Switching Currents Limited by Single Phase Slips in One-Dimensional Superconducting Al Nanowires

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An aluminum nanowire switches from superconducting to normal as the current is increased in an upsweep. The switching current (I_s) averaged over upsweeps approximately follows the depairing critical current (I_c) but falls below it. Fluctuations in I_s exhibit three distinct regions of behaviors and are nonmonotonic in temperature: saturation well below the critical temperature T_c, an increase as $T^{2/3}$ at intermediate temperatures, and a rapid decrease close to T_c. Heat dissipation analysis indicates that a single phase slip is able to trigger switching at low and intermediate temperatures, whereby the $T^{2/3}$ dependence arises from the thermal activation of a phase slip, while saturation at low temperatures provides striking evidence that the phase slips by macroscopic quantum tunneling.

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One of the fundamental questions in one-dimensional (1D) superconductivity is the nature of the current-induced transition from the superconducting to the normal state. Ideally, the maximum current (called the critical current I_c) is set by the depairing mechanism, where the Cooper pairs are destroyed by the electron velocity. Experimentally, however, the depairing limit is difficult to achieve. The maximum current is often limited by either the self-generated field in 3D samples or the motion of magnetic flux in 2D thin films [1]. It was believed depairing I_c may be achievable in a narrow superconducting wire. Nevertheless, fluctuation effects, specifically the spatio-temporal fluctuations of the order parameter known as phase slips (explained below), can induce premature switching [2,3]. A recent study reported phase-slip-induced switching [4]. Unexpectedly, it was found that the switching current (I_s) fluctuations increased monotonically with decreasing temperature. To date, clear evidence is lacking as to whether a single phase slip is capable of inducing switching. Establishing the relationship between individual phase slips and switching provides a tool to study phase slips, to help establish whether they are caused by thermal fluctuations or by macroscopic quantum tunneling [5–12]. Such studies not only elucidate the fundamental physics, e.g., a superconductor-insulator transition caused by quantum-phase slip (QPS) [9,13,14], but also provide the basis for applications from a new current standard to quantum qubits [15,16].

The phase slip is a topological event, during which the superconducting order parameter phase between two adjacent regions of the superconductor changes by 2π over a spatial distance of the order of the coherence length. To understand the destruction of supercurrent in a nanowire, one may gain valuable insight from phase slips in a Josephson junction [1]. In both systems, the motion of the phase is described by a tilted washboard potential [Fig. 1(a)]. Josephson junctions are classified within a resistively and capacitively shunted Junction (RCSJ) model as either under- or overdamped, depending on whether the quality factor, $Q = \sqrt{2eI_cC/\hbar R}$, is greater or less than 1. An underdamped junction is readily driven normal by a single phase slip event; the phase keeps running downhill subsequent to overcoming the free-energy barrier, as damping is insufficient to retrap into a local minimum. Thus, the junction exhibits zero resistance up to I_s and the voltage is hysteretic in a current-voltage (IV) measurement. In an overdamped junction, the phase

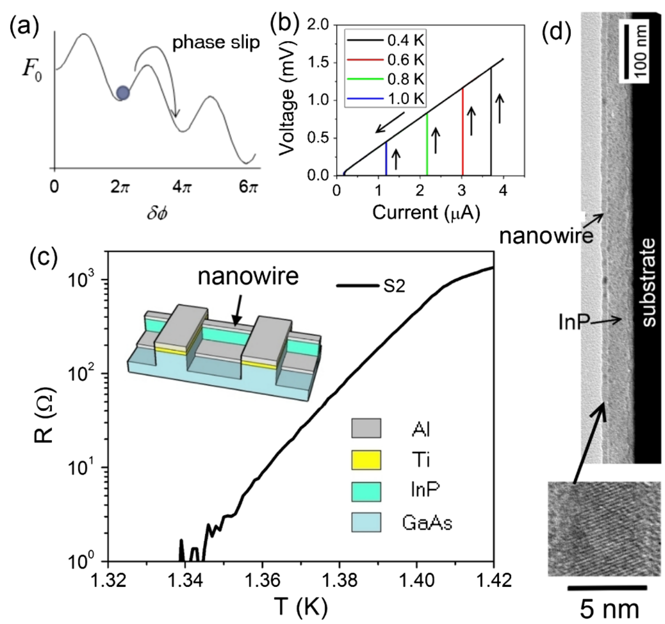

FIG. 1 (color online). (a) Schematic of a washboard potential. (b) Hysteretic IV curves—right to left 0.4–1 K, and (c) resistive transition versus temperature for nanowire $S2$. (d) Transmission electron microscope images of a typical nanowire. The long and uniform nanowire sits on an InP ridge.

moves diffusively between minima; the IV is nonlinear and hysteresis is often not present. The different situations thus lead to differing characteristics.

Because of an extremely small capacitance, a nanowire is believed to be heavily overdamped ($Q \ll 1$). Recently, experimental evidence has accumulated indicating that heating can also lead to hysteretic behavior in overdamped superconducting-normal-superconducting bridges [17]. Here, we report on measuring the fluctuations of I_s in Al superconducting nanowires. In stark contrast to the previously reported monotonic increase with decreasing temperature [4], the fluctuations in our Al nanowires are nonmonotonic with three distinct regions of behaviors. Below $\sim 0.3T_c$, a clear saturation of the fluctuations is observed indicating the switching to be caused by a QPS. At intermediate temperatures ~ 0.3–$0.6T_c$, the fluctuations increase as $T^{2/3}$, signifying that switching is caused by a thermal phase slip (TAPS). At high temperatures above $\sim 0.6T_c$, the rapid decrease of fluctuations points to multiple TAPSs triggering the switching. Although in appearance this behavior is reminiscent of an underdamped Josephson junction, for $T < 0.6T_c$, quantitative estimation demonstrates that heat generated by a single phase slip likely causes a thermal runaway, triggering switching [3]. Because a single QPS or TAPS is sufficient to trigger switching close to I_s, the resistance of the SC state at current levels below I_s can remain zero much of the time, with occasional jumps as a rare single phase slip event occurs.

Five nanowires were studied (Table I). Each end of a nanowire is connected to a large 2D superconducting pad rather than to a normal metal pad [12]. $S3$ was obtained by further oxidizing the surface of $S2$. The fabrication was described previously [18]. The superconducting coherence length at base temperature $\xi_0 \sim 100$ nm. The length of the wires ranges from $15\xi_0$ to $100\xi_0$, while the width is roughly $1/10\xi_0$. The resistivity (4.5 $\mu\Omega$ cm; same as co-evaporated films), along with the inverse proportionality to 20% accuracy between normal resistance per unit length and I_s at base temperature, indicates that there are no resistive tunneling barriers. These nanowires are in the fully metallic limit ($k_F l \sim 60 \gg 1$, where k_F is the Fermi wave number, and l the electron mean free path), in contrast to those studied by Sahu *et al.* [4] which appear to be grainy with $k_F l$ being much closer to 1, and for which Coulomb effects may be important.

TABLE I. Specifics of the samples.

Sample	$S1$	$S2$	$S3$	$S4$	$S5$
Length (μm)	1.5	10	10	10	3
Width (nm)	10.0	9.3	8.4	7.0	5.4
R/L[a] (KΩ/μm)	0.33	0.38	0.50	0.82	1.17
I_m[b] (μA)	4.7	4.1	3.3	2.3	1.4
T_s[c] (K)	1.36	1.33	1.33	1.25	1.03

[a]Resistance per unit length of the nanowires in normal state.
[b]Maximum switching current measured.
[c]Transition temperature, below which the nanowires show zero measured resistance.

$S1$ and $S2$ were measured in a ^{3}He system and $S3$, $S4$, and $S5$ in a dilution refrigerator. To fully remove interference from unwanted noise, each electrical line is equipped with RL filters (1 MHz cutoff) at room temperature, Thermocoax cables (1 GHz cutoff) extending to the mixing chamber of the dilution refrigerator, and RC filters (34 kHz cutoff) at the mixing chamber. For the current sweep a sawtooth waveform was used at a repetition of 10 Hz. The upsweep ramp rate was 50 μA/s for $S1$, $S2$, and $S3$ and 25 μA/s for $S4$ and $S5$. Decreasing the rate by a factor of 10 yielded nearly identical results. Immediately after a voltage jump, the current was turned off, reducing the resistive heating time in the normal state to less than 100 μs, and ensuring adequate time to rethermalize the sample ($\sim 10^{-7}$ s) before the next cycle.

At vanishing current, the nanowires become superconducting below the switching temperature $T_s(I \to 0)$. In Fig. 1(c), the resistive transition is broadened due to TAPSs. To measure I_s fluctuations, we performed $\sim$10 000 I-V sweeps at each temperature, recorded the upsweep I_s and plotted the histogram [Fig. 2(a)]. The probability density function $P(I)$ obeys the expression (suppressing the subscript in I_s):

$$P(I) = \Gamma(I)(dI/dt)^{-1}\left(1 - \int_0^I P(u)du\right), \tag{1}$$

where dI/dt is the ramping rate and $\Gamma(I)$ is the switching rate at current I [19]. If a single phase slip triggers switching, the switching and the phase slip rates are identical, enabling us to extract the phase slip rate from the distribution. Using the single TAPS rate $\Gamma(I) \sim \exp(F(T, I)/k_B T)$, where $F(T, I)$ is free energy barrier, and linearizing the current dependence of $F(T, I)$, the solution for $P(I)$ is in the form of a Gumbel distribution [20]. An example of the fitting to this functional form to data is shown in the inset to Fig. 2(a). For each distribution, we deduce the mean value $\langle I_s \rangle$ and the standard deviation δI_s, as shown in Figs. 2(b) and 2(c), respectively.

In the single TAPS regime, the fluctuation in I_s is approximately proportional to

$$\delta I_s \sim (k_B T/\phi_0)^{2/3} I_c(T)^{1/3} \tag{2}$$

where $I_c(T)$ is the depairing I_c at temperature T. The $T^{2/3}$ is from the exponent in the current dependence of $F(T, I)$, $F(T, I) = F(T)(1 - I/I_c)^{3/2}$ [21]. In all samples, δI_s in the intermediate temperature range ($\sim 0.3T_c$–$0.6T_c$) can be fitted by Eq. (2) very well, shown in Fig. 2(c). The good agreement between data and theoretical fittings indicates that switching is induced by a single TAPS.

When the nanowire has narrower width, it becomes more probable for the phase to undergo a macroscopic

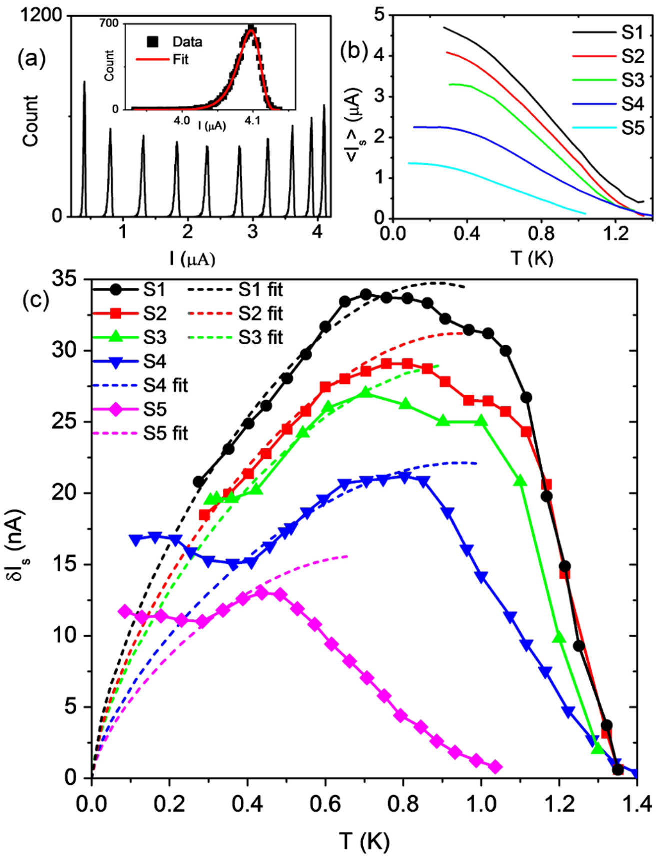

FIG. 2 (color online). (a) I_s distribution for $S2$ at different temperatures: right to left: 0.3 K to 1.2 K in 0.1 K increments. The inset shows the 0.3 K distribution, fitted by the Gumbel distribution [20]. (b) $\langle I_s \rangle$ versus temperature—top to bottom $S1$–$S5$. (c) Symbols: δI_s versus temperature. Dashed lines: fittings in the single TAPS regime using Eq. (2). An additional scale factor of 1.25, 1.11, 1.14, 0.98, and 1.0, for $S1$–$S5$, respectively, (average 1.1 ± 0.1), is multiplied to match the data. Alternatively, a ~6% adjustment in the exponent fits the data without the scale factor.

quantum tunneling process through the barrier. The QPS rate is proportional to $\exp(-\alpha F(T, I)/\Delta)$, where α is constant of order unity, and different possibilities for Δ have been proposed. These include $\hbar/\tau_{\rm GL}$, with $\tau_{\rm GL}$ the Ginzburg-Landau time, and the superconducting gap [4,8,22]. In $S4$ and $S5$, we find a slight increase of the fluctuations with decreasing temperature in the QPS regime (below $\sim 0.3 T_c$), consistent with Δ scaling as the superconducting gap.

To check the approximate linearized expressions for δI_s and $\langle I_s \rangle$ in the single TAPS regime, a full numerical simulation is performed to solve Eq. (1), as shown in Fig. 3. The TAPS rate is given by

$$\Gamma_{\rm TAPS}(T, I) = \Omega_{\rm TAPS} \exp\left(\frac{-F(T, I)}{k_B T}\right), \qquad (3)$$

where $\Omega_{\rm TAPS} = L/\xi_T \sqrt{F(T, I)/k_B T}/\tau_{\rm GL}$ is the attempt frequency, $\tau_{\rm GL} = \pi\hbar/(8k(T_c - T))$, ξ_T is the

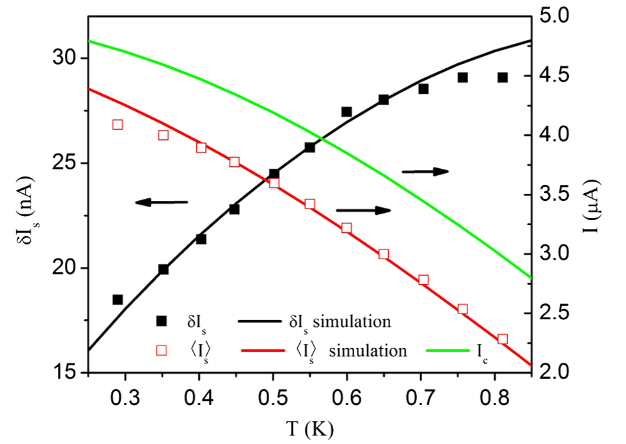

FIG. 3 (color online). δI_s and $\langle I_s \rangle$ fitted by TAPS model: Squares: experimental data for $S2$; solid lines: simulation. $I_c(0) = 5\ \mu$A and $T_c = 1.5$ K are two fitting parameters. A constant scaling factor ~1.15 is multiplied to the calculated δI_s to match the data.

superconducting coherence length at temperature T, and $F(T, I) = \sqrt{6}\hbar/(2e) I_c(T)(1 - I/I_c(T))^{3/2}$ is the free-energy barrier [4]. The zero current free-energy barrier, $F(T) = \sqrt{6}\hbar I_c(T)/2e$, bears similarity to the Josephson energy $E_J = \hbar I_c/2e$ [2]. To extend to the entire temperature range, a phenomenological $I_c(T) = I_c(0)(1 - (T/T_c)^2)^{3/2}$ was employed [23]. Good agreement is achieved up to 0.8 K ($\sim 0.6 T_c$) in $S2$ as shown in Fig. 3. Other samples exhibit a similar agreement.

Above 0.8 K, δI_s falls below the simulated value, at first decreasing gradually, then rapidly. This behavior is associated with the need for more-than-one phase slips to heat up the wire as the current drops [3]. A similar decrease is familiar in Josephson junctions within the phase diffusion regime, where multiple phase slips are required to induce switching [24,25].

To achieve a consistent picture, it must be demonstrated that heat generated by a single phase slip is sufficient to raise the local temperature and trigger switching. A single phase slip deposits an energy $\phi_0 I$ in a time $\phi_0 I/(I^2 R_{\rm core}) \sim 50$ ps, where $\phi_0 = h/2e$ is the flux quantum, and $R_{\rm core}$ is the normal state resistance of the phase slip core. Because of the low $R_{\rm core}$ ($\sim 100\ \Omega$), this energy is deposited predominantly in the normal core rather than removed via plasmon emission [14]. Heat loss through the InP ridge is also ineffective. Immediately after the phase slip, the hot normal electrons are decoupled from the superconducting electrons for a duration the charge-imbalance time $\tau_{\rm imb}$. Within this time, heat diffuses out primarily within the normal electron component to a charge-imbalance distance, $\Lambda_{\rm imb} \sim \sqrt{D\tau_{\rm imb}}$, where D is diffusion coefficient, before it can be transferred to the superconducting electrons to raise their effective temperature to T_f [3]. Following transfer, this entire region of size $2 \times \Lambda_{\rm imb}$ either becomes normal or returns to

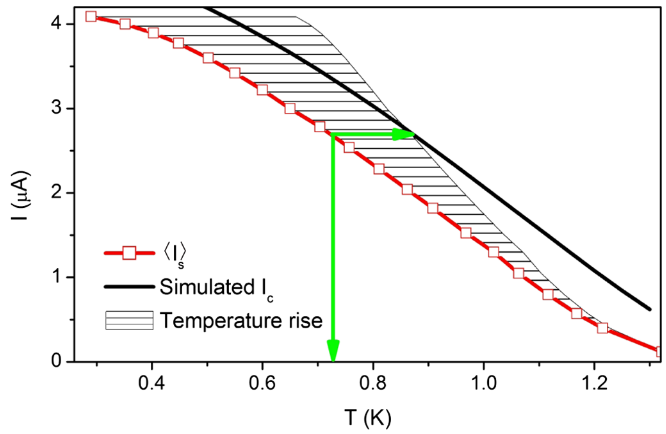

FIG. 4 (color online). Temperature boundary between single- and multiple-phase-slip regimes. The red curve is measured $\langle I_s \rangle$. The thick black curve is simulated depairing I_c. Following the black shaded region toward the right, the thin black curve shows the electronic temperature rise due to a single phase slip event at current I according to Eq. (4). The point at which the thick black curve exits the shaded area marks a change from single- to multiple-phase-slip regimes. The corresponding ambient temperature boundary 0.7 K can be found by moving horizontally to the left to $\langle I_s \rangle$ (shown by green arrows).

superconducting depending on whether I_c at the elevated electronic T_f is exceeded or not [26]. If exceeded, this region becomes normal; its resistance contributes further to heating, causing a thermal runaway. T_f can be estimated as

$$T_f = \sqrt{T_0^2 + \phi_0 I/(\gamma A \Lambda_{\rm imb})}, \quad (4)$$

where $\gamma = C_v/T = 135\ \mathrm{J/(m^3\,K^2)}$, C_v is the specific heat of normal Al [27], A is cross section area, and T_0 is the ambient temperature. Setting $\Lambda \sim 0.8\ \mu$m, Fig. 4 shows that a boundary between the single TAPS regime and the multiple TAPS regime occurs around 0.7 K ($\sim 0.3T_c$), consistent with our experimental result. This value for $\Lambda_{\rm imb}$ is close to the findings in recent experiments on Al wires of sub-μm diameter [28], and is expected to be temperature independent [29]. The same analysis yields a boundary of 0.7 K, 0.6 K, 0.6 K, and 0.45 K for $S1$, $S3$, $S4$, and $S5$, respectively, consistent with Fig. 2(c).

In summary, we demonstrate that 1D Al superconducting nanowires can be switched into the normal state by a single phase slip, over a sizable temperature range. At low T, QPS-induced switching was found in the narrower wires. In the single TAPS regime, I_s fluctuations are proportional to $T^{2/3}$. The fluctuations decrease at higher temperature, where multiple phase slips are needed to trigger switching. Heating by phase slips appears to play a major role in the switching process. The behavior found is likely relevant to nanowires of different materials, due to the commonality of restricted geometry.

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