Effects of Vegetation and Infiltration Feedbacks on Hydrologic Partitioning and Droughts

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Civil and Environmental Engineering in the Graduate School of Duke University
2017
Abstract

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This dissertation addresses feedbacks between vegetation dynamics and land surface response to rainfall events, particularly in Mediterranean climates. Specifically, we ask how a saturated hydraulic conductivity value ($k_s$) that is tied to vegetation biomass affects how water is divided into infiltration and runoff under a range of conditions. The work is divided into three sections.

1. Experiments aimed at obtaining $k_s$ from field measurements. A field campaign in Sardinia was conducted in which a 4 m by 4 m rainfall simulator was constructed and deployed on a number of dates. Measurements of surface runoff from the plot and soil moisture within the plot informed estimates of the effective $k_s$ for each experimental run, and a comparison between $k_s$ and vegetation height measurements revealed a monotonically increasing relationship between the two.

2. Model vegetation and saturation under multiple $k_s$ scenarios. We fit a logistic equation to this relationship and incorporated it into the calculations of a coupled vegetation dynamics and land surface model. Using the model, which is calibrated for the Sardinia field site, we investigated the effect of the variable $k_s$ by comparing the model results of biomass, saturation, and runoff to results using a static $k_s$.

3. Extension of modeling to drought conditions. We used the same model to investigate the effects of a variable $k_s$ on drought recovery by simulating drought
severity through a range of biomass levels relative to a no-drought condition.

We also investigated the effect of the timing of the drought on the relationship between drought severity and recovery.

Our modeling results revealed that the primary result of a variable \( k_s \) is modification of the quantity and mechanism of surface runoff; specifically, runoff increased over the constant \( k_s \) case and shifted from saturation excess runoff to infiltration excess runoff. These effects are more pronounced in drier conditions and when rainfall intensities are in a critical region similar to the \( k_s \) value. We conclude that a dynamic \( k_s \) value is relevant for prediction of surface runoff and may improve the performance of land surface models.
For all those who believed in me when I did not believe in myself.
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List of Abbreviations and Symbols

The following symbols and abbreviations are used commonly throughout the text; others may be context-specific or infrequently used.

Variables

\begin{align*}
  b & \quad \text{Biomass} \\
  \text{LAI} & \quad \text{Leaf area index} \\
  s & \quad \text{Relative soil saturation} \\
  \theta & \quad \text{Volumetric soil moisture} \\
  \text{VPD} & \quad \text{Vapor pressure deficit}
\end{align*}

Fluxes\textsuperscript{1}

\begin{align*}
  d, D & \quad \text{Drainage or leakage} \\
  e & \quad \text{Evaporation} \\
  e_v, E_v & \quad \text{Evaporation of water transpired from vegetation} \\
  e_{bs}, E_{bs} & \quad \text{Bare soil evaporation} \\
  e_t & \quad \text{Total evapotranspiration} \\
  \eta, H & \quad \text{Time step precipitation} \\
  i, I & \quad \text{Infiltration} \\
  p, P & \quad \text{Event or daily precipitation}\textsuperscript{2} \\
  q_I, Q_I & \quad \text{Infiltration excess runoff} \\
  q_S, Q_S & \quad \text{Saturation excess runoff}
\end{align*}
\( q, Q \) Combined runoff

Parameters

- \( k_s \) Saturated hydraulic conductivity
- \( k_s(b) \) Saturated hydraulic conductivity as a function of biomass
- \( n \) Soil porosity or saturated volumetric soil moisture
- \( \omega \) Water use efficiency
- \( z_r \) Root zone depth

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1 For fluxes, lowercase denotes a rate and uppercase is a depth or cumulative depth.

2 \( P \) also represents gross photosynthesis in Chapters 3 and 4.
Acknowledgements

I would first like to thank my funding sources, the National Science Foundation, the Duke University Department of Civil and Environmental Engineering, and the Duke University Graduate School. Next, I am forever indebted to my advising team of Dr. John Albertson and Dr. Amilcare Porporato for their support, ideas, and reassurance.

On a more personal note, I would like to thank members of the Department of Civil and Environmental Engineering, namely Dr. John Dolbow and Dr. Jeffrey Peirce in their role as Director of Graduate Studies and Mrs. Ruby Nell Carpenter as the DGSA. Their unwavering support helped me achieve a goal I at times thought was impossible.

Finally, I am sincerely grateful for the friends and family who have seen me through this eight-year process. My family has kept me grounded in the real world, and my friends have always seemed to give me exactly what I needed. Thank you for inspiring me to continue striving for excellence each and every day.
Water is a critical resource for all aspects of human life. However, despite water covering over seventy percent of Earth’s surface, only 2.5% of it is freshwater, and of that fraction, only 1.2% comprises surface lakes and rivers (Shiklomanov, 1993). In other terms, only 0.03%, or three ten-thousandths, of Earth’s water is readily available for human use. In typical Mediterranean climates, the peaks of temperature/radiation and precipitation oppose one another, leading to hot, dry summers and cool, wet winters. During the winter, runoff from soaking rains fills reservoirs that sustain the population through the dry months. Then, during the relatively short growing season, which is the transition period between winter and summer conditions, grassy vegetation responds quickly to pulses of precipitation. Therefore, accurate quantification and modeling of water in these regions and others are needed for both water resources and vegetation production, especially under scenarios of climate change and increased inter-annual variability of temperature and precipitation.

The Mediterranean region is projected to experience increasing frequency and severity of droughts through the 21st century (Sheffield and Wood, 2008). Studies have already found trends of summer drought starting up to one month earlier (Ruf-
fault et al., 2013) and of larger occurrence probabilities of long dry spells (Sirangelo et al., 2015) throughout the region. Since drought is such a prevalent feature, many studies have focused on the effect of prolonged dryness on vegetation (e.g. Gobron et al., 2005; Pereira et al., 2007).

Furthermore, field studies have shown that Mediterranean vegetation typically has sufficient defense against short-term drought such that it can recover quickly once watering begins again (Lazaro et al., 2001; Xu and Zhou, 2007; Zavalloni et al., 2008; Ceusters et al., 2009; Shinoda et al., 2010). However, severe droughts and droughts at the peak of the growing season can cause reductions in seasonal productivity (Llorens et al., 2004; de Dios Miranda et al., 2009; Jongen et al., 2011; de Dato et al., 2013).

Typically, hydrological models assume that the ability of soil to absorb water depends on static physical properties of the soil and dynamic amounts of available water. However, vegetation and other biological activity may cause the soil characteristics to change on a sub-annual time scale. This idea, building on the current literature in the field, reveals the opportunity to study the intersection of vegetation dynamics and hydrology under increasingly variable climatological conditions. Therefore, the present research aims to improve the accuracy of surface water modeling through a deeper understanding of how vegetation dynamics affect the partitioning of water between, primarily, infiltration into the soil and excess water that flows over the land surface. To make progress in this area, we performed a series of experiments progressing from field work to computational modeling.

Since measured data are often required to inform model development, we begin with the field campaign in Orroli, Sardinia (Italy) beginning in 2010. The team constructed a rainfall simulator to produce artificial rainfall on a 4 m by 4 m plot. Rainfall simulators are frequently used in situations where controlled experiments are desired to elucidate a particular set of information about the land surface and the processes governing it. Accordingly, here we conduct rainfall experiments and
analyze the resulting soil moisture and runoff data to determine effective values for
the parameters that control surface response to precipitation.

Next, the results of the field work are used to estimate the relationship between
vegetation and runoff response. Then, we incorporate resulting relationship into
a coupled land surface and vegetation dynamics model, which determines how the
amount of water in the soil and mass of vegetation change over time. Our hypothesis
is characterized by Figure (1.1). Specifically, we predict that high soil moisture will
allow more vegetation growth, and that this growth will enable additional water to
infiltrate into the soil, generating a positive feedback loop.

Finally, we extend the analysis from behavior under normal conditions to behavior under abnormal conditions. Specifically, we can investigate the effects of the timing and intensity of drought on vegetation and the water balance through the lens of changing land surface properties. If a considerable feedback is present, it may affect recovery from drought, seasonal productivity, and runoff in model results.

Generally, we argue that the response of the land surface to rainfall events can change over time due to reasons other than the amount of water in the soil at the start of a rainfall event. Here, we seek to determine to what extent vegetation controls this response in Mediterranean climates. Armed with the knowledge that runoff response characteristics are not static, we hope to inspire future work in this area with the goal of improving the accuracy of hydrological modeling.

In the remainder of this document, Chapter 2 covers the design and testing of the rainfall simulator, Chapter 3 describes the land surface and vegetation dynamics model along with the effects of a variable saturated hydraulic conductivity on model results, Chapter 4 uses an extension of the same model to investigate the roles

![Figure 1.1: Hypothesized feedback loop.](image)
of drought timing and severity on vegetation growth and water partitioning, and Chapter 5 offers a conclusion and future directions.
Development and testing of a large, transportable rainfall simulator for plot-scale runoff and parameter estimation

Premise
This chapter details the rainfall simulator used to generate estimates for $k_s$ at the Sardinia field site. The data collected from this system inform the $k_s(b)$ relationship used in Chapters 3 and 4.

The following was previously published as follows:

2.1 Introduction

Rainfall simulators have been used extensively to gather runoff, infiltration, and erosion data in both laboratory and field experiments. The results of these experiments are typically used for the purposes of understanding processes such as runoff
and infiltration mechanisms, water routing, and sediment generation and transport at scales ranging from point to hillslope; the emphasis is generally on how surface characteristics such as slope, aspect, soil properties, fire, vegetation, and microtopography affect these processes (e.g., Dunne et al., 1991; Bhardwaj and Singh, 1992; Foster et al., 2000; Stone et al., 2008; Fernandez-Galvez et al., 2008).

More recently, research has started to focus on the effects that changes in surface properties such as land cover or land use can have on the hydrologic cycle (e.g., García-Ruiz et al., 2005; Genxu et al., 2012; Lana-Renault et al., 2011; Maetens et al., 2012; Morán-Tejeda et al., 2010; Romero et al., 2011). These studies often assess how runoff and erosion at the plot and hillslope scale change as vegetation recovers from agriculture, deforestation, grazing, or other disturbances. Therefore, an emerging research need is the further understanding of the interplay between vegetation, additional surface properties, and runoff generation on scales from hillslope to watershed. To begin to answer this question, there is a need for broad experiments that evaluate characteristic land surfaces throughout a watershed and during the course of the year. Specifically, there is currently a lack of studies that estimate soil hydraulic properties at multiple locations in a watershed and at multiple points in time. Indeed, such measurements are needed to improve the runoff response accuracy of watershed hydrologic models. In practice, this requires a large rainfall simulator that is capable of producing high rainfall intensities and can easily be transported between field sites for multiple measurements in space and time.

There are several different types of rainfall simulators, each with its own application, benefits, and shortcomings. The literature on rainfall simulators is extensive; for a more complete review see Battany and Grismer (2000). The standard small- or laboratory-scale (1 m² or smaller) rainfall simulator is a drip tank (Foster et al., 2000; Fernandez-Galvez et al., 2008; Cerdà et al., 1997, and many others). These devices are tanks with a uniform arrangement of holes. They can produce a wide range of
rainfall intensities and typically provide spatially and temporally uniform coverage. The 0.5 m$^2$ simulator of Foster et al. (2000) produced intensities ranging from 7.74 to 28.57 mm h$^{-1}$ with temporal coefficients of variation for intensity ranging from 5.04 to 11.55%, and the 0.24 m$^2$ rainfall simulator of Cerdà et al. (1997) produced 55 mm h$^{-1}$ with a spatial uniformity coefficient of 93%. Larger intensities are also possible; for example Fernandez-Galvez et al. (2008) used a simulator with a range of 0 to 120 mm h$^{-1}$ with an intentionally heterogeneous distribution. The drop size and rainfall intensity in drip tank rainfall simulators are controlled by the diameter of the holes and the pressure in the tank. Another type of small-plot rainfall simulator can use nozzles with a rotating disk that directs water to the plot (originally Grierson and Oades, 1977). These devices also attain uniformity coefficients of at least 75% for a full range of intensities. In general, small-plot rainfall simulators are easily portable due to their small size. However, their limited size (approx. 1 m$^2$) makes them not well suited to capturing plot-scale heterogeneity in surface properties.

For field plots (up to approximately 10 m per side), two standard choices for rainfall simulators are sweeping sprinklers and rows or arrays of nozzles. One of the early simulators of this type, developed by Moore et al. (1983), used oscillating nozzles to obtain intensities of 3.5 to 185 mm h$^{-1}$ with uniformity coefficients between 80.2 and 83.7% over a 4.6 m $\times$ 6.1 m plot. Similarly, the “EMIRE” (Etude et Modélisation de l’Infiltration, du Ruissellement et de l’Erosion) rainfall simulator of Esteves et al. (2000) has a base unit that irrigates a 5 m $\times$ 5 m area with mean intensities from 60 to 76 mm h$^{-1}$ and a mean uniformity coefficient of 80.2%. Fister et al. (2012) developed a rainfall simulator for a 2.2 m$^2$ plot that achieves 85 to 96 mm h$^{-1}$ with a mean uniformity coefficient of 60%; the emphasis was placed on reproducibility rather than uniformity in this case. One of the main advantages of these types of devices is that they can achieve higher drop velocities than drip tanks due to the water pressure in the nozzles. These devices also tend to be expandable
to larger areas by reproducing the base unit. A typical disadvantage of sweeping or oscillating sprinkler rainfall simulators is that the design can include intricate parts and may need to be run by computer, adding to the system expense and complexity. Also, the spray may be intermittent instead of constant as the nozzles sweep back and forth across or rotate around a plot. A typical disadvantage of rows or arrays of spray nozzles is that they tend to have lower uniformity coefficients due to stationary nozzle patterns. The design of Esteves et al. (2000) is close to meeting our needs due to its simplicity and expandability, but the stand pipes are secured with guy wires, making the unit not freestanding and potentially difficult to move or install in desired locations.

Finally, for larger plots on the hillslope scale, the standard design is a rotating boom rainfall simulator, first developed by Swanson (1965). A single unit of this simulator irrigates two parallel plots of 4.3 m × 10.7 m at up to 120 mm h⁻¹ intensity, with two units used to cover plots up to about 23 m long. In addition to covering large areas, these systems can be mounted on a trailer, simplifying transportation between sites. However, trailer-mounted systems cannot be used on steep hillslopes or in other areas that vehicles cannot reach (e.g., forests), which does not make them well suited for use in the wide range of soil and vegetation combinations needed for the present research. Additionally, these systems are usually more complicated than drip tanks or stationary nozzle systems, and, due to the circular spray pattern, these systems are less efficient for covering square or rectangular plots.

Overall, lacking standard designs for rainfall simulators, individual researchers develop devices that suit their particular needs. In this case, the need is for an inexpensive, mechanically simple rainfall simulator system that achieves reasonable uniformity and can be used to estimate soil characteristics such as the saturated hydraulic conductivity in a variety of vegetated locations. Such a device should be easily constructed in areas where access to specialized components is limited, easily
repaired should any components of the system break, and easily transported from one field plot to the next. An additional requirement is that the plot is significantly larger than a single vegetation patch in order to capture the net runoff response that results from the re-infiltration of surface runoff.

To this end, this chapter describes the design and testing of a rainfall simulator using primarily components that can be found in any hardware store. The system irrigates an area of approximately 15 m$^2$ with a range of rainfall intensity rates. In field tests of the system in Sardinia, Italy, data collected included soil moisture throughout the plot and runoff at the downhill edge of the plot.

The intended use of this system is to estimate soil hydraulic properties, primarily the saturated hydraulic conductivity ($k_s$). There is a long history of using field data and numerical inversion to estimate soil parameters (e.g., Kool et al., 1985; Russo et al., 1991; Simunek and Genuchten, 1996; Lassabatere et al., 2006; Ramos et al., 2006; Xu et al., 2012; Rienzner and Gandolfi, 2014). However, these experiments typically use point measurements from instruments such as single ring permeameters or tension disk infiltrometers and therefore do not capture the effective response at the plot scale. Some studies have focused on vertical variability of $k_s$ (Olyphant, 2003; Segal et al., 2008), but few have worked at the plot or hillslope scale. Ram et al. (2012) performed experiments on 4 m wide border strips and used nonlinear optimization to estimate the van Genuchten parameters $\alpha$ and $n$, but fixed $k_s$ based on Guelph permeameter measurements. Also, Verbist et al. (2009) carried out rainfall simulation experiments on 6 m $\times$ 2 m plots and determined that the values of $k_s$ resulting from fitting the two-term Philip infiltration model to measured data were comparable to the values obtained using full nonlinear optimization. Following this approach, we seek to use this new rainfall simulator and simplified parameter estimation methods to calculate physically reasonable values of the saturated hydraulic conductivity of the soil surface and root zone. Overall, this device will improve the
ability of researchers to make more soil hydraulic property measurements over space and time that accurately assess plot-scale runoff response.

2.2 Rainfall simulator system design

Generally, there are 10 important criteria for ideal rainfall simulators (Moore et al., 1983):

1. drop size distribution similar to that of natural rainfall,
2. drop velocity similar to that of natural rainfall,
3. uniform intensity and random drop size distribution over the plot,
4. continuous application over the plot,
5. nearly vertical impact angle,
6. reproducible storm durations and intensities,
7. ability to perform in conditions such as high temperatures and wind,
8. sufficient areal coverage to meet needs of experiment,
9. plot-to-plot and site-to-site portability, and
10. low cost.

Typically, there is some compromise among the criteria as it is difficult to meet all of them. In addition to the primary objective of ease of portability, the choice was made to emphasize low cost, areal coverage, and reproducibility, with the added objective of being able to meet these criteria at at least two different rainfall intensities.
2.2.1 Simulator components

The rainfall simulator consists of several parts: nozzle lines, nozzle supports, a structural frame, and a water delivery system. The four independently operated nozzle lines have either 11 or 12 nozzle assemblies (46 total) connected with 1.5 cm inner-diameter PVC pipe and compression fittings. Each nozzle assembly, as shown in Figure 2.1a, consists of a 0.5 mm opening pressure washing nozzle, threaded hex connector, and threaded hose barb. The barbed ends of the hose barbs were wrapped in teflon tape and gently hammered into a short length of PVC pipe, which was then attached to the compression tees on the main line as shown in Figure 2.1b. The centers of the nozzles are 33.3 cm apart; the lengths of pipe between the nozzles were cut to attain this length and will vary based on the pipe fittings used in other applications. Each line is a total of 4.2 m long with a plugged length of pipe at one end and a 0 to 600 mbar pressure gauge and elbow at the other end that connects to the water delivery system. The nozzle lines are configured as shown in Figure 2.2a, with two sets of nozzles that face each other at a distance of 2.3 m apart.

The nozzle assemblies are supported by L-shaped pieces of metal mounted to stiff metal rods as shown in Figure 2.1b. The nozzles point upwards at alternating angles of 48 and 54° from horizontal; the drops then fall from a height of approximately 3 m. Shorter lengths of the stiff rod support 15 cm bolts that are used to set and maintain the angle. Plastic zip ties hold the nozzles in place on top of the bolts during the experiments. Other than the spacing and angles of the nozzles, other aspects of the support system may be adapted to suit the needs and available materials of other researchers.

The structural frame, shown in schematic in Figure 2.3, consists of six 2 m vertical beams, three 4 m horizontal beams to which the nozzles are mounted, and two 4 m horizontal beams to complete the frame, for overall measurements of 4 m × 4 m
Figure 2.1: Nozzle line setup. (a) Nozzle assembly. The threaded pressure washing nozzle was connected to a short length of PVC pipe using two connectors. (b) Alternating angles of nozzles and pressure gauge used to regulate flow.

The present frame uses 6 cm metal tubing with clamp connectors since it was readily available, but other materials may be used; Schedule 40 PVC is a good inexpensive alternative. Additionally, a plastic mesh attached to these final two beams helped randomize the spray pattern (Foster et al., 2000). The mesh was a heavy-gauge 4 mm grid mesh that was doubled along the edges to prevent ripping and attached to the frame using plastic mesh clips and zip ties to prevent sagging.

The water delivery system conveys water from the 2 m³ tank to the nozzle lines. A submersible pump with a filter, powered by a gasoline generator, pumps water out of the tank via flexible hose. Three tees are used to split the single line into four, and each of these four lines contains a butterfly valve near the pressure gauges so the pressure can be adjusted as necessary.

The overall cost of the rainfall simulator in 2010 was less than USD 1000. This cost includes the simulator components above and the instrumentation discussed below. The modular nature of this unit allows it to be expanded, although attention must be paid to the drop in pressure along the nozzle lines and the resulting drop in
the rate at which water is applied to the plot. Additionally, the components of the system may be changed to suit any particular application.

2.2.2 Instrumentation and data collection

An in-line volumetric flow meter on the tubing between the pump and the nozzle lines measured the total amount of water delivered to the system. The reading on this flow meter was recorded at the beginning and end of each experiment. Time domain reflectometry probes (Campbell Scientific CS616) were used to measure soil moisture. Due to the presence of large rocks in the soil below 20 cm and the desire to take measurements in a regular grid and over a uniform depth, the probes were...
inserted at approximately a 30° angle from horizontal to measure the top 15 cm of the soil. This depth corresponds to the root zone for grassy vegetation. A data logger recorded readings from the probes at 1 s intervals. To estimate the actual amount of water delivered to the plot, two 10 cm simple rain gauges were placed inside the irrigated area.

Runoff from the plot was collected using a tipping bucket flow gauge (Figure 2.4), based on the design of Chow (1976). The metal box is open on the sides and supports two plexiglas buckets that hold approximately 2 L each. The recording mechanism is the magnetic switch component taken from a tipping bucket rain gauge. Water flows into the bucket from a hose attached to the back of the structure (see Figure 2.4b), and the data logger records the time of the pulse created by each tip. At the current field site, the amount of sediment was considered to be negligible compared to the mass of water. Additionally, no sediment appeared to collect in the buckets.
In applications with higher erosion rates, a screen could be placed above the flow gate to filter sediment from the runoff. Finally, it is worth noting that the tipping bucket was developed to take advantage of materials already on hand at the time of construction. Any of a number of other methods of measuring flow may be used to suit the needs of other investigators.

Since the tipping of the bucket is not instantaneous, the amount of water in the bucket can vary with the flow rate. Specifically, while the mass of water needed to make the bucket tip is constant, water continues to collect in the bucket as it tips. Therefore, a higher flow rate can result in a higher volume of water in each bucket. To calibrate the flow gauge for this effect, water with a known flow rate $q$ was directed into the flow gauge and the instrument was video-recorded for the amount of time needed for several tips to occur. The volume in each of the tip of the bucket was inferred from the flow rate and the amount of time the water stream spent in the bucket, $V = q(t_{\text{exit}} - t_{\text{enter}})$. $t_{\text{enter}}$ and $t_{\text{exit}}$ were gathered from the time stamps on the corresponding frames of the video. This process was repeated several times at
different known flow rates. Figure 2.5 shows the data points and relationship of the bucket volume as a function of the tipping frequency as tested in the lab, namely

\[ V = 0.0144f + 2.15, \quad (2.1) \]

where \( f \) is the frequency in tips per minute and \( V \) is the bucket volume in liters. The volume is nearly constant, only ranging between 2.1 and 2.3 L per tip, but the data do show a weak dependence on tipping rate. For each pulse recorded during the experiments, the corresponding number of tips per minute was calculated as \( f = 1/(t_i - t_{i-1}) \), where the pulse time \( t \) is measured in minutes; then Equation (2.1) was used to calculate the expected volume.

**Figure 2.5**: Data and regression for bucket volume as a function of tipping frequency.
2.2.3 Simulator logistics

The components of the rainfall simulator can fit on a small truck with a flat bed approximately 3 to 4 m long. Once on site, it takes four people approximately 90 min to set up the rainfall simulator. Ideally, the water tank will be located close to the plot to reduce the pressure required of the submersible pump, so the tank should be able to be filled on site. If the simulator is to be moved to a nearby plot, it can be picked up by four people and carried to the new location instead of being disassembled and reassembled, leaving only the soil moisture sensors, plot border, and collection system to be re-installed.

Operation of the rainfall simulator can be accomplished with a team of three to four people. Once the simulator and other components are installed, the generator is started and the pump is turned on. The simulator is run for a short time to prime the system, adjust the butterfly vales so each pressure gauge reads 100 mbar \((8 \times 10^3 \text{ Pa})\), and verify that the data logger is working. Then, the starting value of the in-line flow meter and start time are recorded as the system pump is turned back on. During operation, it is useful to have one person monitoring the pressure in the nozzle lines, another monitoring the tipping bucket, and a third person observing the plot to record when and where ponding occurs and to make sure water is draining properly. To turn off the system after the desired experiment time, the pump is turned off and the butterfly valves are closed. The data logger is allowed to continue recording until runoff stops.

2.3 Rainfall simulator testing

Before using the rainfall simulator in the field, laboratory tests were performed to determine the uniformity of the depth of water in the plot under several configurations. Figure 2.2 shows three potential configurations for this particular system. If a nozzle
was not used in a given configuration, it was removed and replaced with a short plugged length of PVC pipe. The first configuration (Figure 2.2b) has 46 nozzles, with 33.3 cm between the center of each nozzle. In the second configuration, which has 31 nozzles, every third nozzle is removed, resulting in nozzle spacings of 33.3 and 66.6 cm. The third configuration has 24 nozzles, with 66.6 cm between all nozzles.

Table 2.1: Summary of nozzle configurations and performance.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>No. of nozzles</th>
<th>Pressure (Pa \times 10^3)</th>
<th>Mean intensity (mm h(^{-1}))</th>
<th>Standard deviation (mm h(^{-1}))</th>
<th>Coefficient of uniformity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>46</td>
<td>8</td>
<td>61.6</td>
<td>18.2</td>
<td>75.7</td>
</tr>
<tr>
<td>2</td>
<td>31</td>
<td>8</td>
<td>43.3</td>
<td>18.9</td>
<td>65.2</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>8</td>
<td>31.8</td>
<td>14.9</td>
<td>62.1</td>
</tr>
</tbody>
</table>

For each configuration, the simulator was run for 30 minutes with water collected in 63 containers on the ground beneath the sprinklers. The 10 cm diameter collection containers were placed in an array with nine containers in the direction of the ground slope and seven containers in the direction perpendicular to the ground slope, both with 50 cm spacing. The uniformity of the applied water was calculated using the Christiansen coefficient of uniformity (CU; Christiansen, 1942):

\[
CU = 100 \cdot \left(1 - \frac{\sum |x - \bar{x}|}{n \bar{x}}\right),
\]

where \(\bar{x}\) is the average of all of the measurements, \(\sum |x - \bar{x}|\) is the sum of the individual deviations from the mean, and \(n\) is the number of measurements taken.

Figure 2.6 shows the rainfall intensity for the three nozzle configurations. The three configurations – with 46, 31, and 24 nozzles – had average rainfall intensities of 61.6, 43.3, and 31.8 mm h\(^{-1}\), respectively. The standard deviations of the three configurations of 18.2, 18.9, and 14.9 mm h\(^{-1}\) are similar to those of the basic unit of the EMIRE rainfall simulator, which covers a similar area using stationary nozzles...
Since each configuration has an operating pressure of $8 \times 10^3 \text{ Pa}$, the average intensity decreases with a decreasing number of nozzles. However, the coefficient of uniformity also decreases with fewer nozzles due to less spatial coverage of the plot. Ideally, one seeks a CU near 80% (e.g., Neff, 1979); however, in the present application there is a trade-off between uniformity and other objectives, such as size, cost, and portability. Accordingly, while the CU values of 75, 65, and 62% are not ideal, they are a reasonable compromise for the objective of emphasizing the applicability and reproducibility of our results over uniformity (Fister et al., 2012). Additionally, rainfall simulators with the highest CU values are typically the smallest (1 m² range); therefore we consider our CU values to be acceptable based on the size of the plot under consideration.

Drop size distributions (DSDs) were measured with a disdrometer for several locations within the irrigation area for both configuration 1 (61.6 mm h⁻¹) and configuration 3 (31.8 mm h⁻¹). In each case, the DSDs for the number of drops in 300 seconds ranged from 0.25 to 3.3 mm drop diameter. The maximum peak frequency occurred at 1 mm, with additional peaks at 2 and 2.6 mm. These results are similar to those documented by Sauvageout and Lacaux (1995). Additionally, drop velocities from the rainfall simulator height were determined to reach terminal velocity for diameters less than 1.5 mm, which covers the peak diameter frequency. The full details of the disdrometer and velocity analyses are shown in Corona (2013).

### 2.4 Field experiments

#### 2.4.1 Field site

The experiments were carried out at an existing field site in Orroli, Italy, on the island of Sardinia (39°41'12.57"N, 9°16'30.34"E) in July and August 2010. The site was accessible by a path leading downhill from the road for approximately 200 m. The simulator components were carried from the road to the site and assembled
around the chosen plot. The water tank was carried to the site while empty and then filled from a water truck located on the road.

In the chosen field plot, tall grasses covered approximately 95 percent of the ground surface, and the land has a gentle slope of approximately 4.6°. The soil is a silt loam with bulk density of 1.38 g cm⁻³ (Montaldo et al., 2008). The choice of a vegetated plot is intentional, as future work with this rainfall simulator will be to assess the effect of vegetation on the estimated soil parameters.
Around the frame of the rainfall simulator, thin sheet metal was inserted approximately 10 cm into the ground to help prevent lateral flow of water across the plot boundaries. To collect the surface runoff, a shallow trench was dug at the bottom edge of the plot, and a plastic ledge was inserted into the ground just below the surface to direct the water onto plastic sheeting. From here, the water flowed by gravity into a tube that was connected to the tipping bucket flow gauge. As shown in Figure 2.7, the soil moisture probes were placed at 1 m intervals in the irrigated section of the plot. Additionally, to verify rainfall intensity, two rain gauges were placed inside the plot. A slight rill already existed in the plot due to an animal footpath, and as a result the outlet of the plot was approximately at the location of soil moisture probe 14.

2.4.2 Experiments

Three experiments were conducted over the course of several days. Each experiment had a different combination of initial soil moisture (\(\theta_0\)), experiment duration (\(T\), minutes), and rainfall intensity (\(p\), mm h\(^{-1}\)). For each experiment period, the rainfall was allowed to continue until the soil was saturated and runoff reached a constant rate. The test conditions are described in Table 2.2.

<table>
<thead>
<tr>
<th>Exp. no.</th>
<th>Date</th>
<th>(\theta_0)</th>
<th>(p) (mm h(^{-1}))</th>
<th>(T) (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29 Jul</td>
<td>0.50</td>
<td>61.6</td>
<td>37</td>
</tr>
<tr>
<td>2</td>
<td>3 Aug</td>
<td>0.32</td>
<td>61.6</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>5 Aug</td>
<td>0.38</td>
<td>31.8</td>
<td>169</td>
</tr>
</tbody>
</table>

2.5 Method of soil parameter estimation

The intended use of this rainfall simulator is to glean surface properties from rainfall experiments. Alberts et al. (1995) demonstrated a sufficient fit of an infiltration
Figure 2.7: Field experiment setup. (a) Field setup, including soil moisture probes, border around edges, rain gauges, nozzles, distribution mesh, and collection system. (b) Details of the plot area, with soil moisture probe location and effective coverage area. The outlet of the plot is at probe 14.

Model to observed data can be found by running the model with a range of values for the saturated hydraulic conductivity and selecting the value that minimizes model error with respect to the observed infiltration. This method can be applied here to obtain $k_{s,\text{bulk}}$ and $k_{s,\text{surf}}$ from the experiment data in a similar manner as Verbist et al. (2009). Here, we define $k_{s,\text{surf}}$ and $k_{s,\text{bulk}}$ to be the saturated hydraulic conductivities of the surface layer and remainder of the root zone, respectively. We use these
two values rather than one $k_s$ value to separate the behavior of the soil surface, which determines the amount of water that infiltrates, from that of the bulk soil, which determines overall soil water content. Based on previous work for this site (Montaldo et al., 2008), the estimated saturated hydraulic conductivity for this soil is approximately 18 mm h$^{-1}$, and, since land use and vegetation cover have not changed at the site, the estimate is still considered accurate to an order of magnitude. Therefore, if $k_s$ values obtained from model fitting are close to this previous estimate, then the rainfall simulator indeed performs as intended.

A common model for infiltration is the two-parameter Philip equation (Philip, 1957):

$$i_t^* = \frac{1}{2} S_i t^{-1/2} + A_t,$$

where $i_t^*$ [L T$^{-1}$] is the infiltration rate at time $t$ under ponded conditions, $S_i$ [L T$^{-1/2}$] is the sorptivity, and $A_t$ [L T$^{-1}$] is a parameter similar to, but not necessarily equal to, the saturated hydraulic conductivity $k_s$. This infiltration equation is used in this analysis because of its simplicity and success on short timescales. In this implementation, the sorptivity expression of Sivapalan et al. (1987) is used:

$$S_i = \left[ \left( 2k_{s,\text{surf}} (\theta_s - \theta_t) \right)^2 \left( -\frac{\psi_a}{\theta_s - \theta_h} \right) \right]$$

$\left( \frac{1}{(2b + 3)} + \frac{\theta_s - \theta_h}{\theta_s - \theta_t} \right)^{1/2}$,

where $k_{s,\text{surf}}$ is the saturated hydraulic conductivity of the surface layer [L T$^{-1}$], $\theta_s$ is the saturated volumetric soil moisture (porosity), $\theta_t$ is the volumetric soil moisture at time $t$, $\theta_h$ is the hydrostatic (minimum) volumetric soil moisture, $\psi_a$ is the air entry suction head [L], and $b$ is the soil water retention curve parameter (Clapp and Hornberger, 1978). Eagleson (1978) showed the application of Philip’s (1960) work
on the diffusion equation on short timescales as

\[
A = \frac{1}{2} \left( k_s + k_0 \right). \tag{2.5}
\]

Using the Campbell (1974) definition of hydraulic conductivity for the surface layer as \(k_{\text{surf}} = k_{s,\text{surf}} \left( \theta/\theta_s \right)^{2b+3}\), Equation (2.5) may be written as a dynamic parameter, namely

\[
A_t = \frac{1}{2} k_{s,\text{surf}} \left( 1 + \left( \theta_t/\theta_s \right)^{2b+3} \right). \tag{2.6}
\]

The unsaturated hydraulic conductivity of the bulk soil is also calculated using the definition of Campbell (1974), namely

\[
k_{t,\text{bulk}} = k_{s,\text{bulk}} \left( \frac{\theta_t}{\theta_s} \right)^{2b+3}, \tag{2.7}
\]

where \(k_{s,\text{bulk}} \, [L \, T^{-1}]\) is the saturated hydraulic conductivity of the bulk soil rather than the soil surface.

The calculation of soil moisture comes from a one-dimensional water balance on a soil layer with thickness \(\Delta z\), beginning with \(\Delta S = V_{\text{in}} - V_{\text{out}}\), where \(S\) is the water storage in the layer, \(V_{\text{in}}\) is the water entering the layer, and \(V_{\text{out}}\) is the water exiting the layer. \(S\) can be represented as \(\theta \Delta z\), so \(\Delta S = \Delta z (\theta_t - \theta_{t-\Delta t})\). \(V_{\text{in}}\) is the water infiltrating from above, so during a small time step \(\Delta t\), \(V_{\text{in}} = i \Delta t\). Neglecting evapotranspiration, \(V_{\text{out}}\) is the drainage of water through the bottom of the layer. Use of Darcy’s law, \(v = -k \frac{dh}{dz}\), with a unit gradient yields \(V_{\text{out}} = k \Delta t\). Therefore, the soil moisture prediction equation is

\[
\frac{\theta_t - \theta_{t-\Delta t}}{\Delta t} \Delta z = i_t - k_{t,\text{bulk}}, \quad \text{or}
\]

\[
\theta_t = \theta_{t-\Delta t} + \left( i_t - k_{t,\text{bulk}} \right) \frac{\Delta t}{\Delta z}, \tag{2.8}
\]

where \(\Delta z \, [L]\) is the soil depth being considered and \(i_t \, [L \, T^{-1}]\) is the actual infiltration rate, defined by

\[
i_t = \min\left(i_t^*, p\right). \tag{2.9}
\]
See Table 2.3 for the parameters used in this analysis.

Infiltration is modeled using the above equations with a time step of $\Delta t = 1\text{ min.}$ First, $A$ and $S$ are calculated according to Equations (2.6) and (2.4). Then, $i^*_t$ and $i_t$ are calculated using Equations (2.3) and (2.9). The current bulk soil hydraulic conductivity is calculated with Equation (2.7), and then the bulk soil moisture $\theta_t$ is calculated using Equation (2.8).

Since Equation (2.3) is defined under ponded conditions, a correction must be made to the time used in the infiltration calculations to account for the period before ponding actually begins. Dingman (2004) accomplishes this using a condensed ponding time ($t_{cp}$), which acts to delay the start of runoff in the Philip model. Without it, modeled runoff begins much before observed runoff. Following Dingman’s approach, $t_s$ is defined as the first time in the original calculations when $p > i^*_t$. The total potential volume that can infiltrate before $t_s$ is

$$I_p = \sum_{t=0}^{t_s} i^*_t, \quad (2.10)$$

and, since $p < i^*$ in this time period, the time it takes for the volume $I_p$ to infiltrate is

$$t_p = \frac{I_p}{p}. \quad (2.11)$$

The condensed ponding time is

$$t_{cp} = t_p - t_s, \quad (2.12)$$

which can be thought of as a correction for when runoff will actually begin compared to when it would start under ponded conditions.

Then, the above calculations for $i_t$, $k_{t,\text{bulk}}$, and $\theta_t$ are repeated using $\hat{t} = t - t_{cp}$ in place of $t$, generating the values $i_{\hat{t}}$, $k_{\hat{t},\text{bulk}}$, and $\theta_{\hat{t}}$, which now account for the ponding correction. Finally, the modeled runoff is calculated as

$$q_{\hat{t}} = \max \left(p - i_{\hat{t}}, 0\right). \quad (2.13)$$
Table 2.3: Parameters used in Philip infiltration model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_s$</td>
<td>0.568</td>
<td>[–]</td>
<td>Field data</td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>0.08</td>
<td>[–]</td>
<td>Montaldo et al. (2008)</td>
</tr>
<tr>
<td>$\psi_{ae}$</td>
<td>0.79</td>
<td>[m]</td>
<td>Clapp and Hornberger (1978)</td>
</tr>
<tr>
<td>$b$</td>
<td>8</td>
<td>[–]</td>
<td>Clapp and Hornberger (1978)</td>
</tr>
<tr>
<td>$z$</td>
<td>0.15</td>
<td>[m]</td>
<td>Field data</td>
</tr>
</tbody>
</table>

Equations (2.3), (2.4), and (2.6) show that the infiltration of water through the surface, and accordingly the runoff, depend strongly on $k_{s,\text{surf}}$ but not on $k_{s,\text{bulk}}$. Likewise, Equations (2.8) and (2.7) show that $k_{s,\text{bulk}}$ affects $\theta_t$ but not $i_t$. Therefore, for a period of total duration $T$ and cumulative runoff $Q_T$,

$$e_{Q_T} = |Q_{T,\text{mod}} - Q_{T,\text{obs}}|$$

(2.14)

and

$$e_\theta = \left(\frac{\sum_{t=0}^{T} (\theta_{t,\text{mod}} - \theta_{t,\text{obs}})^2}{T}\right)^{1/2}$$

(2.15)

can be used as measures of error that, when minimized, indicate the optimal values of $k_{s,\text{surf}}$ and $k_{s,\text{bulk}}$, respectively. $e_{Q_T}$ uses the final $Q$ value to capture the overall behavior of the plot in producing runoff, and $e_\theta$ uses the time series of $\theta$ to capture the evolution of soil moisture during the experiment.

To optimize $k_{s,\text{surf}}$ and $k_{s,\text{bulk}}$, the infiltration model was run using all combinations of the two values ranging from 1 to 30 mm h$^{-1}$ at a step of $\Delta k = 10^{-7}$ m s$^{-1} = 0.36$ mm h$^{-1}$. $e_{Q_T}$ and $e_\theta$ were calculated for each combination, and the combination with the smallest value of

$$e^* = e_{Q_T} + e_\theta$$

(2.16)

was selected for the optimal values of $k_{s,\text{surf}}$ and $k_{s,\text{bulk}}$. 26
2.6 Results and discussion

2.6.1 Soil moisture

While the ultimate goal is to accurately model the infiltration and runoff at the plot scale, we have shown that soil moisture values can be used in the calculation of soil saturated hydraulic conductivity. Since the soil moisture values vary throughout the plot, we use a spatial average of the measured data for the error calculation in Equation (2.15). However, examining the spatial variability within the plot allows for a better understanding of each experiment.

According to previous fieldwork at this site (Montaldo et al., 2008) and examination of the measured soil moisture at saturation, the soil has an average porosity of $\theta_s = 0.568$. However, the saturation value at each individual probe location varies. In order to ease the analysis of the soil moisture, the values were scaled using

$$\hat{\theta}_{x,i} = \frac{\theta_{x,i}}{\max(\theta_x)},$$  \hspace{1cm} (2.17)

where $\theta_{x,i}$ is the reading from probe $x$ at time $t_i$, $\max(\theta_x)$ is the maximum reading for probe $x$ during the experiment, and $\hat{\theta}_{x,i}$ is the scaled value with a maximum value of 1. The initial soil moisture $\theta_0$ as shown in Table 2.2 was then represented as

$$\theta_0 = \text{mean}\left(\hat{\theta}_{x,0}\right) \cdot \theta_s.$$  \hspace{1cm} (2.18)

The first experiment, performed on 29 July (Figure 2.8a), occurred after several test runs of the rainfall simulator, so the soil was close to saturated when the experiment began with $0.82 \leq \hat{\theta}_{x,0} \leq 0.96$ and a mean value of 0.89. Most of the plot reached saturation between 10 and 15 minutes after the experiment began.

The second experiment, performed on 2 August (Figure 2.8b), began with $0.41 \leq \hat{\theta}_{x,0} \leq 0.76$ and a mean value of 0.57. A variety of paths to saturation is present. The probes that take longer to reach saturation are all near the edges of the irrigated area, so the slower increase in soil moisture may be due to either the lateral flow
Figure 2.8: Soil moisture as $\hat{\theta}_{x,t}$ for each TDR probe. (a) Experiment 1. The soil started near saturation due to previous test runs. (b) Experiment 2. (c) Experiment 3.

to, or lack of flow from, adjacent areas of the plot (see Figure 2.7b). For instance, probes 1 and 15, which do not reach saturation during the experiment, are on the border of the irrigated area, as is probe 12, which takes much longer to show an increase in soil moisture than the other probes. Aside from probes 1 and 15, the remainder of the plot reached saturation by approximately minute 90 of rainfall at an intensity of 61.6 mm h$^{-1}$.

Another feature of the measurements for this experiment is an abrupt change in
the time rate of change of soil moisture (i.e., slope of $\hat{\theta}_x$ vs. $t$) for some portions of the plot. This is particularly evident in probes 3, 12, 13, and 14. Considering just the bottom row of the plot, this change occurs first at probe 14, followed by probe 13 then probe 12. This progression seems to suggest that these probe locations change from having water input only from the rainfall simulator to having both rainfall simulator input and overland flow input from upslope over the course of the experiment.

The third experiment, performed on 5 August (Figure 2.8c), started with $0.55 \leq \hat{\theta}_{x,0} \leq 0.78$ and a mean value of 0.67, placing it in between experiments 1 and 2 in terms of initial soil moisture. However, this experiment was performed with nozzle configuration 3, meaning a rainfall intensity of 31.8 mm h$^{-1}$. As a result, it took nearly 3 h for the whole plot to reach saturation. Again, probe 12 takes the longest to reach saturation. There is also a sharp change in the slopes of the soil moisture time series for probes 4, 5, and 6; for each of these three probes, the change in slope occurs soon after the probes immediately downslope (8, 9, and 10, respectively) reach saturation. This feature again seems to indicate a change in the contribution from rainfall only to rainfall plus runoff from nearby areas.

Overall, the time to saturation in each of the three experiments increased as the rainfall intensity and the initial soil moisture decreased, as expected. To investigate the relationship between the location of the soil moisture probes and the spatial variability of the measurements, the soil moisture results have been categorized by location. Probes 2, 3, 5, 6, 9, 10, 13, and 14 are considered interior probes, and the remaining are considered border probes. Figure 2.9 shows the comparison of the averages of the interior and border soil moisture probes compared to the overall average. In each case, the border soil moisture probes overall have lower readings than the interior probes. The difference is particularly pronounced in experiment 2, when the difference between the interior and border probes in the middle of the experiment exceeds the difference in the initial values. This is also true, to a lesser
degree, in experiment 3. A possible explanation for the discrepancy is that the rainfall intensity distribution as shown in Figure 2.6 generally appears to favor the interior of the plot over the edges.

![Graphs showing soil moisture for different experiments](image)

**Figure 2.9:** Soil moisture averaged for all probes, border probes, and interior probes for the three experiments.

### 2.6.2 Runoff

The tips of the tipping bucket flow gauge were converted to time series of runoff rate and cumulative runoff through Equation (2.1). Specifically, with \( t_i \) being the time in minutes of the \( i \)th bucket tip and \( V_i \) the volume of that tip in liters, the runoff rate \( q_i \) in mm h\(^{-1}\) is

\[
q_i = \frac{V_i}{t_i - t_{i-1}} \cdot \alpha, \tag{2.19}
\]
where

\[ \alpha = 60 \text{ min} \cdot 0.001 \text{ m}^3 \text{ L}^{-1} \cdot 1000 \text{ mm m}^{-1} / 15.12 \text{ m}^2 \]

\[ = 3.968 \quad (2.20) \]

is the constant that converts L min\(^{-1}\) to mm h\(^{-1}\). The infiltration rate that results from this runoff is

\[ i_i = p - q_i. \quad (2.21) \]

The cumulative runoff \( Q_i \) is

\[ Q_i = \sum_{j=0}^{i} V_i \cdot \beta, \quad (2.22) \]

where

\[ \beta = 0.001 \text{ m}^3 \text{ L}^{-1} \cdot 1000 \text{ mm m}^{-1} / 15.12 \text{ m}^2 = 0.06614 \quad (2.23) \]

converts L to mm. Since the tipping bucket flow gauge causes \( q_i \) to oscillate around what should be a smooth time series, a four-point moving average was calculated as

\[ \hat{q}_i = \frac{1}{4} \sum_{j=i-3}^{i} q_j. \quad (2.24) \]

Figure 3.7 shows both \( \hat{q}_i \) and \( Q_i \), in addition to the times at which the soil moisture probes reached full saturation. These figures provide a visual check of whether or not the rainfall simulator provides reasonable results. In each of the three experiments, runoff begins approximately when the first probe reaches saturation. The first probe to reach saturation in all three experiments was probe 14, which is next to the outlet of the plot. With \( t_{x,s} \) as the time when probe \( x \) reaches saturation, the average time to saturation for each experiment, \( \bar{t}_s \), is defined as \( \frac{1}{n} \sum_{x=1}^{n} t_{x,s} \) with \( n = 15 \). \( \bar{t}_s \) occurs while the runoff rate is increasing, and the runoff rate changes little after the final probe reaches saturation. The exception is experiment 2, which did not continue beyond time when the final probe reached saturation.
Figure 2.10: Runoff rate and cumulative runoff for the three experiments. The vertical lines represent times when the soil moisture probes reached maximum values.

It is also useful to compare the runoff rates between experiments. For both experiments 1 and 2, the average of the runoff rates near the end of the experiment is approximately 47.6 mm h\(^{-1}\). When compared to the applied rainfall rate of 61.6 mm h\(^{-1}\), the final runoff coefficient \((C_f = q_f/p)\) once the plot is fully saturated is 0.77. For experiment 3, the average rate near the end of the experiment is 24.2 mm h\(^{-1}\), or \(C_f = 0.76\). The similar runoff coefficients emphasize the proportionality between rainfall and runoff rates.

The overall runoff coefficient is

\[
C = \frac{Q_f}{P},
\]  

(2.25)

where \(Q_f\) is the cumulative runoff for the entire experiment and \(P\) is the total precipitation applied during duration \(T\). While the \(C_f\) values are equivalent across the experiments, the \(C\) values differ, with values of 0.61, 0.46, and 0.51. This is expected
due to the varying antecedent soil moisture states as higher $\theta_0$ values correspond to higher $C$ values. Therefore, in this brief analysis of the measured data of soil moisture and runoff, the experiments show that the rainfall simulator produces physically reasonable results.

2.6.3 Parameter estimation

Using the estimation method shown in Section 2.5, the $k_{s,\text{surf}}$ values for the experiments 1, 2, and 3 are 16.5, 8.56, and 4.24 mm h$^{-1}$, respectively, and the $k_{s,\text{bulk}}$ values are 11.8, 7.48, and 8.20 mm h$^{-1}$. Verbist et al. (2009) found a $k_s$ range of approximately 17 to 55 mm h$^{-1}$ for a coarse loamy soil for four replicates at 120 mm h$^{-1}$ precipitation intensity. Since our experiments were performed at two different precipitation intensities, we consider the overall range of 4.24 to 16.5 mm h$^{-1}$ to be acceptable.

The comparison of the observed and modeled time series based on these hydraulic conductivities is shown in Figure 2.11. Optimized hydraulic conductivity values are as follows: experiment 1: $k_{s,\text{surf}} = 16.5$ mm h$^{-1}$, $k_{s,\text{bulk}} = 11.8$ mm h$^{-1}$; experiment 2: $k_{s,\text{surf}} = 8.56$ mm h$^{-1}$, $k_{s,\text{bulk}} = 7.48$ mm h$^{-1}$; and experiment 3: $k_{s,\text{surf}} = 4.24$ mm h$^{-1}$, $k_{s,\text{bulk}} = 8.2$ mm h$^{-1}$.

The comparison of the observed and modeled time series based on these hydraulic
conductivity values are shown in Figure 2.11. Additionally, the observed final infiltration rate for each experiment is defined as

\[ i_f = p - \frac{\sum_{t=t_f}^{T} q_t}{T - t_f}, \]  

(2.26)

where \( t_f \) is the time at which the infiltration and runoff appear to become constant.

The comparison of \( k_{s,\text{surf}} \), \( k_{s,\text{bulk}} \), and \( i_f \) is shown in Figure 2.12. The parameter which shows the largest range over the three experiments is \( k_{s,\text{surf}} \). One possible reason for the reduction in \( k_{s,\text{surf}} \), particularly between the first two experiments, is surface sealing that can occur after water is applied to the surface. Another reason for the difference between \( k_{s,\text{surf}} \) in experiments 1 and 2 and that of experiment 3 is the smaller rainfall intensity in the third experiment. The most stable values are for \( k_{s,\text{bulk}} \). This is likely due to the behavior of the bulk soil being less affected by the conditions that vary between experiments. The literature value of saturated hydraulic conductivity for a silt loam with a mean clay fraction of 0.14 is approximately 25 mm h\(^{-1}\) (Clapp and Hornberger, 1978). The optimized values here are smaller, despite having a clay fraction of 0.05. One possible reason for the disparity is the high silt fraction in this soil (0.76). The most apparent reason, however, is that field values of saturated hydraulic conductivity are highly dependent on rainfall intensity (e.g., Bowyer-Bower, 1993; Stone et al., 2008; Langhans et al., 2010). Larger experimental rainfall intensities are likely to lead to higher \( k_{s,\text{surf}} \) and \( k_{s,\text{bulk}} \) values of this soil.

Since the soil properties do not change between the current experiments, ideally the range of \( k_{s,\text{surf}} \) and \( k_{s,\text{bulk}} \) values should be smaller than is shown in Figure 2.12. The remaining model parameters that have not been determined by field data are \( b \) and \( \psi_{ae} \), we can include them in the optimization to see if the ranges of the \( k_s \) values improves. Values of \( b \) from 5 to 9 with a step of \( \Delta b = 1 \) and values of \( \psi_{ae} \) from 0.3 to 0.8 m with \( \Delta \psi = 0.083 \) m were incorporated into the parameter space.
Figure 2.12: Comparison of infiltration and conductivity rates for the three experiments.

Table 2.4: Parameters for the model optimization (a) using assumed values of $b$ and $\psi_{ae}$, (b) including all four parameters in the optimization, and (c) using $\hat{\psi}_{ae}$ and $\hat{b}$.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$k_{s,\text{bulk}}$</th>
<th>$k_{s,\text{surf}}$</th>
<th>$b$</th>
<th>$\psi_{ae}$</th>
<th>$\theta$</th>
<th>$e_{Qr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mm h$^{-1}$</td>
<td>mm h$^{-1}$</td>
<td></td>
<td>m</td>
<td>[-]</td>
<td>mm</td>
</tr>
<tr>
<td>(a) Optimization with assumed $\psi_{ae}$ and $b$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11.8</td>
<td>16.5</td>
<td></td>
<td>2.84 $\times$ 10$^{-3}$</td>
<td>6.94 $\times$ 10$^{-3}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>7.48</td>
<td>8.56</td>
<td>8</td>
<td>0.79</td>
<td>2.45 $\times$ 10$^{-2}$</td>
<td>6.61 $\times$ 10$^{-5}$</td>
</tr>
<tr>
<td>3</td>
<td>8.20</td>
<td>4.24</td>
<td></td>
<td>8.21 $\times$ 10$^{-3}$</td>
<td>2.10 $\times$ 10$^{-2}$</td>
<td></td>
</tr>
<tr>
<td>(b) Optimization using all parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11.2</td>
<td>18.0</td>
<td>5</td>
<td>0.63</td>
<td>2.69 $\times$ 10$^{-3}$</td>
<td>4.63 $\times$ 10$^{-4}$</td>
</tr>
<tr>
<td>2</td>
<td>13.7</td>
<td>14.8</td>
<td>9</td>
<td>0.38</td>
<td>2.10 $\times$ 10$^{-2}$</td>
<td>8.60 $\times$ 10$^{-4}$</td>
</tr>
<tr>
<td>3</td>
<td>6.52</td>
<td>8.68</td>
<td>7</td>
<td>0.30</td>
<td>5.50 $\times$ 10$^{-3}$</td>
<td>2.12 $\times$ 10$^{-3}$</td>
</tr>
<tr>
<td>(c) Optimization using $\hat{\psi}_{ae}$ and $\hat{b}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11.8</td>
<td>18.6</td>
<td></td>
<td>2.82 $\times$ 10$^{-3}$</td>
<td>2.05 $\times$ 10$^{-3}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12.5</td>
<td>13.6</td>
<td>7</td>
<td>0.44</td>
<td>2.10 $\times$ 10$^{-3}$</td>
<td>6.94 $\times$ 10$^{-3}$</td>
</tr>
<tr>
<td>3</td>
<td>6.04</td>
<td>6.76</td>
<td></td>
<td>6.50 $\times$ 10$^{-3}$</td>
<td>5.56 $\times$ 10$^{-2}$</td>
<td></td>
</tr>
</tbody>
</table>
so each model realization used a unique combination of the four parameters. Again, the parameter assignment that minimized $e^*$ was selected as the optimal set for each equation. These results are shown in the second block of Table 2.4.

$b$ and $\psi_{ae}$ are constant for one plot, so the final step was to average the values of $b$ and $\psi_{ae}$ for the three experiments to get the assumed optimal values $\hat{b} = 7$ and $\hat{\psi}_{ae} = 0.44$ m. The model was then run using $\hat{b}$ and $\hat{\psi}_{ae}$ along with the combinations of $k_{s,\text{surf}}$ and $k_{s,\text{bulk}}$ used in the first optimization. The values that minimize $e^*$ are $k_{s,\text{surf}} = 11.8$, 12.5, and 6.04 mm h$^{-1}$, along with $k_{s,\text{bulk}} = 18.6$, 13.6, and 6.76 mm h$^{-1}$. The modeled $Q$ and $\theta$ are shown in Figure 2.13, the rate comparison is in Figure 2.14, and the overall results are in Table 2.4. The $k_{s,\text{bulk}}$ values for the higher intensity experiments are close to the 18 mm h$^{-1}$ that is expected for this soil, which indicates that, despite the heterogeneity of rainfall intensity throughout the plot, this rainfall simulator can be used to generate reasonable estimates of the primary soil properties that govern infiltration and runoff.

![Figure 2.13](image)

**Figure 2.13:** Observed and modeled $Q$ and $\theta$ using $\hat{b} = 7$ and $\hat{\psi}_{ae} = 0.44$ m. Optimized hydraulic conductivity values are as follows: experiment 1: $k_{s,\text{surf}} = 18.6$ mm h$^{-1}$, $k_{s,\text{bulk}} = 11.8$ mm h$^{-1}$; experiment 2: $k_{s,\text{surf}} = 13.6$ mm h$^{-1}$, $k_{s,\text{bulk}} = 12.5$ mm h$^{-1}$; and experiment 3: $k_{s,\text{surf}} = 6.76$ mm h$^{-1}$, $k_{s,\text{bulk}} = 6.04$ mm h$^{-1}$.

While little difference can be seen between Figures 2.11 and 2.13, a comparison
of Figures 2.12 and 2.14 emphasizes the changes in $k_{s,\text{surf}}$ and $k_{s,\text{bulk}}$ following the optimization process. The ranges of the rates for experiments 2 and 3 are smaller than with the previously assumed values for $b$ and $\psi_{ae}$. Additionally, $k_{s,\text{bulk}}$ is closer to the final infiltration rate, particularly for experiment 2, and the $k_{s,\text{bulk}}$ values for experiments 1 and 2 are more comparable, which is an improvement since these two experiments have the same water application rate. The reduction in $k_{s,\text{surf}}$ from experiment 1 to experiment 2 is still present, which is still likely a result of surface sealing in between experiments. Since the values of $k_{s,\text{surf}}$ and $k_{s,\text{bulk}}$ are more consistent within experiments and across experiments, it appears that this optimization method has improved the estimates for the saturated hydraulic conductivities of the soil in this plot.

2.7 Conclusion

This paper presented the design, construction, and testing of a low-cost rainfall simulator system. This system has several significant advantages. Primarily, it is comprised of readily available and inexpensive materials. Additionally, the 4 m ×
4 m module can be replicated to expand the areal coverage of the simulator. It is fairly easy to change the configuration of the system to produce different intensities, and the low operating pressure means that the power requirements of the system are low as well.

There are, however, some limitations of the system. Primarily, the rainfall intensity pattern is not as uniform as typically desired for rainfall simulators. Additionally, since some water falls on the surface outside the effective irrigated area, the simulator is not as water efficient as it could be. Due to the low operating pressure, the system needs to be close to level to provide even distribution of water, so it cannot be used on steep slopes without adjusting the lengths of the vertical supports. Finally, since the water is sprayed upwards, the system does not work properly in windy conditions without wind shields that are at least 3 m tall in place around the plot frame.

After measuring soil moisture and runoff during three experimental runs of the system, analysis using the Philip infiltration model showed that this rainfall simulator can produce data that allow for the estimation of reasonable values for the saturated hydraulic conductivity of the soil. Specifically, we showed that performing a three-step parameter search optimization that first estimates on $k_s$ with assumed values of $b$ and $\psi_{ae}$, then estimates $b$ and $\psi_{ae}$ using the $k_s$ initial estimates, and finally uses $\hat{b}$ and $\hat{\psi}_{ae}$ to get new values for the $k_s$ estimates allows for the estimation of a total of four soil hydraulic parameters. The implication of this result is the ability to use this rainfall simulator system to estimate parameters needed for hydrologic modeling with less reliance on prior knowledge.

Continued work with this system will be to use the rainfall simulator on plots with different vegetation fractions, types, and patterns. The purpose of gathering the data in these different conditions will be to calibrate model parameters for a coupled hydrologic–vegetation dynamics model. This model will be used to investigate how changes in vegetation growth during the transition from the rainy season to the dry
season in Sardinia will affect overall surface runoff volumes.
Impact of biomass-infiltration feedback on vegetation dynamics and hydrologic partitioning

Premise
This chapter builds on the data collected using the rainfall simulator from Chapter 2. The coupled vegetation and hydrologic model developed here is used to evaluate the effect of a variable $k_s$ on vegetation growth and runoff production in a range of climate conditions in this chapter, and it is modified for use in drought conditions in Chapter 4.

3.1 Introduction

Vegetation and hydrology are intricately linked in nearly all ecosystems. Plants depend on surface and vadose zone water for survival, and in return, they transpire water back into the atmosphere. Additionally, plants can affect hydrological processes by, for example, changing flow patterns of surface water or using soil water before it drains out of the root zone. In typical Mediterranean climates, rising temperatures and decreasing precipitation overlap for the brief growing season, during
which grassy vegetation rapidly increases in biomass when water is available. Thus, understanding the intersection of ecology and hydrology is relevant for studying each field individually.

Accurate characterization of the water balance is required for prediction of both vegetation growth and surface runoff. Soil saturated hydraulic conductivity \(k_s\) has first-order effects on the water balance because, along with other soil hydraulic properties, it determines how rainfall is partitioned into drainage and runoff (Albertson and Kiely, 2001). These soil properties are known to be spatially heterogeneous, varying with factors such as soil type, land use, and biological activity.

Many studies have investigated how the hydrologic response of a plot changes over time, on the order of years, following land use change or recovery from a disturbance (e.g. García-Ruiz et al., 2005; Rulli et al., 2006; Lana-Renault et al., 2011; Genxu et al., 2012). In general, infiltration tends to increase over time following events such as fire and agricultural use. Some of these studies include analyses of \(k_s\); for instance, Li and Shao (2006) found that \(k_s\) increased significantly as recovering farmland progressed through the grassland to forested stages. Additionally, several studies have found that \(k_s\) in irrigated and tilled agricultural plots follows a progression of increasing through the winter months, decreasing at the beginning of irrigation periods, and increasing again through the growing season as a result of wetting and drying cycles, root formation, and biological activity (Bodner et al., 2008; Mubarak et al., 2009; Alletto et al., 2015). Finally, ecologists rely on a positive feedback between vegetation biomass and infiltration to explain the formation of vegetation growth patterns such as bands, rings, and crescents (e.g. HilleRisLambers et al., 2001; Thompson et al., 2010).

However, for locations without tilling or changing land use or land cover, the standard practice is to use one set of soil hydraulic parameters for a given location. There are, fortunately, some studies on how \(k_s\) may change throughout the year.
Rienzner and Gandolfi (2014) performed infiltration experiments on several parcels within one field and found weak seasonal variations in $k_s$ in some cases but not in others. They attributed the changes in $k_s$ to the development of macropores. Das Gupta et al. (2006) found a strong seasonal pattern in unsaturated hydraulic conductivity and attributed the trends to a combination of antecedent soil moisture and activation of macropores near saturation. Olyphant (2003) found a somewhat cyclic pattern of estimated $k_s$ with the highest values occurring in early summer, and Bagarello and Sgroi (2007) found that $k_s$ values were lowest in the winter months. On the other hand, Hu et al. (2012) found that both $k_s$ and bulk density varied seasonally, with $k_s$ increasing from October to March and decreasing from March to June; they suggested that using a constant $k_s$ value measured in the spring months may lead to the underestimation of runoff during the rainy season.

Previously (see Chapter 2), we developed a plot-scale rainfall simulator for estimating infiltration parameters (Wilson et al., 2014). Following the initial experiments, the simulator was deployed in Sardinia (Italy), which has a typical Mediterranean climate, over the course of several months. Results indicated a monotonically increasing relationship between grassy vegetation height and $k_s$ (Corona et al., 2013). Similarly, Niemeyer et al. (2014) found a relationship between leaf area index (LAI) and $k_s$ and suggested that LAI be included in pedotransfer functions for determining $k_s$. Since seasonal variations in $k_s$ do not appear to be consistent, the literature lacks studies on how such a variation can affect modeled hydrologic response in locations where the variation is present.

Thus, based on the observed relationship between the height of grassy vegetation and $k_s$, the present hypothesis is that incorporating a biomass-dependent $k_s$ into a hydrologic and land surface model will have an effect on both the biomass and rainfall partitioning. Compared to the baseline case of having a constant $k_s$, we expect less surface runoff when biomass is high, with a possible feedback of increased vegetation
growth. Conversely, we expect increased runoff during the periods when biomass is low.

To test these hypotheses, we start by analyzing the $k_s$ and vegetation field data to determine four potential forms of $k_s$ as a function of $b$ ($k_s(b)$). Then, we develop a model consisting of two ordinary differential equations (ODEs), one for biomass and one for soil saturation $s$, which are ultimately linked by $k_s$. Finally, we use the model with both constant $k_s$ and variable $k_s(b)$, under a range of parameters related to climate and surface conditions, to observe the differences in $k_s$, $b$, $s$, and water partitioning under the two scenarios.

This investigation will allow us to determine under which conditions it is necessary to implement a variable $k_s(b)$ rather than a constant $k_s$. This distinction may be relevant for those seeking more accurate modeling of vegetation biomass or surface water fluxes on short temporal scales.

### 3.2 $k_s(b)$ relationship

The impetus for the present investigation arose from the work of Corona et al. (2013), in which the rainfall simulator of Wilson et al. (2014) was used a number of times throughout the growing season at an established field site in Sardinia. During these experiments, measured soil moisture and surface runoff were used to estimate effective $k_s$ values for a 4 m by 4 m plot as explained in Chapter 2. The height of the grassy vegetation in the plot was measured before each experiment, and the results yielded a monotonically increasing relationship between grass height and the effective $k_s$.

Experiments typically relate vegetation to infiltration in terms of biomass; accordingly, we converted vegetation height ($h$) to biomass ($b$) using two steps. First, we converted height to LAI assuming a linear relationship between the two ($\text{LAI} = 0.08h$). Second, we converted LAI to biomass using $b = \text{LAI}/c$, where $c$ is the specific leaf area (Hanson et al., 1988; Nouvellon et al., 2000; Arora, 2002). In this
case, \( c = 0.01 \text{ m}^2 \text{ gDM}^{-1} \) (Montaldo et al., 2008). The resulting \( k_s \) and \( b \), along with four possible forms of \( k_s(b) \) to be discussed below, are shown in Figure 3.1.

\[
\begin{align*}
\text{Biomass [g]} & \quad \text{ks [cm/h]} \\
0 & \quad 0 \, \quad 0.5 \\
20 & \quad 1 \\
40 & \quad 1.5 \\
60 & \quad 2 \\
80 & \quad 2.5 \\
100 & \quad 3 \\
120 & \quad 3.5 \\
140 & \quad 4
\end{align*}
\]

\[
\begin{align*}
k_{s,\text{min}} &= k_4 = 0.18 \text{ cm/h} \\
k_{s,\text{max}} &= k_1 + k_4 = 4 \text{ cm/h} \\
k_{s,\text{mean}} &= 0.9 \text{ cm/h}
\end{align*}
\]

**Figure 3.1:** Several possible forms of \( k_s(b) \) shown with the experimental data.

A number of processes contribute to changes in soil infiltration, including, but not limited to: surface sealing, cracking, and macroporosity from biological activity. Practical modeling of changes in infiltration over time thus requires generalization of these processes. Since the grassy vegetation biomass and effective \( k_s \) increase together, in this model we use biomass as a proxy for the undetermined processes by which \( k_s \) changes at this field site.

The best fit to the experimental data is the exponential function of the form

\[
k_s(b) = C_1 \cdot \exp(C_2 \cdot b) \tag{3.1}
\]

with \( C_1 = 0.035 \) and \( C_2 = 0.044 \), \( b \) is in grams, and \( k_s \) is in cm h\(^{-1}\). In the log-linear relationship shown in Equation (3.1), \( k_s \) increases with \( b \) at an increasing rate, which will lead to unreasonably high \( k_s \) values for biomass only slightly higher than the
range shown in Figure (3.1). In practice, there will be an upper limit on $b$ that will limit $k_s$, and we also wish to limit $k_s$ to a known range of values.

A logistic function for $k_s(b)$ allows us to specify the maximum and minimum $k_s$ along with the range of $b$ values over which the increase occurs. Specifically, this second relationship takes the form

$$k_s(b) = \frac{k_1}{1 + \exp(-k_2 \times (b - k_3))} + k_4$$  \hspace{1cm} (3.2)

where $k_1$ is the difference between the maximum and minimum $k_s$ values, $k_2$ is the slope of the linear part of the logistic curve, $k_3$ is the location of the center of the linear part of the curve, offset from $b = 0$, and $k_4$ is the minimum $k_s$ value. Therefore, $k_1$ and $k_4$ determine the range of $k_s$ values and $k_2$ and $k_3$ determine the transitional range of $b$ values. A larger $k_2$ value is a steeper slope, meaning the $k_s$ transition occurs over a smaller range of $b$ values, leaving more of the $b$ range covered by either the maximum or minimum $k_s$ value. Conversely, a small $k_2$ value indicates a larger $b$ range for the transition. Finally, as $k_3$ determines the center of the linear portion of $k_s(b)$, it determines what section of the $b$ range is covered by low, transition, and high $k_s$ values. For example, small $k_3$ indicates a transition occurring at lower values of $b$, so less of the $b$ range is covered by low $k_s$ and more of the range is covered by high $k_s$.

Hence, not only does the logistic equation fit our data well, but its parameters are directly tied to characteristics of the system and can therefore be calibrated easily.

While Equation (4.4) captures the overall behavior of the data, we also wanted to investigate how the form of $k_s(b)$ affects the link between $k_s$ and biomass. Therefore, we chose to compare the model results using the logistic form to those using a linear function and a step function for $k_s(b)$. While a linear $k_s(b)$ also exhibits the characteristic of increasing indefinitely with $b$, the rate is constant, and we can choose parameters such that $b$ does not lead to unreasonable values of $k_s$. The linear
\(k_s(b)\) takes the form

\[
k_s(b) = \frac{k_{s,max}}{b_{lim}} \times b + k_{min},
\]

(3.3)

where \(k_{s,max}\) is equivalent to \(k_1 + k_4\) in Equation (4.4), \(k_{s,min} = k_4\), and the value \(b_{lim}\) is used to specify the slope of Equation (3.3). The linear case may be thought of the low-\(k_2\) extreme of the logistic function, where \(k_3\) is in the middle of the biomass range. We may also impose a limit on \(k_s\), namely \(k_s(b) \leq k_{s,max}\).

Finally, the step function takes the form

\[
k_s(b) = \begin{cases} 
k_{s,min} & b < b_{step} \\
k_{s,max} & b \geq b_{step} \end{cases},
\]

(3.4)

where \(b_{step}\) is the \(b\) value at which \(k_s(b)\) switches from \(k_{s,min}\) to \(k_{s,max}\). The step function may be thought of as the high-\(k_2\) limit of the logistic function.

As a control, we selected three constant \(k_s\) values for comparison, also shown in Figure (3.1). These three values, \(k_{s,min}\), \(k_{s,mean}\), and \(k_{s,max}\), are the minimum, mean, and approximate maximum of the experimental effective \(k_s\) values. We are particularly interested in \(k_{s,mean}\) since it is the value likely to be used in models if one were to select a single value based on the experimental data.

### 3.3 Model Development

In order to see the effect of a variable \(k_s(b)\) on biomass and water partitioning, we developed a model consisting of two ODEs, one for \(b\) and one for \(s\). Since \(k_s\) is a function of \(b\) and has a direct impact on how much water enters the soil column, \(k_s(b)\) serves as the link between the two equations. Accordingly, \(k_s\), \(b\), and \(s\) are the three state variables to which we will devote most of our attention. Furthermore, we use the term “flux variable” to refer to any movement of water, namely infiltration, runoff, drainage, and evapotranspiration. Below we develop the two ODEs and their respective components.
3.3.1 Soil water model

Soil water serves as the link between soil properties and vegetation growth. Here, we represent the relative soil moisture or soil saturation as

\[ s = \frac{\theta - \theta_w}{n - \theta_w}, \]  

(3.5)

where \(\theta\) is the volumetric soil moisture, \(\theta_w\) is the wilting point, and \(n\) is the soil porosity. \(s\) ranges from zero to one, namely \(s_w = s(\theta_w) = 0\) and \(s_s = s(n) = 1\). For convenience, we define \(n' = n - \theta_w\) as the relative porosity. Accordingly, the governing equation for the water balance is

\[ \frac{ds}{dt} = \frac{1}{n'z_r} (i' - e_t - d), \]  

(3.6)

where \(z_r\) is the root zone depth, \(i'\) is the net infiltration rate (precipitation less interception and runoff), \(e_t\) is the total evapotranspiration (ET) including plant ET and bare soil evaporation, and \(d\) is drainage or leakage from the bottom of the root zone (Montaldo et al., 2005; Rigby and Porporato, 2006). The remainder of this section describes each component of Equation (3.6).

In order to model infiltration, we must have precipitation as an input. Often, models use hourly or daily time increments for both precipitation and surface modeling. This discretization, along with calculating fluxes based on soil moisture alone, eliminates the need for more fine-grained precipitation input. On the other hand, since our focus is on the effects of \(k_s\), our model requires sub-hourly precipitation rates. Accordingly, we model precipitation as a rectangular pulse, which is commonly used as an equivalent for more complicated hyetographs. For each rainfall event, the total storm depth (\(\lambda\)) and duration (\(\gamma\)) are randomly drawn from exponential distributions with means \(\bar{\lambda}\) and \(\bar{\gamma}\). The rainfall intensity \(\eta\) is calculated using \(\gamma/\lambda\), and the amount of water that falls during a period of time \(\Delta t\) is \(H = \eta \Delta t\). Finally, the
time from the end of one storm to the start of the next is drawn from an exponential distribution with mean \( \tau \). We have chosen the exponential distribution here for the sake of simplicity, but in practice one should use distributions appropriate for the data at the desired location.

We recognize that a rectangular pulse does not represent the true hyetograph of a typical rainfall event. However, since we are investigating the feedback between vegetation and infiltration rather than developing a predictive model, the rectangular pulse will both aid in the infiltration calculations and simplify the interpretation of model results.

Interception is modeled using the approach of Montaldo et al. (2005), wherein the interception reservoir \( w_r \) has a maximum value \( w_{r,\text{max}} = 0.2 \times f_{\text{veg}} \times c \times b \). Here, \( f_{\text{veg}} \) is the vegetated fraction of the surface, \( c \) is the specific leaf area, and \( b \) is the biomass. The reservoir storage evolves using \( w_{r,t} = \min(w_{r,t-\Delta t} + H, w_{r,\text{max}}) \), meaning the amount of water in the reservoir cannot exceed the maximum value. Similarly, the effective precipitation depth is \( H' = \max(w_{r,t} + H - w_{r,\text{max}}, 0) \), where precipitation only reaches the ground if the interception reservoir is full. The effective precipitation rate is therefore \( \eta' \). These interception relationships represent one way in which vegetation affects how much water infiltrates into the soil, particularly in the beginning of a rainfall event.

For the infiltration of \( H' \) into the soil during a rainfall event, we use a form of Philip’s Equation (Philip, 1957), namely

\[
\begin{align*}
    i &= \begin{cases} 
        \eta & 0 \leq t' \leq t_p \\
        \frac{1}{2} S(s_0)(t' - t_c)^{-1/2} + k_s & t' > t_p
    \end{cases} 
\end{align*}
\]

where

\[
S(s_0) = \left[ \frac{2n(1 - s_0)\Psi_s}{1 + 3m} \left( s_0^{(1+3m)/m} - 1 \right) \right]^{1/2} k_s^{1/2}. \tag{3.8}
\]
Here, \( t' \) is the time since rain first reached the soil \((H' > 0)\), and \( s_0 \) is the soil saturation at \( t' = 0 \). Also note that these equations use \( \eta \) rather than \( \eta' \); this choice is based on the assumptions that the \( \eta' < \eta \) for only the first time step where \( H' > 0 \) and that evapotranspiration during rain events is negligible. With a sufficiently small time step during rain events, the error introduced is negligible compared to the remaining water fluxes.

Additionally, \( t_p \) is the time to ponding for the storm based on \( I(t_p) = \eta t_p = \int_{t'=0}^{t'=t_p} \eta \, idt \). Since Philip’s Equation assumes ponded conditions for the entire event, we must correct the time in the model as we did in Chapter 2. This time, however, since \( \eta \) is constant for each rain event, we follow Rigby & Porporato’s (2006) treatment of the time compression correction:

\[
t_p = \begin{cases} 
\frac{S(s_0)^2(2\eta - k_s)}{4\eta(\eta - k_s)^2} & \eta > k_s \\
\infty & \eta \leq k_s 
\end{cases} 
\quad (3.9)
\]

\[
t_e = \frac{S(s_0)^2}{4(\eta - k_s)^2} 
\quad (3.10)
\]

\[
t_c = t_p - t_e. 
\quad (3.11)
\]

The constant-rate rectangular pulse precipitation is helpful here because it allows us to calculate the ponding time at the beginning of the event and subsequently switch between the two infiltration cases in Equation (3.7) if the ponding time is shorter than the event duration.

Notice that \( k_s \) appears in the expressions for \( i_s \), \( S(s_0) \), \( t_p \), and \( t_e \). Keeping in mind the goal to assess the effect of a variable \( k_s(b) \) on biomass and water partitioning, we can see here that an increase in \( k_s \) should cause an increase in infiltration through a larger maximum infiltration rate, increased sorptivity, a longer time to ponding, and a larger range of intensity values that will not pond at all.
Finally, the infiltration excess runoff rate \( q_I \) is calculated using

\[
q_I = \eta' - i. \tag{3.12}
\]

Since \( s \) has a maximum value of 1 at saturation, the model must check for this condition during rain events. If the calculations yield \( s > 1 \), the excess water is categorized as saturation excess runoff. The rate of this flux during time \( \Delta t \) is

\[
q_S = \begin{cases} 
\frac{n^2 z}{\Delta t} (s - 1) & s > 1 \\
0 & \text{otherwise.} 
\end{cases} \tag{3.13}
\]

These two runoff quantities yield the expression for \( i' \) as it appears in Equation (3.6),

\[
 i' = \eta' - (q_I + q_S). 
\]

The next component of the water balance is evapotranspiration. While several well-developed models for vegetation evapotranspiration rate \( e_v \) exist (e.g. Penman-Monteith), in this analysis we seek a simple representation with a limited number of parameters. First, to isolate the effects of \( k_s \), we adopt an “always ideal” scenario where the radiation and temperature are constant and non-limiting to the vegetation. Then, we assume that a maximum value \( e_{max} \) represents the evapotranspiration in these ideal conditions and under the maximum biomass load. Accordingly, \( e_v \) is determined by a Jarvis-type soil moisture limiting function and the amount of biomass present relative to the maximum potential load, \( b/b_{max} \). The soil moisture limiting function is

\[
f_1(s) = \begin{cases} 
0, & s = 0 \\
\frac{s}{s_{lim}}, & 0 < s < s_{lim}, \\
1, & s_{lim} \leq s \leq 1 
\end{cases} \tag{3.14}
\]

where \( s_w \) and \( s_{lim} \) are the wilting point and limiting soil moisture for vegetation, respectively. These assumptions result in an expression for \( e_v \):

\[
e_v = e_{max} \times \frac{b}{b_{max}} \times f_1(s). \tag{3.15}
\]
In addition to the evaporation of water transpired from vegetation, we must also account for water evaporation from bare soil \((e_{bs})\). To do this, we use the Parlange et al. (1999) approach, whereby bare soil evaporation is

\[ e_{bs} = f_{bs} \times \alpha \times e_{max}. \]  

(3.16)

Here, \(\alpha\) determines the fraction of the Penman potential evaporation that occurs based on the volumetric soil moisture:

\[ \alpha = 1.5 \times (c_1 \times \theta^4 - c_2 \times \theta^3 + c_3 \times \theta^2 - c_4 \times \theta + c_5), \]  

(3.17)

where \(c_1 = 0.705\), \(c_2 = 23.8\), \(c_3 = 22.0\), \(c_4 = 3.52\), and \(c_5 = 0.155\). Combining the expressions for \(e_v\) and \(e_{bs}\) gives us the estimate for total evapotranspiration, \(e_t\):

\[
e_t = e_v + e_{bs} = \left( e_{max} \times \frac{b}{b_{max}} \times f_1(s) \right) + \left( e_{max} \times f_{bs} \times \alpha \right) = e_{max} \times \left( \frac{b}{b_{max}} \times f_1(s) + f_{bs} \times \alpha \right). \]  

(3.18)

The final water loss we calculate directly is drainage, which is water that exits the bottom of the root zone due to gravity when soil moisture is greater than the field capacity \(s_{fc}\). One way to represent drainage is using

\[ d = k_{sb} \times s^{c+1}. \]  

(3.19)

In this equation, \(k_{sb}\) is the saturated hydraulic conductivity at the bottom of the root zone, which we assume is smaller than the surface \(k_s\) in order to prevent excessive drainage losses. \(s\) is the relative soil moisture as defined in Equation (3.5), and \(c = 2(1 + m)/m\), where \(m\) is the exponent in the Brooks and Corey (1964) soil water retention equation.
Finally we combine the water components and arrive at the soil moisture evolution equation

$$\frac{ds}{dt} = \frac{1}{n' z_r} \left( i' - e_{\text{max}} \times \left( \frac{b}{b_{\text{max}}} \times f_1(s) + f_{bs} \times \alpha \right) - k_{sb} \cdot s^{c+1} \right). \quad (3.20)$$

### 3.3.2 Biomass model

The vegetation dynamics model of Montaldo et al. (2005) calculates above ground standing/green, above ground dead, and root biomass for grassy vegetation. Here, we follow their simplified SV3 model, which addresses only the above ground green biomass. This model is

$$\frac{db}{dt} = aP - R - S, \quad (3.21)$$

where $a$ specifies the fraction of the gross photosynthesis $P$ is allocated to above-ground biomass, $R$ is respiration, and $S$ is senescence.

A simple treatment of gross photosynthesis is to use a water use efficiency ($\omega$) to convert the amount of water transpired to mass of biomass generated. This assumption leads us to

$$P = \omega \times e_v. \quad (3.22)$$

Furthermore, $R$ and $S$ are represented by the following functions of $b$:

$$R = m_a f_4(T_m)b + g_a(aP) \quad (3.23)$$

$$S = d_a b, \quad (3.24)$$

where $m_a$, $g_a$, $a$, and $d_a$ are all constants. We also have $f_4(T_m) = Q_{40}^{T_m/10}$, where $T_m$ is the mean daily temperature. For this initial model we will assume a constant temperature $T_m = T_{\text{opt}}$, where $T_{\text{opt}}$ is the optimal temperature for vegetation growth, so $f_4(T_m)$ is also a constant.
By combining the expressions for $P$, $R$, and $S$ and rearranging the terms, we arrive at

$$\frac{db}{dt} = \alpha_1 P - \alpha_2 b$$

(3.25)

where $\alpha_1 = a(1 - g)$ and $\alpha_2 = m_a Q_{10}^{T_{opt/10}} + d_a$.

Since we have an expression for $e_v$ in terms of the state variables (Equation (3.15)), we can re-write $P$ as

$$P = \omega \times e_{max} \times f(s) \times b_{max}$$

(3.26)

Substituting Equation (3.26) into Equation (4.3) allows us to write the biomass equation in terms of the state variables as

$$\frac{db}{dt} = \left( \frac{\alpha_1 \omega e_{max} f_1(s)}{b_{max}} - \alpha_2 \right) b.$$  

(3.27)

Accordingly, the state of increasing biomass is represented by $\frac{db}{dt} > 0$, so

$$\frac{\alpha_1 \omega e_{max} f_1(s)}{b_{max}} > \alpha_2$$

(3.28)

represents the vegetation growth condition. If we further assume that biomass will increase if $s$ is at least a critical value $s_{crit}$, where $0 < s_{crit} \leq s_{lim}$, we can obtain

$$\frac{\alpha_2}{\alpha_1} < \frac{\omega e_{max} f_1(s_{crit})}{b_{max}}$$

(3.29)

This expression aids in model calibration by putting bounds on parameter values.

3.4 Model parameterization and calibration

Table (3.1) shows the precipitation, vegetation, soil, and surface parameters used in this model. The vegetation and soil parameters are generally those from Montaldo.
et al. (2008). The parameters that are new to this model and may require adjustment to obtain accurate behavior are the vegetation parameters $\alpha_1$, $\alpha_2$, $s_{\text{crit}}$, $e_{\text{max}}$, and $b_{\text{max}}$ along with the $k_s(b)$ parameters $k_1$, $k_2$, $k_3$, $k_4$, $b_{\text{lim}}$, and $b_{\text{step}}$. We will briefly explain how the vegetation parameters should be used during model calibration.

As shown in Equation (3.27), the balance of $\alpha_1$ and $\alpha_2$ determines the relationship between soil moisture and biomass. Specifically, smaller values of the ratio $\alpha_2/\alpha_1$ lead to higher biomass growth for any given value of $s$, causing larger overall biomass levels in the model results. Furthermore, if the mean of the biomass is correct but the variance is too small, increasing both $\alpha_1$ and $\alpha_2$, while keeping the ratio constant, will increase the resulting biomass range.

Similarly, reasonable estimates of $s_{\text{crit}}$, $e_{\text{max}}$, and $b_{\text{max}}$ are required for the model to perform correctly. For example, if $b_{\text{max}}$ is too small or $e_{\text{max}}$ is too large, $e_t$ will be overestimated, leading to more rapid decrease in soil moisture and less biomass production. Hence, if $s$ seems to be dropping too quickly, consider decreasing $e_{\text{max}}$. Finally, if biomass is expected to increase after rain events but does not, it is possible that the value of $s_{\text{crit}}$ is too high. Based on the growth condition, higher values of $f_1(s_{\text{crit}})$ mean that the ratio $\alpha_2/\alpha_1$ must be larger, which as previously discussed lowers overall biomass levels.

### 3.5 Design of numerical experiments

After calibrating the model, one can proceed with investigating the effect of a variable $k_s(b)$ on the biomass-infiltration feedback. In the first several numerical experiments, we used the parameters shown in Table (3.1); from here these values will be referred to as the ‘default parameters.’ To begin, the system was allowed to evolve starting from 100 percent biomass density and soil saturation at field capacity. After a number of days ranging from 1 to 100, we applied a single storm of 1 cm at 10 cm h$^{-1}$ then allowed the system to continue evolving. By repeating this experiment for both the
Table 3.1: Model parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\lambda}$</td>
<td>Mean event depth</td>
<td>2 cm</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Mean event duration</td>
<td>3 hr</td>
</tr>
<tr>
<td>$\bar{\tau}$</td>
<td>Mean dry time between storms</td>
<td>5 d</td>
</tr>
<tr>
<td>$c$</td>
<td>Specific leaf area</td>
<td>0.01 [-]</td>
</tr>
<tr>
<td>$s_{\text{lim}}$</td>
<td>Upper bound of plant water stress soil moisture</td>
<td>0.27 [-]</td>
</tr>
<tr>
<td>$s_{\text{crit}}$</td>
<td>Soil moisture above which biomass always increases</td>
<td>0.16 [-]</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>Plant wilting point volumetric soil moisture</td>
<td>0.08 [-]</td>
</tr>
<tr>
<td>$e_{\text{max}}$</td>
<td>Maximum evapotranspiration for $\theta \geq \theta_{\text{lim}}$</td>
<td>0.5 cm d$^{-1}$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Water use efficiency</td>
<td>20 g DM cm$^{-1}$</td>
</tr>
<tr>
<td>$b_{\text{max}}$</td>
<td>Biomass load coefficient</td>
<td>50 g</td>
</tr>
<tr>
<td>$m_a$</td>
<td>Maintenance respiration coefficient</td>
<td>0.012 d$^{-1}$</td>
</tr>
<tr>
<td>$T_{\text{opt}}$</td>
<td>Optimal temperature for photosynthesis</td>
<td>22 deg C</td>
</tr>
<tr>
<td>$g_a$</td>
<td>Growth respiration coefficient</td>
<td>0.22 [-]</td>
</tr>
<tr>
<td>$a$</td>
<td>Biomass allocation to above ground vegetation</td>
<td>0.5 [-]</td>
</tr>
<tr>
<td>$d_a$</td>
<td>Death rate for above ground biomass</td>
<td>0.023 d$^{-1}$</td>
</tr>
<tr>
<td>$Q_{10}$</td>
<td>Temperature coefficient for respiration</td>
<td>2.3 [-]</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Overall growth coefficient, similar to $a(1-g)$</td>
<td>0.39 [-]</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Overall loss coefficient, similar to $\alpha_2 = m_a Q_{10}^{T_{\text{opt}}/10} + d_a$</td>
<td>0.046 d$^{-1}$</td>
</tr>
<tr>
<td>$k_s$</td>
<td>Saturated hydraulic conductivity</td>
<td>Variable cm h$^{-1}$</td>
</tr>
<tr>
<td>$k_{s,b}$</td>
<td>$k_s$ at bottom of the root zone</td>
<td>0.15 cm h$^{-1}$</td>
</tr>
<tr>
<td>$z_r$</td>
<td>Depth of root zone</td>
<td>15 cm</td>
</tr>
<tr>
<td>$\psi_s$</td>
<td>Air entry suction head</td>
<td>-79 cm</td>
</tr>
<tr>
<td>$m$</td>
<td>Brooks and Corey water retention exponent</td>
<td>1.82 [-]</td>
</tr>
<tr>
<td>$n$</td>
<td>Porosity/saturated volumetric soil moisture</td>
<td>0.53 [-]</td>
</tr>
<tr>
<td>$s_{\text{fc}}$</td>
<td>Field capacity</td>
<td>0.33 [-]</td>
</tr>
<tr>
<td>$k_1$</td>
<td>Maximum $k_s$ less minimum $k_s$</td>
<td>3.82 cm h$^{-1}$</td>
</tr>
<tr>
<td>$k_2$</td>
<td>Slope of linear part of $k_s(b)$</td>
<td>0.1</td>
</tr>
<tr>
<td>$k_3$</td>
<td>Location of center of linear part of $k_s(b)$</td>
<td>65 g</td>
</tr>
<tr>
<td>$k_4$</td>
<td>Minimum $k_s$</td>
<td>0.18 cm h$^{-1}$</td>
</tr>
<tr>
<td>$b_{\text{lim}}$</td>
<td>$b$ value used to determine slope of linear $k_s(b)$</td>
<td>100 g</td>
</tr>
<tr>
<td>$b_{\text{step}}$</td>
<td>$b$ value where the step $k_s(b)$ changes values</td>
<td>50 g</td>
</tr>
<tr>
<td>$f_{\text{veg}}$</td>
<td>Fraction vegetated</td>
<td>0.6</td>
</tr>
<tr>
<td>$f_{bs}$</td>
<td>Fraction bare soil, $1 - f_{\text{veg}}$</td>
<td>0.4</td>
</tr>
</tbody>
</table>
constant and logistic cases, we can observe how the response to a rain event changes as soil moisture and biomass decline.

Next, we ran the model with the default parameters and the six previously described $k_s$ scenarios: low constant $k_s$, mean constant $k_s$, high constant $k_s$, linear $k_s(b)$, step $k_s(b)$, and logistic $k_s(b)$. Each model run used the same precipitation series and had a duration of 20 “years,” that is, 20 years of our always-ideal environmental conditions. By looking at the resulting time series and distributions of $k_s$, $b$, and $s$, we can qualitatively assess the mechanisms and characteristics of the effect of $k_s(b)$ on the system.

For quantitative analysis, we focused on model results for one constant $k_s$ value, specifically $k_s, \text{mean}$, and the logistic $k_s(b)$. From here forward, a subscript of $C$ refers to the constant $k_s, \text{mean}$ scenario, a subscript of $L$ refers to the logistic $k_s(b)$ scenario, and $\Delta x$ for any quantity $x$ is $x_C - x_L$. In a series of numerical experiments, we picked one parameter to vary over a range of values to see if and how that parameter affects the differences in biomass and water partitioning between the constant and variable $k_s$ cases. These experiments were as follows:

1. The mean storm depth, $\lambda$, spanned 0.1 cm to 2.5 cm as one way to investigate different climates or rainfall regimes.

2. The mean dry time between storms, $\tau$, spanned 2 d to 30 d as another way to investigate dry and wet climates.

3. The maximum evapotranspiration, $e_{\text{max}}$ spanned 0.2 cm d$^{-1}$ to 2 cm d$^{-1}$ to simulate different vegetation activity levels or different potential evapotranspiration in different climates. In order to maintain the growth condition (Equation 3.28) without changing $s_{\text{crit}}$ or $\omega$, $b_{\text{max}}$ tracked with $e_{\text{max}}$ to keep $e_{\text{max}}/b_{\text{max}}$ constant.
4. The root zone depth, \( z_r \), spanned 4 cm to 80 cm to investigate the effects of \( k_s \) under different vegetation regimes.

We performed several analyses on the results of the above scenarios. First, to investigate long-term biomass behavior, we calculated the 10th percentile, mean, and 90th percentile of the biomass over the entire model period for each parameter value in the experiments described above. Additionally, the normalized difference between the two biomass time series was calculated using

\[
\Delta b' = \frac{\Delta b}{\frac{1}{2}(b_C + b_L)}. \tag{3.30}
\]

The 10th percentile, mean, and 90th percentile of \( \Delta b' \) were also calculated to further highlight the average difference between the constant and logistic biomass series and how that difference varies across parameter values.

The second analysis focused on the long-term partitioning of precipitation into several components: infiltration excess runoff \((Q_I)\), saturation excess runoff \((Q_S)\), gravity drainage \((D)\), vegetation evapotranspiration \((E_v)\), bare soil evaporation \((E_{bs})\), and evaporation of interception storage \((W)\). Each of these fluxes was expressed as a fraction of the total precipitation \((P)\) during the model period, and we compared the partitioning across parameter changes and between the constant and logistic scenarios.

The first two analyses concern long-term statistics, which are helpful for determining the overall feedbacks in the system. In order to assess short-term effects, we turned to the runoff ratio for each rainfall event. For storm \( k \), the total runoff \( Q_k \) is the sum of \( Q_S \) and \( Q_I \) for that storm, and the runoff ratio is defined as

\[
M_k = \frac{Q_k}{P_k}. \tag{3.31}
\]

Following our established notation, the runoff ratio difference for that storm between
the constant and logistic scenarios is $\Delta M_k = M_{k,C} - M_{k,L}$. By observing the values of $M$ and $\Delta M$ against the parameter ranges, rainfall intensity, and biomass, we can quantify how the choice of a constant or variable $k_s$ affects runoff in both different climates and different storm sizes within those climates.

3.6 Results

3.6.1 Feedback Mechanisms

In order to understand the long-term model results, we must first understand the feedback mechanism between biomass, saturation, and $k_s$. Figure (3.2) shows the results of applying a single 1 cm storm at 10 cm h$^{-1}$ at different points in time. In Figure (3.2a), which shows biomass and saturation against time for a 20-day delay, $s_C$ peaks slightly higher than $s_L$. As a result, $s_C$ remains above the saturation required for biomass increase (dashed line) longer than $s_L$. This difference in saturation causes $b_C$ to be slightly higher than $b_L$ for the remainder of the period shown.

The remaining parts of Figure (3.2) show $b$ and $s$ in phase space. For the 1-day delay Figure (3.2b), $s_L$ and $b_L$ peak higher than $s_C$ and $b_C$ due to the logistic $k_s(b)$ being larger than $k_{s,mean}$ at the time of the rain event. This result demonstrates a positive feedback; the high biomass level and corresponding high $k_s(b)$ allow more water to infiltrate into the soil, which in turn allows $b_L$ to increase more than $b_C$. Figure (3.2c) is the phase diagram of the results in Figure (3.2a). Both $s_C$ and $s_L$ increase enough for biomass to grow, but as shown in Figure (3.2a), $s_C$ remains above the growth threshold longer than $s_L$, allowing the increase in $b_C$ to be greater than that of $b_L$. In both Figures (3.2d) and (3.2e), where the rain event occurs after 50 and 100 days, respectively, the low biomass and $k_s(b)$ cause $s_L$ to not cross the growth threshold. As a result, $b_L$ does not increase despite receiving the same amount of rainfall as the constant $k_s$ case. This result shows another positive feedback, but in the opposite direction: once $b_L$ decreases below a certain level, it can get stuck in
Figure 3.2: Time series and phase space for a single storm of 1 cm at 10 cm h$^{-1}$. a) Time series for a delay of 20 days until the rain event. b through e) Phase space of biomass and saturation for storm delays of b) 1 day; c) 20 days; d) 50 days; and e) 100 days. The dashed line indicates the saturation above which the biomass will increase.

After understanding the effects of a single rain event, we can analyze results of multiple storms over a longer period of time. Figure (3.3) shows one year of model results for the constant and logistic scenarios, again using the default parameter set. The left panels show the constant and logistic flux and state variables individually, and the right panels show the difference between the two using the established $\Delta$ notation. Precipitation is shown at the top of both sides for reference.

In the beginning of this model period, starting at point A in Figure (3.3), a number of storms clustered together allow biomass to be high and for the logistic $k_s(b)$ to be greater than $k_{s,\text{mean}}$ as a result. While one might expect more runoff for the constant scenario as a result, the first several rainfall events have nearly identical runoff followed by equal soil moisture and biomass. The reason for the equality is...
that the rain events are small enough such all water infiltrates despite differences in 
k_s, and once water infiltrates the constant and logistic scenarios behave identically.

At point B, a larger rainfall event occurs. This time the precipitation overcomes 
k_s, and the logistic scenario yields more infiltration because \( k_s(b) \) is larger than \( k_s,\text{mean} \) when the storm occurs. Additionally, due to low antecedent soil moisture \( (s_0) \) for 
this event, the difference in infiltration caused the soil moisture at the end of the 
storm \( (s_f) \) to be larger for the logistic case than for the constant case. With a high 
\( s_0 \), the large storm may have fully made up the soil moisture deficit in both cases, 
leading to no difference in \( s_f \).
For this parameter set, the limiting soil moisture for vegetation is \( s_{lim} = 0.27 \). This value is significant because differences in soil moisture will only cause differences in biomass if \( s < s_{lim} \) for one or both scenarios. Notice that while \( \Delta s \) first spikes at point B, \( \Delta b \) doesn’t change until several days later, when \( s_C < s_{lim} \) but \( s_L > s_{lim} \). As a result, \( \Delta b \) continues to grow until the next cluster of storms, when \( \Delta s \) returns to near zero. This result reinforces what we found in Figure (3.2b), that wetter conditions lead to a positive feedback between increasing biomass and infiltration.

In the relatively dry period that follows point B, \( b \) and \( |\Delta b| \) both decrease. The latter occurs because, at a given value of \( s \), \( db/dt \) has a larger magnitude for higher biomass values (see Equation (3.27)). Additionally, the logistic \( k_s(b) \) decreases during this period, eliminating the large infiltration potential gap between the two cases. Hence, dry periods act as a reset for any differences between the state variables of the constant and logistic model scenarios.

Following point C, we see somewhat of a reversal of the behavior between points A and C. Between points C and D, several runoff-producing events occur, but the small \( \Delta k_s \) causes a negligible difference in response between the two scenarios. After point D, low \( s \) values cause \( k_s(b) \) to fall below \( k_{s,mean} \). At point E, the combination of a larger event, low \( s_0 \), and \( k_s(b) < k_{s,mean} \) causes greater infiltration and higher \( s_f \) for the constant case. \( s_L \) barely climbs out of the water stress range, so \( k_s(b) \) remains low and subsequent storms continue the pattern of higher infiltration, soil moisture, and biomass growth for the constant case. It is not until the end of this period, when several storms are clustered together, that \( s_L \) remains high enough for \( b_L \) and \( k_s(b) \) to climb out of this critically low zone. This behavior amounts to a positive feedback between low biomass and low infiltration for the variable \( k_s \) scenario, similar to what is shown in Figures (3.2d) and (3.2e).

This close inspection of Figure (3.3) has revealed the conditions under which a variable \( k_s \) can affect biomass and partitioning of water into infiltration and runoff.
In generally wet conditions, precipitation is less likely to generate a soil moisture
difference that can lead to a difference in biomass. Similarly, small storms are unlikely
to generate runoff in either scenario, which again maintains similar moisture levels.
Large storms during wet periods when biomass is high can favor the logistic case in
terms of infiltration and soil moisture. Large storms following dry periods, however,
favor infiltration for the constant case because of the drop in the logistic $k_s(b)$ during
the dry period. Even moderate storms can enhance this effect when $k_s(b)$ remains
low. Therefore, variable $k_s(b)$ is most likely to affect biomass and water partitioning
during large rain events and when a wet period follows an extended dry period.

3.6.2 Long-term behavior

After understanding the forces at work, we can examine the extended model results.
Figure (3.4) shows two of the primary state variables, $b$ and $k_s$, for 10 years. It also
shows the probability density function (pdfs) of the values for the full 20-year model
period. Figure (3.4a) shows results for the default parameters, and Figure (3.4b)
shows results for the case of $\lambda = 0.9$ cm rather than the default value of 2 cm. There
are some noticeable differences between $b$ values in the six scenarios for both cases;
for instance, during the second year there is a period when the three variable $k_s$
values are all quite high with corresponding high $b$ values. Similarly, the $k_s$ pdf in
Figure (3.4a) shows that the linear case has higher $k_s$ values overall, and there are a
number of instances throughout the model period where the linear case has higher
biomass peaks than the step or logistic cases. However, while some differences in $b$
and $s$ are noticeable, they are not quite as drastic as one might expect based on the
values of $k_s$. This observation leads us to investigate the results in more detail.

From here on, we concentrate only on the $k_{s,\text{mean}}$ constant case and logistic vari-
able case because the former is the one most likely to be used in general practice
and the latter is the only one that covers the full range of $k_s$ values observed in the
Figure 3.4: Ten years of model results for the six original $k_s$ scenarios and the long-term pdfs for the full 20-year model period. (a) $\bar{X} = 2$ cm; (b) $\bar{X} = 0.9$ cm.
data. For the four experiments described in Section 3.5, we will work under the assumption that the values obtained with the logistic \( k_s(b) \) represent reality and the constant case deviates from this standard.

### 3.6.3 Biomass statistics

Figure (3.5) shows the statistics for \( b \) and \( \Delta b' \) for each of the four experiments. These plots show us not only how biomass tracks with each parameter, but also how the choice of the constant or variable \( k_s \) affects these changes. For instance, in the storm depth (\( \bar{\lambda} \)) experiment, we first notice that an increase in \( \bar{\lambda} \) leads to an increase in biomass, which makes sense since larger \( \bar{\lambda} \) values indicate a wetter climate. However, the peak mean \( \Delta b' \) value of approximately 3 percent occurs around \( \bar{\lambda} = 0.9 \) cm, near the middle of the examined range. This result suggests that there exists an intermediate \( \bar{\lambda} \) value for which the constant case will overpredict biomass, but wetter and drier conditions see smaller biomass differences. This idea agrees with our analysis of Figure (3.3): smaller storms are more likely to fully infiltrate for both scenarios, and larger storms are more likely to fully saturate both scenarios, leaving a transition zone where soil moisture differences after a rain event cause differences in biomass.

In the interarrival time (\( \bar{\tau} \)) experiment, we see an inverse relationship between \( b \) and \( \bar{\tau} \), which follows from a larger \( \bar{\tau} \) representing a drier climate. However, unlike the \( \bar{\lambda} \) experiment, both \( \Delta b' \) and the span between the 10th and 90th percentiles of \( \Delta b' \) increase. Additionally, the constant scenario transitions from under-predicting biomass to over-predicting biomass around \( \bar{\tau} = 4 \) d. Again, thinking back to Figure (3.3), larger \( \bar{\tau} \) values mean there is likely to be more drydown time when biomass decreases and \( \Delta b \) “resets.” Concurrently, the logistic \( k_s(b) \) can drop below \( k_{s,\text{mean}} \) more often and is more likely to get stuck there, causing more frequent \( \Delta b \) increases in favor of the constant case. Hence, for the otherwise default conditions, larger \( \bar{\tau} \)
makes the constant case less able to accurately predict biomass.

The $e_{\text{max}}$ experiment displays results similar to the $\overline{\lambda}$ experiment in terms of scale. Recall that $e_{\text{max}}/b_{\text{max}}$ remains constant, meaning that for any pair of $b$ and $s$ values, $e_v$ remains constant with increasing $e_{\text{max}}$ (Equation 3.15) but $e_{bs}$ increases (Equation 3.16). Hence, in general, higher $e_{\text{max}}$ values lead to lower biomass because increased $e_{bs}$ induces drier conditions and leaves less water available for plant use. As we have seen in the previous experiments, drier conditions cause a greater likelihood of the constant scenario over-predicting biomass. Here, $b_C$ is as much as 10 percent higher on average than $b_L$ given otherwise wet conditions (high $\overline{\lambda}$ and low $\overline{\tau}$). Therefore, locations with higher $e_{\text{max}}$ or potential evapotranspiration are more likely to observe a difference between using a variable $k_s(b)$ and a constant $k_s$.

In the root zone depth ($z_r$) experiment, the overall trend is that deeper soils lead to both larger overall biomass levels and a larger under-prediction of biomass for the
constant case. The former occurs because deeper soil can hold more moisture than shallow soil, leaving more water available for plant use and hence larger $b$ values. Additionally, the increased storage capacity of deeper soil causes fewer storms to saturate the soil, leading to more frequent differences in $s_f$ and larger $\Delta b$ values as a result. Finally, we notice that for very shallow soil, the constant case is likely to over-predict biomass compared to the logistic case. This observation aligns with the previous results, as shallow soils are akin to drier conditions and will therefore lead to less infiltration for the logistic case.

Overall, parameter changes that cause drier soil conditions lead to the constant $k_s$ case over-predicting biomass compared to the logistic $k_s(b)$ case. This effect occurs because drier conditions allow the logistic $k_s(b)$ to drop below $k_{s,\text{mean}}$, which in turn favors infiltration for the constant case. The biomass differences for the current parameter set tend to be relatively small, not more than 10 to 15 percent on average, but the large $\lambda$, small $\tau$, and low $e_{\text{max}}$ are all wetter conditions. It appears that a parameter set representing drier conditions could cause greater differences in biomass.

3.6.4 Water partitioning

We have already observed how a variable $k_s$ can cause feedbacks in soil moisture and biomass. Now, we turn to the effects of the choice of $k_s$ on the partitioning of water. In the plots of Figure (3.6), each color represents a water flux and how it varies with parameter changes. By comparing the constant and logistic results, particularly through the lens of the biomass results in Figure (3.5), we can determine if, to what degree, and in what conditions a variable $k_s(b)$ yields different water partitioning than using a constant $k_s$.

In the $\lambda$ experiment, the partitioning results are very similar for the constant and logistic cases with the exception of how water is divided between $Q_s$, $Q_I$, and $D$. Particularly, the logistic case has more infiltration excess runoff than the constant
Figure 3.6: Partitioning of precipitation into component fluxes for the different experiments. In each case, $W_R$ was less than 1%. Dashed vertical lines represent default model parameters.

In each case, which makes sense given the relatively low biomass levels in this experiment (Figure 3.5) and the resulting low value of the logistic $k_s(b)$ (Figure 3.1). Smaller $Q_S$ values for the logistic case mean the soil becomes saturated less often, which leads to slightly less drainage, but as we noted before the soil moisture tends to vary by small amounts and/or for short periods of time between the two scenarios, which leads to little difference between $E_v$ and $E_{bs}$. As $\bar{\lambda}$ increases, the difference between the partitioning of the constant and logistic cases decreases, which is in line with the previous result of smaller $\Delta b$ values in wetter climates.

The $\tau$ experiment results are similar to those of the $\bar{\lambda}$ experiment in that drier conditions cause the logistic scenario to have more infiltration excess runoff than the constant case. Additionally, since drainage is directly related to saturation, large $D$ values are akin to higher overall $s$ values, meaning the logistic case generally experiences lower saturation than the constant case. However, we do see a transition point around $\tau = 4$ d from high to low $Q_S$ for the logistic case; note that this is the same location as the transition of $\Delta b'$ from negative to positive in Figure (3.5).
This transition suggests a regime change from one where, on average, $k_s(b) > p$ to one where $k_s(b) < p$. Therefore, we expect more significant differences in the water partitioning between the constant and variable $k_s(b)$ scenarios when the ranges of realized $k_s(b)$ values and precipitation intensities overlap.

In both the $e_{max}$ and $z_r$ experiments, we again see $Q_I$ increase for the drier cases of the logistic scenario as it remains steady for the constant $k_s$ scenario. This observation, combined with noticing that whichever scenario has higher $Q_I$ values has lower biomass levels, suggests that there is indeed a positive feedback between decreasing biomass and infiltration.

3.6.5 Storm-based runoff

Finally, we turn our attention from long-term statistics to event-based results. Figure (3.7) shows the mean runoff ratio difference, $\Delta M$, as a function of each parameter with the results separated into storm intensity bins. As in the other analyses, we see that drier soil conditions lead to the logistic scenario having more runoff (negative $\Delta M$) than the constant $k_s$ scenario. These plots also reveal that the differences in runoff between the two scenarios can be more drastic than the long-term partitioning suggests. For low intensity storms, which are the most frequent, the runoff ratio is virtually the same for the two scenarios regardless of the experiment. However, as $p$ increases, $|\Delta M|$ increases as well, and again drier conditions lead to more runoff for the logistic scenario. For example, with $\tau = 10$ d, storms with $1 \leq p < 2$ cm h$^{-1}$ have approximately 10 percent more runoff in the logistic case, but this difference increases to 30 percent for storms in the range $3 \leq p < 4$ cm h$^{-1}$. On the other hand, the constant scenario experiences more runoff in wetter conditions. These results do track with the previous observations, as situations where the logistic scenario has more runoff are the same ones with lower biomass and hence lower $k_s(b)$ values.

Continuing to look at increasing storm intensity in Figure (3.7), we notice that
the most extreme storms trend back towards smaller differences in runoff between the constant and logistic scenarios. We summarize this effect in Figure (3.8), where each series represents a constant $\lambda$ view of Figure (3.7). For small rainfall intensities, all rain is likely to infiltrate in both scenarios, regardless of the value of $k_s(b)$; hence we name this region of intensities the infiltration-dominated zone. At the other end of the intensity range, as $p$ becomes quite large, the resulting short ponding times tend to overcome any differences in $k_s$. Accordingly, runoff differences decrease again in this runoff-dominated zone. These two extremes indicate that the choice of a variable
or constant $k_s$ will not affect runoff results for very large or very small storms.

The middle range of Figure (3.8) shows the transition zone from $k_s$ to $p$ being the primary driver of infiltration or runoff. Initially, $\Delta M$ decreases sharply with increasing $p$, particularly for smaller $\bar{\lambda}$ values, indicating that the margin by which $k_s(b) < k_{s,\text{mean}}$ increases in drier conditions. As intensity increases through the transition zone, drier scenarios trend towards $\Delta M = 0$, but the wettest scenario ($\bar{\lambda} = 2.5$ cm) has $\Delta M$ values greater than zero. This result shows that for these model parameters, $k_s(b) > k_{s,\text{mean}}$ only in relatively wet climates. Overall, Figure (3.8) shows that the effect of the high $b$ and $k_s(b)$ feedback is most pronounced during moderately large storms in wet climates and the effect of the low $b$ and $k_s(b)$ feedback is most pronounced during moderately large storms in dry climates.

The common thread through the four experiments is that drier conditions exhibit stronger effects of a variable $k_s$. It would be useful to compare the results of the
experiments to each other, but this is not possible with the experimental variables in their original units. To remedy this issue, we can scale the variables using the following expression:

$$\xi = \frac{e_{\text{max}}}{n \cdot z_r}.$$  

(3.32)

The value $\xi$, which increases with aridity, incorporates all four experimental variables and has units of cm$^{-1}$. The first term in the expression is similar to a standard unitless comparison of potential ET to precipitation, as the numerator is the maximum possible evaporation rate (cm d$^{-1}$) and the denominator is an average amount of precipitation per day (also cm d$^{-1}$). $z_r$ is in the denominator of the second term because a deeper root zone, as previously noted, creates essentially wetter conditions due to the increased water storage capacity.

The results of applying this scaling to Figure (3.8) are shown in Figure (3.9). The most obvious observation is that the $\xi$ ranges in the four experiments are not equal, but the scaling makes the available ranges more similar to one another. Specifically, all have $\Delta M$ values greater than zero for the wettest conditions. The notable exception to the similarities is the $z_r$ experiment, where rather than diverging, the $\Delta M$ curves converge as dryness increases. This feature is due to the fact that shallower soils have larger $\xi$ values, and not only is there a limit on how shallow soil can be, and the value of $k_s$ will make little difference in saturation for a very thin soil. In general, however, Figure (3.9) shows that a method exists to unify the results across experimental variables.

The values of $\overline{\Delta M}$ (Figures 3.7 and 3.8) demonstrate that the effect of intensity on runoff response varies with aridity. Since we are investigating the role of vegetation biomass in water partitioning, it follows to investigate the relationship between biomass and runoff ratio during rain events. Figure (3.10) shows the runoff ratio and biomass for the constant $k_s$ and logistic $k_s(b)$ scenarios with the default model.
parameters. For each intensity group, the constant $k_s$ scenario shows little to no significant relationship between $M$ and $b$, regardless of intensity, which makes sense since $k_s$ is not a function of $b$. For the logistic $k_s(b)$, a negative correlation between $M$ and $b$ emerges. Both the strength of the relationship and the amount of variation in $M$ explained by $b$ increase with intensity; these traits reflect the transition from infiltration-dominated to runoff-dominated response shown in Figures (3.7) and (3.8).

To further investigate the potential influence of biomass on runoff predictability, we can investigate to what extent $b$ explains the variation in $M$ for the logistic scenario. Specifically, we used the fitlm package in Matlab to perform multiple linear regressions for the value of $M$, both with and without $b$ as a regressor, for
Figure 3.10: Runoff ratio and biomass for both the logistic and constant cases, separated by rainfall intensity. $m$ is the slope of the least squares linear regression of $M$ against $b$.

Several groups of precipitation intensities. The two models take the forms

$$M = a_0 + a_1 s_0 + a_2 P + a_3 s_0 P$$

and

$$M = a_0 + a_1 s_0 + a_2 P + a_3 s_0 P + a_4 b,$$  

where $s_0$ is the saturation at the start of the rain event, $P$ is the total event depth, and the $a$ values are regression coefficients. Accordingly, Figure (3.11a) shows the $R^2$ values of each linear model. Once again we see the emergence of the infiltration-dominated, transition, and runoff-dominated zones based on the rainfall intensity. Outside the transition zone, the two models fit $M$ similarly well, meaning the addition of $b$ to the model does not improve the ability to predict $M$ for small or very large rainfall intensities. However, in the transition zone, the model that includes $b$ explains a significantly larger proportion of the variation in $M$ than the model that excludes $b$. 

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Figure (3.11b) further explains the changes in the linear model by showing the mean contribution of each regressor to $M$ in the version including $b$. The mean contribution is defined as the mean value of a regressor in each precipitation group multiplied by the linear model coefficient for that regressor. Biomass has a contribution that is significant in the transition zone and negligible in the infiltration and runoff dominated zones. Additionally, regressors containing $P$ make up the bulk of $M$ in the runoff-dominated zone. These two plot demonstrate that, under the assumption that $k_s$ is a function of $b$, accounting for biomass can improve runoff predictions for moderately large rain events.

![Figure 3.11: a) Comparison of multiple linear regressions for the logistic case: $M = f(s_0, P, s_0P)$ and $M = f(s_0, P, s_0P, b)$. b) Mean contribution of each regressor to the predicted value of $M$.](image)

3.7 Discussion

The preceding analysis demonstrates that, when modeling vegetation dynamics and water partitioning, using a $k_s$ that varies with biomass introduces potential feed-
backs between soil moisture and biomass states. While the long-term statistics of biomass and runoff are not significantly different between scenarios using constant and variable $k_s$ values, the existing differences are more pronounced in drier climate conditions. The most noticeable effect of a variable $k_s$ is observed when investigating runoff during individual rain events of moderately large intensity. Thus, one can see how this dynamic can be overlooked: rather than significantly changing water partitioning for all model scenarios, the variable $k_s$ is most important in drier conditions when large rainfall intensities are possible.

Parameterization is a potential hurdle for widely applying this model. For example, an accurate value of $s_{crit}$ is required since it determines when the system may enter the low biomass/low infiltration feedback state. The relationship between $k_s$ and $b$ is also critical in determining when both the low biomass/low infiltration and high biomass/high infiltration feedbacks occur, so $k_s(b)$ must be as accurate as possible.

While the effects of a variable $k_s$ may be limited to certain conditions, both the concept and application are useful. For instance, the observed feedbacks fit into the narrative of vegetation trapping water and nutrients, resulting in patchy vegetation. Additionally, a variable $k_s$ can improve runoff estimates when needed for purposes such as flood prediction or reservoir management.

3.8 Conclusion

In this analysis, we used a dynamic vegetation and soil moisture model to investigate the effects of a variable $k_s$ on biomass and water partitioning. First, by looking closely at model results for the scenarios of a constant $k_s$ and logistic $k_s(b)$, we pinpointed differences in saturation caused by different $k_s$ values as the primary driver of differences in biomass. A positive feedback between biomass and infiltration helps keep biomass high during wet conditions, and another positive feedback makes
it difficult for vegetation in a variable $k_s$ soil to recover after a dry period.

The effect of $k_s(b)$ on biomass and partitioning was small for the original parameter set. However, upon changing the average storm depth, average interarrival time, maximum ET, and soil depth, we discovered that drier conditions are more likely to lead to a difference between the constant and logistic scenarios. Additionally, the relationship between the ranges of storm intensity and $k_s(b)$ play a critical role in water partitioning, as they control the transition from infiltration-dominated to runoff-dominated rainfall response.

Furthermore, we observed that drier soils are more likely to display a discrepancy between the storm-based runoff ratio of the logistic and constant cases. Specifically, drier conditions with a variable $k_s(b)$ lead to the low biomass/low infiltration feedback. A critical rainfall intensity zone exists for this effect; both small and very large storms have smaller discrepancies between the two cases than moderately intense storms. Again, the relationship between rainfall intensity and the range of $k_s$ values is central to this effect.

Finally, we investigated the specific role of biomass in values of the runoff ratio. While the scenario with constant $k_s$ shows little relationship between $b$ and $M$, the logistic $k_s(b)$ scenario has a negatively correlated relationship between $b$ and $M$ that first increases and then decreases in strength with rainfall intensity. Furthermore, the inclusion of $b$ as a regressor in a linear model to predict $M$ improves model performance by up to approximately 60 percent in the transition zone of rainfall intensity. The contribution of each regressor to the predicted value of $M$ confirms the behavior of the three intensity zones; biomass contribution is greatest during the transition zone and precipitation dominates in the runoff-dominated zone.

Since this analysis focused on changing one parameter at a time, the next logical step is to investigate more complete climate scenarios. For example, how would the constant and logistic scenarios differ with both deeper soil and less rainfall than the
default parameters examined here? Future work will include quantifying biomass and partitioning for a wider range of realistic climate-soil-vegetation systems and further developing a unifying dryness framework for how different parameters affect the system. Despite the limitations of these experiments, we have shown that the effects of a variable $k_s(b)$ can have measurable effects on vegetation biomass and water partitioning. In effect, a variable $k_s(b)$ converts soil from a memoryless system to one with memory of past saturation and biomass conditions. Those interested in modeling vegetation and runoff, particularly on shorter timescales, should consider investigating the existence of a variable $k_s$ on their particular location.
Impact of drought severity and timing on vegetation and water partitioning during recovery

Premise

This chapter extends the work of Chapter 3. Having shown that the effects of a variable $k_s$ are more pronounced in dry conditions, here we explore under what conditions and to what extent the low $k_s$-low infiltration feedback affects vegetation growth and runoff production during the drought recovery period.

4.1 Introduction

Droughts are meteorological and hydrological events with profound impacts on vegetation ecosystems and in turn on society. The lack of water input into a system can cause, for example, loss of rain-fed crops, falling surface reservoir levels, and strain on groundwater resources. In Mediterranean climates specifically, where the warm and wet seasons are out of phase, drought is expected in the summer. However, drought in the winter months can reduce reservoir recharge and drought during the relatively short growing season can reduce plant biomass availability for grazing (Vico et al.,
Furthermore, the expected increase in both short and long term droughts due to climate change (Sheffield and Wood, 2008) causes greater uncertainty in hydrological predictions. Hence, the need exists to understand how periods with lower than average rainfall affect both vegetation and hydrologic characteristics of the land surface.

One existing area of drought study focuses on the effects of dry periods on vegetation and how vegetation recovers following drought in the Mediterranean. Many of these studies (Lazaro et al., 2001; Pereira et al., 2007; Xu and Zhou, 2007; Zavalloni et al., 2008; Ceusters et al., 2009; Shinoda et al., 2010; Jongen et al., 2013, among others) reveal that plant species, particularly grasses, in arid and semiarid environments are fairly resistant to sub-annual drought conditions and can recover biomass soon after normal precipitation resumes. Others have demonstrated that the timing of a drought is critical, with dry conditions during the peak of the growing season having the largest effect on biomass production (e.g. Vicente-Serrano, 2007; Jongen et al., 2011; Aschonitis et al., 2013). Furthermore, depending on the length and severity of the drought, vegetation can take more than one season to recover, indicating that ecosystems have a memory of droughts (e.g. Lloret et al., 2004; Martiny et al., 2005; Yahdjian and Sala, 2006; Kinugasa et al., 2012). Hence, the ability of Mediterranean vegetation to recover from drought conditions depends on both the timing and severity of the drought.

Another area of drought research concerns the effect of dry periods on soil properties and infiltration/runoff response. Several researchers have demonstrated that periods of dry weather, particularly when combined with high temperatures, can lead to increased water repellency and water drop penetration time, ultimately causing increased runoff. For example, Cerda et al. (1998), Lichner et al. (2010), Gimbel et al. (2016), and Siteur et al. (2016) all found increased runoff to be caused by litterfall during droughts that causes soil to become hydrophobic. In another study,
Wang et al. (2015) proposed that air entry into soil pores during dry periods reduces infiltration ability on a sub-annual time scale. Overall, current research suggests that extended dry periods can cause increased runoff response during subsequent rain events.

Relatively recent work highlights the need to integrate vegetation and soil characteristics while studying land surface response. Yildiz and Barros (2007), for instance, used a coupled surface-groundwater hydrological model with satellite land cover data to show that fractional vegetation cover and leaf area index (LAI) are primary controls on hydrologic response, particularly during droughts. Additionally, Parr et al. (2015) and Tesemma et al. (2015) both found that incorporating LAI into the Variable Infiltration Capacity Model (Liang et al., 1994) improved the streamflow predictions of the model; the latter also found a 1.3 to 10.2% increase in runoff during drought conditions relative to not including LAI in the model.

While many researchers have investigated vegetation and soil responses to drought individually, relatively few have carried out work combining the two. However, as suggested by Gimbel et al. (2016), this topic requires our attention:

“The drought history or ... climatic condition in the past had more of an effect on the observed hydrophobicity and infiltration behavior than the actual antecedent soil moisture ... [D]rought effects on infiltration processes need to be considered in hydrological models to obtain realistic predictions regarding water quality and quantity in runoff and groundwater recharge.”

The plot-scale rainfall simulation experiments of Chapter 2 revealed a monotonically increasing relationship between vegetation height and effective soil hydraulic conductivity ($k_s$) at a field site in Orroli, Sardinia (Italy). In a follow-up study (Chapter 3), we used a simplified version of the land surface and vegetation dynam-
ics model (LSM-VDM) developed by Montaldo et al. (2008) to assess how a $k_s$ that is a function of biomass ($k_s(b)$) affects biomass and water partitioning. This work revealed two positive feedbacks, one between high biomass and infiltration and another between low biomass and infiltration, which are particularly relevant for drier climates. We also discovered that introducing a variable $k_s(b)$ leads to a rainfall intensity-dependent inverse relationship between biomass and runoff ratio, where a critical range of moderate rainfall intensities produces a difference in runoff between the constant $k_s$ and variable $k_s$ scenarios.

Clearly, dry conditions affect both vegetation biomass production and the runoff response of the land surface. Now, we seek a deeper understanding of how both components behave in the period following a drought. Specifically, we will investigate how a variable $k_s$ affects the amount of time it takes vegetation to return to normal levels. Similarly, we will analyze any differences in post-drought runoff between the constant and variable $k_s$ scenarios, along with how these effects change based on the timing and severity of the drought.

We approach this investigation using the same constant $k_s$ and variable $k_s(b)$ of the previous chapter along with an updated version of the simplified LSM-VDM and data from the Sardinia field site. Our hypothesis, consistent with the recent work of others noted above and the conclusions from Chapter 3, is that dry conditions and the resulting biomass reduction will cause greater runoff under a variable $k_s(b)$ than with a constant $k_s$; consequently, the reduced infiltration will cause longer biomass recovery times under a variable $k_s(b)$.

4.2 Model changes

The simplified LSM-VDM presented in Chapter 3 models the aboveground green grass biomass ($b$) along with soil saturation ($s$) using the following coupled equations:
\[
\frac{ds}{dt} = \frac{1}{n'z_r} (i' - e_t - d),
\]
\[
\frac{db}{dt} = \left( \frac{\alpha_1 \omega e_{max} f_1(s)}{b_{max}} - \alpha_2 \right) b,
\]

where \(n'\) is relative soil porosity, \(z_r\) is soil depth, \(i'\) is effective infiltration, \(e_t\) is total evapotranspiration, \(d\) is drainage from the bottom of the root zone, \(\omega\) is the plant water use efficiency, \(f_1(s)\) is the moisture-limiting function used in the Jarvis (1976) stomatal conductance model, \(b_{max}\) is the maximum plant biomass per square meter, and \(\alpha_1\) and \(\alpha_2\) are calibration parameters.

This representation only uses variations in moisture to modulate photosynthesis. Since we seek to investigate the seasonality of temperature and precipitation, we must modify the equations to incorporate meteorological forcing. We accomplished this goal by changing the representation of evapotranspiration (ET) from a fraction of a prescribed maximum, \(e_{max}\), to the full Penman-Monteith equation (Penman, 1948; Monteith, 1965):

\[
e_v = f_{\text{veg}} \cdot \frac{(\Delta_{cc} \times R_n (1 - g)) + (\rho c_p VPD/r_a)}{L_v \cdot (\Delta_{cc} + \gamma_{cc} (1 + r_{cc}/r_a))}
\]

This change was possible due to existing measurements of relative humidity, vapor pressure deficit (VPD), incoming radiation, etc.; the lack of these data would require a simpler approach such as the Priestly-Taylor modification. After calculating \(e_v\), we again used the water use efficiency (\(\omega\)) to calculate photosynthesis (\(P\)) using \(P = \omega \times e_v\).

Of note in the Penman-Monteith equation is the stomatal resistance, which is modeled using the method of Jarvis (1976). This method includes three stress functions, \(f_1(s)\), \(f_2(T_a)\), and \(f_3(\text{VPD})\), which modify stomatal conductance through soil moisture, air temperature, and VPD, respectively. With a maximum value of 1, the
product of these three functions (hereafter $\Pi(f_s, f_T, f_V)$) indicates how $e_v$ and $P$ relate to their maximum values.

With the new $e_v$ representation, the differential equation for $b$ reverts to

$$\frac{db}{dt} = \alpha_1 P - \alpha_2 b$$

(4.3)

where $\alpha_1 = a(1 - g)$ and $\alpha_2 = m_a Q_{10}^{T_m/10} + d_a$; mean daily air temperature ($T_m$) is the only variable in these expressions. Since $\alpha_1$ and $\alpha_2$ were previously our calibration parameters, rather than ignore the dependence of biomass loss on temperature we chose to pick a constant by which to scale these values. In the current model, we use multipliers of 2.5 and 0.4 for $\alpha_1$ and $\alpha_2$, respectively.

The remainder of the model is unchanged from the previous version. For more detail see Chapter 3 and Montaldo et al. (2008), the latter in particular contains expressions and parameters required for the Penman-Monteith $ET$ calculation.

### 4.3 Numerical experiments

The location of the data measurement station and original LSM-VDM calibration is a field site in Orroli, Sardinia (39°41’12.57” N, 9°16’30.34” E, 500 m a.s.l.), where the typical Mediterranean climate produces cool, wet winters and hot, dry summers. The water year, which goes from October to September, contains a wet season from November to April and warmer temperatures in June through September. Furthermore, this site in particular is characterized by a shallow silt-loam soil and a combination of grasses, bare soil, and woody vegetation; in this study we only focus on the first two components since grasses are more sensitive to small rainfall inputs.

As previously mentioned, we seek an understanding of the effect that a variable $k_s$ has on drought recovery along with how these effects vary with drought timing and intensity. Accordingly, we ran three numerical experiments comprising model
runs with several different parameters. In the first experiment, we compared model results using a constant \( k_s \) value, \( k_s = 0.9 \text{ cm h}^{-1} \), and a variable value, which uses the following logistic relationship:

\[
k_s(b) = \frac{k_1}{1 + \exp(-k_2 \times (b - k_3))} + k_4 \tag{4.4}
\]

with \( k_1 = 3.82 \text{ cm h}^{-1} \), \( k_2 = 0.11 \), \( k_4 = 92 \text{ g} \), and \( k_4 = 0.18 \). Please see Section 3.2 for more detail on this expression.

The second numerical experiment assessed the effects of drought severity. Since our focus is on the behavior of the soil-vegetation system in the period following a drought, each simulation starts with \( s_0 = 0 \) and \( 0 < b_0/b_{\text{max}} \leq 1 \), where \( b_{\text{max}} \) is the maximum biomass parameter. In this way, the starting value \( b_0 \) indicates the severity or duration of a drought through the loss of aboveground green biomass during the dry period. We use \( b_0 = b_{\text{max}} \) as a baseline scenario for comparison to the varying drought intensities.

For the final experiment, we chose three model starting points, or drought end points, in each water year: the first of December, February, and April. Each of these starting points has different hydrological indications worth studying. A drought ending in December is essentially a late start to the rainy season, so this set of model results will show the effects of eliminating the short burst of growth at the beginning of the wet season before cold temperatures set in. Next, a February start time depicts a drought during the primary recharge period when saturation is high but biomass production is limited by temperature. Finally, an April start indicates dry conditions at the start of the primary growing season. Therefore, with two \( k_s \) scenarios, ten \( b_0 \) values, and three start times, we have 60 sets of model results for each of the six water years of available meteorological data.

A number of analyses will determine the differences between the model scenarios. Specifically, for biomass, we will calculate the number of days \( (T_R) \) it takes for the
biomass of the different drought intensities to converge. Additionally, to determine
the seasonal effect of the drought, we will calculate the difference, in both timing and
magnitude, of spring peak biomass for the different scenarios. Then, we will turn
our attention to runoff, which we will analyze on annual, daily, and sub-daily time
scales.

4.4 Field data and rainfall scaling

The data measurements at the field site include air temperature \((T_a)\), relative hu-
midity \((rh)\), wind speed \((v_a)\), incoming shortwave radiation \((R_{sw})\), photosynthetically
active radiation \((PAR)\), and of course precipitation (Montaldo et al., 2008). We have
these data on a 30-minute time interval from April 2003 through October 2006, with
additional rainfall-only precipitation through January 2010. Hence we have six full
water years of precipitation covering October 2003 through September 2009. We
averaged the existing data across years and applied it to the three years with only
precipitation to generate a complete series of meteorological forcing.

Unfortunately, this measurement period contained smaller than average rainfall
events, with very few producing surface runoff. Therefore, for the purposes of eluci-
dating potential effects on runoff, we scaled the existing rainfall data to artificially
generate larger events. Specifically, we multiplied the depth of each rainfall event
by a random number from the lognormal distribution corresponding to the normal
distribution with \(\mu = 1\) and \(\sigma = 1\), with an additional right-shift of 1 so no depth
would be reduced due to a multiplier less than one. The median multiplier was 1.71,
and the mean event depth increased from 0.15 cm to 0.29 cm. The resulting precipi-
tation, shown in Figure (4.1), has a heavier tail characterized by several large events,
which is more realistic than the limited range of depths measured.
Figure 4.1: Shifted lognormal distribution of precipitation depth multipliers; event depth frequencies for the measured and scaled precipitation.

4.5 Results and Discussion

To understand the general differences between the constant and variable $k_s$ scenarios, we begin with the model results starting on 1 Dec 2003 with $b_0 = 100\%$ Figure (4.2). A full biomass load is akin to a no-drought condition, so we can compare lower $b_0$ scenarios with the $b_0 = 100\%$ scenario. Here, we see essentially no difference between the $s$ and $b$ values of the constant and variable $k_s$ scenarios. This result is in line with our previous work, in which we found that the primary differences between the two $k_s$ implementations are in water partitioning rather than biomass production. Accordingly, we do observe a difference in the cumulative runoff, $Q_T$, between the two scenarios. While small, the difference increases over time, typically making jumps during larger rainfall events that coincide with smaller biomass levels. We attribute this difference in runoff to the lower $k_s(b)$ values that correspond to low biomass.

4.5.1 Optimal growing periods

With a baseline of modeled soil saturation (Figure 4.2), we can begin to make predictions about the effects that drought may have on the system. In the top portion
Figure 4.2: Modeled saturation, vegetation biomass, and cumulative surface runoff.
of Figure (4.3), 10-day moving averages of daily $T_a$ (measured) and $s$ (modeled) are averaged across the six water years. The shaded areas represent the periods when $s \geq s_{lim}$ and $T_a \geq T_{opt}$, or when saturation and temperature are respectively ideal for photosynthesis. The two periods have no overlap, which emphasizes the relatively short growing season that occurs as temperature rises and soil moisture falls.

The bottom portion of Figure (4.3) goes into further detail regarding favorable growing conditions. On the left are the values of the stomatal conductance functions calculated with the average $s$, $T_a$, and VPD for each day in addition to their product $\Pi(f_s, f_T, f_v)$, and the right shows the number of days for which each function is above a given value. The shaded areas represent days of $\Pi(f_s, f_T, f_v) \geq 0.5$; this threshold was chosen because, based on the model results, biomass is more likely to increase than decrease on a day with a $\Pi$ value of at least 0.5. While each function is at least 0.5 for over half the days of the year, the out-of-phase low points for $f_1(s)$ and $f_2(T)$ cause the overall stomatal conductance to be at least 50% for 145 days of the year. These days are split between the two periods of April through June and September through mid-November.

Based on Figure (4.3), we can qualitatively predict potential effects of drought. Specifically, a drought ending in December may mean that the beginning of the water year was dry; the resulting low biomass could mean a lower starting point for the following spring growing season. For February drought endings, it seems that the effect would depend heavily on the length or severity of the dry period and the concentration of rain when the drought is over. For instance, a February return to normal precipitation is likely to recharge soil moisture before temperatures are favorable for biomass production. Finally a drought ending in April has the potential to have significant effects on biomass. During this period, temperature is rising, but conditions will not be favorable for photosynthesis if the soil is dry.
Figure 4.3: (a) Measured air temperature and modeled saturation with optimal ranges of each. (b) Products of the Jarvis stomatal resistance functions reveal limited periods of vegetation growth. Shaded areas represent a product of at least 0.5.
4.5.2 Biomass

Figure (4.4) shows the biomass values for each $b_0$ and start date in the 2006-2007 water year, with the constant scenario on top and the variable scenario on the bottom. These results confirm the previous result of no significant difference between these two $k_s$ types, so from here forward we will focus on only the variable $k_s(b)$ results for biomass. For each start date, two observations emerge: the summer drydown appears to eliminate any major differences in biomass, and a more severe drought (lower $b_0$) causes the peak of spring biomass to be both smaller and later.

![Figure 4.4: Biomass for all start dates and $b_0$ values for the 2006-2007 water year.](image)

For the first of these two observations, we can investigate the biomass recovery times ($T_R$) shown in Figure (4.5). Here, $T_R$ is the average number of days beyond 1 December needed for each $b_0$ scenario to be within 3 g of the no-drought condition; this tolerance is 5 percent of the initial full $b_0$ value of 60 g. $T_R$ is shown as days since 1 December rather than days since the simulation beginning in order to show approximately when in the water year the recovery condition is met. The relationship between $T_R$ and $b_0$ is nearly linear, centered around an average recovery by the end of May for 50 percent $b_0$. The most severe droughts can have recovery times approaching
one year, which indicates the possibility of drought memory that lasts through a full growing season.

![Graph showing biomass convergence](image)

**Figure 4.5:** Average number of days needed for biomass to converge to within 5 percent of the no-drought $b_0$.

Returning to the observation of the shift in the spring peak of biomass, Figure (4.6) shows the average delay in days between the 100 percent $b_0$ peak and the peaks of the drought conditions along with the drought condition peak as a fraction of the 100 percent $b_0$ peak. As previously noted, as the drought becomes more severe, the peak is smaller and occurs later. Interestingly, while the December droughts have slightly higher peak fractions than the other months, it also experiences the longest average delay in the peak occurrence. Looking back at Figure (4.4), it appears that the severe drought biomass is slow to rebound because it starts the primary growing season at a lower value than the high $b_0$ scenarios. Additionally, since lower biomass corresponds to less $ET$, the depletion of soil water is slower and biomass can continue to grow longer than the high $b_0$ conditions. Hence, drought affects the spring peak biomass through altering the initial conditions at the start of the primary growing season.
4.5.3 Daily runoff

Recalling Figure (4.2), we know differences in runoff exist between the constant and variable $k_s$ scenarios. To begin investigating these differences, Figure (4.7) shows the fractional difference in cumulative runoff ($Q_T$) compared to the $b_0$ scenario ($Q_{T0}$).

In both the constant and variable $k_s$ cases, runoff increases as $b_0$ decreases. For a constant $k_s$, scenarios with lower $b$ values lose less water to $ET$ between rain events; the resulting higher antecedent soil moisture when rain begins allows less infiltration and causes more runoff. Additionally, the intervals between the $\Delta Q_T$ values of different $b_0$ starts are fairly constant, which is in line with the regular intervals in $b_0$ causing proportional differences in $ET$. Finally, notice that all changes to $\Delta Q_T$ occur before the 200 day mark. As shown in Figure (4.4), this point marks the beginning of the summer drydown, which equalizes differences in biomass and hence runoff response.

The variable $k_s$ case has increased runoff beyond that of the constant $k_s$ scenario. Since the biomass levels are essentially the same between the two $k_s$ cases, we conclude that the additional runoff is caused by the lower $k_s(b)$ values that occur in the variable $k_s$ case. Indeed, compared to the regular intervals between the $\Delta Q_T$ series in the constant $k_s$ case, the gaps here are irregular due to the nonlinear rela-
Figure 4.7: Fractional differences in surface runoff for the constant and variable $k_s$ scenarios in the 2006-2007 water year.

tionship between $b$ and $k_s$. Furthermore, the difference in runoff response between start months is more pronounced here than in the constant $k_s$ case; in the 2006-2007 example, the maximum fractional $\Delta Q_T$ is approximately 2 percent for a December start but nearly 15 percent for an April start. This difference can also be attributed to biomass, as a large range of $b_0$ values during the rainy season will lead to a large range of $ET$ and antecedent soil moisture values. On the other hand, since $ET$ is low in the winter months, less opportunity exists for different antecedent soil moisture values at the start of rain events during that period. Hence, while the primary effect of the variable $k_s$ is to increase runoff during the recovery period, the effect is enhanced by variations in $ET$ that result from differences in biomass.

Now that we have demonstrated how the variable and constant $k_s$ cases differ in the amount of runoff produced during the biomass recovery period, we will primarily focus on the drought timing and severity effects for the variable $k_s$ scenario alone. Continuing with the 2006-2007 water year example, Figure (4.8) shows the daily runoff ($Q_D$) produced by each $b_0$ scenario and start date combination through the end of the rainy season. The main observation here is that the effect of $b_0$ diminishes
with time. For example, the larger events all occur after 120 days beyond 1 December, meaning the December start scenario has had longer to equilibrate before the ET effect discussed above kicks in. On the other hand, the first storm for the April start occurs only two days into the simulation, meaning a wider range of $k_s(b)$ values are in effect for the different $b_0$ scenarios. Furthermore, the storm on day 157 in the April start set reveals the possible effect of what is not shown in this figure: precipitation intensity. Notice that the response is constant for higher $b_0$ scenarios and then increases steadily for low $b_0$ scenarios. This result suggests that the rainfall intensity of this storm was smaller than the $k_s$ of the large $b_0$ values but larger than that of the low $b_0$ values, meaning the observed runoff was split between infiltration excess runoff and saturation excess runoff.

This potential difference in runoff mechanisms warrants further investigation. Accordingly, Figure (4.9) shows the 30-minute runoff ($Q_{30}$), split into saturation excess runoff ($Q_S$) and infiltration excess runoff ($Q_I$), for several $b_0$ scenarios under a single storm in April 2009. This set of model results are from the April start simulation, so the differences in biomass are quite large at the time of this snapshot. The total event runoff does not vary much between $b_0$ scenarios, however; the runoff mechanism shifts from primarily infiltration excess under low $b_0$ values to saturation excess runoff under high $b_0$ scenarios. This result explains why differences in cumulative runoff are small: the soil becomes saturated regardless of $k_s$, likely because of the shallow (15 cm) root zone. Another interesting feature of this shift is that it alters the timing of runoff. Specifically, the infiltration excess runoff starts sooner for low $b_0$ than the saturation excess runoff does for high $b_0$. Therefore, the biomass state during a rainfall event will determine whether runoff is fast or slow.

Continuing to investigate the split between $Q_S$ and $Q_I$, Figure (4.10) covers a larger time scale to show the division for several storms, this time bringing back the constant $k_s$ case. We see a similar shift from $Q_I$ to $Q_S$ as $b_0$ increases, but only for
Figure 4.8: Responses to individual rain events for varying $b_0$ values and drought end dates.
Figure 4.9: 30-minute saturation excess and infiltration excess runoff for a single storm under varying $b_0$ scenarios.
the variable $k_s$ case. This observation lends credence to the idea that the shift is at least partially due to the precipitation intensity overcoming $k_s$ in some cases but not in others. In fact, for the example shown (water year 2007-2008, April drought end), the variable $k_s$ case has some infiltration excess where the constant $k_s$ case does not. It is likely that the rainfall intensity during that event was greater than the constant $k_s$ but lower than the high-biomass value of $k_s(b)$. Hence, the effects of drought on runoff depend not only on the timing and severity of the drought, but the timing and intensity of the rainfall events that occur following the drought.

Finally, we can summarize the $Q_S/Q_I$ partitioning by observing their cumulative values compared to the no-drought condition. Accordingly, Figure (4.11) shows $Q_I$ and $Q_S$ as fractions of total runoff under the no-drought case. Soon after the drought ends, the $Q_S$ and $Q_I$ partitions vary significantly with $b_0$. As time progresses, dominance shifts towards $Q_I$ and the difference among $b_0$ scenarios decreases. Therefore, taken with the previous runoff results, the days soon after a drought ends are the most likely to produce runoff that varies in mechanism depending on the timing and severity of the drought along with the rainfall intensity.

4.6 Discussion and Conclusion

In this chapter, we used a simplified land surface and vegetation dynamics model to investigate the effects of a biomass-dependent $k_s$, drought timing, and drought severity on the conditions following the dry period. Our primary findings are as follows:

1. The biomass level at the end of a drought, which is an indication of drought severity, determines how long and to what extent biomass will recover during the growing season. More severe conditions lead to peaks in biomass that are both delayed in time and reduced in magnitude.
Figure 4.10: Saturation and infiltration excess runoff for several rainfall events in water year 2007-2008 with an April drought end.
Figure 4.11: $Q_S$ and $Q_I$ partitioning 25, 60, and 180 days following the end of a drought, averaged over all six water years.
2. A variable $k_s$ leads to larger predictions of runoff overall, and this effect is enhanced by the varying $k_s(b)$ and $ET$ corresponding to biomass.

3. While total runoff does not change significantly among drought severities, the vegetation levels soon after the end of the drought will determine how larger rainfall events are partitioned between saturation excess runoff and infiltration runoff.

These findings align with the work of previous researchers, particularly with regards to increased runoff and to drought severity affecting the recovery time.

Limitations of the current model include the lack of multi-component biomass tracking along with an uncertain $k_s(b)$ relationship. However, while the exact causes of changes in $k_s$ for our Sardinian field site are currently unknown, using biomass and effective $k_s$ as a proxy for other processes is a necessary step to justify further work in this area.

From here, this line of research can continue in several directions. One option is to use a more complete version of the LSM-VDM upon which the present model is based to evaluate biomass components including standing dead and root biomass. The model should also be calibrated and validating using the most recent available data. Another direction is to attempt to elucidate the causes of the variable $k_s$ through additional field experiments.

All told, the present analysis contributes to the growing body of work indicating the need for including variable soil and land surface effects in hydrological modeling.
In this thesis, we investigated dynamic soil, vegetation, and runoff response characteristics in a semi-arid climate. Specifically, after designing, building, and testing a rainfall simulator to produce artificial rainfall events up to 60 mm h$^{-1}$, subsequent field experiments revealed a monotonically increasing relationship between vegetation height and the effective saturated hydraulic conductivity.

Then, after investigating several possible forms for the relationship between vegetation and biomass ($k_s(b)$), we determined that a logistic form would be most appropriate due to its good characterization of the field data and ease of modification. Using the logistic $k_s(b)$ and a streamlined land surface/vegetation dynamics model, we investigated the effect of a variable $k_s$ on biomass and water partitioning. We found that while the resulting differences in biomass between the two cases were small, a variable $k_s$ can induce a positive feedback between low biomass and low infiltration along with another positive feedback between high biomass and high infiltration. These feedbacks are more critical for drier ecosystems and for storms of moderate rainfall intensity.

Finally, knowing that a variable $k_s$ can result in storm responses different from
those under a constant $k_s$ scenario, we investigated these effects in the recovery time following droughts. The primary finding related to $k_s$ is that a variable value can lead to increased infiltration excess runoff where the constant $k_s$ case has saturation excess runoff. These effects are more pronounced early in the recovery time and for larger rainfall events. For biomass, the primary discovery is that increasing drought severity causes the spring peak biomass to be lower and delayed from the no-drought condition.

5.1 Implications

The results of this research show that static $k_s$ values may not be ideal for land surface and hydrological modeling in all situations. Rather, a $k_s$ value that changes with regards to other land surface properties may improve runoff predictions for larger precipitation events and soon after extended dry periods end. For regions like the Mediterranean where surface runoff feeds water reservoirs, more accurate prediction of runoff will become more critical as variability in climate conditions increases. Additionally, better characterization of the mechanisms and timing of runoff can aid in the prediction of flash flooding and the design of systems that control storm flows.

5.2 Future directions

The area of research focusing on temporally variable soil properties in unmanaged landscapes is relatively new. Therefore, the first step in continuing this work would be to accurately characterize the relationship between infiltration properties and dynamic land surface properties for both our field site and others. Once we have more knowledge about what properties of the soil change and how vegetation acts to modify them, we can develop general functions for dynamic runoff response.

With additional theoretical knowledge, further research can include incorporating this dynamic response into distributed watershed models to estimate its large scale
effects. Specifically, we can investigate whether or not this model change improves characterizations of runoff/run-on and peak flows.

Ultimately, we have shown that the way in which the land surface responds to rainfall events depends on more than antecedent conditions and static soil properties. Hopefully, this work will convince others of the need to research this area, particularly since water is a critical resource that is becoming more difficult to manage due to climate change.
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Biography

Tiffany Gale Wilson was born on the 26th day of January, 1985, in Somerset, New Jersey. After spending time in Illinois and New Jersey as a child, she earned her Bachelor of Science in Engineering at Princeton University in 2007. Then, a short break from academia left her refreshed for her journey towards earning her Doctor of Philosophy degree in Civil and Environmental Engineering at Duke University in 2017.

During her time at Duke, Tiffany was a Pratt-Gardner Fellow, James B. Duke Fellow, and NSF IGERT-WiSeNet Trainee. She was also the recipient of the 2015 Jeffrey B. Taub Environmental Engineering Graduate Student Award, granted by the Department of Civil and Environmental Engineering.

Tiffany’s work includes three first-author papers, one published and two in preparation, along with a number of posters, presentations, and collaborative papers. After leaving Duke, Tiffany hopes to continue in academia through a postdoctoral research position.