Soft-Targets and Incentive Compensation in Non-Profit Organizations

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Abstract

Monetary targets are highly prevalent in fundraising campaigns. Although some theoretical research has been conducted to explain why fundraising organizations set such targets when charities are raised to fulfill certain capital requirements, there has been no literature that can suitably answer why a target is still announced when such capital requirements are not present. On the other hand, empirical studies have shown that performance-based incentive compensation has become more and more prevalent in the nonprofit sector. Based on the empirical observations, the author theorizes that fundraising organizations implement incentive compensation that is dependent on whether a soft target is reached, in order to motivate the fundraising staff to exert more effort in reaching out to potential donors. This paper presents a theoretical model using a game theory framework to account for the existence of “soft targets” in the fundraising industry.
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1 Introduction

Economic literature on private contribution toward public goods and charities has flourished in the past twenty years. The research, however, has mostly focused on the behavior of the supply side, i.e., the donors. Recently, some economists have made attempts to explore the role of the fundraisers, or the demand side. Andreoni, for instance, proposes a model that captures how fundraisers obtain “seed money” either from a government grant or a group of “leadership givers” prior to launching a full-scale fundraising campaign. He further theorizes that the public goods must meet a minimum quality threshold before yielding any services or practical value (Andreoni 1998). Because of this, Andreoni’s model can suitably explain why many fundraisers would announce their fundraising targets when charities are raised toward projects with certain capital requirements, e.g., the construction of a school. In these cases, there are direct consequences if the targets are not reached—the school cannot be constructed, and thus the charity yields no practical social value.

However, there are also many fundraising campaigns where no repercussions would substantiate, even if the targets are not reached. Consider, for example, Duke University’s Financial Aid Initiative, which aimed to raise $300 million toward the University’s financial aid endowment. The target was announced to the public in 2005 and was eventually reached in November 2008. In retrospect, it appears that there
would not be much punishment, if the University failed to achieve the announced target—it certainly would not have refunded the donors, and would still have used however much was raised toward the financial aid program. It is bewildering as to why the fundraisers would announce these “soft targets,” or targets without clear-cut underlying capital requirements to fulfill, to the general public. To the author’s knowledge, there is no literature that explains this phenomenon, and this paper attempts to fill in the gap.

It is interesting to note that while the general perception of Non-Profit Organizations is that they are institutions that depend heavily on unpaid volunteers, the reality is far different. A recent survey reveals that most Non-Profit Organizations maintain a firm-like structure, with Board of Directors or Trustees, Executive Directors, Chief Operating Officer, Chief Development Officer, fundraising professionals, and so forth (TCS 2008). Figure 1 illustrates the structure of a typical Non-Profit Organization. Moreover, studies have shown that the nonprofit sector has become a major economic force throughout the world. In Maryland, for example, the nonprofit sector accounts for one out of every ten paid workers in 2006, and the 244,086 nonprofit employees earned $10.6 billion in wages and compensation that same year (Salamon & Geller 2008). Similarly, nonprofit is the fourth largest sector in terms of employment in Florida, and the 630,000 employees earned over $14.6 billion in wages and compensation in 2005 (Salamon et al. 2008).
But even more interesting is that more and more fundraising organizations are implementing performance-based compensation. We should point out that such practices are perfectly legal. The Association of Fundraising Professionals (AFP) code of ethics clearly states that members of Non-Profit Organizations are permitted to “accept performance-based compensation, such as bonuses, provided such bonuses are in accord with prevailing practices within the members’ own organizations” (AFP 2007). An interview with Paulette V. Maehara, President and CEO of AFP, in 2004, reveals that
“incentive compensation is a growing trend.” According to the AFP Compensation and Benefits Survey completed in 2003, “19 percent of U.S. members surveyed and 15 percent of Canadian members surveyed received some type of incentive compensation.” Maehara further explained that organizations use this type of compensation not only to retain fundraising professionals, but also as a way to “motivate and reward staff to achieve large goals, such as a billion-dollar campaign” (Watson 2004). Matthew Beem, executive vice president of Hartsook and Associates, made a similar observation, “Until 1960s, virtually all fundraisers were paid solely for service,” but by 1991, “fundraisers were beginning to anticipate receiving both ‘non-cash’ and ‘incentive-based’ rewards for their work” (VanTil 2001).

Based on the empirical evidence, the author, in this paper, theorizes that fundraising organizations use soft targets internally as an instrument to motivate employees to exert more effort in reaching out to potential donors. More specifically, the management designs compensation plans based on whether the soft target is achieved by the end of the fundraising campaign. The target is also made known to the public to signal to the donors as to how hardworking the employees might be, thus influencing the decisions of the donors.

The rest of the paper proceeds as follows. Section 2 reviews relevant literature on Non-Profit Organizations as well as fundraising activities. Section 3 presents a theoretical model that characterizes the strategic interactions amongst the fundraising
manager, employee, and the donors, by explicitly introducing the targeted amount of
cost as a variable. Section 4 then provides a detailed analysis of the model assuming
there are two identical donors. The final section concludes with results and avenues
for future research.

2 Literature Review

Much work has been done in private charity over the past twenty years. Warr, as early
as 1982, explored the Pareto optimal redistribution in private charity. He recognizes
the interdependence in the donors’ utility functions; i.e., either the consumption or
the utility of one individual enters the utility function of another. He points out that
under such conditions, a “free-rider” problem will typically arise, and argues that an
institutional arrangements causing additional transfers to the charity can result in
a welfare improvement (Warr 1982). In a nutshell, Warr uses the following model in
explaining donors’ behavior:

$$\max_{x_i, g_i} u_i(x_i, g_i + G_{-i})$$

s.t. $x_i + g_i = m_i,$

where $g_i$ is individual $i$’s contribution toward the charity, $G_{-i}$ is the sum of all but
individual $i$’s contribution, $x_i$ is individual $i$’s consumption, and $m_i$ represents the
individual’s initial wealth.

We too acknowledge the interdependence in the potential donors’ utility functions, and thus, it should not be surprising that there is a “free-rider” problem arising from our model as well. We also choose a very similar utility maximization problem to that of Warr’s; however, while Warr’s total contribution \( G_i \) is deterministic, ours is a random variable dependent on a donor’s conjecture about the employee’s level of effort, thus reflecting the signaling power of a soft target.

On the demand side (or the fundraisers), we owe our inspiration to Andreoni. One important feature of Andreoni’s model is the introduction of a minimum threshold \( \bar{G} \) that total contributions \( \sum_{i=1}^{n} g_i \) must meet or surpass before any benefits of the public good can be consumed:

\[
G = \begin{cases} 
\sum_{i=1}^{n} g_i & \text{if } \sum_{i=1}^{n} g_i \geq \bar{G}, \\
0 & \text{if } \sum_{i=1}^{n} g_i < \bar{G}.
\end{cases}
\]

The threshold \( \bar{G} \) is what we call a hard target, because it has some particular underlying capital requirements to fulfill. Andreoni shows that both \( G = 0 \) and a \( G^* \) can constitute the Nash equilibria. He further explains how a small grant by government or individuals (“seed money”) received prior to launching the full-scale fundraising campaign can result in a big growth for charity, removing \( G = 0 \) as an equilibrium (Andreoni 1998). Thus, this model explains the importance of a hard target.
The model we are to develop in the following sections introduces a similar $G_T$, representing the targeted charity to raise, but unlike the $\bar{G}$ in Andreoni’s model, $G_T$ does not have an underlying capital requirement to fulfill; in other words, even if $\sum_{i=1}^{n} g_i < G_T$, the value of the donation $G$ is not equal to 0.

In addition to the aforementioned theoretical work, there are also a number of empirical studies that are of interest to us. As we have mentioned in the previous section, an ever increasing number of fundraising organizations have deployed performance-based compensation. In fact, the 2007/2008 Not-For-Profit Compensation Survey reveals that 42% of the participating organizations implement formal incentive plans, and all of these organizations rate their performance-based compensation plans as effective (TCS 2008).

Mesch and Rooney conducted a dedicated empirical study to understand the relationship between compensation, performance, and gender differences in the fundraising industry. Their study reveals that a 0.9% raise in compensation led to a 10% increase in funds raised by the organizations, and this in turn, is associated with a significant increase in bonuses for fundraising professionals, especially the Chief Development Officers (Mesch & Rooney 2008). These results reaffirm the effectiveness of performance-based compensation plans, and provide strong empirical foundation for our theoretical research.
3 The Model

Throughout the analysis, we will make the following fundamental assumption about the fundraising organizations:

**Assumption 1.** *Fundraisers are structured into organizations that are composed of managers and employees.*

This assumption is reasonable, given the empirical evidence introduced in section 1 (TCS 2008).

For simplicity, we will assume that the fundraising organization in question has only one manager and one employee. Outside the organization, there are two potential donors, indexed by 1 and 2.

The strategic interactions will be characterized by the following three-stage dynamic game (see figure 2):

**Stage 1:** The manager sets the targeted charity to raise, $G_T$. He also designs the following incentive-based compensation scheme: the employee receives a wage, $w$, *if and only if the target is achieved*.

**Stage 2:** The employee observes $G_T$ and $w$, and decides on the level of effort to exert in contacting and persuading potential donors, $e$. The more effort the employee exerts, the more likely the potential donor will be informed of the fundraising
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Figure 2: Strategic Interactions of the Model

campaign. We will assume that the probability that a potential donor is informed
is exactly $e$; i.e.,

$$\Pr(\text{Donor is informed}) = e.$$  

Of course, $e \in [0, 1]$ by construction.

Stage 3: If a potential donor is informed, then he contributes $g_i$ to the charity.

4 Theoretical Analysis

Since the model is a three-stage sequential game, we will solve this problem using
backward induction.

4.1 The Donors’ Problem

Throughout the theoretical analysis, the following two assumptions will apply to the
donors:
Assumption 2. The donors are identical.

This assumption guarantees that the Nash equilibria of the Donors’ Problem will be symmetric.

Assumption 3. The donors care about both consumption goods, $x_i$, and how much charity is raised in total, $G$. Here, we will assume that their utility function over $x_i$ and $G$ is Cobb-Douglas:

$$u(x_i, G) = \ln x_i + \alpha \ln G,$$

where a higher $\alpha$ indicates a higher preference for charity relative to consumption goods. We will also normalize the price of the consumption goods to be 1.

We will start by considering the problem from the perspective of Donor 1.

Assuming that Donor 1 is informed, then he will: 1) try to infer how much effort $e$ the employee exerts based on his experience, and 2) make a donation of $g_1$ toward $G$. Notice that in equilibrium, the donor’s conjecture about the employee’s effort, $e^*$, should be correct. Thus, he will expect Donor 2 to contribute $e^* \cdot g_2$ toward the charity.

Case (A1) Suppose that in equilibrium, Donor 1’s donation satisfies $g_1^* \geq G_T$, then the donor knows that as long as he makes the donation, the employee will receive the wage, regardless of the action taken by the other donor. In this case, Donor 1’s total expected donation net of fundraising cost will be $G = g_1 + e^* g_2 - w$. 
Hence, the maximization problem faced by donor 1 is

$$\max_{x_1, g_1} u(x_1, G) = \ln x_1 + \alpha \ln(g_1 + e^* \cdot g_2 - w)$$

s.to. \quad x_1 + g_1 = m.

We can easily solve this maximization problem by using the Lagrangian-multiplier method and obtain Donor 1’s best-response function:

$$g_1(g_2, e^*, m, w) = \frac{-e^* g_2 + \alpha m + w}{1 + \alpha}. \quad (1)$$

Since the donors are identical, we know Donor 2’s best-response function is

$$g_2(g_1, e^*, m, w) = \frac{-e^* g_1 + \alpha m + w}{1 + \alpha} \quad (2)$$

by symmetry.

By solving the system of equations (1) and (2), we can conclude that the Nash equilibrium is

$$g_i^*(e^*, m, w) = \frac{w + m\alpha}{1 + e^* + \alpha}.$$  

Case (A2) Suppose that in equilibrium, Donor 1’s donation satisfies $g_1^* < G_T \leq 2g_1^*$, then the employee will only receive the wage if Donor 2 donates as well, which occurs with probability $e^*$. Hence, Donor 1’s expected total donation net of fundraising cost
is $G = g_1 + e^*g_2 - e^*w$. Solving the following maximization problem:

$$\max_{x_1, g_1} u(x_1, G) = \ln x_1 + \alpha \ln(g_1 + e^*g_2 - e^*w)$$

s.t. $x_1 + g_1 = m,$

gives

$$g_1(g_2, e^*, m, w) = \frac{-e^*g_2 + \alpha m + e^*w}{1 + \alpha}.$$ 

By symmetry,

$$g_2(g_1, e^*, m, w) = \frac{-e^*g_1 + \alpha m + e^*w}{1 + \alpha}.$$ 

Therefore, the Nash equilibrium in this case is

$$g^*_i(e^*, m, w) = \frac{e^*w + m\alpha}{1 + e^* + \alpha}.$$ 

Case (A3) Suppose that Donor 1 chooses $g_1$ such that $2g_1 < G_T$, then $G = g_1 + e^*g_2,$ since the employee will not receive the wage even if the other donor contributes. Following the same procedure shows that the Nash equilibrium is

$$g^*_i(e^*, m, w) = \frac{m\alpha}{1 + e^* + \alpha}.$$ 

Theorem 1. The Nash equilibria of the Donors’ Problem consist of the following three cases:

(A1) If the donor chooses $g_i \geq G_T$, then the Nash equilibrium is

$$g^*_i(e^*, m, w) = \frac{w + m\alpha}{1 + e^* + \alpha}.$$
(A2) If the donor chooses $g_i < G_T \leq 2g_i$, then the Nash equilibrium is

$$g_i^*(e^*, m, w) = \frac{e^*w + m\alpha}{1 + e^* + \alpha}.$$ 

(A3) If the donor chooses $2g_i < G_T$, then the Nash equilibrium is

$$g_i^*(e^*, m, w) = \frac{m\alpha}{1 + e^* + \alpha}.$$ 

We will assume that each $g_i^*$ satisfies its corresponding constraining inequality; otherwise, a Nash equilibrium for that particular case does not exist.

There are a few features that are interesting about the $g_i^*$’s we have obtained. First of all, it should not be surprising that

$$\frac{\partial g_i^*}{\partial m} > 0.$$ 

As the donors’ incomes increase, they will certainly become more generous and will be willing to make a larger contribution toward the charity.

Secondly, notice that

$$\frac{\partial g_i^*}{\partial \alpha} > 0.$$ 

Remember that $\alpha$ signifies how much the donors care about the charity. If they care more about the charity relative to the consumption goods (a higher $\alpha$), they will, of course, contribute more toward the charity.
Finally, we should notice that $g_i^*$ is decreasing in $e^*$, i.e.,

$$\frac{\partial g_i^*}{\partial e^*} < 0.$$ 

This is well explained by Warr: when the donors’ utility functions are interdependent, a free-rider problem arises (Warr 1982). In our model, notice that the donors’ utility function takes the form

$$u(x_i, G) = \ln x_i + \alpha \ln G,$$

where $G$ is dependent on both $g_1$ and $g_2$; i.e., each donor’s contribution toward the charity enters the utility function of the other. As the donor expects the employee to exert more effort, he also expects the donation coming from the other donor to increase, and thus, he will believe that a smaller donation from himself will suffice.

### 4.2 The Employee’s Problem

**Assumption 4.** For simplicity, we will assume that the cost function of the employee is only dependent on effort, and it takes the following form:

$$c(e) = 2e^2.$$ 

We will now discuss three different scenarios corresponding to cases (A1), (A2), and (A3) in the Donors’ Problem.
Case (B1) Given $G_T$, suppose that the employee knows that the donors would set $g_i$ such that $g_i^*(e^*, m, w) \geq G_T$. This corresponds to case (A1) above. Given the optimal choice of the donors, we know that $g_i^* = (w + \alpha m)/(1 + \alpha + e^*) \geq G_T$ must be satisfied in equilibrium. In other words, the employee must be restricting his or her effort within the following range:

$$0 \leq e \leq \left( \frac{w + \alpha m}{G_T} - \alpha - 1 \right)^+. $$

In this case, $g_i^*(e, m, w) \geq G_T$ with certainty; i.e., as long as one of the donors donates, the target will be reached, and the employee will receive the wage $w$. Therefore, the probability of the employee getting the wage $w$ is $(\binom{2}{2} e^2 + \binom{2}{1} e (1 - e) = e^2 + 2e(1 - e)$.

Hence, the employee’s maximization problem is

$$\max_e \pi^e = (e^2 + 2e(1 - e))w - 2e^2$$

s.t. $e \in \left[ 0, \left( \frac{w + \alpha m}{G_T} - \alpha - 1 \right)^+ \right] \cap [0, 1]$.

Notice that the objective function is equivalent to

$$\pi^e = -(w + 2)e^2 + 2we,$$

which is a quadratic, concave function in $e$. The first order condition indicates that the function reaches its peak at

$$e = \frac{w}{2 + w}.$$
Figure 3: Optimal Effort for the Employee in Case (B1)

Accounting for the constraints, we can obtain that the solution to the problem above is

$$e^* = \min \left( \frac{w}{w + 2}, \left( \frac{\alpha m + w}{G_T} - \alpha - 1 \right)^+ \right).$$

In other words, if $$\frac{w}{2 + w} < \frac{w + \alpha m}{G_T} - \alpha - 1$$, i.e., $$G_T < \frac{(2 + w)(\alpha m + w)}{2(1 + w) + \alpha(2 + w)}$$, then

$$e^* = \frac{w}{2 + w},$$

which is independent of $$G_T$$.

On the other hand, if $$\frac{w}{2 + w} \geq \frac{w + \alpha m}{G_T} - \alpha - 1$$, or $$G_T \geq \frac{(2 + w)(\alpha m + w)}{2(1 + w) + \alpha(2 + w)}$$, then

$$e^* = \left( \frac{w + \alpha m}{G_T} - \alpha - 1 \right)^+,$$

which is decreasing in $$G_T$$. 

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The analysis above is captured in figure 3: when $G_T$ is small, the employee knows that the target can be easily reached, and therefore, the level of effort is solely dependent on $w$; i.e., the wage rate is the only source of motivation. Not surprisingly, $e^*$ is increasing in $w$ in this range:

$$\frac{de^*}{dw} = \frac{2}{(w+2)^2} > 0.$$  

A higher wage induces more effort from the employee. However, as $G_T$ increases beyond the threshold, a higher target indicates a lower chance of reaching the target, and thus discourages the employee from exerting as much effort.

Case (B2) If the employee expects the donor to set $g_i$ such that $g_i^*(e, m, w) < G_T \leq 2g_i^*(e, m, w)$, then the only way the target can be reached is if both donors contribute, and the probability of him receiving the wage is $e^2$. The maximization problem boils down to:

$$\max_{e} \pi^e = w \cdot e^2 - 2e^2 = (w - 2)e^2$$

s.t. $g_i^*(e, m, w) < G_T \leq 2g_i^*(e, m, w)$.

For this problem, we have the following subcases:

(B2a) If $w \leq 2$, then $\pi^e$ is nonpositive, and strictly decreases as $e^*$ increases in the interval $[0, 1]$. Hence, $e^* = 0$. 


(B2b) If \( w > 2 \) and \( G_T > 2w \), then the constraint reduces to

\[
e \in \left( \frac{m\alpha - G_T - \alpha G_T}{G_T - w}, \frac{2m\alpha - G_T - \alpha G_T}{G_T - 2w} \right) \cap [0, 1].
\]

When \( w > 2 \), the profit function is strictly increasing in \( e \). We can obtain that the solution to the maximization problem is

\[
e^* = \min \left( 1, \left( \frac{2m\alpha - G_T - \alpha G_T}{G_T - 2w} \right)^+ \right).
\]

(B2c) If \( w > 2 \) and if \( w < G_T \leq 2w \), then we require

\[
e > \frac{2m\alpha - G_T - \alpha G_T}{G_T - 2w}, \quad e > \frac{m\alpha - G_T - \alpha G_T}{G_T - w}, \quad \text{and} \quad 0 \leq e \leq 1.
\]

Again, since the profit function is increasing in \( e \) in this range, we will choose

\[
e^* = 1,
\]

assuming all three inequalities in equation (3) are satisfied. Otherwise, there exists no optimal value for \( e \).

Notice that we are not interested in the case \( G_T < w \), where the announced target is not sufficient to cover the wage—this is neither practical nor credible.

Case (B3) If \( 2g_1^*(e, m, w) < G_T \), then the target cannot be reached. The employee gets a wage of zero however much effort he exerts. Not surprisingly, the effort exerted will be \( e^* = 0 \).

The results from the analysis above are summarized in the following theorem:
Theorem 2. The optimal levels of effort chosen by the employee include the following cases:

(B1) If the employee expects that the donor would choose $g_i^*(e, m, w) \geq G_T$, then

$$e^* = \min \left( \frac{w}{w+2}, \left( \frac{\alpha m + w}{G_T} - \alpha - 1 \right)^+ \right).$$

(B2) If the employee expects that the donor would choose $g_i^*(e, m, w)$ such that $g_i^* < G_T \leq 2g_i^*$, then we have the following subcases:

(B2a) If $w \leq 2$, then

$$e^* = 0.$$

(B2b) If $w > 2$ and if $G_T > 2w$, then

$$e^* = \min \left( 1, \left( \frac{2m\alpha - G_T - \alpha G_T}{G_T - 2w} \right)^+ \right).$$

(B2c) If $w > 2$ and if $w < G_T \leq 2w$, then

$$e^* = 1,$$

if the inequalities in equation (3) are satisfied.

(B3) If the employee expects that the donor would set $2g_i^*(e, m, w) < G_T$, then $e^* = 0$. 
4.3 The Manager’s Problem

We now turn our attention to the fundraising manager. The Manager’s Problem, in essence, is to maximize the expected net donation:

$$\max_{G_r} \pi = \text{Expected Donation} - \text{Expected Fundraising Cost}.$$  

If both donors donate, which occurs with probability \((e^*)^2\), then the fundraising manager receives \(2g_i^*\) in donation. If only one of the donors donates, which happens with probability \(2e^*(1 - e^*)\), then the fundraising manager obtains \(g_i^*\) in donation. Of course, if neither donates, he gets 0. Therefore, the manager’s expected donation is

$$\text{Expected Donation} = 2g_i^*(e^*)^2 + 2e^*(1 - e^*)g_i^* = 2e^*g_i^*.$$  

The expected fundraising cost will be presented shortly.

As it turns out, solving this problem analytically is not a viable option, but it is still possible for us to understand how a soft target plays a role in a fundraising campaign. From this point on, we will proceed with a series of numeric simulations. To make the results as realistic as possible, we will make the following assumptions throughout the simulations:

**Assumption 5.** Each donor has an income of \(m = 4200\).

This number is chosen because the median monthly household income in 2007 is approximately 4186 (U.S. Census Bureau 2008).
Assumption 6. $\alpha = 0.0354$.

Like $m$, the estimate of $\alpha$ is not chosen randomly. As early as 1985, Reece and Zieschang tried to estimate the linear donation functions for a cross section of individuals and estimated that the income effect was 0.0342 (Reece & Zieschang 1985). Andreoni, based on the estimation provided by Reece and Zieschang, chose $\beta = 0.0342$ for his utility function $U = Y^{1-\beta}G^\beta$, where $Y$ is the consumption of private goods and $G$ is the consumption of public goods (Andreoni 1988). (This translates into $\alpha = 0.0354$ in our model.) The same estimate has also been reaffirmed by Pecorino (Pecorino 1999). We thus believe it is a robust estimate for $\alpha$.

4.3.1 When Wage is Low

We will start by analyzing the case where the manager sets $w \leq 2$. In fact, let’s assume $w = 2$. From theorem 2, we know that the manager will never set $G_T$ so that scenarios (B2) and (B3) happen, since in either case, $e^* = 0$. Therefore, the only relevant case is (B1), which corresponds to (A1) in the Donor’s Problem, i.e., $g_i^* \geq G_T$. Since the employee will receive wage as long as one donor donates,

$$\text{Expected fundraising cost} = (e^*)^2 w + 2e^*(1-e^*)w = -e^*(e^* - 2)w.$$
Hence the manager’s maximization problem is

\[
\max_{G_T} \pi = 2e^*g^* + e^*(e^* - 2)w,
\]

\[
= 2e^* \cdot \frac{w + m\alpha}{1 + e^* + \alpha} + e^*(e^* - 2)w, \tag{4}
\]

where \(e^*\) satisfies

\[
e^* = \min \left( \frac{w}{w + 2}, \left( \frac{\alpha m + w}{G_T} - \alpha - 1 \right)^+ \right).
\]

**Case (C1a)** If \(\frac{w}{2 + w} < \frac{w + m\alpha}{G_T} - \alpha - 1\), or \(0 < G_T < 98\), we have \(e^* = 1/2\). In this case, we can see that our objective function \(\pi\) in equation (4) is not dependent on \(G_T\); in fact, given \(m = 4200\), \(\alpha = 0.0354\), and \(e^* = 1/2\), we can calculate the value of \(\pi\):

\[
\pi^* = 2 \times \frac{1}{2} \times \frac{2 + 4200 \times 0.0354}{1 + 1/2 + 0.0354} + \frac{1}{2} \left( \frac{1}{2} - 2 \right) 2 = 97.
\]

In addition,

\[
g^*_i = \frac{2 + 4300 \times 0.0354}{1 + 1/2 + 0.0354} = 98.
\]

The Mathematica code for running this simulation is included in appendix A.1.

**Case (C1b)** If \(\frac{w}{2 + w} \geq \frac{w + m\alpha}{G_T} - \alpha - 1 \geq 0\), or \(98 \leq G_T \leq 146\), we have \(e^* = (w + m\alpha)/G_T - \alpha - 1\), and the profit function can be simplified into

\[
\pi = \frac{2.0708(145.528 - G_T)(150.68 - 3.0354G_T + G_T^2)}{G_T^2}
\]
In this case, the profit function is decreasing in $G_T$ when $G_T \in [98, 146]$ (see figure 4), so choosing $G_T = 98$ is optimal. We can obtain that $\pi^* = 97$, and $g_i^* = 98$.

![Figure 4: Profit Function of the Manager in Case (C1b)](image)

The reader can find the Mathematica simulation code in appendix A.2.

To sum up, when wage is low, the manager might be indifferent about setting the target between 0 and 98, but he will not choose the target beyond 98, as doing so will discourage the employee significantly, leading to a decline in expected net donation (total donation net of total fundraising cost). This result is in accordance with our discovery in case (B1) of section 4.2: for a low range of targets, only the wage, not the target, serves as a motivating factor for the employees. This is why here, the manager is indifferent about setting the target within that range. On the other hand, when the target crosses the threshold (which is 98 in our study), the employee would become
discouraged, resulting in the decline in expected net donation.

4.3.2 When Wage is High

We proceed to inspect the case where \( w > 2 \). In particular, we will set \( w = 80 \) (assuming a $10 hourly wage, and an 8-hour workday.) Our expectation is that as \( w \) increases, optimal choice(s) of \( G_T \) will increase as well, along with the expected net donation. Notice that when \( w > 2 \), both cases (B1) and (B2) become relevant.

The manager, being a profit maximizer, will choose the case for which the expected net donation (the profit) is higher. The procedure for solving cases (C1a') and (C1b'), which correspond (B1), is identical to that for solving cases (C1a) and (C1b), only with \( w = 80 \). We will not belabor the reader with the computational details, but will simply present the results here.

Case (C1a') If \( \frac{w}{2+w} < \frac{w+m\alpha}{G_T} - \alpha - 1 \), or \( 0 < G_T < 114 \), then \( e^* = 0.98 \) and \( \pi^* = 142 \), and \( g_i^* = 114 \).

Case (C1b') If \( \frac{w}{2+w} \geq \frac{w+m\alpha}{G_T} - \alpha - 1 > 0 \), or \( 114 \leq G_T \leq 221 \), we can show that \( \pi \) is decreasing in \( G_T \), so choosing \( G_T = 114 \) is optimal, in which case we have \( \pi^* = 142 \), and \( g_i^* = 114 \).

The reader can find the Mathematica code for these simulations in appendices A.3 and A.4.
We now proceed to work with case (B2). Notice that (B2) represents the case where \( g_i^* < G_T \leq 2g_i^* \) in equilibrium. Hence, the employee will only receive the wage if both donors contribute, which occurs with probability \((e^*)^2\). Therefore, the manager’s problem is

\[
\max_{G_T} \pi = 2g^*(e^*)^2 + 2e^*(1-e^*)g^* - [(e^*)^2]w,
\]

which simplifies to

\[
\max_{G_T} \pi = 2e^*g^* - (e^*)^2w
\]

\[= 2e^* \frac{e^*w + m\alpha}{1 + e^* + \alpha} - (e^*)^2w, \tag{5}\]

as the Nash equilibrium corresponding to \( g_i \leq G_T \leq 2g_i \) is

\[g_i^* = \frac{e^*w + m\alpha}{1 + e^* + \alpha}.\]

Stemming from the results in case (B2b) where

\[e^* = \min \left(1, \left(\frac{2m\alpha - G_T - \alpha G_T}{G_T - 2w}\right)^+\right),\]

we have the following two cases, (C2a) and (C2b). Notice that we are assuming \( G_T \geq 2w \).

**Case (C2a)** Now suppose \( \frac{2m\alpha - G_T - \alpha G_T}{G_T - 2w} > 1 \), this implies \( 160 < G_T < 225 \). Then \( e^* = 1 \). In this case, the profit function is independent of \( G_T \), and we can easily
obtain
\[
\pi^* = 2 \times 1 \times \frac{1 \times 80 + 4200 \times 0.0354}{1 + 1 + 0.0354} - (1)^2(80) = 145,
\]
and
\[
g_i^* = \frac{1 \times 80 + 4200 \times 0.0354}{1 + 1 + 0.0354} = 112.
\]

**Case (C2b)** If \( 0 \leq \frac{2m\alpha - G_T - \alpha G_T}{G_T - 2w} \leq 1 \), then \( 225 \leq G_T \leq 287 \), and we have \( e^* = (2m\alpha - G_T - \alpha G_T)/(G_T - 2w) \). Plugging \( e^* \) into \( \pi \), we can see that the profit function simplifies to
\[
\pi = \frac{1.0354(287.193 - G_T)(-197.573 + G_T)(120.405 + G_T)}{(-160 + G_T)^2}.
\]
In this region, \( \pi \) is also declining in \( G_T \), and thus choosing \( G_T = 255 \) is optimal. In fact, \( \pi^* = 145 \), and \( g_i^* = 112 \).

Similarly, we can work backward from case (B2c):

**Case (C2c)** Since the inequalities in equation (3) must be satisfied, we can easily verify that \( G_T \in [112, 160] \). Then \( e^* = 1 \) and \( \pi^* = 145 \) come naturally.

The reader can find the Mathematica code for these cases in appendices A.5–A.7.

The results from the analysis above are summarized in the first four columns in table 1. The analysis confirms our hypothesis: a higher wage does lead to a higher range of values for the target, and in the process, the expected net donation has increased from 97 to 145—a 49% increase!
Table 1: Summary of Manager’s Problem

<table>
<thead>
<tr>
<th>Case ID</th>
<th>( w )</th>
<th>( G_T )</th>
<th>maximum ( \pi )</th>
<th>( \alpha = 0.0354 )</th>
<th>( \alpha = 0.05 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C1a)</td>
<td>2</td>
<td>(0, 98)</td>
<td>97</td>
<td>135</td>
<td>(0, 136)</td>
</tr>
<tr>
<td>(C1b)</td>
<td>2</td>
<td>98</td>
<td>97</td>
<td>135</td>
<td>136</td>
</tr>
<tr>
<td>(C1a')</td>
<td>80</td>
<td>(0, 114)</td>
<td>142</td>
<td>199</td>
<td>(0, 143)</td>
</tr>
<tr>
<td>(C1b')</td>
<td>80</td>
<td>114</td>
<td>142</td>
<td>199</td>
<td>143</td>
</tr>
<tr>
<td>(C2a)</td>
<td>80</td>
<td>(160, 225)</td>
<td>145</td>
<td>203</td>
<td>(160, 283)</td>
</tr>
<tr>
<td>(C2b)</td>
<td>80</td>
<td>225</td>
<td>145</td>
<td>203</td>
<td>283</td>
</tr>
<tr>
<td>(C2c)</td>
<td>80</td>
<td>[112, 225]</td>
<td>145</td>
<td>203</td>
<td>[141, 160]</td>
</tr>
<tr>
<td>Nash Equilibria</td>
<td>80</td>
<td>[112, 225]</td>
<td>145</td>
<td>203</td>
<td>[141, 283]</td>
</tr>
</tbody>
</table>

We can draw the following important conclusions from the analysis:

**Conclusion 1.** *Setting a reasonable nonzero soft target can increase fundraising organization’s expected net donation.*

This conclusion comes from the fact that \( G_T \) has a nonzero lower bound. Although in the low wage cases (C1a and C1b), the manager can choose the soft target to be 0 (this is equivalent to having no soft targets), we have demonstrated that doing so is not optimal. By choosing a higher wage, coupled with a higher target, the fundraising manager can increase the expected net donation.

However, it is equally important to notice the following conclusion:

**Conclusion 2.** *Setting an unreasonably high soft target decreases expected net dona-
4 THEORETICAL ANALYSIS

Table 2: Summary of Manager’s Problem

<table>
<thead>
<tr>
<th>Case ID</th>
<th>w</th>
<th>$G_T$</th>
<th>$\pi_G$</th>
<th>$\pi$</th>
<th>$G_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C1a)</td>
<td>2</td>
<td>(0, 98)</td>
<td>97</td>
<td>135</td>
<td>(0, 136)</td>
</tr>
<tr>
<td>(C1b)</td>
<td>2</td>
<td>98</td>
<td>97</td>
<td>135</td>
<td>136</td>
</tr>
<tr>
<td>(C1a')</td>
<td>20</td>
<td>(0, 88)</td>
<td>138</td>
<td>193</td>
<td>(0, 117)</td>
</tr>
<tr>
<td>(C1b')</td>
<td>20</td>
<td>88</td>
<td>138</td>
<td>193</td>
<td>117</td>
</tr>
<tr>
<td>(C2a)</td>
<td>20</td>
<td>(40, 166)</td>
<td>146</td>
<td>204</td>
<td>(40, 224)</td>
</tr>
<tr>
<td>(C2b)</td>
<td>20</td>
<td>166</td>
<td>146</td>
<td>204</td>
<td>224</td>
</tr>
<tr>
<td>(C2c)</td>
<td>20</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Nash Equilibria</td>
<td>20</td>
<td>(40, 166)</td>
<td>145</td>
<td>203</td>
<td>(40, 224)</td>
</tr>
</tbody>
</table>

This result is confirmed by the upper bound of $G_T$. Beyond the upper bound, the profit function is decreasing in $G_T$, because setting too high a target discourages the employee from exerting much effort.

We can, of course, run the simulation with various other parameters. The result when $\alpha = 0.05$ is also shown in the last two columns in table 1. As we have mentioned, a higher value for $\alpha$ corresponds to a higher preference for the donation relative to the consumption goods. Not surprisingly, since the donors care more about the charity, the manager could set a higher range for the soft target, and the expected donation net of fundraising cost in equilibrium has also increased.

For the sake of completeness, we have also run the simulations using $w = 20$, and
the results are presented in table 2. As we can see, a lower wage corresponds to a smaller range of values for the target. We have also chosen this case to show that (C2c) might not always have a solution (the “N/A” in the table). This problem arises because the inequalities in equation (3) cannot be satisfied.

5 Conclusion

The theoretical analysis fully justifies the existence and prevalence of soft targets in the fundraising industry, thus filling a gap in current literature in this area. Although soft targets might seem random and unnecessary as they do not have any capital requirements to fulfill, the analyses in the previous sections have shown their value. Indeed, setting a soft-target in a fundraising campaign is by no means arbitrary and is quite a piece of art: when the wage offered to the employee is low, the target should also be set lower, and in those cases, wage might be the only factor the employee considers when deciding on his or her effort. However, the fundraising manager would be far better off choosing a higher wage along with a higher soft target. On the other hand, too high a target is not wise either, as it might discourage the employees from exerting effort at all. A fundraising manager can use these results to design optimal schemes that balance $G_T$ and $w$ so as to raise the largest amount of charity possible.

These results are in accordance with the empirical studies we have quoted in
previous sections. As we have mentioned, performance-based incentive compensation has become ever more prevalent over the past few years, and most Non-Profit Organizations have rated these incentive plans to be effective (TCS 2008). They should be—our theoretical analysis indicates that setting targets in the optimal range, coupled with an increased wage, can increase the net expected donation by a significant amount.

Despite the encouraging results presented in this paper, we are obliged to point out a few limitations to the model. First of all, we did not try to optimize the manager’s profit (net donation) with respect to both target and wage, as wage is not our focus, and doing so only clouds the effect of the soft target. However, future research could be directed to understand the interaction between the two elements to provide insights into optimal incentive mechanism design. Secondly, the model assumes that there are only one employee and two donors. More work can be done to extend the model to a world where there are multiple employees and donors, so as to better reflect the complexity in reality.
A Mathematica Simulation Code

A.1 The case (C1a)

Clear["Global‘*"];

m = 4200; \[Alpha] = 0.0354; w = 2;
e = w/(w + 2)
Reduce[w/(w + 2) < (\[Alpha] m + w)/Gt - \[Alpha] - 1, Gt] // N

gi = (w + m \[Alpha])/(1 + e + \[Alpha]) // N
pi = 2 e*(w + m \[Alpha])/(1 + e + \[Alpha]) + e (e - 2) w // N

A.2 The case (C1b)

Clear["Global‘*"];

m = 4200; \[Alpha] = 0.0354; w = 2;
Reduce[w/(w + 2) >= (\[Alpha] m + w)/Gt - \[Alpha] - 1 >= 0, Gt] // N
e = (\[Alpha] m + w)/Gt - \[Alpha] - 1

gi = (w + m \[Alpha])/(1 + e + \[Alpha]) // N
pi = 2 e*(w + m \[Alpha])/(1 + e + \[Alpha]) + e (e - 2) w // N

A.3 The case (C1a’)

Clear["Global‘*"];

m = 4200; \[Alpha] = 0.0354; w = 80;
e = w/(w + 2) // N
Reduce[w/(w + 2) <= (\[Alpha] m + w)/Gt - \[Alpha] - 1, Gt] // N
gi = (w + m \[Alpha])/(1 + e + \[Alpha]) // N
pi = 2 e*(w + m \[Alpha])/(1 + e + \[Alpha]) + e (e - 2) w // N

A.4 The case (C1b’)

Clear["Global‘*"];

m = 4200; \[Alpha] = 0.0354; w = 80;
Reduce[w/(w + 2) > (\[Alpha] m + w)/Gt - \[Alpha] - 1 >= 0, Gt] // N
\[ e = (\alpha m + w)/Gt - \alpha - 1 \]
\[ gi = (w + m \alpha)/(1 + e + \alpha) // N \]
\[ pi = 2 e*(w + m \alpha)/(1 + e + \alpha) + e (e - 2) w // N \]

A.5 The case (C2a)

Clear["Global'*"];
\[ m = 4200; \alpha = 0.0354; w = 80; \]
\[ e = 1 \]
\[ gi = (e w + m \alpha)/(1 + e + \alpha) // N \]
\[ pi = 2 e*(e w + m \alpha)/(1 + e + \alpha) - e^2 w \]

A.6 The case (C2b)

Clear["Global'*"];
\[ m = 4200; \alpha = 0.0354; w = 80; \]
\[ e = (2 m \alpha - Gt - \alpha Gt)/(Gt - 2 w) \]
\[ gi = (e w + m \alpha)/(1 + e + \alpha) // N \]
\[ pi = 2 e*(e w + m \alpha)/(1 + e + \alpha) - e^2 w \]

A.7 The case (C2c)

Clear["Global'*"];
\[ m = 4200; \alpha = 0.0354; w = 80; \]
\[ e = 1 \]
gi = (e w + m \[Alpha])/(1 + e + \[Alpha]) // N
pi = 2 e*(e w + m \[Alpha])/(1 + e + \[Alpha]) - e^2 w
Bibliography


