Convolution and Volterra Series Approach to Reduced Order Modelling of Unsteady Aerodynamic Loads and Improving Piezoelectric Energy Harvesting of an Aeroelastic System

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Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Mechanical Engineering and Material Science in the Graduate School of Duke University

2020
ABSTRACT

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Abstract

A combined approach of linear convolution and higher order Volterra series to reduced order modelling of unsteady transonic aerodynamic loads is presented. The new approach offers a simple method to determine the memory depth of the system, significantly reduces the effort required to generate a model for a wide range of reduced frequencies, and clearly separates the linear and the non-linear contributions. The generated models are completely separated from any specific input signal or a particular reduced frequency. The models were verified in an aeroelastic simulation of a 2D NACA 0012 airfoil. The results correlate well with wind tunnel tests and previously calculated LCO levels.

Our experimental study sought to answer the question: how to maximize the piezoelectric power extraction of an aeroelastic system? A simple rectangular cantilever plate, which experiences LCO, was used as a basic vibrating system. The plate was covered entirely with piezoelectric elements on both sides. By adding small discrete masses along the plate, we were able to increase the power generation efficiency by 260% while reducing the airspeed required to produce this power by 150%, and the level of vibrations by 320%.
Dedication

To my wife, Nofar, I would not have done this without you.
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1. Introduction

This research program consists of two distinct subjects within the field of aeroelasticity. The first subject is an analytical and computational study of convolution and Volterra series based Reduced Order Modeling (ROM) of unsteady aerodynamic loads. The second topic is an experimental study about improving piezoelectric energy harvesting of an aeroelastic system. The subjects are discussed separately in the following.

1.1 Convolution and Volterra Series Approach to Reduced Order Modelling of Unsteady Aerodynamic Loads

The constant requirements from the aerospace industry to make cheaper, lighter, and more efficient structures have been pushing the common boundaries and standards for aeroelastic and unsteady aerodynamic phenomena to new limits. To address this challenge, among other aspects, the conservative linear unsteady aerodynamic computational models must be improved. An emerging and attractive approach might be using non-linear safety margins as a design criteria. For example, designing for a known Limit Cycle Oscillation (LCO) amplitude instead of conservative linear flutter limit. This approach requires reliable non-linear aerodynamic and structural models for the aircraft development. Moreover, the models will require integration with the methods that are used in the aerospace industry today and in the foreseeable future.
Computing non-linear unsteady aerodynamic loads in general, and in the transonic flight regime in particular, is a long-standing challenge for the practicing aeroelastician. While Computational Fluid Dynamics (CFD) is considered the most reliable method for calculating these loads, it still has major drawbacks. First, it is computationally costly, which makes it impossible to use in the conceptual and preliminary design phases for new aerospace systems, where multiple and rapid calculations are required. Second, it lacks the straightforward transformation into the frequency domain, where most of the aeroelastic mathematical models exist. Finally, it cannot be used efficiently in the Multi-Disciplinary Optimization design phase, which is a major stage in every aircraft development.

To overcome these challenges, Reduced Order Models (ROM) are being developed. These models are used to capture most of the non-linear effects, while being computationally efficient. Throughout the years, many ROM approaches have been developed including Proper Orthogonal Decomposition (POD), Harmonic Balance (HB), and Volterra Series (VS), to name a few and perhaps the most widely studied. The field of ROM is still a major part of ongoing academic aeroelasticity research effort. Among the existing approaches to generate ROM, our work will focus on Volterra Series [1]. Vito Volterra first introduced the series in the late 1800s and Norbert Wiener first successfully used Volterra series for system analysis in the 1940s and 1950s. Several features of the Volterra Series make it an attractive approach for non-linear systems
modeling and identification; it is linearly expandable to a desired degree, the terms are
non-linear convolution products, a fading memory effect can be implemented up to the
desired depth in time, and the series are easily expandable to include multiple inputs.

Due to these elements, Volterra Series has become a widely used method for
non-linear systems modelling and identification, in academia and in various engineering
fields and industries. The following are several examples. A successful non-linear power
amplifier model for mobile communication was implemented with Volterra Series in [2].
The model included the memory effect and improved previously used memoryless
models. An application of Volterra theory for backlash type nonlinearity is covered in
[3]. A good correlation was observed between the experimental data of an inverted
pendulum problem, and Volterra based non-linear approximated model [4]. In [5] a non-
linear model of the interaction of random waves and offshore structure has been
identified. This Volterra based model showed good comparison between numerical and
experimental data. Moreover, the application of this model in future simulation was
straightforward and efficient compared to computational models previously used.

In the aerospace field, Volterra series have been applied successfully for
identification of non-linear aeroelastic systems for several years [6], and it is still an
active research topic. In [7] the authors successfully applied Volterra representation for a
structural nonlinearity in 2D airfoils to analyze the flutter boundary resulting from these
nonlinear terms. This approach resulted in efficient application and an easy to use
extension for more complicated systems. In [8] the recent progress with an experimental setup for identification of nonlinear Volterra kernels was discussed. This setup aims to achieve better correlation between theoretical and experimental methods. In [9] Raveh identified the first two non-linear Volterra kernels with direct CFD analysis of impulse and step responses. The process was then compared to the available experimental data for the AGARD 445.6 Wing. Step response based kernel was better suited for the task. Later, Raveh compared three identification techniques for the non-linear kernels based on CFD analysis input-output data [10]. The identified kernels were successfully used in traditional frequency domain flutter analysis.

The central challenge in the application of the Volterra series is the identification of non-linear kernels. In [6] Silva noted the substantial effort required for kernel identification of higher order terms beyond the 3\textsuperscript{rd} order. However, he also suggests that potential convergence issues might be solved with higher terms, as well as the realization of a better model fit for the highly non-linear systems.

In general, two approaches exist for the identification of non-linear Volterra kernels. The “classic” approach identifies the kernels as a convolution product of the system with impulse or step input signal. In this approach, an explicit form of the kernels is derived, see [6] for example. Explicit expressions up to the 2\textsuperscript{nd} order are popular in literature, and a closed form solution is presented for 3\textsuperscript{rd} order Volterra
kernels in [11] and [12]. Due to increased complexity to isolate kernels with inputs, the models are rarely expanded beyond the 3rd order.

Another approach is using system identification techniques; here we call it the direct method. In this approach a known input-output relation of the system, i.e. a response to a training signal, is used to identify the kernels. The model is not limited to 3rd order kernels and the identification is straightforward. Korenberg [13] discusses the orthogonal expansion of this direct method. Another variation is presented in [14], which utilizes a subset identification technique.

The identification effort can be made more practical and manageable when combined with several simplifying assumptions. First, the kernels can be represented as a weighted set of base functions, as with the Laguerre polynomials [15], [16]. This reduces the number of unknowns and makes the identification process simpler. Second, the basic form of the required terms in the series might be reduced to sparse kernels only, with no cross-memory effects. Balajewicz and Dowell have shown an excellent correlation between CFD analysis and ROM generated data, when using sparse kernels alone [17]. With the direct method, an existing input signal can be used for training and identification purposes, as long as it includes a sufficient frequency bandwidth [18].

The two approaches described above are being used and produce acceptable results up to some level. However, there are still several drawbacks with both. Identifying each kernel separately can be time consuming and being limited in
complexity, might not lead to a satisfactory description of the underlying non-linearities. With the direct approach, the identified model might include dominant higher degree kernels which do not contribute to physical representation and understanding of the underlying system. In both cases, the memory depth of the system is an unknown, one might explore several values before the identification is satisfactory. Finally, a fully identified model of unsteady aerodynamic loads might require a large number of CFD runs to be applicable in all relevant Mach numbers and reduced frequencies.

In this research we focus on generating reduced order models of unsteady transonic aerodynamics. By combining a linear convolution, as in [9] and [19], with higher order Volterra terms, and using a step response analysis as the identification data, we can offer a significantly improved ROM. The resultant model is compact, supports a range of non-linear behavior and is applicable to a wide range of oscillating frequencies. Moreover, the combined approach requires only several simple CFD runs.

1.2 Improving Piezoelectric Energy Harvesting of an Aeroelastic System

Remotely located electronic devices, such as wireless sensors, transmitters and monitoring units, are constantly being developed and deployed. These devices have become more efficient and as a result require less power to operate. This opens possibilities for ambient energy harvesters (EH) to be paired with such devices to make them self-sufficient over long time periods [20], and might be more suitable for these
purposes than batteries which are currently used [21]. In the micro watt range, piezoelectric elements, which convert vibrations into electric energy, have become common [20], [22]. The simplest example of an EH device is a vibrating cantilever beam with piezoelectric elements attached at the root [23], [24], [25]. To achieve favorable energy generation, the physical properties of the beam, i.e. the material, length, cross section dimensions and the tip proof mass, can be tuned according to the available ambient energy sources.

Flow induced vibrations arising from fluid-structure interaction, or aeroelastic effects, are considered as one of the most promising potential sources of energy [21]. These vibrations can be obtained from natural-flow conditions, are self-excited and thus more feasible and scalable than forced vibrations. Combining piezoelectric elements with aeroelastic effects can lead to attractive and efficient EH devices, see [21], [26], [27] for comprehensive reviews.

Aeroelastic effects, such as flutter, are usually undesirable phenomena in any aircraft/structure design. However, while linear flutter is a one-time destructive occurrence, non-linear incidents, such as Limit Cycle Oscillations (LCO) may not cause structural damage. LCO can sustain finite amplitude vibrations for long time periods and thus can be exploited for energy harvesting purposes.

Power generation from LCO can take several forms. The most common is a flexible vibrating plate with piezoelectric element attached to the root, see [28] for an
example. In another variation, a rigid airfoil might be used. The airfoil is mounted on non-linear springs which are tuned to result in LCO. The piezoelectric elements in this case are attached to the leaf springs which vibrate while the rigid airfoil oscillates [29], [30]. An interesting combined approach uses a flexible beam with a pin connected rigid flap [31]. The divergent coupling between the flap in pitch with large rotations angles, and large deflections of the beam in bending drive piezoelectric elements, which are attached to the root of the beam. In [32] a 2D airfoil with 2 Degrees of Freedom (DOF) is investigated. Free play nonlinearity is introduced in the pitch DOF, and the piezo elements are attached to the linear leaf springs in the heave DOF. The model was explored both theoretically and experimentally to optimize for resistive load and the power output. The study shows that the load resistance mainly influences the power output and not the mechanical motion of the system.

Previously mentioned work used discrete piezoelectric elements which are driven by the larger oscillating structure. When the goal is to increase the power output, the location of piezo-elements and the number of such elements must be considered. In recent research [33], newly developed computational models show promising levels of harvested electric power when the entire plate is covered with piezoelectric elements. A theoretical and experimental study which explored the optimal placement of piezoelectric elements along flapping flag is described in [34]. Installing the elements in areas of large curvature led to increased power output. This is especially significant on a
small scale where the stiffnesses of the substrate and the piezo elements are comparable. This was further explored in [35] where segmentation of the piezo elements was studied. The authors found that for increased power output, the elements should be placed outside of the nodal points of the vibrating mode. This will alleviate the effect of cancellation when the element deflects. The authors also noted that finding such points for general oscillatory motion is not an easy task, yet an effort should be made to segment the elements closer to the estimated position of these nodes.

Our work is based on an aluminum plate which experiences LCO in the wind tunnel. The plate is larger compared to [34] or [35], and thus has lower frequency dynamics. In order to exploit the full potential of the structure, the plate is fully covered with piezoelectric elements on both sides. The generated voltages from chordwise located elements are rectified and then combined in parallel to maximize the power output [36]. This is better suited for power extraction from a large number of piezo elements [37], as opposed to locally adjusted segmentation like in [35]. The rectification is realized with a simple full-wave diode bridge [38], [39].

We experimented with changing the dynamics of the plate by adding small discrete masses in several locations, as in [40]. Our experience was consistent with the results for “long” plates, i.e. adding masses destabilizes the system, resulting in lower margins of flutter and LCO [40].
Unlike conventional aeroelastic systems, destabilizing the system can be beneficial for energy harvesting; the power is generated at lower air speeds, the system becomes more efficient and the generator is more sustainable. In this work we explored these effects to improve the energy generation from an existing structure.

This work was presented at the International Forum on Aeroelasticity and Structural Dynamics – IFASD 2019, 9-13 June 2019, in Savannah, Georgia

1.3 Thesis Organization

The two aforementioned topics are completely separated in the following.

Section 2 is dedicated to reduced order modeling of unsteady aerodynamic loads with convolution and Volterra series. This is a theoretical and analytical part.

Section 3 summarizes the experimental part of this thesis. We explore methods to improve the efficiency of energy harvesting of an aeroelastic system which experiences limit cycle oscillations.
2. Convolution and Volterra Series Approach to Reduced Order Modelling of Unsteady Aerodynamic Loads

In this chapter we will discuss the application of convolution and Volterra series to the generation of ROM for unsteady aerodynamic loads. Our goal is to develop a compact and useful model which is physically trackable and easy to use. We will start with the standard Volterra series.

2.1 Background

Volterra series with one input and one output can be described in the following discrete notation. The series are infinite in general but are always truncated to the $N^{th}$ degree for any practical use. $N$ is typically on the order of 2-8.

$$y(t) = h_0 + \sum_{n=1}^{N} H_n[u(t)]$$

We can present below the first three terms in the series:

$$H_1[u(t)] = \sum_{i=1}^{M} h_1(m_i) \cdot u(t - m_i)$$

$$H_2[u(t)] = \sum_{i=1}^{M} \sum_{j=1}^{M} h_2(m_i, m_j) \cdot u(t - m_i) u(t - m_j)$$

$$H_3[u(t)] = \sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{k=1}^{M} h_3(m_i, m_j, m_k) \cdot u(t - m_i) u(t - m_j) u(t - m_k)$$
And in general:

\[ H_n[u(t)] = \sum_{i=1}^{M} \cdots \sum_{i=1}^{M} h_n(m_i, \cdots, m_n) \cdot \prod_{j=1}^{n} u(t - m_j) \]  \hspace{1cm} (3)

Where:

- \( H_n \) – \( n \) degree Volterra operator
- \( h_0 \cdots h_n \) – Volterra kernels
- \( N \) – the highest kernel degree in the series
- \( m_1 \cdots m_M \) – memory lag terms
- \( u(t - m_j) \) – input to the system, shifted in time
- \( y(t) \) – system output

In the equations above, \( h_0 \) is the system’s steady state response. \( H_1[u(t)] \) is the linear response part, where \( h_1 \) is the first order kernel. The linear part represents the convolution of the first order kernel with the system input. Higher order terms result in the system being nonlinear. The time shift and the existence of memory lag terms \( m_1 \cdots m_M \) makes the output dependent on prior time inputs. The number of memory lag terms \( M \), and the memory depth (how far it goes into the past) are independent of the highest degree of the series \( N \). As can be seen from the formulation, the series can be easily expanded to include higher degree kernels and more memory lag terms.
Additional system inputs can be added to the original formulation. This quickly turns the series into a cumbersome formulation, including the cross-kernels in addition to the direct ones. Consider an example with two inputs, i.e. $u(t)$ and $v(t)$:

$$y(t) = h_0 + \sum_{n=1}^{N} H^1_n[u(t)] + \sum_{n=1}^{N} H^2_n[v(t)] + \sum_{n=1}^{N} H^{12}_n[u(t), v(t)] + \sum_{n=1}^{N} H^{21}_n[u(t), v(t)]$$

(4)

Where:

$H^i_n$ – $n$ degree Volterra operator, for single input

$H^{ij}_n$ – $n$ degree Volterra operator, two inputs

$u(t), v(t)$ – two inputs to the system

(Note the added superscript to indicate inputs)

Presenting several lower order terms reveals the complexity of using the series for higher degrees $N$. In general, $h^{12}_2(m_i, m_j) \neq h^{21}_2(m_i, m_j)$ which complicates it even more.
The main challenge with the Volterra series is the identification of nonlinear kernels. This task becomes complicated as the order of the series grow. In practice, Volterra series are rarely used beyond the second degree and a single input, see [2], [3], [4] for several examples. This limits the potential of the method to “lightly non-linear” systems only. On the other hand, expanding the series beyond the second degree might result in better and more accurate models and alleviate convergence issues [6].

Generally, two approaches are available in the literature to perform the task of kernel identification, see [41] for a comparison. With the Lee-Schetzen cross-correlation formula [1], we require a white Gaussian input and isolation of each identified kernel. See also [6], [9]. With the second approach, we use system identification tools to estimate the kernels. Here we do not limit the model to 2\textsuperscript{nd} or 3\textsuperscript{rd} degrees, and the kernels are
identified all at once [13], [14]. In addition, no specific input signal is needed, and the
results are usually considered more accurate [13].

Before starting the identification process, it is worth simplifying the series as
much as possible. Following are several assumptions which will make the identification
simpler.

2.2 Simplifying Assumptions

We assume that the kernels are symmetric, i.e.:

$$h_n(m_1, m_2, \ldots, m_n) = h_n(m_2, m_1, \ldots, m_n) = h_n(m_n, m_1, \ldots, m_2) = \ldots$$

(6)

The above does not necessarily hold for a general nonlinear system, still it is
convenient, and it is a common assumption in the application of Volterra series, see [18]
or [42] for several examples.

Next, we assume that sparse, or diagonal, kernels are sufficient to represent the
system. This follows Balajewicz and Dowell [17] who showed excellent agreement
between the sparse and the full Volterra series in flutter/LCO simulations. See also [43].

We refer to this as no cross-memory effect, and it simplifies the series as follows:

$$h_n(m_i, m_j, m_k, \ldots) = 0 \ \text{for} \ i \neq j \neq k \neq \ldots$$

(7)
To understand how these assumptions make the formulation simpler, let us explore a basic system with \( N = 2, M = 2 \) and two inputs \( u(t), v(t) \). The Volterra representation of this system will be:

\[
y(t) = h_0 + h_1^1(m_1)u(t - m_1) + h_1^2(m_2)u(t - m_2) \\
  + h_2^1(m_1, m_1)u(t - m_1)u(t - m_1) \\
  + h_2^1(m_2, m_2)u(t - m_2)u(t - m_2) + h_2^2(m_1)u(t - m_1) \\
  + h_2^2(m_2)u(t - m_2) + h_2^2(m_1, m_1)u(t - m_1)v(t - m_1) \\
  + h_2^2(m_2, m_2)v(t - m_2)v(t - m_2) \\
  + h_2^{12}(m_1, m_1)u(t - m_1)v(t - m_1) \\
  + h_2^{12}(m_2, m_2)u(t - m_2)v(t - m_2)
\]

(8)

The number of unknown kernels is reduced from 25 to 11 already using (6) and (7). The following kernels still need to be identified:

\[
h_0, h_1^1(m_1), h_1^2(m_2), h_2^1(m_1, m_1), h_2^1(m_2, m_2), \\
h_2^2(m_1, m_1), h_2^2(m_1, m_2), h_2^2(m_2, m_2), \\
h_2^{12}(m_1, m_1), h_2^{12}(m_2, m_2)
\]

(9)

The following section presents a further way to simplify the series.
2.3 Laguerre Polynomials

The kernels can be expanded as a combination of base functions. There are several examples of this approach, e.g. wavelets [44], [45], orthonormal basis function expansion derived from pole location [46], and Laguerre polynomials [1], [15], [16]. Several examples of successful application of Laguerre polynomial approach can be found in [47], [48], [49]. These functions are convenient to use because they form an orthogonal base and are defined by a single parameter. The polynomials have the following form:

\[ L_R(t) = \sqrt{2a} \sum_{k=0}^{R} \frac{(-1)^k R! 2^{R-k}}{k! [(R - k)!]^2} (2at)^{R-k} e^{-at} \]  

(10)

Where:

- \( R \) – the order of the polynomial
- \( a \) – Laguerre time scale factor

The polynomial order indicates the number of zero crossings and \( \frac{1}{a} \) is referred to as the time constant [15]. Figure 1 and Figure 2 show the behavior of these functions with varying parameters:
Figure 1: Laguerre Polynomials with Constant $\alpha$ and Varying Order

Figure 2: Laguerre Polynomials with $R = 3$ and Varying $\alpha$
The Volterra kernels are assumed to be adequately approximated by a weighted sum of Laguerre functions. For the first order kernel it becomes a simple sum:

\[ h_1(m_i) \cong \sum_{r=1}^{R} \theta_r L_r(m_i) \]  

(11)

Where:

\( \theta_r \) – weights of the Laguerre polynomials

\( L_r(\quad) \) – Laguerre polynomial of order \( r \)

Higher order kernels can be treated as separable [48], which means they can be represented by a product of the base functions:

\[ h_n(m_i, \ldots) \cong \prod_{r=1}^{R} \prod_{j=1}^{n} L_r(m_i) \]  

(12)

Using Eq. (11) in the system from Eq. (8) with \( R = 2 \), the first order kernels will be represented in the following way:

\[ h_1^1(m_1) = \theta_1^1 L_1(m_1) + \theta_2^1 L_2(m_1); \quad h_1^2(m_2) = \theta_1^1 L_1(m_2) + \theta_2^1 L_2(m_2) \]  

\[ h_2^1(m_1) = \theta_1^2 L_1(m_1) + \theta_2^2 L_2(m_1); \quad h_2^2(m_2) = \theta_1^2 L_1(m_2) + \theta_2^2 L_2(m_2) \]  

(13)
Where:

$E, E/ –$ Laguerre functions calculated at lag times

$Q, Q+, Q/,$ $Q– –$ Laguerre weights (still unknown)

It is convenient to rewrite the system in matrix notation. For the first order kernels the formulation turns into:

$$
y(t) = h_0 + [u(t - m_1), u(t - m_2), v(t - m_1), v(t)

\begin{bmatrix}
L_1(m_1) & L_2(m_1) & 0 & 0 \\
L_1(m_2) & L_2(m_2) & 0 & 0 \\
0 & L_1(m_1) & L_2(m_1) & 0 \\
0 & L_1(m_2) & L_2(m_2) & 0
\end{bmatrix}

\begin{bmatrix}
\theta_1 \\
\theta_2 \\
\theta_3 \\
\theta_4
\end{bmatrix} = (14)

= h_0 + [U_1][B_1][\theta]

The second order kernels benefit from the orthogonality of Laguerre polynomials and from the separable assumption:

$$
\begin{align*}
h_2^1(m_1, m_1) &= \theta_2^1 L_1(m_1) L_1(m_1) + \theta_2^2 L_2(m_1) L_2(m_1) \\
h_2^1(m_2, m_2) &= \theta_2^1 L_1(m_2) L_1(m_2) + \theta_2^2 L_2(m_2) L_2(m_2) \\
h_2^2(m_1, m_1) &= \theta_2^2 L_1(m_1) L_1(m_1) + \theta_2^2 L_2(m_1) L_2(m_1) \\
h_2^2(m_2, m_2) &= \theta_2^2 L_1(m_2) L_1(m_2) + \theta_2^2 L_2(m_2) L_2(m_2) \\
h_2^{12}(m_1, m_1) &= \theta_2^{12} L_1(m_1) L_1(m_1) + \theta_2^{12} L_2(m_1) L_2(m_1) \\
h_2^{12}(m_2, m_2) &= \theta_2^{12} L_1(m_2) L_1(m_2) + \theta_2^{12} L_2(m_2) L_2(m_2)
\end{align*}

(15)
Where:

\[L_1(\_ )L_1(\_ ), L_2(\_ )L_2(\_ )\) – Laguerre functions products calculated at lag times

\[\theta_{21}^1, \theta_{22}^1, \theta_{21}^2, \theta_{22}^2, \theta_{21}^{12}, \theta_{22}^{12}\) – Laguerre weights (unknown)

Similarly to (14), the second order kernels can be written in the following way:

\[y(t) = [U_2][B_2]\{\theta\} \quad (16)\]

Where:

\[[U_2] = [u(t-m_1)u(t-m_1), u(t-m_2)u(t-m_2), v(t-m_1)v(t-m_1), v(t-m_2)v(t-m_2)]\]

\[[B_2] = \begin{bmatrix} L_1(m_1)L_1(m_1) & L_2(m_1)L_2(m_1) \\ L_1(m_2)L_1(m_2) & L_2(m_2)L_2(m_2) \end{bmatrix} \ldots \]

\[\begin{bmatrix} L_1(m_1)L_1(m_1) & L_2(m_1)L_2(m_1) \\ L_1(m_2)L_1(m_2) & L_2(m_2)L_2(m_2) \end{bmatrix} \quad (17)\]

\[\{\theta\} = \begin{bmatrix} \theta_{21}^1 \\ \theta_{22}^1 \\ \theta_{21}^2 \\ \theta_{22}^2 \\ \theta_{21}^{12} \\ \theta_{22}^{12} \end{bmatrix}\]

Note that the number of unknowns will not grow with additional lag terms.

Beyond that, the benefits of the formulation are not immediately evident for a second
order system but are significant for higher orders. When using Laguerre functions, the number of unknowns, for each input, will grow by $N \cdot R$ for $N$ kernel degree and $R$ Laguerre order (direct kernels). With standard Volterra formulation this number will increase by $M^N$ for $M$ lag terms and $N$ kernel degree.

To represent the full system from Eq. (8), we can combine both orders:

$$y(t) = h_0 + \left[ U_1, U_2 \right] \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \{\theta\}$$

(18)

This can be generalized into a convenient matrix form for an arbitrary degree:

$$y(t) = \left[ 1, U_1, U_2, \ldots \right] \begin{bmatrix} 1 \\ [B_1] \\ [B_2] \\ \vdots \end{bmatrix} \begin{bmatrix} h_0 \\ \theta \end{bmatrix} = [U][B]{\theta}$$

(19)

Where:

$[U]$ – row vector of past inputs and products of past inputs

$[B]$ – block matrix of Laguerre values

${\theta}$ – column vector of Laguerre coefficients and zero order kernel

The $[B]$ matrix can be calculated for a known order of Laguerre functions, and the $[U]$ vector combines the inputs and the products of inputs to the system. The ${{\theta}}$ vector of unknowns need to be identified.
2.4 System Identification

The known input – output values of the system, from CFD analysis for example, and the final form in Eq. (19) can be used to identify the unknown vector \{\theta\}. \([U]\) is calculated for the length of a training signal, thus expanding Eq. (19) into a system of linear algebraic equations. Using linear least squares approach, the vector \{\theta\} can then be calculated as follows:

\[
y = [U][B]\{\theta\}
\]
\[
[UB]^T y = [UB]^T [UB]\{\theta\}
\]
\[
[([UB]^T [UB])^{-1}] [UB]^T y = \{\theta\}
\]

2.4.1 The Orthogonal Approach

The matrix \([U][B]\) can be viewed as a set of column vectors, which are weighted according to the \{\theta\} coefficients in the final expression. Korenberg [13], [50] with co-authors [51], [52] makes a point of orthogonalizing this vector set over the data series, and hence making the estimated terms in \{\theta\} independent. Recursive application of orthogonalization provides unbiased estimates of the coefficients and indicates the contribution of each one [53].

Classic Gram-Schmidt (CGS), or the Modified Gram Schmidt (MGS) algorithms [51] are usually used to orthogonalize the vector set. MGS is better suited for large data
sets and calculations with finite precision [54], [55], because the vectors base is updated with each new orthogonalized vector.

### 2.4.2 Singular Value Decomposition

The singular value decomposition (SVD) of a general matrix \([X]\), \((X \in \mathbb{R}^{c \times d}, c > d)\) is given by the following:

\[
[X] = [U][\Sigma][V]^T
\]  

(21)

Where:

\([U] \in \mathbb{R}^{c \times c}, [V] \in \mathbb{R}^{d \times d}\) – orthonormal matrices, \(U^TU = I_c, V^TV = I_d\)

\([\Sigma] \in \mathbb{R}^{c \times d}\) – diagonal matrix of the singular values of \([X]\) (arranged in non-increasing order), \(\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_d \geq 0\)

The number of singular values which are equal or close to zero (below a certain threshold) is equal to the number of linearly dependent (or very close) rows in matrix \([X]\). These values contribute little to the \([X]\) matrix or to the inverse matrix \([X]^{-1}\). If the first \(e\) singular values are much larger than the last \(d - e\) values, an accurate expression for \([X]\) can be achieved by retaining the first \(e\) values only [56]. We avoid dealing with ill-conditioned matrices, and thus inverting this matrix becomes an easy task:
\[ [X]^{-1} = [V][\Sigma]^{-1}[U]^T = [V] \begin{bmatrix} \frac{1}{\sigma_1} \\ \vdots \\ \frac{1}{\sigma_e} \end{bmatrix} [U]^T \] (22)

Where:

\( \sigma_1, ..., \sigma_e \) – are the retained singular values

In our case, SVD helps with the final estimation of the \( \{\theta\} \) vector, after retaining singular values above certain threshold:

\[
[U][B] = [U][\Sigma][V]^T \\
{\theta} = [V][\Sigma]^{-1}[U]^Ty
\] (23)

### 2.4.3 L1 Regularization

The estimated vector \( \{\theta\} \) will typically have all non-zero terms. An efficient way to add sparsity to the model, and improve the quality of the identification, is by using L1 regularization [56], [57]. The regulated vector is sparse, easily interpreted, and often produces better results [57] as it removes the irrelevant data from the matrix \( [U][B] \). It constitutes an attractive approach as opposed to SVD. In our case, using L1 can result in fewer retained kernels, i.e. a more compact model.
2.5 Numerical Example – 2D NACA 0012 Airfoil

To evaluate the approach so far, we have used the AGARD CT2 and CT5 cases [58] of the NACA 0012 2D airfoil with oscillatory pitch and heave inputs:

\[ \alpha = \alpha_m + \alpha_0 \sin(k\tau) \]
\[ h = h_m + h_0 \cos(k\tau) \]  

(24)

Where:

\( \alpha_m, \alpha_0 \) – steady state and oscillatory pitch angle

\( h_m, h_0 \) – steady state and oscillatory heave motion, normalized to chord

\( k \) – reduced frequency

\( \tau \) – non-dimensional time

Reduced frequency and non-dimensional time are defined in the following way:

\[ k = \frac{\omega b}{U_\infty}; \quad \tau = \frac{tU_\infty}{b} \]  

(25)

Where:

\( \omega \) – dimensional frequency [rad/sec]

\( t \) – time [sec]

\( U_\infty \) - free stream air speed [m/sec]

\( b \) – airfoil half-chord [m]
A Harmonic Balance (HB) [59] based version of the standard Lax-Wendoff scheme [60], [61] for the 2D inviscid, compressible Reynolds averaged Navier-Stokes equations was used as the CFD solver. See the grid in Figure 3. The angle of attack and heave inputs from Eq. (24) were used with numerical values from Table 1. For each of the cases, three input conditions were used: pitch only input, heave only input, pitch + heave input. In each case the analysis was performed for 3 cycles, with 1000 time points for each cycle. Aerodynamic loads, i.e. lift and pitching moment, were calculated. See Figure 4 through Figure 7 for CT2 results, and Figure 8 through Figure 11 for CT5 results.

Figure 3: NACA 0012 Airfoil and Computational Grid
Table 1: Parameters for CFD Calculations

<table>
<thead>
<tr>
<th></th>
<th>CT2</th>
<th>CT5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_\infty$</td>
<td>0.6</td>
<td>0.755</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>$3.16^0$</td>
<td>$0.016^0$</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>$4.59^0$</td>
<td>$2.51^0$</td>
</tr>
<tr>
<td>$h_m$</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>$h_0$</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>$k$</td>
<td>0.0811</td>
<td>0.0814</td>
</tr>
</tbody>
</table>

Where:

$M_\infty$ – free stream Mach number

Figure 4: CFD Results, Lift Coefficient, CT2, Time History
Figure 5: CFD Results, Lift Coefficient, CT2

Figure 6: CFD Results, Moment Coefficient, CT2, Time History
Figure 7: CFD Results, Moment Coefficient, CT2

Figure 8: CFD Results, Lift Coefficient, CT5, Time History
Figure 9: CFD Results, Lift Coefficient, CT5

Figure 10: CFD Results, Moment Coefficient, CT5, Time History
2.5.1 System Identification Results

The CFD simulation was performed with the time step of 0.001sec. In the identification process we used fewer points – steps of 0.8-2sec. Inputs to CFD simulation, Eq. (24), together with the results were used to identify the Volterra kernels, or $\theta$ coefficients in Eq. (19).

Figure 12 through Figure 15 show typical output of the identified model, and the comparison with direct CFD results. The parameters ($N$, $R$, and the length of the training signal) are summarized in the titles. We can observe an excellent fit for all the cases even with a relatively short training signal. The results are shown for a two-variable input, however the procedure works equally well with one variable only.
Better results were achieved with L1 regularization. To understand the difference, compare Figure 13 with Figure 16. Unregulated fit fails short outside of the training signal. Moreover, the resulting model is more compact with less retained kernels. For example, moment coefficient fit of the CT2 case, and $N = 5, R = 3$ we get 7 non-zero kernels for L1 vs. 20 for the Least Squares case, see Figure 17 and Figure 18. On the other hand, the magnitude of the identified kernels is significantly larger for the L1 case.

Figure 12: Volterra ID Comparison to CFD Results, CT2, Lift
Figure 13: Volterra ID Comparison to CFD Results, CT2, Moment

Figure 14: Volterra ID Comparison to CFD Results, CT5, Lift
Figure 15: Volterra ID Comparison to CFD Results, CT5, Moment

Figure 16: Volterra ID Comparison to CFD Results, CT2, Moment

Least Square with SVD
Figure 17: Least Square Identified Kernels, CT2, Moment

Figure 18: L1 Identified Kernels, CT2, Moment
2.6 Challenges with the Direct Volterra ID

The identification results in the above figures display an excellent fit to the CFD data using a short training signal. However, there are several points which might be undesirable when applying the method on a larger scale (more degrees of freedom in an aeroelastic model) and still deriving meaningful results:

1. The ID process might return high degree kernels which result in a better fit, but do not contribute to a physical representation of the problem, see Figure 17 or Figure 18 for example. Moreover, the most important inputs, such as the linear contribution of the angle of attack or the heave velocity, might not get an adequate representation.

2. The memory depth of the Volterra model must be predetermined as one of the parameters. However, there is no clear procedure for identifying this value prior to the ID.

3. A separate ID procedure is required for every $k$ value. This might result in a large set of kernels and a complicated model for any practical application.
2.7 An Alternative Approach: Convolution and the Wagner Effect

The Wagner function, see [62] or [63] for example, expresses the development of unsteady aerodynamic lift on a 2D flat airfoil as a result of step change in the angle of attack. Half of the lift is generated at once and it reaches ~95% of the steady state value after about 30 half chord lengths, see Figure 19. The Wagner function can be used directly to calculate the unsteady lift:

\[
C_L(\tau) = \frac{L}{\rho U_\infty^2 b} = 2\pi \alpha_0 \phi(\tau)
\]  

(26)

Where:

\( C_L(\tau) \) – the lift coefficient

\( \rho \) – air density

\( \alpha_0 \) – angle of attack

\( \phi(\tau) \) – the Wagner function
If a form of a Wagner function (a step response) is known for some geometry and flight conditions, then the unsteady lift can be calculated for any response of the airfoil. Whether in a similar way to Eq. (26), or more generally using convolution. To use the latter, the kernels of the step response function need to be identified, then the lift can be calculated in the following way:

\[
C_l(\tau) = \int_0^\tau h(\tau - \tau^*) \alpha(\tau^*) \, d\tau^* = \int_0^\tau h(\tau^*) \alpha(\tau - \tau^*) \, d\tau^*
\]  

(27)
Where:

\( C_L(\ ) \) - the resulting aerodynamic lift coefficient

\( h(\ ) \) - the convolution kernels, previously identified from step response

\( \alpha(\ ) \) - input angle of attack

\( \tau \) – time, non-dimensional time

\( \tau^* \) - dummy variable

Eq. (27) can be simplified in the discrete time case, the integral turned into a sum and the total time span is determined according to a known number of time steps:

\[
C_L(i\Delta\tau) = \sum_{j=0}^{i} h(j\Delta\tau) \alpha(i\Delta\tau - j\Delta\tau)
\]  

(28)

Where:

\( \Delta\tau \) – time step

\( i = 0,1,2, \ldots \) – integers to account for time point

\( j = 0,1, \ldots ,i \)

Eq. (28) can be used in two ways: first to identify the kernels \( h(\ ) \), with known \( C_L(\ ) \), and \( \alpha(\ ) \) vectors of the step response. Then, second and later with a new \( \alpha(\ ) \) vector, and the identified kernels, to generate the unsteady lift.
2.7.1 Kernel Identification

It is useful to write the first several terms in Eq. (28):

\[ i = 0: C_L(0) = h(0)\alpha(0) \]
\[ i = 1: C_L(\Delta \tau) = h(0)\alpha(\Delta \tau) + h(\Delta \tau)\alpha(0) \]
\[ i = 2: C_L(2\Delta \tau) = h(0)\alpha(2\Delta \tau) + h(\Delta \tau)\alpha(\Delta \tau) + h(2\Delta \tau)\alpha(0) \]  
\[ i = 3: C_L(3\Delta \tau) = h(0)\alpha(3\Delta \tau) + h(\Delta \tau)\alpha(2\Delta \tau) + h(2\Delta \tau)\alpha(\Delta \tau) + h(3\Delta \tau)\alpha(0) \]  

Expanding to any \( i \), we can write the above in a convenient matrix form:

\[
\begin{pmatrix}
C_L(0) \\
C_L(\Delta \tau) \\
C_L(2\Delta \tau) \\
C_L(3\Delta \tau) \\
\vdots
\end{pmatrix} =
\begin{pmatrix}
\alpha(0) & & & \\
\alpha(\Delta \tau) & \alpha(0) & & \\
\alpha(2\Delta \tau) & \alpha(\Delta \tau) & \alpha(0) & \\
\alpha(3\Delta \tau) & \alpha(2\Delta \tau) & \alpha(\Delta \tau) & \alpha(0) \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\begin{pmatrix}
h(0) \\
h(\Delta \tau) \\
h(2\Delta \tau) \\
h(3\Delta \tau) \\
\vdots
\end{pmatrix}
\]

\[ (30) \]

Or:

\[
\begin{pmatrix}
\vdots \\
C_L \\
\vdots
\end{pmatrix} =
\begin{pmatrix}
\vdots \\
\vdots \\
\vdots
\end{pmatrix}
\begin{pmatrix}
\vdots \\
\vdots \\
\vdots
\end{pmatrix}
\]

\[ (31) \]

The kernels vector \( \{h\} \) can be calculated easily, without approximation, by inverting the square, lower triangular matrix \( [\alpha] \):

\[ \{h\} = [\alpha]^{-1}\{C_L\} \]  

\[ (32) \]
Once the kernels are identified, we can use Eq. (28) again with any input to generate the unsteady lift.

In the next section we will use the classic theories of the unsteady lift for 2D flat airfoil to verify this approach.

### 2.7.2 2D Verification Example

This simple example uses a 2D flat airfoil, see Figure 20, with two degrees of freedom (DOF): the pitch angle $\alpha$, and the plunge $h$. The airfoil has a chord of $2b$ with $ba$ as the location of the plunge DOF. For straightforward comparison to classic results in the literature, only the pitch will be considered in this example.

![Figure 20: 2D Flat Airfoil](image)

Theodorsen [64] calculated the loads on oscillating airfoil in potential subsonic flow, resulting in the following expression using two DOF:
\[ L = \pi \rho b^2 \left[ \dot{h} + U_\infty \dot{\alpha} - b a \ddot{\alpha} \right] + 2 \pi \rho U_\infty b C(k) \left[ \dot{h} + U_\infty \alpha + b \left( \frac{1}{2} - a \right) \dot{\alpha} \right] \]  \hspace{1cm} (33)

Where:

\( C(k) \) – the complex Theodorsen function, see Figure 21

The lift generated by the Wagner function can be compared to Theodorsen lift for pitching motion only [65]:

\[ L = \pi \rho b^2 \left[ U_\infty \dot{\alpha} - b a \ddot{\alpha} \right] + 2 \pi \rho U_\infty b C(k) \left[ U_\infty \alpha + b \left( \frac{1}{2} - a \right) \dot{\alpha} \right] \]  \hspace{1cm} (34)
As the input, a general oscillating pitch only motion will be used, with the use of reduced frequency and non-dimensional time:

\[ k = \frac{\omega b}{U_{\infty}}, \tau = \frac{U_{\infty}t}{b} \]

\[ \alpha = \tilde{a}e^{i\omega t} = \tilde{a}e^{ikt} \]  

(35)

Substituting (35) into (34) results in the following expression:

\[ C_L = \frac{L}{\rho U_{\infty}^2 b} = \left\{ \pi jk + ak^2 \right\} + 2\pi C(k) \left[ 1 + \left( \frac{1}{2} - a \right)jk \right] \tilde{a}e^{ikt} \]  

(36)

Unsteady lift using the Wagner function can be calculated as follows [62]:

\[ L = \pi \rho b^2 [\tilde{h} + U_{\infty} \tilde{a} - b \tilde{a} \tilde{a}] \]

\[-2\pi \rho U_{\infty} b \left[ \frac{w_3}{\pi} (0) \phi(\tau) + \int_{0}^{\tau} \frac{dw_3}{\pi} (\sigma) \frac{d\tau}{d\sigma} \phi(\tau - \sigma) d\sigma \right] \]  

(37)

Where:

\[ w_3( ) \] – is the downwash at \( \frac{3}{4} \) chord

\[ \sigma \] – integration variable

The first term is identical to Theodorsen in Eq. (34). We can simplify the second term using integration by parts:
\[
\int_{0}^{s} \frac{dw_{3c}(\sigma)}{d\sigma} \phi(\tau - \sigma)d\sigma = w_{3c}(\sigma)\phi(\tau - \sigma) \bigg|_{0}^{\tau} - \int_{0}^{\tau} w_{3c}(\sigma) \left( \frac{d\phi}{d\sigma}(\tau - \sigma) \right) d\sigma
\]

\[= w_{3c}(\tau)\phi(0) - w_{3c}(0)\phi(\tau) - \int_{0}^{\tau} w_{3c}(\sigma) \frac{d\phi}{d\sigma}(\tau - \sigma)d\sigma \quad (38)\]

Inserting (38) back into (37) will give a simpler expression:

\[L = \pi \rho b^2 \left[ U_{\infty} \dot{\alpha} - ba\ddot{\alpha} \right] - 2\pi \rho U_{\infty} b \left[ w_{3c}(\tau)\phi(0) - \int_{0}^{\tau} w_{3c}(\sigma) \frac{d\phi}{d\sigma}(\tau - \sigma)d\sigma \right] \quad (39)\]

To evaluate the equation, the downwash at 3/4 chord needs to be calculated.

Using the notations in Figure 20, the vertical position and velocity can be defined:

\[z = -h - \alpha(x - ab) \quad (40)\]

\[w(x) = \frac{\partial z}{\partial t} = -\dot{h} - \dot{\alpha}(x - ab) - U_{\infty}\alpha \quad (41)\]

Substituting \(x = b/2\) for 3/4 chord, and using non-dimensional time:

\[w_{3/4c}(t) = -\dot{h} - \dot{\alpha}\left(\frac{b}{2} - ab\right) - U_{\infty}\alpha = -h - \dot{\alpha}b\left(\frac{1}{2} - a\right) - U_{\infty}\alpha \quad (42)\]

\[w_{3/4c}(\tau) = -\frac{U_{\infty}}{b} \frac{\partial h}{\partial \tau} - U_{\infty} \frac{\partial \alpha}{\partial \tau} \left(\frac{1}{2} - a\right) - U_{\infty}\alpha \quad (43)\]

Eq. (43) is simplified for pitch input only:

\[w_{3/4c}(\tau) = -U_{\infty} \left[ \frac{\partial \alpha}{\partial \tau} \left(\frac{1}{2} - a\right) + \alpha \right] \quad (44)\]
Substituting the input from (35):

\[ w_{3/4}^c(\tau) = -U_\infty \left[ jk \left( \frac{1}{2} - a \right) + 1 \right] \bar{a} e^{jk\tau} \]  

(45)

Equation (39) can be evaluated with (45) and using Jones’s approximation of the Wagner function, see [62] for example:

\[ \phi(\tau) \cong 1 - 0.165e^{-0.0455\tau} - 0.335e^{-0.3\tau} \]  

(46)

Remembering \( \phi(0) = 0.5 \), the second term in (39) becomes:

\[ \left[ \begin{array}{c}
\int \\
0
\end{array} \right] = -2\pi \rho U_\infty^2 b \left[ jk \left( \frac{1}{2} - a \right) + 1 \right] \bar{a} \left[ \frac{1}{2} e^{jk\tau} \right. \\
\left. - \int_0^\tau e^{jk\sigma} \left( \psi_1 \varepsilon_1 e^{-\varepsilon_1(\tau-\sigma)} + \psi_2 \varepsilon_2 e^{-\varepsilon_2(\tau-\sigma)} \right) d\sigma \right] \]  

(47)

Finally, solving the integral in (47):

\[
\int = \psi_1 \frac{\varepsilon_1}{(\varepsilon_1 + jk)} e^{-\varepsilon_1\tau} e^{(\varepsilon_1 + jk)\sigma} + \psi_2 \frac{\varepsilon_2}{(\varepsilon_2 + jk)} e^{-\varepsilon_2\tau} e^{(\varepsilon_2 + jk)\sigma} \left. \right|_0^\tau \\
= \psi_1 \frac{\varepsilon_1}{(\varepsilon_1 + jk)} \left( e^{jk\tau} - e^{-\varepsilon_1\tau} \right) + \psi_2 \frac{\varepsilon_2}{(\varepsilon_2 + jk)} \left( e^{jk\tau} - e^{-\varepsilon_2\tau} \right) \\
= \left[ \psi_1 \frac{\varepsilon_1}{(\varepsilon_1 + jk)} \left( 1 - e^{-(\varepsilon_1 + jk)\tau} \right) \\
+ \psi_2 \frac{\varepsilon_2}{(\varepsilon_2 + jk)} \left( 1 - e^{-(\varepsilon_2 + jk)\tau} \right) \right] e^{jk\tau} \]  

(48)

Using (47) and (48) in Eq. (39) will give us the lift with the Wagner effect:
\[ C_L = \frac{L}{\rho U_\infty^2 b} = \left\{ \pi [jk + ak^2] + 2\pi \left[ \frac{1}{2} - a \right] \left[ 1 + \psi_1 \frac{\varepsilon_1}{(\varepsilon_1 + jk)} \left( 1 - e^{-(\varepsilon_1 + jk)\tau} \right) \right] \right\} \bar{a} e^{i\omega \tau} \]  

(49)

Where:

\[ \psi_1 = 0.165; \ \psi_2 = 0.335 \]

\[ \varepsilon_1 = 0.0455; \ \varepsilon_2 = 0.3 \]

In the next sections we compare the unsteady lift from Eq. (28) to the results in Eq. (36) and (49).

2.7.3 Calculation Procedure

1. Use the Wagner function, Eq. (46) and Figure 19, with step input to identify the kernels, vector \( \{h\} \) in Eq. (32).

2. Use the newly identified kernels in Eq. (28) with oscillatory input from (35) to generate the unsteady lift.
2.7.4 Numerical Results

1. Figure 22 shows the results of kernels identification of the Wagner function. “Input Points” with unity step input were used to calculate the kernels using Eq. (32). “Identified” points were calculated with Eq. (30). The matrix $[\alpha]$ is square, therefore the identification is exact.

2. Figure 23 and Figure 24 present a comparison of the lift resulting from oscillatory pitch input. The Wagner lift exhibits a transition phase and matches Theodorsen once steady state is established. Quasi steady response uses Eq. (36) with $C(k) = 1$. Note the convolution which matches the Wagner lift almost completely. Moreover, kernels which were identified once can then be used with any frequency of the input. The only difference will be the recalculated $[\alpha]$ matrix in Eq. (31).

3. The match is also favorable when using richer input, Figure 25. The input consists of sine sum for 10 reduced frequencies: $\tilde{a} \sum e^{ikt}$ for $k = 0.1: 0.1: 1$. The convolution results show a good match with the Wagner output. Only the real part is shown in the figure for clarity and simplicity.
Figure 22: Wagner Kernel Identification

Figure 23: Lift Comparison, $k=0.2$
Figure 24: Lift Comparison, k=0.7

Figure 25: Lift Comparison, k=0.1 to 1 (real part only shown)
The comparisons of the simple formulation above show the potential for using the Wagner effect (convolution) for unsteady aerodynamic load generation. So far, the examples were simple linear and potential flow cases. In the following sections we will expand this approach to more complicated cases, involving CFD generated data and nonlinearities.
2.8 Using the Convolution Integral with CFD Generated Data

In the previous section we have shown the applicability of the approach with a simple analytical case. In this section we will take it further and use CFD generated data for the kernel estimation. First, a step response input-output relation to be used for identification was generated with CFD simulation. Next, aerodynamic responses due to a sinusoidal airfoil motion were calculated with CFD. These simulation results will be used for evaluation purposes of the identification process.

The CFD simulations were performed in FLUENT (a commercial CFD code) using a truncated version of the 2D NASA grid [66] for the NACA0012 airfoil with 257 points on the airfoil. This grid extends 40 chords in each direction. Mach numbers of 0.6 and 0.755 were used, as in Section 2.5. Since buffeting behavior was expected in some cases, the computational model was chosen as per [67]. The mesh motion was implemented using prescribed motion of the airfoil about its quarter-chord from zero angle of attack. The mesh deformation was biased to the outer nodes, meaning that the quality of the mesh in the near field was maintained.

Pitch and heave step responses were analyzed separately. To obtain smoother numerical results, an exponential input was used instead of a pure step for the pitch input, see Eq. (50) below. Vertical velocity, i.e. heave input, was applied directly.

\[ \alpha(t) = \alpha_0 \left(1 - e^{-\tau/\tau_{ref}}\right) \]  \hspace{1cm} (50)
Where:

\( \alpha_0 \) – the desired angle of attack

\( \tau \) – non-dimensional time

\( \tau_{ref} = 0.5 \) – was maintained in the following

2.8.1 Step Simulation Results – Pitch

Step response data was obtained with increasing angle of attack (AOA). The simulations were performed for 1°, 2°, 4°, 6°, 8°, 10°, 12° AOA. An additional AOA of 3° was added for \( M = 0.755 \). The results are shown in Figure 26 and Figure 27 for \( M = 0.6 \) and Figure 28 and Figure 29 for \( M = 0.755 \). We note some numerical irregularities in the figures around \( \tau = 0 \). These could not be eliminated completely. As we will see further, these irregularities do not interfere with the subsequent procedure.

In order to establish the linearity limit of the system, normalized loads were calculated, i.e. \( C_{L_\alpha} = \frac{C_L}{\alpha_0}, C_{M_\alpha} = \frac{C_M}{\alpha_0} \). These are shown in Figure 30 through Figure 33. For \( M = 0.6 \) the response is fairly linear up until 4° AOA and becomes slightly nonlinear for 6°. Then the lift fluctuates in a limit cycle for 8° and 10° due to shock wave boundary layer interaction that leads to “buffet”. At 12° the airfoil loses lift significantly, because of earlier flow separation. For \( M = 0.755 \) the linear response ends earlier at 2°, and an additional 3° step response was calculated to assess the transition from linear to
non-linear response. The lift becomes oscillatory from $4^\circ$ through $8^\circ$ and goes to a lower static limit for lift at higher AOA after one to two cycles.

The moment coefficients throughout this work are calculated around the leading edge of the airfoil.

![CFD Results - Lift Coefficient, Step Input, M=0.6](image)

**Figure 26: Simulation Results – Lift, Step Input, M=0.6**
Figure 27: Simulation Results – Moment, Step Input, M=0.6

Figure 28: Simulation Results – Lift, Step Input, M=0.755
Figure 29: Simulation Results – Moment, Step Input, M=0.755

Figure 30: Simulation Results – Normalized Lift, Step Input, M=0.6
Figure 31: Simulation Results – Normalized Moment, Step Input, M=0.6

Figure 32: Simulation Results – Normalized Lift, Step Input, M=0.755
2.8.2 Kernel ID Determined from CFD Data

The step simulation results can now be used to identify the convolution kernels, similar to Eq. (31) and (32), for both lift and moment coefficients. For identification purposes a larger non-dimensional time step of $\tau = 0.1 - 1$ can be used, instead of the ~0.002 sec step in the CFD simulation. See Figure 34 and Figure 35 for example.

The magnitude of identified kernels decreases as the steady state is approached. At a certain point, adding the next time step kernel to the solution will not contribute significantly to the overall result. Therefore we can omit these kernels entirely, decreasing model size and effectively establishing the memory depth of the system. A fully identified kernel can be neglected below a certain response threshold without loss in
further accuracy. For example, out of 160 identified kernels in Figure 36, 112 are retained above 5e-4 and 100 are retained above 1e-3 limits. Figure 37 shows a comparison of the convoluted step response using these kernel sets.

Once the kernels are identified, they can be used with various inputs to establish the aerodynamic loads at different flight conditions, and for different motions of the airfoil or wing.

Figure 34: Simulation Results and the Data for ID, Lift
Figure 35: Simulation Results and the Data for ID, Moment

Figure 36: Magnitude of Identified Kernels vs. Kernel Number
2.8.3 Convolution with CFD Identified Kernels, Pitch Input

To verify the approach, several CFD analyses with sinusoidal airfoil motion were performed. See Table 2 for the summary of input parameters. The inputs were convoluted with previously identified kernels and compared to CFD results. Figure 38 and Figure 39 show results for 20Hz, Figure 40 and Figure 41 show results for 5Hz motion. The agreement between the convoluted response and CFD results is very good, excluding minimal discrepancy in the beginning of the motion, $\tau < 10$. Convolution cannot follow these numerical anomalies.
Typical results for $M = 0.755$ can be seen in Figure 42 through Figure 45. The agreement between the convolution and CFD results remains good. A small difference at the peaks is evident compared to $M = 0.6$.

The results for different $k$ values were obtained with a single set of step response identified kernel, for each of the Mach numbers. This is a significant reduction of the identification effort compared to Section 2.5. In addition, the retained kernels provide a simple method for estimating the memory depth of the system.

Table 2: Input Parameters for Sinusoidal CFD Simulations - $a_0 \sin(k \tau)$

<table>
<thead>
<tr>
<th>$M_\infty$</th>
<th>0.6</th>
<th>0.755</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$ [Hz]</td>
<td>1, 5</td>
<td>20, 1, 5, 20</td>
</tr>
<tr>
<td>$k$</td>
<td>0.015</td>
<td>0.075</td>
</tr>
<tr>
<td>$\alpha_0$ [deg]</td>
<td></td>
<td>2, 5, 8</td>
</tr>
</tbody>
</table>
Figure 38: CFD vs. Convolution, $M=0.6$, 20Hz Sinusoidal Motion in Pitch, $C_L$

Figure 39: CFD vs. Convolution, $M=0.6$, 20Hz Sinusoidal Motion in Pitch, $C_M$
Figure 40: CFD vs. Convolution, $M=0.6$, 5Hz Sinusoidal Motion in Pitch, $C_L$

Figure 41: CFD vs. Convolution, $M=0.6$, 5Hz Sinusoidal Motion in Pitch, $C_M$
Figure 42: CFD vs. Convolution, $M=0.755$, 1Hz Sinusoidal Motion in Pitch, $C_L$.

Figure 43: CFD vs. Convolution, $M=0.755$, 1Hz Sinusoidal Motion in Pitch, $C_M$. 

65
Figure 44: CFD vs. Convolution, \( M=0.755 \), 5Hz Sinusoidal Motion in Pitch, \( C_L \)

Figure 45: CFD vs. Convolution, \( M=0.755 \), 5Hz Sinusoidal Motion in Pitch, \( C_M \)
2.8.4 CFD Results, ID, and Convolution for Heave Inputs

So far, we looked at aerodynamic loads as a result of pitch input, which was consistent with the analytical foundation in Section 2.7. Here we will add the process for the heave Degree of Freedom (DOF). This serves as a basis for true Multiple Degrees of Freedom (MDOF) implementation of the approach in the future.

The CFD step response analysis was performed with increasing inputs of the vertical velocity, we used 1, 2, 4 m/sec for each of the Mach numbers. The results for $M = 0.6$ are shown in Figure 46 and Figure 47, $M = 0.755$ results are shown in Figure 48 and Figure 49. We can observe a typical transition into a steady state of the lift coefficient for $M = 0.6$, and a more involved oscillating behavior for $M = 0.755$. The moment coefficient, on the other hands, does not end in a finite value in both Mach cases.

Once the coefficients are normalized, a linear behavior is evident, Figure 50 and Figure 51 for $M = 0.6$ and Figure 52 and Figure 53 for $M = 0.755$. Interestingly, the normalized results are nearly identical even for the higher Mach number. This is expected as the maximum (4 m/sec) effective angle of attack is well below the linear limit which we established earlier for pitch motion, see Eq. (51). The results are also consistent with Section 2.5.

\begin{align}
M = 0.6, U &= 208 \text{ m/sec} \rightarrow \alpha_{\text{eff}} = \frac{\dot{h}}{U} \approx 1.1^0 \\
M = 0.755, U &= 262 \text{ m/sec} \rightarrow \alpha_{\text{eff}} = \frac{\dot{h}}{U} \approx 0.9^0
\end{align}

(51)
With the step response data, the process of kernels identification and convolution is similar to pitch inputs. Because we are operating in the linear limit, the ID process is exact and convolution results coincide with CFD data. See Figure 54 and Figure 55 for $M = 0.6$, and Figure 56, Figure 57 for $M = 0.755$. The correlation is very good for $M = 0.6$. For $M = 0.755$ there is a discrepancy between CFD and the convoluted results. The CFD step analysis for $M = 0.755$ stopped early due to distorted mesh and did not reach a steady state, see Figure 48 and Figure 49. As a result the fully developed kernel set cannot be identified, and consequently the convolution with this set will not produce adequate results.

We can expand the model into the non-linear range by adding higher rates of vertical displacement. However, this will also expand it beyond the reasonable values that might be encountered in practice, see previous results in Section 2.5 or the source in [58], and might be challenging with the current mesh settings. Therefore the heave convolution will remain linear in our work. This demonstrates the option of “turning on” non-linearity for some DOFs while others can remain in the linear range.
Figure 46: Simulation Results – Lift, Heave Step Input, M=0.6

Figure 47: Simulation Results – Moment, Heave Step Input, M=0.6
Figure 48: Simulation Results – Lift, Heave Step Input, M=0.755

Figure 49: Simulation Results – Moment, Heave Step Input, M=0.755
Figure 50: Normalized Results – Lift, Heave Step Input, M=0.6

Figure 51: Normalized Results – Moment, Heave Step Input, M=0.6
Figure 52: Normalized Results – Lift, Heave Step Input, M=0.755

Figure 53: Normalized Results – Moment, Heave Step Input, M=0.755
Figure 54: CFD vs. Convolution, M=0.6, 20Hz Sinusoidal Motion in Heave, $C_L$

Figure 55: CFD vs. Convolution, M=0.6, 20Hz Sinusoidal Motion in Heave, $C_M$
Figure 56: CFD vs. Convolution, M=0.755, 5Hz Sinusoidal Motion in Heave, $C_l$

Figure 57: CFD vs. Convolution, M=0.755, 5Hz Sinusoidal Motion in Heave, $C_M$
2.9 Expanding the Model Beyond the Linear Limit

The convolution approach works within the limitations of linear input-output relations. However, outside of the linear range, these kernels will not produce meaningful results. See for example Figure 58 or Figure 59 where the difference between linear convolution and CFD results is evident. Still, our goal is to establish a framework for unsteady loads calculation which includes the non-linear range. In this section we will present the combined approach to accomplish this goal and produce a useful nonlinear model.

Figure 58: $10^\circ$ Step Response vs. Convolution, $M=0.6$
In the beginning of this work, Section 2.3, a convenient Volterra series model was developed. The model is capable of successfully identifying the non-linear relations while being compact and simple to use. In this section, we will use this model to expand the linear one: adding non-linear, higher order Volterra kernels to the established linear convolution model. With this approach, we are keeping physically dominant linear kernels, while adding a correction to the identified system which yields the non-linear behavior.

We propose the following framework:
In the following we designate with $L$ the linear terms and with $NL$ the non-linear ones. Also, the equations will consider lift coefficients for convenience, however the procedure is similar for moment or any other aerodynamic load.

After the linear kernels are identified, $\{h\}$ in Eq. (32), they can be used with any input to estimate the results, for example with inputs beyond the linear limit:

$$\{C_L\}_{High} = [\alpha]_{Big}\{h\}_{L}$$  \hspace{1cm} (52)
The difference between this convoluted value and the CFD results is calculated next:

$$\Delta \{C_L\} = \{C_L\}_\text{CFD} - \{C_L\}_\text{High}$$  \hspace{1cm} (53)

If the above difference is significant, non-linear kernels can be added using Volterra series. The $[U]$ matrix is assembled with $[\alpha]_{Blg}$ and together with $\Delta \{C_L\}$ is used to identify the $\{\theta\}$ vector in Eq. (19). The linear response is generally the dominant portion of the aerodynamic load, therefore the resultant Volterra terms form a smaller, non-linear correction for the convolution:

$$\{C_L\}_\text{NL} = [\alpha]_{Blg}[h]_L + [U]_{Blg}[B]\{\theta\}$$  \hspace{1cm} (54)

The matrix $[U]$ is assembled with powers of AOA, we accounted for the possible symmetry and anti-symmetry of the flow with all non-linear terms in the following way:

$$[U] = [(\text{sign}(\alpha)\alpha^2 \ldots), (\alpha^3 \ldots), (\text{sign}(\alpha)\alpha^4 \ldots), \ldots]$$  \hspace{1cm} (55)

Using the difference for the ID purposes, as in Eq. (53), is beneficial because it allows us to estimate whether the non-linear addition is needed, i.e. if the difference is small, the model can remain linear.

The benefits, which we established with the Laguerre formulation, are still significant. This time however, the entire step response CFD simulation can be included
as the input-output data for training. Moreover, the trimmed kernels become an
effective estimate of the memory depth of the system, rather than a free parameter
defined by user. Furthermore, because the number of unknowns will not grow with added lag
terms in the Laguerre formulation, the model remains compact even when a large number of
linear kernels is used.

The remaining parameters of the series, $N, R, a$, can be chosen to give the best
approximation. The parameters can be selected by trial and error, or with a systematic
approach to yield the best fit for the step response data.

### 2.9.1 Parameter Selection for NL Identification

The Volterra series formulation is guided by several parameters, see Sections 2.3
and 2.4. $N$ sets the highest kernel degree, which in turn defines the product degree of the
convoluted input. $a$ is the time scale and $R$ is the highest order of Laguerre polynomials
and the number of zero crossings. For a defined $a$, or timescale, increasing $R$ will result
in a richer signal near $\tau = 0$. The same is true for increasing $a$ with a defined value for $R$,
see Figure 1 and Figure 2. Combining the two allows for an appropriate representation
of complicated signals.

With our approach, a different set of non-linear kernels is generated for each
change in behavior in the step response simulation, i.e. kernels between $6^0$ and $8^0$ will be
different than between $8^0$ and $10^0$. Each can benefit from a proper adjustment of the
parameters. One way to choose the best parameter set for each application is to calculate the cumulative error between the generated data and CFD result for each of the parameter sets, see Eq. (56). We continue here with $C_L$ for convenience:

$$
\% \text{error} = 100 \sqrt{\frac{1}{N_{\text{points}}} \sum_{i=1}^{N_{\text{points}}} \left[ C_L^{\text{CFD}}(\tau_i) - C_L^{\text{NL}}(\tau_i) \right]^2} \tag{56}
$$

Where:

$N_{\text{points}}$ – number of time points where data is available

$C_L^{\text{CFD}}(\tau_i)$ – simulation result, step response

$C_L^{\text{NL}}(\tau_i)$ – the total identified load from Eq. (54)

After the identification error is known for each parameter set, the most appropriate set can be chosen. Generally we balance between the size of the model and the fit quality, i.e. the error should be minimal as long as the model size is acceptable. Figure 61 and Figure 62 show examples for parameter selection.

The fit is better when the order of Volterra series and Laguerre polynomials is increased, which is expected. At the same time the improvement is not significant as to prefer $N = 5, R = 15$ over $N = 3, R = 10$ for example. The comparison is shown in Figure 63 and Figure 64. This is even less significant for the subsequently convoluted response, as we will see in the next chapters.
Parameter Selection, M=0.755, 6° Step Fit

5.98% Error for N=2 → a=3 R=15
5.49% Error for N=3 → a=3 R=15

5.06% Error for N=4 → a=3 R=10
4.68% Error for N=5 → a=3 R=15

Figure 61: Parameter Selection, M=0.755, 6° Step Fit

Parameter Selection, M=0.6, 8° Step Fit

5.51% Error for N=2 → a=3 R=15
5.48% Error for N=3 → a=3 R=10

5.47% Error for N=4 → a=3 R=15
5.36% Error for N=5 → a=3 R=15

Figure 62: Parameter Selection, M=0.6, 8° Step Fit
Figure 63: Volterra Fit, $M=0.6$, $8^\circ$ Step (1)

Figure 64: Volterra Fit, $M=0.6$, $8^\circ$ Step (2)
2.9.2 Non-Linear Kernel Identification for M=0.6

In this chapter we will show the results for expanding the convolution step response model beyond the linear limit, following the framework in Figure 60 and parameter selection process in the previous chapter. Volterra parameters are summarized in the titles. For $M = 0.6$ the steady state response below $6^\circ$ step is nearly perfectly convoluted with linear kernels; the NL addition in this case is improving the transient, see Figure 65 and Figure 66. For larger AOA responses the NL correction is more significant and is shown in Figure 67 through Figure 70.

![CFD vs. L/NL Convolution, M=0.6: N=3, a=4, R=10](image)

Figure 65: Adding NL Volterra Correction, M=0.6, 2° Step
Figure 66: Adding NL Volterra Correction, M=0.6, 4° Step

Figure 67: Adding NL Volterra Correction, M=0.6, 6° Step
Figure 68: Adding NL Volterra Correction, M=0.6, 8° Step

Figure 69: Adding NL Volterra Correction, M=0.6, 10° Step
2.9.3 Non-Linear Kernel Identification for \( M = 0.755 \)

For \( M = 0.755 \) the linear angle of attack limit is lower and thus requires adding the Volterra terms starting from smaller AOA. To add variety, we will show here the adjustment for moment coefficients, where the fit can be more challenging. The results are shown in Figure 71 through Figure 77. The parameters are summarized in the titles, as before.
Figure 71: NL Volterra Addition, $M=0.755$, 2\textdegree Step

Figure 72: NL Volterra Addition, $M=0.755$, 3\textdegree Step
Figure 73: NL Volterra Addition, M=0.755, 4º Step

Figure 74: NL Volterra Addition, M=0.755, 6º Step
Figure 75: NL Volterra Addition, $M=0.755$, $8^\circ$ Step

Figure 76: NL Volterra Addition, $M=0.755$, $10^\circ$ Step
2.10 Using the Linear and Non-Linear Kernels

At this stage the linear and the non-linear kernels were identified from the CFD calculated step response. With the full kernel set the aerodynamic loads model can be used. The linear input matrix \([\alpha]\), and the product input matrix \([U]\) in Eq. (54) are populated according to the known inputs, and are later convoluted with the identified kernels.

To verify the results we employed the established technique throughout this research. The convoluted results were compared to a full CFD analysis of the sinusoidal motion of the airfoil. We have previously established the linearity of the heave motion, therefore the comparison here is for pitch inputs with parameters from Table 2.
In order to make a proper estimation with the sinusoidal loads, we need to choose an adequate step response data to derive the kernels from. Ideally the levels will match or be close enough. For example, for sinusoidal motion of $2^\circ$, a step response of $2^\circ$ will be used, for a $5^\circ$ motion we will use a $4^\circ$ or $6^\circ$ step response, and for $8^\circ$ oscillations a $8^\circ$ step response will be used. This division is expected due to substantially different behavior of the aerodynamic loads at different angles of attack, see Figure 26 through Figure 33.

### 2.10.1 Unsteady Aerodynamic Loads for M=0.6

At $2^\circ$ the linear convolution is sufficient to represent the oscillating loads, see Figure 78 through Figure 83. The agreement between CFD results and the convolution is very good, excluding a minor discrepancy at 20Hz in the transient portion. This is due an anomaly in the CFD result which cannot be followed by the convolution. The linear kernels were identified once, using the step response data, and then convoluted with different sinusoidal pitching motion inputs.
Figure 78: Results, $M=0.6$, $2^\circ$ Sinusoidal Input, 1Hz, $C_L$

Figure 79: Results, $M=0.6$, $2^\circ$ Sinusoidal Input, 1Hz, $C_M$
Figure 80: Results, $M=0.6$, $2^\circ$ Sinusoidal Input, 5Hz, $C_L$

Figure 81: Results, $M=0.6$, $2^\circ$ Sinusoidal Input, 5Hz, $C_M$
Figure 82: Results, $M=0.6$, $2^\circ$ Sinusoidal Input, 20Hz, $C_L$

Figure 83: Results, $M=0.6$, $2^\circ$ Sinusoidal Input, 20Hz, $C_M$
For $5^\circ$ AOA the non-linear effects become noticeable and the NL kernels should be added using the step response data. We are using a $6^\circ$ step response as the closest available to the desired outcome, see the identified curves in Figure 84 and Figure 85. The $4^\circ$ step response is linear at $M = 0.6$ and therefore cannot be used for NL kernels estimation.

The added NL kernels are used, together with the linear ones, to obtain the loads using the known pitching motion inputs with different frequencies. The results are shown in Figure 86 through Figure 91. The non-linear correction adds a subtle improvement over the linear convolution, even though the difference is apparent in the step response curves.

![Figure 84: $6^\circ$ Step Response ID, $M=0.6$, $C_L$](image.png)
Figure 85: $6^\circ$ Step Response ID, $M=0.6$, $C_M$

Figure 86: Results, $M=0.6$, $5^\circ$ Sinusoidal Input, 1Hz, $C_\ell$
Figure 87: Results, M=0.6, 5° Sinusoidal Input, 1Hz, $C_M$

Figure 88: Results, M=0.6, 5° Sinusoidal Input, 5Hz, $C_l$
Figure 89: Results, $M=0.6$, $5^\circ$ Sinusoidal Input, 5Hz, $C_M$

Figure 90: Results, $M=0.6$, $5^\circ$ Sinusoidal Input, 20Hz, $C_t$
The Volterra results (along with linear convolution and CFD results) for a 8° step change in pitch is shown in Figure 92 and Figure 93. At this AOA the flow self oscillates at 14.6Hz, or \( k = 0.22 \), i.e. the “buffet” phenomena occurs, see Figure 94 for the Fast Fourier Transform (FFT) of the 8° CFD step response.

For a sinusoidal AOA input at 1Hz, Figure 95 and Figure 96, these transient buffet oscillations are evident at the peaks of the sinusoidal motion. The added Volterra terms capture the reduction of the lift and the moment, but do not follow closely after the higher frequency content of the buffet oscillations.

With a 5Hz sinusoidal AOA input the Volterra correction adds forced oscillations, which resemble the step response data, to the underlying motion of the
airfoil, Figure 97 and Figure 98. Furthermore, buffet at 14.6Hz interacts with 20Hz input and decays, see Figure 99 and Figure 100. However at larger time the Volterra model does reach a steady state oscillations for the lift and moment, albeit at higher level than the CFD result.

We believe the source of this phenomenon stems from the oscillatory lift and moment due to a step response. Whenever we encounter buffet in the step response analysis, the convolution + Volterra approach gives less accurate results, more so when the input frequency is close to the buffet oscillations. The interaction between the input oscillations and buffet can result in overfitting and resonant response of the non-linear convolution.

This can be seen in Figure 101 through Figure 103, where the frequency content is shown for each of the inputs. For 1Hz the buffet frequency is not registered at all, at 5Hz the input and the buffet are distinct and for 20Hz the response is amplified over larger frequency range and include both buffet and the AOA input.

It is possible to address the interaction and overfitting within our approach; the Volterra parameters can be chosen in such a way as to eliminate the oscillations from the fit, but still maintain the steady magnitudes of the loads, see Figure 104 and Figure 105 for example. The consequent change in the aerodynamic loads is shown in Figure 106 through Figure 109.
Figure 92: 8° Step Response ID, M=0.6, $C_L$

Figure 93: 8° Step Response ID, M=0.6, $C_M$
Figure 94: Frequency Content of 8° Step Response, M=0.6

Figure 95: Results, M=0.6, 8° Sinusoidal Input, 1Hz, C_l
Figure 96: Results, $M=0.6$, $8^\circ$ Sinusoidal Input, 1Hz, $C_M$

Figure 97: Results, $M=0.6$, $8^\circ$ Sinusoidal Input, 5Hz, $C_l$
Figure 98: Results, M=0.6, 8° Sinusoidal Input, 5Hz, $C_M$

Figure 99: Results, M=0.6, 8° Sinusoidal Input, 20Hz, $C_l$
Figure 100: Results, $M=0.6$, $8^\circ$ Sinusoidal Input, 20Hz, $C_M$

Figure 101: Frequency Content, $M=0.6$, $8^\circ$ Sinusoidal Input, 1Hz
Figure 102: Frequency Content, M=0.6, 8° Sinusoidal Input, 5Hz

Figure 103: Frequency Content, M=0.6, 8° Sinusoidal Input, 20Hz
Figure 104: 8th Step Response ID, Adjusted Parameters, M=0.6, C_L

Figure 105: 8th Step Response ID, Adjusted Parameters, M=0.6, C_M
Figure 106: Results with Adjusted Volterra Parameters, M=0.6, 8°, 5Hz, C_L

Figure 107: Results with Adjusted Volterra Parameters, M=0.6, 8°, 5Hz, C_M
Figure 108: Results with Adjusted Volterra Parameters, $M=0.6$, $8^\circ$, 20Hz, $C_L$

Figure 109: Results with Adjusted Volterra Parameters, $M=0.6$, $8^\circ$, 20Hz, $C_M$
2.10.2 Unsteady Aerodynamic Loads for M=0.755

At $M = 0.755$ the non-linear effect are more substantial, see the step response trends on Figure 28 and Figure 29. As a result the benefit of using the Volterra addition will be noticeable starting from a smaller AOA. Analysis results for $2^\circ$ step input, with the linear convolution and Volterra fit, are shown on Figure 110 and Figure 111. The steady state values are slightly reduced compared to the linear results.

For 1Hz and 5Hz sinusoidal input the NL fix agrees very well with the CFD results, see Figure 112 through Figure 115. Interestingly the lift coefficient for 5Hz is fully linear, Figure 114, and the Volterra addition corresponds to it without additional adjustments.

The 20Hz results follow this trend, excluding the subtle inconsistency in the peak values, Figure 116. This is unexpected because the step response data showed a smooth transition up to the steady state.
Figure 110: 2º Step Response ID, M=0.755, $C_L$

Figure 111: 2º Step Response ID, M=0.755, $C_M$
Figure 112: Results, $M=0.755$, 2° Sinusoidal Input, 1Hz, $C_L$.

Figure 113: Results, $M=0.755$, 2° Sinusoidal Input, 1Hz, $C_M$. 
Figure 114: Results, M=0.755, 2° Sinusoidal Input, 5Hz, C_l

Figure 115: Results, M=0.755, 2° Sinusoidal Input, 5Hz, C_M
Figure 116: Results, $M=0.755$, $2^\circ$ Sinusoidal Input, 20Hz, $C_l$. 
At $5^\circ$ AOA the non-linear effects become significant and the linear terms alone cannot represent the resultant unsteady loads. The step response of $6^\circ$ is used for the estimation and the NL correction, Figure 118 and Figure 119. For this amplitude the flow is self-excited at 22.4Hz, or $k = 0.27$, and the oscillations are damped to a steady state after ~100 non-dimensional time units.

These oscillations are evident from CFD simulation results for 1Hz and 5Hz, Figure 120 through Figure 123. From the figures it is clear that the Volterra addition captures the non-linearity and reduces the modelling error significantly. At 1Hz the linear model overestimates the CFD result by ~50% for both lift and moment at the peaks, Volterra addition brings the error down to 4.7% for lift and 3% for moment. For 5Hz the
correction is more modest; for lift the reduction is from 12% to 10% and for the moment it goes down from 31% to 12%, compared to the linear convolution. The results also suggest that at higher frequencies the non-linear effects are less significant, hence the NL addition might be redundant. The Volterra correction is able to fix the unsteady load levels, however it is not following the higher frequency oscillations of the flow, which is however still sufficient for the load estimation for many applications.

For comparison purposes an additional estimation of $5^\circ$ sinusoidal input analysis was done with the kernels identified from $4^\circ$ step response. At this AOA the flow oscillates at 17.4Hz, or $k = 0.21$, see Figure 124 and Figure 125. The NL correction works slightly better with the $6^\circ$ step response, compare Figure 120 to Figure 126, Figure 121 to Figure 127 for 1Hz, and Figure 122 to Figure 128, Figure 123 to Figure 129 for 5Hz. These two examples demonstrate the robustness of the approach, i.e. the NL addition will be adequate if the step response data used is close enough to the desired value.

The airfoil oscillations at 20Hz, or $k = 0.24$, are very close to the flow oscillations at $k = 0.27$ with the $6^\circ$ step response. The convolution causes an almost resonant amplification and over-fitting of the Volterra model, Figure 130 and Figure 131, for the lift coefficient, and phase shift for the moment. We have previously seen similar behavior at $M = 0.6$. 
We can further observe this through the frequency content of the three cases, Figure 132 through Figure 134. At 20Hz, \( k = 0.24 \), the Volterra convoluted response is amplified with the oscillatory buffet at \( k = 0.27 \).

Adjusting Volterra parameters until we see no oscillations in the step response, like we did with \( M = 0.6 \) results, might be time consuming or impossible in some of the cases. A more attractive approach might be to filter the oscillatory CFD step response through the peaks, i.e. capture the initial response and the steady state without the frequency content. Subsequently fit the Volterra correction through this smoothed CFD step response, see Figure 135 and Figure 136. A different set of Volterra parameters can be used to obtain the best fit. The subsequent convolution with the input is shown in Figure 137 and Figure 138. This adjustment will be further explored in the next section.
Figure 118: 6° Step Response ID, M=0.755, $C_L$

Figure 119: 6° Step Response ID, M=0.755, $C_M$
Figure 120: Results, $M=0.755$, $5^\circ$ Sinusoidal Input, 1Hz, $C_l$

Figure 121: Results, $M=0.755$, $5^\circ$ Sinusoidal Input, 1Hz, $C_M$
Figure 122: Results, $M=0.755$, $5^\circ$ Sinusoidal Input, 5Hz, $C_L$

Figure 123: Results, $M=0.755$, $5^\circ$ Sinusoidal Input, 5Hz, $C_M$
Figure 124: 4º Step Response ID, M=0.755, $C_L$.

Figure 125: 4º Step Response ID, M=0.755, $C_M$. 
Figure 126: Results, $M=0.755$, $5^\circ$ Sinusoidal Input, 1Hz, $C_L$, $4^\circ$ Step Kernels

Figure 127: Results, $M=0.755$, $5^\circ$ Sinusoidal Input, 1Hz, $C_M$, $4^\circ$ Step Kernels
Figure 128: Results, $M=0.755$, $5^\circ$ Sinusoidal Input, 5Hz, $C_l$, $4^\circ$ Step Kernels

Figure 129: Results, $M=0.755$, $5^\circ$ Sinusoidal Input, 5Hz, $C_M$, $4^\circ$ Step Kernels
Figure 130: Results, $M=0.755$, $5^\circ$ Sinusoidal Input, 20Hz, $C_L$

Figure 131: Results, $M=0.755$, $5^\circ$ Sinusoidal Input, 20Hz, $C_M$
Figure 132: FFT, $M=0.755$, 5° Sinusoidal Input, 1Hz

Figure 133: FFT, $M=0.755$, 5° Sinusoidal Input, 5Hz
Figure 134: FFT, M=0.755, 5° Sinusoidal Input, 20Hz

Figure 135: 6° Step Response ID, M=0.755, Cl, Smooth Response
Figure 136: 6° Step Response ID, $M=0.755$, $C_M$, Smooth Response

Figure 137: $M=0.755$, 5° Sinusoidal Input, 20Hz, $C_1$, Smooth Step Response
At $8^\circ$ the oscillations and the steady state levels of the step response are almost identical to the $6^\circ$, see Figure 28, however the steady state is achieved faster, after ~70 non-dimensional time units. The CFD results and the convolution fit are shown in Figure 139 and Figure 140.

The results for $8^\circ$ AOA resemble those for $6^\circ$, see Figure 141 through Figure 144. The NL Volterra correction is capable of capturing the reduced unsteady loads levels, but it does not follow the self-induced flow oscillations. The identification effort is done “offline” with the step response data. The single identified kernel set is used later with inputs with different $k$ values and yields very good estimation for the non-linear loads. The quality of the estimation can be known in advance by comparison of the ID to the step response data.
At 20Hz we see the familiar amplification and phase shifting of the response, Figure 145 and Figure 146, this can be treated similarly to the 6° case. We will address this more in the next section.

Figure 139: 8° Step Response ID, M=0.755, C_l
Figure 140: 8° Step Response ID, M=0.755, C_M

Figure 141: Results, M=0.755, 8° Sinusoidal Input, 1Hz, C_l
Figure 142: Results, $M=0.755$, $8^\circ$ Sinusoidal Input, 1Hz, $C_M$

Figure 143: Results, $M=0.755$, $8^\circ$ Sinusoidal Input, 5Hz, $C_l$
Figure 144: Results, M=0.755, 8° Sinusoidal Input, 5Hz, $C_M$

Figure 145: Results, M=0.755, 8° Sinusoidal Input, 20Hz, $C_\ell$
2.10.3 Frequency Variations for M=0.755, 5° AOA Oscillations

A convolution between an oscillatory step response and an oscillating input can result in a resonant amplification if the frequencies are close. We have seen this before with the 20Hz sinusoidal motion and the buffeting step response. We have also shown a method to alleviate this resonant behavior to obtain a useful Volterra model. In this section we will expand the convolution approach to additional frequencies, track the resonant behavior and present ways to use the approach without adverse effects.

At $M = 0.755$, the step response for $6°$ results in a decaying oscillating behavior at 22.4Hz, or $k = 0.27$, see Figure 118 and Figure 119. In order to cover a wider frequency range, additional CFD analyses were performed over a number of
frequencies, see Table 3. In the following we will limit the comparison to the lift coefficients only, and for an input of 5° AOA.

As before, once the step response kernels are identified, we can convolute those with any desired input. For 1Hz results see Figure 120, 5Hz Figure 122, and Figure 130 for 20Hz. To complete the picture with additional frequencies see Figure 147 through Figure 151. Clearly 20Hz input is not the only case where the non-linear convolution does not correlate well with the CFD results. In general, the Volterra correction has more effect at the lower frequencies, and the linear part seem to suffice at the higher frequencies.

Table 3: CFD Parameters for M=0.755, 5° AOA Simulations

<table>
<thead>
<tr>
<th>$M_\infty$</th>
<th>0.755</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0 \ [deg]$</td>
<td>5</td>
</tr>
<tr>
<td>$f \ [Hz]$</td>
<td>1</td>
</tr>
<tr>
<td>$k$</td>
<td>0.012</td>
</tr>
</tbody>
</table>
Figure 147: Results, $M=0.755$, $5^\circ$ Sinusoidal Input, 8Hz, $C_l$

Figure 148: Results, $M=0.755$, $5^\circ$ Sinusoidal Input, 10Hz, $C_l$
Figure 149: Results, M=0.755, 5° Sinusoidal Input, 15Hz, C_l

Figure 150: Results, M=0.755, 5° Sinusoidal Input, 30Hz, C_l
However, we want to know upfront whether the model is capable of representing the aerodynamic loads. Hence switching between the linear and non-linear models is not an ideal option, and a better approach is needed for the amplified convoluted response.

In the previous sections we addressed this issue by eliminating the oscillatory motion from the step response data, with good results. Here we will expand this approach further over the entire available frequency range.

Two ways of smoothing the step response data were explored: first, the decaying signal through the peaks of the step response data, see Figure 152, second, driving the response into the steady state after the initial peak, Figure 153. In both cases, the new
curves overlay the CFD results of the step response, and then the Volterra kernels are identified for these curves. As before, the parameters for the Volterra fit are summarized in the figure titles. Once the kernels were identified, the convolution can be used in the familiar way.

The amplitudes of the lift coefficients are compared in Figure 154. It is interesting to observe a typical passing through resonance with the full (unsmoothed) convolution. The peak occurs at 20Hz, \( k = 0.24 \), which is the closest to the buffeting frequency of 22.4Hz, \( k = 0.27 \). The amplification at off resonance is lower but still present. *This resonant behavior is a part of the convolution approach and does not represent the aerodynamics of the system for an oscillating airfoil.*

Once the oscillations are smoothed out of the step response, the convolution is used again, with good results. With slight advantage to the CFD Peaks curve (rather than driving the response into the steady state), the largest overestimation is 17% at 15Hz, \( k = 0.18 \), and 11% at 20Hz. Moreover, the off-resonance convolution results are not deteriorated as a result of the smoothing.

With this new knowledge we can update the flow chart in Figure 60, and add the smoothing effect, see Figure 155. *With this update, we have successfully generated unsteady aerodynamic loads in a broad frequency range, \( 0.012 < k < 0.6 \), with a single set of identified kernels.*
Figure 152: $6^\circ$ Step Response ID, $M=0.755$, $C_l$, Peaks

Figure 153: $6^\circ$ Step Response ID, $M=0.755$, $C_l$, Steady State
Figure 154: $C_L$ Amplitude Comparison, $M=0.755$, $5^\circ$ AOA
Figure 155: Updated Framework
2.11 Aeroelastic Simulation

This section is dedicated to an aeroelastic simulation using the aerodynamic models derived throughout this work. We will use the structural parameters from [17] along with the final results from that paper for comparison. A two DOF aeroelastic system, like in our case, can be described by the following two equations, as in [63]:

\[
\begin{align*}
    m \ddot{h} + K_h h + S_\alpha \ddot{\alpha} &= -L \\
    S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + K_\alpha \alpha &= M_y
\end{align*}
\]  

(57)

Where:

- \( m, I_\alpha \) – airfoil sectional mass and the moment of inertia
- \( K_\alpha, K_h \) – airfoil torsional and heave stiffness about the elastic axis
- \( S_\alpha \) – static imbalance
- \( h, \alpha \) – heave and pitch DOF
- \( L, M_y \) – aerodynamic lift and pitch moment

This system of equations can be turned into a non-dimensional form, to make it consistent with our approach and the available data from previous work, following [17], [68], both of which cite the experimental results from [69]:

\[
\begin{bmatrix}
    1 & x_\alpha \\
    x_\alpha & r^2_\alpha
\end{bmatrix}
\begin{bmatrix}
    \ddot{h}/b \\
    \ddot{\alpha}/\alpha
\end{bmatrix} + \frac{4}{V^2}
\begin{bmatrix}
    (\omega_h/\omega_\alpha)^2 & 0 \\
    0 & r^2_\alpha
\end{bmatrix}
\begin{bmatrix}
    h/b \\
    \alpha/\alpha
\end{bmatrix} = \frac{4}{\pi \mu} \begin{bmatrix} -C_L \end{bmatrix} + \begin{bmatrix} 2C_M \end{bmatrix}
\]  

(58)
Where:

\( x_\alpha \) – airfoil static imbalance, \( x_\alpha = \frac{S_\alpha}{mb} \)

\( r_\alpha \) – radius of gyration of the airfoil, \( r_\alpha^2 = \frac{l_\alpha}{mb^2} \)

\( \omega_h, \omega_\alpha \) – uncoupled natural frequencies, \( \omega_h^2 = \frac{K_h}{m}, \omega_\alpha^2 = \frac{K_\alpha}{l_\alpha} \)

\( V \) – reduced velocity, \( V = U_\infty/\omega_\alpha b \)

\( \mu \) – mass ratio, \( \mu = \frac{m}{\pi pb^2} \)

Next, we will define two system states to simplify it further:

\[
q_1 = \begin{bmatrix} h/b \\ \alpha \end{bmatrix}
\]

\[
q_2 = \begin{bmatrix} h/b \\ \alpha \end{bmatrix}
\]

Using the above states and defining \( M = \begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r_\alpha^2 \end{bmatrix}, K = \begin{bmatrix} (\omega_h/\omega_\alpha)^2 & 0 \\ 0 & r_\alpha^2 \end{bmatrix}, \) Eq. (58) can be rewritten in the following way:

\[
\{q_2\} = -\frac{4}{V^2}[M]^{-1}[K]\{q_1\} + \frac{4}{\pi \mu}[M]^{-1}\begin{bmatrix} -C_L \\ 2C_M \end{bmatrix}
\]

Finally it can be formulated in the familiar way which is consistent with the built-in Ordinary Differential Equation (ODE) solvers in MATLAB, like ode45, and makes the implementation of numerical integration easier:
\[
\begin{align*}
\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} &= \begin{bmatrix} -\frac{4}{V^2} [M]^{-1} [K] & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{4}{\pi \mu [M]^{-1}} \end{bmatrix} \begin{bmatrix} 0 \\ -C_L \end{bmatrix} \\
\end{align*}
\]

(61)

The structural parameters are defined as follows: \( x_\alpha = 0, r_\alpha^2 = 1.024, \omega_h/\omega_\alpha = 0.646 \), consistent with [17]. The lift and moment coefficients are calculated using the linear convolution and Volterra series as outlined in previous sections of this thesis.

The linear part of the load coefficients is calculated as follows:

\[
\begin{align*}
C_L(\tau) &= [... \alpha(\tau) ...]h_{\alpha}^{C_L} + [... \dot{h}(\tau) ...]h_{\dot{h}}^{C_L} \\
C_M(\tau) &= [... \alpha(\tau) ...]h_{\alpha}^{C_M} + [... \dot{h}(\tau) ...]h_{\dot{h}}^{C_M}
\end{align*}
\]

(62)

Where:

\( h_{\alpha}^{C_L}, h_{\dot{h}}^{C_L}, h_{\alpha}^{C_M}, h_{\dot{h}}^{C_M} \) – previously identified linear kernels

\[ [... \alpha(\tau) ...], [... \dot{h}(\tau) ...] \] – vectors with simulation results, populated with adequate memory depth for each integration step

The non-linear portion of the loads depends on angle of attack, or pitch only, because of the linearity of the aerodynamic lift and moment due to the heave DOF. The product matrix \([U]\) in Eq. (54) is constructed for each integration step, and used with known kernels:
\[ \Delta C_L(\tau) = [U(\alpha(\tau))][B][\theta]^C_L \]
\[ \Delta C_M(\tau) = [U(\alpha(\tau))][B][\theta]^C_M \] (63)

Where:

\[ [B][\theta]^C_L, [B][\theta]^C_M \] – previously identified NL kernels

The NL kernels were previously identified with step response CFD data for different angles of attack. The unsteady loads behavior is distinct at each AOA level, see Figure 28 for example. Subsequently, using each of the identified kernel sets will result in distinct behavior, as we observed in Section 2.10.2 for example. We should therefore choose the appropriate kernel to use for the particular task or simulation. In the following we will compare several kernel choices. Finally, the total aerodynamic loads are the sum of Eq. (62) and Eq. (63).

2.11.1 Linear Flutter Boundary

Our convolution models compare well to the available experimental data points from [69] for the linear flutter boundary, see Figure 156. The \( V, M \) pairs in this figure are compatible, i.e. flutter match points. To evaluate the flutter boundary the simulation is performed with linear aerodynamic loads only, Eq. (62), and the result is a simple harmonic motion. The system is unstable above the line.
2.11.2 LCO Simulation Results for $M=0.755$

Previously computed results for an LCO of NACA 0012 are available in [17] and [68], these are based on the experimental data from [69]. For the available computational LCO results we look at Figure 157 which is taken from [68]. The data is available for $M = 0.7, 0.8, 0.9$ at points $A, B, C$ respectively. We can expect our results for $M = 0.755$ to fall between points $A$ and $B$. 

![Figure 156: Flutter Boundary Results, Experiment Data [69] and Current Work](image)
We are unstable above the line in Figure 156, and can encounter LCO if the underlying system is non-linear. In the simulation we used the $6^\text{th}$ step identified Volterra kernels, this according to the expected results from [68]. In order to evaluate the simulation robustness, we compare the results to $4^\text{th}$ step identified kernels. See Figure 158 for the comparison. In both cases the LCO amplitudes are comparable with those from [17] and [68].

A better option might be to choose the appropriate NL kernel set according to the instantaneous AOA during the simulation, we call it the “toggling” approach. One possible implementation of this approach is shown in Figure 159. During the simulation
we check the AOA value at each time step, and use the appropriate kernel set accordingly, Figure 159. The memory lag values of AOA are used for the evaluation of the loads. The results of this approach, for varying $\mu$, are shown in Figure 160. The resultant LCO levels are higher compared to Figure 158, which is expected, and still fall within the limits in [68]. Our results are comparable to those in Figure 157, however the LCO amplitudes are not starting smoothly from zero AOA, as expected, but rather jump discretely to a finite LCO amplitude.

Figure 161 and Figure 162 show two examples of the simulation results, both were obtained with the toggling version of the non-linear kernels.

![Simulation Results for M=0.755, V=38.5 with Varying $\mu$](image)

Figure 158: LCO Simulation Results, $M = 0.755, V = 38.5$
Figure 159: NL Kernel Toggle Implementation

Figure 160: LCO Simulation Results, Toggling, $M = 0.755, V = 38.5$
Figure 161: Simulation Results, $V = 38.5$, $1/\mu = 1 \cdot 10^{-3}$

Figure 162: Simulation Results, $V = 45$, $1/\mu = 2.85 \cdot 10^{-4}$
As a final calculation effort we can use the simulation with NL kernel data for $M = 0.6$. The toggling between kernels is set up in a similar manner, but here we can add a linear AOA range, as the non-linear effects are noticeable starting at $6^\circ$ only. See Figure 163 for the implemented scheme. The simulation results are summarized in Figure 164. As expected, the amplitudes are larger compared to $M = 0.755$ due to later engagement of the non-linear effects. The heave amplitudes exceed the airfoil chord (more than twice the half chord), hence the resulting LCO levels are not acceptable for any practical purposes.

Figure 163: Kernel Toggle Implementation for $M = 0.6$
Figure 164: LCO Simulation Results, $M = 0.6, V = 33$
3. Improving Piezoelectric Energy Harvesting of an Aeroelastic System

This section describes the experimental research part of the dissertation. The goal is to explore methods to improve the output and the efficiency of an energy harvester which is based on aeroelastic effects. We will begin with the test setup.

3.1 Experimental Setup

The flexible vibrating plate is cut out of 6061-T6 Aluminum plate, 0.375mm in thickness. The area exposed to the flow has dimensions of 513mm long and 101.6mm wide. The plate is cantilevered from a stiff aluminum base profile, which is attached to the floor of the wind tunnel. See Figure 165. The vertical profile is covered with an aerodynamic 3d printed shroud to reduce vortex shedding over the plate.

3.1.1 Instrumentation

A small accelerometer, PCB 352C22, is located at midspan of the plate tip. The airspeed is measured using a wind tunnel mounted pitot-tube. The plate is covered with 24 piezo elements – DT4-028K/L (12 on each side), manufactured by TE connectivity (www.te.com). The elements are bonded to the plate using epoxy resin. The area of each element measures 171x22mm with an active area of 156x19mm. The elements are
divided into 6 separate “blocks”, with 4 elements in each block. See Figure 166. The elements in each block connect in parallel electrically, with a single output [36].

The measurements in test were acquired using a m+p VibPilot Data Acquisition (DAQ) unit. The instrument is limited to a maximum input of 10V, while our measurements are typically higher. To overcome this limitation, we used a simple voltage divider with identical value resistors. m+p Analyzer software was used for acquisition and data analysis.

### 3.1.2 AC to DC Rectification

Piezoelectric elements generate AC voltage and current. DF04S full wave rectifier with smoothing capacitor was used to obtain DC voltage and measure it over a load resistor. See Figure 167. The output voltage was recorded in two ways: once separately for each block and once with the 6 blocks combined in parallel, after rectification, and then recorded [37]. The two methods gave comparable results.

### 3.1.3 Load Resistor Values

In this work, values of 600-900 kΩ were used for the load resistors. Preliminary tests show favorable power extraction for these values.
3.1.4 Power Output

Once the voltage over the load resistor is recorded, the power output can be calculated as follows:

\[ P = \frac{V^2}{R} \]  

(64)

Where:

- \( P \) – Power [\( \mu W \)]
- \( V \) – Voltage [V]
- \( R \) – Load resistance [kΩ]

Figure 165: Wind Tunnel Test Setup (Fully covered plate shown)
3.2 Preliminary Analysis and Test Correlation

First, we will characterize our mechanical and electrical system, i.e. the modal dynamics and aeroelasticity, both for the empty and the fully covered plates, together with potential power output for the fully covered plate. See Figure 168. Later configurations will be compared to data obtained in this step.

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3.2.1 Numerical and Experimental Modal Analysis

Finite Element (FE) modal analysis was performed to determine the natural frequencies and modes of the structure. We used ANSYS, which is a commercial FE software, for the calculations.

The results were verified with an Experimental Modal Analysis (EMA) of the plate. A PCB 086C01 modal hammer was used to tap the root of the plate, and the response was recorded with the tip accelerometer, PCB 352C22. The recordings were analyzed to determine the natural frequencies and modes of the empty and fully covered plates, see Figure 168. The first four modes are shown in Figure 169, and the results are compared in Table 4.

The values compare well for the empty plate. Note the increased stiffness (and frequency) together with damping for the fully covered plate. Same trend is evident in the Frequency Response Function (FRF) plot. See Figure 170. This change comes from the added piezo elements, wiring, and the epoxy layer which bonds the elements to the plate. A fully covered plate adds 68.7 gr to the empty plate mass of 66.7 gr.
Figure 168: Base Line Configuration – Empty and Fully Covered Plates

Figure 169: Modal Analysis Results – Empty Plate
Table 4: Dynamic Properties of the Plates

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE</th>
<th>Experimental Modal Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Empty Plate</td>
</tr>
<tr>
<td></td>
<td>f [Hz]</td>
<td>f [Hz]</td>
</tr>
<tr>
<td>1B</td>
<td>1.18</td>
<td>1.16</td>
</tr>
<tr>
<td>2B</td>
<td>7.4</td>
<td>7.46</td>
</tr>
<tr>
<td>3B</td>
<td>11.7</td>
<td>12.16</td>
</tr>
<tr>
<td>3B</td>
<td>20.8</td>
<td>21.08</td>
</tr>
</tbody>
</table>

Figure 170: Experimental FRFs for Two Configurations
3.2.2 Flutter Analysis and Test

The results from modal analysis were used as a structural model to perform a preliminary flutter analysis of the plate. A linear, potential flow, commercially available aerodynamic and aeroelastic code, ZAERO, was used.

To verify the results, the plate was placed in the wind tunnel in a flag orientation. See Figure 165. The air speed was increased until the plate started oscillating spontaneously. In several instances the plate was deflected from its initial position, by an external rod, to induce the vibrations. The air speed was increased further until the tip acceleration reached ~100g, or the vibrations became too violent, then the speed was reduced to 0.

The analysis shows a flutter mechanism which involves the first and second bending modes. The system becomes unstable at 14.1 m/sec with second bending being the dominant mode shape. See Figure 171. The flutter frequency is 5.4 Hz. We assumed 1% of structural damping in the flutter computations.

Tip acceleration vs. air speed, for the empty plate, is shown in Figure 172. Limit Cycle Oscillations (LCO) start at 18.1 m/sec, where the acceleration jumps quickly from 4.6g to 53g. This is larger than the predicted 14.1 m/sec, see Figure 171. The difference, here and throughout the paper, can be attributed, in part, to the fact that ZAERO is a linear code, while our experiments show highly non-linear behavior of the plate. In [70] a non-linear structural model of this plate was coupled with linear aerodynamic model.
The analysis in [70] showed closer correlation with the experimental data. We stopped increasing the air speed at ~20 m/sec when the acceleration reached 105g. A small hysteresis was observed while reducing the speed; the LCO stopped at 15.9 m/sec, 2.2 m/sec below the initiation at 18.1 m/sec. This hysteretic behavior was observed throughout the tests, with increased hysteretic velocity increment for the fully covered plate.

Figure 171: VG Plot for the Empty Plate
For the fully covered plate, a different behavior was observed. Two ways of initiating LCO were used: increasing the air speed until the plate vibrates (as in the case with the empty plate) and deflecting the plate from its initial position, with a rod, to initiate the vibrations. In the first case the LCO initiation occurred at 35.9 m/sec, and the flow velocity hysteresis was substantial– 16.1 m/sec, i.e. the vibrations stopped at 19.8 m/sec. See Figure 173. When deflecting the plate, a lower flow velocity for LCO initiation is possible – the plate starts to vibrate at 23.5 m/sec, with a smaller hysteresis increment of 5.2 m/sec for stopping the vibrations. See Figure 174. In both cases the “LCO stop” air speed is comparable, i.e. 19.8 vs. 18.3 m/sec.
The maximum power output for this plate is ~700-1000 μW. Figure 175 shows a combined output from the whole plate, and the block by block output is shown in Figure 176. The significant output occurs when the plate is in LCO, as can be seen from the tip acceleration curve. The maximum power output happens at blocks 2, 5 where the piezo elements experience the largest bending curvature while in LCO. The output from tip blocks 3, 6 is the lowest, due to the minimal bending of 1B and 2B modes at the tip.

![Figure 173: Flutter Test of a Fully Covered Plate, Natural Excitation](image)
Figure 174: Flutter Test of a Fully Covered Plate, Deliberate Excitation

Figure 175: Combined Output from a Fully Covered Plate
3.3 Improving Power Output

To increase the power output from our system and to make it more practical, several changes can be applied:

- Increase the vibration frequency.
- Increase the vibration amplitude.
- Increase curvature in the vibrational motion.
- Use more efficient piezo elements.
- Reduce the required air speed for LCO
We want to maintain the basic aeroelastic system, therefore the LCO mechanism and the frequency will not change significantly in our study. While in LCO, increasing the air speed will increase the vibration amplitude, see Figure 172 or Figure 174 for example. However, this cannot be sustained indefinitely, at some point the vibrations will no longer be limited and structural failure will occur.

Increasing curvature in the LCO motion is possible. 3B frequency is decreasing with increased air speed, see Figure 171, but it is not involved in the flutter mechanism. Our first attempt to increase power output was to reduce this frequency without significantly changing other properties.

This was achieved by adding discrete masses to the plate, similar to [40]. In our case the masses were added at two chordwise and three spanwise locations. See Figure 177. The inward location coincides with the nodal line of 2B mode. Masses added there would not influence this natural mode and frequency. The tip of the plate is a more obvious choice, being the point of maximum deflection. Masses added in the middle of the span, points 478, 58, change the bending modes only. Masses added at the outside points, 74, 53, 2, 42, also influence the torsional modes. Two mass values were used – 5 gr and 10 gr$^1$.

$^1$ The numbers denoting the spatial locations are chosen to be consistent with the node numbering of the finite element model.
3.3.1 Verification – Adding Masses to an Empty Plate

To verify the concept, discrete masses were added to the empty plate, at points 74 and 53, see Figure 178. The modal analysis results are compared in Table 5. ZAERO flutter analysis gives a lower flutter velocity of 10.5 m/sec for 5 gr case and 9.9 m/sec for 10 gr case. See Figure 179 for a typical vg plot. The dominant flutter mechanism remains 1B and 2B. In addition a marginally stable behavior can be observed between 20 and 30 m/sec, which involves 1T and 3B modes. These results can be potentially beneficial to a more significant piezo elements curvature and therefore larger power extraction.

The calculations were verified in wind tunnel tests. A significant increase in tip accelerations was observed, see Figure 180 and Figure 181. For the 5gr case, LCO starts at 15.7 m/sec where the tip acceleration increases up to ~300 g. In the 10gr case, the
acceleration goes over 800 g at 15 m/sec, Figure 181. In the latter case the vibrations became violent enough to snap off the tip mass. See Figure 182.

Note the consistent correlation between the calculated and experimental results. The calculated flutter speed for the empty plate was 14.1 m/sec, Figure 171, with experimental value of 18.1 m/sec, Figure 172. With the added masses the difference is ~5 m/sec, consistent with analysis results being conservative. As mentioned before, the difference can be explained by the non-linear nature of the oscillations as compared to the linear flutter code which was used for the analysis.

The larger amplitude of vibrations can potentially increase the power output from the piezo elements, and the reduced LCO air speed benefits a more practical application of this approach.

Figure 178: Empty Plate with Added Masses (5gr at Points 74, 53 shown)
Table 5: Modal Analysis Results Comparison for an Empty Plate

<table>
<thead>
<tr>
<th>Mode</th>
<th>Empty Plate f [Hz]</th>
<th>5gr at 74, 53</th>
<th>10gr at 74, 53</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>1.18</td>
<td>0.94</td>
<td>0.8</td>
</tr>
<tr>
<td>1B</td>
<td>7.4</td>
<td>6.3</td>
<td>5.8</td>
</tr>
<tr>
<td>1T</td>
<td>11.7</td>
<td>9.2</td>
<td>8.5</td>
</tr>
<tr>
<td>3B</td>
<td>20.8</td>
<td>17.1</td>
<td>15.25</td>
</tr>
</tbody>
</table>

Figure 179: VG Plot for an Empty Plate, 5gr Added at Points 74, 53
Figure 180: Flutter Test – Empty Plate, 5gr at Points 74, 53

Figure 181: Flutter Test – Empty Plate, 10gr at Points 74, 53
3.3.2 Adding Masses to the Fully Covered Plate

The masses were added to the fully covered plate and the test was repeated, see Figure 183. For the 10gr case, the LCO vibrations engaged at 16.1 m/sec and the flow speed was further increased up to 17.2 m/sec, Figure 184, at this point the tip acceleration went over 160g. The maximum power output was 698 μW, Figure 185. In the 5gr case, the vibration started at 18.7 m/sec and the speed was increased slightly until 18.9 m/sec, Figure 186. The tip acceleration at maximum speed was 210g. In this case the power output was improved – 1104 μW, and the largest so far, Figure 187.
We note several interesting differences compared to the base line case. First, the LCO started at lower air speed, which was expected. Potentially the speed can be further increased, and the power output can be compared directly at identical air speeds. Due to the concern with the structural integrity of the plate and the test setup, we chose to avoid this.

Second, compared to Figure 176, the power output from tip blocks 3, 6 exceeded the output from the root blocks 1, 4. This indicates larger bending curvature involvement at the tip of the plate, compared with the base line case. The output from the root blocks was simultaneously decreased, but overall the power output improved, see Figure 187.

Finally, the tip acceleration was lower compared to the empty plate with masses. This can be attributed to significantly larger damping of the fully covered plate. The increased damping can be important in creating a sustainable power generator.
Figure 183: Fully Covered Plate with 5gr Masses at Points 74, 53

Figure 184: Flutter Test for Fully Covered Plate, 10gr Masses at Points 74, 53
Figure 185: Power Output, Fully Covered Plate, 10gr Masses at Points 74, 53

Figure 186: Flutter Test for Fully Covered Plate, 5gr Masses at Points 74, 53
3.4 Discrete Masses Placement

In the previous section we discussed a single configuration with added masses, the power output was slightly increased compared to the base line. We demonstrated the potential of this approach, still it is unclear as to what would be the better location for mass placement. In an attempt to answer this question, we experimented with 14 different cases. See Table 6 for the summary of these cases, including cases 4 and 5 which were already presented. In this section we will address more general trends based on the experiments.
Table 6: Point Masses Configurations

<table>
<thead>
<tr>
<th>Case #</th>
<th>Added Masses at Points [gr]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>74</td>
</tr>
<tr>
<td>Base Line</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td>-</td>
</tr>
</tbody>
</table>

For each case, a modal analysis was performed. For several cases, an additional experimental modal analysis was done. See Table 7. The numerical results are for the empty plate with added masses, the test results are for the plate with piezoelectric elements. The frequencies in test are generally higher than the analytical ones. A subsequent flutter/LCO test was performed for most of the cases. These test results are summarized in Table 8. The LCO start air speed is the speed at which the acceleration changed suddenly to a higher value and the plate started vibrating visibly. The maximum air speed in test is the highest speed at which we could comfortably accept
the level of vibration without endangering the test model. LCO frequency is the dominant frequency of the vibrations.

Table 7: Modal Properties for Various Configurations

| Case # | Natural Frequencies [Hz] | \hline | \hline | \hline |
|--------|---------------------------| \hline | \hline | \hline |
|        | FE Analysis (Empty)       | Experimental (Piezo) | \hline | \hline | \hline |
|        | 1B | 2B | 1T | 3B | 1B | 2B | 1T | 3B | \hline | \hline | \hline |
| Base Line | 1.18 | 7.4 | 11.7 | 20.8 | 1.44 | 9.95 | 11.92 | 22.87 | \hline | \hline | \hline |
| 1      | 0.9 | 7.4 | 11.7 | 18.95 | - | \hline | \hline | \hline |
| 2      | 1  | 7.4 | 8.7 | 20 | - | \hline | \hline | \hline |
| 3      | 0.8 | 5.12 | 7.96 | 18.74 | - | \hline | \hline | \hline |
| 4      | 0.8 | 5.77 | 8.5 | 15.25 | - | \hline | \hline | \hline |
| 5      | 0.94 | 6.33 | 9.23 | 17.1 | - | \hline | \hline | \hline |
| 6      | 0.84 | 5.8 | 9.11 | 16.3 | - | \hline | \hline | \hline |
| 7      | 0.84 | 5.45 | 8.51 | 18.76 | - | \hline | \hline | \hline |
| 8      | 0.77 | 5.5 | 9 | 15.6 | 0.91 | 6.46 | 10.3 | 17.94 | \hline | \hline | \hline |
| 9      | 0.88 | 6.13 | 11.74 | 18 | - | \hline | \hline | \hline |
| 10     | 0.88 | 5.82 | 9.9 | 18.8 | 1.06 | 7.2 | 10.95 | 20.06 | \hline | \hline | \hline |
| 11     | 0.8 | 5.96 | 11.74 | 16.12 | - | \hline | \hline | \hline |
| 12     | 0.94 | 6.47 | 11.74 | 17.68 | 1.11 | 8.67 | 11.43 | 20.46 | \hline | \hline | \hline |
| 13     | 0.74 | 5.77 | 11.74 | 17.47 | 0.85 | 7.29 | 11.4 | 18.75 | \hline | \hline | \hline |
| 14     | 0.77 | 5.8 | 11.74 | 16.5 | - | \hline | \hline | \hline |
An expected outcome from Table 8 is the lower air speed at which LCO start for all the modified cases, compared to the base line case. This extends the practical application of a single aeroelastic system to broader range of air speeds. From the flutter speed at 23.5 m/sec for the base line, we can extend it as low as 15.1 m/sec (case 8 for example). For comparison, below 23.5 m/sec, the base line case experiences no vibrations and therefore no power output.
To directly compare between the cases, we introduce the “Conversion Coefficient” which normalizes the obtained electric power by the invested mechanical energy. This coefficient is dimensionless and is treated as an estimate for power generation efficiency:

\[ CC = \frac{P}{\sum (m_{add} \cdot d) \cdot a_{tip} \cdot f_{LCO}} \]  

(65)

Where:

- **CC** – Conversion Coefficient
- **P** – total power generated from piezo elements [W]
- **m_{add}** – added mass [kg]
- **d** – distance of the added mass from the root* [m]
- **\( \sum (m_{add} \cdot d) \)** – mass loading of the plate
- **a_{tip}** – tip acceleration \([m/sec^2]\)
- **f_{LCO}** – LCO frequency [Hz]

*\( m_{add} \cdot d \) for the base line case is calculated by multiplying the mass of the plate by half chord.
Calculated coefficients for each case are shown in Figure 188 and Figure 189. Separate lines and symbols represent different cases and are plotted against air speed and tip acceleration.

Significantly increased efficiency for the center line placed masses is evident, i.e. larger conversion coefficient at lower tip acceleration. Comparing cases 11 and 4, for example (same mass loading), yields ~290% better conversion for case 11, with comparable LCO speed. Placing masses at the tip adds torsional imbalance to the plate and lowers the frequencies of the torsional modes. While it can result in larger power output, see Table 8, it does not support sustainability of the structure and the generator. For long term generation we would prefer reduced acceleration and mechanical stresses, both of which are supported by center line placed masses.

A direct comparison of the maximum obtained conversion coefficient vs. mass loading is shown in Figure 190, center line placed masses yield better performance. Also, higher efficiency correlates with larger mass loading, however there might be an optimum value with further extension.
Figure 188: Conversion Coefficient vs. Air Speed

Figure 189: Conversion Coefficient vs. Tip Acceleration
3.5 Increased Air Speed

We limited the top air speed in the tests because the plate had to remain structurally functional. Once the data collection was complete, the plate served no further purpose. This was an opportunity to assess the limits of the setup, i.e. increase the speed until structural failure, and it was attempted with case 12 configuration. The maximum air speed in this final test reached 30.7 m/sec with top tip acceleration of 84.1 g, see Figure 191. Power output throughout this test is shown in Figure 192. The plate did not break and remained fully functional until the end of the test.
Figure 191: Case 12 – Final Test

Figure 192: Case 12 – Power Output During Final Test
The acceleration plunges, from 70.8 g to 51.4 g, between 23.3 m/sec and 24.3 m/sec in Figure 191, this plunge is repeated when we decreased the flow, with small hysteresis. At 23.3 m/sec the total power output is 1408 μW. Following this point, power output from blocks 2, 5 is increased, while it is rapidly decreased from the remaining blocks. The next transition occurs at 25.5 m/sec, the power output is increased from all blocks. The final transition occurs at 29.7 m/sec, the power output slopes become higher, and the motion becomes messy. The total power output at the peak is 3995 μW, all time maximum. Note the small output from the root blocks during the entire test.

In our tests, the LCO motion consisted mainly of 2nd bending (2B) mode shape, with small involvement of the 1st torsional mode (1T). This was observed through all the tests, and was attributed to inherent imperfections in the plate, and not perfectly aligned instrumentation and mass placement.

During the last test however, this behavior changed at ~23.3 m/sec – Transition 1. The vibration shape changed into pure 2B without adverse torsion, and the tip acceleration dropped. This continued through 2nd transition, until ~30 m/sec, when the motion became messy again, without definite mode shape. FFT analysis of the tip acceleration confirms our observations. Figure 193 shows the dominant frequencies immediately after LCO starts. The flutter frequency is 5.2 Hz, which is consistent with past flutter analysis. Figure 194 and Figure 195 show the dominant flutter mode, right
after the transition into pure 2B. Finally, in Figure 196 we can see additional frequencies come into play starting at \(~30\) m/sec.

The plate generated power consistently, despite the transitions in vibrating modes. The power output increased steadily with the increase of air speed. The plate maintained structural integrity up until 30.7 m/sec, which is a margin of above 70% over the initial LCO speed of 18 m/sec. This can be attributed to the high non-linearity of the motion and the large damping of the fully covered plate.

Figure 197 shows the calculated Conversion Coefficient for the final test, compared to the initial Case 12 test. The air speed was increased faster, without fully charging the capacitors at LCO start speed. At larger air speeds the transitions and slope changes follow the trends in Figure 192, although these are not fully understood.
Figure 193: Case 12, Final Test – LCO Start

Figure 194: Case 12, Final Test – Before Transition 1
Figure 195: Case 12, Final Test – Transition 1

Figure 196: Case 12, Final Test – Transition 3
3.6 More Efficient Piezoelectric Elements

In previous sections we increased the efficiency of piezo-aeroelastic power generator by changing the dynamics of the basic structure and improving the aeroelastic behavior. In this section we will explore the benefits of using more efficient piezoelectric elements.

An additional plate was assembled. The plate is geometrically identical and is made from the same aluminum strip as the original. The plate was instrumented with the same piezoelectric elements at the root and the tip, i.e. blocks: 1, 3, 4, 6. The area of block 2, where the maximum power output occurs, was covered with different piezo
elements – DuraAct P-876.A11, manufactured by PI (www.piceramic.com). See comparison in Table 9. PI elements have higher piezoelectric strain constant (d31), piezoelectric voltage constant (g31) and electromechanical coupling factors (k31, kt). These are the influencing factors for energy harvesting applications [22]. On the other hand, these elements are substantially stiffer and heavier, and might limit the vibrations. Due to the added stiffness, see Figure 198, we chose to leave the block 5 area uncovered. In addition, the two types of elements have different form factor, see Table 9, therefore an area which was previously covered by 4 TE elements, is now covered by 9 PI elements, Figure 199. After mounting the plate in the tunnel, a basic modal test was performed, the results are summarized in Table 10. The plate is stiffer and more damped compared to the original plates.

<table>
<thead>
<tr>
<th></th>
<th>TE</th>
<th>PI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>DT4-028K/L</td>
<td>P-876.A11</td>
</tr>
<tr>
<td>Type</td>
<td>PVDF</td>
<td>Ceramic</td>
</tr>
<tr>
<td>Dimensions [mm]</td>
<td>171x22</td>
<td>61x35</td>
</tr>
<tr>
<td>Dimensions (Active) [mm]</td>
<td>156x19</td>
<td>50x30</td>
</tr>
<tr>
<td>Thickness [mm]</td>
<td>0.157</td>
<td>0.4</td>
</tr>
<tr>
<td>Elastic Modulus [GPa]</td>
<td>3</td>
<td>16.4</td>
</tr>
<tr>
<td>Density [kg/m3]</td>
<td>1780</td>
<td>7800</td>
</tr>
<tr>
<td>d31 [m/V]</td>
<td>23·10-12</td>
<td>1.867·10-10</td>
</tr>
<tr>
<td>g31 [Vm/N]</td>
<td>216·10-3</td>
<td>1.205·10-2</td>
</tr>
<tr>
<td>k31 [%]</td>
<td>12</td>
<td>37</td>
</tr>
<tr>
<td>kt [%]</td>
<td>14</td>
<td>49</td>
</tr>
</tbody>
</table>
Figure 198: Visual Stiffness Comparison, PI vs. TE

Figure 199: New Plate with PI Elements
Table 10: Modal Test Results for the PI Plate

<table>
<thead>
<tr>
<th>Mode</th>
<th>f [Hz]</th>
<th>Damp [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B</td>
<td>1.41</td>
<td>3.49</td>
</tr>
<tr>
<td>2B</td>
<td>10.97</td>
<td>3.15</td>
</tr>
<tr>
<td>1T</td>
<td>13.78</td>
<td>1.4</td>
</tr>
<tr>
<td>3B</td>
<td>25.75</td>
<td>3.86</td>
</tr>
</tbody>
</table>

For direct comparison, we used case 12 and case 14 mass configuration. Case 12 was chosen due to the extensive testing of the original plate, and case 14 is the most power efficient.

In Case 12, the LCO started at 32.6 m/sec with deliberate deflection. See Figure 200. LCO did not start at lower speeds despite the repeated deflection attempts. This can be due to the increased stiffness and damping of this setup. Note the large hysteresis when decreasing flow velocity, the vibrations stopped completely at 18.4 m/sec, resembling the behavior in initial tests, Figure 173. The power output from this case is shown in Figure 201. Note the significant increase in power output from PI elements at the mid location, the peak output is 4060 μW (for PI elements only). The maximum power output in this case is 5262 μW. The addition of 1202 μW from the remaining TE elements is comparable to our previous experience with case 12, see Table 8 and Figure 192.
In case 14 configuration, the power was generated in two instances, see Figure 202. The LCO started at 22.6 m/sec, then the air speed was reduced until vibrations stopped. Next LCO vibrations started at 17.4 m/sec with deliberate initiation. The maximum tip acceleration during this test remained under 40 g, which resembles the values observed with the original plate, see Table 8. In the end of this test the PI elements were separated from the plate. See Figure 203. No structural failure occurred, and the test ended safely. The initial separation of the elements occurred ~180 sec into the test, see Figure 204, when a plunge in power output was recorded. At ~195 sec the tip acceleration and power output both increased. The plate began vibrating with increased amplitude - 33.6 g (21 g before) and the power output increased again. During these transitions the air speed did not change and remained ~18 m/sec. Before the separation, power output from PI elements was 2875 μW, and the total output resulted in 3681 μW.

The increased amplitude was due to the loss of plate stiffness after elements separation. We believe that the epoxy layer, which bonded the elements to the plate, started failing during case 12 test, and finished during case 14. Interestingly, the power output for PI elements after the separation returned to its levels before the separation. This time however, the power generation was due to elements flapping in the air and not oscillating with the plate.
Plate With PI Elements - Case 12

Figure 200: Flutter Test with PI Elements – Case 12
Figure 201: Power Output, Case 12 with PI Elements
Figure 202: Power Output, Case 14 with PI Elements

Figure 203: PI Plate, End of Case 14 Test
Figure 204: PI Plate, Case 14, Final Test
4. Conclusions

Following the subject separation in our work, the conclusions are discussed separately in the following.

4.1 Convolution and Volterra Series Approach to Reduced Order Modelling of Unsteady Aerodynamic Loads

This research was dedicated to the development of reduced order models for unsteady aerodynamics in the transonic regime. The resultant models can be expanded to higher degree of freedom (DOF) systems, as we might encounter in industrial applications for example. In our approach the unsteady loads are generated from two sources; the linear convolution of the input with the step response kernels, and the addition of non-linear effects via Volterra series – convolution of input products with higher order kernels. So far, the validation included a simple 2DOF example, but we believe there is a potential for further expansion of the approach.

From the beginning, the new models were set to be scalable and intended to be applied easily for higher DOF systems. In Sections 2.1 through 2.5 a compact and easy to use Volterra series formulation was developed. Implementing the Laguerre polynomials allowed us to effectively use low order Volterra terms and still adequately represent complicated behavior of the aerodynamic loads. Using only the sparse Volterra terms eliminated the need to additionally identify the cross-memory kernels. The formulation creates a Reduced Order Model of the unsteady aerodynamic loads using known CFD
results for prescribed airfoil motion. However, it offers no engineering insight into the nature of the created model, i.e. which higher order terms might dominate, and also the memory depth of the system is unknown. Moreover, a separate model is needed for each frequency of the oscillations. These drawbacks were listed in Section 2.6 and are the reason for the initiation of the new approach.

In Section 2.7, we used a textbook example of Theodorsen aerodynamics and the Wagner function to define a convolution integral and use it to generate the unsteady lift of a 2D flat plate. This approach yields an accurate estimation of the linear unsteady aerodynamics for several frequencies using a single model.

In Section 2.8 we expanded the classic example above and showed that the method can be used just as well with data obtained by CFD simulation. The step response input-output data was used to generate unsteady aerodynamic loads with the convolution approach. The linear loads were calculated for pitch and heave DOF.

Section 2.9 combined the previous linear convolution results with the Volterra series formulation from Sections 2.1 through 2.5. Higher order convolution kernels were identified, allowing the representation of the non-linear unsteady aerodynamic loads as well. The non-linear contribution is guided by three user defined parameters only: $N$ – the highest degree of Volterra kernel, $a$ – Laguerre time scale, and $R$ – the highest order of the Laguerre polynomial. The memory depth of the model was determined previously from the linear convolution part, and the non-linear kernels were identified
using CFD calculated step response data. Using varying amplitudes of the step input allowed us to account for substantially different non-linear behavior of the loads at each input level.

In Section 2.10 we identified unsteady aerodynamic loads for sinusoidal in time airfoil motion, linear and non-linear, and compared those to previously calculated CFD results. In the identification phase we use the step response CFD data, which does not require any real-time training signal. The identified model is then used with any known input to generate the actual loads. In this way, the model is completely detached from the oscillating unsteady loads, and it is independent of the oscillation frequency. We showed a good agreement between the CFD calculated loads and the loads generated by the model, except when the step response itself includes an oscillating behavior, like in buffet for example. The model performance worsens if the input frequency is close to the buffet frequency. We also demonstrated that by smoothing the step response data the model once again agrees well with the CFD results. Overall, a single kernel set can effectively generate unsteady aerodynamic loads for a wide frequency range.

Finally, in Section 2.11 we used the model to generate the unsteady aerodynamic loads in an aeroelastic simulation. Linear flutter boundary and the LCO levels compare very well with previous test results and former calculations. As expected, the agreement is better for the flutter boundary per se than the LCO levels.
The following are several highlights of the research:

- The proposed method clearly separates the linear and the non-linear contribution to the unsteady loads. Subsequently, an engineering decision can be made as to what model to use in each case.
- The identification process uses the step response data, as opposed to a dedicated training signal. In this way the model can be generated completely “off-line”. Moreover, the quality of the model can be assessed and adjusted before the aeroelastic simulation.
- Increasing the order of Laguerre polynomials is a convenient way to implement complicated “curvy” behavior of the unsteady loads, without expanding the model to higher order kernels. On the other hand, there is no limitation on the order of the kernel.
- The proposed method allows a substantial reduction in the management effort of a model which includes a large number of reduced frequencies, $k$. The model is detached completely from a particular set of $k$ values.
- As far as we observed, the non-linear correction which is implemented with the Volterra kernels can be significant and not limited to a mild non-linearity.
- The approach takes into account substantially different non-linear behavior by using several levels of step response data and choosing the appropriate kernel bases according to the LCO response levels. This is called the “toggling” approach.
• The method is suitable for oscillatory response, even in the presence of buffet for example, if the airfoil motion frequency is sufficiently different from the buffet frequency.

4.1.1 Future Work

Below are several directions in which this research might proceed further:

• Expanding the model to more DOF and using modal step response data to generate the convolution kernels, linear and non-linear. This might be the easiest way to apply the approach to larger models.

• Standardization of the CFD generated data.

• Systematic management of the resultant kernels. For example, generating a continuous “kernel surface” for smoother toggling between models.
4.2 Improving Piezoelectric Energy Harvesting of an Aeroelastic System

In this work we explored the possibilities of improving the conversion efficiency
of an aeroelastic power generator.

- By placing small discrete masses on the plate, we were able to increase the
  conversion efficiency by ~260%, while simultaneously reducing the required air
  speed from 23.5 m/sec to 15.6 m/sec (Case 14 vs. Base Line for example).

- The non-linear nature of LCO allows us to expand the useful range of air speeds
  from a single onset to a wider range, i.e. 23.5 m/sec in the base line case to 18 m/sec –
  30.7 m/sec for Case 12. The plate survived these tests without structural failure, and
  this opens possibilities for practical energy harvesters based on non-linear aeroelastic
  effects and piezoelectric elements.

- Center line placed masses are preferred for sustainable power generation. Reduced
  frequency torsional modes will subject the structure to higher loads without
  contributing to a better conversion.

Several additional observations we made during the tests are highlighted below:

- The tip area is usually not exploited in vibrational energy harvesting tests. By adding
  small masses towards the tip of the plate, this area can be more significant than the
  root.

- For sustainable power generators, a damped structure is preferred.
• For highly flexible structures, like the plate in our case, low-stiffness PVDF piezo-elements are the preferred choice. Despite their lower efficiency, these elements will not limit the full potential of power generation. PVDF elements are also cost efficient.

**4.2.1 Future Work**

Below are the directions in which this research might continue, towards a viable energy harvesting solution based on an aeroelastic system:

• While in LCO, the plate experiences variations in deflection shapes, final test of Case 12 for example, and as a result variations in power generation. Future work might include an effort to understand these trends.

• An adaptation of this research might include a feedback mechanism which will control the spanwise mass location at several chord locations. This allows one to match the dynamics of the plate to the ambient wind speeds for favorable power generation.
References


Biography