Motivating Wealth-Constrained Actors

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We examine how owners of productive resources (e.g., public enterprises or financial capital) optimally allocate their resources among wealth-constrained operators of unknown ability. Optimal allocations exhibit: (1) shared enterprise profit—the resource owner always shares the operator's profit; (2) dispersed enterprise ownership— resources are widely distributed among operators of varying ability; (3) limited benefits of competition—the owner may not benefit from increased competition for the resource; and, sometimes, (4) diluted incentives for the most capable—more capable operators receive smaller shares of the returns they generate. Implications for privatizations and venture capital arrangements are explored. (*JEL* D82, D44, D20)

Wealth constraints complicate many important economic relationships. For example, entrepreneurs of varying and unobservable ability are often unable to develop and market their inventions independently because they lack the requisite financial resources. Consequently, they turn to venture capitalists for financing. Also, when governments in developing countries seek to privatize state-owned enterprises, they typically face domestic buyers whose wealth is small relative to the value of the privatized enterprise.[1] The purpose of this research is to determine how an owner of productive resources such as financial capital or public enterprises optimally allocates the resources among wealth-constrained operators of differing and unknown ability.

Wealth constraints affect the optimal allocation procedure in many ways. To illustrate their main effects most simply, consider the following stylized example. Suppose the government of a developing country wishes to maximize its financial return from selling a state-owned enterprise to domestic operators. Each potential operator is privately informed about his ability to run the enterprise successfully and has limited wealth. If potential operators were not wealth constrained, the government would optimally auction the enterprise to the highest bidder. The most capable operator in the population would win the auction because expected enterprise profit increases with the operator's ability. Furthermore, the operator's stake in the enterprise (i.e., his share of the profit ultimately generated by the enterprise) would increase with his ability. The operator with the highest possible ability level would pay the highest price for the enterprise in return for receiving all of the profit it generated, leaving the government with no stake in the enterprise. Furthermore, the government's expected returns would increase as the number of potential operators, and thus the expected ability of the most capable operator, increased.[2]

When potential operators have little wealth,

[1] See Maxim Boycko et al. (1995), for instance. Nemat Shafik (1996) reports that foreign investors were involved in only 29 percent of the 2,655 privatizations that occurred between 1988 and 1993. Larry Benveniste et al. (1994) document related problems the Resolution Trust Corporation faced when it sold the massive assets of failed banks in the United States. John McMillan (1994), Peter Cramton (1995, 1997), and Ian Ayres and Cramton (1996) report that the U.S. Federal Communications Commission (FCC) was concerned that liquidity constraints might discourage small businesses, women, and minorities from bidding for radio spectrum rights. Douglas Holtz-Eakin et al. (1994), Patrick Kaufman and Francine Lafontaine (1994), and Jean-Jacques Laffont and Mohamed Matoussi (1995) provide evidence of binding wealth constraints in other settings.

[2] See Laffont and Jean Tirole (1987, 1993), R. Preston McAfee and McMillan (1987), and Michael Riordan and

they are unable to make substantial initial payments for large stakes in the enterprise. Consequently, to increase its financial payoff from the privatization, the government optimally retains a significant stake in the privatized enterprise, even though doing so limits the operator's incentive to run the enterprise efficiently. Thus, wealth constraints result in a pronounced level of *shared enterprise profit.*[3]

Wealth constraints can also lead to particularly *diluted incentives for the most capable* operators. This is because the substantial stake the government typically retains in the privatized enterprise renders it particularly anxious to select the most capable operator to run the enterprise. The government can often distinguish more capable wealth-constrained operators from their less able counterparts by inviting potential operators to bid on the share of realized profit they will ultimately deliver to the government, rather than on an initial payment they will make for the enterprise. More capable operators are often willing to deliver larger shares of realized profit to the government because their costs of running the enterprise are smaller. In return, the more capable operators are promised higher probabilities of being selected to run the enterprise. The government does not always award the enterprise to the most capable operator, however. Doing so would make it unduly attractive for potential operators to exaggerate their capabilities. Since the government awards the enterprise to less capable operators with positive probability, an *ex post* inefficient *dispersion of enterprise ownership* can result.

Although the government may retain larger stakes in the operations of more capable operators, the government will limit its stake so as not to unduly dilute performance incentives for these operators. Consequently, wealth constraints limit the ability of the most capable operators to compete aggressively for the enterprise, which leads to *limited benefits of competition* for the government. The government

often anticipates no benefit at all from increased competition among operators above some threshold level. Thus, expanded participation by wealth-constrained operators will not necessarily increase the government's payoff from privatizations.

Although our formal model abstracts from many real-world features of privatizations, it may help to explain why governments typically retain a sizable share of the enterprises they sell to private investors, even though doing so can reduce ongoing incentives for diligent performance and give rise to problems associated with soft budget constraints. (Other explanations are reviewed in Section IV.) Our findings also suggest a rationale for partitioning government assets and selling the individual components to different investors, even when doing so sacrifices economies of scale or scope.[4] In addition, our analysis indicates why privatized assets may not always be sold to their most efficient operators, even in the absence of politically motivated favoritism or cronyism.

Our analysis may also complement others (see Section IV) in explaining observed properties of venture capital contracts. To illustrate, our model may help to explain why venture capitalists (VCs) typically maintain a significant ownership stake in the ventures they finance (William Sahlman, 1990). Our model also predicts that VCs will not always provide large ownership stakes to even the most capable entrepreneurs. Our model also suggests why, even in the absence of risk aversion, VCs may diversify their investments by funding entrepreneurs of varying ability, and why VCs may limit their search for projects to fund, even when search costs are small.

We discuss the empirical implications of our model and their relevance in venture capital and privatization settings in more detail in the concluding section of the paper. First, though, we analyze a simple formal model in Section I to document the central effects of wealth constraints most transparently. A more general model is analyzed in Section II for the setting where operators have no wealth. Section III extends the model of Section II in three direc-

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Sappington (1987) for developments of these points in related settings.

[3] Shafik (1996) reports that the government stake in privatized enterprises averages 34 percent. Enrico Perotti (1995) provides supporting evidence.

[4] See Yeon-Koo Che and Ian Gale (1996b, 1998) for an analogous prescription.

tions. First, it allows operators to have some, but still limited, wealth. Second, it allows for renegotiation of the reward structure after an operator is selected but before he acts. Renegotiation serves to place a lower bound on the operator's stake in the project and so limits the extent to which the operator's stake varies with his ability. Third, it permits the original resource owner to contribute directly to the production process, as VCs often do, for example, when they direct and monitor the projects they finance. We identify conditions under which the original resource owner optimally contributes more inputs to those projects in which she holds a larger stake, as empirical findings suggest VCs do (Harry Sapienza and Jeffrey Timmons, 1989).[5]

I. A Simple Setting

We begin with a very simple formal model to highlight the central effects of wealth constraints. For concreteness, we emphasize a privatization interpretation of the model. Suppose initially there is a single operator who is uniquely qualified to operate a state-owned enterprise or project. The project either succeeds and provides gross value $V > 0$ or fails and provides a payoff of 0. Success or failure is observed publicly. Success is more likely the more (unobservable) effort e the operator delivers and the greater his (unobservable) ability, $\theta \in \{\theta_L, \theta_H\}$, where $\theta_L < \theta_H$. There are diminishing returns to effort, but the marginal productivity of effort is greater when the operator's ability is higher. We will denote by $p(e, \theta)$ the probability that the project succeeds when the operator with ability θ delivers effort

e.[6] The government and the operator are both risk neutral. The operator requires nonnegative expected utility to operate the project. The government seeks to maximize its expected financial returns.

If the government shared the operator's knowledge of his ability and if the operator were not wealth constrained, the government could ensure its ideal outcome by selling the project to the operator at its maximum expected value, $p(e^*(\theta), \theta)V - e^*(\theta)$, where $e^*(\theta) = \text{argmax}_e\{p(e, \theta)V - e\}$. When the government does not know the operator's ability, it cannot ensure this ideal outcome. Any attempt to do so would induce the high-ability operator (i.e., the one with ability θ_H) to understate his ability to secure the project at a lower sales price. To mitigate this incentive, the government would retain a stake in the project when the operator claims to have low ability, and only allow the operator to retain all the profit he generates when he admits to having high ability. Thus, the operator would secure a larger stake in the project (in return for a larger initial payment), the greater his ability.

An important complication arises when the operator is wealth constrained. To illustrate this complication most vividly, suppose the operator has no wealth, and so cannot make any initial payment to the government. In this case, the government's only useful policy instrument is the operator's stake in the project. Armed with only this one instrument, the government cannot tailor the operator's stake in the project to his unobserved ability (since the operator will always select the highest stake that is offered). Furthermore, the government will retain a stake in the project (even though doing so reduces the operator's effort supply) because this stake constitutes her only source of compensation when the operator cannot pay in advance for the project. Thus, wealth constraints can limit the tailoring of reward structures to the operator's ability, preclude the outright sale of the project to the operator, and result in considerable profit sharing.

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[5] Our analysis extends the work of Riordan and Sappington (1988), Laffont and Jacques Robert (1996), and Che and Gale (1996a, b, 1998, 2000), who examine how wealth constraints affect optimal mechanisms for selling an object to bidders who are privately informed about their valuation of the object and/or their wealth. The operators' wealth levels are common knowledge in our model, but the "object" being sold is a project that requires the unobserved effort of the operator. Lewis and Sappington (2000a) analyze a setting where a single operator is privately informed about his wealth, his ability, and his effort supply. Lewis and Sappington (2000b) examine the interplay between an adverse selection and a moral hazard problem, but presume operators' ability levels to be common knowledge.

[6] Formally, we assume $p_e(e, \theta) > 0$, $p_{ee}(e, \theta) < 0$, $0 < p(e, \theta_L) < p(e, \theta_H) < 1$, and $p_e(e, \theta_H) > p_e(e, \theta_L)$ for all $e > 0$.

Now suppose there are two potential operators, neither of whom has any wealth initially. Also suppose the project is indivisible and can be operated by only one operator at a time.[7] Each operator knows (only) his own ability (θ), which is either low or high. The government cannot observe the ability of either operator, and views the operators as identical *ex ante,* since each operator's ability is the realization of an independent draw of the same random variable.

Finally, suppose the probability of success given effort e and ability θ is $p(e, \theta) = \theta e^{\gamma}$, where $\gamma \in (0, 1)$. It is readily shown that if there were only one operator who could operate the project under this technology and if the operator had no wealth, the government would optimally afford the operator the stake γ in the project regardless of his ability. Thus, with two potential operators, the government could award the stake γ in the project to each operator with probability 0.5. If it did so, though, the government would anticipate a higher return when the high-ability operator operates. Therefore, the government would prefer to assign the project to the high-ability operator more than half the time. The only way it can do so without inducing the low-ability operator to exaggerate his ability is to provide a smaller stake in the project, the more frequently the operator is selected to produce.

This is precisely the optimal strategy for the government. It affords the operators a choice between: (1) a relatively high probability of being selected to operate the project and a relatively small stake in the project; and (2) a smaller probability of operation coupled with a larger stake. The options are designed to induce the high-ability operator to select the higher probability of operation and the smaller stake, and the low-ability operator to select the smaller probability of operation and the larger stake. Since he has no wealth in this setting, the high-ability operator cannot simply outbid his low-ability counterpart by paying more initially for the right to always operate with a large stake in the project. Instead, the high-ability operator distinguishes himself by offering to work for a relatively small stake in the project, provided he is chosen to produce with sufficiently high probability. Production is less profitable for the low-ability operator, so he is less willing to accept a small stake in the project. Instead, the low-ability operator selects the larger stake in the project and the smaller probability of operation.

The key effect of multiple operators is to endow the government with an additional policy instrument: the probability that an operator will be selected to operate the project. With two instruments, the government can tailor equilibrium reward structures to the operators' unobserved abilities. In contrast to the setting with no wealth constraints, though, an operator's stake in the project varies inversely with his ability in the present setting. Furthermore, an operator is chosen to produce the project with positive probability, even when he is less capable than his counterpart. Therefore, the *ex post* allocation of production rights may be inefficient when operators are wealth constrained.

These findings may help to explain why governments often retain substantial stakes in the enterprises they privatize. They also indicate that when it awards an enterprise to other than the most capable operator, a government is not necessarily engaged in corruption or cronyism.

II. *N* Operators with No Wealth

We now demonstrate that these qualitative conclusions persist more generally. We will also show that, in contrast to the setting where operators are not wealth constrained, the government may not gain as more operators bid for the enterprise. For concreteness, we again emphasize the privatization interpretation of the model.

We assume there are $N \geq 1$ operators qualified to operate the government enterprise or project. Operators differ only in their ability, and each operator is privately informed

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[7] Alternatively, the project might be infinitely divisible and characterized by constant returns to scale (as in the analysis in Section III). In this case, the government awards fractions of the project rather than probabilities of operating the project. The qualitative conclusions drawn below hold under this alternative interpretation of our model.

about his ability from the outset. The government views each operator's ability as the realization of an independent and identically distributed random variable with density $f(\theta)$ and distribution function $F(\theta)$.[8] As in Section I, the project either succeeds and produces gross value V or fails and produces 0.[9] For convenience, we assume the elasticity of the probability of success with respect to effort (γ) does not vary with the operator's ability. Formally, we assume

$$(1) \quad p(e, \theta) = \theta e^{\gamma} \qquad \text{for all } \theta \in [\underline{\theta}, \bar{\theta}],$$

$$\text{where } \gamma \in (0, 1).^{10}$$

It can be shown that because the operators are identical *ex ante* the optimal auction will be symmetric and can be characterized by analyzing the allocation offered to the representative operator. The variable $\mu(\theta)$ will denote the equilibrium probability with which an operator of ability θ is selected to operate the project.[11] $T(\theta) > 0$ will denote the equilibrium payment the government makes to the producer of ability θ when the project succeeds. No payment is made when the project fails. $I(\theta)$ is the initial

payment an operator of ability θ delivers to the government in equilibrium. This payment cannot exceed an operator's initial wealth, $W \geq 0$, which is the same for all operators.[12] Each operator's wealth level is common knowledge, as is the entire structure of the model. All operators also have the same opportunity wage, which is assumed to be zero.

The producer chooses his (unobservable) effort to maximize his expected net return, which is the difference between his expected monetary payoff and the cost of his effort. All operators have the same constant marginal cost of effort, which is normalized to unity. Therefore, the effort the producer with ability θ supplies when he is promised T for success is

$$(2) \quad e(T, \theta) \equiv \underset{e}{\text{argmax}}\{p(e, \theta)T - e\}.$$

We will denote by $\{\mu(\theta), T(\theta), I(\theta)\}$ the allocation that an operator with ability θ receives in equilibrium. An operator's allocation consists of the probability that he operates (μ), his payment for success when he produces (T), and the initial payment (I) he delivers to the government. The equilibrium expected profit of an operator with ability θ is

$$(3) \quad \Pi(\theta) = \mu(\theta)[p(e(T(\theta), \theta), \theta)T(\theta)$$

$$- e(\cdot)] - I(\theta).$$

The government seeks to maximize its expected net return.[13] The government's problem, [P], can be represented formally as

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[8] $f(\theta)$ is assumed to be continuously differentiable and to have strictly positive support on $[\underline{\theta}, \bar{\theta}]$. We also impose the standard regularity condition, $(d/d\theta)\{[1 - F(\theta)]/f(\theta)\} \leq 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$, to rule out (uninteresting) regions of pooling in the optimal mechanism. We also avoid corner solutions by assuming that V and $\underline{\theta}$ are sufficiently large that the government always assigns the project to some operator and induces a strictly positive effort supply.

[9] Our findings extend to the case where success gives rise to a stochastic payoff R, with arbitrary distribution $G(R)$. As noted in Section IV, it remains to determine whether our qualitative conclusions change when there are more than two possible performance levels and the entire distribution of performance levels is affected by the operator's effort and ability.

[10] When an operator is unable to deliver transfer payments to the government, standard methods for characterizing incentive-compatible allocations cannot be employed here. However, as the proof of Lemma 1 reveals, equation (1) gives rise to a convenient specification for the operator's profit function that facilitates a complete characterization of incentive-compatible allocations.

[11] If the project exhibits constant returns to scale (as assumed in Section III), then $\mu(\theta)$ can be interpreted as the fraction of the government's resources that the operator with ability θ expects to receive in equilibrium.

[12] The probability that an operator is selected to produce will typically vary in equilibrium with his own ability and with the abilities of other operators. In contrast, payments to and from an operator will vary only with his own ability. Randomized payments are not optimal because aggregate surplus is an increasing, concave function of the producer's effort supply.

[13] Since the government is not wealth constrained, it can lend resources to operators if it is optimal to do so. Consequently, in the present setting, capital markets would not resolve the critical contracting frictions identified below unless financiers had better information than the government about operators' ability or effort levels. An additional role for capital markets might emerge if a richer set of performance levels were admitted. (See Section IV.)

(4)

$$\text{Maximize } N \int_{\underline{\theta}}^{\bar{\theta}} \{\mu(\theta)[p(e(T(\theta), \theta), \theta)V$$
$${\{\mu(\theta),T(\theta),I(\theta)\}}$$

$$- e(\cdot)] - \Pi(\theta)\} \, dF(\theta)$$

subject to, for all θ, $\hat{\theta} \in [\underline{\theta}, \bar{\theta}]$:

(5) $$I(\theta) \le W;$$

(6) $$\Pi(\theta) \ge 0;$$

(7) $$\Pi(\theta) \ge \mu(\hat{\theta})[p(e(T(\hat{\theta}), \theta), \theta)T(\hat{\theta})$$

$$- e(\cdot)] - I(\hat{\theta});$$

(8) $$\mu(\theta) \in [0, 1/N]; \quad \text{and}$$

(9) $$N \int_{\theta}^{\bar{\theta}} \mu(\theta') \, dF(\theta') \le 1 - [F(\theta)]^N.$$

Expression (4) reflects the fact that the government's expected net return is the difference between the total expected surplus from the project and the rent that accrues to the operators. Expression (5) ensures that no operator is ever asked to deliver an initial payment that exceeds his wealth. Expression (6) guarantees the participation of all operators. Expression (7) ensures that $(\mu(\theta), T(\theta), I(\theta))$ is the equilibrium allocation of the operator with ability θ.[14] Expressions (8) and (9) together ensure that the operating probabilities assigned by the government are well defined: (8) bounds these probabilities between 0 and $1/N$ and (9) limits the operating probabilities assigned to operators

[14] In particular, the operator with ability θ prefers his equilibrium allocation to any other allocation $(\mu(\hat{\theta}), T(\hat{\theta}), I(\hat{\theta}))$ that the government might offer. The revelation principle ensures that this formulation is without loss of generality. We will sometimes refer to $(\mu(\hat{\theta}), T(\hat{\theta}), I(\hat{\theta}))$ as the allocation an operator receives when he claims to have ability $\hat{\theta}$. An allocation procedure, or mechanism, is said to be incentive compatible if it eliminates incentives for operators to misrepresent their ability levels.

with ability of at least θ to the probability that they arise in the population.

In this section, we consider the case where the operators have no wealth (so $W = 0$). When he cannot pay the government directly, an operator can convince the government to afford him a greater chance of operating the project only by offering to produce for a smaller stake in the project. This inverse relationship between μ and T is characterized in Lemma 1.

LEMMA 1: *Suppose $W = 0$. Then $T(\theta) = k[\mu(\theta)]^{-(1-\gamma)}$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$ at any feasible solution to [P], where $k > 0$ is a constant.*[15]

Proposition 1 reports that operators with higher ability are optimally selected to produce more often than their less able counterparts, but they receive a smaller stake in the project. The proposition refers to $S(N)$, which is the government's expected surplus [i.e., the value of expression (4)] at the solution to [P].

PROPOSITION 1: *Suppose $W = 0$ and $N \ge 2$.*[16] *Then the solution to [P] is characterized by:*

(i) *Dispersed enterprise ownership, i.e., the project is not always awarded to the operator with the highest ability (so $N \int_{\underline{\theta}}^{\theta} \mu(\theta') \, dF(\theta')$ is not equal to $[F(\theta)]^N$ for all θ), even though the probability that an operator produces increases with his ability (so $\mu'(\theta) > 0$);*

(ii) *Shared enterprise profit, i.e., the government always retains a stake in the project (since $T(\theta) < V$ for all θ) and the government's stake increases with the producer's ability (i.e., $T'(\theta) < 0$ for all θ) resulting in particularly diluted incentives for the most capable operators; and*

(iii) *Limited benefits of competition, i.e., the government's expected welfare does not increase as the number of operators increases beyond some threshold level, N^* (so $S(N)$ does not vary with N when $N \ge N^*$).*

[15] The Appendix provides a sketch of the proof of Lemma 1 and all other formal results. The details of the proofs are available from the authors upon request.

[16] If $N = 1$, then $T(\theta) = \gamma V$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$ at the solution to [P].

COROLLARY 1: *If $W = 0$, $\gamma = 0.5$, and $f(\theta)$ is the uniform density, then $N^* \leq 5$, so the government anticipates no gains from having more than five operators.*

The conclusions in Proposition 1 stem from the following considerations. The government would like to award the project to the most capable operator, ceteris paribus. However, when they have no wealth, more capable operators cannot simply offer to pay more for the right to operate the project. To distinguish themselves from their less capable counterparts, the more capable operators offer to work for a relatively small stake in the project, provided they are selected to operate the project relatively often. Under the presumed technology, less capable operators anticipate particularly small returns from a project in which they hold a small stake, and so do not find it profitable to bid the small stakes that their more capable counterparts do.

This reward structure leaves the government with two reasons to award the project particularly often to operators with high ability. First, their superior ability makes them relatively likely to succeed. Second, their success results in a particularly large payoff $(V - T)$ for the government. However, the government must limit the frequency with which it assigns the project to operators with high ability. Otherwise, it will have to provide particularly small stakes in the project to high-ability operators to deter low-ability operators from exaggerating their capabilities. Such small stakes would diminish the producer's effort unduly and thereby reduce the government's expected welfare. Thus, the least costly way to distinguish more capable from less capable operators is to award the project to less capable operators with positive probability. Consequently, as noted earlier, the assignment of a state-owned enterprise to a less capable private operator is not necessarily indicative of cronyism or politically motivated favoritism.

The government's preferred award schedule can be translated into a cumulative award distribution, which is graphed as a solid line in Figure 1. Also graphed in Figure 1 are two distribution functions that reflect cumulative award probabilities when the project is always awarded to the operator with the highest ability in the population (so $N \int_{\underline{\theta}}^{\theta} \mu(\theta') \, dF(\theta') =$
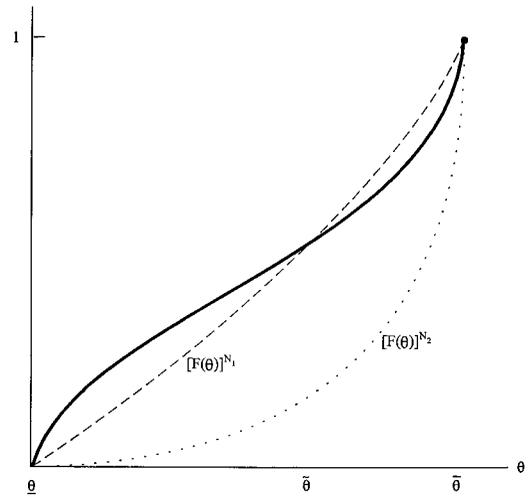

FIGURE 1. FEASIBLE AND PREFERRED DISTRIBUTION FUNCTIONS $(N_2 > N_1)$

$[F(\theta)]^N$). The distribution function drawn with the longer broken segments is the relevant one when there are relatively few (N_1) operators, and the function depicted with the shorter broken segments is the relevant one when there are more (N_2) operators. Notice that as the number of operators increases, the probability that the project is awarded to an operator with less than any specified ability level declines when the project is always awarded to the operator with the highest ability in the population. Since the government can always award the project to the most capable operator in the population, the broken lines in Figure 1 represent feasible award distributions, which vary with the number of bidders.

As Figure 1 illustrates, the government's preferred award distribution is generally not feasible when it faces few operators (e.g., when $N = N_1$). The government generally prefers to assign the project to high-ability operators (e.g., those with ability in excess of $\tilde{\theta}$ in Figure 1) with greater frequency than they arise in the population. In such situations, the government must depart from its preferred award distribution to avoid assigning excessive operating probabilities to more capable operators.

As the number of operators increases, operators with relatively high ability become more numerous in the population. Eventually (as when $N = N_2$ in Figure 1, for example), they

arise with sufficient frequency that if the government were to always assign the project to the most capable operator in the population, it would award the project to the most capable operators too often, and would have to decrease their stake in the project too severely to ensure incentive compatibility. Once this critical number of operators (N^*) is reached, the government's preferred award distribution becomes feasible,[17] and additional operators do not alter the allocations that the government implements. Consequently, in contrast to the setting where wealth constraints do not bind, the government's welfare does not increase as the number of operators increases.[18,19] This finding implies that simply increasing the number of wealth-constrained bidders may not serve to increase a government's returns from privatizations.

Corollary 1 reports that the critical number of bidders can be relatively small. When $\gamma = 0.5$ and ability levels are distributed uniformly, for example, the government never gains from having more than five operators bid for the project.[20]

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[17] In terms of Figure 1, N^* can be viewed as the smallest value of N for which the $[F(\theta)]^N$ function lies everywhere below the government's preferred award distribution function on $(\underline{\theta}, \bar{\theta})$.

[18] Absent wealth constraints, an operator's equilibrium stake in the project increases with his ability. Therefore, as the operator's ability increases, so does the expected surplus from the project. The greater the number of bidders, the greater the likelihood that the chosen operator will have high ability and so make a large payment to the government in return for a large stake in the project. Consequently, the government always gains as the number of bidders increases when the bidders are not wealth constrained.

[19] When few operators ($N < N^*$) are present initially, an increase in the number of operators enables the government to assign the project to the most capable operators with greater frequency. When it does so, the government is forced to reduce the associated reward for success to ensure incentive compatibility. Consequently, the reward structure afforded an operator of specified ability generally varies with the number of operators. The same is not true when operators are not wealth constrained [as Laffont and Tirole (1987, 1993 p. 318), McAfee and McMillan (1987), and Riordan and Sappington (1987) show in related settings]. Absent wealth constraints, increased competition among operators relaxes the critical adverse selection problem but does not affect the moral hazard problem directly. It affects both problems when wealth constraints bind.

[20] Manelli and Vincent (1995) identify other settings where increased competition among suppliers does not benefit a buyer. In one such setting, suppliers are privately informed about their operating costs, and lower costs imply

III. Extensions: Limited Wealth, Renegotiation, and Additional Inputs

We now extend the analysis of Section II in three directions. First, we allow the operators to have some, but still limited, wealth. Second, we permit mutually advantageous renegotiation of the compensation arrangement after a producer is selected, but before he acts. Third, we allow for additional observable inputs to the production process.

A. Limited Wealth

First suppose that all $N \geq 2$ operators have strictly positive, but still limited, wealth. Proposition 2 summarizes our findings for the (expositionally convenient) case where operator wealth is sufficiently large that the wealth constraints (expression (5) in $[P]$) do not bind for the least capable operators.[21] We call this the limited wealth setting.

PROPOSITION 2: *Suppose $W > 0$. Then at the solution to $[P]$ in the limited wealth setting:*

(i) *When their ability is below a critical level, $\theta^L \in (\underline{\theta}, \bar{\theta})$, operators deliver larger initial payments and receive a larger stake in the project as their ability increases (so $I'(\theta) > 0$ and $T'(\theta) > 0$). Furthermore, an operator with ability below θ^L is selected to produce if and only if he is the most capable operator in the population (so $N \int_{\underline{\theta}}^{\theta} \mu(\theta') \, dF(\theta') = [F(\theta)]^N$).*

(ii) *For higher levels of ability ($\theta \in [\theta^L, \bar{\theta}]$), each operator delivers his entire wealth to the government initially (so $I(\theta) = W$) and is afforded a higher operating probability and a smaller stake in the project as his ability increases (so $\mu'(\theta) > 0$ and $T'(\theta) < 0$).*

When operators have intermediate levels of wealth, the government offers only small stakes

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lower product quality. Here, increased bidding competition among suppliers can reduce equilibrium expected quality more than cost, thereby reducing the buyer's welfare.

[21] When W is strictly positive but sufficiently small, the wealth constraints may bind for all operators. In this case, the solution to $[P]$ does not contain the region described in property (i) of Proposition 2.

in the project to low-ability operators to deter more capable operators from understating their ability. Because their stakes are small, the low-ability operators with limited wealth can deliver initial payments that compensate the government adequately for the meager operating profit they anticipate. In contrast, the more capable operators do not have sufficient resources to deliver initial payments that would compensate the government fully for the large stake in the project and the pronounced probability of operation that the government would like to afford them. Once their wealth is exhausted, high-ability operators distinguish themselves from their less able counterparts by offering to work for smaller stakes in the project in return for higher probabilities of being selected to produce, as described in Proposition 1. Thus, the key distortions that arise when operators have no wealth persist for the higher levels of ability in settings where operators have some, but limited, wealth. In particular, shared enterprise profit, dispersed enterprise ownership, and particularly diluted incentives for the most (and perhaps the least) capable operators arise.

B. *Renegotiation*

Now suppose that, as is common in practice, the government's commitment powers are limited. In particular, the government will always renegotiate the operator's stake in the project if it is advantageous to do so. Any renegotiation that takes place occurs before the producer chooses his effort level (*e*). For simplicity, we consider the case where the $N \geq 2$ operators have no wealth (so $W = 0$), and call this the renegotiation setting.

If renegotiation occurs, it will take place when an operator with particularly high ability is selected to produce. Recall that absent renegotiation, an operator with particularly high ability is selected to produce relatively frequently, but is promised a relatively small stake in the project. This stake can be less than the stake (γ) that maximizes the government's expected payoff from the project. Such a small stake will always be increased when renegotiation is possible because a larger stake for the operator increases his expected payoff, and, through the extra effort it induces, also increases the government's expected return. Therefore, the operator's equilibrium stake in the project will always be at least γ.

Any stake above γ (but below 1) will not be renegotiated. A larger stake would reduce the government's expected payoff, and the operator has no resources to compensate the government for this loss. A smaller stake would reduce total expected surplus. Consequently, the compensation the operator would demand to offset the smaller stake would outweigh the increase in the government's expected payoff. Therefore, the optimal mechanism in this setting will exhibit the key properties described in Proposition 1 for the smaller realizations of θ (i.e., those for which $T(\theta) \geq \gamma V$). A constant payment for success ($T(\theta) = \gamma V$) will be implemented for the larger ability realizations, along with a constant probability of being selected to operate the project. These observations are summarized in Proposition 3.

PROPOSITION 3: *At the solution to the government's problem in the renegotiation setting:*

(i) If an operator's ability is below a threshold level, the probability that he is selected to produce increases and his reward for success decreases with his ability (i.e., there exists a $\theta^R \in [\underline{\theta}, \bar{\theta}]$ such that $\mu'(\theta) > 0$ and $T'(\theta) < 0$ for all $\theta \in [\underline{\theta}, \theta^R]$).
(ii) If an operator's ability exceeds this threshold level, neither the probability that he is selected to produce nor his stake in the project varies with his ability. The operator's stake is the one that maximizes the government's expected return from the project (i.e., $\mu'(\theta) = 0$ and $T(\theta) = \gamma V$ for all $\theta \in (\theta^R, \bar{\theta}]$).

Proposition 3 suggests that particularly large government stakes in privatized enterprises are unlikely when the government's commitment powers are weak. It also implies that limited government commitment powers can lead to even more pronounced dispersion of enterprise ownership (i.e., more frequent production by less capable operators) than will arise in the presence of wealth constraints alone.

C. *Productive Inputs from the Resource Owner*

To this point, we have abstracted from any direct contributions the original resource owner (e.g., the government or a venture capitalist)

might make to the production process. Such inputs, however, can be important in practice. For example, the monitoring, oversight, and expertise a venture capitalist contributes to a project she funds can be instrumental in determining its ultimate success or failure. We have also viewed the productive resource as an indivisible asset to this point. But resources like the financial capital a venture capitalist provides are divisible, so we now consider an extension and interpretation of our model that better captures elements of a venture capital arrangement, including a perfectly divisible resource and inputs supplied by the original resource owner.

Suppose there are $N \geq 2$ entrepreneurs, each of whom has a potentially profitable project, but none of the financial capital (wealth) needed to operate the project. Also suppose the venture capitalist's commitment powers are unimpeded (so renegotiation does not occur), and the productive inputs the venture capitalist (VC) provides are observable and contractible.[22] As noted, the VC's essential resource, capital, is perfectly divisible and so can be partitioned and assigned to multiple entrepreneurs. In this setting, $\mu(\theta) \in [0, 1/N]$ will denote the fraction of its capital the VC delivers to an entrepreneur of ability θ. The more capital an entrepreneur receives, the larger the scale (μ) at which he can operate, and thus the larger the gross payoff (μV) when his project succeeds. The entrepreneur's marginal cost of effort when he operates at scale μ is μ.

The productive input K that the VC supplies to an entrepreneur in this environment is assumed to increase the probability $p(\cdot)$ that the project succeeds as follows:

$$(10) \qquad p(e, \theta, K) = K\theta e^{\gamma}.$$

For simplicity, suppose the cost to the VC of supplying K units of the productive input to an entrepreneur who operates at scale μ is $K^{\beta}\mu^{\delta}$, where $\delta \in [0, 1)$ and $\beta > 1$ are parameters. Since $\delta > 0$, it is more costly for the VC to

enhance the probability of project success by any given amount the larger the scale of the project. However, since $\delta < 1$, increasing returns are present, because the VC's cost of delivering the input increases less than proportionately with the scale of operation. Thus, the input might constitute the monitoring, advice, or supervision that VCs often provide, for example (Sahlman, 1990).[23]

It is convenient to assume that the scale economies associated with the input are not too pronounced relative to the cost of supplying the input. Formally, we assume $\beta > \max\{\delta/[1 - \gamma], [1 - \delta]/\gamma\}$ in this setting, which we call the setting with venture capital inputs. Proposition 4 summarizes the main features of the optimal policy in this setting.

PROPOSITION 4: *In the setting with venture capital inputs, entrepreneurs with greater ability operate at larger scales but hold smaller stakes in their projects and receive more inputs from the venture capitalist.*

Proposition 4 suggests that under the specified conditions, a VC will invest most heavily in the projects of the most capable entrepreneurs, and will secure a relatively large stake in their projects. The VC will also devote more personal attention to the projects in which he has invested so heavily.[24]

IV. Conclusions and Implications

We have examined optimal procedures for allocating productive assets to wealth-constrained

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[22] A double moral hazard problem [as in Thomas Hellman (1998)] arises when the government's contributions are not contractible. It is also convenient to assume that N is sufficiently large that the probability constraints (9) do not bind at the solution to the government's problem in this setting.

[23] More generally, the impact of the VC's inputs might vary with observable characteristics of the entrepreneur. For instance, the advice and guidance provided by a VC might prove to be particularly valuable to an entrepreneur with limited experience, as some empirical studies suggest (Harry Sapienza and Jeffrey Timmons, 1989; Sanford Ehrlich et al., 1994).

[24] The proof of Proposition 4 reveals that $K(\theta)$ and $\mu(\theta)$ may decrease with θ, whereas $T(\theta)$ increases with θ if $\delta/[1 - \gamma] < \beta < [1 - \delta]/\gamma$. Thus, the VC may deliver more of the productive input to the least-capable entrepreneurs, who also receive relatively large portions of the VC's resource, but particularly meager rewards for success. In this sense, when her marginal cost of delivering the input does not increase too rapidly with its supply, the VC may essentially take over the projects of the least capable entrepreneurs by substituting her own input for their effort.

operators who are privately informed about their ability. When wealth constraints prevent operators from bidding higher initial payments for the project, they resort to bidding lower stakes in the project, which results in substantial profit sharing. Under certain conditions, more capable operators bid the lowest stakes in return for an increased chance of operating the project, leading to particularly diluted incentives for the most capable producers. Binding wealth constraints can also lead to (*ex post* inefficient) dispersed enterprise ownership, as projects are sometimes awarded to low-ability operators to prevent them from exaggerating their ability. In addition, limited benefits of competition arise, as the project owner does not gain as the number of wealth-constrained operators increases above some threshold level.

Our model was intentionally streamlined to illustrate most clearly the qualitative effects of wealth constraints. Therefore, the model cannot serve as the basis for comprehensive recommendations regarding the optimal design of venture capital contracts or privatization arrangements.[25] Nevertheless, the model may provide an explanation for some common features of these arrangements that complement other explanations in the literature. For example, an entrepreneur typically receives a low wage and is required to invest most of his personal resources in any venture funded by a venture capitalist (Sahlman, 1990), as our model predicts for all except possibly the least capable entrepreneurs. Furthermore, venture capitalists tend to devote the most nonfinancial assistance to those ventures in which they hold the largest ownership stakes (Sapienza and Timmons, 1989), as the venture capitalist does in our setting with venture capital inputs.

Our model also suggests why governments might partition divisible public assets (such as electricity generating capacity) and award the partitions to different wealth-constrained operators, even if scale economies are present and

even if *ex post* competition in the market is not crucial.[26] Moreover, our model explains why ongoing government stakes in privatized enterprises can be optimal, even when such stakes diminish incentives for efficient operation and aggravate problems associated with soft budget constraints (Eric Maskin, 1999).

A government may choose to retain an ownership stake in a privatized firm, even if investors are not wealth constrained. The government may do so to exercise ongoing control over noncontractible dimensions of the firm's performance (John Vickers and George Yarrow, 1991; Oliver Hart et al., 1997) or to secure some immediate revenue while waiting until considerable uncertainty about the ultimate value of the enterprise has been resolved before selling the entire firm (Tim Jenkinson and Colin Mayer, 1988; Olivier Blanchard et al., 1991). By retaining partial ownership in a privatized firm, the government may also credibly signal to potential investors either its intention not to expropriate them in the future (Perotti, 1995) or its favorable private assessment of the innate value of the enterprise (Hayne Leland and David Pyle, 1977).

Factors other than wealth constraints may also explain the stake that venture capitalists retain in the ventures they finance. For example, this stake may help to motivate the venture capitalist to act in the interest of her financial backers by identifying the most promising projects (Sahlman, 1990) or the best individuals to operate the project (Hellmann, 1998). The stake could also help to reduce the risk imposed on a risk-averse entrepreneur (Yuk-Shee Chan et al., 1990).

It would be useful to distinguish among these hypotheses empirically. One way to do so might be to hold control rights constant when examining ownership rights. In settings where control rights can be established independently of ownership rights (through representation on the board of directors, for example), substantial ownership shares may not be necessary to control the noncontractible activities of the privatized firm, but they may help to avoid selling the firm at a price that is far below its market value. A second test of the

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[25] Notice, for example, that we abstracted from the measurement problems that can complicate the implementation of earnings sharing arrangements in practice. McMillan (1994) reports that the FCC decided against earnings sharing arrangements when selling radio spectrum rights, in part, because of the difficulties involved in measuring the earnings that multiproduct firms derive from a single product.

[26] Che and Gale (1996b, 1988) provide similar explanations.

hypotheses might examine the extent to which privatized firms pursue social objectives (such as high labor to capital ratios) over private objectives. A finding that the tendency to do so is largely invariant to the government stake in the firm could suggest that wealth constraints, rather than ongoing control, motivate the government stake. A third test might exploit the fact that if wealth constraints are the primary reason for an ongoing government stake in a privatized enterprise, then this stake should tend to be most pronounced when the bidding process is closed to foreign investors, holding domestic investment resources constant. In contrast, if the stake serves primarily to convince foreign investors that their investments will not be expropriated, then the government stake should be larger when foreign investors bid for the privatized firm, ceteris paribus.

A distinguishing prediction of our model is the inverse relationship between a producer's stake in the project he operates and the probability that he is selected to produce or the fraction of the available resources he is afforded. Thus, a finding that venture capitalists tend to hold the largest ownership stakes in those projects in which they invest the bulk of their investment funds would provide some empirical support for our model. The same would be true of a finding that governments tend to retain the most pronounced stakes in the largest of the many enterprises that they privatize at any point in time.

Our model also suggests that for some production technologies, binding wealth constraints may limit the sensitivity of observed performance to contractual incentives.[27] The limited sensitivity of performance to incentives can arise not because contractual incentives provide little motivation, but because the most pronounced incentives are optimally provided to the least capable producers. Thus, empirical investigations of the impact of incentives on performance must control for relevant wealth constraints and their equilibrium impact on the abilities of selected operators.

In closing, we mention four of the many directions in which our analysis might be fruitfully extended. First, more general production technologies might be studied. In addition to alternative relationships among effort, ability, and performance, more than two distinct performance levels might be admitted. With a richer outcome space, meaningful differences between debt and equity can arise, thereby creating an expanded role for loans to wealth-constrained operators.[28] Second, additional asymmetries among operators might be explored. The precise manner in which relatively wealthy operators are optimally handicapped when competing against less-wealthy operators seems particularly interesting to explore.[29] Third, the merits of encouraging alliances among wealth-constrained operators might be examined. Alliances reduce the effective number of competitors and may facilitate collusion, but they increase the financial resources of allied operators. Fourth, additional intertemporal elements and alternative bargaining structures (including competition among resource owners) might be analyzed. For example, if an entrepreneur can expropriate investments made by a venture capitalist, then gradual, staged financing of the project may be optimal (Neher, 1999). Also, when past performance provides useful signals about an operator's ability, future assignment of ownership and control rights may be linked to past performance in interesting ways.[30]

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[27] In our model with $W = 0$ and $p(e, \theta) = \theta e^{\gamma}$, the equilibrium probability of success does not vary with θ or with the operator's reward structure whenever the number of potential operators is sufficiently large ($N \geq N^*$).

[28] See Lewis and Sappington (2000c) for an illustration of this point in a different context.

[29] McMillan (1994), Cramton (1995, 1997), Ayres and Cramton (1996), and McAfee and McMillan (1996) discuss some of the techniques the FCC employed to encourage participation by selected (often wealth-constrained) groups in the auctions for radio spectrum.

[30] Chan et al. (1990) examine how past performance affects future financial rewards and ownership rights in venture capital relationships.

APPENDIX

Sketch of the Proof of Lemma 1

Let $\Pi(\hat{\theta}|\theta)$ represent the expected profit of the agent with ability θ when he reports his ability to be $\hat{\theta}$. When $W = 0$, $I(\theta) = 0$ for all θ. Consequently,

(A1) $$\Pi(\hat{\theta}|\theta) = \mu(\hat{\theta})[p(e(T(\hat{\theta}), \theta), \theta)T(\hat{\theta}) - e(T(\hat{\theta}), \theta)].$$

Incentive compatibility then provides

(A2) $$\frac{\mu'(\theta)}{\mu(\theta)} = -\frac{T'(\theta)}{T(\theta)}\left[\frac{p(e(T(\theta), \theta), \theta)T(\theta)}{p(e(T(\theta), \theta), \theta)T(\theta) - e(\cdot)}\right].$$

Equations (1) and (2) in the text imply

(A3) $$p(e(T(\theta), \theta), \theta)T(\theta) = \frac{1}{\gamma}\,e(T(\theta), \theta).$$

(A2) and (A3) imply that at any feasible solution to [P]

(A4) $$\frac{\mu'(\theta)}{\mu(\theta)} = -\frac{T'(\theta)}{T(\theta)}\left[\frac{1}{1-\gamma}\right] \qquad \text{for all } \theta \in [\underline{\theta}, \bar{\theta}].$$

Solving the differential equation in (A4) provides the statement in the lemma.

Sketch of the Proof of Proposition 1

After some substitution using equations (1) and (2) and Lemma 1, [P] can be rewritten as

(A5) $$\underset{\{\mu(\theta)\}}{\text{Maximize }} N \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\gamma}\,[\gamma\theta]^{[1/(1-\gamma)]}\{VK_1^{[\gamma/(1-\gamma)]}\mu(\theta)^{1-\gamma} - K_1^{[1/(1-\gamma)]}\}\,dF(\theta)$$

subject to:

(A6) $$z(\theta) \equiv 1 - N \int_{\theta}^{\bar{\theta}} \mu(\theta')\,dF(\theta') - [F(\theta)]^N \geq 0 \qquad \text{for all } \theta \in [\underline{\theta}, \bar{\theta}].$$

Manipulation of the necessary conditions for an optimum reveal that the optimal values of $T(\theta)$, $\mu(\theta)$, and $S(N)$ do not vary with N over regions in which constraint (A6) does not bind at the solution to [P]. It is also readily shown that in these regions

(A7) $$\mu(\theta) = [F(\theta)]^{N-1} \qquad \text{and} \qquad T(\theta) = K_1[F(\theta)]^{-(N-1)(1-\gamma)},$$

where K_1 does not vary with θ or N. (A7) implies that optimal allocations vary with N over some regions in which constraint (A6) binds, and so $S(N)$ increases with N in these regions because constraint (A6) is relaxed as N increases. It is readily shown that constraint (A6) does not bind at the solution to [P] if N is sufficiently large.

Sketch of the Proof of Corollary 1

Constraint (A6) will not bind at the solution to [P] if

(A8) $$\frac{\mu'(\theta)}{\mu(\theta)} \leq \frac{(d/d\theta)[F(\theta)]^{N-1}}{[F(\theta)]^{N-1}} \qquad \text{for all } \theta \in (\underline{\theta}, \bar{\theta}),$$

(A9) $\qquad$ where $\mu(\theta) = \left[N \int_{\underline{\theta}}^{\bar{\theta}} \theta^m \, dF(\theta) \right]^{-1} \theta^m, \qquad$ where $m \equiv \dfrac{1}{\gamma(1 - \gamma)}.$

Using (A9), it is readily shown that when $\gamma = 0.5$, (A8) holds if $N \geq 5$.

Sketch of the Proof of Proposition 2

Using (3) and (A1), and viewing $\mu(\theta)$ and $T(\theta)$ as control variables, $\Pi(\theta)$ and $z(\theta)$ as state variables, $\alpha^{\Pi}(\theta)$ and $\alpha^z(\theta)$ as costate variables, and $\beta^{\Pi}(\theta)$ and $\beta^z(\theta)$ as multipliers, $[P]$ can be formulated as an optimal control problem with the following Hamiltonian:

(A10) $\qquad H = \{\mu(\theta)[p(e(T(\theta), \theta), \theta)V - e(\cdot)] - \Pi(\theta)\}f(\theta)$

$\qquad\qquad + \lambda^L(\theta)\{W - \mu(\theta)[p(e(\cdot), \theta)T(\theta) - e(\cdot)] + \Pi(\theta)\}f(\theta)$

$\qquad\qquad + \alpha^{\Pi}(\theta)\mu(\theta)p_{\theta}(e(\cdot), \theta)T(\theta) + \alpha^z(\theta)Nf(\theta)[\mu(\theta) - [F(\theta)]^{N-1}]$

$\qquad\qquad + \beta^{\Pi}(\theta)\Pi(\theta)f(\theta) + \beta^z(\theta)z(\theta)f(\theta).$

It can be shown that there exists a $\theta^L \in (\underline{\theta}, \bar{\theta})$ such that $\lambda^L(\theta) = 0$ for all $\theta \in [\underline{\theta}, \theta^L)$. This fact and the necessary conditions for an optimum derived from (A10) reveal that

(A11) $\qquad T(\theta) = \dfrac{V}{1 + \left[\dfrac{1 - F(\theta) - \lambda}{\gamma\theta f(\theta)} \right]} \qquad$ and $\qquad \mu(\theta) = [F(\theta)]^{N-1} \quad$ for all $\theta \in [\underline{\theta}, \theta^L),$

(A12) $\qquad$ where $\lambda \equiv \displaystyle\int_{\theta^L}^{\bar{\theta}} \lambda^L(\theta) \, dF(\theta) \leq 1 - F(\theta) \qquad$ for all $\theta \in [\underline{\theta}, \theta^L).$

(A11) and (A12) imply that $T'(\theta) > 0$ for all $\theta \in [\underline{\theta}, \theta^L)$ under the maintained assumptions.

Since $\mu'(\theta) > 0$ and $T'(\theta) > 0$ for all $\theta \in [\underline{\theta}, \theta^L)$, incentive compatibility requires $I'(\theta) > 0$ for all $\theta \in [\underline{\theta}, \theta^L)$, which provides property (i) of the Proposition. The proof of property (ii) is analogous to the proof of Proposition 1.

Sketch of the Proof of Proposition 3

The discussion in the text explains why a renegotiation-proof contract requires $T \in (\gamma V, V]$. The proof parallels the proof of Proposition 1, except that the additional constraint $T(\theta) \in [\gamma V, V]$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$ is appended to the principal's problem. This constraint can be rewritten as

(A13)
$\qquad \mu(\theta) \in [\underline{\mu}, \bar{\mu}], \quad$ where $\underline{\mu} \equiv K_1^{[1/(1-\gamma)]}V^{-[1/(1-\gamma)]} \quad$ and $\quad \bar{\mu} \equiv K_1^{[1/(1-\gamma)]}[\gamma V]^{-[1/(1-\gamma)]}.$

It can be shown that at the (renegotiation-proof) solution to the principal's problem

(A14) $\qquad\qquad\qquad\qquad\qquad T(\theta) = K_1 h^{-(1-\gamma)}\theta^{-(1/\gamma)},$

and

$$(A15) \qquad \qquad \mu(\theta) = \begin{cases} h\theta^m & \text{if } h\theta^m \le \bar{\mu} \\ \bar{\mu} & \text{if } h\theta^m > \bar{\mu} \end{cases},$$

$$(A16) \qquad \text{where } m \equiv \frac{1}{\gamma[1-\gamma]} \qquad \text{and} \qquad N\left[\int_{\underline{\theta}}^{\bar{\mu}/h} h\theta^m \, dF(\theta) + [1 - F(\bar{\mu}/h)]\bar{\mu} \right] = 1.$$

(A14) and (A15) provide the characterization provided in the statement of the proposition.

Sketch of the Proof of Proposition 4

Incentive compatibility requires that at the solution to the principal's problem

$$(A17) \qquad \qquad \frac{\mu'(\theta)}{\mu(\theta)} = -\left[\frac{1}{1-\gamma} \right]\left[\frac{K'(\theta)}{K(\theta)} + \frac{T'(\theta)}{T(\theta)} \right] \qquad \text{for all } \theta \in [\underline{\theta}, \bar{\theta}].$$

(A17) implies that for some constant $\alpha > 0$

$$(A18) \qquad \qquad \mu(\theta) = \alpha[K(\theta)T(\theta)]^{-[1/(1-\gamma)]} \qquad \text{for all } \theta \in [\underline{\theta}, \bar{\theta}].$$

Using (A18), it can be shown that when constraints (A6) do not bind, the principal's problem can be written as

$$(A19) \quad \underset{K(\theta), T(\theta)}{\text{Maximize}} \int_{\underline{\theta}}^{\bar{\theta}} \{ \alpha\theta V[\theta\gamma]^{[\gamma/(1-\gamma)]}[T(\theta)]^{-1} - \alpha[\theta\gamma]^{[1/(1-\gamma)]}$$

$$- \alpha[K(\theta)]^\beta[K(\theta)T(\theta)]^{-[\delta/(1-\gamma)]} - \alpha\gamma\theta^{[1/(1-\gamma)]} - \rho\alpha[K(\theta)T(\theta)]^{[1/(1-\gamma)]} \} \, dF(\theta),$$

where $\rho \ge 0$ is the Lagrange multiplier associated with the constraint $N \int_{\underline{\theta}}^{\bar{\theta}} \mu(\theta) \, dF(\theta) \le 1$.

Pointwise optimization with respect to $T(\theta)$ provides, after some simplification,

$$(A20) \qquad \qquad \rho[K(\theta)T(\theta)]^{(1-\delta)/(1-\gamma)} = K(\theta)^\beta[\beta(1-\gamma) - \delta].$$

(A20) implies that $\beta > [\delta/(1-\gamma)]$ is required to ensure an interior solution. (A20) also implies that

$$(A21) \qquad \qquad K(\theta)^{-\{\beta + [(1-\delta)/(1-\gamma)]\}} = \rho^{-1}[\beta(1-\gamma) - \delta]T(\theta)^{(1-\delta)/(1-\gamma)}.$$

(A21) implies that under the maintained assumptions, $T'(\theta) < 0$ whenever $K'(\theta) > 0$.

Solving (A21) for $T(\theta)$ and employing the necessary conditions for an optimum provide

$$(A22) \qquad \qquad K(\theta)^{[\beta\gamma/(1-\delta)]-1} = k_0 V\theta[\theta\gamma]^{[\gamma/(1-\gamma)]}, \qquad \text{where } k_0 > 0 \text{ is a constant.}$$

(A22) implies that $K'(\theta) > 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$ provided $\beta > [1 - \delta]/\gamma$.

It is readily shown that

$$(A23) \qquad \qquad \mu(\theta) = k_1 T(\theta)^{-\{\beta/[\beta(1-\gamma)+1-\delta]\}}, \qquad \text{where } k_1 > 0 \text{ is a constant.}$$

(A23) implies that $\mu'(\theta) > 0$ whenever $T'(\theta) < 0$.

In summary, we have shown that if $\beta > \max\{[\delta/(1 - \gamma)], [(1 - \delta)/\gamma]\}$, then we will have an interior solution with $K'(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. Consequently, $T'(\theta) < 0$ for all θ from (A21), and so $\mu'(\theta) > 0$ for all θ from (A23).

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