Highly Efficient Wavefront Transformation with Acoustic Metasurfaces

by

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Michael Eric Gehm

Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Electrical and Computer Engineering in the Graduate School of Duke University
2020
Abstract

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Abstract

Metamaterials are artificially engineered materials or structures that exhibit exotic properties that are not found in nature. They have been serving as a primary approach to fully control the behavior of electromagnetic waves, acoustic waves and elastic waves in recent years, and is at present a highly active research area. Metasurfaces, as the 2D version of metamaterials, have opened up unprecedented possibilities for controlling waves at will, offering a solution of molding wave propagation within a thin sheet of structures. Most metasurface designs are based on the so-called generalized Snell’s Law (GSL) which achieves their functionalities by engineering the local phase shift in the unit cells. However, the efficiency of phase-gradient metasurfaces is fundamentally limited by the impedance mismatch and local porer flow mismatch between incident field and reflected/transmitted field, so that part of the energy is scattered into unwanted higher-order diffracted modes, which hinders the applicability in various scenarios. In this work, we approach these issues by exploiting acoustic bianisotropy (Willis coupling for acoustics) as an additional degree of freedom to control waves. We have explored highly efficient wavefront engineering in airborne acoustics, from manipulating simple plane waves and cylindrical harmonics to more complicated fields and finally, arbitrary wavefronts. Then we extended the application of bianisotropic metasurfaces to general impedance matching problems and demonstrated wavefront engineering in underwater acoustics with two examples: an aberration-layer penetration metasurface and a 3D acoustic tweezer.
This dissertation provides a summary of the work undertaken to achieve highly efficient and functional wavefront engineering devices, and briefly outlines some objectives for future work. Firstly, we designed an acoustic bianisotropic unit cell with full control over its scattering properties and demonstrated bianisotropic metasurfaces that overcome the fundamental limits of phase-gradient based metasurfaces. Second, we mapped the approach from Cartesian coordinates into cylindrical coordinates and demonstrated the generation of a pure field with high angular momentum. Third, we introduced surface waves to help power redistribution along the metasurface and achieved highly-efficient beam splitting and reflection. Forth, we further introduced the power-flow conformal metasurface to meet the power balance requirements for an arbitrary perfect wavefront transformation. Then we extended the application of bianisotropic metasurfaces and proposed a general impedance matching strategy, and demonstrated the idea with a case of aberration-layer penetration in water. Last but not least, by shaping the wavefront of underwater ultrasound, a 3D acoustic tweezer is demonstrated for manipulating a wide range of particles in a contact-less manner.
To anyone who is interested in my works.
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List of Abbreviations and Symbols

Symbols

\( c \)  Speed of sound
\( E \)  Electric field
\( H \)  Magnetic field
\( I \)  Intensity field
\( k \)  Wavenumber
\( M \)  Transfer matrix
\( p \)  Pressure field
\( R \)  Global reflection coefficient
\( S \)  Scattering matrix
\( T \)  Global transmission coefficient
\( v \)  Velocity field
\( Z \)  Impedance (Impedance matrix)
\( \rho \)  Density
\( \mu \)  Shear modulus
\( \nabla \)  Gradient operator
\( \nabla \cdot \)  Divergence operator
\( \nabla \times \)  Curl operator
\( \langle \cdot \rangle \)  Average over a period
### Abbreviations

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<tr>
<td>ABS</td>
<td>Acrylonitrile Butadiene Styrene</td>
</tr>
<tr>
<td>BC</td>
<td>Boundary Condition</td>
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<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
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<tr>
<td>PDMS</td>
<td>Polydimethylsiloxane</td>
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<tr>
<td>PEB</td>
<td>Post Exposure Bake</td>
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<tr>
<td>PEC</td>
<td>Perfect Electric Conductor</td>
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<td>PIV</td>
<td>Particle Image Velocimetry</td>
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<td>PMC</td>
<td>Perfect Magnetic Conductor</td>
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<td>PZT</td>
<td>lead zirconate titanate</td>
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<td>SLA</td>
<td>Stereolithography</td>
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<td>SLS</td>
<td>Selective Laser Sintering</td>
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1

Introduction

1.1 Materials and human development

Materials science has shaped the development of civilizations since the dawn of mankind. Better materials have allowed humanity to spread and thrive, and advancements in material processing continue to impact society today. The story of materials is the story of civilization. Indeed, materials have been regarded as such an important aspect of civilizations that entire periods have been defined by the predominant material used: Stone Age, Bronze Age, Iron Age, etc.

Dating back to prehistory, the use of materials begins before the Stone Age. Our ancestors use materials they found in nature, such as bones, wood, stone and animal skin to make weapons, tools, and decorations. Since the Bronze Age, the ability to melt metals and clays allows human to modify the natural materials, leading to an increased hardness that surpass raw materials. Early composite materials also appear: by gluing wood at different angles, our ancestors achieves the materials with better properties than natural wood. In the meantime, interestingly, the development of spinning and weaving represents the early practice of building up the bulk
material with small components like threads and rattan. In the Iron Age, with the development of iron casting techniques, people start to control material properties of steel for various purposes and create materials that do not exist in nature, such as glass.

Since the 20th century, the fast development of chemistry has enabled our ability to synthesize molecules that are not found in nature. Synthetic polymers such as plastics and rubbers started to become the prominent materials for industrial applications, thanks to their vast flexibility in a wide achievable range of material properties.

In modern society, with the help of microscopy, we accumulate knowledge about how material properties are linked to molecule composition and their mesoscale structures. It is then natural to consider the inverse problem: Can we design the molecules, proteins, and their structures on the micro- and nano-scale to create material properties on demand? These types of materials are represented by semiconductors that support the development of Information Age, and structured molecules such as car-

Figure 1.1: The development of materials and civilization.
bon nanotubes, quantum dots and designed proteins that are shaping our future lives.

However, in this age, we are still limited by the ability to mass-produce nanostructures and form bulk materials for industrial use. Luckily, many practical wave-matter interactions typically do not require manipulation of structures on the molecular and atomic levels. The wave behaviors in matters are largely determined by the constituents on the scales below or at interested wavelengths, which can be many orders of magnitude larger than a molecule. With current fabrication technology, it is relatively easy and feasible to build structures that are of the same length scale or fractions of the wavelength of interest. In this regime, the detailed structures inside a unit cell is 'overlooked' by the wave, as if the waves are traveling in a homogeneous bulk material with certain properties.

This category of material by designing structures on the subwavelength scale to form bulks with desired effective properties are termed as 'metamaterials'. ”Meta” means ”beyond” since early works are mostly focused on exploring effective material properties that cannot be found in nature. With the development of this field, the term ”metamaterial” has been broadened to cover the idea of designing artificial materials with properties on demand, not only for waves but also for static systems.

1.2 A brief history of acoustic metamaterials and metasurfaces

The ability to fully control the behavior of classical waves (for example, electromagnetic and acoustic waves) has long been desired and is at present a highly active research area. Among numerous routes, metamaterials have served as a primary approach in recent years. There are several families of metamaterials, corresponding to the waves they target to manipulate: electromagnetic metamaterials, acoustic metamaterials, mechanical metamaterials, thermal metamaterials, etc. We will mainly focus on acoustic metamaterials in this dissertation.
The emergence and early development of acoustic metamaterials are largely inspired by their electromagnetic counterparts. Early electromagnetic metamaterials are motivated by the goal of controlling wave propagation in new ways by exploring exotic material spaces, such as negative permittivity and permeability, then double negative properties. At almost the same time, such motivation spread to the field of acoustic metamaterials: early works are primarily focused on exploring materials with negative effective densities Liu et al. (2000); Yang et al. (2008); Lee et al. (2009a) and bulk modulus Fang et al. (2006); Lee et al. (2009b) for sound isolating applications, followed by materials with both negative effective density and bulk modulus at the same frequency range Li and Chan (2004); Lee et al. (2010); Brunet et al. (2015), leading to negative refractive index that showed promising applications in acoustic imaging Park et al. (2011); Kaina et al. (2015).

An important next step is to expand the knowledge into higher-dimensional cases, represented by the design of anisotropic effective materials. Remarkably, as one of the material properties along the principle axis becomes negative, the material dispersion is characterized by hyperbola instead of ellipse, resulting in the so-called hyperbolic materials. Such exotic anisotropic materials allow evanescent waves to propagate into the far-field, facilitating acoustic far-field imaging beyond the diffraction limit Ao and Chan (2008); Li et al. (2009); Shen et al. (2015).

Another driving force for the development of anisotropic material is the advent of transformation acoustics. The idea emerged from concepts that originated in electromagnetism and optics Pendry et al. (2006); Schurig et al. (2006); Liu et al. (2009). Later, the concept is mapped to other types of waves, such as acoustics Cummer and Schurig (2007); Chen and Chan (2007). It is one of the most powerful tools that bridge functional wave devices with material property distribution with the help of coordinate transformation. The most famous example is acoustic cloaks that hide objects from sound Zhang et al. (2011); Popa et al. (2011); Zigoneanu et al. (2014).
In the meantime, the need for cloaking objects in the underwater environments stimulates the development of inertial and pentamode metafluids Norris (2009).

Metamaterials provide unprecedented ways of wave manipulation. However, the realization typically comes with intricate fabrication, and the devices are subjected to losses. The quest to manipulate waves using the smallest possible amount of space and engineered materials has led to the exploration of acoustic metasurfaces. In contrast to the volumetric modulation using metamaterials, two-dimensional arrangements of subwavelength cells offer an alternative solution of molding wave propagation within a planar or nearly flat geometry, having thicknesses much smaller than the wavelength.

It is worth noting that, the interaction between waves and microstructures on a planar profile is ubiquitous in nature, such as the coloring of bird feather and butterfly wings, and the study of structural coloration is still at present a hot topic. The idea of controlling waves by engineering the phase profile of a planar structure can be traced back to the invention of diffraction gratings in the 18th century. In particular, blazed grating, a special type of diffraction grating, controls the diffraction order by optimizing the geometric profile along a surface. These gratings with designed structures can be viewed as the predecessor of metasurfaces in recent studies.

In both electromagnetic Sun et al. (2012); Yu et al. (2011); Kildishev et al. (2013) and acoustics Zheludev and Kivshar (2012); Cummer et al. (2016); Chen et al. (2016); Glybovski et al. (2016); Yang and Sheng (2017), metasurfaces have facilitated unprecedented possibilities for controlling waves at will. On the one hand, acoustic metasurfaces have been designed to achieve strong sound insulation Yang et al. (2008) or near perfect sound absorption Ma et al. (2014); Li and Assouar (2016); Li et al. (2016a) for noise control. On another hand, one of the most attractive aspects of metasurfaces is the ability to engineer the outgoing wavefronts by manipulating local phase shifts along the metasurface with subwavelength unit cells, in the name of
generalized Snells law (GSL) Xu et al. (2016); Estakhri and Alù (2016a). Such approach has attracted great attention, with many functionalities demonstrated, such as anomalous reflection and refraction Tang et al. (2014); Li et al. (2014, 2013); Zhao et al. (2013); Wang et al. (2016); Xie et al. (2014), surface wave generation Xie et al. (2014), acoustic flat lenses Wang et al. (2014), self-bending beams Zhang et al. (2014), carpet cloaking Esfahlani et al. (2016), sound vortex generation Jiang et al. (2016), source illusion Liu et al. (2017), and acoustic holograms Xie et al. (2016); Melde et al. (2016).

1.3 Active directions for acoustic metamaterials and metasurfaces

Insight into the nature of acoustic responses, development of modern fabrication capability, as well as the simulation and experimental techniques for characterizing metamaterials provide the soil and root for further development of acoustic metamaterials and metasurfaces. Many promising branches have sprouted and thrived on this root. Figure 1.2 illustrates a non-exhaustive map of ongoing efforts for the development of metamaterials. Although it is impossible to exhaust all of them, several important directions are worth mentioning.

Expanding the material space and new phenomena

Early works on acoustic metamaterials have expanded the achievable dynamic density ($\rho$) and bulk modulus ($\kappa$) from both positive to negative value ranges. People have been trying to expand the material property space even further. An important step is to expand $\rho$ and $\kappa$ from real to complex space, where the imaginary parts correspond to the loss and gain inside the material. Loss and gain open the realm of non-Hermitian systems which attracted a lot of attention in recent years. Particularly, by carefully designing the distribution of loss and gain in space, pseudo-Hermitian systems can be designed such that the eigenvalues of the Hamiltonian are purely
real. This is typically achieved in parity-time (P-T) symmetric systems.

Monopole response governed by bulk modulus $\kappa$ and dipole response governed by density $\rho$, are typically considered independent of each other. However, the coupling between monopole and dipole responses, the so-called Willis coupling, are another parameter space that is currently under active exploration since it allows full control over not only the transmission but also the reflection of a unit cell, allowing multifunctional, highly efficient wavefront control, which is the main contribution of this dissertation.

Apart from the full complex space and cross-coupling between the material properties, the time axis can be added as an additional degree of freedom. Time modulation allows coupling between otherwise orthogonal eigenmodes in a dynamic system, facilitates nonreciprocal wave transmission, and provides a means of achieving gain media. However, understanding of the space-time modulated system is still in its infancy. A lot more exciting new phenomena are yet to be explored.

Breaking the time-reversal symmetry also allows people to achieve topological

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**Figure 1.2**: Currently active directions in the field of metamaterials.
insulators, a topic that attracted considerable attention in recent years. The topo-
logical phase, as a new phase of matter, unlocked many favorable properties for wave
control, such as one-way wave guiding without backscattering, and wave propagation
that are immune to impurities and defects.

People in the field of metamaterials are a big fan of breaking the laws. Luckily,
there are many laws in the world of physics. Early metamaterials allow the break of
density law, demonstrating an extraordinary capability for sound insulation; while
metasurfaces allow the violation of Snell’s Law, unlocking arbitrary reflection and
refraction across surfaces. There’s no doubt that violation of the physical laws will
open the door for wave control in unprecedented ways. For example, physicists are
recently challenging the locality, reciprocity, Chu’s limit, and so on.

*Exploring more forms of waves*

Airborne audible sound is great for the demonstration of new ideas. However, there
are other forms of acoustic waves that are of great importance in both science and en-
gineering. For example, airborne ultrasound as a crucial part in sensing and robotics;
ultrasound in fluids has wide range of applications in underwater communication and
biomedical medical imaging and therapy; surface water waves are important for har-
bors, near-sea constructions and off-shore protection; surface acoustic waves (SAW)
are widely used in chips for cellphones, and they are the focus in earthquake protec-
tion; acoustic waves in solids are proven useful for structure health monitoring and
sensing. The idea of metamaterials is starting to play a role in all these fields.

*Active acoustic metamaterials*

Passive and fixed structures have remarkable acoustic properties and performances,
but with active structures, even greater performance can be achieved. To explore
the industrial application of metamaterials and metasurfaces, it is essential to make
devices reconfigurable for various environments. For acoustic waves, active elements are typically achieved with piezoelectric membranes connected to a circuit Popa et al. (2013), or using external fields such as electric Xiao et al. (2015) or magnetic field Chen et al. (2014), the static force Donahue et al. (2014) or geometry change using water Tian et al. (2019a).

*Optimization of the acoustic metamaterial design*

Metamaterials and metasurfaces have demonstrated how extreme manipulation of sound can be achieved in ways that are not possible with conventional materials, but their performances are still far from optimal. Optimization of the acoustic metamaterial design process is thus another promising area. There remains a lot of space to exploit, such as practical transformations that comply with fabrication limitations, fine-tuning of meta-atoms for the control of the effective material properties, and process of designing highly-efficient meta-devices. This branch is the main focus of this dissertation.

It is worth noting that, the directions illustrated in Fig. 1.2 are not independent of others. For example, new degrees of freedom in the material space facilities the realization of optimized structures or new functionalities; expanding the time axis also creates new phenomena in wave propagation, leading to the break of laws and limits in stationary systems; non-linearity and space-time modulation share many features in common. Combinations of these sub-areas can also lead to new directions, which is happening as this dissertation is written, and these fields are beyond the scope of this dissertation.
1.4 Efforts toward highly efficient wavefront engineering

1.4.1 Unit cells optimization

In acoustics, various unit cell topologies have been proposed to achieve a homogenized effective index to control the local transmitted or reflected phase Tang et al. (2014); Li et al. (2014, 2013); Wang et al. (2014, 2016); Xie et al. (2014); Zhu et al. (2016); Li et al. (2015, 2016b). They have been applied to acoustic devices for different functionalities, such as wavefront manipulation Tang et al. (2014); Li et al. (2014, 2013); Zhao et al. (2013); Wang et al. (2014, 2016); Xie et al. (2014), sound absorption Xie et al. (2014); Li et al. (2016b,a), asymmetric transmission Shen et al. (2016) and cloaking Zhang et al. (2013); Esfahlani et al. (2016). To enable better performance, many approaches have been applied to improve the transmission of the unit cells through impedance matching Zhu et al. (2016); Xie et al. (2013a,b); Memoli et al. (2017); Al Jahdali and Wu (2016); Jia et al. (2018). However, the efficiency of phase shift devices is fundamentally restricted by the reflection and scattering into unwanted directions.

1.4.2 Limitations of the GSL

Recent work has shown that even with full transmission and precise phase control with the unit cells, the local phase gradient alone cannot provide full control over the scattered wave Díaz-Rubio and Tretyakov (2017); Wong et al. (2016); Asadchy et al. (2016); Epstein and Eleftheriades (2016); Díaz-Rubio et al. (2017); Estakhri and Alù (2016b); Asadchy et al. (2017). This is because the boundary conditions can not be naively fulfilled by simply shifting the phase of incoming and outgoing waves. We will use the simplest cases in reflection and transmission-type metasurfaces to illustrate their origins.
Limitations for reflection-type metasurface

Consider anomalous reflection as an example, which is the simplest functionality offered by gradient metasurfaces in reflection scenario. The incoming wave at incident angle $\theta_i$ is directed to reflected angle $\theta_r$, with reflection coefficient $R = R_0 e^{j\phi}$. The pressure field below the metasurface can be thus written as

$$ p = p_0 e^{-jk \sin \theta_i x} e^{-jk \cos \theta_i y} + R p_0 e^{-jk \sin \theta_i x} e^{-jk \cos \theta_r y}. \tag{1.1} $$

The pressure and normal velocity field at the position of the metasurface ($y = 0$) writes:

$$ p = p_0 e^{-jk \sin \theta_i x} + R_0 e^{j\phi} p_0 e^{-jk \sin \theta_r x} $$

$$ v_y = \frac{p_0 \cos \theta_i}{Z_0} e^{-jk \sin \theta_i x} - R_0 e^{j\phi} \frac{p_0 \cos \theta_r}{Z_0} e^{-jk \sin \theta_r x} \tag{1.2} $$

An ideal metasurface can reflect incident energy into an angle $\theta_r$ with 100% power efficiency. This condition, equivalent to energy conservation in the normal direction, requires the amplitude ratio of the reflected wave and incident wave to satisfy

$$ \frac{p_0^2}{2Z_0} \cos \theta_i = |R|^2 \frac{p_0^2}{2Z_0} \cos \theta_r \tag{1.3} $$

so that

$$ |R| = \sqrt{\frac{\cos \theta_i}{\cos \theta_r}} \tag{1.4} $$

The impedance requirement along the metasurface can be calculated with $Z(x) = \frac{p(x)}{v_y(x)}$. Fig. 1.3 shows the impedance requirements in the case with $\theta_i = 0^\circ$ and $\theta_r = 70^\circ$. Here $\Lambda = 2\pi/(k(\sin \theta_r - \sin \theta_i))$ is the periodicity of the metasurface. From the figure we can see the real part of the impedance is non-zero, and takes both positive and negative values, meaning that at some part of the metasurface, the unit cells must be lossy, while in other parts, it needs some gain to provide the energy.
Figure 1.3: Impedance requirements for an ideal reflective metasurface in the case of $\theta_i = 0^\circ$, $\theta_r = 70^\circ$.

However, gain media cannot be achieved with passive structures. Therefore, such a requirement can never be achieved using passive unit cells.

To get away with loss and gain, we can ignore the real part and only fulfill the imaginary part using metasurfaces Estakhri and Alù (2016b). However, it will inevitably induce scatterings into undesired modes. Another way of creating scattering-free metasurface is to engineer the loss across the metasurface so that unwanted scatterings are eliminated. However, it will severely impact the overall power efficiency.

Another approach is to design channels between unit cells to allow the exchange of energy within the metasurface. In this case, the response of the unit cell at a specific point is linked not only to the excitation at that point but also to the points around it. Such a non-local response can be designed to achieve near-perfect reflection, and the idea is recently verified with simulation Quan and Alù (2019). However, such a scheme has lots of thin channels, making it hard to fabricate, and sensitive to losses, hindering the experimental demonstration.

Limitations for transmission-type metasurface

Consider anomalous refraction as an example, which is the simplest functionality offered by gradient metasurfaces in the transmission scenario. Similar to the reflection case, in order to create a metasurface that deflects the incident wave with $\%100$
power efficiency, the transmission coefficient $T = |T_0|e^{j\phi}$ must satisfy $|T_0| = \sqrt{\frac{\cos \theta_i}{\cos \theta_o}}$ according to

$$\frac{P_0^2}{Z_0} \cos \theta_i = |T_0|^2 \frac{P_0^2}{Z_0} \cos \theta_i$$

(1.5)

where $\theta_i$ and $\theta_t$ denote incident and transmission angle, respectively. We can directly see that when the transmitted angle is larger than the incident angle, the unit cells must be designed to have a transmission coefficient larger than 1. This feature can never be achieved with any conventional unit cell strategy.

This seemingly counterintuitive result can be explained by looking at the normal wave impedances, which is defined by $Z = \frac{p}{V_{\perp}}$ for acoustics and $Z = \frac{E}{H_{\perp}}$ for electromagnetics, where $v_{\perp}$ and $H_{\perp}$ correspond to the direction where the power flow is normal to the surface. In this sense, the wave impedances for the input and output waves are $Z_i = \frac{Z_0}{\cos \theta_i}$ and $Z_o = \frac{Z_0}{\cos \theta_o}$. Here we use the "output" since our analysis can represent either transmission or reflection. We can see if the waves on both sides are propagating in different directions, the metasurface sees the waves as if they are propagating in different media. Hence, the transmission/reflection coefficient larger than unity is physically allowed.

Now we analyze the power efficiency limit of phase-gradient metasurfaces. When the incident angle and refracted angle are different, only part of the wave energy will be transmitted to the prescribed direction. This is analogous to the case of wave transmission at the interface of two media. The waves that are scattered to other directions (analogous to reflection) are modeled by

$$R = \frac{Z_o - Z_i}{Z_o + Z_i} = \frac{\cos \theta_i - \cos \theta_o}{\cos \theta_i + \cos \theta_o}$$

(1.6)

and the wave transmitted to the desired mode is

$$T = 1 + R = \frac{2 \cos \theta_i}{\cos \theta_i + \cos \theta_o}.$$  

(1.7)
Therefore, the efficiency of the metasurface can be calculated as

\[
\eta = \frac{P_o}{P_i} = \frac{T^2 \cos \theta_o}{\cos \theta_i} = \left( \frac{2 \cos \theta_i}{\cos \theta_i + \cos \theta_o} \right)^2 \frac{\cos \theta_o}{\cos \theta_i}.
\] (1.8)

Fig. 1.4 shows the theoretical efficiency limitation for GSL-based metasurfaces illuminated normally as a function of the angle of refraction. This limitation is inherent to the design approach and does not depend on the unit cell topology. We can see that for low bending angles, the efficiency is not much affected. But it is possible to see how the efficiency of the generalized reflection law metasurfaces dramatically decreases when the refraction angle increases.

1.5 Bi-anisotropy (Willis coupling) in meta-atoms

Rigorous analysis of the problem has shown that the macroscopic impedance matching required for theoretically perfect anomalous refraction of plane waves can be realized if the metasurface exhibits bianisotropy: magneto-electric coupling for electromagnetic metasurfaces Wong et al. (2016); Asadchy et al. (2016); Epstein and
Eleftheriades (2016) and Willis coupling for the acoustic counterpart Díaz-Rubio and Tretyakov (2017); Willis (1981); Sieck et al. (2017). The bianisotropic response can be implemented by asymmetric unit cells, where the scattered fields are different depending on the direction of illumination. For electromagnetic metasurfaces, typical solutions are based on three cascaded impedance layers. By independently controlling the impedance of each layer, the asymmetric response can be fully controlled Lavigne et al. (2017); Chen et al. (2017). These structures have been numerically and experimentally verified. In acoustics, however, practical design or experimental realization of perfect anomalous refractive metasurfaces has remained scarce.

Interest in bianisotropy in acoustics begun recently Koo et al. (2016); Muhlestein et al. (2017); Sieck et al. (2017). Bianisotropy provides two new possibilities for acoustic metasurfaces: independently control the reflection and transmission phases Koo et al. (2016), or the difference in the reflection phases Muhlestein et al. (2017). A deep analysis of the physics behind this phenomenon and clear analogy between electromagnetic and acoustic bianisotropy has been reported Sieck et al. (2017). These results indicate that acoustic bianisotropy could bring new directions for designing efficient metasurfaces, as has already been demonstrated in electromagnetics.

To design bi-anisotropic metasurfaces, one has to deal with three important issues. First, the tangential dimension of the meta-atom must be deeply subwavelength for ensuring a smooth gradient profile in most cases. Secondly, meta-atoms must ensure complete control of the scattered waves. Recent electromagnetic and acoustic studies Wong et al. (2016); Epstein and Eleftheriades (2016); Lavigne et al. (2017); Chen et al. (2017); Díaz-Rubio and Tretyakov (2017) have shown that full control of the bianisotropy requires at least three degrees of freedom in the particle design. Finally, the intrinsic losses associated with the resonant elements can affect the overall efficiency. Therefore, although the three-membrane topology satisfies the minimum requirements for obtaining an arbitrary bianisotropic response, the structures are
resonance-based, inducing tremendous losses inside the structures.

1.6 Contributions

My major contributions to this field are listed as follows:

- We designed a unit cell structure that fully controls its bianisotropic scattering properties by changing the geometrical dimensions. The validity of the bianisotropic particle is tested with the design of three refractive metasurfaces that fully redirect a normally incident plane wave into 60, 70, and 80 degrees on transmission. The power efficiency of the bianisotropic designs is much higher than the corresponding conventional designs. We also experimentally characterize a bianisotropic metasurface for scattering-free acoustic anomalous refraction, where 97% of the transmitted energy goes to the desired direction and less than 2% of the energy is reflected. This work is summarized in Chapter 2. (Li, J. et al., Nature Communications, 2018)(Highlighted in Science 360, Top 50 Nature Communications physics articles published in 2018)

- We extended the bianisotropic metasurface from the Cartesian coordinate into Cylindrical coordinates. We designed the bianisotropic unit cells that fits in Cylindrical coordinates, and performed the theoretical study, simulation, and experimental demonstration of perfect cylindrical wavefront transformation with bianisotropic metasurfaces for both acoustic and electromagnetic waves. The work is summarized in Chapter 3. (Li, J. et al., Physical Review Applied, 2019)(Editors suggestion)

- Bianisotropic metasurfaces achieve near-perfect power efficiency in manipulating plane waves and cylindrical waves, but when applied to more complicated fields, further attention is required while designing the input and output fields so that local power is balanced along the metasurface. We demonstrated that
for arbitrary beam splitting and anomalous reflection, such power balance can be achieved by carefully designing surface waves, and verified it with both simulation and experiments. The power efficiency reaches over 99% in simulations for both cases. In this work, we have also demonstrated a counter-intuitive but interesting fact: for reflection-type metasurfaces, one can improve its efficiency by allowing designed transmission; while for transmission-type metasurfaces, the efficiency can be improved by allowing engineered reflection. This work is summarized in Chapter 4. (Li, J. et al., In preparation, 2020)

- We have designed the power-flow conformal bianisotropic metasurfaces as another versatile strategy to meet local power requirements. Compared with designing surface waves, it provides several advantages: it can easily adapt to arbitrary wave fields while designing surface waves only applies to periodic fields; power-flow conformal metasurface adapts to the power flow with geometric variation, while for surface waves, sometimes the metasurface needs to be carefully discretized. A power-flow conformal bianisotropic metasurface is designed to focus a plane wave in the near field. The design is verified with both simulation and experiments. This work is summarized in Chapter 5. (Peng, X., Li, J. (co-first author) et al., In preparation, 2020)

- We have generalized the application of bianisotropic metasurfaces and investigated using the idea as an advanced impedance matching technique that outperforms conventional strategies. We further designed a possible realization of bianisotropic unit cells in water using the resonance of bubbles. As a demonstration, we have designed a metasurface to help ultrasound in water to transmit through a 10 mm thick steel plate in water. The design is verified with numerical simulations. The transmitted power is boosted by a factor of 11. This work is summarized in Chapter 6. (Li, J. et al., In preparation, 2020)
As an application of wavefront shaping in water, we have demonstrated the first 3D acoustic tweezer that uses a single transducer and combines the radiation force for trapping in two dimensions with the streaming force to provide levitation in the third dimension. The proposed tweezer uses a single piezoelectric transducer and a PDMS lens to generate the required field. Such an idea is demonstrated in both simulation and experiments. The linear acoustic field and the nonlinear streaming field are both measured in experiments, and the achieved levitation force reaches three orders of magnitude larger than previously reported 3D trapping device. This work is summarized in Chapter 7. (Li, J. et al., submitted, 2020)

Main contributions that are not covered in this dissertation are listed as follows:

- We demonstrated a design of impedance-matched sound absorbing surface with a simple construction. By coupling different resonators and generating a hybrid resonance mode, we designed and fabricated a metasurface that is impedance-matched to airborne sound at tunable frequencies with subwavelength scale unit cells. With careful design of the coupled resonators, over 99% energy absorption at a central frequency of 511Hz with a 50% absorption bandwidth of 140Hz is achieved experimentally. The proposed metasurface can be used in many sound absorption applications such as loudspeaker design and architectural acoustics. (Li, J. et al., Applied Physics Letters, 2016)(Top 50 read articles published in 2016)

- We proposed and demonstrated a compact acoustic retroreflector that reroutes incident signals back toward the source with minimal scattering. Gradient refractive index acoustic metamaterials are designed to fulfill the required refrac-
tive index profile. The experiments show that the compact acoustic retroreflector, whose radius is only approximately one wavelength, works in an incident angular range up to 120 over a bandwidth of about 27% of the central frequency. Such compact acoustic retroreflectors can be potentially applied in pulse-echo-based acoustic detection and communication systems. (Fu, Y., Li, J. (co-first author) et al., Physical Review Materials, 2018)

- We have theoretically studied sound propagation in a space-time modulated medium. Finite-difference time-domain simulations are carried out to verify the results. Functionalities such as mode conversion, parametric amplification, and phase conjugation are demonstrated. Breaking time-reversal symmetry facilities the realization of nonreciprocal devices, such as isolators and circulators, which is of fundamental importance in communication systems. (Li, J. et al., Physical Review B, 2019a)

- We proposed and analyzed a waveguide system consisting of two Helmholtz resonators whose backplate is moving in time or two membranes whose surface tensions are time-modulated with a phase difference between them. Strong non-reciprocity and low insertion loss can be achieved for waves transmitted through the structure. An analytic approach is developed to calculate the harmonics generation in the system. The theoretical results are verified by time-dependent finite element simulations. (Zhu, X., Li, J. (co-first author) et al., in preparation, 2020; Chen, S., Li, J. et al., Physical Review B, 2019)

- We have generalized the transfer matrix method and use it to create a general framework to solve wave propagation problems in time-varying acoustic, electromagnetic, and electric circuit systems. The proposed method provides a versatile approach for the study of general space-time-varying systems, which
allows any number of time-modulated elements with an arbitrary modulation profile, facilities the investigation of high-order modes, and provides an interface between space-time-modulated systems and other systems. (Li, J. et al., Physical Review B, 2019b)

For other contributions that are not covered in this list are included in biography.
Scattering-free wave manipulation with bianisotropic metasurface: Cartesian coordinate

2.1 Theoretical requirements for scattering free wave manipulation

The schematics of the system under study is shown in Fig. 2.1. Assuming zero reflection, the pressure fields on the incident and transmitted side of the metasurface

Figure 2.1: The schematics of the system under study. An ideal metasurface converts all the incident energy into the desired mode, with 100% efficiency.
can be written as

\[ p_t(x, y) = p_0 e^{-jk \sin \theta_i x} e^{-jk \cos \theta_i y} \]  \hspace{1cm} (2.1)

\[ p_t(x, y) = A p_0 e^{-jk \sin \theta_i x} e^{-jk \cos \theta_i y} \]  \hspace{1cm} (2.2)

where \( p_0 \) is the complex amplitude of the incident wave, \( \theta_i \) and \( \theta_t \) are the incident and refracted angles, \( k = \omega / c \) is the wavenumber at operation frequency and \( c \) is the speed of sound in the background medium. \( A \) is the coefficient that relates the amplitudes of incident and transmitted waves. In general, \( A \) is a complex number as the amplitudes on both sides are complex values. However, without losing generality, we can always choose an origin such that the initial phase difference between two waves is zero, thus making \( A \) real, measuring the amplitude ratio between incident and transmitted waves. The velocity vectors at both sides of the metasurface can be expressed as

\[ v_i = \frac{p_0}{Z_0} \sin \theta_i e^{-jk \sin \theta_i x} e^{-jk \cos \theta_i y} \hat{x} + \frac{p_0}{Z_0} \cos \theta_i e^{-jk \sin \theta_i x} e^{-jk \cos \theta_i y} \hat{y} \]  \hspace{1cm} (2.3)

\[ v_t = \frac{A p_0}{Z_0} \sin \theta_t e^{-jk \sin \theta_t x} e^{-jk \cos \theta_t y} \hat{x} + \frac{A p_0}{Z_0} \cos \theta_t e^{-jk \sin \theta_t x} e^{-jk \cos \theta_t y} \hat{y} \]  \hspace{1cm} (2.4)

where \( Z_0 = \rho c \) is the characteristic impedance of the background medium. Assuming the metasurface is places at \( y = 0 \). The pressure and velocity vectors at both sides of the metasurface can be related by the following impedance matrix.

\[ \begin{bmatrix} p_t(x, 0) \\ p_t(x, 0) \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \hat{n} \cdot \vec{v}_i(x, 0) \\ -\hat{n} \cdot \vec{v}_i(x, 0) \end{bmatrix} \]  \hspace{1cm} (2.5)

where \( \hat{n} \) is the normal vector of the metasurface, \( Z_{ij} \) are the components of the impedance matrix, and the \( \pm \) sign refers to the fields at both sides of the metasurface. Note that for such a linear time-invariant system under study, reciprocity requires \( Z_{12} = Z_{21} \) and we assume this condition throughout.

An ideal metasurface can refract incident energy into an angle \( \theta_t \) with 100% efficiency. This condition, equivalent to energy conservation in the normal direction,
requires the amplitude ratio of the transmitted wave and incident wave to satisfy

\[ \frac{p_0^2}{Z_0} \cos \theta_i = A^2 \frac{p_0^2}{Z_0} \cos \theta_t \]  

so that

\[ A = \sqrt{\frac{\cos \theta_i}{\cos \theta_t}} \]  

In order to have a lossless and passive solution that satisfies energy conservation, the elements of the impedance matrix must be purely imaginary \((Z_{11} = jX_{11}, Z_{12} = jX_{12}, Z_{21} = jX_{21}, \text{ and } Z_{22} = jX_{22})\). Define \( \Phi = k(\sin \theta_i - \sin \theta_t) \). Putting these assumptions and constraints into Eqn 3.49, we get

\[ 1 = jX_{11} \frac{\cos \theta_i}{Z_0} - jX_{21} A \frac{\cos \theta_t}{Z_0} e^{j\Phi_x} \]  

\[ A e^{j\Phi_x} = jX_{21} \frac{\cos \theta_i}{Z_0} - jX_{22} A \frac{\cos \theta_t}{Z_0} e^{j\Phi_x} \]  

Separating the real and imaginary part of the equation and solving the impedance components, we get

\[ Z_{11} = i Z_0 \cot(\Phi x) \]  

\[ Z_{12} = i \frac{Z_0}{\sqrt{\cos \theta_t \sin(\Phi x)}} \]  

\[ Z_{22} = i \frac{Z_0}{\cos \theta_t} \cot(\Phi x) \]  

The period of the metasurface can be calculated as \( D = 2\pi/\Phi \). Equation (2.12) shows that \( Z_{11} \) is not equal to \( Z_{22} \), so the bianisotropic response is required, and the building blocks of a perfect metasurface cannot be implemented by conventional homogenized high index materials.

The local transmission and reflection coefficients which have to be implemented by the bianisotropic unit cell can be calculated as

\[ t = \frac{2 \sqrt{\cos \theta_t} e^{i\Phi_x}}{1 + \cos \theta_t} e^{i\Phi_x}, \quad r^+ = \frac{1 - \cos \theta_t}{1 + \cos \theta_t} e^{-i2\Phi_x}. \]
and
\[
    r^- = \frac{1 - \cos \theta_t}{1 + \cos \theta_t}.
\] (2.14)

These expressions demonstrate the strict relation, not only in phase but also in magnitude, of the local reflection and transmission coefficients with the desired angle of refraction. Moreover, one can see the difference in phase between the reflection coefficients in forward and backward directions which appear as a consequence of the bianisotropy Muhlestein et al. (2017). It is important to notice that gradient metasurface described by Eq. (2.13) is different from conventional designs based on the generalize refraction law Sun et al. (2012); Yu et al. (2011); Xie et al. (2014) and the gradient bianisotropic metasurface in Koo et al. (2016). This asymmetric behavior can be achieved by using bianisotropic cells proposed in the following section.

2.2 Realization of the bianisotropic unit cells

From the analysis we can see that there are 3 parameters that need to be controlled \((Z_{11}, Z_{12}, \text{and } Z_{22})\), therefore, one needs at least 3 controlling quantities to ensure full control over the impedance matrix.

2.2.1 Three-membrane model

For electromagnetic waves, the required impedance matrix can be fulfilled by three cascaded impedance sheets. For acoustic waves, the three-membrane approach has been proposed, as shown in Figure 2.2

The response of a meta-atom can be expressed in terms of the transmission matrices of membranes and empty spacings between them:

\[
    \begin{bmatrix}
        p_I \\
        \hat{n} \cdot v_I
    \end{bmatrix} = \begin{bmatrix}
        M_{11} & M_{12} \\
        M_{21} & M_{22}
    \end{bmatrix} \begin{bmatrix}
        p_{II} \\
        \hat{n} \cdot v_{II}
    \end{bmatrix} \quad (2.15)
\]
Figure 2.2: Three-membrane approach for both acoustic and EM waves. The right panel shows the equivalent circuit for the model

where $M = M_{Z1} M_T M_{Z2} M_T M_{Z3}$ with

$$M_{Z1} = \begin{bmatrix} 1 & Z_1^z \\ 0 & 1 \end{bmatrix}$$ \hspace{1cm} (2.16)

and

$$M_T = \begin{bmatrix} \cos(kl) & jZ_0 \sin(kl) \\ j \frac{1}{Z_0} \sin(kl) & \cos(kl) \end{bmatrix}$$ \hspace{1cm} (2.17)

Conversion from $M$ matrix to $Z$ matrix is given by

$$M = \begin{bmatrix} Z_{11} & Z_{12} & Z_{12} \left( Z_{22} - Z_{21} Z_{12} \right) \\ Z_{12} & Z_{12} & Z_{12} \\ Z_{12} & Z_{12} & Z_{12} \end{bmatrix}$$ \hspace{1cm} (2.18)

Equating each element of the transfer matrix, the impedances of the three membranes can be defined as

$$Z_1(x) = Z_{11} + Z_{12} + jZ_0 \cot(kl)$$ \hspace{1cm} (2.19)

$$Z_2(x) = j2Z_0 \cot(kl) - \frac{Z_0^2}{Z_{12}} \frac{1}{\sin^2(kl)}$$ \hspace{1cm} (2.20)

$$Z_3(x) = Z_{22} + Z_{12} + jZ_0 \cot(kl)$$ \hspace{1cm} (2.21)
Figure 2.3: Study of a bianisotropic acoustic cell. (a) Geometry of a cell with four side-loaded resonator. The height of the Helmholtz resonators is varied to create different bianisotropic responses. Definition of the forward (+) and backward (-) illuminations. (b) Amplitude and phase of the transmission and reflection coefficients of an arbitrary cell. The dimensions of the cell are: $w = 12$ mm, $h_2 = 1.5$ mm, $w_2 = 1$ mm, $h_1 = 1$ mm $w_1 = 4$ mm, $w_a = 6$ mm, $w_b = 5$ mm, $w_c = 4$ mm, and $w_d = 3$ mm.

2.2.2 Four-resonators approach

The three-membrane model provides an elegant way of controlling the impedance matrix mathematically. However, the surface tension, uniformity and durability for the membranes are extremely hard to control, and it is questionable whether such configuration can be practically realized. Here we propose the four-resonator approach that can be practically implemented while avoiding resonances that produce loss.

The cell architecture that we use to ensure asymmetry, shown in Fig. 2.3, is based on a straight channel with side-loaded resonators. From the analysis of the bianisotropic requirements dictated by the impedance matrix, we can see that with the proposed topology three resonators is the minimum requirement which allows to implement any desired response. However, to obtain extreme asymmetric response
required by some gradient metasurfaces, the resonators have to work near their resonant frequencies and this makes it difficult to control their responses and increase loss. The required resonance also puts constraints on the physical dimensions and cause robustness issues to the practical designs. In order to mitigate these practical limitations, we propose a four side-loaded resonators particle, as shown in Fig. 2.3(a). In this structure: the width and height of the neck, $h_2$ and $w_2$, are fixed in the four resonators; the width of the cavities $h_3$ is also fixed; the height of the air channel $w_1$ and the height of the resonators $w_a, w_b, w_c$ and $w_d$ can be varied to control the asymmetry; and the wall thickness of the unit cell $h_1$ is fixed and will be defined with the fabrication limitations. All the thicknesses are less than half of a wavelength.

A simple way to study the bianisotropic response of the proposed particle is by analyzing the scattering produced by the particle. The scattering of the particle can be expressed in terms of the scattering matrix as

$$
\begin{bmatrix}
p^+_s \\
p^-_s
\end{bmatrix} =
\begin{bmatrix}
r^+ \\
t^+
\end{bmatrix}
\begin{bmatrix}
p^+_i \\
p^-_i
\end{bmatrix}
$$

(2.22)

where $p^\pm_i$ represent the amplitude of the forward and backward incident plane waves, $p^\pm_s$ is the amplitude of the scattered fields at both sides of the particle (that is, $p^+_s = p^+_r + p^-_r$ and $p^-_s = p^-_r + p^+_r$), $t^\pm$ represent the local transmission coefficients, $r^\pm$ are the reflection coefficients.

An analytic expression of the transfer function of the unit cells is developed to facilitate the design of the wavefront transformation metasurface. The geometry of a unit cell is shown in Fig. 2.3(a-b), where $h_1$ is the thickness of the shell, $h_2$ is the width of the neck, $h_3$ is the length of the cavity, $w$ is the height of the unit cell, and $w_1$ and $w_2$ are the height of the channel and neck, respectively. The height of each individual Helmholtz resonator, $w_{a,b,c,d}$, can be different as asymmetric geometry of the unit cell is required by the bianisotropic metasurface.
The relationship for the pressure and volume velocity of the incident and transmitted waves can be expressed as:

\[
\begin{bmatrix}
p^+ \\
\hat{n} \cdot \vec{u}^+
\end{bmatrix} =
\begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
p^- \\
\hat{n} \cdot \vec{u}^-
\end{bmatrix}
\]

(2.23)

where \( \vec{u}^\pm = w\vec{v}^\pm \), and \([M]\) is the total transfer matrix that can be written as:

\[
[M] = [M_{in}] [N_0] [M_a] [N_0] [M_b] [N_0] [M_c] [N_0] [M_d] [N_0] [M_{out}].
\]

(2.24)

Here \([M_a]\) through \([M_d]\) are the transfer matrix of the individual Helmholtz resonator cell, and \([N_0]\) is the transfer matrix relating the Helmholtz resonator cells and the waveguide. The individual transfer matrix of the Helmholtz resonator can be tuned by adjusting the geometry. The transfer matrices of the Helmholtz resonator cells (for example, cell \(a\)) and \(N_0\) can be written as:

\[
[M_a] = \begin{bmatrix}
\frac{2-\alpha_a}{2} & -\frac{\alpha_a}{2} \\
\frac{2-\alpha_a}{2} & \frac{2+\alpha_a}{2}
\end{bmatrix},
\]

(2.25)

and

\[
[N_0] = \begin{bmatrix}
e^{jk\ell_1} & 0 \\
0 & e^{-jk\ell_1}
\end{bmatrix}.
\]

(2.26)

Here \(\alpha_a = R_{w1}/Z_a\) and \(R_{w1} = \rho_0 c_0/w_1\) is the acoustic impedance of the straight channel, \(Z_a\) is the acoustic impedance of the Helmholtz resonator \(a\). The same approach can be applied to the resonators \(b, c,\) and \(d\).

The detailed derivation of \(Z_a\) is given in Li et al. (2016b), and is directly given here for brevity:

\[
Z_a = Z_n \frac{Z_c + jZ_n \tan(kw_2)}{Z_n + jZ_c \tan(kw_2)} + j \text{Im}(Z_d).
\]

(2.27)

Here \(Z_n = \rho_0 c_0/h_2\) and \(Z_c\) are the acoustic impedance of the neck and the cavity of the Helmholtz resonator, respectively. \(\text{Im}(Z_d)\) is the radiation impedance which is
expressed as:

\[
Z_d = \rho_0 c_0 \frac{1 - e^{-jkh_2} - jkh_2}{k^2} + \rho_0 c_0 \sum_{n=1} \frac{1 - e^{-jk'_{zn}h_2} - jk'_{zn}h_2}{k'_{zn}^3}
\]  

(2.28)

with \(k'_{zn} = \sqrt{k^2 - k'_{wn}^2}\) and \(k'_{wn} = n\pi/w_1\). The acoustic impedance of the cavity \(Z_c\) is given by:

\[
Z_c = \sum_n \rho_0 c_0 \frac{k(1 + e^{2jk'_{wn}w_3})\Phi_n^2}{k_{zn}w_3(1 - e^{2jk'_{wn}w_3})},
\]  

(2.29)

where \(\Phi_n = \sqrt{2 - \delta_n \cos(n\pi/2)\text{sinc}(n\pi h_2/2h_3)}\) and \(k''_{zn} = \sqrt{k^2 - (n\pi/h_3)^2}\).

The transfer matrices of \([M_{in}]\) and \([M_{out}]\) are expressed as:

\[
[M_{in}] = \begin{bmatrix}
\frac{1}{2} & \frac{R_{w1}}{2} \\
\frac{1}{2} & -\frac{R_{w1}}{2}
\end{bmatrix},
\]  

(2.30)

and

\[
[M_{out}] = \begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix}.
\]  

(2.31)

By inserting Eqs. (2.27-2.29) into Eq. (2.25), the transfer matrix of an individual Helmholtz resonator unit can be obtained, which can further be combined with Eq. (3.20) to compute the total transfer matrix. The total transfer matrix can be tuned by adjusting the geometrical values of the unit cell. In our design, \(w_1, w_2, h_1, h_2, h_3\) are fixed, the heights of the Helmholtz resonator cells \(w_{a,b,c,d}\) and channel \(w_1\) are put in the genetic algorithm to for the computation of the optimized structure.

Once the transfer matrix has been calculated, we can directly calculate the corresponding impedance matrix as

\[
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix} = \begin{bmatrix}
M_{11} & M_{12} - M_{21}M_{12} \\
M_{21} & M_{22}
\end{bmatrix}.
\]  

(2.32)
These expressions have been used for calculating the actual impedance values in Fig. 2(d). Also, we can calculate the scattering matrix as

\[
\begin{bmatrix}
  r^+ & t^- \\
  t^+ & r^-
\end{bmatrix} = \begin{bmatrix}
  (Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{21}Z_{12} \Delta Z \\
  \frac{2Z_{12}Z_0}{\Delta Z} \\
  \frac{2(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{21}Z_{12}}{\Delta Z} \\
  \end{bmatrix},
\]

(2.33)

where \(\Delta Z = (Z_{11} + Z_0)(Z_{22} + Z_0) - Z_{21}Z_{12}\).

Figure 2.3 shows the transmission and reflection amplitudes and phases for a particle defined by \(h_2 = 1.5\) mm, \(w_2 = 1\) mm, \(h_1 = 1\) mm \(w_1 = 4\) mm, \(w_a = 6\) mm, \(w_b = 5\) mm, \(w_c = 4\) mm, and \(w_d = 3\) mm. For lossless and reciprocal particles, the transmission coefficients and reflection coefficients satisfy \(t^+ = t^- = t\), \(|t|^2 + |r|^2 = 1\), and \(r^*t^* + tr^* = 0\). The analysis of Fig. 2(c-e) shows that only the phase of the reflection is different for opposite directions, and this reflection phase asymmetry is a clear signature of bianisotropy Ra’di et al. (2014); Sieck et al. (2017). Although there is also asymmetry in the orthogonal direction of the unit cells, it can be ignored as long as the width of the channel is significantly smaller than a wavelength. Also, since there are walls between adjacent cells, the wave does not propagate along the orthogonal direction inside the metasurface. Therefore, all the cells in the bianisotropic metasurfaces can be designed individually.

2.3 Designing bianisotropic metasurface with four-resonator approach

2.3.1 Optimization with Genetic Algorithm

For the design of the physical dimensions, genetic algorithm (GA) optimization is used to define \(w_1\), \(w_a\), \(w_b\), \(w_c\) and \(w_d\) so that the calculated impedance matrix matches the theoretical requirements. The population size is 10 and the mutation rate is 0.2. We kept half of the genes for every generation and the best one does not mutate. There is no crossover in the optimization process. The optimization stops after 1500 generations. We ran the algorithm 50 times for each cell to confirm the convergence.
of the optimization process, and to find the best match. The cost function is defined as

\[
\text{cost} = \sqrt{\left| \frac{Z_{s11} - Z_{t11}}{Z_{t11}^*} \right|^2 + \left| \frac{Z_{s12} - Z_{t12}}{Z_{t12}^*} \right|^2 + \left| \frac{Z_{s21} - Z_{t21}}{Z_{t21}^*} \right|^2 + \left| \frac{Z_{s22} - Z_{t22}}{Z_{t22}^*} \right|^2}
\]  

(2.34)

where \( s \) denotes the values achieved by the structure and \( t \) denotes the theoretical requirements.

With the GA algorithm, we can optimize the structural parameters. In the program, the impedance matrix can either be calculated theoretically (as shown in the previous section), or be extracted in a simulation.

### 2.3.2 Parameter retrieval of unit cells in simulation

For the ease of implementation, the method we used to retrieve the impedance matrix in COMSOL is the same as the standard 4-microphone method for acoustic experiments with impedance tubes, whose setups are shown in Figure 4. The positions of 4 microphones are \( x_1, x_2, x_3, x_4 \), respectively. By performing two measurements with different boundary conditions at the end of the tube, we can obtain four independent equations for determination of the four transfer matrix elements. Two different boundaries we used at the end of the tube are plane wave radiation (condition #1) and hard wall (condition #2). The pressure detected by these microphones under these two boundary conditions are noted as \( p_m^{(n)} \) where \( m \) denotes the number of the microphone and \( n \) denotes the number of the boundary condition. They satisfy the condition:

\[
\begin{bmatrix}
  e^{-jkx_1} & e^{jkx_1} \\
  e^{-jkx_2} & e^{jkx_2}
\end{bmatrix}
\begin{bmatrix}
  A^{(1)} & A^{(2)} \\
  B^{(1)} & B^{(2)}
\end{bmatrix}
= \begin{bmatrix}
  p_1^{(1)} & p_1^{(2)} \\
  p_2^{(1)} & p_2^{(2)}
\end{bmatrix}
\]  

(2.35)
Similarly,
\[
\begin{bmatrix}
e^{jkx_3} & e^{jkx_4} \\
e^{-jkx_4} & e^{jkx_4}
\end{bmatrix}
\begin{bmatrix}
C^{(1)} & C^{(2)} \\
D^{(1)} & D^{(2)}
\end{bmatrix}
= \begin{bmatrix}
P_{1}^{(1)} & P_{1}^{(2)} \\
P_{2}^{(1)} & P_{2}^{(2)}
\end{bmatrix}
\] (2.36)

With the measurement of \( p_m^{(n)} \), all the ABCD in the matrices can be calculated. If the metasurface is located at \( x_0 \), then the pressure and velocity at the left side and right side can be written as:

\[
\begin{bmatrix}
p^{(-1)} & p^{(-2)} \\
v^{(-1)} & v^{(-2)}
\end{bmatrix}
= \begin{bmatrix}
e^{-jkx_0} & e^{jkx_0} \\
e^{-jkx_0/Z_0} & -e^{jkx_0/Z_0}
\end{bmatrix}
\begin{bmatrix}
A^{(1)} & A^{(2)} \\
B^{(1)} & B^{(2)}
\end{bmatrix}
\] (2.37)

\[
\begin{bmatrix}
p^{(+1)} & p^{(+2)} \\
v^{(+1)} & v^{(+2)}
\end{bmatrix}
= \begin{bmatrix}
e^{-jkx_0} & e^{jkx_0} \\
e^{-jkx_0/Z_0} & -e^{jkx_0/Z_0}
\end{bmatrix}
\begin{bmatrix}
C^{(1)} & C^{(2)} \\
D^{(1)} & D^{(2)}
\end{bmatrix}
\] (2.38)

Therefore, the transfer matrix of the measured unit cell can be calculated as

\[
M = \begin{bmatrix}
p^{(-1)} & p^{(-2)} \\
v^{(-1)} & v^{(-2)}
\end{bmatrix}
\begin{bmatrix}
p^{(+1)} & p^{(+2)} \\
v^{(+1)} & v^{(+2)}
\end{bmatrix}^{-1}
\] (2.39)

Hence the impedance matrix can be calculated with

\[
Z = \begin{bmatrix}
M_{11} & M_{12} \\
M_{12} & M_{22} - M_{11} M_{12}
\end{bmatrix}
\] . (2.40)

2.3.3 Three design showcases

The first design presented in this work corresponds to \( \theta_t = 60^\circ \). In this case, the required values for the components of the impedance matrix are represented in Fig. 3(d). The operating frequency is chosen to be 3000 Hz that makes the period of the metasurface \( D = 13.2 \text{ cm} \) (\( D/\lambda = 1.15 \)). We use 11 cells along the period for implementing the spatial dependent bianisotropic response, so the width of the unit cell is \( w = D/11 = 12 \text{ mm} \) (\( w/\lambda = 0.10 \)). In the discretization process, we choose the cells to have the impedance values at \( x_n = (n - 0.375)w \), where \( n \) denotes the
Figure 2.4: Bianisotropic metasurfaces for scattering-free anomalous refraction. (a-c) represent the numerical simulation of the total pressure field for bianisotropic metasurfaces and GSL metasurfaces when $\theta_t = 60^\circ$, $70^\circ$, and $80^\circ$. The insets show the phase evolution inside the metasurface. (d-f) represent the impedance matrices profile for $\theta_i = 0^\circ$ and $\theta_t = 60^\circ$, $70^\circ$, and $80^\circ$. Here The denotes theoretical requirements while Opt denotes the impedance values achieved by structure optimization.

index of the cell, to avoid points where the ideal impedance matrix diverges. For the design of the physical dimensions, genetic algorithm optimization is used to define $w_1$, $w_a$, $w_b$, $w_c$ and $w_d$ so that the calculated impedance matrix matches the theoretical requirements. The physical dimensions of the final design and their corresponding transmission/reflection coefficients are summarized in Table 2.1.

From Fig. 2.4 we can see that the required impedance matrix of the perfect metasurface is closely approximated by our unit cells. It should be noted that the metasurface is discretized and approximated with a finite number of cells, and the performance of the metasurface can be possibly enhanced by using a larger number of cells with better spatial resolution.

Full-wave simulations are performed to verify our design. The real part of the simulated acoustic pressure field for our first structure is represented in Fig. 2.4,
Table 2.1: Design parameters of the scattering-free bianisotropic metasurface to steer a normal incident wave toward $\theta_t = 60^\circ$.

<table>
<thead>
<tr>
<th>Cell</th>
<th>$w_1$</th>
<th>$w_a$</th>
<th>$f_a$ (kHz)</th>
<th>$w_b$</th>
<th>$f_b$ (kHz)</th>
<th>$w_c$</th>
<th>$f_c$ (kHz)</th>
<th>$w_d$</th>
<th>$f_d$ (kHz)</th>
<th>$r_r$</th>
<th>$r_i$</th>
<th>$r^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.4</td>
<td>7.6</td>
<td>4.91</td>
<td>6.1</td>
<td>5.21</td>
<td>4.3</td>
<td>6.39</td>
<td>4.9</td>
<td>6.54</td>
<td>-0.24+0.39i</td>
<td>0.46-0.00i</td>
<td>-0.47+0.44i</td>
</tr>
<tr>
<td>2</td>
<td>3.2</td>
<td>8.8</td>
<td>4.78</td>
<td>5.3</td>
<td>5.48</td>
<td>5.6</td>
<td>5.32</td>
<td>5.1</td>
<td>5.59</td>
<td>-0.15+0.14i</td>
<td>0.18-0.06i</td>
<td>-0.95-0.25i</td>
</tr>
<tr>
<td>3</td>
<td>3.5</td>
<td>6.5</td>
<td>4.86</td>
<td>4.2</td>
<td>6.14</td>
<td>4.1</td>
<td>6.22</td>
<td>4.6</td>
<td>5.86</td>
<td>0.28-0.08i</td>
<td>0.16-0.24i</td>
<td>-0.57-0.77i</td>
</tr>
<tr>
<td>4</td>
<td>4.6</td>
<td>5.4</td>
<td>5.24</td>
<td>4.9</td>
<td>5.52</td>
<td>2.8</td>
<td>7.34</td>
<td>4.6</td>
<td>5.71</td>
<td>0.31+0.028i</td>
<td>0.29-0.30i</td>
<td>-0.04-0.91i</td>
</tr>
<tr>
<td>5</td>
<td>4.6</td>
<td>5.4</td>
<td>5.24</td>
<td>3.0</td>
<td>7.10</td>
<td>1.0</td>
<td>11.23</td>
<td>1.9</td>
<td>8.78</td>
<td>0.01+0.45i</td>
<td>0.37-0.25i</td>
<td>0.41-0.79i</td>
</tr>
<tr>
<td>6</td>
<td>6.8</td>
<td>3.2</td>
<td>6.63</td>
<td>3.1</td>
<td>6.73</td>
<td>0.1</td>
<td>16.20</td>
<td>0.1</td>
<td>16.20</td>
<td>-0.20+0.17i</td>
<td>0.25-0.08i</td>
<td>0.85-0.45i</td>
</tr>
<tr>
<td>7</td>
<td>1.4</td>
<td>8.6</td>
<td>4.52</td>
<td>6.1</td>
<td>5.50</td>
<td>6.9</td>
<td>5.13</td>
<td>2.4</td>
<td>8.91</td>
<td>0.68-0.11i</td>
<td>0.18-0.67i</td>
<td>0.48+0.54i</td>
</tr>
<tr>
<td>8</td>
<td>1.9</td>
<td>8.1</td>
<td>4.54</td>
<td>7.8</td>
<td>4.64</td>
<td>7.0</td>
<td>4.94</td>
<td>5.6</td>
<td>5.59</td>
<td>0.01-0.11i</td>
<td>0.08-0.08i</td>
<td>0.50+0.42i</td>
</tr>
<tr>
<td>9</td>
<td>2.1</td>
<td>7.9</td>
<td>4.56</td>
<td>7.6</td>
<td>4.67</td>
<td>6.5</td>
<td>5.10</td>
<td>5.8</td>
<td>5.43</td>
<td>0.46+0.14i</td>
<td>0.28-0.39i</td>
<td>0.28+0.83i</td>
</tr>
<tr>
<td>10</td>
<td>2.1</td>
<td>7.9</td>
<td>4.56</td>
<td>6.8</td>
<td>4.97</td>
<td>4.9</td>
<td>5.95</td>
<td>5.7</td>
<td>5.48</td>
<td>0.37+0.43i</td>
<td>0.47-0.31i</td>
<td>-0.11+0.82i</td>
</tr>
<tr>
<td>11</td>
<td>2.2</td>
<td>7.8</td>
<td>4.58</td>
<td>6.6</td>
<td>5.03</td>
<td>3.9</td>
<td>6.69</td>
<td>4.7</td>
<td>6.06</td>
<td>0.04+0.53i</td>
<td>0.52-0.12i</td>
<td>-0.50-0.68i</td>
</tr>
</tbody>
</table>

where nearly total energy transmission is observed. The simulated amplitude ratio $T$ achieved with our real structure is 1.365, as compared to the theoretically ideal value of 1.414, indicating that 93% of the incident energy is transmitted to the desired direction. This value is higher than the theoretical upper limit of 89% power transmission for conventional GSL based designs. Solid line in Figure 2.5 shows the theoretical efficiency for a metasurface illuminated normally as a function of the angle of refraction. With the purpose of comparison, we use a simulation of a discretized impedance-matched design based on the GSL, confirming that in the conventional metasurfaces only 81% of the input energy is transmitted in the desired direction, with the remainder going into reflection and other diffractive modes. Fig. 2.4 shows the comparison between the responses of both designs, where we can clearly see the improvement obtained with the bianisotropic design.

Despite the piecewise constant and approximate realization of the theoretically ideal impedance profile, this practical structure nearly realizes perfect, lossless transmission of energy in the desired direction. This shows that realistic structures can perform significantly better than conventional metasurfaces. Critically, it also shows that good performance of a wavefront transformation metasurface does not require perfect realization of the ideal impedance profile. A close and piecewise approximation will suffice in our design.

Figure 2.6 depicts the normalized resonance frequency of each individual res-
Figure 2.5: Comparison of the efficiency for anomalous transmission metasurfaces. Bianisotropic designs show great advancement especially for large deflection angles. Realized efficiencies are slightly lower than the theoretical limit as a result of discretization.

Figure 2.6: Normalized resonance frequency of the individual resonators of the scattering-free anomalous refractive metasurface design for $\theta_i = 0^\circ$ and $\theta_t = 60^\circ$. All the resonators are working out of the resonant frequency to avoid high losses.
onators with respect to the operation frequency $f_0$ (3000Hz). It is important to notice that none of the resonators is working near the resonance, so the design will be less sensitive to the losses than other resonant designs, as for example the three-membrane proposal Díaz-Rubio and Tretyakov (2017). The performance of the design is also confirmed in simulation by considering viscous loss since it is the inherent loss of the structure which is inevitable in the experiments. In addition, due to the high resonance frequencies of the resonators, their size allows smaller width of the cells, that is, it is easier to implement gradient metasurfaces with this topology.

To better show the large efficiency enhancement of the bianisotropic metasurface over conventional GSL-based designs, we designed another two cases with larger deflection angles, where the metasurfaces steer the incident beam to $\theta_t = 70^\circ$ and $\theta_t = 80^\circ$, respectively. For these two cases, the metasurfaces are sampled coarsely with only four cells within one period. The theoretical requirements (lines) and the achieved values (dots) of the impedance matrices for both cases are shown in Fig. 2.4(e-f). Detailed dimensions and relative errors can be found in Table 2.2 and 2.3. Fig. 2.4(b-c) show the simulated results of the bianisotropic designs and the corresponding GSL-based designs with ideal impedance matched cells and the same resolution. Energy efficiencies of the bianisotropic designs achieved 96% and 91% for 70° and 80° cases, whereas the corresponding numbers for GSL designs are 58% and 35%, respectively. The imperfect scattered field is caused by the reflection from the metasurface, which is due to non-ideal implementation of the metasurface. We note that, however, the power flow normal to the surface (the conserved quantity that defines energy efficiency) in these reflections is low and contributes little to the overall energy efficiency of the metasurface. Because the deflection angles, that is, 70 deg and 80 deg, are large, the reflection amplitudes of 0.35 and 0.6, respectively, their contributions to normally directed power follow is only 4% and 6%. In other words, the high efficiency is still maintained even though the reflected field amplitudes are
not negligible. Note that GSL-based designs are carried out by impedance matched cells with precise phase control, and the efficiency values are expected to be even lower for real structures. We can see that even with such a coarse representation of the impedance profile and non-negligible relative error, the bianisotropic designs achieved much higher efficiency than the conventional ones. This offers huge advantage for practical realizations, especially in the high frequency or ultrasound range where fabrication capabilities are limited.

Table 2.2: Design parameters of the scattering-free bianisotropic metasurface to steer a normal incident wave toward $\theta_t = 70^\circ$.

<table>
<thead>
<tr>
<th>Cell</th>
<th>cost(%)</th>
<th>$w$</th>
<th>$w_1$</th>
<th>$w_a$</th>
<th>$w_b$</th>
<th>$w_c$</th>
<th>$w_d$</th>
<th>$r^+$</th>
<th>$r^-$</th>
<th>$t^\pm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.42</td>
<td>30.4</td>
<td>10.4</td>
<td>9.1</td>
<td>7.5</td>
<td>11.4</td>
<td>10.2</td>
<td>-0.33+0.39i</td>
<td>0.23-0.45i</td>
<td>-0.48+0.72i</td>
</tr>
<tr>
<td>2</td>
<td>4.50</td>
<td>30.4</td>
<td>11.9</td>
<td>6.5</td>
<td>16.2</td>
<td>8.6</td>
<td>16.0</td>
<td>-0.04-0.51i</td>
<td>0.39-0.34i</td>
<td>0.79+0.32i</td>
</tr>
<tr>
<td>3</td>
<td>1.52</td>
<td>30.4</td>
<td>15.6</td>
<td>9.2</td>
<td>7.2</td>
<td>2.0</td>
<td>3.5</td>
<td>0.07+0.53i</td>
<td>0.38-0.37i</td>
<td>0.27-0.86i</td>
</tr>
<tr>
<td>4</td>
<td>5.94</td>
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<td>10.6</td>
<td>5.2</td>
<td>1.7</td>
<td>-0.06-0.44i</td>
<td>0.33-0.31i</td>
<td>-0.84-0.30i</td>
</tr>
</tbody>
</table>

2.4 Experimental characterization of the design

Measurements were carried out to characterize the design experimentally and confirm its scattering-free property. As an example, we picked the 60$^\circ$ case. The experimental setup and one period of the fabricated sample is shown in the Fig. 2.7(a). The measured transmitted pressure field Fig. 2.7(c) and energy distribution Fig. 2.7(d) are compared with the corresponding simulated fields. Good agreement between simulation and experiment is observed, and the small discrepancies can be attributed to fabrication errors and inevitable losses in the lab environment. The experimental results show that unwanted diffraction orders are greatly suppressed, and all the transmitted energy is concentrated in one direction. To confirm that our metasurface is reflection-free, the reflected field is also measured. The reflection caused by the metasurface is obtained by scanning the reflected region in the empty waveguide and the field with the metasurface, and then calculating the difference between the two
Table 2.3: Design parameters of the scattering-free bianisotropic metasurface to steer a normal incident wave toward $\theta_t = 80^\circ$.

<table>
<thead>
<tr>
<th>Cell</th>
<th>cost(%)</th>
<th>$w$</th>
<th>$w_1$</th>
<th>$w_a$</th>
<th>$w_b$</th>
<th>$w_c$</th>
<th>$w_d$</th>
<th>$r^+$</th>
<th>$r^-$</th>
<th>$t^2$</th>
</tr>
</thead>
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<td>29.0</td>
<td>4.6</td>
<td>9.6</td>
<td>10.7</td>
<td>4.4</td>
<td>4.6</td>
<td>-0.07+0.71i</td>
<td>0.49-0.51i</td>
<td>-0.30+0.64i</td>
</tr>
<tr>
<td>2</td>
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<td>7.1</td>
<td>6.6</td>
<td>19.3</td>
<td>7.4</td>
<td>19.1</td>
<td>-0.15-0.68i</td>
<td>0.53-0.45i</td>
<td>0.68+0.23i</td>
</tr>
<tr>
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<td>15.40</td>
<td>29.0</td>
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<td>9.9</td>
<td>0.3</td>
<td>6.1</td>
<td>0.33+0.67i</td>
<td>0.57-0.49i</td>
<td>0.13-0.65i</td>
</tr>
<tr>
<td>4</td>
<td>9.47</td>
<td>29.0</td>
<td>14.5</td>
<td>7.5</td>
<td>8.7</td>
<td>11.5</td>
<td>9.4</td>
<td>-0.21-0.64i</td>
<td>0.48-0.46i</td>
<td>-0.72-0.18i</td>
</tr>
</tbody>
</table>

Figure 2.7: Experimental setup and results. (a) Schematic representation of the experimental setup and a period of the fabricated sample. (b) Comparison between the normalized scattering of the bianisotropic metasurface (experimental and numerical) and a GSL design. (c-d) Analysis of the real part (c) and magnitude square (d) of the experimental pressure field and the comparison with the numerical simulations.

measured fields. The result shows that only 2% of the energy is reflected.

To demonstrate the performance of the metasurface, the normalized energy distribution on each direction is further calculated by performing Fourier transform along the line right behind the metasurface, and the result is shown in Fig. 2.7(b). The experimental result shows an excellent consistency with simulations, with most of the energy localized in the desired direction (-1 order) and other diffraction modes are severely suppressed, including the high-order evanescent modes, which are visible because we are processing the fields very close to the surface.

The normalized energy distribution of a GSL based metasurface using impedance
matched lossless effective medium computed from the same simulation is also shown in Fig. 2.7(b) for comparison, where undesired diffraction orders can be clearly observed. It should be noted that this number is calculated based on unit cells characterized by matched impedance and ideal refractive indices, and gives the performance limit of conventional designs. The bianisotropic metasurface proposed here therefore provides an alternative route of overcoming the power efficiency limitation and reduce the parasitic energy spread into undesired directions.

2.5 Conclusion

In summary, we design and experimentally demonstrate an acoustic metasurface cell that provides full control of the bianisotropic response and minimizes the implementation losses by ensuring that the individual resonators work below the resonant frequency. The response of the cells, controlled by the physical sides of the four side-loaded resonators and the width of the channel, can be adjusted to provide any scattering requirement. The tangential dimension for the cell is deep subwavelength (1.2 cm for 3000 Hz, \( \lambda/10 \)) compared with previously proposed meta-atoms in Koo et al. (2016) (7 cm for 1300 Hz, \( \lambda/4 \)), such that it is readily to be applied to the cases where more complicated impedance profiles are needed. For a specific asymmetric response, a carefully implemented genetic algorithm optimization method calculates the physical dimensions of the unit cell.

In addition, we have demonstrated the first design and realization of bianisotropic acoustic metasurfaces for scattering-free wavefront manipulations. Three perfect metasurfaces for wavefront modulation (with deflection angles of 60°, 70°, 80°) are designed based on the theory. The performance is validated with numerical simulations, showing great advancement in energy efficiency (93%, 96%, 91%) over conventional GSL-based designs (89%, 58%, 35%), especially at large deflection angles. The scattering-free property of the bianisotropic metasurface is further verified ex-
perimentally. The designed metasurface is shown to be able to steer all the energy to the desired direction with almost no reflection or unwanted scattering.
In analogy to anomalous refraction for flat metasurfaces, one of the possibilities offered by cylindrical metasurfaces is the transformation among different cylindrical waves. This transformation is achieved by locally controlling the phase profile along the surface and contribute to the generation of source illusion Liu et al. (2017).

Source illusion is just an example of many possibilities offered by metasurfaces capable of controlling angular momentum. Recent research has also demonstrated the manipulation of beams for particle trapping Baresch et al. (2016); He et al. (1995) and boosting communication efficiency Shi et al. (2017) with acoustic angular momentum. Passive generation of wave fields with non-zero angular momentum is typically implemented by leaky wave antennas or metasurfaces based on generalized Snell’s law (GSL) Naify et al. (2016); Jiang et al. (2016) for acoustic waves and inhomogeneous anisotropic media Marrucci et al. (2006), spatial light modulator or spiral phase plates Yao and Padgett (2011); Schemmel et al. (2014) for electromagnetic waves. However, from the analysis of beam steering in Cartesian coordinate,
we know that if only the transmission phase profile is controlled, parasitic scattering will appear inevitably, which reduces the efficiency, or even fails to realize the desired functionality, especially for large angular momentum. Similar to the planar wave case for large-angle deflection, generation of large angular-momentum waves using a single layer of GSL based metasurface will not only introduce a large impedance mismatch but will also require a fine discretization of the surface which is not easily achievable by conventional cell architectures. Therefore, generation of wave fields with a large angular momentum still remains challenging. In this Chapter, we will specifically expand the concept of bianisotropic metasurfaces into cylindrical coordinates, and demonstrate its ability to manipulate cylindrical waves with near-perfect efficiency.

3.1 Theoretical requirements for transformation of cylindrical waves

For acoustic waves, the 2D wave equation in the cylindrical coordinates is written as

\[ \nabla^2 p = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 p}{\partial \varphi^2} = \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2}, \tag{3.1} \]

where \( p \) is the acoustic pressure and \( c_0 \) is the sound speed. Just like plane waves in Cartesian coordinates, Bessel-like spinning waves with different angular momentum serve as the bases in cylindrical coordinates. In the general case, the solution to this equation can be written as

\[ p = \sum_n \left[ a_n H_n^{(1)} (kr) + b_n H_n^{(2)} (kr) \right] e^{jn\varphi} e^{j\omega t}, \tag{3.2} \]

where \( H_n^{(1)} \) denotes the Hankel function of the first kind (waves converging to the center) and \( H_n^{(2)} \) denotes the Hankel function of the second kind (waves diverging from the center), index \( n \) represents the angular momentum, \( a_n \) and \( b_n \) are the amplitudes of the waves, and \( k = \omega/c_0 \) is the wavenumber at the frequency of interest.
In this section we will discuss the theoretical requirements for a metasurface to produce perfect transformation between cylindrical waves with different angular momenta, i.e. with different spinning characteristics, as it is shown in Fig. 3.1. The derivation of the solution will be presented considering acoustic waves, however, a similar formulation can be used for electromagnetic waves.

The formulation of the problem starts with the definition of the fields inside and outside the volume bounded by the metasurface. In what follows, the time-harmonic dependency $e^{j\omega t}$ will be omitted. Let us consider the field in Medium I (inside the metasurface) as a divergent wave with the angular momentum $n_1$ that can be expressed as

$$p^I = p_0 H_{n_1}^{(2)}(kr)e^{jn_1\phi} \quad (3.3)$$

where $p_0$ is the wave amplitude. It is important to mention that we only consider a divergent wave inside the metasurface because the objective of the metasurface is to perfectly transform the incident cylindrical wave without reflections. The velocity
The velocity vector can be calculated from the pressure field ($\vec{v} = \frac{1}{\omega \rho} \nabla p$) as

$$\vec{v}^I = \frac{p_0}{Z_0} \left[ j \hat{c}_r H_{n_1}^{(2)}(kr) \hat{\rho} - \frac{n_1}{kr} H_{n_1}^{(2)}(kr) \hat{\phi} \right] e^{jn_1 \varphi}, \quad (3.4)$$

where $Z_0 = \sqrt{\epsilon_0 \mu_0 \rho}$ is the characteristic impedance of air and $\hat{c}_r$ represents the partial derivative with respect to $r$. Following the same approach, we can define the field in Medium II (outside the volume bounded by the metasurface) as a divergent wave with the angular momentum $n_2$ as $p^I = p_t H_{n_2}^{(2)}(kr)e^{jn_2 \varphi}$ with $p_t$ being the amplitude of the transmitted wave. The velocity vector can be expressed as

$$\vec{v}^I = \frac{p_t}{Z_0} \left[ j \hat{c}_r H_{n_2}^{(2)}(kr) \hat{\rho} - \frac{n_2}{kr} H_{n_2}^{(2)}(kr) \hat{\phi} \right] e^{jn_2 \varphi}. \quad (3.5)$$

We assume that the metasurface is a cylindrical tube whose axis is located at the origin, with the inner radius $r_1$ and the outer radius $r_2$. For lossless and scattering-free metasurfaces, the energy conservation condition shall be met. Denoting the time-averaged intensity vector as

$$\mathbf{T} = \frac{1}{2} \text{Re} \{ \mathbf{v}^* \mathbf{v} \} = I_r \hat{\rho} + I_{\varphi} \hat{\phi}, \quad (3.6)$$

this condition can be expressed in terms of the radial components of this vector at the two sides of the metasurface:

$$I_r^I = \frac{p_0^2}{2Z_0} \left[ J_{n_1}(kr) \hat{c}_r Y_{n_1}(kr) - Y_{n_1}(kr) \hat{c}_r J_{n_1}(kr) \right]_{r_1} \quad (3.7)$$

$$I_r^II = \frac{p_t^2}{2Z_0} \left[ J_{n_2}(kr) \hat{c}_r Y_{n_2}(kr) - Y_{n_2}(kr) \hat{c}_r J_{n_2}(kr) \right]_{r_2}, \quad (3.8)$$

where $J_\alpha$ and $Y_\alpha$ represent the Bessel functions of the first and second kind, respectively. These expressions can be simplified as $I_r^I = \frac{p_0^2}{\pi Z_0} \frac{1}{r_1}$ and $I_r^II = \frac{p_t^2}{\pi Z_0} \frac{1}{r_2}$.

To ensure that all the energy of the incident wave is carried away by the transmitted spinning wave, the normal component of the intensity vector crossing a line
element of the inner radius, $S_1 = r_1 d\phi$, has to be equal to the one crossing the corresponding line element in the other radius, $S_2 = r_2 d\phi$. This condition can be written as $I^I_r S_1 = I^II_r S_2$ and yields that the $p_t = p_0$. If we define the macroscopic transmission coefficient as

$$T = \frac{p^H(r_2)}{p^I(r_1)} = \frac{H^{(2)}_{n_2}(kr_2)}{H^{(2)}_{n_1}(kr_1)} e^{j(n_2-n_1)\phi},$$

(3.9)

it is possible to see that the magnitude of the macroscopic transmission coefficient can be greater (smaller) than unity if $n_2$ is greater (smaller) than $n_1$, respectively.

It is noted here that this condition is analogous to the plane-wave case described in Li et al. (2018); Díaz-Rubio and Tretyakov (2017).

The next step towards the realization of perfect transformation between cylindrical waves is to determine the required boundary conditions at both sides of metasurface. At the inner and outer boundaries of the metasurface, for each specific circumferential position, the impedance matrix which models the metasurface is defined as

$$\begin{bmatrix} p^I(r_1, \phi) \\ p^H(r_2, \phi) \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} S_1 \hat{n} \cdot \mathbf{\tilde{v}}^I(r_1, \phi) \\ -S_2 \hat{n} \cdot \mathbf{\tilde{v}}^H(r_2, \phi) \end{bmatrix},$$

(3.10)

where $\hat{n}$ is the unit normal vector to the metasurface. In the most general linear, time-invariant, lossless, and reciprocal case, the impedance matrix is symmetric, $Z_{12} = Z_{21}$, and all its components are purely imaginary, $Z_{ij} = jX_{ij}$ Li et al. (2018).

For compactness, we denote

$$C_{n_1} = H^{(2)}_{n_1}(kr_1)e^{jn_1\phi}$$

(3.11)

$$C_{n_2} = H^{(2)}_{n_2}(kr_2)e^{jn_2\phi}$$

(3.12)

$$C'_{n_1} = \frac{1}{2}[H^{(2)}_{n_1-1}(kr_1) - H^{(2)}_{n_1+1}(kr_1)]e^{jn_1\phi}$$

(3.13)

$$C'_{n_2} = \frac{1}{2}[H^{(2)}_{n_2-1}(kr_2) - H^{(2)}_{n_2+1}(kr_2)]e^{jn_2\phi}$$

(3.14)
and re-write Eq. (3.10) in form of a system of two linear equations:

\[
\begin{align*}
Z_0 C_{n1} &= -S_1 X_{11} C'_{n1} + S_2 X_{12} C'_{n2} \\
Z_0 C_{n2} &= -S_1 X_{12} C'_{n1} + S_2 X_{22} C'_{n2}
\end{align*}
\] (3.15)

After some algebra, the components of the impedance matrix can thus be calculated:

\[
X_{11} = \frac{Z_0 \text{Im}(C_{n1}) \text{Re}(C'_{n2}) - \text{Re}(C_{n1}) \text{Im}(C'_{n2})}{S_1 \text{Im}(C'_{n2}) \text{Re}(C_{n1}) - \text{Re}(C_{n2}) \text{Im}(C'_{n1})}
\] (3.16)

\[
X_{22} = \frac{Z_0 \text{Im}(C_{n2}) \text{Re}(C'_{n1}) - \text{Re}(C_{n2}) \text{Im}(C'_{n1})}{S_2 \text{Im}(C'_{n1}) \text{Re}(C_{n2}) - \text{Re}(C_{n2}) \text{Im}(C'_{n1})}
\] (3.17)

\[
X_{12} = -\frac{Z_0 \text{Im}(C'_{n2}) \text{Re}(C_{n2}) - \text{Re}(C'_{n2}) \text{Im}(C_{n2})}{S_1 \text{Im}(C'_{n2}) \text{Re}(C_{n1}) - \text{Re}(C_{n2}) \text{Im}(C'_{n1})}.
\] (3.18)

For simplicity in the derivations, and to provide another viewpoint for the requirements, the required properties of the metasurface can also be expressed in terms of the transfer matrix, which is defined by

\[
\begin{bmatrix} p^I(r_1, \phi) \\ S_1 \hat{n} \cdot \hat{v}^I(r_1, \phi) \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} p^H(r_2, \phi) \\ S_2 \hat{n} \cdot \hat{v}^H(r_2, \phi) \end{bmatrix}
\] (3.19)

Conversion from the impedance matrix to the transfer matrix is given by

\[
M = \begin{bmatrix} Z_{11} & Z_{12} - Z_{21} Z_{12} \\ Z_{21} & Z_{22} - Z_{12} Z_{21} \end{bmatrix}
\] (3.20)

which indicates that $M_{11}$ and $M_{22}$ are real, while $M_{12}$ and $M_{21}$ are imaginary. Explicit solution for the transfer matrix can thus be written:

\[
M_{11} = \frac{\text{Im}(C'_{n2}) \text{Re}(C_{n1}) - \text{Re}(C'_{n2}) \text{Im}(C_{n1})}{\text{Im}(C'_{n2}) \text{Re}(C_{n2}) - \text{Re}(C'_{n2}) \text{Im}(C_{n2})}
\] (3.21)

\[
M_{22} = -\frac{S_1 \text{Im}(C_{n2}) \text{Re}(C'_{n1}) - \text{Re}(C_{n2}) \text{Im}(C'_{n1})}{S_2 \text{Im}(C'_{n2}) \text{Re}(C_{n2}) - \text{Re}(C'_{n2}) \text{Im}(C_{n2})}
\] (3.22)

\[
M_{12} = \frac{jZ_0 \text{Im}(C_{n2}) \text{Re}(C_{n1}) - \text{Re}(C_{n2}) \text{Im}(C_{n1})}{S_2 \text{Im}(C'_{n2}) \text{Re}(C_{n2}) - \text{Re}(C'_{n2}) \text{Im}(C_{n2})}
\] (3.23)
\[ M_{21} = \frac{j S_1 \text{Im}(C'_{n_2}) \text{Re}(C''_{n_1}) - \text{Re}(C'_{n_2}) \text{Im}(C''_{n_1})}{Z_0 \text{Im}(C''_{n_2}) \text{Re}(C'_{n_2}) - \text{Re}(C'_{n_2}) \text{Im}(C''_{n_2})}. \] (3.24)

It is easy to check that this matrix corresponds to a reciprocal and lossless system.

Note that as long as \(|n_1| \neq |n_2|\), we will always have \(M_{11} \neq M_{22}\), which leads to \(Z_{11} \neq Z_{22}\). This asymmetry is analogous to the plane-wave case in the Cartesian coordinates, meaning that controlling only the transmission phase along the metasurface is not enough for perfect engineering the power flow. Instead, a bianisotropic metasurface with precisely controlled asymmetric response is required.

### 3.2 Realization of the bianisotropic unit cells

For the actual implementation of the metasurface described in the previous section, there are several different possible approaches.

#### 3.2.1 Three-membrane model

**Electromagnetic metasurfaces**

For the electromagnetic case, one can consider a cascade of metallic pattern separated by concentric dielectric substrates [see Figure 3.2]. The patterned metallic sheets can
be modeled as shunt impedances with the following transfer matrix

\[ M_{Zi} = \begin{bmatrix} 1 & 0 \\ Y_i & 1 \end{bmatrix}, i = 1, 2, 3 \]  \hspace{1cm} (3.25)

where \( Y_i = 1/Z_i \) represents the effective impedance of the metallic patterns. On the other hand the transmission matrix of the of a wedge-shaped dielectric sector can be expressed as

\[ M_{Ti} = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}, i = 1, 2 \]  \hspace{1cm} (3.26)

The values matrix elements are functions of the inner and outer radius and the dielectric permittivity \( \varepsilon_d \) (see Supplementary Note 2 for more information). Finally the total transmission matrix can be calculated as

\[ M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = M_{Z1}M_{T1}M_{Z2}M_{T2}M_{Z3} \]  \hspace{1cm} (3.27)

After some algebra, we can obtain the required sheet admittances (\( Y_1, Y_2, \) and \( Y_3 \)) as a function of the required scattering properties (\( M_{11}, M_{12}, M_{21}, \) and \( M_{22} \))

\[ Y_2 = \frac{M_{12} - B_1D_2 - A_1B_2}{B_1B_2} \]  \hspace{1cm} (3.28)

\[ Y_1 = \frac{M_{22} - (D_1D_2 + C_1B_2 + D_1B_2Y_2)}{A_1B_2 + B_1D_2 + B_1B_2Z_2} \]  \hspace{1cm} (3.29)

\[ Y_3 = \frac{M_{11} - (B_1C_2 + A_1A_2 + B_1A_2Y_2)}{A_1B_2 + B_1D_2 + B_1B_2Y_2} \]  \hspace{1cm} (3.30)

At microwave frequency the required sheet admittances can be implemented by using metallic patterns.

*Acoustic metasurfaces*

For the acoustic scenario, the asymmetric response can be obtained as cascade of three different membranes separated a certain distance. The response of a meta-
atom can be expressed in terms of the transmission matrices

$$M = M_{Z1}M_{T1}M_{Z2}M_{T2}M_{Z3}$$

(3.31)

with

$$M_{Zi} = \begin{bmatrix} 1 & Z_i \\ 0 & 1 \end{bmatrix}, \ i = 1, 2, 3$$

(3.32)

and $M_{Ti}, i = 1, 2$ is the transfer matrix of a wedge-shaped dielectric sector, which is a function of its inner and outer radius. Detailed derivation of the explicit expression of $M_{Ti}$ can be found in Supplementary Note 2. Here for simplicity, let us denote

$$M_{Ti} = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}, \ i = 1, 2$$

(3.33)

Then the required impedances for the three membranes can be calculated as

$$Z_2 = \frac{M_{21} - C_1A_2 - D_1C_2}{C_1C_2}$$

(3.34)

$$Z_1 = \frac{M_{11} - (A_1A_2 + B_1C_2 + A_1C_2Z_2)}{C_1A_2 + D_1C_2 + C_1C_2Z_2}$$

(3.35)

$$Z_3 = \frac{M_{22} - (C_1B_2 + D_1D_2 + C_1D_2Z_2)}{C_1A_2 + D_1C_2 + C_1C_2Z_2}$$

(3.36)

3.2.2 Four-resonators approach

By controlling the thickness and in-plane tension of the membranes, one can, in principle, control the impedances to satisfy Eqs. (3.16)-(3.18). However, the surface tension, uniformity and durability for the membranes are extremely hard to control, and it is questionable whether such configuration can be practically realized.

An alternative approach based on a straight channel with four resonators was proposed for flat surfaces Li et al. (2018). The design provides enough degrees of freedom for full control over the bianisotropic response while reducing the loss induced by resonances. Here, we propose the four-resonator design in cylindrical
coordinates for full control over the bianisotropic response of the unit cells. An example cell is shown in Fig. 3.3. In this structure: the width and height of the neck, \( h_{\text{neck}} \) and \( w_{\text{neck}} \), are fixed for the four resonators; the width of the cavities \( w_{\text{cav}} \) is also fixed; the sector angle of the wedge-shaped channel \( \theta_c \) and the height of the resonators \( w_a, w_b, w_c, \) and \( w_d \) can be varied to control the overall impedance response; and the wall thickness of the unit cell is fixed and will be defined by the fabrication limitations. The walls between adjacent cells are assumed to be hard so that the wave does not propagate along the orthogonal direction inside the metasurface. Therefore, all the cells in the bianisotropic metasurfaces can be designed individually.

3.3 Designing bianisotropic metasurface with four-resonator approach

For the design of the physical dimensions, genetic algorithm (GA) optimization is used to define \( \theta_1, w_a, w_b, w_c, \) and \( w_d \) so that the calculated impedance matrix matches the theoretical requirements. The detailed parameters for the GA optimization is the same as in the planar case. The cost function is defined as

\[
\text{cost} = \sqrt{\left| \frac{Z^s_{11} - Z^t_{11}}{Z^t_{11}} \right|^2 + \left| \frac{Z^s_{12} - Z^t_{12}}{Z^t_{12}} \right|^2 + \left| \frac{Z^s_{21} - Z^t_{21}}{Z^t_{21}} \right|^2 + \left| \frac{Z^s_{22} - Z^t_{22}}{Z^t_{22}} \right|^2}
\]  

where \( s \) denotes the values achieved by the structure and \( t \) denotes the theoretical requirements.
3.3.1 GA optimization using theoretical calculation

The transfer matrix of the proposed meta-atom topology can be calculated as

\[ M = M_{TL}M_{H1}M_{T1}M_{H2}M_{T2}M_{H3}M_{T3}M_{H4}M_{TR} \]  

(3.38)

with \( M_{TL}, M_{TR}, \) and \( M_{T1,2,3} \) being the transfer functions of transmission lines at the entrance, exit, and between adjacent resonators, as is shown in Fig. 3.3.

\[ M_{Hi} = \begin{bmatrix} 1 & 0 \\ 1/Z_{Hi} & 1 \end{bmatrix}, \quad i = 1, 2, 3, \]

(3.39)

where \( Z_{Hi} \) are the acoustic impedances for each shunted resonator. The detailed derivation of \( Z_{Hi} \) is given in Li et al. (2016b), and the result is directly given here for brevity: \( Z_a = Z_n Z_c + jZ_n \tan(kw_2) + j \text{Im}(Z_d) \). Here \( Z_n = \rho_0 c_0 / h_2 \) and \( Z_c \) are the acoustic impedance of the neck and the cavity of the Helmholtz resonator, respectively.

\[ \text{Im}(Z_d) \] is the radiation impedance which is expressed as: \( Z_d = \frac{\rho_0 c_0}{w_1 h_2^2} \frac{1-e^{-jkr_2}-jk^2 r_2}{k^2} + \sum_{n=1}^{2} \frac{1-e^{-jkr_2}-jk^2 r_2}{k^2} \) with \( k_{zn} = \sqrt{k^2 - k_{zn}^2} \) and \( k_{zn} = n\pi / w_1 \). The acoustic impedance of the cavity \( Z_c \) is given by \( Z_c = \sum_{n} \frac{\rho_0 c_0}{k_{zn}^2 h_3 (1-e^{-jkr_2})} \). The impedance matrix of an arbitrary meta-atom can then be calculated by converting the transfer matrix using

\[
Z = \begin{bmatrix}
M_{11} & M_{12} & M_{13} & M_{14} \\
M_{21} & M_{22} & M_{23} & M_{24} \\
M_{31} & M_{32} & M_{33} & M_{34} \\
M_{41} & M_{42} & M_{43} & M_{44}
\end{bmatrix}
\]

(3.40)

With the theoretical requirement of the impedance matrix profile for perfect wavefront transformation and the versatility of the meta-atom for full control over the bianisotropic response, the next step is to decide the detailed physical dimensions of the meta-atoms that form the metasurface. Since there are three independent elements in the required impedance matrix \( (X_{11}, X_{12}, X_{22}) \) and five controlling parameters \( (\theta_c, w_a, w_b, w_c, w_d) \), there can be many combinations for a meta-atom
to realize the required impedance matrix. To solve for a practical design within geometrical limitations, a continuous genetic algorithm is adopted for optimization of the design parameters, so that the impedance matrix of the optimized structure matches the theoretical requirements.

3.3.2 Local optimization with simulation

Although theoretical calculation offers a fast and close approximation of the meta-atom behavior, it will also introduce some error due to truncation of the infinite series and the straight channel assumption. On the other hand, extracting the impedance using commercial simulations (for example, COMSOL Multiphysics) offers slow but more precise characterization. Therefore, based on the structure obtained from theoretical optimization, we further optimize it locally using genetic algorithm by slightly perturbing the structure dimensions.

The method used for extracting the impedance matrix from simulation was adopted from the standard “4-microphone” method. The method uses four microphones to measure the pressure at two fixed points on both sides of the tested structure under two different boundary conditions, and the properties can thus be calculated. Based on the same idea, we developed a method to extract the structure properties in cylindrical coordinates.

For the ease of implementation, the method we used to retrieve the impedance matrix in COMSOL is inspired by the standard 4-microphone method for acoustic experiments with impedance tubes, whose setups are shown in Fig. ?. The waves in the upstream and downstream can be written as

\[ p_{\text{up}} = AH_0^{(2)}(kr) + AH_0^{(1)}(kr) \]  \hfill (3.41)

\[ p_{\text{down}} = CH_0^{(2)}(kr) + DH_0^{(1)}(kr) \]  \hfill (3.42)

The positions of 4 microphones are \( x_1, x_2, x_3, x_4 \), respectively. By performing two measurements with different boundary conditions at the end of the tube, we
can obtain four independent equations for determination of the four transfer matrix
elements. Two different boundaries we used at the end of the tube are plane wave
radiation (condition #1) and hard wall (condition #2). The pressure detected by
these microphones under these two boundary conditions are noted as \( p_m^{(n)} \)
where \( m \) denotes the number of the microphone and \( n \) denotes the number of the boundary
condition. They satisfy the condition:

\[
\begin{bmatrix}
H_0^{(2)}(kx_1) & H_0^{(1)}(kx_1)
\end{bmatrix}
\begin{bmatrix}
A^{(1)} & A^{(2)}
\end{bmatrix}
= \begin{bmatrix}
p_1^{(1)} & p_2^{(2)}
p_2^{(1)} & p_2^{(2)}
\end{bmatrix}
\] (3.43)

Similarly,

\[
\begin{bmatrix}
H_0^{(2)}(kx_3) & H_0^{(1)}(kx_3)
\end{bmatrix}
\begin{bmatrix}
C^{(1)} & C^{(2)}
\end{bmatrix}
= \begin{bmatrix}
p_3^{(1)} & p_3^{(2)}
p_4^{(1)} & p_4^{(2)}
\end{bmatrix}
\] (3.44)

With the measurement of \( p_m^{(n)} \) under two different conditions, all the ABCD in
the matrices can be calculated. Therefore, the scattering matrix can be calculated
as

\[
S = \begin{bmatrix}
B^{(1)} & B^{(2)}
C^{(1)} & C^{(2)}
\end{bmatrix}
\begin{bmatrix}
A^{(1)} & A^{(2)}
D^{(1)} & D^{(2)}
\end{bmatrix}^{-1}
\] (3.45)

If the inner radius and outer radius of the metasurface is \( r_1 \) and \( r_2 \), then the pressure
and volume velocity at both sides can be written as:

\[
\begin{bmatrix}
p_i^{(1)} & p_i^{(2)}
S_1 v_i^{(1)} & S_1 v_i^{(2)}
\end{bmatrix} =
\begin{bmatrix}
\frac{H_0^{(2)}(kr_1)}{-\frac{s}{2Z_0}[H_0^{(2)}(kr_1) - H_1^{(2)}(kr_1)] - \frac{s}{2Z_0}[H_0^{(1)}(kr_1) - H_1^{(1)}(kr_1)]}
\end{bmatrix}
\begin{bmatrix}
A^{(1)} & A^{(2)}
B^{(1)} & B^{(2)}
\end{bmatrix}
\] (3.46)

\[
\begin{bmatrix}
p_o^{(1)} & p_o^{(2)}
S_2 v_o^{(1)} & S_2 v_o^{(2)}
\end{bmatrix} =
\begin{bmatrix}
\frac{H_0^{(2)}(kr_2)}{-\frac{s}{2Z_0}[H_0^{(2)}(kr_2) - H_1^{(2)}(kr_2)] - \frac{s}{2Z_0}[H_0^{(1)}(kr_2) - H_1^{(1)}(kr_2)]}
\end{bmatrix}
\begin{bmatrix}
C^{(1)} & C^{(2)}
D^{(1)} & D^{(2)}
\end{bmatrix}
\] (3.47)
The transfer matrix of the measured unit cell can thus be calculated as

\[ T = \begin{bmatrix} p_0^{(1)} & p_0^{(2)} \\ S_2v_0^{(1)} & S_2v_0^{(2)} \end{bmatrix} \begin{bmatrix} p_i^{(1)} & p_i^{(2)} \\ S_1v_i^{(1)} & S_1v_i^{(2)} \end{bmatrix}^{-1} \] (3.48)

Hence the impedance matrix can be calculated as

\[ Z = \begin{bmatrix} \frac{T_{22}}{T_{21}} & -\frac{1}{T_{21}} \\ \frac{T_{12}T_{21}-T_{11}T_{22}}{T_{21}} & \frac{T_{11}}{T_{21}} \end{bmatrix} \] (3.49)

### 3.3.3 A showcase for generation of high acoustic angular momentum

We have designed a metasurface to transform a monopole source \( n_1 = 0 \) located at the center to a spinning field with the angular momentum of \( n_2 = 12 \). In this case, \( r_1 = 15 \text{ cm}, r_2 = 20 \text{ cm}, \) and one period is represented by 6 meta-atoms. In this case, each unit cell occupies a sector of \( \Delta \phi = \pi/36 \), therefore, \( S_1 = \Delta \phi r_1 \) and \( S_2 = \Delta \phi r_2 \). We swept the circumferential positions with a step of 0.1 degrees, and run the GA optimization 50 times at each point to search for the best combination with the lowest relative error.

Based on the structure obtained from theoretical optimization, we further optimize it locally using genetic algorithm by slightly perturbing the structure dimensions within \( \pm 1 \text{ mm} \).

The theoretical requirement for the desired metasurface and the achieved values from the two-step optimization is shown in Fig. 3.4(a). Detailed dimensions of the meta-atoms and their relative errors can be found in Table 3.1. We can see that the required impedance is closely realized by the optimized meta-atoms. Simulation of the obtained structure was performed in COMSOL Multiphysics with the pressure acoustics module. The walls of the unit cells are set to be hard due to the large impedance contrast in the implementation. The background medium is air with density 1.21 kg/m\(^3\) and sound speed 343 m/s. The incident pressure amplitude is 1
Figure 3.4: Theoretical determined and optimized impedances and the simulated fields. (a) Comparison between theoretical requirements and the achieved values using GA optimization. (b) The real part of the simulated acoustic field using real structures. The inset shows the pressure amplitude near the metasurface. (c) The field generated by GSL based metasurface using ideal unit cells as a comparison.

Pa at $r = 2$ cm. The outer edge of the simulated region is connected to a perfectly matched layer. The simulated pressure field and the pressure amplitude are shown in Fig. 3.4(b). We can see that the monopole wavefront is perfectly converted to a field with the angular momentum of 12 without parasitic reflection and scattering. From the pressure amplitude field we can see that the macroscopic transmission coefficient $|T| > 1$, i.e., the pressure on the transmission side is larger than the incident side. The corresponding reference GSL metasurface formed by ideal unit cells with the same
size and the same number of cells period is shown in Fig. 3.4(c) as a comparison. We can see that there is strong reflection and lots of the transmitted energy is scattered to the unwanted modes and the overall wave pattern is corrupted.

3.4 Experimental characterization of the design

The theory and simulations are then verified with experiments. We chose the same example as discussed in the previous section. The experimental setup is shown in Fig. 3.5(a). The sample was fabricated by Selective Laser Sintering (SLS) 3D-printing. The material is Nylon with the density of 950 kg/m$^3$ and sound speed of 1338 m/s, so that the walls can be regarded as rigid due to the large impedance contrast with air. The printed sample has the inner radius of 150 mm and the outer radius of 200 mm, and the height of the sample is 41 mm to fit in the 2D-waveguide.

The monopole source was provided by a 1-inch speaker located at the center, which sends a Gaussian pulse centered at 3000 Hz. The field was scanned by a moving microphone with a step of 1 cm. Then the field is calculated by performing Fourier transform of the detected pulse. Since the overall size of the scanning system is limited, and the field is symmetric, a quarter of the whole field is scanned, as shown in Fig. 3.5(a), and the measured data is then mapped to other regions.

The real part of the scanned field and the phase of the field is plotted in Fig 3.5(b) and Fig. 3.5(c). From the experimental results, we can see that the fabricated meta-

<table>
<thead>
<tr>
<th>Cell</th>
<th>cost(%)</th>
<th>$\theta_c$ (mm)</th>
<th>$w_a$ (mm)</th>
<th>$w_b$ (mm)</th>
<th>$w_c$ (mm)</th>
<th>$w_d$ (mm)</th>
</tr>
</thead>
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<td>7.0</td>
<td>7.0</td>
<td>8.3</td>
<td>6.4</td>
</tr>
<tr>
<td>3</td>
<td>0.16</td>
<td>0.6997</td>
<td>8.4</td>
<td>7.9</td>
<td>2.5</td>
<td>5.2</td>
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<tr>
<td>4</td>
<td>0.55</td>
<td>0.7002</td>
<td>7.4</td>
<td>8.4</td>
<td>0.9</td>
<td>4.3</td>
</tr>
<tr>
<td>5</td>
<td>0.35</td>
<td>1.0221</td>
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<td>8.1</td>
<td>6.7</td>
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</tr>
<tr>
<td>6</td>
<td>0.84</td>
<td>1.3931</td>
<td>8.7</td>
<td>2.1</td>
<td>0.5</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Table 3.1: Design parameters of the meta-atoms in the cylindrical metasurface for generation of a spinning field with angular momentum $n = 12$
Figure 3.5: Experimental setup and results. (a) A photo of the experimental setup. The field is scanned by moving the microphone in the green region. (b) The real part of the measured pressure field. (c) The phase of the scanned field. We can clearly see that the wavefront is nearly perfectly transformed to the field with the angular momentum $n = 12$. (d) The comparison among the bianisotropic metasurface in simulation and experiment, and the ideal GSL-based metasurface in the simulation. In the experiment, $92\%$ of the transmitted energy is concentrated in the desired mode.
surface created the field with much lower unwanted scatterings compared with an ideal GSL-based metasurface shown in Fig. 3.5(d). The small discrepancy is due to the fabrication errors, and the small difference in the air properties between simulation and experiment. In particular, the sound speed was 344 m/s in our lab during the measurement window, while we assumed 343 m/s in the simulation, which will cause the working frequency to increase by about 8 Hz. To quantitatively characterize the results, we extracted the coefficients of contributing modes by taking the data around a circular trajectory and performing a Fourier transform of the fields at $r = 22$ cm to extract the amplitudes of different modes. The power of each mode is calculated and then normalized by the total power. The power distribution over the modes of $n = -30$ to $n = 30$ is plotted in Fig. 3.5(d). For comparison, the same analysis is performed for the simulation of the bianisotropic metasurface and the ideal GSL-based metasurface. We can clearly see that the GSL-based metasurface, even with the perfectly designed cells of full transmission and precise control of the transmitted phase, produces a large component of $n = -12$ mode, so that only 70% of the transmitted energy is in the desired mode, while in the bianisotropic designs, the unwanted scattering is greatly suppressed, showing 99% and 92% of the transmitted energy in the desired mode $n = 12$ in simulation and experiment, respectively. The experimental results show good agreement with the simulation, demonstrating the possibility of near perfect transformation of acoustic wavefronts.

3.5 Conclusion

In this chapter, we have introduced a multi-physics design method for creation of acoustic or electromagnetic bianisotropic metasurfaces of cylindrical shape for perfect generation of waves with arbitrary angular momenta. We first defined theoretically the conditions and requirements, and pointed out that controlling the local phase shift in transmission alone cannot achieve such transformations. Instead, full control
over the bianisotropy is required.

Then we proposed possible realizations for acoustic waves, and verified them with simulations, showing that the proposed metasurface nearly perfectly transforms a monopole source into a spinning wave field with the angular momentum of 12, which is beyond the ability of conventional GSL-based metasurfaces. Then we proposed a systematic and practical way of creating cylindrical bianisotropic acoustic metasurfaces and verified it with experiments. The experimental results show excellent agreement with simulations, with 92% of the transmitted energy concentrated in the desired mode, whereas with the use of an ideal GSL-based metasurface, 30% of the transmitted energy is scattered to other modes. Here we would like to note that the efficiency of the conventional GSL-based design is even lower because the simulation shows that 10% of the energy is reflected indicating that the ideal efficiency can reach only 63%, while our design is free of reflections.
Bianisotropic metasurface for near-perfect reflection and beam splitting with auxiliary surface waves

In Chapters 2 and 3, we have developed the methods to perform near-perfect wavefront transformation for waves in both Cartesian and cylindrical coordinates. Then comes a natural question: can we extend the knowledge and use them to create a more complicated wavefront?

This question seems trivial at first glance because we can always write the incident field and the desired field, calculate the requirements for the impedance matrix profile and realize them using the structure proposed in Chapter 2. However, in many cases, we may arrive at impedance requirements that are impractical. Therefore, the answer is yes but under certain conditions. In this chapter, we will talk about this issue and demonstrate one feasible way to meet such requirements.

4.1 Transmission case: beam splitting

4.1.1 Origin of the power efficiency limit

We first investigate a transmission-type metasurface that splits a normally incident wave into two plane waves with transmitted angles $\theta_1$, $\theta_2$ and transmission coefficients
\(T_{1,2} = t_{1,2} e^{j \phi_{1,2}}\), as illustrated in Fig. 4.2. Here we impose \(\phi_1 = \phi_2\) for simplicity. (Since the waves are propagating in different directions, we can always find a position where \(\phi_1 = \phi_2\) is satisfied, and set that point as the origin). The normally incident wave is simply written as

\[
p_1 = p_0 e^{-j ky}, \quad (4.1)
\]
\[
v_{1,y} = \frac{p_0}{Z_0} e^{-j ky}. \quad (4.2)
\]

The transmitted fields write:

\[
p_2 = T_1 p_0 e^{j k \sin \theta_1 x} e^{-j k \cos \theta_1 y} + T_2 p_0 e^{-j k \sin \theta_2 x} e^{-j k \cos \theta_2 y}, \quad (4.3)
\]
\[
v_{2,y} = \frac{T_1 p_0 \cos \theta_1}{Z_0} e^{j k \sin \theta_1 x} e^{-j k \cos \theta_1 y} + \frac{T_2 p_0 \cos \theta_2}{Z_0} e^{-j k \sin \theta_2 x} e^{-j k \cos \theta_2 y}. \quad (4.4)
\]

The global power flow perpendicular to the metasurface need to be conserved, i.e.

\[
\frac{t_1^2 |p_0|^2 \cos \theta_1}{2Z_0} + \frac{t_2^2 |p_0|^2 \cos \theta_2}{2Z_0} = \frac{|p_0|^2}{2Z_0}, \quad (4.5)
\]

thus

\[
t_1^2 \cos \theta_1 + t_2^2 \cos \theta_2 = 1. \quad (4.6)
\]

On the transmission side, the pressure and normal velocity field at the metasurface \((y = 0)\) are

\[
p_{20} = T_1 p_0 e^{j k \sin \theta_1 x} + T_2 p_0 e^{-j k \sin \theta_2 x}, \quad (4.7)
\]
\[
v_{2,y0} = \frac{T_1 p_0 \cos \theta_1}{Z_0} e^{j k \sin \theta_1 x} + \frac{T_2 p_0 \cos \theta_2}{Z_0} e^{-j k \sin \theta_2 x}. \quad (4.8)
\]

The impedance matrix profile of a metasurface can be calculated by putting the defined fields on both sides of the metasurface into the impedance matrix

\[
\begin{bmatrix}
p_1(x, 0) \\
p_2(x, 0)
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
n \cdot v_1(x, 0) \\
n \cdot v_2(x, 0)
\end{bmatrix} \quad (4.9)
\]
and equate the real and imaginary parts respectively. Note here that for passive and lossless structures, all the components in the impedance matrix shall be purely imaginary Díaz-Rubio and Tretyakov (2017); Li et al. (2018).

As an illustration, we calculated the impedance matrix in the equal splitting case, where \( \theta_1 = \theta_2 = 60^\circ \), \( t_1 = t_2 = 1/\sqrt{2 \cos \theta_{1,2}} \), \( \phi_1 = \phi_2 = \pi/2 \), and the imaginary part of the impedance matrix is plotted in Fig. 4.1(a). Here \( X = \text{Im}[Z] \), denoting the reactance. From Fig. 4.1(a), we can see \( Z_{12} \neq Z_{21} \). Such a result implies that in order to build a metasurface that performs such wavefront transformation, controlled nonreciprocity needs to be introduced into the building blocks, which is extremely difficult if at all possible to realize since reciprocity requires \( Z_{12} = Z_{21} \) in any LTI system.

For further analysis, we can take a look at the intensity field \( I = \frac{1}{2} \text{Re}[p v^*] \). The transmitted intensity field along the metasurface is calculated as

\[
I_{2,y0} = \frac{|p_0|^2}{2Z_0} \{1 + t_1 t_2 (\cos \theta_1 + \cos \theta_2) \cos [k(\sin \theta_1 + \sin \theta_2) x] \}. \tag{4.10}
\]

From Eq. (4.10) we can see the interference between two transmitted beams makes the transmitted intensity along the metasurface non-uniform, as shown in Fig. 4.1(b). However, the incident plane wave creates a uniform intensity profile on the incident side. Such a power flow mismatch shares many similarities with the reflection case Estakhri and Alù (2016b), meaning that the metasurface needs to either embed loss and gain to absorb the energy in some regions and emit energy in others, or provide mechanisms, such as non-locality, to transport power within the metasurface.

### 4.1.2 Designing the field with balanced power

As discussed in the previous section, the difficulty of theoretically perfect wavefront transformation with transmission type metasurfaces lies in that we need a proper
Figure 4.1: Impedance requirements and power distribution for an ideal beam splitting metasurface (a) Required impedance matrix profile along the metasurface, normalized by $Z_0$. $Z_{12} \neq Z_{21}$ implies nonreciprocity within the unit cells. (b) Intensity along both sides of the metasurface, normalized by the incident sound intensity. Such power flow mismatch on both sides requires energy exchange within the metasurface.

In order to match the intensity profile on both sides of the metasurface, we introduce two counter propagating surface waves on the incident side, as illustrated in Fig. 4.2. The pressure field on the incident side writes:

$$p_1 = p_0 e^{-jky} + p_{1s} e^{\alpha_1 y} e^{jk_1 x} + p_{2s} e^{\alpha_2 y} e^{-jk_2 x}, \quad (4.11)$$
Figure 4.2: Illustration of the beam splitting scenario under study. A normally incident wave is ideally splitted into two waves with transmitted angles $\theta_1$ and $\theta_2$ by creating auxiliary surface waves on the incident side.

where $k_1 = \sqrt{k^2 + \alpha_1^2}$, $k_2 = \sqrt{k^2 + \alpha_2^2}$, $p_{1s} = a_1e^{j\theta_1}p_0$, $p_{2s} = a_2e^{j\theta_2}p_0$. Then

$$p_1 = p_0e^{-jky} + a_1p_0e^{\alpha_1y}e^{j(k_1x + \beta_1)} + a_2p_0e^{\alpha_2y}e^{j(-k_2x + \beta_2)}. \quad (4.12)$$

The normal velocity field is

$$v_{1,y} = \frac{p_0}{Z_0}e^{-jky} + \frac{j\alpha_1p_0}{kZ_0}e^{\alpha_1y}e^{j(k_1x + \beta_1)} + \frac{j\alpha_2p_0}{kZ_0}e^{\alpha_2y}e^{j(-k_2x + \beta_2)}. \quad (4.13)$$

At the position of the metasurface ($y = 0$), the pressure and normal velocity field on the incident side are

$$p_{10} = p_0 + a_1p_0e^{j(k_1x + \beta_1)} + a_2p_0e^{j(-k_2x + \beta_2)}, \quad (4.14)$$

$$v_{1,y0} = \frac{p_0}{Z_0} + \frac{j\alpha_1p_0}{kZ_0}e^{j(k_1x + \beta_1)} + \frac{j\alpha_2p_0}{kZ_0}e^{j(-k_2x + \beta_2)}. \quad (4.15)$$

The incident intensity field ($I_y = \frac{1}{2}\text{Re}(pv_y^*)$) is calculated as

$$I_{1,y0} = \frac{|p_0|^2}{2Z_0} \left[ 1 + a_1 \cos (k_1x + \beta_1) - \frac{a_1\alpha_1}{k} \sin (k_1x + \beta_1) + a_2 \cos (k_2x - \beta_2) + \frac{a_2\alpha_2}{k} \sin (k_2x - \beta_2) \right]$$
\[-\frac{a_1a_2(\alpha_1 - \alpha_2)}{k} \sin (k_1 x + \beta_1 + k_2 x - \beta_2)]. \quad (4.16)\]

To realize the local power conservation condition, \(I_{2,y0}\) and \(I_{1,y0}\) should be equal. This equation can have many solutions, but by setting \(\beta_1 = \beta_2 = 0, a_1 = a_2 = a, \alpha_1 = \alpha_2 = \alpha\), we can immediately find one of them. In this case, \(I_{1,y0}\) becomes

\[I_{1,y0} = \frac{|p_0|^2}{2Z_0} \left[ 1 + 2a \cos (\sqrt{k^2 + \alpha^2} x) \right]. \quad (4.17)\]

Comparing Eq. (4.10) and Eq. (4.17), we can see that several conditions shall be met for \(I_{1,y0} = I_{2,y0}\):

\[a = \frac{t_1 t_2 (\cos \theta_1 + \cos \theta_2)}{2}, \quad (4.18)\]

\[\alpha = k \sqrt{(\sin \theta_1 + \sin \theta_2)^2 - 1}. \quad (4.19)\]

Once the field satisfies the local power conservation requirement, the impedance profile can be achieved with passive lossless bianisotropic metasurface designs. The impedance matrix profile can thus be calculated by putting the defined fields into the impedance matrix.

4.1.3 Verification with simulation using the three-layer approach

In the first demonstration, we examine an equal-splitting case where the transmitted angle as \(\theta_1 = \theta_2 = 60^\circ, t_1 = t_2 = 1/\sqrt{2} \cos \theta_{1,2} = 1, \phi_1 = \phi_2 = \pi/2\), incident pressure \(p_0 = 1 \text{ Pa}\). In this case, \(a = \frac{1}{2}, \alpha = \sqrt{2}k\). With the pressure and normal velocity field on both sides of the metasurface, the impedance matrix profile within a period can be calculated, as is plotted in Fig. 4.3(a). We can see that with balanced local power, \(Z_{12} = Z_{21}\) confirms the reciprocity of the unit cells. \(Z_{11} \neq Z_{22}\) indicates that bianisotropic unit cells are needed. For realization in simulation, we chose the
3-layer model proposed in Díaz-Rubio and Tretyakov (2017). The distance between adjacent layers is set as 5mm.

The scattered field from the theoretical calculation and COMSOL simulation are plotted in Fig. 4.3(b) and (c), respectively. In our case, the metasurface is discretized into 10 unit cells per period. From the figures, we can see excellent agreement between theoretical calculation and simulation. Again, there are no unwanted scatterings, and all the incident power is directed to the output field, showing the effectiveness of the proposed metasurface design.

![Figure 4.3](image)

**Figure 4.3:** Design and performance of the beam splitter with equal splitting angles. (a) Comparison between the impedance matrix requirements along the metasurface and realized values from optimization. (b) Theoretical calculated scattered field. (c) Scattered field with 3-layer model. (d) Scattered field with real structure. (e) Mode analysis of the output field.

### 4.1.4 Structure design and verification with simulation and experiments

To realize the desired impedance matrix profile in experiments, we adopted the four-resonator structure described in Li et al. (2018) and Chapter 2. The parameters of the input field and the desired field are kept the same as described in the previous
section. In this case, the periodicity is $\Lambda = 132$ mm. For realization, the metasurface is discretized into 10 unit cells per period, so the width of each unit cell is $w = 13.2$ mm. The wall thickness is $t = 1$ mm, the neck width is $s = 1.5$ mm, the cavity width is $l = 11.25$ mm. The impedance matrix is controlled by tuning the channel width $w_1$, and the height of each cavity $w_a$, $w_b$, $w_c$ and $w_d$. The values of these parameters are determined with two steps. They are first optimized using the Genetic Algorithm (GA). The impedance matrix in GA optimization is analytically calculated and the cost function is defined as

$$\text{cost} = \sqrt{\sum_{i,j=1,2} |Z^{s}_{ij} - Z^{t}_{ij}|^2}. \quad (4.20)$$

where ”s” and ”t” stands for impedance matrix of the four-resonator structure and theoretical requirement, respectively.

In the second step, the parameters obtained from GA is set as the initial values, and are further optimized locally using Pattern Search (PS) algorithm. During the PS optimization, the impedance matrix of a given structure is retrieved with COMSOL simulations. The first step uses analytically calculated values because its computational speed allows a fast and vast amount of random search within a large space, while in the second step, we retrieve the impedances from simulations to guarantee the calculation accuracy. The optimized geometric parameters are given in Table 4.1.

A comparison between the required impedance matrix profile within a period of metasurface and the discretized impedance matrices achieved by structure optimization is given in Fig. 4.3(a). The required impedance profile is closely fulfilled by the optimized structures. The corresponding scattered field with real structure simulation is plotted in Fig. 4.3(d). An excellent agreement can be found between the real structure design, the three-membrane model and the theoretical fields. To quantify
its power efficiency, we calculate the integral of normal intensity on the transmission side and compared the value with the incident intensity. In the simulation, the total transmitted power efficiency reaches 99.94% and only 0.06% of the incident power is reflected. Then we took the complex pressure field along the metasurface on the transmission side and performed Fourier Transform to calculate its far-field radiation pattern. The power scattered into each mode (normal intensity) associated with a certain wavenumber $k_x$ is calculated with

$$I_n(k_x) = \frac{|p(k_x)|^2 \sqrt{1 - (k_x/k_0)^2}}{\sum_{-k_0}^{k_0} |p(k_x)|^2 \sqrt{1 - (k_x/k_0)^2}}$$

and the result is shown in Fig. 4.3(e). On the total transmission side, the power coupled into two desired directions ($k_x/k = \pm \sqrt{3}/2$) are 51.20% and 48.79%, respectively. The slight deviation from an ideal field can be attributed to the finite discretization and the tolerated error in the optimization algorithms.

The sample was then fabricated and tested in a two-dimensional waveguide. The sample was fabricated with Stereolithography (SLA) 3D printing. The fabricated sample and experimental setup are shown in Fig. 4.4(a). In the experiment, the speaker array sends a pulse, which is Gaussian modulated in both space and time,
normally to the metasurface. The signal is recorded by a moving microphone, and the field is mapped by scanning the region of interest. The steady state field is mapped by performing the Fourier transform to the time-gated signal at each position and taking the frequency component of interest.

The simulation and the corresponding experimentally measured fields are shown in Fig. 4.4(b), (c). The fields in the experiment showed good agreement with simulations. In the experiment, the surface wave decaying away from the metasurface can be observed. The far-field radiation was calculated by analyzing the fields along the transmission side of the metasurface. In the experiment, the peak of the output radiation reaches its maximum at $-59^\circ$ and $58^\circ$, with amplitude ratio 0.9256, closely following the design. Several sources for the small discrepancy are: the fabrication error and loss in the air, sound speed change due to the temperature and humidity variation in the environment.

### 4.1.5 Arbitrary beam splitting metasurface design

Then we investigate a metasurface that splits the incident wave into two different splitting angles and arbitrary power ratio. As an example, two transmitted angles are chosen as $\theta_1 = 36.87^\circ$, $\theta_2 = 64.16^\circ$, $\phi_1 = \phi_2 = 3\pi/4$, so the periodicity of the metasurface is $\Lambda = 2\pi/(3k/10) = 381.1$ mm. The reason for choosing these two angles is to make the periodicity not infinite, so the realization is simpler. According to the power conservation condition $t_1^2 \cos \theta_1 + t_2^2 \cos \theta_2 = 1$, we can realize any power distribution between two transmitted waves. In this case, we assume $t_1^2/t_1^2 = 2$, then we can obtain the amplitude of the transmission coefficients are $t_1 = 0.99$, $t_2 = 0.70$, which means the metasurface can theoretically send 78.59% and 21.41% of the incident power into $\theta_1$ and $\theta_2$ directions, respectively. From the above theoretical analysis, $a = 0.43$ and $\alpha = 1.118k$ in this design.

With the pressure and normal velocity field on both sides of the metasurface, the
impedance matrix profile within a period can be calculated and shown in Fig. 4.5(a). It can be observed $Z_{12} = Z_{21}$ with balanced local power, which confirms the reciprocity of the unit cells. And bianisotropic unit cells are needed because of $Z_{11} \neq Z_{22}$. For physical realization, the metasurface is discretized into 30 unit cells per period, so the width of each unit cell is $w = 12.7$ mm. The scattered fields in theory and in simulation with 3-latel model are plotted in Fig. 4.5(b) and (c), respectively. It can

**Figure 4.4:** (a) Sample and experiment setup. (b) Simulated field illuminated by Gaussian beam. (c) Measured field illuminated by Gaussian beam.
be observed that excellent agreement between theoretical calculation and numerical simulation, and no unwanted scatterings distribute in the incident region.

Figure 4.5: Design and performance of the beam splitter with different splitting angles and arbitrary power flow ratio. (a) Comparison between the impedance matrix requirements along the metasurface and realized values from optimization. (b) Theoretical calculated scattered field. (c) Scattered field with the 3-layer model. (d) Scattered field with real structure. (e) Mode analysis of the output field.

Four-resonator structure is also adopted to realize the desired impedance matrix profile. The impedance matrix is controlled by tuning the channel width \(w_1\), and the height of each cavity \(w_a, w_b, w_c\) and \(w_d\). These parameters are determined with GA optimization and PS optimization, the optimized geometric parameters are given in Table 4.2. Figure 4.5(a) shows the required impedance matrix profile and the discretized impedance matrices achieved by structure optimization, which indicates the required impedance profile is closely fulfilled by the optimized structures. The scattered field with real structure simulation is shown in Fig. 4.5(d). It can be found an excellent agreement between the real structure design, the three-membrane model, and the theoretical fields. The slight reflection in simulation with real structure can be attributed to the finite discretization and the tolerated error in the optimization.
algorithms. Then we performed Fourier Transform of the complex pressure field along the metasurface on the transmission side and calculate the power scattered into each mode associated with a certain $k_x$, the result is shown in Fig. 4.5(e). On the transmission side, the power coupled into two desired directions are 76.74% and 21.29%, respectively. The total transmitted power efficiency reaches 98.03%, only 1.97% of the incident power is scattered into unwanted modes.

4.2 Reflection case: redirecting the sound

4.2.1 Designing the field with balanced power

In this case, we aim at fully coupling a plane wave with an angle of incidence $\theta_i$ to a plane wave reflected towards $\theta_r$ with 100% power efficiency, as illustrated in Fig. 4.6. Denote the reflection coefficient as $R = r e^{j\phi}$. The pressure field below the metasurface can be thus written as

$$p_1 = p_0 e^{-jk \sin \theta_i x} e^{-jk \cos \theta_i y} + R p_0 e^{-jk \sin \theta_r x} e^{jk \cos \theta_r y}. \quad (4.22)$$

The normal power conservation requires $r = \sqrt{\cos \theta_i / \cos \theta_r}$. The pressure and normal velocity field at the position of the metasurface ($y = 0$) writes:

$$p_1 = p_0 e^{-jk \sin \theta_i x} + r e^{j\phi} p_0 e^{-jk \sin \theta_i x}, \quad (4.23)$$

$$v_{1,y} = \frac{p_0 \cos \theta_i}{Z_0} e^{-jk \sin \theta_i x} - r e^{j\phi} \frac{p_0 \cos \theta_r}{Z_0} e^{-jk \sin \theta_r x}. \quad (4.24)$$

The intensity field ($I_y = \frac{1}{2} \text{Re}[p v_y^*]$) along the metasurface is calculated as

$$I_{1,y} = \frac{|p_0|^2 r (\cos \theta_i - \cos \theta_r)}{2Z_0} \cos[k (\sin \theta_i - \sin \theta_r) x + \phi]. \quad (4.25)$$

We can see that such an intensity profile requires an exchange of energy between regions of the metasurface. Now we can introduce auxiliary evanescent fields on the transmission side of the metasurface. The interference of the surface waves may
Table 4.2: Design parameters of the individual resonators of the scattering-free bianisotropic metasurface to split a normal incident into two waves with transmitted angles $\theta_1 = 36.87^\circ$, $\theta_2 = 64.16^\circ$ and transmission coefficients $t_1 = 0.99$, $t_2 = 0.70$, implemented with 30 cells within one period.

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provide the mechanism of energy redistribution, but they won’t radiate to the far field. Therefore, they will not carry away the power of the incident wave. The
Figure 4.6: Illustration of the reflective meatsurface under study. A plane incident wave is ideally reflected to an angle \( \theta_r \) by creating surface waves on the transmission side.

The pressure field on the transmission side is defined as

\[
p_2 = p_{1s}e^{-\alpha_1 y}e^{-jk_1 x} + p_{2s}e^{-\alpha_2 y}e^{-jk_2 x},
\]

where \( k_1 = \sqrt{k^2 + \alpha_1^2} \), \( k_2 = \sqrt{k^2 + \alpha_2^2} \), \( p_{1s} = a_1e^{j\beta_1}p_0 \), \( p_{2s} = a_2e^{j\beta_2}p_0 \). Thus

\[
p_2 = a_1p_0e^{-\alpha_1 y}e^{j(\beta_1 - k_1 x)} + a_2p_0e^{-\alpha_2 y}e^{j(\beta_2 - k_2 x)}. \tag{4.27}
\]

The normal velocity field is

\[
v_{2,y} = -\frac{ja_1\alpha_1 p_0}{kZ_0}e^{-\alpha_1 y}e^{j(\beta_1 - k_1 x)} - \frac{ja_2\alpha_2 p_0}{kZ_0}e^{-\alpha_2 y}e^{j(\beta_2 - k_2 x)}. \tag{4.28}
\]

The pressure and normal velocity field at the metasurface \((y = 0)\) is thus

\[
p_{20} = a_1p_0e^{j(\beta_1 - k_1 x)} + a_2p_0e^{j(\beta_2 - k_2 x)}, \tag{4.29}
\]

\[
v_{2,y0} = -\frac{ja_1\alpha_1 p_0}{kZ_0}e^{j(\beta_1 - k_1 x)} - \frac{ja_2\alpha_2 p_0}{kZ_0}e^{j(\beta_2 - k_2 x)}. \tag{4.30}
\]

The normal intensity field writes

\[
I_{2,y} = \frac{|p_0|^2a_1a_2(\alpha_2 - \alpha_1)}{2kZ_0} \cos[(k_1 - k_2)x + \beta_2 - \beta_1 + \frac{\pi}{2}]. \tag{4.31}
\]
Compare Eq. (4.31) with Eq. (4.25), we can see that, in order for them to be matched, we will need:

\[
\beta_2 - \beta_1 + \frac{\pi}{2} = \phi, \tag{4.32}
\]

\[
k_1 - k_2 = k(\sin \theta_i - \sin \theta_r), \tag{4.33}
\]

\[
a_1a_2 = \frac{kr(\cos \theta_i - \cos \theta_r)}{\alpha_2 - \alpha_1}, \tag{4.34}
\]

where \(a_1, \alpha_1, \beta_1, k_1, \) and \(\phi\) can be arbitrary.

Once the local power conservation condition is met, we can design the bianisotropic metasurface with purely passive structures.

4.2.2 Verification with simulation

To verify this scheme, we follow the procedure outlined in the last chapter to design a perfect reflector that couples a normal plane incident wave \(\theta_i = 0\) to a plane wave reflected with angle \(\theta_r = 70^\circ\). Figure 4.7(a) shows the impedance matrix profile within a period \(\Lambda = 2\pi/(k(\sin \theta_r - \sin \theta_i))\). In this case, we set \(|p_0| = 1, |p_{1s}| = 1, \phi = \pi/2\) and \(k_1 = 3k(\sin \theta_r - \sin \theta_i)\). From the figure, we can see that \(Z_{12} = Z_{21}\), confirming the designed field satisfies the local power requirements. The fact that \(Z_{11} \neq Z_{22}\) indicates such a metasurface needs bianisotropic unit cells.

The scheme is verified with simulation in COMSOL. The three-membrane approach is adopted to realize the designed impedance matrix profile. The distance between adjacent membranes is set as 5mm. The calculated impedances for the three membranes are plotted in Fig. 4.7(b). In realization, the metasurface is discretized into \(n = 60\) unit cells per period. The scattered field from the theoretical calculation and the simulation are plotted in Fig. 4.7(c) and (d). From the figures, we can see excellent agreement between theoretical calculation and simulation. There are no unwanted scatterings, showing the effectiveness of the proposed metasurface design.

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4.3 Discussion and future work

With the help of evanescent waves, local power balance on both sides of the metasurface can be achieved. In this chapter, we discussed the power of such a scheme in both reflection-type and transmission-type metasurfaces. We used the perfect reflection case and beam splitting case to demonstrate the highly efficient wavefront transformation with bianisotropic metasurfaces. The power efficiency for such metasurfaces can reach 100% theoretically.

Another interesting and counterintuitive finding is that conventionally while designing reflection type metasurfaces, we typically use hard walls, PEC or PMC to eliminate transmission, whereas, for transmission type metasurfaces, we seek for unit cells withs maximized transmission coefficients. However, in this chapter, we can find that, in order for the metasurface to achieve high power efficiency, sometimes its better to allow controlled transmission for the reflection-type metasurface.
and engineered reflection for the transmission-type metasurfaces.

However, there are a few limitations to such a scheme. First, it can be easily adapted for simple and periodic wavefront transformations, while for non-periodic cases, such as focusing or even holograms, the surface waves that carry energy locally can be complicated which is not straightforward to design. Second, the wavenumber for the surface waves are typically large, so the impedance profile can fluctuate dramatically with space, as we can see in the perfect reflection case. It posts challenges in experimental realization. One possible solution to this situation will be discussed in the next chapter.
Highly-efficient wavefront transformation with power-flow conformal bianisotropic metasurface

In the previous chapter, we have demonstrated near-perfect wavefront transformation strategy for meeting the requirement of local power conservation by designing surface waves along the metasurface, and verified the idea with two representative cases: perfect reflection and beam splitting. However, designing surface wave suffers from several limitations. For example, it is not straightforward to apply the idea for complicated fields; surface waves typically require fine discretization of the metasurface, posting challenges in fabrication; surface waves are bond tightly to the metasurface, so they tend to be more sensitive to the losses within the structures.

Recently, in our collaborative work with Aalto University Díaz-Rubio et al. (2019), we have demonstrated that in reflective metasurfaces, the local power constraint can be resolved by designing a so-called "power flow conformal metasurface". From previous discussions, we understand that the limitations for power efficiency of a metasurface comes from two parts: (i) Mismatch of the power flow for the input field and desired field, and (ii) mismatch of the wave impedances when there are
waves on both sides of the metasurfaces. In power flow conformal metasurfaces, the shape of the metasurface is designed according to the intensity field, so that the power flow normal to the metasurface is always zero. In this case, the metasurface neither absorbs nor emits the energy and thus can be realized with passive structures.

This strategy inspires us to ask: For an arbitrary field, can we take advantage of power-flow conformal metasurface and bianisotropic metasurface so that the local power balance and wave impedance matching are fulfilled simultaneously? Can we provide an "ultimate" solution to theoretically perfect wavefront transformation for arbitrary fields?

In this chapter, we will take a step forward and try to provide one possible solution by designing power flow conformal bianisotropic metasurfaces in the 2D case. We start with the designing principle for an arbitrary field and then apply it to a simple case of near-field focusing. The designed metasurface is verified with both simulation and experiment.

5.1  Design principles for the power-flow conformal bianisotropic meta-surface

As discussed in the previous sections, there are two requirements for building metasurfaces for perfect wavefront transformation: 1. local power conservation and 2. local impedance matching. Once the first condition is met, the second one can be realized using bianisotropic unit cells. Therefore, we will first determine the shape of the metasurface, then calculate the required impedance matrix profile along the metasurface, and finally design structures to meet these requirements.

5.1.1  Determining the shape of the metasurface

We start with the general case where the incident sound pressure field $p_i$ is transformed to the transmitted field $p_t$. For simplicity, we consider time-harmonic waves
in the 2D scenario so that
\[ p_i = p_i(\vec{r})e^{j\omega t}, \]
\[ p_t = p_t(\vec{r})e^{j\omega t}. \]  
(5.1)

where \( \vec{r} = x \cdot \hat{x} + z \cdot \hat{z} \), and \( \omega \) is the angular frequency of the sound. We would drop the time-harmonic term \( e^{j\omega t} \) in the following derivations for conciseness. Using the Newton’s second Law in 2D: \( \rho \frac{\partial \vec{v}}{\partial t} + \nabla p = 0 \), where \( \nabla p = \frac{\partial p}{\partial x} + \frac{\partial p}{\partial z} \), the particle velocity of both the incident and transmitted sound fields can be written as \( \vec{v} = \hat{x} \cdot v_x + \hat{z} \cdot v_z \).

Once the required sound pressure field and particle velocity field are known, the corresponding sound intensity field can be calculated by \( \vec{I} = \frac{1}{2} \text{Re}(p \cdot \vec{v}^*). \)

With the intensity field, we try to find the optimal geometric shape of the metasurface to ensure that the metasurface is power flow-conformal. We assume an infinitely thin acoustic metasurface located at the curve \( z_s = z_s(x) \). If the normal component of the incident sound intensity vector and the normal component of the transmitted sound intensity vector are not the same on both sides of the metasurface, i.e. \( \vec{I}_i(x, z_s) \cdot \hat{n}(x, z_s) \neq \vec{I}_t(x, z_s) \cdot \hat{n}(x, z_s) \), then there must be acoustic energy absorbed or emitted by the metasurface locally, corresponding to loss or gain in the unit cells. Therefore, to design a passive and lossless metasurface, the following condition shall be met:
\[ \vec{I}_i(x, z_s) \cdot \hat{n}(x, z_s) = \vec{I}_t(x, z_s) \cdot \hat{n}(x, z_s). \]  
(5.2)

We define the residual sound intensity field as:
\[ \Delta \vec{I} = \vec{I}_t - \vec{I}_i = \hat{x} \cdot \Delta I_x + \hat{z} \cdot \Delta I_z. \]  
(5.3)

Thus the problem becomes how to find a curve such that the \( \Delta \vec{I} \) field normal to the curve is always zero. We also define an auxiliary vector field and its corresponding scalar potential function as:
\[ \vec{N} = -\hat{x} \cdot \Delta I_x + \hat{z} \cdot \Delta I_z. \]  
(5.4)
It can be proven that if the fields on both sides of the metasurface are source-free, i.e. \( \nabla \cdot I_1 = \nabla \cdot I_2 = 0 \), then we have \( \nabla \times \vec{N} = 0 \), so that \( \vec{N} \) can be written as the gradient of a scalar field,

\[
\nabla g = -\vec{N}
\]  
(5.5)

It can be seen that \( \vec{N} \) is tangential to \( \Delta \vec{I} \). Suppose \( z_g = z_g(x) \) is one of the level curves of the scalar potential function \( g (g[x, z_g(x)] = \text{Const.}) \), then the normal vector along \( z_g \) must be parallel to the \( \vec{N} \) vector, i.e., \( \vec{N}(x, z_g)/|\vec{n}(x, z_g)\). As a result, the power flow-conformal condition \( \Delta \vec{I}(x, z_g) \cdot \vec{n}(x, z_g) = 0 \) is satisfied. In short, the geometric profile of the metasurface should be chosen from the set of \( z_g \), i.e. \( z_s \in \{ z_{g1}, z_{g2}, z_{g3}, \ldots \} \). After choosing one specific \( z_s = z_g|_{z_g(0)=z_0} \), the shape of the metasurface can determined.

5.1.2 Impedance requirement and realization of the metasurface

Once we have \( z_s(x) \) that defines the shape of the metasurface, both the incident field and the transmitted field are determined everywhere on the \( x - z \) plane. We can then derive the impedance matrix profile requirements along the metasurface with the field distribution on both sides. The local response of the acoustic metasurface can be characterized by a 2 \( \times \) 2 surface impedance matrix \( Z = Z(x, z_s) \), whose elements \( Z_{11}, Z_{12}, Z_{21} \) and \( Z_{22} \) are defined by:

\[
\begin{bmatrix}
p_t(x, z_s) \\
p_t(x, z_s)
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
\hat{n}(x, z_s) \cdot \vec{v}_i(x, z_s) \\
\hat{n}(x, z_s) \cdot \vec{v}_t(x, z_s)
\end{bmatrix}.
\]  
(5.6)

Since the local power conservation condition is met, the metasurface can be designed with lossless and passive structures, thus the impedance matrix is purely imaginary, i.e., \( Z_{mn} = jX_{mn} \). We can then re-write the impedance matrix in the
following form so that each element can be determined with the field distribution:

\[
\begin{bmatrix}
\text{Re}(p_i) & \text{Im}(p_i) \\
\text{Re}(p_t) & \text{Im}(p_t)
\end{bmatrix} =
\begin{bmatrix}
X_{11} & X_{12} \\
X_{21} & X_{22}
\end{bmatrix}
\begin{bmatrix}
-\text{Im}(\hat{n} \cdot \vec{v}_i) & \text{Re}(\hat{n} \cdot \vec{v}_i) \\
\text{Im}(\hat{n} \cdot \vec{v}_i) & -\text{Re}(\hat{n} \cdot \vec{v}_i)
\end{bmatrix}.
\quad (5.7)
\]

As discussed in the previous chapters, once the impedance matrix profile is determined, the unit cells can be designed using the three-layer approach or multiple resonator approach.

5.2 A demonstration case: near field focusing

5.2.1 Determining the shape of the metasurface

As an example, a power flow-conformal bianisotropic metasurface is designed for near-field sound focusing. As shown in FIG. 5.1(a), plane incident sound \( p_i = A_i \exp(-jkz) \) is focused to a single point at \((0,0)\) after transmitting through the metasurface. The transmitted sound can be written as \( p_t = A_t H_0^{(1)}(kr) \) accordingly, where \( H_0^{(1)}(x) \) is the zero-th order Hankel function of the first kind. Here \( k = \omega/c \) is the wavenumber in air, \( f = 3000 \text{ Hz} \) is the designed frequency, \( c = 343 \text{ m/s} \) is the sound speed in air. The wavelength is thus \( \lambda = 0.1143m \). \( A_i \) and \( A_t \) are the amplitudes of the incident and transmitted waves, respectively. The particle velocity fields of the incident and transmitted waves are:

\[
\begin{align*}
\begin{cases}
v_{x,i} = 0 \\
v_{z,i} = A_i \exp(-jkz)/Z_0.
\end{cases}
\end{align*}
\quad (5.8)
\]

and

\[
\begin{align*}
\begin{cases}
v_{x,t} = -jA_t H_1^{(1)}(kr) \frac{x}{Z_0 r} \\
v_{z,t} = -jA_t H_1^{(1)}(kr) \frac{z}{Z_0 r}.
\end{cases}
\end{align*}
\quad (5.9)
\]
Here $Z_0 = \rho c$ is the characteristic acoustic impedance of the air, $\rho = 1.225$ kg/m$^3$ is the density of the air at room temperature. $H_1^{(1)}(x)$ is the first order Hankel function of the first kind. The focal length is judiciously chosen as $f = 0.106$ m ($f = 0.93\lambda$). Since the field distribution is symmetric, we expect the metasurface to be also symmetric about the $z$ axis. Therefore, we rescaled the amplitude of the incident wave such that $\Delta \vec{I}(0, -f) = (0, 0)$. The calculated amplitude of the incident and the transmitted field are $A_i = 0.3307$ and $A_t = 1$, respectively. Following the methodology, as mentioned in the previous section, the shape of the power flow-conformal bianisotropic metasurface can be numerically determined. The scalar potential field $g(x, z)$ and the metasurface geometry $z_s = z_s(x)$ is shown in FIG. 5.2. It is worth noting that although we can choose any of the level curves of $g$ in theory, in this case, we selected the only one that is continuous and symmetric about the $z$ axis.
Figure 5.2: Potential field $g(x, z)$ and the shape of metasurface.

5.2.2 Realization of the metasurface

The metasurface is then discretized into 19 unit cells spanning an aperture of 0.31 m. In order to find the optimal geometric parameters for the bianisotropic unit cells, we first run the genetic algorithm (GA) to derive the approximate values of all the parameters with randomized initial population, then use the pattern search (PS) algorithm to find the exact value of each parameter. In GA optimization, the impedance matrix of the structure is calculated analytically for computational efficiency, while in PS optimization, the impedance matrix of a structure is retrieved in COMSOL simulations for accuracy. The method for analytical calculation and parameter retrieval in simulations are outlined in previous sections (or see Li et al. (2018)). Since the sound field and the metasurface is symmetric, we only run the optimization for the first 10 unit cells. For simplicity, only the depths of the cavities of the 4 Helmholtz resonators $w_a, w_b, w_c, w_d$ and the channel width $w_1$ are optimized
Table 5.1: Geometric parameters of the power flow-conformal metasurface

<table>
<thead>
<tr>
<th>n</th>
<th>$x_n$/mm</th>
<th>$z_n$/mm</th>
<th>$w_{1,n}$/mm</th>
<th>$w_{a,n}$/mm</th>
<th>$w_{b,n}$/mm</th>
<th>$w_{c,n}$/mm</th>
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</tr>
<tr>
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<td>7.79</td>
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<td>8.92</td>
<td>9.47</td>
</tr>
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</table>

for, while other geometric parameters are fixed for all unit cells. In our design, $w = 16$ mm, $w_2 = 1$ mm, $h = 50$ mm, $h_1 = 1$ mm, $h_2 = 1.5$ mm, $h_3 = (h - 5h_1)/4 = 11.25$ mm. Other parameters could be found in TABLE 5.1. Here $n$ is the number of the unit cell, $x_n$ and $z_n$ are the $x$ and $z$ coordinates of the center of the $n$th unit cell, respectively.

5.2.3 Verification with Simulation and Experiments

We use the Pressure Acoustic module in COMSOL Multiphysics 5.4 to perform the numerical simulation with our power flow-conformal metasurface design. The sound intensity amplitude $|\vec{I}|$ is shown in FIG. 5.3(c). For comparison, we have also drawn the sound intensity map of the GSL based metasurface in FIG. 5.3(a) and the sound intensity map of the phase shifter metasurface designed with synthetic field distribution method Estakhri and Alù (2016b) in FIG. 5.3(b). We can find that our power flow-conformal metasurface can better localize the sound intensity around the focal point $(0, 0)$ compared with the other two metasurface designs. In addition, as is shown in FIG. ??, the power flow-conformal metasurface has higher power transmittance than the other two designs. We calculate the power transmittance of all three metasurfaces by integrating the sound intensity amplitudes along the $x$ axis.
Figure 5.3: Performance of the conformal bianisotropic metasurface. (a) The field generated with phase gradient metasurface. (b) The metasurface designed with synthetic field method. (c) The intensity field of power-flow conformal metasurface, (d) Experimentally measured intensity field.
Table 5.2: Comparison of the power transmittance.

<table>
<thead>
<tr>
<th>Metasurface type</th>
<th>[-0.15, 0.15] m</th>
<th>[-0.05, 0.05] m</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSL</td>
<td>60.8%</td>
<td>27.8%</td>
</tr>
<tr>
<td>synthetic field</td>
<td>95.4%</td>
<td>76.0%</td>
</tr>
<tr>
<td>power flow-conformal</td>
<td>97.4%</td>
<td>85.0%</td>
</tr>
</tbody>
</table>

Both the large region \((x \in [-0.15, 0.15] \text{ m})\) and the small region \((x \in [-0.05, 0.05] \text{ m})\) results are shown in TABLE 5.2.

For the experiments, we 3D printed the proposed metasurface and tested it in a 2D waveguide. The experimental setup is shown in FIG. 5.1(c). We used a line array of loudspeakers as the sound source. To generate a plane incident wave, all speakers are calibrated so that their output sound pressure amplitudes are the same. A microphone scans with a step size of 5mm across the interested region. Sound absorbing foams are placed around the edges to prevent reflection. We sent a Gaussian pulse centered at 3000Hz to drive the speakers and time-gate the measured signal to eliminate reflection from the boundaries. Fourier transform was performed to extract the frequency component at 3000Hz, then the spatial difference was used to calculate the corresponding sound intensity map. The intensity field in experiments are plotted in FIG. 5.3(d), showing excellent agreement with the simulation. It can be seen that the acoustic energy is focused at the desired location as designed, and parasitic diffractions are greatly suppressed.

5.3 Discussion and future work

In this chapter, we showed the combination of power-flow conformal metasurface and bianisotropic metasurfaces. Taking advantage of the merits of both ideas, a highly efficient wavefront transformation of complicated fields can be achieved with passive structures. This approach can be readily applied to generating more complicated fields. In principle, it provides a solution to perfectly transforming arbitrarily de-
fined wavefronts. In realization, the deviation from perfect wavefront transformation comes from the finite aperture size, discretization, optimization allowed numerical errors and fabrication errors. However, they still outperform the metasurfaces via other approaches since they are theoretically perfect.

For future works, one obvious but non-trivial step is to design perfect metasurface in three dimensions. On this track, we envision building near-perfect holograms without parasitic scatterings. On another hand, we would like to note that there are other approaches to overcome the power constraints in metasurfaces. For example, non-local metasurfaces which engineers power exchange within adjacent unit cells has recently been proposed Quan and Alù (2019). However, they contain extremely long and thin channels, which can subject to high losses, and they are not demonstrated in experiments yet. Another interesting direction to investigate nonreciprocity in unit cells. However, non-reciprocal effects require either strong nonlinearity or breaking the time-reversal symmetry using active elements modulated in both space and time, adding extreme difficulties to experiments. We have done some theoretical and experimental work in this direction, but they are not included in this dissertation. Readers who are interested in these works please see Li et al. (2019a); Shen et al. (2019); Li et al. (2019b); Zhu et al. (2020).
Bianisotropic metasurface for the broad scope of impedance matching

Impedance matching plays an important role in almost all the engineering problems where any form of waves are involved. In the field of electronics, optics, mechanics, and acoustics, impedance matching serves to maximize the power transfer or minimize signal reflection from the load. In optics and acoustics, impedance matching is fulfilled by the quarter-wavelength layer. However, there are limitations to such matching layers. First, the thickness of the matching layer has to be a quarter wavelength. For low-frequency waves, the wavelength is typically large. The matching layer has to be bulky. Second, it requires specific characteristic impedance for the matching layer, while in many cases, such material is not easy to find in nature. Although the material constraint can be mitigated by employing multiple matching layers, the size of these layers will add up and make it even thicker. Third, in some cases, the load impedance can be complex. Designing quarter layers in these cases gets complicated.

From the previous chapters, we can see that bianisotropic metasurfaces are highly-
efficient in wavefront transformation. This is because bianisotropy plays the role of matching the wave impedances on both sides of the metasurface. It is thus natural to ask the question: can we apply the bianisotropic metasurface in the broader scope of impedance matching problems, and provide better control of the wave?

In this chapter, I will generalize the previous derivation and derive the requirement for a metasurface to match an arbitrary impedance. As will be shown, impedance matching with bianisotropic metasurfaces provides many advantages over the traditional methods. First, it doesn’t set a limitation to the thickness of the metasurface. In theory, such a layer can be infinitesimally thin. Second, in addition to maximizing the transmitted power, it also offers control over the transmission phase. Third, it covers any complex impedances, making it fit a broader range of loads.

6.1 Bianisotropic impedance matching for an arbitrary surface

6.1.1 Theoretical formulation

Consider a one-dimensional case where a plane wave traveling in the medium I with impedance $Z_1$ hits a surface with arbitrary impedance $Z_2$. This impedance can be the surface impedance of a structure or a semi-infinite medium. An arbitrary surface impedance is equivalent to a semi-infinite medium with characteristic impedance $Z_2$. Therefore, this case is equivalent to the two-media case shown in Fig. 6.1, where sound incidents from the medium I and transmits into medium II.

The incident and transmitted waves have the complex amplitude $p_0$ and $p_1 =$
$T p_0$, where $T = |T| e^{j\phi}$ is the complex transmission coefficient and $\phi$ represents an arbitrary transmission phase. The characteristic impedance in medium I $Z_1$ and arbitrary impedance can be written as $Z_1 = |Z_1| e^{j\beta_1}$, $Z_2 = |Z_2| e^{j\beta_2}$. The velocity on both incident and transmission side are $v_0 = p_0/Z_1$ and $v_1 = T p_1/Z_2$.

The condition for full power transmission writes:

$$\frac{1}{2} \text{Re}[p_0 v_0^*] = \frac{1}{2} \text{Re}[p_1 v_1^*]$$

which yields the following requirement for the transmission coefficient

$$|T| = \sqrt{\frac{|Z_2|\cos \beta_1}{|Z_1|\cos \beta_2}}$$

(6.2)

The fields on both sides of the metasurface can be related with the transfer matrix $M$

$$\begin{bmatrix} p_0 \\ v_0 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ v_1 \end{bmatrix}.$$  

(6.3)

In the passive lossless case, $M_{11}$ and $M_{22}$ are real while $M_{12}$ and $M_{21}$ are imaginary. Put the prescribed input and output pressure and velocity field into Eq. 6.3, and solve the equation by equating the real and imaginary part respectively, we get the required transfer matrix for the metasurface, written as:

$$M = \begin{bmatrix} \frac{1}{|T|} \cos(\phi - \beta_2) & -i \frac{|Z_2| \sin \phi}{|Z_1||T| \cos \beta_1} \\ -i \frac{|Z_1| \sin(\phi + \beta_1 - \beta_2)}{|Z_1||T| \cos \beta_2} & \frac{1}{|T|} \cos \beta_2 \end{bmatrix}.$$  

(6.4)

The corresponding impedance matrix writes

$$Z = \begin{bmatrix} i|Z_1| \frac{\cos(\phi - \beta_2)}{\sin(\phi + \beta_1 - \beta_2)} & i|T||Z_1| \frac{\cos \beta_2}{\sin(\phi + \beta_1 - \beta_2)} \\ i|T||Z_1| \frac{1}{\sin(\phi + \beta_1 - \beta_2)} & i|Z_2| \frac{\cos \beta_1}{\sin(\phi + \beta_1 - \beta_2)} \end{bmatrix}.$$  

(6.5)

For any desired transmission phase, the impedance can be fulfilled by the three-layer or four-layer model using monopole or dipole resonators, as discussed in the previous chapters.
6.1.2 Sensitivity to errors in impedance matrix

Now we analyze the robustness of this method by adding random perturbation to the impedance matrix of the metasurface. We generate three random numbers within ±10: $e_1$, $e_2$, $e_3$. Then perturb the impedance matrix by:

$$Z_p = \begin{bmatrix} Z_{11}(1 + e_1) & Z_{12}(1 + e_2) \\ Z_{21}(1 + e_2) & Z_{22}(1 + e_3) \end{bmatrix}.$$  \hspace{1cm} (6.6)

Here the subscript ”p” denote ”perturbed”. The scattering matrix can then be calculated using the perturbed impedance matrix. Particularly, for any arbitrary impedance matrix $Z$, the reflection coefficient is

$$S_{11} = \frac{(Z_{11} - Z_1)(Z_{22} + Z_2) - Z_{12}Z_{21}}{(Z_{11} + Z_1)(Z_{22} + Z_2) - Z_{12}Z_{21}},$$  \hspace{1cm} (6.7)

so that the power transmission coefficient can be calculated as

$$T_p = 1 - |S_{11}|^2.$$  \hspace{1cm} (6.8)

![Figure 6.2: Comparison of the power transmission coefficient between the defected metasurface and the case without metasurface.](image)

Figure 6.2 shows a comparison of the power transmission coefficient between the defected metasurface and the case without metasurface. In this case, we set $Z_1$
and $Z_2$ both be real, and transmission phase $\phi = \pi/8$. We can see that without the metasurface, the transmitted power gets low as the impedance contrast ratio ($|Z_2|/|Z_1|$) is increased due to the impedance mismatch between two media. With the help of the metasurface, even the metasurface is defected within an error of $\pm 10\%$, over 90% of the power will be transmitted, even when the impedance contrast is high. Note here that as the contrast ratio gets higher, the values in the required impedance matrix will get more extreme, which sets a barrier for experimental realization.

6.2 Transmission enhancement through a middle layer

In this section, we will look at a specific case where $Z_2$ is the consequence of a medium with a middle layer. This scenario is of interest to many engineering problems. For example, listening to the sound behind a door, preventing light scattering in our glasses, or enhancing Wi-Fi signal in the neighboring room, etc.

6.2.1 Theoretical requirement

Consider a one-dimensional case shown in Fig. 6.3, where sound incidents from medium I ($Z_1 = \rho_1 c_1$) and then hit a middle layer (medium II, $Z_2 = \rho_2 c_2$) of thickness $d$ before transmitting into medium III ($Z_3 = \rho_3 c_3$). Here we assume $Z_1$ and $Z_3$ are both real. In this case, we can directly use Eq. 6.5 by substituting the arbitrary impedance with the form of impedance transfer formula:

$$Z = Z_2 \frac{Z_3 + jZ_2 \tan kl}{Z_2 + jZ_3 \tan kl}.$$  \hspace{1cm} (6.9)

Alternatively, we can first calculate the total transfer matrix for the metasurface and the middle layer using Eq. 6.4. In this case, we have $|T| = \sqrt{Z_3/Z_1}$ and
\( \beta_1 = \beta_2 = 0 \). The total transfer matrix writes:

\[
M = \begin{bmatrix}
\sqrt{\frac{Z_1}{Z_3}} \cos \phi & -i\sqrt{\frac{Z_1 Z_3}{Z_1}} \sin \phi \\
-i\sqrt{\frac{1}{Z_1 Z_3}} \sin \phi & \sqrt{\frac{Z_1}{Z_3}} \cos \phi
\end{bmatrix}.
\]  
(6.10)

The total transfer matrix can also be written as the product of transfer matrices for each part \( M = M_s M_m \) where \( M_m \) represents the middle layer, which can be written directly

\[
M_m = \begin{bmatrix}
\cos(k_2d) & iZ_2 \sin(k_2d) \\
i\frac{1}{Z_2} \sin(k_2d) & \cos(k_2d)
\end{bmatrix}
\]  
(6.11)

Hence we get the theoretical requirement for the metasurface as \( M_s = M M_m^{-1} \). An explicit form reads:

\[
M_s(1, 1) = \sqrt{\frac{Z_1}{Z_3}} \cos \phi \cos k_2d - \frac{1}{\sqrt{Z_1 Z_3}} \sin \phi \sin k_2d
\]  
(6.12)

\[
M_s(1, 2) = -i(Z_2) \sqrt{\frac{Z_1}{Z_3}} \cos \phi \sin k_2d + \sqrt{Z_1 Z_3} \sin \phi \cos k_2d
\]  
(6.13)

\[
M_s(2, 1) = -i\left(\frac{1}{\sqrt{Z_1 Z_3}} \sin \phi \cos k_2d + \frac{1}{Z_2} \sqrt{Z_3} \cos \phi \sin k_2d\right)
\]  
(6.14)

\[
M_s(2, 2) = -\frac{Z_2}{\sqrt{Z_1 Z_3}} \sin \phi \sin k_2d + \sqrt{\frac{Z_3}{Z_1}} \cos \phi \cos k_2d
\]  
(6.15)

By setting \( d = 0 \), it is confirmed that Eq.(6.15) reduces to Eq.(6.10). Conversion
Figure 6.4: Realization of the bianisotropic response in acoustics and the equivalent circuit. (a,b) the series case and the equivalent circuit. (c,d) the parallel case and the equivalent circuit.

From transfer matrix to impedance matrix is given by

\[
Z = \begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix} = \begin{bmatrix}
M_{11}M_{22} - M_{21}M_{12} \\
M_{21}M_{12} - M_{11}M_{22}
\end{bmatrix}.
\]

(6.16)

We can see that \(Z_{11} \neq Z_{22}\), indicating that in order to realize the required transfer matrix, bianisotropic response needs to be incorporated into the metasurface.

6.2.2 Design and realization of the required bianisotropic metasurfaces

As discussed in Chapter 2, an arbitrary bianisotropic impedance matrix can be realized by cascading three components with certain impedance in parallel or in series. In the parallel case, the arbitrary impedance can be achieved with monopole resonators such as Helmholtz resonators, F-P resonators, and Mie-resonators. Whereas in the series case, the arbitrary impedance can be achieved with dipole resonators such as membranes and plates. The realization and their equivalent circuits are shown in Fig. 6.4. The total transfer matrix can be calculated with \(M = M_1M_THM_2M_THM_3\).
where $M_T$ denotes the transmission line between each component, and $M_i$ represents the loads.

**The series approach with membranes**

In this approach, the impedance for each clamped membrane can be modeled as a series $LC$-resonator controlled by its acoustic mass and compliance. Three membranes are divided by a distance of $l$. The transfer matrix of the membrane can be written as

$$M_i = \begin{bmatrix} 1 & Z_i \\ 0 & 1 \end{bmatrix}. \quad (6.17)$$

By equating each element of the total transfer matrix $M$, we can obtain the expressions for the impedance of the three membranes:

$$Z_2 = \frac{2jZ_0 \sin k_0l \cos k_0l - M_{21}Z_0^2}{\sin^2 k_0l} \quad (6.18a)$$

$$Z_1 = \frac{Z_0^2(M_{11} + \sin^2 k_0l - \cos^2 k_0l) - jZ_0Z_2 \sin k_0l \cos k_0l}{2jZ_0 \sin k_0l \cos k_0l - Z_2 \sin^2 k_0l} \quad (6.18b)$$

$$Z_3 = \frac{Z_0^2(M_{22} + \sin^2 k_0l - \cos^2 k_0l) - jZ_0Z_2 \sin k_0l \cos k_0l}{2jZ_0 \sin k_0l \cos k_0l - Z_2 \sin^2 k_0l} \quad (6.18c)$$

where $Z_0$ and $k_0$ are the characteristic impedance and wavenumber of the medium between the membranes.

To show the capability of the proposed approach and validate our calculation, we performed the full-wave simulation with a case where we use the bianisotropic metasurfaces to help the 3000 Hz wave penetrate a 2 mm thick glass wall in air. The metasurface can be designed with arbitrary thickness, and the transmission phase is well-controlled. The components of the impedance matrices for different transmission phase are shown in Fig. 6.5(a), and the corresponding impedance for the three
Figure 6.5: Simulation of the three-layer realization of the proposed metasurface and comparison with the case without metasurface. In this case, we simulated the acoustic wave totally penetrating a 2mm thick glass with controlled phase.

membranes are shown in Fig. 6.5(b). We can see in Fig. (a), \( Z_{11} \neq Z_{22} \), and in Fig. (b), \( Z_{1} \neq Z_{2} \). They both confirm again that for full power transmission through a barrier, the bianisotropic responses are needed. The full-wave simulations for different transmission phases are performed with the acoustics module in COMSOL Multiphysics, and the results are shown in Fig. (c). The transmission coefficients are all above 99.9%, while without the metasurface, less than 0.01% of the power is transmitted.

We would like to note here that this example is used to illustrate the capability versatility of the theory. However, as can be seen in Fig. 6.5(b), the required impedance of the membrane can be large, making it extremely difficult to achieve in experiments.

**The parallel approach with side-loaded resonators**

Similar to the series case, the arbitrary bianisotropic response can be realized with three layers of monopole resonators. The transfer matrix for a shunted impedance is

\[
M_i = \begin{bmatrix} 1 & 0 \\ Y_i & 1 \end{bmatrix}.
\]  

(6.19)
where $Y_i = 1/Z_i$ is the admittance of the load. By equating each element in the total transfer matrix, the admittance for these layers can be calculated as

$$Y_2 = \frac{2jZ_0 \sin k_0 l \cos k_0 l - M_{12}}{Z_0^2 \sin^2 k_0 l} \quad (6.20a)$$

$$Y_1 = \frac{M_{22} + \sin^2 k_0 l - \cos^2 k_0 l - jZ_0 Y_2 \sin k_0 l \cos k_0 l}{2jZ_0 \sin k_0 l \cos k_0 l - Z_0^2 Y_2 \sin^2 k_0 l} \quad (6.20b)$$

$$Y_3 = \frac{M_{11} + \sin^2 k_0 l - \cos^2 k_0 l - jZ_0 Y_2 \sin k_0 l \cos k_0 l}{2jZ_0 \sin k_0 l \cos k_0 l - Z_0^2 Y_2 \sin^2 k_0 l} \quad (6.20c)$$

6.2.3 Metasurface design and simulation

Now we need to design the resonators in water to realize the required impedance. Bubbles in liquids are well known for exhibiting strong resonances with deep subwavelength dimensions. Such a feature makes them an excellent candidate to form a thin sheet that can deliver an arbitrary impedance. The resonance frequency for a single bubble with radius $a$ is given by $\omega_M = \sqrt{(3\beta_g + 4\mu)/(\rho a^2)}$ where $\beta_g$ is the bulk modulus of the trapped gas, $\mu$ and $\rho$ are the shear modulus and density of the host medium. Since the resonance is monopole and the size of the bubble is much smaller than the wavelength, we expect them to only cause a discontinuity in velocity and can be modeled using the parallel approach.

Since the bubbles in water are unstable, we chose Polydimethylsiloxane (PDMS) as the material hosting the bubbles. It has several advantages: (1) PDMS is water-like material whose shear modulus is orders of magnitude smaller than its bulk modulus. (2) It is stable both mechanically and chemically, so the structure can last for a long time. (3) The fabrication technique for small structures in PDMS is relatively mature, and the designed structure can be fabricated with soft lithography. To prepare for the design bought Ecoflex 00-30 Silicone and designed experiments to test
its mechanical properties.

The measured material properties are summarized in Table 6.1. We can see that Shear modulus is more than 4 orders of magnitude smaller than Young’s modulus, so we can safely regard it as fluid.

Table 6.1: Material properties of Ecoflex 00-30 Silicone

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>965</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Sound Speed</td>
<td>$c$</td>
<td>1018</td>
<td>m/s</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E$</td>
<td>898.63</td>
<td>MPa</td>
</tr>
<tr>
<td>Shear modulus</td>
<td>$\mu$</td>
<td>22</td>
<td>kPa</td>
</tr>
</tbody>
</table>

The next step is to build the relationship between bubble size and its corresponding impedance. Here we use a case where the bianisotropic metasurface helps 20 kHz wave to penetrate a 10 mm thick steel plate in water. The steel has density $\rho_{\text{steel}} = 7800\text{kg/m}^3$ and sound speed $c_{\text{steel}} = 5790\text{m/s}$. The structure of the metasurface and unit cell is illustrated in Fig.6.6. Here we choose the lattice constant as $w = 2\text{mm}$, air density $\rho_a = 1.21\text{kg/m}^3$, sound speed $c_a = 343\text{m/s}$. By sweeping the bubble size and searching for the eigenfrequency in COMSOL simulation, we found that a spherical bubble with radius $200\mu\text{m}$ has a resonant frequency around 20kHz. Therefore we choose $r_{\text{ref}} = 200\mu\text{m}$ as the reference radius, and represent the radius
Figure 6.7: The transfer matrix of a single layer of bubble in PDMS host medium as a function of bubble radius $a$.

We swept the radius within the range of $a \in [0.1, 2]$, and retrieved the transfer matrix of the layer. The components in the transfer matrix are plotted in Fig. 6.7. We know that in the transfer matrix $M$, $M_{12}$ has the same unit as the impedance, representing the dipole response, while $M_{12}$ has the same unit as the admittance, representing the monopole response. Both monopole and dipole response of such a bubble layer can be read off from Fig. 6.7. (both normalized with the impedance of water $Z_w = 1.45 \times 10^6 \text{Pa} \cdot \text{s/m}$). It is seen that (1) $M_{12}$ is almost 0, indicating the dipole response of the bubble is negligible. This makes sense since the bubble size is much smaller than the wavelength, so its scattering is not significant. (2) the real part of $1/M_{12}$ is always 0, while the imaginary part varies along with the bubble radius. Such a feature confirms that the bubble response is monopolar. As the bubble size gets large, $M_{11}$ and $M_{22}$ deviate from 1, since their scattering effects cannot be neglected anymore, and dipole mode starts to play a role. However, for small bubbles ($a < 1$), the bubbles can be well represented with an impedance layer. Such a figure serves as a library of bubbles so that we can design the metasurface.
with multiple layers of impedance (admittance) sheets, and choose the corresponding bubble for realization.

Again, we adopted the four-layer model and used GA to find the combination of four impedance sheets. The distance between each layer of the bubble is 3 mm. The cost function in GA algorithm is defined in Chapter 2.3.1 (Eq. 2.34). The theoretical requirements of the impedance matrix and optimized cost function is shown in Fig. 6.8. We can see that there are regions with high cost values so that for some transmission phases, the metasurface cannot be designed. This is because the impedance achieved by bubbles can only cover negative values so that the overall achievable impedance matrix is limited. However, in many cases where the phase is not important, or full coverage of the phase spectrum is not necessary, such an
approach helps wave penetration through the aberration layer.

As a showcase, we pick the transmission phase $\phi = 23$ deg. With the optimized impedance values, we used linear interpolation to find the bubble size. The impedances and corresponding bubble radii are summarized in Table 6.2. The performance of the designed metasurface is then verified with numerical simulations using the Pressure Acoustics Module in COMSOL, and the results are shown in Fig. 6.9(a). The transmission amplitude reaches 0.9827 in simulation, indicating 96.57% of the power is transmitted through the steel plate. As a comparison, we also simulated the case without a metasurface, shown in Fig. 6.9(b). The transmission coefficient amplitude is 0.2860 in this case, so 8.18% of the power is transmitted. Here we can see the transmission with the help of metasurface is 10dB higher than the case without metasurface, and the transmitted power is boosted by a factor of 12. To calculate the bandwidth of the designed metasurface, we also swiped the frequency and calculated the amplitude of transmission coefficients, as shown in Fig. 6.9(c). The half-amplitude working bandwidth reaches 2%.

**Table 6.2: Bubble size and impedances of each layer**

<table>
<thead>
<tr>
<th>Number</th>
<th>a</th>
<th>radius (µm)</th>
<th>impedance (Pa · s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5867</td>
<td>117.34</td>
<td>$-4.16 \times 10^3i$</td>
</tr>
<tr>
<td>2</td>
<td>0.9880</td>
<td>197.60</td>
<td>$-1.98 \times 10^4i$</td>
</tr>
<tr>
<td>3</td>
<td>0.4633</td>
<td>92.66</td>
<td>$-1.02 \times 10^6i$</td>
</tr>
<tr>
<td>4</td>
<td>0.4282</td>
<td>85.64</td>
<td>$-1.35 \times 10^6i$</td>
</tr>
</tbody>
</table>

6.2.4 Sample fabrication

To experimentally realize such metasurface, we adopted the standard PDMS soft lithography. Since the bubbles are much smaller than a wavelength, and we rely only on the monopole mode, to make the metasurface compatible with PDMS molding, the spherical bubbles are mapped to cylindrical ones with the same volume.
The mold is fabricated with photolithography. The steps for photolithography are described as follows: (1) A photomask is laser-printed on transparency. (2) Clean the silicon wafer. (3) Deposit SU-8-2000 with desired thickness using spin-coating, then soft bake at 95°C. (4) Apply the photomask and expose the wafer in ultraviolet for a certain amount of time. (5) Post Exposure Bake (PEB) at 95°C. (6) Immerse in the developer and agitate constantly to remove uncross-linked SU-8 (7) Rinse and dry the mold. (7) The mold is treated with Trichloro(1H,1H,2H,2H-perfluorooctyl)silane vapor in a vacuum chamber to help future separation of PDMS and the mold.

After making the mold, we mixed QSIL 216 clear liquid silicone PDMS parts A and B with a 10:1 mix ratio. The mixture is degassed in a vacuum chamber and poured on the wafer. To control layer thickness, a glass plate (pre-treated with silane) is placed on top of the mold and PDMS, separated by shims with a certain thickness. Then the PDMS and mold are baked in an oven at 120 °C for 2 hours.

By making four layers of thin PDMS sheets, aligning them with microscope and
combining them using O$_2$ plasma, the sample can be fabricated.

However, the bubbles are subject to losses induced by the shear viscosity of PDMS. We have fabricated a single layer of uniform bubble array and tested its sound isolation capability. It is found that although they exhibit resonances that block sound at a certain frequency, the sound insulation with one layer is not significant. Such observation is consistent with previous works. To the best of our knowledge, there are two factors contributing to the significant damping effect: (i) Indeed, we have tested the sound attenuation in PDMS and showed that the loss is small, but these conclusions are compared with the material properties of PDMS, which are generally orders of magnitude larger than that of air. The resonance frequency for a single bubble with radius $a$ is given by $\omega_M = \sqrt{(3\beta_g + 4\mu)/(\rho a^2)}$. These "small" imaginary part of $\mu$ for PDMS is still non-negligible for air. (ii) Our calculation of shear modulus $\mu$ of PDMS is based on static measurements. However, the bubble resonance and corresponding losses are determined by shear modulus and shear viscosity which are almost linearly dependent on the frequency. According to ??, these resonators are almost critically damped, so they can only achieve a limited impedance range. (iii) The frequency dependency of shear modulus also makes resonant frequency to shift. Therefore, the quality factor of such resonance is further lowered.

Another hindering factor is the alignment between different layers, causing fabrication errors that affects interaction among adjacent bubbles.

6.3 Discussion and future work

From the previous discussion, we concluded that the bubble-PDMS approach is subject to a high damping effect induced by shear viscosity in the host medium, and fabrication error due to misalignment. Therefore, even the structure can be designed and works well in simulations, there remain some hurdles for experimental
realization. However, we would like to note here that these difficulties come with the bubble-PDMS approach, not from the theory. So it is necessary to seek other underwater unit cell structures for full control over the impedance matrix.

One possible solution is to use other host media to replace PDMS. For example, hydrogels are a group of water-like materials that attracted a lot of attention in recent years. Especially, researchers have been investigating hydrogels to make them compatible with 3D printing technique.

Another possible solution is to design other types of resonant structures in water, which will be one of the main focuses in future works.
Wavefront shaping for underwater ultrasound: designing a 3D acoustic tweezer

An important step for the development of acoustic metamaterials is to explore their applications. In this chapter, we introduce a lens to shape wavefront of underwater ultrasound, making an acoustic tweezer.

7.1 Introduction: Acoustic tweezers

Precise and contact-free manipulation of physical and biological objects is highly desirable in a wide range of fields that include nanofabrication, micro- and nanorobotics, drug delivery, and cell and tissue engineering. To this end, acoustic tweezers serve as an fast-developing platform for precise manipulation across a broad object size range Baresch et al. (2013a); Ozcelik et al. (2018). There are two primary types of acoustic tweezers under development at present: radiation force tweezers and acoustic-streaming tweezers.

Radiation force tweezers, in which the acoustic radiation force acts as the trap, can be divided into standing-wave tweezers and traveling-wave tweezers. To date,
most demonstrated acoustic tweezers are standing wave tweezers that use counter-
propagating waves to create a mesh of standing-wave nodes and antinodes where the
particles are trapped Shi et al. (2009); Tran et al. (2012); Ding et al. (2012); Foresti
et al. (2013); Ding et al. (2014); Foresti and Poulakakos (2014); Collins et al. (2015);
Augustsson et al. (2016); Collins et al. (2016); Ng et al. (2017); Tian et al. (2019b).
Such systems are particularly suitable for manipulating groups of particles, but the
chessboard-like node network precludes object selectivity. In addition, standing wave
trapping typically requires multiple transducers that surround the trapping region,
which adds complexity and makes it incompatible with some application scenarios,
especially those that involve fixed object inside the trapping region.

Travelling-wave acoustic tweezers, in contrast, form acoustic pressure nodes by
designing the structure of a single beam instead of using interference between beams.
They are typically achieved by controlling the phase patterns across the radiation
aperture Démoré et al. (2014); Marzo et al. (2015); Melde et al. (2016); Marzo and
Drinkwater (2019); Prisbrey and Raeymaekers (2018). Several structured beams are
proposed to date. Particularly, strong localization and creation of acoustic pressure
node can be simultaneously fulfilled by imparting angular momentum into the field
and generating what are known as acoustic vortices Hefner and Marston (1999);
Baresch et al. (2013b); Courtney et al. (2014); Marzo et al. (2015); Riaud et al. (2017);
Marzo et al. (2018); Baudoin et al. (2019). With recent developments in the theory
of acoustic radiation force, acoustic tweezing with vortices has been experimentally
demonstrated Baresch et al. (2013b, 2016); Baudoin et al. (2019).

However, acoustic vortices achieved with either cylindrical or spherical harmonics
create a node line, rather than a point, along the axial direction, limiting its ability
to trap particles in 3D. The ability to obtain a 3D trap and to pick up one particle
independently of its neighbors was only demonstrated recently by Baresch et al.
Baresch et al. (2016). The three-dimensional trapping force is achieved by the dipolar
mode on the propagation axis, which sets limitations on the particle parameters and adds complexity in field shape control. For large particles, it is necessary to increase the acoustic power to overcome the gravity, but nonlinear effects, such as acoustic streaming, will inevitably appear and start to destroy this type of trap. In addition, the gradient force decreases faster for small particles than the drag initiated by acoustic streams, restraining the size range of the particles that can be manipulated. These approaches have used transducer arrays that are expensive and complex, and 3D trapping in fluids with a single transducer has not been reported so far.

Both standing-wave tweezers and traveling-wave tweezers rely on acoustic radiation force to directly manipulate particles, whereas acoustic-streaming tweezers take advantage of the nonlinear acoustic induced fluid flows Friend and Yeo (2011), and thus handle particles indirectly in fluids by creating streaming vortices Ahmed et al. (2016) with oscillating bubbles Hashmi et al. (2012) or rigid structures Huang et al. (2014); Van Phan et al. (2015). These devices tend to be simple devices that are easy to operate, but offer low degree of spatial resolution, because microbubble and microstructure-based phenomena are nonlinear and difficult to control Ozcelik et al. (2018). Particle manipulation has been demonstrated using controlled pumping Zhang et al. (2018), but is limited to 2D, and requires sophisticated control over the source array.

Here we propose a hybrid 3D single beam acoustic tweezer by combining the radiation force and acoustic streaming. We exploit the nonlinear acoustic streaming effect and demonstrate that, instead of being a nuisance, carefully designed acoustic streaming can be embedded in the focused acoustic vortex to create a fully 3D trap. As a proof of concept, we generated a focused acoustic vortex passively with a single piezoelectric transducer and a polydimethylsiloxane (PDMS) lens. The experimental levitation force provided by streaming reaches 3 orders magnitude larger than previously reported Baresch et al. (2016), without restrict limitations to the particle
size, shape and material properties. We demonstrate this three-dimensional acoustic tweezer first by simulation and experimental measurement of the acoustic field. Then the acoustic streaming flow field is measured with particle image velocimetry (PIV). Finally, levitation, trapping and 3D manipulation of a particle is demonstrated in a fluid environment.

7.2 Lens for focused acoustic vortex

An acoustic vortex produces an acoustic node line along the axial direction, therefore, it serves as ideal candidate for a 2D acoustic trap. Focusing an acoustic vortex not only increases its spatial selectivity, but also makes the 2D trap stronger. Therefore the strategy has been applied for trapping particles with airborne ultrasound. However, focused ultrasound in liquids induces nonlinear streaming, especially when lifting heavier particles that require higher wave amplitude. Such streaming can disrupt linear acoustic traps based on radiation forces Baresch et al. (2016). However, a focused acoustic field will also induce a streaming flow localized around its focal point. This localized steady flow, if controlled properly, can also provide the lifting force against gravity to create a 3D trap, as is shown in Fig. 7.1(a). It offers two advantages: i) it doesn’t require resonance modes of the particle to provide lifting force along z axis, as the levitation is provided by the drag in the steady flow; and ii) the drag force can be tuned by controlling the streaming flow velocity. Controlling this streaming force is our strategy for creating a 3D trap.

The focused vortex field is generated by placing a PDMS lens on a circular lead zirconate titanate (PZT) transducer 38 mm in diameter. The lens design principle is based on the combination of approaches previously developed for the acoustic holograms and holographic elementsMelde et al. (2016); Marzo et al. (2015). First, the required 2D phase map of the acoustic wave on the source plane just above the lens is calculated. This phase map has a specific signature pattern in order to
achieve the focused vortex field on the target plane after the wave propagation in the 
\( z \)-direction. It was previously calculated by Marzo et al. Marzo et al. (2015) using 
the BroydenFletcherGoldfarbShanno (BFGS) optimization method that the optimal 
source plane phase field producing the focused vortex trap is a direct sum of the (i) 
focus lens and (ii) simple vortex phase signatures. Therefore, the phase field at the 
exit of the PDMS lens structure is calculated analytically for each pixel of the source 
plane as a sum of the Fresnel lens phase and the simple vortex phase at this pixel 
location. The acoustic wave source amplitude is assumed to be constant here for 
simplicity, as the PDMS has an impedance value close to water. Consequently, the 
2D thickness map of the PDMS lens is calculated from the corresponding 2D phase 
delay map based on the sound wavelength difference in PDMS material and water 

\[
T(r, \theta) = T_0 - \frac{\Delta \phi(r, \theta)}{k_m - k_h}
\]  
(7.1)

where \( T(r, \theta) \) is the thickness of the lens pixel positioned at point with polar coordi-
nates \((r, \theta)\), \(T_0\) is the initial baseline thickness, \(k_h\) and \(k_m\) are the wave numbers in 
the hologram fabrication material and its surrounding medium, and \(\Delta \phi(r, \theta)\) in our 
case is the phase map of the focused vortex sound wave source, calculated from the 
equation:

\[
\Delta \phi(r, \theta) = k_m(\sqrt{r^2 + f^2} - f) + \theta
\]  
(7.2)

where \((r, \theta)\) are the pixel polar coordinates and \(f\) is the focal distance of the lens. The 
calculated thickness profile is then used to construct the 3D solid model (in STEP 
format) of the lens using CAD software Salome 8.3 Ribes and Caremoli (2007); Sal 
with Python 2.7 interface. The STEP solid model of the lens is then ready for 3D 
printing.

The resulting 3D shape of the lens is shown in Figure 7.1(a) and has the height 
contour lines following the Fermat-Archimedes spiral branches.
**Figure 7.1**: Schematics of the design and working principle. (a) Creating a focused acoustic vortex for in-plane particle trapping, and the nonlinear streaming levitates the particle, providing trap in the third dimension. Inset shows a photo of the fabricated device. (b) Evolution of the intensity and phase fields across different cut planes along $z$ axis. The field is gradually focused as it propagates, keeping the spiral phase profile in the central region.

The lens is fabricated with the standard PDMS molding process. A negative mold was fabricated with stereolithography 3D Printing. Part A and part B of Ecoflex 00-30 Silicone is mixed thoroughly by 1:1 weight ratio, degassed with a vacuum chamber and then poured into the mold. The mold is then baked in an oven at 120 Celsius degrees for an hour for the silicone to cure. The lens is then separated from the mold and attached to the piezoelectric patch.

### 7.3 Simulation of the acoustic field and streaming field

The finite element simulations of the acoustic wave propagation were performed in the frequency domain using the open-source finite element solver Code Aster. The simulation domain is a 3D waveguide where the bottom wall represents the wave source and the normal axis ($z$) is the propagation direction. The analytically
calculated phase field at the source plane was used as a boundary condition for the bottom plane \((z = 0)\). The side walls were treated as hard wall boundaries, and the anechoic exit condition was assigned to the top boundary plane. The weak form of the Helmholtz equation is numerically solved by Code Aster in the single-phase acoustics settings, and the acoustic intensity and complex pressure amplitude is calculated in the entire 3D simulation domain.

The results in the \(Oxz\) sectional plane represent the development of the vortex trap pattern in the propagation direction with the maximal amplitude observed near the focal position (see Figure 7.1(b)). The phase signature in the center part of the focal plane indicates the orbital angular momentum characteristic for the focused vortex beams. Figure 7.2 (b-i) shows the numerical and experimental results in two sectional planes: the \(Fxy\) focal plane and the \(Oxz\) central section of the box. The intensity and phase field structure in the focal plane matches the signature of the focused vortex trap described in previous works Melde et al. (2016). The finite element results show good agreement with their experimental counterparts.

The acoustic streaming effect was simulated using another open-source tool, OpenFOAM. The streaming effect modelling was performed in three stages as previously suggested in Catarino et al. (2014): (i) simulation of the wave propagation in time domain using the compressible flow CFD solver, (ii) time-averaging of the effective non-linear equation term to calculate the body force driving the acoustic streaming flow, and (iii) using the incompressible steady-state CFD solver to calculate the streaming velocity field by adding the effective external force equation term calculated in step (ii). All the required solvers are included in the default OpenFOAM distribution with minor additional code modifications required.

In step (ii), the streaming flow effect is achieved by adding an external force term
Figure 7.2: Measurement of the acoustic field. (a) Illustration of the ultrasound scanning system. (b, d) Simulated intensity and phase field of the $x$-$y$ plane at $z = 30$ mm. The ring-shaped intensity profile provides the 2D trap. The spiral phase profile is clearly seen in the central region. (f, h) Simulated intensity and phase field of $x$-$z$ plane at $y = 0$. The intensity forms a node line along $z$ axis, along which the phase is discontinuous. (c, e, g, i) The corresponding measured intensity and phase field in the experiment, in great agreement with the simulation.
into the second stage compressible flow equations:

$$ F = \langle -\rho \frac{\partial u}{\partial t} - \rho_0 (u \cdot \nabla) u \rangle $$

(7.3)

where $u$ is the acoustic particle velocity and $\rho$ is the compressible fluid density calculated during the first acoustic simulation stage, $\rho_0$ is the equilibrium constant density, and $\langle , \rangle$ indicates the time-averaging of the term over a significant number of iterations.

The results of the streaming fluid flow simulation are presented in Figure 7.3. Figures 7.3(b-c) show the streaming velocity magnitude distribution in the focal plane ($z = 30$ mm) and the sectional plane in the sound wave propagation direction ($Oxz$). The results in the $Oxz$ plane show a significant outward fluid flow away from the sound source near the focal point (Figure 7.3(c)). The flow converges and increases in magnitude towards the focal plane and carries simulated particles from the periphery of acoustic lens region towards the axis of symmetry. The focal plane section results indicate that the simulated flow magnitude is actually weaker along the axis itself but reaches the maximum in the surrounding cylindrical region, forming a fluid vortex where the acoustic vortex is located (Figure 7.3(b)). These combined effects will not only trap the particle in the $x$-$y$ plane, but provide a strong localized drag flow for levitation and thus trapping in the $z$ direction.

7.4 Experimental verification

Measurements confirm that the design is capable of 3D particle trapping can be used to move a particle along a prescribed trajectory in three dimensions. The lens in the experiment was fabricated with (PDMS) molding. It is then attached to a PZT 38 mm in diameter and 4.1 mm in thickness, with a 500 kHz resonance frequency.
Figure 7.3: Streaming field measurement setup and results. (a) Illustration of the streaming field measurement setup. (b) Simulation of streaming velocity field along $x$-$y$ plane at $z = 30$ mm. The streaming flow is focused in the central region, forming a streaming vortex. (c) Simulation of streaming velocity field along $x$-$z$ plane at $y = 0$ mm. The velocity gradually increases towards the focal point, and the low velocity along the node line is the result of the streaming vortex. (d) Measured streaming velocity field along $x$-$z$ plane at $y = 0$ mm, in good agreement with the simulation.

7.4.1 Measurement of the acoustic field in water

The acoustic field created by the lens is first measured by a hydrophone attached to a 3D positioning stage, as illustrated in Fig. 7.2(a). The measurement was performed in a 40-gallon water tank. For the measurement of the linear acoustic field, the computer-controlled function generator (RIGOL DG4102) generates a Gaussian-modulated pulse centered at 500 kHz. The signal is amplified by ENI 2100L RF power amplifier and drives the PZT disk. A hydrophone (ONDA HNR-0500) is attached to a 3D scanning stage to scan the field. The output signal from the hydrophone is recorded by AlazarTech ATS 9440 waveform digitizer at a sample rate 125MS/s. The signal at each scanned position is averaged over 1024 measurements to eliminate noises. Then Fourier transform is performed to extract the amplitude
and phase for 500 kHz to generate the field map.

The scanned acoustic pressure and phase profile across $x$-$y$ plane and $x$-$z$ plane in Figure 7.2(c, e, g, i) shows excellent agreement with the corresponding simulations. The acoustic node along the $z$ axis where the particles can be trapped is clearly seen. Compared with a cylindrical vortex that generates a non-negligible secondary ring, the spatial selectivity of such a focused vortex is greatly enhanced Baresch et al. (2013b); Baudoin et al. (2019).

7.4.2 Measurement of the streaming field

The acoustic streaming field is measured with particle image velocimetry (PIV). The experimental setup is illustrated in Fig. 7.3(a). Polyamide seeding particles with density 1.03 g/cm$^3$ and mean size 60 µm are dispersed in water. A 532 nm laser line generator emits a fan-shaped beam and the light plane is aligned with $x$-$z$ plane. Light scattered by the seeding particles is then captured and recorded with the slow motion mode of a cellphone camera, therefore the flow field can be indirectly measured by tracking the particles using ensemble correlation PIV algorithm. The video is processed with PIVlab, an open source toolbox in MATLAB. Since the particle is sparsely dispersed in the fluid, the ensemble correlation PIV algorithm is adopted, where 2000 frames are used as an ensemble.

The measured streaming field is shown in Fig. 7.3(d), where it is observed that the upward flow converges to the focal point, reaching maximum velocity at the focused region, and diverges after passing the focal plane. The drag force provided by such a localized steady flow as the serves as levitating force and enables the 3D trapping and manipulation.
7.4.3 3D particle manipulation

To demonstrate the 3D trapping capability of the proposed acoustic tweezer, we first demonstrated lifting a cellulose acetate polymer sphere as shown in Fig. 7.4 and mov. S1. The particle has a diameter 1.5 mm with density 1.3g/cm$^3$. The calculated levitation force provided by the tweezer reaches 5.2 $\mu$N, which is 3 orders of magnitude larger than the previously reported tweezer that relies on radiation force with dipole mode Baresch et al. (2016). It is also worth noting that, since the levitation is provided by the drag force in flow instead of radiation force, the upward levitation force is less sensitive to the shape and material properties of the particle.

As a demonstration, we have also shown the trapping of cylindrical particles with diameter 1.3 mm and height 1.3 mm, and total weight 6 mg, as shown in mov. S2 and mov. S3. In this case, the calculated levitation force reaches 41.8 $\mu$N. For comparison, we dropped another particle outside the focused region, and it sinks quickly, as is shown in mov. S3. Note that the applied voltage on the transducer can be further increased for faster streaming, while the first-order linear field shape is preserved. Therefore, the tweezer can lift the heavier particle without sacrificing its in-plane trapping capability. We also demonstrate the ability to move the particle along a prescribed three dimensional trajectory by scanning the source transducer, as shown in the inset of Fig. 7.4 and mov. S4, where the trapped particle closely follows the designed path.

Compared with the 3D trap demonstrated by Baresch et al. Baresch et al. (2016) where the trapping force along $z$ direction relies on the dipole mode in a sphere, levitating particles with drag in the streaming flow offers several advantages. First, it is able to levitate heavier particles since the drag in streaming flow can provide larger upward force than using radiation forces. Second, it removes the dependence of trapping on the shape and material properties of the levitated particle. Third,
Figure 7.4: **Snapshot of the levitated particle.** The red particle is stably trapped in 3D (Supplementary movie S1). The inset shows the trace of a particle moving along a designed path outlined by the yellow arrows. The particle closely follows the 3D motion of the lens. (Supplementary movie S4)

the wave field is generated by a single transducer and passive lens instead of the transducer array, which provides an inexpensive and reliable route for contact-free particle and fluid manipulation.

### 7.5 Discussion and future works

Radiation force acoustic tweezers are a versatile platform for object manipulation capable of handling a wide range of applications in biology, chemistry, and medicine, owning to their simplicity and biocompatibility. However, for particle manipulation in the three dimensional space, there is a trade off between particle parameters and acoustic amplitude because of the stability issue and nonlinear streaming per-
turbation induced by finite amplitude acoustic waves. Here we have described and
demonstrated a hybrid 3D single beam acoustic tweezer by combining the radiation
force and acoustic streaming. We show that instead of being a nuisance, carefully
designed acoustic streaming can be harnessed to help control particles in fluids and
create a fully 3D trap. As a proof of concept, we have (i) designed a focused acoustic
vortex lens that facilitates acoustic vortex trapping and localized upward streaming
flow simultaneously, (ii) verified the designed acoustic field and the corresponding
streaming field in both simulation and experiments, and (iii) demonstrated three
dimensional trapping and manipulation of particles in fluid.

Compared with previously reported 3D acoustic tweezers in fluids, using acoustic
streaming to defy gravity provides several benefits. First, since streaming tweezers
rely on streaming drag instead of radiation force, they can accommodate a broader
range of material properties and particle shapes, especially for heavier particles where
dipole mode radiation force is not sufficient. For example, in the case shown in this
paper, the levitation force provided by streaming drag is about 3 orders of magni-
tude larger than previously reported radiation force tweezers. Second, the levitation
force can be tunable, so the same tweezer can adapt to a wide range of particles
by simply tuning the ultrasound amplitude, and thus offers great versatility. Third,
for biomedical applications where trapped particles are small, radiation force tweez-
ers require scaling up the frequency to keep a radius-to-wavelength ratio sufficiently
large. However, sound absorption and non-linearity become more significant for
higher frequencies, so that streaming becomes unavoidable, making radiation tweez-
ers less reliable. In contrast, streaming tweezers are expected to take advantage of
high frequencies.

There are also several limitations to the proposed tweezers. First, since the
streaming force is nonlinear, the precision in the axial direction is not as good as the
radial direction. This is a common issue for single-sided three dimensional acoustic
tweezers, and how to improve its axial resolution remains an open question. Second, due to the nonlinear nature of acoustic streaming, the relation between the required levitation force and applied acoustic amplitude is not linear. In addition, the precise characterization of the levitation force requires coupling between the nonlinear acoustic field, fluid mechanics, and fluid-structure interaction, which adds difficulty for analytical calculation and full simulation. Therefore, in the current setup, adaptation to various particles is achieved by careful tuning of the input amplitude. Other possible ways to tune the levitation force may include applying short pulsed signals with different duty cycle Marzo et al. (2018). Nevertheless, we have shown that streaming offers an additional degree of freedom for acoustic tweezers that can greatly extend its versatility.
This last chapter briefly summarizes the main content and conclusions of this dissertation and discusses several potential future paths for the metamaterial and metasurface based highly-efficient wavefront shaping systems.

8.1 Summary

In this dissertation, we first went through a brief history of acoustic metamaterials and metasurfaces and looked at some of the directions under active investigation. Specifically, we envision wavefront shaping to create functional devices as an important next step for the development of acoustic metamaterials. To this end, our effort is focused on (i) designing metasurfaces with maximized power efficiency and (ii) designing metamaterials and metasurfaces for wavefront shaping in a fluid environment.

To design metasurfaces with maximized power efficiency, we first analyzed the fundamental limitations of metasurfaces and the origin of such limitation. For reflection-type metasurfaces, interference between the incident field and desired field reshapes the power flow, requiring the metasurfaces to contain either loss and gain,
or mechanisms to facilitate local power redistribution. For transmission-type meta-
surfaces, wave impedance mismatch is the main reason for unwanted scatterings.

To solve this problem, we analyzed the theoretical requirement for a perfect meta-
surface and found that acoustic bianisotropy (Willis coupling) can be incorporated
into unit cells to match the wave impedances on both sides. We proposed a system-
atric design procedure for unit cells with full control over their scattering properties.
Based on the proposed unit cell, we designed the metasurfaces to steer waves with
near-unity power efficiency and verified the designs with both simulations and ex-
periments.

We then extended the knowledge in Cartesian coordinates into cylindrical coor-
dinates. We proposed the unit cell structure that works for cylindrical coordinates
and demonstrated the design procedure for the metasurface. As a demonstration, we
designed a metasurface that converts a monopole source to a field with large angular
momentum $n = 12$ with nearly 100% power efficiency. Such functionality cannot
be achieved with conventional phase gradient metasurfaces due to the unwanted
scatterings. The design is verified with both simulation and experiments.

Then we tried to apply the metasurface to wavefront shaping problems other
than beam steering. We found that such an extension is not as trivial as it seems.
For transmission-type metasurfaces, as the output field gets complicated, the normal
component of power flow on both sides of the metasurface is unmatched for most
cases. Such local power mismatch shares similarities with reflection-type metasur-
faces, and thus requires either gain and loss, or power exchange within the metasur-
face. To meet local power requirements, we can also design the fields carefully such
that local power conservation is met. To this end, we introduced evanescent waves
to carry the energy along metasurfaces so that local power is matched locally. We
designed both transmission-type bianisotropic metasurface for beam splitting and
reflection-type bianisotropic metasurface for efficient beam steering with the auxil-
Evanescent waves provide an approach for matching local power flow, but they are not suitable for more complicated (non-periodic field) applications, and sometimes are sensitive to losses in metasurfaces. To overcome these difficulties, we demonstrated the power-flow conformal metasurface for both the reflection case. In Chapter 5, we combined bianisotropy and power-flow conformal strategy and demonstrated power-flow conformal metasurfaces as a general method to construct highly efficient metasurfaces for arbitrary fields. The strategy is verified with both simulation and experiments in the case of near-field focusing.

From the analysis of metasurfaces for highly-efficient wavefront transformation, we know that bianisotropy takes care of wave impedance matching on both sides of the metasurface. In Chapter 6, we generalized the application of bianisotropic metasurfaces to a broad scope of impedance matching problems. We have shown that bianisotropic metasurfaces can help match any impedance with arbitrary thickness and full control over the phase. Then we investigated a particular case of helping waves penetrate a middle layer. We designed a unit cell structure in water to control the bianisotropic responses, and then a metasurface to help 20kHz wave penetrate a 10mm thick steel plate in water. The design was verified in simulation. Impedance matching is of fundamental importance in a wide range of engineering scenarios, and we anticipate bianisotropic metasurfaces can serve as a versatile platform for designing impedance matching layers.

As another example of wavefront engineering in water, we demonstrate the first 3D acoustic tweezer that uses a single transducer and combines the radiation force
for trapping in two dimensions with the streaming force to provide levitation in the third dimension. The idea is demonstrated in both simulation and experiments, and the achieved levitation force reaches three orders of magnitude larger than for previous 3D trapping. This hybrid acoustic tweezer that integrates acoustic streaming adds a new twist to the approach and expands the range of particles that can be manipulated.

8.2 Future works on wavefront shaping with engineered structures

This section discusses several potential future directions for wavefront shaping using metamaterials and metasurfaces. After reviewing the works discussed in this dissertation, we can refer back to Figure 1.2 and see where we are now.

First, on the highly efficient wavefront transformation, although power-flow conformal metasurface provides a general solution in the 2D case, extension into the 3D case is not trivial. This is because we cannot write a field that is always tangential to the intensity field and calculate its scalar potential field, due to the fact that tangential direction to an arbitrary vector forms a plane, instead of a vector. If we follow the power flow, we will get a streamline instead of a surface. Therefore, how to construct a power flow conformal metasurface in 3D remains an open question. It will not only enable holograms with 100% power efficiency but also provide a route for constructing extreme fields that are not achievable with conventional metasurfaces. We anticipate that the Method Of Characteristics (MOC) could be a viable route to solve this problem.

Secondly, designing structures for the underwater environment remains challenging, since the contrast between water and available materials in the real world is not significant so that multi-physical coupling is typically needed. However, acoustic waves have many advantages over other forms of waves in water. For example, due to the conductivity of seawater, electromagnetic waves decays fast in the sea, so that
sound is almost the only choice for communication systems. Therefore, designing simple, versatile, reliable, stable, and fabrication-friendly unit cells to control their scattering properties in water will be an important next step towards application.

Apart from the lens, transformation acoustics, gradient metasurface, and impedance matrix-based metasurfaces, new platforms to engineer material distribution could be an interesting topic since they may unlock new possibilities for controlling wave-front. In this regard, supersymmetry can be a promising candidate to design material profiles with identical scattering properties, or creating waveguides with only fundamental modes.

Time-varying structures have opened new possibilities for wave manipulation and is under fast development in recent years. However, the investigation into scattering properties of a time-varying structure is still at its infancy, and there remains a huge space for exploration. We the interaction between space-time variation, asymmetry, nonlocality, and loss/gain are of particular interest for further exploration.
Appendix: The setup of the two-dimensional waveguide acoustic field-mapping measurement in air

The samples in Chapters 2-6 were fabricated with Stereolithography (SLA) or Fused Deposition Modeling (FDM) 3D printing. The materials used are Nylon or Acrylonitrile butadiene styrene (ABS). They both have a characteristic impedance 4 orders of magnitude higher than that of air, so that the boundaries can be treated as rigid.

The samples were tested in a two-dimensional waveguide. The experimental setup is shown in Fig. 1. In the experiment, the speaker array consists of 28 identical speakers, sending out a pulse, which is a sinusoidal wave modulated with Gaussian function in both space and time, normally to the metasurface. The signal is recorded by a moving microphone, and collected by data acquisition (NI PCI-6251). The field is mapped by scanning the region of interest. The steady state field is mapped by performing the Fourier transform to the time-gated signal at each position and taking the frequency component of interest. The reflected signal is calculated by scanning the empty field and the field with samples, and then calculate their differences to remove the incident signal.
Figure 1: Experimental setup of 2D field mapping
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Biography

Junfei Li attended Beijing Institute of Technology, Beijing, China from 2011 to 2015, where he earned a Bachelor degree of Science, majoring in Mechanical Engineering. He was matriculated at Duke University, Department of Electrical and Computer Engineering in 2015. In 2018, Junfei won the Fitzpatrick Institute for Photonics (FIP) Chambers Scholar Award. He is a member of the Acoustical Society of America (ASA) and Materials Research Society (MRS). He has been serving as a reviewer for many journals, such as Physical Review Letters, Physical Review Applied, Physical Review Materials, Nanophotonics, Wave Motion, Annalen der Physik, Physics Letters A, and Physica Status Solidi A. Following is a list of his publications:

Journal publications:


isotropic acoustic metasurface. *In preparation*


