Factor-Hoarding and the Propagation of Business-Cycle Shocks

BY CRAIG BURNSIDE AND MARTIN EICHENBAUM

This paper analyzes the role of variable capital-utilization rates in propagating shocks over the business cycle. The model on which our analysis is based treats variable capital-utilization rates as a form of factor-hoarding. We argue that variable capital-utilization rates are a quantitatively important source of propagation to business-cycle shocks. With this additional source of propagation, the volatility of exogenous technology shocks needed to explain the observed variability in aggregate U.S. output is significantly reduced relative to standard real-business-cycle models. (JEL E32, E22, C52)

This paper analyzes the role of variable capital-utilization rates in propagating shocks over the business cycle. To this end, we formulate and estimate an equilibrium business-cycle model in which cyclical capital-utilization rates are viewed as a form of factor-hoarding (see Burnside and Eichenbaum [1994] for a brief review of different factor hoarding models). We find that variable capital-utilization rates substantially magnify and propagate the impact of shocks to agents’ environments. The strength of these propagation effects is evident in the dynamic response functions of various economy-wide aggregates to shocks in agents’ environments, in the statistics that we construct to summarize the strength of the propagation mechanisms in the model, and in the volatility of exogenous technology shocks needed to explain the observed variability in aggregate U.S. output. Other authors have argued that standard real-business-cycle (RBC) models fail to account for certain features of the data because they do not embody quantitatively important propagation mechanisms. These features include the observed positive serial correlation in the growth rate of output (Timothy Cogley and James M. Nason, 1995) and the correlation between the forecastable component of real output and various other economic aggregates (Julio J. Rotemberg and Michael Woodford, 1996). Allowing for variable capital-utilization rates substantially improves the ability of the model to account for these features of the data.

It is well known that standard real-business-cycle models do not embody quantitatively important propagation mechanisms (see e.g., Lawrence J. Christiano, 1988; Cogley and Nason, 1995). As a result, these models must rely on highly variable, exogenous, aggregate technology shocks to account for the observed fluctuations in aggregate economic activity. The primary evidence for this type of shock consists of interpreting movements in the Solow residual as reflecting stochastic movements in the aggregate production technology. But, by now, there is abundant evidence casting doubt on this interpretation. In addition to reviewing this evidence, John H. Cochrane (1994) discusses the difficulty in finding quantitatively large sources of aggregate shocks (monetary or real) to the postwar U.S. economy. This serves as our motivation for understanding how the shocks that do occur are magnified and propagated over time. In this paper we pursue one obvious source of propagation, variable capital utilization (as well as labor-hoarding), and investigate its quantitative importance in an otherwise standard RBC model.

To model variable capital utilization, we assume that the aggregate technology for producing goods depends on effective capital

* Burnside: The World Bank, 1818 H Street N.W., Washington, DC 20433; Eichenbaum: Department of Economics, Northwestern University, Evanston, IL 60208; National Bureau of Economic Research, and Federal Reserve Bank of Chicago. We thank two anonymous referees for helpful comments and suggestions. Any views expressed here are ours and not necessarily those of the World Bank or any part of the Federal Reserve System.
services and effective hours of work. The latter is defined as labor effort times total hours of work. The former is defined as the capital-utilization rate times the stock of capital. The rate at which capital depreciates is assumed to be a function of the capital-utilization rate. This is consistent with the assumptions made in Jeremy Greenwood et al. (1988), among others. In equilibrium, firms will, on average, choose to "hoard" capital; that is, they will set capital-utilization rates to less than capacity. Because of this, they can immediately increase the effective stock of capital in response to shocks that raise the marginal product of capital. In the standard model, firms would have to wait at least one period to raise the stock of capital. To the extent that capital takes time to build, they would have to wait even longer.

Our main quantitative findings can be summarized as follows. First, according to our estimates, capital utilization moves in a systematic way over the business cycle, rising by about 1.8 percentage points during NBER expansions and falling by 2.1 percentage points during NBER recessions. Second, capital utilization rates are very volatile relative to the stock of physical capital. Specifically, we estimate that the standard deviation of the growth rate of effective capital services is roughly 4.5 times higher than the standard deviation of the growth rate of the stock of physical capital. Concentrating on cyclical movements in the stock of capital gives a misleading picture of cyclical movements in effective capital services. Third, we find that allowing for variable capital-utilization rates and labor-hoarding leads to roughly a 33-percent reduction in the estimated standard deviation of the innovation to technology shocks. Most of this reduction is attributable to cyclical movements in the capital-utilization rate. Fourth, we find that variable capital-utilization rates substantially magnify and propagate the impact of shocks to agents' environments. The magnification and propagation effects induced by factor-hoarding are sufficiently large that the model does as well as standard RBC models in accounting for the volatility of output. This is true despite the fact that the estimated volatility of technology shocks is much smaller in our model than in the standard RBC model. What distinguishes the models is not whether, but how they account for the volatility of output. Virtually all of the output movements in standard RBC models reflect the direct impact of technology shocks on the aggregate production technology. With variable capital-utilization rates, a substantially smaller proportion of the standard deviation of output is due to the direct impact of technology shocks.

Fifth, we find that our model is able to account for various features of postwar U.S. business cycles that standard RBC models cannot account for. Cogley and Nason (1995) highlight the weak propagation mechanisms in standard RBC models by focusing on the autocorrelation function of the growth rate of output. They show that, to a first approximation, output movements in the model reflect only exogenous technology shocks. Therefore, when these shocks are modeled as a random walk, output is very close to being a random walk. This implication is counterfactual: unlike the Solow residual, U.S. output growth is positively serially correlated. With variable capital-utilization rates, the model can simultaneously account for the univariate time-series properties of the growth rate of output and the Solow residual.

Rotemberg and Woodford (1996) have highlighted the weak propagation mechanisms in standard RBC models by focusing on the correlation between expected output growth (at various horizons), with objects like the growth rates of output and average productivity, and the logarithms of hours worked and the ratio of consumption to output. In Burnside and Eichenbaum (1994), we show that the model considered in this paper does a substantially better job of accounting for these statistics than the standard RBC model.

The remainder of this paper is organized as follows. Section I displays our model. Section II discusses our econometric methodology. Our empirical results are reported in Section III, and concluding remarks are contained in Section IV.

I. The Model

In this section, we present a variant of Gary D. Hansen's (1985) indivisible-labor model, modified to incorporate factor-hoarding in the form of variable capital-utilization rates and varying labor effort. Different approaches to modeling time-varying capital utilization have
been pursued in the literature (see Susanto Basu and Miles Kimball [1995] for a critical overview). Here we pursue a particularly simple approach, which has the feature that units of effective labor input co-move positively with capital utilization because the two are complements in production. Below, we discuss the robustness of our major results to an alternative specification in which capital utilization is defined as the workweek of capital.

The model economy is populated by a large number of infinitely-lived individuals. To go to work, an individual must incur a fixed cost of $\zeta$ hours. Once at work, an individual stays for a fixed shift length of $f$ hours. The time-$t$ instantaneous utility of such a person is given by

$$\ln(C_t) + \theta \ln(T - \zeta - W_t f).$$

Here, $T$ denotes the individual's time endowment, $C_t$ denotes time-$t$ privately purchased consumption, $\theta \equiv 0$, and $W_t$ denotes the time-$t$ level of effort. According to (1) labor suppliers care about effective hours of work. The time-$t$ instantaneous utility of a person who does not go to work is given by $\ln(C_t) + \theta \ln(T)$.

Time-$t$ output is produced according to

$$Y_t = (K_t U_t)^{1-\alpha} (N_t f W_t X_t)^{\alpha},$$

where $0 < \alpha < 1$, $K_t$ denotes the beginning of time $t$ capital stock, $U_t$ represents the capital utilization rate, $N_t$ denotes the number of individuals at work during time $t$, and $X_t$ represents the time-$t$ level of technology. According to (2), what matters for producing output is the total amount of effective capital, $K_t U_t$, and total effective hours of work, $N_t f W_t$. We think of (2) as capturing, in a parsimonious way, the notion that capital services and labor input are complements in production.

Production function (2) can also be rationalized as the reduced-form representation of a particular technology relating the capital-utilization rate to the workweek of capital. Specifically, suppose that $N_t$ and $K_t$ are predetermined from the perspective of time $t$. Also, let $U_t$ denote $f$ times the number of shifts that the firm operates during time $t$. For simplicity, assume that $U_t$ has continuous support. Suppose that the amount of output produced by a given worker in a unit of time is given by the function $F[K_t/(N_t f U_t), W_t]$ where $F(\cdot)$ is homogeneous of degree one, concave, and differentiable. The variable $K_t/(N_t f U_t)$ denotes the amount of capital at the disposal of the typical individual during his shift. For example, if the firm runs one shift, then $U_t = f$, and the amount of capital at the disposal of each worker equals $K_t/N_t$. If the firm runs two shifts, then $U_t = 2f$, and the amount of capital at the disposal of a given individual equals $2K_t/N_t$. Given these assumptions, total time-$t$ output, $Y_t$, produced by $N_t$ workers, each working $f$ hours, is equal to $Y_t = N_t f K_t/(N_t f U_t), W_t$. When the function $F(\cdot)$ is Cobb-Douglas, we obtain production-function (2). Thus, with this particular rationalization of (2), changes in capital utilization correspond to changes in the number of shifts that firms employ.

As in Greenwood et al. (1988), we suppose that using capital more intensively increases the rate at which capital depreciates. Specifically, we assume that the time-$t$ depreciation rate of capital, $\delta_t$, is given by

$$\delta_t = \delta U_t^\phi,$$

where $0 < \delta < 1$ and $\phi > 1$. The stock of capital evolves according to

$$K_{t+1} = (1 - \delta_t) K_t + I_t,$$

where $I_t$ denotes time-$t$ gross investment. Under our assumptions, firms will not, in general, find it optimal to fully utilize the stock of capital, preferring to “hoard” some capital so that they can use it more intensively when the returns to doing so are unusually large.

We assume that the level of technology, $X_t$, evolves according to

$$X_t = X_{t-1} \exp(\gamma + \nu_t),$$

where $\nu_t$ is a serially uncorrelated process with mean 0 and standard deviation $\sigma_v$. The aggregate resource constraint is given by

$$C_t + I_t + G_t \leq Y_t,$$

where $G_t$ denotes the time-$t$ level of government consumption. We assume that $G_t$ evolves according to

$$G_t = X_t \exp(g_t)$$
where \( g_t \) evolves according to
\[
(8) \quad g_t = \mu (1 - \rho) + \rho g_{t-1} + \varepsilon_t,
\]
where \( \mu \) is a scalar, \( |\rho| < 1 \), and \( \varepsilon_t \) is a serially uncorrelated process with mean 0 and standard deviation \( \sigma_\varepsilon \).

In the presence of complete markets, it is straightforward to show that the competitive equilibrium of this economy corresponds to the solution of the following social-planning problem:
\[
(9) \quad \text{Max} \left\{ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln (C_t) + \theta N_t \ln (T - \zeta - W_t \delta) + \theta (1 - N_t) \ln (T) \right] \right\}
\]
subject to \( (2), (3), \) and \( (5)-(8) \) by choice of contingency plans for \( \{ C, K_{t+1}, N_t, U_t, W_t; t \geq 0 \} \). Here we have normalized the number of agents in the economy to 1.

To allow for a simple form of labor hoarding, we proceed as in Burnside et al. (1993) and assume that \( N_t \) must be chosen before \( X_t \), and \( g_t \), are seen. Let \( \Omega^* \) denote the information set that includes the lagged values of all time-\( t \) variables in the model as well as \( K_t \). Let \( \Omega^* \) consist of the union of \( \Omega \) and \( (X_t, g_t) \). We assume that \( N_t \) is chosen on the basis of \( \Omega^* \), while \( \{ C_t, K_{t+1}, U_t, W_t \} \) is chosen on the basis of \( \Omega^*_t \). This formulation of the problem incorporates the ideas that firms must make employment decisions conditional on their views about the future state of demand and technology and that firms cannot adjust the number of employees instantly in response to the shocks affecting their environment.

With this specification of the information set there are a variety of market structures that can support the allocation emerging from the social planner’s problem. Here we briefly discuss the one most similar to the market structure underlying the Hansen (1985) and Richard Rogerson (1988) indivisible-labor models. In particular we assume that there is a market for lotteries, in which agents choose their probability of employment and an associated state-contingent amount of effort. The probability of employment chosen depends on the elements of \( \Omega^*_t \), whereas effort is a state-contingent function of the elements of \( \Omega^* \). In the Rogerson economy when agents participate in the lottery market they face a known wage rate to be received if they become employed. In our economy, firms and labor suppliers face a state-contingent wage rate schedule specifying the wage rate as a function of the elements of \( \Omega^*_t \). This means that the equilibrium wage rate depends upon effort, which depends upon the realization of \( (X_t, G_t) \). We assume that agents can buy and sell contingent claims to consumption that depend upon, among other things, their \textit{ex post} employment status. Finally, we assume that households own the capital stock. Households and firms trade state-contingent rental contracts on capital which specify the quantity traded and the rental rate as functions of the elements of \( \Omega^*_t \). With this specification, the \textit{ex post} rental rate depends upon the realized capital-utilization rate through the dependence of \( U_t \) and \( \delta \) on \( (X_t, G_t) \).

Variations in capital utilization involve trade-offs between the effects on output and the effects on the depreciation rate of capital. We can examine these trade-offs by considering the planner’s Euler equation for \( U_t \):
\[
(10) \quad (1 - \alpha) \frac{Y_t}{U_t} = \phi \delta U_{t-1}^{\gamma} K_t.
\]
According to (10) the optimal plan for \( U_t \) sets the marginal product of an increase in the utilization rate equal to the marginal change in depreciation of the capital stock. Other things equal, factors that increase the marginal product of capital utilization, such as \( X_t \) or \( N_t \), lead to an increase in the rate of capital utilization. (See Section III for a more detailed discussion.)

A number of related models are of interest. First, when the rate of capital utilization cannot be varied, the model reduces to that of Burnside et al. (1993). For convenience we refer to this as the labor-hoarding model. Second, if in addition \( N_t \) is chosen after \( X_t \), and \( g_t \), are seen, then effort will be chosen to be a constant, and the model becomes observationally equivalent to the standard RBC indivisible-labor model, modified to incorporate government consumption into the analysis, which is analyzed in Christiano and Eichenbaum (1992). For convenience we
refer to this specification as the *benchmark model*.

Finally, it is of interest to examine the robustness of our results to different ways of modeling capital utilization. One natural alternative, pursued by Mark Bils and Jang-Ok Cho (1994) and Basu and Kimball (1995), among others, is to suppose that firms use only one shift but vary the number of hours in that shift. Under this assumption, \( U_t \) is equal to the work-week of capital, defined as the number of hours in the single shift. The simplest way to pursue this formulation in our setup is to suppose that effort is a constant, say 1. In addition, denote the variable shift length by \( W_f \) and impose the constraint that \( U_t \) is identically equal to \( W_f \), so that changes in the capital utilization are linked in a one-to-one way with changes in shift length. With the other elements of the model unchanged, the depreciation rate of capital is still a function of \( U_t \), and the competitive equilibrium of this alternative hoarding model corresponds to the solution of the factor-hoarding model, subject to the additional constraint that \( U_t = W_f \). Later, we comment on the relative empirical plausibility of the two hoarding models.

In general it is not possible to solve any of these models analytically. Here, we use the log-linear solution procedure discussed in Burnside (1995) to obtain an approximate solution to the planning problem.

II. Econometric Method

In this section we discuss our methodology for estimating and evaluating the empirical performance of the factor-hoarding model.

A. Measuring Capital, Depreciation, Capital Utilization, and Labor Effort

We face several problems in implementing our model. First, we do not have data on labor effort. In addition, existing measures of capital utilization are sector-specific and subject to substantial measurement error (see Matthew D. Shapiro, 1989).

Second, our model implies that technology shocks cannot be measured by the Solow residual. The conventional method of calculating the Solow residual (at least in the RBC literature) begins from the assumption that output is produced via the Cobb-Douglas production function:

\[
Y_t = S_t K_t^{1-\alpha} H_t^\alpha
\]

where \( H_t \), total time \( t \) hours worked, equals \( N_t f_t \). Given a consistent estimate of \( \alpha \), the log of the Solow residual can be computed using the relationship

\[
\ln(S_t) = \ln(Y_t) - (1 - \alpha) \ln(K_t) - \alpha \ln(H_t).
\]

But, abstracting from measurement error in capital, our model implies that

\[
\ln(S_t) = \alpha \ln(X_t) + (1 - \alpha) \ln(U_t) + \alpha \ln(W_t).
\]

Consequently shocks that cause capital utilization or labor effort to vary over time drive a wedge between the actual technology shocks and the Solow residual.

Third, we cannot measure capital using the official government time series on the stock of capital, which we denote by \( \hat{K}_t \). According to our model, the depreciation rate on capital varies as a function of the capital-utilization rate whereas the official government data are constructed under the assumption of approximately straight-line depreciation over fixed service lives for each type of capital.\(^1\)

Our strategy for dealing with these problems is as follows. Given a vector of parameters, and the equilibrium conditions implied by our model, we can obtain measures of the capital stock, the utilization rate, effort, and the level of technology. This subsection describes these measures in detail. The next subsection describes how the parameters and these measures may be simultaneously determined.

To obtain a measure of the stock of capital we proceed as follows. The Euler equation for

\(^1\) The official data do imply a time-varying depreciation series. However, this is an artifact of changes in the distribution of the stock of capital across different types of capital. For details, see U.S. Department of Commerce (1994).
utilization, equation (10), implies that the depreciation rate is

\[ \delta_i = \frac{(1 - \alpha) Y_i}{\phi K_i}. \]

Equation (4) then implies that

\[ K_{t-1} = K_t - \frac{(1 - \alpha)}{\phi} Y_t + I_t. \]

Given values of \( \alpha, \phi, \) and \( K_1, \) which we parameterize, equation (15) describes a recursive procedure for determining the capital stock. In a separate appendix (available from the authors upon request), we document that our quantitative findings are neither sensitive to setting \( K_1 = \bar{K}_1 \) nor to setting \( K_t = \bar{K} \) for all \( t. \)

Next consider the problem of measuring capital utilization. The planner’s first-order condition for \( U_t \) implies that

\[ U_t = \left( \frac{(1 - \alpha) Y_t}{\phi \delta K_t} \right)^{1/\alpha}. \]

Relation (16) allows us to deduce a time series for \( U_t \) given time series for \( Y_t \) and \( K_t \) and values for \( \alpha, \phi, \) and \( \delta. \)

To identify effort, \( W_t, \) we exploit the fact that the linearized equilibrium law of motion for \( W_t \) can be expressed as

\[ \ln(W_t) = \pi_0 + \pi_1 \ln(K_t) + \pi_2 \ln(H_t) + \pi_3 \ln(G_t) + \pi_4 \ln(X_t). \]

The scalars \( \pi_i \) are functions of the model’s underlying structural parameters. The production function implies that

\[ \ln(X_t) = \alpha^{-1} \left[ \ln(Y_t) - (1 - \alpha) \ln(K_t) - (1 - \alpha) \ln(U_t) - \alpha \ln(H_t) - \alpha \ln(W_t) \right]. \]

\footnote{In our empirical analysis we only need to identify \( \ln(U_t) \) up to a constant. As a result, we do not identify the parameter \( \delta. \)}

Given the time series on \( U_t \) implied by (16), relations (17) and (18) can be solved to obtain time series for \( W_t \) and \( X_t \) as functions of the time series for \( Y_t, K_t, H_t, \) and \( G, \) and the model’s structural parameters.

### B. Estimation and Diagnostic Procedures

In order to estimate and diagnose the performance of our model we use the generalized method of moments (GMM) procedure (Lars P. Hansen, 1982) discussed in Christiano and Eichenbaum (1992). Model parameters and various second moments of the data are estimated using an exactly identified GMM estimator. The procedure imposes no restrictions from the model in the equations that identify the second moments of the data. To test the model, we compute alternative estimates of the same second moments as functions of the estimated model parameters. We test the hypothesis that these two sets of estimates are the same in population by using a Wald statistic discussed in Christiano and Eichenbaum (1992). We only test single hypotheses in this paper. Our decision to do this reflects results in Burnside and Eichenbaum (1996) pertaining to the small-sample properties of GMM-based Wald test statistics. In the remainder of this subsection we describe the unconditional moment restrictions underlying our GMM estimator.

We did not estimate the parameters \( \tau, \beta, f, \) and \( \zeta. \) Instead we fixed \( T \) at 1,369 hours per quarter and \( \beta \) at 1.03\(^{-1/4}. \) The parameter \( f, \) the number of hours worked by an employed person, was set to 324.8 so as to imply a nonstochastic steady-state value of effort equal to 1. We experimented with various values of \( \zeta \) and found that our results were insensitive to choices between 20 and 120. The results reported here correspond to a value of \( \zeta \) equal to 60.

The vector of remaining model parameters, \( \Psi_t, \) is given by

\[ \Psi_t = \{ \theta, \bar{\delta}, \alpha, \bar{K}_t, \gamma, \sigma_t, \mu, \tau_g, \rho, \sigma_e, g/y \} \]

where \( \bar{\delta}, \tau_g, \) and \( g/y \) are defined in what follows. The measures of the capital stock and the level of technology we described in the previous section are functions of \( \Psi_t, \) the
observable data (which we denote by $Z$), and the fixed parameters given above. We denote these functions by $K_i = K_i(Z, \Psi_i)$ and $X_i = X_i(Z, \Psi_i)$.

We estimate the model parameters, and simultaneously obtain measures of the capital stock and the level of technology, by imposing the following exactly identifying restrictions whose interpretation is discussed below:

(19) $E \ln(H_t) = \ln[ N(\Psi_t) f ]$

(20) $E \ln(\delta_t) = \ln(\delta)$

(21) $\left\{ E \left[ \beta \left( \frac{C_t}{C_{t+1}} \right) \left( 1 - \alpha \right) \left( 1 - \frac{1}{\phi(\Psi_t)} \right) \times \frac{Y_{t+1}}{K_{t+1}(Z, \Psi_t)} + 1 \right] \right\} = 1$

(22) $E \Delta \ln[Y_t / K_t(Z, \Psi_t)] = 0$

(23) $E \Delta \ln[X_t(Z, \Psi_t)] = \gamma$

(24) $E \left\{ (1 - \rho L) \left[ g_t(Z, \Psi_t) - \mu - \tau_e \right] / T = 0 \right\}$

(25) $E \left\{ \left[ g_t(Z, \Psi_t) - \mu - \tau_e \right] \right\} = 0$

(26) $E \left\{ \left[ (1 - \rho L) \left[ g_t(Z, \Psi_t) - \mu - \tau_e \right] \right\} \right\} = 0$

(27) $E \left\{ (1 - \rho L) \left[ g_t(Z, \Psi_t) - \mu - \tau_e \right] \right\} = 0$

(28) $E \left\{ \left[ g_t(Z, \Psi_t) - \mu - \tau_e \right] \right\} = 0$

(29) $E \left[ \ln(G_t) - \ln(Y_t) - \ln(g/y) \right] = 0$.

In (19), the function $N(\Psi_t)$ represents the nonstochastic steady-state level of employment implied by the vector of model parameters. Thus, (19) simply imposes the restriction that the mean of the log of hours worked in the data should be equal to that in the model.

To calculate the first- and second-moment properties of our linearized model, we need values for the curvature parameter $\phi$ and the nonstochastic steady-state value of the depreciation rate, $\delta = \delta U^\phi$. As detailed in Burnside and Eichenbaum (1994), the first-order conditions of the planner’s problem evaluated in nonstochastic steady state imply that

$$\phi = \phi(\Psi_t) = \frac{\beta^{-1} \exp(\gamma) - 1}{\delta} + 1.$$  

To estimate $\delta$ we construct a depreciation series using the official capital series, $\hat{K}_t$, which is given by

$$\delta_t = 1 + (I_t - \hat{K}_{t+1}) / \hat{K}_t.$$  

Equation (20) corresponds to the assumption that, while the official measure of the capital stock, $\hat{K}_t$, does not match the actual capital stock on a quarter-to-quarter to basis, the Bureau of Economic Analysis does measure the average rate of depreciation correctly.

Equation (21) is an implication of the planner’s Euler equation for capital. Equation (22) imposes the restriction of our model that output and capital have the same growth rate. The law of motion for $X$, described in Section I implies that (23) and (24) should hold for our measure of the level of technology.

We found that in practice $g_t$ exhibited a time trend in our sample. Consequently, we allowed it to depend on a constant, a time trend, and one lag of itself; that is, we adopted the specification

$$g_t \equiv g_{t-1} - \mu - \tau_e (t-1) + \nu_t$$

where $\tau_e$ is a scalar parameter. Equations (25)–(28) identify the parameters entering this specification.

Finally, to solve our model, we parameterize the steady-state share of government consumption in output, denoted by $g/y$. Equation (29) imposes the restriction that the logarithm of this parameter should be equal to the mean of the logarithm of the share of government consumption in output.

To diagnose the empirical performance of our model we must estimate various moments of the data in ways that do not involve the first-order conditions of the model. Let $\Psi_t$ denote a vector of population moments to be estimated. We focus on four types of moments:
(a) the mean of a variable \( z_t \), denoted \( \mu_z \), (b) the standard deviation of a variable, \( z_t \), denoted \( \sigma_z \), (c) the standard deviation of one variable, \( s_t \), relative to that of another variable, \( z_t \), denoted \( \sigma_s / \sigma_z \), and (d) the autocorrelations of a variable \( z_t \), denoted by \( \rho_z \).

To estimate these moments we use the following restrictions:

\[
E(z_t - \mu_z) = 0
\]

\[
E \left( (z_t - \mu_z)^2 - \sigma^2_z \right) = 0
\]

\[
E \left[ (s_t - \mu_s)^2 - \left( \frac{\sigma_s}{\sigma_z} \right)^2 (z_t - \mu_z)^2 \right] = 0
\]

\[
E \left[ (z_t - \mu_z) (z_{t-1} - \mu_z) - \rho_z \sigma_z^2 \right] = 0.
\]

One problem with specifying the elements of \( \Psi_2 \) is that variables like \( C_t, Y_t, K_t \), and average productivity exhibit a marked trend. For the objects in \( \Psi_2 \) to be meaningful, they must refer to the moments of stationary time series. Since our model implies that these variables are difference-stationary stochastic processes, we work with the growth rates of these variables. For comparability with much of the RBC literature, we also report a limited set of results obtained with the stationary inducing transformation of the data discussed in Robert J. Hodrick and Edward C. Prescott (1980). Thus, depending on the context, the moments in \( \Psi_2 \) pertain to first-differenced time series or Hodrick-Prescott (HP)-filtered time series.

We refer the reader to the available separate appendix for further details regarding our estimation and testing procedures.

**C. Data**

The official capital stock, \( \dot{K}_t \), was measured as the sum of the net stocks of consumer durables, producer structures and equipment, and government and private residential capital plus government nonresidential capital. These data were obtained from the U.S. Department of Commerce (1994).\(^1\) Private consumption, \( C_t \), was measured as the sum of private-sector expenditures on nondurable goods plus services plus the imputed service flow from the stock of consumer durable goods. The first two measures were obtained from the National and Income Product Accounts (NIPA). The third measure was obtained from the Board of Governors of the Federal Reserve System and is derived from the Board's quarterly model of the U.S. economy (for details see mnemonic YCD in Flint Brayton and Eileen Mauskopf [1985]). Government consumption, \( G_t \), was measured by real government (federal, state, and local) purchases of goods and services (obtained from NIPA) minus real government investment, measured as the change in the gross stock of government capital (obtained from U.S. Department of Commerce). Gross investment, \( I_t \), was measured as purchases of consumer durables, gross private nonresidential (structures and equipment) and residential investment (all obtained from NIPA), as well as the change in the gross stock of government capital (obtained from U.S. Department of Commerce [1994]). Output, \( Y_t \), was measured as \( C_t + G_t + I_t \) plus net exports and time-\( t \) inventory investment (both from NIPA). Our measure of hours worked is the seasonally adjusted household hours series obtained from Citibase (mnemonic LHOURS).\(^2\) Our data cover the period 1955:1–1992:4 and were converted to per capita terms using the civilian noninstitutional population aged 16 and over from Citibase (mnemonic P16).

The measure of hours worked used in many RBC studies is the one constructed by Hansen (1985). Hansen’s series is based on the household survey of hours worked, adjusted for quality of labor input. Since Hansen’s sample covers the period 1955:3–1984:1, that is the data period used in many RBC studies. In Burnside and Eichenbaum (1996), we document that the main differences between our measure of hours worked and Hansen’s measure pertain to their low-frequency behavior and the fact that the Hansen series sample is the greater degree of very high-frequency variation. In

\(^1\) We constructed quarterly capital stock data (both net and gross) by interpolating the annual data using the RATS procedure INTERPOL (Thomas A. Doan, 1995b).

\(^2\) Although it is seasonally adjusted, this series displays marked seasonal autocorrelations at multiples of four lags.

As a result, we passed the series through Estima’s Census XII approximation program (Doan, 1995a).
TABLE I—MODEL PARAMETERS $\Psi$, ESTIMATES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark (i)</th>
<th>Factor-hoarding (ii)</th>
<th>Alternative (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>4.60 (0.0374)</td>
<td>3.89 (0.0408)</td>
<td>3.73 (0.0441)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.682 (0.0047)</td>
<td>0.674 (0.0093)</td>
<td>0.670 (0.0118)</td>
</tr>
<tr>
<td>$\bar{\delta}$</td>
<td>0.0195 (0.0002)</td>
<td>0.0195 (0.0002)</td>
<td>0.0195 (0.0002)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0028 (0.0010)</td>
<td>0.0034 (0.0008)</td>
<td>0.0038 (0.0006)</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>0.0107 (0.0008)</td>
<td>0.0072 (0.0005)</td>
<td>0.0071 (0.0007)</td>
</tr>
<tr>
<td>$g/l$</td>
<td>0.185 (0.0022)</td>
<td>0.185 (0.0030)</td>
<td>0.185 (0.0038)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>5.21 (0.0235)</td>
<td>1.87 (0.0645)</td>
<td>-0.890 (0.104)</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>-0.0014 (0.0001)</td>
<td>-0.0021 (0.0002)</td>
<td>-0.0024 (0.0004)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.954 (0.0233)</td>
<td>0.956 (0.0241)</td>
<td>0.948 (0.0278)</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.0155 (0.0011)</td>
<td>0.0146 (0.0012)</td>
<td>0.0137 (0.0011)</td>
</tr>
<tr>
<td>$K_t$</td>
<td>43.3 (5.15)</td>
<td>46.4 (8.23)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The parameters are defined in Section I, and the moment restrictions which identify them are described in Subsection II-B. The benchmark model is from Christiano and Eichenbaum (1992). The factor-hoarding model is described in Section I, and the alternative model is described in Subsection III-D. The estimates and the standard errors (in parentheses) are computed using the GMM estimator described in Section II (described in more detail in the separate appendix, available from the authors upon request).

In Basu and Kimball’s (1995) notation, the parameter $\phi$ is equal to $\Delta + 1$. They report a point estimate of $\phi$ equal to about 7 with a standard error equal to 8.

Burnside and Eichenbaum (1996) we also assess the sensitivity of the results reported in this paper to the measure of hours worked and sample period. The key result is that for all of the models considered, results obtained using the different hours measures over the sample period 1955:3–1984:1 are very similar.

III. Empirical Results

Columns (i) and (ii) of Table I report parameter estimates for the benchmark and factor-hoarding models, respectively. A number of interesting results can be seen here. First, with the exception of the standard deviation of the technology shock, the parameter estimates for the models are very similar. This greatly facilitates comparisons across the models. Note, in particular, that $\alpha$ is estimated to equal 0.674 and 0.682 in the factor-hoarding and benchmark models, respectively. Thus, the time series on the Solow residual that emerge from the two models are very similar to those used in existing RBC analyses (see e.g., Hansen, 1985).

Second, the estimated value of $\bar{\delta}$, 0.0195, implies that $\phi$ is roughly equal to 1.56. We do not know of any plant-level studies that provide evidence on this parameter. However, Basu and Kimball (1995) have recently provided an independent estimate of this parameter using manufacturing-industry data. While their point estimate is substantially larger than ours, it is well within one standard error of 1.56. The imprecision of their estimate is due to the more general functional form for $\delta$, that they employ. As they note, their approach is equivalent to assuming that $\delta = \delta_0 + \delta_1 U_s^\phi$. We interpret their results as indicating that there is little evidence against the hypothesis that $\delta_0 = 0$, a maintained assumption of our analysis. Still, it is important to note that if we had imposed a value of $\delta_0$ greater than zero then the value of $\phi$ consistent with $\bar{\delta} = 0.0195$ would have been larger. Other things equal, this would have reduced the quantitative importance of variable capital-utilization rates as a propagation mechanism.

Third, the expected value of the rate of depreciation, $\bar{\delta}$, is equal to 0.0195 (or an expected annual rate of depreciation of 8 percent). The implied two-standard-deviation band for quarterly depreciation is (0.0181, 0.0208).

Figure 1 displays the time series on the capital-utilization rate implied by our model. We also display the Federal Reserve’s measure of capacity utilization for the manufacturing sector. There are at least two reasons why these time series might differ: (i) our
measures of output and capital cover a broader segment of the economy than the Fed’s, and (ii) the Fed’s measure is subject to substantial measurement error (see Shapiro, 1989). Still, it is comforting that the series track each other reasonably well. As reference points, we have also included, in the shaded areas, NBER recessions (peaks to troughs). Note that, according to our estimates, capital utilization rises during recoveries and falls during recessions. For example, capital utilization fell from a peak of around 80 percent in 1989 to a low of around 77 percent in 1991, after which it began to increase. During the eight NBER recoveries in our sample, capital utilization rose by about 2 percentage points, on average. In contrast, during the seven NBER recessions, it fell by about 2 percentage points, on average. Next, notice that capital utilization is quite volatile. In particular, our point estimates imply that the standard deviation of the growth rate of the effective stock of capital \((U/K)\) equals 0.0057, whereas the standard deviation of the growth rate of the stock of capital equals 0.0012. Focusing on the physical stock of capital would give a very misleading picture of the volatility of effective capital input.

Figure 2 displays the time series on technology shocks emerging from the benchmark and factor-hoarding models, along with NBER recession reference points. Note that the broad movements in the two series are similar, with the growth rate of technology tending to fall during recessions and rise during recoveries, with two important differences. First, the slowdown in the growth rate of technology that occurs in the mid to late 1960’s, is less pronounced with the measure of technology shocks that is based on the factor-hoarding model. Second, incorporating factor-hoarding into the analysis leads to a substantially smaller estimate of the volatility of technology shocks. Relative to the benchmark model, correcting for factor-hoarding leads to a 33-percent reduction in the standard deviation of the innovation to technology shocks.\(^6\) It also mitigates, but does not eliminate, some embarrassing features of the standard measure of technology shocks. For example, in the 1982 recession, the standard measure of technology falls by 2.8 percent whereas our measure only falls by 2.2 percent. Since we view technological regress in the postwar United States as a priori implausible, we view this result as somewhat encouraging, in the sense that our correction helps mitigate the problem. However, it clearly does not fully resolve it.

\(^6\) Correcting for labor-hoarding alone leads to a 11-percent reduction in the standard deviation of the innovation to technology shocks.
A. Some Simple Diagnostics

Before discussing the quantitative role of variable capital utilization in propagating shocks, we briefly assess the performance of the benchmark and factor-hoarding models in terms of their ability to account for some of the standard statistics emphasized in the RBC literature. Columns (ii) and (iii) of Table 2 present the implications of the two models for the volatility of consumption, investment, hours worked, and average productivity relative to output \( \sigma, \sigma_c, \sigma_i, \sigma_h, \sigma_y, \text{ and } \sigma_{un}/\sigma_y \). Panel A reports results for the full sample, while panel B reports results for the sample period 1955:3–1984:1. In all cases, results pertain to moments of the log first-differenced data. Column (i) reports non-model-based estimates of these moments. Overall the factor-hoarding model does substantially better than the benchmark model on these dimensions of the data. Comparing panels A and B, we see that the results are very robust to the choice of sample period.

B. Propagation of Shocks

We now consider the implications of factor hoarding for the propagation of shocks to agents' environments and the volatility of output. Panel A of Table 3 reports a variety of statistics pertaining to the volatility of the growth rate of output. Columns (i) and (ii) report our estimate of \( \sigma_y \) in the data and the value of \( \sigma_y \) implied by the different models, respectively. Panel B refers to the same statistics for HP-filtered data.

According to panel A, the benchmark model generates a counterfactually large amount of volatility in the growth rate of output. Here we can reject the hypothesis that the benchmark model and sample moments are the same at extremely low significance levels. In contrast, both the labor-hoarding and factor-hoarding models do very well in accounting for this feature of the data. Interestingly, panel B reveals that it is very difficult to distinguish between the models once we work with HP-filtered data, because of the improved performance of the benchmark model. Viewed overall, all three of the models do reasonably well at accounting for the volatility of aggregate output.

The key distinguishing feature of the models is not whether but how they generate volatility in output. The estimated volatility of technology shocks is very different in the three models; but all of them generate substantial volatility in output. Since the estimated volatility of technology shocks is smallest in the factor-hoarding model, factor-hoarding must somehow act to magnify and propagate those shocks.

Before analyzing how factor-hoarding does this, we first quantify the relative strength of
## Table 2—Tests of the Models

<table>
<thead>
<tr>
<th>Moment</th>
<th>U.S. data (i)</th>
<th>Benchmark (ii)</th>
<th>Factor-hoarding (iii)</th>
<th>Alternative (iv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 1955:1—1992:4:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{w}/\sigma_{c}$</td>
<td>0.523 (0.042)</td>
<td>0.449 (0.002)</td>
<td>0.590 (0.011)</td>
<td>0.476 (0.009)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.072]</td>
<td>[0.134]</td>
<td>[0.260]</td>
</tr>
<tr>
<td>$\sigma_{w}/\sigma_{c}$</td>
<td>2.617 (0.206)</td>
<td>2.383 (0.047)</td>
<td>2.522 (0.071)</td>
<td>2.315 (0.059)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.288]</td>
<td>[0.636]</td>
<td>[0.132]</td>
</tr>
<tr>
<td>$\sigma_{w}/\sigma_{c}$</td>
<td>0.791 (0.055)</td>
<td>0.636 (0.026)</td>
<td>0.800 (0.047)</td>
<td>0.492 (0.039)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.006]</td>
<td>[0.891]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>$\sigma_{w}/\sigma_{c}$</td>
<td>0.796 (0.068)</td>
<td>0.449 (0.002)</td>
<td>0.956 (0.009)</td>
<td>0.582 (0.010)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.000]</td>
<td>[0.017]</td>
<td>[0.002]</td>
</tr>
<tr>
<td>B. 1955:3—1984:1:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{w}/\sigma_{c}$</td>
<td>0.755 (0.059)</td>
<td>0.610 (0.023)</td>
<td>0.774 (0.050)</td>
<td>0.472 (0.035)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.017]</td>
<td>[0.781]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>$\sigma_{w}/\sigma_{c}$</td>
<td>0.782 (0.075)</td>
<td>0.449 (0.003)</td>
<td>0.960 (0.010)</td>
<td>0.586 (0.013)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.000]</td>
<td>[0.015]</td>
<td>[0.006]</td>
</tr>
</tbody>
</table>

Notes: The benchmark model is from Christiano and Eichenbaum (1992). The factor-hoarding model is described in Section I, and the alternative model is described in Subsection III-D. The estimates and the standard errors (in parentheses) are computed using the GMM estimator described in Section II (described in more detail in the separate appendix, available from the authors upon request). Numbers under the heading U.S. data are the sample moments computed without reference to any model. Numbers under the heading “Model” are the corresponding estimates computed using the estimated parameters of the indicated model. All moments refer to first-differenced logarithms over the indicated sample periods; $\sigma_{w}$, $\sigma_{s}$, $\sigma_{r}$, $\sigma_{c}$, and $\sigma_{w}$ are the standard deviations of output, private consumption, gross investment, hours worked, and average labor productivity, respectively. Numbers in square brackets are probability values associated with Wald statistics for testing the hypothesis that the model and data population moments are the same.

the internal propagation mechanisms in the different models. One simple way to do this is to compare the variability of output when the propagation mechanisms in the models are operative with that when they are not operative. According to all of the models, the log level of output can be represented as

$$
(30) \quad \ln(Y_t) = \ln(X_t) + y,
$$

where $y$ represents the time-$t$ stationary deviation of the log of output away from its trend path. We can think of $y$, as corresponding to the part of output variation that is explained by the models’ internal mechanisms for propagating shocks, as distinct from the variation that is due directly to exogenous movements in $X_t$.

Suppose that we shut down these propagation mechanisms and set hours worked, effort, and capital utilization equal to their constant nonstochastic steady-state values. Also let $K_t$ vary in direct proportion to $X_{t-1}$, with $K_t = kX_{t-1}$. Here $k$ denotes the nonstochastic steady-state value of $K_t/X_{t-1}$. Then output, $Y^*$, evolves according to

$$
(31) \quad \ln(Y^*) = \ln(X_t) + y
$$

where $y$ denotes the nonstochastic steady-state value of $y_t$. 

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
<table>
<thead>
<tr>
<th>Model</th>
<th>U.S. data (i)</th>
<th>Model (ii)</th>
<th>$\sigma_x$ (iii)</th>
<th>$\sigma_y/\sigma_x$ (iv)</th>
<th>$\delta_0/\sigma_x$ (v)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. First-Differenced Data, 1955:1–1992:4:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.0093</td>
<td>0.0120</td>
<td>0.0107</td>
<td>1.127</td>
<td>1.101</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
<td>(0.0008)</td>
<td>(0.0008)</td>
<td>(0.015)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Labor-hoarding</td>
<td>0.0093</td>
<td>0.0089</td>
<td>0.0096</td>
<td>0.935</td>
<td>0.920</td>
</tr>
<tr>
<td></td>
<td>(0.0009)</td>
<td>(0.0007)</td>
<td>(0.0007)</td>
<td>(0.011)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Factor-hoarding</td>
<td>0.0093</td>
<td>0.0089</td>
<td>0.0072</td>
<td>1.233</td>
<td>1.183</td>
</tr>
<tr>
<td></td>
<td>(0.0009)</td>
<td>(0.0006)</td>
<td>(0.0005)</td>
<td>(0.026)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Alternative</td>
<td>0.0093</td>
<td>0.0083</td>
<td>0.0071</td>
<td>1.170</td>
<td>1.146</td>
</tr>
<tr>
<td></td>
<td>(0.0009)</td>
<td>(0.0008)</td>
<td>(0.0007)</td>
<td>(0.023)</td>
<td>(0.005)</td>
</tr>
<tr>
<td><strong>B. HP-Filtered Data, 1955:1–1992:4:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.0155</td>
<td>0.0155</td>
<td>0.0138</td>
<td>1.126</td>
<td>1.100</td>
</tr>
<tr>
<td></td>
<td>(0.0014)</td>
<td>(0.0011)</td>
<td>(0.0010)</td>
<td>(0.015)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Labor-hoarding</td>
<td>0.0155</td>
<td>0.0133</td>
<td>0.0123</td>
<td>1.076</td>
<td>1.052</td>
</tr>
<tr>
<td></td>
<td>(0.0016)</td>
<td>(0.0010)</td>
<td>(0.0010)</td>
<td>(0.016)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Factor-hoarding</td>
<td>0.0155</td>
<td>0.0142</td>
<td>0.0093</td>
<td>1.530</td>
<td>1.465</td>
</tr>
<tr>
<td></td>
<td>(0.0016)</td>
<td>(0.0010)</td>
<td>(0.0007)</td>
<td>(0.035)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>Alternative</td>
<td>0.0155</td>
<td>0.0112</td>
<td>0.0092</td>
<td>1.223</td>
<td>1.185</td>
</tr>
<tr>
<td></td>
<td>(0.0017)</td>
<td>(0.0010)</td>
<td>(0.0009)</td>
<td>(0.037)</td>
<td>(0.007)</td>
</tr>
<tr>
<td><strong>C. First-Differenced Data, 1955:3–1984:1:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.0103</td>
<td>0.0131</td>
<td>0.0116</td>
<td>1.129</td>
<td>1.114</td>
</tr>
<tr>
<td></td>
<td>(0.0010)</td>
<td>(0.0008)</td>
<td>(0.0009)</td>
<td>(0.014)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>Factor-hoarding</td>
<td>0.0103</td>
<td>0.0096</td>
<td>0.0079</td>
<td>1.219</td>
<td>1.182</td>
</tr>
<tr>
<td></td>
<td>(0.0010)</td>
<td>(0.0007)</td>
<td>(0.0006)</td>
<td>(0.026)</td>
<td>(0.012)</td>
</tr>
<tr>
<td><strong>D. HP-Filtered Data, 1955:3–1984:1:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.0159</td>
<td>0.0169</td>
<td>0.0150</td>
<td>1.127</td>
<td>1.112</td>
</tr>
<tr>
<td></td>
<td>(0.0015)</td>
<td>(0.0013)</td>
<td>(0.0012)</td>
<td>(0.0124)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>Factor-hoarding</td>
<td>0.0159</td>
<td>0.0154</td>
<td>0.0102</td>
<td>1.510</td>
<td>1.463</td>
</tr>
<tr>
<td></td>
<td>(0.0016)</td>
<td>(0.0011)</td>
<td>(0.0008)</td>
<td>(0.036)</td>
<td>(0.021)</td>
</tr>
</tbody>
</table>

**Notes:** The benchmark model is from Christiano and Eichenbaum (1992). The labor-hoarding model is from Burnside et al. (1993). The factor-hoarding model is described in Section I, and the alternative model is described in Subsection III-D. The estimates and the standard errors (in parentheses) are computed using the GMM estimator described in Section II. Numbers under the heading “U.S. data” are the sample moments computed without reference to a model. Numbers under the heading “Model” are computed using the estimated parameters of the indicated model. Moments refer to either first-differenced logarithms or HP-filtered data (as indicated) over the indicated sample periods; $\sigma_x$ is the standard deviation of output, $\sigma_y$ is the sample standard deviation of the level of technology, $\sigma_y/\sigma_x$ is the ratio of these standard deviations implied by the model, and $\delta_0/\sigma_x$ is the same ratio if the variance of shocks to government consumption is set at zero. Numbers in square brackets are probability values associated with Wald statistics for testing the hypothesis that the model and data population values of $\sigma_y$ are the same.
One measure of propagation is the standard deviation of \( \ln(Y_t) \) relative to the standard deviation of \( \ln(X_t) \). Since the latter equals the standard deviation of \( \ln(X_t) \), we denote this ratio by \( \sigma_x / \sigma_y \). This statistic can be calculated for all three models by applying a stationary-inducing filter to both \( \ln(Y_t) \) and \( \ln(X_t) \). Columns (iii)–(v) of Table 3 report \( \sigma_x, \sigma_x / \sigma_y \), and \( \tilde{\sigma}_x / \sigma_y \), respectively. The variable \( \sigma_x \) denotes the volatility of model output allowing for shocks to government purchases and technology, while \( \hat{\sigma}_x \) denotes the volatility of model output allowing only for technology shocks.

A number of interesting results emerge here. As is well known, shocks to government purchases do not contribute substantially to the volatility of output, so that the value of \( \sigma_x / \sigma_y \) is quite close to \( \tilde{\sigma}_x / \sigma_y \), regardless of which model we consider. Second, working with HP-filtered data, the benchmark model yields a value of \( \tilde{\sigma}_x / \sigma_y \) approximately equal to 1.10; that is, the propagation mechanisms embedded in that model generate only a 10-percent increase in the volatility of output. In contrast, the propagation mechanisms embedded in the factor-hoarding model lead to a 47-percent increase in the volatility of output (\( \tilde{\sigma}_x / \sigma_y \approx 1.47 \)). Second, \( \tilde{\sigma}_x / \sigma_y \) is actually lower in the labor-hoarding model than in the benchmark model (1.05 vs. 1.10). Third, comparing panels A and B, we see that the results are qualitatively the same, but less dramatic, when we work with growth rates, with \( \tilde{\sigma}_x / \sigma_y \) equal to 1.10, 0.92, and 1.18 in the benchmark, labor-hoarding, and factor-hoarding models, respectively. To economize on space, panels C and D of Table 3 report results for the sample period 1955:3–1984:1 for the benchmark and factor-hoarding models only. The key result here is that our results are very robust to the choice of sample period.

A different way to assess the importance of factor-hoarding as a propagation mechanism is to consider the dynamic response functions of different variables in the factor-hoarding and benchmark models to shocks in \( X_t \) and \( g_t \). Figure 3 reports the dynamic...
response of the log level of output to 1-percent shocks in $X$, and $g$.

Consider the effect of a 1-percent shock to $X$. In the impact period, the response of output is quite similar in the two models, with $Y$ rising by 1.10 percent in the benchmark model and 1.09 percent in the factor-hoarding model. Thereafter, the response path is different. In the benchmark model, output smoothly declines to its new steady-state growth path, approaching it from above. In the factor-hoarding model, the one-period-ahead effect is larger than the impact effect (1.54 percent vs. 1.09 percent). Thereafter output smoothly declines to its new steady-state growth path, approaching it from above. Notice that the rate of convergence is reasonably slow, with output up by more than 1.14 percent 24 quarters after the technology shock. Row 2 of Figure 3 reveals a similar, if less dramatic, pattern in the case of a shock to $g$.

Figure 4 displays the dynamic response functions of hours worked in the benchmark and factor-hoarding models to 1-percent shocks in $X$, and $g$. In the impact period of the shock to $X$, hours worked rise by roughly 0.62 percent in the benchmark model and then smoothly return to their (unaffected) nonstochastic steady-state level. In the factor-hoarding model, by construction, hours worked do not respond contemporaneously to a shock in $X$. In the period after the shock they rise by 0.81 percent and then slowly converge to their (unaffected) nonstochastic steady-state level. Notice that, after the impact period of the shock, the rise in hours worked in the factor-hoarding model exceeds the corresponding rise in the benchmark model along the entire adjustment path to nonstochastic steady state. A similar pattern is observed in the response of the two models to a shock in $g$.

The "hump-shaped" response of output to shocks in the factor-hoarding model can be understood by considering the dynamic response functions of effort and capital utilization in that model. These are displayed in Figure 5. In the impact period of the shock to $X$, effort rises by 0.28 percent and then immediately
reverts to its nonstochastic steady-state level. The absence of a persistent response in effort reflects our assumption that it is costless to adjust hours worked in the period after the shock. By contrast, capital utilization initially rises by 0.70 percent and then climbs by even more (to a total of 0.97 percent) in the next period. Thereafter capital utilization smoothly approaches its (unaffected) steady-state level from above. To understand the hump-shaped response of capital utilization, notice that the impact-period response of effort is small relative to the second-period response of hours worked. This reflects the relative curvature of agents’ preferences over effort and employment. Therefore, the response of the log of effective labor input, being the sum of the responses of the log of effort and the log of hours worked, displays a hump-shaped pattern. It is not surprising that utilization, being a complement to labor input, displays a similar hump-shaped response to shocks. Since variations in effort, hours, and utilization account for most of the transitory movement in output, the level of output also displays a hump-shaped response. Similar patterns arise in response to shocks to $\sigma$.

Figure 6 depicts the dynamic response of the Solow residual in the factor-hoarding model to 1-percent shocks in $X_t$ and $g_t$. The key point here is that factor-hoarding induces a large persistent deviation between the actual technology shock and the measured Solow residual. In the impact period of the shock, both effort and capital utilization rise. Since the Solow residual is calculated using measured hours worked and the stock of capital, it rises by more than is implied by the shock to technology (1.09 percent vs. 0.68 percent). In the periods after the shock, effort reverts to its steady-state level, but capital utilization continues to be high relative to its steady-state level. Consequently, the Solow residual remains above its steady-state growth path which is 0.68-percent higher than before. As capital utilization slowly declines to its steady-state level, the Solow residual approaches its steady-state growth path from above. Naive Solow-residual accounting, which attributes all of the movements in total factor productivity...
to movements in $X_t$, clearly overstates the volatility of technology shocks. In contrast to the labor-hoarding model considered in Burnside et al. (1993), the deviation between the Solow residual and the technology shock is highly persistent, reflecting the persistent deviation of capital utilization from its steady-state level. This is the basic reason why our model generates a much smaller estimate of the volatility of technology shocks than do the labor-hoarding and benchmark models.

Finally, note that in response to a 1-percent shock in $g$, the Solow residual initially rises (by roughly 0.08 percent) and then slowly approaches its unchanged steady-state value from above, as capital utilization reverts to its unchanged steady-state value. Again, naive Solow-residual accounting overstates the volatility of technology shocks. Indeed, here the analyst would infer that a sequence of technology shocks has occurred even though there has only been a one-time shock to aggregate demand and no shock to technology whatsoever.

C. Persistence of Output Growth

Cogley and Nason (1995) discuss the propagation mechanisms in RBC models by focusing on the autocorrelation function of output growth. They show that many RBC models imply that the growth rate of output is close to being a white-noise process. In contrast, the actual growth rate of U.S. output displays positive persistence. Cogley and Nason interpret this discrepancy as reflecting the weakness of the propagation mechanisms embedded within standard RBC models. The key question addressed in this subsection is whether the propagation mechanisms in the factor-hoarding model are sufficiently strong so as to allow it to account for the autocorrelation function of $\Delta \ln(Y_t)$.

The first row of Figure 7 illustrates our unconstrained estimates of the autocorrelation function of $\Delta \ln(Y_t)$, as well as those implied by the benchmark and factor-hoarding models. As noted above, the actual growth rate of U.S. output is positively autocorrelated: specifically
the first two autocorrelation coefficients are positive and significant. Though it is difficult to discern from Figure 7, the benchmark model implies that all the autocorrelations are negative. In contrast, the factor-hoarding model does well at accounting for the autocorrelation function of $\Delta \ln(Y_t)$. The joint hypothesis that the first two autocorrelation coefficients implied by the factor-hoarding model are the same as those in the data cannot be rejected at conventional significance levels. This hypothesis is strongly rejected for the benchmark model.

The impulse-response functions discussed in the previous subsection provide some intuition for these results. In the benchmark model, $\Delta \ln(Y_t)$ is significantly affected only in the impact period of a technology shock. It follows that, since innovations to technology are a white-noise process, output growth closely resembles white noise. In the factor-hoarding model, a 1-percent shock to technology causes output to rise by 1.09 percent in the period of the shock. In the second period after the shock, output climbs by an additional 0.46 percent. Thus, the initial spurt in output growth is a signal of further growth in output. A similar pattern is observed in response to shocks to $g$. Not surprisingly, the growth rate of output is predicted to display positive serial correlation.

Notice from the second row of Figure 8 that both models predict the correct pattern of autocorrelation in the growth rate of the Solow residual. Thus, the factor-hoarding model is able to generate positive serial correlation in output growth without generating counterfactual predictions for the growth rate of the Solow residual. The model does this without generating an internally inconsistent law of motion for technology shocks. Recall that we began by assuming that $X_t$ is a random-walk process. Given the observed Solow residual and the estimated time series on effort and capital utilization, the model implies an estimated time series on technology shocks. Our
estimation procedure need not result in a technology-shock process that is a random walk. However, we could not reject the individual null hypotheses that the model-based measure of $\Delta \ln X_t$ is uncorrelated with $\Delta \ln X_{t-\tau}, \tau = 1, \ldots, 9$. The benchmark model must choose to match the correlation structure of $\Delta \ln (S_t)$ or of $\Delta \ln (Y_t)$. As things stand, it matches the correlation structure of $\Delta \ln (S_t)$ but not $\Delta \ln (Y_t)$. We could improve the model’s performance regarding $\Delta \ln (Y_t)$ by assuming that $\Delta \ln (X_t)$ is positively serially correlated. But this would imply that $\Delta \ln (S_t)$ is positively serially correlated and would simply substitute one counterfactual implication for another.

D. A Brief Comparison with an Alternative Capital-Utilization Model

In the remainder of this section we briefly comment on the robustness of our main qualitative results to the way we modeled capital utilization. Recall that in Section I we outlined an alternative hoarding model in which (i) effort is constant and (ii) changes in capital utilization are linked one-to-one with changes in the length of the single shift that firms use. Here we document that the factor-hoarding and alternative hoarding models have very similar qualitative implications for the response of capital utilization and the observed Solow residual to shocks in agents’ environments. The main difference is quantitative.

Column (iii) of Table 1 reports our estimates of the structural parameters of the alternative hoarding model. Comparing these to the corresponding estimates for the factor-hoarding model in column (ii), we see that they are very similar. Column (v) of Table 2 reports the performance of the alternative capital-utilization model with respect to the diagnostic moments {\(\sigma_o/\sigma, \sigma_i/\sigma, \sigma_n/\sigma, \sigma_{an}/\sigma\)}. The key point to note here is the deterioration of the alternative model’s performance with respect to $\sigma_n/\sigma$, relative to the
factor-hoarding model. Here we can easily reject the hypothesis that the model and data
population moments are the same. The intuition for this result is straightforward. By rigidly
linking changes in total hours worked to changes in capital utilization, the alternative
model induces an additional direct cost of varying hours: higher capital depreciation
rates. The net result is a substantial reduction in the relative volatility of hours worked. From
Table 3 we see that this translates into a lower volatility of output than that which emerges
from the factor-hoarding model, along with weaker internal propagation mechanisms. To
conserve on space, we do not report the impulse-response functions for the alternative
model; but these are qualitatively very similar to those of the factor-hoarding model. Positive
technology shocks and shocks to government purchases continue to generate persistent in-
creases in capital utilization rates and persistent deviations of the Solow residual from the
level of technology. The major difference is that the size of these responses is smaller
than that emerging from the factor-hoarding model. In sum, while the models have qualiti-
atively similar implications, our model embeds stronger propagation mechanisms and does
a better job of accounting for the aggregate facts.

IV. Conclusion

This paper formulated and estimated an equilibrium business-cycle model in which
capital-utilization rates vary over the business cycle. We argue that cyclical movements in
effective capital services are substantially more volatile than cyclical movements in the stock
of capital. In addition we argued that variable capital-utilization rates are a quantitatively im-
portant source of propagation to business-cycle shocks.

A virtue of our model is that it represents a minimal perturbation of the standard RBC
model. This greatly simplifies comparisons with existing work in the literature. However,
this simplicity is purchased at the cost of descriptive realism. For example, using data from
the auto industry, Timothy F. Bresnahan and Valerie A. Ramey (1994) argue that, of the
multiple margins used by the managers of an automobile assembly plant, varying regular
hours by shutting the plant down for a week is the most important. Second most important is
adding or dropping a shift. We view the model considered in this paper as approximating these
sorts of adjustments. We conjecture that the aggregate capital-utilization rate that would
emerge from explicitly modeling those richer environments would also respond positively to
technology shocks and innovation to government purchases. In this sense, we expect the
qualitative features of our model to be quite robust to alternative ways of modeling variable
capital utilization. However, the robustness of our quantitative results remains very much an
open issue to be addressed in future research.

REFERENCES

Basu, Susanto and Kimball, Miles. "Cyclical Productivity with Unobserved Input Varia-

Bils, Mark and Cho, Jang-Ok. "Cyclical Factor Utilization." Journal of Monetary Econom-

Brayton, Flint and Mauskopf, Eileen. "The Federal Reserve Board MPS Quarterly Eco-
170–292.

Bresnahan, Timothy F. and Ramey, Valerie A. "Output Fluctuations at the Plant Level.":

Burnside, Craig. "Notes on the Linearization and GMM Estimation of Real Business Cy-

Burnside, Craig and Eichenbaum, Martin. "Factor Hoarding and the Propagation of
Business Cycle Shocks." National Bureau of Economic Research (Cambridge, MA )

———. "Small Sample Properties of GMM-Based Wald Tests." Journal of Economic

Burnside, Craig; Eichenbaum, Martin and Rebelo, Sergio. "Labor Hoarding and the Business

Christiano, Lawrence J. "Why Does Inventory Investment Fluctuate So Much?" Journal