Nanosecond Shock Wave-Induced Surface Acoustic Waves and Fracture at Fluid-Solid Boundaries

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Mechanical Engineering and Materials Science in the Graduate School of Duke University

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ABSTRACT

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Abstract

Shock wave lithotripsy revolutionized kidney stone disease treatment after its introduction in the early 1980s. However, researchers still have not fully understood the mechanism how a shock wave pulverizes kidney stones. Recent research shows known mechanisms working in the early stage of the treatment may not be as effective in the later stage of the treatment due to the shrinkage of the size and irregular geometry of the kidney stone fragments. Some pioneering studies have indicated that surface acoustic waves (SAWs) may contribute to stone fragmentation especially in the later stage of the treatment. In this study, the mechanism whereby a lithotripsy shock wave induces SAW and causes damage is investigated. Specifically, the generation and propagation characteristics of leaky Rayleigh waves (LRWs) by a spherically divergent shock wave incident on a glass-water boundary are investigated both experimentally and numerically. In the experiment, the spherical divergent shock wave was generated by a nano pulse lithotripter (NPL). The rise time and pulse duration of this shock wave are approximately 60 ns and 300 μs. The numerical study shows that the maximum tensile stress produced in the solid reaches almost 400 MPa when the standoff distance between the NPL probe tip and the glass/water boundary was set to 0.5 mm. The maximum tensile stress produced in the solid is formed on the boundary, which is caused by the dynamic interaction between the LRWs and an evanescent wave generated concomitantly along the
fluid-solid interface. The resultant tensile stress field drives the initiation of pre-existing microcracks and their subsequent extension along a circular trajectory, confirmative with the direction of the principal stress on the boundary. We have further demonstrated that this unique ring-like fracture, prevalent in damage produced by high-speed impact, can be best described by the Tuler-Butcher criterion for dynamic brittle failure, and the orientation of the ring fracture extension into the solid also follows closely the trajectory of the local maximum tensile stress distribution. A Tuler-Butcher criterion described damage threshold was to by around $2.4 \times 10^{-4}$ MPa·s. This study demonstrates quantitatively for the first time that the ring-like fracture is induced by the SAW and, in addition, uncovers the mechanism whereby impulsive SAWs produce dynamic fracture in brittle (glass) materials.
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1. Introduction

1.1 Shock wave lithotripsy (SWL)

Shock wave lithotripsy is a medical technology that utilizes extracorporeally generated and focused shock waves to break up kidney stones under the guidance of bi-fluoroscopic imaging without traumatizing surrounding tissues (C. Chaussy et al., 1982). Since its introduction in the early 1980s, SWL has revolutionized the surgical management of urolithiasis (C. G. Chaussy & Fuchs, 1989; Lingeman, McAteer, Gnessin, & Evan, 2009). Contemporary shock wave lithotripters have evolved progressively within the technology for shock wave generation, focusing, patient coupling, imaging and overall functionality of the system (Rassweiler et al., 1992). Despite this, the efficiency of shock wave lithotripsy has not improved appreciably in the past two decades. The first generation, Donier HM3 electrohydraulic lithotripter, is still considered as the “gold standard” lithotripter in terms of the treatment efficiency (Lingeman, 2003).

1.2 Stone comminution mechanisms in SWL

The lack of progression in stone treatment efficiency has been attributed, in part, to an incomplete understanding of the fundamental mechanisms and associated dynamic processes responsible for stone comminution. Multiple mechanisms of stone fragmentation have been proposed, which can be categorized into cavitation related and stress wave driven stone fragmentation mechanisms. The tensile tail of the SWL pulse
initiates and drives the growth of bubbles from nucleation sites (Church, 1989; Lokhandwalla & Sturtevant, 2000). The bubble activities were captured by high-speed photography (Sass et al., 1991). The asymmetric collapse of bubbles adjacent to a stone surface leads to small jet of liquid to impinge onto the stone, inducing a local transient compressive stress field in the stone (Crum, 1988; Lomonosov & Hess, 2002; P Zhong & Chuong, 1993; P. Zhong, C. J. Chuong, & G. M. Preminger, 1993) This compressive stress field and its propagation may cause surface pitting and possibly stone fragmentation (Lomonosov & Hess, 2002). In addition, different types of stress waves inside the stone and at the fluid-solid interface can be generated by the direct shock wave-stone interaction, depending on the geometry and size of the stone, as well as the beam width of the lithotripter field. These elastic waves propagate and then interfere with each other after their reflection from the lateral and back surface of the stone. This interference creates a transient and non-uniform stress field with high stress concentration built up towards the backside of the stone. This high stress concentration may initiate the opening and propagation of pre-existing micro-cracks inside the stone (Zhang, Nault, Mitran, Iversen, & Zhong, 2016). In brittle materials, such as kidney stones, three primary modes of stress-induced failure modes exist: mode I, opening mode due to tensile stress applied normal to the plane of the crack; mode II, sliding mode due to shear stress applied normal to the plane of the crack front; mode III, tearing mode due to shear stress acting parallel to the
plane of the crack and the crack front (Lawn, 1993). Generally, a single shock wave can hardly fracture a kidney stone; instead, multiple shocks are required to initiate fracture and a few thousands of shock waves are needed to pulverize the stone to fine fragments for spontaneous discharge. The stone comminution process is best described by the concept of dynamic fatigue based on, for example, the cohesive zone model (Lomonosov & Hess, 2002; Ortiz, 1988) in which tensile stress can facilitate the opening of pre-existing micro-cracks, which eventually coalesce with each other and with adjacent macro-cracks to form a fracture (Lokhandwalla & Sturtevant, 2000). Strong tensile stress inside a kidney stone could be induced by different mechanisms. For example, a reflected longitudinal (P) wave in tension from the posterior interface of a stone may superimpose with the tailing tensile component of the forward propagating longitudinal wave to induce a high tensile stress near the posterior surface of the stone to cause spalling damage (Xi & Zhong, 2001). Beside longitudinal waves, shear (S) waves generated at curved surface may also interfere with the reflected P wave to induce a strong tensile stress along the central axis of the stone and near the posterior surface (Cleveland & Sapozhnikov, 2005; Sapozhnikov, Maxwell, MacConaghy, & Bailey, 2007). After the incident angle of the shock wave exceeds the critical angle for the S wave, evanescent waves are generated at the interface, which produce a moving compression zone within the subsurface area. This compression zone was postulated to squeeze the stone and lead to stone crash. This evanescent waves
related stone fragmentation mechanism has been referred to as “quasi-static squeezing “(Eisenmenger, 2001).

1.3 Surface Acoustic Waves (SAWs) in SWL

The stress waves related mechanisms mainly contribute in the early stage of the SWL treatment. In the later stage of the treatment, as a stone becomes irregular and its size decreases, the stress waves induced internal tensile stress drop dramatically due to destructive interference (Zhang, Nault, Mitran, Iversen, & Zhong, 2016). This varying stone fragmentation mechanisms in the early stage and later stage of the SWL treatment is also manifested by the changes in the rate of stone comminution throughout the treatment. In particular, the SWL treatment has been shown to be comprised of two distinctive phases: the early accelerating phase and the later decelerating phase (Pei Zhong, 2013).

Surface acoustic wave induced damage was proposed as a potential stone fragmentation mechanism in the later stage of the treatment (Pei Zhong, 2013). In theory, surface acoustic wave can be generated in SWL by either a direct shock wave impact or a cavitation induced jet impingement. A surface acoustic wave propagating at the fluid-stone boundary may interact with surface defects produced by cavitation pitting. This interaction will cause local stress concentration, and may lead to crack extension from the exterior surface to the interior of the stone.
Up to now, limited evidence is available to demonstrate the existence of SAW in SWL. Based on numerical simulations, it was suggested that SAW could be produced either in a spherical or cylindrical stone by an incident lithotripter pulse (Cleveland & Sapozhnikov, 2005). Moreover, a tensile stress hot spot observed in the posterior apex of the spherical stone was attributed to the SAW (Cleveland & Sapozhnikov, 2005). Besides numerical evidence, ring fractures were observed on the front surface of a slab stone phantom subjected to highly focused shock waves. This ring fracture was hypothesized to be created by the interaction between a SAW and a reflected P wave from the back surface, both of which were generated during the shock wave impact (Pei Zhong, 2013). However, there is still no comprehensive investigation into the generation of SAWs in stones by a shock wave impact and how SAWs may contribute to stone fracture.

1.4 Summary of this study

This study uncovers the mechanism whereby a lithotripsy shock wave induces leaky Rayleigh waves (LRWs) at a fluid-solid (glass) boundary, which, upon propagation, cause ring-like fractures on the glass surface. The generation of LRWs by a spherical shock wave on a glass-water boundary was investigated both experimentally and numerically. The glass sample has similar acoustic properties and a fracture toughness an order of magnitude higher than those of hard kidney stones (P. Zhong, C. Chuong, & G.
Preminger, 1993). Compared to kidney stones or artificial kidney stones, glass samples have controllable quality and are optically transparent, which facilitates the experimental study of the characteristics of crack initiation and extension produced by the shock wave impact. A concrete correlation has been established between the LRW induced characteristic tensile stress field and the crack patterns. These results will provide insights into future studies on SAW in SWL.

Photoelastic/shadowgraph images were used to collect experimental evidence for the generation of SAWs by shock wave impact. The generation of LRWs was confirmed experimentally, and furthermore, validated by a finite element model analysis of the shock wave interaction at the fluid-solid interface. Thereafter, the model was utilized to investigate the characteristics of LRW generation by an incident spherical shock wave at various standoff distances from the fluid-solid boundary. The characteristic distribution of the LRW induced tensile stress field at the glass-water surface was found to be attributed to the counteractive interaction between LRWs and the evanescent waves. The surface fracture patterns and inner structure of the cracks were examined by optical microscopy and laser confocal microscopy, respectively. The typical ring-like fracture patterns were found to be induced by LRW induced tensile stress field. The initiation of this ring-like fracture was found to be a dynamic fatigue process and is best described by the Tuler-Butcher criterion.
2. Stress waves in solids

Various types of stress waves are generated during LSW-stone interactions. The stress waves that penetrate into the interior of the stone after their generation are called bulk waves, including P wave and S wave. Other stress waves propagating along the fluid-solid interface are known as surface acoustic waves (SAWs). Tensile or shear stresses induced by the bulk waves and SAWs may initiate cracks and drive their extension and propagation inside the stone, eventually leading to fracture.

2.1 Solid mechanics

The concept of strain $S_{ij}$ describes the deformation of an elastic body induced by mechanical stress in general, or by acoustically excited vibration more specifically in this dissertation work. The strain is related to the particle displacement in the material $u_i$ through the following equation

$$ S_{ij} = \frac{\partial u_i}{\partial x_j} \quad (2-1) $$

The restoring force for the deformation is defined in terms of the stress field $\sigma_{ij}$, which is related to the strain field by the Hook’s law

$$ \sigma_{ij} = c_{ijkl}S_{kl} \quad (2-2) $$
in which $c_{ijkl}$ is the elastic stiffness constant. This equation is also called the elastic constitutive equation. In addition, the equation of motion, complements the elastic constitutive equation to lay out the foundations of wave theory in solids

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2}$$

(2-3)

in which $\rho$ is the density of the medium. From Eq. 2-1, 2-2, 2-3, we can derive the general equation for elastodynamics.

$$\frac{\partial}{\partial x_j}(c_{ijkl} \frac{\partial u_k}{\partial x_l}) = \rho \frac{\partial^2 u_i}{\partial t^2}$$

(2-4)

### 2.2 P and S waves

In an isotropic elastic medium, Eq. 2-4 is given by:

$$(\lambda + 2\mu)\nabla \cdot \mathbf{u} - \mu \nabla \times \nabla \times \mathbf{u} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}$$

(2-5)

where $\lambda, \mu$ are the Lame constants, and $\rho$ is the density of the material. Using the Helmholtz’s theorem, $\mathbf{u}$ can be expressed in terms of a scalar potential ($\Phi$) and a vector potential ($\Psi$).

$$\mathbf{u} = \nabla \Phi + \nabla \times \Psi$$

(2-6)

By substituting Eq. 2-6 into Eq.2-5 and with additional manipulations, we can obtain two wave equations.

$$\nabla^2 \Phi - \frac{1}{c_t^2} \frac{\partial^2 \Phi}{\partial t^2} = 0, c_t = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

(2-7)
In this way, we decompose the particle displacement in the medium into dilatational displacement and rotational displacement corresponding to the disturbance induced by a longitudinal (L or P) wave and a transverse (T or S) wave, respectively, which propagate independently. In Eqs. 2-7 and 2-8, \( c_l \) and \( c_t \) represent the longitudinal and transverse wave speeds, respectively. For a harmonic motion, the solutions of Eq. 2-7, 2-8 have the form

\[
\Phi(x, y, z) = \Phi(x, y, z) \exp(-i\omega t) \quad (2.9)
\]

\[
\Psi(x, y, z) = \Psi(x, y, z) \exp(-i\omega t) \quad (2.10)
\]

in which \( x, y, z \) are the coordinates in a Cartesian coordinate system, \( \omega \) is the angular frequency of the harmonic motion.

Substituting Eqs. 2-9 and 2-10 into Eqs. 2-7 and 2-8, respectively, we obtain the Helmholtz equations

\[
\nabla^2 \Phi(x, y, z) + k_l^2 \Phi(x, y, z) = 0 \quad (2.11)
\]

\[
\nabla^2 \Psi(x, y, z) + k_t^2 \Psi(x, y, z) = 0 \quad (2.12)
\]

in which \( k_l = \frac{\omega}{c_l}, k_t = \frac{\omega}{c_t} \) are the wave number of the longitudinal wave and transverse wave, respectively. The general solutions of Eqs 2-11, 2-12 corresponding to plane longitudinal and transverse waves in an infinite medium are:

\[
\Phi(x, y, z) = Ae^{iklr} \quad (2.13)
\]
\[ \psi(x, y, z) = Be^{ikr} \]

in which \( A, B \) are arbitrary constants, \( r \) is the position vector in the Cartesian coordinate system.

P and S waves are the two most fundamental modes of waves in isotropic elastic media. The displacement of the medium excited by a P wave is parallel to the propagation direction of the wave; the displacement of the medium excited by an S wave is perpendicular to the wave traveling direction.

### 2.3 Surface Acoustic Waves

Lord Rayleigh investigated another type of wave propagating along a solid-vacuum boundary, whose disturbance is largely confined to the neighborhood of the boundary. This type of surface wave, named after Lord Rayleigh, is called Rayleigh wave. Let us first consider a plane harmonic Rayleigh wave propagating along the \( x \) axis (Fig.1).

\[ \text{Figure 1. The coordinate system and the solid-vacuum boundary.} \]
Since the plane wavefront of the Rayleigh wave is parallel to the y axis, the motion is independent of the y coordinate. The disturbance of the Rayleigh wave is confined to the boundary; therefore, its magnitude decays along the z axis. Generally, this wave can be expressed as (neglecting the harmonic term)

\[ \Phi(x, z) = F(z)e^{ikx} \] (2-15)

\[ \Psi(x, z) = G(z)e^{ikx} \] (2-16)

in which \( k \) is the wave number in the X direction.

Substituting these expressions into Eqs. 2-15 and 2-16, we obtain the expressions for \( F(z) \) and \( G(z) \)

\[ F(z) = A e^{-qz} \] (2-17)

\[ G(z) = B e^{-sz} \] (2-18)

in which \( A, B \) are arbitrary constants; \( q^2 = k^2 - k_l^2 \), \( s^2 = k^2 - k_t^2 \). Thus, for a plane harmonic Rayleigh wave propagating along the X direction,

\[ \Phi(x, z, t) = A e^{-qz} e^{i(kx - \omega t)} \] (2-19)

\[ \Psi(x, z, t) = B e^{-sz} e^{i(kx - \omega t)} \] (2-20)

At a solid-vacuum boundary, there is no traction force in the vacuum side; therefore, boundary conditions are:

\[ \sigma_{zz} = 0, \sigma_{xz} = 0 \] (2-21)

The stress field could be determined from the displacement field using the constitutive relations of the elastic medium.
\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{zz} \\
\sigma_{xz}
\end{bmatrix} = 
\begin{bmatrix}
\lambda + 2\mu & \lambda & \lambda \\
\lambda & \lambda + 2\mu & \lambda \\
\lambda & \lambda & \lambda + 2\mu
\end{bmatrix}
\begin{bmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial w}{\partial z} \\
\frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)
\end{bmatrix} \tag{2-22}
\]

\(u, w\) are the components of the displacement \(u\) in the \(x\) and \(z\) direction, respectively.

From Eq. 2-6, \(\sigma_{zz}, \sigma_{xz}\) can be represented by \(\Phi, \Psi\) as:

\[
\sigma_{zz} = \lambda \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) + 2\mu \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial x \partial z} \right) \tag{2-23}
\]

\[
\sigma_{xz} = \mu \left( \frac{\partial^2 \Phi}{\partial x \partial z} + \frac{\partial^2 \Phi}{\partial x^2} - \frac{\partial^2 \Psi}{\partial z^2} \right) \tag{2-24}
\]

Applying the boundary conditions (Eq. 2-21), we have

\[
\sigma_{zz} = \lambda \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) + 2\mu \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial x \partial z} \right) = 0 \tag{2-25}
\]

\[
\sigma_{xz} = \mu \left( \frac{\partial^2 \Phi}{\partial x \partial z} + \frac{\partial^2 \Phi}{\partial x^2} - \frac{\partial^2 \Psi}{\partial z^2} \right) = 0 \tag{2-26}
\]

Substituting the expressions of \(\Phi\) and \(\Psi\) (Eqs 2-25, 2-26), we obtain relations between \(A, B\), and the wave number \(k\). Replacing \(B\) by \(A\), the expressions of \(\Phi\) and \(\Psi\) become:

\[
\Phi(x, z, t) = -A e^{-aqz} e^{i(kx - \omega t)} \tag{2-27}
\]

\[
\Psi(x, z, t) = iA \frac{2kq}{k^2 + s^2} e^{-sz} e^{i(kx - \omega t)} \tag{2-28}
\]

By plugging Eqs. 2-27,2-28 into Eqs. 2-25, 2-26, we obtain the characteristic equation for \(k\)

\[
\left( \frac{k}{k_t} \right)^6 - 8\left( \frac{k}{k_t} \right)^4 + 8 \left[ 3 - 2\left( \frac{k}{k_t} \right)^2 \right] \left( \frac{k}{k_t} \right)^2 - 16\left[ 1 - \left( \frac{k}{k_t} \right)^2 \right] = 0 \tag{2-29}
\]
This equation has six roots in total. The value of one of these roots is larger than \( k_l \) and \( k_t \), corresponding to the wave number of the Rayleigh wave, \( k_R \). This root always exists for any values of Lame constants for elastic media. Since \( q = \sqrt{k_R^2 - k_l^2} \) and \( s = \sqrt{k_R^2 - k_t^2} \) are both positive real numbers, the longitudinal and transverse mode of the Rayleigh wave decay in depth by \( e^{-qz} \) and \( e^{-sz} \), respectively. The speed of the Rayleigh wave, \( c_R (= \frac{\omega}{k_R}) \), is smaller than the \( c_l (= \frac{\omega}{k_l}) \) and \( c_t (= \frac{\omega}{k_t}) \).

At an inviscous fluid-solid boundary, there is slippage but no penetration of materials from one side to the other. Therefore, the particle displacement is continuous only along the boundary normal direction, i.e.,

\[
w_o = w
\]  

(2-30)
in which \( w_o \) and \( w \) are the component of the particle displacement in the \( z \) direction in the fluid and solid, respectively (Fig.2). The balance of force at the two sides of the boundary demands
To describe the fluid field, we also introduce a velocity potential $\varphi$. In the fluid, the displacement

$$\mathbf{v} = \nabla \varphi$$

In the fluid, the displacement potential also satisfies the wave equation

$$\nabla^2 \varphi - \frac{1}{c_0^2} \frac{\partial^2 \varphi}{\partial t^2} = 0$$

Here we consider one of the general solutions of Eq.2-33, corresponding to a plane harmonic plane wave propagating obliquely.

$${\sigma}_{zz} = -p, {\sigma}_{xz} = 0 \quad (2-31)$$
\[ \varphi = C e^{i(kx-hz-\omega t)} \]  \hspace{1cm} (2-34)

in which \( k, h \) is the wave number along the x and z direction, respectively; and \( k^2 + h^2 = k_o^2 \), \( k_o = \frac{\omega}{c_o} \) is the wave number in the fluid. To utilize the boundary conditions, we first represent \( p, w_o, w \) by potentials as

\[ p = -i \rho_o \omega \varphi \]  \hspace{1cm} (2-35)
\[ w_o = -i \frac{\partial \varphi}{\partial z} \]  \hspace{1cm} (2-36)
\[ w = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x} \]  \hspace{1cm} (2-37)

Substituting the representations of \( p, w_o, w, \sigma_{xz}, \sigma_{zz} \) by potentials into the boundary conditions (Eqs. 2-30, 2-31)

\[ -i \frac{\partial \varphi}{\partial z} = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x} \]  \hspace{1cm} (2-38)
\[ \lambda \left( \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} \right) + 2\mu \left( \frac{\partial^2 \varphi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x \partial z} \right) = i \rho_o \omega \varphi, \mu \left( \frac{\partial^2 \varphi}{\partial x \partial z} + \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2} \right) = 0 \]  \hspace{1cm} (2-39)

Substituting the expressions of \( \Phi, \Psi, \varphi \) (Eqs. 2-19, 2-20, 2-34) into Eqs. 2-38, 2-39, we obtain the relations for \( A, B, C, \) and \( k \). The characteristic equation for \( k \) is given by:

\[ \left( \frac{k_o}{k} \right)^6 - 8 \left( \frac{k_o}{k} \right)^4 + 8[3 - 2 \left( \frac{k_l}{k} \right)^2] \left( \frac{k_o}{k} \right)^2 - 16[1 - \left( \frac{k_l}{k} \right)^2] = i \frac{\rho_o}{\rho} \sqrt{\frac{k^2 - k_l^2}{k_o^2 - k^2}} \]  \hspace{1cm} (2-40)

in which \( \rho_o \) is the fluid density. This characteristic equation differs from the characteristic equation Eq. 2-29 for a Rayleigh wave only by the presence of the additional term on the right hand side. If the fluid density \( \rho_o \to 0 \), this characteristic equation will degenerate into Eq. 2-29. Therefore, this equation could have a root \( k_R' \), which tends to \( k_R \) as \( \rho_o \to 0 \).
The condition for this root to exist is that the speed of the transverse wave, \(c_t\), in the solid is larger than the speed of the sound, \(c_o\), in the fluid. \(k'_R\) is always a complex number. Therefore, the associated wave also decays as it propagates along the fluid-solid boundary. This wave is thus called the leaky Rayleigh wave or "generalized Rayleigh wave", with a propagation speed, \(c'_R = \frac{\omega}{\text{Re}(k'_R)}\), satisfying

\[ c_o < c_R < c'_R < c_t < c_l \]  

(2-41)

Accordingly, the wave number of the leaky Rayleigh wave \(k'_R\) is found to be

\[ k_i < k_t < k'_R < k_R < k_o \]  

(2-42)

We can also replace \(B, C\) in Eqs.-13, 14, 34 by \(A\) and obtain

\[ \Phi = -A e^{i(k'_R x - \omega t)} e^{-\sqrt{(k'_R)^2 - k_i^2} z} \]  

(2-43)

\[ \Psi = iA \frac{2k'_R}{2(k'_R)^2 - k_t^2} e^{i(k'_R x - \omega t)} e^{-\sqrt{(k'_R)^2 - k_R^2} z} \]  

(2-44)

\[ \varphi = A\omega \frac{\sqrt{(k'_R)^2 - k_t^2}}{\sqrt{k_o^2 -(k'_R)^2 [2(k'_R)^2 - k_t^2]}} e^{i(k'_R x - \sqrt{k_o^2 -(k'_R)^2} z - \omega t)} \]  

(2-45)

The term \(e^{-\sqrt{(k'_R)^2 - k_i^2} z}\), \(e^{-\sqrt{(k'_R)^2 - k_R^2} z}\) in Eqs.2-43, 2-44 indicate that the leaky Rayleigh wave decays exponentially into the depth of the solid. In the fluid, however, the component of the leaky Rayleigh wave propagates obliquely into the interior of the fluid. This fluid component is called "Schmidt head wave" or "Mintrop wave" (H.UBERALL, 1973). This
leaking head wave leads to the decaying of the leaky Rayleigh wave during its propagation along the boundary.

Besides the complex root corresponding to the wave number of the leaky Rayleigh wave, the characteristic equation always has another real root, \( k_s \), which corresponds to the wave number of another surface acoustic wave, known as “Scholte wave”. A real wave number of the Scholte wave indicates that it propagates along the fluid-solid boundary without decaying. The speed of the Scholte wave, \( c_s \), is smaller than \( c_o \), \( c_t \) and \( c_l \). Correspondingly, \( k_s > k_o > k_t > k_l \). The three potentials for a Scholte wave have similar forms as those for a leaky Rayleigh wave

\[
\Phi = -A e^{-i(k_s x - \omega t)} e^{-\sqrt{k_s^2 - k_l^2} z} \tag{2-46}
\]

\[
\Psi = iA \frac{2k_s \sqrt{k_s^2 - k_l^2}}{2k_s^2 - k_t^2} e^{-i(k_s x - \omega t)} e^{-\sqrt{k_s^2 - k_t^2} z} \tag{2-47}
\]

\[
\phi = A\omega \sqrt{k_s^2 - k_l^2 \sqrt{k_s^2 - k_t^2}} e^{i(k_s x - \omega t)} e^{\sqrt{k_s^2 - k_o^2} z} \tag{2-48}
\]

The terms \( e^{-\sqrt{k_s^2 - k_l^2} z} \), \( e^{-\sqrt{k_s^2 - k_t^2} z} \), \( e^{\sqrt{k_s^2 - k_o^2} z} \) in Eq. 2-46, 2-47, 2-48 indicate that the Scholte wave attenuates exponentially with depth in both the solid and fluid. For a dense solid-light fluid combination \( (\rho \gg \rho_o) \), most of the energy of the Scholte wave is localized in the fluid (Zhu & Popovics, 2006). In contrast, most of the energy of the leaky Rayleigh wave resides in the solid (Zhu & Popovics, 2006).
2.4 Generation of $P$, $S$ and LRW waves at a fluid-solid boundary by an incident acoustic plane wave

In this section, we discuss the excitation of stress waves at a fluid-solid boundary by an incident acoustic wave (Brekhovskikh & Godin, 1990). First, we consider a plane incident wave with a unit amplitude as shown in Fig.3, given by:

$$\varphi = \exp(ikx + ihz - i\omega t)$$

in which $\varphi$ is the displacement potential associated with this plane wave, and $k$, $h$ is the component of the wave number, $k_o$, in the $x$ and $z$ positive direction, respectively. The incident angle of this plane wave is

$$\theta_i = \arcsin\left(\frac{k}{k_o}\right)$$

Figure 3. Stress waves excited by an incident plane wave from the fluid.
Upon reaching the fluid-solid boundary, this incident plane wave generates a reflected plane wave, potentially a plane longitudinal wave and a plane transverse wave in the solid. To satisfy the boundary conditions (Eqs.2-30,2-31), the component of the wave number in the x direction of all the four waves has to be continuous. Thus, the three plane waves can be expressed as

\[ \varphi' = V \exp(ikx - ihz - i\omega t) \]  
\[ \Phi = W_t \exp(ikx + iqz - i\omega t) \]  
\[ \Psi = W_t \exp(ikx + isz - i\omega t) \]  

in which \( \varphi' \) is the displacement potential associated with the reflected wave, \( \Phi, \Psi \) is the scalar and vector displacement potential; \( k, q \) is the component of the wave number of the longitudinal wave, \( k_t \),and \( k, s \) is the component of the wave number of the transverse wave, \( k_t \). \( V, W_t, W_t \) is the amplitude of the reflected wave, refracted longitudinal wave, and the refracted transverse wave, respectively. Since the incident wave has a unit amplitude, \( V, W_t, W_t \) also equals to the reflected coefficient for the reflected wave, and the transmitted coefficient for the longitudinal wave and transverse wave, respectively. The reflected angle equals to the incident angle. The refraction angles are

\[ \theta_t = \arcsin\left(\frac{k}{k_t}\right) \]  
\[ \theta_t = \arcsin\left(\frac{k}{k_t}\right) \]
To match the component of the wave number in the x direction in the incident wave and the refracted longitudinal, transverse waves, the two refraction angles have to satisfy the relations

\[ \sin(\theta_i) k_o = \sin(\theta_l) k_l = \sin(\theta_t) k_t \]  

(2-56)

This is the Snell’s law, which can also be expressed in a more common form as

\[ \frac{\sin(\theta_i)}{c_o} = \frac{\sin(\theta_l)}{c_l} = \frac{\sin(\theta_t)}{c_t} \]  

(2-57)

By substituting expressions of \( \varphi', \Phi, \Psi \) into the boundary conditions (Eqs. 2-30, 2-31) We can obtain the expressions for \( V, W_l, W_t \):

\[ V = \frac{Z_l \cos^2 \theta_l + Z_t \sin^2 \theta_l - Z}{Z_l \cos^2 \theta_l + Z_t \sin^2 \theta_l + Z} \]  

(2-58)

\[ W_l = \frac{-2 \rho Z_l \cos \theta_t}{Z_l \cos^2 \theta_l + Z_t \sin^2 \theta_l + Z} \]  

(2-59)

\[ W_t = \frac{-2 \rho Z_t \sin \theta_t}{Z_l \cos^2 \theta_l + Z_t \sin^2 \theta_l + Z} \]  

(2-60)

in which \( Z = \frac{\rho_o c_o}{\cos(\theta_i)} \), \( Z_l = \frac{\rho c_l}{\cos(\theta_i)} \), \( Z_t = \frac{\rho c_t}{\cos(\theta_i)} \) are the impedances of the acoustic wave, longitudinal wave, and transverse wave, respectively; \( \rho \) is the density of the solid.

Normally, we have \( c_o < c_t < c_l \). According to the Snell’s law (Eq.2-57), as the incident angle \( \theta_i \) increases, it will first reach a critical angle, \( \theta_L = \arcsin(\frac{c_o}{c_l}) \), where the refracted angle of the longitudinal wave becomes \( 90^\circ \) and the longitudinal wave travels parallel to the boundary. Thereafter the increased \( \theta_i \) will reach another critical angle, \( \theta_T = \arcsin(\frac{c_o}{c_T}) \), where the refracted transverse wave will travel parallel to the boundary. At the
critical angles, the refracted longitudinal and transverse wave are also called “lateral waves”. During their propagation along the boundary, they will launch acoustic waves propagating into the interior of the fluid at the corresponding critical angles. These acoustic waves are called “head waves”. Beyond the critical angle, the refracted angle becomes complex and the corresponding acoustic impedance becomes pure imaginary. A pure imaginary impedance (reactance) indicates that there is no propagation of energy. Physically, the corresponding refracted wave will be an inhomogeneous wave “gliding” along the boundary and decays in depth (Brekhovskikh & Godin, 1990).

An incident plane acoustic wave can also generate a LRW at a fluid-solid boundary. According to the Snell’s law, the incident angle, leaky Rayleigh wave angle, should be:

\[
\theta_{LRW} = \arcsin \left( \frac{c_o}{c_{LRW}} \right)
\]

in which \(c_{LRW} = \frac{\omega}{Re(k_{LRW})} \) is the speed of LRW. At \(\theta_{LRW}\), the projected speed of the incident wave along the x-axis equals to the speed of the LRW; therefore, the incident wave constantly reinforces the LRW. Thus, the \(\theta_{LRW}\), represents a resonant condition.
The principle for generating a LRW by an incident plane wave from the fluid is similar to the principle for launching a Rayleigh wave by the conventional wedge method (Fig.4). Thus, the fluid is also called a “water wedge” (Dransfeld & Salzmann, 1970a). In the wedge method, a longitudinal wave propagating through the wedge converts to a Rayleigh wave or a LRW by the periodic perturbation at the boundary induced by the incident longitudinal wave. In theory, the incident longitudinal wave can be approximated by a periodic stress at the boundary

\[
\sigma_{zz} = \sigma z e^{ik_0 x} e^{-i \omega t} \text{ for } |x| < b
\]

\[(2-62)\]
\[ \sigma_{xz} = \sigma_x e^{i k_o x} e^{-i \omega t} |x| < b \]  
(2-63)

in which \( k_o = k_L \sin(\theta_l) \), \( k_L \) is the wave number of the longitudinal wave in the wedge, \( \sigma_z, \sigma_x \) are the amplitude of the stress. This approximation is permissible only under the condition that the wave impedance of the solid medium is greater than the wave impedance of the wedge material. In the reference book (Viktorov, 1967b), the asymptotic expressions of the displacement induced in the far field by the wedge method are

\[
w = -A_1 \left( i \frac{q_R}{k_R} \sigma_z + \frac{k_R^2 + s_R^2}{2 k_R^2} \sigma_x \right) \frac{\sin(k_o - k_R)b}{k_o - k_R} e^{i k_R x} + (B_1 \sigma_z + B_2 \sigma_x) \frac{\sin(k_o - k_R)b}{k_o - k_R} \frac{e^{i k_R x}}{k_L^2 x^2} + \\
(C_1 \sigma_z + C_2 \sigma_x) \frac{\sin(k_o - k_R)b}{k_o - k_R} \frac{e^{i k_R x}}{k_L^2 x^2} + 0 \left[ \frac{1}{k_L^2 x^2} \right] \alpha \frac{\sin(k_o - k_R)b}{k_o - k_R} \]  
(2-64)

\[
u = A_2 \left( \frac{k_R^2 + s_R^2}{2 k_R^2} \sigma_z - i \frac{s_R}{k_R} \sigma_x \right) \frac{\sin(k_o - k_R)b}{k_o - k_R} e^{i k_R x} + (B_3 \sigma_z + B_4 \sigma_x) \frac{\sin(k_o - k_R)b}{k_o - k_R} \frac{e^{i k_R x}}{k_L^2 x^2} + \\
(C_3 \sigma_z + C_4 \sigma_x) \frac{\sin(k_o - k_R)b}{k_o - k_R} \frac{e^{i k_R x}}{k_L^2 x^2} + 0 \left[ \frac{1}{k_L^2 x^2} \right] \alpha \frac{\sin(k_o - k_R)b}{k_o - k_R} \]  
(2-65)
in which $A_1, A_2, B_1, B_2, B_3, B_4, C_1, C_2, C_3, C_4$ are constants. The first term in these expressions corresponds to the displacement induced by a Rayleigh wave, and the second and third terms describe the displacement induced by a longitudinal and transverse wave, respectively. This theoretical predicition was qualitatively confirmed by the experimental measurement (Fig. 5).

$$\sin\left(\frac{k_0 - k_R}{k_0 - k_R}\right)$$ in both Eqs. 2-64, 2-65 maximize at $k_0 = k_R$, the Rayleigh wave induced displacement resonates at

$$k_0 = k_R \quad (2-66)$$

Since $k_0 = k_L \sin(\theta_i)$, this equation equals to

$$\sin(\theta_i) = \frac{k_R}{k_L} = \frac{c_L}{c_R} \quad (2-67)$$
This particular incident angle $\theta_R = \arcsin\left(\frac{c_L}{c_R}\right)$ is called a Rayleigh angle, which is analogous to $\theta_{LRW}$ for LRW generation. Thus, the resonate condition confirms that $\theta_R$ is the optimal condition to generate a Rayleigh wave. Qualitatively, the $\frac{\sin(k_0-k_R)b}{k_0-k_R}$ term in the displacement (Eqs 2-64, 2-65) indicate the amplitude of the Rayleigh wave and the sharpness of the maximum is proportional to the beam width of the incident longitudinal wave. Thus, for an ideal plane wave incidence with an infinite beam width, the amplitude of the induced Rayleigh wave is theoretically unbounded and the Rayleigh wave is only generated at the Rayleigh angle, $\theta_R = \arcsin\left(\frac{c_L}{c_R}\right)$. For the generation of a LRW, only the normal periodic boundary perturbation (Eq. 2-62) retains because a fluid exerts no shear stress onto the boundary. Thus, for the displacement induced by the generated LRW in the far field, any term in Eqs 2-64, 2-65 including $\sigma_z$ is expected to retain while any term including $\sigma_x$ is expected to disappear since the whole model mathematically is a linear system. The displacement induced by an LRW is also proportional to $\frac{\sin(k_0-k_R)b}{k_0-k_R}$; therefore, the conclusion for the Rayleigh wave still holds true for LRW generation.
3. Materials and Methods

Experimentally, we used a dynamic high-speed photoelastic/shadowgraph imaging setup to visualize the generation of LRWs on glass surfaces impinged by a NPL-generated shock wave in water. A finite element (FE) model of the problem was also constructed to analyze the shock wave interaction at the fluid-solid boundary. The photoelastic/shadowgraph images were compared to the numerical results for model validation. Thereafter, the FE model was used to calculate the dynamic stress field induced in the solid, in relation to the propagating LRWs on the fluid-solid boundary, and to facilitate the understanding of the mechanism responsible for the ring-like fractures produced on the glass surface.

3.1 Dynamic photoelastic imaging technique

A dynamic brightfield photoelastic/shadowgraph imaging system (Xi & Zhong, 2001) was adapted to visualize the generation and propagation of SAWs at a fluid-solid boundary. Combined with an ultra high-speed camera, this imaging system is capable of simultaneously resolving the time evolution of the wavefronts for various stress waves in the photoelastic sample and the pressure waves in the adjacent fluid.

A photoelastic material, such as glass and epoxy, is optically isotropic when it is free of stress. However, under stress, the material becomes optically anisotropic.
Consequently, a polarized light will be decomposed into three rays of polarized light travelling along the axes of three principal stresses inside the material. Maxwell in 1852 (Kuske & Robertson, 1974) first formulated the relationship between the refraction index and the principal stress along each direction:

\[ n_1 - n_0 = c_1 \sigma_1 + c_2 (\sigma_2 + \sigma_3) \]  
\[ n_2 - n_0 = c_1 \sigma_2 + c_2 (\sigma_3 + \sigma_1) \]  
\[ n_3 - n_0 = c_1 \sigma_3 + c_2 (\sigma_1 + \sigma_2) \]

where \( \sigma_1, \sigma_2, \sigma_3 \) are the principal stresses; \( n_0 \) is the unstressed refraction index; \( n_1, n_2, n_3 \) are the principal indices of refraction which coincide with the principal stress directions; \( c_1, c_2, c_3 \) are the stress optic coefficients of the material.

Equations 3-1 to 3-3 describe the stress-optic law for a material in a three dimensional stress state. The three rays of the polarized light experiencing different refraction indices will propagate at different speeds when traveling through the material. The differences in the propagation speeds will lead to relative retardations of the light rays, leading to the formation of interference fringe patterns when they combine together outside the material. These interference fringe patterns manifest the status of the stress field inside the material.
The simplest polariscope is composed of a polarizer and an analyzer, whose axis of polarization is orthogonal to each other. To demonstrate the principle of a polariscope, we consider a photo-elastic sample under a plane stress condition (Fig. 6), in which Eqs. 3-1 and 3-2 reduces to

\[ \Delta = \frac{2\pi hc}{\lambda} (\sigma_1 - \sigma_2) \]  

(3-4)
in which $\Delta = \frac{2\pi(n_2-n_1)h}{\lambda}$ the angular retardation, $h$ is the thickness of the sample, $c = c_2 - c_1$ is the relative stress-optic coefficient, $\lambda$ is the wavelength of the illumination monochromatic light. As shown in Fig.6, for the illumination monochromatic light, $E_0 \cos \omega t$, only the component $E_0 \cos(\omega t) \cos \alpha$ passes the polarizer and reaches the sample. The sample under stress will decompose this light into two components

\[ E_1 = E_0 \cos \alpha \cos \beta \cos \omega t \]  
\[ E_2 = E_0 \cos \alpha \sin \beta \cos \omega t \] 

Figure 7. A polariscope composed of a polarizer, an analyzer, and a pair of quarter wave plates.
E₁ vibrates parallel to σ₂ while E₂ vibrates parallel to σ₁. The propagation inside the sample adds additional angular phase. The two light components become

\[ E_1 = E_0 \cos \alpha \cos \beta \cos(\omega t - \Delta_1) \]  
(3-7)

\[ E_2 = E_0 \cos \alpha \sin \beta \cos(\omega t - \Delta_2) \]  
(3-8)

in which \( \Delta_1 = \frac{2\pi n_1}{\lambda} (n_1 - 1) \), \( \Delta_2 = \frac{2\pi n_2}{\lambda} (n_2 - 1) \). These two light components combine after passing the analyzer, yielding

\[ E = E_0 \cos \alpha \sin \beta \sin \left( \frac{\Delta_2 - \Delta_1}{2} \right) \sin(\omega t - \frac{\Delta_2 + \Delta_1}{2}) \]  
(3-9)

The intensity of the combined light, which the camera is sensitive to, is given by

\[ I = (E_0 \cos \alpha)^2 \sin^2 \beta \sin^2 \left( \frac{\Delta_2 - \Delta_1}{2} \right) \]  
(3-10)

If I decreases to zero, black fringes occur. Equation 3-10 indicates that both the orientation and the amplitude of the principal stresses will affect the fringe pattern. When \( \sin^2 \beta \) equals to zero, dark fringe patterns appear. Those fringe patterns, related to the direction of the principal stress field, are called isoclinic fringe patterns. When \( \sin^2 \left( \frac{\Delta_2 - \Delta_1}{2} \right) \) equals to zero, dark fringe patterns also appear. Those fringe patterns, related to the principal stress difference, are called isochromatic fringe patterns (Kuske & Robertson, 1974). To eliminate the isoclinic fringe patterns, we can add two half wave plates between the polarizer and analyzer to compose a circular polariscope. As shown in Fig 7, the angle between the axis of polarization of the polarizer and the fast axis of the first half wave plate should be 45 degrees. If the fast axis of the first half wave plate is parallel to the slow
axis of the second quarter wave plate, the intensity of the light after passing the analyzer will be

\[ I = \frac{1}{2} E_0^2 \sin^2 \left( \frac{\Delta_2 - \Delta_1}{2} \right) \]  \hspace{1cm} (3-11)

Equation 3-11 shows that the camera receives no light if the sample is under no stress (i.e., no relative retardation). Thus, the photoelastic image exhibits a dark background. A dark background means that the field of view is always dark except for the area occupied by the photoelastic sample. In this study, however, a bright background mode was adopted by rotating the second quarter wave plate until its fast axis is parallel to the fast axis of the first quarter wave plate. In contrary to the dark background mode, the camera receives all the light passing the polarizer in the bright background mode if the sample is under no stress. In the bright background mode, still, the light passing the polarizer will be decomposed to two lights when impinging onto the first half wave plate.

\[ E_{f1} = \frac{\sqrt{2}}{2} E_0 \cos \alpha \cos \omega t \]  \hspace{1cm} (3-12)

\[ E_{s1} = \frac{\sqrt{2}}{2} E_0 \cos \alpha \cos \omega t \]  \hspace{1cm} (3-13)

The quarter wave plate will introduce a relative angular phase to the light vibrating along the slow axis. Thus, after passing the half wave plate, the two lights become
\[ E_{f1}' = \frac{\sqrt{2}}{2} E_0 \cos \alpha \cos \omega t \] (3-14)

\[ E_{s1}' = \frac{\sqrt{2}}{2} E_0 \cos \alpha \sin \omega t \] (3-15)

When these two lights impinge onto the photoelastic sample, each of them will be decomposed into two components along the two principal stress directions, respectively. The two light vibrating along a principal stress direction will combine into

\[ E_1 = E_f' \cos \left( \frac{\pi}{4} - \beta \right) + E_s' \cos \left( \frac{\pi}{4} + \beta \right) = \frac{\sqrt{2}}{2} E_0 \cos \alpha \cos (\omega t - \frac{\pi}{4} + \beta) \] (3-16)

\[ E_2 = -E_f' \sin \left( \frac{\pi}{4} - \beta \right) + E_s' \cos \left( \frac{\pi}{4} - \beta \right) = \frac{\sqrt{2}}{2} E_0 \cos \alpha \sin (\omega t - \frac{\pi}{4} + \beta) \] (3-17)

The propagation inside the sample will introduce a relative angular phase \( \Delta \) to the light vibrating along the slow axis.

\[ E_1' = \frac{\sqrt{2}}{2} E_0 \cos \alpha \cos (\omega t - \frac{\pi}{4} + \beta) \] (3-18)

\[ E_2' = \frac{\sqrt{2}}{2} E_0 \cos \alpha \sin \left( \omega t - \frac{\pi}{4} + \beta - \Delta \right) \] (3-19)

When hitting the second quarter wave plate, these two lights will be decomposed and then combined to two, vibrating along the fast and slow axis, respectively.

\[ E_{f1} = E_1' \cos \left( \frac{\pi}{4} - \beta \right) - E_2' \sin \left( \frac{\pi}{4} - \beta \right) \] (3-20)

\[ E_{s2} = E_1' \sin \left( \frac{\pi}{4} - \beta \right) + E_2' \cos \left( \frac{\pi}{4} - \beta \right) \] (3-21)
The second quarter wave plate introduce another $\frac{\pi}{2}$ relative angular phase to the light vibrating along the slow axis; therefore, after passing the second quarter wave plate the two lights become

\[
E_{f2}' = \frac{\sqrt{2}}{2}E_0 \cos \alpha \cos \left( \omega t - \frac{\pi}{4} + \beta \right) \sin \left( \frac{\pi}{4} - \beta \right) + \frac{\sqrt{2}}{2}E_0 \cos \alpha \sin \left( \omega t - \frac{\pi}{4} + \beta - \Delta \right) \cos \left( \frac{\pi}{4} - \beta \right) \tag{3-22}
\]

\[
E_{s2}' = \frac{\sqrt{2}}{2}E_0 \cos \alpha \sin \left( \omega t - \frac{\pi}{4} + \beta \right) \sin \left( \frac{\pi}{4} - \beta \right) - \frac{\sqrt{2}}{2}E_0 \cos \alpha \cos \left( \omega t - \frac{\pi}{4} + \beta - \Delta \right) \cos \left( \frac{\pi}{4} - \beta \right) \tag{3-23}
\]

The analyzer combines these two lights as

\[
E = \frac{\sqrt{2}}{2}E_{f2}' - \frac{\sqrt{2}}{2}E_{s2}' = E_0 \cos \left( \omega t - \frac{\Delta}{2} \right) \cos \frac{\Delta}{2} \tag{3-24}
\]

The intensity received by the camera will be

\[
l = E_0^2 \cos^2 \alpha \cos^2 \frac{\Delta}{2} \tag{3-25}
\]

When the sample is free of stress, the intensity reaches its maximum; therefore the photoelastic images exhibit a bright background. This means that for the area surrounding the photoelastic sample in the field of view, the photoelastic imaging system has no effect except for further polarizing the laser illumination. For our study, the photoelastic sample
was surrounded by water; the imaging system captured the shadowgraph images of flow occurring in the surrounding fluid.

A dynamic photoelastic imaging system is needed to resolve the dynamic stress field induced by propagating stress waves. To capture a transient stress field, either the illumination time of the light source or the exposure time of the camera has to be sufficiently short. Within such a short time period, stress waves will be technically “frozen” as they only propagate an unobservable distance in the image.

An inherent limitation of the dynamic photoelastic imaging technique is that it cannot determine the stress distribution inside the solid based on photoelastic images. Even in a 2D plane stress condition, as described by Eq. 3-4, the photoelastic images only provides information regarding the difference between the two in-plane principal stresses, without additional information on either the magnitude or the direction of each individual principal stress.
In general, the experimental condition used in this study does not rigorously fulfill the requirement for a 2D plane stress condition. The photoelastic images we obtained is a projection of a 3D dynamic stress field onto a 2D image plane. Reconstruction of the 3D dynamic stress field along the light path from a 2D photoelastic image is still a challenging mathematical problem (Kuske & Robertson, 1974). Thus, we cannot deduce the complete stress distribution from photoelastic images. Instead, we employed the photoelastic imaging technique to confirm the generation of LRWs in the experiment and to validate the setup of our numerical model, from which we could then calculate the stress field distribution.

Figure 8. A schematic diagram of the experimental setup.
3.2 Experimental setup

Figure 8 shows the experimental setup. A shock wave was produced in water by an 3.6 Fr Nano Pulse lithotripsy (NPL) probe (Lithotech Medical Israel) operated at 10 kV. The propagation of the shock wave and its subsequent interaction with a glass sample (50x50x3.3 mm in LxWxH BOROFLOAT® Borosilicate Windows, Edmund Optics Inc.) was captured by the dynamic photoelastic/shadowgraph imaging system (Fig. 8). Briefly, a low coherence illumination laser (640 nm±6 nm, SI-LUX-640, Specialised Imaging) with 10-ns pulse duration was first focused by a microscope objective (10×/0.5, Carl Zeiss AG) and then spatially filtered by an iris (Ø0.7 mm, ID8, THORLABS). The laser beam was collimated by a Ø4.25" Schlieren mirror (#71-014, Edmunds Optics). Two quarter-wave plates (model, manufacturer) were placed in the light path with their fast axis parallel to each other and aligned at +45° and −45°, respectively, to the polarization direction of the polarizer and analyzer. After passing the polariscope and the tank, the beam was focused by another Schlieren mirror before projected onto an ultra high-speed camera (Kirana-M5, Specialised Imaging), operated at 5 million frames per second (fps). The illumination laser and the camera were synchronized and controlled by a complimentary software (Kirana control). The imaging system was further synchronized with the NPL using a photodetector (DET110, THORLABS) and an oscilloscope (HDO6104, Teledyne LeCroy Inc.).
The photo-detector picked up the spark discharge produced by the NPL probe, and the signal was used to trigger the oscilloscope, and subsequently the imaging system.

### 3.3 Numerical modeling

The numerical model was constructed in COMSOL Multiphysics 5.3 (Burlington, MA). A 2D axisymmetrical model was built by using tools provided in the “Acoustic-
Solid interaction, Transient” physics in the acoustics module. Figure 9 shows a schematic diagram of the numerical model and all relevant material properties. The geometry of the numerical model is a 20 mm diameter circle, which is evenly split into two parts corresponding to the fluid domain (top) and the solid domain (bottom). The wave propagation in the fluid domain is described by the acoustic wave equation

\[
\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = Q_m
\]

in which \( p \) is pressure, \( c \) is the speed of sound in the fluid, and \( Q_m \) is the monopole strength. The elastodynamics in the solid is described by

\[
\rho_s \frac{\partial^2 u}{\partial t^2} = \nabla \cdot \sigma + f
\]

\[
\sigma = C : s = C : \nabla_s u
\]

in which \( \rho_s \) is the density of the solid, \( u \) is the particle displacement vector, \( \sigma \) is the stress tensor, \( f \) is the external force per unit volume, \( C \) is the elastic stiffness constants (Auld, 1990), \( s \) is the strain tensor, and \( : \) represents the dot product of a fourth order tensor and a second order tensor. The fluid domain and the solid domain are coupled by the Acoustic-Structure Boundary conditions

\[
\mathbf{n} \cdot \frac{1}{\rho} \nabla p = -\mathbf{n} \cdot \frac{\partial^2 u}{\partial t^2}, p = -\sigma_{z z}, \sigma_{rz} = 0
\]
In this dissertation, the NPL probe was modeled as a monopole in water, located at a standoff distance ($S_d$) from the glass boundary along the symmetrical axis (Fig. 9). In COMSOL, this monopole source is mathematically described by:

\[ \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 4\pi S(t)\delta(r - r_0) \]  

(3-30)
in which $S(t)$ is the strength of the monopole, and $\delta(r - r_0)$ is the delta function, indicating that the monopole is at $r = r_0$. By solving Eq. 3-30, we obtain the acoustic pressure induced by the monopole at distance $r$ given by:

$$p(r, t) = \frac{s(t - \frac{r}{c_0})}{r}$$  \hspace{0.5cm} (3-31)

Figure 10 shows the monopole strength $S(t)$ calculated from hydrophone measured pressure waveforms for $r = 9$ mm, $r = 12$ mm, and $r = 15$ mm. At each radial
location, the waveform was obtained by averaging 8 measurements. The agreement between the fitted $S(t)$s suggest that the shock wave maintains a spherical wavefront during its propagation; therefore, it justifies the approximation of the shock wave source as a monopole. As shown in Fig. 10, the $S(t)$ from measurements closely fit into an analytical model describing shock waves (ref)

$$S(t) = 10.31 \left(1 + \tanh \left(\frac{t-t_0}{\frac{1}{t_{RT}}}\right)\right) e^{-\frac{t-t_0}{t_L}} \cos\left(2\pi f_L(t - t_0) + \frac{\pi}{3}\right) H(t - t_1)$$  \hspace{1cm} (3-32)

in which $t_0 = 0.2 \mu s$, $t_{RT} = 10 ns$, $t_L = 1.1 \mu s$, $f_L = 110 kHz$, $t_1 = 0.2 \mu s$ and $H(t)$ is a Heaviside function. Equation 3-32 was used to setup the strength of the monopole in the COMSOL model.

![Figure 11. The spectrum of the monopole strength signal.](image_url)
To implement the finite element method to solve this problem, the whole geometric domain is first discretized into small sub-domains using quadratic triangular elements. Compared to another commonly used quadrilateral elements, triangular elements introduce no artificial anisotropy to an acoustic problem. As recommended by COMSOL, to resolve a harmonic wave with sufficient accuracy, the element size should be at least one sixth of the wavelength of the harmonic wave. Figure 11 shows the spectrum of a signal described by Eq. 3-32. As we can see, most of the spectrum contents are confined in the frequency range below 25 MHz. Thus, the maximum mesh element dimension ($h_{\text{max}}$) was set as:

$$h_{\text{max}} = \frac{\lambda_{\text{min}}}{6} \quad (3-33)$$

in which $\lambda_{\text{min}}$ is the minimum wavelength of the sound wave in the fluid that the numerical model is designed to resolve, which is 10 $\mu$m in this simulation.

The default implicit second-order accuracy generalized-$\alpha$ method was adapted to solve the transient acoustic problem. This iteration method introduces smaller numerical damping of high frequencies than other methods provided by COMSOL (COMSOL®, 2016). A Courant-Friedrichs-Lewy (CFL) = 0.2 was used to define the time-step size as

$$\Delta t = \frac{CFL \cdot h_{\text{max}}}{c_0} = \frac{0.2 h_{\text{max}}}{c_0} \quad (3-34)$$
At CFL = 0.2, the temporal discretization error produced in the numerical simulation is roughly the same as the spatial discretization error for the generalized-\( \alpha \) method.
4. Generation of Surface Acoustic Waves

4.1 Shock wave emission produced by NPL

In this study, a SAW was generated at the glass-water boundary by the impact of a shock wave produced by the underwater electrical spark discharge at the tip of an NPL probe. The NPL probe tip is comprised of a central anode and a peripheral sheath. During the discharge, an electric spark is produced between the central anode and the peripheral sheath along the shortest/weakest path. If multiple weakest paths coexist, multiple electrical sparks can be generated. Typical examples of either one or two sparks produced at the probe tip are shown in Fig 12. Each spark produces a spherically diverging shock wave. Figure 13 shows the wavefront of the shock wave captured by shadowgraphs at 6

![Figure 12](image.png)

*Figure 12. (a) Single spark produced at the NPL probe tip (b) Dual-sparks produced at the NPL probe tip.*
µs and 8 µs after the spark. Because of the dual-spark potential of the NPL device, we measured the pressure waveform of the shock wave at three different radial distances (r = 9 mm, 12 mm, 15 mm) along five orientations (θ = 15°, 30°, 45°, 60°, 90°). Figure 14 shows the averaged pressure waveforms at r = 9 mm in different orientations based on eight individual measurements at each orientation. As the angle from the NPL probe axis increases, the measured waveform generally broadens in the full width at half maximum (FWHM) pulse duration while the peak pressure amplitude decreases. At small angles, the two potential shock waves will superimpose closely with each other to produce a

Figure 13. The wavefront of NPL-generated shock wave captured at different times after an electrical spark discharge in water.
single pressure peak of high amplitude. At large angles (> 60°), we can discern two peaks in the pressure waveform due to the significant phase delay of the two individual shock waves along the propagation direction.

![Image of pressure waveforms at various angles](image)

**Figure 14.** The averaged pressure waveforms of NPL-generated shock waves measured by a fiber optic probe hydrophone (FOPH) at various orientations.

### 4.2 LRW captured by photoelastic/shadowgraph images

Figure 15a shows the propagation of a SAW along the water-glass boundary with the arrow indicating the location of the wavefront of the SAW in each frame. The wavefront of this SAW in the solid is comprised of two branches, which connects to a
A straight wavefront in the water. The speed of the SAW was determined by the scope of a linearly fitted line between the radial distance of the wavefront from the probe axis and the corresponding time (Fig. 15 b). The experimentally calculated SAW speed (3024 m/s) is closely matched with the theoretical wave speed of a leaky Rayleigh wave (3163 m/s) at the glass-water boundary, further confirming the generation of a LRW. Figure 15 c shows that the wavefront of the SAW in the glass is connected tangentially by a straight line to

**Figure 15.** (a) The propagation of a SAW induced by the incident shock wave at the water-glass boundary, (b) The linear fitting curve between the radial location of the wavefront and the corresponding time, (c) The inclination angle of the wavefront of the leaky pressure wave in water at $t = 6.2 \mu s$. 
the reflected wave in the water. This straight line represents the wavefront of a leaky pressure wave in water, which is generated by the LRW-induced disturbance at the interface. The inclination angle of the leaky wave, $\theta$, is determined by the ratio of the LRW speed, $c_{LRW}$, to the sound speed in water, $c_0$, as

$$\theta = \arcsin \left( \frac{c_p}{c_{LRW}} \right) = \arcsin \left( \frac{1500}{3163} \right) = 28^\circ$$

(4-1)

This feature is also confirmed by the inclination angle from the shadowgraph image (i.e., 28 degrees).

Furthermore, Figure 16 shows the generation of LRWs by the NPL-generated shock wave at different standoff distances ($S_d$) of 0.5 mm, 1.0 mm, and 2.0 mm. Besides the LRW, the photoelastic images also capture the propagation of a transverse ($T$) wave inside the glass sample. Because the speed of the transverse wave ($c_T$) in the borosilicate glass is slightly greater than $c_{LRW}$ the wavefront of the transverse wave initially overlaps with that of the LRW. During propagation, the two wavefronts gradually separate from each other. After the separation, the shadowgraph images clearly show that the wavefront of the LRW in the glass is comprised of two branches - a leading transverse branch and a lagging longitudinal branch, as described by the theory of LRW (Brekhovskikh & Godin, 1990). For a larger $S_d$ both the LRW and the $T$ wave propagated a shorter distance at the same elapsed time after the spark discharge since the shock wave impacted at the
boundary later. LRWs induced at different $S_d$ propagate at the same speed while the inclination angles of the corresponding leaky waves also remain the same.

The generation of a LRW also depends on the material properties of the solid boundary. Theoretically, a LRW can only exist at a boundary between a fluid and a stiff
solid, in which the transverse wave speed is greater than the sound speed in the fluid (Zhu & Popovics, 2006). Figure 17 shows the evolution of the fringe patterns in an epoxy sample induced by NPL under similar test conditions ($S_d = 0.5$ mm, 1.0 mm, and 2.0 mm). Since the speed of the $T$ wave in the epoxy sample is smaller than the sound speed in water, no
LRW was generated. Instead, the two ends of the wavefront of the $T$ wave always attach to the two ends of the wavefronts of the direct and reflected shock waves in the fluid. In contrary, the speed of the $T$ wave in glass is greater than the speed of sound in water; once the projection of the phase velocity of the incident wave $P_i$ onto the boundary becomes

Figure 17. Fringe patterns induced in an epoxy sample by incident shock waves produced by the NPL probe tip at different $S_d$
smaller than the speed of $T$ wave, the wavefront of the $T$ wave starts to detach from the wavefront of $P$. 
5. LRW induced stress field

According to the principle of photoelastic imaging, the fringe patterns exhibited by the polariscope is produced by the temporal stress field in the solid. Thus, the photoelastic images have demonstrated that the stress field within the subsurface area of the solid induced by the LRW propagating along the fluid-solid boundary. To overcome the inherent limitations of the photoelastic imaging regarding the details of the stress field, I validated a finite element model and applied it to quantitatively analyze the LRW-induced stress field in the solid.
5.1 Validation of the COMSOL model

![Diagram showing material properties and setup of the mathematical problem for numerical model validation.]

Before relying the COMSOL model for any quantitative study, I first tested the numerical accuracy of this model and validated its applicability to NPL-induced stress field. Numerical errors may rise from the numerical implementation of the governing equations and the boundary conditions (2-26 – 2-29). These numerical errors are inherent in the software and can hardly be reduced. Other numerical errors may be caused by

\[
Ricker\ \text{wavelet} = (1 - 2[\pi f_0 \left(t - \frac{1.2}{f_0}\right)]^2) e^{-\left(\pi f_0 (t - \frac{1.2}{f_0})^2\right)}
\]

\[f_0 = 1.43 \text{ (MHz)}\]

Figure 18. Setup of the mathematical problem for numerical model validation. The temporal profile of the monopole is defined by the Ricker wavelet. The red dots indicate the locations of the source and the receiver.
incorrect selection of numerical parameters, such as the mesh size and time step. The inherent numerical errors are assessed by comparing a numerical solution from COMSOL with an analytical solution obtained using the de Hoop method (de Hoop & Van der Hijden, 1984) for the same mathematical problem (Fig. 18). My experience suggests that most numerical errors occur in the implementation of the boundary conditions, which involve transportation of energy and momentum at the boundary between a fluid and a

Figure 19. The comparison between the analytical solution and the numerical solution.
solid medium. For the mathematical model in Fig. 18, all types of bulk waves (\(L, T\)
reflected, and head waves) and surface waves (\(LRW\), Scholte wave, lateral wave, and
evanescent waves) with different polarization are expected to be generated and detected.
Thus, this mathematical problem is a benchmark case to test the capability of the
numerical model to capture the complex physical phenomenon occurring at the

![Image: Figure 20. The convergence test result by shrinking the maximum element size from \(h = 16.7 \mu m\) to \(12.5 \mu m\).]

boundary. As shown in Fig.19, a Ricker wavelet wildly used in seismology, was used as
the excitation signal. Figure 20 shows the pressure signals detected by the receiver in the
COMSOL model (dashed lines) and that calculated by the de Hoop model (solid line). The
perfect agreement between the numerical solution and the analytical solution demonstrate the high fidelity of the numerical model. On the other hand, the de Hoop method (de Hoop & Van der Hijden, 1984) does not provide an explicit solution of the stress field generated in the solid. Therefore, after the validation the COMSOL model is used to calculate the stress field produced by the NPL.

As described in Chapter 3, the maximum mesh element size was set to resolve all frequency contents below 25 MHz. To further justify this choice, I did a convergence test by further shrinking the maximum mesh element from 16.7 µm down to 14.3 µm and 12.5 µm and examine the convergence of signals detected in both fluid and solid. Figure 19 shows pressure signals detected in the fluid from three settings have already converged either in the fluid or in the solid. This convergence further justifies the choice of 12.5 µm as the maximum mesh element size to resolve most of frequency contents in the shock wave.

As described in Chapter 4, the NPL probe tip may generate one or two electrical sparks during its energy discharge. In the two sparks case, two shock waves produced superimpose to form the spherical shock wave captured in the shadowgraph image (Fig. 13). Despite the complexity in the generation of the shock wave, the source of the shock wave is simply modeled as a monopole. The accessible computational resource prohibits a more sophisticated model to resemble the complete pressure distribution around the
wavefront. The monopole source in an axial symmetric model produces a spherical shock wave with a uniform pressure distribution along all directions. As shown in Fig. 21, the hydrophone measured waveforms along 30° is used to define the monopole source strength. This selection of the waveform is based on the consideration that this numerical study is focused on the generation of the LRW and associated stress field. Theories suggest

Figure 21. Hydrophone measurement along the 30° orientation.
that a $LRW$ is mainly generated at the critical incident angle $\theta_i = \theta_{LRW} = 28^\circ$. Thus, the monopole model based on measurement along $30^\circ$ will capture the magnitude of the $LRW$ more closely even though the magnitude of other waves could be either under- or over-estimated. As we will show later in the results, the strongest tensile stress produced in the solid is indeed generated by the $LRW$.

Figure 22 shows the comparison between photoelastic/shadowgraph images and numerical simulations results. For the numerical simulation results, the solid side exhibits the distribution of the maximum tensile stress $\sigma_T$ (i.e., the first principal stress) and the fluid side exhibits the distribution of pressure. Excellent agreements were observed regarding the wavefront positions of the incident ($P_i$) and reflected ($P_r$) shock (or pressure) waves in water, the transverse ($T$) wave in the glass, and the $LRW$. Furthermore, the wavefront of the $LRW$ is composed of dual branches in both the photoelastic/shadowgraph images and the distribution of $\sigma_T$ from the numerical simulation. These excellent agreements confirm the applicability of the monopole model.
for this study. In Fig. 23, the solid side in the first column exhibits the distribution of curl displacement and the second column exhibits the distribution of the divergence displacement. Curl displacement only shows transverse movement associated with $T$ wave while divergence displacement shows dilatational movement associated with $L$ wave. Figure 23 shows that the wavefront of $LRW$ is comprised of a leading transverse branch ($LRW^T$) jointed asymptotically by a trailing longitudinal branch ($LRW^L$) on the boundary, which is a unique and characteristic feature of the $LRW$ (Brekhovskikh & Godin, 1990).

### 5.2 Stress field

The impact of a spherical wave at a flat water-glass boundary generates various types of waves depending on the incident angle ($\theta_i$). $L$ and $T$ waves are generated

*Figure 22. The comparison between the photoelastic/shadowgraph images and the numerical simulations results.*
simultaneously if $\theta_i$ is within the critical angle of the $L$ wave ($\theta_L$). As shown in Fig. 24a, the $L$ wave only induces a compressive stress. At $\theta_L$, the bulk $L$ wave turns into a lateral $L$ wave while the incident spherical wave continues to generate the $T$ wave. Since the speed of the $L$ lateral wave is greater than the projection of the speed of the incident wave along the boundary, the wavefront of $L$ lateral wave detach from the wavefront of incident
wave while propagating along the boundary (Fig. 24c). The lateral $L$ wave induced disturbance on the boundary generates an $L$ head wave. The wavefront of this $L$ head
wave is too weak to be seen in this contour plot. As shown in Fig. 24a, the $T$ wave propagating behind the L wave induces tensile stresses in the solid. Similarly, at the critical angle of the $T$ wave ($\theta_T$), the bulk $T$ wave converts into a lateral $T$ wave, launching a $T$ head wave propagating along the boundary while detaching from the wavefront of the $P_i$ in water. The wavefronts of the $T$ lateral wave and $T$ head wave are barely distinguishable from the wavefront of the $LRW$ and the Schmit head wave since their corresponding speeds are close to each other (H.UBERALL, 1973). As the incident angle exceeds $\theta_T$, the $P_i$ induces an evanescent wave. This evanescent wave is comprised of a transverse component ($E^T$) (Fig. 24b) and a longitudinal component ($E^L$) (Fig. 24c) Beyond Figure 24. (a) An exhibit of the maximum tensile stress in the solid showing various types of stress waves generated by the impact of a spherical wave. (b) an exhibit of the curl displacement in the solid showing the shear components (c) an exhibit of the divergence displacement in the solid showing the longitudinal components.
$\theta_T$, the $P_i$ generates a $LRW$. Similar to other lateral waves, this $LRW$ also induces a Schmidt head wave while propagating along the boundary. The generation of the Schmidt head wave leads to the amplitude decaying of the $LRW$ during its propagation. Figure 24a shows that a strong tensile stress is induced within the subsurface area near the $LRW$.

Figure 25 shows four snapshots of the $\sigma_T$ distribution before and after the wavefront of $LRW$ detaches from the wavefront of the $T$ wave. The area in strong tension travels with the wavefront of the $LRW$ after it detaches from the wavefront of the $T$ wave. This result indicates that the maximum of $\sigma_T$ is mainly produced by the $LRW$ even before the detachment occurs. However, the $T$ wave may contribute to the initial buildup of $\sigma_T$ to reach its maximum.
To dissect the contribution of LRW and T to the maximum of $\sigma_T$, I used the numerical model to quantify the entire transient stress field excited by the incident shock wave. Receivers were embedded along five refraction rays to detect $\sigma_T$ induced by the stress waves passing by (Fig. 26). Figure 27 shows the variations in $\sigma_T$ profile along the five refraction rays at progressively increased radial distances from the boundary. As the refraction angle ($\theta$) increased from 0° (Ray 1) to 5° (Ray 2), I observed a transition from compressive stress produced by the L-wave ($c_L = 5498$ m/s) to tensile stress generated by the T-wave ($c_T = 3479$ m/s). As $\theta$ increased further to 42° (Ray 3), the tail ends of the two LRW branches started to emerge. The leading $LRW^T$ had a shorter pulse duration than the trailing $LRW^L$; yet both were comparable in $\sigma_T$ magnitude to that produced by the T-wave sandwiched between them. At $\theta = 85°$ (Ray 4), the dominance of LRW-induced $\sigma_T$ with significantly increased magnitude became clear when the two branches gradually merged and obscured the T-wave induced $\sigma_T$. Finally, the $LRW^T$ and $LRW^L$ jointed together at the

*Figure 26. A schematic drawing showing the direction of the 5 refraction rays.*
glass-water boundary (Ray 5), producing the maximum $\sigma_T$ (i.e., $\sigma_{T,max}$) an order of magnitude stronger than its counterpart induced by the $T$-wave in the bulk of the glass at small $\theta$.

By quantitatively analyzing the entire transient stress field, I found that the global maximum $\sigma_T$ (i.e., the maximum of $\sigma_T$ occurs during the whole simulation period) is almost 20 times greater than the $\sigma_T$ induced by $T$ along the 45° refraction ray. With an estimation of the magnitude of $\sigma_T$ induced by the $T$ wave along the boundary, I can roughly dissect the contribution of LRW and $T$ to the maximum $\sigma_T$. 


The estimation of the magnitude of $\sigma_T$ induced by the $T$ wave along the boundary is based on the analysis of a harmonic plane $T$ wave generated by a harmonic plane $P_i$.

![Figure 27](image)

**Figure 27.** The COMSOL model-calculated maximum tensile stress $\sigma_T$ (i.e., the first principle stress) pulses detected by receivers placed along five $T$ wave rays (Ray 1 - 5) of different refraction angles, in which Ray 1 overlaps with a $L$ wave ray at 0° refraction angle, while Ray 5 overlaps with a LRW at 90° refraction.

The results are produced at $S_d = 0.5$ mm.

The estimation of the magnitude of $\sigma_T$ induced by the $T$ wave along the boundary is based on the analysis of a harmonic plane $T$ wave generated by a harmonic plane $P_i$. 
The transmission coefficient between the displacement potential associated with the $T$ wave in the glass and the displacement potential in the water is

$$\psi = \frac{-\frac{\rho_s Z_t \sin(2\theta_t)}{\rho_0}}{Z_1 \cos^2 2\theta_t + Z_2 \sin^2 2\theta_t + Z}$$

(5-1)

For a plane harmonic $T$ wave, $\psi$ is

$$\psi = \Psi \exp(i k \cos(\theta_t) z + i k \sin(\theta_t) r)$$

(5-2)

The three stress components produced by this plane harmonic $T$ wave are

$$\sigma_{rr} = -2\mu \frac{\partial^2 \psi}{\partial r \partial z} = 2\mu k^2 \sin \theta_t \cos \theta_t \psi$$

(5-3)

$$\sigma_{zz} = 2\mu \frac{\partial^2 \psi}{\partial \theta \partial z} = -2\mu k^2 \sin \theta_t \cos \theta_t \psi$$

(5-4)

$$\sigma_{rz} = \mu \left( \frac{\partial^2 \psi}{\partial r^2} - \frac{\partial^2 \psi}{\partial z^2} \right) = \mu \left( -k^2 \sin^2 \theta_t \psi + k^2 \cos^2 \theta_t \psi \right)$$

(5-5)

in which $\mu$ is the shear modulus of the glass. The $\sigma_T$ induced by this stress in a cylindrical coordinate is

$$\sigma_T = \frac{\sigma_{xx} + \sigma_{rr}}{2} + \sqrt{\frac{\sigma_{rr} - \sigma_{xx}}{4}}$$

(5-6)

The transmission coefficient ($T$) is defined by:

$$T = \frac{\sigma_T}{p} = \frac{\mu k^2 \psi}{\rho_0 \omega^2 \phi} \sqrt{(\cos^2 \theta_t - \sin^2 \theta_t)^2 + \frac{(4 \sin \theta_t \cos \theta_t)^2}{4}} =$$

$$\frac{\mu}{\rho_0 c_T} \sqrt{(\cos^2 \theta_t - \sin^2 \theta_t)^2 + \frac{(4 \sin \theta_t \cos \theta_t)^2}{4}} \left| \frac{-\frac{\rho_s Z_t \sin(2\theta_t)}{\rho_0}}{Z_1 \cos^2 2\theta_t + Z_2 \sin^2 2\theta_t + Z} \right|$$

(4-7)
Figure 28 shows the transmission coefficient $T$ at $\theta_T = 45^\circ$ is around 8 while the $T$ at $\theta_T = 90^\circ$ is around 15. This observation indicates that the $\sigma_T$ induced by the T wave at the boundary is around two times of the $\sigma_T$ induced by the T wave at $45^\circ$. This result indicates the LRW primarily contributes to the $\sigma_T$ on the boundary and the effect of T wave is negligible.

\[\text{Figure 28. The transmission coefficients (T) from T wave induced } \sigma_T \text{ to the amplitude of the incident pressure wave at different refraction angles.}\]
5.3 The $\sigma_T$ at the boundary

Since \textit{LRW} induced $\sigma_T$ on the boundary is significantly greater than \textit{T} waves induced internal $\sigma_T$, the rest of study will focus on the \textit{LRW} induced $\sigma_T$ on the boundary. Figure 29 shows the $\sigma_T$ detected by receivers at various radial distance at the boundary. I observe four features characterizing the \textit{LRW} induced $\sigma_T$ field. First, along the radial direction, the temporal profile of the maximum $\sigma_T$ ($\sigma_{T,max}$) shows a rapid buildup and
then a gradual decay process. Second, the wave profile broadens while its amplitude decays. Third, for individual wave profiles produced in the decaying stage, as indicated

Figure 29. $\sigma_T$ induced at the boundary.
by the red arrow, the stress all suddenly drops from tension to compression. Fourth, the global maximum of $\sigma_T$ is produced beyond the $\theta_{LRW}$.

Figure 30. The variations of LRW-induced stress components with normalized depth.

Figure 30 shows the depth-dependent variations of the six stress components produced by the LRW near the solid surface at $r = 0.5$ mm under $S_d = 0.5$ mm. In an axial symmetric case, the $\sigma_T$ can be calculated from stress components by

$$\sigma_T = \frac{\sigma_{rr} + \sigma_{zz}}{2} + \sqrt{\sigma_{rz} + \left(\frac{\sigma_{rr} - \sigma_{zz}}{2}\right)^2}$$  \hspace{1cm} (5-7)
As shown in Fig.30, at the glass/water boundary, \( \sigma_{rz} = \sigma_{zz} = 0 \); therefore, \( \sigma_T = \sigma_{rr} \) at the glass water boundary. As the depth (in the \(-z\) direction) increases, \( \sigma_{rr} \) decays exponentially and converts from tension to compression at about 0.08 \( \lambda_0 \) where \( \lambda_0 = c_{LRW}^* t_p \). Within the same depth, \( \sigma_{\varphi\varphi} \) decreases monotonically while \( \sigma_{zz} \) and \( \sigma_{rz} \) increase from zero to a maximum, which is an order of magnitude lower than the peak value of \( \sigma_{rr} \), before a gradual decay. The distribution of the six stress components in depth generally agrees with the distribution of stress components produced by a Rayleigh wave (Viktorov, 1967b). This agreement further confirms that the maximum \( \sigma_T \) is produced by the LRW.

Figure 31 shows that the \( \sigma_{T, \text{max}} \) at the boundary produced at various \( S_d \) all experiences a buildup process and a decaying process. However, the buildup and
decaying rate varies from different cases. The buildup and decaying rate increase with decreased $S_d$. 

Figure 31. Distribution of $\sigma_{T,max}$ at the boundary for different $S_d$. 
5.4 Generation of $\sigma_T$ at the boundary

Figure 32 shows the changes in the temporal profiles of $\sigma_T$ and the corresponding pressure in the fluid near the boundary at various radial distances. For $S_d = 0.5$ mm, the $\sigma_T$ first appears at $r = r_i = 0.16$ mm, which is smaller than $r_{LRW}$ (= 0.27 mm) - the corresponding radial distance when the shock wave incident angle ($\theta_i$) equals to $\theta_{LRW}$. In

![Figure 32. Temporal profiles of $\sigma_T$ in the solid and corresponding pressure on the fluid side produced by the advancing evanescent wave along the boundary at various radial distances.](image-url)
addition, $\sigma_{T,\text{max}}$ is achieved at $r = r_m = 0.5$ mm, far exceeding $r_{LRW}$. In comparison, under $S_d = 3.0$ mm, $r_i$, $r_{LRW}$ and $r_m$ shift to 0.89, 1.62 and 2.2 mm, respectively.

Several interesting features can be observed in Fig. 33 where $\sigma_{T,\text{max}}$ normalized by $P_i^*$, the peak acoustic pressure incident on the fluid-solid boundary at $\theta_{LRW}$, is plotted against $\frac{r}{S_d}$ (or $\theta_i$). First, the initiation of $\sigma_T$ appears at $\theta_i = \sim 17^\circ$, independent of $S_d$.  

\[ \frac{\sigma_{T,\text{max}}}{P_i^*} = \frac{1.16}{(\frac{r}{S_d} - 0.53)^{0.91}} \]

\[ \alpha \sim \frac{1}{r^{1.63}} \]

\[ \alpha \sim \frac{1}{r^{1.12}} \]

\[ \theta_1 (\text{degrees}) \]

$\theta_T^* = 25.7^\circ$ \hspace{1cm} $\theta_{LRW}^* = 28.3^\circ$

\[ 17 \quad 26 \quad 60 \quad 75 \]

\[ 10 \quad 8 \quad 6 \quad 4 \quad 2 \quad 1 \quad 0 \]

\[ \frac{0.5}{1} \quad \frac{r}{S_d} \]

\[ \frac{\sigma_{T,\text{max}}}{P_i^*} \]

\[ \theta_i (\text{degrees}) \]

\[ \theta_1 = \sim 17^\circ \]

$\theta_{LRW} = 28.3^\circ$

\[ S_d \]

\[ r \]

\[ P_i^* \]

\[ 1.16 \]

\[ (\frac{r}{S_d} - 0.53)^{0.91} \]

\[ \alpha \sim \frac{1}{r^{1.63}} \]

\[ \alpha \sim \frac{1}{r^{1.12}} \]

\[ \frac{r}{S_d} \]

Figure 33. The distribution of $\frac{\sigma_{T,\text{max}}}{P_i^*}$ along the normalized radial distance $\frac{r}{S_d}$ (bottom x-axis) and the corresponding incident angle $\theta_i$ (top x-axis) at various $S_d$. 

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Second, while the peak of $\frac{\sigma_T^{max}}{p_i}$ is produced significantly beyond $\theta_{LRW}$, it will rise with the corresponding $\theta_i$ shifting asymptotically toward $\theta_{LRW}$ when $S_d$ increases. Third, beyond the peak, $\frac{\sigma_T^{max}}{p_i}$ decays initially at a rate (e.g., $r^{-1.63}$ at $S_d = 3$ mm) varying with $S_d$. However, as $\theta_i$ increases, the decay of $\frac{\sigma_T^{max}}{p_i}$ converges to the same rate of $r^{-1.12}$, independent of $S_d$. In comparison, all these decay rates are greater than the corresponding decay rate of $r^{-0.5}$ associated with Rayleigh waves produced by a point source (F.Graff, 1975), which is consistent with the gradual re-radiation (or loss) of LRW energy by the Schmidt head wave into the surrounding fluid (H.UBERALL, 1973). Also importantly, the characteristic dual branch feature of the LRW first appears at about $\theta_i = 26^\circ$ (Fig. 34), shortly beyond the critical incident angle for the $T$ wave ($\theta_T^* = 25^\circ$). Therefore, the $\sigma_T$ is initiated by the $T$ wave at $\theta_i = 17^\circ$, and boosted subsequently by the ensuing LRW. Altogether, these results suggest that $\theta_{LRW}$ represents a resonant condition, around

![Figure 34. The contour plots of $\sigma_T$ showing the generation and propagation of LRW with characteristic dual branch feature and $T$ wave at various incident angles ($\theta_i$).](image)

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which LRW of varying launching efficiency will be generated by the sweeping incident shock wave to gradually build up the intensity (Viktorov, 1967b). Since \( \theta_{LRW}^* \) represents a resonant condition (H.UBERALL, 1973), an incident spherical shock wave will launch a LRW of sufficient efficiency within an incident angle bandwidth of 2 \( \Delta \theta \) around \( \theta_{LRW}^* \) (Fig.35) (Viktorov, 1967b). The dimension \( (D) \) of the projection area at the water-glass
boundary covered by the resonant band is proportional to the standoff distance \((S_d)\) given by:

\[
D = S_d\left[\tan(\theta_{LRW}^* + \Delta\theta) - \tan(\theta_{LRW}^* - \Delta\theta)\right]
\]

Equation 5-8 indicates that the area covered by the resonant bandwidth of the LRW will extend to infinity if \(S_d\) of the monopole source becomes infinitely large. Within this area, the incident wave keeps reinforcing each newly generated LRWs (H.UBERALL, 1973).

Figure 35. A schematic diagram showing the area covered within the resonant band at an \(S_d\).
Consequently, the resultant tensile stress produced by the LRW will approach asymptotically to infinity when \( S_d \) increases indefinitely. Further, since the propagation distance of the newly generated LRW on the boundary increases with \( S_d \), more high frequency components of the LRW will be re-radiated into the fluid (Dransfeld & Salzmann, 1970b; H.UBERALL, 1973), leading to a longer rise time in the resultant \( \sigma_T \) pulse, as shown in Fig. 31.

Also importantly, a compressive stress is produced by the advancing evanescent wave (EW) along the boundary on the fluid side when \( \theta_i \) exceeds \( \theta^*_T \) (see Fig. 24). This evanescent wave moves along the boundary together with the \( P_i \) and \( P_r \). The radial coordinate of the contact point of the \( P_i \) and \( P_r \) is given by:
\[
R = S_d \tan(\theta_i)
\]

The time the incident wave reaching this contact point is:
\[
t = \frac{S_d}{c_0 \cos \theta_i}
\]
in which \( c_0 \) is the sound speed in water.

From Eq. 5-9 and 5-10, the expanding speed of the contact point can be derived as:
\[
v = \frac{dR}{dt} = \frac{dR}{d\theta_i} \cdot \frac{d\theta_i}{dt} = \frac{S_d \cos \theta_i}{c_0 \cos \theta_i} \cdot \frac{S_d \tan \theta_i}{c_0 \sin \theta_i} = \frac{c_0}{\sin \theta_i}
\]
The speed of the advancing EW on the boundary decreases progressively with \( \theta_i \), reaching eventually the sound speed in water \( (c_0) \) at glancing incidence. Therefore, within
the resonant bandwidth of $\theta_{LRW}$ two simultaneously generated waves (i.e., $LRW$ and $EW$) will superimpose destructively with each other until the faster $LRW$ has advanced sufficiently to move out of the compressive region covered by the EW (see Fig. 24). Consequently, this interaction is also manifested by a rapid decay of $\frac{\sigma_{T,max}}{P_i}$ initially after the peak, compared to a gradual decay downstream when the characteristics of the $LRW$ propagation on the boundary become clear. The difference in decaying rates in suggests that the exact interaction mechanism between the evanescent wave and $LRW$ depends on $S_d$.

The evanescent wave is produced to counteract the load from $P_i$ and $P_r$ to satisfy the boundary conditions (2-30 – 2-31). As shown in Fig 32, at $r = 1.5$ mm and $r = 3.2$ mm for $S_d = 0.5$ mm and $S_d = 3$ mm, respectively, the magnitude of the compressive stress produced by the trailing evanescent wave is close to the magnitude of the pressure produced by $P_i$ and $P_r$ in the fluid. Thus, the contribution of the evanescent wave to $\frac{\sigma_{T,max}}{P_i}$ will not significantly change for different $S_d$ cases. In contrast, the tensile stress produced by the $LRW$ will keep increasing as $S_d$ increases due to a more efficient superposition of individual $LRW$ wavelets. This increase in the tensile stress will lead to a greater constructive contribution to the $\frac{\sigma_{T,max}}{P_i}$. As shown in Fig. 33, the peak of $\frac{\sigma_{T,max}}{P_i}$ will approach asymptotically to infinity when $S_d$ increases indefinitely. This interpretation is
consistent with the theory that for a plane wave incidence (equivalent to $S_d \rightarrow \infty$) $\frac{\sigma_{T,\text{max}}}{p_i^*}$ will approach infinity at $\theta_{LRW}$ (H. UBERALL, 1973).
6. Ring fractures

6.1 Formation of a ring fracture

Under the spherical shock wave impact, the LRW induced $\sigma_T$ at the glass-water boundary is generally greater than the typical tensile strength of a glass sample ($20 – 200$ MPa) (SCHOTT, 2007). Thus, such a strong $\sigma_T$ was expected to create fractures from the glass-water boundary. Figure 36 shows the characteristic ring-like fractures produced by the LRW-induced transient stress field on the surface. A crack was initiated after the 18th shock at a radial distance $R_1 = 1.6$ mm from the center where the probe axis projected through the glass surface. Since tensile stresses are primarily responsible for the failure of brittle materials, this crack is likely initiated by the tensile stress imposed on a pre-existing flaw of critical size on the glass surface. The presence of flaws tends to weaken the brittle material in tension. After its initiation, the crack extended circumferentially in both directions by a short arc length on the order of $\lambda_0$. Thereafter, the crack extended progressively along a circular path, driven by subsequent shock impacts. Eventually, the crack developed into a ring-like fracture after the 39th shock. The initiation and progression of the crack were likely to follow a dynamic fatigue process. Pre-existing micro-flaws were driven by successive shocks to extend and coalesce with adjacent ones until an observable macro-crack was formed. Figure 36 also shows the initiation of another
crack after the 23\textsuperscript{th} crack. This crack then was driven by subsequent shocks to propagate along a different circular path and eventually also developed into a circumferential fracture with a smaller radius ($R_2 = 1.4$ mm). These observations indicate that the pre-existing flaws primarily dictate the
Figure 36. The initiation and development of ring-like fractures on the glass sample surface.
location of the fracture initiation since its presence significantly lowers the required critical tensile stress for crack initiation through stress concentration at the tip of the crack (Lawn, 1993). The circular growth path of the crack after its initiation indicates that the extension of the crack was driven primarily by the local $\sigma_T$ on the surface. The ring-like fracture formation suggests that the first mode of brittle fracture (Lawn, 1993) dominates this dynamic fracture process (Fig. 37). The circular propagation path further suggests that the principle of local symmetry (Goldstein & Salganik, 1974) or equivalently, the

![Figure 37. The direction of $\sigma_T$ to the crack plane.](image)
maximum hoop stress principle (Bower, 2009) that favors crack extension along arbitrary paths with a symmetric stress distribution/no shear stress near their tips, also holds true in this impulsive fracture process.

6.2 Features of the ring-like fracture on the surface

![Figure 38. Multiple ring-like fractures induced at $S_d = 0.5$ mm.](image)

The overall increase in the shock loading on the glass surface will enhance the possibilities of crack initiation. At small $S_d (= 0.5$ mm), $\sigma_T$ was strong over a large area on the glass surface (see Fig. 31). Consequently, the required stress for crack initiation will be
exceeded over a large area on the glass surface, leading to the formation of multiple fracture rings with different radii (Fig. 38). The discrete distribution of fracture rings still suggests the dominate role of pre-existing flaws in the crack initiation process. Otherwise, fracture rings would have been created everywhere within a certain area, within which sufficient $\sigma_T$ is produced. In contrast, at large $S_d$ ($S_d = 2.5\, \text{mm}$), $\sigma_T$ became weak with only a single ring-like fracture produced (Fig. 39). At even larger $S_d$ ($\Rightarrow 3.0\, \text{mm}$), no fracture could be produced up to 100 shocks (Fig. 39). Together, these observations suggest the critical role of $\sigma_T$ in the crack initiation process. Despite the presence of micro-flaws, sufficient local $\sigma_T$ has to be supplied by the incident shockwave for the crack initiation.

The tensile strength of glass measured under practical conditions varies from 20 to 200 MPa (SCHOTT, 2007). The $\sigma_{T,max}$ in the glass sample calculated by the numerical mode varied in the range of 140 MPa (the maximum of $\sigma_{T,max}$ produced at $S_d = 3.0\, \text{mm}$) and 160 MPa (the maximum of $\sigma_{T,max}$ produced at $S_d = 2.5\, \text{mm}$). Since the tensile strength of a material under impact loading depends on the stress rate and sample quality, these results suggest that the tensile strength estimated from the model calculation falls within the high end of the practical tensile strength range for glasses.
Figure 39 shows that the radii of the ring-like fracture increased as $S_d$ becomes larger. Since the distribution of pre-existing flaws vary from sample to sample, the initiation location of the ring fracture is random even with the assumption that the variation in shock impact is negligible. Figure 40 shows that all the ring-like fractures from six tests at the same $S_d$ were created within a circumferential zone of fracture area. Overall, as $S_d$ became larger, the width of the fracture area and the total number of fracture rings decrease. This observation suggests the existence of a critical threshold to initiate fracture. As $S_d$ became larger, the area covered by sufficient $\sigma_T$ exceeding the threshold shrinks accordingly, resulted in a decrease in the width of the fracture area. If we assume a
uniform distribution of pre-existing flaws on the glass sample surface, the total number of flaws will drop as the area shrinks. This drop of the number of flaws leads to the decrease in the number of fracture rings.

6.3 Features of the ring fracture

Once the crack is initiated, in addition to its extension along a circular path on the surface, the crack will also extend into the interior of the glass. Labels $E_1$ and $E_2$ in Fig. 36 point to the images of the crack planes resulted from the extension of the crack into the

![Figure 40. The dimension of all ring fractures induced at various $S_d$.](image)
interior of the glass. Figure 41 shows the cross section view of a 3D constructed confocal image of a ring-like fracture. Clearly, the crack was first initiated perpendicularly to the surface. The initiation direction of the crack is perpendicular to the direction of the LRW-induced $\sigma_T$, suggesting that the crack was induced by the LRW induced $\sigma_T$ based on the principle of local symmetry. After its initiation, the crack propagated perpendicularly into the surface for a small distance and then changed its direction of extension with depth. Figure 42a shows that the LRW-induced $\sigma_T$ is parallel to the surface normal at the

Figure 41. The extension of the ring-like fracture into the interior of the glass sample.
However, with increased depth (in the negative z direction) the direction of $\sigma_T$ bifurcates into two branches (a leading LRW$^T$ and a trailing LRW$^L$), which rotates in the opposite directions, as shown in Fig. 42b ($S_d = 0.5$ mm, $r = 0.5$ mm). The direction of the $\sigma_T$ produced by the trailing LRW$^L$, generally agrees with the propagation direction of the crack into the interior of the glass. The agreement between the propagation direction of the crack and the direction of LRW$^L$ induced $\sigma_T$ also suggests that the principle of local symmetry holds for this impulsive fracture process. The crack should be first initiated by the $\sigma_T$ produced on the surface, where the magnitude of $\sigma_T$ is maximal. After its initiation, the $\sigma_T$ produced by the subsequent LRW$^L$ drives the crack into the interior.
extension into the glass obliquely. Under further LRW impact, the $\sigma_T$ produced by the leading $LRW^L$ branch is parallel to the crack plane, thus having little effect on its propagation. The $\sigma_T$ produced by the trailing $LRW^T$ branch is perpendicular to the crack plane, and thus will continue to drive the extension of the crack.

Figure 43 shows that the trajectory of the ring fracture first decreases and then increase with radial distance. This feature is generally consistent with the $z$-axis variation of the $LRW^L$ produced $\sigma_T$ (Fig. 44). This agreement further confirms the ring-like fractures were created by the $LRW^L$ produced $\sigma_T$ and the propagation of the crack in this impulsive dynamic fracture process generally follows the principle of local symmetry.

\[
\frac{r}{S_d} = \begin{align*}
0.7 & \quad 1.0 \\
1.4 & \quad 1.8 \\
2.3 & \quad 175 \mu m \\
63^\circ & \quad 49^\circ \\
41^\circ & \quad 62^\circ \\
61^\circ & \quad S_d = 0.5 \text{ mm}
\end{align*}
\]
Figure 44. The comparison between the observed orientation of the ring fractures and the $\sigma_T$ induced cracks along the radial direction.
6.4 Ring-like fracture initiation criterion

Figure 45a shows the distribution of $\sigma_{T,\text{max}}$ within the fracture area for various $S_d$. Since no ring fracture was produced at $S_d = 3.0$ mm, all the $\sigma_{T,\text{max}}$ produced at $S_d = 3.0$ mm should be below a critical threshold while the $\sigma_{T,\text{max}}$ produced within the fracture area at smaller $S_d$ should be above this threshold. However, the ring-like fracture results summarized in figure 45a suggest that such a threshold based on $\sigma_T$ does not exist. In contrast, the results suggest that the NPL-produced damage in glass samples can be better predicted by a stress integral (SI) threshold based on the Tuler–Butcher criterion for dynamic fracture of brittle materials (Fig. 45b). Specifically, the calculated SI values at various radial locations under $S_d = 2.5$ mm or less where fractures were observed.

![Graphs showing the distribution of $\sigma_{T,\text{max}}$ and SI within the fracture area at various $S_d$.](image)

Figure 45. (a) The distribution of $\sigma_{T,\text{max}}$ within the fracture area at various $S_d$. (b) The distribution of SI within the fracture area at various $S_d$. 

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experimentally were found to be higher than the threshold value of SI calculated at $S_d = 3.0 \text{ mm}$ (i.e. $2.4 \times 10^{-4} \text{ MPa} \cdot \text{s}$). Since SI incorporates both the magnitude and duration of the tensile stress pulse, this finding further confirms that the tensile strength of the glass under dynamic loading is stress rate dependent. As the NPL probe tip moves closer to the glass surface, the magnitude of the SI will increase, with a concomitant broadening of the range of radial distance above the damage threshold. This prediction is consistent with the higher number of ring fractures and wider damage zone observed experimentally at small $S_d$ (Fig. 40).
6.5 Dynamic fatigue in ring fracture formation

Figure 46. The correlation between the number of shocks required for the initiation of the ring fracture and the SI induced at the initiation location.

Figure 46 shows that generally a high number of shocks are required for the initiation of a ring-like fracture at a location where the SI produced by the LRW induced tensile stress pulse is low. This observation suggests that the initiation of the ring-like fracture is a dynamic fatigue process (Lokhandwalla & Sturtevant, 2000). Each tensile stress pulse facilities the extension of a pre-existing micro-flaw, and their subsequent coalescence with adjacent ones, eventually leading to the formation of an observable crack.
under an optical microscope (Lokhandwalla & Sturtevant, 2000), which is similar to the initiation of the ring-like fracture in this work.
7. Conclusion

7.1 Summary of the dissertation

The generation of LRWs by spherical shock waves produced by NPL and the surface fractures created by the LRW-induced $\sigma_T$ have been comprehensively investigated in this dissertation project. The first chapter of this dissertation reviews the development of SWL technology with a focus on stone comminution mechanisms. In addition, the first chapter discusses the motivation to study SAW induced fracture process in lithotripsy. The second chapter lays the theoretical foundation on solid mechanics and wave dynamics that are used in this study. The discussion of wave dynamics covers the characteristics of bulk and surface acoustic waves and their generation by an incident plane wave at the fluid-solid boundary. The third chapter discusses the principle of photoelastic imaging technique, and establishment of a light field circular polariscope and associated high-speed shadowgraph system that can be used to simultaneously capture fluid dynamics and photoelastic imaging. Besides, the third chapter covers the construction of the finite element model in COMSOL Multiphysics. The numerical model is validated against an analytical solution and then used to quantify the stress field in the solid produced by LRWs. The fourth chapter discusses features of the LRWs observed by the photoelastic/shadowgraph imaging. In particular, the wavefront of the LRW is found to
be comprised of two branches, propagating along the glass-water boundary at a speed close to the theoretical value. In addition, the inclination angle of the wavefront of the Schmidt head wave is close to the Rayleigh angle as predicted by theory. The results of a parametric test suggest that these features of LRWs are independent of the $S_i$ between the NPL probe tip and the glass boundary. In the fifth chapter, the numerical errors inherent to the COMSOL model was first assessed by comparison between the numerical results and the computational results from a known analytical solution. The agreement between the numerical and analytical results demonstrates the numerical accuracy of the COMSOL model. The applicability of the monopole model was tested by a comparison between contour plots from the numerical model and corresponding photoelastic images. The close agreement between the shapes and locations of various stress and pressure waves between the two models demonstrates that the numerical model is capable of capturing the main physical features in the experiment. Furthermore, the numerical results suggest that the accumulation of LRWs generated by the impact of the incident shock wave in the vicinity of the $\theta_{LRW}$ leads to the $\sigma_T$ buildup at the glass-water boundary. The interaction between the simultaneously generated $LRW$ and an evanescent wave creates a characteristic tensile stress field on the glass-water boundary. Along the radial direction, the distribution of this tensile stress field is characterized by a rapid rise to build up the maximum tensile stress on the boundary, followed by a gradual decay. Results of the
fracture tests are presented and discussed in the sixth chapter. The shock wave impacts initiated a crack at a radial distance away from the central axis of the probe tip. This crack was driven by subsequent shock wave impacts to propagate along a circular path and eventually form a ring-like fracture. The locations and geometrical features of the ring-like fracture were found to be highly correlated with the characteristic $\sigma_T$ field at the boundary and subsurface area. The dose-dependency in the fracture initiation further suggests that the formation of the ring-like fracture is a dynamic fatigue process.

### 7.2 Future work

In this study, the generation of LRWs and fracture at a flat glass-water boundary by a spherical shock wave impact was investigated experimentally and numerically. It is concluded that LRW-induced $\sigma_T$ drives the initiation and the propagation of the ring-like fracture. However, in SWL kidney stones usually have a curved and irregular geometry. The characteristics of LRW and other types of SAW generation and propagation on a fluid-solid boundary with curved geometry may be different than on a flat surface. This difference will lead to a different $\sigma_T$ distribution in the subsurface area, and may correspondingly produce different fracture patterns. In addition, cavitation in SWL creates surface pitting. The formation and accumulation of cavitation-induced surface defects have been speculated to influence stone fracture and eventually the fragmentation
process in SWL (Zhong, 2013). In the future, the overall investigational approach and insights from this study can be leveraged to study the mechanism of SAW generation, propagation and potential contribution to stone fragmentation in SWL.

In particular, a parametric numerical study should be carried out to investigate the effects of stone geometry on the generation and propagation of SAW. Ellipsoids with different eccentricities can be selected to evaluate how the curvature of the stone may affect the generation and propagation of SAW, and more importantly, the $\sigma_T$ field. Different material properties can be used in the ellipsoidal model to assess the effects of various kidney stone compositions to the generation and propagation characteristics of SAW in SWL.

Moreover, additional experiments should be conducted to investigate the effects of cavitation on SAW-induced fracture process. For example, a micro-indentation system can be used to create well-controlled surface damage patterns (MARSHALL, 1984) to simulate cavitation induced surface defects. The presence of a surface defect is expected to reduce the SI threshold required for crack initiation under subsequent shock wave impact. By creating surface defect on the glass surface, for example, it is anticipated that the initiation of ring-like fractures may be observed in the region beyond the fracture zoom shown in Fig. 40 and at $S_d = 3.0$ mm. Incorporation of surface defects in the
numerical model of shockwave-stone interaction should be considered (Sapozhnikov et al., 2007).
Reference


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