COMMUNICATIONS

Implicit Tax Rate Reductions with Growth, Progressive Taxes, Constant Progressivity, and a Fixed Public Share

The presence of progressive elements in the tax system implies that increases in per capita income will produce more than proportional increases in tax revenue. This well-known aspect of our tax system has been discussed most frequently, until recent years, in terms of contracyclical fiscal policy and "automatic stabilizers." More recently, with the widespread shift of interest to the question of economic growth, the effect of a progressive revenue structure is more likely to be discussed in terms of a persistent contractionary force during periods of rising income and output. The "full employment surplus" concept developed by the Council of Economic Advisers has been used to make this point.

More specifically, let us assume that the share of public sector spending in total spending is to remain constant over a period of sustained growth in total and per capita income. Then a tax system with progressive components will generate ever-rising full employment budget surpluses. If full employment is to be maintained under such circumstances, and if private consumption spending maintains its roughly constant share of disposable income, then private investment must continually rise as a share of total output,[1] tax rates must be periodically reduced, or progressive taxation must be abandoned.[2] Still more precisely, if planned private full employment saving and investment remain roughly equal over time, the pattern of periodic tax rate reduction must produce tax revenues proportional over time to national income, the required proportion being the desired share of public spending in income and output.

It is the purpose of this paper to state with rigor and precision the pattern of tax rate reductions which will meet these requirements. The question is approached in Part I by means of a theoretical model with highly restrictive assumptions, as indicated below. After the required pattern of tax rate reductions is established for the simple model, the results are generalized sufficiently to talk about the present situation of the United States. In Part II, the 1964–65 personal income tax cut is evaluated in the light of the

[1] This possibility perhaps implies falling interest rates, and/or accelerating technological advance, and an accelerating rate of growth. We shall sidestep all these knotty issues and related questions of liquidity traps, the interest elasticity of investment and the like by concentrating our attention on the tax reduction case. More simply, we may observe that balance-of-payments considerations rule out for the present any possibilities of the easy-money-tight-fiscal-high-investment policy mix.

[2] We shall sidestep the controversy over progressive taxation by restricting our attention to the case in which progressive taxes are retained. This appears to represent a long-standing consensus in the United States; in fact, political pressures are such that changes in the degree of progressivity are likely to be small in either direction. This observation lends further relevance to our model's concentration on the constant-progressivity (defined below) case.

theoretical analysis of Part I. Part III explores some further implications of the ideas discussed in Parts I and II.

I. *An Aggregate Tax Model*

Assume an economy with the following characteristics: (1) constant population, (2) steadily rising income and output per capita at rate z per year, (3) an unchanged distribution of income over time, (4) no taxes other than a progressive personal income tax, (5) government spending is a constant share of total expenditure, and (6) planned private saving and investment are equal at full employment so that (7) the appropriate behavior of aggregate tax revenues over time is to maintain the share of income given by the desired share of public sector spending in total output. The last assumption implies a continually balanced public budget.

In the model which follows, households (or tax-paying units) are arrayed by income from the lowest to highest; the resulting array may be described by the income function

$$(1) \qquad\qquad y = f(x)$$

where y refers to an individual household's income and x to its place in the array. The ith household, for example, has income $f(x_i)$. As explained above, the number of households, n, for the moment, is held constant. If we assume a continuous function (as we easily may with n large), aggregate income, Y, is seen to be

$$(2) \qquad\qquad Y = \int_0^n f(x)dx$$

It also follows that aggregate income is rising at rate z if the income of the household occupying each position in the array is rising through time at rate z, and the distribution of income among households remains unchanged.

The income of the household occupying the ith place in the array is then, at time t,

$$(3) \qquad\qquad y_t = e^{zt}f(x_i)$$

and aggregate income at time t is

$$(4) \qquad\qquad Y_t = e^{zt}\int_0^n f(x)dx$$

Further, it is not strictly required that every household's income rise at rate z; households may "switch places" as long as the occupant of place i has an income e^z times that of the occupant one period before.

Suppose now that the progressive income tax in question is of the form

$$(5) \qquad\qquad r = ay^b$$

where r is the tax rate. This function, apparently first fitted to U.S. data in [1], gives a rather good fit to U.S. personal income tax data for 1962, as is

shown in Part II of this paper. This function has a number of useful properties; the tax amount per household, for example, is simply $y \cdot ay^b$ or ay^{b+1}. Aggregate tax revenue, T, is given by

$$(6) \qquad\qquad T = a \int_0^n [f(x)]^{b+1} dx,$$

A further important property is the following: the elasticity of T with respect to Y (Y changing only via increases in y, distribution unchanged as above) is simply $b+1$.[3]

The tax is, of course, progressive if $b>0$, proportional if $b=0$. With a and b unchanged, T/Y rises through time if Y rises through time by means of rising per capita income.

Now consider the problem of keeping the share of national income paid in taxes constant through time while per capita income rises; and doing so in such a way as to keep the progressivity of the tax system unchanged.[4] We have already shown that aggregate income is rising at rate z per year, while aggregate tax revenues are rising at rate $z(b+1)$ per year. Our problem, then, is to reduce tax rates steadily in such a way that tax revenues rise only at rate z per year. Referring to (6), we observe that this may be accomplished by steadily lowering the parameter a at rate zb per year.[5] Each household's income after tax thus rises at rate z, so that the progressivity of the tax structure remains unchanged as required.[6] This result is consistent with the observation that the parameter b is, after all, the "progressivity" parameter.

Aggregate tax revenues might also be maintained at a constant share of national income by steadily lowering the parameter b; that is, diminishing the degree of progressivity through time. The appropriate time path for values of b is not as simple as is the case for a, but tax rates may be reduced (or increased) by changing b through time.

Our model may now be generalized in a number of ways. First, consider a tax system with proportional and progressive components. It will behave over time as our model suggests in that aggregate tax revenues will rise at a faster rate than national income, though less rapidly than in our model. Revenue proportionality through time is easily achieved by applying our

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[3] A formal proof is given in [1] or is available from the author. Briefly, aggregate income is rising at rate z per year, while tax revenues are rising at rate z $(b+1)$ per year. Thus $dT/dY \cdot Y/T$ is $b+1$. The cross-sectional tax-income elasticity is also the aggregate time series tax income elasticity. The result holds regardless of the form of the income distribution function, provided only that it remains unchanged over time.

[4] By "keeping progressivity unchanged," we mean that the after-tax distribution of income among families does not change through time. Bear in mind that, by assumption, the distribution of before-tax income is also unchanged through time.

[5] Since the whole expression (6) is linear in a and would otherwise be rising at rate $z(b+1)$, dropping a at rate zb reduces the rate at which (6) rises to $z \cdot 1 = z$.

[6] By (3) and (5) the after-tax income of the ith household (at time $t=0$) is $f(x_i) - a_0 [f(x_i)]^{b+1}$. Its before-tax income, growing steadily at rate z, is (at time t) $y_t = e^{zt} f(x_i)$, and its after-tax income at time t (with the parameter a falling at rate bz from its initial value a_0) is $y_t = e^{zt} f(x_i) - a_0 e^{-bzt} [e^{zt} f(x_i)]^{b+1} = e^{zt} f(x_i) - a_0 e^{zt} [f(x_i)]^{b+1}$. This is simply e^{zt} times the initial after-tax income which is therefore growing at rate z.

results to the progressive components of the tax structure.[7] Next, consider an economy in which both the number of taxpayer families and family income rise through time. The first factor tends to produce rising tax revenue in proportion to rising national income, while the second factor introduces the effect treated in our model. Again, revenue proportionality may be maintained as suggested in our model.

Cases in which the public sector grows at some rate other than z may again be readily handled; the tax parameter a (or b, if progressivity changes are desired) is simply lowered slower or faster than rate bz, depending on whether the desired public sector growth rate exceeds or falls short of z. Insofar as contracyclical fiscal policy is desired, rate reductions are accelerated in downswings, and delayed in upswings (subject, of course, to the usual forecasting and lag problems). Moreover, the a and b parameters may be varied in offsetting ways. Progressivity may be increased (by raising b) while tax rates in general are reduced (by more than compensating reductions in a). As we shall see, this appears to be the case with respect to the recent tax cut.

II. *The Federal Personal Income Tax in the United States*

The federal personal income tax in the United States may be approximated very closely by the function $t = ay^{b+1}$ where y refers to each taxpayer's adjusted gross income and t to his tax liability. As noted above, this function has already been fitted to U.S. data for 1953. I have calculated mean adjusted gross income and mean tax liability in 1962 for each of the 29 income classes reported in [2] by dividing the number of returns in each class into total adjusted gross income and total tax liability for the class (data reproduced in Appendix I). Regressing logarithms of mean tax liability on logarithms of mean adjusted gross income gives the following estimates of the a and $b+1$ parameters in the above tax function (standard errors in parentheses)[8]

(7) $\log t = -5.4034 + 1.3589 \log y$ ($r^2 = .9936$)

                    (.04615)   (.02189)

The bottom two income classes (0–$600 and $600–1000) were omitted since the first has a zero tax liability (impossible with an exponential function) and the second yields negligible tax revenue. As the small standard errors

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[7] We ignore regressive elements since they are swamped quantitatively in the United States by the federal personal income tax. The reader may easily further generalize our model to cover tax systems with larger regressive components.

[8] The standard error in this case must be qualified in that we are regressing mean taxes on mean incomes for each income group, and all measures of error apply to these means. Individual taxes are much more widely dispersed, but this dispersion is, in effect, removed by averaging. This qualification is of no particular moment when one deals, as we do, with aggregate tax revenues. It should not be inferred that the standard error of estimate for any individual's tax liability is as low as the standard error of estimate for each class mean.

indicate, estimates of both parameters are significantly $\neq 0$ by the ordinary t test.[9]

The fit is improved, and the estimate of the b+1 parameter raised, if the top four income classes are omitted, as follows:

(8) $\log t = -6.6011 + 1.4943 \log y$ $r^2 = .9996$

                    (.00424   (.00715)

The parameter estimates are again (much more) significantly $\neq 0$ by the t test. Equation (8) compared to (7) indicates that the cross-section tax income elasticity declines in the very highest income classes. Actually, the rate structure climbs so steeply up to an income level of $100,000 that rates in the highest brackets would have to exceed 100 per cent to maintain the pattern of rate increases found below the $100,000 income level.

Adjusted gross income, to be sure, is not conceptually identical to personal income in the national income and product accounts. Transfer payments are omitted, capital gains are included, and tax-free interest payments (significant only in the high tax brackets) are omitted, to name only a few of the more obvious discrepancies. It is not easy to determine the extent to which our conclusions would be modified if good data on personal income and tax liability arrayed by the same income classes were available. There is the further complicating issue of capital gains (realized and unrealized). Realized gains are included in our adjusted gross income figures, whereas they would not be included in officially estimated aggregate personal income. One might argue that capital gains in part reflect reinvested corporate earnings so that they might properly be regarded to some extent as income. In any event, it is difficult to imagine that the parameter estimates would be drastically affected, especially the comparisons of parameters before and after the 1964–65 tax cuts discussed below.

The next question concerns the parameters of the income tax function after the 1964–65 rate reductions. I have approached this question by taking the 1962 mean adjusted gross income for each of the 29 classes and estimating for each class the mean tax liability which would result from the 1965 rates, with further adjustments for the removal of the dividends-received credit and the establishment of the minimum standard deduction.[10] Other changes in the tax code appear to have little effect on the overall progressivity of the tax and were not considered in estimating 1965 taxes for each income level. The estimates are given in Appendix I.

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[9] Mishan and Dicks-Mireaux in [1] estimated the coefficients as
$\log t = -2.49801 + 1.42483 \log y$ $(r^2 = .9983)$
for 1953 taxes. Their result differs from ours principally in the estimate of a, a result not surprising, given the temporary Korean War 10 per cent rate increase still in effect in 1953. The subsequent reduction is reflected in the 1962 rates which were the same in 1962 as in 1954. Various deductions and exemptions had, of course, changed but rates in general had not. Dropping the 10 per cent rate increase (as was done in 1954) would affect a, not b.

[10] More specifically, the application of 1965 rates to mean 1962 adjusted gross income for each income class involved the following steps: (1) for 1962 mean taxable income in each class, calculate the tax liability using both single and married taxpayer rates; (2) select the calculation which most closely approximates the 1962 reported mean tax liability, and calculate the

When estimated mean tax liabilities are regressed on mean incomes (again omitting the two bottom groups), parameter estimates are:

(9) $\log t = - 6.2081 + 1.4174 \log y$ $(r^2 = .9914)$

            $(.06737)$ $(.02644)$

As before, the fit is improved and the estimate of $b+1$ raised if the top four income groups are omitted:

(10) $\log t = - 7.6538 + 1.5809 \log y$ $(r^2 = .9986)$

            $(.01468)$ $(.01330)$

Again, a glance at the standard errors indicates the significance of the parameters.

We may now examine the tax cut in the light of our earlier analysis. As expected, the parameter a is reduced as we compare (7) and (9) or (8) and (10). The hypothesis that a did not change is easily rejected at the .01 significance level in both cases. It appears that a fell about 55 per cent (or, omitting the highest brackets, 65 per cent). If the rate reductions had been confined solely to this parameter, tax revenues from 1962 income would have fallen by about 55 per cent. Using our results of Part I, such a reduction would have absorbed the increase in revenues—over and above a constant proportion of national income—for a period of over 65 years. Actually the reduction in revenues (income held constant) was about 20 per cent; the reduction in a was in part offset by an increase in b. The tax reduction, therefore, disposed of the increase in revenues (beyond a growth proportional to national income) for about 20 years.

Now we consider the hypothesis that the parameter $b+1$ was increased by the 1964 legislation. If we omit the 4 highest income classes and compare (10) with (8), the increase in $b+1$ is significant at the .01 level by the t test. Comparing (9) with (7), the results are less certain. At the .01 level, the difference is not significant; one would have to adopt a significance level slightly above 10 per cent before the increase in $b+1$ would meet the test. We may conclude that the progressivity of the tax clearly increased for income levels up to $100,000, a result partly explainable by the adoption of a minimum standard deduction and the elimination of the dividends-received credit. For the very highest income groups, the new tax rates remain strongly progressive, but one can give no clear-cut answer as to whether the tax function in that area is more or less progressive than before. We should also mention the probability that errors in estimating tax liabilities on the basis of the 1965 rates are largest in the highest income brackets.

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percentage deviation; (3) use 1962 taxable income as the base for estimating the tax liability at 1965 rates (this implies the same deductions and exemptions except as noted below); (4) apply the 1962 single or married rate for each income class as determined by (2), and adjust by the same percentage deviation; (5) recalculate for each class, beginning with the lowest; apply an estimated minimum standard deduction; continue until the results do not differ from the previous method (in this case, the 4000–4500 income bracket); (6) in higher income returns, separately calculate the tax on capital gains, and the dividends-received credit.

III. *Summary and Further Implications*

The foregoing analysis has suggested that federal personal income tax revenues will rise about 1.4 per cent for every 1 per cent increase in income per taxpayer. In a world of continual full employment private saving-investment equality, full employment would require a rising government share in total spending, or periodic tax cuts. If a constant tax-income relation, and a constant degree of progressivity are desired, the indicated tax cut pattern, as noted above, is a reduction in a at the rate bz per annum. In a cyclical, balanced-budget-over-the-cycle world, tax cuts would have to average rate bz per annum, but could be integrated into the pattern of contracyclical fiscal policy by delaying rate reductions during upswings, and accelerating them during downswings. If there appears to be some chronic

APPENDIX

MEAN INCOME, AND MEAN TAX LIABILITIES FOR 1962
(ACTUAL) AND AT 1965 RATES (Estimated)

Income Class	Mean Adjusted Gross Income	Mean Tax Liability 1962 Rates	Mean Tax Liability 1965 Rates
(1)	(2)	(3)	(4)
0–600	326	—	—
600–1,000	798	14	8
1,000–1,500	1,241	52	30
1,500–2,000	1,747	91	60
2,000–2,500	2,252	135	90
2,500–3,000	2,749	190	130
3,000–3,500	3,251	247	172
3,500–4,000	3,750	307	217
4,000–4,500	4,249	374	267
4,500–5,000	4,749	439	320
5,000–6,000	5,495	538	400
6,000–7,000	6,482	671	509
7,000–8,000	7,474	831	645
8,000–9,000	8,479	1,012	803
9,000–10,000	9,472	1,203	968
10,000–11,000	10,470	1,389	1,129
11,000–12,000	11,468	1,599	1,310
12,000–13,000	12,467	1,813	1,493
13,000–14,000	13,468	2,031	1,678
14,000–15,000	14,469	2,244	1,858
15,000–20,000	17,006	2,842	2,360
20,000–25,000	22,220	4,230	3,512
25,000–50,000	33,350	8,016	6,677
50,000–100,000	65,688	22,089	18,585
100,000–150,000	119,375	47,403	42,693
150,000–200,000	171,150	71,498	65,160
200,000–500,000	283,564	122,145	113,010
500,000–1,000,000	666,052	295,811	279,777
over 1,000,000	2,020,222	875,761	819,659

Source: Cols. 1–3, [2, Table 1, p. 13]. Col. 4, Author's estimates.

tendency for private investment to fall short of full employment saving, the whole process (average rate cuts at rate bz per annum with cyclical adjustment) would have to take place at a lower level of aggregate tax revenues. A further conclusion is that the process of tax reduction is likely, over the long pull, to be a periodic occurrence unless government outlay is to rise consistently at a rate considerably greater than the rate at which total output increases. We should also consider whether smaller, more frequent tax cuts might be more desirable than large reductions of the 1964–65 variety.

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The Myth of Absolute Advantage

Perhaps the oldest, and certainly one of the most viable, myths in all of economics is that of "absolute advantage." Although absolute advantage is a logical impossibility in the context of interregional and international trade theory (as I hope to show in what follows), that has not prevented it from surviving to its one hundred and fiftieth birthday (or beyond if we credit Torrens rather than Ricardo with its origin).

I. *Ricardo*

The myth begins with Ricardo's exposition in his *Principles* [6, p. 71]. His familiar example is as follows:

To produce an equal quantity of cloth requires: in England, 100 man-years of labor; in Portugal, 90 man-years of labor. To produce an equal quantity of wine requires: in England, 120 man-years of labor; in Portugal 80 man-years of labor. And Portugal is said (in modern terms) to have an "absolute advantage" over England in production of both commodities.[1]

Suppose we ask this question: Why does it take 100 man-years to produce in England an amount of cloth which requires only 90 man-years in Portugal? Ricardo says, "It would undoubtedly be advantageous to the capitalists of

[1] Ricardo's example is sometimes turned around with other writers by using an equal number of units of labor in both countries and deriving therefrom different amounts of product. Thus, Ricardo's example could be expressed by saying that with an equal quantity of labor: England will produce 90 yards of cloth or 80 gallons of wine; Portugal will produce 100 yards of cloth or 120 gallons of wine. This, obviously, does not alter the basic question raised, namely, why labor in Portugal is more productive than labor in England.