Seasonal Volatility of Corn Futures Prices*†‡

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Abstract

This paper examines the seasonal patterns evident in the volatility of corn futures prices. It adds to the high-frequency volatility literature by exploring large price discontinuities (jumps), and both the continuous and discontinuous portions of volatility in a commodity, corn. Furthermore, the paper analyzes how the growing cycle of corn influences volatility over the course of a year. After identifying a distinct pattern in the volatility of corn over the growing season, this paper articulates some of the potential causes of the observed trend. The amount of information available to the market seems to drive the volatility of corn prices, and the changes in volatility occur almost entirely in the continuous portion of volatility which led to jumps being evenly distributed throughout the year.
1. Introduction

Volatility is an essential tool for both pricing assets and managing risk. Although there have been many studies regarding the characteristics of asset price volatility, all previous research has focused on equities or currencies and skipped over the qualities of commodity volatility. Corn is dramatically different than typical financial assets such as equities since the commodity is grown and harvested in a seasonal fashion. Every year the government allocates billions of dollars to corn subsidies, and the Energy Policy Act of 2005 requires that corn based ethanol production be doubled by 2012. Additionally, price volatility and the ability to hedge against future price movements is vital to the livelihoods of individual farmers across the nation (Henrique). To understand the impact the price fluctuations of corn have on American consumers, farmers, and motorists the volatility of corn must be examined.

Studies such as Park (2000) examine the effect that certain trading restrictions have on volatility in commodities markets. Yang, Haigh, and Leatham (2001) investigates the volatility of corn futures prices at daily frequencies to understand the impact of changing growing regulations on corn volatility. Manfredo et al. (2000) use daily data to calculate implied volatility to predict future price movements in corn markets. Despite these studies, the seasonal growing cycle of the commodity creates a unique impact on volatility that has not been studied before using high-frequency, intra-day price data to determine how the growing seasons affect both the continuous and discontinuous portions of volatility. Figure 1 demonstrates the cyclical movement of remaining corn stocks change over the course of a year. Corn is planted in the spring and harvested in early October. The crop is then stored or converted into products beginning in October, and used over the course of the next year. As shown in Figure 1, prior to the harvest of corn, the remaining stocks from the previous year have almost been completely
used up. Given that the crop is grown and harvested over the course of a year, and the remaining corn stocks vary with the seasons, it is natural to question the impact this seasonality has on volatility. Therefore, one should examine the time-varying, seasonal nature of the volatility to accurately assess the risk of corn futures prices. This paper aims to examine whether there are seasonal and monthly patterns of corn futures volatility.

To examine seasonal and monthly patterns in volatility this paper uses a prevalent assumption in modern finance theory, that financial asset prices follow a continuous, or smooth, underlying price function. Papers as early as Merton (1976), however, motivate the idea that the price movements not only occur in discrete time, but also tend to exhibit large discontinuous positive and negative changes which suggest that a price model that assumes a continuous sample path may not be a reasonable approximation of the observed data. This new model would imply that asset prices follow a typically smooth path with infrequent but large price movements scattered stochastically over time. Anderson, Bollerslev, and Diebold (2006) note that the assumption of a continuous sample path in theoretical models for asset pricing is clearly violated in practice. Recently, financial economists have begun to identify and quantify these random jumps in returns using high-frequency asset data. Barndorff-Nielsen and Shephard (2004) introduced bipower-variation, a non-parametric statistic that provides a consistent estimator for the volatility not including jumps. Their work provides the theoretical framework to study jumps. Huang and Tauchen (2005) validate this idea through extensive Monte Carlo analysis.

First, in section 2 this paper will begin with a brief discussion of the model of asset prices that is used to develop the tests for statistical discontinuities in prices. Next, in section 3 it will outline the methods used to conduct an examination of both the volatility of corn futures prices and the prevalence and timing of jumps over the course of the year. Section 4 explains the data that are used in the paper. Then, in section 5 these
results will be analyzed compared to the growing and harvesting pattern of corn in order to better understand how the seasonal nature of corn production impacts the volatility of the commodity.

2 Stochastic Model of Returns

It is important to understand the theoretical model of asset price movements and the concept of market microstructure noise that are used to derive the results of this paper. To motivate our discussion of jump discontinuities and seasonal volatility in corn futures prices we will begin by investigating a standard model of asset price evolution. This paper considers a log price, $p(t)$, that changes over time as

$$dp(t) = \mu(t)dt + \sigma(t)dW(t), \quad 0 \leq t \leq T \tag{1}$$

where $\mu(t)dt$ represents the time-varying drift component of the asset. The time-varying volatility of the price movement is represented by $\sigma(t)$ where the $dW(t)$ term is standardized Brownian motion. The equation defined above is the continuous time interpretation of a Binomial model in which the price can either more up or down over small time intervals. The drift term and the standard Brownian motion are the consequences of adding independent identically distributed log-returns over infinitesimally small time periods.

Recent literature has suggested that the addition of jumps in the price process is important for theoretical and empirical modeling. Merton (1976) introduced the following equation to include a jump process:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + \kappa(t)dq(t), \quad 0 \leq t \leq T \tag{2}$$
where the non-continuous portion of the price movement is added with the term \( \kappa(t)dq(t) \). In this equation \( q(t) \) is a counting process and \( k(t) \) is the magnitude of the jump. Here, while the price paths still take place in continuous time, the paths are discontinuous.

3. Methods

To determine whether a statistically significant price discontinuity, or jump, occurs during a trading period, high frequency measures of variance must be calculated. Barndorff-Nielsen and Shephard (2004) introduced a test that uses high frequency price data to determine if there is a jump over the course of a day. The test compares two different measures of variance: realized variance and bipower variation. The variance is calculated daily in unit \( t \), and intraday geometric returns are defined as

\[
r_{t,j} = p(t - 1 + \frac{j}{M}) - p(t - 1 + \frac{j-1}{M}), \quad j = 1, 2, 3, \ldots M
\]

where \( p \) is the log price and \( M \) is the number of observations in the given day. The first measure of quadratic variation is the realized variance (RV) which is defined as:

\[
RV_t = \sum_{j=1}^{M} r_{t,j}^2
\]

and the alternate measure is realized bipower variation (BV) which is defined:

\[
BV_t = \mu - 2 \left(\frac{M}{M-1}\right) \sum_{j=2}^{M} \left| r_{t,j} \right| r_{t,j-1}
\]

where \( \mu_\alpha = E(|Z|^{\alpha}), \ Z \sim N(0, 1), \ \alpha > 0. \)
Both of these two measures were thoroughly investigated in Barndorf-Nielsen and Shephard (2005) to produce asymptotic results that allow for the separate identification of the continuous and jump components of the quadratic variation. While the RV measures both the variation of the continuous process and the jump process, the BV only measures the variation of the continuous process. The BV is robust to jumps because it multiplies adjacent returns. Thus any singular, large price move will be canceled out when it is followed or preceded by, and thus multiplied by, a small return. Therefore, the difference between the RV and the BV isolates the jump component of the daily volatility.

Furthermore, to measure of the percentage of total variance caused by the jump process this paper uses the relative jump:

\[ RJ_t = \frac{RV_t - BV_t}{RV_t} \]  \hspace{1cm} (6)

This result can be used to test the hypothesis that no jump occurred on any particular day. The test can be expressed as a z-statistic:

\[ Z_{TP, rm, 1:T} = \frac{RJ_{1:T}}{\left( v_{bb} - v_{qq} \right)^{1/2} \max(TP_{1:T})} \frac{TP_{1:T}}{BV_{1:T}^2} \]  \hspace{1cm} (7)

where TP is the tripower quarticity defined as:

\[ TP_t = M_1 \left( \frac{M}{M-2} \right) \sum_{j=3}^{M} r_{t,j-2}^{4/3} r_{t,j-1}^{4/3} r_{t,j}^{4/3} \]  \hspace{1cm} (8)

The TP is used in this test because, as shown in Barndorf-Nielsen and Shephard (2004), it converges to the integrated quarticity of the price process. And Huang and Tauchen (2005) discovered that the best test statistic for jump detection is the Z-Tripower-Max.
test statistic as represented in equation 5. The statistic, $Z_t$, approaches a standard normal distribution, $N(0, 1)$, as $M$ approaches infinity and the data get denser.

The test operates under the assumption that there are no jumps. This means that higher values $Z$ suggest the presence of jumps on a particular day. When $Z_t$ is sufficiently high then we can reject the null hypothesis that there are no jumps. Throughout this paper, the $Z$-statistic is tested at the .001 confidence level to distinguish a day with jumps from a day without jumps.

Another method of computing variance, Realized Semi-Variance, is also investigated. RS is defined by Barndorff-Nielsen, Kinnebrock, and Shephard (2008) in order to separate the negative and positive variation. Realized Semi-Variance is the sum of squared negative returns and for the purposes of this paper will be referred to as down-variance, DV.

$$RS = \sum_{j=1}^{M} r_{t,j}^2 * 1_{r_{t,j} \leq 0}$$ (9).

Up-Variance, also developed in the same paper, is the sum of squared positive returns, but for purposes of simplicity is calculated by:

$$UV = RV - DV$$ (10).

Realized DV and UV were initially developed to examine the predictive power of negative and positive returns on volatility, but are used here to examine potential causes of the trends observed in the seasonal variance of corn futures prices.

4. Data
This paper examines the prices of corn futures from the start of trading in January 1983 to the end of trading in December 2007. The high-frequency corn futures data used for this research were purchased from http://tickdata.com. The data were delivered in one-minute intervals, but the returns used in this paper are calculated at five minute intervals in an attempt to find a balance between the micro-structure noise, and a significant amount of data. The fundamental value for corn futures is determined by forward-looking supply and demand. The predicted intersection of supply and demand at the delivery date is constantly changing. Microstructure-noise results from bid-ask bounce, the time it takes information to reach the market, and other high-frequency factors that cause the price to deviate slightly from this fundamental value. Market microstructure noise is noticeable when estimating variance using high frequency data, and can have a significant impact on tests for jumps in asset prices.

Corn was chosen for this study since it has a cyclical, seasonal growing pattern, and is the most widely produced feed grain in the United States, accounting for more than 90 percent of total value and production of feed grains. Around 80 million acres of land are planted to corn, with the majority of the crop grown in the aptly named Corn Belt. The Corn Belt includes the states of Michigan, Minnesota, South Dakota, Wisconsin, Ohio, Illinois, Indiana, Iowa, Missouri, Kansas, and Nebraska. Most of the crop is used as the main energy ingredient in livestock feed. Hogs, cattle, sheep, and poultry eat more than half of the corn grain grown each year. Corn is also processed into a multitude of food and industrial products including starch, sweeteners, corn oil, beverage and industrial alcohol, and fuel ethanol. Some corn is used for silage. Corn silage is livestock food that is made from the parts of the corn plant that are left after the roots and ears of corn have been taken off.

The commodity futures are traded on the Chicago Board of Trade from 9:30 am to 1:15 pm central time through both open-outcry and, more recently, electronic methods.
Each contract is a dollar denominated physical delivery contract of 5000 bushels, approximately 127 metric tons. The prices are quoted in cents and quarter cents per bushel. Corn futures provide a way to participate in price discovery, and manage exposure to price risk for corn producers, food processors, livestock operators, and other market participants have related to the purchase and sale of corn.

5. Results

The results are split into three sections. First, the trends of both RV and BV are examined. Next, DV is studied in an attempt to understand potential causes of the trends observed in the variance. Thirdly, the amount and time of occurrence of jumps is assessed to further understand the seasonal trends in corn futures price volatility.

5.1 RV and BV

Over the 25 years from which the data were collected, 1983-2007, the RV when expressed in standard deviation and annualized implies an average annualized volatility of 17.07%. When BV is expressed in the same manner the implied annualized volatility is slightly lower at 15.93%. The average volatility of the stock market during this same time was about 16.00%. This suggests that there are discontinuities in the price moves throughout each year which are expressed in the RV but not picked up by the BV. Additionally, the volatility was not constant throughout the year. Figure 3 shows both the average RV and average BV on a month by month basis over the 25 years. The RV increases linearly from low volatility in February to a peak in July before decreasing over the later months of the year.

Remarkably, the remaining corn stocks at the end of each quarter (Figure 1) follow a striking seasonal pattern that can explain the observed shifts in RV and BV during the course of a year. Immediately following a harvest, when remaining corn stocks are at their peak, the volatility of corn prices are at the lowest point, the RV represented
as annualized volatility is 13.42%. Right before the harvest, however, with corn stocks at their lowest level, the volatility of futures prices is at its highest point, the RV implies an annualized volatility of 21.63%. The difference between the trough of RV and the peak of the RV is statistically significant at a .01 level. The disappearance of corn stocks, shown in Figure 2, follows a similar trend, but the stocks remaining at the end of August are not enough to sustain the same levels of consumption. Since futures markets are intended to aid in price discovery, the price seems most volatile when the supply is at a very low level, and the future supply is not fully known.

The timing of the crop reports released by the USDA provides further evidence that quantity of information possessed by market participants extensively contributes to the observed pattern of volatility. At the end of each March (the beginning of corn growing season) the Prospective Plantings report indicates how many acres of corn producers are expecting to plant. At precisely this time, the RV and BV begin to increase as information about the future supply of corn reaches the market. Then, as the summer progresses, monthly crop production reports are issued to give an updated estimate of the supply and demand for corn. In June, the USDA releases a more concrete plantings report, shortly after which the volatility of corn prices reaches its peak. Once this information about the harvest has been absorbed by the market the volatility begins to decrease steadily. Finally, after the October harvest there the volatility reaches its lows as market participants wait until next March for any supply information to be calculated.

Interestingly, the BV also exhibits the same pattern. This suggests that the time varying nature of the volatility is largely explained by the continuous portion for the price evolution function. If the seasonal nature of the harvest caused statistically significant discontinuities in returns then there would be a difference between the patterns of the RV and the BV over the course of the year. However, when analyzed at either a monthly or weekly interval, the RV and BV demonstrate the same single peaked pattern over the course of the year.
5.2 Realized Down-Variance

DV can be used to better understand potential causes of the seasonal trend that is observed in both RV and BV. DV is the variance of only negative returns. If corn price volatility rises only due to natural disasters or other sudden shocks that unexpectedly hurt the future supply of corn then there would likely be larger price increases than price decreases. Thus, DV would be constant throughout the year while UV would display the same significant single peaked shape that is seen in both the RV and the BV in Figure 3. However, as seen in Figure 4, this is not the case.

The DV and UV both follow the same pattern as the RV and BV in figure 2. Not only is this pattern qualitatively the same for RV, BV, DV and UV on a monthly and weekly interval throughout the year, but also UV is always slightly greater than UV. This indicates that there is no significant trend in returns that suggests disasters, or other supply constraining events, are driving the volatility of corn futures prices. If that were the case, it would be indicated by positive returns dominating negative returns during the late spring and summer months of each year. This indicates that price shifts occur in both directions with similar magnitude. Producers can plant more acres than initially expected just as easily as they may plant fewer acres than were predicted by the USDA surveys. This supports the idea that the volatility changes of corn futures prices over the course of each year are largely continuous and not caused by discontinuous price jumps which may be caused by unexpected supply shocks.

5.3 Jumps

For the entire sample there were 372 total days that were identified by the BNS test as containing jumps. This is 5.8% of the total number of days in the sample. Since the
test was calculated at the .001 confidence level this suggests that the null hypothesis that price is completely described by a continuous function should be rejected.

While the volatility of corn prices was observed to fluctuate over the course of the year there are not necessarily more jumps during any given period of the year. The key number in the calculation of the BNS Z statistic is the relative jump. Figure 6 shows how the relative jump statistic varies over the course of the year on average over the 25 years. The figure includes 95% confidence bands to demonstrate statistical significance.

Figure 5 illustrates the difference between RV and BV on average over 1983 to 2007. This difference displays a similar pattern to the raw moves in RV and BV which means that as RV grows, so is the amount of the volatility that is caused by jumps. However, Figure 6 shows that the relative jump statistic does not follow the pattern that was observed in the volatility. Qualitatively, it appears that the cyclical nature of corn production does not cause a prevalence of jumps on a seasonal basis. In fact, jumps contribute relatively more to the total volatility during the winter months, periods of low volatility, than the periods of higher volatility. It is possible that this difference in the relative importance of jumps can be due to the nature of the changes in volatility at the different times of year. During the late summer, when RV is at its peak, most changes in the price of corn are caused by uncertainty in the future harvest and supply of corn. On the other hand, during the winter months the supply is almost certain, and instead changes in demand lead to changes in the price of corn. The fact that jumps are more relatively important during these winter months may suggest that shifts in demand are more likely to cause discontinuous movements in corn price volatility than the changes in supply that are seen during the pre-harvest, summer months.

Once the jumps are calculated this observation is seen to hold. Table 2 shows how jumps were distributed by season. While the summer months do have a high number of jumps compared to the fall and spring, the winter is the season where the most jumps occur. Most significantly, no season has more that 28% or less than 21% of the jumps. A
two proportion Z test between the amount of winter jumps (the season with the most jumps) and the spring jumps (the season with the least jumps) gives a test Z-statistic of 2.20. This means that at the .01 confidence level the null hypothesis that the proportions are equal cannot be rejected. Therefore, that while the number of jumps in each season is not the same, the differences are not statistically significant. Furthermore, separating the jumps into warm/growing weather and cold weather fails to demonstrate even a minor difference in total jump days. This data seem to suggest that jump days are not due to the growing and harvesting patterns of corn.

Figure 7 continues to support the idea that the pattern of jumps is unrelated to the pattern of the planting and harvesting of corn. The jumps are scattered throughout the year and occur just as often in January as in July.

6. Conclusion

Overall, this paper is the first to apply high frequency jump tests to the corn futures market. It explores the monthly trends of the volatility of corn futures prices. Just as corn production obeys seasonal patterns and a cyclic production cycle, so does the volatility of the prices. The yearly pattern in corn volatility is distinct. As corn stocks dwindle corn price volatility rises to a peak in July. This pattern is observed in both RV and BV which suggests that it is part of the continuous portion of volatility, not the discrete jump portion of the volatility. Additionally, jumps are evenly distributed throughout the year when divided into months, seasons, or growing and non-growing periods. This data indicate that typical seasonal increases in the volatility are not due to discrete jumps in price that would be cause by unexpected events.

Furthermore, this suggests that volatility rises due to known, predictable events such as changes in the quantity of information available to the market. The uncertainty about the next year’s crop shifts throughout the course of the growing season. More information about the amount of corn planted and shifting growing conditions are likely to blame for
increases in volatility. Neither natural disasters nor large unexpected shifts in the size of the harvest are to blame for the rise in volatility. This explains why the continuous volatility increases as corn stocks dwindle, and follows a distinct seasonal pattern in line with the cyclical production cycle of the commodity.
A. Figures

Figure #1

Ending Corn Stocks

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Millions of Bushels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sept.-Nov.</td>
<td>0.00</td>
</tr>
<tr>
<td>Dec.-Feb.</td>
<td>2,000.00</td>
</tr>
<tr>
<td>Mar.-May</td>
<td>4,000.00</td>
</tr>
<tr>
<td>Jun.-Aug.</td>
<td>6,000.00</td>
</tr>
<tr>
<td>2001</td>
<td>8,000.00</td>
</tr>
<tr>
<td>2002</td>
<td>10,000.00</td>
</tr>
<tr>
<td>2003</td>
<td>12,000.00</td>
</tr>
<tr>
<td>2004</td>
<td>14,000.00</td>
</tr>
<tr>
<td>2005</td>
<td>16,000.00</td>
</tr>
<tr>
<td>2006</td>
<td>18,000.00</td>
</tr>
</tbody>
</table>
Figure #2

Total Corn Disappearance

Millions of Bushels

Quarters:
- Sept.-Nov.
- Dec.-Feb.
- Mar.-May
- Jun.-Aug.

Years:
- 2001
- 2002
- 2003
- 2004
- 2005
- 2006
Figure #3

RV by month 1983-2007

BV by month 1983-2007
Figure #4

Average Monthly Semi-Variance 1983-2007

Average Monthly Upward Variance 1983-2007
Figure #5

RV - BV by month 1983-2007
Figure #6

Relative Jump by month with 95% confidence lines 1983-2007

Standard Error: .00813273
Figure #7

Average Jumps Per Month 1982-2007
### B. Tables

#### Table #1

**USDA Corn Yearbook Table**

<table>
<thead>
<tr>
<th>Mkt year</th>
<th>Quart</th>
<th>Beginning stocks</th>
<th>Production</th>
<th>Imports</th>
<th>Total supply 2/</th>
<th>Food, alcohol, and industrial use</th>
<th>Seed use</th>
<th>Feed and residual use</th>
<th>Total domestic use</th>
<th>Exports</th>
<th>Total disappearance</th>
<th>Ending stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td></td>
<td>1,899.11</td>
<td>9,502.58</td>
<td>10.14</td>
<td>11,411.83</td>
<td>2,026.31</td>
<td>20.10</td>
<td>5,864.22</td>
<td>7,910.63</td>
<td>1,904.77</td>
<td>9,815.40</td>
<td>1,596.43</td>
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<tr>
<td>2002</td>
<td></td>
<td>1,596.43</td>
<td>8,966.79</td>
<td>14.45</td>
<td>10,577.66</td>
<td>2,320.24</td>
<td>20.01</td>
<td>5,562.85</td>
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<td>1,587.89</td>
<td>9,490.99</td>
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<td>2003</td>
<td></td>
<td>1,086.67</td>
<td>10,089.22</td>
<td>14.08</td>
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<td>2,516.55</td>
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<td>8,332.06</td>
<td>1,899.82</td>
<td>10,231.88</td>
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<td>2004</td>
<td></td>
<td>958.09</td>
<td>11,807.09</td>
<td>10.83</td>
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<td>8,434.98</td>
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<td>8.81</td>
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<td>2006</td>
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<td>11.98</td>
<td>12,514.01</td>
<td>3,466.50</td>
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<td>5,594.74</td>
<td>9,085.00</td>
<td>2,125.37</td>
<td>11,210.37</td>
<td>1,303.65</td>
</tr>
</tbody>
</table>

**Million bushels**

**Sources:** USDA, National Agricultural Statistics Service, Crop Production and Grain Stocks; USDA, World Agricultural Outlook Board, World Supply and Demand Estimates; and U.S. Department of Commerce, Bureau of the Census.
Table #2
Jumps by Season

<table>
<thead>
<tr>
<th>Season</th>
<th>Total Jumps</th>
<th>Percent of Total Jumps in Season</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter</td>
<td>105</td>
<td>28.2</td>
</tr>
<tr>
<td>Spring</td>
<td>78</td>
<td>20.97</td>
</tr>
<tr>
<td>Summer</td>
<td>101</td>
<td>27.15</td>
</tr>
<tr>
<td>Fall</td>
<td>88</td>
<td>88</td>
</tr>
</tbody>
</table>

Warm Weather Jumps: 193 (51.88%)
Cold Weather Jumps: 179 (48.12%)
8. References
