The Method of Constructal Design in Heat Exchangers

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Mechanical Engineering and Materials Science in the Graduate School of Duke University

2018
ABSTRACT

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Abstract

This dissertation documents the usefulness of applying the Constructal design methodology of heat exchangers to the thermal and flow performance of such architectures. Based on the method of Constructal design, the system performance can be enhanced by providing its architecture or configuration more freedom to morph. Throughout this dissertation, each chapter deals with different design aspects that are relevant to heat exchangers.

Chapter 2 documents the joint performance of heat pumps that are served by a common loop (heat exchanger) buried in the ground, and which operate simultaneously: one heat pump absorbs heat from the buried loop whereas the other one rejects heat. The findings show that the flow and thermal performance are affected by the way in which the two heat pumps are connected to the buried loop.

Chapter 3 illustrates the free search for the optimal geometry of flow channel cross-sections that meet two objectives simultaneously: reduced resistances for heat transfer and fluid flow. Two elemental cross-sections are considered, squared and equilateral triangular. The results show that the two objectives are best met when the solid wall thickness is uniform, i.e., when the wetted perimeters are square and triangular, respectively. Arrays of square elements and triangular elements are also considered, and the results show that the array of triangular elements meets the two objectives better than the array of square elements.
In chapter 4, the relation between the efficiency of a power plant and its size is determined by relying on thermodynamics. The results show that the power plant efficiency should increase nonlinearly with the size, with a concave profile, i.e. with diminishing returns.

Chapter 5 shows numerically how the freedom in morphing a one stream cooling system embedded in a squared plate influence the thermal performance. Several configurations are considered in order to establish the evolutionary direction in which the thermal performance of the cooling system is improving. Examples are serpentine shapes, with various number of meanders, and loops cooling configurations (square, circular and clover leaf). The minimum peak temperature within the heated plate is achieved with the clover leaf designs.

To summarize, a primary feature of this dissertation is the ability to predict how the heat exchanger performance changes with morphing its flow configuration, varying the flow cross-sectional shape, and scaling up and scaling down the size of the flow design.
Dedicated to

Dear parents (Salman & Fawzia)

and

Beloved wife (Sara)
## Contents

Abstract.............................................................................................................................. iv

List of Tables ..................................................................................................................... ix

List of Figures ..................................................................................................................... x

Nomenclature................................................................................................................... xiii

Acknowledgements........................................................................................................ xviii

1. Introduction..................................................................................................................... 1

2. Energy design for dense neighborhoods ......................................................................... 4
   2.1 Model ...................................................................................................................... 5
   2.2 Thermal performance analysis .............................................................................. 13
   2.3 Pumping power ..................................................................................................... 24
   2.4 Discussion ............................................................................................................. 28

3. Arrays of flow channels with heat transfer embedded in conducting walls ................. 30
   3.1 Square element ...................................................................................................... 30
      3.1.1 Flow resistance ................................................................................................. 33
      3.1.2 Thermal resistance .......................................................................................... 37
   3.2 Triangular element ................................................................................................ 45
      3.2.1 Flow resistance ................................................................................................ 45
      3.2.2 Thermal resistance ......................................................................................... 48
   3.3 Square array or triangular array? .......................................................................... 50
   3.4 Discussion ............................................................................................................. 54
4. Economies of Scale............................................................................................................... 55
  4.1 Allocation of size ........................................................................................................... 60
  4.2 Allocation of irreversibility .......................................................................................... 65
  4.3 Allocation of fluid flow volume ................................................................................... 66
  4.4 Discussion .................................................................................................................... 73
5. Evolutionary design of cooling a plate with one stream ..................................................... 76
  5.1 Serpentines .................................................................................................................. 77
  5.2 Numerical method ........................................................................................................ 79
  5.3 The effect of serpentine shape .................................................................................... 80
  5.4 Loops ............................................................................................................................. 87
  5.5 Discussion .................................................................................................................... 94
6. Conclusions and Recommendations ................................................................................. 95
References ............................................................................................................................ 98
Biography ............................................................................................................................ 105
List of Tables

Table 2.1: The coefficients of Equations (2.19) and (2.20). .................................................. 23
Table 3.1: Mesh accuracy test................................................................................................. 41
Table 4.1: Calculated flow volume ratio for various power plants................................. 72
Table 5.1: Grid independent test.......................................................................................... 80
Table 5.2: Optimal designs summary. .................................................................................. 93
List of Figures

Figure 2.1: Two heat pumps on the same loop (HT, LT), one rejecting heat and the other absorbing heat: mixing occurs downstream of each heat pump. ........................................ 7

Figure 2.2: Two heat pumps on the same loop: mixing downstream of HT, and upstream of LT. .......................................................................................................................... 7

Figure 2.3: Two heat pumps on the same loop: mixing upstream of both HT and LT.... 9

Figure 2.4: Two heat pumps on the same loop: mixing upstream of HT and downstream of LT. .......................................................................................................................... 9

Figure 2.5: Slender loop for connecting the two heat pumps in a shorter trench. ........ 12

Figure 2.6: Why the variable $\varepsilon = e^{-N}$ increases monotonically in time, and marks the direction of time................................................................. 12

Figure 2.7: Results for the configuration designed in Fig. 2.1 ................................... 16

Figure 2.8: Results for the configuration designed in Fig. 2.2 ................................... 18

Figure 2.9: Results for the configuration designed in Fig. 2.3 ................................... 19

Figure 2.10: Results for the configuration designed in Fig. 2.4 ............................... 21

Figure 2.11: The effect of the total pumping power on the rate of heat rejection from the HT pump. .......................................................................................................................... 26

Figure 2.12: The effect of the total pumping power on the rate of heat absorption at the LT pump......................................................................................................................... 27

Figure 3.1: Three designs of flow channels embedded in a square cross-section. .... 32

Figure 3.2: The variation of fluid flow resistance for different flow channels, represented by area factor $\phi$, of square arrays, (a) square and (b) circular flow channel.................. 36

Figure 3.3: Velocity and temperature profiles of a flow channel embedded in square solid................................................................................................................................. 38

Figure 3.4: The mean temperature of the fluid stream which flows through hot contacting walls. .................................................................................................................. 40
Figure 3.5: The variation of overall thermal resistance with area factor $\phi$, (a) square flow channel and (b) circular flow channel................................................................. 43

Figure 3.6: The overall thermal resistance versus the fluid flow resistance of (a) square flow channels, (b) circle flow channels. ................................................................. 44

Figure 3.7: Flow channels embedded in a triangular cross section. ........................................ 45

Figure 3.8: The flow resistance variation for triangular arrays with the area factor $\phi$, (a) triangle flow channel and (b) circular flow channel. .................................................. 47

Figure 3.9: For triangular cross-section; the overall thermal resistance versus area factor $\phi$. ................................................................................................................. 49

Figure 3.10: The results of fluid flow resistance and thermal resistance for the triangular and circular flow channels embedded in triangle cross-section......................... 49

Figure 3.11: Square and triangle arrays with the same flow length and mass flow rate per unit of cross-section area. ..................................................................................... 51

Figure 4.1: The power output effect on the efficiency of several kinds of power plants [57]. .......................................................................................................................... 57

Figure 4.2: The effect of size on turbine efficiency [58]. ......................................................... 58

Figure 4.3: The efficiency loss decreases as the condenser area increases [59]. ................. 58

Figure 4.4: The efficiency of ocean thermal power plants versus the base power [60]. .. 59

Figure 4.5: The correlation between helicopter engine efficiency and engine size [61]. .. 59

Figure 4.6: (a) Power plant model as a closed system and (b) power plant model with two temperature gaps and a middle portion that operates reversibly.......................... 63

Figure 4.7: The effect of the size of total heat transfer surface (C) on the efficiency of the power plant........................................................................................................... 64

Figure 4.8: (a) Power plant as closed system operating irreversibly; (b) model with fluid flow irreversibilities at the hot end and cold end; and (c) the scales of the duct model. .. 68

Figure 5.1: Solid plate with heating from above, and with embedded serpentine cooling ($N = 3$ and $L_c = 3L$). ............................................................................................................ 77
Figure 5.2: (a) How the serpentine shape changes, and (b) the corresponding temperature fields as the number of elbows N changes while $L_c = 3L$.................................................. 82

Figure 5.3: The dimensionless temperature difference $\tilde{T}$ as function of N and $L_c$ for the serpentine design .............................................................................................................. 83

Figure 5.4: (a) Three cases showing how the position (x) of the inlet and the outlet changes, and (b) the corresponding temperature fields. ................................................................. 85

Figure 5.5: The effect of the inlet and outlet position on the dimensionless temperature $\tilde{T}$ ........................................................................................................................................ 86

Figure 5.6: How the cooling stream changes for the squared and circular loops. .............. 87

Figure 5.7: Temperature fields of the heat generating plate for the square and circular loops with various lengths. ........................................................................................................ 88

Figure 5.8: The clover leaf designs as cooling systems; (a) thinner, and (b) wider lobes. 89

Figure 5.9: Temperature fields of the heat generating plate for the two clover designs with various loop lengths. ........................................................................................................ 90

Figure 5.10: The dimensionless temperature difference $\tilde{T}$ as function of $L_c/L$ for the squared, circular and clover leaf loops. ................................................................. 92
Nomenclature

A  contact surface, m²

Aₙ  cross section of duct, m²

Aₙ,ₙ cross sectional area, m²

c  specific heat of solid, J kg⁻¹ K⁻¹

C  total heat transfer conductance inventory, W K⁻¹

Cₕ  heat transfer conductance of high temperature gap, W K⁻¹

Cₙ  heat transfer conductance of low temperature gap, W K⁻¹

ç  specific heat of the fluid at constant pressure, J kg⁻¹ K⁻¹

cₚ  factor, Eq. (2.28)

D  diameter of the buried pipe, m

Dₙ  stream diameter, m

Dₙ  hydraulic diameter, m

f  friction factor for the flow in pipe

h  conduction equivalent heat transfer coefficient, W m⁻² K⁻¹

h  effective heat transfer coefficient, W m⁻² K⁻¹

kₙ  solid thermal conductivity, W m⁻¹ K⁻¹

kₙ  fluid thermal conductivity, W m⁻¹ K⁻¹

kₙ  thermal conductivity ratio, kₙ = kₙ/kₙ

K  flow expression, Eq. (4.25) and Eq. (4.26), W m⁹

L  length of duct, m
L  side length of heated body, Fig. 5.1, m
L_c  distance between centers, Fig. 3.1c, m
L_c  stream length, Fig. 5.1, m
L_{s,t}  side length, m
L_x  length of channel, m
m  mass flow rate, kg s\(^{-1}\)
m_0  background mass flow rate, kg s\(^{-1}\)
m_a  total mass flow rate through the T_a pipe, kg s\(^{-1}\)
m_b  total mass flow rate through the T_b pipe, kg s\(^{-1}\)
m_H  mass flow rate of HT pump, kg s\(^{-1}\)
m_L  mass flow rate of LT pump, kg s\(^{-1}\)
m''  mass flow rate per unit area of array cross-section, kg s\(^{-1}\) m\(^{-2}\)
N  number of heat transfer units
N  number of meanders, Fig. 5.2
Nu  Nusselt number
p  witted perimeter of duct, m
P  pressure, Pa
Po  Poiseuille constant
Pr  Prandtl number
q_{HT}  heat rejected by HT pump, W
\( q_L \) heat absorbed by LT pump, W

\( q'' \) heat flux, W m\(^{-2}\)

\( Q \) heat generation, W

\( \dot{Q}_H \) heat input to the power plant, W

\( \dot{Q}_L \) heat rejected by the power plant, W

\( r \) mass flow rate ratio, \( \frac{\dot{m}_L}{\dot{m}_H} \)

\( r_H \) mass flow rate ratio, \( \frac{\dot{m}_0}{\dot{m}_H} \)

\( r_L \) mass flow rate ratio, \( \frac{\dot{m}_0}{\dot{m}_L} \)

\( R \) radius of rounded corner, Fig. 3.1c, m

\( Re \) Reynolds number

\( R_f \) dimensionless fluid flow resistance

\( R_{th} \) dimensionless thermal resistance

\( \dot{S}_{gen,H} \) entropy generation rate in high temperature gap, W K\(^{-1}\)

\( \dot{S}_{gen,L} \) entropy generation rate in low temperature gap, W K\(^{-1}\)

\( \tilde{T} \) dimensionless temperature difference, Eq. (5.2)

\( T_c \) coolant temperature, K

\( T_H \) high temperature, K

\( T_{H,\text{end}} \) temperature of hot reservoir, Fig. 4.6, K

\( T_{H,in} \) fluid temperature at the end of \( T_a \) pipe (Fig. 2.2 and 2.3), K

\( T_{H,in} \) temperature of fluid which enters the loop from HT pump, K
\( T_{\text{H, out}} \) temperature of fluid that exists the loop toward HT pump, K
\( T_{\text{in}} \) inlet temperature, K
\( T_L \) low temperature, K
\( T_{L\text{, end}} \) temperature of cold reservoir, Fig. 4.6, K
\( T_{L\text{, in}} \) fluid temperature at the end of \( T_b \) pipe (Fig. 2.3 and 2.4), K
\( T_{L\text{, out}} \) temperature of fluid which enters the loop from LT pump, K
\( T_{\text{max}} \) temperature of fluid that exists the loop toward LT pump, K
\( T_{\text{max}} \) peak temperature, K
\( T_{\text{out}} \) outlet temperature, K
\( T_w \) external wall temperature, K
\( T_x \) soil temperature, K
\( U \) average longitudinal fluid velocity, m s\(^{-1}\)
\( V \) total duct volume, m\(^3\)
\( V_H \) duct volume of high temperature gap, m\(^3\)
\( V_L \) duct volume of low temperature gap, m\(^3\)
\( \dot{W}_C \) reversible power output, W
\( \dot{W}_H \) required work to circulate the high temperature flow, W
\( \dot{W}_L \) required work to circulate the low temperature flow, W
\( x \) coordinate, m
\( y \) heated plate thickness, m
Greek symbols

\( \Delta P \) pressure difference, Pa

\( \Delta T \) temperature difference, K

\( \varepsilon \) dimensionless factor, Eq. (2.8)

\( \phi \) dimensionless flow area, Eq. (3.6)

\( \eta \) power plant efficiency

\( \nu \) kinematic viscosity, m\(^2\) s\(^{-1}\)

\( \rho \) flow density, kg m\(^{-3}\)

\( \tau \) dimensionless temperature difference, Eq. (2.17)

\( \psi \) dimensionless perimeter, Eq. (3.6)

Subscripts

\( f \) fluid

HT high temperature heat pump

in inflow

lam laminar

LT low temperature heat pump

min minimum

out outflow

s square element

t triangular element

turb turbulent
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Chapter 1

Introduction

The energy production and supply has been increasing for decades in order to match the growth in population and consumption. This growth was addressed by fuel switching in which an alternative cleaner and economical source has replaced the conventional ones like solar, geothermal and nuclear energy. Another way to respond to this demand increase is by improving the performance of current power plants and energy systems. Nowadays, the heat exchanger is an essential component in most, if not all, of the power plants. Boilers, condensers and cooling systems attached to solar panels or electronic devices consist of heat exchangers. So, the advancement in designing heat exchangers leads to an improvement in overall performance of heat generation systems.

In this dissertation, the key merit is the freedom in designing the heat exchangers based on Constructal law which states that “For a finite-size flow system to persist in time (to live), its configuration must change in time such that it provides easier and easier access to its currents (fluid, energy, species, etc.)” [1].

Chapter 2 presents an original analysis of coupling two ground heat pumps which are used for alternating purposes (heating or cooling) buildings. The way of coupling the two neighboring heat pumps by using common ground heat exchanger (buried loop) impacts the amount of the heat exchanged between the ground and the buildings. Additionally, the shape of the flow channel cross-section plays an important role in the thermal-flow performance of heat exchangers. In chapter 3, the main purpose of
optimizing the shape of the flow channel is to simultaneously enhance the heat transfer rate and to reduce the pumping power requirement.

Another aspect considered in this research work is the “economies of scale” in flow systems. The relation between the efficiency of thermal fluid systems (power plants) and their size is presented in chapter 4. The expression “economies of scale” is part of common language, and it means savings (reductions) that are registered when a small job is handled in bulk by a bigger entity. This expression has an old history, and much of its use is rooted in the industrial revolution. For example, savings in effort and expenditure were made possible in the unloading of ships when many workers with sacks on their backs were replaced by one conveyor belt.

The empirical evidence is clear: the bigger are more efficient carriers (per unit) than the smaller. This natural tendency is observed across the board, from animal design to technology, logistics and economics. In chapter 4, we rely on physics (thermodynamics) to determine the relation between efficiency and size. The analysis is performed in two ways, while using the thermodynamic model of a power plant. First is the tradeoff between the ‘external’ irreversibilities due to the temperature differences that exist above and below the temperature range occupied by the circuit executed by the working fluid. Second is the allocation of the fluid flow irreversibility between the hot and cold portions of the fluid flow circuit.

Chapter 5 shows numerically the effect of evolving the configuration of one stream cooling system toward better heat transfer performance. The cooling system
embedded inside the heated plate were morphed freely from serpentes with various design features (length, location and number of meanders) to loops with several configurations (square, circle and clover leaf). The method of Constructal design highlights the importance of the evolution in the heat exchanging system design.
Chapter 2

Energy design for dense neighborhoods

Ground coupled heat pumps draw heat from and reject heat to the ground, depending on diurnal and seasonal conditions and geographic orientation of the building. Even on the same day and in the same neighborhood, a building facing the sun may need cooling, while a neighboring building facing away from the sun may need heating. The thermal coupling between the heat pump flow system and the ground is influenced by a buried loop through which the heat pump fluid circulates. Heat pumps that provide cooling to the building reject heat to the loop and the ground. Heat pumps that heat the building extract heat from the loop and the ground.

This is an active field of research. Specifically, ground heat exchangers can work in open or closed loops. Here in this chapter, the focus is only on the closed loop design, which consists of a stream flowing in buried pipes installed horizontally or vertically in the available terrain [2–8]. Ground heat exchangers are traditionally designed as single pipes or channels with arrays of loops or serpentinaes [7–12]. Closely related to this field, and with impact on a greater scale, is the extraction from geothermal energy from shallow subsurface flow loops. The state of the art in this domain is presented by Vienken et al. [13]. Optimization methods for such designs are described by de Paly et al. [14], Bayer et al. [15] and Daróczy et al. [16]. The energy policy significance of this field is outlined by Hähnlein et al. [17].
The growth of thickly settled neighborhoods pushes the ground coupled design toward multiple heat pumps that are served by the same loop buried in a shared land area. When the heat pumps operate in unison, all rejecting or all absorbing, the design challenge is to accommodate all the heat pumps by using the same loop. As the density of urban settlement increases further, the multiple heat pump assembly acquires a new feature: some heat pumps reject heat to the ground while at the same time neighboring heat pumps absorb heat from the ground. In this chapter, the focus is on this feature of thickly settled design, including the analysis on the simultaneous operation and performance of heat pumps that reject and absorb heat from the ground.

2.1 Model

The simplest design for ground coupled heat pumps is shown in Figs. 2.1-2.4. This model is chosen in order to highlight the goal of this work, which is to discover the relationship between the flow configuration and the performance of the combination of two heat pumps. To begin with, a horizontal heat exchanger loop (subsurface) is assumed, not a vertical hole, even though in high density neighborhoods the available ground area is scarce. An infinite medium is also assumed: the effect of the boundaries of the round area is neglected, and so are the diurnal and seasonal temperature trends of the surroundings.

Two heat pumps operate with heat transfer to and from a single-loop heat exchanger buried in the ground. The soil is modeled as a homogeneous solid with constant, uniform and isotropic properties. The thermal resistance of the wall of the
buried pipe is assumed negligible relative to the thermal resistance posed by the solid around the pipe. This heat transfer model is discussed further in conjunction with Fig. 2.6.

The two heat pumps are situated at the opposite ends of the loop. The high temperature heat pump (HT) rejects heat and is located at $x = 0$, where $x$ is measured along the loop. The low temperature heat pump (LT) absorbs heat and is located at $x = L$. The objective of the work described in this chapter is to identify the design (the flow configuration) such that the overall performance ($q_H, q_L$) is increased.

In the absence of the heat pumps, an auxiliary pump circulates fluid with the flow rate $m_0$ in the clockwise sense as shown in the Figs. 2.1-2.4. There are four ways to connect the pumps relative to one another. In Fig. 2.1, mixing occurs downstream of each heat pump. The LT pump draws its stream ($m_L$) from the stream that comes from the outlet of the HT pump, namely $\dot{m}_a = \dot{m}_0 + \dot{m}_H + \dot{m}_L$. Right after this connection, the LT pump rejects its $\dot{m}_L$ stream to the loop, so that along the returning leg of the loop the flow rate is $\dot{m}_b = \dot{m}_a$.

In Fig. 2.2, the order in which the connections of the LT pump are made is reversed. First, the LT pump rejects its $\dot{m}_L$ stream ($T_L$) to the loop, and then draws it back at a higher temperature, $T_{L,\text{out}}$. Mixing occurs downstream of HT, and upstream of LT. The flow rates in the long ducts of the loop are the same, $\dot{m}_a = \dot{m}_b = \dot{m}_0 + \dot{m}_H$. 

6
Figure 2.1: Two heat pumps on the same loop (HT, LT), one rejecting heat and the other absorbing heat: mixing occurs downstream of each heat pump.

Figure 2.2: Two heat pumps on the same loop: mixing downstream of HT, and upstream of LT.
The new feature of the design shown in Fig. 2.3 is that the background flow ($\dot{m}_0$) is the only flow presented in the two long stretches of the loop. The LT and HT pumps reject their streams ($\dot{m}_L$, $\dot{m}_H$) to the loop and extract them almost immediately, after the mixing with background flow with higher temperature $T_{L,end}$ and lower temperature $T_{H,end}$, respectively.

The fourth feasible way to connect the two heat pumps to the same loop is shown in Fig. 2.4. New relative to the other configurations is that the circulating fluid through the long legs of the loop contains the background flow $\dot{m}_0$ and the flow rate $\dot{m}_L$. The HT pump flow rate $\dot{m}_H$ enters the loop with $T_H$ and then is extracted after the mixing with lower temperature $T_{L,end}$. Mixing occurs upstream of HT and downstream of LT heat pump.

The shape of the loop is not important as long as the ground volume affected by heat transfer with the loop has a thermal penetration distance that is much smaller than the loop length $L$. The two legs of the loop are in counterflow. They have the same length ($L$) because when placed in such a counterflow, the two legs require a single trench in which to be buried, regardless of the shape of the counterflow, Fig. 2.5. If the two legs are independent ($L_1$, $L_2$) then the total trench length would be greater, $L_1 + L_2$, as shown in the lower part of Fig. 2.5.
Figure 2.3: Two heat pumps on the same loop: mixing upstream of both HT and LT.

Figure 2.4: Two heat pumps on the same loop: mixing upstream of HT and downstream of LT.
In the design of Fig. 2.1, the HT pump adds fluid of temperature $T_H$ and mass flow rate $\dot{m}_H$ to the loop. The LT pump adds cold fluid of temperature $T_L$ and mass flow rate $\dot{m}_L$ to the same loop. The heat transfer from the loop to the ground is time dependent, in accord with the solutions for transient heat conduction from a buried cylinder in an infinite isothermal medium [18]. This process can be modeled as heat transfer from a cylinder to an isothermal conducting medium through a heat transfer coefficient $h$ that decreases monotonically in time, Fig. 2.6. The loop has two long sections. On the warm side (a), the mass flow rate is $\dot{m}_a$ and the fluid changes from $T_{H,in}$ to $T_{L,out}$. On the cold side (b), the temperature of the flow $\dot{m}_b$ is changed from $T_{L,in}$ to $T_{H,out}$. The objective is to analyze the relationship between the heat rejected by one heat pump and the heat absorbed by the other,

$$\dot{q}_H = \dot{m}_H c_p (T_H - T_{H,out}) \quad (2.1)$$

$$\dot{q}_L = \dot{m}_L c_p (T_{L,out} - T_L) \quad (2.2)$$

When the ratio $\dot{q}_H/\dot{q}_L$ is known, the designer knows the relative size of LT vs HT, i.e., how to match the two heat pumps that are served by the same loop. The conservation of mass requires

$$\dot{m}_a = \dot{m}_H + \dot{m}_L + \dot{m}_o = \dot{m}_b \quad (2.3)$$

The conservation of enthalpy at the two inlet corners of the loop requires

$$\dot{m}_H c_p T_H + (\dot{m}_o + \dot{m}_L) c_p T_{H,in} = (\dot{m}_o + \dot{m}_H + \dot{m}_L) c_p T_{H,in} \quad (2.4)$$
\[ \dot{m}_L c_p T_L + (\dot{m}_0 + \dot{m}_H) c_p T_{L,\text{out}} = (\dot{m}_0 + \dot{m}_H + \dot{m}_L) c_p T_{L,\text{in}} \]  

(2.5)

Next, the \( T_a \) stream is cooled by contact with the ground of uniform far-field temperature \( T_\infty \). The temperature drop along stream leg (a) is [19]:

\[ T_{L,\text{out}} - T_\infty = (T_{H,\text{in}} - T_\infty) \, e^{-N} \]  

(2.6)

where \( N \) is the number of heat transfer units of the buried pipe of length \( L \),

\[ N = \frac{h p L}{\dot{m}_a c_p} \]  

(2.7)

\( p \) is the perimeter of contact between the \( T_a \) pipe and the ground, and \( h \) is the conduction-equivalent heat transfer coefficient. Likewise, the \( T_b \) stream is heated by contact with the \( T_\infty \) ground, and it experiences the temperature rise:

\[ T_\infty - T_{H,\text{out}} = (T_\infty - T_{L,\text{in}}) \, e^{-N} \]  

(2.8)

where \( N \) is the same as in Eq. (2.7). The factor \( e^{-N} \) accounts for the lifetime of the loop heat transfer process. As the time increases, \( h \) decreases (from infinity at \( t = 0 \)), and so does \( N \). This means that \( e^{-N} \) increases from zero at \( t = 0 \), to values that approach 1 as the time increases. The factor \( e^{-N} \) marks the passing of time, as shown in Fig. 2.6. In the remaining analysis, the notation \( \varepsilon = e^{-N} \) will be used with the consideration of \( \varepsilon < 1 \).
Figure 2.5: Slender loop for connecting the two heat pumps in a shorter trench.

Figure 2.6: Why the variable $\varepsilon = e^{-N}$ increases monotonically in time, and marks the direction of time.
2.2 Thermal performance analysis

The objective here is to determine the ratio \( \frac{q_L}{q_H} \) for the four configurations, which, from Eqs. (2.1) and (2.2), is

\[
\frac{q_L}{q_H} = \frac{r_H}{r_L} \cdot \frac{T_{L,\text{out}} - T_L}{T_H - T_{H,\text{out}}} \tag{2.9}
\]

where

\[
r_H = \frac{\dot{m}_0}{\dot{m}_H}, \quad r_L = \frac{\dot{m}_0}{\dot{m}_L}, \quad r = \frac{\dot{m}_L}{\dot{m}_H} \tag{2.10}
\]

For configuration shown in Fig. 2.1, the outlet temperatures \( T_{H,\text{out}} \) and \( T_{L,\text{out}} \) follow from Eqs. (2.4) – (2.6) and (2.8), which can be rewritten as

\[
T_H + \left( r_H + \frac{r_H}{r_L} \right) T_{L,\text{out}} = (1 + r_H + \frac{r_H}{r_L}) T_{H,\text{in}} \tag{2.11}
\]

\[
T_L + \left( r_L + \frac{r_L}{r_H} \right) T_{H,\text{out}} = (1 + r_L + \frac{r_L}{r_H}) T_{L,\text{in}} \tag{2.12}
\]

\[
T_{L,\text{out}} - T_\infty = (T_{H,\text{in}} - T_\infty) \varepsilon \tag{2.13}
\]

\[
T_\infty - T_{H,\text{out}} = (T_\infty - T_{L,\text{in}}) \varepsilon \tag{2.14}
\]

First, we find that

\[
T_{L,\text{out}} = (1 - \varepsilon) T_\infty + \frac{\varepsilon r_L}{r_L + r_L r_H + r_H} T_H + \frac{\varepsilon (r_H r_L + r_H)}{r_L + r_H r_L + r_H} T_{H,\text{out}} \tag{2.15}
\]
\[ T_{H,\text{out}} = (1 - \varepsilon)T_\infty + \frac{\varepsilon r_H}{r_L + r_H r_{H} + r_{H}} T_L + \frac{\varepsilon(r_H r_L + r_L)}{r_L + r_L r_H + r_H} T_{L,\text{out}} \] (2.16)

Next, we solve the systems (2.14) – (2.15) for \( T_{H,\text{out}} \) and \( T_{L,\text{out}} \) and introduce the notation

\[ T_H = T_\infty + \tau \Delta T \] (2.17)

\[ T_L = T_\infty - (1 - \tau) \Delta T \] (2.18)

where \( \Delta T = T_H - T_L \), and \( \tau \) is a number between 0 and 1, which marks the position of \( T_\infty \) between \( T_H \) and \( T_L \). The analysis concludes with

\[ \frac{T_{L,\text{out}} - T_L}{\Delta T} = a_1 \tau + b_1 (1 - \tau) \] (2.19)

and

\[ \frac{T_H - T_{H,\text{out}}}{\Delta T} = a_2 \tau + b_2 (1 - \tau) \] (2.20)

for which the coefficients are listed in the first line of Table 2.1. The ratio \( \frac{q_L}{q_H} \) emerges as a function of mass flow rate imbalance \( \frac{\dot{m}_L}{\dot{m}_H} \) and time \( \varepsilon \), cf. Eq. (2.9),

\[ \frac{q_L}{q_H} = a_1 \tau + b_1 (1 - \tau) \]

\[ \frac{\dot{m}_L}{\dot{m}_H} = a_2 \tau + b_2 (1 - \tau) \] (2.21)

Broadly speaking, \( \frac{q_L}{q_H} \) should increase when \( \frac{\dot{m}_L}{\dot{m}_H} \) increases: this is why \( \frac{\dot{m}_L}{\dot{m}_H} \) is the denominator on the left side of Eq. (2.21). The right-side accounts for the
effect of $\dot{m}_L/\dot{m}_H$ and $\varepsilon$ on the presumed proportionality between the size ratio $q_L/q_H$ and the ratio of mass flow rates, $\dot{m}_L/\dot{m}_H$. As time increases, $\varepsilon$ approaches 1, and the right side of Eq. (2.21) approaches $\dot{m}_H/\dot{m}_L$ which means that $q_L/q_H$ approaches 1. This is confirmed in Fig. 2.7a, in which Eq. (2.21) is plotted. Generally, the ratio $q_L/q_H$ is smaller than 1, and this means that the LT pump extracts from the ground only a fraction of the energy deposited by the HT pump.

In the limit where $\dot{m}_0$ vanishes, the ratios $r_L$ and $r_H$ approach zero, and the coefficients for Eqs. (2.19) and (2.20) reduce to

$$a_1 = \frac{\varepsilon (1 + r)}{(1 + r)^2 - \varepsilon^2} \quad b_1 = 1 - \frac{\varepsilon^2 r^2}{(1 + r)^2 - \varepsilon^2}$$

(2.22)

$$a_2 = 1 - \frac{\varepsilon^2}{(1 + r)^2 - \varepsilon^2} \quad b_2 = \frac{\varepsilon r (1 + r)}{(1 + r)^2 - \varepsilon^2}$$

(2.23)

where $r$ is the mass flow rate imbalance between the two heat pumps, cf. Eq. (2.10). Figure 2.7b illustrates the effect of the primary flow amount $\dot{m}_0$ which is circulated in the underground loop in the absence of the heat pumps. When $\dot{m}_0$ vanishes, there is a minor decline in the relative performance of the two heat pumps; however, when $r_H$ is raised from 0.1 to 0.5, the relative performance slightly increases. Therefore, the performance of the two heat pumps is proportional to $\dot{m}_0$; however, the power needed to the auxiliary pump is also increased.
Figure 2.7: Results for the configuration designed in Fig. 2.1: (a) the relative performance $q_L/q_H$ of the two heat pumps, (b) the influence of $\hat{m}_0$ on the relative performance and (c) the time variation of the dimensionless heat rates.
The analysis of the design of Fig. 2.2 proceeds along the same steps as in Eqs. (2.1) – (2.20). The new coefficients for Eqs. (2.19) and (2.20) are listed in the second entry of Table 2.1. The performance of the two-pump assembly in configuration 2 is illustrated in Fig. 2.8a. The figure shows that as the time ($\varepsilon$) increases, the ratio $q_L/q_H$ approaches to 1. In addition, the performance $q_L/q_H$ increases significantly with increasing the mass flow rate ratio, $\dot{m}_L / \dot{m}_H$. Figure 2.8b shows a remarkable impact in the relative performance of the two heat pumps when $\dot{m}_0$ is changed. For the three ratios, there is an increase in the performance $q_L/q_H$ when the ratio $\dot{m}_L / \dot{m}_H$ increases.

For the design of Fig. 2.3, the analysis concludes again with Eqs. (2.19) and (2.20), for which the coefficients are reported in the third entry of Table 2.1. Figure 2.9a shows the relative performance ratio $q_L/q_H$ changes linearly as the time ($\varepsilon$) increases because $\dot{m}_L$ and $\dot{m}_H$ are not pumped through the long-buried stretches in which the temperature varies exponentially. When the ratio $\dot{m}_L / \dot{m}_H$ is less than 1, the relative performance increases as time ($\varepsilon$) increases. In contrast, the performance decreases with the time when the ratio $\dot{m}_L / \dot{m}_H$ is greater than 1. The influence of varying $\dot{m}_0$ is shown in Fig. 2.9b in which $q_L/q_H$ increases as the background stream increases. The heat transfer presence of the underground loop disappears when $\dot{m}_0$ vanishes, because in that limit, $a_1 = b_1 = a_2 = b_2 = 0$. 

17
Figure 2.8: Results for the configuration designed in Fig. 2.2: (a) the relative performance $q_L/q_H$ of the two heat pumps, (b) the influence of $m_0$ on the relative performance and (c) the time variation of the dimensionless heat rates.
Figure 2.9: Results for the configuration designed in Fig. 2.3: (a) the relative performance $q_L/q_H$ of the two heat pumps, (b) the influence of $m_0$ on the relative performance and (c) the time variation of the dimensionless heat rates.
For the configuration shown in Fig. 2.4, the analysis produces the coefficients shown in the bottom entry of Table 2.1, which are to be used in Eqs. (2.19) and (2.20). The performance of the two heat pumps is reported in Fig. 2.10a, which shows that the ratio \( q_L/q_H \) decreases, over time, regardless of flow rate ratio \( r \). The relative performance ratio \( q_L/q_H \) is significantly larger in Fig. 2.10a than in Figs. 2.7a, 2.8a, and 2.9a. Figure 2.10b shows that the ratio \( q_L/q_H \) decreases as \( \dot{m}_0 \) (e.g., \( r_H \)) increases, in contrast with the other three configurations.

The analysis of the relative performance between the two heat pumps shows the overall ratio of the heat that could be extracted or rejected to the loop. However, the amount of heat absorbed by the LT pump or rejected by HT pump from the underground coupled loop is not represented by the relative performance ratio.

So, to complete the description of the performance for the system, we calculated \( q_L \) and \( q_H \) and expressed them in dimensionless terms. The reference heat transfer rate was selected as \( \dot{m}_H c_p (T_{\infty} - T_L) \). The dimensionless version of \( q_L \) as a function of \( \tau \) is

\[
\tilde{q}_L = \frac{\dot{m}_L c_p (T_{L,out} - T_L)}{\dot{m}_H c_p (T_{\infty} - T_L)} = \frac{r (T_{L,out} - T_L)}{(1-\tau)(T_H - T_L)} = \frac{r}{(1-\tau)}\left[ a_1 \tau + b_1 (1-\tau) \right] \tag{2.24}
\]

Similarly, the dimensionless heat current rejected by the HT heat pump is

\[
\tilde{q}_H = \frac{\dot{m}_H c_p (T_H - T_{H,out})}{\dot{m}_H c_p (T_H - T_{\infty})} = \frac{T_H - T_{H,out}}{\tau(T_H - T_L)} = \frac{a_2 \tau + b_2 (1-\tau)}{\tau} \tag{2.25}
\]
Figure 2.10: Results for the configuration designed in Fig. 2.4: (a) the relative performance $q_L/q_H$ of the two heat pumps, (b) the influence of $r$ on the relative performance and (c) the time variation of the dimensionless heat rate.
The variation of the dimensionless heat rate for the design of Fig. 2.1 is shown in Fig. 2.7c. As the mass flow rate ratio (r) increases, $\tilde{q}_L$ increases with approximately the same order of (r). However, $\tilde{q}_H$ decreases in time (ε) when r < 1, and it increases when r > 1. Figure 2.8c shows that the dimensionless heat transfer rate rejected from the HT pump ($\tilde{q}_H$) is higher than the heat transfer rate gained by the LT pump ($\tilde{q}_L$), despite the increase of ratio $m_L/m_H$.

In the design of Fig. 2.3, the heat transfer rates absorbed or rejected to the underground loop are much smaller compared with three other configurations, as shown in Fig. 2.9c. The variation of $\tilde{q}_H$ and $\tilde{q}_L$ is almost the same as time (ε) increases. Furthermore, as the ratio (r) increases, the change of $\tilde{q}_L$ is insignificant. Figure 2.10c shows that the value of $\tilde{q}_L$ increases over time, until ε reaches to approximately 0.4, and then it decreases. The design of Fig. 2.4 provides a higher heat transfer rate $\tilde{q}_L$ than the other configurations.
Table 2.1: The coefficients of Equations (2.19) and (2.20).

<table>
<thead>
<tr>
<th>Figure</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>(a_1 = \frac{\varepsilon r_L (r_L + r_L r_H + r_H)}{(r_L + r_L r_H + r_H)^2 - \varepsilon^2 (r_H r_L + r_L r_H + r_H r_H)^2 + r_L r_H)})</td>
<td>Figure 2.1</td>
</tr>
<tr>
<td></td>
<td>(b_1 = 1 - \frac{\varepsilon^2 (r_H r_L + r_L^2)}{(r_L + r_L r_H + r_H)^2 - \varepsilon^2 (r_H r_L + r_L r_H + r_H r_H)^2 + r_L r_H)})</td>
<td>Figure 2.1</td>
</tr>
<tr>
<td>2.2</td>
<td>(a_1 = \frac{\varepsilon r_L}{r_L + r_L r_H + r_H - \varepsilon^2 r_L r_H})</td>
<td>Figure 2.2</td>
</tr>
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<td></td>
<td>(a_2 = 1 - \frac{\varepsilon^2 r_L}{r_L + r_L r_H + r_H - \varepsilon^2 r_L r_H})</td>
<td>Figure 2.2</td>
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<td></td>
<td>(b_1 = 1 - \frac{r_H}{r_L + r_L r_H + r_H - \varepsilon^2 r_L r_H})</td>
<td>Figure 2.2</td>
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<td></td>
<td>(b_2 = \frac{\varepsilon r_H}{r_L + r_L r_H + r_H - \varepsilon^2 r_L r_H})</td>
<td>Figure 2.2</td>
</tr>
<tr>
<td>2.3</td>
<td>(a_1 = \frac{\varepsilon r_L}{(1 + r_L)(1 + r_H) - \varepsilon^2 r_L r_H})</td>
<td>Figure 2.3</td>
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<td></td>
<td>(a_2 = 1 - \frac{1 + r_L}{(1 + r_L)(1 + r_H) - \varepsilon^2 r_L r_H})</td>
<td>Figure 2.3</td>
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<td></td>
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<td></td>
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<td>Figure 2.3</td>
</tr>
<tr>
<td>2.4</td>
<td>(a_1 = \frac{\varepsilon r_L}{r_L + r_L r_H + r_H - \varepsilon^2 r_L r_H})</td>
<td>Figure 2.4</td>
</tr>
<tr>
<td></td>
<td>(a_2 = 1 - \frac{r_L}{r_L + r_L r_H + r_H - \varepsilon^2 r_L r_H})</td>
<td>Figure 2.4</td>
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<td></td>
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<td>Figure 2.4</td>
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<td></td>
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<td>Figure 2.4</td>
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2.3 Pumping power

The analysis of the pumping power required by the underground stream is discussed in this section. We assume that the flow through the buried pipe is in the fully turbulent and fully rough regime such that $\Delta P$ is proportional to $\dot{m}^2$,

$$\Delta P = \frac{8f}{\rho} \frac{L}{\pi D^5} \dot{m}^2$$ (2.26)

The pumping power associated with the pressure difference $\Delta P$ and the total length of the duct is

$$W = \frac{\dot{m}_a \Delta P_a}{\rho} + \frac{\dot{m}_b \Delta P_b}{\rho}$$ (2.27)

Equations (2.26) and (2.27) yield

$$W = c_{turb} \dot{m}_a^3 + c_{turb} \dot{m}_b^3$$ (2.28)

where the constant is $c_{turb} = 8fL \left( \rho_0^2 \pi D^5 \right)$. The reference pumping power is based on $\dot{m}_H$, namely $\dot{W}_{ref} = c \dot{m}_H^3$. The dimensionless pumping power is $\hat{W} = W / \dot{W}_{ref}$, which for the designs of Figs. 2.1-2.4 become

$$\hat{W}_1 = \frac{\dot{m}_a^3}{\dot{m}_H^3} = (1 + r_H + r)^3$$ (2.29)

$$\hat{W}_2 = \frac{(\dot{m}_0 + \dot{m}_H)^3}{\dot{m}_H^3} = (1 + r_H)^3$$ (2.30)

$$\hat{W}_3 = \frac{\dot{m}_0^3}{\dot{m}_H^3} = r_H^3$$ (2.31)
\[ \bar{W}_4 = \left( \bar{m}_0 + \bar{m}_L \right)^3 / \bar{m}_H^3 = (r_H + r)^3 \]  

(2.32)

Figure 2.11 shows the variation of the heat current \( \bar{q}_{\text{H}} \) rejected by HT pump as the pumping power and the background mass flow rate increase. For clarity, the ratio \( \bar{m}_L / \bar{m}_H \) was set equal to 1, and the time parameter \( \epsilon \) was set at 0.2. The four designs divide themselves into two groups. In designs of Figs. 2.1 and 2.2, \( \bar{q}_{\text{H}} \) decreases as \( \bar{W} \) increases. However, in designs of Figs. 2.3 and 2.4, the trend is the opposite. In conclusion from this section, to reject more heat with using less power, the superior designs are the ones of Figs. 2.1 and 2.2.

Similarly in Fig. 2.12, we plotted the relationship between the heat current absorbed by the LT heat pump and the required pumping power. This time, the four designs group themselves differently: only in Figs. 2.1 and 2.4 is a greater \( \bar{q}_{\text{L}} \) achievable with less \( \bar{W} \). Together, Figures 2.11 and 2.12 show that only the design of Fig. 2.1 offers greater \( \bar{q}_{\text{H}} \) and \( \bar{q}_{\text{L}} \), simultaneously, with less \( \bar{W} \).
Figure 2.11: The effect of the total pumping power on the rate of heat rejection from the HT pump.
Figure 2.12: The effect of the total pumping power on the rate of heat absorption at the LT pump.
2.4 Discussion

In this chapter, the merits of a design consisting of two heat pumps buried underground and connected to the single loop-shaped ground heat exchanger were documented. The goal of the analysis presented in this chapter was to discover how the changes in the configuration of the flow system affect the performance of the assembly.

Four different ways were considered in which the two heat pumps are connected to the loop. We reported the individual heat transfer rates experienced by the two heat pumps, and the relative performance ratio $q_L/q_H$. The results show that the best design is the one defined in Fig. 2.1, where mixing occurs downstream of both LT and HT, in which case a higher heat ($\tilde{q}_{H}$) could be rejected to the ground and simultaneously absorbed by LT pump ($\tilde{q}_{L}$). On the other hand, the results for the design of Fig. 2.2 showed higher capability to reject heat from the HT pump to the underground loop. Specifically, configuration 2 is more applicable when the sole need is to cool a building, while configuration 4 should be used when the demand is to warm up the neighboring buildings. The effect of increasing the background flow rate $m_0$ on the heat gained or rejected to the underground loop was also analyzed. The design of Fig. 2.4 offers a performance that is superior to the designs of Figs. 2.1-2.3, as shown in Figs. 2.7a, 2.8a, 2.9a and 2.10a.

So here we conclude that if we endow the configuration with freedom to change, and if we compare the resulting alternatives on the same basis, it is possible to identify not only the better configurations but the key design feature that influences the
performance. That feature is the position of the mixing process relative to its closest heat pump.
Arrays of flow channels with heat transfer embedded in conducting walls

Recent studies have focused on optimizing the flow architecture in order to enhance the heat transfer rate [19–26]. Several studies have also demonstrated the effect of the wetted perimeter shape on the flow performance [27–34]. Additionally, the longitudinal configuration of the flow channel such as zigzag, curvy, step [35], single-layer, double-layer, tapered [36], wavy [37] and converting channels [38] for the same cross-sectional shape was investigated and compared with the conventional straight channel with uniform cross-sectional shape. On this background, the comparison between different cross-sectional shapes to accomplish the two objectives simultaneously, lower pressure drop along the flow channel and higher heat transfer rate between the fluid flow and the heated wall, is a new point of view that deserves to be investigated. In this chapter, we seek to simultaneously decrease the longitudinal flow resistance and the transversal thermal resistance for the square, circle and triangle cross-sectional channels. This work is based on two analytical models, i.e., scale analysis, and computational fluid dynamics (CFD) models.

3.1 Square element

We begin with a square array, with these known features: side length $L_s$, loop cross section $L_s^2$, flow cross section $A_f$, solid cross section $A_s = L_s^2 - A_f$, fluid properties: $(k_f, \mu, \nu)$, and solid properties: $(k_s, \text{etc})$. This search is to find the best shape of the wetted
perimeter, such that the two objectives are met at the same time: small transversal thermal resistance between the fluid stream and the square perimeter, and small longitudinal flow resistance, $\Delta P/\dot{m}$. The mass flow rate $\dot{m}$ is fixed; it is the stream allocated to one element with square cross section as shown in Fig. 3.1. The shape of the wetted perimeter varies between the two extremes shown in the upper part of Fig. 3.1: (a) square flow cross-section, and (b) round flow cross-section.
Figure 3.1: Three designs of flow channels embedded in a square cross-section.
3.1.1 Flow resistance

Assume that the flow is laminar and fully developed (Poiseuille). This means that the longitudinal flow resistance is [19]

\[ \frac{dP}{dx} = f \frac{4}{D_h^2} \frac{1}{\rho} U^2 \]  \hspace{1cm} (3.1)

where \( U \) is the average fluid velocity,

\[ U = \frac{\dot{m}}{\rho A_f} \]  \hspace{1cm} (3.2)

\( D_h \) is the hydraulic diameter,

\[ D_h = \frac{4A_f}{p} \]  \hspace{1cm} (3.3)

\( p \) is the wetted perimeter and \( f \) is the friction coefficient

\[ f = \frac{Po}{Re_{D_h}}, \hspace{1cm} Re_{D_h} = \frac{UD_h}{\nu} \]  \hspace{1cm} (3.4)

and \( Po \) is the Poiseuille constant. The extreme values of \( Po \) for cases (a) and (b) in Fig. 3.1 are [19], respectively;

\[ Po_a = 14.2 \hspace{2cm} Po_b = 16 \]  \hspace{1cm} (3.5)

The effect of the shape of the wetted perimeter is brought into the analysis by introducing the dimensionless factors \( \phi \) and \( \psi \):
Combining Eqs. (3.1) – (3.6), we write the dimensionless fluid flow resistance as

$$R_f = \frac{dP/dx}{\dot{m}v/D_h^2 A_f} = 2\text{Po} \frac{\psi^2}{\phi^3}$$

(3.7)

One objective is to decrease $R_f$. We already knew that if this is the only objective, then the best cross section is (b), the round one. In this chapter, however, there is also a second objective (to minimize the transversal thermal resistance), and for this reason we continue with the general case where $\phi$ and $\psi$ may vary.

When the rule of morphing (a) into (b) is simple, as in Fig. 3.1 where all the corners are identical, there is a relation between $\phi$ and $\psi$. To see this, we assume that $\phi$ is specified, therefore $\phi_a = \phi_b = \phi$. Next for the round perimeter (b), we have $p_b = \pi D_b$ and $A_b = (\pi/4)D_b^2$ from which we deduce

$$\psi_b = \frac{\pi^{1/2}}{2} \phi^{1/2}$$

(3.8)

In case (a), the solid cross section is the same as in case (b), but $p_a = 4L_a$ and $A_a = L_a^2$, therefore

$$\psi_a = \phi^{1/2}$$

(3.9)

Another factor that could have been varied is the ratio $R/L_c$ (i.e., case c) while keeping the minimum wall thickness fixed. Varying the radius of curvature was not
considered as both the dimensionless fluid flow resistance and thermal resistance would fall within the limits given by cases (a) and (b). To draw case (c), we assume that the radius of the rounded corners is \( R = 0.1L_c \), so that 
\[
\begin{align*}
p_c &= 4L_c + 2\pi R \\
A_c &= L_c^2 + 4RL_c + \pi R^2,
\end{align*}
\]
therefore
\[
\psi_c = 0.967\phi^{1/2} \quad (3.10)
\]

In summary, the values of \( R_{f,a} \), \( R_{f,b} \) and \( R_{f,c} \) depend only on the assumed value for \( \phi \). By varying \( \phi \), the fluid cross-section varies in size while the area of the square solid is kept constant. These functions are plotted in Fig. 3.2. Note that case (b) exists where \( \phi \) does not exceed the value that corresponds to the circle touching the sides of the square in Fig. 3.1 (b):
\[
\psi_{\text{max,b}} = \frac{(\pi/4)L_s^2}{L_s^2} = 0.785 \quad (3.11)
\]

Similarly, case (c) exists when \( \phi \) does not exceed 0.994.
Figure 3.2: The variation of fluid flow resistance for different flow channels, represented by area factor $\phi$, of square arrays, (a) square and (b) circular flow channel.
3.1.2 Thermal resistance

The thermal objective of the flow system can be simulated numerically for the model shown in Fig. 3.3. The system is sufficiently long parallelepiped with a square cross section in which we can fit any of the drawings shown in Fig. 3.1. Each of such drawing is characterized by its $\phi$ and $\psi$.

The fluid flow is laminar and fully developed, which means that the longitudinal velocity distribution ($u$) does not vary from one $x$ cut to another. The temperature field is also fully developed, therefore the meniscus shape of the $T$ distribution over the flow cross section does not vary from one $x$ to another.

Assume that the long external walls of the parallelepiped are isothermal at $T_w$, and that the average temperature of the fluid entering through the $x = 0$ end is $T_{in}$. Also assume that the mass flow rate ($\dot{m}$) along the duct is specified. The continuity of heat flux at the solid-fluid interface (in the same plane as the cross section) requires

$$k_s \frac{\partial T}{\partial n} = k_f \frac{\partial T}{\partial n}$$

(3.12)

where $n$ is the normal to the solid/fluid interface. Equation (3.12) reveals the role of $\tilde{k} = k_s / k_f$ as a dimensionless parameter of the flow system. Another parameter is the fluid Prandtl number, Pr, set equal to 1, which is the order of magnitude of common fluids.
Figure 3.3: Velocity and temperature profiles of a flow channel embedded in square solid.
The overall thermal contact is measured between the outer surface of the system ($T_w$) and the bulk temperature of the fluid in the channel. At the outlet, $x = L_x$, the bulk fluid temperature is $T_{out}$. The fluid temperature change from inlet to outlet is given by (Fig. 3.4) [19]

$$\ln\frac{T_w - T_{in}}{T_w - T_{out}} = \frac{\bar{h}4L_s L_x}{mc_p} \tag{3.13}$$

where $4L_s$ is the perimeter of the square cross section, and $\bar{h}$ is an effective (overall) heat transfer coefficient that accounts for the thermal contact between $T_w$ and the bulk temperature $T(x)$. Note that $T(x)$ increases from $T_{in}$ to $T_{out}$. The right side of Eq. (3.13) is the number of heat transfer units of the fluid and solid system, which is defined as

$$N = \frac{\bar{h}4L_s L_x}{mc_p} \tag{3.14}$$

where $\psi 4L_s$ is the perimeter of contact, and $N$ is greater when the thermal contact is stronger.

The non-isothermal flow COMSOL model was used to simulate the temperature field of the channel flow. This non-isothermal model is also used for the fluid flow and heat transfer, i.e., in which the fluid properties such as density and viscosity are temperature dependent. The fluid properties at the duct entrance are $c_p = 4192 \text{ J/(kg K)}$, $\rho = 999.7 \text{ kg/m}^3$, $\nu = 10^{-6} \text{ m}^2/\text{s}$, and $m_0 = 0.01\text{kg/s}$ is considered. The solid wall properties are $c = 475 \text{ J/(kg K)}$, $\rho = 7850 \text{ kg/m}^3$, $k_s = 44.5 \text{ W/(m K)}$. The flow regime is
laminar as the Reynolds number \( \text{Re} = \frac{U D_h}{\nu} \) does not exceed 2300. Here, \( D_h \) is either the diameter of the cylinder or the hydraulic diameter of the fluid flow channel.

![Diagram](image)

**Figure 3.4:** The mean temperature of the fluid stream which flows through hot contacting walls.

The governing equations for the non-isothermal 3-D flow are mass flow continuity, momentum and energy equations for the fluid in the duct plus the heat conduction equation for the solid wall [39]. The boundary conditions are as follows. At the inlet of the duct, a uniform temperature \( T_{\text{in}} \) (300 K) is assumed and the mean velocity of the channel flow is specified as \( \bar{U} = \bar{m} / \rho A \) in which \( \bar{m} \) is fixed. The entrance region for laminar pipe flow is calculated as \( L_e / D_h \approx 0.06 \text{Re} \), where \( D_h \) is the hydraulic diameter. At the duct outlet, the pressure is constant, and the heat conduction condition is \( n \cdot (k_r \nabla T_r) = 0 \). No slip boundary condition \( (u = 0) \) is applied on the side walls of the flow duct. A constant temperature boundary condition (350 K) is specified at the outer
surfaces of the solid walls. Table 3.1 shows the computational results for one case where $\phi = 0.7$, in square arrays. The results show that the average temperature at the channel outlet becomes almost independent of the mesh size. Therefore, the mesh with $2 \times 10^5$ elements is used in this chapter’s study because the computational results converge as the mesh is refined, i.e., the effect of the mesh on the computational results becomes negligible as the grid size is decreased beyond the $2 \times 10^5$ elements.

Table 3.1: Mesh accuracy test.

<table>
<thead>
<tr>
<th>Number of elements</th>
<th>$T_{out,avg}$ (K)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>51,924</td>
<td>325.7256</td>
<td>------</td>
</tr>
<tr>
<td>143,936</td>
<td>325.1887</td>
<td>0.165</td>
</tr>
<tr>
<td>204,771</td>
<td>325.1928</td>
<td>0.0013</td>
</tr>
<tr>
<td>311,025</td>
<td>325.1934</td>
<td>0.00018</td>
</tr>
<tr>
<td>577,148</td>
<td>325.1938</td>
<td>0.00012</td>
</tr>
</tbody>
</table>

In this numerical work, $N$ is a measure of the degree of thermal contact, because it is proportional to the thermal conductance, cf. Eq. (3.14). The value of $N$ is determined after numerical simulations, i.e., after calculating $T_{out}$ and substituting this value (along with $T_w$ and $T_{in}$) on the left side of Eq. (3.13). There will be one $N_a(\phi)$ curve for the configuration of Fig. 3.1a, one $N_b(\phi)$ for Fig. 3.1b and another $N_c(\phi)$ for Fig. 3.1c.
Keeping all the other parameters fixed, such as $\dot{m}$, $\text{Pr}$, $\text{k}$ and $c_p$, and varying only $\phi$. It is expected that $N$ would decrease when the fluid flow dimensionless cross-section ($\phi$) would increase.

Qualitatively, the overall thermal resistance $R_{th}$ varies monotonically with the inverse of $N$, as reported in Fig. 3.5,

$$R_{th} = \frac{1}{N} \quad (3.15)$$

The trend in Fig. 3.5 is the opposite of what was anticipated for $R_f$ in Fig. 3.2. This conflict is the basis for the two-objective search sketch in Fig. 3.6, which is obtained by eliminating $\phi$ between Figs. 3.2 and 3.5. The search for better configurations [(a), (b), or (c)] is toward low $R_f$ and low $R_{th}$ simultaneously. The better configuration is the one represented by a curve that comes closer to the origin, namely, the curve for the cross-section shown in Fig. 3.1a.
Figure 3.5: The variation of overall thermal resistance with area factor $\phi$, (a) square flow channel and (b) circular flow channel.
Figure 3.6: The overall thermal resistance versus the fluid flow resistance of (a) square flow channels, (b) circle flow channels.
3.2 Triangular element

The analysis of the two-objective performance \((R_f, R_{th})\) of a triangular array follows the same steps as in the preceding section. We focus on the extreme configurations, (a) triangular flow channel and (b) round flow channel, while the volume of wall material is fixed as shown in Fig. 3.7. The volume fraction occupied by the flow cross-section and channel is \(\phi\). The flow cross-section and the wetted perimeter are \(A_i = 0.433\phi L_i^2\) and \(p = \psi 3L_i\), such that \(\psi_a = \phi^{1/2}\) and \(\psi_b = 3^{-3/4} \pi^{1/2} \phi^{1/2}\).

![Figure 3.7: Flow channels embedded in a triangular cross section.](image)

3.2.1 Flow resistance.

In place of Eq. (3.7), we obtain the following dimensionless fluid flow resistance

\[
R_f = \frac{dP / dx}{\bar{m}v / D_i^2 A_f} = \frac{9}{8} \text{Po} \frac{\psi^2}{\phi^3}
\]  
(3.16)
The mass flow rate $\dot{m}$ is fixed: In this case, $\dot{m}$ is the stream allocated to one element with triangular cross section, Fig. 3.7. For the triangular duct, Fig. 3.7a, we substitute $P_o = 13.3$ and conclude that

$$R_{fa} = 14.96\phi^{-2} \quad (3.17)$$

Similarly, for the round cross section, Fig. 3.7b, where $P_o = 16$, we obtain

$$R_{fb} = 10.88\phi^{-2} \quad (3.18)$$

Equations (3.17) and (3.18) are plotted in Fig. 3.8, which shows that the flow resistance of the round channel is smaller than the resistance posed by the triangular channel. Note the similarity between Fig. 3.8 and Fig. 3.2. For the round cross-section (b), the $R_f (\phi)$ curve ends at $\phi = 0.6$, which represents the round duct that touches the triangular perimeter of the available space.
Figure 3.8: The flow resistance variation for triangular arrays with the area factor $\phi$, (a) triangle flow channel and (b) circular flow channel.
3.2.2 Thermal resistance.

The effect of duct cross-sectional shape on the thermal resistance was determined by using the same numerical method as in section 3.1.2 and Fig. 3.2. The effective number of heat transfer units of the fluid-solid flow system that occupies the volume with triangular cross section and length $L_x$. The number of heat transfer units is based on the perimeter of the triangular cross section of the volume,

$$N = \frac{\psi 3L_{t}L_{x}\bar{h}}{mc_p}$$  \hspace{1cm} (3.19)

The inverse of $N$ is regarded as a measure of the degree of thermal contact between the fluid and the external perimeter ($3L_t$), namely $R_{th} = N^{-1}$, as shown in Fig. 3.9. The thermal resistance of the design with triangular flow cross section (a) is smaller than the thermal resistance of the design with round flow cross section.

Figures 3.8 and 3.9 were combined into the $R_f - R_{th}$ plot shown in Fig. 3.10. For designs with two simultaneous objectives, small $R_f$ and small $R_{th}$, configuration (a) is better than configuration (b). This conclusion corresponds to what was observed in Fig. 3.6 for square cross sections.
Figure 3.9: For triangular cross-section; the overall thermal resistance versus area factor $\phi$.

Figure 3.10: The results of fluid flow resistance and thermal resistance for the triangular and circular flow channels embedded in triangle cross-section.
3.3 Square array or triangular array?

The similarities between the performance of the square elemental volume (Figs. 3.2, 3.5, 3.6) and the triangular elemental volume (Figs. 3.8-3.10) are evident. They invite the question of which cross-section would perform better, the square or equilateral triangle.

To answer this, first we recognize that the whole assembly is an array of many elements of the same kind (Fig. 3.11) in which the two flow elements are arranged in a staggered manner and the heat exchanger is of counterflow type. The whole assembly has a purpose and a flow rate, which are the same regardless of the choice made for the shape of the elemental cross-section. In other words, the mass flow rate per unit of array cross sectional area, \( \dot{m}^* \) is specified and fixed.

Second, in the study of square elements (Fig. 3.1), \( \dot{m} \) was fixed because the size (the cross section) of the square element was fixed. The comparison was between three flow cross sectional shapes (a, b, c), each carrying the same \( \dot{m} \). This is why the flow resistance defined in Eq. (3.7) depended on only \( \phi, \psi \) and \( \text{Po} \). The same is true for the flow resistance defined in Eq. (3.16) for the triangular element: the comparison was between Figs. 3.7a and 3.7b, and \( \dot{m} \) was the same for both configurations.
Figure 3.11: Square and triangle arrays with the same flow length and mass flow rate per unit of cross-section area.

As we move to the whole arrays shown in Fig. 3.11, we observe not only that \( \dot{m}'' \) is the same for the two arrays, but also that the \( \dot{m} \) that appears in Eq. (3.7) is not necessarily the same as the \( \dot{m} \) in Eq. (3.16). For clarity, we compare the two arrays in terms of pressure drop, not flow resistances. For the square array, Eq. (3.7) can be rewritten as

\[
\left( \frac{dP}{dx} \right)_{\text{s}} \frac{L_s^2}{\dot{m}'' v} = \frac{2P_{o_s}}{\phi^4 \tilde{L}_s^2}
\]  

(3.20)

where \( \tilde{L}_s = L_s/L_x \) is the dimensionless side of the square element, and \( L_x \) is the length of the array in the flow direction.

For the triangular array, Eq. (3.16) can be rewritten as

\[
\left( \frac{dP}{dx} \right)_{\text{t}} \frac{L_t^2}{\dot{m}'' v} = \frac{27}{8} \frac{P_{o_t}}{\phi^4 \tilde{L}_t^2}
\]  

(3.21)
where $\bar{L}_t = L_t / L_s$ is the dimensionless side of the triangular element. Dividing Eqs. (3.20) and (3.21), and recalling $Po_s = 14.2$ and $Po_t = 13.3$, we obtain

$$\frac{(dP/dx)_s}{(dP/dx)_t} = 0.633 \left( \frac{L_t}{L_s} \right)^2$$

(3.22)

The ratio $L_t/L_s$ is a degree of freedom in the drawing shown in Fig. 3.11:

(i) If the sides of the elements are equal, $L_t = L_s$, then the pressure drop along the square array is 36.7 percent smaller than along the triangular array.

(ii) If the cross-sectional areas of the elements are equal, $L_t^2 = \left( \frac{3^{1/2}}{4} \right) L_s^2$, then the right side of Eq. (3.22) is equal to 1.461. In this case, the pressure drop along the triangular array is one third smaller than along the square array.

The thermal resistances of the two arrangements based on square and triangular cross-sections can also be compared on an equal basis. To start with, as noted already en route to Eq. (3.15), the thermal resistance of one element is large when $N$ is small. The relation between thermal resistance and $1/N$ becomes a proportionality in the limit of $N \ll 1$. This can be shown from Eq. (3.13), after writing $N$ for the right hand side, and

$$\Delta T = T_{out} - T_{in}, \text{ and also, } q = \dot{m} c_p (T_{out} - T_{in})$$

where $q$ is the total heat transfer rate from the element of length $L_x$ to the stream $\dot{m} \bar{n}$ allocated to that element. In the limit $N \ll 1$, Eq. (3.13) becomes

$$\frac{\Delta T}{q/\dot{m} c_p} = \frac{1}{N}$$

(3.23)

and, after returning to the notation of Eq. (3.13),
\[
\left( \frac{\Delta T}{q} \right)_s = \frac{1}{h_s \psi 4L_s L_x}
\]  

(3.24)

The corresponding formula for the triangular element, derived from Eq. (3.19), is

\[
\left( \frac{\Delta T}{q} \right)_t = \frac{1}{h_t \psi 3L_t L_x}
\]  

(3.25)

Dividing Eq. (3.24) and (3.25), we arrive to the ratio of the two thermal resistances,

\[
\frac{\left( \frac{\Delta T}{q} \right)_s}{\left( \frac{\Delta T}{q} \right)_t} = \frac{3}{4} \frac{h_t}{h_s}
\]  

(3.26)

Next, for fully developed laminar flow in channels with constant temperature [as in Eq. (3.13)], we have from Ref. [19], Table 3.2,

\[
\text{Nu}_s = \frac{h_s D_{n,s}}{k_f} \approx 2.89 \quad \text{Nu}_t = \frac{h_t D_{n,t}}{k_f} \approx 2.35
\]  

(3.27)

where \( D_{n,s} = \psi L_s \) and \( D_{n,t} = \psi 3^{-1/2} L_t \), cf. Table 3.1 in Ref. [19].

The right side of Eq. (3.26) turns out to be equal to 1.056, independently of the choice of \( L_t/L_s \). In conclusion, from a thermal contact standpoint, the triangular array (Fig. 3.7a) is 5.3 percent superior to the square array (Fig. 3.1a).
3.4 Discussion

The analysis of finding the optimum cross-section of the flow in an assembly with identical array of tubes in counterflow was demonstrated in this chapter. The optimum cross-section was determined after considering two purposes, simultaneously, which are: decrease the fluid flow resistance (or the pressure drop) and reduce the heat transfer resistance.

For the square elements, we found that the geometry that serves the two objectives best is the one with constant wall thickness, i.e., square wetter perimeter (Fig. 3.1a). The design with circular wetted perimeter (Fig. 3.1b) is the worst. The designs with wetted perimeters shaped as squares with rounded corners are in between. It was also found to that the triangular wetted perimeter cross-section (Fig. 3.7a) outperforms the one with circular flow cross section (Fig. 3.7b).

Important and interesting is the comparison between the two arrays consisting of square and triangular elements (Fig. 3.11). The comparison was done based on a specified fluid mass flow rate per unit of array cross sectional area, such that the ratio \( L_s/L_t \) is a degree of freedom. We considered two such ratios, \( L_s/L_t = 1 \), and \( L_s/L_t = 3^{1/4}/2 \).
Chapter 4

Economies of Scale

Why is size so important? This fundamental question dominates theoretical thought throughout science, from animal design [40–51] and river flow [52–54], to vehicle technology [40,55], logistics and economics. The empirical observations are of the same kind, regardless of their realm of origin, animate or inanimate. Their summary is that the bigger are more ‘efficient’ in their respective movement, or movers of animal weight, pumps of blood and water, movers of freight, manufacturer of goods, and communications. It is ‘cheaper’ to move something as a component of a larger mover, as opposed to moving it alone, against its environment. This universal empirical fact (and source of wisdom in human design) is widely recognized as ‘economies of scale’.

Why are economies of scale a universal phenomenon? To answer this question is the purpose of this chapter. To answer the question in a most fundamental way, we formulate the argument in physics, more precisely, in thermodynamics.

In constructing the physics argument for economies of scale, we benefit from recent advances in thermodynamics that unify the design and functioning of all moving (flow) systems. In every example, the physics is simple [40,56]; (i) the system moves against its environment because it is being pushed, (ii) the pushing is provided by power produced in ‘engines’ (natural, including the human made), and (iii) the power is dissipated instantly during movement relative to a resisting environment.

In this physics framework, it is evident that the physics of ‘economies of scale’ lies in the effect of size on the efficiency of the engine that drives the movement. This is
the direction that the argument takes, and for even greater clarity, the argument is constructed in term of the simplest models of engines and efficiency concepts.

Performance data show that in eight classes of power generation installations, the larger size is associated with greater efficiency (Fig. 4.1) [57]. The concept of efficiency here is used in the thermodynamics sense, as the ratio between the power output of an engine divided by the rate of heat input to that engine (or divided by the rate of fuel consumption). The efficiency of hydro-turbines exhibits the same size effect: greater efficiency is linked with greater power output, and greater power is associated with larger physical size (Fig. 4.2) [58]. The design analysis of a condenser for concentrated solar power plant showed that the efficiency loss decreases as the condenser face area increases (Fig. 4.3) [59]. In ocean thermal power plants, the efficiency increases as the installed power output (base power) increases [60], as shown in Fig. 4.4, the efficiency increases from 2.5% to 3.5% when the base power increases from 5 MW to 40 MW.

Figure 4.5 shows that the efficiency (η) of the engines for helicopters increases in proportion with the engine size (M_e, kg) raised to a power comparable with 1/4. The H factor is the heating value of the fuel. The data are from the helicopter models adopted throughout the history of helicopter evolution [61]. The data in Fig. 4.5 document the efficiencies of the power plants of all helicopter models, not the efficiencies of helicopters as vehicles.

In summary, the phenomenon of economies of scale is present across the board. Even though in Fig. 4.2 the efficiency data are not aligned completely monotonically with size, they support the broad message of Figs. 4.1–4.5, which is that the bigger are
more efficient. To uncover the physics basis for the phenomenon of economies of scale is the objective of this chapter.

Figure 4.1: The power output effect on the efficiency of several kinds of power plants [57].
Figure 4.2: The effect of size on turbine efficiency [58].

Figure 4.3: The efficiency loss decreases as the condenser area increases [59].
Figure 4.4: The efficiency of ocean thermal power plants versus the base power [60].

Figure 4.5: The correlation between helicopter engine efficiency and engine size. In the indicated correlation, the military helicopter data (the black circles) were not included. If the military data are included, the correlation becomes $\eta_H = 0.53 M_e^{0.25}$, with $R^2 = 0.79$ [61].
4.1 Allocation of size

In Fig. 4.6a, the power plant is modeled [62] as a closed system that operates in steady state between two temperature reservoirs, $T_H$ and $T_L$. The heat input is $\dot{Q}_H$ and the power output $\dot{W}$. In thermodynamics, there is already a sizable literature that is based on simple models of this kind [63]. The irreversibility of power plant operation is made explicit on Fig. 4.6b, where the power plant is viewed as a sandwich of three subsystems. The temperature difference $T_H - T_{HC}$ drives the heat input $\dot{Q}_H$, while the temperature difference $T_{LC} - T_L$ drives the rejected heat current $\dot{Q}_L$:

$$\dot{Q}_H = C_H (T_H - T_{HC}) \quad (4.1)$$

$$\dot{Q}_L = C_L (T_{LC} - T_L) \quad (4.2)$$

$C_H$ and $C_L$ represent the heat transfer conductance of the two temperature gaps, and are proportional to the respective sizes of the heat transfer surfaces that line each gap. The irreversibility of the power plant is concentrated in these two temperature gaps. For simplicity, it is assumed that the intermediate compartment is operating reversibly,

$$\frac{\dot{Q}_H}{T_{HC}} = \frac{\dot{Q}_L}{T_{LC}} \quad (4.3)$$

The total size of the two heat transfer surfaces is represented by the sum of the heat transfer conductances as follows:

$$C = C_H + C_L \quad (4.4)$$
The power output $\dot{W}$ is the same as the power production from the middle compartment:

$$\dot{W}_C = \dot{Q}_H \left(1 - \frac{T_{LC}}{T_{HC}}\right)$$  \hspace{1cm} (4.5)

The efficiency of the power plant, $\eta$, depends on the size of each heat exchanger, $C_H$ and $C_L$,

$$\eta = \frac{\dot{W}_C}{\dot{Q}_H} = 1 - \frac{T_{LC}}{T_{HC}},$$  \hspace{1cm} (4.6)

where

$$T_{HC} = T_H - \frac{\dot{Q}_H}{C_H},$$  \hspace{1cm} (4.7)

$$T_{LC} = T_L - \frac{1 - \frac{\dot{Q}_H}{C_H T_H}}{1 - \left(1 + \frac{C_H}{C_L} \frac{\dot{Q}_H}{C_H T_H}\right)}.$$  \hspace{1cm} (4.8)

Next, we account for the allocation of the conductance inventory ($C$) between the two ends of the power plant ($C_H$ versus $C_L$) by introducing the fraction $x = \frac{C_H}{C}$, therefore

$$C_H = x C \text{ and } C_L = (1 - x) C.$$  \hspace{1cm} (4.9)

Combining Eqs. (4.6) – (4.8) with using Eq. (4.9), we find
\[
\eta = 1 - \frac{T_L / T_H}{1 - \frac{\dot{Q}_H}{T_H C} \left( \frac{1 + 1}{x + 1 - x} \right)}
\]

which shows that the efficiency is maximum at \( x = 1/2 \). In this configuration where the thermal conductance inventory is allocated equally to the hot end and the cold end, the efficiency is

\[
\eta = 1 - \frac{T_L / T_H}{1 - 4Q_H / (T_H C)}
\]

Equation (4.11) shows that the maximized efficiency increases when the size \( C \) increases. This is illustrated in Fig. 4.7. The efficiency versus size is a concave curve: the efficiency increases at a diminishing rate as the size increases. Figure 4.7 also shows that \( \eta \) increases when \( T_H / T_L \) increase. In the limit of infinite size, the irreversibility of the two temperature gaps vanishes, and \( \eta \) approaches the Carnot efficiency, \( 1 - T_L / T_H \).
Figure 4.6: (a) Power plant model as a closed system and (b) power plant model with two temperature gaps and a middle portion that operates reversibly.
Figure 4.7: The effect of the size of total heat transfer surface (C) on the efficiency of the power plant.
4.2 Allocation of irreversibility

Here we consider the question of whether the uniform allocation of size \((C_H = C_L)\) translates into uniform distribution of irreversibility between the hot and cold ends of the power plant. The rate of entropy generation in the hot compartment is

\[
\dot{S}_{\text{gen},H} = \dot{Q}_H \left( \frac{1}{T_{HC}} - \frac{1}{T_H} \right) = \dot{Q}_H \frac{\dot{Q}_H/C_H}{T_H \left( T_H - \dot{Q}_H/C_H \right)} \tag{4.12}
\]

The rate of the entropy generation in the cold compartment is

\[
\dot{S}_{\text{gen},L} = \dot{Q}_L \left( \frac{1}{T_L} - \frac{1}{T_{LC}} \right) = \dot{Q}_H \frac{\dot{Q}_H/C_H}{T_H \left( T_H - \dot{Q}_H/C_H \right) \left( T_H - 2\dot{Q}_H/C_H \right)} \tag{4.13}
\]

The ratio

\[
\frac{\dot{S}_{\text{gen},H}}{\dot{S}_{\text{gen},L}} = 1 - \frac{\dot{Q}_H}{T_H C_H} < 1 \tag{4.14}
\]

shows that when the two compartments are of equal size, the cold compartment generate more entropy than the hot compartment. The total entropy generation rate

\[
\dot{S}_{\text{gen}} = \dot{S}_{\text{gen},H} + \dot{S}_{\text{gen},L} = \frac{\dot{Q}_H/T_H}{CT_{H1}} - 1 \tag{4.15}
\]

Equation (4.15) shows that if the size increases indefinitely, the irreversibility of the power plant vanishes, in agreement with the conclusion reached under Eq. (4.11).
4.3 Allocation of fluid flow volume

Next, we consider the fluid flow irreversibility associated with power generation. The model is presented in Fig. 4.8. This time, instead of heat transfer across finite temperature differences (Fig. 4.6), we isolate the effect of fluid flow friction, or pressure drop. This effect is presented at the hot and cold ends of the power plant, in the two ducts that line the heat transfer surfaces which was discussed in section 4.1.

The irreversible power plant model (Fig. 4.8b) is a sandwich of three compartments: the duct at the hot end, the middle compartment that is assumed to operate reversibly, and the duct located at the cold end. The reversible compartment generates the power output \( \dot{W}_c = \dot{Q}_H (1 - T_L / T_H) \). The actual power output is \( \dot{W} \), and it is smaller than \( \dot{W}_c \),

\[
\dot{W} = \dot{W}_c - \dot{W}_H - \dot{W}_L
\]

This is because of the mechanical power needed to push the working fluid (of flow rate \( \dot{m} \)) through the hot and cold ducts.

The model for estimating \( \dot{W}_H \) and \( \dot{W}_L \) is based on Fig. 4.8c. The duct has the flow cross section \( A_f \), wetted perimeter \( p \), hydraulic diameter \( D_h = 4A_f / p \), swept length \( L \), contact (heat transfer) surface area \( A \), and volume

\[
V = LA_f
\]
For illustration, assume first that the flow is in a fully developed laminar regime. In this case, the pressure drop along the duct is

\[
\Delta P_{\text{lam}} = \frac{4L}{\frac{D_h}{2}} f \frac{1}{2} \rho U^2
\]  

(4.18)

where \( f = \frac{P_o}{Re} \), \( Re = U D_h / v \), \( U = \dot{m} / (\rho A_f) \), and \( P_o \) is the Poiseuille constant, which depends on the shape of \( A_f \) cross-section [19]. Then, Eq. (4.18) becomes

\[
\Delta P_{\text{lam}} = \frac{2L P_o v \dot{m}}{D_h^2 A_f}
\]  

(4.19)

On the other hand, if the flow is fully developed and in fully rough turbulent regime, then Eq. (4.18) holds with \( f \approx \) constant, and yields:

\[
\Delta P_{\text{turb}} = \frac{2Lf \dot{m}^2}{\rho D_h A_f^2}
\]  

(4.20)

Next, we use scale analysis to recognize the \( L \) and \( A_f \) scales

\[
L \sim A^{1/2}, \ A_f \sim D_h L
\]  

(4.21)

such that Eqs. (4.19) and (4.20) become

\[
\Delta P_{\text{lam}} \sim 2P_o v \dot{m} \frac{A^3}{V^3}
\]  

(4.22)

\[
\Delta P_{\text{turb}} \sim \frac{2f \dot{m}^2 A^{5/2}}{\rho V^3}
\]  

(4.23)
Figure 4.8: (a) Power plant as closed system operating irreversibly; (b) model with fluid flow irreversibilities at the hot end and the cold end; and (c) the scales of the three dimensional duct model.
The similarities between Eq. (4.22) and (4.23) are evident. The pumping power dissipated driving fluid flow through the duct flow is \( \dot{W} = \dot{m} \Delta P / \rho \), which means

\[
\dot{W}_{\text{lam}} \sim \frac{2}{\rho} \text{Po} v m^2 \frac{A^3}{V^3} = \frac{K_{\text{lam}}}{V^3}
\]  
(4.24)

where \( K_{\text{lam}} \sim \frac{2}{\rho} \text{Po} v m^2 A^3 \), and

\[
\dot{W}_{\text{turb}} \sim \frac{2}{\rho^2} f m^3 \frac{A^{5/2}}{V^3} = \frac{K_{\text{turb}}}{V^3}
\]  
(4.25)

where \( K_{\text{turb}} \sim \frac{2}{\rho^2} f m^3 A^{5/2} \).

In summary, regardless of the flow regime, the pumping power required by one duct has the form

\[
\dot{W}_{\text{duct}} \sim \frac{K}{V^3}
\]  
(4.26)

for which the expression for \( K_{\text{lam}} \) and \( K_{\text{turb}} \) are given under Eqs. (4.24) and (4.25). Equation (4.26) holds for each of the two ducts shown in Fig. 4.8b,

\[
\dot{W}_H \sim \frac{K_H}{V_H^3}
\]  
(4.27)

\[
\dot{W}_L \sim \frac{K_L}{V_L^3}
\]  
(4.28)

where \( V_H \) and \( V_L \) are the respective duct volumes, the sum of which is the total flow volume constraint,
\[ V = V_{H} + V_{L}, \text{ constant} \]  

Defining the duct volume allocation fraction \( y = V_{H}/V \), such that \( 1 - y = V_{L}/V \), the total pumping power becomes

\[
(\dot{W}_{H} + \dot{W}_{L}) V^{3} \sim \frac{K_{H}}{y^{3}} + \frac{K_{L}}{(1 - y)^{3}}
\]  

(4.30)

This quantity is minimum when

\[
y = \frac{1}{1 + (K_{L}/K_{H})^{1/4}}
\]  

(4.31)

and has the scale

\[
(\dot{W}_{H} + \dot{W}_{L})_{\min} \sim \left( K_{H}^{1/4} + K_{L}^{1/4} \right)^{4} V^{-3}
\]  

(4.32)

This new result reinforces the conclusion that the phenomenon of economies of scale is predictable, because the minimized pumping power loss [Eq. (4.32)] decreases very sharply as the size \( V \) increases. In this configuration, the duct volume \( V \) is allocated according to the fraction

\[
\frac{V_{H}}{V_{L}} = \frac{y}{1 - y} = \left( \frac{K_{H}}{K_{L}} \right)^{1/4}
\]  

(4.33)

The irreversibility, or the dissipation of useful power, is allocated according to the same fraction:
\[
\frac{\dot{W}_H}{\dot{W}_L} = \frac{K_H}{K_L} \cdot \left( \frac{V_H^3}{V_L^3} \right)^{1/4}
\]

(4.34)

In conclusion, the fluid fraction irreversibility is allocated between the two ends of the power plant in the same proportion as the available duct volume. Which end has more duct volume (and pumping power) depends on the ratio \( K_H / K_L \). Four different samples of power plants are considered in Table 4.1. For example, if at the hot end, the \( \dot{m} \) stream is supercritical water at \( T_H = 600^\circ C \) and \( P_H = 27 \) MPa, and if the flow regime is turbulent with \( f \approx 10^{-2} \), then

\[
K_H \approx \left( 3.4 \times 10^{-6} \cdot \dot{m}^3 \cdot A^{5/2} \right) W.m^9
\]

(4.35)

where \( \dot{m} \) and \( A \) are expressed in kg/s and m\(^2\), respectively. If at the cold end the \( \dot{m} \) stream is saturated steam at \( 40^\circ C \), and the flow is turbulent with \( f \approx 10^{-2} \), then

\[
K_L \approx \left( 2 \times 10^{-8} \cdot \dot{m}^3 \cdot A^{5/2} \right) W.m^9
\]

(4.36)

Next, assume that the contact (heat transfer) surface \( A \) is allocated equally between the hot and cold ends, so that the efficiency is maximum. Then, the duct volume allocation ratio \( (V_H / V_L) \) is approximately 3.6. In power plants with superheated steam, the hot-end volume should be roughly 6 times larger than the cold-end volume, cf. cases 3 and 4 in Table 4.1.
Table 4.1: Calculated flow volume ratio for various power plants.

<table>
<thead>
<tr>
<th>Case</th>
<th>Temperature (°C)</th>
<th>Pressure (MPa)</th>
<th>Density (kg/m³)</th>
<th>State</th>
<th>$K \times 10^{-6}$ [W m⁹]</th>
<th>$V_H/V_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>600</td>
<td>27</td>
<td>76.56</td>
<td>Supercritical</td>
<td>3.4</td>
<td>3.60</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>25</td>
<td>89.8</td>
<td>Supercritical</td>
<td>2.5</td>
<td>3.32</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>10</td>
<td>26.06</td>
<td>Superheated</td>
<td>29</td>
<td>6.17</td>
</tr>
<tr>
<td>4</td>
<td>500</td>
<td>8</td>
<td>24.27</td>
<td>Superheated</td>
<td>34</td>
<td>6.39</td>
</tr>
<tr>
<td>Cold end</td>
<td>40</td>
<td>0.0074</td>
<td>992.17</td>
<td>Saturated steam</td>
<td>0.02</td>
<td>-----</td>
</tr>
</tbody>
</table>
4.4 Discussion

This chapter is concluded with a few observations regarding the model used to unveil the physics origin of the phenomenon of economies of scale. This simple model which was used in this analysis is in consistency with the internal combustion engine basics (Fig. 4.1). The engine has a high-temperature and a low-temperature sources, otherwise it would not be an engine. The exergy input to the boiler comes from combustion, and it implies the existence of a “high temperature.” In thermodynamics, the concept of exergy transfer by heat transfer ($E_Q$) is based on three features of the device that experiences exergy transfer:[62] a heat interaction, $Q$, a high temperature, $T_f$, and a low temperature, $T_L$,

$$E_Q = Q \left( 1 - \frac{T_L}{T_f} \right)$$ \hspace{1cm} (4.37)

The exergy “input” to the boiler is the Carnot work that is associated with the heat input and the two temperatures across a Carnot engine. Alternatively, if we know the exergy input, the heat input, and the low temperature, then we know the high temperature $T_f$, which is the “effective flame temperature of combustion as an exergy source”[62]. Furthermore, the high-temperature heat exchanger of internal combustion engine is the piston cylinder that serves as a combustion chamber, and inside this chamber the burning fuel plus air mixture and the cold air intake constitute a “direct contact” heat exchanger. So, the larger size of this direct-contact heat exchanger goes with a more powerful and more efficient engine. This is the size effect indicated for IC engine in Fig. 4.1.
Similarly for the gas turbine power plant (or a car engine), the low temperature heat exchanger is the atmosphere, “the big sewer in the sky,” which receives the hot exhaust from the engine, cools it by mixing (by direct contact, again), and makes it available as cold air at the inlet to the compressor of the gas turbine power plant, or at the inlet to the cylinder of the car engine.

The fact that in some engines some components are not made by humans does not mean that such features do not exist. The discipline of thermodynamics allows us to see all the components, even in devices where nature provides, for free, the missing hardware. In the earth-size engine that drives the atmospheric and oceanic circulation, all the hardware components are “missing” [40].

Back to the analysis in this chapter, the simplest possible models for how heat and fluid flow through a closed system that generates power were chosen. The simplest models, shown in Figs. 4.6 and 4.8, are complicated enough so that they capture the physics, which is the effect of size on efficiency. With these models, it does not mean to suggest that we are representing one of the power plants of Fig. 4.1. The objective of theory is to show how to predict the natural effect in the simplest manner, from the point of view from which the phenomenon is most visible.

An application of the present work is this: if the natural evolutionary trend is toward architectures that flow more easily, and larger architectures are more efficient, then why are not all the flow systems evolving toward being larger and larger?

The reason is that all flow in nature is on an area or in volume, as in Fig. 4.8(c): one stream in, one stream out, and a finite volume that is bathed fully, i.e., vascularized.
The flow is between one point and infinity of points (area, volume). Even though the bigger streams are more efficient carriers, the finite-size area or volume cannot be bathed everywhere by big channels. Small channels are necessary in order to bathe the interstices completely. To sweep the area, few large movers and many small movers must flow together, hierarchically,[40] because this is how movement is facilitated the most on the area or volume.

The chief conclusion is about the physics basis of the economies of scale phenomenon: the bigger should function with fewer losses per unit of size. It is not about predicting the efficiency vs. size curve of a particular flow system that generates power. All the examples compiled in Fig. 4.1 share the features that were included in the models of Figs. 4.6 and 4.8. They have fluid currents that flow through small or large duct cross sections, and heat and mass currents that pass through small or large transfer surfaces.
Chapter 5

Evolutionary design of cooling a plate with one stream

Evolutionary design is a universal physics phenomenon, bio and non-bio [40,64]. It is particularly evident in the evolution of technology. For example, the designs of compact heating or cooling systems evolve incessantly since they have a significant impact on the performance of numerous technologies. Specifically: higher peak temperatures in photovoltaic modules cause a decrease in conversion efficiency [65,66]. The efficiency of monocrystalline and polycrystalline silicon solar cells decreases by 0.45 percent per one-degree temperature increase. The efficiency of amorphous silicon cells also decreases by 0.25 percent per 1 K increase [67]. In package of electronics, the peak temperature limits the operation and survival of the apparatus. The thermal performance of geothermal heat pumps depends on the design of its heat exchanger, which can be a single pipe or channels with arrays of serpentes and meanders [7–12,26,68,69]. Tree-shaped designs for conduction cooling were also proposed for decreasing peak temperatures [70–76].

In this chapter, we determine the relationship between peak temperature and heat flow configuration in a variety of morphing designs with the objective of cooling a heat generating plate with one continuous stream. The design process is evolutionary. We treat this as a basic heat conduction problem, which is a simple model for PV modules cooling systems and for the ground heat exchanger design. The objective is to analyze the influence of the shape and layout of the cooling stream on the peak temperature inside the
heated domain. The ultimate objective is to determine the design direction in which thermal performance increases. A numerical approach is used throughout this chapter.

5.1 Serpentes

Consider the cooling of a heat generating plate, as shown in Fig. 5.1. The first design type is a serpentine-shaped fluid stream embedded in the plate, where the side length is \( L \), and the plate thickness is \( y = 0.1L \). The diameter of the serpentine duct is fixed to \( D_c = 0.01L \), and its length \( L_c \) is assigned to values from 2\( L \) to 4\( L \). The shape of the serpentine is free to morph.

![Figure 5.1: Solid plate with heating from above, and with embedded serpentine cooling (\( N = 3 \) and \( L_c = 3L \)).](image)
We vary the shape of the serpentine by changing the number of elbows N, as shown in Fig. 5.2a. The flow rate through the meandering duct is assumed to be high enough so that the duct can be modeled as isothermal at $T_c$. A uniform heat flux is applied on the upper surface of the square plate. The remaining surfaces of the plate are insulated. The temperature distribution is governed by the heat conduction equation

$$\rho_s c u \nabla T = \nabla \cdot (k_s \nabla T) + Q$$  \hspace{1cm} (5.1)

where $\rho_s$, $c$, and $k_s$ are the solid density, the specific heat and the thermal conductivity, respectively. The convection term is neglected due to the isothermal duct flow assumption.

The peak temperature and its location within the conductive domain change as the serpentine configuration changes. In order to measure the thermal performance for various designs, we introduce the dimensionless temperature difference

$$\tilde{T} = \frac{\Delta T}{\Delta T_{\text{scale}}}$$  \hspace{1cm} (5.2)

where $\Delta T = T_{\text{max}} - T_c$. The scale of the temperature difference is

$$\Delta T_{\text{scale}} = \frac{q y}{k_s}$$  \hspace{1cm} (5.3)

where $y = 0.1L$ is the thickness of the plate.
5.2 Numerical method

Heat is conducted from the solid body to the embedded serpentine stream. The temperature field in the solid is simulated by numerically solving the conduction equation using a package based on the finite element method [39]. The solid plate properties used in the analysis are $c = 678 \text{ J/(kgK)}$, $\rho_s = 2320 \text{ kg/m}^3$, and $k_s = 34 \text{ W/(mK)}$. The boundary conditions are as follows. The fluid in the serpentine duct is isothermal at $T_c = 293 \text{ K}$. The solid volume is fixed. Constant heat flux is applied to the top surface of the plate as $-n \cdot (-k_s \nabla T) = q^*$, where $q^* = 200 \text{ W/m}^2$. The remaining boundaries are insulated.

Table 5.1 was constructed in order to check the mesh accuracy. These computational results are for the serpentine with three turns ($N = 3$) and $L_c = 3L$. They were simulated using a three-dimensional free tetrahedral mesh. The table shows that the peak temperature of the heated domain becomes independent of the mesh size when the number of mesh elements exceeds $5 \times 10^4$. Therefore, the mesh with $10^5$ elements was used throughout this chapter. The numerical procedure consisted of calculating $T_{\text{max}}$ for various serpentine configurations in order to find the dimensionless temperature difference ($\tilde{T}$). The inverse value of $\tilde{T}$ measures the global thermal conductance of the serpentine cooling design.
Table 5.1: Grid independent test.

<table>
<thead>
<tr>
<th>Number of mesh elements</th>
<th>$T_{\text{max}}$(K)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23,605</td>
<td>296.993</td>
<td>---</td>
</tr>
<tr>
<td>28,731</td>
<td>297.086</td>
<td>0.031</td>
</tr>
<tr>
<td>50,287</td>
<td>297.134</td>
<td>0.016</td>
</tr>
<tr>
<td>81,784</td>
<td>297.140</td>
<td>0.002</td>
</tr>
<tr>
<td>111,254</td>
<td>297.140</td>
<td>0.00008</td>
</tr>
</tbody>
</table>

5.3 The effect of serpentine shape

In the first set of numerical simulations, the coolant inlet and outlet were positioned at $x = 0.5L$, as shown in Fig. 5.1. The number of turns varied from the smallest value of $N$, which is $N = 1$ at $L_c = 2L$, to $N = 2$ at $L_c = 3L$, and $N = 3$ at $L_c = 4L$, all the way to $N = \infty$ where the serpentine reduces to a straight band. Figure 5.2a shows that in case of $L_c = 3L$, the smallest value of $N$ is 2. The temperature fields that correspond to Fig. 5.2a are shown in Fig. 5.2b, in which the temperature uniformity increases as $N$ increases to $N = 4$, but then starts decreasing where the hot spot area begins to spread. The locations of the hot spots are indicated with arrows.

Figure 5.3 shows the effect of varying the number of elbows of the serpentine. The total length of the serpentine also changes, $L_c = 2L$, $3L$, and $4L$. In all cases, the minimum $\bar{T}$ occurs when $N$ is around 4. For $N$ values greater than 4, $\bar{T}$ gradually increases as $N$ increases, and the effect of increasing $N$ becomes negligible. Figure 5.3
also shows that increasing the serpentine pipe length $L_c$ leads to better performance, as the dimensionless temperature $\tilde{T}$ decreases.
Figure 5.2: (a) How the serpentine shape changes, and (b) the corresponding temperature fields as the number of elbows $N$ changes while $L_c = 3L$. 

82
Figure 5.3: The dimensionless temperature difference $\tilde{T}$ as function of $N$ and $L_c$ for the serpentine design.
Consider next the effect of changing the location of the duct inlet and outlet. Of interest is the optimal inlet and outlet location, which varies from the center of the heated plate (x = 0.5L) to its side (x = 0). For illustration, in Fig. 5.4a, the number of elbows is fixed at N = 2 and the serpentine length is Lc = 2L. The optimal location of the serpentine inlet and outlet is determined such that the dimensionless temperature difference $\tilde{T}$ is minimum. Figure 5.4b shows the temperature fields that correspond to Fig. 5.4a. Among the configurations shown in Fig. 5.4, the serpentine configuration with x = 0.2L provides greater temperature uniformity.

Figure 5.5 shows the effect that the position of the inlet and outlet has on the dimensionless temperature difference $\tilde{T}$. The minimal $\tilde{T}$ corresponds to x = 0.2L for both Lc = 2L and Lc = 3L. This means that when the inlet and the outlet are located at x = 0.2L, the serpentine cools the domain more uniformly.
Figure 5.4: (a) Three cases showing how the position (x) of the inlet and the outlet changes, and (b) the corresponding temperature fields.
Figure 5.5: The effect of the inlet and outlet position on the dimensionless temperature $\tilde{T}$. 
5.4 Loops

The freedom to morph the cooling configuration provides the opportunity to consider an entirely different class of designs, such as the one-loop design of Fig. 5.6. Three loop shapes are considered, square, circular and clover leaf. The loop length $L_c$ varies from a small loop in the center of the plate, to the largest loop, which touches the edges of the plate. The inlet and the outlet of the stream are located in the middle of one of the plate sides.

Figure 5.7 shows the temperature fields associated with the square and the circular loops. The stream length varied from $L_c = L$ to the possible maximum length $L_{c,\text{max}}$, which for the square loop is $4L$, and for the circular loop is $\pi L$. As the loop length ($L_c$) increases, the hot spots are initially confined to the corners of the plate and then jump suddenly to the plate center.

Figure 5.6: How the cooling stream changes for the squared and circular loops.
Figure 5.7: Temperature fields of the heat generating plate for the square and circular loops with various lengths.
The cooling system shaped as clover leaf is shown in Fig. 5.8. The first design (Fig. 5.8a) is a clover leaf shape that has thinner lobes than in the second design (Fig. 5.8b). The main constraint for both clover designs is the total stream length $L_c$ which varies from $L$ to $L_{c,\text{max}}$. The maximum loop length is 4.5$L$ for the first clover design and approximately 4.2$L$ for second one. To recall, the objective is to make the temperature distribution more uniform throughout the heated plate. Consequently, the overall dimensionless temperature difference is minimized by morphing the design.

The effect of clover evolution is shown in Fig. 5.9, where the hot spot regions became narrower as the stream length $L_c$ increases. From the contours of the isotherms on the right side of Fig. 5.9, we see that the temperature distribution uniformity is associated to a loop length $L_c$ of approximately 3$L$. Increasing the loop length further causes the hot spots to move from the corners to the center of the heated plate.

![Diagram showing clover leaf designs as cooling systems; (a) thinner, and (b) wider lobes.](image)

**Figure 5.8:** The clover leaf designs as cooling systems; (a) thinner, and (b) wider lobes.
Figure 5.9: Temperature fields of the heat generating plate for the two clover designs with various loop lengths.
The critical loop length associated with this sudden move is also linked to the best design, where $\tilde{T}$ is minimum. Indeed, Fig. 5.10 shows that the minimum peak temperature obtained with four loop designs (circular, square, clover design 2, clover design 1) occurs when $L_c/L = 2.5, 2.8, 3.2$ and 4.1, respectively. Interesting is that the minimum peak temperature is gradually decreasing as the cooling system configuration is morphing from the circular loop to the first clover design. We found that the peak temperature for the first clover design is the lowest among all loop designs, namely $\tilde{T} \approx 3$. This minimum is also lower than the minima achieved with serpentines of the various length in Fig. 5.3 and Fig. 5.5. The optimal designs of the four configurations are summarized in Table 5.2.
Figure 5.10: The dimensionless temperature difference $\tilde{T}$ as function of $L_c/L$ for the squared, circular and clover leaf loops.
Table 5.2: Optimal designs summary.

<table>
<thead>
<tr>
<th>Configuration type</th>
<th>$L_c / L$</th>
<th>$\hat{T}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serpentine with $N = 4$</td>
<td>2</td>
<td>9.44</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6.48</td>
</tr>
<tr>
<td>Serpentine with deviated stream inlet and outlet (x = 0.2 L)</td>
<td>2</td>
<td>5.56</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3.64</td>
</tr>
<tr>
<td>Circular loop</td>
<td>2.5</td>
<td>5.07</td>
</tr>
<tr>
<td>Squared loop</td>
<td>2.8</td>
<td>5.05</td>
</tr>
<tr>
<td>Clover design 2</td>
<td>3.2</td>
<td>3.95</td>
</tr>
<tr>
<td>Clover design 1</td>
<td>4.1</td>
<td>2.98</td>
</tr>
</tbody>
</table>
5.5 Discussion

In this chapter, we considered various designs of the serpentine cooling system. First, the pipe inlet and outlet were located at the center of the heated plate and the number of turns varied for different values of serpentine length. The best serpentine configuration which has lower peak temperature was found to be at $N = 4$. This finding was the same for other serpentine lengths such as $L_c = 2L, 3L, 4L$. Second, the stream inlet and outlet locations were changed from the center of the plate to its side. The results show that the thermal performance increased when the stream inlet and outlet position was located at $x = 0.2L$. This configuration shows better temperature uniformity across the heated plate.

Additionally, square, circular and clover leaf loops were considered, and their lengths varied. The inlet and the outlet of the stream were located in the middle of the plate side. The results show that each design has a particular loop length such that the peak temperature is minimal. The circular loop was able to maintain the minimal peak temperature ($T \cong 5$) with a shorter loop length than in the other configurations. With a longer loop length, the clover design 1 reduced the peak temperature to $T \cong 3$. Consequently, the first clover configuration with $L_c/L = 4.1$ is the superior design.
Chapter 6

Conclusions and Recommendations

The implications of the work presented in this dissertation are of fundamental character. The conclusion that morphing the flow configuration freely is the key to higher performance and is generally applicable, especially in the field of ground-coupled heat pumps for thickly settled urban areas as shown in chapter 2. Future studies may employ the used method in more realistic models that incorporate the features that were neglected in the present model. Options for making the model more realistic are the inclusion of temperature changes in the environment, and boundary effects such as the presence of additional one-loop assemblies in the close proximity of the system.

In the third chapter, we illustrated the search for optimum cross-section of the flow in an assembly with identical array of tubes in counterflow with two objectives at the same time: reduced fluid flow resistance and reduced heat transfer resistance. We performed the scale analysis for individual elements (pipe flow). The fluid flow and heat transfer were simulated using a 3-D non-isothermal COMSOL model. The element cross sections which were considered in this chapter are square, circular, and equilateral triangular. The element wall material was fixed, while the shape and length of the wetted perimeter of the flow channel were free to change. In summary, the array of triangular channels meets the two objectives better than the array of square channels.

In the fourth chapter, we showed that the phenomenon of economies of scale is predictable from pure physics, and consequently it is present in all flow systems that experience evolutionary changes in their configurations toward greater global...
performance. The physics basis was demonstrated by considering a power plant model with heat transfer and fluid flow irreversibility distributed between the hot end and the cold end. The total heat transfer surface inventory (the thermal conductance, C) and the total fluid flow volume (V) were fixed. We presented that the overall efficiency increases at decreasing rate as the thermal conductance inventory increases. This means that the predicted efficiency-size curve must be concave, in accord with the empirical data observed in the literature [57–61]. We also found that the total pumping power loss decreases proportionally with $V^{-3}$ as total flow volume increases. The convexity of the power loss vs size curve means that the corresponding effect of the volume on efficiency is $\eta$-V curve which is also concave.

The fundamental path outlined in chapter 4 deserves to be explored in greater detail, especially in applied physics, animal design, economics and technology evolution.[40,51,52]. In brief, the flow design is an architecture that changes predictively according to the size of the system Magnifying or miniaturizing a known design is not the way to discover the proper configuration and performance of a larger and, respectively, smaller flow system.

In sum, the thermodynamics presented in Chapter 4 showed that the economies of scale phenomenon is a fundamental feature of all flow (moving) systems, animate, inanimate, and human made. The size effect on efficiency manifests itself in other features of design, distinct from the power generation considered in our analysis. For example, the rate of heat loss from a furnace, a cooking vessel, or the body of an animal is smaller per unit of body size (mass, volume).
Chapter 5 demonstrates numerically the effect of evolving the design of the cooling system on the temperature distribution of a three-dimensional conducting body. The dimensionless temperature difference, which is associated with the peak temperature, was calculated for various serpentine and loop configurations. The temperature uniformity of these configurations was investigated. The conclusions of this work are applicable to improving the thermal performance of PV modules, solar panels and electronics packaging.
References


Biography

Abdulrahman Almerbati was born in 1987 in Al-Khobar, Saudi Arabia. He received his B.S. (2009) and M.S. (2012) from King Fahd University of Petroleum & Minerals, KFUPM, in Mechanical Engineering. At KFUPM, he started his career life as a research assistant in 2009 and was promoted to a lecturer in 2012. After that, he earned a scholarship to study abroad, based on which his PhD study at Duke University was funded. He worked as a teaching assistant for several graduate and undergraduate in the department of Mechanical Engineering and Material Science at Duke University. His research accomplishments are represented by the one patent and six peer reviewed journal articles:


