Oligopoly and Financial Structure: The Limited Liability Effect

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We argue that product markets and financial markets have important linkages. Assuming an oligopoly in which financial and output decisions follow in sequence, we show that limited liability may commit a leveraged firm to a more aggressive output stance. Because firms will have incentives to use financial structure to influence the output market, this demonstrates a new determinant of the debt-equity ratio.

The literature on financial structure and the literature on oligopoly have at least one common feature: they both place relatively little emphasis on the strategic relationships between financial decisions and output market decisions. In financial theory, the product market is typically assumed to offer an exogenous random return which is unaffected by the debt-equity positions of the firms in the market. Correspondingly, in the economic analysis of oligopoly, the firm's obligations to debt holders and the possibility of financial distress are usually ignored in modeling the strategic interaction between producers in the output market.

This approach of focusing separately on financial and output decisions is clearly useful in understanding certain aspects of both financial structure and strategic output market behavior. It seems equally clear, however, that there are important linkages between financial and output decisions.

The choice of financial structure can affect output markets in the following way, which we refer to as the limited liability effect of debt financing. As firms take on more debt, they will have an incentive to pursue output strategies that raise returns in good states and lower returns in bad states. The basic point is that shareholders will ignore reductions in returns in bankrupt states, since bondholders become the residual claimants. As debt levels change, the distribution of returns to shareholders over the different states changes, which in turn changes the output strategy favored by shareholders.1

A second possible linkage between output and financial markets is the strategic bankruptcy effect. Any one firm's susceptibility to financial distress depends on its financial structure, and its fortunes will usually improve if one or more of its rivals can be driven into financial distress. Therefore, firms might make output market decisions that raise the chances of driving their rivals into insolvency. Since the possibility of financial distress for each firm is contingent on its financial structure, this is a second channel for finances to affect output markets.

In this paper we examine the relationship between financial and output decisions in a formal structure capturing essential aspects of both modern financial and oligopoly the-

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1 The idea that limited liability creates a conflict of interest between bondholders and equity holders is described in Michael Jensen and William Meckling (1976) and in Stewart Myers (1977), and has been examined by Jeremy Bulow and John Shoven (1978), Jerry Green and Shoven (1983), and Varouj Aivazian and Jeffrey Callen (1980). These studies do not, however, explicitly consider the output market, which is the focus of this paper.
ory. Our model focuses on the “limited liability” effect of debt financing. The strategic bankruptcy effects of financial decisions are analyzed in our companion paper (1985). The analysis we offer here illustrates two important points. First, output market behavior will, in general, be affected by financial structure. Second, foresighted firms will anticipate output market consequences of financial decisions; therefore, output market conditions will influence financial decisions.

In the analysis to follow, we deliberately abstract from the physical capital investment decision by assuming that the firm’s capital stock is fixed (at least temporarily). We adopt this approach so as to isolate the linkage between the firm’s financial position and its behavior in the output market. If physical investment was allowed to vary endogenously with the firm’s financial position, certain readily identifiable effects of financial decisions on output behavior would emerge. First, firms would have an incentive to undertake capital investments that lower the marginal cost of production in output markets so as to commit themselves to more aggressive positions in the output market. (This effect has been investigated by Avinash Dixit, 1980; Curtis Eaton and Richard Lipsey, 1980; and Brander and Barbara Spencer, 1983; among others.)

In addition, as emphasized by Michael Jensen and William Meckling and Stewart Myers (1977), the firm’s debt-equity mix would affect the optimal investment strategy. A referee suggested that heavily leveraged firms might lower their level of physical capital investments in order to minimize the salvageable assets that would be lost to debt-holders in the event of bankruptcy. This would in turn affect costs and the output market in the obvious way. These are interesting issues, but they are logically separate from the effects we identify; accordingly we abstract from the investment decision by assuming it is made before the debt-equity mix is determined. Output adjustments are therefore to be thought of as resulting from changes in variable inputs.

The basic model we investigate is a two-stage sequential duopoly game. In stage 1, the two firms decide upon financial structure. In stage 2, they select output levels taking as given the financial composition determined in stage 1. The equilibrium concept is the sequentially rational Nash equilibrium in debt levels and output levels. In other words, the second-stage outcome is a Cournot equilibrium in output which is correctly anticipated by the firms when choosing debt levels in the first stage.2

The output decisions of firms are made before the realization of a random variable reflecting variations in demand or costs. Once profits are determined, firms are obliged to pay debt claims out of operating profits, if possible. If profits are insufficient to meet debt obligations, the firm goes bankrupt and its assets are turned over to the bondholders. This simplified depiction of the relation between financial distress and output markets is not universally descriptive, but it does yield a tractable model that can be readily compared with similar models of output market rivalry in the oligopoly literature where financial structure is ignored.

Before proceeding it is important to place our analysis in context in both the financial and oligopoly literatures. A large part of the literature on financial structure can be regarded as a response to Franco Modigliani and Merton Miller (1958), who assert that, under certain conditions, the value of the firm is independent of financial structure. Our paper falls in the class of exceptions to the Modigliani-Miller Theorem. Following Alan Kraus and Robert Litzenberger (1973), among others, we appeal to a background of imperfect markets to allow departures from the Modigliani-Miller world. The exact nature of the market imperfections should become clear as we proceed. The standard treatment of choice of financial structure, as presented in Kraus and Litzenberger, in-

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2In our context, sequential rationality is equivalent to what Reinhardt Selten (1975) has referred to as subgame perfection. The most important implication of subgame perfection is that players are restricted to credible threats; players cannot have equilibrium strategies that would call for them to carry out actions that would be against their best interests at the time the action is to be taken. This basic idea goes back at least to Thomas Schelling (1956).
volves a tradeoff between financial distress costs and the tax advantages of debt. Our model, which abstracts from both bankruptcy costs and taxes, points to the interaction between output and financial markets as a determinant of financial structure.

Of the existing published literature in financial theory, our paper is closest in spirit to Sheridan Titman (1984) in which financial structure influences potential profits for suppliers of a durable good. Titman argues that low levels of debt constitute a commitment by a producer to stay in the market, which raises the value of the durable good because it means that the firm will be available to service the good in the future. (This effect operates as long as each firm has some advantage over rivals in servicing its own products.) In our model, financial structure serves as a commitment to a particular output strategy, and is therefore related to the recent literature on commitment in oligopoly theory, in which physical capital, location, product choice, or R&D choices constitute a first stage which influences the Nash equilibrium in the subsequent output market. Standard references in this literature include Eaton-Lipsey and Dixit. The equal opportunity duopoly structure we use is similar in form to Brander-Spencer.

An outline of the paper follows. Section I sets out the basic model. Section II is devoted to the output market equilibrium and, in particular, shows the dependence of output equilibrium on financial structure and compares the output equilibrium in the (base) case, in which equity holders control the firm, with the case in which debtholders control the firm. Section III examines the selection of debt levels and describes how output market considerations influence capital structure. Section IV contains concluding remarks and discusses extensions to the paper.

I. The Model

Firms 1 and 2 are rivals in an output market where they produce competing products $q_1$ and $q_2$, respectively. For concreteness, we assume there is Cournot quantity competition in the output market. Other forms of market rivalry involving advertising, R&D, or Bertrand price competition could be analyzed just as well using our model. The operating profit for firm $i$, which is defined as the difference between revenue and variable cost, is denoted by $R^i(q_i, q_j, z_i)$. The random variable $z_i$ reflects the effects of an uncertain environment on the fortunes of firm $i$. It is assumed to be distributed over the interval $[z, \bar{z}]$ according to density function $f(z)$. For simplicity we assume that $z_i$ and $z_j$ are independent and identically distributed.

We assume that $R^1$ satisfies the usual properties: $R^1_i > 0$, $R^1_j < 0$, and $R^1_{ij} < 0$ (where subscripts denote partial derivatives). We adopt the convention that high values of $z_i$ lead to higher operating profits: $R^1_i > 0$, meaning that higher realizations of $z_i$ correspond to better states of the world. The effect of $z_i$ on marginal profit turns out to be very important in our analysis. We consider two possibilities.

(i) $R^1_{iz} > 0$. This corresponds to a situation where marginal profits are higher in better states of the world. This would arise, for example, if higher realizations of $z$ correspond to downward shifts of the marginal cost schedule, or to upward shifts in the marginal revenue schedule facing the firm. We take this to be the normal case.

(ii) $R^1_{iz} < 0$. This means that good states of the world are correlated with low marginal returns to extra sales. This case seems less likely to arise but it is possible.5

3 We would also like to mention a Ph.D. thesis by Vojislav Maksimovic (1986) and a discussion paper by Franklin Allen (1985) which address the same fundamental issue as this paper: the strategic relationship between oligopoly and financial markets. Maksimovic represents simultaneous development of a modeling approach that is similar in some respects to ours and establishes some of the same insights. It also examines some interesting repeated game extensions of the basic model.

4 See Eaton and Mukesh Eswaran (1984) for a very helpful synopsis of the strategic commitment paradigm applied to industrial organization.

5 A rather contrived example can be constructed as follows. Suppose that firms $i$ and $j$ each have fixed
Another argument for considering the $R'_{iz} < 0$ case arises when we assume that firms engage in other forms of competition besides quantity or price competition. For example, suppose that firms primarily compete through advertising, as is the case in some retail markets. Let $q_i$ be firm $i$'s expenditure on advertising and assume that higher realizations of $z_i$ correspond to cases where market demand is high. It seems plausible that, in some cases, when conditions are good, there might be little need for advertising, whereas advertising would be more effective in increasing sales in a market with sagging demand. In this case we would find that marginal returns from advertising, $R'_{iz}$, are inversely related to the state of nature, $z_i$, leading to the case $R'_{iz} < 0$.

An alternative way of thinking of the sign condition on $R'_{iz}$ is to view $q_i$ and $z_i$ as separate “inputs” which go into making profits. The case $R'_{iz} > 0$, which is the standard case under quantity competition, means that the two inputs are complementary, while the other case is the case of substitute inputs.

The financial structure of the firm is summarized by the variable $D_i$, which represents the debt obligation of firm $i$. As indicated in the introduction, we assume that the capital investment is made before the financing mix is decided upon. Choosing the debt level, with the total financing requirement fixed, would then fix the equity financing level by default. This interpretation is not necessary, however, and the firm could just as easily be viewed as giving the borrowed money directly to shareholders. Debt levels, in turn, are assumed to be chosen before output decisions are decided upon and are taken as given when output levels are chosen. Output decisions are then made before the uncertainty over demand or cost is resolved. It is not essential that there be no uncertainty resolved before output decisions are made. What is important is that there be some residual uncertainty left to be resolved after output decisions are made. Certainly this is true of most industries.

After production occurs and the uncertainty regarding firms’ profits is settled, the firm is obliged to pay creditors $D_i$ out of current profits. If the firm is unable to meet its debt obligations, its creditors are paid whatever operating profits are available.7

Given debt levels $(D_1, D_2)$, the firm is assumed to choose output levels with the objective of maximizing the expected value of the firm to the shareholders.8 This is what an owner-manager would choose to do, and is certainly what wealth-maximizing shareholders would want the firm to do. The value to the shareholders is referred to as the equity value and is represented by the letter $V$:

\[
V^i(q_i, q_j) = \int_{z_i}^{\hat{z}_i} (R_i^i(q_i, q_j, z_i) - D_i) f(z_i) dz_i,
\]

where $\hat{z}_i$ is defined by

\[
R_i^i(q_i, q_j, \hat{z}_i) - D_i = 0,
\]

assuming $z < \hat{z}_i < \hat{z}$. When $z_i = \hat{z}_i$, firm $i$ can just meet its debt obligations with nothing left over. The expression in (1) represents expected current-period profits net of debt obligations in good ($z_i \geq \hat{z}_i$) states of the

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7For simplicity we assume that the asset value of the firm is zero, as if assets are completely used up in the production of output. Creditors can, therefore, collect only current operating profits if the firm becomes insolvent.

8The interesting possibilities that the rival firms may somehow be connected through interlocking directorships, or that they are both owned by a common group of shareholders are not considered here.
world. In bad states \((z_i < \hat{z}_i)\), the firm earns zero as all of its earnings are paid to debtholders. (Note that we are assuming that shareholders of the firm are risk neutral with respect to the firm's returns and therefore have their interests served by maximization of expected equity value.)

Expression (2) shows the implicit dependence of \(\hat{z}_i\) on \(D_i\), \(q_i\), and \(q_j\). As this relationship is important in establishing the principal results of the paper, it is useful to report the following derivatives:

\[
\begin{align*}
(3a) \quad & d\hat{z}_i / dD_i = 1 / R^*_i(\hat{z}_i) > 0 \\
(3b) \quad & d\hat{z}_i / dD_j = 0 \\
(3c) \quad & d\hat{z}_i / dq_i = - R^*_i(\hat{z}_i) / R^*_{ii}(\hat{z}_i) \\
(3d) \quad & d\hat{z}_i / dq_j = - R^*_i(\hat{z}_i) / R^*_{ij}(\hat{z}_i) > 0.
\end{align*}
\]

The natural assumption, abstracting from agency problems between managers and shareholders, is that managers maximize equity value in this stage of the game when debt levels are taken as given. Later in the paper we examine the earlier decision of how much debt the firm should take on. At this earlier stage, managers are assumed to maximize total value. Once the debtholders are captive, however, the managers have no subsequent incentive to act in the debtholders' interests. For purposes of comparison it is, nevertheless, useful to consider the problem of maximizing the debt value of the firm in the output phase, as if debtholders were running the firm. In this case, the maximand, denoted \(W^*(q_i, q_j, \ldots)\), is given by

\[
W^*(q_i, q_j, \ldots) = \int_{\hat{z}_i}^{z} R^*(q_i, q_j, z_i) f(z_i) dz_i + D_i(1 - F(\hat{z}_i)).
\]

(We are assuming that \(z_i\), the lowest possible value of \(z_j\), generates positive operating profits for all relevant values of the choice variables. This is a convenient but inessential assumption.) The first term in (4) represents the operating profit of the firm in states of the world when this profit is insufficient to completely cover debt obligations. The second term represents those states of the world in which the creditors of the firm are paid in full.

The model presented here is the simplest model we could develop to explore possible connections between financial decisions and oligopolistic output markets. Generalizations to include more firms, other forms of market rivalry, and correlated random disturbances yield the same qualitative results.

II. Output Market Equilibrium

This section examines how the limited liability aspects of financial leverage affect the strategic output decisions by firms. Taking existing debt levels \(D_i\) and \(D_j\) as predetermined, the management of each firm chooses output to maximize either \(V\) or \(W\), depending on whether it acts in the interest of shareholders or debtholders.

A. Equity Value Maximization

We take the case of equity value maximization as our standard case. Assuming an interior solution, the choice of output for firm \(i\) is obtained by setting the derivative of (1) with respect to \(q_i\) equal to zero:

\[
V^*_i = \int_{\hat{z}_i}^{z} R^*_i(q_i, q_j, z_i) f(z_i) dz_i.
\]

The second-order condition is

\[
V^*_{ii} < 0.
\]

The Nash output (or Cournot) equilibrium is obtained from the simultaneous solution of (5) for \(i, j = 1, 2\). In addition we also require that

\[
V^*_{ij} < 0,
\]

\[
V^*_i V^*_j - V^*_j V^*_i > 0,
\]

\(^9\text{Besides the expression in (5), the derivative of } V^*_i \text{ with respect to } q_i \text{ also includes another term, }
- d\hat{z}_i / dq_i (R^*_i(\hat{z}_i) - D_i), \text{ which vanishes by (2).}\)
which are standard conditions in Cournot-type models. Equation (8) is equivalent (given second-order conditions) to reaction functions being downward sloping. It tends to hold if expected marginal revenue declines when the output of the other firm rises. Equation (8), if it holds everywhere, implies uniqueness of the equilibrium and reaction function stability. It is well-known that even in the simplest Cournot models, conditions analogous to (7) and (8) can be violated by feasible demand and cost structures, and that is certainly true here. One case in which these conditions do hold, however, is if \( z_i \) is uniformly distributed, demand is linear, and marginal cost is constant.

While this paper focuses nearly exclusively on the Cournot duopoly market structure, the first central insight we offer applies quite generally. This central insight is that higher debt levels tend, in the standard case \( R'_{iz} > 0 \), to increase a firm’s desired output. The intuition is as follows.

In bad states of the world, the firm’s profits are insufficient to meet its debts. The equity holders’ claims on the firm go to zero, and the debtholders become the residual claimants on the firm’s profits. In other words, limited liability implies that debtholders become residual claimants in bad states: specifically, when \( z_i \leq \hat{z}_i \). Equity holders are residual claimants in good states of the world, as illustrated in Figure 1.

An increase in debt causes \( \hat{z}_i \) to rise, meaning that the range of states over which the firm becomes bankrupt is expanded. In particular, with \( R'_{iz} > 0 \), it is states with low marginal returns to output that are moved from the region in which equity holders are residual claimants to the bankrupt region, where debtholders are the residual claimants. In other words, these low marginal profit states are no longer relevant to equity holders and equity holders would therefore want output to rise. Thus an increase in debt tends to make equilibrium output rise.

A football team that is behind late in the game will take chances that it would not normally take. The reasoning is that bad realizations are irrelevant, for the worst the team can do is lose, and it will do that anyway if it does not take chances. The more the team is behind, the more aggressive it will become. Our firm, representing equity holders, reacts in the same way. As debt rises, low marginal value states become irrelevant, for in those states the firm is turned over to the debtholders, and the equity holders get zero in any case. Since the firm restricts attention to higher marginal profit states, it adopts a more aggressive stance. (Note that the reasoning is precisely reversed if \( R'_{iz} < 0 \).)

Presenting this intuition rigorously requires a slightly different treatment for each different market structure. The case of monopoly is easiest. The monopoly case is obtained by letting \( q_j = 0 \) in expression (5) and by assuming that \( q_j \) remains at zero throughout the comparative static exercise. Total differentiation of (5) with respect to \( q_i \) and \( D_i \) then yields the comparative static formula:

\[
dq/dD = -V'_{ID}/V'_{ii}.
\]

The denominator is negative by second-order condition (6), which means that \( dq/dD \) has the same sign as \( V'_{ID} \): Output rises with debt if increases in debt cause marginal expected profits to rise.

The expression for \( V'_{ID} \) is given by

\[
V'_{ID} = -R'_i(\hat{z}_i)d\hat{z}_i/dD,
\]

or, using (3a),

\[
(9) \quad V'_{ID} = -R'_i(\hat{z}_i)/R'_z(\hat{z}_i)f(\hat{z}_i).
\]

The intuition associated with this expression is central to our paper. The denominator of (9) is obviously positive. Therefore, \( V'_{ID} \) (and \( dq/dD \)) have the opposite sign to \( R'_i(\hat{z}_i) \).
This term is just marginal profit evaluated at the worst state of nature relevant to equity holders. With \( R'_i \) increasing in \( z \), it follows that \( R'_i \) evaluated at \( \hat{z} \) must be negative, since a weighted average of \( R'_i \) over \( \hat{z} \) and strictly better states is zero from first-order condition (5). Expression (9) implies that \( V'_{ij} \) and, therefore, \( dq_i/dD \) must be positive in this case. The point is that \( \hat{z} \) is precisely the marginal state that is being removed from consideration (by equity holders) as \( D \) rises. Consequently, the associated value of marginal profit, \( R'_i(\hat{z}) \), is also thrown out of the marginal calculation determining output. Since this is the lowest value of marginal profit, optimal output must rise.

This discussion proves that output is rising in debt (for \( R'_{iz} > 0 \)) in the case of monopoly. This can also be proven for perfect competition and for other market structures as well. For reasons to be discussed later, it turns out that the cases of perfect competition and monopoly are not very interesting when the full two-stage game is considered. The duopoly case is interesting in the full game. We return to the duopoly case now. Occasionally in what follows it will be helpful to examine circumstances in which the firms are symmetric in the sense that \( D_i = D_j = D \) and operating profits \( R_i \) and \( R_j \) are identical functions. In this case, given (8), the Nash equilibrium in outputs will be symmetric and unique. Proposition 1 is a formal statement, for the symmetric duopoly case, of the result just described for monopoly.

**PROPOSITION 1:** Assume firms 1 and 2 are symmetric. Then the Nash equilibrium output level \( q = q_i = q_j \) is increasing in the debt level \( D = D_i = D_j \) when \( R'_{iz} > 0 \) and decreasing when \( R'_{iz} < 0 \).

For this and all subsequent proofs, we present the analysis only for the \( R'_{iz} > 0 \) case, as the other case follows easily in a similar manner.

**PROOF:**

(1) \( R'_{iz} > 0 \) The comparative static effect of an increase in the common debt level, \( D \), is determined by totally differentiating first-order condition (5) \( V'_{ij} = 0 \) with respect to \( q_i, q_j, \) and \( D \). Normally one would have to differentiate both first-order conditions (for \( i \) and \( j \)) and solve the resulting simultaneous system for comparative static effects \( dq_i/dD \) and \( dq_j/dD \). Here, however, we can exploit the symmetry of the model and totally differentiate just one of the first-order conditions, incorporating the constraint that \( dq_i = dq_j = dq \). This yields

\[
V'_{ii}dq_i + V'_{id}dq + V'_{id}dD = 0.
\]

Solving for \( dq/dD \) then yields

\[
(10) \quad dq/dD = -V'_{id}/(V'_{ii} + V'_{ij}).
\]

The denominator in (10) is negative by (6) and (7). Just as for the monopoly case, \( V'_{id} \) is given by

\[
(11) \quad V'_{id} = -R'_i(\hat{z}_i)/R'_i(\hat{z}_i)f(\hat{z}_i),
\]

and, since \( R'_j \) is increasing in \( z \), \( R'_j(\hat{z}_i) \) must be negative (once again using (5)), implying that the expression in (11) is positive. Combining this with (10) yields

\[
(12) \quad dq/dD > 0,
\]

as was to be shown.

Two important corollaries follow directly from Proposition 1.

**COROLLARY 1:** Assume firms 1 and 2 are symmetric. A completely equity-financed industry \( (D = 0) \) will produce a lower output than the corresponding leveraged industry.

\[11\] The statement of Proposition 1, and Corollaries 1 and 2 are strictly correct provided \( z_i > z \) whenever \( D_i > 0 \). For simplicity we assume this to be the case in all that follows. If \( z_i < z \) for some range of positive debt levels, then the output of firm \( i \) would remain unchanged over that range.
(D > 0) when \( R'_{iz} > 0 \). If \( R'_{iz} < 0 \), the equity-financed industry will produce more than the leveraged industry.

COROLLARY 2: A necessary and sufficient condition for financial structure to have no effect on the output market is \( R'_{iz} = 0 \) for \( i = 1, 2 \).

The intuition underlying Proposition 1 and Corollary 1 is just as described in the monopoly case, and is not offset by the interaction between firms.

It has been noted by several authors, particularly Jensen-Meckling and Myers (1977), that if investments are chosen after financial structure is set (in contrast to the assumption of fixed capital maintained in our paper), then increasing the debt level should cause equity holders to undertake more risky investments, since they can declare bankruptcy in bad states of the world, while earning high rates of return in good states.\(^{12}\) Our results concerning the output decision are conceptually similar. The normal case, \( R'_{iz} > 0 \), corresponds to a situation in which increasing output increases the variance in the firm's profit stream because the marginal returns from extra production are positively correlated with the overall fortunes of the firm: the firm benefits most from increased production in good states of nature, and is harmed most by overproduction in bad states of nature. Thus increasing output is, in our model, analogous to a risky investment in the Myers framework in that it tends to be more attractive to shareholders when the firm is partially debt financed. When \( R'_{iz} < 0 \), increasing output is risk reducing. A debt-financed firm will, in this case, reduce output, as it has less need to avoid risk because it can declare bankruptcy in bad states of the world.

Corollary 2 confirms the assertion set out in the introduction that one cannot legitimately treat the financial and real sides separately, except in special cases. Our next result indicates the strategic commitment aspects of financial decisions on the firms' behavior in the output market.

PROPOSITION 2: Given \( R'_{iz} > 0 \), a unilateral increase in firm \( i \)'s debt, \( D_i \), causes an increase in \( q_i \) and a decrease in \( q_j \). If \( R'_{iz} < 0 \), then \( dq_i / dD_i < 0 \) and \( dq_j / dD_i > 0 \).

PROOF:

\( (R'_{iz} > 0) \) The method of proof is to totally differentiate first-order conditions (5) to generate the following system:

\[
\begin{align*}
(13) & \quad V'_{ij} dq_i + V'_{ij} dq_j + V'_{ij} dD_i = 0, \\
(14) & \quad V'_{ij} dq_i + V'_{ij} dq_j + V'_{ij} dD_i = 0.
\end{align*}
\]

The first point to note is that \( V'_{ij} \) does not depend on \( D_j \). Putting (13) and (14) in matrix form and using Cramer's rule to solve for comparative static effects \( dq_i / dD_i \) and \( dq_j / dD_i \) yields

\[
\begin{align*}
(15) & \quad dq_i / dD_i = -V'_{ij}V'_{ij} / B \\
(16) & \quad dq_j / dD_i = V'_{ij}J_{ij} / B,
\end{align*}
\]

where \( B = V'_{ii}V'_{ij} - V'_{ij}V'_{ji} > 0 \) from (8). Since \( B \) is positive, while \( V'_{ij} < 0 \) by (6) and \( V'_{ji} < 0 \) by (7), all that is needed is to sign \( V'_{ij} \).

\[
(17) \quad V'_{ij} = -R'_{ij}(\hat{z}_i) f(\hat{z}_i) d\hat{z}_i / dD_i = -R'_{ij}(\hat{z}_i)J_{ij} / R'_{ij} f(\hat{z}_i),
\]

by (3a). Notice that (17) is identical to (11) which we have already established is strictly positive in the proof of Proposition 1. It follows, therefore, that

\[
(18) \quad dq_i / dD_i > 0; \quad dq_j / dD_i < 0,
\]

which completes the proof.

Proposition 2 represents the key insight to be brought out in this analysis. Notice that first-order condition (5) is the reaction function for firm \( i \) in implicit form indicating

\(^{12}\) See also D. Galai and R. W. Masulus (1976) and Joseph Stiglitz and Andrew Weiss (1983) for related consideration of conflict of interest issues.
B. Debt Value Maximization

The idea that managers of a firm might be controlled by debtholders cannot be taken as a serious representation of many North American firms, but it may have some empirical significance.\textsuperscript{13} What we wish to do here is compare the objectives of debtholders with those of the equity holders.

Proceeding as before, we characterize the Nash equilibrium in the output market, where now firms act to maximize their debt value as given by (4). The Nash equilibrium output levels are given by the simultaneous solution to

\begin{equation}
W_i^j = \int_{z_i}^{z_j} R_i^j(q_i, q_j, z_i) f(z_i) dz_i = 0,
\end{equation}

for \( i = 1, 2 \).\textsuperscript{14} In addition to (19) we require

\begin{align}
W_i^j &< 0, \\
W_i^j &< 0, \\
W_i^j W_j^i - W_i^j W_j^i &> 0.
\end{align}

These conditions carry the same interpretation as (6)--(8) for the case of equity maximization. According to Proposition 1, equity-managed firms tend to produce a level of output above the level that maximizes overall firm value (debt value plus equity value).

As noted by a referee, it follows almost directly that a firm managed by debtholders would choose an output level below the output maximizing level. We state this result without proof\textsuperscript{15} as Proposition 3.

**PROPOSITION 3:** Assume firms 1 and 2 are symmetric and \( D_1 = D_2 > 0 \). Equilibrium output under equity value maximization is

\textsuperscript{13}For example, in some situations, control by debtholders, especially banks, might be a precondition for obtaining financing.

\textsuperscript{14}Besides the term in (19), \( W_i^j \) also contains the term \((d_i^j / d_q_j)(R_i^j(z) - D_i)\), which is zero by (2).

\textsuperscript{15}A rigorous proof of Proposition 3 is contained in an earlier version of our paper available as UBC Economics Discussion Paper 85–10, or from the authors.
greater than (less than) equilibrium output under debt value maximization when \( R'_{iz} > 0 \) \((R'_{iz} < 0)\).

The results of Proposition 3 indicate the conflict of interest that exists between bondholders and equity holders in the firm. The different preferences of debt and equity holders for output levels are easily explained by noting that equity holders are residual claimants in some states, while debtholders are residual claimants in other states, as illustrated in Figure 1. When \( R'_{iz} > 0 \), equity holders prefer larger output levels than do debtholders, because the equity holders are the residual claimants in states of nature when marginal returns are high. Debtholders become the residual claimants in those states when the firm cannot cover its debt obligation, which happen to be states in which the marginal returns to extra output are very low.

Rather clearly, if \( R'_{iz} < 0 \), so that high marginal returns to output are associated with bad states of the world in which the debtholders are residual claimants, the results is reversed. In such a case, debtholders would prefer higher output levels than equity holders.

An interesting implication of the foregoing discussion is that when \( R'_{iz} > 0 \), equity holders in the industry might well be better off if firms in the industry were controlled by bondholders. The reason for this is that equity-managed firms will tend to produce more than the joint profit-maximizing output. If the firms could agree to act as a cartel, they could increase combined profits. Transferring ownership to debtholders would cause the firms to move toward the cartel output level. Of course, no single firm would by itself have an incentive to transfer control, for if one firm did and the other did not, the firm controlled by the debtholders would do even worse than at the original equilibrium. In effect equity-controlled leveraged firms suffer a magnified version of the usual “prisoner's dilemma” aspect of oligopolistic rivalry, with the amount of magnification increasing with leverage.

It is intuitively reasonable that in the standard case \((R'_{iz} > 0)\), debtholders would prefer lower output levels than equity holders. What is perhaps more surprising is that debtholders, like equity holders, will in this case increase their desired output level as debt levels increase. In general comparative static effects for equity-controlled industries and debt-controlled industries are qualitatively identical.

**PROPOSITION 4:** A debt-controlled industry has the following comparative static properties: (a) If firms are symmetric, equilibrium output is increasing in the common debt level \( D \) for \( R'_{iz} > 0 \), and it is decreasing in \( D \) for \( R'_{iz} < 0 \). (b) With \( R'_{iz} > 0 \), a unilateral increase in the debt of firm \( i \), \( D_i \), causes an increase in the output of firm \( i \) and a decrease in the output of firm \( j \). If \( R'_{iz} < 0 \), the signs are reversed.

**PROOF:**

Proofs are obtained by totally differentiating first-order conditions (19) with respect to output levels and debt levels and, solving for the comparative static effects, then using (20), (21), and (22), along with (3) to obtain signs. The details of the proofs are not reported here as they are virtually identical to the details in the proofs of Propositions 1 and 2.

Although these results may seem counterintuitive at first glance, the interpretations are straightforward. Consider the \( R'_{iz} > 0 \) case. Looking at Figure 1 it is clear that an increase in debt levels raises the critical value of \( z_i \) at which bankruptcy occurs. Debtholders become the residual claimants over a wider range of states of the world, as higher levels of \( z \) are added to this range. Because \( R'_{iz} > 0 \), the expected marginal profit of extra output goes up in the range relevant to debtholders. Therefore, on the margin it is profitable for the debtholders to increase output. As pointed out by a referee, in the limit, as the debt level becomes so large that debtholders become residual claimants in all states of the world, the firm will be managed just as if the firm were completely equity financed. Part b of Proposition 4 is explained by the same phenomenon that un-
derlies the behavior of equity holders. For example, when $R'_{1i} > 0$, a unilateral increase in leverage shifts the firm’s reaction function outwards, increasing its equilibrium output and lowering the output of its rival.

III. Selection of Debt Levels

The preceding sections have examined the dependence of industry output levels on debt structure, treating debt as a predetermined or exogenous variable. In this section we describe the determinants of the debt structure. The existing literature on capital structure examines several important factors influencing the amount of debt chosen. The most standard treatment involves trading off the tax advantages of debt against bankruptcy costs in determining the optimal debt-equity position (as in Kraus and Litzenberger). Also, some analysts have stressed the use of capital structure to signal information about the firm to investors. In this section we abstract from these well-understood determinants of financial structure and focus instead on an additional motive for holding (or not holding) debt that derives from the strategic commitment aspects of leverage in relation to output markets.

In what follows we assume that the manager of the firm is free to choose whatever output level he desires after debt is issued. In particular, bond covenants, which would restrict the manager’s strategy decisions, are not considered. Bond covenants and other precommitment devices might, of course, be used for strategic purposes, but here we focus exclusively on strategic commitment through financial structure.

The equilibrium concept we use for the selection of debt levels is the Nash equilibrium in debt levels, subject to the constraint that firms and bondholders correctly anticipate the resolution of the Nash equilibrium in the output market. The equilibrium is, therefore, sequentially rational.

More specifically, firms and investors understand that equilibrium output levels are determined by debt levels as implied by the simultaneous solution to first-order conditions (5). We denote the functional dependence of output levels on debt levels as follows: $q_i = q_i(D_i)$, where $D = (D_1, D_2)$.

The objective of the firm’s owners when making the debt decision is to maximize the total value of the firm, which is the sum of the equity value $V^i$ and the debt value $W^i$. The basic point is that if potential debtholders are foresighted, then owners of the firm can sell bonds which promise to pay $D_i$ only for their true value, taking into the account the possibility of bankruptcy. This true value is $W^i$. Hence the total value of the firm, denoted by $Y^i$ is

$$Y^i(q_i(D), q_j(D), D)$$

$$= \int_{\tilde{z}_i}^{\tilde{z}_i} R(q_i(D), q_j(D)) f(z_i) dz_i$$

$$+ \int_{\tilde{z}_i}^{\tilde{z}_i} R(q_i(D), q_j(D)) f(z_i) dz_i.$$ 

This expression is obtained by adding (1) and (4) and noting that

$$\int_{\tilde{z}_i}^{\tilde{z}_i} D_i f(z_i) dz_i = D_i (1 - F(\tilde{z}_i)),$$

leaving the two terms shown in (27). This combined value is, as one would expect, the expected value of operating profits over all states of the world. Because we abstract from bankruptcy costs and from the tax advantages of debt, issuing debt is strictly a break-even transaction for the firm, except for the fact that equilibrium output levels will depend on debt levels. If, by way of contrast, planned output levels were exogenously fixed (by, for example, quota allocations) then issuing debt would be purely neutral, having no effect on total value, $Y^i$, as in a Modigliani-Miller world. However, $q_i$ and $q_j$ are written as functions of $D$, the industry debt structure, to indicate that output levels are in fact functions of the debt levels. A particular debt structure for the
industry is a commitment to a particular output structure.

The marginal effect of an increase in $D_i$ on the value of firm $i$ is given by

$$Y_{Di}^i = \left[ \int_{z}^{z_1} R_i'(z_i)f(z_i)dz_i \right] dq_i/dD_i$$

$$+ \left[ \int_{z}^{z_1} R_i'(z_i)f(z_i)dz_i \right] dq_j/dD_i$$

$$+ \left[ \int_{z}^{z_1} R_i'(z_i)f(z_i)dz_i \right] dq_j/dD_i$$

$$+ \left[ \int_{z}^{z_1} R_i'(z_i)f(z_i)dz_i \right] dq_j/dD_i.$$ 

The first term is zero by (5). The second term measures the effect of an induced change on $q_i$ on the debt value of the firm. Notice that when $R_i'(z_i) > 0$, (5) implies that $R_i'(z_i) < 0$ for all $z \leq z_1$, reflecting the fact that equity holders will choose higher output levels than debtholders would like. A similar argument serves to establish that $R_i'(z_i) > 0$ for all $z \leq z_1$ whenever $R_i'(z_i) < 0$. Taken together these two conditions imply

$$\int_{z}^{z_1} R_i'(z_i)f(z_i)dz_i < 0( > 0)$$

if $R_i'(z_i) > 0(< 0).$ 

Using Proposition 2 ($dq_i/dD_i > (< 0)$ if $R_i'(z_i) > (< 0)$, combined with (29), allows us to sign the second term in (28):

$$\int_{z}^{z_1} R_i'(z_i)f(z_i)dz_i < 0.$$

This term indicates that the induced change in output caused by taking on more debt exacerbates the conflict of interest between debt and equity holders and lowers the debt value of the firm. The third and final term of expression (28) represents the strategic effect of debt. A higher debt level for firm $i$ induces a change in the equilibrium output of firm $j$. Specifically, if $R_i'(z_i) > 0$, a higher level of debt for firm $i$ implies lower output for firm $j$. This effect, taken by itself, raises both the debt value and the equity value of firm $i$ because $R_i'(z_i) < 0$. A lowered output by firm $j$ is unambiguously good for firm $i$.

Thus there are two conflicting effects of increasing debt on the value of the firm. Extra debt worsens the conflict of interest between debt and equity holders, tending to lower the value of the firm, as reflected in (30). In addition, however, extra debt has the value-increasing strategic effect we have just described. Despite these partially offsetting effects of debt, we are able to prove that the strategic effect dominates for sufficiently small levels of debt, insuring an interior ($D^i > 0$) solution to the firm’s value maximization problem. Specifically, if $D_i = 0$ there is no conflict between bondholders and equity holders (there are no bondholders) implying that the second term of (28) is zero.\(^{17}\) The first term is of course equal to zero, while the third term remains strictly positive implying that $Y_{Di}^i$ is strictly positive at $D_i = 0$.

The other case, $R_i'(z_i) > 0$, has strikingly different implications. In this case, $dq_i/dD_i$ is positive, which implies that increases in $D_i$ will cause the rival firm to increase output, which in turn lowers profits for firm $i$. Therefore the third term of (28) is strictly negative. Expression (28) as a whole is strictly negative at all feasible (nonnegative) levels for debt and we obtain a corner solution at $D^i = 0$. These results are expressed in Proposition 5.

**PROPOSITION 5:** Industry debt levels will be strictly positive if $R_i'(z_i) > 0$. If $R_i'(z_i) < 0$, firms will be entirely equity financed.

Proposition 5, together with Proposition 1, implies that if firms hold any debt, they will produce more output than in the traditional industrial organization version of oligopoly, in which firms are assumed to be 100 percent equity financed.

The intuition underlying Proposition 5 is derived from our earlier results in Proposition

\(^{17}\)Strictly speaking $z = z_1$ in this situation, rendering the integral equal to zero.
tions 2 and 3. When marginal returns from output are positively correlated with the state of the world ($R^i_{iz} > 0$), Proposition 2 indicates that taking on more debt confers a strategic advantage of the firm, in that it causes the rival’s equilibrium output to fall. This strategic benefit from debt financing must be traded off against the resulting decrease in the debt value of the firm as debt increases (as indicated by expression (31)). We have shown that the strategic effect must dominate for sufficiently low debt levels, leading to an internal solution for the debt-equity ratio. In the case in which good states of the world are correlated with low marginal returns to output, debt has only value-reducing effects. The strategic effect of debt causes rival output to rise, and the conflict of interest between debtholders and equity holders remains. A corner solution with no debt is the solution in this case.

It also follows from the discussion above that a monopolist or a perfectly competitive firm would choose a corner solution with no debt. In either case, the strategic effect of debt financing is not relevant and firms would have no reason, in our model, to use debt. Therefore, strategic commitment through financial structure does not apply to monopoly and perfect competition in the same way that it applies to oligopoly.

Proposition 5 should be interpreted with some care, since it is derived from a model in which certain empirically important determinants of financial structure, such as taxes, are explicitly ignored. Our analysis isolates the strategic output market effects as an influence on financial structure.

Our final result makes the point that, at least in the symmetric case, equilibrium debt levels do not maximize value for the firms taken together. While this is not surprising, given that firms behave noncooperatively in financial markets, it does suggest certain collusive financing arrangements that firms might profitably engage in.

**PROPOSITION 6:** Under symmetric conditions the value of the industry, $Y^1(D) + Y^2(D) = 2Y(D)$, is not maximized in equilibrium. In particular, $dY(D)/dD < 0$ if $R^i_{iz} > 0$, and $dY(D)/dD > 0$ if $R^i_{iz} < 0$.

**PROOF:**

($R^i_{iz} > 0$) If (6) and (8) hold globally, then in the special case where $D_j = 0$, $\hat{z} = \tilde{z}$, and $q_i = q_j = q$, then (6) and (8) imply

$$\int_{\tilde{z}}^{z} R'(q, q, z_i) f(z_i) dz_i$$

is strictly concave in $q$. Proposition 6 then follows directly from the fact that completely equity-financed firms fail to maximize joint profits because of excess production. This tendency on the part of firms to overproduce is exacerbated as they take on debt.

The basic point of Proposition 6 is straightforward. A noncooperative oligopoly produces more output than a profit-maximizing cartel or monopoly would. When $R^i_{ij} > 0$, increases in debt beyond zero cause output to rise still further: the use of debt lowers profits. In other words, debt is actually procompetitive. An interesting observation is that standard Cournot oligopoly corresponds exactly to the case of complete equity financing. Ignoring the interaction between financial and output markets causes the competitiveness of such oligopologies to be understated.

This structure also suggests that central control of financing arrangements might be an attractive collusive practice. If credit markets for a particular industry are quite concentrated, then lenders would have incentives to act as facilitating agents for collusion.

**IV. Concluding Remarks**

This paper makes the basic point that product market decisions and financial decisions will normally be related. We have analyzed this relationship for a particular industry structure in which financial decisions and product market decisions follow in sequence. In this situation, the limited liability provisions of debt financing imply that changes in financial structure alter the distribution of returns between debt and equity holders, and therefore change the output strategy favored by equity holders.
Because financial structure influences the output market equilibrium, foresighted owners of the firms will have incentives to use financial structure precisely so as to influence the output market in their favor. Given the behavior of the rival firm, a firm which ignored the strategic effect of financial decisions would have lower total value than a firm which took advantage of these effects.\textsuperscript{18} These strategic uses of financial structure are purely predatory, and the net effect when both firms use them is that both firms are worse off. In our model, symmetric firms use financial structure as a commitment variable to influence the output market equilibrium. The symmetry is, however, not fundamental. One could examine an asymmetric market in which, for example, an incumbent firm used financial structure to preempt possible entry by a rival, just as capital or \( R&D \) can be used to deter entry.

Our analysis abstracts from empirically important aspects of the financial structure decision. In any empirical work one would certainly have to incorporate the tax advantages of debt, and the possibility of bankruptcy costs. In addition, the so-called agency aspects of financial structure could be very important. Specifically, if the firm is run not by shareholders but by imperfectly monitored managers, then very different results could emerge. Managers, presumably, would be very concerned about losing their jobs, especially if the outside world cannot tell whether a bankrupt firm has suffered bad luck or bad management. If so, then high debt levels might normally make the management of a firm extremely cautious and might tend to reduce industry output, in contrast to certain results in our model.

The analysis by Michael Bradley et al. and others indicates that there are systematic differences across industries and similarities within industries with respect to financial structure. Our analysis suggests that these variations in financial structure might be explained by industry-specific factors. For example, the mode of competition within an industry: price competition, quantity competition, \( R&D \) races, competitive advertising, and so on, would all have significant and different implications for financial structure. In addition, as shown in our model, the pattern of random returns (as reflected, for example, by the sign of \( R_{ij}^{2} \)) is likely to have an effect on financial structure.

Another implication of our analysis is that the institutional structure of credit markets can have an important impact on the economic performance of output markets. In our model, we identify an opportunity for credit institutions with monopoly power to act as a facilitating agent for collusion in the output market, but this is only one of several possible links. Finally, our model suggests that the public finance aspects of interest deductability might well include the output effect that is induced by the resulting higher debt levels.

The overall point that we wish to emphasize is that opening the linkage between financial markets and the "real" side of firms' decisions for analysis suggests a number of possibly important and certainly interesting economic consequences, of which our model provides some examples.

\textbf{REFERENCES}


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