Web-based Supporting Materials for "Robust Bayesian hierarchical model using normal/independent distributions" by Geng Chen and Sheng Luo

BUGS code for fitting model Indep-CN

```
model
{
  for (i in 1:obs) # obs: number of total observations
  {
    # Construct contaminated Normal distribution
    u.contam2[i] ~ dbern(nu2)
    w2[i] <- gamma2*u.contam2[i] + (1-u.contam2[i])
    Y.conti[i] ~ dnorm(mu.conti[i], taudy[i]) # continuous outcome
    taudy[i] <- w2[i]*tau.conti
    # K.ordi: number of ordinal variables, K=2 in DATATOP study
    for (k in 1:K.ordi) { Y.ordi[i, k] ~ dcat(prob.y[i, k, 1:n[k]]) }
  }

  # construct mean of the continuous variable
  for (i in 1:obs)
    {
      mu.conti[i] <- a.conti + b.conti * theta[i]
    }

    # Construct the probability vector for the ordinal variables
    for (i in 1:obs)
    {
        for (k in 1:K.ordi)
            {
            for (l in 1:(n[k]-1)) { logit(psi[i, k, l]) <- a.ordi[k,l]
                                             - b.ordi[k]*theta[i] }
            prob.y[i, k, 1] <- psi[i, k, 1]
            for (l in 2:(n[k]-1)) { prob.y[i, k, l] <- psi[i, k, l]
                                             - psi[i, k, l-1] }
            prob.y[i, k, n[k]] <- 1 - psi[i, k, (n[k]-1)]
            }
    }

  # construct random effects
  for (i in 1:N)
  {
     u[i, 1:2] ~ dmnorm(zero[], precision[i, ,])
     u.contam1[i] ~ dbern(nu1)
     w1[i] <- gamma1*u.contam1[i] + (1-u.contam1[i]) # Contaminated Normal
```

```
 # construct the variance-covariance matrix for random effects
     precision[i,1:2,1:2] <- inverse(Sigma[i,,])
     Sigma[i,1,1] <- 1/w1[i]
     Sigma[i,1,2] <- rho*sig/w1[i]
     Sigma[i,2,1] <- Sigma[i,1,2]
     Sigma[i,2,2] <- sig*sig/w1[i]
  }

  # construct theta, the latent variable of subject i at time j
  for (i in 1:obs)
  {
     theta[i] <- u[subject[i], 1] + (beta[1] + beta[2]*treat[i]
                + u[subject[i], 2])*time[i]
  }

  # prior for regression coefficients
  for (i in 1:2) { beta[i] ~ dnorm(0, 0.01) }

  # specify prior distributions
  rho ~ dunif(-1, 1)
  sig ~  dgamma(0.01, 0.01)

  # prior for continuous variable's parameters
  b.conti ~ dgamma(0.001,0.001)
  a.conti ~ dnorm(0, 0.0005)
  tau.conti ~ dgamma(0.001,0.001)
  sd.conti <- 1/sqrt(tau.conti)

  # prior for ordinal variables' parameters
  for (k in 1:K.ordi)
  {
    b.ordi[k] ~ dgamma(0.001,0.001)
    a.ordi[k, 1] ~ dnorm(0,0.001)
    for (l in 2:(n[k]-1)) { a.ordi[k, l] <- a.ordi[k, l-1] + delta[k, l-1] }
    for (i in 1:(n[k]-2)) {delta[k, i] ~ dnorm(0,0.01)I(0,) }
  }

# prior for contaminated normal tuning parameters
    nu1 ~ dbeta(1,1)
    gamma1 ~ dbeta(1,1)
    nu2 ~ dbeta(1,1)
    gamma2 ~ dbeta(1,1)
}
```

Table 1: Simulation results from models Indep-T, Indep-SL, and Indep-CN to recover their true parameters.

	Indep-T					Indep-SL					Indep-CN				
	True	Bias	SD	SE	CP	True	Bias	SD	SE	CP	True	Bias	SD	SE	CP
a_1	25.00	0.12	0.55	0.60	0.93	25.00	−0.01	0.71	0.62	0.91	25.00	−0.04	0.5 9	0.59	0.98
b_1	10.00	−0.11	0.58	0.55	0.93	10.00	−0.03	0.70	0.62	0.88	10.00	−0.06	0.49	0.48	0.95
a_2	80.00	0.25	1.06	1.14	0.94	80.00	−0.08	1.40	1.18	0.91	80.00	−0.07	1.16	1.12	0.97
b_2	18.00	−0.20	1.07	1.00	0.91	18.00	−0.01	1.27	1.13	0.89	18.00	−0.16	0.87	0.88	0.97
a_{31}	−2.70	−0.04	0.16	0.15	0.91	−2.70	−0.01	0.17	0.16	0.95	−2.70	0.00	0.15	0.15	0.92
a_{32}	−0.60	−0.03	0.13	0.14	0.93	−0.60	0.01	0.15	0.14	0.93	−0.60	−0.00	0.12	0.13	0.97
a_{33}	2.00	−0.03	0.14	0.14	0.95	2.00	0.02	0.17	0.15	0.93	2.00	−0.01	0.13	0.14	0.99
a_{34}	2.80	−0.03	0.15	0.15	0.98	2.80	0.01	0.20	0.16	0.89	2.80	−0.01	0.15	0.15	0.98
a_{35}	5.00	−0.01	0.20	0.20	0.95	5.00	0.02	0.28	0.20	0.86	5.00	0.01	0.19	0.20	0.97
a_{36}	6.00	−0.00	0.20	0.22	0.99	6.00	0.02	0.27	0.22	0.91	6.00	0.01	0.22	0.22	0.95
b_3	2.00	−0.01	0.13	0.12	0.95	2.00	0.00	0.16	0.14	0.93	2.00	−0.01	0.10	0.11	0.97
a_{41}	−0.10	−0.01	0.05	0.05	0.94	−0.10	0.00	0.06	0.05	0.95	−0.10	−0.00	0.05	0.05	0.95
a_{42}	1.00	−0.01	0.06	0.06	0.91	1.00	0.02	0.06	0.06	0.91	1.00	−0.00	0.06	0.06	0.93
a_{43}	1.80	−0.01	0.07	0.07	0.98	1.80	0.01	0.07	0.07	0.95	1.80	0.00	0.07	0.07	0.92
a_{44}	2.60	−0.01	0.09	0.09	0.95	2.60	0.02	0.08	0.09	0.95	2.60	0.01	0.09	0.09	0.99
a_{45}	3.30	−0.00	0.11	0.11	0.94	3.30	0.02	0.10	0.11	0.95	3.30	0.01	0.12	0.11	0.91
a_{46}	4.00	0.02	0.15	0.14	0.93	4.00	0.02	0.14	0.14	0.95	4.00	0.03	0.13	0.14	0.97
b_4	0.40	−0.00	0.03	0.03	0.93	0.40	0.00	0.03	0.03	0.89	0.40	−0.00	0.03	0.03	0.95
β_{10}	0.40	0.03	0.11	0.11	0.95	0.40	0.01	0.12	0.11	0.91	0.40	0.01	0.11	0.10	0.92
β_{11}	−0.50	−0.04	0.15	0.15	0.91	−0.50	−0.03	0.17	0.15	0.91	−0.50	−0.03	0.14	0.14	0.93
ν_σ	2.00	0.12	0.40	0.38	0.94	3.00	1.12	2.07	2.78	0.88	—	—	—	—	—
ν_y	3.00	0.26	0.69	0.70	0.93	4.00	1.42	2.60	3.17	0.88	—	—	—	—	—
γ_1	—	—	—	—	—	—	—	—	—	—	0.10	0.01	0.02	0.02	0.97
γ_2	—	—	—	—	—	—	—	—	—	—	0.12	0.01	0.02	0.02	0.88
ν_1	—	—	—	—	—	—	—	—	—	—	0.10	0.02	0.03	0.03	0.91
ν_2	—	—	—	—	—	—	—	—	—	—	0.15	0.02	0.03	0.03	1.00
ρ	0.40	−0.01	0.05	0.05	0.94	0.40	−0.01	0.04	0.05	0.96	0.40	−0.00	0.05	0.05	0.93
σ_u	1.30	0.02	0.08	0.07	0.89	1.30	0.02	0.07	0.07	0.93	1.30	0.01	0.07	0.07	0.93

Table 2: Simulation results from models Dep-T, Dep-SL, and Dep-CN to recover their true parameters.

	Dep-T					Dep-SL					Dep-CN				
	True	Bias	SD	SE	CP	True	Bias	SD	SE	CP	True	Bias	SD	SE	CP
a_1	25.00	0.00	0.54	0.57	0.95	25.00	−0.02	0.60	0.61	0.97	25.00	−0.08	0.57	0.55	0.91
b_1	10.00	−0.05	0.47	0.47	0.95	10.00	−0.02	0.49	0.47	0.92	10.00	−0.00	0.45	0.38	0.91
a_2	80.00	0.00	1.10	1.11	0.96	80.00	−0.09	1.16	1.16	0.92	80.00	−0.20	1.10	1.06	0.90
b_2	18.00	−0.10	0.84	0.86	0.96	18.00	−0.01	0.89	0.87	0.94	18.00	−0.02	0.80	0.70	0.88
a_{31}	−2.70	−0.00	0.17	0.15	0.92	−2.70	−0.00	0.15	0.15	0.93	−2.70	0.03	0.15	0.14	0.92
a_{32}	−0.60	−0.00	0.13	0.13	0.95	−0.60	−0.01	0.14	0.14	0.94	−0.60	0.02	0.13	0.13	0.95
a_{33}	2.00	0.00	0.14	0.14	0.94	2.00	0.01	0.14	0.15	0.93	2.00	0.02	0.15	0.14	0.96
a_{34}	2.80	−0.00	0.15	0.15	0.95	2.80	0.02	0.14	0.16	0.97	2.80	0.02	0.15	0.15	0.96
a_{35}	5.00	0.01	0.20	0.20	0.94	5.00	0.03	0.19	0.20	0.97	5.00	−0.00	0.20	0.19	0.96
a_{36}	6.00	0.01	0.22	0.22	0.94	6.00	0.04	0.22	0.22	0.94	6.00	0.04	0.23	0.22	0.96
b_3	2.00	−0.00	0.12	0.11	0.91	2.00	0.00	0.11	0.11	0.98	2.00	0.00	0.10	0.10	0.94
a_{41}	−0.10	0.00	0.05	0.05	0.98	−0.10	0.00	0.05	0.05	0.92	−0.10	0.00	0.05	0.05	0.90
a_{42}	1.00	−0.00	0.05	0.06	0.97	1.00	0.00	0.06	0.06	0.91	1.00	−0.00	0.06	0.06	0.96
a_{43}	1.80	−0.00	0.07	0.07	0.96	1.80	0.00	0.07	0.07	0.94	1.80	0.01	0.07	0.07	0.97
a_{44}	2.60	0.01	0.09	0.09	0.94	2.60	0.01	0.10	0.09	0.92	2.60	0.00	0.07	0.09	0.96
a_{45}	3.30	0.01	0.12	0.11	0.94	3.30	0.01	0.12	0.11	0.98	3.30	0.00	0.11	0.11	0.94
a_{46}	4.00	0.05	0.16	0.14	0.91	4.00	0.01	0.14	0.14	0.96	4.00	0.02	0.15	0.14	0.95
b_4	0.40	−0.00	0.02	0.03	0.97	0.40	−0.00	0.03	0.03	0.90	0.40	0.00	0.02	0.02	0.96
β_{10}	0.40	0.00	0.11	0.10	0.97	0.40	0.02	0.11	0.11	0.92	0.40	0.02	0.11	0.10	0.94
β_{11}	−0.50	0.01	0.14	0.14	0.94	−0.50	−0.01	0.16	0.15	0.97	−0.50	−0.01	0.14	0.13	0.92
ν	4.00	0.13	0.47	0.44	0.91	3.00	0.20	0.58	0.94	0.97	—	—	—	—	—
γ_1	—	—	—	—	—	—	—	—	—	—	0.10	0.00	0.01	0.01	1.00
ν_1	—	—	—	—	—	—	—	—	—	—	0.10	0.01	0.01	0.02	0.96
ρ	0.40	0.00	0.04	0.05	0.95	0.40	0.00	0.04	0.05	0.99	0.40	−0.00	0.05	0.05	0.95
σ_u	1.30	0.03	0.09	0.07	0.86	1.30	0.01	0.07	0.07	0.93	1.30	0.01	0.07	0.06	0.94

Table 3: Simulation results from models Indep-CN, Dep-CN and Indep-N using half normal distribution priors in setting II in which there were 5% outliers in both the continuous outcome and random effects.

		Indep-N				Dep-CN				Indep-CN			
	True	Bias	SD	SE	CP	Bias	SD	SE	CP	Bias	SD	SE	CP
a_1	25.000	1.300	0.639	0.888	0.761	0.172	0.607	0.531	0.908	0.147	0.574	0.530	0.930
b_1	10.000	6.442	0.767	0.642	0.000	−0.123	0.416	0.383	0.882	0.040	0.393	0.383	0.924
a_2	80.000	3.771	1.095	1.587	0.283	0.033	1.140	1.037	0.941	0.069	1.081	1.034	0.936
b_2	18.000	12.655	0.936	1.181	0.000	−0.180	0.756	0.707	0.921	0.056	0.713	0.697	0.930
a_{31}	−2.700	−0.438	0.147	0.198	0.333	−0.025	0.144	0.142	0.941	−0.028	0.148	0.142	0.962
a_{32}	−0.600	−0.408	0.131	0.180	0.302	−0.006	0.134	0.124	0.914	−0.011	0.123	0.124	0.962
a_{33}	2.000	−0.368	0.146	0.184	0.465	0.007	0.155	0.135	0.901	0.006	0.148	0.135	0.911
a_{34}	2.800	−0.354	0.147	0.191	0.509	0.018	0.160	0.144	0.947	0.012	0.157	0.144	0.930
a_{35}	5.000	−0.366	0.189	0.229	0.660	0.014	0.197	0.191	0.928	0.023	0.195	0.191	0.949
a_{36}	6.000	−0.356	0.228	0.254	0.736	0.029	0.233	0.219	0.947	0.040	0.222	0.219	0.962
b_3	2.000	1.341	0.157	0.167	0.000	−0.022	0.106	0.096	0.901	0.019	0.106	0.096	0.892
a_{41}	−0.100	−0.080	0.051	0.059	0.792	0.003	0.052	0.053	0.941	0.000	0.054	0.053	0.943
a_{42}	1.000	−0.079	0.052	0.063	0.805	0.001	0.051	0.058	0.967	0.002	0.055	0.058	0.968
a_{43}	1.800	−0.075	0.060	0.073	0.874	0.010	0.061	0.069	0.974	0.008	0.065	0.069	0.968
a_{44}	2.600	−0.077	0.087	0.090	0.855	0.015	0.089	0.088	0.934	0.011	0.091	0.087	0.924
a_{45}	3.300	−0.073	0.105	0.113	0.893	0.024	0.110	0.112	0.954	0.021	0.113	0.111	0.955
a_{46}	4.000	−0.075	0.143	0.144	0.918	0.030	0.151	0.144	0.941	0.028	0.161	0.144	0.917
b_4	0.400	0.278	0.038	0.042	0.000	−0.002	0.025	0.025	0.954	0.005	0.025	0.025	0.962
β_{10}	0.400	0.019	0.063	0.074	0.975	−0.003	0.105	0.096	0.908	0.006	0.097	0.092	0.930
β_{11}	−0.500	0.113	0.082	0.088	0.780	0.017	0.131	0.126	0.947	−0.001	0.124	0.125	0.943
ρ	0.400	0.354	0.021	0.025	0.000	0.041	0.044	0.045	0.862	0.031	0.041	0.046	0.911
σ_u	1.300	−0.109	0.056	0.048	0.346	−0.004	0.066	0.063	0.921	−0.001	0.065	0.064	0.949