Fast, Nondestructive Quantum-state Readout of a Single, Trapped, Neutral Atom

by

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Mark C. Kruse

Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Physics in the Graduate School of Duke University
2018
**Abstract**

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Abstract

Experimental systems that trap single, neutral atoms have recently emerged as a promising platform for experiments in a range of disciplines such as quantum information science, quantum simulation and fundamental light-atom interaction. In this thesis, I build such a system and use it to trap and study a single, neutral atom of $^{87}$Rb. I confront and overcome several experimental challenges while designing and building the system. For example, I develop a MOT of unusual geometry with which to load the single-atom trap and also a detection scheme that robustly detects the trapped atom nondestructively, that is, without pushing it out of the trap. The result of this design and construction process is a system that stably traps a single atom in an optical dipole trap. I achieve trap lifetimes of over 1 minute in the absence of near-resonant laser light.

In addition to the experimental apparatus, I develop a thorough rate-equation model to predict the population dynamics of the trapped atom’s internal quantum state when probed by near-resonant light. This model gives unique insight into the influence of the atom’s internal dynamics on the detected scattering rate. I use this model to predict several important experimental parameters and compare it to the experimental data. This allows me to characterize the parameters that govern how the atom interacts with near-resonant laser light and how that interaction affects the experimental data. For example, I perform an absolute calibration of the collection efficiency of the experimental system, a first for a single, neutral-atom trap.
Using these experimental and modeling tools, I investigate the scattering rate of an atom in the presence of near-resonant linearly-polarized laser light. This is of great interest to the field because it is used to measure the atom’s internal quantum state, in a process known as quantum-state readout. Fast and accurate quantum-state readout is crucial to the success of many protocols in quantum information science and quantum simulation. Using the tools described here, I achieve quantum-state readout with an average fidelity of $97.6\pm0.2\%$ using a linearly-polarized probe beam. The readout requires a measurement time of $160\pm20\ \mu s$, and the atom remains in the trap after the readout in $97.1\pm0.1\%$ of the trials. I use linearly-polarized light instead of circularly-polarized light because it makes the readout less sensitive to the atom’s occupation of a specific magnetic sublevel, and hence does not require sublevel-specific state preparation. It also allows for a more flexible experimental geometry. This is the fastest and highest-fidelity nondestructive readout of a single neutral atom performed with a linearly-polarized probe beam reported to date.

In addition, I identify a decay in the atom’s scattering rate over the course of the readout time that limits the quantum-state readout fidelity. I investigate possible sources of this decay using the rate-equation model and a model of the readout protocol, and I conclude that it is likely caused by a combination of Raman transitions and heating. The heating is related to the near-resonant probe light and also to the optical dipole trap that holds the atom. I discuss ways that this decay can be avoided, but point out that these possible solutions result in longer readout times. This investigation has applications across a wide variety of experiments that require fast quantum-state readout.
To my grandparents
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List of Abbreviations and Symbols

Abbreviations

AOM  Acousto-optic modulator
APD  Avalanche photodiode
CCD  Charge-coupled device
CCTV Closed-circuit television
DDS  Direct digital synthesizer
DFB  Distributed feedback
ECDL External cavity diode laser
EMCCD Electron-multiplying charge coupled device
FORT Far-off-resonant trap
FPGA Field-programmable gate array
HWP  Half-waveplate
IR   Infrared
LC   Inductor-capacitor
MM   Multimode
MOT  Magneto-optical trap
NA   Numerical aperture
NIST National Institute of Standards and Technology
ODT  Optical dipole trap
PBS  Polarizing beamsplitter
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<td>p/c</td>
<td>Probe/cool</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional integral derivative</td>
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<tr>
<td>PM</td>
<td>polarization-maintaining</td>
</tr>
<tr>
<td>P/N</td>
<td>Part number</td>
</tr>
<tr>
<td>QWP</td>
<td>Quarter-waveplate</td>
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<tr>
<td>RF</td>
<td>Radio frequency</td>
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<tr>
<td>SBR</td>
<td>Signal to background ratio</td>
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<tr>
<td>SM</td>
<td>Single-model</td>
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<td>SPCM</td>
<td>Single photon counting module</td>
</tr>
<tr>
<td>TTL</td>
<td>Transistor-transistor logic</td>
</tr>
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<td>UHV</td>
<td>Ultra-high vacuum</td>
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Acknowledgements

I thank my family for their continued support of me, my choices, and my ambitions. My parents, Tom and Kathy Shea, and my brother Joe have had my back for as long as I can remember. I could not be where I am now without the loving environment they provided during my childhood and that they continue to foster today. I wrote my college admissions essay about how my family gave me the courage to pursue my education and passions. It’s still true.

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Though the world we experience every day appears to obey the laws of classical physics, we know that it consists of atoms and photons that are fundamentally quantum mechanical in nature. The theory of quantum mechanics was developed in the 1920s to explain the behavior of these tiny systems. Since then, interest in understanding the quantum nature of the world has continued to grow along with the dream of experimentally observing, trapping, and controlling individual quantum systems, particularly individual atoms.

Significant experimental progress has been made and single atomic ions were first trapped and observed in 1978 [84], [125]. Single neutral-atom trapping is a much newer field and individual neutral atoms were first trapped in a cavity in 1999 [129] and in free space in 2001 [103]. I focus on free-space single-atom traps in this thesis.

Alongside these experimental gains has grown a theoretical and conceptual interest in using individual quantum systems as information-theoretic resources and to simulate other systems in nature. The first of these interests has grown into the field of quantum information science [87] and the second into the field of quantum simulation [33].
The work presented in this thesis has applications in both of these fields so I introduce them briefly in this chapter. I then discuss existing work in single neutral-atom traps, aiming to provide context and motivation for the work in this thesis. I highlight the importance of fast and nondestructive quantum-state readout and discuss previous effort in this area. I finish with a brief overview of the work presented in this thesis.

1.1 Applications of single, controllable quantum systems

1.1.1 Quantum information science

Quantum information science has experienced significant growth since its start in the mid-1980s and can be separated into the fields of quantum computing and quantum communication. Quantum computing has the goal of building a computer that uses quantum systems to encode information, which promises efficient computation of certain, hard problems. A large-enough quantum computer can solve problems such as prime factorization [110] and unlabeled search [38] faster than a classical computer.

Quantum communication exploits the fact that quantum systems also hold the promise of fundamentally secure communication [10] even in the presence of an eavesdropper. Quantum communication schemes rely on photons as their quantum bits (qubits) of choice as photons are natural carriers of information. Quantum computers, on the other hand, tend to utilize “matter” (non-photonic) qubits such as trapped ions, superconducting circuits, or trapped neutral atoms.

At the intersection of quantum computing and quantum communication is the idea of a quantum network [58]. In a quantum network, quantum nodes process and store quantum information and are connected by quantum channels that transport quantum states and distribute entanglement within the system. The work of this thesis has potential future applications to quantum computing and to quantum networks.
The field of experimental quantum computing has long been dominated by single trapped ions as they are the most mature technological platform in the field [124]. Trapped ions are the best storage medium we have for quantum information and set the record for longest qubit coherence time at 10 minutes in 2017 [121]. They also currently hold the record for the highest-fidelity quantum gates with a fidelity of $>99.99\%$ for single-qubit gates and $99.9\%$ for two-qubit gates [7].

Superconducting quantum circuits have been adopted by most industry interests in quantum computing because of their natural compatibility with current manufacturing technology and their fast gate times (tens of ns [42]). These platforms are promising and the community is aiming to build ever larger devices with IBM recently announcing a 50-qubit device [120], Intel a 49-qubit device [86], and Google a 72-qubit device [56].

Trapped neutral atoms also provide a promising platform for quantum computing protocols [100]. The all-optical trapping technology used for neutral atoms allows complex geometries of qubits to be created [8]. This potentially enables multiqubit gates to be implemented more easily than in the 1D and 2D geometries of trapped ions and superconducting circuits.

In addition, neutral atoms are good candidates to form quantum networks of nodes connected by photonic links [94]. Trapped neutral alkali-metal atoms are resonant with near-infrared photons rather than the ultraviolet and green photons that interact with most trapped ions or the microwave photons that drive superconducting circuits. Near-infrared photons can be detected with higher efficiency and can travel farther in optical fiber than these other wavelengths. They are also much closer to the telecommunications wavelengths of 1550 and 1310 nm which experience the lowest loss in silica optical fibers.
1.1.2 Quantum simulation

In addition to their uses for quantum information protocols, neutral atoms have long been excellent platforms for analog quantum simulation, which aims to use a simple quantum system to mimic a more complex one. An ensemble of atoms trapped in an optical lattice was used to simulate the superfluid to Mott insulator transition in 2002 [36] and since then similar setups have studied many systems in condensed matter physics and beyond. “Quantum gas microscopes,” where single atoms are imaged at individual lattice sites of an optical lattice, have been used to study both bosonic [6],[107] and fermionic [16] dynamics. The complex geometries achievable with all-optical traps are particularly suited to the simulation of other complex quantum systems.

With continued experimental progress, it has become possible to bring neutral-atom systems from the regime of analog quantum simulation to that of gate-based quantum simulation, where systems are simulated by implementing quantum logic gates on individual qubits. Such systems are also suited to quantum computing. The key to making this transition has been the ability to trap neutral atoms in arrays of traps that are spatially separated enough to be individually addressable. This is in contrast to trapping in lattices where the atoms are separated by a fraction of the lattice-light wavelength. In 2017, an array of neutral atoms was used to demonstrate a 51-qubit quantum simulation of an Ising-type spin model with tunable interactions [12]. This was quickly followed by a 53-qubit simulation of a multibody phase transition with a trapped ion system [130]. Trapped neutral-atoms systems are coming into their own and beginning to compete with more mature technologies.

1.1.3 Protocol structure and figures of merit

It’s important to note that quantum information and quantum simulation protocols rely on the same basic structure of preparation, interaction, and measurement. First
the system has to be prepared into a quantum state. Then, the interaction of interest (e.g., a one- or two-qubit gate or a change in the system Hamiltonian) is performed, and finally the resulting quantum state is measured, or read out of the system. The figures of merit that generally interest the field are the speed at which these individual steps can be done, the length of time the quantum state of interest can survive (its coherence time), and the accuracy with which the measured quantum state matches the desired state, known as the fidelity. A more precise definition of fidelity is given in Ch. 6. Since quantum information is collected through measurement, the readout of quantum states is of paramount importance to the success of both quantum computing and quantum simulation protocols.

My contribution to the field of atomic physics lies in building a new experimental system to stably trap and study a single, neutral atom and carefully studying the atom’s scattering rate in the presence of near-resonant laser light. I design and test experimental protocols geared towards minimizing experimental complexity. I also develop a thorough rate-equations model to explain the atom’s quantum-state population dynamics in the presence of near-resonant readout light and the AC-Stark shifts induced by the trap. This model allows me to characterize and understand the system and yields insights into how to optimize quantum-state preparation and readout protocol. With this system, I experimentally realize fast quantum-state readout of a single, trapped, neutral atom without losing the atom from the trap. The results presented here can be applied to many single-atom set-ups existing in the field and the experimental system described is a flexible foundation for several types of future experiments.

I now present a brief review of existing work in single neutral-atom traps. I then highlight the importance of fast and nondestructive quantum-state readout and discuss previous work in this area. I finish with a brief overview of the work presented in this thesis.
The first single-atom trap. In Schlosser et al.’s experiment, dipole trap light (yellow) enters the vacuum chamber and is focused by a custom-built lens into the microscopic ODT. The same lens also collects fluorescence light (red) from the atom. This figure is a modification of Figure 1 in Ref. [103].

1.2 Single, neutral-atom traps

The first experiment to successfully trap a single, neutral atom in free-space was reported by Schlosser et al. in 2001 [103]. They used a tightly-focused, red-detuned, Gaussian beam to create a microscopic optical dipole trap (ODT) for a single atom. The crucial feature of their setup was a custom-built lens inside the vacuum chamber that focused the dipole trapping light to create the microscopic ODT. The mechanism of the trapping technique is described in Ch. 2. The single atom was loaded into the dipole trap from a background magneto-optical trap (MOT). Figure 1.1 depicts the general experimental setup and is a modification of Figure 1 of Ref. [103].

Schlosser et al. found that the atom loaded into the ODT with sub-Poissonian statistics and that it was possible to reach a regime, which they named the “collisional blockade regime,” where there was always either one atom or zero atoms in the trap [104]. This proved to be a robust (though probabilistic) method for achieving single-atom trapping and is discussed in Sec. 2.4. Tightly-focused Gaussian-beam traps
are the most common single-atom traps in use today, and the type of trap used in this thesis. In this introduction, I focus on work undertaken in such traps.

1.2.1 Rydberg-atom systems for quantum protocols

A large portion of the single-atom-trapping field is interested in implementing quantum computing protocols. This requires implementing two-qubit gates between individual atoms [114], a non-trivial task for weakly-interacting neutral atoms. In 2000, Jaksch and colleagues proposed a mechanism for achieving two-qubit gates in neutral atoms via the Rydberg-interaction [47], [69] and the field of Rydberg-based quantum computing was born. A thorough review of the field is offered in Ref. [100].

Within a few years of Jaksch et al.’s proposal and Schlosser et al.’s demonstration of single-atom trapping, several groups demonstrated controllable single qubit gates in neutral-atom traps [105], [128], [51]. A two-qubit Rydberg gate was first demonstrated in 2010 by Isenhower et al. [45] and by Wilk et al. [123]. This was a milestone achievement for the neutral-atom quantum computing community.

Since this initial demonstration, two-qubit Rydberg gates have been demonstrated in arrays of microtraps in 2015 [72] and with improved fidelity in 2016 [48]. Multiqubit gates have been formed in an experiment where several atoms are trapped in the same ODT and entangled via the Rydberg blockade [28]. Significant progress has also been made in using Rydberg-based systems for quantum simulation in both two-dimensional [63] and a one-dimentional [12] simulators. Rydberg-based entangled states with an entanglement fidelity of 0.97 were reported in June 2018 [67], the highest neutral-atom entanglement reported to date.

The 5+ year gap between the first demonstration of a two-qubit gate in 2010 and subsequent experiments (and the 10-year wait between proposal and demonstration) hints at the experimental challenges that these systems face. These challenges are detailed in Mark Saffman’s review of the field from 2016 [98] and require significant
effort to overcome. Investigating these systems and challenges was the motivation for the work in this thesis.

1.2.2 Non-Rydberg-based systems

Rydberg-based platforms form the majority of the single-atom trapping systems being used today, but they are not the only platform available. Two-qubit gates can also be implemented with controlled collisions between individual atoms [39]. This technique was used to entangle atoms in optical lattices in 2007 [3] and was extended to single-atom traps in 2015 [55]. The same set-up that entangled two neutral atoms in 2015 also demonstrated two-particle interference in an atom Hong-Ou-Mandel experiment in 2014 [54] and produced spin-singlet states of single atoms in 2018 [66].

As previously mentioned, single-atom traps are also used to interface atoms with photons for atom-photon entanglement and building up quantum networks. This has been given a lot of attention for trapped ions [26] and for neutral atoms trapped inside optical cavities [94] but has also been demonstrated with free-space traps. In 2006 a single atom trapped in a microscopic dipole trap was entangled with a single photon [119] and in 2012 two such atoms, independently trapped 20 m apart, were remotely entangled with each other in a heralded way via entanglement swapping by Hofmann et al. [40]. This was a crucial step towards creating a quantum network of distant nodes and was, at the time, the longest-distance creation of heralded entanglement between matter qubits. As impressive as this achievement is, it had a low remote-entanglement rate (\(~0.4 \, \mu\text{Hz}\) compared to a similar experiment with trapped ions in 2008 whose remote entanglement rate was \(~26 \, \text{mHz}\) [76]. A later trapped-ion system increased the remote entanglement rate to \(4.5 \, \text{Hz}\) in 2015 [43]. One of the main causes of this low rate was the quantum-state readout protocol used in the system. Hofmann et al., like most other groups at the time, used a quantum-state readout technique that pushed their atom out of the trap. This necessitated
re-loading an atom into their trap before repeating the experiment. Since their trap took several seconds to load, they were limited to sub-Hz repetition rates. This is the problem that motivated the work in this thesis.

1.2.3 Basic atomic physics and quantum science

In addition to the quantum information and quantum simulation protocols outlined above, single, neutral-atom traps can further our understanding of fundamental quantum science and atomic physics. The remote entanglement experiment of Hofman et al. was extended in 2017 to a full loop-hole-free test of Bell’s inequality [96]. Also in the last five years, Rydberg-based systems have directly measured the van der Waals force between two atoms [9], created atomic Fock states of known atom number [27], and studied the nonlinearity of the Jaynes-Cummings ladder [64]. Similar traps have measured the Wigner time-delay in a packet of light scattered by a single atom [14] and the coherent scattering of light by small, dense clouds of atoms [49].

1.3 Quantum state readout

Almost all of the experiments mentioned in this chapter rely on the ability to determine the atom’s internal quantum state. Achieving this readout is challenging, so I address it here. Experiments that use qubits usually use specific magnetic sublevels of the hyperfine ground states of the atom as the qubit states. Discriminating between specific magnetic sublevels of a single hyperfine state is very difficult because the sublevels are degenerate unless perturbed by something like an external magnetic field. Thus, the two qubit states are usually located in different hyperfine ground states. The $m_F = 0$ sublevels are generally regarded as the best choice of qubit states because they are insensitive to fluctuations in magnetic field. For qubits encoded in $m_F \neq 0$, such fluctuations are a major source of decoherence. In $^{87}$Rb, the $|F = 1, m_F = 0\rangle$ sublevel and the $|F = 2, m_F = 0\rangle$ sublevel of the $5S_{1/2}$ ground
Fig. 1.2 **Level diagram of $^{87}$Rb** a) The full level diagram of $^{87}$Rb shows two typical qubit states $|0\rangle$ and $|1\rangle$ located in the $m_F = 0$ sublevels of the $5S_{1/2}$ ground state hyperfine levels. Relevant level splittings and quantum numbers are given on the diagram. b) Any readout protocol that distinguishes between the $F = 1$ and $F = 2$ hyperfine levels also distinguishes between the two qubit states. This is the type of “quantum-state readout” protocol discussed in this thesis.

state for a good qubit, as depicted in Fig. 1.2a. This is the qubit utilized by many Rydberg-based protocols [100]. Since the qubit states are located in different hyperfine ground states, a readout protocol that distinguishes between the two hyperfine levels of the ground state also distinguishes between the two qubit states. In this way, the qubit states can be mapped to the hyperfine levels, as depicted in Fig. 1.2b. This is the type of protocol I describe here and name “quantum-state readout.”
Fig. 1.3  **Quantum state readout of $^{87}$Rb**  The atom is probed (blue arrow) on the $^{5}S_{1/2} \ F = 2 \rightarrow ^{5}P_{3/2} \ F'3$ cycling transition. On the left, an atom in the $F = 2$ ground state fluoresces (red arrow) at the rate of $\sim 10^6$ decays/s and appears “bright”. On the right, an atom in the $F = 1$ ground state has a much lower scattering rate ($\sim 1$ decay/s) and appears “dark” in the presence of this probe.

Distinguishing between $^{87}$Rb atoms in the $F = 1$ and $F = 2$ hyperfine level of the $^{5}S_{1/2}$ ground state is done via fluorescence detection, a technique developed in ion traps [126]. Fluorescence detection takes advantage of the difference in energy of the ground states in order to address atoms in the $F = 2$ ground state but not in the $F = 1$ ground state using a probe beam resonant to the $^{5}S_{1/2} \ F = 2 \rightarrow ^{5}P_{3/2} \ F'3$ transition, depicted in Fig. 1.3.

Any atom that is in the $^{5}S_{1/2} \ F = 2$ state when the probe beam is turned on is excited to the $^{5}P_{3/2} \ F' = 3$ state and decays, emitting fluorescence. This is depicted in Fig. 1.3, with the blue arrow signifying the probe and the red arrow the subsequent atomic fluorescence. Since quantum mechanical selection rules dictate that the $^{5}P_{3/2} \ F' = 3$ state only decays to $^{5}S_{1/2} \ F = 2$, this process repeats and the atom continues to scatter photons from this transition. This picture is valid as long as the probe beam is tuned close to the atomic resonance and weakly excites the atom. The rate at which the multi-level atom scatters photons is determined by the detuning of
the probe beam from the various atomic transitions and the intensity of the probe beam, as discussed in Chs. 3 and 5. For typical experimental parameters, it is \( \sim 10^6 \) decays/s.

If, on the other hand, the atom is in the \( 5S_{1/2} F = 1 \) ground state, the same, weak probe beam is detuned from the atomic transition by the hyperfine splitting of the ground states \( \Delta_{hf} = 6.8 \) GHz and the scattering rate is typically only \( \sim 1 \) decay/s. Thus, an atom in \( F = 2 \) scatters roughly a million more photons than an atom in \( F = 1 \) in the presence of this probe beam in a given readout time. Fuller calculations of these scattering rates are given in Ch. 6 and the theory behind them is discussed in Ch. 2.

This large difference in scattering rate allows fluorescence state detection. If the photons that the atoms scatters are collected and detected, an atom in the \( F = 2 \) state appears “bright” in the presence of the probe beam while an atom in \( F = 1 \) appears “dark.” Generally the atom is illuminated by the probe beam for a set “probe time” and if the number of photons collected is above a chosen threshold \( n_{thresh} \), the atom is declared to be in the bright state. If fewer than \( n_{thresh} \) photons are detected the atom is declared to be in the dark state [83]. This scheme is known as threshold detection. The details of predicting the detection fidelity are discussed in Ch. 6.

This state detection scheme is conceptually simple but experimentally challenging. The main challenge is that the probe beam used to excite the atom and cause the fluorescence which needs to be detected can also heat the atom. The mechanism behind this heating is discussed in Ch. 2. If the probe beam heats the atom too much, it will be lost from the trap, potentially before enough photons are collected to determine if the atom is in the bright state or the dark state. Increasing the probe power increases the rate of photon scattering but also increases the heating.

For trapped ion systems this heating does not matter that much because the ions traps are very deep, with a depth of \( \sim 10^3 \) K [112]. Since the energy imparted to
Fig. 1.4  **Destructive Quantum State Readout** i) three atoms are trapped and imaged in a state-insensitive way (black circles).  ii) All of the atoms are prepared into quantum state $|1\rangle$ (green). iii) One atom is moved to quantum state $|0\rangle$ (blue). iv) All state $|1\rangle$ atoms are pushed out of the trap.  v) The remaining atom(s) are imaged in a state-insensitive way. This figure is based on Figure 2 in Ref. [105].

the atom by a single photon is generally equivalent to a few hundred nK, the ion can scatter billions of photons before being heated out of the trap, and there is plenty of time to collect enough photons to tell if the atom is bright or dark. This allows trapped ion experiments to use a strong beam for state detection and achieve very high-fidelity quantum-state readout of $> 99.9\%$ in only a few $\mu$s without losing the atom from the trap [88].

For neutral atoms trapped in ODTs, heating is a much bigger problem. The ODTs are typically $\sim 1$ mK deep, meaning the atom can only scatter a few thousand photons before it is lost, as explained in Sec. 2.5. This makes performing state detection without heating the atom out of the trap challenging, which is why the first state readout schemes were destructive and took advantage of the heating caused by the probe beam [77],[105]. I explain this destructive method below before moving to discussing current nondestructive schemes.
### 1.3.1 Destructive state readout

Figure 1.4 depicts an experiment that uses the destructive state readout technique developed in 2003 [60]. This figure is based on Figure 2 of Ref. [105]. In step i, several atoms of unknown quantum state (depicted in black) are trapped in an array of traps and imaged with light that also cools the atoms down, ensuring they are not lost from the trap. Unfortunately, this type of imaging is insensitive to the atomic ground state. Next (step ii), all of the atoms are prepared into ground state $|1\rangle$ (depicted in green). This preparation is usually done by a laser pulse, depicted with the green rectangle in the figure. In step iii, one of the atoms is addressed separately (blue rectangle) and moved to ground state $|0\rangle$ (depicted in blue). This is an example of a single-qubit gate, but could also represent a more complex procedure. Step iv is the destructive push-out step and all of the state $|1\rangle$ atoms are pushed out of the trap with a strongly resonant laser beam (dark green rectangle). This is done with a probe tuned to the atom’s cycling transition, like that of Fig. 1.3. In this way, the destructive readout scheme takes advantage of the heating caused by the probe beam. Finally (step v), the remaining atoms are imaged again in a state-insensitive way showing that only one of the atoms was in state $|0\rangle$ and remained in the trap. Using this technique, Ref. [77] discriminated between atoms of different quantum states with a contrast of 200:1, or an error of $1/201 = .00498$. Defining $fidelity = 1 - error$, this gives a state readout fidelity of $> 99.5\%$, the record for single, neutral atoms. The total state readout time constitutes steps iv and v of Fig. 1.4 and took 1 ms for step iv and 150 ms for step v. Other experiments have lowered the duration of these steps to 100 $\mu$s for step iv and 20-60 ms for step v [95] mainly by increasing the power of the push-out beam and collecting more light out of the atom during the state-insensitive detection step.
Fig. 1.5  **Typical experimental sequence** The trap is probabilistically loaded from the background MOT, during the loading phase which is followed by state preparation, experiments, and state detection. The loading phase is the longest step by at least one, usually two of three orders of magnitude. State detection is usually the second longest step.

### 1.3.2 Nondestructive state readout

The destructive state-readout scheme developed in Ref. [60] is very accurate (fidelity $>99.5\%$), but requires reloading the trap after $\sim 50\%$ of the experiments for typical sequences. This drastically reduces the rate at which experiments can be run, seriously limiting the computation rates for quantum computing protocols and the remote entanglement rates for quantum repeater protocols. An example experimental sequence is shown in Fig. 1.5 with typical durations for the trap loading, quantum-state preparation, experiments, and quantum-state readout steps. The trap is probabilistically loaded from the background MOT, usually taking a few seconds. The quantum-state preparation can be as fast as a few $\mu$s, the experiments as fast as tens of ns, and the quantum-state readout usually takes a few ms. Consequently, both avoiding the need to reload the trap and decreasing the length of the state detection step can increase the rate at which experiments can be repeated.

The time-scales of these rates are generally set by the type of interaction governing each step. A natural timescale of the system is the excited state lifetime of the atom. This is typically tens of ns. Another time scale is dictated by the coupling strength between the atom and the laser. This governs how quickly the atom scatters
photons from the laser and cannot be faster than twice the spontaneous emission
time. State-preparation can be achieved by using relatively strong lasers to move
the atomic population into the desired quantum state. This typically requires the
atom to absorb and emit hundreds of photons. Even with the strongest-possible
atom-light coupling, this takes a few microseconds. “Experiments” can be as simple
as the emission of a single photons, so this step can be on the order of a few times
the excited state lifetime. Quantum-state readout is usually performed with near-
resonant light that interacts weakly with the atom. The light scattered by the atom
must be collected and experimental systems can only collect a small fraction (~1%)
of the scattered light. Thus, the atom typically needs to scatter thousands of photons
before enough are collected for the readout to be successful, which takes several ms
with a weakly-resonant probe.

As an aside, I note in Fig. 1.5 that the trap is probabilistically loaded from the
MOT. This is the loading scheme demonstrated by Schlosser et al. [103] that has a
maximum efficiency of 50% and is still used by most groups, including me. However,
there have been efforts to replace this probabilistic loading by deterministic loading
that have achieved loading with 90% efficiency in 170 ms [65] and 80% efficiency in
60 ms [31]. These deterministic loading schemes are promising but add significant
experimental complexity to the system as they require two additional frequencies
of light present during the loading sequence. I do not discuss them further in this
thesis.

Even if deterministic loading is used and the loading procedure sped up, it is
still desirable to avoid reloading an atom into the trap after each experiment. With
this in mind, Fuhrmanek et al. [30] and Gibbons et al. [35] first demonstrated non-
destructive quantum-state readout in 2011. Nondestructive state readout schemes
rely on collecting a large percentage of the photons scattered by the atom. This allows
fewer photons to be scattered from the atom before enough are collected to surpass
threshold and make a decision about the atom’s quantum state. Nondestructive schemes also usually illuminate the atom with two counter-propagating probe beams and use probe light that is tuned to a slightly lower frequency than the atomic resonance. In this way the probe beams can cool the atom while probing it, extending its lifetime in the trap [62]. This cooling mechanism is described in Ch. 2. Cooling with two beams at a single frequency is not as efficient as the state-insensitive cooling that is used in the destructive technique. Thus, great experimental care must be taken in determining the probe power and frequency and collecting a lot of photons from the atom is crucial. I use both of these techniques in this thesis. I also show that careful choice of the probe beam frequency, power, and polarization has a drastic affect on the readout fidelity and speed.

In both Refs. [30] and [35], the $^{87}$Rb atom was probed on the $5S_{1/2} F = 2 \rightarrow 5P_{3/2} F'3$ transition, depicted in Fig. 1.3. To collect the photons, Fuhrmanek et al. put a high numerical aperture (NA) lens inside their vacuum chamber and used it both to focus down the trap light and collect the atomic fluorescence. The collected fluorescence was sent to an avalanche photodiode (APD) for detection [30]. A diagram of their setup is down in Fig. 1.6a, which is modified from Figure 1 in Ref. [30]. Gibbons et al. collected the atomic fluorescence with a high-NA lens outside of their vacuum chamber. Their ODT was also created by lenses outside the chamber and was retroreflected to create a 1D optical lattice in which they only trapped one atom at a time [34]. Figure 1.6b depicts their setup and is a modified version of Figure 1 in Ref. [34].

Using the scheme outlined above, Fuhrmenek et al. achieved quantum-state read-out with a fidelity of 98.6% in 1.5 ms of probe time. The atom was retained in the trap >98% of the time after their readout scheme, demonstrating its nondestructive nature [30]. Gibbons et al. achieved 95% fidelity in 300 µs of readout time, with 98% retention [35].
The two experiments were slightly different. Fuhrmanek et al. probed the atom with circularly polarized light. This scheme can decrease readout errors (increasing fidelity) but requires some additional experimental complexity such as careful alignment of the probe beam to the chosen quantization axis (set by the magnetic field in Fig. 1.6a). Gibbons et al. used linearly polarized light, which is less protected from errors but does not require a magnetic field. I have investigated the pros and cons of these schemes and discuss them in Ch. 6.

Since this initial work in 2011, two groups have extended the scheme of Fuhrmanek et al. to quantum-state readout in multi-atom arrays using a camera to detect the fluorescence. In 2017, Kwon et al. demonstrated 98.6% detection fidelity with a 6 ms readout time and with 98% of the atoms retained in the array [62]. Their readout fidelity of >98% included corrections for imperfect state preparation and background losses. Without that correction, their mean readout fidelity was 97%. Almost simultaneously, Martinez-Dorantes et al. demonstrated 98% readout fidelity with a 10 ms readout time and with 99% of the atoms retained in the array [74]. For their experiment, they developed a novel Bayesian inference method to weight their readout towards those pixels on the camera on which an atom had previously
been detected. This algorithm is nontrivial to develop and implement, but without it their state readout fidelity was 93% [74]. Both of these experiments used a camera for the state readout, so, in addition to the readout time listed, there was at least 100 ms of wait time while the data was transferred from the camera to the computer.

The work described in this thesis uses an experimental setup similar to that shown in Fig. 1.6a and an experimental protocol similar to that used by Gibbons et al. described in Ref. [35]. My goal is to achieve fast, nondestructive quantum-state readout in order to improve the rate at which experiments can be repeated. Ultimately, I achieve a quantum-state readout fidelity of 97.6 ± .2% in 160 ± 20 µs of probe time. The average retention of the atoms in the trap is > 97%. These numbers include no correction for atom loss and no algorithmic adjustments.

My experimental protocol is quite a bit simpler than that of Refs. [30], [62], and [74]. My work is distinct from that reported in [35] in that I develop a rate-equations model for the system and optimize the frequency at which I drive the atom. I also determine the error rates in the system using data, a mathematical model previously developed for a trapped-ion system [88], and the rate-equation model developed here. This yields information about the factors limiting the fidelity. Finally, I compare quantum-state readout using linearly- vs. circularly-polarized light both experimentally and using the rate-equation model. Table 1.1 compares the quantum-state readout results reported in this thesis to those reported in the experiments described above. To the best of my knowledge, there has been no other work reported in nondestructive quantum-state readout in neutral-atom traps.

1.4 Thesis overview

This thesis describes my work in building an experimental system to stably trap a single, neutral atom and closely investigate it’s interaction with near-resonant laser light. I describe the experimental system and the theoretical model I have developed
Table 1.1: Comparison of quantum-state readout in neutral-atom traps


<table>
<thead>
<tr>
<th>Experiment</th>
<th>Fidelity</th>
<th>Time</th>
<th>Retention</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Destructive schemes</td>
<td>&gt; 99.5%</td>
<td>20-150 ms</td>
<td>50%</td>
<td></td>
</tr>
<tr>
<td>Ref. [30]</td>
<td>98.6%</td>
<td>1.5 ms</td>
<td>&gt; 98%</td>
<td>[1], [2]</td>
</tr>
<tr>
<td>Ref. [35]</td>
<td>95%</td>
<td>300 µs</td>
<td>&gt; 98%</td>
<td>[3]</td>
</tr>
<tr>
<td>Ref. [62]</td>
<td>98.6%(97%)*</td>
<td>6 ms+</td>
<td>&gt; 98%</td>
<td>[1], *[4], +[5]</td>
</tr>
<tr>
<td>Ref. [74]</td>
<td>&gt; 98%*</td>
<td>10 ms+</td>
<td>99%</td>
<td>[1], +[5], +[6]</td>
</tr>
<tr>
<td>This thesis</td>
<td>97.6 ± 0.2%</td>
<td>160 ± 20 µs</td>
<td>&gt; 97%</td>
<td>[3]</td>
</tr>
</tbody>
</table>

to describe the atom’s population dynamics in the presence of near-resonant probe light. I use this system and model to demonstrate fast, nondestructive quantum-state readout of a single neutral atom, achieving readout fidelities comparable to those seen in other systems in a shorter measurement time. The thesis is organized as follows.

In Ch. 2, I present an overview of the theory needed to understand single-atom optical-dipole traps. I begin with the basic theory of an electromagnetic wave interacting with a two-level atom. I use this theory to describe the forces that light exerts on atoms, focusing on the dipole trapping force that is used to create the single-atom ODT and the radiation pressure force that cools atoms in the ODT and the MOT. I then discuss how the ODT is loaded from the MOT and describe the “collisional blockade regime” identified by Schlosser et al. [104] that allows for single-atom trapping. I finish with a discussion of the heating and loss mechanisms that can cause a single atom to be lost from the ODT.

In Ch. 3, I extend some of the two-level atom theory of Ch. 2 to the case of multilevel atoms such as $^{87}\text{Rb}$, which is the atom trapped in this thesis. I focus on the $5S_{1/2}$ ground state and $5P_{3/2}$ excited state of $^{87}\text{Rb}$ which collectively have 24
magnetic sublevels. I define the multilevel dipole matrix element and discuss how it gives rise to level-specific shifts to the atom’s sublevels when it is trapped in the ODT. These are known as AC-Stark shifts and can have a significant affect on the atom’s resonance frequency and the way it interacts with near-resonant probe light. The calculation I present at the beginning of Ch. 3 was published in Physical Review A as a Comment in 2017 [106].

The rest of Ch. 3 is devoted to the development of a multilevel rate-equations model that describes the atomic population dynamics of the 24-level $^{87}$Rb atom in the presence of the AC-Stark shifts and near-resonant laser light. This model predicts the population of each atomic sublevel as a function of time for various probe-light parameters. I explain how the model is assembled and implemented in Wolfram Mathematica, which I use to solve the system of differential equations. I explain the limitations of the model and show some results to demonstrate it’s power. The rate-equation model developed here does not include any new atomic physics but it is unusual in that it covers all of the atom’s magnetic sublevels, accounts for any (pure) laser polarization and frequency, and includes the AC-Stark shifts caused by the dipole traps. Most groups do not model the full system of the atom this thoroughly, as the model is labor intensive to implement.

In Ch. 4, I describe the experimental setup that I build from scratch to trap a single $^{87}$Rb atom. A schematic of the system is shown in Fig. 1.7. I discuss the design choices I made in building the system, specifically the choice to use an unusual MOT geometry and to put a lens inside the vacuum chamber. The trap is created with 852-nm light that the in-vacuum aspheric lens focuses down into a microscopic ODT. The same asphere also collected the 780-nm atomic fluorescence and directs it down an imaging path where it is detected by both an electron multiplying CCD (EMCCD) camera and a single-photon counting module (SPCM) APD. The quick response of the SPCM enables the fast quantum-state readout experiments described
in Ch. 6.

Once the experimental system is described, I show results of single-atom trapping. Figure 1.8 depicts single atoms loading into the ODT from the MOT in real time. The stair-step fluorescence signal that indicates single-atom loading is clearly visible. There are two distinct fluorescence levels: one corresponding to 0-atoms in the trap and the other to 1-atom in the trap. Very occasionally, two atoms enter the trap and are ejected due to the effect of the collisional blockade regime. This is evidenced by the spike in the middle of the time trace indicated by the arrow.

In addition to detecting atoms loading from the MOT, I also explain how I detect the atoms in the ODT without the MOT present. This detection is separate from the quantum-state readout described in Sec. 1.3.2. When I use the phrase “atom detection,” I refer to detection of any atom that is in the trap, regardless of its quantum state. The phrase “quantum-state readout” refers to determining which hyperfine ground state the atom’s valence electron occupies. I describe three
different detection methods and compare their efficacy. I find that the best detection method is a probe/cool detection scheme similar to that described in Ref. [109] but optimized and adapted to the setup detailed here. With this scheme, I obtain very good discrimination between cases of 1 atom in the trap or 0 atoms in the trap. Example results are shown in Fig. 1.9. This scheme is nondestructive, the atom is not lost from the trap after detection, and can be detected multiple times.

I finish Ch. 4 by using the probe/cool detection scheme to measure the lifetime of the atom in the trap in the absence of near-resonant laser light. This lifetime is only limited by collisions between the trapped atom and atoms in the background gas of the vacuum chamber and any heating caused by the ODT light itself. I measure the trap lifetime to be $> 60$ s, which is long for these types of systems, where lifetimes of 5-10 s are often reported [62].

In Ch. 5, I report a set of experiments that characterize the experimental system described in Ch. 4 and the atom-light interaction predicted by the rate-equation model. I begin with a measurement of the AC-Stark shifts caused by the ODT
that manifest as a shift to the atom’s resonance frequency. In the system described here, the atomic resonance is shifted $\sim 46$ MHz to the higher-frequency side of the untrapped atomic resonance. Comparing this shift to that predicted by the equations given in Ch. 3, I determine that the atom’s ground states are shifted $\sim 27$ MHz lower in energy, corresponding to an ODT trap depth of 1.28 mK. This is typical of a single-atom trap system.

In Ch. 5, I also develop a method to compare the predictions of the rate-equation model of Ch. 3 to data measured using the experimental setup. This allows me to extract very useful experimental parameters and gain insight into the system. In order to do this comparison, I predict the expected collection efficiency of the in-vacuum asphere for the $\pi$- and $\sigma$-polarized dipole radiation emitted by the atom. I use this prediction and knowledge of the atomic transitions to convert the output of the rate-
equation model to a number of counts expected to be detected by the SPCM. This allows me to compare the rate-equation model predictions to experimental results for several experiments.

One such experiment is a saturation experiment, the results of which are shown in Fig. 1.10. This experiment measures how the atom’s scattering rate saturates with increasing probe power. Fitting the rate-equation model to the data allows me to extract the parameter that describes how the probe power is related to the atomic saturation parameter and also the overall collection efficiency of the system. To the best of my knowledge, this type of saturation measurement has been reported for a trapped ion system [88], but not for a neutral atom. This is likely because neutral atoms are generally detected with lower fluorescence levels than ions, making it harder to investigate low-light-level phenomena such as the low-power region of the saturation curve. The probe/cool sequence described in Ch. 4 allows me to overcome this difficulty.

I also compare the rate-equation model to several spectroscopy experiments run with the probe at different powers. These experiments yield insight into possible heating in the system, which affects the results of Chs. 6 and 7. I finish Ch. 5 with an analysis of the collection efficiency of the experimental system, which is lower than expected. I examine the measured imperfections and discuss possible causes and solutions.

Chapter 6 reports on the initial results of the quantum-state readout experiments undertaken in this thesis. I begin with a discussion of the quantum-state readout protocol for classifying atoms as either in the “bright” quantum state that scatters photons from the probe or the “dark” quantum state that does not. I focus on sources of error and the calculation of fidelity and provide a mathematical model for this process initially developed by Dr. Geert Vrijsen of Professor Jungsang Kim’s research group and reported in the thesis of Dr. Stephen Crain [21]. I show some
initial results for state detection and discuss how the state detection scales with probe beam frequency and power.

In Ch. 7, I examine the result reported in Ch. 6 more closely. I show time-of-arrival data of the photons during the readout pulse. This data illustrates that there is a decay in the scattering rate during the readout that affects the readout fidelity. I investigate several possible sources of this decay and conclude that it is likely due to a combination of off-resonant pumping and heating. For the fast readout protocol described here, both are difficult to avoid.

I then show the final, improved results, and discuss the errors that are currently limiting the system. Figure 1.11 shows some typical results where the bright vs. dark quantum states are discriminated with $97.4 \pm .3\%$ fidelity in $144 \mu s$.

Chapter 7 finishes with a discussion of the results of quantum-state readout performed with linearly-polarized light and circularly-polarized light. Data is presented as well as the rate-equation model of each type of protocol. I discuss and advan-
Fig. 1.11  **Fast, nondestructive quantum-state readout.** Atoms are either prepared in the bright quantum state (red) or the dark quantum state (black) and are then detected. Declaring all events with >2 photons bright and all with <2 photons dark, the state readout fidelity is $97.4 \pm 0.3\%$ fidelity achieved in 144 µs.

tages and disadvantages of both. In general, circularly-polarized readout schemes have a lower-possible error rate (and thus a higher fidelity) than linearly-polarized readout schemes. However, they require a more complex experimental setup and I present evidence that they are inherently slower, unless additional state preparation is undertaken. Furthermore, the linearly-polarized readout scheme is more naturally compatible with the most desirable hyperfine qubit, which uses the $m_F = 0$ sublevels of the ground states. Averaging several data sets together, I achieve quantum-state readout with an average fidelity of $97.6 \pm 0.2\%$ in an average time of $160 \pm 20$ µs. This is faster than other reported schemes (see Tbl. 1.1) and uses a significantly simpler experimental setup and protocol than the circularly-polarized readout schemes that report a higher fidelity.

In Ch. 8 I summarize the results and discuss possible future directions for the experimental setup.
An introduction to single-atom traps

In this chapter, I provide an introduction to single neutral atom traps such as the one described in this thesis. I begin with the basic physics of a two-level atom in an electromagnetic (optical) field. This introduction lays the ground work for much of the theory needed to understand the experiments and described in later chapters. Next, I cover the optical dipole force and how it is used to trap atoms. The atoms must be cooled before they can be trapped, so I also cover the radiation pressure force and its use for cooling. I finish with a discussion of loading single-atom traps and the heating and loss mechanisms that must be considered.

2.1 Introductory theory of atom-light interaction

I begin by describing the interaction of a single two-level atom with an electromagnetic field. This is the physics underlying almost all of the work in this thesis. I present it here to introduce the formalism and some basic results that are needed to understand the theory discussed in Ch. 3. I use the semi-classical formulation of this theory, following Refs. [15] and [78].
2.1.1 A two-level atom in an optical field

Consider a single atom with a ground state $|g\rangle$ of energy $E_g = \hbar \omega_g$ and an excited state $|e\rangle$ of energy $E_e = \hbar \omega_e$. Figure 2.1 depicts the system of interest. The atom has resonance frequency $\omega_{eg} = \omega_e - \omega_g$ and excited state decay rate $\gamma_e = 1/\tau$ where $\tau$ is the excited state lifetime. The atom is probed by a coherent electromagnetic field $\vec{E}(t) = \vec{E} e^{-i\omega t} + \vec{E}^* e^{i\omega t}$ of frequency $\omega$ and field amplitude $\vec{E}$; $\vec{E}$ indicates that the electric field is a vector. The frequency difference between the probe field and atomic resonance is known as the detuning $\Delta$ and is described by $\Delta = \omega - \omega_{eg}$. The system is closed; there are no other energy levels or fields to consider. The Hamiltonian of this system is given by

$$\hat{H} = \hat{H}_0 + \hat{V}(t) = \begin{pmatrix} E_g & -\vec{\mu}_{ge} \cdot \vec{E}(t) \\ -\vec{\mu}_{eg} \cdot \vec{E}(t) & E_e \end{pmatrix},$$

(2.1)

where $\hat{H}_0$ is the quantum operator of the unperturbed atomic Hamiltonian and $\hat{V}(t) = -\vec{\mu} \cdot \vec{E}(t)$ describes the interaction between the atom and the optical field under the dipole approximation [15]. The dipole approximation neglects the spatial variation of $\vec{E}(\vec{r}, t)$ over the region in which the electric field interacts with the atom. Since the wavelength of the electric field is on the order of hundreds of nanometers and the wavefunction of the atom is typically contained within 1 nm, this assumption is valid for all of the experiments discussed in this thesis [78]. In this case, $\vec{\mu} = e\vec{r}$ is the dipole moment of the atom ($e$ is the elementary charge and $r$ the position coordinate) and $\mu_{ge} = \langle g|e\vec{r}|e\rangle = \mu_{eg}^*$ is the dipole matrix element.

Under the perturbation of the electromagnetic field, the atomic energies $E_g$ and $E_e$ are no longer the eigenvalues of the full Hamiltonian of Eq. 2.1. Diagonalizing this Hamiltonian, it can be shown that when the incident optical field is far from the atomic resonance ($\Delta \gg \gamma_e$), the new energy levels of the atom-photon system are
Fig. 2.1 Two Level Atom. An atom has ground state \( |g\rangle \) and excited state \( |e\rangle \) and resonance frequency \( \omega_{eg} \). The excited state decays at rate \( \gamma_e \) and has a transition linewidth \( \Gamma_{eg} \).

shifted by \[37\]

\[
\delta E_{g,e} = \pm \frac{\langle e | \mu | g \rangle^2}{\hbar \Delta} |E|^2.
\] (2.2)

These light-induced shifts are known as the AC-Stark shifts since they arise from the AC electric field of the optical beam. They can greatly affect the behavior of the atom under different forms of illuminations and will be discussed in detail in Ch. 3.

2.1.2 Two-level atom population dynamics

The full dynamics of the system can be calculated using the density matrix formulation of the Schrödinger equation

\[
\dot{\rho}_{mn} = \frac{i}{\hbar} [\hat{H}, \rho]_{mn}
\] (2.3)

and introducing phenomenological dephasing terms [15]. The diagonal elements of the density matrix represent the populations of the ground (\( \rho_{gg} \)) and excited (\( \rho_{ee} \)) states, while the off-diagonal terms describe the coherence between these states. The two dephasing terms of interest are the decay in the excited state population \( \gamma_e \) and the decay in the atomic coherence. The coherence decay term \( \Gamma_{eg} \) represents the dephasing of the atomic dipole moment and manifests as a transition linewidth for the atom. These two decay terms are depicted diagrammatically in Fig. 2.1. From here on, I assume that there is no additional dephasing in the system. In the
work described in this thesis, the trapped atom is relatively well-isolated from the environment and other dephasing mechanisms (collisions with other atoms, variations in background magnetic fields, etc.) do not need to be considered. Under this assumption, $\Gamma_{eg} = \gamma_e/2$ [15].

Separating out the time dependence, I rewrite the density matrix elements as $\rho_{ee} = \sigma_{ee}$, $\rho_{gg} = \sigma_{gg}$, and $\rho_{eg} = \sigma_{eg} e^{-i\omega_{eg} t}$, and apply the rotating wave approximation to find that the dynamics of the two-level system

$$
\dot{\sigma}_{eg} = (i\Delta - \Gamma_{eg})\sigma_{eg} - \frac{i}{2\hbar}\mu_{eg}\vec{\sigma}(\sigma_{ee} - \sigma_{gg}),
$$

$$
\dot{\sigma}_{ee} = -\gamma_e\sigma_{ee} - \frac{i}{2\hbar}(-\mu_{ge}\vec{\sigma}^*\sigma_{eg} + \sigma_{ge}\mu_{eg}\vec{\sigma}),
$$

$$
\dot{\sigma}_{gg} = \gamma_e\sigma_{ee} + \frac{i}{2\hbar}(-\mu_{ge}\vec{\sigma}^*\sigma_{eg} + \sigma_{ge}\mu_{eg}\vec{\sigma}),
$$

where $\Delta = \omega - \omega_{eg}$ is the detuning defined above [78].

To find the steady state populations of the system, the above equations can be rearranged to give the Optical Bloch Equations (OBE)

$$
0 = -\gamma_e\sigma_{ee} - \frac{\Gamma_{eg}}{\Delta^2 + \Gamma_{eg}^2} \frac{\Omega_{eg}^2}{2}(\sigma_{ee} - \sigma_{gg}),
$$

$$
0 = \gamma_e\sigma_{ee} + \frac{\Gamma_{eg}}{\Delta^2 + \Gamma_{eg}^2} \frac{\Omega_{eg}^2}{2}(\sigma_{ee} - \sigma_{gg}),
$$

$$
\sigma_{ee} + \sigma_{gg} = 1,
$$

where I have defined the on-resonance Rabi frequency $\Omega_{eg} = \vec{\mu}_{eg} \cdot \vec{\sigma}/\hbar$ [78]. The third equation of Eq. 2.5 describes the population conservation condition dictated by the closed nature of the system. An expanded version of these equations will be used in Ch. 3 to calculate the full population dynamics of the trapped atom.

By solving these equations, the excited state population can be written compactly
as
\[
\rho_{ee} = \frac{s/2}{1 + s}, \tag{2.6}
\]

where the saturation parameter \(s\) has been defined as
\[
s \equiv \frac{|\Omega_{eg}|^2/2}{\Delta^2 + \gamma_e^2/4}. \tag{2.7}
\]

This expression clearly shows that the excited state population will saturate to 1/2 for \(s \gg 1\) [78]. Hence, \(s\) is a useful measure for how strongly the optical field is interacting with the atom. This makes sense when we recall that \(s\) contains both \(\Omega_{eg}\) and \(\Delta\) which describe the coupling of the optical field to the atom’s dipole matrix element.

Experimentally, the atomic population dynamics are expressed through the atom’s scattering of photons. The total scattering rate of a two-level atom \(\gamma_{sc}\) is simply \(\gamma_e\) multiplied by the population of the excited state
\[
\gamma_{sc} = \gamma_e \rho_{ee} = \frac{s\gamma_e/2}{1 + s} = \frac{s_0 \gamma_e/2}{1 + s_0 + (2\Delta/\gamma_e)^2}, \tag{2.8}
\]

where \(s_0 = 2|\Omega_{eg}|^2/\gamma_e^2\) is known as the on-resonance saturation parameter [78]. We use \(s_0\) to define a new quantity the saturation intensity \(I_{sat}\), such that \(s_0 = I/I_{sat}\) where \(I = 2c\varepsilon_0|\delta'|^2\) is the intensity of the applied field. The concept of the saturation intensity and Eqs. 2.6 and 2.8 are useful for estimating the atom’s behavior under various illumination conditions and will be used throughout this thesis.

### 2.1.3 The mechanical effect of light on atoms

In addition to affecting the internal state of an atom, optical fields impose mechanical forces on the center of mass motion of an atom. Physically, this can be understood because photons carry moment and we know that force \(F\) is defined as
\[
F = \frac{d}{dt} \langle \hat{p} \rangle \tag{2.9}
\]
where $\langle \hat{p} \rangle$ denotes the expectation value of the quantum mechanical momentum operator [78]. Reference [78] is a good foundation for understanding optical forces on atoms and works through some of the details of deriving specific forces. In the end, it is found that two forces naturally arise from this treatment: the dipole force and the radiation pressure force.

The dipole force originates from the light-shifts described by Eq. 2.2. When the optical field is far from resonance ($\Delta \gg \gamma$), the dipole force dominates over the radiation pressure force and can be used for trapping. It is described in detail in Sec. 2.2. Near resonance, the radiation pressure force is dominant. It arises from scattered photons imparting a momentum kick to the atom. The radiation pressure force can both heat and cool an atom and is described in Sec. 2.3.

In this section I develop the tools needed to accurately describe the atomic state of a closed two-level atom as it interacts with a coherent optical field. I expand some of this formalism to the case of multi-level atoms in Ch. 3.

2.2 The dipole force and trapping

The details of the dipole trapping force are thoroughly covered by [37] and I summarize them here. The physical origin of this force can be understood by considering the energy level shifts described by Eq. 2.2. These shifts change the energy of the atom and depend on the squared amplitude of the optical field. Thus, as the squared amplitude changes, so do the energy shifts. A focused laser beam is a natural example of this case. The spatially varying light-shift corresponds to a spatially varying potential felt by the atom and generates the dipole force, which is conservative [78]. For atoms in the ground state, this force can be described as

$$F_{dip} = -\nabla(\delta E_g) = -\nabla U_{dip},$$  \hspace{1cm} (2.10)
where $\delta E_g$ is the ground state light shift and $U_{\text{dip}}$ is the corresponding ground state dipole potential [37]. When a laser beam is far detuned, in the absence of other fields, this force dominates the atom-light interaction.

Given this, let us consider again the light-shifts defined by Eq. 2.2. We now know that these shifts form a potential felt by the atom. Note that the sign of the shift depends on the sign of $\Delta$. Red-detuned light ($\Delta < 0$) shifts the ground state to a lower energy, providing a potential well that attracts the atom, as shown in Fig. 2.2. If the atom’s kinetic energy is low enough, it will be trapped in this potential well. Blue-detuned light ($\Delta > 0$), on the other hand, repels the atom. The excited state is shifted the opposite direction to the ground state. The implications of this will be explored in detail in Ch. 3.

Since $|\vec{E}|^2$ is related to beam intensity by $I = 2\epsilon_0 c |\vec{E}|^2$, the trap profile follows the beam intensity profile of the incident field [37]. The atom is trapped with a red-detuned, focused Gaussian beam with an intensity profile given by $I(r, z) = I_0 \exp[-2r^2/(w_0^2(1+(z/z_R)^2))]$ where $r$ ($z$) is the radial (axial) dimension and $w_0$ and $z_R$ are the minimum Gaussian waist and the Rayleigh length, respectively [101]. A diagram of the Gaussian beam is shown in Fig. 2.3a. The trap can be approximated...
as a cylindrically-symmetric harmonic oscillator,

\[ U(r, z) \approx -U_0 \left[ 1 - 2 \left( \frac{r}{w_0} \right)^2 - \left( \frac{z}{z_R} \right)^2 \right], \quad (2.11) \]

where \( U_0 \) is the maximum trap depth given by the light shift calculation [37]. Figure 2.3b depicts the radial Gaussian trapping potential and its approximating harmonic potential. The radial trap frequency is given by \( \omega_r = (4U_0/mw_0^2)^{1/2} \) and the axial by \( \omega_z = (2U_0/mz_R^2)^{1/2} \) where \( m \) is the mass of the atom. The radial trap frequency is much higher than the axial frequency, reflecting much tighter confinement in the radial direction. The exact calculation of the trap depth \( U_0 \) depends on the atomic dipole matrix element \( |\langle e | \mu | g \rangle|^2 \) and must be treated carefully. Further examination of this calculation and the light shifts are discussed in Ch. 3.

As shown in Fig. 2.3b, trap depths are typically expressed in the units of temperature, \( U_0/k_B \) where \( k_B \) is Boltzmann’s constant. Most single-atom traps are around 1 mK in depth. Since room temperature atoms have a temperature \( \sim 300 \) K, atoms must be cooled if they are to remain confined in a typical dipole trap. The cooling mechanism based on the radiation pressure force that allows this to occur is discussed in the next section.

2.3 The radiation pressure force and cooling

The field of laser cooling and trapping atoms has a long and productive history [19],[20],[90]. Here, I describe the most basic type of laser cooling, Doppler cooling, arising from the radiation pressure force. This force originates in the atom’s absorption and spontaneous reemission of photons. Each photon imparts a momentum kick to the atom when it is absorbed. The subsequent spontaneous emission is isotropic and averages to zero over many events. The absorption events, however, are directed along the wavevector, \( \vec{k} \) of the incoming laser beam and exert a force on the atom.
known as the radiation pressure force [78]. This force is described by

\[ \vec{F} = \hbar \vec{k} \gamma_e \rho_{ee}, \]  

(2.12)

where \( \vec{k} \) is the photon wavevector, \( \hbar \vec{k} \) is the momentum kick from each photon, \( \gamma_e \) is the decay rate of the excited state, and \( \rho_{ee} \) is the excited state population [78].

The magnitude of this force is dependent on the excited state population of Eq. 2.5 and hence on the laser frequency and detuning. This force is dissipative, so it can be used for cooling [78].

To understand how the force can be used to cool, we must consider the motion of the atom. The atoms begin at room temperature (\( \sim 300 \) K), not at rest. When the atom’s motion is taken into account, it can be shown that the radiation pressure force contains a velocity-dependent term \(-\beta \vec{v}\) [78]. This term is proportional to the detuning of the incident laser beam and for red-detuned light (\( \Delta < 0 \)) it slows the atom. This can be intuitively understood by considering the atom’s motion relevant to the laser beam and the Doppler effect. If the laser beam is tuned to below the atomic resonance, atoms moving towards the beam see light that is Doppler-shifted.
towards their resonance. Atoms moving away from the beam see light that is farther detuned. Consequently, atoms preferentially absorb photons from the laser beam that opposes their motion and their motion is damped. This phenomenon is known as Doppler cooling [78]. With counter-propagating red-detuned beams, Doppler cooling acts in both directions and atoms can be slowed towards zero [78]. It is important to note that the radiation pressure force from blue-detuned light ($\Delta < 0$) heats atoms. This heating mechanism is discussed in Sec. 2.5.

Doppler cooling can provide a reservoir of cold atoms that can be trapped. By using multiple pairs of red-detuned, counter-propagating laser beams, atoms can be cooled in all three dimensions in an optical molasses [78]. The temperature of such a molasses has a lower bound known as the Doppler limit [78]. For $^{87}$Rb, this temperature is $\sim 150$ µK [116], cold enough that atoms will remain trapped in a $\sim 1$ mK deep optical trap. Some groups load their single-atom trap directly from an optical molasses [115]. More often, however, the molasses is paired with an inhomogeneous magnetic field that provides a trapping potential to create a magneto-optical trap (MOT) [93]. Most single-atom traps, including the one described in this thesis, are loaded from MOTs.

As an aside, I note that the discussion of laser cooling given here has focused on the case of a two-level atom and only requires one laser frequency. In reality, the atoms that are trapped, such as $^{87}$Rb, are multi-level and require multiple frequencies for effective laser cooling. Specifically, $^{87}$Rb has two ground states, and lasers resonant with both ground states are required to create an optical molasses or a MOT. The lasers needed are often described as the cooling laser and repump laser, terminology which is briefly used in Sec. 2.4. I do not explain the details of MOT operation here; the specifics of the MOT used for the experiments in this thesis are described in App. A.
2.4 Loading single-atom traps

A thorough study of loading dipole traps from MOTs was conducted by Kuppens et al. [61] in 2000 and was extended to single-atom traps by Schlosser et al. [104] in 2002. Kuppens et al. came up with a phenomenological model to describe the number of atoms $N$ in the dipole trap during loading

$$\frac{dN}{dt} = R_{\text{load}} - \gamma_{\text{coll}}N - \beta'N^2,$$

(2.13)

where $R_{\text{load}}$ is the loading rate that depends on the background atom density of the MOT, $\gamma_{\text{coll}}$ is the single body decay rate governed by collisions with the atoms in the background gas, and $\beta'$ is the two-body decay rate, called the collisional term [61]. Reference [104] adapts Eq. 2.13 to the case of very few atoms by replacing $N^2$ with $N(N - 1)$ and identifies two regimes: “weak loading” and “strong loading”. In weak loading, the loading rate $R_{\text{load}}$ is small and $\langle N \rangle \sim R_{\text{load}}/\gamma_{\text{coll}}$ because the collisional term is negligible. In strong loading, the collisional term dominates and $\langle N \rangle \sim \sqrt{R_{\text{load}}/\beta'}$.

The crossover between these two regimes is the “collisional blockade regime” of loading where two-body losses dominate. In this regime, if the trap initially contains an even (odd) number of atoms, pairs of atoms are lost until 0 (1) are left. Hence, operating in this regime allows one to trap just a single atom [103]. This regime occurs when $\gamma_{\text{coll}}/2 < \beta'/4$ [104]. Thus, a good understanding of these parameters is necessary for appropriate trap design.

The single-body decay rate depends on collisions with the background gas and thus on the overall pressure in the chamber. The inverse of this rate gives a background-limited lifetime for the atom in the trap $\tau_{\text{coll}}$. In 2005 Saffman and Walker calculated this value to be

$$\gamma_{\text{coll}} = \frac{1}{\tau_{\text{coll}}} = \sqrt{\frac{3T_b}{m}} n_b \sigma_{Rb - Rb},$$

(2.14)
where $T_b$ is the temperature of the background atoms (typically $\sim 300$ K), $m$ is the mass of $^{87}$Rb, $n_b$ is the background number density of atoms, which can be related to the pressure, and $\sigma_{Rb-Rb}$ is the Rb-Rb scattering cross-section [99]. For reasonable experimental pressures of a few nanoTorr, lifetimes of several seconds are possible [99]. In state-of-the-art single-atom traps, lifetimes are often in the range of tens of seconds [91] whereas dipole traps in general have reached lifetimes of several minutes [89].

The two-body decay rate $\beta'$ mainly stems from light-assisted collisions leading to radiative escape [61]. In light-assisted collisions, an excited state atom and a ground state atom form a long range molecule and attract each other. As they move towards each other they pick up kinetic energy. Once the excited state atom decays, the attraction is switched off and the two atoms move past each other. In the case of radiative escape, they have gained enough kinetic energy that they both exit the trap, causing a pair-wise loss [32, 61].

Given this description of the loss-mechanism, it makes intuitive sense that $\beta'$ is related to the fraction of atoms in the excited state, as that governs the likelihood of radiative escape. The fraction of atoms in the excited state is determined by the details of the cooling and repump lasers that trap the MOT, and can be described as $I_R/(I_R + aI_M)$ where $I_{M(R)}$ is the intensity of the MOT cooling (repump) light and $a$ is a constant related to the relative optical pumping rates of the cooling and repump transitions [61]. This quantity will be further described in Ch. 4.

The frequency of these collisions is density dependent, so the trap volume $V$ must also be taken into account [61]. Indeed, $\beta'$ can be described by

$$\beta' = \frac{KI_M}{V} \frac{I_R}{I_R + aI_M},$$  \hspace{2cm} \text{(2.15)}$$

where $K$ is a constant [61]. The volume can be approximated as a cylinder whose size is determined by the trapping beam waist $w_0$ and Rayleigh length $z_R$ and the
atomic temperature $T$. Under these assumptions, the volume is

$$V = \pi w_0^2 z_R \ln \left( \frac{1}{1 - \eta} \right) \sqrt{\frac{\eta}{1 - \eta}},$$  \hspace{1cm} (2.16)$$

where $\eta = k_B T/|U_0|$ was shown to be a constant $\eta = 0.4$ [61]. Given these equations, it is possible to design a trap that can reach the collisional blockade regime and achieve single-atom loading. The details of such a trap are discussed in Ch. 4.

2.5 Heating and loss in single-atom traps

Once a single atom is loaded into the trap, it must remain trapped for long enough to complete the experiments of interest. Realistically, the longest possible lifetime of the trap is governed by the background-limited lifetime described in Eq. 2.14. This lifetime, however, does not account for mechanisms which impart sufficient energy to the atom to allow it to escape the trap. Such heating mechanisms are important to consider experimentally and are discussed here.

2.5.1 Heating from scattering

As was mentioned in Sec. 2.3, radiation pressure can cause heating if the applied radiation is on-resonance or blue-detuned from the atom. In these scenarios, scattered photon imparts an amount of energy to the atom determined by the photon’s momentum. The recoil energy is defined as

$$E_{\text{rec}} = \frac{\hbar^2 k^2}{2m},$$  \hspace{1cm} (2.17)$$

where $m$ is the mass of the atom and $k$ the photon wavenumber [30]. Another way to think about this heating is by considering the effective temperature of an atom with the momentum $\hbar k$ of a resonant photon. This temperature is known as the recoil temperature and is $T_{\text{rec}} = 2E_{\text{rec}}/k_B$ where $k_B$ is Boltzmann’s constant. For
Table 2.1: Loss mechanisms in single-atom traps

<table>
<thead>
<tr>
<th>Loss mechanisms</th>
<th>Timescale (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Background gas collisions</td>
<td>$\mathcal{O}(10 - 10^2)$</td>
</tr>
<tr>
<td>Dipole trap photon scattering</td>
<td>$\mathcal{O}(10^2)$</td>
</tr>
<tr>
<td>Resonant photon scattering</td>
<td>$\mathcal{O}(10^{-4})$</td>
</tr>
<tr>
<td>Trap intensity noise</td>
<td>$\mathcal{O}(10 - 10^2)$</td>
</tr>
<tr>
<td>Trap pointing noise</td>
<td>$\mathcal{O}(10)$</td>
</tr>
</tbody>
</table>

$^{87}$Rb atoms, $T_{\text{rec}} \approx 360$ nK. Considering that each resonant photon scattered by the atom raises its energy, the atom can only scatter a limited number of resonant photons before it is heated out of the trap. If we consider the number of scattered photons required to raise the atom’s energy to equal a trap depth of $U/k_B = 1$ mK, we calculate $U/2E_{\text{rec}} \approx 2800$ scattering events [30]. The rate at which this heating occurs is dependent on the scattering rate. For resonant illumination with a beam equal to the saturation intensity, the scattering rate is $\sim 10$ MHz and the atom heats out of the trap after only $\sim 280$ µs. The scattering rate from the off-resonant dipole beam, on the other hand, is only $\sim 10$ Hz and limits the lifetime to $\sim 280$ s.

2.5.2 Other heating mechanisms

In addition to heating from scattering, a few other heating mechanisms have been identified for dipole traps. Trap laser intensity noise causes a form of parametric heating as fluctuations in intensity cause fluctuations in the trap spring constants [102]. Trap pointing fluctuations also cause parametric heating by physically shaking the trap in space [102]. For reasonable experimental parameters in modern traps, this heating leads to a lifetime on the order of tens or hundreds of seconds and does not dominate for most experimental procedures [99].

The loss mechanisms discussed in this chapter are summarized in Table 2.1. Clearly, the lifetime of the atom in the trap is heavily dominated by the charac-
ter of the illumination to which it is exposed. Resonant illumination can easily blow the atom out of the trap in milliseconds or less due to radiation pressure. Experimental protocols must be designed to minimize this heating and preserve the trapped atom, as will be discussed in Chs. 4 and 6.

2.6 Summary

In this chapter I provide an introduction to neutral single-atom traps. I describe the basic physics of atom-light interaction and the origin of the dipole force which is used for trapping. I discuss how the radiation pressure force can be used to cool atoms and create a magneto-optical trap (MOT) from which to load a single-atom trap. I also explain the loading mechanisms and the collisional blockade regime which guarantees single-atom trapping. Understanding the details of this mechanism are necessary for designing a suitable experimental system, as is discussed in Ch. 4. Finally, I discuss heating mechanisms in single-atom traps which can limit trap performance. Consideration of these mechanisms, particularly radiation pressure, is critical to the experimental discussions of Chs. 4 and 6. Some of this theory is expanded to consider multilevel atoms in Ch. 3. Specifically, the light shifts that provide the trapping potential and the population dynamics of the atomic state under perturbation from electromagnetic fields are covered in greater detail.
In this chapter, I extend some of the two-level-atom theory of Ch. 2 to the case of multilevel atoms. The theory developed here is necessary to understand the experimental results presented in Chs. 5, 6, and 7. It allows me to model the experimental system of a single atom in an optical dipole trap, which facilitates experimental control over the atom’s quantum state.

The theory for multilevel atoms is well known and covered in many texts such as references [24], [78], [116]. I primarily follow the conventions defined in [24]. I begin with a discussion of the dipole matrix element and proceed to a calculation of the multilevel AC-Stark shifts. I end by developing a rate-equation model of the atomic state dynamics for the 24-levels of the $^{87}$Rb atom relevant to the work in this thesis. Some of the theory work in Secs. 3.1 and 3.2 was presented by Shih and Chapman in 2013 [109]. However, there was a mistake in their work that I found and corrected in a Comment published in 2017 [106]. I note the relevant work in the text below.
3.1 Hyperfine structure and the multilevel dipole matrix element

The alkali-metal atom $^{87}\text{Rb}$ is used for the experiments described in this thesis. $^{87}\text{Rb}$’s electronic structure consists of a closed-shell core of electrons and one valence electron [78]. Interactions between the valence electron and optical fields are what enable laser cooling, dipole trapping, and the experiments in this thesis. The state of the valence electron is defined by its orbital angular momentum $\ell$ and spin angular momentum $s$ [78]. These momenta couple through the fine-structure interaction to form the total electronic angular momentum $J$. The hyperfine interaction couples $J$ to the nuclear spin $I$ to form the total atomic angular momentum $F$ [78]. For the experiments in this thesis, $F$ is a good quantum number and the basis used to describe the interactions.

The energy levels of the valence electron relevant to this thesis are its 5$S_{1/2}$ ground state and 5$P_{3/2}$ excited state. The transition between these levels is known as the D2 transition [116]. The hyperfine interaction leads to the ground state splitting into the $F = 1$ and $F = 2$ levels and the excited state splitting into the $F = 0$, $F = 1$, $F = 2$, and $F = 3$ levels. Each $F$ level has $2F + 1$ magnetic sublevels (called Zeeman sublevels) labeled by the $m_F$ quantum numbers. There are 24 levels total. The full D2 level structure of $^{87}\text{Rb}$ is depicted in Fig. 3.1a.

Similar to the two-level case, each pair of levels in a multilevel atom has a dipole matrix element $\mu_{ij}$ that describes the coupling between them, where $i, j$ are subscripts denoting the levels involved. The multilevel dipole matrix element is given by $\mu_{ij} = \langle i | \hat{\epsilon} \cdot \vec{r} | j \rangle$ where $\hat{\epsilon}$ is a unit vector describing the polarization of the incident electromagnetic field and is always perpendicular to the field’s wavevector $\vec{k}$ [78]. The three components of the polarization vector are denoted $\hat{\epsilon} = (\epsilon_{-1}, \epsilon_0, \epsilon_{+1})$ and correspond to the $\sigma^-$, $\pi$, and $\sigma^+$ polarizations, respectively. These polarizations are defined in the spherical basis with respect to a chosen quantization axis; and I label
their subscripts \( q = -1, 0, 1 \). \( \pi \)-polarized light \((q = 0)\) is linearly polarized along the quantization axis while \( \sigma^{+(-)} \)-polarized light \((q = \pm 1)\) is circularly polarized. The quantization axis is chosen for mathematical convenience.

The coupling of the polarization vector to the angular momentum of the electron is due to conservation of angular momentum. The \( \pi \)-polarized light drives transitions where the z-projection of the electron’s angular momentum is unchanged and \( \Delta m_F = 0 \). Likewise, \( \sigma^{+(-)} \)-polarized drives transitions where \( \Delta m_F = \pm 1 \) because the photon’s non-zero spin angular momentum has been imparted to the atom [78].

Moving forward, I consider the coupling between two states \( i = |\zeta,F,m_F\rangle \) and \( j = |\zeta',F',m_{F'}\rangle \) that are labeled by their \( F \) and \( m_F \) quantum numbers with \( \zeta \) denoting all of the additional quantum numbers needed to identify each state. The primes are used to differentiate one state from the other. The matrix element between the states can be written as a product of a reduced matrix element and a Clebsch-Gordon coefficient

\[
|\mu_{i,j}|^2 = d_{i,j}^2 \times |CG_{i,j}|^2
\]  

(3.1)

where the reduced matrix element is defined by

\[
d_{i,j}^2 = \langle \zeta J|e\mathbf{r}|\zeta' J'\rangle^2,
\]  

(3.2)

and the Clebsch-Gordon coefficient by

\[
|CG_{i,j}|^2 = (2F + 1)(2F' + 1) \left( \begin{array}{cc} F & 1 \\ -m_F & -q \end{array} \right) \left( \begin{array}{cc} F' & 1 \\ m_{F'} & m_F \end{array} \right)_3^2 \left\{ \begin{array}{ccc} J & J' & 1 \\ F & F' & I \end{array} \right\}_6^2,
\]  

(3.3)

[106]. I emphasize that I have used the normalization convention adopted by the U.S. National Institute of Standards and Technology (NIST) [24], which tabulates many reduced matrix elements. Under the NIST convention, \( d_{i,j}^2 \) is symmetric under exchange of indices: \( |\langle \zeta J|e\mathbf{r}|\zeta' J'\rangle|^2 = |\langle \zeta' J'|e\mathbf{r}|\zeta J\rangle|^2 \). There is another normalization convention present in the field [116], that results in additional level degeneracy factors...
appearing in the reduced matrix element. I define the reduced matrix element of this other convention as $d'$ and note that $d'^2 = d^2/(2J + 1)$ where $J$ is the $J$-quantum number of the unprimed state and $(2J+1)$ describes its degeneracy. Under this other convention, $d$ is not symmetric under the interchange of the primed and unprimed states because of the introduction of the degeneracy factor. One must be careful not to mix these two normalization conventions. Mixing these conventions caused the mistake in [109] that our Comment corrected [106]. The matrix element described by Eq. 3.1 is a direct analog to that of the two level system in Sec. 2.1 and it propagates through all calculations of the atomic system.

3.2 Calculating AC-Stark shifts in optical traps

Just as an incident laser beam causes energy level shifts in a two-level atom, shifts also arise in multilevel atoms. Correctly calculating these energy-level shifts is necessary because they provide the optical dipole potential that traps the atom. This section details the calculation of these shifts.

In the two-level atom, the shift of the ground state $\delta E_g$ is determined by the coupling of the incident optical field to the dipole matrix element between the ground and excited states, and is given by

$$\delta E_g = \pm \frac{|\langle r | \mu | g \rangle|^2 h \Delta}{|\delta'|^2}. \quad (3.4)$$

Analogous to Eq. 3.4, for the multilevel case, the level shift can be described as the sum of the shifts from each pair of interacting levels. I only consider the shifts caused by the linearly-polarized far-detuned trapping light. The perturbation caused by the trapping beam dominates the other perturbations in the system, so I choose to set the quantization axis along the direction of the trap polarization. Thus, the trap light is $\pi$-polarized and $q = 0$. 

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The shift calculation is known \[108\] and found to be

\[
\delta E_{\zeta,F,m_F} = -\frac{I}{2\epsilon_0 c \hbar} \sum_{\zeta',F',m'_{F'}} \Delta_{\zeta',F',\zeta,F} |\langle \zeta', F, m_F | \text{er} | \zeta', F', m'_{F'} \rangle|^2,
\]

where

\[
\Delta_{\zeta',F',\zeta,F} = \frac{1}{\omega_{\zeta',F',\zeta,F} + \omega} + \frac{1}{\omega_{\zeta',F',\zeta,F} - \omega}
\]

is the detuning between the incident field and the atomic resonance, and I use the fact that \(I = 2\epsilon_0 |\xi|^2 \) \[106\]. It is important to note that \(\omega_{\zeta',F',\zeta,F} = \omega_{\zeta',F'} - \omega_{\zeta,F} \) is the difference in the bare (unshifted) frequencies of the states and the detuning factor has the property that \(\Delta_{\zeta',F',\zeta,F} = -\Delta_{\zeta,F,\zeta',F'} \) \[106\]. Furthermore, I note that these shifts are symmetric under interchange of the primed and unprimed levels.

I stress that this shift calculation is valid when the following statements are true. The trapping beam is far-detuned from the atomic resonance, meaning that the detuning between the trap light and the atomic transitions \(\Delta_{\text{ODT}}\) is much larger than the hyperfine splitting of the atom’s excited state \(\Delta_{\text{HFS}}\) and ground state \(\Delta_{\text{HFS}}\). On the other hand, \(\Delta_{\text{ODT}}\) is on the order of the fine-structure splitting \(\Delta_{\text{FS}}\) so the hyperfine quantum number \(F\) is used in this calculation. Furthermore, the dipole trap light is weak. It’s scattering rate \(\gamma_{\text{ODT}}\) is much less than the natural linewidth of the atom, \(\gamma_{\text{ODT}} < \gamma\) \[37\].

The sublevels of the 5S\(_{1/2}\) and 5P\(_{3/2}\) levels of \(^{87}\text{Rb}\) are the 24 magnetic sublevels relevant to this thesis. To calculate the level-shifts of these levels, I consider interactions between these states and all of the atomic states up to those with orbital number \(n = 9\), including the \(S, P,\) and \(D\) orbitals. The higher lying states make a very small contributions to the shift but are included for completeness. The states considered are tabulated in App. B along with their reduced matrix elements, wavelengths, and \(J\) quantum numbers. The reduced matrix elements are taken from reference \[4\] and the wavelengths from the NIST atomic and spectral database \[59\].
In applying Eq. 3.5 to the energy levels depicted in Fig. 3.1a, I find that the ground state Zeeman sublevels shift uniformly, but the degeneracy of the excited states is lifted by the dipole trap light, as depicted in Figure 3.1b. For red-detuned trapping light, the ground states shift to lower energy, providing a trapping potential. The excited states shift to higher energy and in the $F = 1$ and $F = 3$ excited states, the level degeneracy is lifted and a differential AC-Stark shift appears. The values given in Fig. 3.1b are typical of those used in the experiments of this thesis. Though the AC-Stark shifts are small compared to the level splitting, they are large.
compared to the atomic linewidth of 6 MHz and have a significant effect on the atom’s interaction with near-resonant optical fields. This effect is illustrated in Sec. 3.7 and plays a major role in explaining the experimental results presented in Chs. 5, 6, and 7.

3.3 Determining the reduced dipole matrix element

There are a few details not discussed above that are worth mentioning here. First, I note that the value of the reduced matrix element $d$ can be determined from the spontaneous emission rate of a higher energy state $|u\rangle = |\zeta_u, F_u, m_{F_u}\rangle$ decaying to a lower energy state $|\ell\rangle = |\zeta_\ell, F_\ell, m_{F_\ell}\rangle$ using the relation

$$A_{u \rightarrow \ell} = \frac{\omega_{u\ell}^3}{3 \pi \epsilon_0 \hbar c^3 (2J_u + 1)} d^2$$

(3.7)

in SI units, or

$$A_{u \rightarrow \ell} = \frac{2.02613 \times 10^{18}}{\lambda^3} \frac{d^2}{(2J_u + 1)}$$

(3.8)

with $A_{u \rightarrow \ell}$ in s$^{-1}$, $d$ in atomic units ($ea_0$, with $a_0$ the Bohr radius), and the transition wavelength $\lambda$ in Å [109]. Here, $A_{u \rightarrow \ell}$ is the Einstein A coefficient and $A_{u \rightarrow \ell} = \gamma_{u \rightarrow \ell} = 1/\tau$ where $\tau$ is the excited state lifetime. In these expressions, the degeneracy factor $J_u$ appears and is always taken as the quantum number for the higher-energy state. Given the degeneracy factors in this expression, it is important to keep the procedure for finding $d$ from $A_{u \rightarrow \ell}$ separate from determining the energy level shifts of Eq. 3.5 to avoid any misunderstanding in interpreting $J_u$ in Eq. 3.8. The work of Shih and Chapman [109] combined these results, mixing normalization conventions, which led to the error that we corrected in our Comment [106]. I discuss the procedure for finding the reduced matrix element more fully in Sec. 3.5.
3.4 An alternative approach

It is also worth noting that the method for calculating the shifts outlined in this chapter is not the only method available. I have elected to use the dipole-matrix-element-based formalism to describe the light-atom interaction but there exists another formalism that treats the atom as an entity with different polarizabilities that govern its behavior in the presence of light [85]. The shifts discussed here depend on the scalar and vector polarizability of the atom (the tensory polarizability vanishes since the trap light is $\pi$-polarized [85]). The polarizability-approach is very powerful but relies on theoretically intensive calculations of the polarizabilities at the specific wavelength under consideration. The work of Ritter et al. [85] studied a single $^{87}$Rb atom in a very deep optical trap inside a cavity. They used the polarizability-approach to explain their findings and were provided the polarizability for $^{87}$Rb at their trapping wavelength by theoretical models [85]. I used their expression to validate Eq. 3.5 and I compare it to their work and to the work of reference [109] in our Comment [106]. I also note that some recent work by Hu et al. has calculated and measured the vector and tensor polarizabilities of the ground states of $^{87}$Rb by studying the AC-Stark shifts induced by a pair of Raman lasers [41]. Calculation of the light-shifts, like that presented here, has received renewed attention recently as experimental control of trapped neutral atoms has improved. The vector light shift has been found to negatively affect schemes that aim to cool the atom to its motional ground state [117]. This renewed interest motivates developing methods like the one described here to correctly calculate these shifts. I do not implement such cooling in this thesis but it would be a natural extension of this work.
3.5 Multilevel population dynamics: rate equations

Just as I extend the AC-Stark shift calculation to multiple levels, I can also extend the dynamics equations of Eq. 2.5 to the multilevel case. This calculation provides a model for the dynamics of the atomic populations in the presence of near-resonant optical fields. Since the atom is detected via its interaction with light, understanding the dynamics of this interaction is crucial for the experiments described in this thesis. The model of the multilevel population dynamics developed here is used extensively to explain the experimental observations of Chs. 5, 6, and 7 but has some inherent limitations that I detail below.

I stress that this models the atom’s interaction with near-resonant fields, in contrast to the far-detuned field discussion above. For near-resonant fields, the laser’s detuning from the atomic resonance is much smaller than the ground-state hyperfine splitting, $\Delta_L \ll \Delta_{HFS}$, and generally resolves the excited state hyperfine splitting as well. Similar to the previous section, these calculations are valid when the laser weakly drives the atom and the scattering rate is on the order of or smaller than the spontaneous emission rate.

I begin by revisiting Eq. 2.5 and re-writing it as

$$
\dot{\sigma}_{ee} = -\gamma_e \sigma_{ee} - R_{eg} \sigma_{ee} + R_{eg} \sigma_{gg} \\
\dot{\sigma}_{gg} = \gamma_e \sigma_{ee} + R_{eg} \sigma_{ee} - R_{eg} \sigma_{gg} \\
\sigma_{ee} + \sigma_{gg} = 1
$$

(3.9)

where I define the rate

$$
R_{eg} = \frac{\Gamma_{eg}}{\Delta^2 + \Gamma_{eg}^2} \frac{|\Omega_{eg}|^2}{2}.
$$

(3.10)

I note that this equation does not contain the atomic coherences $\sigma_{eg}$ discussed in Sec. 2.1. This equation, being developed for the two-level atom, also does not include the AC-Stark shifts calculated above. I revisit both of these issues shortly.
Examining Eq. 3.9, I see that the first two equations each have a spontaneous emissions term $\gamma_e \sigma_{ee}$ followed by a stimulated emission term $R_{eg} \sigma_{ee}$ and absorption term $R_{eg} \sigma_{gg}$. For the excited state, spontaneous emission and stimulated emissions deplete its population (the terms are negative), while absorption increases its population. The opposite is true in the ground state. The strength of these processes is dictated by the rates at which they happen, $\gamma_e$ and $R_{eg}$, and the populations of the levels $\sigma_{gg}$ and $\sigma_{ee}$. The third equation in Eq. 3.9 describes the population conservation in the system. I refer to equations such as Eq. 3.9 as rate equations and they form the basic structure of the model described in this chapter.

It is important to note that the rate equation model has some inherent limitations. It only includes the atomic population terms that arise in the light-atom interaction and ignores the coherence terms ($\sigma_{eg}$ terms in Eq. 2.4) between the ground and excited states. The coherence between two levels governs the collective behavior of the atomic population in those levels. This coherence gives rise to oscillations in the populations of these levels known as Rabi oscillations [78]. The coherence also has a natural dephasing rate that causes the Rabi oscillations to decay over time (assuming that the incident light is only weakly driving the atom) [24]. The timescale for this decay is usually tens of ns, a few times the spontaneous emission time. As long as I restrict myself to investigating the atomic population at longer time scales, Rabi oscillations and the coherences which cause them can be ignored. A more formal justification of the rate-equation approach is given for the case of $n$-photon absorption in Ref. [2], [79], [81], [80].

3.5.1 An example four-level system

Extending the rate equation treatment to multiple levels is relatively straightforward, but labor intensive. Before describing the behavior of the entire system of $^{87}$Rb, I establish my notation conventions using a toy-system. Consider a four-level system
with two ground states and two excited states. I describe the population of level \( i \) as \( n_i \), replacing the \( \sigma \) notation of Eq. 3.9. I label the ground states \( n_1 \) and \( n_2 \) and the excited states \( n_3 \) and \( n_4 \). The levels are depicted in Fig. 3.2.

Full treatment of this four-level system generates population terms as well as coherence terms between all four states. Some of the coherence terms are the \( \sigma_{eg} \) type addressed previously but there are also coherences between the equal-energy ground states and the equal-energy excited states. Such coherences between degenerate states are known as Zeeman coherences and the rate equation model does not account for them. Zeeman coherences can give rise to interesting physical phenomena such as the formation of dark states due to coherent population trapping [5]. These dark states can be useful for state preparation of a specific magnetic-sublevel, but such preparation is not used in this thesis. Furthermore, dark states affect whether an atom appears bright or dark in the presence of a probe beam. This is detrimental to the fidelity of the quantum-state readout experiments reported in this thesis.

Zeeman coherences can arise when Zeeman sublevels are coupled by the driving field. For a single frequency field, this coupling comes from a mixture of polarizations driving the atom. Consequently, I only use the rate equation model when the quantization axis of the problem can be chosen in such a way to eliminate the coherence terms between the atomic Zeeman sublevels [5]. This is possible when a single-frequency optical field that is purely \( \pi \)- or \( \sigma \)-polarized drives the atom. In this case, the axis defined by the laser polarization can be taken as the quantization axis and the density matrix of the system can be written in a basis where the Zeeman sublevels are the basis states and the coherences vanish [5]. If multiple polarizations are present the light is generally elliptically polarized, the choice of quantization axis is not so trivial, and the coherence terms cannot be removed. Indeed, elliptically polarized light has been used to form and manipulate atomic coherences in several studies by Milner et al. [79], [81], [80].
A four-level system A four-level system consists of two ground states $|n_1\rangle$ and $|n_2\rangle$ and two excited states $|n_3\rangle$ and $|n_4\rangle$. a) $\sigma^+$-polarized light couples $|n_1\rangle$ and $|n_4\rangle$ via rate $R_{1;4}$. Spontaneous emission couples $|n_4\rangle$ to both ground states. b) $\pi$-polarized light couples $|n_1\rangle$ and $|n_3\rangle$ via rate $R_{1;3}$ and $|n_2\rangle$ and $|n_4\rangle$ via rate $R_{2;4}$. Spontaneous emission couples both excited states to both ground states.

Zeeman coherences can also arise in the presence of two laser fields of different frequencies when the two frequencies are tuned such that they can drive a two-photon resonance in the atom. This occurs when the two frequencies address the same atomic state with 0 detuning and can give rise to very interesting and useful effects such as coherent population trapping [111]. However, it cannot be modeled by the rate equations so I avoid this situation.

Figure 3.2 depicts the effects of a $\sigma^+$-polarized field and a $\pi$-polarized field separately because such fields are applied separately in the model to prevent the formation of Zeeman coherences. I do not consider cases where ground states couple to each other or cases where excited states couple to each other.

As seen in Fig. 3.2a, a $\sigma^+$-polarized field couples states $n_1$ and $n_4$ with rate $R_{1;4}$ defined by Eq. 3.10. Spontaneous emission couples $n_4$ to both ground states. Assuming that there is no initial population in the excited states, level $n_3$ does not
participate in the population dynamics in the presence of a $\sigma^+$ driving field. Level $n_2$ does participate in the dynamics because it is fed population via spontaneous emission from $n_4$. The population dynamics depicted in Fig. 3.2a are described by

$$
\begin{align*}
\dot{n}_1^{\sigma^+} &= \gamma_{4;1} n_4 + R_{1;4} (n_4 - n_1), \\
\dot{n}_2^{\sigma^+} &= \gamma_{4;2} n_4, \\
\dot{n}_4^{\sigma^+} &= -(\gamma_{4;1} + \gamma_{4;2}) n_4 - R_{1;4} (n_4 - n_1),
\end{align*}
$$

(3.11)

where $\dot{n}_i^q$ denotes the time derivative of population $n_i$ driven by polarization $q$ and $\gamma_{ij}$ is the spontaneous emission rate between two particular sublevels. In this notation, $\gamma_{ij} = |\mu_{ij}|^2 \gamma$ where $\gamma = 1/\tau$ is the total excited state decay rate.

In Eq. 3.11, the first term in each equation describes coupling due to spontaneous emission and adds population to the ground states while removing population from the excited states. The second term describes the stimulated emission caused by the driving field and also adds population to the ground states while removing population from the excited state. The final term in each equation describes the absorption caused by the driving field and has the opposite effect as the stimulated emission; it removes population from the ground states and adds it to the excited state.

Though the spontaneous emission expression in the $\dot{n}_4$ equation of Eq. 3.11 is technically correct, we know that all excited states of a single atom decay at the same rate $\gamma$ [116]. Indeed, $\sum_g \gamma_{eg} = \gamma$, and I rewrite Eq. 3.11 as

$$
\begin{align*}
\dot{n}_1^{\sigma^+} &= \gamma_{4;1} n_4 + R_{1;4} (n_4 - n_1), \\
\dot{n}_2^{\sigma^+} &= \gamma_{4;2} n_4, \\
\dot{n}_4^{\sigma^+} &= -\gamma n_4 - R_{1;4} (n_4 - n_1),
\end{align*}
$$

(3.12)

This is the system of rate equations that describes the population dynamics of this four-level system in the presence of a $\sigma^+$-polarized field.
A π-polarized field has a different effect on the four-level system. It couples states $n_1$ and $n_3$ and states $n_2$ and $n_4$, shown in Fig. 3.2b. Spontaneous emission couples both excited states to both ground states and all four levels participate in the population dynamics. The rate equations for this system are written

\[
\begin{align*}
\dot{n}_1^\pi &= \gamma_{3;1} n_3 + \gamma_{4;1} n_4 + R_{1;3} (n_3 - n_1), \\
\dot{n}_2^\pi &= \gamma_{3;2} n_3 + \gamma_{4;2} n_4 + R_{2;4} (n_4 - n_2), \\
\dot{n}_3^\pi &= -\gamma n_3 - R_{1;3} (n_3 - n_1), \\
\dot{n}_4^\pi &= -\gamma n_4 - R_{2;4} (n_4 - n_2),
\end{align*}
\]  

(3.13)

where again the spontaneous emission terms are followed by stimulated emission and absorption terms. Having explained this toy-model and developed additional notation, I now turn to the full system of rate equations for $^{87}\text{Rb}$.

### 3.5.2 Assembling the full system of rate equations for $^{87}\text{Rb}$

There are 24 atomic sublevels in $^{87}\text{Rb}$. Figure 3.3 depicts the levels numbered 1 through 24. This is the numbering scheme I use in the theory developed here. The system of equations is assembled by considering the individual couplings between each pair of levels in the presence of polarized driving fields. I note that I include the AC-Stark shifts calculated in Sec. 3.2 in the level structure used here. I insert these shifts manually and discuss that process in more detail in Sec. 3.6.3.

To illustrate how I assemble the system, Fig. 3.3 depicts the couplings that the ground state 1 and the excited state 17 have with the rest of the system. The couplings caused by π-polarized light are depicted with solid arrows, σ⁻-polarized light with dashed arrows, and σ⁺-polarized light with dot-dashed arrows. State 1, $|S_{1/2}, F = 1, m_F = -1\rangle$, has non-zero matrix elements with states 9, 10, 11, 13, 14, and 15 depending on the polarization present. State 17, $|P_{3/2}, F = 2, m_F = +2\rangle$, couples with states 3, 7, and 8. I reiterate that the rate equations model is only
Fig. 3.3  Rate equations labels The levels of $^{87}\text{Rb}$ are numbers 1-24. The couplings between level 1 and level 17 and the rest of the system are depicted. Couplings caused by $\pi$-polarized light are solid arrows, $\sigma^-$-polarized light dashed arrows, and $\sigma^+$-polarized light dot-dashed arrows.

valid in the absence of Zeeman coherences, so the considered optical field must have a pure polarization and different polarizations must be considered separately.

The rate equations for the $n_1$ and $n_{17}$ states can be written using the notation
defined above. They are given by

\[
\dot{n}_1^{\sigma^+} = \sum_{k \in \{9,11,15\}} \gamma_{1,k} n_k + \sum_{k \in \{9,11,15\}} R_{1,k}(n_k - n_1),
\]

\[
\dot{n}_{17}^{\sigma^+} = -\gamma n_{17} - \sum_{k \in \{3,7\}} R_{17,k}(n_{17} - n_k),
\]

\[
\dot{n}_1^\pi = \sum_{k \in \{10,14\}} \gamma_{1,k} n_k + \sum_{k \in \{10,14\}} R_{1,k}(n_k - n_1),
\]

\[
\dot{n}_{17}^\pi = -\gamma n_{17} - R_{17,8}(n_{17} - n_8),
\]

\[
\dot{n}_1^{\sigma^-} = \gamma_{1,13} n_{13} + R_{1,13}(n_{13} - n_1),
\]

where the superscript denotes the applied polarization. The first term in each equation describes the spontaneous emission in the system. The second term describes the stimulated emission and absorption.

Since the equations are linear, it is natural to express them in matrix form. For ease, I split the equations into two matrices, one for spontaneous emission and one for stimulated emission and absorption. The general formulation of the set of equations is

\[
\dot{\vec{n}} = \mathbf{A}^\gamma \vec{n} + \mathbf{R}^{st\text{/abs};q} \vec{n},
\]  

(3.15)

where \(\vec{n}\) is the population vector of the levels \(n_k\) indexed by \(k \in \{1...24\}\) and \(\dot{\vec{n}}\) is the time derivative of the population vector, indexed by \(j \in \{1...24\}\) for a driving polarization \(q\). Here, \(\mathbf{A}^\gamma\) is the matrix of spontaneous emission terms and has components \(A_{j,k}^\gamma\). The spontaneous emission matrix couples all excited states to all ground states; it has no dependence on the driving fields or their polarizations. The only way that an excited state level is not coupled by the spontaneous emission matrix is if its population value is zero (as was the case for \(n_3\) in Eq. 3.12). All ground state levels receive population via the spontaneous emission matrix as long as the excited states that decay to them have non-zero population. \(\mathbf{R}^{st\text{/abs};q}\) is the matrix of stimulated
emission and absorption terms in the presence of a driving field of polarization $q$ and it has components $R_{j,k}^{st/abs}$.$^q$. There are three different $R_{j,k}^{st/abs}$ matrices, one for each polarization.

The symmetry of the equations described by Eq. 3.15 makes the system under-constrained, so an additional constraint is required to find a unique solution. This constraint is provided by population conservation

$$\sum_{k=1}^{24} n_k = 1. \quad (3.16)$$

In theory, one could include population conservation to solve the entire system of 25 coupled equations. I solve the rate equations in the program Wolfram Mathematica, however, and its NDSolve function cannot enforce the population conservation equation properly. I enforce it manually by removing level 24, the $|5P_{3/2}, F = 3, m_F = 3\rangle$ level, from the system and replacing it with $\left(1 - \sum_{k=1}^{23} n_k\right)$. This is further discussed in Sec. 3.6.4.

3.6 Implementing the rate equations in Wolfram Mathematica

Now that the system of equations has been formulated, I describe how the equations are implemented in Wolfram Mathematica (Mathematica) to find a solution. I begin by verifying that the calculations done in Mathematica are properly normalized. This leads to a discussion of the spontaneous emission matrix $A^\gamma$. I then consider the specifics of the optical interactions modeled and the stimulated emission/absorption matrix $R_{j,k}^{st/abs}$ of the laser-dependent terms. I finish by describing how I account for population conservation.

3.6.1 Normalizing the calculations

As discussed in Sec. 3.1, the reduced dipole matrix element $d_{i,j}^2$ can be normalized in multiple ways. I adhere to the NIST convention in order to utilize the values of $d_{i,j}$
reported in [4] when calculating the AC-Stark shifts. Unlike in the AC-Stark shift calculations, however, the rate equations only model interactions between the $5S_{1/2}$ and $5P_{3/2}$ levels of the D2 line of $^{87}$Rb. Thus,

$$d_{D2}^2 = \langle n = 5, L = 0, J = 1/2 | \vec{e} \vec{r} | n' = 5, L' = 1, J' = 3/2 \rangle^2$$  \hspace{1cm} (3.17)

is the only reduced dipole matrix element that appears in the rate equations and it appears in every single term. Consequently, it is helpful to normalize the calculations so that $d_{D2}^2$ can be set to one and does not need to be carried around the equations.

To ensure that the correct normalization is chosen, I turn to our physical understanding of the system. Recall that the dipole matrix element appears in the $\gamma_{ij} = |\mu_{ij}|^2 \gamma$ excited state decay terms. We know that the sum of the spontaneous decay terms out of a given excited state must equal the total decay rate $\gamma = 1/\tau$. Thus, the individual $\gamma_{ij} = d_{D2}^2 \times CG_{i,j}^2$ terms associated with each excited state must describe the fraction of the excited state population that decays to each ground state. Since the reduced dipole matrix element is the same in each term, this information must be encoded in the Clebsch-Gordon coefficients.

To use this fact, I consider the $|F' = 3, m_{F'} = 3\rangle$ state, where the prime denotes that it is one of the excited states. This state only decays to one ground state, the $|F = 2, m_F = 2\rangle$ state, where the lack of primes indicates a ground state. Thus, we know that $|\mu_{3,3,2,2}|^2 \gamma = \gamma$ and

$$|\mu_{3,3,2,2}|^2 = d_{D2}^2 \times |CG_{33,22}|^2 = 1$$  \hspace{1cm} (3.18)

where I have replaced subscript $i$ with subscripts corresponding to $F', m_{F'}$ and subscript $j$ with $F, m_F$, to indicate the quantum states $|F' = 3, m_{F'} = 3\rangle$ and $|F = 2, m_F = 2\rangle$. Using the definition of Eq. 3.3, I find

$$|CG_{33,22}|^2 = (2 \ast 2 + 1)(2 \ast 3 + 1) \begin{pmatrix} 2 & -1 & 3 \\ -2 & 1 & 3 \end{pmatrix}^2 \begin{pmatrix} 1/2 & 3/2 & 1 \\ 3 & 2 & 3/2 \end{pmatrix}_{6j}^2,$$  \hspace{1cm} (3.19)
where I have inserted the nuclear spin, \( I = 3/2 \). Eq. 3.19 allows me to use Mathematica to calculate \(|CG_{33,22}|^2\), and in doing so I find that

\[
|CG_{33;2}^2|_{\text{Mathematica}} = \frac{1}{4} = \frac{1}{2J' + 1},
\]

(3.20)

where the subscript \( \text{Mathematica} \) indicates the result of the calculation done with the Wolfram Mathematica Program and \( J' = 3/2 \) is the \( J \)-quantum number of the excited state. I expect this result to be equal to 1, not 1/4. Recalling the discussion of normalization conventions in Sec. 3.2, I see that the Mathematica program uses the normalization convention of [116], not the NIST normalization convention I use in this thesis. Thus, the matrix element expression \(|\mu_{ij}|^2\) must be multiplied by \((2J + 1)\) in the Mathematica code to enable me to set \( d^2 = 1 \). To be clear, in the Mathematica code, I use the equation

\[
|\mu_{F^*,m_{F^*};F,m_F}|^2_M = (2J' + 1)(2F + 1)(2F' + 1) \left( \begin{array}{ccc} F & 1 \\ -m_F & m_F - m_{F'} & m_{F'} \end{array} \right)^2 \left\{ \begin{array}{c} J \\ J' \\ F \\ I \end{array} \right\}_6^2 
\]

(3.21)

where the subscript \( M \) has replaced \( \text{Mathematica} \) for brevity and I have inserted \( q = m_{F'} - m_F \) to avoid the need to define \( q \) for each transition.

3.6.2 Spontaneous emission terms

Now that proper normalization is ensured, I assemble the spontaneous emission matrix \( A^\gamma \). For the sake of space, I do not write the full \( A^\gamma \) matrix here. Instead, I take again the two illustrative levels, 1 and 17, and write the 1st and 17th rows of \( A^\gamma \) as

\[
A^\gamma_{1,k} = (0, ..., |\mu_{1;9}|^2_M \gamma, |\mu_{1;10}|^2_M \gamma, |\mu_{1;11}|^2_M \gamma, 0, |\mu_{1;13}|^2_M \gamma, |\mu_{1;14}|^2_M \gamma, ..., |\mu_{1;15}|^2_M \gamma, ...) \\
A^\gamma_{17,k} = (0, ..., -\gamma, ...) 
\]

(3.22)
where the ellipses indicate elements equal to 0. The first row, $A^\gamma_{1,k}$, governs the spontaneous emission terms in the $\hat{n}_1$ equation and shows the appropriate excited states feeding population into this level. Likewise, the 17th row $A^\gamma_{17,k}$ shows the excited state decay rate removing population from the $n_{17}$ level.

3.6.3 Laser-dependent terms

The spontaneous emission matrix $A^\gamma$ is independent of the optical field(s) applied to the atom and is relatively simple in structure. The $R_{st/absq}$ matrix, on the other hand, contains the rate $R_{j;k}$ defined in Eq. 3.10 and depends on the laser(s) applied to the atom. Looking again at this rate and using the definition of Rabi frequency, $R_{j;k}$ can be written

$$R_{j;k} = \frac{\Gamma}{\Delta_{L-j;k}^2 + \Gamma^2} \frac{|\hat{\mu}_{j;k} \cdot \hat{E}_L|/\hbar|^2}{2},$$ (3.23)

where $\Gamma = \gamma/2$ and the subscript $L$ refers to the applied laser. $\hat{E}_L$ describes the laser field’s amplitude, and $\Delta_{L-j;k}$ denotes the detuning of the applied field from the selected transition. Each transition generally has a unique detuning because of the $m_F$-level-dependent AC-Stark shifts caused by the ODT and calculated in Sec. 3.2. I manually insert these shifts here because they arise from the far-detuned trapping beam, not the near-resonant laser described by the rate equations. Furthermore, I note that these shifts actually arise due to the $\sigma_{eg}$ coherence terms ignored in this rate-equation treatment. If the model included the $\sigma_{eg}$ terms and considered the trapping beam as well as the near-resonant beam, I would not need to manually insert the AC-Stark shifts. However, such a calculation is quite complex and can be difficult to solve analytically [2]. The rate-equation model is a useful approximation. The approach taken here is similar to that described in Ref. [2]. There, the authors develop a rate-equation model for $n$-photon absorption and manually insert the Stark shifts that arise from coherences.
The Rabi frequency term depends on the laser field amplitude $\mathcal{E}_L$, the laser polarization vector $\vec{\epsilon}_L$, and the atomic transition matrix element. Since the field amplitude is the same for every transitions addressed by the laser, it is helpful to separate it from the matrix element. To this end, I define a reduced Rabi frequency $|\tilde{\Omega}_L|^2 = |\mathcal{E}_L|^2/\hbar^2$ and rewrite Eq. 3.23 as

$$R_{j;k} = \frac{1}{2} \frac{|\tilde{\Omega}_L|^2 |\epsilon_L|^2 |\mu_{j;k}|^2_M}{1 + \Delta_{L-j;k}^2/T^2},$$

(3.24)

where $|\mu_{j;k}|^2_M$ is the matrix element defined in Eq. 3.21 and $\epsilon_L^q$ is the amplitude of the polarization component indexed by $q$ that is relevant to the $j;k$ transition. I explicitly include $\epsilon_L^q$ here to enable me to write the notation more compactly later, as is discussed at the end of this section.

To illustrate the details of calculating this rate, I consider a laser $L_1$ that is π-polarized ($q = 0$) and addresses the $j = 1 \to k = 10$ transition. This is the transition between the $F = 1, m_F = -1$ ground state and the $F' = 1, m_{F'} = -1$ excited state. If the laser has frequency $\omega_{L1}$, then the rate is written

$$R_{1:10}^\pi = \frac{1}{2} \frac{|\tilde{\Omega}_{L1}|^2 |\epsilon_{L1}^0|^2 |\mu_{1:10}|^2_M}{1 + (\omega_{L1} - (\omega_{11b} + \delta_g + \delta_{e11}))/T^2},$$

(3.25)

where the detuning $\Delta_{L-j;k}$ has been explicitly written. In this notation, $\omega_{11b}$ is the frequency of the unshifted (bare) $F = 1 \to F' = 1$ transition, $\delta_g$ is the AC-Stark shift of the ground state, and $\delta_{e11}$ is the AC-Stark shift of the $F' = 1, m_{F'} = -1$ excited state.

Sometimes, it is helpful to use the rate equations to model the atomic population dynamics in the presence of two optical fields of different frequency. This is valid as long as the fields do not drive two-photon resonances within the atom. As an example, I consider two lasers: $L_1$, which addresses the sublevels of the $F = 1$ ground states and $L_2$, which addresses the $F = 2$ ground states. Both lasers are
\(\sigma^+\)-polarized \((q = 1)\). As before, I give the row \(R^{st/abs;\sigma^+}_{17,k}\) as illustrative of the matrix.

\[
\begin{align*}
R^{st/abs;\sigma^+}_{17,k} &= (0, 0, \frac{1}{2} \frac{\tilde{\Omega}_{L1}}{\Gamma^2} \frac{|\epsilon_{L1}|^2 |\mu_{17;3}|_M^2}{1 + (\omega_{L1} - (\omega_{12b} + \delta_g + \delta_e2))^2/\Gamma^2}, \ldots; \frac{1}{2} \frac{\tilde{\Omega}_{L2}}{\Gamma^2} \frac{|\epsilon_{L2}|^2 |\mu_{17;7}|_M^2}{1 + (\omega_{L2} - (\omega_{22b} + \delta_g + \delta_e2))^2/\Gamma^2}, \ldots, \nonumber
\end{align*}
\]

\[
-(R^{st/abs}_{17,3} + R^{st/abs}_{17,7}), \ldots),
\]

(3.26)

where there are non-zero entries at \(R^{st/abs}_{17,3}\) and \(R^{st/abs}_{17,7}\), and \(R^{st/abs}_{17,17}\). The \(R^{st/abs}_{17,17}\) term is the negative of the sum of the other non-zero entries. These rates can be very different in strength depending on the Rabi frequency and detuning of each laser. This structure illustrates the transcendental nature of the equations that is addressed by enforcing population conservation. The method of achieving this is discussed below.

I note here that inclusion of \(\epsilon^q_L\) in Eq. 3.24 allows me to combine the three \(R^{st/abs;q}\) that depend on different polarizations into a single matrix with serious caveats. In this treatment

\[
R^{st/abs;total} = R^{st/abs;\pi} + R^{st/abs;\sigma^+} + R^{st/abs;\sigma^-},
\]

(3.27)

where each polarization-specific matrix is already weighted by \(|\epsilon^q_L|^2\). Thus, as long as pure polarization is used, two of the matrices are equal to 0 and coherences are not formed. Occasionally this notation is useful in its compactness, but it is dangerous because it obscures the need to keep the polarizations separate to avoid coherences. That is why I have not used it up to now. I introduce it here because its compactness if useful for writing the full system of equations with population conservation enforced.
3.6.4 Enforcing population conservation

As mentioned in Sec. 3.5.2, population conservation is enforced by manually removing level 24, the \( |5P_{3/2}, F = 3, m_F = 3 \rangle \) level, and replacing it with \( (1 - \sum_{k=1}^{23} n_k) \).

Level 24 is removed because it only interacts with level 8, so only the equation for \( \dot{n}_8 \) needs to be changed. For compactness, I combine the effects of different lasers and polarizations here, though they must be treated carefully and separately as discussed above. The equation for \( \dot{n}_8 \) is given by

\[
\dot{n}_8 = \sum_{k \in \{12,16,17,22,23,24\}} \gamma_{8,k} n_k + \sum_{k \in \{12,16,17,22,23,24\}} R_{8,k} (n_k - n_8). \tag{3.28}
\]

Making the substitution, the equation becomes

\[
\dot{n}_8 = \sum_{k \in \{12,16,17,22,23\}} \gamma_{8,k} n_k + \gamma_{8,24} \left( 1 - \sum_{k=1}^{23} n_k \right) + \sum_{k \in \{12,16,17,22,23\}} R_{8,k} n_k + R_{8,24} \left( 1 - \sum_{k=1}^{23} n_k \right) - \sum_{k \in \{12,16,17,22,23,24\}} R_{8,k} n_8, \tag{3.29}
\]

where the replaced \( n_{24} \) terms are pulled out of the sums. Rearranging terms, I write

\[
\dot{n}_8 = \sum_{k \in \{12,16,17,22,23\}} (\gamma_{8,k} + R_{8,k}) n_k - \sum_{k \in \{12,16,17,22,23,24\}} R_{8,k} n_8 + \gamma_{8,24} + R_{8,24} - \sum_{k=1}^{23} (\gamma_{8,24} + R_{8,24}) n_k. \tag{3.30}
\]

The substitution results in two constant terms \( \gamma_{8,24} \) and \( R_{8,24} \) appearing in the \( \dot{n}_8 \) equation. A term that contains every population in the system also appears \( \sum_{k=1}^{23} (\gamma_{8,24} + R_{8,24}) n_k \). The constant terms can be accounted for with a column vector, \( \vec{C}_\text{popcons} \) indexed by \( k \in \{1...23\} \) with zeros in every row except \( k = 8 \), where the entry is \( \gamma_{8,24} + R_{8,24} \). The \( \sum_{k=1}^{23} (\gamma_{8,24} + R_{8,24}) n_k \) term can be accounted for by
a matrix $P_{\text{popcons}}$ with zeros in every row except the 8th. The 8th row has identical entries of $-(\gamma_{8,24} + R_{8,24})$ in every column.

3.6.5 Full rate equation model

Using the notation developed above, the entire system of equations is written

$$\dot{\vec{n}} = (A_{\gamma} + R_{\text{st/abs;total}} + P_{\text{popcons}}) \vec{n} + \vec{C}_{\text{popcons}},$$  \hspace{1cm} (3.31)

where $\vec{n}$ is the population vector of the levels $n_k$ indexed by $k \in \{1...23\}$ and $\dot{\vec{n}}$ is the time derivative of the population vector indexed by $j \in \{1...23\}$. I reiterate that $R_{\text{st/abs;total}}$ must be treated carefully to ensure that it is not used to model a situation where coherences can arise. This system of equations is solved in Mathematica using the NDSolve function and an appropriate choice of initial populations.

3.7 Results of the model

To verify that the model is implemented properly, I investigate a simple but experimentally relevant situation. I consider the case of an atomic population initially distributed equally among the $F = 1$ $m_F$ sublevels under the influence of light tuned to the $F = 1 \rightarrow F' = 2$ transition. The light is $\sigma^-$-polarized and has a reduced Rabi frequency $\tilde{\Omega}_{L1} = \gamma$. In the situation modeled here, the atomic population is excited to the $F' = 2$ excited state from which it decays to $F = 1$ and $F = 2$. Once in the $F = 2$ ground state, the atom is not addressed by the incident laser beam. Consequently, it remains in this state and over time the atomic population is cleaned out of $F = 1$ and moved into $F = 2$. This situation partially models the effect of MOT repump light, which contains both $\sigma^+$ and $\sigma^-$ light.

Initially, I model a system with no AC-Stark shifts as though the atom is not in the ODT. A schematic of the situation is shown in Fig. 3.4a. The solution to the rate equations yields the atomic sublevel populations as a function of time and the
Fig. 3.4  **Rate equations model for** $F = 1 \rightarrow F' = 2$ **σ–polarized light**  

a) A σ–polarized field (green arrows) addresses the $F = 1 \rightarrow F' = 2$ transition.  

b) The rate equations model the atomic population initially in the $F = 1$ ground state being moved to the $F = 2$ ground state.

ground state populations are given in Fig. 3.4b. As expected, the $F = 1$ ground state population (in states 1, 2, and 3) is quickly removed and transferred to $F = 2$ (states 4 through 8). This process happens on the order of 1 µs, which corresponds to $\sim 40$ spontaneous emission times. The spontaneous emission time (excited state lifetime) is $\sim 27$ ns. We see that the population is moved towards states 4 and 5 due to the polarization of the driving field.
Fig. 3.5 Rate equations model for $F = 1 \rightarrow F' = 2 \sigma^+$-polarized light a) A $\sigma^+$-polarized field (green arrows) addresses the $F = 1 \rightarrow F' = 2$ transition. b) The rate equations model the atomic population initially in the $F = 1$ ground state being moved to the $F = 2$ ground state.

When the same situation is modeled with $\sigma^+$ light, the behavior is similar to that of Fig. 3.4 but for the $m_F$ levels of opposite sign. The population is moved towards states 7 and 8 under the influence of $\sigma^+$ light. The results are shown in Fig. 3.5. The MOT repump light is a combination these two situations, weighted by the Rabi frequencies of the $\sigma^+$ and $\sigma^-$ fields. Since the atom experiences a magnetic field in the MOT, coherences are destroyed and do not affect the behavior described here.
For an atom in the ODT, the MOT repump light is detuned from the $F = 1 \rightarrow F' = 2$ transition by the AC-Stark shifts. This lowers the excitation rate and slows the process of cleaning out the $F = 1$ state but does not otherwise affect the results. The output of the rate equation model for $\sigma^-$ light with the shifts included is given in Fig. 3.6.

3.8 Summary

In this chapter I develop the theory that is needed to understand the interactions of $^{87}$Rb with an optical field. I calculate the AC-Stark shifts of the $^{87}$Rb $5S_{1/2}$ and $5P_{3/2}$ levels caused by a linearly-polarized far-detuned optical field like that which creates a dipole trap. This work has already been published as a Comment [106]. I also build a full rate-equation model of the atomic system to calculate how the populations of the atom’s magnetic sublevels change in the presence of various near-resonant optical fields and the AC-Stark shifts. Such a thorough rate-equation model as the one developed here is unusual in the field of single-atom trapping. I find this model crucial for gaining insight into the dynamics of the atomic population and for predicting the results of experiments. This rate equation model is used heavily in Chs. 5, 6, and 7 of this thesis and is a major contribution to the work presented here.
Fig. 3.6 Rate equations model for $F = 1 \rightarrow F' = 2$ $\sigma^-$-polarized light with AC-Stark shifts a) The $\sigma^-$ light is detuned from the shifted transitions. The relevant shifts and level splittings are given on the diagram, which is not to scale. b) The $F = 1$ ground state population is still moved to the $F = 2$ ground state but the process happens more slowly than in the unshifted case.
Having covered the theory of single-atom traps and the rate equation model, I now describe the single-atom trap that I build and use for the experiments in this thesis. I begin by briefly describing the vacuum system in which the atom is trapped and the magneto-optical trap (MOT) that loads the single-atom optical dipole trap (ODT). I then describe the design choices made for the single-atom trap and the basic trap operation. I finish with results demonstrating the successful trapping of an atom, a discussion of the detection of a single atom, and the measurement of the atom’s lifetime in the trap. This chapter aims to give an outline of the experiment and discussion of relevant experimental choices and procedures. More experimental details are given in Ch. 5 and App. A.

4.1 Vacuum system and magneto-optical trapping

The experiments described in this thesis rely on the ability to reliably trap single atoms and manipulate their quantum state in a controlled manner. The atoms are trapped within a vacuum chamber to isolate them from the environment, necessary for stable trapping. The vacuum chamber I use is a Kimball Physics spherical cube
The atoms are trapped inside in a spherical cube vacuum chamber attached to an ion pump.

that has six 2.75”-diameter ports and eight 1.33”-diameter ports. The 2.75” ports are oriented around the faces of the cube and the 1.33” ports on the cube corners. This is a small, nearly all-metal chamber, making it suitable for very clean, ultra-high vacuum (UHV) operation. One of the 2.75” ports holds the $^{87}$Rb dispensers and is connected to an Agilent 40L/s VacIonPlus Starcell ion pump. The other 2.75” ports and all of the 1.33” ports are sealed with windows to allow optical access to the chamber. Figure 4.1 shows the chamber. A fuller description of the vacuum system is given in Sec. A.1.

Inside the vacuum chamber, a magneto-optical trap (MOT) creates a cold cloud of $^{87}$Rb atoms. It acts as a cold reservoir of atoms from which the single-atom optical dipole trap (ODT) loads. Standard MOTs use a quadrupolar magnetic field and six laser beams to create a three-dimensionally cooled and trapped cloud of atoms [93]. Initially I built a 6-beam MOT but could not achieve the ideal geometry because of the constraints imposed by the vacuum chamber. The 6-beam MOT was not stable.
and did not lead to reliable single-atom trapping. Therefore, for the experiments in this thesis I develop an unusual eight-beam MOT. The eight beams enter the vacuum chamber through the 1.33” corner viewports of the vacuum chamber. A full explanation of the MOT and the experimental choices behind this unusual geometry is given in Sec. A.2.

The eight MOT beams contain $\sigma^+$- and $\sigma^-$-polarized light nearly resonant with the $F = 2 \rightarrow F' = 3$ transition of the $5S_{1/2} \rightarrow 5P_{3/2}$ line of $^{87}$Rb. This light Doppler cools the atoms in the MOT and is known as cooling light. Though the $F = 2 \rightarrow F' = 3$ transition is closed, occasionally an atom pumps through the $F' = 2$ excited state and decays into the $F = 1$ ground state in what is known as an off-resonant transition. Over time, population accrues in the $F = 1$ ground state and ceases to interact with the cooling light. Thus, light resonant with the $F = 1 \rightarrow F' = 2$ transition is used to clean population out of the $F = 1$ ground state and ensure the atoms are continuously cooled. This is known as the repump light. The MOT frequencies are depicted in Fig. 4.2, along with the optical dipole trap (ODT) frequency and an additional probe frequency. The ODT system is detailed in Sec. 4.2 and the probe in Sec. 4.3.

The MOT cooling and repump frequencies are generated from independent lasers that I stabilize to saturated absorption setups. A more complete laser frequency scheme, including laser lock-points and applied frequency shifts is given in Fig. A.20. Section A.2.7 describes the slightly-unusual saturated-absorption laser-frequency stabilization technique I use in this thesis.

4.2 Trapping system design

Once the MOT is formed, it acts as a cold reservoir of atoms from which the optical dipole trap (ODT) loads. As discussed in Sec. 2.4, reliable single-atom trapping is best achieved in a trap that operates in the collisional-blockade regime with $\gamma_{coll}/2 <$
β'/4 [104]. Thus, a small background collision rate ($\gamma_{\text{coll}}$) and a very small trap volume (which is inversely related to $\beta'$) are required. The background collision rate is determined by the pressure in the vacuum chamber (hence the desire for UHV) and is calculated with Eq. 2.14. For typical experimental pressures of low $10^{-9}$ Torr, $\gamma_c \sim O(0.5) \text{ s}^{-1}$. For a tightly focused, Gaussian-beam optical dipole trap (ODT) with an $\sim O(\mu\text{m})$ beam waist, $\beta' \sim O(1) \text{ s}^{-1}$. This meets the design requirements for collisional blockade. The tight beam waist required for trapping is created using a high numerical aperture (NA) lens. To better understand this design choice, I define the numerical aperture and explain its relation to beam waist and light collection.

4.2.1 High numerical apertures lenses for single-atom traps

The numerical aperture (NA) of a lens characterizes the range of angles over which the lens focuses and accepts light. The NA is defined as

$$NA = n \sin \theta,$$

(4.1)
Fig. 4.3  **Numerical aperture** A lens of radius $D$ and focal length $f$ has a Numerical Aperture (NA) defined by the angle $\theta$.

where $n$ is the refractive index of the medium in which the lens is placed and $\theta$ is the half-angle of the cone of light that can enter the lens [13]. This cone is defined by the lens radius $D$ and focal length $f$ such that $\tan \theta = D/f$, as shown in Fig. 4.3. Higher $\theta$ leads to higher NA. The NA is defined using the ray-optics picture of light.

The Gaussian-beam trap, however, is better described using wave-optics, in which case the half-angle $\theta$ is replaced with the beam divergence $\theta_0$. A beam entering the lens is focused with a divergence approximately given by $\theta_0 \approx \theta$. Beam divergence is inversely related to beam waist by

$$2\theta_0 = \frac{4}{\pi} \frac{\lambda}{2w_0},$$  \hspace{1cm} (4.2)

where $\lambda$ is the wavelength of the light and $w_0$ is the beam waist defined in Sec. 2.2 [101]. Thus, a high NA leads to the small beam waist desired for creating an ODT.

The NA of a lens also characterizes the amount of light the lens accepts. If a point source is placed at the focus of the lens, the NA is a measure for how much light is collected and collimated by the lens. For an isotropic point source, the full solid angle of emission $\Omega$ is a sphere of surface area $\Omega = 4\pi$. The lens collects the light emitted into the cone subtended by $2\theta$. Thus, the solid angle collected by a lens is

$$\Omega_{\text{lens}} = 2\pi(1 - \cos \theta).$$  \hspace{1cm} (4.3)
Fig. 4.4 **Cross-section of the chamber during MOT trapping.** The MOT beams (red cylinders) enter the chamber through the 1.33” corner ports. The lens (blue) is mounted in a lens tube and groove grabbers (yellow) in one of the 2.75” ports. The lens’s focal plane (yellow line) overlaps with the MOT.

Larger NA and higher $\theta$ lead to a larger fraction of the light emitted from a point source at the lens focus being collected by the lens.

Consequently, a high-NA lens is useful both for creating a tightly-focused ODT and for collecting light from the trapped atom. Collecting a lot of light from the atom greatly facilitates studying its interaction with near-resonant laser light and achieving fast, nondestructive quantum-state readout. This leads us to use the highest-NA lens possible in designing the trapping system. Unfortunately, high-NA lenses often have short focal lengths, since $\tan \theta$ is inversely related to $f$. Since the atoms must be trapped inside a vacuum chamber, this presents a challenge.

One way to overcome this challenge is to use a custom-built lens with both a high NA and a long focal length. Such lenses are expensive (several thousand dollars to tens of thousands of dollars) and must be carefully designed. I do not use such a lens. Instead, I use a simple, single-element, aspheric lens mounted inside the vacuum chamber. The lens is held in the chamber in such a way that its focal plane overlaps
with the beams which create the MOT, as shown in Fig. 4.4. More details of the mounting mechanism are given in Sec. A.3. The lens is used to focus trap light into the MOT to trap a single atom and to collects light from the trapped atom. I use an NA = 0.54 lens with a focal length of 15 mm and a working distance of 11.5 mm for the experiments described in this thesis. The lens is from Thorlabs, P/N AL1815-B. This high-NA lens collects 7.9% of the fluorescence from an isotropic point source at its focus. A fuller discussion of the reasons for choosing this lens is given in Sec. A.3.

4.2.2 Creating the optical dipole trap

To create the optical dipole trap, the in-vacuum lens focuses an 852-nm-wavelength, linearly-polarized, Gaussian beam to a small waist. The trapping light is red-detuned of the 780-nm-wavelength atomic resonance, trapping the atom at the focus of the beam. A diagram of the basic setup is shown in Fig. 4.5. Linearly polarized trap light is chosen because linearly-polarized traps exhibit better loading characteristics than elliptical and circularly polarized traps [61]. Circularly-polarized trap light also lifts the degeneracy of the magnetic sublevels in the ground state of the trapped atom [37]. For the experiments performed in this thesis, such degeneracy lifting is an unwanted complication.

For a collimated input beam of diameter 3.2 mm, the asphere focuses the trapping beam to a 2.54-μm-radius beam waist, as calculated by a Zemax simulation of the system. The beam waist is measured as the radial distance from the beam center at which intensity reaches $1/e$ its peak value. Zemax OpticStudio is an optical design software in which I model the experimental system. I use the software license shared between Professor Jungsang Kim and Professor David Brady’s groups. A trap waist of 2.54 μm yields $\beta' = 4.3 \text{ s}^{-1}$ for typical MOT-beam intensities. To reach the collisional-blockade regime with this $\beta'$, a $\gamma_{\text{coll}} < 2.15 \text{ s}^{-1}$ corresponding
Fig. 4.5 **ODT Diagram.** The 852 nm trapping light enters the vacuum chamber and is focused by an aspheric lens to create the ODT which loads from the MOT. The same lens collects the 780 nm atomic fluorescence, which is sent down an imaging path by a dichroic beamsplitter.

to a pressure < $8.4 \times 10^{-9}$ Torr is required.

It is worth noting that the tightness of the trapping waist is highly dependent on the alignment of the trapping beam through the asphere and the size of the input beam. Misalignment cants the focus and can increase the waist, as does a smaller-diameter input beam. Both of these cases decrease $\beta'$. Hence, the trap design must have sufficient error tolerance that the collisional-blockade regime can still be reached. Given the reasonably achievable $\gamma_{\text{coll}}$ described above, this criterion is met in the setup described here. Since alignment sensitivity makes it difficult to accurately predict the trap depth, the trapped atom itself is used to interrogate the trap behavior and provide quantitative values for $\gamma_{\text{coll}}$ and $\beta'$. The results of such interrogation are detailed in Ch. 5.
4.3 Detection of a single atom

Once an atom is trapped, it must be detected. To this end, an optical path for imaging the atom is designed. The in-vacuum asphere collects fluorescence from the trapped atom and acts as the first optic in the imaging path. Ideally, the atomic fluorescence would be collimated by the asphere as it exited this chamber. A collimated beam of 780-nm-wavelength light would be least aberrated and easiest to focus onto the detection optics. However, the atom is trapped at the focus of the 852-nm-wavelength trapping beam and this focal position is not the same as the focal position for 780 nm light. Consequently the fluorescence is not collimated by the asphere and the 780-nm-wavelength light exiting the asphere is slightly converging and spherically aberrated. These issues are discussed in detail in Ch. 5.

The 852 nm trap light and the 780 nm fluorescence light are separated by a dichroic beamsplitter. Reflection by the dichroic directs the 780 nm fluorescence
down an imaging path towards two detectors. The fluorescence is focused to an intermediate image at an aperture. This aperture acts as a spatial filter to block stray scatter in the system from entering the detectors. After the aperture, the light is collimated by a lens and enters a 90:10 beamsplitter that transmits 10% of the light to a lens which focuses it onto the sensor of an electron-multiplying CCD (EMCCD) camera. The beamsplitter reflects 90% of the light to a lens that focuses it onto the tip of an multimode fiber that is connected to a single photon counting module (SPCM) avalanche photodiode. A band-pass filter is placed before the fiber to ensure that only 780 nm fluorescence is sent to the SPCM. A diagram of the imaging path is given in Fig. 4.6 and it is explained in detail in Sec. A.7.

The EMCCD camera is an Andor iXon897 and is mainly used for initial alignment of the trap. It can be run continuously, acting as a video camera, to give real-time information about the MOT and ODT. It can also be triggered with a TTL pulse to take a pre-programmed exposure at the input of the trigger pulse. The SPCM is a Perkin-Elmer AQR14+. I am indebted to Professor Paul Kwiat of the University of Illinois Urbana-Champagne for the loan of the SPCM. It provides the quantitative data presented in this thesis.

The SPCM is activated with what’s known as a TTL “gate” pulse. When the gate is high, the SPCM is active and detects photons incident on its sensor. For each photon detected, the SPCM outputs a TTL detect pulse. When the gate pulse is low, the SPCM is turned off and does not output detect pulses. This gating capability allows me to switch the SPCM on only when the atom is illuminated and should be fluorescing. This decreases the likelihood of detector dark counts and saturation effects contaminating the data.

Considering the specified transmittivity of the optical elements in the imaging path (collectively 74%), the solid angle of the high-NA asphere (7.9%), and the detection efficiency of the SPCM at 780nm (66%), this imaging system is capable
of detecting $\sim 3.8\%$ of the light emitted by an isotropic point source at the asphere focus. This calculation does not account for misalignments in the system, less-than-perfect fiber coupling efficiency, or the polarization of the emitted radiation. Those details are discussed in Ch. 5.

I pause here to define some terminology. The experiments in this thesis require both determining if an atom is trapped in the ODT and determining the atom’s quantum state, \textit{i.e.}, which hyperfine ground state the atom occupies. I use the word “detection” for the process of determining the atom’s presence in the ODT. I describe the process of determining the atom’s quantum state as quantum-state “readout.” This section and Sec. 4.4 describe the detection of the atom in the ODT. Quantum-state readout is covered in Chs. 6 and 7.

Now I present results demonstrating successful loading of single atoms into the ODT and successful single-atom detection. The first results show single atoms loading from the MOT into the ODT with the MOT present in the background. Detection with both the EMCCD and the SPCM is described. I then describe detection of the single-atom in the ODT without a MOT present in the background. The final section in this chapter gives a more detailed comparison of different detection schemes.

\subsection*{4.3.1 Loading from the MOT}

Initially, the ODT is loaded from the MOT and detected on the EMCCD camera with the MOT still present in the background. When the ODT is in the strong-loading regime described in Sec. 2.4, the ODT appears on the camera as a bright, dense spot in the background MOT, see Fig. 4.7. This strong-loading regime is used to align the SPCM to the ODT. The imaging alignment is fully described in Sec. A.7.2. Though this image demonstrates that the ODT is loading and overlapped with the MOT, it does not provide evidence of single-atom trapping. For that, we need to see a discrete signal that shows clearly either 0 or 1 atoms in the ODT.
To achieve single-atom trapping, the strong-loading regime must be left and the collisional-blockade regime entered. To achieve this, the loading rate is decreased by decreasing the MOT density. This is done by lowering the intensity of the MOT beams, which weakens the MOT, and also by moving the location of the MOT with respect to the ODT. The MOT is moved using shim magnetic fields, which translate the magnetic zero at which the MOT sits. Thus, the MOT is moved to the side so that the ODT overlaps with a less-dense MOT region. Strong loading is seen with typical MOT powers of $\sim 15$ mW of cooling light and $\sim 3$ mW of repump light. For the collisional-blockade regime, those powers are decreased to $\sim 7$ mW and $2.2$ mW, respectively. When the collisional-blockade regime is reached, the bright spot of Fig. 4.7 disappears. In this regime, the SPCM avalanche photodiode becomes more useful than the camera.

The short integration times of the SPCM allow it to detect the discrete, stair-step fluorescence signal of single atoms entering and exiting the trap. This signal is characteristic of the collisional-blockade regime. Figure 4.8 shows a time trace of single atoms loading from the background MOT in the collisional-blockade regime.

Clear fluorescence steps are visible showing 0 or 1 atom present in the trap. In the middle of the trace, a spike indicates the entrance of a second atom into the trap and
is immediately followed by a drop to the 0-atom level, indicating two-body loss. This is the collisional-blockade regime in action. The trace is noisy and the background is high due to the presence of the MOT, but the signal is unmistakable. The 0-atom signal is \( \sim 1575 \) counts and the 1-atom signal \( \sim 1675 \), yielding a signal-to-background ratio (SBR) of \( 100/1575 = .06 \). I use SBR as a simple way to characterize and compare the single-atom signals presented in this thesis. Higher SBR generally means a better signal and improved detection scheme. Figure 4.8 represents one of the first single-atom signals measured in the system. In the next section, I detail how I change the detection technique to improve the SBR.

4.3.2 Detecting without the MOT

Though Fig. 4.8 clearly indicates the presence of a single atom in the ODT, the MOT is on in the background and the atoms are continuously moving in and out of the ODT. This situation is not conducive to control over the atom’s quantum state. To achieve that control, the atom must be isolated from the MOT more
Fig. 4.9 **Example experimental sequence.** *(not to scale)* The ODT is loaded from the MOT in the loading phase. The MOT beams and B fields are then switched off allowing the MOT to fall away before the atom in the ODT is detected and experiments performed.

To this end, after a loading phase, the MOT laser beams and magnetic fields are switched off and the MOT is allowed to fall away for 10 ms. Any atom that has loaded into the ODT remains in the trap. The atom remaining in the ODT is illuminated with a laser beam causing it to fluoresce. The SPCM gate is activated and the fluorescence is detected to verify that an atom has been loaded into the trap. A typical experimental sequence is shown in Fig. 4.9. After atom detection, experiments are performed followed by another fluorescence detection to see if that atom has been lost from the trap. Different detection methods have been investigated during the work done in this thesis. I detail those methods in the next section.

Note that in most of the timing diagrams given in this thesis, I do not specify that the SPCM gate is turned on. I omit the gate pulse for the sake of space and diagram clarity. When the specifics of the gate pulse are necessary, they are included in the diagram. Otherwise, it should be assumed that when the SPCM gate is on when the atom is being detected.
4.4 Comparing different detection schemes

It is possible to detect the atom with various detection methods because the atom fluoresces under many different conditions. Atomic fluorescence is caused by the MOT beams that scatter light from both the $F = 1$ and $F = 2$ ground states of the atom and by a probe beam resonant with the $F = 2$ ground state. Figure 4.2 gives a sense of the laser frequencies I use in the experiments but it is oversimplified. We know that the ODT causes AC-Stark shifts to the energy levels of the trapped atom and those shifts cannot be ignored when considering the atomic fluorescence. A more accurate level scheme showing the MOT and probe lasers in the presence of the AC-Stark shifts is given in Fig. 4.10.
The repump light is depicted by a green arrow to distinguish it from the MOT cooling light depicted in red. The probe beam is depicted in blue. The colors are consistent across all diagrams. Only the $F' = 2$ and $F' = 3$ excited states are shown here for brevity because the $F' = 0$ and $F' = 1$ states are not directly addressed by any of the lasers in the experiment. The atom occasionally pumps into those levels through an off-resonant transition; that process is described in Ch. 6. The AC-Stark shifts (red numbers) are given for an ODT created by 40 mW of trapping light, as is typical of the experiments presented in this thesis.

The MOT cooling laser is tuned to 13 MHz below the unshifted $F = 2 \rightarrow F' = 3$ transition. The repump laser is tuned to 2 MHz above the unshifted $F = 1 \rightarrow F' = 2$ transition. The unshifted atomic energy levels are depicted by black dashed lines. The probe laser is tuned to different frequencies between experiments, generally very close to the AC-Stark-shifted levels, and one typical detuning is given here. The MOT and repump lasers are a mixture of $\sigma^+$ and $\sigma^-$ polarizations, so they drive $m_F \rightarrow m_{F'} \pm 1$ transitions. The probe laser is $\pi$-polarized and drives $m_F \rightarrow m_{F'}$ transitions, as shown in the diagram. I first describe atom detection via the MOT beams. I then discuss detection via the probe beam before finishing by describing a combination probe/cool sequence.

### 4.4.1 Detection with MOT beams

For detection with the MOT beams, an experimental sequence like that given in Fig. 4.11a is performed. After the MOT has fallen away, the MOT beams and SPCM gate are turned on to detect the atom. Detecting with the MOT beams is appealing because it uses lasers already present in the setup. However, it is challenging in part due to a large amount of background scatter from the MOT beams themselves. Scatter raises the background level of counts in the detection (as seen in Fig. 4.8) and can swamp the signal. Indeed the data of Fig. 4.8 was taken with an additional
aperture inserted in the imaging path between the dichroic beamsplitter and the vacuum chamber. This aperture is necessary for imaging with the MOT beams even when the MOT is off because it blocks most of the light they scatter into the detector. Without this aperture, the detector is overwhelmed with scatter and no signal is visible above the background. Detection with the MOT beams is also challenging because of the beams’ detuning from the AC-Stark shifted resonance of the atoms. This detuning means that a trapped atom does not interact with the MOT beams very strongly and the signal is relatively weak.

The results of detection with the MOT beams after the MOT has fallen away are given in Fig. 4.11b. The figure shows the results of 500 attempted load and detect sequences and there is evidence of two bands of events. The events in the band averaging 2600 counts are those in which a single atom was not loaded into the ODT; this is the 0-atom background caused by scattered light. The events in the band averaging 3100 counts represent successful single-atom loadings and detection by the MOT beams; this is the 1-atom signal. The signal to background ratio of this data is ∼0.2, an improvement over the SBR of 0.06 shown in Fig. 4.8. This improvement is mainly due to a longer exposure time compared to Fig. 4.8 and improved imaging alignment.

It is very clear that these bands are not stationary in time. There is a slow drift visible across all of the events due to slow fluctuations in the power of the MOT beams. This power fluctuation is not detrimental to the MOT operation and loading of the dipole trap but it clearly affects the use of the MOT beams for atom detection. Without considering the full time trace, for example, it is difficult to tell if an event generating 2900 counts came from the 0-atom band around event 0 or the 1-atom band around event 250.

Furthermore, the additional aperture inserted to collect this data blocks some of the signal from the ODT. This is extremely detrimental to the quantum-state readout
**Fig. 4.11**  **Fluorescence detection with MOT beams**  
a) (not to scale) After loading, the MOT is allowed to fall away and then the MOT light is turned on for 100 ms. This causes any atom in the ODT to fluoresce.  
b) The results of 500 attempted trap loadings and MOT-light detections are shown. The successful loads (avg counts $\sim 3100$) are distinguishable from the unsuccessful loads (avg counts $\sim 2600$. The SBR is 0.2.
described in Chs. 6 and 7. Despite the improved SBR in Fig. 4.11b compared to Fig 4.8, I conclude that the MOT beams are not suitable for atom detection in the experiments described here.

4.4.2 Detection with probe beam only

The problem of detection light causing excessive background scatter is ameliorated by turning off the MOT beams during detection and using a separate probe beam for detection. The probe beam passes through the vacuum chamber perpendicular to the imaging path, a geometry that minimizes background scatter into the detector (see Fig. 4.12). The probe beam is generated from a different laser than the MOT beams, enabling me to independently control its frequency. This allows it to be tuned to the AC-Stark-shifted resonance of the atom in the ODT, as shown in Fig. 4.10, increasing the signal from the atom. Hence, the probe operates at lower power than the MOT beams and for a shorter amount of time. Both of these facts help to minimize unwanted background scatter. More details of the probe beam system are given in Section A.4.
Unfortunately, there is a complication to detecting with the probe beam that must be addressed. The probe beam only interacts with atoms in the $F = 2$ ground state. This is very useful for quantum-state readout and serves the basis of the scheme described in Chs. 6 and 7, but it is not ideal for atom detection. For atom detection, we need to know if an atom is in the trap regardless of the atom’s quantum state. If the probe beam is used for detection, any atom that is in the $F=1$ ground state is not detected. Hence, an additional state preparation step is required before the atom is detected by the probe beam. This step involves shining MOT repump light on the atom to clean out any atomic population in the $F = 1$ ground state. The atom is then detected with the probe light. A timing diagram of this scheme is shown in Fig. 4.13a.

A histogram of the results of 500 runs of this experimental sequence is shown in Fig. 4.13b. The probe beam is incident on the atom for 200 $\mu$s during which counts are collected. There is clear a peak at $\sim 0$ counts, which corresponds to the 0-atom background events. The 1-atom signal events are spread out but appear to be centered around 9 counts. Calculating a rough estimate of the 0-atom background by taking the ratio of the first two bins, the background is $50/260 \approx 0.2$ counts. This makes the SBR of this data 44, a clear improvement over the case of using MOT light for detection.

To accurately evaluate the usefulness of this detection scheme, however, we need a more technical measure than SBR. Namely, we need to quantify the discrimination error, which is the likelihood of mis-labeling an event as either signal or background. To this end, I develop a procedure to calculate the discrimination error associated with this data. I start by considering the normalized histogram of all 500 events binned in 1 count bins (see Fig. 4.14). The data is fit (red line in figure) to the sum
Fig. 4.13  Atom detection with probe light a) (not to scale) After loading, the MOT is allowed to fall away. Then an 100 µs state preparation pulse of MOT repump light is applied to the atom followed by a 200 µs probe detect pulse. b) The results of 500 attempted load and detect sequences are shown. There is a clearly a peak around 0 showing the case where no atom was loaded. The 1-atom signal is centered around 8 counts and is broad.
Fig. 4.14  **Probe detection results with fit.** The results of the probe-light detection are fit by a Poissonian and a Gaussian. The details are given in the text.

of a Poissonian, for the 0-atom background, and a Gaussian, for the 1-atom signal.

\[
Fit = Ae^{-\lambda \frac{x^2}{2}} + B \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}},
\]

(4.4)

where \(\lambda\) is the mean of the background Poissonian and \(\mu\) and \(\sigma\) are the mean and standard deviation of the signal Gaussian. The fit parameters are \(A = 0.61 \pm 0.01\), \(\lambda = 0.17 \pm 0.01\) and \(B = 0.42 \pm 0.03\), \(\mu = 7.3 \pm 0.5\), and \(\sigma = 5.3 \pm 0.5\). The coefficient \(B\) shows that the trap was loaded in \(\sim 40\%\) of the events. The fitted distributions are found to cross at 2.8 counts. Taking that as a discrimination threshold \(c_d\), I define the discrimination error \(\epsilon\) for each distribution as follows

\[
\epsilon_{\text{sig}} = \int_0^{c_d} B \frac{1}{\sqrt{2\pi}\sigma^2_{\text{sig}}} e^{-\frac{(x-\mu_{\text{sig}})^2}{2\sigma^2_{\text{sig}}}},
\]

(4.5)

\[
\epsilon_{\text{bg}} = A \left( 1 - \sum_{x=0}^{c_d} e^{-\lambda \frac{x^2}{2}} \right).
\]
The full error is given by the average of the two: $error = 0.5 \times (\epsilon_{\text{sig}} + \epsilon_{\text{bg}})$. For the data presented here, the discrimination error is 1.4%.

The discrimination error is lowered by decreasing the background and increasing the signal. This separates the two distributions. Through better alignment of the imaging system and suppression of scatter with shielding and beam blocks, I decrease the average 0-atom counts from $\lambda = 0.17$ to $\lambda = 0.09$. This is an improvement of almost a factor of two. Increasing the atom signal is more challenging. The naive approach is to increase the power in the probe beam. Unfortunately, the probe beam can lead to radiation-pressure heating, as discussed in Sec. 2.5, which causes loss from the trap. This is undesirable since we aim to repeat multiple experiments on each atom. In addition, increasing the probe beam power increases the likelihood that an atom undergoes an off-resonant transition and pump into the $F = 1$ ground state. When this happens, the atom stops fluorescing. This limits the amount of fluorescence that can be collected from the atom and causes the fluorescence rate to drop over time. Consequently, the signal distribution is not stationary in time which violates the assumptions of the detection discrimination procedure adopted above. The ramifications of the off-resonant pumping are explored in detail in Chs. 6 and 7.

To evaluate the usefulness of the probe detection scheme described here, we must step back and consider the one of the goals of the work presented in this thesis, which is to achieve accurate quantum-state readout. “Accurate” in this case means low error and state-readout errors on the level of a few percent are desirable. This discrimination error of 1.4% in the atom detection translates into the quantum-state readout, potentially limiting the process. More importantly, as the parameters of the probe beam are changed, the signal distribution is not stationary in time, negating this discrimination process. Both of these situations are undesirable and better atom detection needs to be achieved.
4.4.3 Probe/cool detection sequence

As we have seen, detecting the atom with MOT light is challenging because of large background and blocked signal. Probe-light detection is problematic because it only detects atom in the $F = 2$ ground state and can suffer from atom heating leading to loss of the atom from the trap and off-resonant pumping making the signal non-stationary in time. Given this, I found that the best detect sequence was a probe/cool sequence similar to that described in [109], but modified for the experimental setup described here.

In this sequence the probe beam and MOT beams are toggled on and off out of phase with each other and the SPCM is gated to collect photons only when the probe beam is on. Generally, the probe and SPCM gate are turned on for a very short time (e.g., 5 µs) and then turned off and followed by a longer cooling pulse (e.g., 100 µs) of the MOT beams. The sequence is repeated, 100-1000 times, and the counts output by the SPCM are summed to yield e.g., 5 ms of integrated probing time. The probe light causes any atom in the $F = 2$ state to fluoresce and this fluorescence is registered by the SPCM. The MOT beams contain cooling light detuned from the

Fig. 4.15 Probe/cool detection sequence. The probe beam and MOT beams are toggled on and off out of phase with each other. The SPCM is gated on only during the probe pulses.
The $F = 2 \rightarrow F' = 3$ transition which Doppler cools the atom to prevent loss from the trap. The MOT beams also contain repump light on the $F = 1 \rightarrow F' = 2$ transition which pumps any atom in the $F = 1$ ground state to the $F = 2$ ground state. The atom then fluoresces under the next probe pulse. Consequently, any atom present in the ODT fluoresces under this probe/cool scheme. Since the probe pulse is so short, off-resonant transitions do not manifest in this scheme like they do in the probe-only scheme, and the signal distribution is relatively stationary in time. A detailed timing diagram is depicted in Fig. 4.15. Additional delays are included between steps to account for the switching time of the acousto-optic modulators (AOMs) which control the beams. The optimal delays shown in the diagram are empirically determined.

Using this sequence greatly improves the detection of the single atom. Figure 4.16b depicts a histogram of 500 attempted load and probe/cool (p/c) detect events. The histogram clearly shows a 0-atom background peaked at $\sim 5$ counts and a 1-atom signal peaked at $\sim 260$ counts. This yields an SBR of 51, a vast improvement over that seen previously. This data was taken with 1000 repetitions of the p/c cycle, yielding 5 ms of total probe integration time and 247 ms of total cycle time. The p/c cycle was also repeated to show that the atom is retained in the trap and not lost. The first detection (called first image in the figure legend) is shown in blue and the second detection (called retention image) is shown in green. A timing diagram is given in Fig. 4.16a. Under this scheme, the atom is generally retained in the trap in $\sim 98\%$ of the successful load events.

Using the same discrimination procedure described in Sec. 4.4.2 and the fit given by Eq. 4.5, the results are fit with parameters $A = 0.581 \pm 0.005$, $\lambda = 4.75 \pm 0.03$ and $B = 0.41 \pm 0.02$, $\mu = 262.4 \pm 0.8$, and $\sigma = 17.2 \pm 0.8$. The coefficient $B$ shows that the trap was loaded in $\sim 40\%$ of the events. The fitted distributions are shown in Fig. 4.17 and $c_d$ is found to be 52 counts. For this data, the discrimination error is
**Fig. 4.16  Probe/cool detection**  

a) The p/c sequence is repeated to demonstrate that the atom is not lost from the trap.  
b) A histogram of probe/cool detection events clearly shows a 0-atom background peaked at 5 counts and a 1-atom signal peaked at 260 counts.
Fig. 4.17  **1000 x p/c sequence detection results with fit.** The results of the p/c sequence are fit by a Poissonian and a Gaussian.

The results of the p/c sequence are fit by a Poissonian and a Gaussian. Found to be $< 10^{-37}$, demonstrating that the probe/cool sequence provides excellent discrimination between the 0-atom and 1-atom distributions.

The data in Fig. 4.16b shows detection achieved with 5 ms of integrated probe time. Because of the interleaved cool pulses, however, this sequence takes 247 ms of total time. I decrease that time by decreasing the number of repetitions of the p/c cycle. Data showing the result of a p/c cycle repeated 100 times instead of 1000 is shown in Fig. 4.18. The fit parameters are $A = 0.723 \pm 0.002$, $\lambda = 0.508 \pm 0.003$ and $B = 0.276 \pm 0.007$, $\mu = 25.73 \pm 0.16$, and $\sigma = 5.11 \pm 0.16$, yielding an SBR of 50. Going through the discrimination procedure described above, I find that $c_d = 3.9$ and the discrimination error is 0.09 %. This is achieved with only 500 $\mu$s of integrated probe time and 24.75 ms of total time.

Decreasing the number of p/c cycles not only decreases the experimental time required for detection, it also decreases the chance that an atom is lost during the
Fig. 4.18  100 x p/c sequence detection results with fit The results of the p/c sequence repeated 100 times are fit by a Poissonian and a Gaussian.

detection. This increases the probability of retaining the atom in the trap and the number of detections that can be performed on a single atom. Table 4.1 summarizes the effect that shortening the probe/cool sequence has on the number of detections the atom can undergo before it is lost. For the data in the final row of the table, taken with the p/c cycle repeated 100 times, the atom is retained in the trap in > 99.6% of the trials. The probe/cool detection sequence described in this section provides excellent detection of the atom without heating the atom out of the trap. It also probes the atom’s behavior in the absence of off-resonant pumping. This allows me to characterize the atom-laser interaction and verify the rate equation model, as I describe in Ch. 5.
Table 4.1: Retention in probe/cool detection.

<table>
<thead>
<tr>
<th>p/c repeats</th>
<th>Time (ms)</th>
<th>average detections/atom before loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>248</td>
<td>15.0</td>
</tr>
<tr>
<td>500</td>
<td>124</td>
<td>18.3</td>
</tr>
<tr>
<td>250</td>
<td>62</td>
<td>47.5</td>
</tr>
<tr>
<td>100</td>
<td>25</td>
<td>83.4</td>
</tr>
</tbody>
</table>

4.5 Trap lifetime measurement via p/c detection

For successful completion of the experiments presented in this thesis, it is vital that the atom remain trapped long enough for experiments to be performed. Generally, trap lifetimes of several seconds are desirable. As discussed in Sec. 2.4, the trap lifetime is limited by atoms from the background vapor in the chamber colliding with the atom in the trap. These collisions cause atom-loss at the rate \( \gamma_{\text{coll}} \) and are governed by the pressure in the vacuum chamber. In theory, the lifetime can be estimated by measuring the chamber pressure directly. Chamber pressure is measured by monitoring the current drawn by the ion pump that pumps on the chamber [82]. This current is proportional to pressure and vacuum companies publish calibration curves for each pump. In the experiment described here, however, the pressure is low and the current is generally very small (\( \mu A \)), making it hard to measure. Consequently, it is more accurate to measure the lifetime of the atom in the trap directly.

The lifetime is measured with the experimental sequence given in Fig. 4.19. The ODT is loaded from the MOT and then the MOT is allowed to fall away. A probe/cool detection sequence detects the presence of an atom. All fields except the ODT light are then turned off and the atom sits in the trap unperturbed for a variable wait time \( t \) before a second p/c detection sequence determines if the atom is still present. The wait time varies from 5 \( \mu s \) to 60 s. During the wait time \( t \) the atom
Fig. 4.19  Experimental sequence for measuring trap lifetime An atom is loaded in the trap and detected with a probe/cool detection sequence. Then, all fields except the ODT are turned off for a variable time $t$ before the atom is detected again.

can be lost from the trap due to collision with a background atom or heating from the trap light, as discussed in Sec. 2.5. Both of these loss mechanisms are present in all experiments in this thesis, and I call such loss single-body loss (sbl). The fraction of atoms retained in the trap decays exponentially with wait time at the rate $\gamma_{sbl}$.

The results of the experiment described above are given in Fig. 4.20. The error bars on the data points are the standard error. The fit is shown on the graph along with the 95% confidence bands. The decay rate of the fit is $0.0156 \pm 0.0011 \text{ s}^{-1}$, where the error is the standard error. This decay rate corresponds to a lifetime of $64.5 \pm (\pm)5(4) \text{ s}$. The 95% confidence interval for the lifetime is [56, 76] s and the coefficient of determination of the fit is $R^2 = 0.998$.

This lifetime is long compared to typical single atom traps where lifetimes tend to be in the $5 - 10$ s range [62]. Using

$$\gamma_{coll} = \frac{1}{\tau_{coll}} = \sqrt{\frac{3T_b}{m} n_b \sigma_{Rb-Rb}},$$

(4.6)
The fit to the experimental data shows a single body decay rate of \(0.0156 \text{ s}^{-1}\), corresponding to a trap lifetime of 64 s.

with \(\sigma_{Rb-Rb} = 2.5 \times 10^{-13} \text{ cm}^2\), I calculate that the pressure in the vacuum chamber is \(6.1 \times 10^{-11} \text{ Torr}\) \[99\]. This low pressure indicates that I achieve a clean UHV environment.

The low \(\gamma_{sbl}\) rate extracted from the fit of Fig. 4.20 tells us that the chance of losing an atom due to single-body loss during an experimental sequence is very low. Typically, the experimental steps between p/c atom detections only last \(\sim 30\) ms, during which time there is a .05% chance that the atom is lost from the trap in the absence of near-resonant fields.

4.6 Summary

In this chapter I describe the basic experimental pieces of the single-atom dipole trap I develop for the work in this thesis. I explain the choice to use an unusual 8-beam MOT to load the trap and to put a high-NA asphere inside the vacuum chamber. I present evidence of single-atom trapping and detection and detail three different detection schemes. I argue that the probe/cool scheme developed in Sec. 4.4.3 gives
the best single-atom detection. Using this scheme I measure the lifetime of the atom in the trap and extracted the single-body loss rate. This scheme also enables me to characterize the atom-probe interaction and verify the rate equation model of Ch. 3.

The experimental system and protocols developed here are a major result of this thesis work. They enable me to closely characterize the scattering rate of a single, trapped atom in the presence of near-resonant laser light. That characterization leads to achieving fast, nondestructive quantum-state readout and is the subject of Ch. 5. The protocols and system developed here have applications for a wide range of single-atom-trapping experiments.
Experimentally characterizing the system with the rate-equation model

Now that the basic experimental setup has been described and the rate-equation model developed, I turn to experiments that combine these two. In order to achieve the quantum-state control and readout that is the main result of this thesis, it is necessary to characterize the atom-light interaction and the AC-Stark shifts experienced by the atom in the ODT. The AC-Stark shifts have a significant effect on the atom’s interaction with near-resonant fields, as discussed in Ch. 3. The differential shifts of the excited states make the situation particularly complicated.

Consequently, many groups have developed elaborate schemes to avoid detecting in the presence of these differential shifts. Some groups trap at a so-called “magic”-wavelength, where the shifts are all uniform [68],[70]. This is challenging because it limits the choice of laser used, which limits the available trapping power and trap depth. Other groups develop complicated detection schemes in which the trap is chopped on and off while the atom is detected, similar to the probe/cool (p/c) sequence, but with probe and trap light [44],[50]. This is can lead to heating and
atom loss unless great care is taken and often requires $>1$ MHz chop rate, which can be experimentally challenging to implement. Many groups avoid the issue of the differential Stark shifts by detecting with $\sigma$-polarized light, which only excites the $|2,2\rangle \rightarrow |3,3\rangle$ cycling transition [30],[62],[74]. In this case, only a single excited-state sublevel comes into play. There are several drawbacks to this scheme, which I detail in Ch. 7.

I avoid the necessity for these complicated schemes by carefully characterizing the AC-Stark shifts present in the experiment. This allows me to detect the atom using $\pi$-polarized light in the presence of the ODT. Once the atom is trapped, there are several characterization experiments that I perform and describe here. I begin by performing spectroscopic measurements of the trapped atom to determine the precise AC-Stark shifts caused by the trapping light. From this I extract information about the trap depth and the transverse diameter of the ODT, the beam waist. I also measure the saturation of the atom as a function of probe power and fit it to the rate-equation model developed in Ch. 3. I extract both the saturation parameter $s$ and the collection efficiency of the system from this fit. Finally, I use the extracted saturation parameter to retake the spectroscopic data at a known $s$ and compare it to the model. This gives me information about possible sources of heating in the system. The saturation measurement also points to significant loss of signal in the system. This issue is explored at length at the end of this chapter.

5.1 Single-atom spectroscopy

As discussed in Ch. 3, the trapping light shifts the atomic frequency due to the AC-Stark effect and the size of the shift determines the trap depth. It is important to know the depth of the trap so that I know how much heating the atom can experience before it is lost from the trap. In addition, the depth provides information about the shifted resonance frequency of the atom. Knowing the precise resonance of the atom
is necessary for achieving good control over the atom’s quantum state, particularly for fast protocols. The shifts can be probed experimentally and also calculated theoretically.

To determine the frequency shift experimentally, I perform an experiment similar to that reported by Shih and Chapman in 2013 [109] using the p/c sequence described in Sec. 4.4.3. The probe is $\pi$-polarized and drives the $\Delta m_F = 0$ transitions of the $5S_{1/2}F = 2 \rightarrow 5P_{3/2}F' = 3$ line. The frequency of the probe beam is changed in between runs to determine the $5S_{1/2}F = 2 \rightarrow 5P_{3/2}F' = 3$ resonance. Each run consists of 250 attempted loads. The data presented in Fig. 5.1 represents the average counts detected during a p/c cycle for those events in which an atom was loaded into the trap. The error bars are the standard deviation of the events at each frequency. A clear shift is evident, with the atom’s resonance frequency occurring at $\sim+46$ MHz of the bare atomic resonance. There is a $\sim5\%$ uncertainty on this shift due to the uncertainty in the laser lock point and the trap power. This shift is observed for a power of $40 \pm 1$ mW of trapping light entering the chamber. The full-
Table 5.1: Calculated AC-Stark shifts for a trapping beam of 40 mW focused to a 2.54 µm waist.

<table>
<thead>
<tr>
<th>$m_F$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5S_{1/2}F = 2$</td>
<td>-38 MHz</td>
<td>-38 MHz</td>
<td>-38 MHz</td>
<td></td>
</tr>
<tr>
<td>$5P_{3/2}F' = 3$</td>
<td>+29 MHz</td>
<td>+26 MHz</td>
<td>+18 MHz</td>
<td>+4 MHz</td>
</tr>
<tr>
<td>total shift</td>
<td>+67 MHz</td>
<td>+64 MHz</td>
<td>+56 MHz</td>
<td></td>
</tr>
</tbody>
</table>

width-at-half-maximum of this curve is $\sim 13$ MHz, larger than the atomic linewidth of 6MHz. This is due to the AC-Stark shifts of the excited state and is discussed again in Sec. 5.2.1. There is a slight asymmetry apparent in the data of Fig. 5.1. This is discussed in Sec 5.5.

To extract information about the trap depth from this shift, it is compared to a calculation. The shifts are calculated using Eq. 3.5. Taking the expected trap waist of 2.54 µm and the measured trap power of 40 ± 1 mW, the expected ground-state shift is $-34$ MHz for all ground-state magnetic sublevels. The excited-state shifts are non-uniform but symmetric around $m_F = 0$ and the calculated expected shifts are given in Tbl. 5.1. I list the $m_F$ shifts for the $5S_{1/2}F = 2$ ground and $5P_{3/2}F' = 3$ excited states. I also list the total shift of the $\Delta m_F = 0$ transition probed by the $\pi$-polarized probe beam.

The calculated total shifts in Tbl. 5.1 are $\sim 30\%$ larger than the measured shift shown in the Fig. 5.1. This discrepancy cannot be explained by the 5% experimental uncertainty on the measurements of the trapping beam power and probe lock-point. This implies that the beam waist is larger than the ideal 2.54 µm for which the shifts are calculated. The size of the beam waist influences the trap depth the atom experiences and also the ease with which the atom can be imaged. The atom can be present anywhere within the trap, so the imaging system must be able to image the full trap spot-size (twice the beam waist) for good light collection from the atom.
The imaging issues are discussed in Sec. 5.6. The experimental discrepancy in beam waist observed here motivates further investigation to determine the beam waist actually present in the experiment.

5.2 Beam waist estimation and trap depth

Since the asphere that creates the trap is inside the vacuum chamber, the beam waist cannot be measured directly. The beam waist can be estimated, however, from the spectroscopic data of Fig. 5.1. This is done by using the AC-Stark calculation and the rate-equation model to predict the spectroscopic data and comparing the prediction to the measured results. The beam waist can also be estimated using Zemax. Below I describe the results of these two estimates.

5.2.1 Waist estimation and trap depth from spectroscopy and rate-equation model

To model the spectroscopic experiment, I first predict the expected AC-Stark shifts for a given beam waist and insert those shifts into the rate-equation model of Ch. 3. Figure 5.2a depicts the shifted used to model the 2.54 \( \mu \text{m} \) waist. The atomic population is initially distributed evenly between the \( 5S_{1/2} F = 2 \) magnetic sublevels to model the likely population of an atom that has been loaded from the MOT. I then run the rate-equation model for a \( \pi \)-polarized probe beam at a given frequency, changing the frequency between each run to mimic the spectroscopic experiment.

The rate-equation model calculates the atom’s \( m_F \) level populations as a function of time. Since the atom is probed with the p/c sequence that has probe pulses of 5 \( \mu \text{s} \), I consider the atomic population at \( t = 5 \mu \text{s} \). The cool pulses that are interleaved with the probe pulses effectively re-initialize the atomic state into an even mixture of the \( F = 2 \) sublevels before each probe pulse. Thus, taking \( t = 5 \mu \text{s} \) is valid even though the total integrated probe time is longer, often 5 ms. The sum of the total excited-state population at \( t = 5 \mu \text{s} \) is proportional to the scattering rate of
the atom and, hence, to the counts collected during the p/c sequence. By varying the frequency of the probe light in the rate-equation model, I determine how the excited-state population varies with probe frequency. This models the spectroscopic experiment described in Sec. 5.1.

The model can be run for any desired beam waist and beam power. I run it at several different waists for a power of 40 mW, since that power is used experimentally. The results are displayed in Fig. 5.2b. The model is run for a probe with a reduced Rabi frequency of $\tilde{\Omega} = 0.5\gamma$. The right-most curve (in blue) corresponds to a waist of 2.54 $\mu$m and the shifts depicted in Fig. 5.2a. The other curves correspond to waists of 2.75 $\mu$m and 3.0 $\mu$m. As we can see, the measured shift of 46 MHz is recovered for a waist of $\sim$3 $\mu$m, implying that this is a better estimate of the waist that forms the trap. The ground-state shift at this waist is calculated to be $-27.5$ MHz, which corresponds to a peak trap depth of 1.28 mK.

It is worth noting at the peak excited-state population does not occur at the largest $m_F$ shift. For example, the largest total shift given in Tbl. 5.1 is +67 MHz but the peak of the corresponding modeled spectroscopic curve (blue in Fig. 5.2b) occurs at +65 MHz. This is because the atomic response is a mixture of the interactions depicted in 5.2a. I find that as a general guide, the peak of the spectroscopic curve occurs around the $5S_{1/2}F = 2|m_F| = 1 \rightarrow 5P_{3/2}F' = 3|m_F'| = 1$ transition for a $\pi$-polarized probe. The response is weighted towards the $m_F = 0$ and $m_F = \pm 1$ shifts due to the Clebsch-Gordon coefficients of the transitions.

Figure 5.2b also shows that, for larger trap shifts, the spectroscopic curve peak lowers and the curve broadens. This broadening is due to the increasing differential shifts in the excited state affecting the atom’s interaction with the probe beam, as reported experimentally by Shih and Chapman in 2013 [109]. Shih and Chapman measured the shifted resonance of a trapped atom at various trap powers and calculated the AC-Stark shifts expected at those powers. They did not formulate a
Fig. 5.2  Modeled spectroscopic data at various trap waists  a) The rate-equation model the spectroscopic experiment by considering the probe beam of varying frequency interacting with the AC-Stark shifted levels. The level shifts given here are for an ideal trapping waist of 2.54 µm. b) The rate equations calculate the expected excited-state population as a function of probe frequency for different trapping waists. This models the spectroscopic data of Fig. 5.1.
rate-equation model of the experiment like the one I present here.

5.2.2 Waist estimation from Zemax

The rate-equation model estimates that a trapping beam waist of 3 µm matches the experimentally observed spectroscopic data of Fig. 5.1. However, Zemax predicts a focused waist of 2.54 µm for a collimated 3.2 mm 1/e-diameter trapping beam entering the asphere. Either the Zemax prediction is wrong or the assumptions made in generating that prediction are false.

To investigate this issue, I return to the experimental setup of the trapping beam. The trapping beam exits a single mode fiber and is nominally collimated by a lens. Measuring the 1/e beam diameter after the fiber, the diameter is found to be 3.2 mm. The beam diameter is measured with a Thorlabs beam profiler (P/N BP209-VIS). When the beam diameter is measured again closer to the vacuum chamber, the diameter is found to be 3.0 mm, indicating that the beam is converging entering the vacuum chamber. Thus, the assumption that the input beam is collimated is false and the Zemax prediction of 2.54 µm is not relevant to the actual experimental system. When the Zemax simulation is adjusted to account for the measured convergence in the trapping beam, the predicted waist is 2.96 µm, matching the estimation from the rate-equation model.

The beam waist not only governs the trap depth experienced by the atom, it is also related to the two-body decay rate $\beta'$, which is crucial to reaching the collisional-blockade regime for single atom loading. With an experimentally estimated waist of 3 µm, Eq. 2.15 can be used to find $\beta' = 2.2 \text{ s}^{-1}$. This is a smaller value than that calculated in Sec. 4.2.2 for a waist of 2.54 µm. Nevertheless, given the experimentally measured single-body decay rate of $\gamma_{coll} = \gamma_{sbl} = .0156 \text{ s}^{-1}$ from Sec. 4.5, it is clear that $\gamma_{coll}/2 \ll \beta'/4$ in this setup, satisfying the requirement for collisional blockade.

The single-atom spectroscopic measurement presented here gives important in-
formation about the beam waist and trap depth experienced by the atom. The peak trap depth is estimated to be 1.28 mK. Since the atom is trapped from the MOT and there is no sub-Doppler cooling purposefully applied, I assume that the atom begins experiments at the Doppler temperature of 150 µK. This means that the atom can undergo ~ 2800 scattering events without additional cooling before being lost from the trap, as calculated in Sec. 2.5.

To determine the rate at which such scattering becomes a problem, the exact nature of the coupling between the trapped atom and the probe beam needs to be understood. Understanding this is crucial to achieve fast and nondestructive quantum-state readout. The atom-probe coupling information is encoded in the rate-equation model. Thus, the model should be matched precisely to the experimental system. This requires considering the dipole emission pattern of the atom and is covered in the next section.

5.3 Collection of the atom’s dipole emission

As described in Chs. 2 and 3, the atom can be described as a dipole. Thus, it exhibits dipole radiation and radiates π- and σ-polarized light into different emission patterns. This phenomena is well known and the radiation patterns are given by

\[
\frac{dP_\pi}{d\Omega} \propto \frac{3}{8\pi} \sin^2 \theta, \\
\frac{dP_\sigma}{d\Omega} \propto \frac{3}{16\pi} (1 + \cos^2 \theta),
\]

where \( dP_q/d\Omega \) denotes the angular distribution of the power radiated into a given direction for a polarization \( q \) and \( \theta \) is the polar angle between the quantization axis and the direction of emission [46]. The emission patterns are depicted in Fig. 5.3 with the π-polarized emission in blue and the σ-polarized emissions in red.
Fig. 5.3  **Dipole emission patterns**  The emission patterns of a) $\pi$-polarized and b) $\sigma$-polarized light radiated by the atomic dipole. Note that the scales are the same on both plots and the $z$-axis is the quantization axis.

### 5.3.1 Collection efficiency of each polarization

Calculating the collection efficiency of an optical system for a given emitted polarization is a known problem. It has been addressed for trapped-ion systems in the context of designing quantum repeater networks by Luo *et al.* in 2009 [71] and Kim *et al.* in 2011 [57]. Both works used parabolic mirrors as reflection optics to create their high-NA systems. Kim *et al.* also considered coupling into a single mode fiber by calculating the overlap between the collected mode and an ideal Gaussian mode [57].

Here, I use transmission optics and a multimode fiber. Thus, I do not consider the overlap between the emitted radiation and an ideal Gaussian mode, I simply predict the fraction of emission collected by the asphere. To do this, the dipole emission pattern of the asphere is integrated over the full NA of the lens. This calculation yields the collection efficiency $CE$ of the lens for each polarization. This calculation has been done recently for $\sigma$-polarization by Kwon *et al.* [62] and Martinez-Dorantes
Fig. 5.4  **Lens collection of dipole emission.** The asphere optical axis $y$ is perpendicular to the quantization axis $z$.  

a) A polar plot of the $\pi$-polarized (in blue) and $\sigma$-polarized (in red) emission patterns show the fraction emitted into the lens NA in the $yz$ plane.  

b) A 3D-plot of the $\pi$ emission is shown with the relevant spherical coordinates for an arbitrary emission direction.

*et al.* [75]. Neither of these works addresses $\pi$ polarization because both experiments only use $\sigma$-polarized light.

The high-NA lens that collects the atomic emission is oriented with its optical axis perpendicular to the quantization axis. The emission patterns, plotted in Fig. 5.3, do not have equal amplitudes along the optical axis, and the lens collects different fractions of each polarization. This is made clear in Fig. 5.4a, where a polar plot of the emission patterns is given with the asphere position indicated.
In spherical coordinates, the collection efficiency is given by

\[
CE_\pi = \int_{\theta_i}^{\theta_f} \int_{\phi_0}^{\phi_f} \frac{3}{8\pi} \sin^2 \theta \sin \theta d\theta d\phi,
\]

\[
CE_\sigma = \int_{\theta_i}^{\theta_f} \int_{\phi_0}^{\phi_f} \frac{3}{16\pi} \left(1 + \cos^2 \theta \right) \sin \theta d\theta d\phi,
\]

where

\[
\theta_{i(f)} = \frac{\pi}{2} \mp \arcsin (NA),
\]

\[
\phi_0 = \arcsin \left( \sqrt{\frac{NA^2}{\sin^2 \theta} - \frac{1}{\tan^2 \theta}} \right),
\]

and NA is the NA of the asphere [62], [75]. A 3D plot of the \(\pi\)-polarized emission is given in Fig. 5.4b to show the relevant variables. The limits of integration are found by considering a point \(P\), where the radiation emitted along \(\vec{r}\) intersects the edge of the lens. Figure 5.5 depicts the situation. The limits of integration for \(\theta\) are straightforward. The limits for \(\phi\) depend on \(\theta\) are not immediately apparent so I discuss them briefly here.

It is clear from Fig. 5.5 that \(\sin \phi = x'/\rho\) where \(x'\) is the x-coordinate of point\(\ldots\)
$P$ and $\rho$ is the projection of $\vec{r}$ into the $xy$-plane. Considering the face of the lens, depicted in the $xz$-plane on the right of Fig. 5.5, we see that $\rho = D - r^2 = D^2 - z'^2$, where $D$ is the lens radius and $D = r \sin \alpha = r \times NA$. Closer examination of Fig. 5.5 shows that $\rho = r \sin \theta$ and $\rho = r \cos \theta$. These relations can be combined to yield the limits of integration given in Eq. 5.3.

Using $NA = 0.54$ and solving Eq. 5.2 numerically, I find $CE_\pi = 0.11$ and $CE_\sigma = 0.064$. The lens collects slightly more of the $\pi$-polarized emission and slightly less of the $\sigma$-polarized emission than the isotropic collection efficiency of 7.9% calculated in Sec. 4.2.1.

### 5.3.2 Collection efficiency for the light emitted by a given excited-state sublevel

In order to use this $CE$ prediction in the model, the specific transitions being probed must be considered. I probe the atom on the $5S_{1/2} F = 2 \rightarrow 5P_{3/2} F' = 3$ transition using $\pi$-polarized light. The probe beam addresses all of the sublevels of the $5S_{1/2} F = 2$ ground state and excites the atomic population in these levels to the $5P_{3/2} F' = 3$ excited state from which it decays, generating the fluorescence collected by the asphere. Though each excited sublevel decays at the same total rate $\gamma$, most can decay to via multiple paths to different ground-state sublevels and each decay path emits either $\pi$-polarized or $\sigma$-polarized light. The fraction of decays from a particular excited-state sublevel into a particular ground-state sublevel is given by the Clebsch-Gordon coefficient (CG) of that transition. Thus, the fraction of $\pi$- vs. $\sigma$-polarized light emitted by decay from a given excited-state sublevel depends on the CGs for the transitions out of that sublevel. The CGs for the $5P_{3/2} F' = 3$ excited-state sublevels are given in Fig. 5.6, where the $\pi$-polarized transitions are depicted in blue and the $\sigma$-polarized transitions in red. The total CGs out of each excited-state sublevel sum to 1, and the total $\pi$ and total $\sigma$ decay is symmetric around $m_{F'} = 0$.

To estimate the fraction of photons collected from each excited-state sublevel, I
sum the CGs of Fig. 5.6 and multiply by the CEs determined with Eq. 5.2. For example, $5P_{3/2} F = 3$ $m_F = 3$ only emits $\sigma^+$-polarized light, so its scattering is collected with 6.4% efficiency. The $5P_{3/2} F = 3$ $m_F = 2$ state, on the other hand emits $2/3 \sigma^+$ light and $1/3 \pi$ light, so its emission is collected with $100(2/3 \times .064 + 1/3 \times .11) = 7.9\%$ efficiency. The predicted total collection efficiency $CE_{tot}$ for each excited-state sublevel is given in Tbl. 5.2. It’s apparent that the asphere collects the most light from the $m_F = 0$ decay and the least from the $|m_F| = 3$ decays. Thus, the total collected scattering rate $\gamma_{CE}$ from the $5P_{3/2} F = 3$ excited state can be
Table 5.2: $CE_{tot}$ for the emission from each $5P_{3/2} F = 3$ $m_F$ sublevel.

<table>
<thead>
<tr>
<th>$m_F$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CE_{tot}$</td>
<td>9.14%</td>
<td>8.83%</td>
<td>7.91%</td>
<td>6.38%</td>
</tr>
</tbody>
</table>

modeled as

$$\gamma_{CE} = \sum_{k=18}^{24} (CE_{tot})_k n_k \gamma,$$

(5.4)

where $n_k$ is the atomic population defined in Sec. 3.5, $k$ indexes the excited-state sublevels, $(CE_{tot})_k$ is the total collection efficiency for each sublevel, and $\gamma$ is the excited-state decay rate. Using this method, the excited-state populations predicted by the rate equations $n_k$ are translated into the measured scattering rate. This same procedure can be applied to all of the excited-state sublevels if desired. For the experiments presented here the atom is always probed on the $5S_{1/2} F = 2 \rightarrow 5P_{3/2} F = 3$ transition, so only the $CE_{tot}$ values given in Tbl. 5.2 are needed.

5.4 Single-atom saturation

Now that the dipole emission patterns are properly accounted for, the model can be used to predict accurately the collected atomic emission for a given probe field. Comparing such predictions to experimental results accesses the atom-probe coupling information that is needed for fast quantum-state readout. Considering the atom’s saturation under increasing probe beam power yields insight into both the atom-probe coupling and the total photon detection efficiency of the system.

As discussed in Sec. 2.1.2, an atom’s scattering rate saturates as a function of probe intensity with the form

$$\gamma_{sc} = \gamma_{ee} = \frac{s^2/2}{1 + s},$$

(5.5)
where $s$ is the saturation parameter and $\gamma$ the excited-state decay rate. As defined in Eq. 2.7, $s$ is proportional to the Rabi frequency of the transition being considered and there is a different $s$ for each transition in $^{87}$Rb. Since I am interested in describing the behavior of the entire atomic system, I define a reduced saturation parameter $\tilde{s} = s|\tilde{\Omega}|^2/\gamma^2$, where $\tilde{\Omega}$ is the reduced Rabi frequency defined in Sec. 3.6 that does not include the matrix elements but does include the optical field amplitude.

To relate Eq. 5.5 to the rate-equation model, I predict the outcome of an experimentally relevant saturation-type experiment. The model accounts for an ODT with 40 mW of trapping power and a 3 $\mu$m waist. The peak atomic fluorescence occurs at a shift of +46 MHz from the bare resonance, so this is the probe frequency I use in the simulation. The probe beam’s $\tilde{\Omega}^2$ value is varied to model atomic saturation. A summing procedure similar to that used to obtain Eq. 5.4 gives the weighted total excited-state population, which is proportional to the collected fluorescence. Since the atom is probed with a p/c sequence that uses 5 $\mu$s probe pulses, the population values for $t = 5 \mu$s are used. The results are plotted in Fig. 5.7 for $\tilde{\Omega}^2$ values of $0.1\gamma$ to $10\gamma$.

Figure 5.7 clearly shows the excited-state population saturating as probe Rabi frequency increases. This output is fit with

$$\text{weighted excited-state population} = A \frac{\alpha|\tilde{\Omega}|^2/2}{1 + \alpha|\tilde{\Omega}|^2}, \quad (5.6)$$

where $\alpha|\tilde{\Omega}|^2 = \tilde{s}$ is the reduced saturation parameter. The fit is depicted on Fig. 5.7 and yields $A = 0.0886 \pm 0.000225$, and $\alpha = 0.738 \pm 0.00566$, where the error is the standard error. The 95% confidence interval for $A$ is [0.0880, 0.0891] and for $\alpha$ is [0.725, 0.752]; it is not plotted because it is indistinguishable from the fit line. The coefficient of determination of the fit is $R^2 = 0.99999$. The data follows the functional form of the theoretical curve very well.
The output of this fit converts the 2-level atom equation of Eq. 5.5 to a multilevel equation based on the full rate-equation model and collected dipole emission pattern of the system. The parameter $A$ encodes information about the total amount of fluorescence collected by the lens and shows that the lens should collect $\sim 8.86\%$ of the atomic fluorescence. The parameter $\alpha$ give insight into how the multilevel system is driven by a probe beam. The results of the fit can be recast as a multilevel saturation equation of the form

$$multilevel\ saturation = 0.0886\frac{0.738s}{\frac{\gamma}{2}}\frac{s^2}{1 + 0.738s^2},$$

(5.7)

where $s$ is the saturation parameter predicted by the rate equations.

Equation 5.7 can be compared directly to data. To do this, the atom is probed with a beam operating at the shifted atomic resonance ($+46$ MHz from the bare atomic resonance). In each set of runs, several loads are attempted and the atom is probed using the p/c sequence. The probe power is changed between each set of runs. The experimental results are shown in Fig. 5.8. The points are the average

Fig. 5.7  **Rate equation model of atomic saturation.** The rate-equation mode output (blue dots) shows excited-state population saturating with probe Rabi frequency. The results are fit by an equation explained in the text.
counts detected for all of the successful loads at a given probe power and the error bars are the standard deviation of the averaged data.

This data is fit with an equation of the form

\[ \text{counts} = C \times 0.0886 \times \frac{0.738 \times aP/2}{1 + 0.738 \times aP}, \]  

(5.8)

where \( aP = s \) is the saturation parameter that is proportional to the probe power \( P \). The prefactor \( C = \eta \times \gamma \times t \) includes the excited-state decay rate \( \gamma = 2\pi \times 6 \text{ MHz} \), the total integrated probe time \( t = 5 \text{ ms} \), and additional optical losses in the system \( \eta \). These losses are discussed in detail in Sec. 5.6. The data is fit to Eq. 5.8 with the fit weighted by the data error. The results of the fit are plotted on Fig. 5.8 along with the 95% confidence interval. I find that \( a = 5.07 \pm 0.267 \) and \( C = 13400 \pm 370 \) with the standard error. The 95% confidence interval is \([4.42, 5.72]\) for \( a \) and \([12490, 14290]\) for \( C \). The coefficient of determination of the fit is \( R^2 = 0.9997 \).

The fitted value of \( a \) allows me to convert between the power in the probe beam and the saturation parameter of the atom. Thus, I can drive the atom at a known saturation parameter and investigate its behavior. Section 5.5 uses this information...
to further characterize the atom-probe interaction. The value of $C$ allows me to extract information about the optical losses in the system. Taking this value and using $C = \eta \times \gamma \times t$, I find that $\eta = .07$ or only 7% of the light collected by the lens is detected by the SPCM. This value is much lower than expected. I discuss this issue in detail in Sec. 5.6. After much work, explained in Sec. 5.6, I increased the collection efficiency to yield $C = 20500 \pm 350$ which converts to $\eta = 0.108$. This implies that 11% of the light collected by the lens is detected and in total $0.108 \times 0.0889 = 0.0096$, ~1% of the atomic fluorescence is detected. This is an improvement by a factor of 1.6.

The type of saturation experiment described here has been done for trapped ion systems by Noek et al. in 2013 [88]. To the best of my knowledge, it has not been done in neutral atom systems.

5.5 Spectroscopy at known saturation parameter

In the previous section, I describe how a measurement of the atom’s saturation can be fit by the rate-equation model. This fit yields a value for the coupling constant $a$ that defines the relationship between the probe power and the atom’s saturation parameter. With this information in hand, it is possible to drive the atom at a known saturation parameter and compare the outcome to the rate-equation model. This yields insight into non-ideal behavior that occurs in the system and is not captured by the model. Understanding such behavior is important for optimizing the fast state-readout protocol presented in Chs. 6 and 7.

To investigate possible non-ideal behavior, I return to the spectroscopic measurements of Sec. 5.1. Completing a saturation measurement like that of Fig. 5.8, I find the value of $a$. I then use this value of $a$ to determine the probe power that yields $s = 1$ and perform a spectroscopic measurement on the atom at this probe power. The results are shown in Fig. 5.9 with the rate-equation-model prediction.
The vertical error bars on the data are statistical. There is also a 5% error in the horizontal direction due to uncertainty in the laser-lock point. The errors on the model are due to the 10% uncertainty in the value of $a$ extracted from the saturation measurement.

The model and the data agree quite well to within the uncertainty. There is very little broadening in the experimental data compared to the rate equations model, indicating that there are not many broadening mechanisms such as heating occurring at this saturation parameter. There is a little asymmetry in the data at frequencies tuned below the shifted resonance. This asymmetry may be due to minor heating dynamics in the trap and is discussed below. As an aside, I note that when taking the data presented in this section, I do a spectroscopic measurement each day because the value of $a$ can change slightly from day to day as the probe alignment drifts, AOM powers drift, etc. The variations are $\sim 10\%$.

Ultimately, I am interested in achieving fast quantum-state readout of the trapped atom. Thus, I need to probe the atom with relatively high power comparable to
With this in mind, I repeat the above experiment at various probe powers and consider how the AC-Stark shifts changes with power. I do three spectroscopic experiments at probe powers of 50, 200, and 450 $\mu$W. These correspond roughly to $s = 0.26$, $s = 1.5$, and $s = 3.4$. I generally do quantum-state readout with $\sim 200 \mu$W of probe power. The experimental results are plotted in Fig. 5.10a. It is very clear that at high powers, the atomic resonance shifts to a lower frequency. The output of the rate-equation model is plotted with the data in Fig. 5.10b where I have also added the $s = 1$ data of Fig. 5.9. The $s = 1$ data has been shifted by 1 MHz in Fig. 5.10b compared to Fig. 5.9 to account for the fact that this data set was taken at slightly different trap power than the other three data sets. The output of the rate-equation model for $s = 0.26$, $s = 1$, $s = 1.5$, and $s = 3.4$ is plotted along with the data. Each model output is scaled using the collection efficiency extracted from the saturation measurement taken for each data set.

Figure 5.10b shows that the rate-equation model does not predict the resonance shift observed at high probe power. Given the good agreement between the model and the data at lower powers, it is likely that this shift arises from dynamics not captured in the model. This shift may be evidence of heating/cooling dynamics within the trap. Probe light that is on-resonance and tuned above resonance causes radiation-pressure heating of the atom, as discussed in Sec. 2.5. This heating could suppress the level of counts collected because it causes the atom to sample shallower trap depths where it experiences a smaller AC-Stark shift. Thus, the on-resonance or above-resonance light is farther detuned from the shallower resonance, the atomic scattering rate decreases during the probe-pulse, and the atom scatters fewer photons in total. For light tuned below resonance, however, the atom is cooled. It stays near the Doppler-temperature for longer and samples the same trap depth, continuing to scatter photons at a constant rate during the probe pulse. Thus, more total photons are scattered. This asymmetry in behavior shifts the apparent resonance lower. This
Fig. 5.10 *Spectroscopic data varies with probe power.*  

a) The shifted resonance of the atom is probed at various powers. The resonance clearly shifts to the red for higher powers.  
b) The data is matched with the rate-equation model of the experimental parameters. The data taken at 125 µW is shifted to account for the different trap depths in the different experimental runs.

The same argument could explain the line-shape asymmetry seen in the $s = 1$ data of Fig. 5.9. The scattering decreases more quickly above resonance than below resonance.

The spectroscopic data presented here gives valuable insight into possible heating in the experiment. This issue is revisited in Ch. 7. This insight was made possible by the saturation measurement of Sec. 5.4, which yields the parameter $a$ that couples
probe power to saturation parameter; a measurement that is novel for neutral single atom traps. The saturation measurement also indicates that the overall detection efficiency of this system is low and that issue is addressed in the remainder of this chapter.

5.6 Detection efficiency challenges and improvement

The fit to the saturation data of Fig. 5.8 yields $C = 13400$. Since then, the imaging system has been improved to yield $C = 20500$. Dividing this by the $\gamma/2$ and the collection time yields $\eta \sim 11\%$ where $\eta$ is an efficiency that accounts for other optical losses in the system. Given the CE calculated in Sec. 5.3, this yields a total detection efficiency of $\sim 0.96\%$. This is lower than expected and warrants explanation. This section details the investigations into extremely low coupling efficiency seen in Fig. 5.8 that led to the improvement quoted above. I also detail possible avenues for future improvements.

5.6.1 Expected detection efficiency

The number of photons detected by the SPCM is never equal to the number of photons collected by the high-NA lens. The light collected by the lens is directed down an imaging path where it passes through several optical elements each of which causes a small loss to the signal. In addition, the SPCM itself has a imperfect detection efficiency that is characterized and specified by the manufacturer. The losses of the elements in the system, as specified by the manufacturers, are given in Tbl. 5.3; they yield a total expected transmission of $52\%$, $\eta = 0.52$. Taken with the asphere collection efficiency calculated in Sec. 5.3, the expected total detection efficiency is $4.6\%$. This is a best-case-scenario for perfect conditions and does not include any loss from misalignment or imperfect coupling into the fiber. Those are major sources of loss that are discussed in detail below.
As discussed above, the measured collection efficiency in the system is $\sim 1\%$, much lower than the ideal case of 4.6%. This implies that the coupling efficiency into the multimode fiber is only $\sim 20\%$, far from ideal. The coupling cannot be increased drastically through changing the beam pointing and centering into the fiber, despite repeated attempts to do so. This suggests that there are aberrations in the collected light that are preventing high-efficiency fiber coupling. I call the collected light the atom image and describe an investigation into its aberrations below.

### 5.6.2 Experimental evidence of aberrations

Ideally, the aberrated image of the atom would be detected with a camera to provide two-dimensional information of the aberrations. If the camera were also moved through the image focus, then full three-dimensional information could be obtained. Matching this data with a Zemax simulation gives precise information of the aberrations, allowing their correction. This procedure has been used in trapped-ion systems to provide the most precise measurement of atomic position ever [127]. The proce-
Fig. 5.11  **Layout of imaging system** The asphere (AS) collects the atomic fluorescence and directs it through the vacuum viewport (VP) to the dichroic beamsplitter (DB). It then transits two lenses (L1 and L2) which create an intermediate image at an aperture that is used for blocking out scatter. The final lens, L3, focuses it onto the fluorescence onto the tip of a multimode fiber, “imaging” the atom.

dure requires high-fidelity images of the faint aberrated image so that the structures of the different aberrations are clear.

Unfortunately, this procedure is extremely difficult to implement in the system described in this thesis. Since the ODT is much shallower than an ion trap, less light can be scattered from the atom before it is heated out of the trap compared to the case of an ion. Thus, a very powerful camera is needed to detect it; particularly if the image is aberrated. I have an EMCCD camera which can detect the atom but its pixels are 16 µm in size, making it difficult to detect the fine features that are the hallmark of different aberrations. Furthermore, increasing the gain of the EMCCD to detect the weak, aberrated image also increases the noise of the sensor, potentially washing out the fine features that need to be detected. Finally, for this procedure to be accurate, the camera should be placed at the same position as the fiber that connects to the SPCM since that is where the aberrated image is collected. Unfortunately, the EMCCD is large and cannot fit at the fiber location in the current setup. Thus, the EMCCD cannot give accurate images of the aberrations and I develop a procedure to gain some aberration information from detection with the SPCM.

To obtain aberration information using the SPCM, I scan the fiber tip position in the plane transverse to the optical axis. A diagram of the relevant parts of the
imaging path is given in Fig. 5.11. The coordinate system is the same as that used for defining the quantization axis of the polarization. The fiber tip is mounted on an two-axis translation stage, making the scan precise and repeatable. I find the fiber tip location that provides peak signal and call this the $(0, 0)$ location. To scan in the vertical ($z$) direction, I leave the horizontal knob of the two-axis translation stage at the $0$ location and scan the vertical knob in known increments. I run a p/c detection cycle at each scan location to record the fluorescence detected by the SPCM as a function of fiber tip position. This yields a vertical profile of the fluorescence signal. I then return the tip to the $(0, 0)$ position and scan in the horizontal ($x$) direction to build up a horizontal profile.

This procedure provides two line-scans of the beam profile convolved with the aperture of the multimode fiber. I call these signal profiles. For simplicity, I assume that the $(0, 0)$ position, which yields peak fluorescence is at the center of the multimode fiber. Thus, along the axes of the scan, the 100 $\mu$m-diameter fiber can be treated as a rectangular aperture of 100 $\mu$m width. If the imaging system were ideal, the $3 \mu$m waist of the trap would be magnified to a 40 $\mu$m-diameter spot at the fiber tip (the total system magnification is $6.7x$). The expected signal profile from this ideal case is shown in blue in Fig. 5.12. An ideal spot of 40 $\mu$m-diameter is smaller than the radius of the 100 $\mu$m-diameter fiber, yielding a flat-topped signal profile when the fiber position is scanned.

The measured signal profiles are shown in red in Fig. 5.12; the error bars are statistical. It is clear that the measured results do not match the ideal case because the fluorescence does not flatten across the fiber. In the vertical direction (Fig. 5.12a), the signal profile is relatively symmetric around 0 and approximates the signal of an ideal 80 $\mu$m-diameter Gaussian spot (given in green). The horizontal profile, shown in Fig. 5.12b, is must more distorted. The beam is larger and asymmetric, exhibiting a definite tail in one direction. This type of tail is indicative of coma,
**Fig. 5.12** Atom image profiles at fiber tip. The fiber tip is scanned in the plane transverse to the optical axis to recover profiles of the atom image. The profile of the ideal 40 µm-diameter spot is shown in blue.  
a) The vertical profile is relatively symmetric but corresponds to a Gaussian spot of 80 µm in diameter.  
b) The horizontal profile is asymmetric showing a distinct tail. The center of the profile approximates that of the 200 µm-diameter Gaussian spot.
which is caused by tilt and decenter in the imaging path. The central portion of the
beam is relatively well-fit by the signal of an ideal 200 \( \mu \)m-diameter Gaussian beam,
larger than the vertical profile. This could be due to coma in the imaging path but
it could also be due to astigmatism.

The beam profiles shown in Fig. 5.12 indicate that there is defocus, coma, and
possibly astigmatism in the image of the atom. Defocus is relatively easy to correct
experimentally but coma and astigmatism are much more difficult to address. To
determine how each aberration affects the collection efficiency, I use Zemax’s encircled
energy calculation to estimate the fiber-coupling efficiency as the aberrations are
modeled. Figure 5.11 shows the Zemax model of the system. The relevant lenses are
marked. I discuss the results of the Zemax modeling in the next sections.

5.6.3 Defocus

Defocus in the imaging system makes the atom image larger than expected at the
fiber. The final focus at the fiber is most strongly affected by the distance between
the fiber tip and the final lens, L3 in Fig. 5.11, and this lens can be used to correct
it. In Zemax, the optimized imaging system shows that the spot-size at the fiber is
\(~ 40 \) \( \mu \)m in diameter and 98.4% of the energy intercepted by the asphere is contained
within a circle of 50 \( \mu \)m-radius at the fiber tip. This is the encircled energy and
simulates the energy that can be coupled into the 100 \( \mu \)m-diameter fiber. A plot of
the encircled energy for the optimized image is shown in Fig. 5.13 where the fiber
radius of 50 \( \mu \)m is marked by a dashed line. The encircled energy is given in the
red and diffraction-limited case in black. The optimized image is almost diffraction
limited.

If the distance between L3 and the fiber is not ideal, the energy encircled within a
50 \( \mu \)m-radius drops drastically. For ease of language, I will refer to this figure as the
predicted fiber coupling. Fig. 5.14 depicts the encircled energy at the fiber tip as L3
Fig. 5.13  **Zemax calculation of encircled energy at fiber for optimized imaging system.** The encircled energy for the optimized imaging system is shown in red and the diffraction-limited case in black. The fiber diameter is shown with the black dashed line.

Moves closer and farther from the fiber tip. The 0-position on the plot is the location that gives the ideal predicted fiber coupling of Fig. 5.13. Highlighted on the plot are the locations where the image is a 80 µm spot at the fiber tip. If the lens is too close to the fiber by 1.25 mm, the predicted fiber coupling drops to ~20% and the beam profile is ~80 µm in diameter. This matches relatively well with the observations for the vertical signal profile and the total coupling efficiency. However, it does not address the issue of the coma/astigmatism that the horizontal signal profile displays.

5.6.4  **Coma/astigmatism**

TILT/DECENTER OF IMAGING PATH WITH RESPECT TO ASPHERE

Coma and astigmatism are both caused by tilt and decenter in the optical path. These aberrations can be much more difficult to address than defocus. In this setup, it is possible that the entire imaging system outside of the vacuum chamber is tilted or decentered with respect to the asphere inside the vacuum chamber. The imaging path is mounted in a cage system to keep the optics rigid and centered with respect to each other. This limits the tilt and decenter between optical elements in the
Fig. 5.14 **Effect of defocus on encircled energy.** The encircled energy drops drastically as the lens position is tuned along the optical axis, $y$. The ideal lens position is $y = 0$. The circled points are those lens positions which yield a 80 µm-diameter spot at the fiber tip.

imaging path. However, the cage system is free-standing on the table and is not mounted to the vacuum chamber that houses the asphere. Consequently, there is no guarantee that it is not tilted or decentered with respect to the asphere. I model such possible tilt and decenter in Zemax. The optical path outside of the vacuum chamber is moved as a unit to simulate either a decenter or a tilt.

Decenter causes a tail in the image, as seen in the Zemax spot diagram of Fig. 5.15. The spot diagram depicts the ray positions in the plane of the fiber tip. A perfectly focused spot would be circular here. The spot in Fig. 5.15 shows evidence of coma. The Airy disk, which represents a diffraction limited spot, is shown in black.

Though Fig. 5.15 clearly shows coma, the decenter has to be relatively large to greatly affect the encircled energy. A 5 mm decenter for an ideally positioned L3 shows the tail of Fig. 5.15 but still yields 97.7% predicted fiber coupling. If the position of L3 is also wrong, then the predicted fiber coupling drops to 75% when L3 is 1 mm too far from the fiber but remains at 97% when L3 is 1 mm too close to the fiber. Neither of these fiber coupling calculations are low enough to explain
Fig. 5.15  **Spot diagram of an image with 5 mm of decenter in the path.**
The spot diagram of the ray at the fiber tip illustrate the presence of coma when
the imaging path is decentered by 5 mm. The scale on the diagram is 20 mm per
square. The airy disk is depicted in black.

Table 5.4: Encircled energy at fiber tip varies with tilt and defocus. CnC stands for
can not calculate and indicates failure of Zemax to converge to a solution within a
reasonable time.

<table>
<thead>
<tr>
<th>Tilt</th>
<th>0°</th>
<th>0.5°</th>
<th>0.75°</th>
<th>1°</th>
</tr>
</thead>
<tbody>
<tr>
<td>L3 + 1 mm from ideal</td>
<td>.5954</td>
<td>.4986</td>
<td>.1024</td>
<td>CnC</td>
</tr>
<tr>
<td>L3 at ideal position</td>
<td>.9839</td>
<td>.8413</td>
<td>4595</td>
<td>CnC</td>
</tr>
<tr>
<td>L3 - 1 mm from ideal</td>
<td>.5097</td>
<td>.9786</td>
<td>7624</td>
<td>.3266</td>
</tr>
</tbody>
</table>

the results of the saturation measurement. It is unlikely that the imaging path is
decentered by much more than 5 mm or it would have been noticed and corrected
when the path was being built.

Though decenter does not make a large difference to the encircled energy, the
affect of tilt is drastic. Table 5.4 summarizes the encircled energy calculations for
4 values of tilt at three different positions of L3. Clearly, even a tilt of less than 1°
can cause the encircled energy to drop drastically. The asymmetry in the behavior
as the lens is moved towards and away from the fiber could be explained by either
coma or astigmatism. Tilt also causes a tail in the beam profile. Figure 5.16 depicts
Fig. 5.16  **Spot diagram of an image with 0.5° of tilt in the path.** The spot diagram shows the presence of coma when the imaging path is tilted by 0.5°. The scale on the diagram is 40 mm per square. The airy disk is depicted in black.

the spot diagram in the presence of a 0.5° tilt. The scale on this spot diagram is twice that of the diagram in Fig. 5.15.

**TILT/DECENTER OF ASPHERE WITH RESPECT TO IMAGING PATH**

So far, I have addressed the possibility that the imaging system is decentered or tilted with respect to the asphere. There is also the possibility that the asphere is tilted or decentered with respect to the rest of the imaging system. The asphere is mounted inside the vacuum chamber. The lens is mounted inside a custom lens tube and was press-fit by hand into the tube upon mounting. There could be a small tilt to the lens. The lens tube is held in groove grabbers (MCF275-GrvGrb-CYL0750) that are mounted to the vacuum chamber. The groove grabbers are press-fit by hand into grooves in the vacuum chamber so they could be slightly asymmetric in their fitting. The groove grabbers are a split-tube design that is tightened onto the lens tube with screws. The pieces must be assembled inside the vacuum chamber where there is very limited clearance. Consequently, it is hard to get them assembled at all let alone verify that they are properly centered, everything is of equal tension, etc. Thus, there could a small decenter in the mounting system.
If the asphere and the trapped atom are decentered with respect to the rest of the imaging system, the situation is analogous to that described above where the imaging system is decentered with respect to the the asphere. Thus, the simulations described above remain valid. Likewise, if the asphere is tilted with respect to the imaging system. If the asphere is tilted with respect to the vacuum viewport, some distortion can creep into the beam. In this case, Zemax shows that a tilt of 1° yields a fiber coupling of 78% and a tilt of 2° yields a coupling of 57%.

**DECENTER OF ATOM WITH RESPECT TO ASPHERE**

Another and potentially more serious issue is the possibility that the trapped atom is not centered on the asphere. If this is the case, the atom is decentered from the entire optical system. This is possible if the trapping beam going into the asphere is tilted or decentered with respect to the asphere. I model this situation in Zemax and describe it here.

Due to the large NA of the asphere, any decenter of the atom with respect to the asphere is greatly magnified by the imaging system. A 100 µm decenter of the atom leads to a clear tail in the image and 78.8%, a ~60 µm spot at the fiber, and 79% predicted fiber coupling. A 200 µm decenter yields 40% fiber coupling and a >100 µm spot.

Such decenter can be caused by misalignment of the trapping beam into the asphere. The trapping beam could be either decentered or tilted. A several mm decenter of the trapping beam is required to produce a 100 µm decenter at the atom position. This is because the high NA of the lens actually helps in this case by minimizing changes of the input beam location. It is unlikely that the trapping beam is decentered by that much since I verify that it is centered on the vacuum window going into the chamber and coming out of the chamber after the asphere. The alignment procedure is described in App. A. Tilt of the trapping beam has a much larger effect and 0.5° of tilt causes 100 µm of decenter of the trap location.
Tilt is more likely the cause of any decenter of the atom with respect to the asphere.

5.6.5 Experimental improvements

Based on the discussion above, the most likely causes of aberrations in the imaging system are defocus in the imaging path, tilt of the imaging path with respect to the asphere, and tilt of the input trapping beam causing the trap to be decentered on the asphere. There is also the possibility of the mounting of the asphere causing tilt and decenter but that cannot be addressed without a major overhaul of the system. In this section, I discuss the experimental difficulties that currently exist in addressing the above issues and suggestions for future improvements.

DEFOCUS

Since the defocus is mostly governed by the placement of the final lens before the fiber L3, in theory it should be easy to correct. Indeed, L3 is held in a translation stage with longitudinal travel for this purpose. Experimentally, I find that when the lens is moved away from the fiber, the coupling drops. When the lens is moved closer to the fiber, the coupling improves to a certain extent but the translation stage actuator reaches the end of its translation before optimal coupling is reached. I have tuned the foci of the other lenses in the system to allow me to move the final lens out of its extreme position. This helps somewhat but not very much, and I usually find the best coupling when the lens is at the end of its translation. I have seen improvement of collection efficiency using this procedure.

Unfortunately, the translation stage holding the lens must be fixed to the two-axis translation mount in which the fiber sits to enable the fiber to be translated with respect to the lens. This limits the spacing between the lens and the fiber, particularly as there also needs to be a bandpass filter mounted as close to the lens as possible. To address this spacing issue, a new solution for mounting the fiber must be found, perhaps in something custom-machined. However, since this is labor
intensive and would only address the defocus but not the tilt and decenter concerns, I have not addressed it for the experiments in this thesis. It would be a good thing to address in the next-generation of this experiment.

There is some evidence that fixing the defocus would help narrow the signal profiles but would not change the asymmetric nature of the beam. This evidence is given by the EMCCD camera in the setup which is located at a different (but nominally similar) position to the fiber. The EMCCD camera has a translatable lens directly in front of it whose spacing is not limited in the way that the spacing of L3 before the fiber is limited. Thus, the lens before the camera can produce relatively sharp images of the atom. Three such images are shown in Fig. 5.17. The EMCCD camera sits on its side in the setup, so the vertical dimension in these images is the equivalent of the horizontal signal profile taken by the fiber. At the position of tightest focus, the atom image is \( \sim 3 \) pixels wide and \( \sim 6 \) pixels long. Since the pixels are 16 \( \mu \text{m} \) in size, this corresponds to a 48x96 \( \mu \text{m} \) spot. In the narrower dimension, this is close to the ideal 40\( \mu \text{m} \) spot. However, there is still the broader dimension, which supports the evidence of coma or astigmatism seen by the fiber. The aspect ratio of the image does not change as the lens is translated. This implies that the tail is due to coma, not astigmatism.

**COMA/ASTIGMATISM**

As explained above, the observed coma is likely due to tilt and decenter in the imaging path. I have tried to address this using the mirrors in the path but have not seen much change. If the entire imaging path is tilted or decentered with respect to the asphere, the best way to fix the issue is to redesign the imaging system. Since this system was built, Thorlabs has released a new product (P/N VFA275) that is an adapter designed to mount cage-mounted optomechanics to a vacuum viewport. This equipment would provide the structure needed to tie the cage-mounted imaging path to the vacuum chamber. This would ensure that the imaging path was centered and
un-tilted with respect to the chamber. This equipment should definitely be included in the next generation of this experiment.

Without this adapter, or something like it, any change to the orientation of the imaging path with respect to the chamber is a guess and there is no guarantee that it would lessen the aberrations. Furthermore, such a change could easily totally decouple the imaging system from the atom, requiring major work to get the image back. Consequently, I have not made this change in the setup as of writing this thesis.

The Thorlabs adapter mentioned above could also be used to address the possible issue of tilt of the trapping beam with respect to the asphere. The trapping beam path is currently not cage mounted. If it were rebuilt in a cage mount that was fixed to the vacuum chamber, then centering would be assured and tilt would be much easier to manage. There is some work that can be done without this, however. Using the images on the camera, I can tweak the position of the trapping beam mirrors to change its tilt and decenter with respect to the asphere. I have very carefully done this with the aim of tightening the image on the EMCCD. Since the EMCCD pixels are so large, it is difficult to do this with high precision but I have seen some success.
The investigations described here identify several possible causes of aberrations in the image leading to loss of collection efficiency. Several possible avenues for improvement are identified and the steps taken towards improved alignment are described. Future possible avenues of improvement are also discussed. The work detailed here led to the total collection efficiency improving from 0.6% to \( \sim 1\% \), a factor of 1.6 improvement. Though the collection efficiency is still lower than expected, it is sufficient to achieve fast nondestructive state readout, as detailed in the next chapter.

5.7 Summary

In this chapter, I combine the rate-equation model of Ch. 3 and the power of the p/c sequence developed in Ch. 4 to characterize the experimental system. I develop a method for translating the output of the rate-equation model into an experimental prediction that is very power and is used for the remainder of this thesis.

The characterization experiments I perform allow me to extract coupling between the probe beam power and saturation parameter and to absolutely calibrate the overall collection efficiency of the system. I discuss this collection efficiency and suggest avenues for future improvement.
Nondestructive quantum-state readout protocol and initial results

Now that I have described and characterized the experimental system, I describe my work on nondestructive quantum-state readout. I begin this chapter with a general discussion of the readout protocol similar to that given in the introduction. I next discuss sources of error in the protocol and briefly present a mathematical model of the readout protocol [88], [21]. I name this the protocol model to distinguish it from the rate-equation model developed in Ch. 3. I use the protocol model to discuss how experimental changes affect the state readout fidelity.

I then turn to the specific readout protocol and experimental sequence used in this thesis and present results for readout conducted with a probe beam tuned to the frequency that causes peak atomic fluorescence. I finish by describing how the protocol fidelity scales with probe laser frequency and power, which leads to an improvement in the readout fidelity.
Fig. 6.1 *Quantum state readout of $^{87}\text{Rb}$* The atom is probed (blue arrow) on the $5S_{1/2} \rightarrow 5P_{3/2}$ cycling transition. On the left, an atom in the $F = 2$ ground state fluoresces (red arrow) at the rate of $\sim 10^6$ decays/s and appears “bright”. On the right, an atom in the $F = 1$ ground state has a much lower scattering rate ($\sim 1$ decay/s) and appears “dark” in the presence of this probe.

6.1 Nondestructive quantum-state readout protocol

As discussed in Ch. 1, most quantum information and quantum simulation protocols require determining which hyperfine level of the ground state the atom’s valence electron occupies. The qubit is usually mapped to two specific magnetic sublevel of the ground state that are located in different hyperfine levels. Consequently, hyperfine level readout is sufficient for reading out the qubit state. For $^{87}\text{Rb}$ atoms, this readout requires distinguishing between atoms in the $F = 1$ and $F = 2$ hyperfine level of the $5S_{1/2}$ ground state, and is done via fluorescence detection [126] by probing the atom on the $5S_{1/2} \rightarrow 5P_{3/2}$ $F' = 3$ transition.

Any atom that is in the $5S_{1/2} \rightarrow 2$ state when the probe beam is turned on scatters photons and appears “bright” in the presence of the probe. This is depicted in Fig. 6.1 with the blue arrow indicating the probe-beam frequency and the red arrow the atomic fluorescence. A rough estimate of the atom’s scattering rate can
predicted by Eq. 2.8. This two-level equation is an approximation of the multilevel atomic system, so I only use it to get an order-of-magnitude estimation for the rate. This is helpful for a back-of-the-envelope calculation of the difference in scattering rate for an atom in the bright state vs. one in the dark state. For an on-resonance readout pulse of saturation parameter \( s = 1 \), the bright-state atom scatters photons at a rate \( R_0 \) given by

\[
R_0 = \frac{s_0 \times \gamma/2}{1 + s_0} = \frac{\gamma}{4} \approx 10 \times 10^6 \text{ decays/s}. \tag{6.1}
\]

This is indicated approximately on Fig. 6.1.

For an atom in the \( 5S_{1/2} F = 1 \) state, the same probe beam is detuned from the atomic transition by the hyperfine splitting of the ground states \( \Delta_{hf} = 6.8 \text{ GHz} \). The scattering rate of an atom in this dark state in the presence of the probe beam is given by

\[
R_d = \frac{s_0 \times \gamma/2}{1 + s_0 + (2\Delta_{hf}/\gamma)^2} \approx 3 \text{ decays/s}, \tag{6.2}
\]

where the subscript \( d \) indicates that this is the dark state. The difference in scattering rate is roughly six orders of magnitude, as depicted in the figure.

The atom is illuminated by the probe beam for a set “probe time” and the scattered photons are detected. If the number of photons detected equals or exceeds a threshold \( n_{\text{thresh}} \), the atom is declared to be in the bright state. If fewer than \( n_{\text{thresh}} \) photons are detected, the atom is declared to be in the dark state. This scheme is known as threshold detection [83] and is used to determine the atom’s unknown quantum state.

In this thesis, I characterize the accuracy of such a quantum-state readout scheme. To do this, I prepare the atom in a known quantum state, run the readout protocol, and compare the results of the readout to those expected given the prepared state of the atom. This allows me to characterize the errors in the readout scheme and
determine its fidelity. For example, if the atom is prepared in the bright state and fewer than $n_{\text{thresh}}$ photons are detected, it is mis-identified as a dark-state atom. For a set of measurements on atoms prepared in the bright state, the fraction of measurements for which fewer than $n_{\text{thresh}}$ photons are detected is the bright-state error $\epsilon_b$. Likewise, any atom in the dark state for which $n_{\text{thresh}}$ or more photons are detected is mis-identified as bright, and the ensemble of such measurements constitutes the dark-state error $\epsilon_d$. The fidelity of the readout protocol is given by

$$\mathcal{F}_n = 1 - \frac{1}{2}(\epsilon_b + \epsilon_d),$$

where the subscript $n$ denotes the value of $n_{\text{thresh}}$. The physical sources of these errors are describe in Sec. 6.2.

To design an effective, nondestructive protocol, it is necessary to consider the heating effects due to the presence of the probe beam, which induce atom loss. If the probe beam is unidirectional, and on-resonance with or tuned to a higher energy than the atomic transition, it heats the atom via radiation-pressure heating. The atom is assumed to begin at the Doppler temperature of 150 $\mu$K. From the investigations described in Ch. 5, we know that the trap depth is $\sim$1.28 mK. As described in Sec. 2.5, the recoil temperature of a single resonant photon is 360 nK. Converting all temperatures to $\mu$K, the atom can scatter $(1280 - 150)/36 \sim 3100$ photons before its kinetic energy equals the trap depth and it escapes [30]. Thus, in designing a readout protocol, the atom must scatter much fewer than 3100 photons if the protocol is to be non-destructive. I note that this is a conservative estimate that may underestimate the number of photons that can be scattered from the atom. Using counter-propagating beams can limit the heating if the beams are well-aligned and truly balanced. In this case, the heating effects of the beams can counteract each other and heating is suppressed. Such an effect is hard to estimate or quantify, however, as it is highly dependent on the alignment of the experimental system.
Thus, I use the 3100 photons calculated above as a benchmark for this system, even though I probe with counter-propagating beams.

6.2 Sources of error in readout

The fidelity measurement procedure described above is determined by the errors in the readout protocol. These errors manifest in the bright-state readout and the dark-state readout and each has different sources. I discuss them separately here.

6.2.1 Bright-state errors

Bright-state errors occur when an atom that is in the $F = 2$ bright state is mis-identified as dark. In other words, when too few photons are detected during the probe time to classify it as bright. Some of these errors arise from experimental imperfections but some arise from inherent physical processes that cannot be avoided.

Experimental imperfections in the system, such as poor photon-collection efficiency or a probe beam that only couples weakly to the atom can lead to bright-state errors. If the photon collection efficiency is poor, too many photons are lost from the imaging system to allow for a low error rate. Likewise, if the probe beam is very low power or very far off-resonance, then the scattering rate will be low and the atom will not scatter enough photons to keep the errors low. Errors in state preparation also lead to error in the bright-state detection. If the atom is not properly prepared into the bright state, it will fail to scatter photons. Such state-preparation errors are not separated from the state-detection errors in this work.

Let us assume that the atom starts in the bright state and let us neglect experimental imperfections that could limit the photon detection. In this case, the atom fails to scatter enough photons to be detected only if it transitions from the bright state to the dark state during the probe time. Since the atom is detected on the $F = 2 \rightarrow F' = 3$ transition, this change from bright state to dark state can only occur
if the atom undergoes an off-resonant transition, what I call off-resonant pumping or ORP. This type of transition is depicted in Fig. 6.2. This physical process is unavoidable for $\pi$-polarized probe light and leads to bright-state errors.

The atom starts in the $F = 2$ ground state and the probe beam (solid blue arrow) excites it to $F' = 3$ from which it decays back to $F = 2$ (solid red arrow), as desired. Occasionally, however, the atom is excited to the $F' = 2$ or $F' = 1$ states and decays to the $F = 1$ ground state. These situations are depicted by the dashed blue and red arrows and are labeled $\epsilon_b$ signifying a bright-state error. Once in the $F = 1$ ground state, the atom is far-detuned of the probe beam and ceases to scatter photons. Transitions through the $F' = 0$ excited state are not considered because they are forbidden by quantum mechanical selection rules.

To calculate the rate at which these off-resonant transitions occur, I consider an atom in the $F = 2$ ground state with a scattering rate $R_0$ in the presence of a probe beam. The rate at which it is excited to the $F' = 2$ level is given by the same equation as that for $R_d$ except the hyperfine splitting of the ground states is replaced
by the hyperfine splitting of the excited states. The splitting between $F' = 3$ and $F' = 2$ is $\Delta_{3,2} = 266$ MHz. Likewise $\Delta_{3,1} = 423$ MHz. I ignore the AC-Stark shifts for this estimation because they are small compared to the excited state splittings. Using Eq. 6.2 with the appropriate detunings inserted, $R_{2\rightarrow 2} \approx 10^{-4} \times R_0$ and $R_{2\rightarrow 1} \approx 5 \times 10^{-5} \times R_0$, where the subscripts indicate the rate of pumping from $F = 2$ to $F' = 2$ and from $F = 2$ to $F' = 1$, respectively. Once the atom is excited to one of these states, the likelihood that it decays to the $F = 1$ ground state is given by the branching ratio of the excited state decay to $F = 1$ [24]. These numbers are shown as fractions in Fig. 6.2. Using this information, the total rate of off-resonant pumping that can cause bright-start errors is given by

$$R_{b \rightarrow d} = \frac{1}{2} R_{2\rightarrow 2} + \frac{5}{6} R_{2\rightarrow 1},$$

(6.4)

where $R_{b \rightarrow d}$ indicates the rate at which a bright state atom transitions to the dark state. Given the estimates for $R_{2\rightarrow 2}$ and $R_{2\rightarrow 1}$ given above and a rate of $R_0 = \gamma/4 = 9.6 \times 10^6$ decays/s, $R_{b \rightarrow d} \approx 10^{-4} R_0 \approx 960$ decays/s.

### 6.2.2 Dark-state errors

Off-resonant pumping can also lead to an atom in the dark state pumping to the bright state. When this occurs, the dark state atom begins to scatter photons in the presence of the probe and is mis-identified as bright, leading to a dark-state error. The pathways which enable this are depicted in Fig. 6.3.

The rate at which each transition occurs is calculated in the same way above except that the detuning factors now include the ground state hyperfine splitting $\Delta_{hf} = 6.8$ GHz. Using Eq. 6.2, $R_{1\rightarrow 2} \approx 2 \times 10^{-7} R_0$ and $R_{1\rightarrow 1} \approx 2 \times 10^{-7} R_0$, where the subscripts denote excitation from the $F = 1$ ground state to the $F' = 2$ and $F' = 1$ excited states, respectively. The two rates are approximately equal because the 6.8 GHz splitting of the ground state dominates the $\sim 150$ MHz splitting between
Fig. 6.3 Off-resonant pumping leading to dark-state errors The atom starts in the $F = 1$ ground state and does not scatter photons from the probe (solid blue arrow). Occasionally, it off-resonantly pumps through the $F' = 2$ or $F' = 1$ state to decay to the $F = 2$ ground state (dashed blue arrows and think red arrows). These transitions lead to dark-state errors $\epsilon_d$.

the two excited states. Taking into consideration the branching ratios of the excited state decay,

$$R_{d\rightarrow b} = \frac{1}{2} R_{1\rightarrow 2} + \frac{1}{6} R_{1\rightarrow 1},$$

and $R_{d\rightarrow b} = 10^{-7} \times R_0 \approx 2$ decays/s. The ratio of the two off-resonant pumping rates is $R_{b\rightarrow d}/R_{d\rightarrow b} \approx 500$, indicating that a bright state atom is much more likely to pump dark than a dark state atom is to pump bright because the excited state are much closer in energy than the ground states. This is different than the case of ions where $R_{b\rightarrow d}/R_{d\rightarrow b} \approx 16$ for the commonly used ion $^{171}$Yb$^+$ [88], mainly because the excited state hyperfine splitting is 2.1 GHz and the ground state hyperfine splitting is 12.6 GHz.

Given that the rate at which a bright-state atom pumps dark is much larger than that at which a dark-state atom pumps bright, one might assume that the dark-state errors are negligible compared to the bright-state errors, particularly if the detection is accomplished in a short time. Unfortunately, this is not true due to experimental
realities. A dark state error occurs when photons are detected and those photons may not be scattered by the atom. A large source of these counts is stray background light entering the detector. This can be drastically reduced with shielding and other techniques that are discussed in Sec. 7.3. However, an intrinsic source of background counts comes from the detector itself. All photodetectors have a non-zero probability to register a photon dark count caused by non-photon induced electrons triggering the avalanche that registers the count [52]. The total rate at which background counts are detected $R_{bg}$ is the sum of the scattered light and the detector dark counts and for most experimental situations it dominates $R_{d\rightarrow b}$. The measured $R_{bg}$ for typical experimental conditions in this thesis range from $R_{bg} \sim 750$ Hz to $\sim 1$ kHZ, with $\sim 150$ Hz of that arising from detector dark counts. This background-count rate dominates the off-resonant pumping rate of 2 decays/s described above and must be considered when modeling the system. Imperfect dark-state preparation also leads to dark-state errors. These errors are not separated from dark-state detection errors in this work.

6.3 Mathematical model of the readout protocol

In this section, I describe a mathematical model for fluorescence-based state readout using the photon-threshold discrimination method described above [88], [21]. The details of this protocol model are given in App. C. The model predicts the probability for an atom to scatter $n$ photons in time $t$.

The model assumes that the atom starts in initial state $i$ with scattering rate $R_i$ and then transitions (e.g. via ORP) to state $f$, which has scattering rate $R_f$. This transition occurs at the rate $R_{orp}$. We assume that once in state $f$, the atom does not transition back to state $i$. This assumption is justified if we consider short detection times, as is the case in the experiments described here. I note that the subscripts I use are different from those used in Ref. [21].
The total probability of the atom scattering \( n \) photons during time \( t \) from an atom in state \( R_i \) is given by

\[
P_{\text{Tot}}(n; t, R_i, R_{f}, R_{\text{orp}}) = e^{-(R_{\text{orp}} t)} P_{\text{ph}}(n; R_i t) + P_{\text{tr}}(n; t, R_i, R_{f}, R_{\text{orp}}),
\]

where the first term is the probability that atom scatter \( n \) photons in state \( i \) and does not undergo a transition and the second term is the probability of scattering \( n \) photons while undergoing the sort of ORP transition described above. The probabilities \( P_{\text{ph}}(n; R_i t) \) and \( P_{\text{tr}}(n; t, R_i, R_{f}, R_{\text{orp}}) \) are given and explained in App. C.

### 6.3.1 Probability of detecting \( n \) photons in time \( t \)

To convert Eq. 6.6 into a probability of detecting \( n \) photons during time \( t \), we must consider the physical rates present in the experiment. An atom that is initially in the bright state \( F = 2 \) scatters photons at rate \( R_i = R_0 \). The number of photons detected is only a fraction of these photons, determined by the total detection efficiency \( \eta \) of the system. There is also the possibility that the photons detected come from background scatter and detector dark counts at rate \( R_{bg} \). Thus, \( R_i = \eta R_0 + R_{bg} \). The rate at which the atom transitions to the dark state \( F = 1 \) is given by \( R_{\text{orp}} = R_{b\rightarrow d} \) and once in state \( F = 1 \), photons are scattered at rate \( R_f = 10^{-6} \times R_0 \). Again, background scatter must be considered and we see that photons are detected at a rate \( R_f = 10^{-6} \times R_0 + R_{bg} \). Since \( 10^{-6} \times R_0 \approx 4 \text{ decays/s} \) and \( R_{bg} \approx 750 \text{ Hz} \), the background scatter dominates the scattering from the dark state and I find that \( R_f = R_{bg} \). Consequently, the total probability for detecting \( n \) photons in time \( t \) for an atom initially in the bright-state is given by

\[
P_{\text{Tot}; b}(n; t) = e^{-(\eta R_0 + R_{bg} + R_{b\rightarrow d}) t} \left( \frac{\eta R_0 + R_{bg}}{\eta} \right)^n t^n \sum_{k=0}^{n} \frac{(\eta R_0 + R_{b\rightarrow d})^k}{k!} \eta R_0^k
\]

\[
= e^{-(\eta R_0 + R_{bg} + R_{b\rightarrow d}) t} \sum_{k=0}^{n} \frac{(\eta R_0 + R_{b\rightarrow d})^k}{k!} \eta R_0^k
\]

\[
\times \left[ \sum_{k=0}^{n} \frac{(\eta R_0 + R_{b\rightarrow d})^k}{k!} \left( \frac{\eta R_0}{\eta R_0 + R_{b\rightarrow d}} \right)^n \right] + \left( \frac{\eta R_0}{\eta R_0 + R_{b\rightarrow d}} \right)^n e^{-R_{bg} t} \sum_{k=0}^{n} \left( \frac{\eta R_0 + R_{b\rightarrow d}}{\eta R_0} \right)^k \frac{R_{bg} t^k}{k!}
\]

\[
(6.7)
\]
For an atom beginning in the dark state \((F = 1)\) the initial photon collection rate is dominated by the background scatter and \(R_i = R_{bg}\). The atom transitions to the bright state at rate \(R_{orp} = R_{d\rightarrow b}\) and once in the bright state photons are collected with rate \(R_f = \eta R_0 + R_{bg}\). The total probability of obtaining \(n\) photons from a dark-state atom is

\[
P_{Tot:d}(n; t) = e^{-(R_{bg} + R_{d\rightarrow b})t} \left(\frac{R_{bg}t^n}{n!}\right) + \left(\frac{R_{d\rightarrow b} \cdot e^{-R_{bg}t}}{\eta R_0 - R_{d\rightarrow b}}\right)^n \left[ e^{-R_{d\rightarrow b}t} \sum_{k=0}^{n} \frac{(\eta R_0 - R_{d\rightarrow b})^k (R_{bg}t)^k}{k! (\eta R_0)^k} - e^{-R_0t} \sum_{k=0}^{n} \frac{(\eta R_0 - R_{d\rightarrow b})^k (\eta R_0 + R_{bg})^k t^k}{k! (\eta R_0)^k} \right].
\]  

\((6.8)\)

For the threshold detection method, the decision to classify the atom as bright or dark is based on receiving a certain number \(n_{thresh}\) of photons in a given time \(t\). Any bright-state atom for which fewer than \(n_{thresh}\) photons are detected yields an error. Using Eq. 6.7, I calculate the bright state error probability to be

\[
E_b(t) = \sum_{k=0}^{n_{thresh}-1} P_{Tot:b}(k; t).
\]  

\((6.9)\)

An atom that is in the dark state for which \(n_{thresh}\) or more photons are detected constitutes a dark state error. The dark state error probability is given by

\[
E_d(t) = 1 - \sum_{k=0}^{n_{thresh}-1} P_{Tot:d}(k; t).
\]  

\((6.10)\)

These error rates can be combined into a fidelity using Eq. 6.3. These equations are used to fit the experimental data in Sec. 7.1.

6.3.2 Model predictions

The model described here can be used to predict the fidelity of a given readout scheme as a function of probe-pulse duration. Taking estimates of \(\eta R_0 = 50\) kHz,
Fig. 6.4  **Model for quantum-state readout**  a) The model described in the text predicts the fidelity as a function of probe pulse duration for a given set of readout protocol parameters. The peak fidelity occurs for a 2-photon threshold. b) The bright-state and dark-state errors used to calculate $F_2$ are plotted as a function of probe pulse duration.

$R_{b\rightarrow d} = 960 \text{ Hz}, R_{d\rightarrow b} = 2 \text{ Hz},$ and $R_{bg} = 1 \text{ kHz},$ the fidelities predicted by the model are given in Fig. 6.4a. The fidelities based on a 1-photon, 2-photon, and 3-photon threshold are plotted.

To interpret these fidelities, recall that they are calculated by averaging the
bright-state and dark-state error at each time and subtracting the average error from one. Thus, each fidelity curve is calculated from two other curves, the bright-state and dark-state errors.

The bright-state and dark-state errors predicted for detecting with a 2-photon threshold are given in Fig. 6.4b. These are the curves the are combined to generate $\mathcal{F}_2$ in Fig. 6.4. The bright-state errors fall over time, which makes sense. As the atom continues to scatter photons, the likelihood of having detected fewer than two photons (the bright-state error) falls. The speed of the initial drop is primarily governed by the atom’s total scattering rate. At long times, the bright-state error bottoms out. This level is governed by the ratio between $\eta R_0$ and $R_{b\rightarrow d}$. The dark-state errors are dominated by $R_{bg}$ and grow over time.

The fidelity peaks when the bright-state errors (in blue) are no longer dropping more rapidly than the dark-state errors (in black) are increasing. This occurs around the kink in the bright state error curve where the bright-state errors begin flattening out. We see that the peak $\mathcal{F}_2$ value occurs around this point. The 2-photon threshold gives the highest fidelity for these rates and for most of the experiments presented in this thesis. Generally the thresholding on 1 photon (calculating $\mathcal{F}_1$) happens too quickly, before the bright-state errors have dropped enough to raise the fidelity. Likewise, thresholding on three photons takes too long and the dark-state errors erode the fidelity.

6.4 Readout experiment with an on-resonance probe beam

Now that the basic scheme has been described, I turn to the experiment that characterizes the quantum-state readout protocol. The quantum-state readout is performed using a single pulse of the probe beam described in Ch. 4 and depicted in Fig. 6.5a. The quantization axis is taken along the direction of the linear-polarization of the trapping beam, denoted by $z$ in the figure. The probe beam is linearly polarized
along the same axis, so it probes the $\Delta m_F = 0$ transitions of the $F = 2 \rightarrow F' = 3$ transition, as depicted in Fig. 6.5b. The probe beam is retro-reflected to ensure that there are two counter-propagating beams probing the atom. This helps to reduce radiation-pressure heating that can lead to the atom being lost from the trap. The probe beam depicted here is tuned to the frequency that yields the highest scattering rate according to spectroscopic data, like that discussed in Ch. 5. As discussed in Ch. 5, that location is approximately equal to the shifts experienced by the $m_F = \pm 1$ transitions. I note here that, for the sake of brevity, I refer to the frequency at which the trapped atom’s fluorescence peaks as the “shifted resonance” of the atom. All frequency detunings are referenced to this frequency.

6.4.1 State preparation

For the characterization experiment, the atom is first prepared into either the bright state $F = 2$ or the dark state $F = 1$ with the MOT beams. This state-preparation scheme is chosen because it uses laser frequencies already present in the experimental setup. This simplifies the experimental protocol. More complex state-preparation schemes are briefly discussed in Sec. 7.4.

The MOT repump light is used to prepare the atom into the $F = 2$ bright state. As demonstrated in Sec. 3.7, this light transfers any atom that is in the $F = 1$ state to the $F = 2$ state. Figure 6.6 depicts MOT repump light addressing the relevant atomic transition. The repump light is resonant with the $F = 2 \rightarrow F' = 2$ un-shifted atomic resonance. The resonance of the trapped atom is shifted $\sim +40$ MHz due to the AC-Stark shifts, meaning that the MOT repump light is detuned of the atom’s resonance. Even so, I find that 100 $\mu$s of repump light is sufficient to prepare the atom in the $F = 2$ bright state, in keeping with the predictions of the rate-equation model that are depicted in Fig. 3.6. I have run experiments with longer state preparation steps and see no improvement in the resulting detection fidelity.
**Fig. 6.5 Nondestructive state readout scheme**

a) The probe beam enters the chamber perpendicular to the collection axis and is retroreflected. b) The probe is $\pi$ polarized with respect to the $z$ quantization axis and drives the $\Delta m_F = 0$ transitions of the $F = 2 \rightarrow F' = 3$ transition.

Shorter state preparation times, however, degrades the fidelity, suggesting that the shorter pulse did not prepare the atom into the bright state in some of the trials. It is important to note that the fidelity calculated for the state readout protocol does not distinguish between errors in the state readout and errors in state preparation. The two types of errors are not separated in the experiments presented in this thesis.
The atom is prepared in the bright state by the MOT repump light (green arrows).

The dark state is prepared using MOT cooling light, depicted in Fig. 6.7. This state preparation takes longer than the bright-state preparation because the MOT cooling light is tuned to the $F = 2 \rightarrow F' = 3$ transition, which nominally is closed due to quantum mechanical selection rules. In order to transfer an atom from the $F = 2$ ground state to the $F = 1$ ground state, the atom must undergo ORP through the $F' = 2$ or $F' = 1$ excited states [78]. These transitions are discussed in detail in Sec. 6.2. The rate at which ORP occurs depends on the detuning of the cooling light from the cycling transition. In this case, the detuning caused by the AC-Stark shifts helps the state preparation because the MOT light is tuned to a lower energy than the cycling transition, making it closer to the $F = 2 \rightarrow F' = 2$ transition and increasing the rate at which the ORP occurs. I find that 5 ms of MOT cooling light is sufficient for dark-state preparation.

The experimental sequence for quantum-state readout of a bright-state atom is depicted in Fig. 6.8. The experiment begins with loading an atom into the
Dark state preparation

The atom is prepared in the dark state by the MOT cooling light (red arrows).

trap. A probe/cool (p/c) detection sequence is performed to determine if the load is successful. Next, the atom is prepared into the bright state using 100 µs of MOT repump light. Once prepared, a single pulse of probe light illuminates the atom and the photons scattered by the atom are detected. Finally, the p/c sequence is repeated to verify that the atom is not lost from the trap during detection. The experimental sequence for preparing a dark-state atom is identical except that the 100 µs pulse of MOT Repump light is replaced with a 5 ms pulse of MOT cooling light. The state preparation and detection can be repeated several times before the atom is lost.

6.4.2 On-resonance readout

I have investigated several different sets of probe-beam parameters (frequency, power, polarization) to achieve high-fidelity, fast, nondestructive state readout. In this section, I give a set of results for an on-resonance probe pulse. It turns out that such a pulse is not optimal for readout, but I use it as illustrative of the data collected for this readout scheme and the issues which can arise.
The data presented here is collected with a 150 $\mu$W probe beam on-resonance with the AC-Stark shifted atom. This corresponds to a saturation parameter of $s \sim 1$. To predict the effectiveness of readout with this beam, I consider the scattering rate of the atom. The rate at which the multi-level atom scatters photons is determined by the detuning of the probe beam from the various atomic transitions and the intensity of the probe beam, as discussed in Chs. 3 and 5. However, often a rough estimate of the rate is calculated from the two-level scattering Eq. 2.8. Using Eq. 6.1, I calculate $R_0 \approx 9.5 \times 10^6$ decays/s. Given this scattering rate the atom scatters an average of $\sim 1900$ photons during a 200 $\mu$s probe pulse. This is below the 3100-photon heating-limit calculated in Sec. 6.1. This initial data was taken before the collection-efficiency improvements described in Sec. 5.6 and the collection efficiency was 0.6%. Given this, an average of 11 photon counts are expected for a bright-state atom readout with this probe pulse.

For an atom in the dark state, the same probe beam causes a scattering rate
of $R_d \approx 3.4$ decays/s, as calculated with Eq. 6.2. In a probe time of 200 $\mu$s, the dark state atom scatters an average of 0.0007 photons, effectively zero. During this time the SPCM registers more than zero photons, however, due to stray light in the system and dark-counts leading to false detection events. For the data described here, this background count rate is $\sim 1$ kHz. Thus, the dark-state detection is limited by background scatter and in 200 $\mu$s, the detector registers an average of 0.2 photons.

Using these rates and the ORP rates estimated in Sec. 6.2, I use the model of Sec. 6.3 to predict the readout fidelity with this probe pulse. The model predicts that a peak fidelity of $\mathcal{F}_2 = 97.6\%$ will be reached with 150 $\mu$s of probe time.

Figure 6.9 shows the results of the readout protocol with this probe pulse. A set of 616 experiments where the atom is prepared in the bright state is depicted in red. A total of 368 experiments where the atom is prepared in the dark state is depicted in black. Clearly, the bright-state atom scatters photons from the probe, as expected. The average number of photons detected is $5.64 \pm 0.89$, where the error is the standard deviation of the results. A dark-state atom does not scatter photons at the same rate, and the average number of photons detected is $0.2 \pm 0.2$. The agreement between these numbers and the predictions from the two-level scattering rate are discussed at the end of this section.

The discrimination threshold is set at $n_{\text{thresh}} = 2$ photons (shown on the plot). The bright-state error is the sum of the 0 and 1 photon bins of the red histogram. The dark-state error is the sum of all bins with $> 1$ photon of the black histogram. For this data, $\mathcal{F}_2 = 94.0\% \pm 0.8\%$, where the subscript indicates the value of $n_{\text{thresh}}$ and the error is the standard error. This fidelity calculation is not corrected to account for imperfect state preparation. The uncertainty on the fidelity is large because the data set is relatively small.

The fidelity calculation quoted above is for the data collected during the entire 200 $\mu$s probe pulse. However, I can also calculate the fidelity for the data at each
Initial results of quantum-state readout The red histogram shows the readout results for atoms prepared in the bright state. The black histogram is the readout of atoms prepared in the dark state. The readout pulse is 200 µs long. The threshold is chosen at 2 photons.

Time point during the probe pulse, matching the prediction of the model described in Sec. 6.3. The SPCM registers the photon counts and outputs TTL pulses that are counted by the field programmable gate array (FPGA) which runs the control program for the experiment. The FPGA records the time of arrival of the photons, allowing me to calculate the bright-state and dark-state errors and the fidelity for each time point (I bin the data in 1 µs increments). The details of this control program are given in Sec. A.8.

The errors and fidelity calculation at each time point are given in Fig. 6.10. I plot the fidelity calculation for a threshold of $n = 1$, $n = 2$, and $n = 3$ in orange, red, and green, respectively. We see that the peak fidelity is reached with $n = 2$ in 140 µs, indicating that the probe could be shortened without loss of fidelity. This matches the predictions of the model but the overall fidelity is lower than expected.
The peak fidelity occurs at 140 μs because that is the time at which the bright state errors are no longer falling faster than the dark state errors are rising, as shown in Fig. 6.10b. The type of error data depicted in Fig. 6.10b is very useful for gaining insights into how to improve the protocol and for determining the limitations of the system. Those are discussed in Sec. 7.1 and 7.3, respectively.

The fidelity calculated from the data in Fig. 6.9 is lower than the estimate of 97.6% described above, as is the average number of photons scattered from a bright-state atom. That estimate used the two-level atom equations and predicted a scattering rate of 0.25γ for the bright-state atom. Using the rate-equation model developed in Ch. 3, I calculate the total excited state population of the atom in the presence of this probe beam to be 0.15, yielding a scattering rate of 0.15γ and an expected photon collection of 6.84 photons, much closer to what is detected in the experiment.

This significant decrease compared to the two-level prediction is due to the broadening and lowering of the resonance for the trapped atom, depicted in Fig. 5.2. This shows that the two-level model is not sufficient to capture the behavior of the atom in the presence of the AC-Stark shifts. The average number of photons collected in the experiment (5.64) is still lower than the estimation from the rate equations model. Some possible reasons become evident with more data and are discussed in Sec. 7.1.

It is also important to consider the nondestructive nature of this probe scheme by calculating the fraction of atoms retained in the trap after the readout. The data plotted in Fig. 6.9 only contains events where an atom was present in the second p/c sequence. For both histograms, 100 attempted loads were run. For the bright-state preparation, 55 atoms are loaded and 368 experiments performed, an average of 6.7 experiments/atom. Twenty experiments are attempted on each atom and the atom survives all 20 experiments in 4 of the trials. Thus, I calculate that
Fidelity as a function of time for the initial results of quantum-state readout. 

a) $F_1$, $F_2$, and $F_3$, are calculated for each time point and depicted in orange, red, and green, respectively. The peak fidelity is found to be $F_2 = 0.94 \pm 0.008$ at 140 $\mu$s. 

b) The bright state errors (blue) and dark-state errors (black) used to calculate $F_2$ are shown.
368 + 55 – 4 = 419 experiments were attempted on an atom. This is the number of measured experiments plus the number of atom loads (to account for those trials in which an atom was lost) minus the number of times the atom survives all the trials. Therefore, the bright-state atom retention is 368/419 = 0.878 or 87.8%. For the dark-state atoms, 54 atoms are loaded and 616 experiments run, an average of 11.4 experiments per atom. The retention of dark-state atoms is 94.2%. The average retention for all atoms is 91%. This retention calculation is not corrected for atom-loss due to background collisions or trap heating.

The fraction of atoms retained in this experiment is significantly lower than in other experiments, where average retention is > 98% [30], [35], [62], [74]. This poor retention suggests that the on-resonance probe is causing atom loss. The fidelity of the readout protocol presented here is also lower than that reported previously for a similar scheme by Gibbons et al.. They reported a fidelity of 95% in a probe time of 300 µs [35]. The result presented here is faster than that reported by Gibbons et al., mainly due to the fact that this data was taken by probing the atom on-resonance while they probed off-resonance. Since the atom is probed on-resonance, the scattering rate is higher and the bright-state errors fall faster, allowing a faster detection. However, the fidelity is not sufficiently high to be useful.

The fidelity can be improved by decreasing the dark-state errors and the bright-state errors. Decreasing the dark state errors corresponds to narrowing the black histogram in Fig. 6.9. Decreasing the bright-state errors requires moving the red histogram in Fig. 6.9 to higher photon number. This motivates investigating other frequencies and powers for the probe beam to try to increase the scattering rate.

6.5 Readout fidelity scaling with power and frequency

To increase the scattering rate of the atom, an easy step to take is to increase the probe-beam power. I increase the probe beam power for the on-resonance probe
from 150 µW to 220 µW, corresponding to $s \sim 1.5$. This yields histograms like those of Fig. 6.11. The peak $F_2 = 94.3 \pm 0.5\%$ is achieved in 203 µs. Comparing Fig. 6.11 to Fig. 6.9, we see that the bright-state histogram does have a higher average photon count-rate of 8.8 ± 1.8 photons, but it is a broader distribution and has a pile-up of events at lower photon number so the overall fidelity does not improve. This supports the idea of ORP limiting the fidelity. I investigate higher powers as well and do not see great improvement in the fidelity.

In addition to trying a higher-power probe beam, I also investigate how the fidelity changes as the probe beam frequency is changed. The atomic linewidth is 6 MHz and the scattering rate depends on size of the probe detuning compared to the linewidth. The probe frequency is scanned between -8 MHz and +6 MHz detuned of the shifted resonance. The probe pulse power is fixed at 220 µW. The maximum fidelity for each scan is shown in Fig. 6.12 and the time at which the fidelity was reached is noted on the graph. For those frequencies at which multiples scans were run, the average
Fig. 6.12  **Fidelity of quantum-state readout with changing probe frequency.** The probe beam frequency detuning from the shifted resonance is changed between -6 MHz and +6 MHz. The probe pulse duration is 200 µs and the power 220 µW.

time is given with the statistical error. The fidelity peaks at -6 MHz detuned of the shifted resonance frequency. Tuning slightly closer to resonance (-3 MHz) speeds up the probe time a little bit but at the expense of fidelity. I do have one data-point at a lower detuning (-9 MHz) which suggests higher fidelity (98%) is achievable for longer probe times (475 µs). This is suggestive but has not been reproduced more than once, so I do not include it here.

The average retention of the near-resonance data plotted in Fig. 6.12 is shown in Fig. 6.13. For those frequencies with several data points, the retention was averaged for all the data and the error bars are the combined standard errors. As expected, the retention is lowest for the on-resonance probe and increases at larger detunings.

With the probe at the optimal detuning of -6 MHz from the shifted resonance, I scan the probe power. The results are given in Fig. 6.14. The lower power probe
Fig. 6.13  **Average retention of quantum-state readout.** The average retention of the quantum-state readout in Fig. 6.12 is given.

requires a longer probe pulse to reach peak fidelity and that fidelity is still lower than for the shorter, higher intensity probe. This is because as the detection time lengthens, dark-state errors become more problematic and lower the fidelity. Also, at lower probe powers, the bright-state errors decreases more slowly because the scattering rate is lower. At higher probe power, the detection pulse can be shortened by almost a factor of two without loss of fidelity. Greater than a factor of two increase in probe power is required to achieve this because the atom is saturating, as discussed in Sec. 5.4. The fidelity suffers a little, however. The errors bars on this data point are large because fewer experiments are run at this high power. The average retention for this data is relatively constant at ~95%.

The best fidelity occurs at a detuning of -6 MHz, for a probe power of 200 µW and a probe time of 200 µs. The average fidelity of all experiments taken with those parameters (and plotted in Fig. 6.12) is $96.4 \pm 0.3\%$ where the error bar is calculated from the errors of the individual scans. The average time taken to reach this fidelity is $200 \pm 14 \mu s$, with a statistical error bar. Combined histograms of the bright-state and
dark-state data are given in Fig. 6.15. The bright-state histogram consists of 3798 events and the dark-state histogram of 5080 events. Comparing these histograms to those of Fig. 6.11 makes it clear that the bright-state preparation at lower frequency yields a tighter distribution of photons that is better separated from zero than the on-resonance probe beam. Possible explanations for this are investigated in the next chapter.

I note here that detecting with a probe beam that is -6 MHz detuned of the shifted resonance yields results which surpass those reported by Gibbons et al. in Ref. [35]. Gibbons et al. detected at a frequency that was farther detuned from resonance, which could explain why they required a longer probe time.

6.6 Summary

In this chapter I describe the nondestructive quantum-state readout protocol investigated in this thesis. I discuss sources of error in the protocol and a mathematical
model which can predict the protocol’s fidelity [21]. I report initial experimental results using both an on-resonance probe beam and probe beams detuned from atomic resonance. I demonstrate that a detuning of -6 MHz yields optimal fidelity and report results that surpass those of previous experiments which use this detection scheme [35]. I also present data which suggests physical mechanisms in the system, such as ORP, that limit the fidelity. Chapter 7 further investigates this behavior and presents improved quantum-state readout results.

**Fig. 6.15** Quantum state readout at -6 MHz. The probe beam frequency is set at -6 MHz of the unshifted resonance, the probe power at 200 µW and the probe time at 200 µs. The bright-state histogram is given in red and the dark-state histogram in black.
Further investigation and improvement of the quantum-state readout protocol

The previous chapter presented promising results of a quantum-state readout scheme using a probe beam detuned -6 MHz from the location of the trapped atom’s peak fluorescence. These results are a marked improvement over those achieved with a probe beam tuned to the location of the peak fluorescence. They also improve on existing results for $\pi$-polarized quantum-state readout [35]. With the aim of improving the result further, I undertake an investigation into the scattering rate of the atom. This investigation yields insights into the dynamics of the atomic system during quantum-state readout. I reiterate here that the “shifted resonance” of the atom refers to the frequency at which the trapped atom’s fluorescence peaks. All frequency detunings are referenced to this frequency.

I begin this chapter by carefully considering the result of Ch. 6. In doing so, I document that the atom’s scattering rate decays over the course of the probe pulse. This decay is undesirable as it limits the number of photons scattered from the atom and, hence, the quantum-state readout fidelity. I investigate the decay’s dependence
on probe power and frequency and use the protocol model presented in Sec. 6.3 to
fit the experimental data and quantify the relevant rates.

I consider several possible causes for this decay and present experimental evidence
investigating each one. I conclude that the scattering-date decay is likely caused by
a combination of off-resonant pumping (ORP) and heating. This work is relevant to
improving quantum-state readout fidelity and shortening readout time.

After investigating the scattering rate-decay, I make experimental improvements
to the system and present final, improved results for the nondestructive quantum-
state readout protocol. I discuss current limitations of the readout and suggest
methods for future improvement.

I end this chapter with a comparison of the readout protocol presented here,
which uses linearly-polarized readout light, to the protocol most commonly used in
the field, which uses circularly-polarized readout light. I investigate these protocols
experimentally and using the rate-equation model. I argue in favor of performing
readout with linearly-polarized light.

7.1 Evidence of scattering-rate decay during readout

To better understand the results of Ch. 6 and to try to improve them, I consider
more carefully the atom’s scattering rate during the probe pulse. The scattering
rate is investigated experimentally by considering the time of arrival of the photons
scattered by the bright-state atom during readout. The number of photons arriving
in an interval $t$ to $t + \Delta t$ gives a snapshot of the atom’s scattering rate at time $t$.
For this discussion, I consider one of the -6 MHz data sets that contributes to the
bright-state histogram of Fig. 6.15. In Figure. 7.1, I bin and plot the time-of-arrival
of all the photons collected during the probe pulse of the quantum-state readout.
The photons are grouped into 1 $\mu$s bins. This data set is representative of those
taken at this probe frequency and power.
From the data in Fig. 7.1, it is clear that the number of photons collected decreases over the course of the probe pulse. This decrease is discussed quantitatively below and points to a decay in the scattering rate. If this decay were not present, the atom would scatter more total photons, increasing the separation between the bright-state and the dark-state histogram and increasing the readout fidelity. Consequently, a decaying scattering rate lowers the readout fidelity.

To quantify this decay, I fit the data using an exponential fit, shown in Fig. 7.2 and given in Tbl. 7.1. The error bars in the table are the standard error of the fit. In the figure, the data is given in blue, the fit in red, and the 95% confidence bands in orange.

The time-of-arrival data given in Fig. 7.3 and the scattering-rate decay it illustrates has not been shown in previous publications of nondestructive state-readout experiments [30], [35], [62], [74]. After exhaustively searching the literature, I find that scattering rate decay was mentioned once in the thesis of the first author of
Fit to the scattering rate decay. The scattering-rate decay is fit with an exponential curve. Data is in blue, the fit in red, and the 95% confidence bands in orange.

Table 7.1: Fitting the fluorescence decay of Fig. 7.1. The data of Fig. 7.1 is fit with an exponential curve.

<table>
<thead>
<tr>
<th>Fit</th>
<th>A (photons)</th>
<th>b (photons/s)</th>
<th>Fit $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Fit = A e^{bt}$</td>
<td>$32 \pm 1$</td>
<td>$-1.6 \pm 0.2 \times 10^3$</td>
<td>0.9690</td>
</tr>
</tbody>
</table>

Ref. [30], but only to state that they detuned far-enough away from the resonance that the scattering rate was relatively constant [29]. Since this scattering rate decay is detrimental to the detection fidelity, I investigate how it depends on probe power and frequency.

7.1.1 Decay variation with probe power

I begin by investigating how the fluorescence decay varies with power. For this, I use the data sets already reported in Figure 6.14. I consider the time of arrival of the photons in these data sets and fit them to a decaying exponential, using the method
Table 7.2: **Fluorescence decay variation with power.** Several scans are taken with a -6 MHz-detuned probe beam at various powers. The time of arrival of the photons is considered and the fluorescence decay fit to an exponential.

<table>
<thead>
<tr>
<th>Probe power</th>
<th>A (photons)</th>
<th>b (photons/s)</th>
<th>Fit $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 µW</td>
<td>16 ± 1</td>
<td>$-0.15 \pm 0.3 \times 10^4$</td>
<td>0.9210</td>
</tr>
<tr>
<td>100 µW</td>
<td>23 ± 1</td>
<td>$-0.93 \pm 0.08 \times 10^3$</td>
<td>0.9379</td>
</tr>
<tr>
<td>200 µW</td>
<td>32 ± 1</td>
<td>$-1.6 \pm 0.2 \times 10^3$</td>
<td>0.9690</td>
</tr>
<tr>
<td>200 µW</td>
<td>40 ± 1</td>
<td>$-1.3 \pm 0.2 \times 10^3$</td>
<td>0.9687</td>
</tr>
<tr>
<td>200 µW</td>
<td>18 ± 1</td>
<td>$-3.0 \pm 0.4 \times 10^4$</td>
<td>0.9231</td>
</tr>
<tr>
<td>600 µW</td>
<td>64 ± 1</td>
<td>$-3.5 \pm 0.1 \times 10^4$</td>
<td>0.9725</td>
</tr>
</tbody>
</table>

described above. Several data-sets are taken at 200 µW of power, because this power yields the best quantum-state readout fidelity. The output of the fits is given in Tbl. 7.2.

The decay rates $b$ given in Tbl. 7.2 show that as probe power increases, so does the rate at which the fluorescence decays. We know that increasing the probe power increases the scattering rate, so this suggests that the fluorescence decay is correlated with the initial scattering rate.

### 7.1.2 Decay variation with probe frequency

In order to investigate how the scattering rate changes with probe frequency, I take three sets of data on the same day to ensure that the experimental conditions are as similar as possible. I take one set at a detuning of -6 MHz from the shifted resonance, one on-resonance, and one at +6 MHz from the shifted resonance. The time-of-arrival data is plotted in Fig. 7.3 and the scattering-rate decay is clearly evident. The atom’s scattering rate decays in all three data sets. The change is small for the -6 MHz detuned probe, but is pronounced for the on-resonance and +6 MHz probes. I fit the binned data of Fig. 7.3 to a decaying exponential, with the same form as that reported in Tbl. 7.1. The output of the fits is summarized in Tbl.
Table 7.3: **Fitting the fluorescence decay of Fig. 7.3.** The data of Fig. 7.3 is fit to an exponential decay curve described in the text.

<table>
<thead>
<tr>
<th>Probe detuning</th>
<th>A (photons)</th>
<th>b (photons/s)</th>
<th>Fit $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6 MHz</td>
<td>40 ± 1</td>
<td>$-1.3 ± 0.2 \times 10^3$</td>
<td>0.9687</td>
</tr>
<tr>
<td>0 MHz</td>
<td>63 ± 1</td>
<td>$-5.7 ± 0.2 \times 10^3$</td>
<td>0.9646</td>
</tr>
<tr>
<td>+6 MHz</td>
<td>21 ± 1</td>
<td>$-6.5 ± 0.4 \times 10^{-3}$</td>
<td>0.910</td>
</tr>
</tbody>
</table>

7.3 and indicates that the fluorescence decay increases with probe frequency.

7.1.3 **Fitting the data with the state-readout model**

I can also fit the experimental data with the model of the readout protocol described Sec. 6.3 and detailed in App. C. I name this the protocol model, to distinguish it from the rate-equation model. This model enables me to extract the collected scattering rate $\eta R_0$ and the nominal ORP rate $R_{b\rightarrow d}$ for each data set. In order to perform the fits, I calculate the readout fidelity for each of the three data sets presented in Sec. 7.1.2. For this discussion, it is helpful to consider the readout fidelity calculated by thresholding on 1-photon instead of 2. That is, I consider $F_1$ rather than $F_2$. $F_1$ is more sensitive to the atom’s scattering rate than $F_2$, because it is determined by the first photons scattered by the atom. The $F_1$ curves for each frequency are plotted in Fig. 7.4. As expected, the curve for an on-resonance probe beam rises faster than the others, since this probe beam causes the highest scattering rate.

The $\eta R_0$ and $R_{b\rightarrow d}$ rates are extracted by fitting the measured average bright-state-error values that correspond to these fidelity curves. These average values are the fraction of atoms for which fewer than 1 photon has been detected by a given time point. The measured average bright-state errors are given in Fig. 7.5. The errors bars on the plot are the standard error of each average value. The fits are depicted in the figure as black lines. The fit 95% confidence levels are depicted as
Fig. 7.3  **Bright-state atom scattering at different probe detunings.** The time of arrival of the photons scattered by the atom for a 200 µs, 220 µW probe beam that is a) -6 MHz b) 0 MHz and c) +6 MHz detuned from the shifted atomic resonance are given.
Fig. 7.4 $\mathcal{F}_1$ at each time point for different probe beam frequencies. $\mathcal{F}_1$ as plotted for each time point in the 200 $\mu$s probe pulse for three different probe detunings. The on-resonance probe (0 MHz) curve rises fastest, as expected.

Table 7.4: Results of fitting the bright-state errors to the model of App. C. The fits of Fig. 7.5 yield the collected scattering rate $\eta R_0$ and off-resonant pumping rate $R_{b\rightarrow d}$ at each probe frequency. The measured $R_{bg}$ is also included for completeness.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>$R_{bg}$ (kHz)</th>
<th>$\eta R_0$ (kHz)</th>
<th>$R_{b\rightarrow d}$ (kHz)</th>
<th>Fit $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6 MHz</td>
<td>1.05</td>
<td>39.4 ± 0.2</td>
<td>1.31 ± 0.04</td>
<td>0.9997</td>
</tr>
<tr>
<td>0 MHz</td>
<td>1.13</td>
<td>58.7 ± 0.5</td>
<td>4.1 ± 0.1</td>
<td>0.9970</td>
</tr>
<tr>
<td>+6 MHz</td>
<td>1.12</td>
<td>33.6 ± 0.3</td>
<td>3.63 ± 0.1</td>
<td>0.9990</td>
</tr>
</tbody>
</table>

blue, dashed lines. From the fits, I extract the collected scattering rate $\eta R_0$ and $R_{b\rightarrow d}$ for each frequency. The model also contains the background scattering rate $R_{bg}$. I calculate this rate for each data set using the rate of counts collected during those probe pulses where an atom was not trapped. This is the equivalent of the background count rate. I enter this calculated background rate into the model before fitting it to the data. The results of the fitting procedure are summarized in Tbl. 7.4 with the 95% confidence level errors of the fits. The coefficients of determination
Fig. 7.5  $F_1$ **bright-state-error curve at each time point for different probe beam frequencies.** The measured bright-state error is plotted for each time point in the 200 µs probe pulse for three different probe detunings. The error bars are the standard error on each measured value. The data are fit with the readout model, depicted as a black line. The dashed blue lines are the 95% confidence levels of the fits.

for all of these fits are quite high, suggesting that the functional form of the model is a good match for the data.

The highest $\eta R_0$ is found for the on-resonance probe, as expected. The scattering rate decrease for the detuned probe beams but is not symmetric about the resonance frequency, as it would be an untrapped atom. This makes sense when we consider the differential AC-Stark shifts of the excited state, pictured in Fig. 7.6. The peak fluorescence (what I call the shifted resonance) occurs at the frequency of the $m_F = \pm 1 \rightarrow m_{F'} = \pm 1$ transitions. Thus, the probe beam that is detuned -6 MHz from this resonance is tuned to the $m_{F'} = \pm 2$ transitions while the probe that is tuned +6 MHz from the resonance is detuned from all of the transitions. Therefore, it is not surprising that the scattering rate is slightly higher for the -6 MHz probe than it is for the +6 MHz probe. I also note that the fits to the fluorescence decay of this...
Fig. 7.6  \( \pi \)-polarized probe beam. The probe is \( \pi \) polarized with respect to the \( z \) quantization axis and drives the \( \Delta m_F = 0 \) transitions of the \( F = 2 \to F' = 3 \) transition.

Data, given in Tbl. 7.3, do not follow the same trend as these scattering rates. This is contrary to the correlation posited in Sec. 7.1.1. The data of Sec. 7.1.1 tracked a change in scattering rate due to the probe power while the change seen here is due to probe frequency. This suggests that scattering rate decay has a different dependence on probe frequency than it does on probe power.

Compared to the extracted \( \eta R_0 \), the trend in the extracted \( R_{b\to d} \) more closely follows the trend seen in the experiment. The \( R_{b\to d} \) value is smaller for the -6 MHz data than for the other two. However, the readout model uses \( R_{b\to d} \) to describe ORP and the trend observed here does not have the frequency-dependence expected for ORP. I discuss this in the next section.
7.2 Possible explanations for scattering-date decay

7.2.1 Off-resonant pumping

One possible source of the scattering-rate decay is the off-resonant transitions described in Sec. 6.2. In this section, I discuss how these transitions lead to scattering-rate decay and provide experimental evidence that they are occurring. I then discuss the expected frequency dependence for the rate of ORP and compare it to the measured data. This illustrates that ORP is not the only source of this scattering-rate decay.

Off-resonant transitions occur when a bright-state atom transitions through the $F^\prime = 2$ and $F^\prime = 1$ excited state and decays to the dark-state. When this happens, the atom ceases scattering light from the probe. For an ensemble of bright-state atoms, or an ensemble of measurements on one bright-state atom, this translates into a decrease in photon scattering over time. Given that, it seems likely that ORP is contributing to the scattering-rate decay so I investigate this experimentally.

Before describing the experiment that tests for these transitions, I note that ORP transfers atomic population from the $F = 2$ ground state to the $F = 1$ ground state. As they transitions occur, atomic population accrues in the $F = 1$ dark state. The dark state is addressed by the MOT repump light and this light can be used to transfer its population back to the $F = 2$ bright state. Indeed, this is the mechanism I use to prepare the atom into the bright state in the first place.

To test for the presence of off-resonant transitions, I run a readout sequence like that depicted in Fig 7.7a. I prepare the atom into the bright-state, probe the atom and track the scattering-rate decay. After a certain amount of time, I shutter the probe and quickly shine MOT repump light onto the atom. This moves any atomic population that has accrued in the dark state back into the bright state. I then unblock the probe light and monitor the scattering rate. The results are shown in
Fig. 7.7b. The pulses indicated by the arrow are artifacts of turning on the SPCM and should be ignored. The scattering rate increases after this procedure, indicating that population was accruing in the $F = 1$ state due to ORP. However, the scattering rate is not fully recovered, hinting that off-resonant transitions may not be the only mechanism affecting the scattering rate. I also note that this data is taken with a much longer probe pulse than I use during quantum-state readout. I choose this long pulse to exaggerate the effects of ORP.

The rate at which ORP occurs is dependent on the overall scattering rate of the atom and the detuning of the probe beam from the $F' = 2$ and $F' = 1$ transitions. For a higher scattering rate, ORP is more likely to occur. ORP is also more likely to occur for a probe beam tuned closer to the $F' = 2$ and $F' = 1$ transitions, like the -6 MHz detuned probe. Thus, in the data presented in Fig. 7.3, off-resonant transitions are more likely to occur for the -6 MHz detuned probe than for the +6 MHz detuned probe.

Considering the $R_{b\rightarrow d}$ values reported in Tbl. 7.3, we see that $R_{b\rightarrow d}$ is larger for the +6 MHz data than for the -6 MHz data, contrary to our expectation. The disagreement between our physical understanding of the frequency-dependence of the off-resonant transitions and the $R_{b\rightarrow d}$ values extracted from the data using the protocol model motivates me to investigate the validity of the protocol model.

The model was formulated to account for ORP that moves the atom from the bright state to the dark state during the probe pulse. These are captured in the model by the rate $R_{b\rightarrow d}$. As discussed above, such off-resonant transitions lead to a loss of scattering rate. This means that the protocol model is a mathematical description that accounts for a loss of scattering rate. Such scattering-rate decay is seen in the experimental data. The decay process described by the model is assumed to be Poissonian. That may not be true for the unknown process present in the data. Nevertheless, the general idea of using the model, which contains a term that
Evidence of off-resonant pumping transition. a) An experimental sequence is run that uses MOT repump light to transfer atomic population from $F = 1$ to $F = 2$ in the middle of the probe pulse. b) The resulting signal shows that this revives the scattering rate, indicating that there was population accruing in the $F = 1$ state. The counts indicated by the arrow are spurious gate pulses, not true photons counts. They should be ignored.

7.2.2 Magnetic-sublevel dependent scattering

Given that the observed decay in scattering rate is dependent on the probe detuning from the shifted atomic resonance, I hypothesize that it is related to the level structure of the AC-Stark shifted atom. The probe pulse is $\pi$-polarized and drives the $\Delta m_F = 0$ transitions depicted in Fig. 7.8. Since the excited-state AC-Stark shifts
are not uniform in $m_F$ level, the atom experiences different probe-beam detunings as it moves between different $m_F$ ground states. These detunings can be larger than the 6 MHz natural linewidth of the atom, and so they affect the scattering rate. I call this $m$-state pumping.

Figure 7.8 depicts the hypothesized atomic behavior during $m$-state pumping for a probe beam detuned +6 MHz of the shifted resonance. This probe is tuned to a slightly higher frequency than all of the $m_F$ transitions and is farthest detuned from the $m_F = \pm 2$ transitions. Therefore, under this probe beam, I expect the population to accrue in the $m_F = \pm 2$ states. Once in these states, the atom has a lower scattering rate, causing the overall scattering rate to drop. The atomic population is depicted in the figure by gray circles in the $F = 2$ ground state. Darker color indicates higher fractional population.

To try to avoid $m$-state pumping, I develop an experimental scheme to drive the atom with two frequencies of light simultaneously. I probe the atom with light
resonant with the $m_F = 0$ transition and with light resonant with the $m_F = \pm 2$ transition. This is intended to drive all of the transitions together, avoiding m-state pumping and the different scattering rates that could lead to fluorescence rate decay. To do this, I drive the acousto-optic modulator (AOM) that sets the frequency of the probe beam with two different frequencies. This allows me to put frequency sidebands on the probe light at the desired frequencies, which I verify using a beat-note setup to measure the probe-beam frequency.

Figure 7.9 shows the result of this experiment. The data shown here is sparse because it is from an experimental run containing relatively few experiments. The run was repeated and the observation confirmed multiple times. Even though the probe beam contains light resonant with both the $m_F = 0$ and $m_F = \pm 2$ transitions, the data looks very similar to that seen with a one-tone probe. The scattering-rate decay is still evident and it does not appear that the addition of a second frequency has had a profound effect on the data. This is puzzling until we look closely at the atomic population dynamics using the rate-equation model.

I use the rate-equation model to predict the $m_F$ ground-state populations during a 200 $\mu$s probe of various frequencies. I model a single-frequency probe resonant with the $m_F = 0$ transition, a single-frequency probe resonant with the $m_F = \pm 2$ transitions and a two-tone probe that includes light at both frequencies. The results are shown in Fig. 7.10 and the legend given in the first plot is valid for all. I label the states using the $|F, m_F\rangle$ notation.

In addition to plotting the $F = 2$ ground state sublevels, I plot the total $F' = 3$ excited state population in orange. This population governs the atomic scattering rate and, therefore, the experimental data. Considering the excited state population, we see that the total population for the two-tone probe is very nearly the average of the single-tone probe values. This explains why the experimental data of Fig. 7.9 looks so similar to data taken with a single-probe beam and reinforces the conclusion.
that the two-tone probe has little effect on the scattering rate.

There is evidence of \( m \)-state pumping dynamics occurring at short time scales. This is most clearly seen in Fig. 7.10b where it is indicated with an arrow. However, the atomic population reaches a steady-state equilibrium very quickly, within a few spontaneous emission cycles of the excited state. Since the excited state lifetime is \( \sim 27 \) ns, this occurs within \( \sim 1 \) \( \mu \)s and is not visible in the experimental data.

The most interesting aspect of this result is that the ground state population is accruing in \( m_F = 0 \) in all three plots. This is particularly surprising for Fig. 7.10a where the probe beam is resonant with the \( m_F = 0 \) transition. In this case, I would expect the \( m_F = 0 \) sublevel to be cleaned out by the probe and that is not happening. This result becomes explicable when we consider the Clebsch-Gordon coefficients (CG’s) that govern the decay of the \( F' = 3 \) excited state sublevels to the \( F = 2 \) sublevels. They are depicted in Fig. 7.11.

The CG’s govern the rate at which a particular excited state sublevel decays to a
Fig. 7.10  *Rate-equation predictions for two-tone probe.* Predicted sub-level populations of the $F = 2$ ground state and the total $F' = 3$ excited state population for a single frequency probe tuned to the a) $m_F = 0$ transition and b) $m_F = 0$ transition (the arrow indicates $m$-state pumping), and a c) two-tone probe tuned to the $m_F = 2$ and $m_F = 0$ transitions.
Fig. 7.11  **CGs for the** $5P_{3/2}F' = 3 \rightarrow 5S_{1/2}F = 2$ **decays** The Clebsch-Gordon coefficients for the a) $\sigma^-$, b) $\pi$, and c) $\sigma^+$ decays from the excited state $5P_{3/2}F' = 3$ to the ground state $5S_{1/2}F = 2$.

particular ground-state sublevel. For example, the $|3, 0\rangle$ excited state decays to the $|2, 0\rangle$ ground state $3/5$, or 60%, of the time and to each of the $|2, \pm 1\rangle$ ground states 20% of the time. The CG’s out of each level sum to one, as they must. Considering these CG’s, we see that the $F' = 3$ excited state decay is weighted towards the central $m_F$ sublevel of the $F = 2$ ground state. Thus, very little population accrues in the stretched states, even for light tuned to the $m_F = 0$ transition. This explains why the peak of the fluorescence curve is so close to the $m_F = 0$ and $m_F = \pm 1$ shifts. This is a surprising result (though it was hidden in the CG’s the whole time) that has profound implications for the speed at which atomic population can be manipulated.
This is revisited in Sec. 7.4.

7.2.3 Heating during readout

The discussion in Sec. 7.2.1 suggests that off-resonant transitions are not solely responsible for the fluorescence decay seen in Fig. 7.3. The fact that the decay is much worse for light that is tuned to the higher-energy side of the shifted transition (blue-detuned) than for light that is tuned lower than the transition warrants further investigation.

To pursue this, I measure the average number of photons received during a 200 µs readout probe of several frequencies. This is the same experiment as the single-atom spectroscopy measurements of Ch. 5 except that the data is taken with a 200 µs readout pulse instead of with the probe/cool sequence. Thus, the atom is not cooled during this data and population can accrue in the the $F = 1$.

I predict the results of the experiment using the rate-equation model. The model predicts the total excited state population for the duration of the probe pulse. Three of these predictions are shown as the orange curves in Fig. 7.10. I see that the population decreases slightly over time due to off-resonant transitions, which are allowed in the rate-equation model. The predicted decrease is linear for all frequencies of interest. Therefore, at each frequency, I average the predicted population over the course of the probe and convert it into a count rate by multiplying by the scattering rate, the probe time, and the collection efficiency of the system. The result of this process yields a predicted spectroscopic curve for the 200 µs probe pulse.

The experimental data and the rate-equation prediction are plotted in Fig. 7.12, where the data is shown in blue and the rate-equation prediction in orange. The error bars on the data points are the standard error. Comparing the data to the prediction, illustrates that the experimental scattering rate does not follow the rate-equation model as probe frequency changes. The apparent peak of the data is shifted
lower than expected. This is similar to the behavior seen in Ch. 5 for a high-power probe beam used during the probe/cool sequence. In the discussion of that data, I hypothesized that heating was causing the shift to the fluorescence peak. Since there is no MOT cooling light on the atom during the probe readout, I consider heating again here.

Radiation pressure heating is a known effect that displays asymmetric behavior in probe-detuning. It only occurs for light that is on-resonance with or blue-detuned of the atomic resonance. This heating mechanism is known to affect the atomic resonance through Doppler broadening of the resonance line [78]. Atoms at increased temperature move faster, leading to a larger Doppler shift in the face of resonant light, broadening their resonance line.

To understand the significance of resonance-line broadening, I note that atomic resonance lines have a Lorentzian shape with a non-zero width. For a back-of-the-envelope calculation, I use the fact that the ratio of the peak heights of a broadened and an un-broadened Lorentzian is roughly inversely proportional to the ratio of
their widths. That is, broadening the width by a factor of 2 changes the height by a factor of $1/2$.

The natural linewidth of $^{87}$Rb is 6 MHz and for a perfectly cold, un-perturbed atom, the predicted spectroscopic curve has a 6 MHz width. For the rate-equation prediction shown in Fig. 7.12, this is not the case. The shape of the predicted fluorescence curve is not distorted, but it has a full width at half max (FWHM) of $\sim$12 MHz, larger than the 6 MHz natural linewidth of an untrapped atom. This predicted larger width is due to the AC-Stark shifts lifting the degeneracy of the excited states, as discussed in Ch. 5.

The data taken with the probe beam is not Lorentzian in shape. The distortion is explained by the frequency-dependent scattering-rate decay seen in the time-of-arrival data. The scattering rate decay suppresses the expected fluorescence for frequencies near-resonance. The suppression is strongest for frequencies on-resonance (45 MHz on the plot) and above resonance (>45 MHz). Here, I investigate the likelihood that heating is Doppler-broadening the atom. This broadening is in addition to that caused by the AC-Stark shifts and predicted by the rate-equation model.

I begin by estimating the amount of additional resonance-line broadening required to account for the suppression seen in the data. I take the value predicted by the rate-equations $C_{RE}$ as the expected value for the atom. I then calculate the ratio of the counts measured in the experiment $C_{data}$ to $C_{RE}$ to find the suppression. One over this ratio yields the broadening required to generate this suppression. Table 7.5 shows the results of this calculation for the near-resonant frequencies. At the most-suppressed point, the atom’s resonance is supposed by almost a factor of $1/2$. Thus, the atomic resonance would need to be a broadened by a factor of two to account for the observed suppression.

A quick calculation demonstrates that Doppler-broadening is not a large-enough effect to account for the observed suppression. The trapped atom is assumed to be
Table 7.5: Broadening required to account for observed fluorescence rate suppression

The ratio between the data and rate-equation mode prediction of Fig. 7.12

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>$C_{\text{data}}$</th>
<th>$C_{\text{RE}}$</th>
<th>$C_{\text{data}}/C_{\text{RE}}$</th>
<th>$C_{\text{RE}}/C_{\text{data}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>7.3</td>
<td>7.5</td>
<td>1.03</td>
<td>0.97</td>
</tr>
<tr>
<td>43</td>
<td>9.8</td>
<td>7.0</td>
<td>0.71</td>
<td>1.4</td>
</tr>
<tr>
<td>45</td>
<td>11.5</td>
<td>6.7</td>
<td>0.58</td>
<td>1.7</td>
</tr>
<tr>
<td>47</td>
<td>10.8</td>
<td>6.1</td>
<td>0.56</td>
<td>1.8</td>
</tr>
<tr>
<td>49</td>
<td>8.4</td>
<td>4.5</td>
<td>0.54</td>
<td>1.9</td>
</tr>
<tr>
<td>51</td>
<td>6.0</td>
<td>3.9</td>
<td>0.65</td>
<td>1.5</td>
</tr>
<tr>
<td>53</td>
<td>4.2</td>
<td>3.6</td>
<td>0.85</td>
<td>1.2</td>
</tr>
<tr>
<td>55</td>
<td>3.0</td>
<td>3.0</td>
<td>0.99</td>
<td>1.0</td>
</tr>
</tbody>
</table>

at the Doppler temperature at the beginning of the probe pulse. This temperature is $T = 150 \ \mu K$ for an $^{87}$Rb atom. As discussed in Sec. 2.5, each scattered photon imparts $\sim 360$ nK of energy (in temperature units) to the atom. For an on-resonance probe pulse of $s = 1.5$, the atom’s collected scattering rate is $\sim 60$ kHz, as described in Tbl. 7.4. Given the $\sim 0.6\%$ collection efficiency of the system, this corresponds to a total scattering rate of $\sim 10^7$ decays/s. During the 200 $\mu s$ probe pulse, the atom scatters $\sim 2000$ photons, causing a temperature increase of 720 $\mu K$. This makes the atom’s final temperature 0.87 mK. This temperature can be converted into a frequency shift using the equation

$$\omega_D = kv_D = \frac{2\pi}{\lambda} \sqrt{\frac{k_B T_D}{m}},$$

(7.1)

where $k_B$ is Boltzmann's constant, $T$ is temperature, $m$ is the atomic mass, and $\lambda$ is the wavelength of the scattered photon [78]. A temperature of 0.87 mK yields a frequency shift of $\omega_D = 2.3 \times 10^6$ rad/s, corresponding to a resonance line broadening of $\sim 4.6$ MHz. This is smaller than the atom’s natural linewidth of 6 MHz and the trapped atom’s resonance FWHM of 12 MHz. This broadening is not large enough to explain the suppressed scattering rate seen in the experiment.
Though the fluorescence decay is not due to Doppler broadening of the atomic resonance line, it could still be related to heating. As the atom is heated, it moves about in the trap. The interplay between the atom’s motion in the trap and heating was recently investigated by Martinez-Dorantes et al. in 2018 [75]. Martinez-Dorantes et al. identify two distinct heating mechanisms. One is radiation-pressure heating, which I describe above. The other they call dipole force fluctuations (DFF) and is the main focus of their work.

DFF heating is dependent on the interaction between the atom’s motion in the trap and the energy levels of its internal quantum states. These are coupled by the nature of the trapping potential. Recall that the trapping potential is created by AC-Stark shifts induced by a tightly-focused Gaussian trapping beam. For any given atomic level, the magnitude of the AC-Stark shift is proportional to the intensity of the trapping beam. Thus, the AC-Stark shifts vary in space, following the shape of the Gaussian beam. Figure 7.13 depicts the situation. For simplicity, I only depict the shifts for a single magnetic sublevel of the ground state and a single sublevel of the excited state. The ground state sublevels shift uniformly, so the trapping potential is the same for each of them. The excited state shifts are $m_F$ sublevel dependent, so in reality the atom experiences a slightly different potential for each of the excited state sublevels. For the 40 mW, 3 $\mu$m waist trapping beam used in this thesis, an atom at the center of the trap experiences a total AC-Stark shift of 47 MHz while an atom 2 $\mu$m from center only experiences a 16 MHz shift.

Note that the ground-state atom sits in a potential well. It is attracted to the location of highest intensity as this corresponds to the largest negative shift to the ground state. The excited state, on the other hand, is shifted to above the atomic resonance. This corresponds to a repulsive potential. Therefore, when the atom is in the excited state, it is briefly repelled from the center of the trap and pushed towards the trap edges. As Martinez-Dorantes et al. note, this leads to heating [75].
Fig. 7.13  Spatia70-dependent AC-Stark shifts. The AC-Stark shifts felt by the atom depend on its location in the trap. For example, for the 3 \, \mu m waist trapped used in this thesis, an atom at the center of the trap feels a 47 MHz total shift and an atom 2 \, \mu m from center only feels a 16 MHz shift.

Martinez-Dorantes et al. demonstrate that the absorption and emission of weakly-resonant light corresponds to jumps between different locations in the trap and therefore different trap potentials. They simulate this process for weak, near-resonant light with a toy model and show that it causes an exponential energy gain during fluorescence scattering [75]. Their investigation was aimed at identifying different mechanisms of atom-loss from the trap and did not address the scattering rate of the atom. Thus, they advocate for detecting with light that is far-detuned from the
atomic resonance to avoid this heating and the resultant atom-loss. They suggest tuning to above the atomic resonance to avoid the off-resonant transitions which cause population loss. Since I aim to readout the atom’s quantum state quickly, detuning far from resonance hinders me.

Here, I provide a general discussion arguing that the heating described in Ref. [75] could explain the suppressed scattering rate observed in the experiment. The toy model developed by Martinez-Dorantes et al. for weakly-resonant light predicts an exponential gain in energy of the atom due to DFF heating [75]. This gain in energy raises the average temperature of the atom, and therefore increase it’s average velocity. As the atom moves more quickly in the trap, it samples a larger range of trap locations and, thus, a larger range of AC-Stark shifts. Away from the center of the trap, the atom experiences a smaller AC-Stark shift than it does at the center. Consequently, a probe beam tuned to the frequency of the atom at the center of the trap, is detuned from an atom elsewhere in the trap. This spatially-dependent detuning suppresses the scattering rate of atoms that are not located at the center of the trap. Consequently, it preferentially suppresses the scattering rate of hot atoms.

To roughly estimate the size of this effect, I return to the suppression results presented in Tbl. 7.5. For near-resonant light, the scattering rate is suppressed to ~60% of the “peak value” predicted by the rate-equation model. To consider how this peak value may change with probe detuning, I look again at the rate-equation model prediction. The model predicts the scattering rate for an atom in the presence of a given set of AC-Stark shifts. It shows that the scattering rate is highest when the probe beam is tuned on-resonance with the shifted transition, and the scattering rate drops as the probe is detuned. Looking at Fig. 7.12, I see that a 5 MHz detuning from the frequency of peak fluorescence yields a scattering rate of 60% of its peak value.

The 5 MHz detuning noted above corresponds to a probe beam that is a 10%
detuned from the peak shift of 45 MHz. A 10% change in the trap depth experienced by the atom will have the same effect. The curvature of the Gaussian beam-trap is steep and a small change in location can easily correspond to a 10% change in trap depth. For the 3 µm waist trap used in this thesis, the trap depth drops by 10% of its peak value at 0.6 µm from the trap center. For the atom to move 0.6 µm over the course of the 200 µs readout time, it must be traveling 0.3 cm/s. This is well below the ~12 cm/s speed of an atom at the Doppler temperature.

Of course, the atom does not travel in a straight line for the full 200 µs. It is constantly buffeted by photons from the probe beam and is being pulled back to the center of the trap by the attractive dipole force. Nevertheless, the atom does not have to be very far from the center of the trap for its scattering rate to be suppressed. As shown Fig. 7.13, if the atom is 2 µm from the center of the trap, the AC-Stark shift is only ~30% of the peak shift. Consequently, an atom that spends 50% of its time at 2 µm, will experience a 60% suppression in scattering rate.

Furthermore, the excited state repulsion preferentially pushes the atom towards the wings of the trap where it experiences a smaller shift and is more detuned from the probe. Given that the DFF mechanism investigated by Ref. [75] predicts an exponential increase in energy and, hence, a vast increase in velocity, it is not unreasonable to think that as the atom heats, its scattering rate will lower.

7.3 Improved results and current limitations

Given the discussion of Sec. 7.1, it seems that the scattering-rate decay is due to a combination of ORP and heating from the probe beam. Both can be ameliorated by moving off-resonance but that lowers the overall scattering rate, leading to a longer detection time. For fast detection, detecting closer to resonance is necessary, suggesting that this scattering-rate decay is unavoidable. However, it is possible to improve the fidelity by decreasing the background scatter and increasing the collection effi-
ciency of the system. This decreases $R_{bg}$ and increases $\eta R_0$ without affecting $R_{d\rightarrow d}$, as described in Sec. 6.3.

With additional shielding of the imaging path and putting apertures in the probe beam path to block stray reflections, I decrease $R_{bg}$ from $\sim 1$ kHz to $\sim 700$ Hz. I expect this to improve the fidelity by lowering the dark-state errors. Taking the rates of the first row of Tbl 7.3, the protocol model predicts that this change in $R_{bg}$ will decrease the dark-state error from 1.78% to 1%, yielding an increase in fidelity of 0.39%.

Going through the procedure described in Sec. 5.6, I increase the total collection efficiency of the system from $\sim 0.6\%$ to $\sim 1\%$. This increases the collected scattering rate $\eta R_0$. Considering the size of this increase, the model predicts that the bright-state error will decrease from 5.9% to 3.6%. Improving the fidelity by $\sim 1\%$. Furthermore, the bright-state errors will drop faster for higher collection efficiency, decreasing the time required to achieve the highest fidelity readout.

I also increase the retention by using only 500 repetitions of the probe and cool pulses in the p/c detection sequence rather than 1000 repetitions. This decreases the likelihood that the p/c detection itself causes the atom to be lost from the trap, as discussed in Sec. 4.4.3. In addition, I insert a cooling pulse of MOT light after the quantum-state readout step [62]. I find that 10 ms of cooling light greatly decreases the likelihood that the atom is lost from the trap during the p/c sequence after quantum-state readout. The adjusted experimental sequence is given in Fig. 7.14.

Using these experimental improvements, I repeat a state-readout experiment with a 220 $\mu$W probe beam detuned -6 MHz from the shifted atomic resonance. The results of one such run are given in Fig. 7.15. The peak fidelity achieved in this data is $\mathcal{F}_2 = 97.4 \pm 0.3\%$ and is reached after 144 $\mu$s of probe pulse. This follows the improvements predicted by the protocol model. The bright-state atoms survive for an average of 11.8 experiments/atom, leading to a bright-state retention of 95.5$\pm 0.3\%$. 

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Quantum state readout experimental sequence with cooling pulse. Not to scale. The atom is loaded into the trap and detected using the p/c detection scheme. The atom is then prepared into the bright state with 100 µs of MOT repump light before being probed with the probe beam for 200 µs. A 10 ms cooling pulse of MOT light cools the atom after the state readout. Finally, the p/c detection is repeated to ensure that the atom has not been lost from the trap.

The dark-state atoms survive for an average of 17.6 experiments/atom, corresponding to a 98.7 ± 0.3% retention rate. Thus, the average retention rate for this data is 97.1 ± 0.2%, a great improvement over that reported in Ch. 6. Running a similar sequence with a 1 ms cooling pulse after the state-readout yields an average retention of 95.7%, indicating that the longer cooling pulse improves the retention.

Figure 7.15 depicts $F_2$ and $F_1$ for each time point of the probe pulse duration. $F_1$ reaches a fidelity of 95.1 ± 0.4% in 80 µs, indicating that a slight compromise on fidelity can significantly increase the speed of the readout. Fitting the $F_1$ bright-state errors with the protocol model, the extracted rates are $\eta R_0 = 52.7 ± 0.2$ kHz and $R_{b→d} = 940 ± 30$ Hz and the coefficient of determination of the fit is $R^2 = .9993$. The $R_{bg}$ measured during this data is 700 Hz. The increase in $\eta R_0$ compared to the results of Tbl. 7.4 is due to the increase in collection efficiency. The extracted $R_{b→d}$ value is a little smaller than that of Tbl. 7.4 but not drastically so.

The errors for $F_1$ and $F_2$ give insight into what is currently limiting the readout. The bright-state and dark-state errors for thresholding on both 1 and 2 photons are
Fig. 7.15  **Improved quantum-state readout at -6 MHz.** The fidelity for the improved quantum-state readout is plotted for each time point of the probe pulse. $\mathcal{F}_1$ peaks at $95.1 \pm 0.4\%$ and $\mathcal{F}_2$ at $97.4 \pm 0.3\%$.

shown in Fig. 7.16. From these curves, we see that $\mathcal{F}_1$ is dominated by the dark-state errors that quickly rise to overtake the bright-state errors. $\mathcal{F}_2$ is limited by speed of the fall of the bright-state errors and the level at which they flatten out.

The protocol model predicts that a factor of two increase in $\eta R_0$ for the same $R_{bg}$ and $R_{b\rightarrow d}$ would yield a peak fidelity of 98.8\% in $\sim 75 \ \mu s$ of detection time, a significant improvement. As discussed in Sec. 5.6, this is nontrivial to achieve and would constitute a natural continuation of the work in this thesis. If the collection efficiency were improved by a factor of two and $R_{bg}$ were decreased to $\sim 200 \ \text{Hz}$ (near the level of the dark-counts of the detector), then the model predicts a peak fidelity of 99.5\%. This would rival the highest-reported fidelity for neutral atoms, which was achieved with a destructive readout protocol [77]. $R_{bg}$ can be decreased by focusing the probe beam that performs the quantum-state readout. This allows a lower power probe to be used for readout [30], [35], [62]. Aligning a focused probe to the single atom is difficult however, particularly if counter-propagating beams are used. The
Fig. 7.16  Errors of improved quantum-state readout  The errors of a typical improved readout-sequence are shown for thresholding on a) 1 photon and b) 2 photons. The $\mathcal{F}_1$ calculation based on plot a is limited by the dark-state errors. The $\mathcal{F}_2$ calculation based on plot b is limited by the bright-state errors.
Fig. 7.17  **Improved quantum-state readout at -6 MHz.** The probe beam frequency is set at -6 MHz of the unshifted resonance, the probe power at 200 $\mu$W and the probe time at 200 $\mu$s. The bright-state histogram is given in red and the dark-state histogram in black.

The experiment becomes very sensitive to probe-beam alignment.

I repeat the experiment several times and the combined histograms are plotted in Fig. 7.17. The bright-state histogram contains 3583 total experiments and the dark-state histogram 3550 total experiments. This is more experiments than reported in Refs. [35] and [62]. The average peak fidelity is $F_2 = 97.6 \pm 0.2\%$ where the error bar is a combined standard error from the individual runs. This fidelity is reached in 160 $\pm$ 20 $\mu$s, where the error bar is statistical. Looking for a shorter detection time, the average $F_1 = 95.0 \pm 0.3\%$ is reached in $84 \pm 6 \mu$s. Comparing Fig. 7.17 to Fig. 6.15, we see that the bright-state histogram has a higher average, as expected for the improved collection efficiency. For the runs with a 10 ms probe pulse, the average retention is 97.1 $\pm$ 0.1\%.

The data presented here represents an improvement over the previous study of nondestructive quantum-state readout using $\pi$-polarized light [35]. That work re-
ported 95% readout fidelity achieved in 300 $\mu$s of readout time. I report 97.6 $\pm$ 0.2% fidelity in 160 $\mu$s of readout time. Furthermore, > 95% fidelity can be achieved in as a little as 80 $\mu$s of readout time by thresholding on 1 photon instead of 2. Other studies of quantum-state readout have used $\sigma$-polarized readout light and report readout fidelities of > 98% achieved in 1.5 to 10ms of readout time [30], [62], [74]. In the next section, I compare the $\pi$-polarized protocol used here and in Ref. [35] to the $\sigma$-polarized protocols reported in Refs. [30], [62], [74].

7.4 Comparing $\pi$ to $\sigma$ readout

The quantum-state readout protocol I report here uses $\pi$-polarized readout light to achieve a readout fidelity of >97% in <200 $\mu$s. Most other groups use $\sigma$-polarized light for quantum state readout [30],[62],[74]. Therefore, I consider the advantages and disadvantages of $\sigma$-polarized readout compared to the protocol presented here. I discuss driving with $\sigma^+$-polarized light here, though the arguments are identical for $\sigma^-$ light.

7.4.1 The promise of $\sigma$-polarized readout light

I first consider the advantages that driving the atom with $\sigma$-polarized light offers over driving with $\pi$-polarized light. The main advantage of $\sigma^+$-polarized light (of the appropriate frequency) drives the $F = 2, m_F = 2$ ground state to $F' = 3, m_{F'} = 3$ excited state transition, depicted in Fig. 7.18. These states are known as the stretched states of the atom and represent a true cycling transition. Any atom in the excited state $F' = 3, m_{F'} = 3$ is forbidden by quantum mechanical selection rules from decaying to any state other than the $F = 2, m_F = 2$ state.

One advantage of driving the atom on this transition is that it represents a true two-level system. Thus, all of the simple two-level equations of Ch. 2 apply. As I have demonstrated in this thesis, the two-level atom equations are not sufficient
Fig. 7.18  **Improved quantum-state readout at -6 MHz.** The probe beam frequency is set at -6 MHz of the unshifted resonance, the probe power at 200 µW and the probe time at 200 µs. The bright-state histogram is given in red and the dark-state histogram in black.

to describe the atomic system when driving with π-polarized light. This is mainly because of the magnetic-sublevel-dependent AC-Stark shifts in the excited state. Driving the atom on the cycling transition allows one to (mostly) ignore these shifts since the atom is only ever in one of two levels.

More importantly, for a truly closed cycling transition, ORP to the $F = 1$ ground state is forbidden by quantum mechanical selection rules. This lack of off-resonant transitions is a significant advantage for σ-polarized readout, as it greatly decreases the bright-state error. To illustrate, I use the model of Sec. 6.3 to predict the bright-state errors for the experiment reported in this chapter if the off-resonant transition rate were zero. The results are plotted in Fig. 7.19. For this prediction, I run the model with rates that match those of the experiment described here: $\eta R_0 = 50$ kHz, $R_{bg} = 750$ Hz, and $R_{d\rightarrow d} = 2$ Hz. I consider both the ORP rate fit from the experimental data $R_{b\rightarrow d} = 960$ Hz (blue curve) and the rate $R_{b\rightarrow d} = 0$ Hz (black
Fig. 7.19  **Predicted bright-state errors for** $R_{b \rightarrow d} = 0$.  The bright-state errors predicted for typical experimental values are plotted in blue.  The same prediction for no off-resonant transitions is plotted in black.

At 200 $\mu$s, the bright-state errors of the $R_{b \rightarrow d} = 960$ Hz curve have flattened out at the 3.4% level.  Thus, even if the dark-state errors were zero (experimentally infeasible because of detector dark counts), the fidelity would be limited to 98.3%.  For $R_{b \rightarrow d} = 0$, the bright-state error is .04% at 200 $\mu$s and is still decreasing.  Indeed, it never stops decreasing and does not limit the fidelity.  At 200 $\mu$s, the fidelity could be as high as 99.96%.

In reality, perfect $\sigma$ polarization is very difficult to achieve experimentally and without it, the stretched states do not form a true cycling transition [62].  Therefore, off-resonant transitions still occur in readout schemes that use $\sigma$-polarized light at a rate that depends on the detuning of the probe beam from the $F' = 2$ and $F' = 1$ excited states and the error in the polarization.  The difficulty in achieving perfect $\sigma$-polarization has been described by Kwon *et al.* in the supplemental material of Ref. [62] and by Martinez-Dorantes in his PhD thesis [73].  Speaking with one of the authors of Ref. [62], Dr. Matthew Ebert, I know that this alignment was not a
trivial undertaking. One of the main hurdles is that detecting with $\sigma$-polarized light limits your choices of experimental geometry. The mathematical description of the system is much simpler if the $\sigma$-polarized beam travels along the quantization axis of the system. Since the AC-Stark shifts caused by the linearly-polarized trapping light are such a strong perturbation to the atom, it is almost always easiest to take the quantization axis along the direction of this linear polarization. Thus, the $\sigma$-polarized probe light must travel along this axis to. This constrains the experimental geometry and the alignment of the probe beam’s polarization to this axis is hard.

Kwon et al. achieve a polarization purity of $I_{\sigma^+}/I_{\sigma^-} = 1600$ where $I_i$ represents the intensity of the polarization $i$ present in the probe beam. With this polarization purity, they estimate that the off-resonant transitions in their system are suppressed by a factor of 20 compared to those measured during readout with un-polarized light.

With this level of suppression, Kwon et al. report reaching a fidelity of 98.7% when their calculation is corrected for imperfect state preparation and background loss of the atoms and 97% when it is uncorrected [62]. This fidelity was achieved with 6 ms of detection time plus 100 ms of time to transfer the data from the camera to the computer. Using a similar scheme, Martinez-Dorantes et al. report achieving a fidelity of $>98\%$ within 10 ms of probe time. They also used a camera for readout, so require time to transfer the data from the camera to the computer. Martinez-Dorantes et al. also developed a complex Bayesian algorithm to weight their readout protocol towards pixels where an atom had already been detected. Without this algorithm, their state readout fidelity was 93% [74]. An earlier experiment by Fuhrmanek et al. reported achieving 98.6% readout fidelity in 1.5 ms of readout time [30]. They also used $\sigma$-polarized readout.

In the experiments reported in this thesis I achieve an average fidelity of 97.6% in 160 $\mu$s using $\pi$-polarized readout light. The fidelity is not corrected for imperfect state preparation or background atom loss and does not utilize any unusual algo-
rithm. The fidelity is lower than that achieved in protocols with $\sigma$-polarized light that suppresses ORP. If the $R_{b\rightarrow d}$ rate in this experiment were suppressed by a factor of 20, like that reported in Ref. [62], the model of Sec. 6.3 predicts a bright-state error of 0.2% at 200 $\mu$s given the other rates measured. With a corresponding dark-state error of 1%, this suggests that readout fidelity of $> 99\%$ could be achievable using $\sigma$-polarized light.

7.4.2 Experiments comparing readout with $\sigma$-polarized and $\pi$-polarized light

To test this, I run a set of experiments with $\sigma$-polarized light as the quantum-state readout light. The experimental setup I use is depicted in Fig. 7.20a. There are three ways in which the setup differs from that used for the other experiments reported in this thesis. First, the polarization-axis of the trapping light is rotated to be aligned with the x-axis instead of the z-axis. Second, a magnetic field is applied along this axis during the readout. Third, a quarter waveplate is inserted in the probe-beam path to change the polarization of the probe beam from linear to circular.

The experimental sequence is slightly different than for $\pi$-polarized readout as the magnetic field is applied to the atoms during readout. I apply the magnetic field to mimic the experiments of Refs. [30], [62], and [74], all of which apply a magnetic field. The experimental sequence is given in Fig. 7.20b. In addition, the frequency of the probe beam used for readout is different than that used in the $\pi$-polarized case. It has been shown that for atoms trapped in ODTs like the one used here, the peak fluorescence rate when driving with $\sigma$-polarized light is reached at a lower frequency than when driving with $\pi$-polarized light [109]. This is because the stretched states experience much smaller AC-Stark shifts compared to the central $m_F$ states, as seen in Fig. 7.18. Since the cycling transition of the stretched state is the strongest transition under $\sigma$-polarized light, it contributes most the atomic scattering. Hence, peak fluorescence is seen near this stretched-state shift.
Fig. 7.20  **Experimental setup and sequence for σ-polarized readout**  
a) The experimental setup is altered by rotating the trapping-light polarization, applying a magnetic field $B_x$ and inserting a quarter waveplate (QWP). b) The experimental sequence is altered by applying $B_x$ to the atom during the state readout.

Driving the atom with near-resonant, σ-polarized light, the best fidelity I achieve is $F_3 = 98.1 \pm 0.4\%$ in 390 $\mu$s of probe light, achieved by thresholding on three photons. The fidelity curves of this data set are shown in Fig. 7.21a. This is a slightly higher fidelity than I see with the π-polarized beam but it takes longer to reach by a factor of two. For better comparison to the π-polarized data, I consider the $F_2$ fidelity. The peak $F_2 = 97.7 \pm 0.4\%$ is reached in 326 $\mu$s. The corresponding bright-state and dark-state error curves for the $F_2$ fidelity are shown in Fig. 7.21b.
Fig. 7.21  **σ-polarized quantum-state readout results**  
a) The fidelity of the σ-polarized readout protocol reaches a peak fidelity of $F_3 = 98.1 \pm 0.4\%$ in 390 $\mu$s of probe light  
b) The $F_2$ errors show that the bright-state error is lower than that for $\pi$-polarized readout.
Comparing this bright-state error to that of Fig. 7.16b shows that the bright-state error is flattening out at a lower level, suggesting that there are fewer off-resonant transitions occurring. This is part of what contributes to the higher fidelity.

Though this data looks promising, the retention is very poor which represents a major problem. The bright-state atoms are only retained 79.4% of the time in this data. The dark-state atoms are retained 98.6% of the time yielding an average retention of only 89%. The poor retention of the bright-state atoms suggests that I am driving the atom too hard during this readout, leading to heating and loss. Lowering the probe-beam power would fix this problem but also lengthen the collection time. Since this scheme already takes twice as long as the π-polarized scheme for a small increase in fidelity, the advantage that it yields is debatable.

Considering the atom’s scattering-rate during this probe pulse, I note another important difference between this readout scheme and the π-polarized scheme. The time-of-arrival of the photons is plotted in Fig. 7.22a. Unlike for the linearly-polarized probe beams, here there is a slow increase to the scattering rate at the beginning of the pulse. This is a marked difference from the π-polarized schemes where the scattering rate is immediately high. This slow increase is tied to the longer time it takes to reach peak fidelity. Since the atom is scattering fewer photons early in the probe pulse, it takes longer to surpass $n_{thresh}$. Turning to the rate-equation model, I see that this behavior is predicted for a σ-polarized probe in this experimental setup. The rate-equation prediction is shown in Fig. 7.22b.

To understand this predicted behavior, recall that the atom is prepared into the $F = 2$ ground state by the MOT repump light. This means that is has roughly equal probability of being in any one of the $F = 2$ ground-state sublevels at the beginning of the readout pulse. For an ensemble of experiments, this translates to an equal distribution of atomic population among the $F = 2$ ground-state sublevels. The situation is depicted in Fig. 7.23 with the gray circles representing atomic population.
**Fig. 7.22** Scattering rate of $\sigma$-polarized quantum-state readout  

a) The time-of-arrival of the photons collected during the quantum-state readout shows the scattering rate slowly increasing.  

b) The behavior is predicted by the rate-equation model.
The probe-light is tuned to the cycling transition, meaning that it only resonant with atoms in the $F = 2, m_F = 2$ ground state sublevel. Any atoms in the other sublevels are detuned from this probe beam by the AC-Stark shifts of the excited state. Consequently, they experience a lower scattering rate, are less likely to be excited, and are less likely to be moved into a different $m_F$ sublevel. The probe-beam is also fighting against the CG coefficients which cause the excited state to preferentially decay in the $m_F = 0$ sublevel. This also slows down the process of pumping into the stretched state. The $\sigma^+$-polarized probe-beam eventually pumps the atomic population into the stretched state, at which point it is resonant with the probe. Hence, the scattering rate slowly increases it becomes constant.

The rate-equation model predicts this process to take tens of microseconds, at least an order of magnitude longer than the $m$-state pumping with $\pi$-polarized light.
Furthermore, the model predicts the pumping to take longer for impure polarizations. I do not work hard to ensure the purity of the polarization in the setup, so polarization error could also be contributing to this slow increase.

The slow-pumping effect described here can be avoided by preparing the atom into the stretched state before the readout pulse, as was done by Fuhrmanek et al. in Ref. [30]. This likely contributed to their ability to achieve faster readout (1.5 ms) compared to the subsequent experiments of Refs. [62], [74]. However, state preparation into the stretched state is not experimentally trivial because it requires a different laser-frequency than any other steps in the protocol. Fuhrmanek et al. did their state preparation with a $\sigma^+$-polarized beam tuned to the $F = 2 \rightarrow F' = 2$ transition combined with MOT repump light. The state preparation beam, since it is $\sigma$-polarized, must be sent down the same optical path as the readout beam. Both the additional frequency requirement and geometry constraint, require careful planning to overcome.

Furthermore, the Rydberg-based protocols used for most experiments require the atomic qubit to be located in the $m_F = 0$ hyperfine levels of the ground state. Thus, the qubit cannot be moved to the cycling transition without the risk of disturbing it. This is why Kwon et al. do not pump the atom into the stretched state before readout. Failure to do so, however, leads to the slow increase to the scattering rate seen here. It also opens the door to the possibility of ORP occurring during readout before the atom is pumped into the stretched state. Kwon et al. address this possibility, which they term “transient depumping,” in the supplemental material of Ref. [62].

For the $\pi$-polarized readout protocol used in this thesis, there is no slow increase to the fluorescence rate limiting the speed of the readout. The readout is insensitive to the initial distribution of population between the magnetic sublevels, so no special state preparation is needed. Furthermore, the strongest transition for $\pi$-polarized
light is the $m_F = 0$ transition, which why the atomic fluorescence peaks so close to the shifted resonance of this transition. Thus, the $\pi$-polarized readout protocol is naturally compatible with the ideal $m_F = 0$ qubit state and does not require complex state preparation.

In summary, the $\pi$-polarization readout protocol presented in this thesis has several advantages over the $\sigma$-polarized protocols reported elsewhere. The experimental procedure and setup reported here is much simpler than that required for $\sigma$-polarized protocols. The $\pi$-polarized protocol is also naturally compatible with the most desirable qubit state, unlike the $\sigma$-based protocol. Finally, the $\pi$-polarization protocol reaches peak fidelity much faster than similar $\sigma$-based protocols by at least an order of magnitude. I demonstrate this experimentally and predict it with the rate-equation model.

7.5 Summary

In this chapter, I thoroughly investigate the nondestructive quantum-state readout protocol presented in this thesis. I identify a generally unreported decay to the scattering rate of the atom over time. I investigate this decay experimentally, and show evidence that this decay is related to ORP and heating.

I also describe experimental improvements in the system that enable me to achieve the highest fidelity and fastest quantum-state readout yet reported for $\pi$-polarized readout light. On average, I achieve a fidelity of $97.6 \pm 0.2\%$ in a readout time of $160 \pm 20 \mu s$. This is a marked improvement on the previously reported result of $95\%$ fidelity achieved in $300 \mu s$ of readout time [35]. I discuss the current limitations of the readout reported here and suggest avenues for future improvement.

Finally, I compare the readout protocol and result used in this thesis to those reported for $\sigma$-polarized readout. The $\sigma$-polarized protocols report detection fidelities of $> 98\%$ achieved in $> 1$ ms of detection time [30], [62], [74]. The result described
here is of slightly lower fidelity but is an order of magnitude faster. It does not
include corrections to the fidelity calculation. Furthermore, the linearly-polarized
readout protocol is insensitive to the initial population of the magnetic sublevels and
it only requires that the probe’s polarization be oriented in the same direction as
the trapping light’s polarization. This makes it experimentally simpler than $\sigma$-based
protocols.
In this thesis, I describe a single-atom trapping system that achieves fast, nondestructive quantum-state readout of a trapped, neutral atom. In this conclusion I highlight three major results of this work. First, I build experimental apparatus to stably trap a single atom of $^{87}\text{Rb}$. Second, I develop a rate-equation model that describes the atom’s quantum-state dynamics in the presence of near-resonant laser light and the trap-induced AC-Stark shifts. Third, I combine these two achievements to investigate and characterize that atom’s scattering rate in the presence of linearly-polarized, near-resonant laser light. The tools I have developed and the insights I have gained are brought to bear on the problem of nondestructive quantum-state detection, resulting in the fastest, highest-accuracy quantum state readout yet reported using linearly-polarized readout light. In this chapter I summarize the main results of my work and discuss possible directions for future experiments in the system. This experimental system is suited to many future experiments and the results reported here are applicable to similar systems investigating quantum information and quantum simulation protocols and basic quantum science phenomena.
8.1 Summary

This thesis describes the experimental apparatus that I develop to trap a single, neutral atom of $^{87}$Rb. In building this apparatus from scratch, I encounter several experimental challenges and generate new knowledge as I address them. For example, using a small, mostly-metal vacuum chamber enables me to maintain the clean, ultra-high vacuum environment required for a long-lived atom trap. However, to use this chamber and successfully and stably load the single-atom trap, I must develop an unusual 8-beam MOT geometry. The result of this choice is a system in which the atom’s background-limited lifetime in the trap is $> 1$ minute, longer than in many other experiments. This long lifetime is fundamental to the rest of the experimental work in this thesis as it guarantees that the atom remains in the trap for the extent of the experiments unless perturbed by the experimental sequence.

In addition the MOT geometry and vacuum chamber work, I also develop several protocols for detecting the presence of the atom in the trap. I compare three specific protocols and choose to use a probe/cool sequence that robustly detects the atom without pushing it out of the trap. This detection sequence is very effective and allows me to detect the atom even when it is interacting very weakly with the probe beam. This sequence generates such clean data, that I would encourage anyone working in such traps to use this sequence whenever possible.

In order to characterize the internal population dynamics of the trapped atom under the influence of near-resonant laser light, I develop a rate-equation model that includes all 24 magnetic sublevels of $^{87}$Rb’s D2 transition and the sublevel specific AC-Stark shifts induced by the trap. I use this model to predict spectroscopic data for the trapped-atom, enabling me to identify possible heating at high probe power and long probe readout time. I also use this model to perform an absolute calibration of the collection efficiency of the experimental system. To the best of my knowledge, this
is the first such calibration performed for a neutral atom trap and is an experimental
tool very useful to the field.

Using all of the knowledge described above, I experimentally investigate and
theoretically predict the atom’s scattering rate in the presence of linearly-polarized
probe pulse used for quantum-state readout. Perhaps the most surprising predic-
tion of the rate-equation model is that the atom’s population preferentially accrues
in the $m_F = 0$ magnetic sublevel during readout. In order words, pumping to
the stretched states is suppressed. This is due to the Clebsch-Gordon coefficients
that govern the excited state decay and has significant implications for the speed
of quantum-state readout achievable with circularly-polarized light compared to
linearly-polarized light.

I also use the experimental system to identify that the atom’s scattering rate
decays over the duration of the readout. This decay is dependent on both the probe
power and frequency and limits the fidelity of the quantum-state readout. Using a
model of the readout protocol, I extract quantitative rates that govern this decay.
With experimental investigation and predictions from the rate-equation model, I
identify that this decay is likely caused by a combination of off-resonant transitions
and heating. The heating is due to the scattering of the readout light and the
interaction of the atom with the trapping-potential. This scattering-rate decay has
profound implications for the speed and accuracy of quantum-state readout in single-
atom traps.

Building on the knowledge described above, I demonstrate fast, nondestructive
quantum-state readout using linearly-polarized probe light. I demonstrate a readout
fidelity of $97.6 \pm 0.2\%$ achieved in a readout time of $160 \pm 20 ~\mu s$. The atom remains
in the trap after readout in an average of $>97\%$ of the experiments. This readout
is faster and of higher fidelity than previously reported results for linearly-polarized
readout [35]. It is also an order-of-magnitude faster than readout schemes reported
that use $\sigma$-polarized light. The readout protocol demonstrated here is experimentally simple compared to those reported elsewhere. It is also independent of the atom’s initial distribution among the $m_F$ sublevels of the ground state. This protocol is applicable across a wide range of experiments that require fast, nondestructive quantum-state readout.

8.2 Future Directions

The main results presented here are the experimental demonstration of a system that stably traps a single, neutral atom, the development of a rate-equation model to predict the atomic population dynamics in the presence of AC-Stark shifts and near-resonant laser light, and the demonstrated ability to quickly read out the atom’s quantum state without losing the atom from the trap. The rate-equation model and quantum-state readout presented here are applicable across a wide-range of experimental setups which utilize single-atom optical dipole traps for quantum information and quantum simulation protocols. The experimental system developed here is flexible enough to use for several future experiments in these fields. In this section I briefly discuss possible future applications of the work described here beyond the possible improvements discussed in the preceding chapters.

First, I note that the experimental system and rate-equation model reported here provide a good system for investigating the dynamics of trapped neutral atoms in the presence of near-resonance laser light. Fully understanding such dynamics, particularly in the presence of the AC-Stark shifts caused by the dipole trap, is of continued interest to the field as it aims to improve and optimize quantum protocols. For example, this system could be used to further explore heating in dipole traps [75], [18]. Demonstration and improvement of efficient cooling schemes in the presence of the light-shifts is also of continued interest [17], [117], [53].

Furthermore, I note that the field of atom trapping has developed many interest-
ing techniques for controlling the quantum state of atoms in ensembles and MOTs. Investigating the efficacy of these techniques for single atoms in dipole traps could greatly aid quantum computing and simulation protocols that require fast and precise quantum control. For example, extending shortcut-to-adiabatic passage techniques [25] from MOTs to single atom traps could increase the speed of single-qubit gates in these systems. Such investigation could be undertaken in this system.

I note that this experiment also forms the basis for a simple quantum device. With added control to prepare the atom into specific magnetic sublevels of the ground state, this system could realize an atomic qubit, opening up the door to many possible experiments. Such control is non-trivial to implement but the strategies for achieving it are known. It can be achieved by using a magnetic field to lift the degeneracy of the ground-state sublevels and microwave photons tuned to the ground-state splitting [105]. It can also be achieved using two-photon Raman transitions through stimulated Raman adiabatic passage [119].

One such experiment of particular interest is using a single atom to generate random numbers certified to be quantum mechanical [118]. Generating a string of true random numbers is of great interest for cryptographic and security applications. The experiment developed here could generate a physical string of random numbers with no modification. It has also been shown that quantum mechanics can guarantee the randomness of a bit-string generated by a single quantum system that exhibits certain properties [1]. With added control over the atom’s quantum state, the system developed here could generate such a certified-random string. Furthermore, seeding the system with the random number output of a physical (non-quantum) random number generator, such as that developed by Rosin et al. [97], opens the door to explore the interplay between classical physical random number generation and quantum random number generation.

Finally, entangling the atomic qubit with a photon would allow this system to
become a node in a quantum network [119]. Such a node could be used in a hybrid network that aimed to interface neutral atoms with, for example, trapped ions [113]. Such an interface would be the beginnings of a device that could take advantage of the long lifetime and coherence times of trapped ion systems as well as the near-infrared photons of neutral atoms.
Appendix A

Experimental Apparatus

In this appendix, I detail the system that I use to stably trap, image, and address a single atom of Rubidium-87. The atom is confined in an optical dipole trap (ODT) that is loaded from a magneto-optical trap (MOT). Here I describe the components of both of these trapping systems. I also discuss the imaging system and control system used for acquiring data.

A.1 UHV system

All of the experiments described in this thesis are conducted in an ultra-high vacuum (UHV) environment. The UHV system consists of a Kimball Physics 2.75” expanded spherical cube vacuum chamber (P/N MCF275-SphCube-C6A8) (Fig. A.1a). The vacuum chamber has six 2.75” ports on the faces of the spherical cube and eight 1.33” ports at the corners. One of the 2.75” ports is connected via a metal-T to an Agilent VacIon Plus 40 Starcell ion pump and also contains Rb dispensers from SAS Getters. The 40 L/s ion pump is connected to a Duniway controller (P/N IPC-SO62) and an all-metal valve, as shown in Fig. A.1.
Fig. A.1 Vacuum Chamber and Pump System. 

a) The Kimball Physics spherical cube chamber used Figure adapted from Kimball Physics Website. 
b) The assembled vacuum system.

The five remaining 2.75” vacuum ports and all of the 1.33” ports are sealed with windows (viewports). Three of the 2.75” viewports and four of the 1.33” viewports are anti-reflection (AR) coated for a broadband range of 780-1064 nm wavelengths. The AR-coated viewports are from MDC Vacuum and are P/N 450041 for the 2.75” flange size and P/N 450040 for the 1.33” flange size. The un-coated viewports are
a)

Fig. A.2  Vacuum chamber and viewports The vacuum chamber ports are sealed with windows. The AR-coated windows are orange/brass in color and the un-coated windows are gray. a) A y-z view of the chamber. b) An x-z view of the chamber.

from Kurt Lesker and are P/N VPZL-265 for the 2.75” flange size and P/N VPZL-133 for the 1.33” flange size. Figure A.2 shows the viewports on the vacuum chamber. The AR-coated viewports are orange/brass in color while the un-coated viewports are gray. The AR-coated ports are located on those windows through which MOT beams propagate in the initial MOT-geometry described in Sec. A.2.2.

A.1.1 Bake-out procedure for achieving UHV operation

To achieve UHV operation, I assemble the parts listed above, as shown in Fig. A.1, and connect the all metal valve to the bellows on the lab’s pumpdown station. The pumpdown station consists of a rotary pump and a turbomolecular pump (Varian P/N Turbo-V70LP) and includes an ion gauge to monitor pressure during the pumpdown. To get the system under vacuum the chamber must be pumped out by various vacuum pumps and also baked at a high temperature.

The bake procedure drives water and other contaminants out of the chamber
walls to allow the chamber to reach the UHV regime. The desired temperatures are reached by heating the system with heat tape connected to variable voltage supplies and wrapping the system in fiberglass insulation and oil-free aluminum foil. The system is heated such that the highest temperature is at the vacuum chamber with the temperature slowly decreasing towards the all-metal valve and the bellows. The temperature gradients in the system must be carefully monitored during the bake because large gradients can cause leaks to develop. The glass-to-metal transitions of the vacuum viewports are particularly delicate. The basic procedure is to heat the cell, heat the pump-down station, cool the cell, and then cool the pump-down station. I have baked the system several times over the course of the work detailed in this thesis. In the hottest bake, the chamber reached 280 °C. The maximum allowed temperatures for the various components can be found on the manufacturer’s websites.

The pumpdown procedure is described below.

**Step 1:** Assemble the vacuum chamber and connect the free port of the all-metal valve to the bellows on the pumpdown station. So not open the valve yet, just connect it to the bellows.

**Step 2:** Wrap the chamber and pump-down station in preparation for the bake. The chamber is wrapped using a combination of fiberglass insulation, oil-free aluminum foil, and heating tape with thermocouples placed around the system to measure the temperatures at relevant points. The wrapping order varies depending on the part of the system being wrapped. For the windows, the wrapping order is: thermocouple, insulation, foil, heat tape, insulation, foil. The order for glass/metal transitions is: foil, heat tape, insulation, foil. The wrapping order for metal is: thermocouple, heat tape, insulation, foil. Be sure to leave air around Rb-dispersers inside their foil wrapping to insulate them from the heat. If the getters are heated directly, they release too much Rb into the system. Also, do not heat the glass ion...
gauge on the pumpdown station. I generally measure the temperature at two of the windows, at the body of the chamber, at the metal-T which connects the chamber to the ion pump, at the ion pump, at the all-metal-valve connecting the pump to the bellows, and then along the bellows.

**Step 3**: Make sure all valves are closed. Turn on the rotary pump. Wait \(\sim 30\) s and listen for bubbling in the system. Once the bubbling has subsided, open the turbo valve on the pump-down station, and wait \(\sim 30\) s listening for bubbling. Once the bubbling has subsided, open the gate valve on the pump-down station. The red dot on the gate valve disappears when the valve is fully open. Once the bubbling has subsided, open the all-metal valve. This opens the chamber to the atmosphere.

**Step 4**: Once the bubbling has subsided, turn on the turbo pump, listening to it slowly rev-up. The turbo and roughing pumps together quickly pull the system down to a pressure in the \(10^{-5}\) Torr range and if left overnight, the system can reach a pressure of low \(10^{-7}\) Torr.

**Step 5**: Begin the bake. Attach the heat tape to variable voltage supplies (vari-acs). Slowly increase the voltages of the variacs to slowly increase the temperature. I generally increase the temperature by \(\sim 20\) °C per hour. It usually takes over a day to reach the final bake temperature. As the temperature is increased, the pressure increases as well. If the system is left at a certain temperature for a long time, the pressure begins to drop. The hottest bake applied to the system in December 2015 raised the chamber temperature to 280-300 °C, the all-metal valve to \(\sim 260\) °C, and the bellows to \(\sim 250\) °C. Subsequent bakes raised the temperature of the chamber to 180 °C, the valve to 160 °C, and the bellows to 150 °C.

**Step 6**: Once the system is at its final temperature, it is left for a time and the pressure is monitored as it falls. Once the pressure has stabilized, which can take over 24 hours, the ion pump is turned on. This usually occurs at pressures in the low \(10^{-7}\) Torr range. The ion pump should not be turned on if the pressure is above
10^{-7}\text{Torr. When the ion pump is turned on, the pressure spikes before dropping back down again. Leave the system at temperature as the pressure falls.}

**Step 7:** Once the pressure has stabilized, which can take over 24 hours, begin cooling the system down. This is the reverse of the heating procedure described above and takes several hours. As the system is cooled back to room temperature, the pressure drops drastically, by 1 or 2 orders of magnitude.

**Step 8:** Once the system is back at room-temperature, the pressure should be in the 10^{-9} \text{Torr range. Close the all-metal valve using a torque-wrench. Do not over-tighten the valve, the proper torque-seating can be found on the manufacturers website. For the all-metal valve used in this thesis, the proper torque is 70 in-lbs. With the all-metal valve closed, the pressure in the chamber drops as the ion-pump is pumping on a smaller volume.}

**Step 9:** Close the gate valve and then close the turbo-valve. Turn off turbo-pump, opening the leak-valve to help the turbo spin-down. Turn off roughing pump.

**Step 10:** Unwrap the chamber and disconnect it from the pump-down station.

Sometimes, it is helpful to gently run the Rubidium dispensers during baking to ensure that nothing builds up on them during bake-out. I run the dispenses occasionally for a brief time (<2 minutes) while the chamber is at peak temperature, using currents <2 A. When the dispensers are run, the pressure spikes before dropping. I turn off the dispensers when the pressure levels out. I do not run the dispensers again until the pressure has dropped.

If the pump-down station has sat unused for a long time, it may be necessary to bake-out the pump down station before baking out the full system. To do this, cap-off the bellows and then start up the roughing and turbo pumps as described above. The two together should be able to pull the pump-down station into the low 10^{-7} \text{Torr range or lower in less than 24 hours. Baking may be required to reach this pressure.}
A.2 Magneto optical trap

Inside the UHV chamber, the magneto-optical trap (MOT) is created by applying a combination of optical and magnetic forces to the atoms. An inhomogeneous magnetic field creates a trapping potential while laser beams use radiation pressure to cool the atoms so that their kinetic energy is below the trap depth. The physics of MOT creation has been studied in depth and is detailed in several publications such as [78] and [122]. This section covers the creation of the inhomogeneous magnetic field and the optical layout of the laser beams that create the MOT needed for the experiments described in this thesis. I conclude by discussing the alignment procedure I use for overlapping the MOT with the optical dipole trap.

A.2.1 Basic MOT geometry

Traditionally, a MOT is created by 3 pairs of counter-propagating laser beams oriented perpendicular to each other. The lasers propagate along the principle axes of a quadrupolar magnetic field created by a pair of coils in the Anti-Helmholtz configuration. The lasers apply radiation pressure in the same direction as the linear field gradient of the trapping field. Gradients in the range of 10 G/cm are used for trapping [122]. The standard MOT geometry is depicted in Fig. A.3.

The six 2.75” ports on the vacuum chamber depicted in Fig. A.1a form three pairs of ports oriented around three perpendicular axes. They are the only six ports which satisfy this geometry. Thus, using these ports for MOT beams is the only way to achieve the standard MOT geometry in the vacuum chamber used here. Unfortunately, this cannot be done because one of the 2.75” ports connects to the ion pump and holds the Rb dispensers. Another of the 2.75” ports holds the in-vacuum lens that is used to create the single atom trap, as detailed in Sec. A.3. The in-vacuum lens can be seen in Fig. A.2. Given these constraints, some of the MOT
Fig. A.3  **Ideal MOT geometry.** a) A MOT requires counter-propagating pairs of beams (red arrows) and a quadrupolar magnetic field created by coils of wire in the Anti-Helmholtz configuration (orange rings). b) The magnetic field has a linear gradient near the field zero. The gradient along the z axis is roughly twice that along the radial direction. The values in these plots are purely illustrative.

beams must propagate through the 1.33” corner-windows of the chamber.

A.2.2  **Initial MOT**

Initially, I create a MOT with one pair of beams propagating through 2.75” ports (called beam pair A) and two pairs of beams propagating through 1.33” windows (called B and C). The ports used are the AR-coated ports depicted in Fig. A.2. The Anti-Helmholtz coils are oriented around the large 2.75” ports on the x-axis. I use the axis orientation consistent with the experimental diagrams in the rest of this thesis. The beam and magnetic field coil geometry and beam labeling is depicted in Fig. A.4.

I choose this beam geometry because the beam pair A is oriented along one of the principle axes of the coils, as required for the standard MOT geometry. Beam pairs B and C form a plane and in that plane the angle between them is 70.53°. It is known that MOT trapping is possible with non-perpendicular beams, even with beams angles as acute as XX. The plane defined by beam pairs B and C is not...
Initial MOT CAD The initial MOT uses one pair of beams (A) propagating through 2.75” windows and two pairs (B and C) through 1.33” windows.

Initial MOT geometry a) The magnetic field coils and beam pair A (green) are oriented around the y-axis. The plane defined by the B (red) and C (blue) beams is canted from the A beams. b) The B and C beams form an acute angle.

perpendicular to pair A, it is canted 54.74° from perpendicular. The beam geometry and angles is depicted in greater detail in Fig. A.5. Unfortunately, this geometry forms an unstable MOT that is unsuitable for loading a single atom trap. After much investigation, I determine that the instability is due to the orientation of the laser beams with respect to the magnetic field coils. Unlike the ideal situation depicted in
Fig. A.6  **Initial MOT B fields** The beams along the axis of the coils (pair A) experience a linear gradient (blue curve). The other beams encounter a near-zero field gradient (red curve). The values in these plots represent the coil geometry and current used.

Fig. A.3, in the orientation of Fig. A.4, beam pairs B and C propagate along an axis with almost zero field gradient for a relatively large distance, as shown in Fig. A.6. Consequently, there is very little trapping along these beams and the atoms are not well confined in three dimensions. Furthermore, the lack of spherical symmetry in this beam geometry makes radiation pressure force balance (beam balance) difficult to achieve.

### A.2.3 Final MOT

To fix the instability noted above, I change the beam geometry to use four pairs of beams all going through the corner windows (see Fig. A.7a). The spherical symmetry of this geometry makes beam balance much easier to achieve. Each pair of beams is canted 54.74° from the z-axis and the in-plane angle between any two beams is 70.53°. These angles are the same as those shown for beam pairs B and C in Fig. A.5.

I also move the magnetic field coils to be oriented around one of the pairs of beams. The new orientation of the coils is shown in Fig. A.7a where the coils are
Fig. A.7  **Final MOT geometry.**  a) Four pairs of beams (labeled A-D) traverse the chamber. One pair (A) passes through the center of the quadrupole coils.  b) Each beam experiences a linear field gradient. The values in these plots represent the coil geometry and current used in the experiments of this thesis.

shown in copper. The rotation of the field coils necessitates some additional work to mount the coils that is detailed in Sec. A.2.4. This four-beam MOT has linear field gradients along all four beam paths, as shown in Fig. A.7b. This enables the stable MOT trapping needed for reliable single-atom trapping. All of the data presented in the body of this thesis is collected in this MOT geometry.

This new MOT geometry slightly complicates the optical layout of the beams going into the chamber, since no beams propagate in the plane of the table. It also means that two of the pairs of beams propagate through windows that are not AR-coated. The extra loss these windows cause is addressed in the optical setup, which is discussed in Sec. A.2.6.

**A.2.4  MOT magnetic coils and mount**

The MOT uses the standard quadrupolar magnetic field geometry where the field is created by an pair of wire coils carrying equal but opposite currents [11]. The size of the B field and B-field gradient that the coils produce is determined by their radius, separation, and the total current they carry. Generally, larger gradients are
desirable as they produce a tighter MOT. Larger gradients are produced by larger
coil radius, smaller coil separation, and larger current. In designing the coils for this
system, however, experimental realities limit the available design parameters. The
coils must be small enough in radius and far enough apart to physically fit around
the chamber without blocking optical access. These requirements generally lower
the B-field gradient achievable with a given current. The current can be increased in
order to increase the B-field gradient but not too much or it heats the coils, requiring
water-cooling that complicates the experimental setup. Furthermore, the coils are
situated around the small corner windows of the chamber, so a custom mount is
designed and built to hold them. The mount must hold the coils rigid with respect
to each other and to the chamber without blocking optical access to the windows.
It also must be strong enough to hold the weight of the coils, which varies as the
coil parameters are changed. The full design process required several iterations of
changing the coil design in CAD and re-calculating the resulting B-field gradients.

The mount is depicted in Fig. A.8. The first part of the mount is an aluminum
mold for winding and holding each coil (Fig. A.8a). The mold consists of a rect-
angular backplate, a cylindrical core, and a circular baseplate. Each piece contains
a 1.5”-diameter central hole that provides optical access to the window. Three # 8
screw holes allow the pieces of the mold to be bolted together with 8-32 bolts. In
the original CAD design, all of these are depicted as through-holes. However, after
production, the bolt heads proved to be too large, so I added tapped holes to the
backplate to use instead. These extra holes are visible in the photograph in Fig.
A.8d3. The bolt holes are equally spaced around a 1.82”-diameter circle centered in
each piece. The orientation is chosen to avoid interfering with the chamber windows.
The relevant dimensions are given in Fig. A.8a.

A heat-resistant PEEK plastic alignment sleeve (Fig. A.8b) fits closely into the
1.5”-diameter holes in the coil mold to orient the coil to the chamber window. The
Fig. A.8  MOT Coil Mount. a) The coil mold consists of a backplate (1), a coil core (2) and a coil baseplate (3). b) A window sleeve acts to align the coil to the window. c) The coil mount and window sleeve fit together (1) and slide over the corner window (2). d) The finished mount as designed (1 and 2) and produced (3).
sleeve fits snugly on the chamber windows (of outer diameter 1.33”). The sleeve has a collar that sits flush with the backplate to ensure that the coil-assembly is held the correct distance from the chamber. There are cutouts in the collar to allow for the bolts that hold everything together. The end of the sleeve is stepped to allow for clearance between the chamber viewports. The assembly is shown in Fig. A.8c.

The backplate of the mold also contains several arced slots oriented in a 6”-diameter circle centered on the plate. The slots are 0.17” wide, to allow a medium-fit of a # 8 bold. These arcs allow 6.724”-long threaded stainless steel support rods to be bolted to the mold (with 8-32 bolds) to hold the coils rigid with respect to each other and the define the axis through the center of the coils. The 3/8”-diameter support rods are tied to the optical table and surrounding optical assembly to hold the coils in place with respect to the chamber. This is done using mountable shaft collars (McMaster P/N 5878T17) with set-screws that tighten around the rods. The collars have 8-32 threaded mounting-holes to which either 1/2” optical posts or angled blocks can be attached. The angled bocks are Thorlabs P/Ns AM547 (54.7°) and AM353 (35.3°). The coil backplate also contains a radially-oriented slot through which the wire is threaded for winding the coil. The finished assembly is shown in Fig. A.8d.

The coils are 360 turns each, wound within the designed form in 15 layers of 12 turns; they are hand-wound on a lathe. The coil mount-assembly is my design and the pieces were machined by Richard Nappi in the Physics Machine Shop. Once the pieces were finished, I ground down some of the corners to better fit around the chamber, added the tapped holes to the backplate for bolting the coil mold together, and added mounting holes for the coil banana plugs. I also added extra tapped holes to the backplates in case I needed to mount cooling-fins to them. When assembled the coils fit right around the chamber, as designed.

In addition to the quadrupole coils used to create the trapping field, three pairs of
Helmholtz coils create uniform magnetic fields along the principle axes of the system. These coils shift the location of the magnetic field zero and tweak the position of the MOT. Each pair consists of two square 13 x 13 inch coils connected such that their currents travel in the same direction. The 3 pairs of coils create a cube around the chamber. The x-axis coils are 20 turn/coil, the y-axis coils are 40 turns/coil, and the z-axis coils are 20 turns/coil. The y-axis coils are more turns to enable higher possible fields to be achieved in this direction. This is necessary because this is the axis along which the ODT is formed, so overlapping the MOT with the ODT in the y-direction is important.

The quadrupole coils are typically driven with \( \sim 1.46 \) A of current and do not require the use of cooling fins. They are driven with an Agilent E315A power supply. The currents which drive the Helmholtz coils vary daily but typical currents are 1.95 A for the x-axis coils, 0.53 A for the y-axis coils, and 1.80 A for the z-axis coils. The x-axis coils are driven with an Agilent E3646A power supply, the y-axis coils with an MPJA 9333-PS power supply, and the z-axis coils with an Agilent E3610A supply.

### A.2.5 MOT laser system

The MOT requires both cooling and repump light to operate. The laser frequencies that I use can be found in Fig. A.20 at the end of this section. A diagram of the optical layout of the lasers used is depicted in Fig. A.9.

The cooling light is provided by a Toptica BoosTA tapered amplifier pumped by an AOSense external cavity diode laser (ECDL). The AOSense ECDL (S/N AOS-0008) generates 45 mW of linearly polarized light with a specified linewidth of <300 kHz. The laser is driven by an SRS LDC501 driver/TEC controller with current setpoint 164.15 mA. The temperature is stabilized by the TEC to 20.77 °C.

As shown in Fig. A.9, the first optic the beam encounters is a \( \sim 35 \) dB optical isolator that protects the diode laser from feedback into the cavity. The isolator is
Fig. A.9  **Schematic of MOT Lasers.** Diagram showing the MOT laser system as the beams are combined for transfer to the vacuum chamber.

from Thorlabs, P/N IO-3D-780-VLP. The beam exiting the laser is elliptical, so I use an anamorphic prism pair to reshape it before some is picked off for a saturated absorption spectroscopy setup. The spectroscopy setup is used for laser stabilization, as discussed in Sec. A.2.7. After the pickoff, the cooling light is sent through a second 35 dB isolator (same P/N) and coupled into a polarization maintaining (PM) single-mode (SM) fiber. The two isolators and fiber coupling are required to feed this cooling light into the tapered amplifier. After the tapered amplifier, 15 mW of input light has become 500 mW of PM SM fiber-coupled output and serves as the workhorse of the system.

The repump light is provided by a second AOSense ECDL (S/N AOS-0007). The laser is driven by an SRS LDC501 driver/TEC controller with current setpoint of 163.54 mA. The temperature is stabilized by the TEC to 21.4 °C. Since the repump transition is much weaker than the cooling transition, less repump power is required and this laser is not amplified. The AOSense laser that supplies this light is identical to the one that supplies the cooling light except that the repump laser has a circularizing lens installed inside the laser cavity. Thus, it does not require dramatic beam reshaping and after an isolator (same P/N as for the cooling laser)
and pickoff into the spectroscopy setup, is ready to be used. As an aside, I note that the AOSense ECDLs used in this work are some of the first lasers developed by that company.

The two laser beams described above are combined and manipulated to form the beams necessary for creating the MOT, as depicted in Fig. A.9. The experiment requires precise timing control, so I implement Acousto-Optic Modulators (AOMs) to controllably adjust the beam pointing. Since the beams are SM fiber-coupled, beam pointing adjustment effectively shutters the beams on and off. The AOMs also allow me to precisely control the beam frequencies. The AOMs are driven by custom-built drivers described in Sec. A.8.

The fiber-coupled cooling light from the tapered amplifier is attenuated by a HWP and PBS before being sent through an Isomet 1260C AOM operated at 120 MHz. The spare cooling light is sent to a high-power beam block (Thorlabs LB1). The repump light is sent through an Isomet 1205C AOM operated at 80 MHz before being fiber-coupled and brought to the same breadboard that contains the cooling light. Once the repump light is on the same breadboard as the cooling light, they are combined at a polarizing beam splitter (PBS) and again fiber-coupled into a PM SM fiber. This ensures that the two frequencies are co-propagating in all of the MOT beams, as required for ideal MOT behavior. I typically have \( \sim 10 \) mW of available repump light and \( \sim 30 \) mW of available cooling light at the fiber. I usually operate with \( 2.3 \) mW of repump and \( 7 \) mW of cooling light. The SM fiber directs the combined beams from the side of the table that houses the lasers to the vacuum chamber. A barrier is erected between the side of the table with the lasers and the side with the vacuum chamber to limit laser light scatter into the imaging optics.
A.2.6 MOT optical system

As described in Section A.1, the MOT is created in a non-standard geometry using four pairs of beams. Here I focus on the optical layout of the MOT beams, depicted in Fig. A.10. Beam intensity balance is critical to MOT stability, so eight equal-intensity beams are required for the MOT. I am assured equal size, Gaussian-profiled beams since all of the light comes from the same single-mode fiber. The light out of the fiber is collimated to ~1 cm in diameter. I then use PBSs and half waveplates (HWPs) to split the fiber output into four equal-power beams. These beams enter the chamber from below (beams I, II, III, and IV in Fig. A.10) and are retro-reflected by mirrors outside of the upper corner windows (beams V-VIII in Fig. A.10). To ensure beam balance between the forward propagating and retro-reflected beams, I put telescopes in the beam paths with controllable detuning. I make the beams slightly converging so that the retroflected beams are a smaller than the forward propagating beams to account for the power lost to reflections off the vacuum chamber windows and the quarter waveplates (QWPs). Apertures in the beam paths aid in alignment and allow me to aperture down the beams once the MOT has initially been found. Smaller beams scatter less light into the imaging optics.

A.2.7 MOT laser frequency stabilization

The MOT cooling and repump lasers are stabilized with an active feedback loop that “locks” the laser frequency to an atomic reference. The general optical layout is depicted in Fig. A.11. A vapor cell of Rb atoms in a standard saturated absorption spectroscopy setup provides the atomic reference [92]. The natural linewidth of Rubidium is 6MHz, but the linewidth measured in the saturation absorption setups is ~10 MHz. This broadening is due to imperfect alignment in the Doppler-free setup and power broadening.

As the laser frequency is scanned across resonance (by an SRS DS335 function
generator), the photo-detector in the setup outputs a Lorentzian atomic reference line. However, laser frequency stabilization requires dispersive feature. The standard way to turn a Lorentzian feature into a dispersive one is to add modulation to the signal and use a lock-in amplifier for phase sensitive detection, effectively taking a derivative of the Lorentzian [22]. I use an SRS SR830 DSP lock-in amplifier for locking the cooling laser and an SRS SR530 lock-in amplifier for the repump laser. Most groups introduce modulation into their laser beam by current-modulating the laser itself or using an acousto-optic or electro-optic modulator in the saturated absorption setup. I take a different approach and modulate the atomic reference.

To do this, I use a home-built solenoid that houses the vapor cell. I apply an AC
Fig. A.11  **Laser frequency stabilization.** Diagram showing the optical layout of the saturated absorption setup and the electronics needed for laser frequency stabilization.

Voltage to the solenoid to produce an AC magnetic field on the atoms, modulating the atomic frequencies. Since the atomic response to the B field is magnetic sublevel-dependent, I only probe the atoms with circularly polarized light to avoid broadening the resonance signal [22]. I use a 15 nF capacitor in parallel with the solenoid to create an LC circuit with a resonance at the desired modulation frequency. The solenoid and LC circuit were designed to have a resonance at the desired frequency of 68 kHz. This frequency is high enough to avoid low-frequency 1/f noise but lower than the 100 kHz bandwidth of the lock-in amplifiers. The driving voltage of 3 Vpp is supplied by an SRS DS335 function generator. The solenoid is 237 turns, hand-wound on a lathe. An example error signal generated by this system is shown in Fig. A.12.
Once the error signal is created, it is fed into a PID controller that provides a feedback signal for the laser. The digital PID controllers used for the cooling and repump lasers were designed by Professor Jungsang Kim’s group. The output of the lock-in amplifier is fed to a Xilinx field-programmable gate array (FPGA) (P/N XEM6010-LX45) on which is programmed the digital logic to implement the PID loop. The FPGA output signal is sent to the piezo-actuator that controls the cavity length of the AOSense laser. Beat note measurements show that the linewidth of this locking system is less than 2MHz. Since the natural linewidth of Rubidium is 6MHz, this is sufficient for the experiments described in this thesis. The lasers remain locked for hours at a time as I run experiments.

A.2.8 MOT imaging

The MOT is primarily imaged using three CCTV cameras connected to televisions. One camera images from the bottom, one from the back of the chamber, and one from the side. Thus, I have information about the MOT position and motion in all
three directions. The cameras are from PixelLink and Marshall (Model 20111).

Figure A.13 shows the creation of a stable MOT using the MOT system described in this section. A bright, large MOT is shown in the left and center images. This is the type of MOT I use for initially aligning the system and is created with $\sim 20$ mW of cooling power and 6 mW of repump power. For finding the ODT, however, a weaker MOT is used like the one in the right hand image (visible as a dim cloud in the center of the lens). The MOT is made with $\sim 5$ mW of cooling like and 2 mW of repump light.

When the MOT is first trapped, and for a strong MOT, the Rb dispensers must be run. They are run at $\sim 2$ A of current generated by an Agilent E3631A power supply. During the initial alignment of the system, a strong MOT is often helpful and the dispensers may need to be run every few days. Once everything is aligned and the single atom trap is loading, I find that a very weak MOT is sufficient for ODT operation. The MOT is so weak that it is barely visible on the CCTV cameras. I have even achieved reliable ODT operation when no MOT is visible on the CCTV cameras and the MOT is barely visible on the EMCCD camera. The weaker MOT can aid in ODT operation because weaker MOTs are generally colder, allowing the atom in the ODT to begin at a lower initial temperature. When the MOT is very weak, I also do not run the dispensers very often. This keeps the background pressure in the chamber low, improving the trap lifetime. I have achieved daily ODT operation without running the dispensers for 8-16 weeks at a time.

A.3 Optical dipole trap system

To create the optical dipole trap (ODT), I use an Eagleyard DFB laser (P/N EYP-DFB-0852-00150-1500-TOC03-000) with 150mW of total output power and a center wavelength of 852 nm. For ideal trap behavior, the trapping beam must have very low intensity noise [102] and be a clean, stable, Gaussian mode. To ensure low
Fig. A.13  **MOT CCTV Images.** Three CCTV cameras image the MOT from below (left image), the side (center image) and the back (right image). The invacuum lens is clearly visible in all images. The left image also shows alignment markings drawn on the screen. The MOT is indicated by a red arrow in each image. The left and center images are of strong MOTs while the right image is of a weak MOT.

Intensity noise, I control the laser with an Arroyo controller with low current noise (P/N 6305). The current set-point is 242.2 mA and the temperature is stabilized to 25 °C.

The beam is sent through an IntraAction AOM (P/N ATM-2301A2) operated at 230 MHz that shutters the beam on and off, as dictated by the experimental sequence. This AOM is not driven by a custom driver but by an IntraAction driver Model DE. The beam is shuttered on and off with a TTL pulse sent to the modulation input on the driver. To generate a Gaussian mode, I reshape the beam with cylindrical lenses before coupling it into a SM PM fiber for delivery to the chamber.

Inside the vacuum chamber, a high numerical aperture lens focuses the trapping beam to a small waist. A schematic of the basic system is shown in Ch. 4 and for convenience is reproduced in Fig. A.14. I use a single element aspheric lens from Thorlabs (P/N AL1815-B) to create the ODT. The lens has a diameter of 18 mm and is mounted inside the vacuum chamber in a custom lens tube held in standard groove grabbers with an inner diameter of 0.75” (Kimball Physics P/N MCF275-GrvGrb-CYL0750) that are mounted to one of the chamber windows. The setup is
Fig. A.14  **ODT Diagram.** The 852 nm trapping light enters the vacuum chamber and is focused by an aspheric lens to create the ODT that loads from the MOT. The same lens collects the 780 nm atomic fluorescence, which is sent down an imaging path by a dichroic beamsplitter. This imaging system is described in detail in Section A.7.

depicted in Fig. A.15. The lens is press-fit into the lens tube and epoxied in place via three small holes drilled in the end of the lens tube (not shown in the CAD drawings). The lens tube has a back collar that sits flush to the groove grabbers to hold the lens in position.

This is a simple design that allows me to reach NA=0.54 without a complicated, custom, multi-element objective. Higher single-element NA lenses are available but generally have very short working distances, which makes it difficult to trap a MOT near them for loading the dipole trap. Initially, I put a NA=0.83 asphere in the chamber but its working distance of 5 mm made it impossible to overlap the MOT with the ODT and load the ODT. Thus, the NA=0.54 lens with a working distance of 11.5 mm is a good compromise between high numerical aperture and ease of trapping.

The ODT beam exits the single mode fiber and is directed into the chamber and through the high-NA lens. I usually operate with 40 mW of power in the ODT beam. Targets are placed on the front and rear vacuum chamber windows to ensure that the beam is passing through the chamber and the lens centered and straight. The
Fig. A.15  **In-vacuum lens tube** a) The asphere (blue) is held inside the chamber by a custom lens tube (gray) and off-the-shelf groove grabbers (yellow). b) and c) The lens tube is made of aluminum with dimensions to that hold the lens inside the groove-grabbers.

beam can be seen on the targets with an IR-viewer. The circular targets are made of paper and fit the diameter of the windows. They have crossed lines on them to indicate the center of the windows. On the rear window, a well-positioned beam is centered and circular. If the beam is misaligned through the chamber, it appears decentered and elliptical on the back window target. In addition to these targets, I use an IR-viewer to locate the weak back-reflections from the beam passing through the first window. By ensuring that these back-reflections are overlapped with the
input beam all the way down the beam path back to the fiber, I am assured that the
beam is entering the window at normal incidence.

A.4 Probe beam

In addition to the MOT and ODT lasers, I also use a Toptica DLPro laser in the
setup. The DLPro laser is driven by a Toptica SYS DC110 controller with a set
laser current of 135 mA and temperature of 18.1 °C. This laser generates the probe
beam for the ODT. The laser is stabilized to an atomic reference using a saturated
absorption setup similar to that depicted in Fig. A.11. Unlike that setup, however,
the DLPro setup does not utilize a solenoid. The DLPro has a Toptica DigiLock
module attached to it that modulates the laser itself to provide the modulation
needed for creating the error signal. Similarly, the DigiLock module provides the
phase sensitive detection, generates the error signal, and creates the feedback signal.
The lock frequency is depicted in Fig. A.20.

The probe beam is sent through an AOM (Isomet P/N 1205C) operated ~84
MHz before being fiber coupled for delivery to the vacuum chamber. Unlike the
AOM setups described previously, this setup utilizes a double-pass configuration,
allowing the AOM frequency to be changed without shifting the beam position and
affecting the SM fiber coupling [23]. This is useful since the probe beam frequency
is often changed, e.g. in the spectropscopy experiments described in Chap. 5. After
fiber coupling, the probe beam is sent through a tunable 1:1 telescope before being
sent through the chamber and retroreflected. The telescope allows power tuning of
the retroreflected beam. The beam must be reflected so that power balance can be
maintained and the probe beam does not heat the atom out of the ODT.
A.5 MOT and ODT alignment

Initially, I center all of the MOT beams on the vacuum windows to ensure that they are intersecting in the center of the chamber. The quadrupole coils are mounted to the chamber in such a way that the field zero is also in the center of the chamber, and that is roughly where the initial MOT forms.

Ultimately, however, I need the MOT overlapped with the location of the ODT, which may not be precisely centered. I can address this issue because I can see the asphere inside the chamber, as shown in Fig. A.13. I know the working distance and the diameter of the asphere, so by measuring its width on the CCTV image, I estimate the location of the focal plane inside the chamber. Then, I tweak the beam alignment and the shim coils to walk the MOT to the estimated location of the ODT. I also use the CCTV cameras to ensure that the MOT is staying centered with respect to the lens. For fine alignment, I send 780nm light down the ODT path and use it to blow the MOT away. By lowering the blow-away power and tweaking the MOT position, I optimize the blow-away. This provides rough alignment of the MOT to the ODT. This procedure works fairly well for aligning the MOT in the plane transverse to the asphere but is less precise for alignment along the optical axis. That alignment is optimized using the single atom imaging system described Sec. A.7.

A.6 Probe beam alignment

The probe beam alignment is done in three stages. First, the forward-going beam is aligned to the MOT and ODT. This is achieved by looking at the beam on the bottom-viewing and side-viewing CCTV cameras that image the MOT (the left and center images in Fig. A.13). The beam can be see traversing the chamber and is roughly aligned with the MOT. Alignment is optimized by using the beam to blow
the MOT away and minimizing the power required to do so. Once the forward-going beam is aligned, the retroreflection is aligned in two stages. First, apertures are placed in the beam path and the closed to almost obstruct the forward-going beam. The retroreflection is then overlapped with the forward-going beam on these apertures. Three apertures are used, one on either side of the vacuum chamber and one in the middle of the telescope that is used for beam balance. Since the beam is small in the middle of the telescope, making sure the retroreflection goes through this aperture ensures pretty good alignment.

Pretty good is not good enough, however, because the retroreflection alignment greatly affects beam balance, which affects how much the atom is heated by the probe beam and whether or not the atom is retained in the trap after experiments. To achieve ideal alignment, I use an IR-viewer to look at the probe beam optical path. When the retroreflection is optimal, I can see the reflected beam propagating back down the path and coupling into the cladding of the SM fiber, causing the fiber to glow in the IR viewer. I have seen that when the glow is not present, the retention of the atom in the trap is noticeably worse. The alignment can wander off over the course of a couple of days of experiments, but this trick of using the IR viewer to look for the glowing fiber enables me to keep it peaked up easily.

A.7 Single-atom imaging

Once the atom is trapped in the ODT, I must image it to ensure that the experimental protocols are successful. The imaging system and its alignment to the ODT are non-trivial. I describe them in detail in this section.

A.7.1 Imaging system

I use two types of detector to image the ODT: an Andor IXon 897 electron-multiplying CCD (EMCCD) camera and a Perkins-Elmer avalanche photodiode fitted with read-
out electronics to make it a single photon counting module (SPCM) (Model SPCM-AQR). I am indebted to Professor Paul Kwiat for the loan of this detector. The EMCCD provides good initial imaging as the large sensor (8.2 × 8.2 mm) makes it relatively easy to align to the atom. The SPCM provides fast, low-background readout. The optical system is designed to produce images of the atom in the ODT on both the EMCCD camera sensor and the SPCM sensor. The SPCM is fiber coupled with a 100 µm-core, black-jacketed, multi-mode fiber. The full imaging system is described below and shown diagrammatically in Fig. A.16.

The first element in the imaging system is the high-NA asphere inside the vacuum chamber. This lens collects the 780-nm fluorescence from the atom and nearly collimates it out of the vacuum chamber. I specify that the fluorescence is nearly collimated because there is some defocus caused by the difference in wavelength between the trapping beam and the atomic fluorescence. A trapping beam of 852 nm light is sent into the chamber and focused by the high-NA lens. The atoms are trapped at that location, the 852 nm focus, which means that the 780 nm fluorescence of the atoms, since it is a shorter wavelength, is slightly converging as it exits the chamber. Zemax simulations of the lens as these wavelengths show that the focal shift between 780 nm and 852 nm light is 44 µm, while the Rayleigh range at this NA is 23.6 µm. Thus, the 780 nm fluorescence collected from the 852 nm focus is slightly converging. This defocus is corrected by the downstream optics. Chapter 5 explains the possible aberrations in this system in greater detail.

After exiting the chamber, the 780 nm fluorescence encounters a dichroic filter (Semrock P/N FF801-Di02-25x36) that separates the imaging path from the ODT path. From the dichroic onwards, the imaging path is cage mounted for stability. After the dichroic, a telescope brings the fluorescence to an intermediate image at a zero-aperture iris that provides spatial filtering to eliminate background scatter. The iris is on an x-y translation stage, allowing its center to be well-overlapped with
Fig. A.16 Schematic of Single Atom Imaging System. Not to scale. Diagram showing the single-atom imaging system. All distances are in mm and are approximate. The red line indicates the path of the 780-nm atomic fluorescence. The yellow line indicates the path of the 852-nm ODT light.

The FORT image. The telescope consists of two best-form lenses from Thorlabs. The first has a 200 mm focal length (P/N LBF254-200-B) and the second a 100 mm focal length (P/N LBF254-100-B). Both lenses of the telescope are on z-translation stages, allowing fine tuning of the fluorescence collection.

After the telescope, a 90:10 R:T beamsplitter separates the imaging path into the two arms. The reflected arm (90% of the fluorescence) passes through a focusing lens on an z translation stage and is imaged onto the tip of the multimode (MM) fiber held
in an x-y-translation stage and connected to the SPCM. This final lens provides the majority of the fine tuning for the SPCM coupling. The transmitted fluorescence also passes through a final focusing lens on a z-translation stage that focuses it down onto the camera. This final lens provides the primary focusing correction for the Andor image. Both final lenses are best-form lenses with a 50 mm focal length (Thorlabs P/N LBF254-050-B). Before each final lens, is a bandpass filter that only passes 780 nm light to the fiber or camera. The filter before the fiber is from Semrock, P/N LL01-780-25. The filter before the camera is from Thorlabs, P/N FL780-10.

A.7.2 Imaging system alignment

First, the EMCCD is aligned to the MOT. The EMCCD arm of the imaging path is built with the EMCCD replaced by a SM fiber and the lenses are all placed in each other's conjugate planes to make alignment easier. The fiber sends 780nm light down the imaging path. That light is used to fine-tune the lens positions with respect to each other to ensure focusing and collimation where appropriate and to set the location of the aperture. Additional apertures are used as needed to make sure the beam remains centered and straight through the optics. Those apertures are later removed. Ultimately, the light is overlapped with the 852nm light going into the chamber and is used to blow-away the MOT. Optimal blow-away while maintaining overlap with the 852nm ODT beam signals sufficient alignment that the MOT should be imaged on the camera.

Once the MOT is imaged on the camera, the image of the ODT is found by moving the MOT with the shim fields and tweaking the final lens before the camera. Moving the MOT changes the ODT loading, thus changing its brightness. It is easiest to see the ODT when it is in the strong loading regime discussed in Sec. 2.4. In this regime, the ODT appears as a bright, dense spot in the MOT. The lenses in the system affect the ODT image focus on the camera. I find that it is easiest to
the see the ODT with a very weak MOT. The MOT is weakened by decreasing the laser power down to $\sim 5 - 10 \text{ mW}$ of cooling light and $\sim 2.5 \text{ mW}$ of repump light. Though imaging the ODT initially is very challenging, the imaging system is robust once the ODT image is found.

Once the ODT is imaged on the camera, I set up an alignment beam for coupling the single atom fluorescence into the SPCM. Some of the spare light from the tapered amplifier SM fiber coupled and brought to the chamber. For this alignment beam, I focus $780\text{nm}$ light into the chamber through the back window. The Zemax simulation of this setup is shown in Fig. A.17. Once the beam exits the chamber, I overlap it with the $852\text{nm}$ ODT beam and the image of the ODT on the camera. When imaging onto the camera, I use very low power so as not to saturate the camera. Once this “fake” ODT is aligned, I close the shutter to the camera and turn up the power in the beam. At high power, I couple it into the MM fiber that will be sent to the SPCM. After the alignment, the SPCM is coupled to the ODT signal. I turn off the alignment beam and look again at the ODT on the camera; making sure that it is strongly loading. By chopping the ODT on and off, I see a chopped signal on the SPCM. I then optimize the size of this chopped signal using the SPCM imaging optics, thus optimizing the collection of fluorescence from the ODT. Once this fake ODT is aligned, it is very helpful for future imaging modifications.

As described in Ch. 5, this alignment procedure does not remove all aberrations from the system. Chapter 5 suggests some improvements. I also suggest taking
greater care over aligning the back-propagating “fake” ODT. Maximizing the coupling of the fake ODT into the SM fiber that delivers the ODT light would greatly improve the alignment of the fake ODT, making it a much more useful tool for the imaging alignment.

A.8 Experimental control electronics

A final, crucial piece of the experimental system is the electronics that control the experimental sequence and acquire data. The control system I use was designed by Dr. Peter Maunz for the group of Professor Jungsang Kim. I assembled and implemented this system myself but am very grateful to Dr. Maunz and Professor Kim’s students and postdocs for helping me get it running and adapt it for these experiments.

The system is built around a Xilinx XEM6010-LX45 FPGA and custom software written in python by Dr. Maunz. The FPGA, interfaced with a digital to analog converter (Texas Instruments P/N DAC8568) and analog to digital converter (Analog Devices P/N AD7606), provides a flexible platform for implementing the necessary controls in a manner that is not tied to the global clock of a personal computer. A block-diagram of the system is shown in Fig. A.18. The FPGA directly provides several TTL outputs, considered “triggers”, that serve various purposes in the system. One trigger remotely activates the EMCCD to acquire images. Several others go to solid state relays that switch on and off the current supplied to the magnetic field coils. Another modulates the IntraAction AOM driver for the AOM that controls the ODT beam. A final trigger gates the SPCM on and off during the experiment. The FPGA also reads and counts the TTL input from the SPCM.

In addition to these direct digital inputs and outputs, the FPGA also interfaces with custom drivers for the AOMs on the repump, cooling, and probe beams. These drivers are built around DDS chips (Analog Devices P/N AD9912) that provide the
The FPGA interfaces with various other electronics to provide the necessary experimental control. In addition to the digital input, the DDS’s require a 1 GHz master clock that is provided by an SRS frequency reference (Model SG382) providing -8 dBm of power at 1 GHz. The output of each DDS is sent through an RF switch (Mini Circuits P/N ZASWA-2-50DR+) and amplifier (Mini Circuits P/N ZHL-1-2W+) before reaching the AOM. All of these electronics except the RF amplifiers are housed in a control box as depicted in Fig. A.19.

**Fig. A.18** *Control System Block Diagram.* The FPGA interfaces with various other electronics to provide the necessary experimental control.

**Fig. A.19** *Control Box.* Diagram (left) and picture (right) showing the layout of the control electronics inside the control box.
A.8.1 *Experimental control code*

This section provides the pulse program I wrote to run the experiment. The first section sets relevant experimental parameters such as AOM frequencies and amplitudes, times for various steps, etc. The next section sets internal control parameters to differentiate the different DDS’s, set the order of the various TTL pulses requires, etc. The third section defines the two functions that control the timing of the experiment. The final section runs the functions.

```
# PulseProgramPlus input
# Author: Margaret E. Shea
# Date: 05/22/2018

const DDSCooling = 2
const DDSRepump = 1
const DDSDetectRepump = 0
const DDSDLPro = 3
const DACAOM = 1
#const SPCMChannel = 0

# frequencies and amplitudes for DDS’s
parameter <AD9912_FRQ> DetectRepumpFreq = 110 MHz
parameter <AD9912_FRQ> RepumpFreq = 80 MHz
parameter <AD9912_FRQ> RepumpLoadFreq = 90 MHz
parameter <AD9912_FRQ> CoolingFreq = 125 MHz
```
parameter <AD9912_FRQ> MolassesFreq = 120 MHz
parameter <AD9912_FRQ> DLProFreq = 80 MHz
parameter <AD9912_FRQ> DLProLoadFreq = 105 MHz
parameter <AD9912_FRQ> DLProOtherFreq = 80 MHz
parameter <AD9912_FRQ> DetectFreq = 130 MHz
parameter <AD9912_FRQ> PushFreq = 94 MHz

parameter DetectRepumpAmp = 500
parameter RepumpAmp = 500
parameter CoolingAmp = 100
parameter DetectCoolingAmp = 200
parameter CoolingTestAmp = 500
parameter DLProAmp = 500
parameter DLProAmpHigh = 500
parameter PushAmp = 500
parameter dummy = 0
parameter MaxInitRepeat = 10
parameter AtomThreshold = 1050

#parameter LowRepumpAmp = 0
#parameter LowCoolingAmp = 0

#parameter <DAC8568_VOLTAGE> AOMDAConvoltage
#parameter <DAC8568_VOLTAGE> AOMDACoffvoltage

# times
parameter LoadTime = 100 ms
parameter DarkTime = 100 ms
parameter WaitTime = 10 ms
parameter MolassesTime = 10 ms
parameter CoolingTime = 1 ms
parameter DelayTime = 10 ms
parameter ProbeTime = 2 us
parameter probecoolcycle = 1000
parameter probecoolcycleR = 500
parameter ProbeCoolTime = 1 ms
parameter SPCMTime = 3 us
parameter PrepTime = 20 ms
parameter PumpTime = 200 us
parameter SingleProbeTime = 80 us
parameter HighTime = 80 us
parameter PushTime = 80 us

# control parameters
parameter experiments = 100
counter FORTDetectCounter = 0
counter DetectCounter = 0
counter FORTCounter1 = 0
counter FORTCounter2 = 0
counter Counter1 = 0
counter Counter2 = 0
trigger ddsCoolingApply = 2
trigger ddsRepumpApply = 1
trigger ddsDetectRepumpApply = 0
trigger ddsDLProApply = 3
trigger DACTrigger

# parameter PresenceThreshold = 75

# excitcodes
exitcode ReloadExitCode = 0xffffe000000000001
exitcode endLabel = 0xffffffffffffffff

# internal variables
var experimentsleft = 100
var initRemaining = 0
var currentexperiment = 0
var cyclenumber = 0
var FirstImagePhotons = 0
var RetainImagePhotons = 0

# masks and shutters (controls the TTL pulses)
shutter InitializationShutter
masked_shutter MOTO
masked_shutter CoilsOn
masked_shutter QuantCoilOn
masked_shutter DetectOn
masked_shutter SPCMGate
masked_shutter FORTOn
masked_shutter CameraOn1
masked_shutter CameraOn2
masked_shutter ChopOn
masked_shutter CoolingOn
masked_shutter RepumpOn

### Functions ###

def LoadImage():
    set_dds( channel=DDSCooling, freq=CoolingFreq, amp=CoolingAmp )
    set_dds( channel=DDSRepump, freq=RepumpFreq, amp=RepumpAmp )
    # set_dds( channel=DDSDLPro, freq=DLProLoadFreq, amp=DLProAmp )
    set_dds( channel=DDSDLPro, freq=DLProFreq, amp=DLProAmp )
    set_dds( channel=DDSDetectRepump, freq=DetectRepumpFreq, amp=DetectRepumpAmp )
    set_trigger( ddsCoolingApply )
    set_trigger( ddsRepumpApply )
    set_trigger( ddsDLProApply )
    set_trigger( ddsDetectRepumpApply )

### Turn on the MOT and ODT to load the ODT(FORT) ###

set_shutter( MOTOn )
set_shutter( CoilsOn )
set_shutter( FORTOn )
update( LoadTime )
set_inv_shutter( MOTOn )
set_inv_shutter( CoilsOn )
update( WaitTime )
    # set_inv_shutter( MOTOn )
    # update( WaitTime )
set_dds( channel=DDSCooling, freq=DetectFreq, amp=DetectCoolingAmp )
set_trigger( ddsCoolingApply )
update()
clear_counter()
update( WaitTime )
set_counter( FORTDetectCounter )
update()
set_dds( channel=DDSDLPro, freq=DLProFreq, amp=DLProAmp )
set_trigger( ddsDLProApply )
### Probe/cool cycle ###
cyclenumber = 0
while cyclenumber < probecoolcycle:
    set_shutter( SPCMGate )
    set_shutter( DetectOn )
    update( ProbeTime )
    set_inv_shutter( DetectOn )
    update( DelayTime )
    set_inv_shutter( SPCMGate )
    update( SPCMTime )
    set_shutter( MOTOn )
    update( CoolingTime )
    set_inv_shutter( MOTOn )
    update( SPCMTime )
    update()
    cyclenumber += 1
    clear_counter()
    update()

def AdaptivePrepDetectImage():
    set_dds( channel=DDSCooling, freq=CoolingFreq, amp=CoolingAmp )
set_dds( channel=DDSRepump, freq=RepumpFreq, amp=RepumpAmp )
set_dds( channel=DDSDLPro, freq=DLProFreq, amp=DLProAmp )
set_trigger( ddsCoolingApply )
set_trigger( ddsRepumpApply )
set_trigger( ddsDLProApply )

### Repeat this process MaxInitRepeat times ###

initRemaining = MaxInitRepeat
while initRemaining>0:
    update( DarkTime )
    set_dds( channel=DDSCooling, freq=MolassesFreq, amp=DetectCoolingAmp )
    set_trigger( ddsCoolingApply )

### State preparation ###

    set_shutter( RepumpOn )
    # set_shutter( CoolingOn )
    update( PumpTime )
    set_inv_shutter( RepumpOn )
    # set_inv_shutter( CoolingOn )
    update()
    clear_counter()
    update(DarkTime)

### State readout ###

    set_dds( channel=DDSDLPro, freq=DLProOtherFreq, amp=DLProAmp )
    set_trigger( ddsDLProApply )
    set_counter( FORTCounter1 )
    set_shutter( SPCMGate )
    update( DelayTime )
    set_shutter( DetectOn )
update( SingleProbeTime )
set_inv_shutter( DetectOn )
update( DelayTime )
set_inv_shutter( SPCMGate )
clear_counter()
update( WaitTime )

### Cooling after state-readout ###
set.dds( channel=DDSCooling, freq=DetectFreq, amp=DetectCoolingAmp )
set_trigger( ddsCoolingApply )
    set_shutter( CoolingOn )
    set_shutter( RepumpOn )
    update( MolassesTime )
    set_inv_shutter( RepumpOn )
    set_inv_shutter( CoolingOn )
    update( WaitTime )
update( DarkTime )
set_counter( DetectCounter )
update()

### Probe/cool retention image ###
cyclenumber = 0
while cyclenumber < probecoolcycleR:
    set_shutter( SPCMGate )
    set_shutter( DetectOn )
    update( ProbeTime )
    set_inv_shutter( DetectOn )
    update( DelayTime )
    set_inv_shutter( SPCMGate )

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update( SPCMTime )
set_shutter( MOTOon )
update( CoolingTime )
set_inv_shutter( MOTOon )
update( SPCMTime )
update()
cyclenumber += 1
clear_counter()
initRemaining -= 1
update( WaitTime )

#################################################
### This code runs the above functions in sequence ###
#set_shutter(InitializationShutter)
update( DarkTime )
while not pipe_empty():
    apply_next_scan_point()

currentexperiment = 0
while currentexperiment < experiments:
    if LoadTime>0:
        LoadImage()
    if ProbeCoolTime>0:
        AdaptivePrepDetectImage()
currentexperiment += 1
update()

exit( endLabel )
Fig. A.20 Experimental Laser Frequencies. Diagram showing the laser frequencies used in the experiments of this thesis. The lockpoints are indicated as well as the AOM frequencies that shift the lasers.
Appendix B

Relevant atomic structure data

Table B.1: Transitions considered for calculating shifts in the $5S_{1/2}$ level of $^{87}$Rb using Eq. 3.5.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$\lambda$ (Å) [59]</th>
<th>$d$ (ea$_0$) [4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5S_{1/2} \rightarrow 5P_{1/2}$</td>
<td>7949.8</td>
<td>4.227</td>
</tr>
<tr>
<td>$5S_{1/2} \rightarrow 5P_{3/2}$</td>
<td>7802.4</td>
<td>5.977</td>
</tr>
<tr>
<td>$5S_{1/2} \rightarrow 6P_{1/2}$</td>
<td>4216.7</td>
<td>0.342</td>
</tr>
<tr>
<td>$5S_{1/2} \rightarrow 6P_{3/2}$</td>
<td>4203.0</td>
<td>0.553</td>
</tr>
<tr>
<td>$5S_{1/2} \rightarrow 7P_{1/2}$</td>
<td>3592.6</td>
<td>0.118</td>
</tr>
<tr>
<td>$5S_{1/2} \rightarrow 7P_{3/2}$</td>
<td>3588.1</td>
<td>0.207</td>
</tr>
<tr>
<td>$5S_{1/2} \rightarrow 8P_{1/2}$</td>
<td>3350.8</td>
<td>0.061</td>
</tr>
<tr>
<td>$5S_{1/2} \rightarrow 8P_{3/2}$</td>
<td>3348.7</td>
<td>0.114</td>
</tr>
<tr>
<td>$5S_{1/2} \rightarrow 9P_{1/2}$</td>
<td>3229.2</td>
<td>0.046</td>
</tr>
<tr>
<td>$5S_{1/2} \rightarrow 9P_{3/2}$</td>
<td>3228.0</td>
<td>0.074</td>
</tr>
</tbody>
</table>
Table B.2: Transitions considered for calculating shifts in the $5P_{3/2}$ level using Eq. 3.5. *This transition wavelength if listed as negative because the $5S_{1/2}$ state of lower in energy than the $6P_{3/2}$ state.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$\lambda$ (Å) [59]</th>
<th>$d$ (ea$_0$) [4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5P_{3/2} \rightarrow 4D_{3/2}$</td>
<td>15292.6</td>
<td>3.65</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 4D_{5/2}$</td>
<td>15293.7</td>
<td>10.89</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 5S_{1/2}$</td>
<td>-7802.4*</td>
<td>5.977</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 5D_{3/2}$</td>
<td>7761.6</td>
<td>0.59</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 5D_{5/2}$</td>
<td>7759.8</td>
<td>1.76</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 6S_{1/2}$</td>
<td>13668.8</td>
<td>6.048</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 6D_{3/2}$</td>
<td>6301.0</td>
<td>0.48</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 6D_{5/2}$</td>
<td>6300.1</td>
<td>1.42</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 7S_{1/2}$</td>
<td>7410.2</td>
<td>1.363</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 7D_{3/2}$</td>
<td>5724.6</td>
<td>0.355</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 8S_{1/2}$</td>
<td>6161.3</td>
<td>0.714</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 8D_{3/2}$</td>
<td>5431.8</td>
<td>0.272</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 8D_{5/2}$</td>
<td>5431.5</td>
<td>0.81</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 9S_{1/2}$</td>
<td>5653.7</td>
<td>0.355</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 9D_{3/2}$</td>
<td>5260.2</td>
<td>0.212</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 9D_{5/2}$</td>
<td>5260.2</td>
<td>0.593</td>
</tr>
<tr>
<td>$5P_{3/2} \rightarrow 10S_{1/2}$</td>
<td>5390.6</td>
<td>0.330</td>
</tr>
</tbody>
</table>
Appendix C

Mathematical model of state readout protocol

In this section, I describe a mathematical model for fluorescence-based state readout using the photon-threshold discrimination method described in this thesis. This model calculates the probability for an atom to scatter a certain number of photons \( n \) in a time \( t \). The model was initially developed by Dr. Geert Vrijsen in Professor Jungsang Kim’s research group to support their efforts in fast state readout of a single, trapped ion [88] and is reported in the thesis of Dr. Stephen Crain [21]. I detail it here as it is used in Ch. 6 to fit experimental data. The model is very general and I keep it so until the end of this section when I insert the rates discussed above.

I begin by considering the probability that an atom in a certain state \( i \) scatters \( n \) photons in time \( t \). This probability is described by a Poissonian distribution with a mean \( \lambda \) equal to the expected number of photons. The scattering rate \( R_i \) is multiplied by the time interval to give \( \lambda = R_i t \). The probability \( P_{ph}(n; t, R_i) \) is given by

\[
P_{ph}(n; t, R_i) = e^{-R_i t} \frac{(R_i t)^n}{n!}.
\]  

(C.1)
I assume that the atom starts in initial state $i$ with scattering rate $R_i$ and then at some point transitions (e.g. via off-resonant pumping) to state $f$, which has scattering rate $R_f$. This transition occurs at the rate $R_{orp}$ where the subscript indicates off-resonant pumping. I assume that once in state $f$, the atom does not transition back to state $i$. This assumption is justified if I consider short detection times, as is the case in the experiments described here. I note that the subscripts I use are different from those used in Ref. [21].

During this process, the atom scatters $n$ total photons in time $t$. It scatters $k$ photons while in state $i$ and then at time $\tau$ transitions to state $f$ and scatters the remaining $n-k$ photons while in state $f$. The probability of getting $k$ photons while the atom is in state $i$ is given by $P_{ph}(k; \tau, R_i)$ and the probability of getting $n-k$ photons from state $f$ is $P_{ph}(n-k; t-\tau, R_f)$. The off-resonant transition from state $i$ to state $f$ is an event that is also drawn from a Poissonian probability distribution. The probability of this transition occurring after any number of photons can be described by the probability density of a non-zero number of Poissonian events. If the transition occurs at time $\tau$, this probability is

$$\bar{p}_\tau(\tau, R_{orp}) = \frac{\partial}{\partial \tau} \sum_{n=1}^{\infty} \frac{e^{R_{orp} \tau} (R_{orp} \tau)^n}{n!} = \frac{\partial}{\partial \tau} (1 - e^{-R_{orp} \tau}) = R_{orp} e^{-R_{orp} \tau}. \quad (C.2)$$

Allowing the transition to occur at any time $\tau$ within the range 0 to $t$ and after any number of photons $k$ up to the total number collected $n$, the total probability of collecting $n$ photons in time $t$ from the process described here is given by

$$P_{tr}(n; t, R_i, R_f, R_{orp}) = \sum_{k=0}^{n} \int_{0}^{t} d\tau P_{ph}(k; \tau, R_i) \bar{p}_{orp}(\tau, R_{orp}) P_{ph}(n-k; t-\tau, R_f). \quad (C.3)$$
Inserting Eqs. C.1 and C.2 into Eq. C.3, the probability is found to be

\[
P_{tr}(n; t, R_i, R_f, R_{orp}) = \sum_{k=0}^{n} R_{orp} e^{-R_f t} \int_0^t \left( \frac{(R_i \tau)^k (R_f \tau)^{n-k}}{k!} \right) \frac{k!}{(n-k)!} e^{-(R_i+R_f+R_{orp}) \tau} d\tau.
\]

\[\text{(C.4)}\]

Equation C.4 is evaluated using integration by parts. Considering the limits of integration, we see that only the \(k = 0\) and \(k = n\) terms of the sum remain. However, given it’s structure, \(n\) iterations of integration by parts are required to find the solution and additional sums arise from the integration. The final solution is

\[
P_{tr}(n; t, R_i, R_f, R_{orp}) = \left( \frac{R_{orp}}{R_i - R_f + R_{orp}} \right) \left( \frac{R_i - R_f}{R_i - R_f + R_{orp}} \right)^n \times \left[ e^{-R_f t} \sum_{k=0}^{n} \frac{(R_i - R_f + R_{orp})^k (R_f t)^k}{k!} (R_i - R_f)^k - e^{-(R_i + R_{orp}) t} \sum_{k=0}^{n} \frac{(R_i - R_{orp} + R_f)^k (R_i t)^k}{k!} (R_i - R_f)^k \right].
\]

\[\text{(C.5)}\]

This is the probability of detecting \(n\) photons in time \(t\) when the atom undergoes a transition from state \(i\) to state \(f\). The sum of this probability over all possible photons numbers \(n\) is \(\sum_{n=0}^{\infty} P_{tr}(n; t, R_i, R_f, R_{orp}) = 1 - e^{-R_{orp} t}\), which is 1 minus the probability of undergoing zero transitions.

To calculate the full probability of collecting \(n\) photons in time \(t\), we must also consider the case where the atom not undergo a transition. Keeping this in mind, we find that the total probability is

\[
P_{Tot}(n; t, R_i, R_f, R_{orp}) = e^{-(R_{orp} t)} P_{ph}(n; R_i t) + P_{tr}(n; t, R_i, R_f, R_{orp}), \quad \text{(C.6)}
\]

where the first term is the probability of the atom scattering \(n\) photons while in state \(i\) and not undergoing a transition and the second term is the probability of scattering \(n\) photons while undergoing the sort of transition described above.
Bibliography


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Biography

Margaret Eileen Shea (Meg) was born in Boston, MA on January 8, 1986. She was raised in Chapel Hill, NC where she attended lower and high school before attending the North Carolina School of Science and Mathematics (NCSSM). Meg graduated from NCSSM in 2004 and matriculated at Yale University. While enrolled at Yale, she spent a semester at Peking University in Beijing, China. In 2008, Meg graduated from Yale with a B.S. cum Laude in Physics with departmental honors. She then moved to Oundle, England where she spent three years teaching physics and math to 13-18 year-old students at Oundle School. While there, Meg also did particle physics phenomenology research at Cambridge University with Prof. W.J. Stirling.

In 2011, Meg moved back to NC and matriculated at Duke University. She spent her first year working in experimental high energy physics with Prof. Al Goshaw before joining Prof. Daniel Gauthier’s quantum electronics laboratory in 2013. While at Duke, Meg was name a University Scholar in Physics by the Graduate School and received the Townes-Perkins-Elmer Fellowship from the Physics Department. She was the Secretary and then President of the Physics Department’s Graduate Student Organization, a Graduate Mentor for the University Scholar’s Program, and Vice President of Duke’s Student Chapter of the Optical Society of America.

Meg earned her M.S. in Electrical and Computer Engineering in 2016, under the supervision of Prof. Jungsang Kim. She received her PhD in Physics in 2018 for her work in single atom trapping and neutral atom quantum state readout.