ABSTRACT

Essays on the Economics of Higher Education: Investigating College Major Choice

by

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Seth Sanders

An abstract of a dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Economics in the Graduate School of Duke University 2017
Abstract

This dissertation consists of two separate essays on major choice in higher education. In the first chapter, I investigate how differences in information affect students’ major choices over time. Since college has such a short time horizon, the amount of information students have before coming in may play a big role in how well they are matched to their final major. They may also choose their initial major based on how uncertain they are about their match with that major, especially since they have the option to switch in future periods. This paper discusses students’ search process in finding a major, and how information impacts behavior and ultimate outcomes. I set up a tiered structure where the student must first choose a field (either STEM or Non-STEM) and then choose a major within that field. This allows for matches within a particular field to be correlated, thus providing information on non-chosen majors within the same field. The student makes decisions based on the choices that will maximize her expected utility over her entire college career. Since her current choices and information set depend on past decisions, and since there are a finite number of periods, I can solve the dynamic decision problem using backwards recursion.

Once I solve for the student’s optimal decision path, I estimate the model using data from the Campus Life and Learning Survey from Duke University. The CLL data allows me to observe students’ expected majors at multiple points throughout their college career. I attempt to find the model parameters that best match partic-
ular moments in the data. The first key type of moment involves overall switching patterns, that is, the probability of choosing a particular field in the initial period, and then the probabilities of later decisions conditional on the first choice. The second key type of moment I match captures which students are making which decisions. I look at how academic ability, as measured by SAT Math scores, and gender affect the choice probabilities in the data.

I find that STEM has a lower average match value than Non-STEM, even though it attracts more than half the students in the first period. The uncertainty in matches is higher in STEM, and it is more costly to switch into STEM in later periods. Thus, students have an incentive to see if they get a good match early. Students with higher math ability are more likely to choose STEM in the first period, but the sorting by ability greatly increases in the later period. It is costly to switch into STEM from non-STEM in the second period, while the reverse move is virtually costless.

In the second chapter, co-authored with Thomas Ahn, Peter Arcidiacono, and James Thomas, we estimate an equilibrium model of grading policies. On the supply side, professors offer courses with particular grading policies. Professors set both an intercept and a return to studying and ability in determining their grading policies. They make these decisions, attempting to maximize their own utility, but taking into account all other professors’ grading policies. On the demand side, students respond by selecting a bundle of courses, then deciding how much to study in each class conditional on enrolling. We allow men and women to have different preferences over different departments, how much they like higher grades, and how costly it is to exert more effort in studying.

Two decompositions are performed. First, we separate out how much of the differences in grading policies across fields is driven by differences in demand for courses in those fields and how much is due to differences in professor preferences across fields. Second, we separate out how much differences in female/male course
taking across fields is driven by i) differences in cognitive skills, ii) differences in the valuation of grades, iii) differences in the cost of studying, and iv) differences in field preferences.

We then use the structural parameters to evaluate restrictions on grading policies. Restrictions on grading policies that equalize grade distributions across classes result in higher (lower) grades in science (non-science) fields but more (less) work being required. As women are willing to study more than men, this restriction on grading policies results in more women pursuing the sciences and more men pursuing the non-sciences.
To my parents, for their encouragement and support every step of the way
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Abbreviations

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<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>NM</td>
<td>New Major</td>
</tr>
<tr>
<td>NFM</td>
<td>New Field and Major</td>
</tr>
<tr>
<td>B1M</td>
<td>Back to initial major</td>
</tr>
<tr>
<td>B1F1M</td>
<td>Back to Initial Field and Initial Major</td>
</tr>
<tr>
<td>B1FNM</td>
<td>Back to Initial Field with New Major</td>
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This dissertation consists of two separate essays on major choice in higher education. In the first chapter, I investigate how differences in information affect students’ major choices over time. Since college has such a short time horizon, the amount of information students have before coming in may play a big role in how well they are matched to their final major. They may also choose their initial major based on how uncertain they are about their match with that major, especially since they have the option to switch in future periods. This paper discusses students’ search process in finding a major, and how information impacts behavior and ultimate outcomes. I set up a tiered structure where the student must first choose a field (either STEM or Non-STEM) and then choose a major within that field. This allows for matches within a particular field to be correlated, thus providing information on non-chosen majors within the same field. The student makes decisions based on the choices that will maximize her expected utility over her entire college career. Since her current choices and information set depend on past decisions, and since there are a finite number of periods, I can solve the dynamic decision problem using backwards recursion.
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We then use the structural parameters to evaluate restrictions on grading policies. Restrictions on grading policies that equalize grade distributions across classes result in higher (lower) grades in science (non-science) fields but more (less) work being required. As women are willing to study more than men, this restriction on grading policies results in more women pursuing the sciences and more men pursuing the non-sciences.
2 Dynamic Major Choice Patterns

2.1 Introduction

About 30% of students who entered college in 2012 officially changed majors at least once by 2014, according to the most recent cohort in the Beginning Postsecondary Students Longitudinal Survey. One third of those students changed majors more than once. One of the most common themes in the major choice literature is students’ uncertainty about their preferences, abilities, and future earnings associated with various fields of study. The first goal of this paper is to model the college major search process, incorporating uncertainty both at the broad field-level and at a narrower level of individual majors. An important component of this is determining how limited information affects students’ initial choices and subsequent switching behavior. A second goal is to compare the importance of uncertainty against the cost of switching into particular majors, and how these two factors combine in determining students’ major choice patterns throughout college.

There is a large literature that documents the differences in labor market earn-

1 The BPS is a nationally representative longitudinal survey. The 2012 cohort included approximately 25,000 respondents.
ings by major.\textsuperscript{2} These results have proven consistent, even after controlling for selection due to ability (Arcidiacono (2004)) and occupation choice (Grogger and Eide (1995)). While expected earnings have been found to contribute to major choice, non-pecuniary factors such as ability and preferences over the subject matter are even more important for the student’s decision.\textsuperscript{3} Arcidiacono even finds that students care more about their preferences for the major itself than for the job that it will lead to although both are important.

Models starting with Bamberger (1988), Altonji (1993), and Arcidiacono (2004) have incorporated uncertainty into the college major choice problem. More recently, subjective expectations methods have been used to clarify and quantify the level of uncertainty students experience and how they respond to it. Arcidiacono et al. (2012a) shows that students have significant forecast errors in their expected earnings and abilities by major, and that 7.5\% of students would switch majors if their forecasts were accurate. Zafar (2011) finds that students respond to information in a way consistent with Bayesian updating. The study also finds that learning has both general and major-specific components, and that students who switch are responding primarily to information about their own major.

Empirical work has demonstrated that information is actually transferred through the education process. For example, grades carry information about both general and major-specific ability (Arcidiacono (2004)), a result that is echoed in Zafar (2011). This allows for education to have a role in information acquisition as well as human capital accumulation.

In this paper, I will focus on the informational role of education and will incorpo-


\textsuperscript{3} See Arcidiacono (2004), Beffy et al. (2012), and Wiswall and Zafar (2014). This supports the results found in Carneiro et al. (2003) that non-pecuniary factors are an important part of schooling choices.
rate both field- and major-specific information. This means I will model learning with a common component across majors within fields, but not across fields. Although some studies have acknowledged the general and specific nature of information production, it has not been explicitly modeled in the two-tier structure that I propose. I will start with the basic framework undertaken in Neal’s paper on job mobility. Neal (1999) assumes workers have both career- and job-specific matches and have no initial information about any match. He finds that workers search in a two-stage process in which they first settle on a career and only then search for a specific job within that career. This paper will apply the idea to the choice of college major, and will extend it further by assuming limited information. We might think that students have at least some idea of their relative strengths and preferences, as opposed to Neal’s no information model. Students also have a short time horizon in college, limiting their ability to switch many times in order to find a good match.4

These and other papers have looked at particular factors that influence students’ major choices, such as the student’s ability in the subject, his expected future earnings from that degree, and his relative preferences for future labor market considerations and current enjoyment of the topic. In the theoretical model, I abstract from including any particular match inputs and combine everything into a single parameter describing overall match quality. This simplification allows me to isolate the role played by information in students’ decisions over time. When I estimate the model, I allow the student-field match to be a function of ability (as measured by SAT scores) and gender.

4 Also notice that there is a trade-off between experimenting to find the best match versus specializing in one field, which could lead to higher wages or which might be required for certain occupations. In his 2010 paper, Malamud (2010) explores the effects of timing of specialization in college and finds that early specialization is associated with more costly switches in the labor market but the effects do not persist over time.
2.2 Data and Descriptive Evidence

Data Sources

The data for this analysis come from two sources at Duke University: the Campus Life and Learning project, plus registrar data. The main source of students’ expectation and major data is the Campus Life and Learning (CLL) survey performed at Duke University. The survey includes a sample of students from the incoming 2001 and 2002 classes (2005 and 2006 graduating classes). It was designed as a panel with four waves for each student: a pre-college survey to determine details about the students’ background prior to entering college, plus surveys at the end of the freshman, sophomore, and senior years.

I use one primary question in two survey waves to look at major switching behavior over time. The pre-college CLL survey asks students to give their expected major.\(^5\) This survey, conducted during the summer before students matriculated, indicates students’ expectations before they arrive at Duke. Second, the survey at the end of the sophomore year asks students to report their current major at that time.\(^6\) Along with the CLL data, I use registrar data of official major(s) and degree(s) (AB, BS, or BSE) conferred at graduation. I limit my sample to students who responded to those two questions (registrar data is available for all students). This means I have 3 data points for each student, spread equally through time beginning just before matriculation and ending at graduation.

Institutional Set-Up

Students are not required to declare a major until the end of sophomore year, so I use the term ”major” to denote a self-reported focus in a certain department. Thus,

\(^5\) The survey question reads: ”What do you expect will be your eventual college major? (If don’t know, list ”Don’t Know.”)”

\(^6\) The survey question reads: ”What is (are) your current major(s) (declared or expected)?”
switches between majors as I define them may not reflect official registrar data, but rather track students’ expectations at defined points throughout college.

One important note regarding the institutional structure at Duke is that there are two undergraduate schools: the Pratt School of Engineering, and the Trinity School of Arts and Sciences. These schools have different admissions procedures, and it is possible, although potentially difficult, to switch between the two. The Pratt School offers four majors, all resulting in a B.S.E. degree, while Trinity offers 47 majors in the Sciences, Social Sciences, and Humanities. Engineering students have much more structured graduation requirements, and behave differently from other students in terms of switching behavior, so I present data analyses both including and excluding these students.

Data Construction

The most important aspect of setting up the data for analysis is being able to classify each student’s major and corresponding field in each time period. There are three challenges involved in translating the data from the survey responses: (i) double majors, (ii) “Don’t Know” responses, and (iii) open-ended responses.

To simplify the number of possible transition paths through time, I only allow for a single major in each period. This restriction affects almost 50% of the students in my sample. In the pre-college survey, 15% of the students in my sample expect to double major. By the end of sophomore year, that number climbs to 32%, and 38% of students graduate with a double major. The details of how I choose which major to keep in each period are in the appendix, but most cases are covered by the following two rules: (1) Favor consistency over time (fewer switches), and (2) Favor earlier switches. An example of Rule (1) would be if a student lists multiple majors

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7 Percentages refer to students in my sample, not all survey respondents.

8 I apply these same rules to students who respond with expected major X or Y, not only X and Y. 22 students in the initial survey and 3 students in the second wave fall into this category.
in any period, but has one major that is constant through all periods. An example of Rule (2) would be a student who expects to major in X in the initial survey, expects to double major in X and Y in the 2nd year survey, and who graduates with major Y. Rule (2) says to choose major Y in the second period.

These rules capture the notion that majors as defined in this paper indicate student interest in a particular major. More importantly, it emphasizes when a student reverses a previous decision. In that sense, the rules set a high bar in defining a switch in majors. The goal is to capture indications that the student is shifting her trajectory towards a different end than when she started at Duke.

The second issue is that 30% of students who would otherwise be included in my sample responded “Don’t Know” in the pre-college survey. Since I am interested in tracking changes at both the field and major level, I have to drop these students. While it is possible to back out their initial field interest using their first-semester course schedule, predicting at the finer level of particular majors is too error-prone. The only other option would be to classify all these students as switching majors (either within or across fields) in Period 2.

The third issue in preparing the survey responses for analysis arises because the relevant survey questions are open-ended. This means I need to match students’ responses with the specific majors that are offered at Duke. In the pre-college survey, just over 6% of responses are professional tracks or groups of majors that cross departmental boundaries. The primary offenders here are “Pre-Med” (4.3%) and “Engineering” (2.1%). While many resources for pre-professional students are available (including suggestions for useful courses), there is no official link between any particular major and interest in Pre-Med, Pre-Law, Business, or other graduate-level programs. I drop these students, along with several more who create individualized courses of study outside the majors offered by Duke.9 See Appendix A for more

9 The Duke Undergraduate Course Bulletin describes this alternative path: “As degree candi-
details on how I classified majors and selected the sample for analysis.

The final issue is how to classify the majors themselves as STEM or Non-STEM. In particular, Economics and Psychology offer a B.S. degree, and a sizable fraction of students within those departments are somewhat similar to STEM majors in their course choices. I present results with these majors classified as Non-STEM.

**Descriptive Patterns**

The phenomenon underlying this study is pictured in 2.1 and 2.2. There is a striking difference in the switching patterns between students who start in STEM and those who start in Non-STEM. If Engineering students are included in the sample, we see about the same fraction of students staying in their initial field choice. For those who do switch, however, only a tiny fraction switch out of Non-STEM into STEM, while almost 2/3 of the switchers starting in STEM move into Non-STEM. If Engineering students are excluded, this pattern is exacerbated even further. The transitions between sophomore and senior year follow a similar, although much more attenuated, pattern.

Table 2.1 presents the evolution of major choice/concentration in the 10 most popular majors. Although the group of majors remains (mostly) constant over time, the composition changes drastically between the first and second periods.

We see a drop in the number of STEM majors of almost 30% by the sophomore year. The fraction of students in STEM remains relatively stable in the last two periods. We can also see that the shifts are not equally distributed across majors. The biggest exodus occurs in biology, which loses almost half of its students in the first two years. Psychology, on the other hand, more than doubles its student population. One final thing to note is that, while the concentration of students in dates in Program II, students separate themselves from the requirements and options of Program I including the requirement for a major and the options of multiple majors and minors.”
the five most popular STEM majors remains stable throughout all periods, students spread out in Non-STEM after the initial period. The fraction of students outside the top five Non-STEM majors doubles in the later periods. This could indicate that students are more aware of the options in STEM, but not how well they will fit with them, compared to students learning about new options in Non-STEM once they

**Figure 2.2: Transition Probabilities: Sophomore Year to Senior Year**
Table 2.1: Percentage of Students in Top 10 Majors over Time

<table>
<thead>
<tr>
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<th>Pre-College</th>
<th>Sophomore</th>
<th>Graduation</th>
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<tbody>
<tr>
<td><strong>STEM</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Biology</td>
<td>19.6%</td>
<td>9.1%</td>
<td>9.1%</td>
</tr>
<tr>
<td>Biomedical Eng</td>
<td>15.4%</td>
<td>12.4%</td>
<td>12.4%</td>
</tr>
<tr>
<td>Chemistry</td>
<td>4.3%</td>
<td>2.5%</td>
<td>2.5%</td>
</tr>
<tr>
<td>Electrical Eng</td>
<td>4.9%</td>
<td>5.1%</td>
<td>5.0%</td>
</tr>
<tr>
<td>Mathematics</td>
<td>3.1%</td>
<td>2.5%</td>
<td>2.3%</td>
</tr>
<tr>
<td>Other</td>
<td>8.9%</td>
<td>9.0%</td>
<td>8.4%</td>
</tr>
<tr>
<td><strong>Non-STEM</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Economics</td>
<td>9.9%</td>
<td>10.3%</td>
<td>10.3%</td>
</tr>
<tr>
<td>History</td>
<td>3.7%</td>
<td>4.9%</td>
<td>5.6%</td>
</tr>
<tr>
<td>Political Science</td>
<td>9.1%</td>
<td>7.8%</td>
<td>8.0%</td>
</tr>
<tr>
<td>Psychology</td>
<td>6.4%</td>
<td>13.0%</td>
<td>13.0%</td>
</tr>
<tr>
<td>Public Policy</td>
<td>7.4%</td>
<td>8.9%</td>
<td>8.6%</td>
</tr>
<tr>
<td>Other</td>
<td>7.4%</td>
<td>14.6%</td>
<td>15.0%</td>
</tr>
</tbody>
</table>

Note: N=486 students

arrive at Duke.

Table 2.2 presents logit estimates for whether a student switched majors within their first two years. We see that it is females and students with low Math SAT scores who are switching out of STEM. Without further structure on the problem, however, we can’t distinguish their underlying reasons for starting in STEM and the subsequent changes. The model I present in the next section is one way of teasing out the role of uncertainty in students' decisions across time.

2.3 Model

*Model Set-Up*

Assume that in each period \( t \in \{1, \ldots, T\} \), a student, \( i \), chooses a major by first choosing a field and then choosing from the possible majors within that field. The choice of the field/match pair is denoted \( d_{it} = (k, j(k)) \) for field \( k \in \{1, 2\} \) and major \( j(k) \in \{1, \ldots, J(k)\} \). Define \( H_{it} = \{d_{i1}, \ldots, d_{i,t-1}\} \) as the set of \( i \)'s choices before
Table 2.2: Estimates of Probability of Switching to a New Major

<table>
<thead>
<tr>
<th></th>
<th>With Engineering</th>
<th></th>
<th>Without Engineering</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coeff.</td>
<td>Std. Error</td>
<td>Coeff.</td>
<td>Std. Error</td>
</tr>
<tr>
<td>Female</td>
<td>0.263</td>
<td>0.315</td>
<td>0.281</td>
<td>0.320</td>
</tr>
<tr>
<td>Start in STEM</td>
<td>-0.160</td>
<td>0.309</td>
<td>0.401</td>
<td>0.368</td>
</tr>
<tr>
<td>Female × STEM</td>
<td>0.894**</td>
<td>0.423</td>
<td>0.632</td>
<td>0.491</td>
</tr>
<tr>
<td>Math SAT</td>
<td>-0.001</td>
<td>0.165</td>
<td>-0.024</td>
<td>0.170</td>
</tr>
<tr>
<td>Math SAT × STEM</td>
<td>-0.612***</td>
<td>0.230</td>
<td>-0.431</td>
<td>0.262</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.146**</td>
<td>0.453</td>
<td>-1.077**</td>
<td>0.487</td>
</tr>
<tr>
<td>Observations</td>
<td>486</td>
<td></td>
<td>367</td>
<td></td>
</tr>
<tr>
<td>Demographic</td>
<td></td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>characteristics</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.092</td>
<td></td>
<td>0.082</td>
<td></td>
</tr>
</tbody>
</table>

*Note: Simple logit regression where dependent variable is an indicator for whether the student changed majors between the pre-college and sophomore year surveys.

Before the initial decision, the student is uncertain about the match values for all fields and majors. I assume that his prior beliefs can be modeled as normal.
distributions in the following way:

\[ \theta_k \sim N(\mu_{\theta_k}, \sigma_{\theta_k}^2) \]

\[ \gamma_{j(k)} \sim N(\mu_{\gamma}, \sigma_{\gamma}) \]

I interpret these distributional parameters as describing the student’s “best guess” about each match, and the amount of uncertainty about that guess. There are several things to notice here. First, the prior distribution of field matches is field-dependent. That is, the two fields may have different priors, in both the mean and variance parameters. This is a key difference from the models in Neal (1999) and Pavan (2010). In Neal (1999), all priors at the occupation level (analogous to the field level here) are ex ante identical. Pavan (2010) loosens this restriction by allowing for the means to differ by occupation, but continues to assume identical variances. Allowing for differences in both parameters means that the student may not only have different expected match values for different fields, but may also have different levels of uncertainty regarding the outcomes. This generalization enables me to shed light on the importance of uncertainty in the timing and outcomes of students’ major choices throughout the college experience.

Second, note that the prior distribution for the major match is equivalent across all majors. It is neither major- nor field-dependent, so all majors are ex ante equivalent from the student’s perspective.

Finally, note that the matches are perfectly observed once chosen by the student (there is no error in the utility function), and that the values are constant over time (the match values are not indexed by \( t \)). Neal makes the same assumptions, while Pavan assumes that the matches follow a random walk. This means the match values are not constant through time, but also that more time spent in any occupation or employer does not provide any additional information about a “true” match value. So in a sense, all information that could be known by the agent is discovered in the
first period. Miller (1984) focuses more on the informational capacities of jobs and so allows for learning over time. However, he does present the case of immediate revelation of the true match in his paper.

One final assumption is that all distributions are independent. Any particular draw does not make any subsequent draws more or less likely to be favorable to the student. There is no information about alternative draws contained within any particular choice. The student must actually choose a major to get any more information about that particular match. The value of the tiered structure is that, even though the draws are independent, information about the expected utilities of other choices is gained through the common information provided by the field draw.

Intuitively, you can think about the field match as describing the common characteristics of all the majors within it. If majors were divided into STEM and non-STEM, all majors within the STEM field might focus on technical knowledge, analysis, and numerical data, while non-STEM majors may focus on linguistic knowledge, writing, and verbal data. Major matches within each field contain characteristics particular to that major, after the common field characteristics have been taken out. For example, computer science majors will learn particular advanced coding skills that math majors will not.

Define $I_{t-1}$ as the cumulative set of information known to the student before the decision in Period $t$ is made. The initial information set, $I_0$ simply contains the distributional parameters that make up the student’s priors: $I_0 = \{\mu_{\theta_k}, \sigma_{\theta_k}, \mu_{\theta_1}, \sigma_{\theta_1}, \mu_\gamma, \sigma_\gamma\}$.

**Student’s Problem**

The student’s problem is to find the optimal decision rule, $d^*$ that will maximize the total discounted expected utility across all periods. Thus, for any period $t$, the student solves the following problem:
\[ V_t(d_t)|I_{t-1} = \max_{d_t} \mathbb{E}\left[ \left( U_t(d_t) + \beta \mathbb{E}[V_{t+1}(d_{t+1}|I_t)] \right) | d_t, I_{t-1} \right] \]

where \( d_t^* \) is the optimal decision rule in Period \( t \).

I solve the problem using backwards induction. Normally, dynamic programming problems are too large and unwieldy to solve analytically. \(^{10}\)

Figure 2.3 shows the choices available to the student in each period. Throughout the paper, I pool elements within a choice set which have the same expected outcome from the student’s perspective. For example, if a student switches majors from major A to major B in Period 2, the elements making up the choice sets in Period 3 are different than if he had switched from A to C. However, all the same aggregate choice options still exist. Keeping track of past choices and outcomes is sufficient to determine the expected value of each pooled group of field/major combinations. Without loss of generality, denote the student’s Period 1 choice as \( (\theta_k, \gamma_{j(k)}) \). Additional choices resulting in a new major or field are denoted by primes.

Note that information provides no direct utility; it is simply a tool, a resource that the student can use to achieve his ultimate goal, which is a good match. If he has very little information, but ends up with a high enough match, he will be satisfied. This is true even when switching is costless.

One of the key features of this finite, imperfect information problem is that it has much to say about the initial choice of major. If everything looks the same to the student ex ante, he will pick randomly, but when he has different initial priors, the initial choice probabilities change. This is important in determining why certain fields are popular initial choices but have large attrition in later years. The differences

\(^{10}\) Increasing the number of periods and/or the number of alternatives in the choice set quickly blows up the number of potential outcomes that must be considered. For example, if the number of choice alternatives, \( J \), is constant through time, the number of potential outcomes can be described by the geometric sequence \( J^T \) \(^{(1-J)^T} \). So even with only 3 periods and 3 choices in each period, there are 39 potential outcomes.
in information available to students could be a factor in explaining differences in initial interest that random choice is not able to explain. The cost of switching into particular fields may also affect students’ decisions. The field that is more costly to switch into may be more popular initially because those students on the margin have an incentive to try that field first to avoid paying the higher penalty in the next period.

**Imperfect Information Model**

**Period 3**

In the final period, the student simply wants to maximize his current expected utility since there is no possibility of further switches. The student’s problem will depend on his previous choices and draws in Periods 1 and 2.

Table 2.3 gives the optimal Period 3 policy. The first column denotes the Period 2 choice, which describes the particular problem the student faces in the last period. The second column shows the elements in the information set conditional on $d_2$.\(^{11}\)

---

\(^{11}\) All information sets include the prior distributional parameters, so I have left those out for the
sake of visual clarity.
| Per 2 Choice | $I_3$ | Per 3 Choice Set | $\mathbb{E}(U_3|I_3)$ | $\theta$ Condition | $\gamma$ Condition(s) | $\theta + \gamma$ Condition(s) |
|--------------|-------|-----------------|---------------------|--------------------|---------------------|-------------------------------|
| Stay | \{\theta_k, \gamma_j(k)\} | Stay | $\theta_k + \gamma_j(k)$ | $\gamma_j(k) > \mu_\gamma$ | $\theta_k + \gamma_j(k) > \mu_{\theta, \nu} + \mu_\gamma$ | |
| | | NM | $\theta_k + \mu_\gamma$ | $\gamma_j(k) < \mu_\gamma$ | $\theta_k + \gamma_j(k) > \mu_{\theta, \nu} + \mu_\gamma$ | |
| | | NFM | $\mu_{\theta, \nu} + \mu_\gamma$ | $\theta_k < \mu_{\theta, \nu}$ | $\theta_k + \gamma_j(k) < \mu_{\theta, \nu} + \mu_\gamma$ | |
| New Major | \{\theta_k, \gamma_j(k), \gamma_j'(k)\} | Stay | $\theta_k + \gamma_j'(k)$ | $\gamma_j'(k) > \mu_\gamma$ | $\theta_k + \gamma_j'(k) > \mu_{\theta, \nu} + \mu_\gamma$ | |
| | | NM | $\theta_k + \mu_\gamma$ | $\gamma_j'(k) < \mu_\gamma$ | $\theta_k + \gamma_j'(k) > \mu_{\theta, \nu} + \mu_\gamma$ | |
| | | B1M | $\theta_k + \gamma_j(k)$ | $\gamma_j(k) > \gamma_j'(k)$ | $\theta_k + \gamma_j(k) > \mu_{\theta, \nu} + \mu_\gamma$ | |
| | | B1F | $\theta_k + \mu_\gamma$ | $\gamma_j(k) < \mu_\gamma$ | $\theta_k + \gamma_j(k) < \mu_{\theta, \nu} + \mu_\gamma$ |  |
| New Field and Major | \{\theta_k, \gamma_j(k), \gamma_j'(k), \theta_{k'}\} | Stay | $\theta_{k'} + \gamma_j(k')$ | $\gamma_j(k') > \mu_\gamma$ | $\theta_{k'} + \gamma_j(k') > \theta_k + \gamma_j(k)$ | |
| | | NM | $\theta_{k'} + \mu_\gamma$ | $\gamma_j(k') < \mu_\gamma$ | $\theta_{k'} + \gamma_j(k') > \theta_k + \gamma_j(k)$ | |
| | | B1F1M | $\theta_k + \gamma_j(k)$ | $\gamma_j(k) > \mu_\gamma$ | $\theta_k + \gamma_j(k) > \theta_{k'} + \gamma_j(k')$ | |
| | | B1FNM | $\theta_k + \mu_\gamma$ | $\gamma_j(k) < \mu_\gamma$ | $\theta_k + \gamma_j(k) < \theta_{k'} + \mu_\gamma$ | |
Period 2

Given the optimal decision rule for the final period, we can move back to Period 2. At this point in the problem, the student knows the value of his period 1 choices. I use that information, plus the knowledge of the final period decision rule, to determine the student’s best choice given his Period 1 decision.

The Period 2 choice set is not path dependent (though different particular Period 1 draws will clearly result in different particular elements within the choice set). In every case, $I_1 = \{\theta_k, \gamma_{j(k)}\}$ and $C_2 = \{S, \text{NM}, \text{NFM}\}$.

I can plug in the expected value functions from Period 3 into the second term of $V_2$. The potential choices in the future term will still depend on the Period 2 choice, so I match the expressions accordingly.
\[ \mu_{\theta k'} + \mu_\gamma \]

Stay if \( \gamma_{j(k')} > \gamma_{j(k)} \)

B1M if \( \gamma_{j(k')} < \gamma_{j(k)} \)

New Field and Major

\[ \mu_\gamma \]

New Major if \( \gamma_{j(k)} < \mu_\gamma \)

B1M if \( \gamma_{j(k)} > \mu_\gamma \)

\[ \mu_{\theta k'} \]

\( \theta_k + \mu_\gamma \)

\( \theta_k + \gamma_{j(k)} \)

Stay

Back to 1st Field and 1st Major

\( \theta_k \)

\( \theta_k + (\gamma_{j(k)} - \mu_\gamma) \)

New Major

\[ \mu_\gamma \]

Figure 2.5: Period 3 Solution if \( d_2 = \text{NM} \)

\[ \theta_k + \mu_\gamma \]

\( \theta_k + \gamma_{j(k)} \)

Stay

\[ \mu_\gamma \]

\( \theta_k \)

\( \theta_k + (\gamma_{j(k)} - \mu_\gamma) \)

New Major

Figure 2.6: Period 3 Solution if \( d_2 = \text{NFM} \) and \( \gamma_{j(k)} > \mu_\gamma \)
In order to facilitate the characterization of the optimal policy, I first prove that some alternatives in Period 3 are strictly dominated, so the size of the choice sets can be reduced.

**Proposition 1.** (i) If $d_2 = \text{NM}$, then $d_3 \neq \text{NFM}$. (ii) If $d_2 = \text{Stay}$, then $d_3 = \text{Stay}$.

**Proof.** (i) Since $\theta_k$ is known at this point in the problem, it is not uncertain whether
\( \theta_k \) is greater than or less than \( \mu_{\theta_k'} \). So \( P(d_3=\text{NM}|d_2=\text{NM}) \) and \( P(d_3=\text{NFM}|d_2=\text{NM}) \) cannot both have positive probability. If \( \theta_k > \mu_{\theta_k'} \), then NFM would never be chosen after picking a new major in Period 2. If \( \theta_k < \mu_{\theta_k'} \), then \( d_2=\text{NM} \) would never be chosen in favor of \( d_2=\text{NFM} \). We can see this by comparing \( \mathbb{E}(V_2|\text{NM}_2, \text{NFM}_3) = \theta_k + \mu_\gamma + \beta(\mu_{\theta_k'} + \mu_\gamma) \) with \( \mathbb{E}(V_2|\text{NFM}_2, \text{B1FNM}_3) = \mu_{\theta_k'} + \mu_\gamma + \beta(\theta_k + \mu_\gamma) \). The expected utilities are the same, but in different time periods, so for any \( \beta < 1 \), \( \mathbb{E}(V_2|\text{NFM}_2, \text{B1FNM}_3) > \mathbb{E}(V_2|\text{NM}_2, \text{NFM}_3) \). If \( c_3^* = \text{B1FNM} \), then this relationship holds. If \( c_3^* \neq \text{B1FNM} \), then \( \mu_{\theta_k'} + \mu_\gamma + \beta(U(c_3^*)) > \mu_{\theta_k'} + \mu_\gamma + \beta(\theta_k + \mu_\gamma) > \theta_k + \mu_\gamma + \beta(\theta_{k'} + \mu_\gamma) \). So the expected utility of choosing \( d_2 = \text{NM} \), \( d_3 = \text{NFM} \) is strictly less than \( d_2 = \text{NFM} \) for all \( d_3 \in d_3 \).

(ii) Similarly, all of the expected utilities in Period 3 are known for \( d_2 = \text{Stay} \). If \( \gamma_{j(k)} > \mu_\gamma \), then \( P(\text{NM}_3|\text{Stay}_2) = 0 \). If \( \gamma_{j(k)} < \mu_\gamma \), then the minimum expected value of choosing a new major in Period 2 is \( \theta_k + \mu_\gamma + \beta(\theta_k + \mu_\gamma) > \theta_k + \gamma_{j(k)} + \beta(\theta_k + \mu_\gamma) \). So, if \( \gamma_{j(k)} > \mu_\gamma \), then \( d_2 = \text{Stay} \), \( d_3 = \text{NM} \) is dominated by \( d_2 = \text{Stay} \), \( d_3 = \text{Stay} \), and if \( \gamma_{j(k)} < \mu_\gamma \), then it is always better to choose \( d_2 = \text{NM} \).

Finally, using the same logic, if \( \theta_k + \gamma_{j(k)} > \mu_{\theta_k'} + \mu_\gamma \), then \( P(d_3 = \text{NFM}|d_2 = \text{Stay}) = 0 \). If \( \theta_k + \gamma_{j(k)} < \mu_{\theta_k'} + \mu_\gamma \), then it would always be better to choose NFM in Period 2 instead of Stay. If \( \gamma_{j(k)} < \mu_\gamma \), then a low field draw would still result in \( \theta_k + \mu_\gamma \) in Period 3. If \( \gamma_{j(k)} > \mu_\gamma \), then a low field draw would result in \( \theta_k + \gamma_{j(k)} \). In either case, since \( \theta_k + \gamma_{j(k)} < \mu_{\theta_k'} + \mu_\gamma \), the total expected utility for the two periods would be higher by choosing NFM in Period 2, even if the only difference is the discount factor, as in part (i).

Since the student can return to any previous field and major, the initial matches are a lower bound for the Period 3 match values. Therefore, any switches in Period 2 can only increase the Period 3 expected utility relative to the initial draws. There is potentially a tradeoff between lower expected values in Period 2 and higher expected
values in Period 3. If the expected utility in Period 2 is at least as large as the Period 1 utility, though, then switching can only be beneficial because it is potentially better in every period and never worse (in expectation).

Returning to the student’s problem, I conjecture that the solution can be characterized in the following way: There exist cutoff points $\theta^*$ and $\gamma^*$ such that a student will make the following decision:

1. Stay if $\theta_k + \gamma_{j(k)} > \theta^* + \gamma^*$ and $\gamma_{j(k)} > \gamma^*$

2. New major if $\theta_k < \theta^*$ and $\gamma_{j(k)} < \gamma^*$

3. New field and major if $\theta_k < \theta^*$ and $\theta_k + \gamma_{j(k)} < \theta^* + \gamma^*$

Now consider a student who draws $\theta_k = \theta^*$ and $\gamma_{j(k)} = \gamma^*$ in Period 1. This puts her right at the nexus where, according to Proposition 1, she is indifferent between all 3 choices in Period 2. If I can find a $(\theta^*, \gamma^*)$ pair such that the indifference conditions across all choices hold, then I will have found a solution to the student’s Period 2 problem.

I start by assuming that the student drew a high enough $\theta_k$ so they are only deciding whether to stay or draw a new major. Set $E_{\gamma}(\text{Stay}_2|\gamma_{j(k)} = \gamma^*) = E_{\gamma}(\text{NM}_2|\gamma_{j(k)} = \gamma^*)$. I want to prove that there exists a unique $\gamma^*$ that solves this equation. This will give me a $\gamma^*$ at which the student is indifferent between staying in their current major or drawing a new option.

For a high enough field match, the expected values for staying or switching majors can be expressed in the following way:
\[ E_\gamma(\text{Stay}_2 | \gamma_{j(k)} = \gamma^*) = \gamma^* + \beta \gamma^* \]
\[ E_\gamma(\text{NM}_2 | \gamma_{j(k)} = \gamma^*) = \mu_\gamma + \beta E_\gamma(V_{3,\gamma} | \text{NM}_2, \gamma_{j(k)} = \gamma^*) \]
\[ = \mu_\gamma + \beta \left[ P(\gamma_{j'(|k)} < \gamma^*)\gamma^* + P(\gamma_{j'(|k)} > \gamma^*)E(\gamma_{j'(|k)} > \gamma^*) \right] \]
\[ = \mu_\gamma + \beta \left[ F_\gamma(\gamma^*)\gamma^* + (1 - F_\gamma(\gamma^*)) \left( \mu_\gamma + \frac{\sigma_\gamma^2 f_\gamma(\gamma^*)}{1 - F_\gamma(\gamma^*)} \right) \right] \]
\[ = \mu_\gamma + \beta \left[ \mu_\gamma + F_\gamma(\gamma^*)(\gamma^* - \mu_\gamma) + \sigma_\gamma^2 f_\gamma(\gamma^*) \right] \]

**Proposition 2.** There exists a unique \( \gamma^* \) that solves:
\[ E_\gamma(\text{Stay}_2 | \gamma_{j(k)} = \gamma^*) = E_\gamma(\text{NM}_2 | \gamma_{j(k)} = \gamma^*) \]

**Proof.** Define \( \Delta_\gamma E(S.NM) = E_\gamma(S_2 | \gamma_{j(k)} = \gamma^*) - E_\gamma(\text{NM}_2 | \gamma_{j(k)} = \gamma^*) \). Then \( \gamma^* \) is the root of the equation:
\[ 0 = \Delta_\gamma E(S.NM) \]
\[ = \gamma^*(1 + \beta) - \mu_\gamma - \beta \left[ \mu_\gamma + F_\gamma(\gamma^*)(\gamma^* - \mu_\gamma) + \sigma_\gamma^2 f_\gamma(\gamma^*) \right] \]

If I rearrange some terms and divide through by \( \sigma_\gamma \), I get:
\[ 0 = \frac{\gamma^* - \mu_\gamma}{\sigma_\gamma} - \frac{\beta}{1 + \beta} \left[ F_\gamma(\gamma^*) \left( \frac{\gamma^* - \mu_\gamma}{\sigma_\gamma} \right) + \sigma_\gamma f_\gamma(\gamma^*) \right] \]

I can rewrite \( F_\gamma(\gamma^*) \) and \( f_\gamma(\gamma^*) \) into standard normal notation (\( \Phi(\cdot) \) and \( \phi(\cdot) \)).\(^{12}\)
\[ 0 = \frac{\gamma^* - \mu_\gamma}{\sigma_\gamma} - \frac{\beta}{1 + \beta} \left[ \Phi(\frac{\gamma^* - \mu_\gamma}{\sigma_\gamma}) \left( \frac{\gamma^* - \mu_\gamma}{\sigma_\gamma} \right) + \phi(\frac{\gamma^* - \mu_\gamma}{\sigma_\gamma}) \right] \]

Let \( X = \frac{\gamma^* - \mu_\gamma}{\sigma_\gamma} \). Then,
\[ 0 = X - \frac{\beta}{1 + \beta} \left[ X \Phi(X) + \phi(X) \right] \quad \text{(2.2)} \]
\[ 0 = X - \frac{\beta}{1 + \beta} \left[ X \Phi(X) + \phi(X) \right] \quad \text{(2.3)} \]

\(^{12}\) \( F_\gamma(\gamma^*) = F(\gamma | \mu_\gamma, \sigma_\gamma^2) = \Phi(\frac{\gamma^* - \mu_\gamma}{\sigma_\gamma}) \) and \( f_\gamma(\gamma^*) = f(\gamma | \mu_\gamma, \sigma_\gamma^2) = \frac{1}{\sigma_\gamma} \phi(\frac{\gamma^* - \mu_\gamma}{\sigma_\gamma}) \)
I can easily reclaim \( \gamma^* \) by setting \( \gamma^* = \mu_\gamma + \sigma_\gamma X \). To prove a unique solution exists, first take the derivative with respect to \( X \):

\[
\frac{d}{dx} \left( X - \frac{\beta}{1 + \beta} \left[ X \Phi(X) + \phi(X) \right] \right) \nonumber
\]

\[
= 1 - \frac{\beta}{1 + \beta} \left[ X \Phi'(X) + \Phi(X) + \phi'(X) \right] \nonumber
\]

\[
= 1 - \frac{\beta}{1 + \beta} \left[ X \phi(X) + \Phi(X) - X \phi(X) \right] \nonumber
\]

\[
= 1 - \frac{\beta}{1 + \beta} \Phi(X) > 0 \quad (2.4) \nonumber
\]

We see that \( \Delta_\gamma \mathbb{E}(S.NM) \) is increasing in \( X \) for all values of \( X \), and \( \frac{\partial X}{\partial \gamma} = \frac{1}{\sigma_\gamma} > 0 \), so the difference is also increasing in \( \gamma^* \). Now evaluate the function at \( \gamma^* = \mu_\gamma \) and \( \gamma^* = \mu_\gamma + \sigma_\gamma \):

At \( \gamma^* = \mu_\gamma \), \( X = 0 \) and \( \Delta_\gamma \mathbb{E}(S.NM) = 0 - \frac{\beta}{1 + \beta} \left[ 0 + \phi(0) \right] < 0 \).

At \( \gamma^* = \mu_\gamma + \sigma_\gamma \), \( X = 1 \) and \( \Delta_\gamma \mathbb{E}(S.NM) = 1 - \frac{\beta}{1 + \beta} \left[ \Phi(1) + \phi(1) \right] > 0 \).

We see from above that at \( \mu_\gamma \), \( \Delta_\gamma \mathbb{E}(S.NM) < 0 \), which means the expected value from picking a new major is higher than that of staying. If we move to a point one standard deviation above the mean, we see the opposite result. Since we know \( \Delta_\gamma \mathbb{E}(S.NM) \) is monotonically increasing in \( \gamma^* \), there must be a unique point in \( (\mu_\gamma, \mu_\gamma + \sigma_\gamma) \) such that \( \Delta_\gamma \mathbb{E}(S.NM) = 0 \).

Given the existence of an optimal \( \gamma^* \), I now attempt to find a unique \( \theta^* \) such that
\[ \mathbb{E}_\theta(\text{Stay}_2|\theta^*, \gamma^*) = \mathbb{E}_\theta(\text{NFM}_2|\theta_k = \theta^*, \gamma_{j(k)} = \gamma^*) : \]

\[ = \mu_{\theta_{k'}} + \mu_{\gamma} + \beta \left[ P(\theta_{k'} > \theta^* + \gamma^* - \mu_{\gamma})P(\gamma_{j(k')} < \mu_{\gamma})\left[ \mathbb{E}(\theta_{k'}|\theta_{k'} > \theta^* + \gamma^* - \mu_{\gamma} + \mu_{\gamma}) \right] 
+ P(\theta_{k'} < \theta^* + \gamma^* - \mu_{\gamma})P(\theta_{k'} + \gamma_{j(k')} < \theta^* + \gamma^*|\theta_{k'} < \theta^* + \gamma^* - \mu_{\gamma})(\theta^* + \gamma^*) 
+ P(\gamma_{j(k')} > \mu_{\gamma})P(\theta_{k'} + \gamma_{j(k')} > \theta^* + \gamma^*|\gamma_{j(k')} > \mu_{\gamma}) \right] \times \mathbb{E}(\theta_{k'} + \gamma_{j(k')}|\theta_{k'} + \gamma_{j(k')} > \theta^* + \gamma^*, \gamma_{j(k')} > \mu_{\gamma}) \right] \]

The \( \theta^* \) value is where the expected discounted future benefit of getting a potentially better match exactly outweighs the expected cost in the current period. That is, the expected values of choosing a new field and major are lower than the certain values of staying. But, the realized values of the new draws could be higher than the current matches, leading to an increase in overall utility. If the new matches are not better than the current ones, the student can switch back in the last period.

If there are cutoff values \( \theta^* \) and \( \gamma^* \) that solve the Period 2 maximization problem in the manner characterized in Proposition ??, the solution can be drawn according to Figure 2.8.

**Period 1**

As a reminder,

\[ V_1(d_1)|I_0 = \max_{d_1} \left( \mathbb{E}\left[U_1(d_1)|I_0\right] \right. + \beta \mathbb{E}\left[U_2(d_2^*)|I_1\right] \left| I_0 \right] + \beta^2 \mathbb{E}\left[ \mathbb{E}\left[U_3(d_3^*)|I_2\right] | I_1 \right] \right| I_0 \]

\[ = \max_{d_1} \mathbb{E}\left[ \left( U_1(d_1) + \beta \mathbb{E}[V_2(d_2^*)|I_1] \right) | I_0 \right] \]

See Appendix B for details on constructing the expression. This expression effectively determines which field the student will choose in the initial period. Since all majors are equivalent to the student ex ante, the initial choice of major has no effect.
on the difference in expected total utilities between choices in different fields (or within the same field).

2.4 Estimation

2.4.1 Empirical Implications for Switching Patterns

Recall from Lemma 2 that the model had two empirical implications: (1) A student will never switch fields after already switching majors within a field, and (2) If a student does not change majors in a period, he will never switch in any future period. Table 2.4 documents the percentages of students in each possible set of choices over both periods. The highlighted cells are those patterns that do not adhere to the model implications.

We see that almost 94% of students behave in a way that is consistent with the theory. All students chose paths that follow the first implication. That is, no student who switched majors within her original field in the first two years later switched fields. This holds for students starting in both STEM and non-STEM. Some students
do switch in the later period after not switching in the first period, contrary to the second theoretical prediction. A smaller percentage of students who start in STEM act against the predictions (2.2%) than those who start in Non-STEM (4.2%). Note that these cell sizes are so small that the total number of students whose behavior doesn’t match the theory is 15.

Table 2.4: Dynamic Switching Patterns

For initial STEM majors:

<table>
<thead>
<tr>
<th>Soph Year → Graduation</th>
<th>Stay</th>
<th>NM</th>
<th>NFM</th>
<th>B1M</th>
<th>B1F</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-College</td>
<td>Stay</td>
<td>53.1%</td>
<td>0.7%</td>
<td>1.5%</td>
<td>-</td>
<td>55.3%</td>
</tr>
<tr>
<td></td>
<td>NM</td>
<td>12.8%</td>
<td>0.4%</td>
<td>0.0%</td>
<td>0.4%</td>
<td>13.6%</td>
</tr>
<tr>
<td></td>
<td>NFM</td>
<td>30.4%</td>
<td>0.4%</td>
<td>-</td>
<td>0.4%</td>
<td>31.1%</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>96.3%</td>
<td>1.5%</td>
<td>1.5%</td>
<td>0.4%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

For initial Non-STEM majors:

<table>
<thead>
<tr>
<th>Soph Year → Graduation</th>
<th>Stay</th>
<th>NM</th>
<th>NFM</th>
<th>B1M</th>
<th>B1F</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-College</td>
<td>Stay</td>
<td>55.4%</td>
<td>4.2%</td>
<td>0.0%</td>
<td>-</td>
<td>59.6%</td>
</tr>
<tr>
<td></td>
<td>NM</td>
<td>34.3%</td>
<td>0.9%</td>
<td>0.0%</td>
<td>0.9%</td>
<td>36.2%</td>
</tr>
<tr>
<td></td>
<td>NFM</td>
<td>2.8%</td>
<td>0.5%</td>
<td>-</td>
<td>-</td>
<td>4.2%</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>92.5%</td>
<td>5.6%</td>
<td>0.0%</td>
<td>0.9%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Estimation

According to the theory presented above, student $i$’s utility of choosing a STEM field is:

$$U_{i,S,j(S)} = \theta_iS + \gamma_{ij(S)}$$

To facilitate estimation, I restrict the model to two periods. The vast majority of switches takes place in the initial two years, with just over 5% of students making a switch in the final two years (in the sample including Engineering students). I also
incorporate an explicit switching cost that is specific to the initial/final field pair. The details of this version of the model are spelled out in Appendix C.

The theoretical model presented gives the optimal decision rule an individual will follow in each period, given particular distributional parameters. I now want to turn to the data to estimate those parameters within a given context (and to determine how well the model fits students’ behavior). To move from the economic model to an empirical estimation, I need to consider (1) which features of the data are most important for the model to capture, and (2) what additional modeling assumptions are necessary to make the best use of the data.

The main empirical features of interest are the initial choices of students, how the students transition over time (which fields or majors students switch out of or into, and when these switches occur), and how these choice paths differ by particular student characteristics. Here I focus on math ability (as measured by SAT Math scores) and gender.

Regarding additional assumptions, first note that the theoretical model already makes parametric assumptions concerning students’ prior beliefs, namely, that each student’s beliefs over her match values follow a normal distribution with mean $\mu_{i\theta}$ and variance $\sigma^2_{i\theta}$. I will work within that framework. Some additional factors must be taken into account to make this estimable. First, how do I aggregate the matches beyond the individual level? Most importantly, which observable factors can be used to determine the match value, and what assumptions will I make regarding heterogeneity in student preferences? These need to be considered in light of the method of estimation, and finally, all factors need to be considered in the context of the overall goal of the model.

For the first point, since I only observe each student in one particular instance of ”going to college,” there is no way to estimate individual distributional parameters. The most we can say at the individual level has already been determined in the
theory—that is, the *range* of possible parameters leading to particular outcomes. For example, if an individual chose non-STEM in the first period, we know that her expected utility of non-STEM is higher than that of STEM. This relationship is characterized by the expression of the initial value function detailed in Appendix B. For now, I assume that the distribution parameters for the major match are identical for all students. I also assume that the variance of the field matches are the same across students. In the following paragraphs, I discuss how I treat the expected match value at the field level.

By aggregating the students, the parameters can take on a slightly different meaning. Instead of interpreting the parameters as individual-level indices of expected utility and uncertainty, they can instead be attached to the fields themselves. That is, the aggregated mean and variance for each field can be viewed as descriptive measures of the actual historical matches associated with the field across all students who have chosen it. So if \( \mu_1 > \mu_2 \), then, on average, students receive higher matches in Field 1. If \( \sigma_{\theta_1}^2 > \sigma_{\theta_2}^2 \), then matches in Field 1 have had a higher dispersion of realized matches.

For point two, notice that nothing in the current model is observable except students’ decisions over time. Thus, I will need additional assumptions about how the match values and switching costs are constituted. First, I assume that the expected value of the field match is made up of three types of inputs: a population-level mean match value for the field, individual-level student characteristics, and a student-specific preference parameter for the field. I further assume that the preference parameter is distributed normally in the population. Of these, only the student characteristics are observable to the econometrician. The field-specific mean and the student’s private preferences are known by each student, but are not observable to the researcher.

Three comparisons of interest:
1. The variance of the STEM versus Non-STEM matches

2. Within each field, the portion of the variance that is known to the student versus unknown

3. The relative importance of switching costs versus uncertainty in explaining switching behavior

2.4.2 Specification and Parameterization

Recall from 2.1 that student $i$’s utility of choosing field $k$ and major $j(k)$ in time $t$ is:

$$U_{it}(d_{it} = (k, j(k))) = \theta_{ik} + \gamma_{ij(k)} - c_{it}(d_{it}, H_{it})$$

I specify the field match as consisting of 3 parts: (i) a field-specific mean, (ii) an individual $\times$ field-specific component, and (iii) an error term.

$$\theta_{ik} = \mu_{\theta_{ik}} + \alpha_{ik} + \varepsilon_{\theta_{ik}}$$

where the individual-specific component can be further broken down into an observable portion based on student characteristics, and an unobserved individual-specific preference for field $k$:

$$\alpha_{ik} = X_i \beta_k + \eta_{ik}, \quad \text{where } \eta_{ik} \sim N(0, \sigma_{\eta_k}^2)$$

For the $X_i$ term, I include SAT Math scores and an indicator for whether student $i$ is female.

The $\eta_{ik}$ term is known by the student ex ante and is constant over time. It allows for each student to have a “private preference” for field $k$ that shifts their expected match value up or down relative to the baseline mean conditional on demographic characteristics. By construction, the preference term is uncorrelated with the other covariates, and it has a zero mean. Any correlation between the preference term and $X_i$ will already be picked up by the $\beta_k$ coefficients, and any non-zero mean will get absorbed by the $\mu_{\theta_{ik}}$ term.
The value of including this term is that it differentiates between the amount of uncertainty attached to each field versus the amount of inherent variability in preferences that is not captured by the student characteristics included in the model. A high value for $\sigma^2_{\eta_k}$ indicates that students are using other factors—*which they already know beforehand*—beyond the characteristics captured by $X_i$ to calculate their expected match values. A high value for $\sigma^2_{\theta_k}$, on the other hand, indicates that students face a high amount of uncertainty about their match values. Basically, it breaks the variance of the field match into two parts: the part known by the student and the part unknown by the student.

I specify the major match simply as the realized value of the draw from the distribution of majors:

$$\gamma_{ij(k)} = \epsilon_{\gamma_{ij(k)}}$$

Switching costs in Period 1 are zero, and I let the Period 2 switching costs vary by initial and final major, as well as student characteristics:

$$c_{i2}(d_{i2}, H_{i2}) = \begin{cases} 0 & \text{if } d_{i1} = d_{i2} \\ c0_{k,j^j} + \beta_{c_k,j^j'}X_i & \text{if } d_{i1} = (k, j(k)) \text{ and } d_{i2} = (k, j'(k)) \\ c0_{kk'} + \beta_{c_{kk'}}X_i & \text{if } d_{i1} = (k, j(k)) \text{ and } d_{i2} = (k', j(k')) \end{cases}$$

As mentioned in the model section, the students know all potential switching costs ex ante. Thus, the costs effectively change the expected utility for each choice in the relevant period(s). This helps to capture a potentially important factor in students’ initial major choices. If it is relatively costly to switch into a field but relatively cheap to switch out of it, that gives students an incentive to try it in the first period. Along with the match covariates, these costs help to determine which students are moving into or out of particular fields and majors.

The field and major match error terms are distributed as in the theoretical model, where

$$\epsilon_{\theta_k} \sim N(0, \sigma^2_{\theta_k}) \text{ and } \epsilon_{\gamma_{ij(k)}} \sim N(0, \sigma^2_{\gamma})$$
The utility of student $i$ choosing major $j(S)$ in STEM can thus be written as:

$$U_{1Sj(S)} = \mu_{\theta_S} + \beta_S X_i + \eta_{i\theta_S} + \varepsilon_{i\gamma_{j(S)}}$$

$$U_{2Sj(S)} = \mu_{\theta_S} + \beta_S X_i + \eta_{i\theta_S} + \varepsilon_{i\gamma_{j(S)}} - c_{i2}(d_{i2} = (S, j(S)), H_{i2})$$

### 2.4.3 Identification

Since only differences in utility are relevant for decision-making, I normalize the expected match value for Non-STEM across all students to be 0, and I normalize the expected match value across majors to 0. I also set the switching cost within Non-STEM majors to be constant across all students and equal to 1/4 of the standard deviation of Non-STEM matches. Finally, I set $\beta_{c2SS} = -c_{0SS}$. This means that for females, the cost of switching within STEM comes entirely from the coefficient on SAT Math scores.

The scale of the utility functions must also be normalized, so I set $\sigma_{\theta_N} = 1$ and $\sigma_{\eta_N} = 1$. Finally, I set the discount factor to 0.75.

Utility of choosing Non-STEM is then expressed as:

$$U_{1Nj(N)} = \eta_{i\theta_N} + \varepsilon_{i\eta_N}$$

$$U_{2Nj(N)} = \eta_{i\theta_N} + \varepsilon_{i\eta_N} - c_{i2}(d_{i2} = (N, j(N)), H_{i2})$$

The STEM parameters are thus interpreted as being relative to non-STEM:

(i) $\mu_{\theta_S} =$ average mean for STEM - average mean for non-STEM

(ii) $\beta_S =$ relative effects of student characteristics on the probability of choosing STEM versus non-STEM

(iii) $\sigma^2_{\theta_S} =$ variance of STEM match values relative to the variance of Non-STEM match values

(iv) $\sigma^2_{\eta_S} =$ variance of individual preferences for STEM relative to Non-STEM

A positive estimate of $\mu_{\theta_S}$ implies that STEM matches for males with SAT Math
scores at the mean are higher, on average, than the equivalent Non-STEM matches. Similarly, a positive estimate of $\mu_{\theta_s} + \beta_{2S}$ implies that STEM matches for females with SAT Math scores at the mean are higher, on average, than the Non-STEM counterparts. If $\sigma_{\theta_s}^2 > 1$, then the variance of STEM matches is greater than the variance of Non-STEM matches. Finally, if $\sigma_{\eta_s}^2 > 1$, then the variation in individual unobserved preferences for STEM is higher than for Non-STEM.

With these normalizations, I can estimate 14 parameters: all parameters related to STEM matches ($\mu_{\theta_s}, \beta_{1S}, \beta_{2S}, \sigma_{\theta_s}, \sigma_{\eta_s}$), the variance of major match values ($\sigma_{\gamma}$), all switching cost coefficients for changing fields (starting in both STEM and Non-STEM), and the baseline cost plus the coefficient on math ability for the cost of switching within STEM.

**Method of Moments**

The goal of estimation here is to find the parameters that best match the overall decision patterns seen in the data. I use a (partially-simulated) generalized method of moments approach to solve for the parameters that minimize the distance between moments observed in the data and those estimated by the model. In particular, I’m looking to match the probabilities of starting in a particular field and then progressing along each possible choice path (aggregated at the level of the theoretical model), conditional on gender and math ability.

As an example, let $\mathbb{E}[I(d_{i1} = S|X)] = m_1(\Psi_0; X)$, where $\Psi_0$ is a vector of true parameter values, and $X$ includes student demographic characteristics. We can rearrange the equation so that the moment equals 0: $\mathbb{E}[I(d_{i1} = S|X)] - m_1(\Psi_0; X) = 0$. This is the true (according to the model) expected value of the expressed indicator function at the population level. This is equal to the probability of a student who is randomly chosen from the population picking STEM in Period 1, conditional on demographic characteristics $X_i$. These population-level moment conditions are given
by the model.

The empirical counterpart to the population moment is:

\[ \frac{1}{N} \sum_{i=1}^{N} I(d_{i1} = S|X_i) - m_1(\hat{\Psi}; X) = 0 \]

The first term on the left hand side is easily calculated from the data as the number of students in the sample who chose STEM in the pre-college survey divided by the total number of students in the sample.

Now suppose that the theory gives \( L \) moment conditions, each of which has a sample counterpart that can be obtained from the data. Define \( g_L(\Psi) \) as an \( L \times 1 \) vector of moment conditions, each of which equals 0:

\[ g_L(\Psi) = \frac{1}{N} \sum_{i=1}^{N} g(X_i; \Psi) \]

Then the GMM estimator is the vector of parameters that minimizes

\[ g_L(\Psi)'\hat{W}g_L(\Psi) \]

where \( \hat{W} \) is an \( L \times L \) symmetric and positive definite weighting matrix. The weighting matrix is only relevant for the efficiency of the estimator. I use \( W = I \), noting that this will produce consistent, but inefficient, results. I calculate 15 moments to estimate 14 parameters. The moments I use are all linearly independent choice probabilities, plus the mean path probabilities across student characteristics (female and Math SAT score).

2.4.4 Simulation and Computation

I note two important factors in obtaining estimates. First, I use a mixture of analytical expressions and simulation in calculating the moments. I simulate the choice probabilities by taking draws for the individual preference term. I use the following procedure in calculating the objective function to be minimized:
1. For each student, draw $R$ values for $\eta_{iN}$ and $\eta_{iS}$, where $\eta_{iN} \sim N(0, 1)$ and $\eta_{iS} \sim N(0, \sigma_{\eta_s}^2)$. Label the draws $\eta^r_{iN}$ and $\eta^r_{iS}$, for $r = 1, \ldots, R$.

2. Calculate $\mu^r_{i\theta_S}$ and $\mu^r_{i\theta_N}$ for each student, where $\mu^r_{i\theta_k} = \mu_{\theta_k} + X_i \beta + \eta^r_{ik}$.

3. Compute all moments using $\mu^r_{i\theta_S}$ and $\mu^r_{i\theta_N}$. Label the vector of moments $\mathbf{g}_L^r(\Psi)$.

4. Repeat Steps 2 and 3 $\forall r \in \{1, \ldots, R\}$

5. The simulated moments are the average moments across all draws: $\bar{\mathbf{g}}_L(\Psi) = \frac{1}{R} \sum_r \mathbf{g}_L^r(\Psi)$.

6. The simulated method of moments estimator is then the vector of parameters that minimizes $\bar{\mathbf{g}}_L(\Psi) / \bar{\mathbf{g}}_L(\Psi)$.

Second, the objective function I’m minimizing is not continuous in parameters, which means that gradient-based optimization algorithms cannot be used. Instead of using a direct search method, which doesn’t require the gradient, I use a smoothing technique to make the function continuous. In the two-period version, the level of smoothing required is minimal. I only need to smooth out the choice in the initial period from an indicator function to a probabilistic one. This carries into all the other moments since the later choice probabilities are conditional on the initial choice.

In Step 3 above, I calculate whether the student starts in STEM using an indicator function: $d_{i1} = (S, j(S))$ if $I(\mathbb{E}(U_{iS,j(S)}) > \mathbb{E}(U_{iN,j(N)}))$. This means that, for each student, the outcome is a step function in the parameters. If the utility for STEM is increasing in a particular parameter, then the indicator function will equal zero for low values and will jump to one once the parameter crosses some minimum threshold. The gradient is therefore either zero or undefined, and optimization algorithms which use gradient-based methods won’t know which direction to search. To overcome this problem, I replace the indicator function with a logistic function (proposed by ?). This gives each student a probability $\in (0, 1)$ of choosing STEM:

$$P(d_{i1} = (S, j(S))) = \frac{\exp(\mathbb{E}(U_{iS,j(S)})/\lambda)}{\exp(\mathbb{E}(U_{iS,j(S)})/\lambda) + \exp(\mathbb{E}(U_{iN,j(N)})/\lambda)}.$$
where $\lambda$ is a smoothing parameter. Lower values of $\lambda$ give probabilities closer to the 0-1 choice (and thus closer to the actual observed data), while higher values of $\lambda$ increase the smoothness of the function (potentially helping the search algorithms, but at the expense of a worse approximation). I set $\lambda = 0.1$.

2.4.5 Results

Table 2.5 gives the resulting parameter estimates. We can start by looking at the three comparisons of interest mentioned at the beginning of this section. First, note that $\sigma_{\theta_s} > 1$, meaning that the unknown component of STEM matches is larger than that of Non-STEM. Second, the individual preferences for STEM have lower variance than their Non-STEM counterparts, evidenced by $\sigma_{\eta_S} < 1$.

As for the switching costs, we see that switching majors within STEM or switching into STEM from Non-STEM are both very costly. Switching out of STEM is basically costless for a male with average math ability. Females have negative switching costs when moving from STEM to Non-STEM, and when moving from Non-STEM to STEM.

2.5 Conclusion

The major that students choose is an important factor in their enjoyment of college, as well as their future labor market opportunities and earnings. Students are switching majors at a fairly high rate, and this varies depending on student characteristics, which may be related to differences in information.
Table 2.5: Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coeff.</th>
<th>Parameter</th>
<th>Coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean Parameters</strong></td>
<td></td>
<td><strong>Switching Cost Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>$\mu_{\theta S}$</td>
<td>-0.34</td>
<td>STEM $\rightarrow$ STEM</td>
<td></td>
</tr>
<tr>
<td>$\beta_{1S}$ (SAT Math)</td>
<td>0.90</td>
<td>$c_{0SS}$</td>
<td>1.08</td>
</tr>
<tr>
<td>$\beta_{2S}$ (female)</td>
<td>0.23</td>
<td>$\beta_{1eSS}$ (SAT Math)</td>
<td>0.66</td>
</tr>
<tr>
<td><strong>Variance Parameters</strong></td>
<td></td>
<td>Non-STEM $\rightarrow$ STEM</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\theta S}$</td>
<td>1.86</td>
<td>$c_{0NS}$</td>
<td>0.78</td>
</tr>
<tr>
<td>$\sigma_{\gamma}$</td>
<td>1.05</td>
<td>$\beta_{1eNS}$ (SAT Math)</td>
<td>-0.08</td>
</tr>
<tr>
<td>$\sigma_{\theta S}$</td>
<td>0.48</td>
<td>$\beta_{2eNS}$ (female)</td>
<td>-0.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>STEM $\rightarrow$ Non-STEM</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c_{0SN}$</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta_{1eSN}$ (SAT Math)</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta_{2eSN}$ (female)</td>
<td>-0.46</td>
</tr>
</tbody>
</table>

**Normalizations**

- $\mu_{\theta N} = 0$
- $\beta_{1N} = 0$
- $\beta_{1S} = 0$
- $\sigma_{\theta N} = 1$
- $\sigma_{\theta N} = 1$
- $\mu_{\gamma} = 0$
- $\beta = 0.75$

$c_{0NN} = \sigma_{\theta N}/4$
<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(S_1)$</td>
<td>0.570</td>
<td>0.572</td>
</tr>
<tr>
<td>$P(\text{Stay}_2, S_1)$</td>
<td>0.295</td>
<td>0.316</td>
</tr>
<tr>
<td>$P(\text{NFM}_2, S_1)$</td>
<td>0.085</td>
<td>0.079</td>
</tr>
<tr>
<td>$P(\text{Stay}_2, \text{NS}_1)$</td>
<td>0.248</td>
<td>0.250</td>
</tr>
<tr>
<td>$P(\text{NFM}_2, \text{NS}_1)$</td>
<td>0.160</td>
<td>0.158</td>
</tr>
<tr>
<td>mean(SATM, $S_1$)</td>
<td>0.180</td>
<td>0.102</td>
</tr>
<tr>
<td>mean(SATM, $\text{Stay}_2, S_1$)</td>
<td>0.191</td>
<td>0.148</td>
</tr>
<tr>
<td>mean(SATM, $\text{NM}_2, S_1$)</td>
<td>-0.009</td>
<td>0.004</td>
</tr>
<tr>
<td>mean(SATM, $\text{Stay}_2, N_1$)</td>
<td>-0.109</td>
<td>-0.049</td>
</tr>
<tr>
<td>mean(SATM, $\text{NM}_2, N_1$)</td>
<td>-0.077</td>
<td>-0.057</td>
</tr>
<tr>
<td>mean(Female, $S_1$)</td>
<td>0.310</td>
<td>0.307</td>
</tr>
<tr>
<td>mean(Female, $\text{Stay}_2, S_1$)</td>
<td>0.117</td>
<td>0.123</td>
</tr>
<tr>
<td>mean(Female, $\text{NM}_2, S_1$)</td>
<td>0.074</td>
<td>0.570</td>
</tr>
<tr>
<td>mean(Female, $\text{Stay}_2, \text{NS}_1$)</td>
<td>0.136</td>
<td>0.132</td>
</tr>
<tr>
<td>mean(Female, $\text{NM}_2, \text{NS}_1$)</td>
<td>0.088</td>
<td>0.090</td>
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Equilibrium Grade Inflation with Implications for Female Interest in STEM Majors

Additional authors for this chapter: Thomas Ahn, Peter Arcidiacono, and James Thomas

3.1 Introduction

Even after accounting for selection, substantial earnings differences exist across majors. Majors in engineering and the sciences, as well as economics and business, pay substantially more than other fields.\(^1\) Further, earnings disparities across majors have increased substantially over time (Altonji et al. (2014) and Gemici and Wiswall (2014)). Despite their value in the marketplace, STEM (Science, Technology, Engineering, and Mathematics) fields are perceived to be undersubscribed. A report by the President’s Council of Advisors on Science and Technology (2012) suggests substantial needs to increase the number of STEM majors. Florida has proposed freezing tuition for STEM majors (Alvarez (2012)), and the state of New York is offering free tuition for high performing students who enroll in public institutions as

\(^1\) See Altonji et al. (2012) and Altonji et al. (ming) for reviews.
STEM majors, conditional on working in the state for at least five years (Chapman (2014)).

But many more students enroll in college expecting to major in a STEM field than actually finish in a STEM field (Arcidiacono (2004), Arcidiacono et al. (ming), Stinebrickner and Stinebrickner (2014)). This is not just due to students dropping out: many students switch from STEM to non-STEM fields, particularly in comparison to those who switch from non-STEM to STEM fields. Further, it is predictable who will switch. Those who have relatively weak academic preparation (e.g. SAT scores or HS grades) are much more likely to leave STEM fields. While relatively high levels of academic preparation are associated with persisting in STEM majors, there is little evidence that high levels of academic preparation are more rewarded in the labor market for STEM majors than for non-STEM majors. Women are also more likely to switch: Arcidiacono et al. (2012b) show with data from Duke University, that differences in academic preparation can account for the large differences in switching behavior across races but is unable to explain the substantial gender gap.

A potential channel for influencing the number and composition of STEM majors are grading policies. Should grading policies prove to be an important predictor of major choice, they may serve as a relatively cheap way of increasing STEM majors. While other means such as increasing pre-college academic preparation or the share of underrepresented groups in STEM fields may also be effective, these methods are also very costly with the benefits coming much later. Shifting the way teachers teach the sciences and introducing more laboratory-based curricula are both expensive. Altering training, hiring, and promotion in academia, government agencies, and firms is also costly, as are adjusting long-standing cultural attitudes in the home, school, and workplace.

There is evidence that grades affect sorting into majors. The same majors that pay well also give (on average) significantly lower grades (Sabot and Wakeman-Linn
(1991), Johnson (2003)) and are associated with more study time (Brint et al. (2012), Stinebrickner and Stinebrickner (2014)). Lower grades and higher study times deter enrollment. Sabot and Wakeman-Linn (1991) show that the absolute level of grades was a far more important indicator of taking further courses in the subject than their ranking within the class. Butcher et al. (2014) showed that Wellesley’s policy of capping the fraction of A’s given resulted in shifts towards science classes and science majors. There is also evidence that students enter unaware of the extent of cross-department differences in grading standards. Stinebrickner and Stinebrickner (2014) show that the over-optimism regarding performance at Berea College is primarily driven by students over-predicting their performance in the sciences. As students take more classes, students generally revise their expected performance in the sciences downward. This holds true even for students who persist in the sciences who ought to have received relatively positive grade realizations.

With students responding to grading practices through their choice of courses, departments may set their grading policies in order to deter or encourage enrollment. Those with low enrollments may find it difficult to increase or maintain their faculty size. Hence, incentives exist to raise grades in order to encourage enrollment in these departments. On the other hand, departments that are flush with students may have incentives to lower grades to keep their enrollments to a more manageable size. Within any given department, individual professors may also seek to influence enrollment up or down for his or her class to minimize teaching effort or maximize student learning.²

Differences in grading policies may have differing effects for males and females. In principle female students should be particularly interested in STEM fields. Women report studying substantially more than men (Stinebrickner and Stinebrickner (2014),

² This issue is becoming even more salient as more universities move toward a fiscal model where departmental budgets are more directly determined by enrollment size or credits generated.
Arcidiacono et al. (2012b)), and they should be undeterred by the higher study requirements of these classes. Yet, females are substantially less likely to graduate with a STEM major than males. Ideas for why this might happen have been numerous, including role model effects (Rask and Bailey (2002), Hoffmann and Oreopoulos (2009), Carrell et al. (2010)) and future labor market considerations (Gemici and Wiswall (2014), Bronson (2014)) among many others. In addition to these channels, women may study more in part because they value the benefits of studying—higher grades—more than their male counterparts (Rask and Bailey (2002), Rask and Tiefen-thaler (2008)). Good grades may yield direct psychic benefit, or they may impact time to graduation or ability to qualify for grants and scholarships. For example, if female students are more risk averse or pessimistic about attrition probability compared to their male counterparts, grades may hold more value. Again, the advantage to focusing on grading policies is that it may be relatively cheap to do so compared to alternative programs.

We propose to estimate an equilibrium model of student course enrollment and effort decisions as well as professor decisions regarding grading standards. How professors set grades affects enrollment and how much students study, though differentially for men and women. The professor objective function includes enrollments, so part of how professors set grades is determined by course demand. With the estimates of the equilibrium model, we will be able to evaluate how differences in grading practices across fields affect, partly as a result of demand, the share of courses taken in different fields. Further, we can see whether cross-departmental differences in professor preferences over enrollment either exacerbate or mitigate the differences in grading across fields.

3 The gender gap is not uniform across STEM fields. Indeed, in some STEM fields, such as biosciences, women receiving BA’s actually outnumber men.
3.2 Data

Estimating such a model requires rich data on student course taking, study hours, and grades. We use a detailed student enrollment data set from the University of Kentucky (UK). UK, the state’s flagship public post-secondary institution, has a current undergraduate enrollment of approximately 21,000. The school was ranked 119 out of approximately 200 ‘National Universities’ by U.S. News & World Report (U.S. News & World Report 2013). This places UK in the middle of the distribution of large post-secondary institutions, and the student body serves as a good cross-section of college students nationwide.

The data set contains student demographic and course enrollment information. Each semester, the entire student body’s course selections and grades are recorded by the Registrar’s Office. This data set is particularly valuable because every student outcome in every class is captured, allowing us to estimate a rich model of student and professor interactions. Furthermore, we can analyze course selection and performance explicitly modeling the student choosing his or her semester course-load (which may be contain one to five courses) from the entire course catalog. We also collect information on course pre-requisites. Restricting the student’s choice set using this data aids computation as well as creating a more true-to-life representation of the decisions a student faces when selecting his or her courses. For this study, we focus on student enrollment observations from one semester, Fall 2012.4

In addition, we have access to class evaluation surveys completed by students at the end of the semester. We note that coverage is not complete, as some departments chose not to make evaluation data available. Linking the class evaluation data to the enrollment data is complex, as rules for identifying the course (or sections within the

---

4 We have the capability to link student data across multiple semesters, spanning Fall 2008 to Spring 2013. This yields approximately 1.4 million student/class observations. We restrict ourselves to one semester due to computational constraints.
course) and instructor (or sub-instructor - frequently a graduate student - teaching under the supervision of a head-instructor) are defined independently by the department. As students do not identify themselves in the evaluation forms, we aggregate the data up to the class level. We are able to match 76 percent of classes successfully. We then restrict the data to classes with at least a 70 percent response rate and drop classes with small numbers of respondents to prevent possible identification. Most critically for our research, students are asked about the number of hours per week they spend on studying for this particular course. This information makes the identification of structural parameters on study effort possible, allowing us to analyze how differences in average study time relates to course and student characteristics across courses.\footnote{The survey asks 20 questions on the value of the course and instructor to the student on a five-point Likert scale. Each student reveals what year of school he or she is in, how valuable he or she finds the course and instructor, expected final grade, and whether the course was a major requirement.}

Our Fall 2012 sample yields 89,582 student/class observations. There are 19,527 unique undergraduates, implying that on average, each student enrolls in (but not necessarily completes) four to five courses.\footnote{We also observe withdrawal data. Withdrawal rate of undergraduates is approximately 5.4%. Of these, approximately 45% withdraw from the course prior to the midterm examination. We speculate that many of these students were ‘shopping around’ for courses at the start of the semester and realized that they needed to drop a course they had not been attending part-way through the semester.} Table 3.1 provides demographic summary statistics, separated by gender. Overall, women and men look similar when entering college. Women have slightly higher high school grades and slightly lower standardized ACT scores.\footnote{SAT scores are converted to equivalent ACT scores, and the math and verbal sections are averaged.} Women also have higher grades while in college. Sharp differences show up in major selection. While women comprise a slight majority at UK overall, the ratio between men and women in STEM majors is approximately 1.6. In contrast to students from more selective institutions (seen in many other stud-
ies of higher education outcomes), over 30 percent of students at UK are part-time students, taking less than 12 credits during the semester.

Table 3.2 summarizes class-level characteristics separated by STEM-status of the course. STEM classes are substantially larger and give significantly lower grades compared to non-STEM courses. As implied by Table 3.1, female students are the minority in STEM classes. This is despite the fact that they perform better, on average, than their male counterparts in these courses. On average, each STEM course requires one more hour of study time per week (or 30 percent more time) than a non-STEM course. The study time difference actually understates the true gap across STEM and non-STEM courses. On average, students with higher academic ability will select more often into STEM courses, so each hour spent studying should yield more learning. Yet, STEM classes average much lower grades. A student attempting to generate an equivalent grade across a STEM and non-STEM course will have to invest significantly more than an extra 30 percent in study time in the STEM course.

Table 3.3 presents simple OLS results showing the relationship between individual and class characteristics with grades and study hours after controlling for a large number of academic background measures.\(^8\) The grades regression sample is at the student/class level, and the study hours per week regression sample is at the class level. The first column gives the results for grades. The patterns are consistent with those in Table 3.2, STEM classes give lower grades and females have higher grades. Classes that have a higher fraction of female students also give higher grades. This is consistent with there not being a grade curve that is common across STEM or non-

---

\(^8\) We restrict our sample to standard classes with at least 16 students. The total number of classes in the data set is 2,026. From this we exclude nearly half of the classes from the analysis. Many of the excluded classes can be categorized into: non-academic classes (e.g. “academic orientation” or “undergraduate advising”), advanced and remedial independent student courses (including tutoring), classes in fine arts requiring individualized instruction (e.g. “voice”, “jazz ensemble”, or “art studio”), and graduate-level classes taken by very advanced undergraduate students.
Table 3.1: Descriptive Statistics by Gender

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>High school GPA</td>
<td>3.13</td>
<td>3.34</td>
</tr>
<tr>
<td></td>
<td>(1.20)</td>
<td>(1.16)</td>
</tr>
<tr>
<td>ACT Score</td>
<td>25.2</td>
<td>24.4</td>
</tr>
<tr>
<td></td>
<td>(4.42)</td>
<td>(4.18)</td>
</tr>
<tr>
<td>Fall 2012 GPA</td>
<td>3.02</td>
<td>3.24</td>
</tr>
<tr>
<td></td>
<td>(0.713)</td>
<td>(0.665)</td>
</tr>
<tr>
<td>Fall 2012 Credits</td>
<td>11.7</td>
<td>12.0</td>
</tr>
<tr>
<td></td>
<td>(4.29)</td>
<td>(4.22)</td>
</tr>
<tr>
<td>STEM Major</td>
<td>38.0%</td>
<td>23.8%</td>
</tr>
</tbody>
</table>

Note: Fall 2012 University of Kentucky undergraduate students, 9,729 men, 9,798 women. Standard deviations in parentheses.

STEM departments else the higher grades females receive would translate into lower grades for everyone else. Class size has a negative effect on grades. The coefficient on class size confounds two effects that work in opposite directions. On the one hand, students prefer higher grades so higher enrollments should be associated with higher grades. On the other, courses that have high intrinsic demand may have lower grades since these courses do not need to have high grades to attract students.

The second column on Table 3.3 shows regressions of study hours on the average characteristics of the class. STEM classes are associated with an extra half hour of study, slightly less than what is seen in the descriptive statistics. This suggests that STEM classes are attracting students who are willing to study more, with the grading policies in the STEM classes further spurring on these students to commit more time to study. Classes that have more women also study more, consistent with the previous literature (DiPrete and Buchmann 2013). But perhaps the most interesting coefficient is that on average grades. Courses that give higher grades have less study time, suggesting grades should be interpreted as relative, not absolute, measures of accomplishment, as well as suggesting grade inflation may have negative consequences for learning.
Table 3.2: Descriptive Statistics by Course Type

<table>
<thead>
<tr>
<th></th>
<th>STEM</th>
<th>Non-STEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class Size</td>
<td>78.1</td>
<td>46.3</td>
</tr>
<tr>
<td>(101.1)</td>
<td>(64.0)</td>
<td></td>
</tr>
<tr>
<td>Average Grade</td>
<td>3.03</td>
<td>3.31</td>
</tr>
<tr>
<td>(0.50)</td>
<td>(0.46)</td>
<td></td>
</tr>
<tr>
<td>Average Grade</td>
<td>Female</td>
<td>3.11</td>
</tr>
<tr>
<td>(0.59)</td>
<td>(0.46)</td>
<td></td>
</tr>
<tr>
<td>Study Hours</td>
<td>3.61</td>
<td>2.70</td>
</tr>
<tr>
<td>(1.68)</td>
<td>(1.12)</td>
<td></td>
</tr>
<tr>
<td>Percent Female</td>
<td>37.0%</td>
<td>55.9%</td>
</tr>
</tbody>
</table>

Note: Fall 2012 University of Kentucky courses with enrollments of 16 or more students, 379 STEM courses, 1,164 non-STEM courses. For study hours, 293 STEM courses and 793 non-STEM courses. Standard deviations in parentheses.

Table 3.3: Regressions of Grades and Study Time on Characteristics of the Individual and/or Class

<table>
<thead>
<tr>
<th>Dependent Var.</th>
<th>Grade</th>
<th>Study hours per week</th>
</tr>
</thead>
<tbody>
<tr>
<td>STEM Class</td>
<td>-0.325</td>
<td>0.520</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.148)</td>
</tr>
<tr>
<td>Female</td>
<td>0.140</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td></td>
</tr>
<tr>
<td>Percent Female</td>
<td>0.395</td>
<td>0.547</td>
</tr>
<tr>
<td></td>
<td>(0.203)</td>
<td>(0.191)</td>
</tr>
<tr>
<td>Average Grade</td>
<td>-0.635</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.089)</td>
<td></td>
</tr>
<tr>
<td>ln(Class Size)</td>
<td>-0.116</td>
<td>-0.396</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.048)</td>
</tr>
<tr>
<td>Observations</td>
<td>72,449</td>
<td>1,085</td>
</tr>
</tbody>
</table>

Note: Additional controls for grades regression include, minority status, freshman, STEM major, pell grant, in-state student, ACT score, HS gpa, percent minority, percent freshman.
Additional controls in study hours regression include percent freshmen, percent STEM major, percent pell grant, percent in-state, average ACT score, average HS gpa, percent minority.
3.3 Model

Individual $i$ chooses $n$ courses from the set $[1, \ldots, J]$. Let $d_{ij} = 1$ if $j$ is one of the $n$ courses chosen by student $i$ and zero otherwise. The payoff associated with a bundle of courses is given by the sum of the payoffs for each of the individual courses where the payoffs do not depend on the other courses in the bundle. We specify the payoff for a particular course $j$ as depending on student $i$’s preference for the course, $\delta_{ij}$, the amount of study effort the individual chooses to exert in the course, $s_{ij}$, and the expected grade conditional on study effort, $\mathbb{E} [g_{ij} | s_{ij}]$:

$$U_{ij} = \phi_i \mathbb{E} [g_{ij} | s_{ij}] - \psi_i s_{ij} + \delta_{ij} \quad (3.1)$$

Students then solve the following maximization problem when choosing their optimal course bundle:

$$\max_{d_{i1}, \ldots, d_{iJ}} \sum_{j=1}^{J} d_{ij} U_{ij} \quad (3.2)$$

subject to: $\sum_{j=1}^{J} d_{ij} = n$, $d_{ij} \in \{0, 1\} \forall j$

The grade student $i$ receives in course $j$, $g_{ij}$, depends on the academic preparation of student $i$ for course $j$, $A_{ij}$, the amount of study effort put forth by the student in the course, $s_{ij}$, the grading policies of the professor, and a shock that is unknown to the individual at the time of course enrollment, $\eta_{ij}$. We specify the grading process as:

$$g_{ij} = \beta_j + \gamma_j (A_{ij} + \ln(s_{ij})) + \eta_{ij} \quad (3.3)$$

Grading policies by the professors are then choices over an intercept, $\beta_j$, and a return to academic preparation and effort, $\gamma_j$.\footnote{For example, if there is a university-wide (or department-level) mandated/recommended grade distribution, we will be able to capture such a policy, as $\beta_j$ and $\gamma_j$ will have lower variance.} Gains from study effort enters in as a log
to capture the diminishing returns to studying. Along with the linear study effort cost defined in the utility function, this ensures an interior solution for the optimal amount of study time.

Students are assumed to know the professors’ grading policies. Substituting in for expected grades in (3.1) yields:
\[
U_{ij} = \phi_i (\beta_j + \gamma_j [A_{ij} + \ln(s_{ij})]) - \psi_i s_{ij} + \delta_{ij}
\]
(3.4)
The optimal study effort in course \(j\) can be found by differentiating \(U_{ij}\) with respect to \(s_{ij}\):
\[
0 = \frac{\phi_i \gamma_j}{s_{ij}} - \psi_i
\]
(3.5)
Substituting the optimal choice of study time into (3.4) yields:
\[
U_{ij} = \phi_i (\beta_j + \gamma_j [A_{ij} + \ln(\phi_i) + \ln(\gamma_j) - \ln(\psi_i) - 1]) + \delta_{ij}
\]
(3.6)
Those who have lower study costs, low \(\psi_i\), and higher levels of academic preparation, high \(A_{ij}\), find courses with higher \(\gamma_j\)’s relatively more attractive all else equal. Those who place a relatively high weight on expected grades, high \(\phi_i\), study more conditional on choosing the same course, but are more attracted to courses with higher grade intercepts, high \(\beta_j\).

Substituting the expression for optimal study time into the grade process equation yields:
\[
g_{ij} = \beta_j + \gamma_j (A_{ij} + \ln(\phi_i) + \ln(\gamma_j) - \ln(\psi_i)) + \eta_{ij}
\]
(3.7)

---

10 Students have a number of formal and informal resources to learn about grading policies. Informally, they may rely on friends who have previously taken the course and other information social networks. Professors may send out preemptive signals by posting syllabi online. More formally, course evaluations, which also reveal the (anonymous) responders’ own expected final course grades, are on-line and publicly available. In addition, several websites curate online “reviews” of professors and courses.
Professors who set relatively higher values of $\gamma_j$ see more study effort because higher $\gamma_j$’s induce more effort and because higher $\gamma_j$’s attract students with lower study costs.

The key equations for estimation are then given by:

(i) the solution to the students maximization problem where (3.6) is substituted into (3.2),

(ii) the grade production process given in (3.7), and

(iii) the optimal study effort given in (3.5).

The next section describes the parameterizations used to estimate the model as well as the assumptions necessary to overcome the fact that our measures of study effort from the course evaluations are not linked to the individual’s characteristics.

3.4 Estimation

We first describe our estimating strategy under the assumption that there is no student-level unobserved heterogeneity. Under this assumption, the log likelihood function is additively separable, and we are able to estimate the parameters in three stages. We then show how to adapt our estimation method to handle student-level unobserved heterogeneity.

In the first stage, we estimate a reduced form version of the grade production process (Eq. (3.7)). The relationship between student characteristics and grades gives estimates for the reduced form parameters. The returns to effort are also identified, up to a normalization at the department level, by how student characteristics translate into grades relative to the normalized course.

In the second stage, we relate the optimal study effort given in Eq. (3.5) to the student evaluation data. The evaluations are collected for each class, and students
report how many hours they spent studying in that class. This helps us recover some of the study effort parameters, as well as unravel some of the normalizations on the $\gamma$s required in the first stage. We are able to relate all $\gamma$s across departments, but still cannot only identify the $\gamma$'s relative to one course.

In the last stage, we estimate the choice problem given by Eqs. (3.6) and (3.2). We use the estimates from the first stage to calculate expected grades for each student and the estimated effort from the second stage. Combining the estimates from these three stages allows us to identify all the grading policy parameters, the grade preference parameters, the effort cost parameters, and the course preference parameters.

### 3.4.1 Parameterizations

To estimate the model, we need to place some structure on course preferences, $\delta_{ij}$, the value of grades, $\psi_i$, and the cost of effort, $\phi_i$. Further, we must relate academic preparation, $A_{ij}$, to what we see in the data. Denote $w_i = 1$ if individual $i$ is female and zero otherwise. Denote $X_i$ as a row vector of explanatory variables such as ACT scores, high school grades, race, etc.$^{11}$ Denote $Z_i$ a a row vector of explanatory variables that affect preferences for particular departments or levels of courses within departments. Hence $Z_i$ includes gender as well as year in school, allowing women to have a preference for classes in particular departments and the attraction of upper-division versus lower-division classes to vary by department and year in school. Preference shocks for courses are represented by $\epsilon_{ij}$. Finally, we partition courses into $K$ departments, $K < J$, where $k(j)$ gives the department for

$^{11}$ The majority of students at the University of Kentucky submit ACT scores in their college applications.
the $j$th course. We then parameterize the model as follows:

\begin{align*}
A_{ij} &= w_i \alpha_{1k(j)} + X_i \alpha_{2k(j)} \tag{3.8} \\
\delta_{ij} &= \delta_{0j} + w_i \delta_{1k(j)} + Z_i \delta_{2k(j)} + \epsilon_{ij} \tag{3.9} \\
\psi_i &= \exp(\psi_0 + w_i \psi_1 + X_i \psi_2) \tag{3.10} \\
\phi_i &= \phi_0 + w_i \phi_1 \tag{3.11}
\end{align*}

There is no intercept in $A_{ij}$ as it can not be identified separately from the $\beta_j$’s. Note that the same variables enter into academic preparation, preferences, and effort costs, only with different coefficients. Preferences for courses allow for both course fixed effects as well as students with particular characteristics preferring courses in particular departments, $\delta_{1k(j)}$. Note also that the effort costs are exponential in the explanatory variables. This ensures that effort costs are positive. Finally, preferences for grades are only allowed to vary by gender. In principle, we could allow them to vary with $X_i$ as well, but this would substantially complicate the model.

Having separate estimates by gender across all the relevant parameters will help uncover some of the driving forces behind the gender gap in STEM. For example, if female intrinsic demand for courses in STEM departments is relatively low ($\delta_{1k(j)}$ negative) while preferences for grades and cost of effort are relatively equal across males and females ($\psi_1$ and $\phi_1$ close to zero), then changing grading policies will have no effect on the gender gap in STEM. In this case, it would require figuring out why females are not interested in STEM fields, and policies would have to be geared more towards early education about opportunities in STEM for females, or changing cultural attitudes towards females in STEM. On the other hand, if females have significantly different preferences over grades and study effort than males, then altering grading policies could affect the gender distributions within classes and departments. For example, if females have higher preferences for grades ($\phi_1$ positive) and lower cost of effort ($\psi_1$ negative) than males, then increasing $\gamma_j$ and correspondingly changing
\( \beta \) to keep enrollments in STEM courses the same would result in an increase in the fraction of females in STEM.

### 3.4.2 Estimation without Unobserved Heterogeneity

**Grade parameters**

Substituting the parameterizations for academic preparation, \( A_i \), the value of grades, \( \phi_i \), and study costs, \( \psi_i \), into (3.7) yields the following reduced form grade equation:

\[
g_{ij} = \theta_{0j} + \gamma_j \left( w_i \theta_{1k(j)} + X_i \theta_{2k(j)} \right) + \eta_{ij} \tag{3.12}
\]

where:

\[
\begin{align*}
\theta_{0j} &= \beta_j + \gamma_j (\ln(\phi_0) + \ln(\gamma_j) - \psi_0) \\
\theta_{1k(j)} &= \alpha_{1k(j)} + \ln(\phi_0 + \phi_1) - \ln(\phi_0) - \psi_1 \\
\theta_{2k(j)} &= \alpha_{2k(j)} - \psi_2
\end{align*}
\] (3.13, 3.14, 3.15)

We estimate the reduced form parameters \( \{\theta_{0j}, \theta_1, \theta_2\} \) as well as the structural slopes, the \( \gamma_j \)'s, using nonlinear least squares. A normalization must be made for every department as scaling up the \( \theta \)'s by some factor and scaling down the \( \gamma \)'s by the same factor would be observationally equivalent. We set one \( \gamma_j \) equal to one for each department.\(^{12}\) Denote \( C_k \) as the normalization for department \( k \). We then estimate \( \gamma^N_j \) where \( \gamma_N = \gamma_j / C_{k(j)} \). Similarly, we estimate \( \theta^N_{1k(j)} \) and \( \theta^N_{2k(j)} \) where

\[
\theta^N_{1k(j)} = \theta_{1k(j)} C_{k(j)} \quad \text{and} \quad \theta^N_{2k(j)} = \theta_{2k(j)} C_{k(j)}.
\]

The variation in the data used to identify \( \{\theta^N_1, \theta^N_2\} \) comes from the relationship between student characteristics and grades in each department. The variation in the data used to identify the \( \gamma^N_j \)'s is how these characteristics translate into grades relative to the normalized courses.

\(^{12}\) The study effort analysis allows us to recover the normalizations for all the departments but one, as we will show in section 3.4.2. The final normalization is undone in the estimation of the utility parameters, shown in section 3.4.2.
Study parameters

We next turn to recovering some of the study effort parameters as well as undoing the normalization made on all the $\gamma$’s but one. To do so, we use (3.5). The issue with using (3.5) is that we do not directly observe study effort. However, the course evaluation data give reported study hours for each individual in the classroom. This information cannot be linked to the individual data on grades, academic preparation, and course choices. But the evaluation data does provide information about the year in school of the evaluator (e.g., freshman, sophomore, junior, or senior).

To link study hours to study effort, we assume that the relationship is log-log with measurement error $\zeta_{ij}$:

$$\ln(p_{hi}) = \mu \ln(s_{ij}^*) + \zeta_{ij}$$  \hspace{1cm} (3.16)

Substituting in for $s_{ij}^*$ yields:

$$\ln(h_{ij}) = \mu (\ln(\mu_1) + \ln(\phi_1) + \ln(\gamma_j) - \ln(\psi_1)) + \zeta_{ij}$$  \hspace{1cm} (3.17)

$$= \kappa_0 + w_i \kappa_1 - X_i \kappa_2 + \ln(\gamma_j) + \zeta_{ij}$$  \hspace{1cm} (3.18)

where:

$$\kappa_0 = \mu (\ln(\phi_0) - \psi_0)$$  \hspace{1cm} (3.19)

$$\kappa_1 = \mu (\ln(\phi_0 + \phi_1) - \ln(\phi_0) - \psi_1)$$  \hspace{1cm} (3.20)

$$\kappa_2 = \mu \psi_2$$  \hspace{1cm} (3.21)

The coefficient on $\ln(\gamma_j)$ then gives the curvature of the relationship between study effort and hours.

Recall that we had to normalize one $\gamma_j$ for every department in the grade equation. Substituting in with our estimate $\hat{\gamma}_j$ and making the appropriate adjustments yields:

$$\ln(h_{ij}) = \bar{\kappa}_0 + w_i \kappa_1 - X_i \kappa_2 + \kappa_{3k(j)} + \mu \ln(\hat{\gamma}_j^N) + \zeta_{ij}$$  \hspace{1cm} (3.22)
where \( \kappa_{3k(j)} = \mu \ln(C_{k(j)}/C_1) \) and \( \bar{\kappa}_0 = \kappa_0 + \mu \ln(C_1) \). Here \( C_1 \) is the normalized course for the base department.

Since we can only link characteristics of the students to the evaluation data by year in school, the observations we use in estimating the study parameters are at the class-year level. Let \( l_i \) indicate the year in school of student \( i \). Our estimating equation for students of level \( l \) is then:

\[
\frac{\sum_i(l_i = l)d_{ij} \ln(h_{ij})}{\sum_i(l_i = l)d_{ij}} = \bar{\kappa}_0 + w_{jl}\kappa_1 - X_{jl}\psi_2 + \kappa_{3k(j)} + \mu_2 \ln(\hat{\gamma}_N) + \zeta_{jl} \tag{3.23}
\]

where \( w_{jl} \) and \( X_{jl} \) are the averages of these characteristics for those of year level \( l \) enrolled in course \( j \). We correct for potential bias due to measurement error in \( \gamma_j \) using instrumental variables. We use the share of freshmen, sophomores, and juniors in each class as instruments for \( \log(\gamma_j) \), and estimate using two stage least squares.

Estimates of (3.23) allow us to recover the elasticity of hours with respect to study effort, \( \hat{\mu} \), as well as an estimate of \( \psi_2 \), how observed characteristics affect study costs, as \( \hat{\psi}_2 = \hat{\kappa}_2/\hat{\mu}_2 \). We can also partially undo the normalization on the \( \gamma \)'s, solving for \( \gamma \)'s that are normalized with respect to one course rather than one course in each department. Namely, let \( \gamma_j^P = \gamma_j^N \exp(\hat{\kappa}_{2k(j)}/\hat{\mu}_2) \). \( \gamma_j^P \) provides an estimate of \( \gamma_j/C_1 \).

The last normalization—the returns on preparation and study time in the normalized course—will be recovered in the estimation of the utility function parameters. The remaining structural parameters embedded in (3.23) can be recovered after estimating the parameters of the utility function, described in the next section.

**Utility parameters**

We now turn to estimation of the parameters of the utility function. Given our estimates of the grade equation, equation (3.12), we can calculate expected grades in each of the courses given optimal study choices:

\[
E[g_{ij}|s^{*}_{ij}] = \hat{\theta}_{ij} + \gamma_j^N \left( w_{ij}\hat{\theta}_{1k(j)}^N + X_{ij}\hat{\theta}_{2k(j)}^N \right) \tag{3.24}
\]
Given the estimates of the unnormalized returns to study and ability, \( \hat{\gamma} \), we can express the utility \( i \) receives from choosing course \( j \) and studying optimally as:

\[
U_{ij} = \delta_{0j} + w_i\delta_{1k(j)} + Z_{ij}\delta_{2k(j)} + \left( E[g_{ij}|s_{ij}^*] - \gamma_j \right) (\phi_0 + w_i\phi_1) + \epsilon_{ij} \tag{3.25}
\]

We then substitute in for \( \gamma_j \) with \( C_1\hat{\gamma}_j^P \) which, after rearranging terms, yields:

\[
U_{ij} = \delta_{0j} + w_i\delta_{1k(j)} + Z_{ij}\delta_{2k(j)} + E[g_{ij}|s_{ij}^*](\phi_0 + w_i\phi_1) + C_1\hat{\gamma}_j^P(\phi_0 + w_i\phi_1) + \epsilon_{ij} \tag{3.26}
\]

The goal is then to recover the course fixed effects, \( \delta_{0j} \), the value women place on courses in particular departments, \( \delta_{1k(j)} \), other department-specific preferences as well as preferences over instructor characteristics, \( \delta_{2k(j)} \), preferences over grades, \( \phi \), and the returns to ability and study time in the normalized course, \( C_1 \).

The variation in the data that identifies \( \phi_0 \) and \( \phi_1 \) comes from how individuals sort based on their comparative advantage in grades. Someone who is strong in mathematics will be more likely to sort into classes where the returns to ability in mathematics is high. To the extent that women are more or less likely to sort based on where their abilities are rewarded then identifies \( \phi_1 \).

More subtle is the identification on the returns to the normalized course. If separate course fixed effects were estimated for both men and women then \( C_1 \) would not be identified as it would be subsumed into the course fixed effects. But by allowing females preferences to vary at the department rather than the course level, the extent to which sorting happens beyond the effect through grades themselves identifies \( C_1 \).

We assume that \( \epsilon_{ij} \) is distributed Type 1 extreme value. If individuals were choosing one course, estimation of the parameters in (3.26) would follow a multinomial logit. Students, however, choose bundles of courses. Even though the structure of preferences over grades, \( \phi \), and the returns to ability and study time in the normalized course, \( C_1 \).

\[13\] Note that females are also allowed to value having a female professor more or less than males.
the model is such that there are no complementarities for choosing particular combinations of courses, the probability of choosing a particular bundle does not reduce to the probabilities of choosing each of the courses separately.

*Simulated maximum likelihood*

We use simulated maximum likelihood to estimate the choice parameters. To illustrate the approach, denote $K_i$ as the set of courses chosen by $i$. Denote $M_i$ as the highest payoff associated with any of the non-chosen courses:

$$M_i = \max_{j \notin K_i} \delta_0 + w_i \delta_{1k(j)} + Z_{ij} \delta_{2k(j)} + \left( \frac{E[g_{ij}]}{g_{ij}} - \gamma_j \right) (\phi_0 + w_i \phi_1) + \epsilon_{ij}$$

Suppose $K_i$ consisted of courses \{1, 2, 3\} and that the values for all the preference shocks, the $\epsilon_{ij}$’s, were known with the exception of those for \{1, 2, 3\}. The probability of choosing \{1, 2, 3\} could then be expressed as:

$$Pr(d_i = \{1, 2, 3\}) = Pr(\overline{U}_{i1} > M_i, \overline{U}_{i2} > M_i, \overline{U}_{i3} > M_i)$$

$$= Pr(\overline{U}_{i1} > M_i) Pr(\overline{U}_{i2} > M_i) Pr(\overline{U}_{i3} > M_i)$$

$$= (1 - G(M_i - \overline{U}_{i1}))(1 - G(M_i - \overline{U}_{i2}))(1 - G(M_i - \overline{U}_{i3}))$$

where $G(\cdot)$ is the extreme value cdf and $\overline{U}_{ij}$ is the flow payoff for $j$ net of $\epsilon_{ij}$.

Since the $\epsilon_{ij}$’s for the non-chosen courses are not observed, we integrate them out of the likelihood function and approximate the integral by simulating their values from the Type I extreme value distribution. Denoting $M_{ir}$ as the value of $M_i$ at the $r$th draw of the non-chosen $\epsilon_{ij}$’s and $R$ as the number of simulation draws, estimates of the reduced form payoffs come from solving:

$$\max_{\phi, \delta} \sum_i \ln \left( \frac{\left[ \sum_{r=1}^R \prod_{j=1}^J (1 - G(M_{ir} - \overline{U}_{ij}))^{d_{ij}} \right]}{R} \right)$$  \hspace{1cm} (3.27)$$

*Recovering the remaining structural parameters*
Given $\hat{\phi}_0$, $\hat{\phi}_1$, and $\hat{C}_1$, we are now in a position to recover the remaining structural parameters. The normalizing constants for each department where $k \neq 1$ can be recovered using $\hat{C}_k = \exp(\hat{\kappa}_{2k(j)}/\hat{\mu}_2)\hat{C}_1$. Estimates of the unnormalized $\gamma_j$’s are given by $\hat{\gamma}_j = \hat{\gamma}_j^N\hat{C}_{k(j)}$.

The remaining structural parameters from the study effort estimation, equation (3.23), are the study cost intercept, $\psi_0$, and the (relative) female study costs, $\psi_1$. These can be recovered using:

\[
\hat{\psi}_0 = \ln(\hat{C}_1) + \ln(\hat{\phi}_0) - \frac{\hat{\kappa}_0}{\hat{\mu}}
\]
\[
\hat{\psi}_1 = \ln(\hat{\phi}_0 + \hat{\phi}_1) - \ln(\hat{\phi}_0) - \frac{\hat{\kappa}_1}{\hat{\mu}}
\]

The remaining structural parameters of the grade equation, equation (3.12), are the course intercepts, $\beta_j$, and the returns to observed abilities $\alpha_{1k(j)}$ and $\alpha_{2k(j)}$. These can be recovered using:

\[
\hat{\beta}_j = \hat{\theta}_{0j} - \hat{\gamma}_j(\ln(\hat{\phi}_0) + \ln(\hat{\gamma}_j) - \hat{\psi}_0)
\]
\[
\hat{\alpha}_{1k(j)} = \frac{\hat{\theta}_{1j}}{\hat{C}_{k(j)}} - \ln(\hat{\phi}_0 + \hat{\phi}_1) + \ln(\hat{\phi}_0) + \hat{\psi}_1
\]
\[
\hat{\alpha}_{2k(j)} = \frac{\hat{\theta}_{1j}}{\hat{C}_{k(j)}} + \hat{\psi}_2
\]

3.4.3 Estimation with Unobserved Heterogeneity

We now consider the case when one of the components of $X_i$ is unknown to take into account correlation across outcomes for the same individual. We assume that this missing component takes on $S$ values where $\pi_s$ is the unconditional probability of the $s$th value. Let $X_{is}$ be the set of covariates under the assumption that individual $i$ is of type $s$. The components of the unobserved heterogeneity are identified through
the correlation of grades in each of the courses as well as the probabilities of choosing different course combinations.

Integrating out over this missing component destroys the additive separability of the log likelihood function suggesting that the estimation of the three sets of parameters (grades, course choices, and study time) can no longer be estimated in stages. However, using the insights of Arcidiacono and Jones (2003) and Arcidiacono and Miller (2011), it is possible to estimate some of the parameters in a first stage.

In particular, note that the selection problem occurs because students select into courses. By focusing just on the grade estimation as well as a reduced form of the choice problem, we can greatly simplify estimation, recovering the grade parameters as well as the conditional probabilities of being each of the types. These conditional type probabilities can then be used as weights in the estimation of the choice and study parameters.

First consider the parameters of the grade process and the course choices. With unobserved heterogeneity, we now need to make an assumption on the distribution of \( \eta_{ij} \), the residual in the grade equation. We assume the error is distributed \( N(0, \sigma^2) \). We then specify a flexible choice process over courses that depends on an parameter vector \( \varphi \). The integrated log likelihood is:

\[
\sum_i \ln \left( \sum_{s=1}^{S} \pi_s L_{igs} (\theta, \gamma) L_{ics} (\varphi) \right)
\]

(3.28)

where \( L_{igs} (\theta, \gamma) \) and \( L_{ics} (\varphi) \) are the grade and choice (of courses) likelihoods respectively conditional on \( i \) being of type \( s \).

We apply the EM algorithm to then estimate the grade parameters and course choice parameters in stages. We iterate on the following steps until convergence, where the \( m \)th step follows:

1. Given the parameters of the grade equation and choice process at step \( m - 1 \),
\{\theta^{(m-1)}, \gamma^{(m-1)}\} \text{ and } \{\varphi\} \text{ and the estimate of } \pi^{(m-1)}, \text{ calculate the conditional probability of } i \text{ being of type } s \text{ using Bayes rule:}

\[q_{is}^{(m)} = \frac{\pi_s^{(m)} \mathcal{L}_{igs} (\theta^{(m-1)}, \gamma^{(m-1)}) \mathcal{L}_{ics} (\varphi^{(m-1)})}{\sum_s \pi_{s'}^{(m)} \mathcal{L}_{igs'} (\theta^{(m-1)}, \gamma^{(m-1)}) \mathcal{L}_{ics'} (\varphi^{(m-1)})}\] (3.29)

2. Update \(\pi_s^{(m)}\) using \((\sum_{i=1}^N q_{is}^{(m)}) / N\).

3. Using the \(q_{is}^{(m)}\)'s as weights, obtain \(\{\theta^{(m)}, \gamma^{(m)}\}\) by maximizing:

\[\sum_i \sum_s q_{is}^{(m)} \ln [\mathcal{L}_{igs} (\theta, \gamma)]\] (3.30)

4. Using the \(q_{is}^{(m)}\)'s as weights, obtain \(\varphi^{(m)}\) by maximizing:

\[\sum_i \sum_s q_{is}^{(m)} \ln [\mathcal{L}_{ics} (\varphi)]\] (3.31)

Once the algorithm has converged, we have consistent estimates of \(\{\theta, \gamma, \varphi\}\) as well as the conditional probabilities of being in each type. We can use the estimates of \(q_{is}\) as weights to form the average type probabilities of students of year in school \(l\) in class \(j\) to then estimate the parameters in (3.23). Finally, we use the estimates of \(q_{is}\) as weights in estimating the structural choice parameters using (3.27).

3.4.4 Implications from the Demand-Side Estimation

Even without estimating professor preferences, much can be learned from the demand-side estimates. First, we can explain some of the persistent gender gap in STEM majors. Demand-side estimates allow us to decompose differences in course choices, grades, and study effort between males and females into parts due to:

(i) differences in preferences \((\delta_{ij})\),
(ii) differences in value of grades ($\phi_i$),

(iii) differences in study costs ($\psi_{ij}$).

The estimates of the model can also be used to see how enrollment in STEM courses by both men and women would be affected by changes in grading practices. First, we can adjust the intercepts in the grading equation such that the average student’s expected grade is the same across courses, isolating the role of the level of the grade from the differences in the slopes, and therefore return to effort. Second, we can forecast course choices if all professors were to have the same grading practices.

3.5 Results

3.5.1 Preference estimates

Table 3.4 presents the preference parameters with the exception of the study costs, the class-specific intercepts, and the coefficients on year in school cross department cross level of the course. Recall that the parameter on expected grades is identified from variation in how abilities are rewarded in different classes. Both men and women value grades, with the estimates suggesting that women value grades around 18% more than men. The estimate of female preferences for female professors is positive, with the estimate suggesting that women be indifferent between a class that had a female professor and one that had a male professor who gave grades that were a little under 0.3 points higher. This coefficient is likely biased upward due to the aggregation of departments. To the extent that female professors are more likely to be in departments that females have a preference for and there is variation within our aggregated groups, we may be picking up within-group preferences for departments.

The second set of rows of Table 3.4 shows female preferences (relative to male preferences) for different departments. The omitted category is Agriculture. The largest difference in preferences is between Engineering and English: 1.48 points,
which translates into 3 grade points. Engineering, however, is an outlier with all the
other gaps smaller that one point (or 2 grade points).

Table 3.4: Estimates of Preference Parameters

<table>
<thead>
<tr>
<th>Preference for:</th>
<th>Coeff.</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected grades ($\phi$)</td>
<td>0.4157</td>
<td>(0.0199)</td>
</tr>
<tr>
<td>Female $\times$ expected grades</td>
<td>0.0759</td>
<td>(0.0157)</td>
</tr>
<tr>
<td>Female $\times$ female professor</td>
<td>0.1455</td>
<td>(0.0186)</td>
</tr>
<tr>
<td>$C_1$ (normalizing constant)</td>
<td>0.9581</td>
<td>(0.2593)</td>
</tr>
</tbody>
</table>

Female preferences for Departments

<table>
<thead>
<tr>
<th>Department</th>
<th>Coeff.</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engineering</td>
<td>-1.0615</td>
<td>(0.0737)</td>
</tr>
<tr>
<td>Econ., Fin., Acct.</td>
<td>-0.5091</td>
<td>(0.0593)</td>
</tr>
<tr>
<td>Social Sciences</td>
<td>-0.2862</td>
<td>(0.0545)</td>
</tr>
<tr>
<td>Communication</td>
<td>-0.1528</td>
<td>(0.0537)</td>
</tr>
<tr>
<td>Chemistry &amp; Physics</td>
<td>-0.1482</td>
<td>(0.0599)</td>
</tr>
<tr>
<td>Languages</td>
<td>-0.1033</td>
<td>(0.0582)</td>
</tr>
<tr>
<td>Mathematics</td>
<td>-0.0072</td>
<td>(0.0688)</td>
</tr>
<tr>
<td>Mgmt. &amp; Mkting.</td>
<td>0.1153</td>
<td>(0.0662)</td>
</tr>
<tr>
<td>Regional Studies</td>
<td>0.2216</td>
<td>(0.0698)</td>
</tr>
<tr>
<td>Biology</td>
<td>0.2546</td>
<td>(0.0638)</td>
</tr>
<tr>
<td>Education &amp; Health</td>
<td>0.3287</td>
<td>(0.0581)</td>
</tr>
<tr>
<td>Psychology</td>
<td>0.3758</td>
<td>(0.0659)</td>
</tr>
<tr>
<td>English</td>
<td>0.4167</td>
<td>(0.0769)</td>
</tr>
</tbody>
</table>

3.5.2 Study effort estimates

Estimates of the study effort parameters are presented in Table 3.5.\(^{14}\) Lower study
costs result in women studying a little over 7% more than men conditional on taking
the same class. However, women also study more because they value grades more,
with this effect at over 18\%,\(^{15}\) again conditional on taking the same class resulting
in an overall effect of over 25%.

All of the measures of academic preparation (ACT scores and high school grades)

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\(^{14}\) Because of measurement error in the $\gamma$'s that is compounded by it entering as a log in the study
effort equation, we drop classes in the bottom 5% of the $\gamma$ distribution. Parameters of the study
effort equation stabilize after this point.

\(^{15}\) This number comes from the difference in the log of the preferences for grades: $\ln(\phi_0 + \phi_1) -
\ln(\phi_0) = \ln(.4916) - \ln(.4157)$. 

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have the expected sign but are small in magnitude. While blacks and first generation students have higher study costs, Hispanics and miscellaneous minorities have lower study costs. Those who are the second unobserved type have higher study costs but, as we will see in subsequent tables, are more able.\footnote{The population probability of being the second unobserved type is 0.213. The information on grades and course-taking does a good job of sorting individuals into types. See Appendix Figure X for a histogram of the conditional type probabilities.} Finally,

The second set of columns shows how the returns to study effort vary across classes, taking the median $\gamma$ class for each course grouping. The heterogeneity is quite large. A ten percent increase in study effort would translate into over a third of a grade point increase in mathematics but would translate into less than a tenth of a grade point in agriculture, management and marketing, and education.

Table 3.5: Estimates of Study Effort and Departmental Returns to Studying

<table>
<thead>
<tr>
<th>Study Effort</th>
<th>Std. Error</th>
<th>Department</th>
<th>Median $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>-0.0737</td>
<td>(0.0799)</td>
<td>Mathematics</td>
</tr>
<tr>
<td>ACT read</td>
<td>-0.0022</td>
<td>(0.0097)</td>
<td>Engineering</td>
</tr>
<tr>
<td>ACT math</td>
<td>-0.0182</td>
<td>(0.0112)</td>
<td>Biology</td>
</tr>
<tr>
<td>HS GPA</td>
<td>-0.0041</td>
<td>(0.0872)</td>
<td>English</td>
</tr>
<tr>
<td>Black</td>
<td>0.2342</td>
<td>(0.1724)</td>
<td>Chemistry &amp; Physics</td>
</tr>
<tr>
<td>Hispanic</td>
<td>-0.3174</td>
<td>(0.2519)</td>
<td>Psychology</td>
</tr>
<tr>
<td>Other Min.</td>
<td>-0.2687</td>
<td>(0.2609)</td>
<td>Econ., Fin., Acct.</td>
</tr>
<tr>
<td>First Gen.</td>
<td>0.1068</td>
<td>(0.1149)</td>
<td>Regional Studies</td>
</tr>
<tr>
<td>Unobs. Type</td>
<td>0.1979</td>
<td>(0.0842)</td>
<td>Communication</td>
</tr>
<tr>
<td>Effort elasticity</td>
<td></td>
<td></td>
<td>Languages</td>
</tr>
<tr>
<td>$\ln(\gamma)$</td>
<td>0.5445</td>
<td>(0.2055)</td>
<td>Social Sciences</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Agriculture</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mgmt. &amp; Mkting.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Education &amp; Health</td>
</tr>
</tbody>
</table>

3.5.3 Grade estimates

The estimated $\alpha$'s, the department-specific ability weights, are given in Table 3.6. These are calculated by taking the reduced-form $\theta$'s, undoing the normalization on
the $\gamma$'s, and subtracting off the part of the reduced form that $\theta$'s that reflect the study time (taken from $\psi$). The departments are sorted such that those with the lowest female estimate are listed first. Note that in all departments the female estimate is negative. This occurs because females study substantially more than males yet receive only slightly higher grades. Given that sorting into universities takes place on both cognitive and non-cognitive skills and that women have a comparative advantage in non-cognitive skills, males at Kentucky have higher cognitive skills than their female counterpart even though in the population cognitive skills are similar between men and women.

Negative estimates are also found for Hispanics. While Hispanics have higher grades than African Americans, our estimates of the study costs suggested that they also studied substantially more. Given the very high estimate of Hispanic study time we would have expected Hispanics to perform even better in the classroom than they actually did if their baseline abilities were similar to African Americans.
Table 3.6: Estimates of Department-Specific Ability Weights ($\alpha$)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>-0.297</td>
<td>-0.267</td>
<td>-0.240</td>
<td>-0.225</td>
<td>-0.225</td>
<td>-0.224</td>
<td>-0.224</td>
<td>-0.209</td>
<td>-0.198</td>
<td>-0.195</td>
<td>-0.190</td>
<td>-0.150</td>
<td>-0.145</td>
<td>-0.145</td>
<td>-0.145</td>
<td>-0.145</td>
<td>-0.075</td>
<td></td>
</tr>
<tr>
<td>ACT read</td>
<td>0.000</td>
<td>0.005</td>
<td>0.005</td>
<td>-0.003</td>
<td>-0.007</td>
<td>0.000</td>
<td>0.011</td>
<td>0.000</td>
<td>0.009</td>
<td>0.013</td>
<td>0.003</td>
<td>0.010</td>
<td>0.000</td>
<td>0.004</td>
<td>0.004</td>
<td>0.004</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>ACT math</td>
<td>0.011</td>
<td>0.011</td>
<td>-0.002</td>
<td>0.000</td>
<td>-0.007</td>
<td>-0.008</td>
<td>0.005</td>
<td>-0.010</td>
<td>-0.006</td>
<td>-0.003</td>
<td>-0.004</td>
<td>-0.001</td>
<td>-0.014</td>
<td>0.016</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HS GPA</td>
<td>0.226</td>
<td>0.212</td>
<td>0.187</td>
<td>0.097</td>
<td>0.198</td>
<td>0.064</td>
<td>0.472</td>
<td>0.416</td>
<td>0.226</td>
<td>0.260</td>
<td>0.192</td>
<td>0.231</td>
<td>0.443</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Black</td>
<td>0.115</td>
<td>0.094</td>
<td>0.091</td>
<td>0.196</td>
<td>0.217</td>
<td>0.124</td>
<td>0.034</td>
<td>-0.122</td>
<td>0.106</td>
<td>0.126</td>
<td>0.075</td>
<td>0.171</td>
<td>0.085</td>
<td>-0.121</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hispanic</td>
<td>-0.326</td>
<td>-0.276</td>
<td>-0.323</td>
<td>-0.286</td>
<td>-0.306</td>
<td>-0.453</td>
<td>-0.285</td>
<td>-0.293</td>
<td>-0.421</td>
<td>-0.363</td>
<td>-0.180</td>
<td>-0.335</td>
<td>-0.387</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Misc. Min.</td>
<td>-0.305</td>
<td>-0.261</td>
<td>-0.383</td>
<td>-0.304</td>
<td>-0.334</td>
<td>-0.349</td>
<td>-0.334</td>
<td>-0.214</td>
<td>-0.455</td>
<td>-0.285</td>
<td>-0.363</td>
<td>-0.350</td>
<td>-0.341</td>
<td>-0.534</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st Gen</td>
<td>0.038</td>
<td>0.057</td>
<td>0.033</td>
<td>0.073</td>
<td>0.010</td>
<td>0.102</td>
<td>0.017</td>
<td>0.040</td>
<td>0.023</td>
<td>0.041</td>
<td>0.057</td>
<td>0.057</td>
<td>0.041</td>
<td>0.033</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>0.913</td>
<td>0.982</td>
<td>0.897</td>
<td>0.575</td>
<td>1.221</td>
<td>0.630</td>
<td>0.939</td>
<td>1.989</td>
<td>1.418</td>
<td>1.199</td>
<td>1.018</td>
<td>1.143</td>
<td>1.186</td>
<td>1.724</td>
<td>1.724</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
With the estimates of the grading equation, we can report expected grades for an average student. We do this for freshmen, separately by gender, both unconditionally and conditional on taking courses in that department in the semester we study. Results are presented in Table 3.7. Three patterns stand out. First, there is positive selection into STEM courses: generally those who take STEM classes are expected to perform better than the average student. This is not the case for many departments. Indeed, the second pattern is that negative selection is more likely to occur in departments with higher grades. Finally, women are disproportionately represented in departments that give higher grades for the average student. Of the seven departments that give the highest grades for the average student (female or male), all have a larger fraction female than the overall population. In contrast, of the seven departments that give the lowest grades, only three have a fraction female that is larger than the overall population.

Table 3.7: Expected Freshmen GPA for Median Classes By Department, Unconditional and Conditional on Taking Courses in that Department

<table>
<thead>
<tr>
<th>Department</th>
<th>EGPA Females Unconditional</th>
<th>EGPA Females Conditional</th>
<th>EGPA Males Unconditional</th>
<th>EGPA Males Conditional</th>
<th>Share Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education &amp; Health</td>
<td>3.71</td>
<td>3.67</td>
<td>3.54</td>
<td>3.16</td>
<td>0.70</td>
</tr>
<tr>
<td>Agriculture</td>
<td>3.49</td>
<td>3.28</td>
<td>3.39</td>
<td>2.96</td>
<td>0.56</td>
</tr>
<tr>
<td>Communication</td>
<td>3.42</td>
<td>3.39</td>
<td>3.16</td>
<td>3.17</td>
<td>0.56</td>
</tr>
<tr>
<td>Mgmt. &amp; Mkting.</td>
<td>3.30</td>
<td>3.41</td>
<td>3.14</td>
<td>3.27</td>
<td>0.52</td>
</tr>
<tr>
<td>Languages</td>
<td>3.26</td>
<td>3.28</td>
<td>3.09</td>
<td>3.12</td>
<td>0.55</td>
</tr>
<tr>
<td>Regional Studies</td>
<td>3.20</td>
<td>3.31</td>
<td>2.97</td>
<td>3.11</td>
<td>0.66</td>
</tr>
<tr>
<td>Social Sciences</td>
<td>3.10</td>
<td>3.11</td>
<td>2.94</td>
<td>2.90</td>
<td>0.51</td>
</tr>
<tr>
<td>English</td>
<td>3.07</td>
<td>3.12</td>
<td>2.93</td>
<td>3.00</td>
<td>0.65</td>
</tr>
<tr>
<td>Psychology</td>
<td>2.98</td>
<td>2.98</td>
<td>2.75</td>
<td>2.73</td>
<td>0.67</td>
</tr>
<tr>
<td>Engineering</td>
<td>2.96</td>
<td>3.01</td>
<td>2.82</td>
<td>3.03</td>
<td>0.18</td>
</tr>
<tr>
<td>Econ., Fin., Acct.</td>
<td>2.78</td>
<td>2.93</td>
<td>2.77</td>
<td>2.91</td>
<td>0.37</td>
</tr>
<tr>
<td>Mathematics</td>
<td>2.72</td>
<td>2.78</td>
<td>2.59</td>
<td>2.71</td>
<td>0.47</td>
</tr>
<tr>
<td>Biology</td>
<td>2.69</td>
<td>2.82</td>
<td>2.59</td>
<td>2.78</td>
<td>0.60</td>
</tr>
<tr>
<td>Chem &amp; Physics</td>
<td>2.47</td>
<td>2.62</td>
<td>2.51</td>
<td>2.74</td>
<td>0.47</td>
</tr>
<tr>
<td>Overall</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.51</td>
</tr>
</tbody>
</table>
3.5.4 Drivers of the STEM gap

Given the estimates of the student’s choices over classes and effort and the estimates of the grading process, we now turn to examining sources of the male-female gap in choice of STEM classes. Table 3.8 shows share of STEM classes taken for males and females as well as how that share changes for women as we change different characteristics. The baseline share of STEM classes for men and women is 0.400 and 0.284, respectively. The first counterfactual changes female preferences for grades to be the same as male preferences for grades. This increases the share of STEM courses for women by almost one percentage point, closing the gender gap by almost eight percent.

Turning off observed ability differences such as differences in ACT scores and high school grades has smaller effects on the gap (row 3), though larger effects are found for unexplained gender differences in ability (row 2). Note that these effects are not driven by women being weaker academically per se, but in part due to women being relatively stronger in non-STEM courses.

Counterfactuals (4) and (5) look at differences in tastes. Counterfactual (5) turns off taste differences for departments, which increases the share of women to 0.3, closing the STEM gap by 13 percent. These taste differences may be a mixture of pre-college experiences and the culture of different departments. Hence anything the university can do to close the STEM gap on this end is likely bounded above by this number. Counterfactual (6) turns off female preferences for female professors. One way of closing the gender gap in STEM would be to hire more female professors. However, even representation across fields would only close the gap by a little over three percent.

Finally, we examine how changing expected grades across departments affects the gender gap. Namely, we equalize mean grades across courses by increasing (or
Table 3.8: Decomposing the Gender STEM Gap

<table>
<thead>
<tr>
<th></th>
<th>PE Female</th>
<th></th>
<th>PE Male</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Share</td>
<td>Increase</td>
<td>Share</td>
<td>Increase</td>
</tr>
<tr>
<td>Baseline</td>
<td>28.40%</td>
<td>0.00%</td>
<td>40.02%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Turn off grade prefs (1)</td>
<td>29.34%</td>
<td>0.94%</td>
<td>40.02%</td>
<td>0.94%</td>
</tr>
<tr>
<td>Turn off gender ability (2)</td>
<td>32.17%</td>
<td>3.78%</td>
<td>40.02%</td>
<td>3.78%</td>
</tr>
<tr>
<td>Turn off observed ability (3)</td>
<td>28.89%</td>
<td>0.50%</td>
<td>40.02%</td>
<td>0.50%</td>
</tr>
<tr>
<td>Turn off tastes (4)</td>
<td>29.94%</td>
<td>1.54%</td>
<td>40.02%</td>
<td>1.54%</td>
</tr>
<tr>
<td>Turn off female prof pref (5)</td>
<td>28.79%</td>
<td>0.40%</td>
<td>40.02%</td>
<td>0.40%</td>
</tr>
<tr>
<td>Equalize expected grades for average student (6)</td>
<td>33.40%</td>
<td>5.00%</td>
<td>44.00%</td>
<td>3.98%</td>
</tr>
</tbody>
</table>

decreasing) the course-specific intercepts. However, there is still heterogeneity in grades due to the relative difference in $\gamma$’s and $\alpha$’s, the former being especially important as it dictates the returns to studying. This counterfactual raises the share of STEM courses taken by females to about a third, higher than any of the counterfactuals. The reason the effects are larger here on the gender ratio than in the first counterfactual is that the returns to studying are much higher in STEM courses and women are willing to study more than men, due both to valuing grades more and having lower study costs. Note that the effect of the STEM gap is mitigated because men also shift towards STEM under this policy.

While the patterns here suggest a potentially cheap way of closing the gender gap is to equalize average grades across fields, professors are likely to respond to restrictions on grading policies. However, the response may further reduce the gender gap. The reason is that, if STEM courses are forced to give higher grades, they are likely to assign more work to deter entry. More work translates into higher $\gamma$’s which make STEM courses relatively more attractive to women. The reverse holds for departments that are forced to lower their grades: in order to attract more students, they must lower workloads, implying lower values of $\gamma$ which makes these courses relatively less attractive to women.
3.6 Equilibrium Grading Policies

In Section 3.5, we show grading policy parameters $\beta_j$ and $\gamma_j$ differ significantly across departments. In particular, STEM courses generally have lower grading intercepts $\beta_j$ but higher returns on effort $\gamma_j$ than non-STEM courses. One principle goal of this paper is to analyze how these grading differences influence course choices and the implications for the gender gap in STEM.

However, this finding also prompts an additional question: Why do grading policies vary across courses? In particular, why do STEM courses have lower intercepts but higher returns on effort than non-STEM courses? Understanding how professors choose grading policies is crucial to anticipate equilibrium responses to changes in the environment. For example, increasing STEM preparation in the hopes of increasing the number of STEM majors may be partially undone by how professors change their grading policies in response to the new environment.

In this section, we develop a model which describes how professors choose grading policies and propose a method for estimating the professor preference parameters of this model. The model assumes professors care about three factors: (i) the number of students in their class, (ii) the cost of assigning work, and (iii) grades given. Professors set policy parameters $\beta_j$ and $\gamma_j$ to influence these factors.

The model allows for grading policies to arise from differences in intrinsic demand of students. Heterogeneity in non-grade preferences $\delta_{ij}$ and abilities $A_{ij}$ imply that some courses would be more popular than others even with homogenous grading policies. These differences in intrinsic demand imply that the relationship between grading policies and the composition and outcomes of enrolled students differs across courses. A professor teaching an intrinsically popular course will need to grade especially harshly to achieve the same class size as a less popular course with average grading standards.
Because grading policies in all courses affect the choices of students, the composition of students in each course depends on the grading policies of all professors. This general equilibrium feature means that each professor’s optimal grading policy depends on the grading policies of all other professors. We assume professors do not collude when choosing grading policies implying policies are set in a non-cooperative game between professors.

To estimate professor preference parameters, we solve for parameter values which explain why observed grading policies were optimal for professors. First, we estimate grading policy parameters and student preference parameters using the methods described in Section 3.4. Second, we derive the set of first order conditions which describe a pure-strategy equilibrium to the non-cooperative grade policy setting game. This system of first order conditions describes how professor preference parameters, grading policy parameters, and student parameters relate to one another when all professors are setting grading policy parameters optimally. Finally, we solve for professor preference parameters which satisfy the set of first order conditions given estimates of grading policy parameters and student preference parameters.

3.6.1 The Professor’s Problem

We assume professors choose grading policy parameters $\beta_j$ and $\gamma_j$ to maximize an objective function which depends on both the number and outcomes of students who take their class. This objective function needs to reconcile the following patterns in the data:

- the correlation between $\beta_j$ and $\gamma_j$ is extremely negative at -0.96,

- $\delta_{0j}$ is negatively correlated with $\beta_j$ (-0.42) and positively correlated with $\gamma_j$ (0.42), and

- STEM classes have even stronger negative (positive) correlations with $\beta_j$ ($\gamma_j$)
To capture these features of the data, we set up the professor’s objective function to depend on (i) the number of students in their class, (ii) the cost of assigning work, and (iii) grades given. First, we specify the professor’s objective function as quadratic in log expected enrollment. This allows for professors to initially desire students to take their class but at some point utility may fall as additional students are added. Second, there is a quadratic cost in assigning work, $\gamma$. This cost of assigning work varies across classes resulting in some departments having lower costs of assigning work than others. This would be the case if, for example, math problems are easier to grade than essays. Finally, professors have an ideal expected grade for their class and deviating from this ideal is costly. In this way some departments may have cultures for harsher grading than other department irrespective of demand.

Denote $G_j(\beta, \gamma)$ as the expected average grade in class $j$ given the vector of grading policies for all courses $\beta$ and $\gamma$. The dependence on $\beta$ and $\gamma$ comes through the composition of the students that take the course. Denoting $P_{ij}(\beta, \gamma)$ as the probability $i$ takes course $j$ given the vector of grading policies, $G_j(\beta, \gamma)$ is given by:

$$G_j(\beta, \gamma) = \beta_j + \gamma_j \left[ \frac{\sum_i P_{ij}(\beta, \gamma) [A_{ij} + \ln(\phi_i) - \ln(\psi_i)]}{\sum_i P_{ij}(\beta, \gamma)} \right] + \ln(\gamma_j)$$

The average ability of the students in the class is given by:

$$A_j(\beta, \gamma) = \frac{\sum_i P_{ij}(\beta, \gamma) [A_{ij} + \ln(\phi_i) - \ln(\psi_i)]}{\sum_i P_{ij}(\beta, \gamma)}$$

Then the objective function the professors maximize is:

$$V_j(\beta, \gamma) = 2\lambda_1 \ln \left[ \sum_i P_{ij}(\beta, \gamma) \right] - \lambda_2 \left( \ln \left[ \sum_i P_{ij}(\beta, \gamma) \right] \right)^2$$

$$+ 2\lambda_3 A_j(\beta, \gamma) - 2\epsilon_{2j} \gamma - \lambda_4 \gamma^2 - \left( \ln[G_j(\beta, \gamma)] - e_{1j} \right)^2$$

$$= 2\lambda_1 \ln \left[ \sum_i P_{ij}(\beta, \gamma) \right] - \lambda_2 \left( \ln \left[ \sum_i P_{ij}(\beta, \gamma) \right] \right)^2$$

$$+ 2\lambda_3 A_j(\beta, \gamma) - 2\epsilon_{2j} \gamma - \lambda_4 \gamma^2 - \left( \ln[G_j(\beta, \gamma)] - e_{1j} \right)^2$$  (3.32)
where \( e_{j1} \) represents the professor’s ideal grade for the average student taking the class and \( e_{j2} \) allows for heterogeneity in the cost of assigning work. Note that we specify the ideal average grade in logs. Deviating from one’s ideal grade is more costly when grades are lower than ideal then when grades are higher than ideal due to, for example, student complaints.

### 3.6.2 Estimation

Denote the \( W_j \) as a vector that includes indicators for whether the course is in each of the fourteen departments\(^{17}\), whether the course is an upper division, and whether the course is upper division and in STEM. We parameterize \( e_{j1} \) and \( e_{j2} \) such that they vary with \( W_j \) as well as unobserved preference terms \( \bar{e}_{j1} \) and \( \bar{e}_{j2} \) according to:

\[
e_{j1} = W_j \Psi_1 + \bar{e}_{j1}
\]
\[
e_{j2} = W_j \Psi_2 + \bar{e}_{j2}
\]

\( \bar{e}_{j1} \) and \( \bar{e}_{j2} \) are assumed to be orthogonal to the department and the level of the course.

Taking the first order condition with respect to \( \beta \) and rearranging terms yields:

\[
\ln \left( \frac{G_j(\beta, \gamma)}{e^3} \right) = \left( \lambda_1 - \lambda_2 \ln \sum_i P_{ij}(\beta, \gamma) \right) \left( \frac{\sum_i \frac{\partial P_{ij}(\beta, \gamma)}{\partial \beta} G_j(\beta, \gamma)}{\sum_i P_{ij}(\beta, \gamma) \frac{\partial G_j(\beta, \gamma)}{\partial \beta}} \right) + \left( \frac{\lambda_3 \partial A_j(\beta, \gamma)}{\partial \beta} \right) \left( \frac{G_j(\beta, \gamma)}{\partial G_j(\beta, \gamma)} \right) + W_j \Psi_1 + \bar{e}_{j1}
\]

Given our estimates of student demand, grading policies, and study effort, the only unknowns in (3.34) are \( \lambda_{j1}, \lambda_{j2}, \) and \( \bar{e}_{j1} \). However, \( \bar{e}_{j1} \) is correlated with \( \beta_j \) and \( \gamma_j \).

To deal with the endogeneity of \( \beta_j \) and \( \gamma_j \), we make an additional assumption that \( \bar{e}_{j1} \) and \( \bar{e}_{j2} \) are orthogonal to innate demand, that is preferences for the course

\(^{17}\) Recall that classes are only counted as belonging to one department.
outside of those dealing with expected grades. The assumption is then that innate course demand once expected grades and effort are conditioned out is uncorrelated with professor grade preferences and workload targets.

With this assumption we instrument for $\beta_j$ and $\gamma_j$ with functions of the innate demand for the course. In particular, for each course we calculate the probability of taking class $j$ for each individual when expected grades and workloads are the same in all courses. Summing across all individuals gives demand for course $j$ that is only a function of preferences, not grading policies. Label this innate demand $D_j$.

We then (separately) regress our estimates of $\beta_j$ and $\gamma_j$ on $D_j$, $D_j^{1/2}$, $D_j^2$, and $W_j$. Label the predicted values from this regression as $\hat{\beta}_j^{IV}$ and $\hat{\gamma}_j^{IV}$. Following we then substitute in for $\beta_j$ and $\gamma_j$ with $\hat{\beta}_j^{IV}$ and $\hat{\gamma}_j^{IV}$. We then use (3.34) to obtain estimates of $\lambda_1$, $\lambda_2$, $\lambda_3$, and $\Psi_1$. Given these estimates, we can then back out estimates of $e_{1j}^*$.

The remaining structural parameters are $\lambda_4$, $\Psi_2$ and $e_{2j}^*$. We obtain these using the first order condition of (3.32) with respect to $\gamma_j$:

$$0 = \left( \lambda_1 - \lambda_2 \ln \left[ \sum_i P_{ij}(\beta, \gamma) \right] \right) \left( \frac{\sum_i \frac{\partial P_{ij}(\beta, \gamma)}{\partial \gamma}}{\sum_i P_{ij}(\beta, \gamma)} \right) \right)$$

$$- \left( \ln \left[ G_j(\beta, \gamma) \right] - W_j \Psi_1 - e_{1j}^* \right) \left( \frac{\frac{\partial G_j(\beta, \gamma)}{\partial \beta}}{G_j(\beta, \gamma)} \right)$$

$$+ \frac{\lambda_3 \frac{\partial A_j(\beta, \gamma)}{\partial \gamma}}{\Psi_2 - \lambda_3 \gamma_j - e_{2j}^*} \right) \right)$$

(3.34)

Note that we have estimates of all the terms in the first lines. Let $B$ denote the estimated values of the first line, given by:

$$B = \left( \hat{\lambda}_1 - \hat{\lambda}_2 \ln \left[ \sum_i P_{ij}(\hat{\beta}, \hat{\gamma}) \right] \right) \left( \frac{\sum_i \frac{\partial P_{ij}(\hat{\beta}, \hat{\gamma})}{\partial \gamma}}{\sum_i P_{ij}(\hat{\beta}, \hat{\gamma})} \right)$$

$$- \left( \ln \left[ G_j(\hat{\beta}, \hat{\gamma}) \right] - W_j \hat{\Psi}_1 - \hat{e}_{1j}^* \right) \left( \frac{\frac{\partial G_j(\hat{\beta}, \hat{\gamma})}{\partial \beta}}{G_j(\hat{\beta}, \hat{\gamma})} \right)$$

$$+ \frac{\lambda_3 \frac{\partial A_j(\hat{\beta}, \hat{\gamma})}{\partial \gamma}}{\hat{\Psi}_2 - \lambda_3 \hat{\gamma}_j - \hat{e}_{2j}^*} \right) \right)$$

(3.35)
We then instrument for $\gamma_j$ to get $\lambda_4$ with $\gamma_j^{IV}$ where our estimating equation is now:

$$B = W_j \Psi_2 + \lambda_3 \gamma + e_2^* \quad (3.36)$$

Given estimates of $\Psi_2$ and $\lambda_3$, we can then back out estimates of $e_2^*$.

### 3.6.3 Professor preference estimates

Table 3.9 shows estimates of the professor parameters. The first panel shows the estimates of how professors value student demand as well as the quadratic cost of assigning work. All coefficients have the expected signs. Namely, the estimates show that professors value enrollment at first but then additional enrollment becomes costly. The optimal enrollment for the demand portion of the professor’s objective function is 79 students, but note that, because of the log form, deviations are more costly moving to lower enrollments than higher enrollments. The coefficient on $\gamma^2$ is negative, consistent with assigning work eventually being more and more costly.

The second panel shows the parameters that are heterogeneous across departments. These include the parameters related to the ideal grade as well as heterogeneity in the cost of assigning work. All coefficients are relative to those for Agriculture. The first column shows the ideal grade. Since the specification is in logs, these should be interpreted as percents. Coefficients are sorted with those that have the highest ideal grades at the top. Despite STEM classes being associated with lower grades, their ideal grades actually higher than their non-science counterparts. This is especially true for Biology which has an ideal grade that is 20% higher than Agriculture. Weighting these intercepts by department enrollment shows that ideal grades in STEM classes are almost 4% higher than non-STEM classes. The last two entries show how ideal grades vary by level of the course. Ideal grades are lower in upper division classes and lower still in upper division STEM classes. Note that this does not imply that upper division courses give lower grades as demand for these
classes is also lower, which exerts upward pressure on grades.

Since grades are on average lower in STEM classes, student demand must be the driver of the lower grades. This is confirmed in the second column that shows estimates of log grades when we do not account for demand. Namely, we regress log expected grades on department indicators as well as indicators for upper division and upper division cross STEM. The coefficients are substantially different. Ignoring demand shows Education with the highest ideal grades with STEM classes making up four of the bottom six departments. Similarly, ignoring demand paints a different picture for ideal grades in upper division courses where absent controlling for demand grades are higher in upper division classes and higher still in upper division STEM classes.

One check on whether these estimates are reasonable is to compare the (weighted-by-department-enrollment) variance in the department intercepts with and without controls for demand. Controlling for demand reduces the variance in the department intercepts by almost 38%. The fact that controlling for demand reduces variation in these intercepts provides some credibility the results.

The third column of the second panel shows the cost of assigning work, again relative to Education. The coefficients are sorted such that the departments with the lowest costs of assigning work are at the top. Consistent with it being easier to grade more objective work, the lowest costs of assigning work are in Mathematics and Engineering. While it is somewhat surprising that English is next, the overall patterns are associated with more objective classes having lower costs of assigning work. Upper division non-STEM classes see a reduction in the cost of assigning work, but the reverse is true for upper division STEM classes.

The heterogeneity in the cost of assigning work show leads to different vehicles for responding to high demand. When demand for a course with a low cost of assigning work is high, this professor can mitigate demand by assigning more work. Assigning
more work deters students from taking the class and affects the type of students who
take the class. With more able and harder working students taking the class, the
professor can lower his grading intercept while still maintaining grades that are close
to his or her ideal.

Table 3.9: Estimates of Professor Preferences

<table>
<thead>
<tr>
<th>Demand Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(Enroll) $(\lambda_1)$</td>
</tr>
<tr>
<td>ln(Enroll)$^2$ $(-\lambda_2)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ideal Grade Parameters</th>
<th>Cost of Assigning Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biology</td>
<td>0.1948 (0.0395)</td>
</tr>
<tr>
<td>Psychology</td>
<td>0.1347 (0.0325)</td>
</tr>
<tr>
<td>Education &amp; Health</td>
<td>0.1055 (0.0200)</td>
</tr>
<tr>
<td>Mgmt. &amp; Mkting.</td>
<td>0.1010 (0.0242)</td>
</tr>
<tr>
<td>Chem &amp; Physics</td>
<td>0.0417 (0.0300)</td>
</tr>
<tr>
<td>Engineering</td>
<td>0.0414 (0.0268)</td>
</tr>
<tr>
<td>Econ., Fin., Acct.</td>
<td>0.0365 (0.0276)</td>
</tr>
<tr>
<td>Communication</td>
<td>0.0072 (0.0170)</td>
</tr>
<tr>
<td>Mathematics</td>
<td>0.0047 (0.0267)</td>
</tr>
<tr>
<td>Social Sciences</td>
<td>-0.0102 (0.0198)</td>
</tr>
<tr>
<td>Languages</td>
<td>-0.0646 (0.0191)</td>
</tr>
<tr>
<td>Regional Studies</td>
<td>-0.0740 (0.0219)</td>
</tr>
<tr>
<td>English</td>
<td>-0.1263 (0.0239)</td>
</tr>
<tr>
<td>Upper class</td>
<td>-0.0818 (0.0112)</td>
</tr>
<tr>
<td>Upper class×STEM</td>
<td>-0.0618 (0.0201)</td>
</tr>
<tr>
<td>$\gamma^2 (-\lambda_4)$</td>
<td>-0.1415 (0.0082)</td>
</tr>
</tbody>
</table>

3.6.4 Calculating equilibrium counterfactuals

Conditional on estimates of the $\beta$'s and $\gamma$'s, it is relatively simple to recover estimates
of the professors’ preference parameters. However, the reverse is not true. Given $\lambda$'s
and a counterfactual environment, it is computationally very demanding to find the
2,100-plus counterfactual $\beta$’s and $\gamma$’s.

To circumvent the computational demands, we approximate grading policies
based on a set of covariates. Namely, we project the $\beta$’s and ln($\gamma$)'s on a set of
covariates including a flexible function of the professor parameters, baseline preferences for the course ($\delta_0$), department fixed effects that are allowed vary by upper and lower level classes, and whether the professor was female.\(^{18}\) These covariates explain over 90% of the variation in for both the $\beta$’s and the $\gamma$’s. We then use this set of covariates–plus the residual of this regression–to estimate counterfactual grading policies as functions of these covariates. Note that since the residual is included the professor first order conditions will be exactly satisfied in the baseline scenario.

Given this set of covariates, we search for coefficients that minimize the sum of squared errors associated with the first order conditions, minimizing over 70 parameters rather than over 2100. Although this is an approximation, we know exactly how well the approximation is performing and can additional covariates if the fit is poor.\(^{19}\) Further, given the counterfactual grading policies, we can reestimate the professor parameters in the counterfactual environment. If the estimated professor preferences in the counterfactual are close to the values from the baseline scenario then we can be at least somewhat assured that we have a reasonable approximation to the counterfactual grading policies.

### 3.7 Conclusion

The lack of graduates in STEM majors–particularly among under-represented groups–has been of some policy concern. We show that there is a potentially cheap way to change the number and composition of STEM majors. Namely, grading policies have a substantial effect on sorting into STEM classes. We show that a substantial portion of the gender STEM gap can be removed by having STEM classes give grades that are on average similar to those in non-STEM classes.

\(^{18}\) The flexible function of the professor parameters includes the level of both parameters, the log of both parameters, and, in the case of $\beta$, the preference for studying taken to the $-0.5$ power.

\(^{19}\) Note we can see how much the fit improves relative to the fit provided by the baseline grading policies.
These grading policies, however, are in part choices by professors. Hence administrative policies designed to change how professors grade will elicit responses by professors on other dimensions such as workload. These responses by professors may result in an even further closing of the STEM gender gap. Namely, if classes across departments are forced to give similar grades on average, then STEM (non-STEM) classes will employ alternative means to deter (encourage) enrollment in their courses by changing workloads. This will result in STEM classes assigning even more work and non-STEM classes assigning less work. Since women are willing to study more, the increased STEM workload works as less of a deterrent to women taking STEM courses.
Conclusion

The lack of graduates in STEM majors—particularly among under-represented groups—has been of some policy concern. In the first paper, I find that STEM majors have a lower average match value than non-STEM, but a higher variance in matches. Thus, students are less certain about how well they might match with STEM. Students with higher math ability are more likely to choose STEM in the first period, but the sorting by ability greatly increases in the later period. It is costly to switch into STEM from non-STEM in the second period, while the reverse move is virtually costless. Females are more likely to choose STEM initially, but they have much lower switching costs in the latter period. Overall, they switch more often than males, but they also switch out of STEM at a higher rate than the reverse. All of these results support the theoretical result that students will choose the field with more uncertainty in the early periods (given similar expected match values) because of the option to switch later if they get a bad match. This is especially true when the more uncertain field is also more costly to switch into in later periods, as in the case of STEM.

In the second paper, we show that there is a potentially cheap way to change
the number and composition of STEM majors. Namely, grading policies have a substantial effect on sorting into STEM classes. We show that a substantial portion of the gender STEM gap can be removed by having STEM classes give grades that are on average similar to those in non-STEM classes.

These grading policies, however, are in part choices by professors. Hence administrative policies designed to change how professors grade will elicit responses by professors on other dimensions such as workload. These responses by professors may result in an even further closing of the STEM gender gap. Namely, if classes across departments are forced to give similar grades on average, then STEM (non-STEM) classes will employ alternative means to deter (encourage) enrollment in their courses by changing workloads. This will result in STEM classes assigning even more work and non-STEM classes assigning less work. Since women are willing to study more, the increased STEM workload works as less of a deterrent to women taking STEM courses.
Appendix A

Data: Campus Life and Learning

Table A.1: Sample Selection

<table>
<thead>
<tr>
<th>Selection Criterion</th>
<th>Students Remaining in Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLL Respondents</td>
<td>1,182</td>
</tr>
<tr>
<td>Major not available in all periods</td>
<td>872</td>
</tr>
<tr>
<td>SAT Math score not available</td>
<td>764</td>
</tr>
<tr>
<td>No Degree</td>
<td>756</td>
</tr>
<tr>
<td>Answered ”Don’t Know”</td>
<td>496</td>
</tr>
<tr>
<td>Major in Program II</td>
<td>491</td>
</tr>
<tr>
<td>Cannot Classify Major</td>
<td>487</td>
</tr>
<tr>
<td>Ever in Engineering</td>
<td>367</td>
</tr>
</tbody>
</table>

In my sample of students at Duke, 49% (240/487) of students had a double major in at least one period. 15% (72/487) of students had a double major in the first period, 32% (158/487) in the second period, and 38% (184/487) in the final period. If I exclude engineers, 43% of students ever had a double major. The percentages per period also fall slightly to 14%, 26%, and 31% in Periods 1, 2, and 3. This is a significant fraction of students, so here I will present my method of choosing a student’s primary major in each period. I followed a few main rules in selecting...
Table A.2: Rules for Selecting a Single Major & Number of Students Affected

<table>
<thead>
<tr>
<th>Rule</th>
<th>Number of students affected</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ever Engr = 0</td>
</tr>
<tr>
<td><strong>Rule: Favor Consistency</strong></td>
<td></td>
</tr>
<tr>
<td>Only one major stays constant in all periods</td>
<td>73</td>
</tr>
<tr>
<td>Only one major stays constant in last 2 periods</td>
<td>38</td>
</tr>
<tr>
<td><strong>Rule: Favor Earlier Switches</strong></td>
<td>9</td>
</tr>
<tr>
<td><strong>Rule: Favor Official Order</strong></td>
<td>19</td>
</tr>
<tr>
<td>Consistent double major across all periods</td>
<td>6</td>
</tr>
<tr>
<td>Consistent double major across last 2 periods</td>
<td>13</td>
</tr>
</tbody>
</table>

These are the number of students with a double major in each period (and thus affected by the rule).

Table A.3: Double Majors

<table>
<thead>
<tr>
<th></th>
<th>Ever Engr = 0</th>
<th>Ever Engr = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-college Only</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>Sophomore Year Only</td>
<td>19</td>
<td>5</td>
</tr>
<tr>
<td>Graduation Only</td>
<td>35</td>
<td>15</td>
</tr>
<tr>
<td>Pre-College and Sophomore Year</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Pre-College and Graduation</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>Sophomore Year and Graduation</td>
<td>52</td>
<td>41</td>
</tr>
<tr>
<td>All Periods</td>
<td>19</td>
<td>12</td>
</tr>
</tbody>
</table>

one major per period. The table below lists the rules and provides the number of students affected in turn.
Appendix B

Period 1 Value Function

\textbf{Period 1:} \ \max_{c_1} \mathbb{E}\left[ \left( U_1(c_1) + \beta \mathbb{E}[V_2(c_2) | I_1] \right) | I_0 \right]

\text{Stay}

\begin{align*}
P(S_2 | \gamma_{j(k)} > \mu_\gamma) &= 2 \int_{\gamma^*}^{\infty} f_\gamma(x) \left( 1 - F_{\theta_k}(\theta^* + \gamma^* - x) \right) dx \\
\mathbb{E}\left[ V_2(S) | c_2^* = S, c_1, I_0, \gamma_{j(k)} > \mu_\gamma \right] &= \frac{1 + \beta}{P(S_2 | \gamma_{j(k)} > \mu_\gamma)} \int_{\gamma^*}^{\infty} f_\gamma(x) \left( 1 - F_{\theta_k}(\theta^* + \gamma^* - x) \right) (\mu_{\theta_k} + x) \\
&\quad + \sigma_{\theta_k}^2 f_\gamma(x) f_{\theta_k}(\theta^* + \gamma^* - x) dx
\end{align*}
New Major

\[ P(\text{NM}_2|\gamma_{j(k)} > \mu_\gamma) = (2F_\gamma(\gamma^*) - 1)(1 - F_{\theta_k}(\theta^*)) \]

\[ P(\text{NM}_2|\gamma_{j(k)} < \mu_\gamma) = 1 - F_{\theta_k}(\theta^*) \]

\[ \mathbb{E}\left[V_2(\text{NM})\mid c_2^* = \text{NM}, c_1, I_0, \gamma_{j(k)} > \mu_\gamma\right] \]

\[ = \left(\mu_{\theta_k} + \sigma_{\theta_k} \frac{f_{\theta_k}(\theta^*)}{1 - F_{\theta_k}(\theta^*)} + \mu_\gamma\right) \left(1 + \beta\right) \]

\[ + \beta \left[ \int_{\mu_\gamma}^{\gamma^*} \frac{f_\gamma(x)}{F_\gamma(\gamma^*) - F_\gamma(\mu_\gamma)} \left[F_\gamma(x)(x - \mu_\gamma) + \sigma_\gamma^2 f_\gamma(x)\right] dx \right] \]

\[ \mathbb{E}\left[V_2(\text{NM})\mid c_2^* = \text{NM}, c_1, I_0, \gamma_{j(k)} < \mu_\gamma\right] \]

\[ = \left(\mu_{\theta_k} + \sigma_{\theta_k} \frac{f_{\theta_k}(\theta^*)}{1 - F_{\theta_k}(\theta^*)} + \mu_\gamma\right) \left(1 + \beta\right) + \beta \frac{\sigma_\gamma}{\sqrt{2\pi}} \]
New Field and Major

\[ P(NFM_2 | \gamma_{j(k)} > \mu_\gamma) = 2 \int_{\gamma^*}^{\infty} f_\gamma(x) F_{\theta_k}(\theta^* + \gamma^* - x) \, dx + (2 F_\gamma(\gamma^*) - 1) F_{\theta_k}(\theta^*) \]

\[ P(NFM_2 | \gamma_{j(k)} < \mu_\gamma) = F_{\theta_k}(\theta^*) \]

\[
\mathbb{E}\left[V_2(NFM) \mid c_2^* = NFM, c_1, I_0, \gamma_{j(k)} > \mu_\gamma \right] \\
= (\mu_{\theta_{k'}} + \mu_\gamma)(1 + \beta) + \beta \left[ \frac{\sigma_\gamma}{\sqrt{2\pi}} \right. \]

\[ + \frac{1}{2} \sigma_{\theta_{k'}}^2 \int_{-\infty}^{\theta^*} \int_{\mu_\gamma}^{\theta^* + \gamma^* - x} \frac{f_{\theta_k}(x) f_\gamma(y)}{P(NFM_2 | \gamma_{j(k)} > \mu_\gamma)} f_{\theta_{k'}}(x + y - \mu_\gamma) \, dy \, dx \]

\[ + \sigma_{\theta_{k'}}^2 \int_{-\infty}^{\theta^*} \int_{\mu_\gamma}^{\theta^* + \gamma^* - x} \int_{\mu_\gamma}^{\infty} \frac{f_{\theta_k}(x) f_\gamma(y)}{P(NFM_2 | \gamma_{j(k)} > \mu_\gamma)} f_\gamma(z) f_{\theta_{k'}}(x + y - z) \, dz \, dy \, dx \]

\[ + \frac{1}{2} \int_{-\infty}^{\theta^*} \int_{\mu_\gamma}^{\theta^* + \gamma^* - x} \frac{f_{\theta_k}(x) f_\gamma(y)}{P(NFM_2 | \gamma_{j(k)} > \mu_\gamma)} F_{\theta_{k'}}(x + y - \mu_\gamma)(x + y - (\mu_{\theta_{k'}} + \mu_\gamma)) \, dy \, dx \]

\[ + \int_{-\infty}^{\theta^*} \int_{\mu_\gamma}^{\theta^* + \gamma^* - x} \int_{\mu_\gamma}^{\infty} \frac{f_{\theta_k}(x) f_\gamma(y)}{P(NFM_2 | \gamma_{j(k)} > \mu_\gamma)} f_\gamma(z) F_{\theta_{k'}}(x + y - z)(x + y - (\mu_{\theta_{k'}} + \mu_\gamma)) \, dz \, dy \, dx \]

\[
\mathbb{E}\left[V_2(NFM) \mid c_2^* = NFM, c_1, I_0, \gamma_{j(k)} < \mu_\gamma \right] \\
= (\mu_{\theta_{k'}} + \mu_\gamma)(1 + \beta) + \beta \left[ \frac{\sigma_\gamma}{\sqrt{2\pi}} + \frac{1}{2} \sigma_{\theta_{k'}}^2 \int_{-\infty}^{\theta^*} \frac{f_{\theta_k}(x)}{F_{\theta_k}(\theta^*)} f_{\theta_{k'}}(x) \, dx \right. \]

\[ + \sigma_{\theta_{k'}}^2 \int_{-\infty}^{\theta^*} \frac{f_{\theta_k}(x)}{F_{\theta_k}(\theta^*)} f_\gamma(y) f_{\theta_{k'}}(x + \mu_\gamma - y) \, dy \, dx \]

\[ + \frac{1}{2} \int_{-\infty}^{\theta^*} \frac{f_{\theta_k}(x)}{F_{\theta_k}(\theta^*)} F_{\theta_{k'}}(x)(x - \mu_{\theta_{k'}}) \, dx \]

\[ + \int_{-\infty}^{\theta^*} \int_{\mu_\gamma}^{\theta^* + \gamma^* - x} \int_{\mu_\gamma}^{\infty} \frac{f_{\theta_k}(x)}{F_{\theta_k}(\theta^*)} f_\gamma(y) F_{\theta_{k'}}(x + \mu_\gamma - y)(x + \mu_\gamma - (\mu_{\theta_{k'}} + \mu_\gamma)) \, dy \, dx \]
Period 1 Value Function:

\[ V_1(c_1)|I_0 = \max_{c_1} \mathbb{E}\left[\left(U_1(c_1) + \beta \mathbb{E}[V_2(c^*_2)|I_1]\right)|I_0\right] \]

\[ \mathbb{E}\left[\mathbb{E}[V_2(c^*_2)|I_1]|I_0\right] \]

\[ = \left\{ P(\gamma_{j(k)} > \mu_\gamma) \left\{ P\left(c^*_2 = S|c_1, I_0, \gamma_{j(k)} > \mu_\gamma \right) \mathbb{E}\left[V_2(S)|c^*_2 = S, c_1, I_0, \gamma_{j(k)} > \mu_\gamma \right] \right. \right. \]

\[ + P\left(c^*_2 = NM|c_1, I_0, \gamma_{j(k)} > \mu_\gamma \right) \mathbb{E}\left[V_2(NM)|c^*_2 = NM, c_1, I_0, \gamma_{j(k)} > \mu_\gamma \right] \]

\[ + P\left(c^*_2 = NFM|c_1, I_0, \gamma_{j(k)} > \mu_\gamma \right) \mathbb{E}\left[V_2(NFM)|c^*_2 = NFM, c_1, I_0, \gamma_{j(k)} > \mu_\gamma \right] \}

\[ \left\} \{ P(\gamma_{j(k)} > \mu_\gamma) \left\{ P\left(c^*_2 = S|c_1, I_0, \gamma_{j(k)} < \mu_\gamma \right) \mathbb{E}\left[V_2(S)|c^*_2 = S, c_1, I_0, \gamma_{j(k)} < \mu_\gamma \right] \right. \right. \]

\[ + P\left(c^*_2 = NM|c_1, I_0, \gamma_{j(k)} < \mu_\gamma \right) \mathbb{E}\left[V_2(NM)|c^*_2 = NM, c_1, I_0, \gamma_{j(k)} < \mu_\gamma \right] \]

\[ + P\left(c^*_2 = NFM|c_1, I_0, \gamma_{j(k)} < \mu_\gamma \right) \mathbb{E}\left[V_2(NFM)|c^*_2 = NFM, c_1, I_0, \gamma_{j(k)} < \mu_\gamma \right] \} \} \]
\[
\frac{1}{2} \left\{ (1 + \beta) \int_{\gamma^*}^{\infty} f_\gamma(x)(1 - F_{\theta_k}(\theta^* + \gamma^* - x)) (\mu_{\theta_k} + x) + \sigma_{\theta_k}^2 f_\gamma(x) f_{\theta_k}(\theta^* + \gamma^* - x) \, dx \right.
\]
\[
+ (2 F_\gamma(\gamma^*) - 1)(1 - F_{\theta_k}(\theta^*)) \left[ (\mu_{\theta_k} + \sigma_{\theta_k} f_{\theta_k}(\theta^*) + \mu_\gamma) (1 + \beta) \right] \\
+ \beta \left[ \int_{\mu_\gamma}^{\gamma^*} \frac{f_\gamma(x)}{F_\gamma(\gamma^*) - F_\gamma(\mu_\gamma)} [F_\gamma(x)(x - \mu_\gamma) + \sigma_\gamma^2 f_\gamma(x)] \, dx \right] \\
\left. + \left[ 2 \int_{\gamma^*}^{\infty} f_\gamma(x) F_{\theta_k}(\theta^* + \gamma^* - x) \, dx + (2 F_\gamma(\gamma^*) - 1) F_{\theta_k}(\theta^*) \right] \times \left[ (\mu_{\theta_k} + \mu_\gamma)(1 + \beta) + \beta \left[ \frac{\sigma_\gamma}{\sqrt{2\pi}} \right] \right] \right\}
\]
\[
+ \frac{1}{2} \sigma_{\theta_k}^2 \int_{-\infty}^{\theta^*} \int_{\mu_\gamma}^{\theta^* + \gamma^* - x} \frac{f_{\theta_k}(x) f_\gamma(y)}{P(\text{NFM}_2 | \gamma_j(k) > \mu_\gamma)} f_{\theta_k}(x + y - \mu_\gamma) \, dy \, dx \\
+ \sigma_{\theta_k}^2 \int_{-\infty}^{\theta^*} \int_{\mu_\gamma}^{\theta^* + \gamma^* - x} \int_{\mu_\gamma}^{\infty} \frac{f_{\theta_k}(x) f_\gamma(y)}{P(\text{NFM}_2 | \gamma_j(k) > \mu_\gamma)} f_\gamma(z) f_{\theta_k}(x + y - z) \, dz \, dy \, dx \\
+ \frac{1}{2} \left\{ 0 + \left[ 1 - F_{\theta_k}(\theta^*) \right] \left[ (\mu_{\theta_k} + \sigma_{\theta_k} f_{\theta_k}(\theta^*) + \mu_\gamma)(1 + \beta) + \beta \left[ \frac{\sigma_\gamma}{\sqrt{2\pi}} \right] \right] \right\} \\
+ F_{\theta_k}(\theta^*) \left[ (\mu_{\theta_k} + \mu_\gamma)(1 + \beta) + \beta \left[ \frac{\sigma_\gamma}{\sqrt{2\pi}} \right] + \frac{1}{2} \sigma_{\theta_k}^2 \int_{-\infty}^{\theta^*} \frac{f_{\theta_k}(x)}{F_{\theta_k}(\theta^*)} f_{\theta_k}(x) \, dx \\
+ \sigma_{\theta_k}^2 \int_{-\infty}^{\theta^*} \int_{\mu_\gamma}^{\infty} \frac{f_{\theta_k}(x)}{F_{\theta_k}(\theta^*)} f_\gamma(y) f_{\theta_k}(x + \mu_\gamma - y) \, dy \, dx \\
+ \frac{1}{2} \int_{-\infty}^{\theta^*} \frac{f_{\theta_k}(x)}{F_{\theta_k}(\theta^*)} F_{\theta_k}(x) (x - \mu_{\theta_k}) \, dx \\
\left. + \int_{-\infty}^{\theta^*} \int_{\mu_\gamma}^{\infty} \frac{f_{\theta_k}(x)}{F_{\theta_k}(\theta^*)} f_\gamma(y) F_{\theta_k}(x + \mu_\gamma - y)(x + \mu_\gamma - (\mu_{\theta_k} + y)) \, dy \, dx \right\} \right\}
\]
Appendix C

Two-Period Model

Imperfect Information Model: Two-Period Version

Here, I write out the student’s problem and solution if there are only two periods. I also allow for switching costs in the second period that can differ according to whether it’s a within-field or across-field switch as well as the destination field. In all the notation, I label the student’s initial choice of field as $k$ and their initial choice of major as $j(k)$. So $d_1 = (k, j(k))$ without loss of generality. The Period 2 choices are represented in the following way:

$$d_2 = \begin{cases} 
(k, j(k)) & \text{if Stay} \\
(k, j'(k)) & \text{if New Major} \\
(k', j(k')) & \text{if New Field and Major}
\end{cases}$$

I use the following notation for the switching costs:

$c_{jj'}$ if New Major

$c_{kk'}$ if New Field and Major
\[ \mu_{\theta_{k'}} + \mu_{\gamma} - c_{jj'} - (c_{kk'} - c_{jj'}) \]

\( \left( = \mu_{\theta_{k'}} + \mu_{\gamma} - c_{kk'} \right) \)

**Figure C.1: Period 2 Solution**

**Period 2**

**Student’s Problem**

\[ \max(\theta_{ik} + \gamma_{ij(k)}, \theta_{ik} + \mu_{\gamma} - c_{jj'}, \mu_{\theta_{k'}} + \mu_{\gamma} - c_{kk'}) \]

**Solution**

- **Stay** if \( \gamma_{ij(k)} > \mu_{\gamma} - c_{jj'} \) and \( \theta_{k} + \gamma_{ij(k)} > \mu_{\theta_{k'}} + \mu_{\gamma} - c_{kk'} \)
- **NM** if \( \gamma_{ij(k)} < \mu_{\gamma} - c_{jj'} \) and \( \theta_{k} - c_{jj'} > \mu_{\theta_{k'}} - c_{kk'} \)
- **NFM** if \( \theta_{k} - c_{jj'} < \mu_{\theta_{k'}} - c_{kk'} \) and \( \theta_{k} + \gamma_{ij(k)} < \mu_{\theta_{k'}} + \mu_{\gamma} - c_{kk'} \)

**Period 1**

**Student’s Problem**
\[
\begin{align*}
\max & \ E(U_1|I_0) + \beta E(U_2|d_1, I_0) \\
& = \max \mu_{\theta_k} + \mu_\gamma + \beta \sum_{d_2} P(d_2|d_1)E(U_2|d_2, d_1, I_0) \\
& = \max \mu_{\theta_k} + \mu_\gamma + \beta \left[ P(S_2|d_1, I_0)E(U_2|S_2, d_1, I_0) \\
& \quad + P(NM_2|d_1, I_0)E(U_2|NM_2, d_1, I_0) + P(NFM_2|d_1, I_0)E(U_2|NFM_2, d_1, I_0) \right]
\end{align*}
\]

The probability and expected value functions are expressed as follows.

\[
P(\text{Stay}_2|I_0) = P(\gamma_{j(k)} > \mu_\gamma - c_{jj'}, \theta_k + \gamma_{j(k)} > \mu_{\theta_{k'}} + \mu_\gamma - c_{kk'}) \\
= P(\gamma_{j(k)} > \mu_\gamma - c_{jj'})P(\theta_k + \gamma_{j(k)} > \mu_{\theta_{k'}} + \mu_\gamma - c_{kk'}|\gamma_{j(k)} > \mu_\gamma - c_{jj'}) \\
= (1 - F_\gamma(\mu_\gamma - c_{jj'})) \int_{\mu_\gamma - c_{jj'}}^{\infty} \frac{f_\gamma(x)}{1 - F_\gamma(\mu_\gamma - c_{jj'})} \left(1 - F_{\theta_k}(\mu_{\theta_{k'}} + \mu_\gamma - x - c_{kk'}) \right) dx \\
= \int_{\mu_\gamma - c_{jj'}}^{\infty} f_\gamma(x) \left(1 - F_{\theta_k}(\mu_{\theta_{k'}} + \mu_\gamma - x - c_{kk'}) \right) dx \\
= \int_{\mu_\gamma - c_{jj'}}^{\infty} \frac{1}{\sigma_\gamma} \Phi \left( \frac{x - \mu_\gamma}{\sigma_\gamma} \right) \left(1 - \Phi \left( \frac{\mu_{\theta_{k'}} + \mu_\gamma - x - c_{kk'} - \mu_{\theta_k}}{\sigma_{\theta_k}} \right) \right) dx
\]

\[
P(NM_2|I_0) = P(\gamma_{j(k)} < \mu_\gamma - c_{jj'}, \theta_k - c_{jj'} > \mu_{\theta_{k'}} - c_{kk'}) \\
= P(\gamma_{j(k)} < \mu_\gamma - c_{jj'})P(\theta_k > \mu_{\theta_{k'}} - (c_{kk'} - c_{jj'})) \\
= F_\gamma(\mu_\gamma - c_{jj'}) \left(1 - F_{\theta_k}(\mu_{\theta_{k'}} - (c_{kk'} - c_{jj'})) \right) \\
= \Phi \left( \frac{-c_{jj'}}{\sigma_\gamma} \right) \left(1 - \Phi \left( \frac{\mu_{\theta_{k'}} - (c_{kk'} - c_{jj'}) - \mu_{\theta_k}}{\sigma_{\theta_k}} \right) \right)
\]
\[ P(\text{NFM}_2|I_0) = P(\theta_k - c_{jj'} < \mu_{\theta_{k'}} - c_{kk'}, \theta_k + \gamma_{j(k)} < \mu_{\theta_{k'}} + \mu_\gamma - c_{kk'}) \]
\[ = P(\gamma_{j(k)} < \mu_\gamma - c_{jj'})P(\theta_k < \mu_{\theta_{k'}} - (c_{kk'} - c_{jj'})) \]
\[ + P(\gamma_{j(k)} > \mu_\gamma - c_{jj'})P(\theta_k + \gamma_{j(k)} < \mu_{\theta_{k'}} + \mu_\gamma - c_{kk'}|\gamma_{j(k)} > \mu_\gamma - c_{jj'}) \]
\[ = F_\gamma(\mu_\gamma - c_{jj'})F_{\theta_k}(\mu_{\theta_{k'}} - (c_{kk'} - c_{jj'})) \]
\[ + \int_{\mu_\gamma - c_{jj'}}^{\infty} f_\gamma(x)F_{\theta_k}(\mu_{\theta_{k'}} + \mu_\gamma - x - c_{kk'})dx \]
\[ = \Phi\left(\frac{-c_{jj'}}{\sigma_\gamma}\right) \Phi\left(\frac{\mu_{\theta_{k'}} - (c_{kk'} - c_{jj'}) - \mu_\theta}{\sigma_{\theta_k}}\right) \]
\[ + \int_{\mu_\gamma - c_{jj'}}^{\infty} \frac{1}{\sigma_\gamma} \phi\left(\frac{x - \mu_\gamma}{\sigma_\gamma}\right) \Phi\left(\frac{\mu_{\theta_{k'}} + \mu_\gamma - x - c_{kk'} - \mu_\theta}{\sigma_{\theta_k}}\right) dx \]
\[ \mathbb{E}(U_2|S_2, I_0) = \mathbb{E}(\theta_k + \gamma_{j(k)}|\gamma_{j(k)} > \mu_\gamma - c_{jj'}, \theta_k + \gamma_{j(k)} > \mu_{\theta_k'} + \mu_\gamma - c_{kk'}) \]

\[ = \int_{\mu_\gamma - c_{jj'}}^{\infty} f_\gamma(x|x > \mu_\gamma - c_{jj'}, \theta_k + x > \mu_{\theta_k'} + \mu_\gamma - c_{kk'}) \times \mathbb{E}(\theta_k|\theta_k > \mu_{\theta_k'} + \mu_\gamma - c_{kk'} - x) + x \, dx \]

\[ = \int_{\mu_\gamma - c_{jj'}}^{\infty} \frac{f_\gamma(x)(1 - F_{\theta_k}(\mu_{\theta_k'} + \mu_\gamma - c_{kk'} - x))}{\mathbb{P}(S_2)} \times \left[ \mu_{\theta_k} + \sigma_{\theta_k}^2 \frac{f_{\theta_k}(\mu_{\theta_k'} + \mu_\gamma - c_{kk'} - x)}{1 - F_{\theta_k}(\mu_{\theta_k'} + \mu_\gamma - c_{kk'} - x)} + x \right] \, dx \]

\[ = \int_{\mu_\gamma - c_{jj'}}^{\infty} \frac{1}{\sigma_\gamma} \Phi \left( \frac{x - \mu_\gamma}{\sigma_\gamma} \right) \left( 1 - \Phi \left( \frac{\mu_{\theta_k'} + \mu_\gamma - c_{kk'} - x - \mu_{\theta_k}}{\sigma_{\theta_k}} \right) \right) \frac{1}{\mathbb{P}(S_2)} \times \left[ \mu_{\theta_k} + \sigma_{\theta_k}^2 \frac{1}{\sigma_{\theta_k}} \Phi \left( \frac{\mu_{\theta_k'} + \mu_\gamma - c_{kk'} - x - \mu_{\theta_k}}{\sigma_{\theta_k}} \right) + x \right] \, dx \]

\[ \mathbb{E}(U_2|\text{NM}_2, I_0) = \mathbb{E}(\theta_k + \mu_\gamma|\theta_k > \mu_{\theta_k'} - (c_{kk'} - c_{jj'})) \]

\[ = \mu_{\theta_k} + \sigma_{\theta_k}^2 \frac{f_{\theta_k}(\mu_{\theta_k'} - (c_{kk'} - c_{jj'}))}{1 - F_{\theta_k}(\mu_{\theta_k'} - (c_{kk'} - c_{jj'}))} + \mu_\gamma - c_{jj'} \]

\[ = \mu_{\theta_k} + \sigma_{\theta_k}^2 \frac{1}{\sigma_{\theta_k}} \Phi \left( \frac{\mu_{\theta_k'} - (c_{kk'} - c_{jj'}) - \mu_{\theta_k}}{\sigma_{\theta_k}} \right) + \mu_\gamma - c_{jj'} \]

\[ \mathbb{E}(U_2|\text{NFM}_2, I_0) = \mu_{\theta_k'} + \mu_\gamma - c_{kk'} \]
Appendix D

University of Kentucky Data Details
<table>
<thead>
<tr>
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<th>STEM</th>
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<tbody>
<tr>
<td>Aerospace Studies</td>
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</tr>
<tr>
<td>Agr Economics</td>
<td>Linguistics</td>
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<tr>
<td>Agricultural Biotechnology</td>
<td>Merchand, Apparel &amp; Textile</td>
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<tr>
<td>Agricultural Education</td>
<td>Mil Sci and Leadership</td>
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<td>Modern &amp; Classical Lang</td>
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<td>Allied Health Ed &amp; Rsrch</td>
<td>Nursing</td>
</tr>
<tr>
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<td>Philosophy</td>
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<tr>
<td>Anthropology</td>
<td>Plant Pathology</td>
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<tr>
<td>Appalachian Studies</td>
<td>Plant and Soil Sciences</td>
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<td>Arts Administration</td>
<td>Political Science</td>
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<td>Accountancy</td>
<td>Psychology</td>
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<tr>
<td>Economics</td>
<td>Public Health</td>
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<tr>
<td>Biosystems &amp; Agr Engineering</td>
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<tr>
<td>Business and Economics</td>
<td>Schl Of Journalism &amp; Telecom</td>
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<td>Schl of Art and Visual Studies</td>
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<td>Communication Disorders</td>
<td>Schl of Human Env Sci</td>
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<td>Communication &amp; Info Studies</td>
<td>Schl of Interior Design</td>
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<td>Community &amp; Leadership Dev</td>
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<td>Dept of Management</td>
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</tr>
<tr>
<td>Dept of Gerontology</td>
<td>Sustainable Agriculture</td>
</tr>
<tr>
<td>Dept of Fin &amp; Quant Methods</td>
<td>Latin American Studies</td>
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<table>
<thead>
<tr>
<th>Biological</th>
<th>Chemical &amp; Materials Engineering</th>
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<td>Biology</td>
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<td>Civil Engineering</td>
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<td>Computer Science</td>
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<tr>
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<td>Physics and Astronomy</td>
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<td>School of Architecture</td>
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Table D.2: Aggregation of Departments

<table>
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<tr>
<th>Categories</th>
<th>Departments</th>
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<tbody>
<tr>
<td><strong>Agriculture</strong></td>
<td>Agricultural Biotechnology, Agricultural Economics, Agricultural Ed, Agriculture General, Animal &amp; Food Sciences, Biosystems &amp; Agr Engineering, Environmental Studies, Forestry, Landscape Architecture, Plant Pathology, Plant &amp; Soil Sciences, Sustainable Agriculture</td>
</tr>
<tr>
<td><strong>Regional Studies</strong></td>
<td>Appalachian Studies, Family Sciences, Gender &amp; Women’s Studies, Hispanic Studies, Latin American Studies</td>
</tr>
<tr>
<td><strong>Communication</strong></td>
<td>Arts Admin, Communication, Communication &amp; Info Studies, Fine Arts - Music, Fine Arts - Theatre Arts, Schl Of Journalism &amp; Telecomm, Schl of Art &amp; Visual Studies, Schl of Interior Design</td>
</tr>
<tr>
<td><strong>Engineering</strong></td>
<td>Chemical &amp; Materials Engineering, Civil Engineering, Computer Science, Electrical &amp; Computer Engineering, Engineering, Mechanical Engineering, Mining Engineering, Schl of Architecture</td>
</tr>
<tr>
<td><strong>Languages</strong></td>
<td>Linguistics, Modern &amp; Classical Languages, Philosophy</td>
</tr>
<tr>
<td><strong>English</strong></td>
<td>English</td>
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<tr>
<td><strong>Biology</strong></td>
<td>Biology, Entomology</td>
</tr>
<tr>
<td><strong>Mathematics</strong></td>
<td>Mathematics, Statistics</td>
</tr>
<tr>
<td><strong>Chem &amp; Physics</strong></td>
<td>Chemistry, Earth &amp; Environmental Sciences, Physics &amp; Astronomy</td>
</tr>
<tr>
<td><strong>Psychology</strong></td>
<td>Psychology</td>
</tr>
<tr>
<td><strong>Social Sciences</strong></td>
<td>Anthropology, Geography, History, Political Science, Schl of Human Environmental Sciences, Sociology</td>
</tr>
<tr>
<td><strong>Mgmt. &amp; Mkting.</strong></td>
<td>Aerospace Studies, Department of Mgmt, Dept of Mkt &amp; Supply Chain, Merchand, Apparel &amp; Textiles, Mil Sci &amp; Leadership</td>
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<td>Accountancy, Economics, Dept of Finance &amp; Quantitative Methods</td>
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Table D.3: Students with and without ACT scores

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</tr>
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Observations | 17,664 | 2,540 |
Bibliography

(2012), Report to the President: Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics, President’s Council of Advisors on Science & Technology.


Biography

Amy Kathleen Hopson was born on August 22, 1985, in Nacogdoches, TX. She graduated *magna cum laude* from Texas A&M University in 2007 with a B.S. in Economics and Applied Mathematical Sciences, earning Honors in Mathematics. After graduating, she worked for a year as a Research Associate at Private Enterprise Research Center at Texas A&M.

She started graduate school at Duke University in Fall 2008 and earned an MA in Economics from Duke in 2009. She voluntarily left the PhD program in Spring 2011, and worked as an Economist at Cardno ENTRIX from August 2011-July 2012. She returned to the PhD program at Duke in Fall 2012 and will earn a PhD in Economics in 2017.