and the second term due to the unsteady pressure. However, the steady
pressure contribution almost always has negligible size in comparison to the
unsteady pressure contributions as showed by Kielb [31], and therefore it can
typically be ignored, although CYCLOPS3D has the internal ability to com-
pute and add the steady pressure work contribution of cyclic symmetric modes
if the user activates this option. If we consider only the work-per-cycle from
the unsteady pressure term, \( W_{up} \), Eq. 5.12 further reduces to
\[ W_{cyc} = W_{up} = \pi \iint \left( \phi_c p_i + \phi_s p_r \right) \cdot \bar{N}_{std} dA \] (5.13)
for a forward traveling wave, i.e., \( \phi = \phi_c - i\phi_s \). If the wave was backward
traveling, i.e., \( \phi = \phi_c + i\phi_s \), the addition sign in Eq. 5.13 would become a
subtraction sign. This mathematical exercise is left for the reader.

Considering here the forward traveling wave, the unsteady pressure is \( p = p_{cr} +
\end{document}
changes sign, the cross-interaction terms will switch sign due to the change in
definition of $\vec{\phi}$; therefore, Eq. 5.14 becomes $W_{up} = W_{cc} + W_{ss} + W_{cs} - W_{sc}$.

CYCLOPS3D applies this cyclic symmetric work formulation using the super-
position of the unsteady pressure work terms from the three fundamental mode
shapes. The following equations are used to calculate the 2x2 cyclic symmetric
work matrix terms:

$$W_{cc} = [h_{\xi c}h_{\eta c}\alpha_c] \begin{bmatrix} W_{\xi \xi} & W_{\xi \eta} & W_{\xi \alpha} \\ W_{\eta \xi} & W_{\eta \eta} & W_{\eta \alpha} \\ W_{\alpha \xi} & W_{\alpha \eta} & W_{\alpha \alpha} \end{bmatrix} \begin{bmatrix} h_{\xi c} \\ h_{\eta c} \\ \alpha_c \end{bmatrix}, \quad (5.15)$$

$$W_{ss} = [h_{\xi s}h_{\eta s}\alpha_s] \begin{bmatrix} W_{\xi \xi} & W_{\xi \eta} & W_{\xi \alpha} \\ W_{\eta \xi} & W_{\eta \eta} & W_{\eta \alpha} \\ W_{\alpha \xi} & W_{\alpha \eta} & W_{\alpha \alpha} \end{bmatrix} \begin{bmatrix} h_{\xi s} \\ h_{\eta s} \\ \alpha_s \end{bmatrix}, \quad (5.16)$$

where, for example, $h_{\xi s}$ is the sine mode component of the $\xi$ displacement, and

$$W_{mn} = \pi \iint_A \left( \vec{\phi}_m \cdot \vec{N}_{std} \right) dA, \quad (5.17)$$

where $m$ and $n$ substitute for the any combination of $\xi$, $\eta$, or $\alpha$, and $\text{GF}_ni$ is
the imaginary part of the complex generalized force (as defined in Section 3.3)
for the $n$-mode. The cross-interaction, or “off-diagonal”, terms are calculated
slightly differently using

$$W_{cs} = [h_{\xi c}h_{\eta c}\alpha_c] \begin{bmatrix} \tilde{W}_{\xi \xi} & \tilde{W}_{\xi \eta} & \tilde{W}_{\xi \alpha} \\ \tilde{W}_{\eta \xi} & \tilde{W}_{\eta \eta} & \tilde{W}_{\eta \alpha} \\ \tilde{W}_{\alpha \xi} & \tilde{W}_{\alpha \eta} & \tilde{W}_{\alpha \alpha} \end{bmatrix} \begin{bmatrix} h_{\xi s} \\ h_{\eta s} \\ \alpha_s \end{bmatrix}, \quad (5.18)$$

and

$$W_{sc} = [h_{\xi s}h_{\eta s}\alpha_s] \begin{bmatrix} \tilde{W}_{\xi \xi} & \tilde{W}_{\xi \eta} & \tilde{W}_{\xi \alpha} \\ \tilde{W}_{\eta \xi} & \tilde{W}_{\eta \eta} & \tilde{W}_{\eta \alpha} \\ \tilde{W}_{\alpha \xi} & \tilde{W}_{\alpha \eta} & \tilde{W}_{\alpha \alpha} \end{bmatrix} \begin{bmatrix} h_{\xi c} \\ h_{\eta c} \\ \alpha_c \end{bmatrix}, \quad (5.19)$$

where

$$\tilde{W}_{mn} = \pi \iint_A \left( \vec{\phi}_m \cdot \vec{N}_{std} \right) dA, \quad (5.20)$$

and $\text{GF}_{nr}$ is the real part of the complex generalized force for the $n$-mode.

9. Lastly, given the individual section thickness, CYCLOPS3D radially integrates
to obtain total work and kinetic energy, and from this calculates aerodynamic
damping using Eq. 5.2.
Numerous Aero Files exist that span many different airfoil shapes and loading conditions. As an example, four Baseline Aero files for two blades, referred to as EPFL and LM, are shown in Fig. 5.2. As indicated by their respective exit Mach numbers, each Baseline Aero case has four exit Mach numbers which produce a unique set of airfoil loading distributions. One of the first steps in running CYCLOPS3D is to select the Baseline Aero case that exhibits the closest possible match to the test case’s steady aerodynamic loading conditions. It is especially important to match the shock location and strength because of the highly concentrated local work contribution that shocks contribute to the total work-per-cycle.

At last, the predictive capabilities of CYCLOPS3D for an industrial power turbine blade is shown in Fig. 5.3. After comparing the steady loading distributions, the LM Baseline Aero files were selected as they closely matched this particular LPT blade’s loading profile and shock characteristics. As evident in Fig. 5.3, the amplitude and shape of the damping vs. IBPA curves agree quite well with the commercial CFD code for both modes. The only difference is the commercial CFD code may take days to converge a single interblade phase angle, whereas CYCLOPS3D takes minutes to setup the input files and seconds to compute damping for all interblade phase angles.

Figure 5.2: Coefficient of pressure profiles of two LPT airfoils at various exit Mach numbers from Meingast [40]. Left: EPFL; Right: LM.
Figure 5.3: Damping vs. IBPA CYCLOPS3D results for an industrial LPT blade compared with full scale URANS CFD results for two mode shapes (primarily bending and primarily torsion).

5.2 FECLOPS3D

FECLOPS3D is the pre-processor code which automatically formats the primary CYCLOPS3D input file given blade geometry and mode shape inputs (typically UIF/UOF files from ANSYS). For a user-defined number of spanwise sections, FECLOPS3D calculates the mass, mass-moment, LE/TE positions/deflections, and Mach number/pressure ratio interpolation markers. With these spanwise quantities and interpolation points, CYCLOPS3D integrates spanwise work contributions by computing the 2D rigid body pitching axes and interpolating between scaled work values from the Baseline Aero files. Sample FECLOPS3D input and output files along with their full descriptions and how to use them are given in Appendix A.

The general process of FECLOPS3D is now described.

1. First, FECLOPS3D reads the entire 3D airfoil mesh data.

2. Next, it reads the control file, and divides the blade into a user-specified number of spanwise sections before finding the corresponding values of the leading and trailing edge coordinates. It is important here that the user specifies an optimal number of spanwise sections. Assign a low number of blade cuts, and the user will lose the added three-dimensional effects that FECLOPS3D
and CYCLOPS3D is designed to capture. Assign too many spanwise sections, and the user will find incomplete airfoil sections at certain radii due to mesh refinement limitations. Typically, 20 spanwise sections are used as this has historically yielded acceptable solution refinement/convergence without coming close to running into over-refinement issues.

3. Then, a user-generated input file is read, which allows FECLOPS3D to calculate interpolated values of the relative exit velocity, \( v_2 \), exit relative Mach number, \( M_2 \), and critical pressure ratio, \( \Pi_c \) as a function of radius.

4. After this, the user is prompted to specify the number of axial sections, \( N_{ax} \), in which to segment the blade for the purpose of spanwise mass and mass-moment approximations. These calculations were incorporated into FECLOPS3D to eliminate any reliance on modal mass value inputs for each mode shape. The reason for this is to circumvent the unreliability of various commercial finite element solvers to output a consistent modal mass value. Therefore, to ultimately produce aerodynamic damping values in CYCLOPS3D, the total aerodynamic work is normalized by a physical kinetic energy according to Equation 5.2.

The spanwise mass calculation is a constant density-weighted integral of the enclosed areas of \( N_{ax} - 1 \) distinct sections approximated as parallelograms with the exception of the first and last sections near the leading and trailing edges, which are approximated as triangles. The parallelograms are defined by a set of \( N_{ax} \) axial coordinates and \( 2N_{ax} \) circumferential coordinates as depicted by Fig. 5.4.

To calculate the mass-moment about the leading edge of each radial section, the integral form of the parallel-axis theorem, given by Eq. 5.21 as

\[
I_{LE} = \sum_{i=1}^{N_{ax}-1} I_{cm,i} + m_i d_{LE,i}^2, \tag{5.21}
\]

is used, where \( m_i \) and \( d_{LE,i} \) are the \( i \)th section’s mass and straight-line distance to the leading edge, respectively. \( I_{LE} \) is the total airfoil mass-moment with respect to rotation about the leading edge and \( I_{cm,i} \) is the \( i \)th section mass-moment about its own center of mass defined as

\[
I_{cm,i} = m_i(\Delta x_i^2 + \Delta y_i^2)/12, \tag{5.22}
\]

since the parallelogram slices are being approximated as flat plates, where \( \Delta x_i \) and \( \Delta y_i \) are the axial and circumferential dimensions of the \( i \)th section. As a last note, all mass and mass-moment calculations are done on a per-unit-span basis as required by CYCLOPS3D.

---

1 There are two sets of circumferential coordinates. One each associated with the minimum and maximum circumferential points at each axial section.
Figure 5.4: Depiction of a 2D airfoil section being discretized into $N_{ax} - 1$ sections for the mass and mass-moment calculations. $N_{ax} = 17$.

5. An output file ("sections.txt") is created that stores the axial and minimum/maximum circumferential coordinates used for the mass/mass-moment calculation, which is generated for troubleshooting purposes only.

6. Lastly, a user-specified number of mode shapes are read in a loop (typically the same mode with different nodal diameters and frequencies). During each loop, the new spanwise leading and trailing edge axial and circumferential displacements are recomputed and appended to the main CYCLOPS3D input file.
6

Summary and Future Work

The primary objective of this work was to improve physical understanding, reveal steady aerodynamic influences, and develop a preliminary design tool for the suppression of low-pressure turbine blades found in turbomachines. With the new discoveries uncovered by this dissertation, key aspects behind the steady aerodynamic loading effect on LPT flutter are much better understood. Today’s researchers are learning that many crucial design parameters have been ignored in the past. Further, this thesis has conclusively demonstrated that the design of flutter-free LPT blades must fully consider mode-dependent loading and shock effects in addition to pervasively used reduced frequency guidelines through the design cycle. This revelation of the complicated, non-monotonic nature of the mode-dependent load sensitivities has demonstrated how significantly shock structures might alter the least stable aerodynamic damping. This chapter concisely summarizes the main conclusions and provides suggestions for future work in the field of LPT flutter.

6.1 Summary of Work

Regarding LPT flutter parameters, it is well known that the reduced frequency and mode shape are critical for safe, flutter-free design. Specifically, raising the reduced frequency tends to stabilize turbine cascades, whereas changing the mode shape can completely modify the flutter boundary as seen in the Tie-Dye plot examples included in this work. Digging deeper into this issue, a thin, computationally efficient quasi-2D grid slice of Standard Configuration 4 was subjected to rigorous RANS CFD analysis across multiple reduced frequencies, rigid body mode shapes, and exit Mach number conditions to validate and highlight these well-established LPT flutter assertions. The topic of increasing physical understanding of where instability-inducing forces arise was then pursued by inspecting the unsteady CFD results from an influ-
ence coefficient perspective. Several helpful conclusions were reached and regarding the unsteady aerodynamic influences of neighboring blades as a function of reduced frequency and mode shape. For instance, at low reduced frequencies, it was shown that the neighboring suction side blade dominates the unsteady aerodynamic forces seen by the reference blade, which was true regardless of mode shape or exit Mach number. This claim by now has been supported by multiple studies and authors; therefore, this result may be generalized and applied to other LPT airfoil shapes. Pitching modes were also shown to have higher harmonic influence coefficient content compared with bending modes at low \( k \)-values explaining the pitching mode’s tendency to exhibit non-sinusoidal damping vs. interblade phase angle curves. Conversely, at higher values of reduced frequency, the neighboring pressure side blade displays a relatively dominant influence on the unsteady aerodynamic forces seen by the reference blade. However, a higher reduced frequency also means higher self-influence stabilization which curtails neighboring unsteady effects. This was shown in Fig. 3.11 where the reference blade (blade 0) \( \omega_{cyc} \) contribution formed a linear relationship with reduced frequency. Further, the linear relationship between reduced frequency and acoustic resonance interblade phase angles was also developed for the quasi-2D cascade in Eqs. 3.17-3.20. These acoustic perturbations were more pronounced at higher reduced frequencies as the unsteady pressures propagated farther away from the reference blade suction side.

This dissertation also explored the effect of steady aerodynamic loading on LPT flutter stability. These Chapter 4 studies, which scrutinized various combinations of loading, reduced frequency, mode shape, airfoil geometry, and IBPA, were segmented into the four categories seen in Table 6.1:

<table>
<thead>
<tr>
<th>Category</th>
<th>Airfoil(s)</th>
<th>Mode Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading: Variable ( k )</td>
<td>SC4</td>
<td>Bend: Flex</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pitch Axis: LE, MC</td>
</tr>
<tr>
<td>Loading: Constant ( k )</td>
<td>SC4, Hannover (3D)</td>
<td>Bending: Flex, Edge</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pitch Axis: LE, MC</td>
</tr>
<tr>
<td>Mode Dependency</td>
<td>SC4</td>
<td>Bend: All Directions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pitch Axis: Chordline</td>
</tr>
<tr>
<td>Geometric Dependency</td>
<td>SC4, KTH, BP2, Flat Plate</td>
<td>Bend: All Directions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pitch Axis: Chordline</td>
</tr>
</tbody>
</table>

In the first study segment, the reduced frequency was not held constant across various loading conditions despite the change in back pressure altering the exit relative velocity. The conclusion of this first study contradicted an existing notion among some industrial turbine manufacturers that increased loading monotonically decreases LPT aeroelastic stability. The mid-chord pitch mode, however, did see an
approximately monotonic stability decrease with load in this variable reduced frequency case. Another interesting result arose where certain regions of loading saw sharp positive and negative stability gradients, especially in the case of the leading edge pitching mode, which foreshadowed subsequent discussions on shock influence later in Chapter 4. Therefore, this initial study proposed that steady aerodynamic loading’s effect on LPT flutter was a strong function of mode shape, and suggested that certain combinations of loading and mode shape inherently have greater stabilization potential.

The constant reduced frequency loading study was necessary to confirm that the stability effects of loading in the previous study were primarily due to the loading changes, and not the well-established reduced frequency parameter adjustments. This new reduced frequency control parameter was enforced constant by proportionally adjusting the unsteady CFD’s physical blade vibration frequency proportionally with steady CFD’s mass-averaged exit relative velocity. The eventual findings verified that the loading indeed may stabilize or destabilize an LPT blade and was a function of the four rigid body mode shapes investigated. Unlike reduced frequency, however, the relationship between LPT flutter and blade loading is non-monotonic. For example, the least stable damping for the $k = 0.3$ mid-chord pitching mode, which closely approximates a spanwise cut of a 3D first-torsion mode, traversed both sides of the neutral aerodynamic damping line several times as the load increased. This unusual discovery actually served to numerically verify some experimental data captured long ago by Cardinale [8]. Therefore, if a jet engine or power turbine blade designer plans to increase blade loading in any way (e.g., redistributing load toward tip, removing stages), this study showed it is crucial to carefully consider the sensitivity of the current or new mode shape to steady loading variations, especially if stage pressure ratios change significantly either due to atmospheric uncertainties, variable stator vane schedules, or various engine operating points.

Furthermore, it was shown that when the back pressure drops below the critical choking pressure ratio, $\Pi_c = p_2/p^* < 1$, a qualitative change occurs effecting how steady loading affects flutter, and it was immediately hypothesized that this was a direct result of a new shock formation when the mass flow goes into and out of choked conditions. To verify this hypothesis, contour maps of the $k = 0.1$ flex mode unsteady pressure amplitudes and phases on the airfoil surface were visualized as a function of critical choking pressure ratio. From this image, the key driver controlling LPT flutter’s mode-dependent loading sensitivity was clearly the passage shock formation, which primarily influenced the suction side of the blade. The unsteady pressures on the blade’s pressure side showed negligible variations with load, thus confining the influence of steady aerodynamic loading to the suction side of the reference blade.

Several other main findings of this study segment are reiterated in the list below for convenience:

1. For quasi-2D Standard Configuration 4, increasing the blade loading stabilized the flex mode and destabilized the edgewise mode for supercritical values of
back pressures, i.e., \( p_2 > p^* = p_0 \left( \frac{2}{\gamma+1} \right)^{\gamma/(\gamma-1)} \).

2. The unsteady pressure characteristics sharply change on the blade’s suction side surface below the critical value of \( \Pi_c = p_2/p^* = 1 \). These bifurcations can abruptly increase or decrease the minimum aerodynamic damping.

3. From an alternative perspective of the LPT airfoil designer, the steady blade loading dictates allowable mode shapes that are aeroelastically stable.

4. By varying the back pressure, the inlet and exit Mach numbers shift which affects the acoustic resonance conditions of the entire cascade. Therefore, the least stable IBPA shifts with loading and the qualitative shape of the damping vs. IBPA curves transform.

The third study segment thoroughly investigated a wide range of mode shapes in an attempt to define which modes benefited or were hurt by loading increases. The Quasi-2D Standard Configuration 4 LPT was retained for this analysis, and the critical reduced frequencies were mapped for various bending and pitching modes using a linear superposition approach of the three fundamental mode shapes. Therefore, all possible translation directions (i.e., \( 0^\circ \leq \theta \leq 180^\circ \)) and all pitching axes along the chordline (i.e., \( 0 \leq a \leq 1 \)) were examined. The variations of \( k_{\text{crit}} \) as a function of pitching mode and load were divided into four distinct qualitatively behaving modal regions which are:

**Pitch Region I**: \( (0.0 \leq a < 0.46) \) → Most stable region. High, non-monotonic load sensitivity and high sensitivity to mode shape. \( k_{\text{crit}} < 0.3 \)

**Pitch Region II**: \( (0.46 \leq a < 0.55) \) → Very low sensitivity to load and very high, linear sensitivity to mode. \( 0.1 < k_{\text{crit}} < 0.6 \)

**Pitch Region III**: \( (0.55 \leq a < 0.85) \) → Least stable region. High sensitivities to both load and mode. Higher loads stabilize. \( k_{\text{crit}} > 0.5 \)

**Pitch Region IV**: \( (0.85 \leq a \leq 1.0) \) → Medium load sensitivity and low, linear sensitivity to mode. \( 0.4 < k_{\text{crit}} < 0.7 \)

The bending modes were also divided into four modal behavior zones, which will be summarized shortly when discussing the next and final loading study segment: geometric dependencies. For this study, three new quasi-2D LPT airfoil test cases were carefully selected such that they exhibited diverse geometric and flow characteristics including thickness, camber, stagger angle, solidity, inlet flow velocity, specific heat ratio, and baseline isentropic Mach number distribution around the airfoil. The goal was to determine which LPT flutter loading influences could be generalized to other airfoil shapes, and which were geometry-dependent. In general, the sensitivity of critical reduced frequency to loading and bending mode variations were classified into four unique modal zones defined in Table 6.2.

Several main points from this last loading study category are now listed:

1. Zone I bending modes (\( \theta = 0^\circ - 30^\circ \)) represent a safe design region with relatively low values of \( k_{\text{crit}} \) regardless of mode shape and loading condition.
Table 6.2: Critical Reduced Frequency Sensitivity Regions for Bending Modes: SC4/KTH

<table>
<thead>
<tr>
<th>Zone: (θ range)</th>
<th>Sensitivity to Increased Load</th>
<th>Sensitivity to Bend Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: θ = 0° − 30°</td>
<td>Non-monotonic</td>
<td>Low: possible design region</td>
</tr>
<tr>
<td>II: θ = 30° − 100°</td>
<td>Stabilizes</td>
<td>High: ≈ linear</td>
</tr>
<tr>
<td>III: θ = 100° − θξ + 10°</td>
<td>Low, ≈ load independent</td>
<td>High gradient surrounds θξ</td>
</tr>
<tr>
<td>IV: θ = θξ + 10° − 180°</td>
<td>Destabilizes</td>
<td>High: ≈ linear</td>
</tr>
</tbody>
</table>

2. For Zone II modes (θ = 30° − 100°), which include the flex and circumferential bending modes, stability is a function of loading as \( k_{crit} \) changes monotonically with pressure ratio. However, loading may act as a stabilizing or destabilizing agent depending on airfoil geometry. For blades like SC4 & KTH featuring thicker fore sections with higher aft turn, loading was shown to monotonically stabilize the cascade. On the other hand, loading destabilized cascades with flat-plate-like blades with relatively thin, straight aft portions like the flat plate and BP2 configurations.

3. Independent of loading or airfoil geometry, the flex mode is more stable than the tangential bending mode. Critical reduced frequency varies linearly between these modes.

4. In bending Zone III (θ = 100° − θξ + 10°), there is a load-independent linear stabilization as the bend angle transitions from tangential to edgewise bending.

5. Zone IV bending modes (θ = θξ + 10° − 180°) generally grow less stable with increased loading and are highly sensitive to changes in bend angle.

6. The critical reduced frequency as a function of bend angle reaches a local maximum at approximately θ = 100°, which corresponds to a translation slightly beyond a pure tangential bending mode. Conversely, \( k_{crit} \) reaches a local minimum near the axial bending mode, but the exact bend angle of this minimum varies with loading and geometry.

7. The relationship between loading, mode shape, and critical reduced frequency shares greater similarities across all airfoils for bending modes as compared to pitching modes.
8. The critical IBPA shifts approximately 180 degrees as the pitching axis moves fore and aft of a single, normal shock.

9. Locating the pitching axis near the shock location for design conditions minimizes destabilizing local work contributions from the shock.

10. The blade portion just aft of the shock contributes a larger integrated quantity to the local work when compared

Several practical design guidelines follow from the conclusions of this work. The first is that, regardless of pitching or bending mode, an LPT blade designer wishing to push into higher loading regimes should avoid specific modal zones where loading monotonically decreases stability for a given cascade. Additionally, modal regions exhibiting high sensitivity, i.e., unpredictability, to critical reduced frequency as a function of mode or load, such as the Zone II pitching modes near mid-chord in Figs. 4.39a & 4.39b, should also be avoided where possible in design. Next, from a practical viewpoint, the engineer should compare the blade’s actual $k$ value to $k_{\text{crit}}$ after calculating the approximate rigid body pitching axis near 80% span, where the spanwise damping contributions are roughly largest. Then, if $k < k_{\text{crit}}$, depending on the modal zone, modify the airfoil geometry to move the mode shape in the direction of decreasing $k_{\text{crit}}$ while maintaining roughly the same steady aerodynamics and shock position along the chordline. From the opposite approach angle, the designer may just as easily maintain the current mode shape while altering the airfoil steady loading distribution to push the shock aft/fore if the modal zone shows loading monotonically increases/decreases stability.

The following provides a practical example of potential ways to apply design decisions based on actual values of reduced frequency, bending mode, and loading condition for a fluttering LPT blade. For the example at hand, we consider the BP2 blade’s least stable first-bending nodal diameter. Refer to Fig. 6.1, which is a zoomed-in version of Fig. 4.40b from Chapter 4 that details the critical reduced frequency as a function of bending angle in the region of Zone II. Upon analyzing the flow conditions and rigid body displacements at 82.5% span, which is approximately the radius of maximum aerodynamic work contribution, the actual reduced frequency and bending mode shape of the blade were determined to be $k = 0.19$ and $\theta = 84^\circ$ as indicated on Fig 6.1 by the horizontal dashed-red and vertical solid-green lines, respectively. The loading condition at this spanwise location is $\Pi_c = 0.525$. Linearly interpolating the critical reduced frequency value between $\Pi_c = 0.48$ and $\Pi_c = 0.55$, a designer would determine $k_{\text{crit}} = 0.242$ and that this blade will flutter because $k = 0.19 < k_{\text{crit}} = 0.242$. A redesign is necessary and there are multiple different options the engineer may pursue. They are:

- **Design Option 1**: Change loading; maintain mode & $k$: It is clear from Fig. 6.1 that the lowest loading conditions (highest values of $\Pi_c$) are less prone to flutter. However, changing the loading to these supercritical values
Figure 6.1: Critical reduced frequency vs. bend angle for the BP2 blade. Actual design conditions at 82.5% span: $\Pi_c = 0.525$, $k = 0.19$, $\theta = 84^\circ$. Interpolation between $\Pi_c = 0.48$ and $\Pi_c = 0.55$ yields $k_{\text{crit}} = 0.242$.

of $\Pi_c$ is most likely out of the question for this design ($\Pi_c = 0.525$) in terms of pure engine performance. An alternative option is to take advantage of the non-monotonic relationship between loading and $k_{\text{crit}}$ that the BP2 blade endures near its design condition. On one hand, it would be a poor decision to slightly decrease the loading as values in the range $\Pi_c = 0.62 - 0.76$ possess higher critical reduced frequency values. However, by increasing the loading slightly to $\Pi_c = 0.48$, which corresponds to $k_{\text{crit}} = 0.20$, the value of critical reduced frequency quickly approaches the actual $k$ value and this design change would result in an approximately neutral flutter stability at this particular combination of mode ($\theta = 84^\circ$) and reduced frequency ($k = 0.19$).

• **Design Option 2: Change mode shape; maintain load & $k$:** This original bending mode shape design is approximately purely tangential (or circumferential); therefore, in this example, there are two equally beneficial ways to modify the bending mode shape to increase flutter stability. First, the designer may decide to reshape the airfoil or blade stacking to move the actual mode shape closer to the flex direction (indicated by the teal dashed line at $\theta = 54^\circ$).
Regardless of loading, the trends in Fig. 6.1 exhibit a well-defined monotonic reduction in critical reduced frequency as the bend angle decreases from tangential bending to flex bending. For this specific design example which has loading of $\Pi_c = 0.525$, linear interpolation states once the bend angle reaches or drops below $\theta \leq 64^\circ$, then $k_{\text{crit}} \leq k_{\text{actual}}$. Of course, the steady aerodynamics must not significantly change during the new redesign for this to hold true. Alternatively, the designer may just as easily take advantage of the sharp drop in critical reduced frequency in Zone III by moving the bend angle direction toward $\theta = 110^\circ - 120^\circ$. In other words, even if the steady aerodynamic loading and actual reduced frequency remain unchanged, stability can be achieved by moving the bend angle roughly $20^\circ$ in either direction for this specific design.

- **Design Option 3: Increase $k$; maintain mode & load**: This design option is the equivalent of raising the red-dashed line in Fig. 6.1. Since $k$ is comprised of 3 parameters ($k = \omega b/V^2$), the reduced frequency can be increased by either increasing the modal frequency of vibration, lengthening the blade chord, or decreasing the relative exit velocity. Again, these changes must be completed without significantly changing the steady aerodynamic loading or mode shape.

- **Design Option 4: Simultaneously change $k$, mode, & load**: In reality, this will be the option that most designers must implement. Sometimes it may be impossible to independently change one of these parameters without effecting the others. However, by intelligently leveraging the unique qualitative behaviors identified in each modal region defined in Chapter 4, an optimal design-specific decision of how much to increase $k$, shift the mode shape, or vary $\Pi_c$ can be deduced.

Lastly, this dissertation introduced a preliminary design tool package consisting of two Fortran subroutines: CYCLOPS3D and FECLOPS3D. An instruction manual is also provided in Appendix A. The ultimate purpose of this tool was introduce a way for engineers in the preliminary design stage of turbine blades to rapidly assess how hypothetical changes in airfoil geometry, stacking, frequency, mode shape, or even loading might impact overall cascade stability.

The main theory leverages the fact that LPT blade aerodynamics are similar across many different geometries by using preexisting “Baseline Aero” matrices that are composed of a multitude of complex work matrices derived from unsteady CFD of the three fundamental two-dimensional rigid body modes at various reduced frequencies and interblade phase angles. After converting spanwise LE/TE cyclic symmetric rigid body displacement data into fundamental mode components, CYCLOPS3D continues its analysis by interpolating on $k/\sigma$ combinations from a set of Baseline Aero files, integrating/scaling spanwise work quantities to obtain total work, and then normalizing by a kinetic energy factor to obtain aerodynamic damping. Finally, an example of the CYCLOPS3D damping vs. IBPA results were compared.
directly with the actual CFD results\textsuperscript{1} with a remarkable prediction accuracy. Not only were the most and least stable IBPAs identified for each mode shape, but the actual level of damping magnitude was nearly spot on throughout the entire traveling wave domain.

6.2 Future Work Recommendations

This section provides recommendations for topics of study still in need of future investigation within the field of low-pressure turbine flutter analysis.

Future research of unsteady parameters should continue deciphering the underlying physics behind LPT flutter. Mathematically, we know flutter comes from “bad” phase. But what physically causes the unsteady pressure phase to align with the modal velocity? Future studies should investigate how the phase of the influence coefficients change as a function of reduced frequency and other parameters while aiming to identify novel correlations with unsteady pressure phase. In addition, it is proposed to investigate three-dimensional mode shapes to form generalities and a widespread understanding of unsteady parametric effects on LPT flutter.

This study has presented evidence supporting the hypothesis that the manner in which loading affects flutter margin is a function of mode shape. There are three major follow-up steps needed in the immediate future: 1) Generalization across all blade/cascade geometries and 3D mode shapes 2) Developing a novel monotonic design parameter for how loading/shocks influence stability 3) Preliminary design tools.

Several comments related to future work ideas within these categories are now listed in addition to a few other specific research topics deserving attention in the field of LPT flutter.

**Geometric Studies and Three-Dimensional Generalizations:**

1. It would be interesting to conclusively determine how various cascade solidity values influence the effect of blade loading on LPT flutter margins. A sound hypothesis is that high-solidity cascades are more easily influenced by shock passages as a function of interblade phase angle. A deeper understanding of how the variation of the passage shape and throat location change as a function of mode shape and interblade phase angle for specific airfoil shapes (and solidity levels) is needed. The shock strength, relative position of the shock to the pitching axis, and the variation in passage throat as a function of mode and IBPA all play a crucial role in solving the underlying physics. Figure 6.2 shows how the ratio of the max-to-min throat distance normalized by the mean throat distance varies as a function of mode shape and IBPA for

\textsuperscript{1} The industrial frequency domain CFD code used the actual (elastic body) three-dimensional mode shapes
Standard Configuration 4 modes\(^2\). For the bending modes of Fig. 6.2a, the largest variation in throat distance comes at \(\theta = 100^\circ\), which is a translation nearly perpendicular to the chordline when the adjacent airfoils are moving in anti-phase. This result is expected. However, for the pitching modes in Fig. 6.2b, it is more complicated. The largest variations from the mean throat distance (located near 67\% chord on the suction side of STCF 4) occur at higher values of IBPA and pitching axes between the leading edge and quarter-chord. Once the pitching axis moves aft of steady throat location, there is little change in mean throat for all IBPAs.

![Figure 6.2](image)

**Figure 6.2:** Visualization of peak-to-peak variation of minimum throat distance normalized by mean throat distance as a function of IBPA and mode shape.

2. Future investigations should focus on higher loadings in the 3D investigation to fully capture the development and evolution of a three-dimensional shock structure and its overall influence on cascade stability.

3. Documenting the effect of tip clearance on free-standing 3D geometries should be addressed in future research. The tip leakage vortices unload portions of the blade near the outer diameter. The induced velocity creates a disturbance in the loading and shock structure, which may significantly alter the aeroelastic stability of the blade for some mode shapes. Additionally, the aeroelastic consequences of the flow through the seal cavity at the tip for tip-shrouded blades should be studied.

**Design Tools:**

1. A priority should be the determination of a three-dimensional mode shape parameter which would be analogous to the 2D pitching axis sensitivity seen

---

\(^2\) In Fig. 6.2a, \(\theta = 0^\circ\) and \(\theta = 90^\circ\) correspond to the edgewise and flex bending modes, respectively.
in the Tie-Dye plots. For the 2D pitching axis, it is trivial to apply calculus to show the precise gradient vector in which to optimally relocate the pitching axis to increase flutter stability. It is proposed to develop analogous 3D coefficients for real mode shapes with the purpose of revealing the optimal way to modify the 3D mode shape to increase stability.

2. It is recommended to construct an optimization chain that simultaneously iterates airfoil geometry under certain geometric and steady loading constraints to optimize unsteady pressure distributions such that maximum aerelastic stability achieved as a function of mode shape. Sample geometric and flow constraints include blade chord, exit metal angle, inlet/exit velocity, and or inlet/exit flow angle. Using numerical schemes that implement Lagrangian multipliers and constrained optimization of an analytical source/sink airfoil model may provide sound starting points. Airfoil pressure/suction side parameters like max camber may be defined using cubic polynomials to control inlet/exit metal angle [3], or by simply use NACA-like constraints. Alternatively, a Genetic Algorithm scheme may be used for optimization.

3. Another idea is to create Tie-Dye plots where a novel, monotonic, critical loading parameter is mapped as a function of rigid body mode shape for a specific reduced frequency value (instead of mapping critical reduced frequency at a specific loading value).

4. A useful design tool would be to combine the ideas of Cardinale [8] and Micalef [44] by generating an interpolated flutter map that is superimposed on a loading vs. engine RPM diagram. This would enable identification of regions equivalent to the “flutter bite” in fans and compressors.

**Other Topics:**

1. Coupled Mode Flutter: Coupled mode flutter analyses should be conducted to determine how this loading phenomenon influences airfoils exhibiting characteristics that are susceptible to mode-coupling such as low solidity, low mass ratio, or a small frequency margin between neighboring eigenfrequencies.

2. Turbulence Models: This dissertation used only the Spalart-Allmaras turbulence model for its CFD results. Therefore, a study should be conducted verifying the effect of using different turbulence models is negligible.

3. Mistuning: A simple mistuning analysis for various levels of steady aero dynamic loading is recommended to determine if mistuning can benefit certain shock conditions more than others. Furthermore, an amendment to CY-CLOPS3D should be included that incorporates an automatic calculation of a user-specified level of cascade mistuning.
4. Aeroacoustics: Future research regarding LPT aeroacoustics should attempt to predict three-dimensional acoustic resonance conditions. A 3D computational or analytical model incorporating swirl and three-dimensional variations of flow field characteristics like the blade chord, flow velocity, and Mach number should be constructed. Furthermore, a small research effort should focus on determining if there are prominent effects from the cut-on upstream and cut-off downstream waves for Low-Pressure Turbines in a similar mechanism as the one identified for wide-chord fans by Vahdati[67].

5. Multi-Row Effects: One potential research effort would be to model the effects of a turbine rear frame or the upstream stator vane shock-impingement location. Additionally, an exhaustive multi-stage study should resolve the significance of neighboring rows altering the shock structure and unsteady pressure phase dynamics seen by a reference rotor blade.

6. Nonlinearity: Although many different research efforts have shown that low-pressure turbine CFD is highly linear within reasonable limits of vibration amplitude, it is still recommended to conduct a nonlinear study for different mode shapes and cascade geometries. The hypothesis to test here is that some mode shapes will feature higher sensitivities to nonlinearity for certain cascade geometries.
Appendix A

Instruction Manual for
FECLOPS3D/CYCLOPS3D

This Appendix provides a detailed instruction manual in how to generate Baseline Aero files, and how to run CYCLOPS3D and its pre-processor code, FECLOPS3D, which is first described.

A.1 FECLOPS3D

FECLOPS3D is the pre-processor to CYCLOPS3D which automatically formats the primary CYCLOPS3D input file given blade geometry and mode shape inputs, which typically have UIF and UOF file extensions if they originate from ANSYS software, respectively. For a user-defined number of spanwise sections, Feclops3D calculates the mass, mass-moment, LE/TE positions/deflections, and Mach number/pressure ratio interpolation markers. With these spanwise quantities and interpolation points, Cyclops3D integrates spanwise work contributions by computing the 2D rigid body pitching axes and interpolating (pressure ratio scaled) work values from the Baseline Aero files. The main inputs listed first, followed by a discussion of running the executable, and lastly the main outputs are discussed.

A.1.1 Input 1: Control File

The first input in this analysis chain is a simple control text file to be named “control.txt”. Figure A.1 contains a sample control file as well as a line-by-line description of the entries.

Note that it may be necessary to negate one or more of the first three control file lines depending on the UIF/UOF geometry sign and numbering convention. All coordinates should be increasing in the following directions:
Figure A.1: Sample FECLOPS3D control.txt file with line-by-line description below.

- Axial: Positive Engine Axis (fore looking aft)
- Circumferential: Blade PS to SS
- Radial: Hub to Tip

Note that Lines 4-5 indicate the minimum and maximum radial dimensions to integrate on the 3D blade, respectively. It is recommended to “cut” off a small portion of the hub and tip to avoid spurious section data as airfoil geometric data is not typically as refined here. The impact should for cutting off the hub section will be negligible as the low displacement values here lead to small values of spanwise work contributions; however, the user should carefully check the sensitivity in the damping vs. nodal diameter results for small changes in Line 5 (max tip radius to investigate).

Some UIF and UOF files, like the image in Fig. A.2, have been known to “flip”
the circumferential coordinate, for instance, such that increasing circumferential co-
ordinates go from the SS to PS. In that case, we would account for this in Line 2 of the control file with a negative sign in front of the appropriate column number to “alert” FECLOPS3D.

**Figure A.2:** Image of undeformed airfoil section (UIF) superimposed with its displaced cosine and sine mode shapes (UOF) that all have a “flipped” positive circumferential sign convention.

Most line number descriptions in Fig. A.1 are self-explanatory with the exception of Line 8, which is the offset number separating the node identifiers between the primary and duplicate sectors in the UIF (grid file) data. Figure A.3 diagrams out exactly how to calculate this Line 8 input.

### A.1.2 Input 2: Characteristic File

A sample characteristic file is shown in Figure A.4. This file contains spanwise flow data which is interpolated on to assign values of relative exit velocity, relative exit Mach number, and pressure ratio as a function of radius.

The text file should contain no more than 100 rows of the four columns which are defined as:

- Column 1: Radial dimension (inches) - starting with hub and increasing toward tip. Note: the first and last value should be lower and higher than control.txt Lines 4 and 5, respectively, to avoid extrapolation.
- Column 2: Relative exit velocity (feet/second)
Figure A.3: Diagram explaining how to calculate the Line 8 input of the FE-CLOPS3D control.txt file.

- Column 3: *Relative* exit Mach number
- Column 4: Critical pressure ratio, $\Pi_c = \frac{p_2}{p^*}$, where $p_2$ is the exit static pressure and $p^*$ is critical choking pressure defined by the isentropic relation

$$p^* = p_{0,1} \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}}, \quad \text{(A.1)}$$

where $p_{0,1}$ is the inlet total pressure and $\gamma = C_p/C_v$ is the specific heat ratio.

A.1.3 Input 3: Grid and Mode Files

The UIF (grid) and UOF (mode shape) inputs are formatted text files automatically output from ANSYS software. The UIF file contains the blade’s hot or cold shape...
A.1.4 Running the FECLOPS3D Executable

There are four easy steps to run the FECLOPS3D executable:

1. Ensure all listed FECLOPS3D input files are properly formatted, named, and located within the executable directory. Then, run feclopsV34.exe.

2. In the command window that pops up, enter the desired CYCLOPS3D input file name. Internal calculations now begin.

3. The executable will pause to prompt the user to enter the desired number of axial sections to use in the airfoil mass and mass-moment calculations. It is recommended to start somewhere between 25-40 axial sections. A higher number of axial sections means greater approximation accuracy. Of course, there is an exception to this rule if the refinement is too high (a limit near 50 is common) as nonsensical mass calculations (e.g., negative mass) will result due to UIF node refinement issues.
Figure A.5: Sample beginnings of a UIF (grid data) and UOF (eigenvector data) file, which are inputs to FECLOPS3D.

4. FECLOPS3D will calculate and integrate the spanwise mass and output this to the command window before once again pausing. The user will check if the total airfoil mass quantity is reasonable. Enter 1 to proceed with final program execution, or enter 0 to re-do mass/mass-moment calculations with a different number of axial sections.

A.1.5 Output 1: Summary File

The summary output file is a text file named “feclops3d.out”. Examples of data found in the summary output file (that may be used for troubleshooting purposes) include:

- Spanwise mass/mass-moment values
- LE/TE locations, Cosine/Sine mode LE/TE deflections, & pitching axes out to span
- Interpolated values of exit relative velocity, Mach number and critical pressure ratio
- Section data: chord, stagger, reduced frequency/velocity

It is recommended to plot the spanwise axial/circumferential LE/TE coordinates to visualize the smoothness of the blade outline that FECLOPS3D calculated in addition to the LE/TE displacements to ensure a smooth transition from hub to tip.
The user may have to increase or decrease the number of spanwise sections specified in the “control.txt” file to obtain desired refinement/smoothness. Figure A.6 shows a small portion of the feclops3d.out output and how it might be plotted to check the spanwise continuity of displacements in both the axial and circumferential directions. This also gives you a sense from which mode contributes to a higher percentage of the total kinetic energy, and where the largest work contributions originate (roughly 70% span in this example with the sine mode dominating).

A.1.6 Output 2: Sections File

The “sections.txt” output file lists the axial and minimum/maximum circumferential coordinates that FECLOPS3D locates from the UIF data to use for its internal spanwise mass and mass-moment calculations. Note that FECLOPS3D uses a hard-wired blade density value of 0.3 lbm/in$^3$; however, the user may easily change this value to the actual test blade’s density in the source file if desired. This output file serves as a troubleshooting tool when performing sanity checks of the mass and mass-moment calculations. An example of the output from one spanwise section for 40 axial sections is shown in Fig. A.7. Note that the leading and trailing edge points are not shown because these locations are assumed to be triangles for the mass calculations as opposed to all other sections approximated as parallelograms.

A.1.7 Output 3: Main CYCLOPS Input File

This output file is the main input file to CYCLOPS3D, and should be relocated to the CYCLOPS3D executable’s working directory. For a description, refer to Appendix A.2.1.
Figure A.6: Sample portion of feclops3d.out file output and a corresponding check of spanwise modal smoothness. User-specified number of spanwise sections = 20.
Figure A.7: Sample beginning of a sections.txt output file listing the axial and min/max circumferential coordinates FECLOPS3D finds to use calculate and integrate mass and mass-moment. One spanwise section with 40 axial sections shown.
A.2 CYCLOPS3D

Cyclops3D converts the LE/TE cyclic symmetric rigid body displacement data into fundamental mode shape components, interpolates between reduced frequency and IBPA, computes spanwise work, and then integrates/scales these interpolated quantities to obtain total work. Finally, this value is normalized by a kinetic energy factor yielding aerodynamic damping vs. IBPA. The main inputs and outputs are described as well as a step by step guide on running the executable.

A.2.1 Input 1: Main Input File

The main input file to CYCLOPS3D (the main output file from FECLOPS3D) passes on geometric information such as the spanwise leading/trailing edge coordinates and displacements for all mode shapes and nodal diameters and section mass/mass-moments, as well as spanwise flow information like the exit Mach number and loading condition which are used to interpolate between and scale the Baseline Aero file selections listed near the top of the input file. For convenience, a sample image of the beginning of the main input file to CYCLOPS3D is shown in Fig. A.8.

A description of the first 20 rows of the main CYCLOPS3D input file is now given.

- Line 1: Desired name of Cyclops3D output file
- Lines 2-7: File names of 6 Baseline Aero files. Order must be monotonically increasing (i.e., from lowest to highest exit Mach number)
- Line 8: Number of blades in the cascade
- Line 9: Number of nodal diameter cases to read
- Line 10: Nodal Diameter corresponding section data immediately following (in this example, ND=2)
- Line 11: Number of spanwise sections to read for each nodal diameter (loops through this many sections before proceeding to next nodal diameter)
- Line 12: Min Radius, Max Radius (inches)
- Line 13: Vibration frequency of this nodal diameter (Hz)
- Line 14: Mode Shape Scale Factor (usually set to 1.0)
- Line 15: Section-specific radius value (inch), mass/unit span (lbm/in), mass-moment about the leading edge/span (lbm-in²/in)
- Line 16: Section-specific relative exit velocity (ft/s), exit relative Mach number, and Π₁, as defined in Eq. 4.5
Figure A.8: Sample format for the beginning of the primary CYCLOPS3D input file, which is the output from FECLOPS3D.

- Line 17: LE axial coordinate, LE tangential coordinate, TE axial coordinate, TE circumferential coordinate
- Line 18: Cosine Mode Only: LE axial displacement, LE tangential displacement, TE axial displacement, TE tangential displacement
- Line 19: LE axial coordinate, LE tangential coordinate, TE axial coordinate, TE circumferential coordinate
- Line 20: Sine Mode Only: LE axial displacement, LE tangential displacement, TE axial displacement, TE tangential displacement
This pattern would repeat for the remaining sections 4-20 not shown in Fig. A.8, and then according to Line 9, repeats for the two remaining nodal diameter cases also not shown in Fig. A.8, starting fresh at Line 10 for each nodal new nodal diameter.

A.2.2 Input 2: Baseline Aero Files

The Baseline Aero file is a collection of the fundamental complex work matrices (see Eq. 5.1) for various combinations of reduced frequency and interblade phase angle at a given loading condition for a given airfoil geometry. A three-dimensional turbine blade has variable mode shape, chord length, exit relative flow velocity, and loading out to span, which means the rigid body pitching axis, reduced frequency, and loading will change at each radial cut. However, CYCLOPS3D easily interpolates between various Baseline Aero files and integrates spanwise contributions to work-per-cycle after calculating rigid body displacement vectors \([h_\xi, h_\eta, \alpha]\) and applying Eq. 4.8 at every user-defined radial cut.

For convenience and formatting reference, the start of a sample Baseline Aero file format is shown in Fig. A.9.

The description of the first 16 rows of the Baseline Aero file is now given.

- Line 1: Reference name of Baseline Aero file
- Line 2: Chord length of baseline airfoil (inches)
- Line 3: Mass/span (lbm/in) of baseline airfoil
- Line 4: Mass-moment/span (lbm-in^2/in) about the leading edge of the baseline airfoil.
- Line 5: Relative exit Mach number
- Line 6: Critical pressure ratio, Πc, as defined in Eq. 4.5
- Line 7: Number of reduced frequency values, number of interblade phase angles in file
- Line 8: Corresponding reduced frequency, IBPA for following 3 lines of complex work values
- Line 13-15: Real and Imaginary Work Values for the 3x3 fundamental Work matrix of Eq. 5.1 (lbf-in/in)
- Line 16: Same k, next value of IBPA

After the first fundamental work matrix is listed on lines 13-15, all other k/σ combinations are listed directly after their corresponding header line which lists the corresponding values of reduced frequency and interblade phase angle, respectively. The text file format loops through all 24 interblade phase angle values before looping to the next value of k. That is, thinking of it in terms of nested for-loops, the IBPA for-loop is embedded within the k for-loop.

The CFD analysis chain and procedure in which to generate the Baseline Aero files is now described.

It is highly recommended to automate the below Baseline Aero file generation procedure using bash scripts or another form of macro automation software. This is extremely important for two reasons. First, there are roughly 576 unsteady CFD runs per Baseline Aero file if you do not include the 0-iteration dot products to obtain the six off-diagonal work terms per k/σ combination. Two, the user theoretically will create numerous Baseline Aero files for various loading conditions and this equates to a large number of repetitive tasks. Not only will automation expedite Baseline Aero file generation, but it eliminates chances for human error on the input and post-processing side.

Here are the steps:

1. Create a quasi-2D CFD grid with thickness Δz, which will be later used to normalize the fundamental work matrix in terms of unit span.

2. Generate and save the three fundamental rigid body mode shapes which are:
   - ξ: 1-chord translation parallel to the chordline (positive in direction of LE to TE)
   - η: 1-chord translation perpendicular to chordline (positive in direction of PS to SS)

\footnote{CYCLOPS3D allows up to six different Baseline Aero files for interpolation purposes.}
• \(\alpha\): 1-radian rotation about the leading edge (positive with TE initially rotating toward SS)

3. For your desired loading condition (i.e., critical pressure ratio, \(\Pi_c\), or exit Mach number), run, converge, and save the steady CFD solution.

4. After the steady solution is converged, it is time to set-up the unsteady CFD runs, which serve to create the fundamental work matrices for combinations of 8 different reduced frequencies and 24 interblade phase angles. The user should note that these number of \(k/\sigma\) values were selected because most Baseline Aero files generated thus far use 8 \(k\)-values ranging from 0.0-0.7 in \(k = 0.1\) increments and 24 \(\sigma\)-values ranging from \(-165^\circ\) to \(180^\circ\) in \(15^\circ\) increments. However, in the current version of CYCLOPS v3.4, the user may specify up to 12 reduced frequencies and 32 IBPAs for their if that level of refinement is necessary.

5. To solve for the blade vibration frequency to enforce at each value of \(k\), which uses the semi-chord definition, use Equation A.2 below:

\[
f = \frac{kV_2}{\pi c}.
\]  
(A.2)

Here, \(f\) is the vibration frequency (in Hz) corresponding to the semi-chord reduced frequency, \(k\), \(c\) is the full chord length (in feet), and \(V_2\) is the relative exit velocity (in feet/second).

6. For all 8 \(k\)-values, run, converge, and save unsteady CFD for all combination of the 24 interblade phase angles and 3 fundamental mode shapes. It is recommended to have a constant automatic convergence criteria set for these cases, such as some user-defined level of residual error or a maximum number of iterations once the generalized force is known to have settled down.

7. Check unsteady CFD convergence. When evaluating all 576 CFD solutions, there may be convergence issues at certain IBPA values. While there are many reasons for non-convergence, such as a poor steady state solution, more often than not the primary culprit is acoustic resonance. It is recommended to create an automated convergence-checking criteria which notifies the user of problematic \(k/\sigma\) combinations so the issue can either be manually or automatically be resolved. Some troubleshooting methods that have worked in the past include: a) Slightly changing the IBPA by \(\pm 0.25^\circ - 1.5^\circ\) to mitigate the acoustic resonance effect or b) reducing the CFL number starting with the steady solution.

8. Calculate and save the other 6 off-diagonal fundamental work matrix work-values for each \(k/\sigma\) combination: These terms include: \(W_{\xi\eta}, W_{\xi\alpha}, W_{\eta\xi}, W_{\eta\alpha}, W_{\alpha\xi}, W_{a\eta}\). This can be automated two ways. Consider, for example, the off-diagonal term \(W_{\xi\eta}\). The first automation method is to have the unsteady
CFD code read in the $xi$-mode shape and the $\eta$ unsteady pressures, and have the code restart to run zero iterations such that it only calculates the work-per-cycle. In MUSTANG, the generalized force is computed and converted to work-per-cycle using the following conversion equations:

$$GF = A + iB,$$  \hspace{1cm} (A.3)

where $A$ and $B$ are the real and imaginary parts of the generalized force, respectively, and

$$W_{\text{cyc}} = -i\pi GF = \pi(B - iA).$$  \hspace{1cm} (A.4)

The other automation method is simply outsourcing this surface integral of the dot product between the $\xi$ eigenvector and $\eta$ unsteady pressures to a program like MATLAB or similar. Either way, the six off-diagonal complex work terms must be calculated and saved for all $k/\sigma$ combinations.

9. Normalize all saved complex work values by the quasi-2D grid thickness, $\Delta z$, such that all work values are in per unit span (inch) basis. Finally, convert the work/span dimensions to units of lbf-inch/inch.

10. Format the Baseline Aero text file according to the description above.

\textbf{A.2.3 Running the CYCLOPS3D Executable}

Once all input files have been created and are properly formatted, the CYCLOPS3D executable will generate entire damping vs. nodal diameter curves for a three-dimensional blade within a couple seconds. The program itself is easy to use when following these steps:

1. Ensure the updated primary CYCLOPS3D input file and baseline aero files are located within the CYCLOPS3D executable’s working directory.

2. \textit{Optional:} change desired output file name or baseline aero files at the top of the main input file.

3. Run cyclopsV34.exe

4. In the command prompt, select verbose (y) if troubleshooting, or non-verbose output (n).

5. Enter CYCLOPS3D input file name. The code then echoes back the number of text lines it reads, and asks the user if he or she is certain the input file name is correct. Enter ‘y’ to continue code execution, or ‘n’ to reenter the input file name.

6. Enter ‘s’ to exit the command window.
7. Evaluate results in output file and damping_vs_nd.txt.

Note: If running CYCLOPS3D with 1 nodal diameter in the input file, the code will automatically prompt the user if they wish to run a full IBPA sweep at constant frequency.

A.2.4 Output 1: Summary File

The summary output text file has the user-defined name located at the top of the primary input file. Examples of data found in a non-verbose summary output file include:

- The input file echoed
- Which Baseline Aero files are used for interpolation at each spanwise section
- Interpolated values of relative exit velocity, relative exit Mach, and critical pressure ratio.
- Spanwise distribution of work/span (forward/backward traveling wave), kinetic energy, damping/span (FTW/BTW)
- Radially integrated work and total damping vs. nodal diameter

A verbose output file produces much more text for the user to sift through for troubleshooting purposes. Some extra data points that are found in verbose output files include:

- Spanwise reduced frequency
- Chord scale factor between baseline aero and test blade
- Kinetic energy split between cosine and sine mode
- Pitching axis location for each section/mode and values of \([h_\xi, h_\eta, \alpha]\)
- Breakdown of the 2x2 cyclic symmetry work matrix for each spanwise section
- Sensitivity to changes in pitching axis including max gradient direction and level

A.2.5 Output 2: Damping vs. Nodal Diameter

This file simply lists all aerodynamic damping vs. nodal diameter data in a convenient format for quick result interpretation. A sample image of a case being evaluated over three nodal diameters (including both forward and backward traveling waves) is shown in Fig. A.10. The damping values listed are commonly referred to as \(\zeta\)-values, or critical damping ratios which are dimensionless. Multiply these values by 100 to obtain percent-damping.
Figure A.10: Sample damping_vs_nd.txt file.
Bibliography


151

Biography

Joshua Joseph Waite was born in 1987 in Virginia Beach, Virginia. Together with his mother, father, and sister, Joshua moved to Maine at an early age where he attended Lawrence High School and competed nationally on the math team, and regionally for golf and drama. By 2010, Joshua graduated magna cum laude from the University of Maine earning a B.S. in Mechanical Engineering. Desiring a higher education, Joshua finally left Maine for the warmer climates of North Carolina to pursue a Ph.D. at Duke University. His Duke education began studying nonlinear dynamics under adviser Dr. Lawrence Virgin and mentors Dr. Ted Lyman and Dr. Richard Wiebe. In 2013, Joshua received his M.S. degree for his experimental and numerical investigations of a single-degree-of-freedom snap-through buckling dynamical system. Shortly thereafter, he was the recipient of the 2013 National Defense Science and Engineering Graduate (NDSEG) fellowship. This was a major turning point in Joshua’s educational path, as he switched focus entirely to research turbo-machinery aeroelasticity under his adviser Dr. Robert Kielb and mentor Dr. Stephen Clark.

A list of Joshua’s publication now follows:


