SWITCHING OF TECHNIQUES AND CONSUMPTION PER HEAD: AN ECONOMIC CLARIFICATION

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Using the notation of our original paper, we summarize different techniques by \((n \times n)\) input matrices \(a, b, c, \ldots\) and the corresponding labor requirement vectors \(a_0, b_0, \ldots\). When one sets the wage \(w = 1\) as a normalization, the price vector \(p_a\), associated with any technique \(a\), is a function of the interest rate \(r\): 

\[
p_a(r) = a_0 [I - (1+r)a]^{-1}.
\]

[In equilibrium, prices exactly cover costs, \(p_a = a_0\) (direct labor costs) + \((1+r)p_a\) (cost, including interest, of inputs, i.e., circulating capital); (1) gives the solution for \(p_a\).] The nonsubstitution theorem assures that, given any \(r\), say \(r_0\) (smaller than the maximum possible rate), there is one technique, say \(a\), which minimizes all prices, i.e.,

\[
P_a(r_0) = a_0 [I - (1+r_0)a]^{-1} \leq b_0 [I - (1+r_0)b]^{-1} = p_b(r_0)
\]

for any other technique we choose to label \(b\).

Consider now the steady-state consumption possibility frontier of this economy. Suppose that the economy grows at a rate \(g\), and initial labor is normalized at 1. Then, when technique \(a\) is employed, per capita consumption possibilities are given by

\[
a_0 [I - (1+g)a]^{-1} c = 1
\]

where \(c\) is the consumption vector. This can also be written

\[
p_a(g)c = 1.
\]

Ozga's point is the following. Suppose that, for the rates of interest \(r_0\) and \(r_1 (r_1 > r_0)\), the corresponding optimal techniques are \(a\) and \(b\). Is it true that the consumption possibility frontier of technique \(b\), associated with the higher interest rate \(r_1\) and given by

\[
p_b(g)c = 1,
\]

is necessarily lower than that associated with technique \(a\), the optimal technique at the lower interest rate \(r_0\)? The answer is no. For this to be true, every element (price) in the vector \(p_a(g)\) must be lower than the corresponding element in \(p_b(g)\). Examples to the contrary can be easily constructed. Diagrammatically, the con-

sumption possibility frontiers in two dimensions can appear as in Figure I.

This phenomena can be further exhibited by looking at the factor-price frontier for the two goods (see Figure II). In each diagram the vertical height at \( r=g \) gives the maximum steady-state consumption of that good. In both diagrams, as one moves from \( r_0 \) to \( r_1 \), technique \( b \) becomes optimal, giving a higher real wage in terms of both goods (by the nonsubstitution theorem). But this switch reduces the maximum consumption of good 2 and increases it for good 1! Where the economy ends depends on consumption habits. It is clear that one cannot say unequivocally that "consumption possibilities" have increased or decreased.
It should be noted, however, that the above indeterminateness disappears once we confine the discussion to the consumption possibility frontiers of laborers only. The consumption vector of laborers, denoted by $c_L$, is given by

$$p_a(r) c_L = 1 \text{ or } p_b(r) c_L = 1$$

according to the technique employed. Since prices are monotonically increasing with the rate of interest, we know that

$$p_a(r_0) < p_b(r_1) \text{ if } r_1 > r_0,$$

and consequently the consumption possibility curves of $c_L$ never intersect (see Figure III).

![Diagram](image)

**Figure III**

The indeterminateness pointed out by Ozga is closely related to the "Wicksell effect." A change in the rate of interest changes the prices and quantities of all goods, including capital goods. While the direction of the change in prices is clear, the change in quantities depends on consumption habits of the economy and cannot be predicted a priori. Thus, if $X$ is the vector of the amounts of goods used as (circulating) capital, the value of the capital stock is $p_aX$ (with technique $a$). As $r$ increases, all $p$'s increase, but the change in the production basket $X$ is unpredictable. Thus the consumption possibilities of the capitalists, which depend on their profits $r(p_aX)$, can change in either direction.