

SUFFICIENT CONDITIONS FOR EXTRACTING LEAST COST
RESOURCE FIRST

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KEMP AND LONG [1] DEMONSTRATE that it may be preferable to exploit high and low cost resource deposits simultaneously and not in sequence as is typically assumed in the resources literature. They show that it is desirable to delay extraction from low cost pools in order to smooth consumption over time, if the resource in the ground is society's only store of wealth. This paper considers a model in which extracted resources can be converted into capital which may either be consumed or stored to provide for consumption later on. We find that a sufficient condition for the strict sequencing of extraction to be optimal is that stored capital be productive so that it can be used to produce additional capital.

Following Kemp and Long, we assume the economy has N deposits, \bar{q}_i , of oil ($i = 1, 2 \dots N$), a constant labor force \bar{l} , and a higher cost substitute for oil. A unit of labor can extract a_i units of oil from the i th deposit, or it can produce one equivalent unit of energy from the perfect substitute. The deposits are ordered so that $a_1 > a_2 > \dots > a_N > 1$. Denote $e_i(t)$ as the amount extracted from deposit i , $x_1(t)$ as the amount extracted from all deposits, $x_2(t)$ as the amount produced by the substitute, and $l_i(t)$ as the labor allocated to deposit i . Then,

$$(1) \quad x_1(t) = \sum_i e_i(t) = \sum_i a_i l_i(t),$$

$$(2) \quad x_2(t) = \bar{l} - \sum_i l_i(t) = \bar{l} - \sum_i e_i(t)/a_i$$

We also assume the economy can convert energy directly into capital which may either be consumed or stored. For simplicity we assume the rate of change in the capital stock is

$$(3) \quad \dot{K}(t) = x_1(t) + x_2(t) + \delta K(t) - C(t),$$

where $K(t) \geq 0$ is the stock of capital, $C(t)$ is the rate that society consumes the stock, and δ is the exogenous rate of capital growth (or depreciation).²

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²In a different context Solow and Wan [3] also demonstrate that extracting low cost resources first is optimal when capital is productive. In their model capital grows according to

$$\dot{K} = F(K(t), R(t), L(t)) - C(t)$$

where F is a production function for new capital, whose inputs are, respectively, the services of capital, the flow of the extracted resource, and labor. In this case capital is productive if $\partial F(\cdot)/\partial K(t) > 1$.

Let $u(C(t))$ be the utility derived from consumption, and assume that u is a smooth, strictly increasing, and concave function. Then the planner seeks to

$$(4) \quad \text{maximize}_{\{e_i(t)\}, \{C(t)\}} \int_0^{\infty} e^{-\rho t} u(C(t)) dt$$

subject to (1)–(3), $\dot{q}_i = -e_i$, $q_i(0) = \bar{q}_i$, $K(0) = 0$, $\rho > \delta$, and the non-negativity of $e_i(t)$, $C(t)$, and $K(t)$. A solution to (4) exists and is characterized by the following proposition.³

PROPOSITION: For $\delta > 0$, the optimal program involves maximizing production by exploiting the least cost energy sources first.

For the case where $\delta \leq 0$, it can be shown (see Lewis [2, pp. 10–14]) that it is optimal for $K(t) = 0$ for all t , though other optimal programs involving $K(t) > 0$ may exist when $\delta = 0$. In this situation, efficient production does not require storing the resource in the form of capital so that the Kemp–Long results, which are derived assuming that storage is not possible, continue to hold.

PROOF: Contrary to our assertion, assume there is some interval (t_1, t_2) where the rate of output is not maximized. Consider a small continuous variation in output, $\Delta x \equiv \Delta x_1 + \Delta x_2$, during (t_1, t_2) that shifts production from the end to the start of the interval such that

$$(5) \quad \Delta x(t) \begin{cases} \geq 0 \\ < 0 \end{cases} \quad \text{for } t \begin{cases} \in (t_1, \hat{t}) \\ \in (\hat{t}, t_2) \end{cases} \quad \text{and} \quad \int_{t_1}^{t_2} \Delta x(z) dz = 0$$

with $\Delta q_i(t_2) = 0$ for all i . Suppose for now that $\Delta C(t) = 0$ during (t_1, t_2) . The feasibility of this assumption is verified below. Hence $\Delta \dot{K}(t) = \Delta x(t) + \delta \Delta K(t)$ by (3). Since whenever the capital stock is positive it grows at the rate δ , an increase or decrease in the output flow of $\Delta x(z)$ at time z changes the stock at time $t \geq z$ by the amount $\Delta x(z)e^{\delta(t-z)}$. Clearly $\Delta K(t) > 0$ for $t \in (t_1, \hat{t})$. For $t \in (\hat{t}, t_2)$, equation (5) implies

$$(6) \quad \begin{aligned} \Delta K(t) &= \int_0^{\hat{t}} \Delta x(z) e^{\delta(t-z)} dz + \int_{\hat{t}}^t \Delta x(z) e^{\delta(t-z)} dz \\ &> e^{\delta(t-\hat{t})} \left[\int_0^{\hat{t}} \Delta x(z) dz + \int_{\hat{t}}^t \Delta x(z) dz \right] \geq 0, \end{aligned}$$

since $\Delta x(z) > (<) 0$ for $z < \hat{t}$ ($> \hat{t}$). Hence $\Delta K(t) > 0$ for $t \in (t_1, t_2)$, so that $\Delta C(t) = 0$ is in fact feasible. But according to (5) and (6) $\Delta K(t_2) > 0$ and $\Delta q_i(t_2) = 0$ without changing consumption during (t_1, t_2) . Hence consumption could be increased at t_2 , while maintaining it at the same level at all other times, by allowing society to eat the extra capital, $\Delta K(t_2)$. This shows that the original program could not have been optimal. But, since an optimal program exists, it must involve extracting at a maximum rate, thus completing our proof.

CONCLUSION

In partial equilibrium models of resource production, it seems to be almost transparent that it is efficient to exploit low cost deposits first. Implicit in such models is the

³Lewis [2] contains a more complete description of such optimal programs.

assumption that the money saved today from exploiting low cost deposits can be profitably invested to yield future income. Our results suggest that in a general equilibrium analysis, the desirability of extracting from the least cost energy sources first, depends on whether or not the extracted resource can be converted into productive capital that earns a positive return.

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