Essays in Empirical Macroeconomics

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Economics in the Graduate School of Duke University 2010
Abstract
(Economics)

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Abstract

This dissertation consists of three essays in empirical macroeconomics. In the first essay, I explore the dynamic effects of aggregate news about future technology improvements on sectoral fundamentals. I document that the durable goods sector responds significantly more to news shocks than the nondurable goods sector. By looking at the behavior of inventories, which have been largely neglected in the news literature, I show that aggregate news propagates the business cycle mainly through the durable goods sector. My theoretical framework is a two-sector, two-factor, real business cycle model augmented with the following three real rigidities: habit persistence in consumption, variable capacity utilization, and investment adjustment costs in both sectors. In addition, I introduce inventories as a factor in the production of durable goods. The model is successful in replicating the empirical responses of the US economy to news shocks. It reproduces the stronger response of the durable goods sector and can perfectly match the responses of inventories.

The second essay, which is joint work with Roberto Pancrazi, evaluates the effects of a change in monetary policy on the decline of the volatility of real macroeconomic variables, and on its redistribution from high to medium frequencies during the post-1983 period. By using a dynamic stochastic general equilibrium model, we find that the monetary policy alone cannot account for the observed changes in the spectral density of output, investment, and consumption. However, when we also consider a change in the exogenous processes, a different monetary policy accounts for 40
percent of the decline in the high-frequency volatilities and partially accounts for the redistribution of the variance toward lower frequencies.

In the third essay, I study exchange rate dynamics. In particular, I investigate the main features of a rich theoretical model that are necessary to explain exchange rate volatility and persistence. As a theoretical framework, I use a small open economy dynamic stochastic general equilibrium (DSGE hereafter) model. The model is estimated using Bayesian techniques. I use post Bretton-Woods data for the following three countries: Australia, Canada, and the United Kingdom (UK hereafter). The performance of the benchmark model in replicating both real exchange rate persistence and volatility is rather good. I show that the domestic and importing sector price stickiness and indexation parameters are the most important features of the model for a successful replication of the real exchange rate dynamics. The importance of the importing sector price stickiness and indexation parameters is increasing in the share of importing goods in the consumption basket. The most important shocks for explaining the exchange rate volatility at business cycle frequency are the investment specific technology shock, monetary policy shock, and labor supply shock, among domestic economy shocks, and the shock to the interest rate among the foreign shocks.
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A Sectoral Approach to News Shocks

1.1 Introduction

The idea that the expectations about future changes in productivity represent an important driving force of the business cycle has received a great deal of attention in the recent literature in economics, after being abandoned for more than half a century. Beaudry and Portier [2005, 2006], Christiano, Ilut, Motto, and Rostagno [2008], Jaimovich and Rebelo [2009], Beaudry and Lucke [2009], and Schmitt-Grohé and Uribe [2009], among others, provide compelling evidence that news about future technological developments accounts for the bulk of business-cycle fluctuations.

A different stream of the literature investigates the role of the durable goods industries in the propagation of business cycles. In particular, Mankiw [1985] concludes that durable goods industries play an essential role in the business cycle and that explaining fluctuations in the durable goods sector is vital for understanding aggregate economic fluctuations. The purpose of this paper is to link these two ideas and analyze through which sectors news shocks propagate the business cycle.

1 Pigou [1927] was one of the first authors to propose that agents’ expectations about the future are an important source of business cycle fluctuations.
Understanding the nature of the aggregate news shock is a crucial first step in my analysis since the effects of aggregate news on sectoral fundamentals obviously depend on whether aggregate news contains information about the nondurable, durable, or both sectors. For instance, consider the following three simple scenarios: aggregate news being news about only the nondurable sector, about only the durable sector, or about both sectors simultaneously. In the first situation, an aggregate news shock would represent a demand shock to the durable sector and a supply shock to the nondurable sector, since investment in capital in the nondurable sector increases as a result of higher productivity and higher consumer demand. In the second case, a news shock represents both a supply and a demand shock to the durable goods sector, and, presumably, would imply significantly different behavior on the part of durable goods producers in response to the shock. Finally, the third scenario represents a combination of the first two.

By examining the behavior of a variety of fundamentals at the sectoral level, I show that the second scenario is the dominant one. Aggregate news appears, mainly, to be news about productivity in the durable goods sector. Although the dynamics of all sectoral level fundamentals are altered in the three scenarios, the behavior of investment and inventories in the durable goods sector is essential to revealing which of the three scenarios is most likely. In particular, my empirical analysis shows that the response of the inventories to a unit aggregate news shock is statistically significant; this would not be the case if the dominant scenario were the first one (aggregate news being news only about the nondurable sector), since, as I show, a demand shock to the durable goods sector would not affect inventories of the durable sector in a quantitatively significant way. My analysis, therefore, confirms that the behavior of inventories carries essential information for understanding the propagation of news shocks and business cycles.

To identify news shocks, I use the two-step identification procedure proposed by
Beaudry and Portier [2006]. This identification procedure sequentially uses short-run and long-run identification schemes to recover news shocks. It assumes that news shocks are ones which are orthogonal to current total factor productivity (TFP hereafter) innovations but permanently raise TFP in the long run. On the other hand, news shocks represent innovations to the stock price index and therefore instantaneously affect it. Whereas Beaudry and Portier [2006] recover aggregate news shock, I recover, in addition, sector-specific news shocks. Specifically, I retrieve news shocks for the manufacturing sector, the durable goods sector, and the nondurable goods sector. In order to do so, I create a data set composed of quarterly TFP (corrected for variation in capacity utilization) and stock price indices at the level of these three sectors. To do so, I mimic the procedure utilized by Beaudry and Portier [2006], but implement it at the sectoral level.

After obtaining measures of aggregate and sectoral news shocks, I investigate how they affect the behavior of the following sectoral fundamentals: productivity, consumption, hours, output and investment. I show that there is a great deal of comovement among these variables at the sectoral levels. However, the data indicate that there are much higher percentage responses of relevant variables in the durable goods sector than in the nondurable goods sector. In particular, after a 1 percent aggregate news shock, durable sector productivity responds by approximately 3 percent after ten quarters, while the response of productivity in the nondurable goods sector is approximately one half of one percent. The percentage responses of other durable goods sector fundamentals are also significantly higher than those of the same fundamentals in the nondurable goods sector. Furthermore, when I investigate the effects of sector-specific news on the same sectoral fundamentals this basic pattern prevails.

My results suggest that aggregate news shocks are propagated primarily through the durable goods sector. Within the durable goods sector, the percentage responses
are the highest in the following two-digit standard industrial classification code (SIC) industries: primary metals, industrial machinery, instruments, and electronic equipment. These are, in fact, the industries with the highest share in the value added of the manufacturing sector in the United States. The percentage responses of the nondurable goods sector variables are much less than those of the durable goods sector variables; among the nondurable goods sector industries, the industries that respond the most are those that produce goods having some level of durability, even though they are classified as nondurable industries.

My investigation of inventories is motivated by an obvious and key difference between the durable and nondurable sectors. Nearly a century ago, Pigou [1927] proposes that the possibility of holding stocks of inventories explained the fact that business cycle fluctuations are more pronounced in durables industries than in nondurables industries. Early research in the real business cycle tradition (see Blinder [1986], Christiano and Eichenbaum [1987], Eichenbaum [1984], Ramey [1989]) focused considerable attention on the importance of explaining the behavior of inventories. Recent research has suggested that improved inventories management contributed to the great moderation (see Kahn [2008, 2009]). I re-establish the role and importance of inventories with new empirical evidence concerning the response of inventories to news shocks. To do so, I use the two standard inventories indicators: the inventories-

---

2 In his book *Industrial Fluctuations*, Pigou says the following: "When for any reason the aggregate demand is increased in commodities that are durable and are not destroyed in the act of use, the resultant extra production of these commodities in the years of high demand involves the existence of a correspondingly enlarged stock, and so gives rise to a smaller demand for new production of these commodities than it used to give rise to before. Thus, the upward fluctuation of industrial activity above the normal carries with it a subsequent downward fluctuation below the normal when the stimulus is removed, and not merely a subsequent return to the normal... The same thing holds good of those consumption goods which are destroyed in a single act of use, provided that they are durable in their own nature and are of such a sort that they can be held in store without great cost of risk: for dealers pile up stocks of them in booms, and in depressions are forced to offer them out of their stocks in competition with the current output of industry... Here, then, we have a second reason for expecting that instrumental industries will fluctuate more than others, even though it is in the others that the cause of fluctuations lies."
to-sales ratio and the change in the inventories-to-output ratio. Whereas the percentage responses of both inventories indicators to news are statistically significant in the durables sector, the percentage responses of inventories in the nondurables sector are statistically insignificant.

Finally, I design a theoretical model that is consistent with the empirical evidence on how the economy responds to news shocks. Specifically, I build a two-sector, two-factor, real business cycle model which follows Baxter [1996] in its basic structure. Sector 1 produces a pure consumption (nondurable) good, whereas sector 2 produces consumer durables and the capital good used in producing both goods. Both sectors use capital and labor as their factor inputs. The key difference between the two sectors is that a good produced in sector 2 can be stocked, whereas the output of sector 1 is perishable. I model this feature by adding inventories into the production function of sector 2, following Christiano [1988] and Kydland and Prescott [1982]. These authors argue that the stock of inventories, as the stock of fixed capital, provides a flow of services to a firm.

My model is augmented by the following four real rigidities: habit persistence in consumption, investment adjustment costs in both sectors, as well as adjustment costs associated with changes in the stock of consumer durables, and variable capacity utilization. I introduce habit persistence in consumption in order to obtain a hump-shaped consumption response. Following Jaimovich and Rebelo [2009], I introduce investment adjustment costs in order to obtain comovement between hours, consumption, output and investment. Variable capacity utilization assures that the model-based and empirical measures of TFP coincide. Finally, news shocks are introduced by feeding the empirical responses of TFP in the two sectors into the model.

The model is successful in mimicking the empirical responses to news shocks at the sectoral level. First, the response of the inventories-to-sales ratio can be perfectly matched. Second, the model is able to reproduce the stronger overall response of the
durable goods sector to news shocks. For example, responses of hours, consumption and output are all higher in the durable goods sector. Third, the model reproduces the comovements observed among sectoral level fundamentals. That is, the model reproduces comovement among hours, consumption, output and investment in both sectors. Even though it cannot perfectly match the responses of all the variables (for example, investment response in the nondurable goods sector), overall the model fits the data rather well.

The remainder of this chapter is structured as follows: In Section 1.2, I describe some of the main findings in more detail, the data used in the analysis, as well as all empirical findings. In particular, I first explain how I obtain measures of aggregate and sectoral news shocks and how I recover their effects on sectoral fundamentals. The two-sector model is presented in Section 1.3. Section 1.4 explains my model calibration and estimation procedures. Quantitative findings using the model are presented in Section 1.5. Section 1.6 concludes.

1.2 Empirical Evidence

In this paper, I investigate, analyze and define the nature of aggregate news shocks. I first explore the effects of aggregate news shocks on sectoral TFPs and stock price indices. Specifically, I analyze the manufacturing sector, the durable goods sector, and the nondurable goods sector (see responses in Figure 1.1). According to the analysis, whereas TFPs do not respond immediately to news, stock prices do in all sectors. This observation confirms that news is immediately reflected in the variables that express agents’ expectations about the future, such as stock prices. Another striking observation is the different quantitative behavior of durable and nondurable goods sectors. Both the percentage responses of TFP and stock prices in the durable goods sector are significantly greater than in the nondurable goods sector. Specifically, after ten quarters, the response of durable sector TFP is almost
five times greater than the response of nondurable sector TFP, whereas the impact response of the durable sector stock price index is 2 percent greater than the impact response of the nondurable sector stock price index. This might suggest that the information contained in an aggregate news shock is more about future productivity in the durable sector than about nondurable sector productivity.

A key difference between durable and nondurable producers is that the former are more able to respond quickly to demand shocks by running down stocks of inventories.³ My exploration of the role of inventories, which have not previously been studied in the news literature, is motivated by this key difference between the producers of durable and nondurable goods. My empirical evidence suggests that the response of inventories-to-sales ratio to an aggregate news shock is almost nonexistent in the nondurable industries, whereas there is a noticeable and significant drop in the durable industries. The average responses of the inventories-to-sales ratio in the manufacturing, durable, and nondurable sectors are presented in Figure 1.2. Specifically, in the durables sector the ration drops by 0.4 percent within one year of a news shock, whereas the average response in the nondurables sector is small and insignificant.

In order to achieve a more complete analysis, I extend my investigation to other sectoral-level fundamentals, such as hours, output, investment, and consumption. A discussion of this follows a detailed explanation of how aggregate news shock and sectoral news shocks are identified.

³ This is not to say that stocks of inventories cannot be held in the nondurable sector industries, but their volume is lower than in the durable sector industries. In particular, more than 60 percent of manufacturing sector inventories are carried by the durable sector. Also, the inventories of durable producers are more durable in nature.
1.2.1 Identification of News Shocks

When the effects of a particular shock are discussed in macroeconomics, an important first step is to clearly communicate the validity of the identification scheme. The primary purpose of this paper is not to propose a new identification scheme but to extend a standard existing identification scheme to the sectoral level. News shocks are typically defined as the arrival of new information about future productivity growth that ends up being reflected instantaneously in forward-looking variables, but does not have an instantaneous impact on the current TFP.\footnote{See, for example Beaudry and Portier [2006], Christiano et al. [2008] and Jaimovich and Rebelo [2009].} Rather, the effects on TFP are realized only after a certain number of quarters. Although it is relatively straightforward to think about this phenomenon in the theoretical framework, recovering its empirical analog is more challenging.

In my empirical analysis, I follow the two-step procedure proposed by Beaudry and Portier [2006]. Their identification procedure takes, as a starting point, the definition of a news shock given above. The most natural choice of variables on which to base the procedure are a measure of productivity and some forward-looking variable that contains information about future developments. Like Beaudry and Portier, I use TFP as the measure of productivity, and a stock price index as the forward-looking variable. I start with a bivariate time series model for these variables. In order to recover news shocks, I sequentially impose two separate identification restrictions, described as the short-run and long-run, on the model.

To describe the short-run restriction, I assume that the two variables can be represented in log first differences, by the Wold representation:

\[
\begin{bmatrix}
\Delta TFP_t \\
\Delta SP_t
\end{bmatrix} = \Gamma(L) \begin{bmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t}
\end{bmatrix}
\]

where \( \Gamma(L) = \sum_{i=0}^{\infty} \Gamma_i L^i \), and the two shocks, \( \varepsilon_{1t} \) and \( \varepsilon_{2t} \), are mutually orthogonal.
and have unit variance. The short-run restriction imposes that \( \varepsilon_2 \), has no short-run effect on TFP. More formally, this restriction is imposed by setting the 1, 2 element of the matrix \( \Gamma_0 \) to zero.

The long-run restriction is based on an alternative Wold representation

\[
\begin{bmatrix}
\Delta TFP_t \\
\Delta SP_t 
\end{bmatrix} = \tilde{\Gamma}(L) \begin{bmatrix}
\tilde{\varepsilon}_{1t} \\
\tilde{\varepsilon}_{2t}
\end{bmatrix}
\]

where \( \tilde{\Gamma}(L) = \sum_{i=0}^{\infty} \tilde{\Gamma}_i L^i \), and the two shocks, \( \tilde{\varepsilon}_{1t} \) and \( \tilde{\varepsilon}_{2t} \), are mutually orthogonal and have unit variance. The long-run restriction is that only \( \tilde{\varepsilon}_{1t} \), has a long-run effect on TFP. This restriction is imposed by setting the 1, 2 element of the matrix \( \tilde{\Gamma}(L) = \sum_{i=0}^{\infty} \tilde{\Gamma}_i L^i \) to zero.\(^5\)

As such these two identification schemes are purely ad-hoc. However, suppose that it happens to be the case that the two recovered disturbances, \( \varepsilon_2 \) and \( \tilde{\varepsilon}_1 \), are extremely highly correlated, or effectively the same. This suggests that the procedure has recovered a single shock that, since it satisfies the short-run restriction, does not have an immediate effect on the productivity and affects it only with a delay, and, since it satisfies the long-run restriction, captures all important long-run information about productivity. Given these characteristics, the shock satisfies the two characteristics of news described above. Of course, the procedure only delivers plausible measures of news if the two shocks happen to be highly correlated.

1.2.2 Data

The data used in this paper can be broadly divided into aggregate and sectoral-level series. Specifically, I use data from the manufacturing which allow me to distinguish

---

\(^5\) If the two series are found to be cointegrated, all elements in the second column equal zero, i.e. \( \tilde{\varepsilon}_1 \) is the only permanent shock. This is because the rank of the matrix is one in the presence of the one cointegrating relation. Hence, in this case, a simple Cholesky decomposition cannot be used to recuperate the structural news shock, as in Blanchard and Quah [1989]. In fact, to recover disturbance \( \tilde{\varepsilon}_1 \), I follow the procedure proposed by King et al. [1991]. This procedure allows one to impose the long-run restrictions using the fact of the existence of the cointegrating relations.
between the durable and nondurable goods sectors. As a further level of disaggregation, I also use data on nineteen two-digit SIC code manufacturing industries. Data for the manufacturing, durable goods, and nondurable goods sectors are available at the quarterly frequency for the sample period 1972:I - 2005:III. Data for the two-digit industries are also available at the quarterly frequency, but for a shorter sample period, 1972:I - 1997:III. Finally, aggregate data, for the entire US economy, are available at the quarterly frequency for the period 1949:I - 2006:IV.

In order to recover news shocks at all levels of aggregation, I create a data set composed of TFP and stock prices for these three sectors and nineteen two-digit industries. I construct TFP following Burnside, Eichenbaum, and Rebelo [1995]. Rather than considering their three different technology specifications, I use the model that allows me to measure TFP at the two-digit level, despite the absence of observations on material inputs. Specifically, I assume that time \( t \) gross output is produced using a Leontief production function

\[
Y_t = \min (M_t, V_t),
\]

where \( M_t \) denotes time \( t \) materials and \( V_t \) denotes value-added at time \( t \), which, itself, is produced using hours of work \( (N_t) \), the stock of capital \( (K_t) \), and time varying capital utilization, measured by electricity use \( (E_t) \). As Burnside, Eichenbaum and Rebelo show, this specification allows gross output in sector \( i \) to be written as

\[
Y_t^i = A_t^i F \left( N_t^i, \frac{E_t^i}{\phi} \right),
\]

where \( \phi \) represents the assumed fixed proportion between electricity consumption and capital services, with the latter being the product of the capital stock and its work week. I further assume that the function \( F (\cdot) \) takes the Cobb-Douglas form.

\[6\] In 1997 the SIC was replaced by the North American Industry Classification System (NAICS), and it is not possible to extend the two-digit SIC series further than 1997. On the other hand, historical data have not been transformed to match NAICS.
After assuming that the labor and electricity markets are perfectly competitive, the expression for TFP in a sector or industry $i$ can be obtained using a first-order log-linear approximation of the production function:

$$\Delta Y_t^i = (1 - \alpha_1) \Delta N_t^i + \alpha_1 \Delta E_t^i + \Delta A_t^i,$$

where $\Delta A_t^i$ is assumed to be the growth rate of TFP, $\Delta Y_t^i$ the growth rate of output, $\Delta N_t^i$ the growth rate of labor input and $\Delta E_t^i$ the growth rate of electricity use in sector or industry $i$.\(^7\) Data on output (measured using the relevant indices of industrial production) and electricity consumption (my proxy for capital services) at the sectoral level are obtained from the Federal Reserve Board. As emphasized by Beaudry and Portier [2006] and Sims [2009], TFP measures that are adjusted for capacity utilization are preferable since they lead to more realistic, substantially delayed, productivity responses to news shock. As a sectoral labor measure, I use the quarterly averages of monthly production workers, which is constructed as the product of the following two time series: average weekly hours of production workers and the number of production workers, both obtained from the Bureau of Labor Statistics (BLS hereafter). Finally, my assumption of a constant returns to scale Cobb-Douglas technology allows me to calibrate the parameter $1 - \alpha_1$, using labor’s share of income.\(^8\) I compute labor’s share as the ratio of labor compensation and nominal income in each sector or industry. Both series needed for this calculation are obtained from the Bureau of Economic Analysis (BEA hereafter).

Sectoral stock price data are extracted from Kenneth French’s website.\(^9\) Data

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\(^7\) Since in my model I consider inventories to be a factor of the production in the durable goods sector, the durable sector measure of TFP will be slightly altered in order to account for this:

$$\Delta A_t^{dur} = \Delta Y_t^{dur} - (1 - \alpha_{dur}) \Delta N_t^{dur} - \alpha_{dur} (1 - \rho) \Delta E_t^{dur} - \alpha_{dur} \rho \Delta I_t^{dur},$$

where $\rho$ controls for the role of inventories $I_t$ in the production function of the durable goods sector.

\(^8\) Burnside [1996] concludes that "the typical U.S. manufacturing industry displays constant returns with no external effects."

\(^9\) The data are available for download at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french
on S&P500 were obtained from Robert Shiller’s website.\textsuperscript{10} Using the consumer price index (CPI) as the inflation measure and the civilian noninstitutional population over 16, I convert these data to real per capita measures. Besides using TFP and stock price indices data in my empirical analysis, I also use data on real per capita consumption, hours, investment, output and several inventories indicators, at the aggregate and sectoral level. Detailed explanations of these data and all other data used in the paper, as well as their sources, are provided in the Appendix A.1. Log levels of the sectoral data on TFP, stock prices, consumption, hours, output, investment, capital stock and inventories-to-sales ratios are given in Figures 1.3 and 1.4.

1.2.3 Aggregate and Sectoral News

The news literature has focused primarily on aggregate news and its effects on aggregate variables, and has remained largely silent concerning the effects of news on sectoral fundamentals, and concerning the possibility that some shocks are news specific to a particular sector of the economy. In order to understand these sectoral aspects of news shocks, in this section I describe news shocks recovered at the sectoral level, and I explore the dynamic responses of sectoral fundamentals to aggregate and sector-specific news.

Beaudry and Portier [2006] show that, for the aggregate US economy, the two disturbances, $\varepsilon_{2t}$ and $\tilde{\varepsilon}_{1t}$, obtained from the short-run and long-run identification schemes are highly correlated, and induce nearly identical dynamic responses of TFP and stock prices. This feature of the identified shocks is preserved when a third variable, such as consumption, output, investment, or hours is added to the system. Figure 1.5 shows the responses of aggregate TFP and the stock price index to these

\textsuperscript{10} Available at http://www.econ.yale.edu/~shiller/data.htm
two disturbances. In both panels of the graph the impulse responses produced by the two orthogonalization schemes nearly coincide. Furthermore, the short-run and the long-run impulse responses lie entirely within the 95% confidence interval of the long-run impulse response, which is represented by the shaded area. Looking at these responses, one can reasonably conclude that the recovered shock has the two characteristics of a news shock. First, it has no immediate effect on TFP. This is true, by construction, in the case of the short-run identification, whereas, in the case of the long-run identification, the response of TFP is not restricted, but is estimated to be very close to zero. After no immediate effect, the TFP response begins to increase in the second quarter and remains at the level of approximately 0.6 percent in the long run. Second, the stock price index response is a large 6 percent. This observation suggests that, as the news about the future productivity is received, agents alter their expectations of future productivity, and this brings about a positive response of the stock prices. Recovered aggregate news shock series together with the National Bureau of Economic Research (NBER) recession dates are shown in Figure 1.6.

Next I turn to identifying news shocks in the manufacturing, durable goods and nondurable goods sectors. I also compare the recovered sectoral level shocks to the aggregate news shocks.

I identify manufacturing news shock using a two-variable vector autoregressive system in the log first differences of TFP and stock price index for the manufacturing sector. According to a Likelihood Ratio Test, the hypothesis of cointegration between TFP and the stock price index cannot be rejected at the 5% level.\(^{11}\) Therefore, a vector error correction model (VECM) is the appropriate specification. In the case of the manufacturing sector, the correlation between the two identified disturbances, \(\varepsilon_2\) (short-run) and \(\tilde{\varepsilon}_1\) (long-run), is very high, 0.994.

The responses of the manufacturing sector TFP and stock price index to a 1

\(^{11}\) See Hamilton, page 648.
percent increase in $\varepsilon_2$ and $\tilde{\varepsilon}_1$ are displayed in Figure 1.7. As in the aggregate economy, the dynamics induced by the two disturbances are very similar. As before, the short-run disturbance, $\varepsilon_2$, permanently affects TFP. Moreover, the long-run disturbance, $\tilde{\varepsilon}_1$, has nearly no impact effect on TFP. These two facts suggest that the identification procedure has correctly isolated news shocks in the manufacturing sector. The impulse response functions for TFP have a somewhat different shape in the manufacturing sector than they do for the full economy. In the aggregate case, TFP increases soon after the shock, and does so sharply, whereas manufacturing sector TFP increases slowly after an initial (statistically insignificant) drop in the first two quarters after the shock.\textsuperscript{12} The response of the manufacturing stock price index is very similar to the response of its aggregate counterpart to aggregate news. It increases on impact by 7.5 percent and remains at roughly the same level indefinitely.

I also identify durable-sector-specific news shocks and nondurable-sector-specific news shocks by adopting the same identification procedure. I use a two-variable VECM in the log first differences of sectoral TFP and stock prices for the durable and nondurable goods sectors respectively.\textsuperscript{13} Figure 1.8 displays the impulse response functions of sectoral TFPs and stock prices.

As in the case of the aggregate economy and manufacturing sector, the disturbances originating from the short-run and long-run identification procedures induce quite similar dynamics. However, in both sectors, the long-run shock, $\tilde{\varepsilon}_1$, has an immediate effect on TFP; in the durable sector it rises by approximately 2 percent, whereas in the nondurable sector it rises by 1 percent. But, as the time horizon expands, the two responses almost converge in both sectors. Furthermore, the two

\textsuperscript{12} Nevertheless, when aggregate TFP is corrected for capacity utilization, its qualitative response more closely resembles the response of the manufacturing TFP response. In this case, aggregate TFP does not increase for several years; the diffusion process appears to be slower than when unadjusted measure of TFP is used.

\textsuperscript{13} In both sectors, a Likelihood Ratio Test suggests the presence of one cointegrating vector.
responses lie within the 95% long-run confidence bands, suggesting statistical similarity.

Evidently, the long-run response of durable sector TFP is considerably greater than that of nondurable goods sector TFP; after 40 quarters, the effect of a 1 percent durable sector news shock on durables TFP is approximately 4 percent, whereas the effect of a 1 percent nondurable sector news shock on nondurables TFP is less than 1 percent. This greater response of durables TFP suggests that the relatively large 1.5 percent response of manufacturing TFP to a news shock is mostly driven by the behavior of the durable sector. In addition, the response of durables TFP to durables news qualitatively resembles the response of its manufacturing counterpart to manufacturing news. Both the higher durables TFP response and its resemblance to the manufacturing TFP response suggest that manufacturing news is mainly about the future growth of durables TFP, rather than about the future growth of nondurables TFP.

Even though the long-run disturbances have an immediate effect on sectoral TFPs, the correlations between the two disturbances in both sectors are still high (0.934 in the durable goods sector and 0.989 in the nondurable goods sector). Scatter plots of the two disturbances, \( \varepsilon_2 \) and \( \tilde{\varepsilon}_1 \), at the manufacturing, durables, and nondurables levels are plotted in Figure 1.9. In addition, the correlation between the aggregate and nondurable news shocks (0.738), is slightly lower that the correlation between the aggregate and durable news shocks (0.758). This result also holds when manufacturing news is compared to durable and nondurable news; the correlation between the durables and manufacturing news is 0.885, whereas the correlation between the nondurable and manufacturing news shocks is 0.837. These facts indicate that aggregate and manufacturing news shocks carry more information concerning the durable sector developments, than they do concerning the nondurable sector developments.
In order to investigate whether the higher durable sector responses are due only to some aggregation effect, in the next section I extend the analysis to a more disaggregated level. I consider two-digit SIC industries and their responses to various types of news shocks.

**Two-digit SIC Level**

I perform the same identification procedure at the level of nineteen two-digit SIC manufacturing industries. SIC represents a numerical system developed by the Federal Government for classifying all types of economic activity within the United States economy. Specifically, ten of these industries are classified as durable goods industries, and the remaining nine as nondurable goods industries. For each two-digit SIC industry, I construct a vector autoregressive system composed of the TFP and stock price index of that particular industry, sequentially performing short-run and the long-run identifications. However, at the more disaggregated level, impulse response functions implied by the two disturbances follow rather different dynamics, contrary to the aggregate or sectoral levels. Thus, the correlation between the two recovered shocks is rather small, on average. This evidence suggests that the identification scheme fails to recover news shocks that are specific to the two-digit SIC industries. One plausible explanation is that classifying firms according to two-digit SIC codes is somewhat difficult. In particular, SIC classifies establishments by their primary type of activity - the activity that contributes the most to the value added of the establishment. Since firms often operate in more than one industry, the information that is contained in their TFP or stock price index is not necessarily tied to only one two-digit SIC industry. Hence, as the level of disaggregation deepens, it becomes more difficult to extract industry-specific news shocks from the data.

Even though I am not able to identify news specific to the two-digit SIC industries using the procedure of Beaudry and Portier, the behavior of the two-digit SIC
productivity, stock prices, and other fundamentals in response to recovered aggregate or sectoral news may carry important information. Conditional upon the recovered shocks representing news shocks, one can run the following regression:

\[ \Delta X_t = \sum_{j=0}^{J} \psi_j \varepsilon_{2,t-j} + \mu_t, \]  

(1.1)

in order to infer the effects of news shocks on a variable \( X_t \). Here, \( \varepsilon_{2,t} \) represents the recovered structural news shock at time \( t \) coming from the short-run identification procedure.\(^{14}\) Specifically, it can represent either the aggregate news shock or one of the three recovered sectoral news shocks. The point estimate impulse response of variable \( X_t \) to a particular news shock after a horizon \( n \) is measured as the cumulative sum of the regression coefficients \( \psi_j \): \[ \sum_{j=1}^{n} \psi_j. \]

I first investigate the effects of an aggregate news shock on the productivity of the nineteen two-digit SIC industries. Figure 1.10 displays the responses of durables and nondurables industries to a unit aggregate news shock. First, the impact response of productivity is statistically equal to zero in all industries, satisfying the condition that news shocks do not affect productivity immediately. Second, durables industries are, on average, more responsive to aggregate news in the long-run than nondurable industries. Within the durable goods sector, industries with the highest percentage responses are the following: primary metals, industrial machinery, instruments and electronic equipment. These industries have the highest shares in the total value added of the durable goods sector. On average, the responses in the nondurable sector industries are much smaller. Industries that consistently respond the most, among the nondurable goods industries, are chemicals, petroleum, and textile mills, which are industries whose output has a somewhat durable character.\(^{15}\)

\(^{14}\) Since I showed that the correlations between the long-run and short-run shocks are very high (both at the aggregate and sectoral levels), the effects of \( \tilde{\varepsilon}_1 \) and \( \varepsilon_2 \) on \( X_t \) are nearly identical.

\(^{15}\) Beaudry and Portier [2005] use annual Multifactor Productivity Trends sectoral data in order
When I repeat a similar exercise - recovering industry responses to sectoral news - I obtain similar conclusions. First, durable goods industries respond more to all types of news shocks. Second, the response of each two-digit SIC industry is the highest to a news shock specific to the sector to which the industry belongs (according to the SIC). For instance, nondurable goods industries respond more to nondurable news shocks than to durable news shocks, whereas durable goods industries respond more to durable news shocks than to nondurable news shocks. This result is somewhat expected, since the sector-specific news carries more information about the TFP in that particular sector than, for example, aggregate news.

After documenting that aggregate and sectoral news do not have an impact effect on TFP in the two-digit industries, I also examine the responses of the two-digit SIC stock price indices. If the price indices rise on impact this supports the view that the recovered shocks represent news. The responses of stock prices in the nineteen two-digit manufacturing industries to a 1 percent aggregate news shock are shown in Figure 1.11. Stock prices respond significantly on impact in each industry, confirming that news shocks are immediately reflected in forward-looking variables. The average response of the stock price indices in the durable goods sector is higher than that of the stock price indices in the nondurable goods sector. Specifically, stock prices of each durable sector two-digit industry respond by more than 7 percent to an aggregate news shock, whereas the response in four nondurable goods industries is less than 7 percent. The initial large response of stock prices is followed by decreases during the following ten quarters. To explain this decline, Haertel and Lucke [2008] argue that by the time new technology is diffused, competition reduces profits and stock prices adjust to a lower level.

to analyze the effects of aggregate news shocks on two-digit SIC industries. They also find higher responses in the durables sector. In particular, they find that the industries with the highest growth rate among the durable sector industries are those with the highest long-run responses to the aggregate news.
1.2.4 Effects of News on Other Sectoral Fundamentals

As previously shown, the percentage response of durable goods sector TFP to news is significantly larger than that of nondurable goods sector TFP. In particular, the long-run durable sector TFP response is almost five times greater than the response of the nondurable sector TFP. In order to complete the empirical analysis, I explore whether this result is also present in the case of other sectoral fundamentals. Specifically, I examine the responses of sectoral output, consumption, hours, and investment.

Figure 1.12 displays the responses of sectoral fundamentals to a 1 percent aggregate news shock. The shaded area represents 90% confidence bands of the manufacturing sector responses. There are several features worth noticing. First, there is evidence of the comovement among the sectoral fundamentals. Second, the responses of all durable sector fundamentals are significantly greater than the responses of nondurable sector fundamentals. Specifically, after ten quarters, the percentage response of durable sector output is four times higher than that of nondurable sector output. Also, the responses of durable consumption, hours and investment are higher than those of the same variables in the nondurable goods sector. Third, only consumption responses are statistically positive on impact, whereas the impact responses of other fundamentals are not statistically different from zero. After ten periods, all durable sector fundamentals remain above their initial levels and these responses are statistically different from zero. In contrast, in the nondurable goods sector only the responses of hours and output are significantly different from zero after 10 quarters; the responses of consumption and investment are not statistically significant over this time horizon.

I also examine the effects of sectoral news on the same sectoral fundamentals. In line with the previous results for TFP, the effects of sectoral news on sectoral fundamentals are larger than the effects of aggregate news. In particular, durable
goods sector fundamentals respond most to the durable news shock, while nondurable goods sector fundamentals respond most to the nondurable news shock.

Finally, since I document an overall larger response of durable sector TFP to aggregate news shock, there is an implied increase in the relative productivity of the durable and nondurable goods sectors. Hence one would expect a decline in the relative price of durables goods in response to an aggregate news shock. Furthermore, if consumption for both types of goods occurs on impact, due to a positive wealth effect, the lack of inventories of nondurables could create a scarcity effect that would reinforce the decline in the relative price of durables. When I analyze the response of the relative price of durable goods (the ratio between durable sector and nondurable sector consumer price indices) to an aggregate news shock, the above intuition turns out to be correct. As Figure 1.13 displays, the relative price of durable goods decreases by 0.8 percent after ten quarters.

Overall, the evidence presented in this section further supports the idea that durable goods sector fundamentals are more responsive to news shocks, and that aggregate news shocks propagate business cycles mainly through the durable goods sector.

1.2.5 Inventories

A key difference between durable and nondurable goods is that producers of durables can stock inventories and use them as a buffer to news shocks. This feature of durable goods has potentially important implications for how the sector responds to news shocks. For this reason, I devote this section to examining the behavior of the two most commonly used inventories indicators: the inventories-to-sales ratio and the change in the inventories-to-output ratio (see Blinder and Fisher [1981] and Lovell [1961]). To obtain the responses of these variables to news shocks, I add the inventories indicator to the benchmark two-variable system. When the third variable
is added, the identification process becomes more involved.\textsuperscript{16} Figures 1.14 displays the responses of inventories-to-sales ratio in three sectors (manufacturing, durables, and nondurables) to the sectoral level news shocks in those sectors (identified with either the short-run or long-run restriction). The view commonly accepted in the literature is that inventories-to-sales ratio is countercyclical.\textsuperscript{17} Firms accumulate their inventories when demand is weak, and liquidate them when demand is high. Also, if there is uncertainty about the sales in the future, firms may hold inventories against the contingency that demand will be unexpectedly high. Therefore, one would expect the inventories-to-sales ratio to decrease when good news about future productivity arrives. My analysis confirms this view. Specifically, the inventories-to-sales ratio in manufacturing decreases by 0.25 percent over six quarters, remaining at that level in the longer run. As expected, the response of inventories in the durables sector is twice as large as that in the manufacturing sector (a decline of almost 0.6 percent), whereas the response of the inventories-to-sales ratio in the nondurables sector is statistically insignificant over the whole time horizon.

Figure 1.15 displays the corresponding responses of the change in the inventories-to-output ratio. After an initial decrease, this indicator increases, reaching a peak after five to six quarters. Although the response is significant at the level of manufacturing and durable sector, there is, again, no statistically significant response in the nondurable sector. The rationale behind this qualitative response is as follows: as the good news about the future productivity is realized, producers decrease the inventories levels in order to meet increased demand. However, as time passes, they increase production and inventories levels return to some optimally determined,\textsuperscript{16} A detailed explanation is provided in the Appendix A.2.\textsuperscript{17} For example, Blinder [1981] argues: "The most commonly used indicator of the state of inventory equilibrium or disequilibrium is the ratio of inventories to sales in manufacturing and trade. This ratio moves countercyclically, rising in recessions."
long-run level of inventories. This indicator is not permanently affected by the news, but this is explained by the fact that it measures the change in inventories (scaled by output) as opposed to the level.

*Inventories at the two-digit SIC level*

After documenting that inventories respond much more in the durables sector than in the nondurables sector, I investigate in which two-digit industries inventories respond the most to news shocks, and explore whether these coincide with the industries whose productivities respond most to news shocks. In Figure 1.16, I present the responses of the inventories-to-sales ratio at the two-digit SIC level, to a unit aggregate news shock. The responses of both durable and nondurable sector industries are presented.

The percentage responses of the inventories-to-sales ratios of durable sector industries are much larger than those of nondurable sector industries. Results are robust when sectoral news shocks are used instead of aggregate news shocks. Again, durable goods industries are more responsive than the nondurable goods industries. The responses of durables industries are largest with respect to durable news, just as the responses of nondurables industries are greatest with respect to nondurable news. Responses of all industries, on average, are larger in the case of a unit manufacturing news shock than in that of a unit aggregate news shock. Additionally, the percentage responses of durable sector inventories are higher, on average, in the case of a unit durable news shock than in the case of a unit manufacturing news shock. Nondurable goods inventories do not respond significantly to any of the news shock, with one exception: petroleum inventories respond significantly to nondurable news shock. On the other hand, the responses of several durable goods industries are statistically significant. Finally, the durable industries with biggest declines in inventories are the same as those with the largest responses of productivity to news.
At first, this result might seem surprising. If the increased demand for goods, as a result of the wealth effect, were distributed evenly across industries, then one might expect the sectors with the biggest TFP responses to be more able to meet demand without running down inventories. But there are two important considerations that work against this argument. First, the different responses of TFP in the different sectors imply changes in relative prices. Industries with larger TFP responses will have falling relative prices and there will be substitution towards their products. Second, news about productivity leads to an increase in investment demand that, in the short run, should put additional pressure on inventories in the durables sector but not in the nondurables sector.

1.3 The Model

I use a two-sector, two-factor, real business cycle model as a theoretical framework to study sectoral business cycles. As in Baxter [1996], sector 1 produces a nondurable consumption good, and sector 2 produces a consumer durable good and the capital good used as an input in the production of both sectors. The main difference between the two sectors is that a good produced in sector 2 can be stocked. In the model, the reason that durable goods are held as stocked inventories, is that inventories are an argument of the production function of sector 2, following Christiano [1988] and Kydland and Prescott [1982]. These authors argue that the stock of inventories, as the stock of fixed capital, provides a flow of services to a firm.

1.3.1 Households

The economy is populated by a large number of identical, infinitely-lived agents who derive utility from the consumption of the nondurable consumption good, the service flow from the durable consumption good, and leisure. The agent’s lifetime utility is
\[ U = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{\left( C_t^* - hC_{t-1}^* \right) v (1 - N_t)}{1 - \sigma} \right) ^{1-\sigma} - 1, \]

where \( \beta \) represents a subjective discount factor, \( h \) is the coefficient of habit persistence in consumption, \( \sigma \) is the inverse of the elasticity of intertemporal substitution defined in terms of \( \left( C_t^* - hC_{t-1}^* \right) v (1 - N_t) \), and where \( N_t \) represents hours worked at time \( t \). Two types of preferences are considered: preferences proposed by Greenwood, Hercowitz, and Huffman [1988], which I refer to as GHH, and preferences proposed by King, Plosser, and Rebelo [1988], which I refer to as KPR.\(^{18}\) In the case of GHH preferences, utility function is separable in consumption and leisure, taking the following form:

\[ U^{GHH} = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{\left( C_t^* - hC_{t-1}^* \right) - \psi N_t^\theta}{1 - \sigma} \right) ^{1-\sigma} - 1. \]

In the case of KPR preferences, utility is given by

\[ U^{KPR} = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{\left( C_t^* - hC_{t-1}^* \right) (1 - \psi N_t^\theta)}{1 - \sigma} \right) ^{1-\sigma} - 1. \]

The composite consumption good \( C_t^* \) is given by the constant elasticity of substitution function:

\[ C_t^* = \left[ \chi_1 C_{1t}^{\mu} + \chi_2 C_{2t}^{\mu} \right]^{\frac{1}{\mu}}, \]

\(^{18}\) Jaimovich and Rebelo [2009] show that these two types of preferences induce qualitatively different responses of the main macroeconomic variables, most importantly hours, to news about future TFP increase. The main characteristic of GHH preferences is that the optimal number of hours worked depends only on the contemporaneous real wage, and therefore news about a future TFP increase produces neither substitution effect, nor a wealth effect on hours. Consequently, hours do not decrease on impact as the result of news. This is not the case with KPR preferences where the optimal number of hours worked responds to changes in lifetime income as well as the current wage. Given good news about future changes in TFP, agents reduce today’s supply of labor, because they perceive a higher level of lifetime income, and therefore want to enjoy more leisure.
where \( C_{1t} \) and \( C_{2t} \) represent period \( t \) consumption of the nondurable consumption good and consumption of the service flow from the durable consumption good, respectively, and parameters \( \chi_1 \) and \( \chi_2 \) determine the weight of these two goods in the composite consumption good. The parameter \( \mu \) is equal to \( (1 - 1/\varrho) \), where \( \varrho \) is the constant elasticity of substitution between \( C_{1t} \) and \( C_{2t} \). If the elasticity of substitution is greater than 1 in absolute value (i.e., \( 0 < \mu < 1 \)), goods are substitutes, whereas, if the elasticity of substitution is less than 1 in absolute value, goods are complements. Finally, I assume that the service flow from the durable consumption good is proportional to the stock of the durable consumption good \( S_t \):

\[
C_{2t} = \gamma S_t, \quad \gamma > 0.
\]

### 1.3.2 Producers

Two final goods are produced in the economy: a perishable consumption good, produced in sector 1, and a capital good, produced in sector 2. A good produced in sector 2 can be used either as an investment good in both sectors or as a consumer durable. Production processes in both sectors require the use of labor and capital. In addition, the production function of sector 2 has inventories as an argument.\(^{19}\) In both sectors, I model capital services as the product of the capital stock and the level of capital utilization. The cost of increasing utilization is additional depreciation of the capital stock. This feature is introduced through the depreciation rate that depends on the rate of capacity utilization, and takes the form

\[
\delta^j(u_{jt}) = \delta^j_0 + \delta^j_1 (u_{jt} - 1) + \frac{\delta^j_2}{2} (u_{jt} - 1)^2,
\]

where \( \delta^j_0, \delta^j_1, \delta^j_2 > 0 \) and \( j = 1, 2 \)

---

\(^{19}\) As Christiano [1988] argues: that "all other things being equal, larger inventory stocks probably do augment society’s ability to produce goods. For example, spatial separation of the stages of production and distribution, together with economies of scales in transportation, implies that labor inputs can be conserved by transporting goods in bulk and holding inventories.” Also, Kydland and Prescott [1982] suggest that "with larger inventories, stores can economize on labor resources allocated to restocking.” Therefore, adding inventories into the production function does not seem unreasonable.
responding to the two sectors. Parameter $\delta_j^t$ represents the steady state depreciation rate of the $j$th sector, since $\delta_j^t$ is calibrated such that, in the steady state, the utilization rate in both sectors is equal to one. Sector 1’s production technology is a standard Cobb-Douglas function:

$$Y_{1t} = F_{1t}(K_{1t}, N_{1t}) = A_{1t}(N_{1t})^{1-\alpha_1} (u_{1t}K_{1t})^\alpha_1,$$

where $N_{1t}$ and $K_{1t}$ represent labor and capital input at time $t$, $u_{1t}$ represents the rate of capacity utilization in sector 1, $A_{1t}$ represents the technology process in sector 1, and $0 < 1 - \alpha_1 < 1$ is labor’s share in sector 1. Sector 2’s production function is assumed to have the form:

$$Y_{2t} = F_{2t}(K_{2t}, N_{2t}, I_t) = A_{2t}(N_{2t})^{1-\alpha_2} [(1 - \rho)(u_{2t}K_{2t})^{-\nu} + \rho I_t^{-\nu}]^{-\alpha_2},$$

where $N_{2t}$ and $K_{2t}$ represent labor and capital used in the production of sector 2 output at time $t$, $u_{2t}$ is the capacity utilization rate in sector 2, $I_t$ denotes stock of inventories at time $t$, and $0 < 1 - \alpha_2 < 1$ is labor’s share in sector 2. The parameter $\rho$ controls the role of inventories in the production function of sector 2. If $\rho = 0$ we are back to the standard Cobb-Douglas production function case. Finally, the elasticity of substitution between capital and inventories is $\frac{1}{1+\nu}$; this elasticity is arguably less than one which is why $\nu$ is required to be positive.\(^{20}\)

Households are assumed to own the physical capital used in both sectors. Labor is assumed to be mobile across sectors; at the same time, I assume that the adjustment cost is incurred when the level of investment changes over time. In addition, changing stocks of consumer durables is also subject to an adjustment cost.\(^{21}\) The capital stocks in both sectors, $K_{1t}$ and $K_{2t}$, and the stock of consumer durables $S_t$ evolve

\(^{20}\) See Kydland and Prescott [1982].

\(^{21}\) I follow Bernanke [1985], Startz [1989], and Baxter [1996] in assuming that the stock of consumer durables is subject to the adjustment cost, although my specification is different from theirs.
over time following laws of motion:

\[ K_{1,t+1} = (1 - \delta^1 (u_{1t})) K_{1t} + X_{1t} \left( 1 - \phi_1 \left( \frac{X_{1t}}{X_{1t-1}} \right) \right), \]

\[ K_{2,t+1} = (1 - \delta^2 (u_{2t})) K_{2t} + X_{2t} \left( 1 - \phi_2 \left( \frac{X_{2t}}{X_{2t-1}} \right) \right), \]

\[ S_{t+1} = (1 - \delta_S) S_t + D_t \left( 1 - \phi_3 \left( \frac{D_t}{D_{t-1}} \right) \right), \]

where \( X_{1t} \) and \( X_{2t} \) denote gross investment in sectors 1 and 2 at time \( t \), while \( D_t \) denotes purchases of new consumer durables. Function \( \phi_j (\cdot) \) represents the adjustment cost function, which is chosen so that it satisfies the condition of no adjustment costs in the steady state; i.e. \( \phi_j (1) = \phi_j' (1) = 0, (j = 1, 2, 3) \). Also, \( \phi_j' (\cdot), \phi_j'' (\cdot) > 0 \). This function does not necessarily need to be identical across the sectors, and, therefore, can take different forms.  

1.3.3 Resource Constraints

Since an individual’s allocation of time is normalized to 1, the hours in both sectors cannot exceed the total available hours \( N_t \) that are equal to \( 1 - L_t \), where \( L_t \) denotes time allocated to leisure at time \( t \). Therefore, a unit of time is allocated as follows:

\[ N_{1t} + N_{2t} + L_t \leq 1. \]

The resource constraint for the sector producing the pure consumption good is

\[ C_{1t} \leq Y_{1t}. \]

For the sector producing the capital good the resource constraint is

\[ D_t + X_{1t} + X_{2t} + \Delta I_t \leq Y_{2t}. \]

22 See Appendix A.3 for the exact forms used.
1.3.4 Introducing News Shocks into the Model

To analyze the theoretical effects of news, I introduce news shocks into the model by making reference to my estimates in Section 1.4. In particular, I assume that the news shock corresponds to an aggregate news shock as identified by my estimation procedure. I assume that the response of TFP in model sector 1 corresponds to the response of TFP in the nondurables sector to an aggregate news shock in the data. I assume that the response of TFP in model sector 2 corresponds to the response of TFP in the durables sector to an aggregate news shock in the data. I first estimate the regression coefficients \( \psi'_s \) in (1.1); in this case, \( X_t \) represents the productivity of sector \( k \) (\( k = 1, 2 \)). The sequence \( \sum_{i=0}^{n} \psi^k_i \) represents a point estimate of the impulse response function of sector \( k \)'s TFP to a news shock (aggregate or sectoral), after \( n \) periods. In the regressions I set \( J = 8 \) so that the implied impulse response functions of the levels of TFP in the two sectors are constant for \( n > J = 8 \).

These estimated productivity responses are then introduced into the model. Figure 1.1 shows the responses of sectoral TFPs to an aggregate news shock; the responses are smoother than productivity processes commonly used in the theoretical news literature, where news about future technology developments is often introduced as follows. The economy is assumed to be in the steady state in period 0, when a signal arrives suggesting that in \( s \) periods a positive technology shock will occur.\(^{23}\) In this case, the productivity process remains at its steady-state level until period \( s \), when the productivity increase is realized. TFP then rises by 1 percent and follows its exogenous law of motion afterwards.\(^{24}\)

In my analysis, the productivity process is at its steady state level in period

\(^{23}\) Christiano et al. [2008] and Jaimovich and Rebelo [2009], among others, allow for the possibility of this signal to be false or noisy. That is, after \( s \) periods the expected increase in technology does not happen, or is smaller than previously expected.

\(^{24}\) See, for example Christiano et al. [2008], Beaudry and Portier [2004, 2005], Jaimovich and Rebelo [2009], and Schmitt-Grohé and Uribe [2009].
0, after which it starts to increase, with TFP increasing by 1 percent in the long-run. In both approaches, in period 0 households learn the expected future path of the technology process. The key difference is that in my analysis the productivity increase begins in period 1 and occurs smoothly over time, whereas in the typical theoretical analysis it is delayed (s periods) and abrupt.

1.4 Calibration and the Estimation Procedure

Before obtaining quantitative predictions from the model, I assign numerical values to its parameters. I calibrate some of the structural parameters of the model in a standard fashion; the rest of the parameters are estimated. Table 1.1 summarizes values of the calibrated parameters.

The time unit is defined to be a quarter. I assign a value of 0.9902 to the subjective discount factor $\beta$, which is consistent with an annual real interest rate of 4 percent. I calibrate utility parameters, $\chi_1$ and $\chi_2$, such that the steady-state shares of nondurable goods and durable goods in composite consumption equal the average over the sample period ($C_1/C^* = 0.723$ and $C_2/C^* = 1 - 0.723$). The preference parameter $\psi$ is chosen so that the agents allocate one-third of their time-endowment to work. Following Bernanke [1985] and Baxter [1996], annual capital depreciation rates in the two sectors are 7.1 percent, which leads to the quarterly depreciation rates, $\delta_0^1$ and $\delta_0^2$, being 1.73 percent. The annual depreciation rate of the stock of durables is 15.6 percent (following Baxter [1996]). I calibrate the parameters $\delta_1^1$ and $\delta_1^2$ to ensure that steady-state capital utilizations in both sectors, $u_1$ and $u_2$, equals unity. The labor share coefficients, $1 - \alpha_1$ and $1 - \alpha_2$, are chosen to match the mean of labor’s share over the sample period. The parameter $\rho$, which determines the role of inventories in the production function of sector 2, is chosen to match the share of inventories in output. Finally, I choose the parameter $\nu$ so that the elasticity of substitution between capital services and inventories matches the value used by
I use impulse response function matching to estimate the remaining parameters: \( \sigma \) (the inverse of the elasticity of intertemporal substitution), \( h \) (the habit persistence parameter), \( \varrho \) (the elasticity of substitution between \( C_{1t} \) and \( C_{2t} \)), the three coefficients of the investment adjustment cost functions (\( \kappa_1, \kappa_2, \) and \( \kappa_S \)), and the two coefficients of the rate of capacity utilization functions (\( \delta_1^2 \) and \( \delta_2^2 \)). Let \( \zeta \) denote parameters that I estimate, \( \Phi (\zeta) \) denote the model impulse responses that are functions of the structural parameters \( \zeta \), and \( \hat{\Phi} \) denote the corresponding estimated empirical impulse responses. The estimator for \( \zeta \) is the solution to the following minimization problem:

\[
\hat{\zeta} = \arg \min_{\zeta} \left( \hat{\Phi} - \Phi (\zeta) \right)^\prime W^{-1} \left( \hat{\Phi} - \Phi (\zeta) \right),
\]

where \( W \) is the diagonal matrix with the sample variances of the \( \hat{\Phi} \)'s along the diagonal.\(^{25}\) I match the following impulse response functions: composite consumption, aggregate hours, output, and investment in the durable goods sector. The point estimates of the parameters are given in Table 1.2. In the following section, I discuss the predictions of the model.

### 1.5 Predictions of the Estimated Model

One of the main challenges the news literature has faced is building a model that can generate Pigou cycles, a comovement between consumption, hours, output, and investment, in response to news about higher future TFP. Beaudry and Portier [2004] were the first authors to recognize that the standard real business cycle model, with KPR preferences, fails to meet this challenge. In particular, good news increases consumption and leisure on impact through the wealth effect. Since leisure increases,

\(^{25}\) I follow Altig, Christiano, Eichenbaum, and Lindé [2005] who argue that with this choice of the weighting matrix \( W \), \( \hat{\zeta} \) is the value of \( \zeta \) which ensures that theoretical IRFs lie as much as possible inside the confidence bands of estimated IRFs.
hours worked and output decrease. The only way for consumption and hours (or output) to move in opposite directions is through a decrease of investment. To solve this problem, Beaudry and Portier propose a three-sector model, in which consumption is given as a composite of nondurable and durable goods. Both of these goods are produced with labor and a fixed production factor. The model is capable of generating Pigou cycles. However, in a more recent paper, Jaimovich and Rebelo [2009] formulate a one-sector model that is able to generate Pigou cycles. This is a standard real business cycle model, augmented with the investment adjustment costs and variable capital utilization. Furthermore, the model features a new type of preferences that do not induce a wealth effect on leisure/labor when news is received. Therefore, hours does not decrease on impact and the desired comovement between output, consumption and investment can be obtained. Several other authors have been able to obtain the desired comovement between these variables using a one-sector model (see den Haan and Kaltenbrunner [2009], Christiano et al. [2008], Schmitt-Grohé and Uribe [2009]). In most of these papers news is introduced as described above: in period $t$ agents learn that there will be a one-percent permanent increase in TFP beginning in period $t + j$, where $0 < j \leq n$. Therefore, the productivity process features a kink in period $t + j$.

In my empirical analysis I make a distinction between the durable and nondurable goods sectors. One obvious difference between the producers in these two sectors is the possibility of the durable sector producers holding stocks of inventories. This channel can help my model replicate comovement between consumption and investment, since holding stocks of inventories is one way that the durable goods sector producers can meet higher consumer demand without necessarily having to decrease investment.

Finally, there are two main distinctions between my approach and the approaches taken in the aforementioned papers. First, as indicated before, I introduce news
shocks by feeding the empirical responses of TFPs in the two sectors into the model. Thus, the productivity process in this case does not feature a kink, but is rather a much smoother process. Second, the focus of my analysis is not only on obtaining the right qualitative comovement between the main macroeconomic variables (hours, consumption, output and investment), but also on obtaining quantitatively good fit of the model impulse response functions to those in the data, while considering additional implications of the model for the behavior of inventories, and behavior of sectoral level data. In order to make progress on these quantitative dimensions, it was necessary to use a richer model than the ones found in the literature.

1.5.1 Benchmark Model

Figures 1.17 and 1.18 display the theoretical and empirical responses of five macroeconomic time series (consumption, hours, investment, output and inventories-to-sales ratio) to a unit aggregate news shock. Theoretical responses are computed using the benchmark model described in Section 3. The shaded areas represent 90% confidence intervals of the empirical responses, and the dashed lines represent point estimates of the empirical impulse responses. The solid lines represent theoretical responses under GHH preferences, and the starred lines represent responses under KPR preferences. There is no significant difference between the model responses when preferences take the KPR or GHH form. This result is not surprising, considering the nature of the technology process. In particular, the technology process does not feature the kink as it does in the most of the rest of the theoretical literature on news shocks; instead, it starts increasing slowly from period 1. This representation of technology is more consistent with the notion that the technology process diffuses slowly over time, and, therefore, the main distinction between KPR and GHH preferences described above is almost eliminated.

Comovement between total hours, consumption, output, and investment is evi-
dent. Aggregate hours worked increase; at the same time, the benchmark model is not able to match the response of hours worked in the nondurable sector. In particular, the benchmark model wrongly predicts a decrease of nondurable hours. My intuition is as follows: hours can move freely between the sectors, and, therefore, hours worked tend to increase in the sector in which the productivity is higher. In order to solve this problem I introduce adjustment costs in labor in the nondurable sector. Adding this feature improves the model fit in this dimension; the model can generate an increase of hours in the nondurable goods sector. Moreover, the model matches the response of hours in the durable goods sector very well. Since this increase is larger than the decrease of hours in the nondurables sector, the benchmark model is still able to obtain increase of the total hours.

The model does a good job in replicating the consumption responses. It can generate positive initial responses of consumption in the durable goods sector and in composite consumption, whereas the initial response of nondurable consumption is below the confidence bands for several periods. The response of durable goods consumption lies inside the confidence bands in all periods.

The model cannot generate a negative initial response of investment in the nondurable goods sector, which is observed in the data. In order to drive down investment in the nondurable sector, a high value of the coefficient in the investment adjustment cost function is needed. This is also true in the case of durable goods sector investment; the estimated investment adjustment cost must be quite large. Had these adjustment costs not been introduced, the responses would be much larger and occur more quickly, because changing the level of investment would not be costly for producers. The response of output in the durable goods sector is matched very well; the model response of durables sector output lies within the 90% confidence bands of

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26 Also Jaimovich and Rebelo [2009] introduce this cost in order to obtain better response of the total hours.
the empirical response almost all the time, whereas nondurables sector output is inside the confidence bands only in the longer run. That is, the hump-shaped response of output in the nondurable goods sector cannot be obtained, which is primarily the result of the qualitative response of output following the shape of the productivity process in the nondurable sector. Rather than being hump-shaped, the response of nondurable goods sector output is very small over the first four quarters, after which it starts slowly to increase, reaching the confidence band of the empirical response after six quarters.

Finally, the model is able to replicate the response of the inventories-to-sales ratio, which is not one of the responses that is being matched in the estimation. That my model works well in this dimension is very encouraging, particularly since it is able to replicate the response of the variable which, as mentioned above, is crucial for understanding differing extent to which news shocks are propagated in durable and nondurable goods sectors.

I conclude by arguing that by examining a model with distinct durable and nondurable goods sectors, with an explicit role for inventories, and by modeling news shocks using an approach that departs from most of the theoretical literature, I am able to replicate some of the key characteristics of the empirical responses of the economy to news about future productivity. The model performs relatively poorly in explaining the behavior of the nondurable goods sector, but is quite successful in explaining the behavior of the durable goods sector and some aggregate variables.

1.6 Conclusions

In this paper, I present evidence that the responses to news shocks of the fundamentals in the durable goods sector are different from the responses of these fundamentals in the nondurable goods sector. In particular, the responses of durable goods sector fundamentals are significantly greater than the responses of nondurable goods
sector fundamentals. After a 1 percent aggregate news shock, durable sector productivity rises by approximately 3 percent after ten quarters, whereas the response of productivity in the nondurable goods sector is approximately one half of one percent. The percentage responses of other durable goods sector fundamentals are also significantly higher than those of the same fundamentals in the nondurable goods sector. In order to explain these different behaviors of the durable and nondurable goods sectors, I introduce inventories into the analysis. By looking at the behavior of this variable, which has been largely neglected in the news literature, I am able to conclude that inventories play an important role in how aggregate news shocks are propagated through the durable goods sector.

As a theoretical framework, I use a two-sector, two-factor, real business cycle model. I also introduce inventories into the production function of durable goods producers. My model is consistent with the empirical evidence on how the economy responds to news shocks. Specifically, the model is successful in mimicking the empirical responses to news shocks at the sectoral level. First, the model reproduces the comovement among hours, consumption, output and investment in both sectors. Second, the model is able to reproduce the stronger overall response of the durable goods sector to news shocks. Finally, the model is able to perfectly match inventories responses.
1.7 Tables and Figures
Table 1.1: Calibrated Parameters of the Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.04^{-1/4}</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.7</td>
<td>Service flow from durables</td>
</tr>
<tr>
<td>$1 - \alpha_1$</td>
<td>0.74</td>
<td>Labor share in the nondurable goods sector</td>
</tr>
<tr>
<td>$1 - \alpha_2$</td>
<td>0.60</td>
<td>Labor share in the durable goods sector</td>
</tr>
<tr>
<td>$\delta^0_1$</td>
<td>0.0173</td>
<td>Steady-state depreciation rate in the nondurable goods sector</td>
</tr>
<tr>
<td>$\delta^2_1$</td>
<td>0.0173</td>
<td>Steady-state depreciation rate in the durable goods sector</td>
</tr>
<tr>
<td>$\delta^S_1$</td>
<td>0.0358</td>
<td>Depreciation rate of the stock of durables</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.0003</td>
<td>Determines the role of inventories in the production function</td>
</tr>
<tr>
<td>$\chi_1$</td>
<td>1.185</td>
<td>Composite consumption good parameter with sector 1</td>
</tr>
<tr>
<td>$\chi_2$</td>
<td>0.053</td>
<td>Composite consumption good parameter with sector 2</td>
</tr>
<tr>
<td>$\delta^1_1$</td>
<td>0.0303</td>
<td>Nondurable goods sector depreciation rate parameter</td>
</tr>
<tr>
<td>$\delta^2_1$</td>
<td>0.0247</td>
<td>Durable goods sector depreciation rate parameter</td>
</tr>
<tr>
<td>$\nu$</td>
<td>3.671</td>
<td>Elasticity of substitution between inventories and capital</td>
</tr>
</tbody>
</table>
Table 1.2: Estimated Parameters of the Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_1$</td>
<td>9.48</td>
<td>Sector 1 investment adjustment cost function parameter</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>5.31</td>
<td>Sector 2 investment adjustment cost function parameter</td>
</tr>
<tr>
<td>$\kappa_S$</td>
<td>8.73</td>
<td>Stock of durables adjustment cost function parameter</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.64</td>
<td>Intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$h$</td>
<td>0.78</td>
<td>Habit persistence in consumption parameter</td>
</tr>
<tr>
<td>$\delta_1^1$</td>
<td>0.17</td>
<td>Parameter of the depreciation rate in the nondurable sector</td>
</tr>
<tr>
<td>$\delta_2^2$</td>
<td>0.14</td>
<td>Parameter of the depreciation rate in the durable sector</td>
</tr>
</tbody>
</table>
**Figure 1.1**: Impulse responses of the sectoral TFPs and Stock Price Indices to a unit aggregate news shock

*Note*: Impulse responses of sectoral TFPs to a unit aggregate news shock are given in the left panel; the responses of sectoral stock price indices are given in the right panel. Shaded areas represent 5% and 95% confidence bands of the responses at the level of the manufacturing sector, which are obtained using the Monte-Carlo experiments with 1000 replications.

**Figure 1.2**: Impulse Response of the Sectoral Inventories-to-Sales Ratio to a Unit Aggregate News Shock

*Note*: Point estimates of the responses of manufacturing, durables and nondurables sector inventories to sales ratio to a unit aggregate news shock are displayed. Shaded areas represent 5% and 95% confidence bands of the responses at the level of the manufacturing sector, which are obtained using the Monte-Carlo experiments with 1000 replications.
Figure 1.3: Durable Goods Sector Data

Figure 1.4: Nondurable Goods Sector Data
Figure 1.5: Impulse Responses to Shocks $\varepsilon_2$ and $\tilde{\varepsilon}_1$ in the (TFP, SP) VAR at the Aggregate Level

Note: Left panel represents the response of the aggregate TFP to the two disturbances; the response of the aggregate stock price index to these disturbances is given on the right panel. Dotted lines represent point estimates of the responses to a unit $\varepsilon_2$ shock, whereas solid lines represent the responses to a unit $\tilde{\varepsilon}_1$ shock; the borders of the shaded area represent 5% and 95% confidence bands of the impulse response functions in the case of long-run identification. Confidence bands are obtained using the Monte-Carlo experiments with 5000 replications.
Figure 1.6: Recovered Aggregate News Shock and NBER Recession Dates

Note: This figure shows the time series of recovered aggregate news shock; data are smoothed using a one year moving average.
Figure 1.7: Impulse responses to shocks $\varepsilon_2$ and $\bar{\varepsilon}_1$ in the (TFP, SP) VAR at the manufacturing sector level

Note: Left panel represents the response of the manufacturing TFP to the two disturbances; the response of the manufacturing stock price index to these disturbances is given on the right panel. Dotted lines represent point estimates of the responses to a unit $\varepsilon_2$ shock, solid lines represent the responses to a unit $\bar{\varepsilon}_1$ shock, whereas the borders of the shaded area represent 5% and 95% confidence bands of the impulse response functions in the case of long-run identification. Confidence bands are obtained using the Monte-Carlo experiments with 5000 replications.
Figure 1.8: Impulse Responses to Shocks $\varepsilon_2$ and $\tilde{\varepsilon}_1$ in the (TFP, SP) VAR at the Durable Sector Level (the upper panel) and Nondurable Sector Level (the lower panel)

Note: The top panel represents the responses of the durable goods sector TFP and stock price index, whereas the lower-panel represents the responses of the nondurable goods sector TFP and stock price index. On both panels, left graph represents the responses of TFPs to the two disturbances, whereas the right graph represents the responses of SP to these disturbances. Dotted lines represent point estimates of the responses to a unit $\varepsilon_2$ shock, solid lines represent the responses to a unit $\tilde{\varepsilon}_1$ shock, whereas the borders of the shaded area represent 5% and 95% confidence bands of the impulse response functions in the case of long-run identification. Confidence bands are obtained using the Monte-Carlo experiments with 5000 replications.
Figure 1.9: Correlation between $\varepsilon_2$ and $\tilde{\varepsilon}_1$ in the Manufacturing, Durable and Nondurable Goods Sectors

Note: Two disturbances, $\varepsilon_2$ and $\tilde{\varepsilon}_1$, are plotted against each other at the level of the manufacturing sector, durable goods sector and nondurable goods sector.
Figure 1.10: Responses of Two-Digit SIC Durables (left panel) and Nondurables Industries (right panel) TFPs to a Unit Aggregate News Shock

Figure 1.11: Responses of Two-Digit SIC Durables (left panel) and Nondurables Industries (right panel) Stock Prices to a Unit Aggregate News Shock
FIGURE 1.12: Responses of Sectoral Fundamentals to a Unit Aggregate News Shock
Figure 1.13: Impulse Response of the Relative Price of Durable Goods to a Unit Aggregate News Shock

Note: Solid lines represent point estimates of the relative price of durable goods to a unit aggregate news shock. Shaded areas represent 5 percent and 95 percent confidence bands.
Figure 1.14: Impulse Responses of the Inventories-to-Sales Ratio to a Unit Sectoral News Shock

Note: Left panel represents the response of the manufacturing inventories-to-sales ratio to the two disturbances coming from the three-variable (TFP, stock price index, inventories-to-sales ratio) VECM at the manufacturing sector level. Middle panel represents the response of the durables inventories-to-sales ratio to the two durable sector disturbances originating from the three-variable VECM at the durable goods sector level. Right panel represents the response of the nondurable inventories-to-sales ratio to the two nondurable sector disturbances originating from the three-variable VECM at the nondurable sector level. In all three panels the dotted lines represent the point estimates of the response to the short-run sectoral disturbance, whereas the solid lines represent the point estimates of the responses to the long-run sectoral disturbance. Finally, shaded areas represent 5% and 95% confidence bands of the long-run sectoral responses. Confidence bands are obtained using the Monte-Carlo experiments with 5000 replications.
**Figure 1.15:** Response of the Change in the Inventories-to-Output ratio to a Unit Sectoral News Shock

*Note:* Left panel represents the response of the change in inventories-to-output ratio in the manufacturing sector to the two disturbances originating from the three-variable (TFP, stock price index, change in inventories-to-output ratio) VECM at the manufacturing sector level. Middle panel represents the response of the durables change in inventories-to-output ratio to the two durable sector disturbances (originating from the three-variable VECM at the durable goods sector level). Right panel represents the response of the nondurable change in inventories-to-output ratio to the two nondurable sector disturbances (originating from the three-variable VECM at the nondurable goods sector level). In all three panels the dotted lines represent the point estimates of the response to the short-run sectoral disturbance, whereas the solid lines represent the point estimates of the responses to the long-run sectoral disturbance. Finally, shaded areas represent 5% and 95% confidence bands of the long-run sectoral responses. Confidence bands are obtained using the Monte-Carlo experiments with 5000 replications.
Figure 1.16: Impulse Response of Inventories-to-Sales Ratios to a unit Aggregate News Shock at the Level of Two-Digit SIC Industries
Figure 1.17: Model and Empirical Responses to a Unit Aggregate News Shock I

Consumption and Hours Responses

Note: The shaded areas represent 90% confidence intervals of the empirical responses, and the dashed lines represent point estimates of the empirical impulse responses. The solid lines represent theoretical responses under GHH preferences, and the starred lines represent responses under KPR preferences.
**Figure 1.18**: Model and Empirical Responses to a Unit Aggregate News Shock II

**Investment, Output and Inventories Responses**

*Note*: The shaded areas represent 90% confidence intervals of the empirical responses, and the dashed lines represents point estimates of the empirical impulse responses. The solid lines represent theoretical responses under GHH preferences, and the starred lines represent responses under KPR preferences.
2

Medium-Run Effects of Monetary Policy During the Great Moderation

2.1 Introduction

The large reduction in macroeconomic volatility that occurred after the early 1980s has attracted an enormous amount of consideration in the last decade. Stock and Watson [2003] introduced the term “Great Moderation” to indicate this period of significant stabilization in economic fluctuations. Kim and Nelson [1999], McConnell and Perez-Quiros [2000], Blanchard and Simon [2001], Stock and Watson [2003], and many others, contributed formal tests of the presence of such moderation. Moreover, several authors investigated what are the sources of the reduction in macroeconomic volatility during the last three decades.\footnote{Giannone, Lenza, and Reichlin [2008] provide a detailed summary of the literature about the sources of the Great Moderation.} Whereas the majority of macroeconomists (see e.g. Stock and Watson [2002, 2003], Ahmed, Levin, and Wilson [2004], Primiceri [2005], Galí and Gambetti [2009], Liu, Waggner, and Zha [2009] attribute the decline in macroeconomic volatility to a reduction in the variance of exogenous shocks, others have focused on changes in the policy conducted by the monetary authority.
In fact, Clarida, Galí, and Gertler [2000], Cogley and Sargent [2001, 2005], Lubik and Schorfheide [2004], and Boivin and Giannoni [2006] have argued that monetary policy has become more aggressive since the early 1980s and that this change of attitude could have induced the observed changes in macroeconomic volatility.

The vast majority of the studies on the Great Moderation isolate the cyclical component of macroeconomic variables using high frequency filters, such as the first difference filter or the Hodrick and Prescott [1997] filter. These filters exclude a large portion of the total volatility of the variables from the analysis of cyclical behavior. More recently, Pancrazi [2009] documents that the Great Moderation is mainly a high-frequency phenomenon. By using a broader set of filters, Pancrazi [2009] shows that whereas the high-frequency variance of macroeconomic variables declined after the early 1980s, the medium-frequency variance did not. This implies that during the Great Moderation the spectral shape of real variables substantially changed, since the decline in volatility was not uniform at all frequencies. This result appears to be in contrast with Ahmed, Levin, and Wilson [2004], who conclude that the spectral density of output growth before and during the Great Moderation period differs only by a proportional factor, and Stock and Watson [2002] who conclude that the coefficient of the univariate autoregressive model for GDP growth is time invariant. However, as pointed out, the first difference filter has high power mainly at high frequencies and therefore might be missing relevant information at medium frequencies.

The first contribution of this paper is, therefore, to explore the main sources of both observed properties of the Great Moderation: the reduction of high-frequency volatility of the real macroeconomic variables, and its observed spectral redistribution from high to medium frequencies. Specifically, we investigate whether these two facts are caused by the altered monetary policy or by different statistical properties of the shocks in the post-1983 period. Using a medium-scale DSGE model, we find that
the change in the variance and persistence of the exogenous shocks are the main contributors to the change in the level of the spectral density (i.e. on the variance) of consumption, output, and investment, as well as to the redistribution of their spectral density (i.e. on the shape of the spectrum). In fact, if the exogenous shocks had been the same as in the pre-1983 period, a more aggressive monetary policy would have had no effect on the volatility of the real variables. However, when we consider the statistical changes of the variance and persistence of the shocks as estimated in the post-1983 period, the different monetary policy contributes to approximately 40 percent of the total decline in the variance of the three macroeconomic variables. Moreover, whereas the changed monetary policy affects mainly the high-frequency volatility, it only slightly influences the medium-frequency volatility.

In order to provide some intuition about how a change in monetary policy could effect the spectral density and spectral distribution of the real variables, we first consider a simple New Keynesian model, as in Galí [2008]. The model is characterized by two rigidities: imperfect competition in the goods market and price stickiness. The dynamics of the model are driven by two shocks: a technology shock and a monetary policy shock. Using this simple model, we show that a change of the monetary policy rule toward a more aggressive inflation targeting has a large effect on the shape of the spectrum of output, causing a decline in its level and a redistribution of its density from high to medium frequencies. In particular, we show that a change of the monetary policy affects the weights of the two shocks in the total variance of output; a larger response of monetary policy to inflation increases the relative weight of the technology shock with respect to the monetary shock. Since, in our calibrated, model the technology shock is more persistent than the monetary shock, its increased weight causes the redistribution of the volatility of output toward lower frequencies.

Even though this model is very useful for providing intuition, it is rather unre-
alistic, since it abstracts from the investment sector, it has few rigidities, and it is driven only by two shocks. For quantitative analysis of the effects of monetary policy and exogenous shocks, a rich medium-scale model is more appropriate. Therefore, as a theoretical framework, we use a fairly standard, DSGE model in the spirit of Christiano, Eichenbaum, and Evans [2005] (CEE hereafter) and Smets and Wouters [2003]. The model is augmented by a number of real and nominal rigidities. The nominal rigidities include price and wage stickiness, and indexation to past inflation. The real rigidities stem from habit formation in consumption, monopolistic competition in factor and product markets, and investment adjustment costs. The model is driven by four shocks: a neutral technology shock, an investment-specific shock, a fiscal policy shock, and a monetary policy shock.²

We consider two subsamples. The first subsample covers the period 1947:I-1978:IV, whereas the second subsample covers the period 1983:I-2007:IV. Following Ahmed, Levin, and Wilson [2004], we eliminate the four years from 1978 to 1982 from the sample, since it is generally believed that the monetary policy rule followed in that period was rather different from the monetary policy rules used in all other sub-periods. We estimate the parameters governing the four exogenous processes separately in the two subsamples, using their data counterparts. Since the goal of this paper is to assess effects of different shocks on the change of variance of the real variables during the Great Moderation, we assume that the structural parameters of the model are constant throughout the whole sample. The structural parameters of the model are calibrated, using corresponding data statistics or conventional wisdom.

Our model, when driven by the estimated exogenous processes, generates realistic high-frequency dynamics of the macroeconomic variables, in both subsamples. The performance of the model at medium frequencies is less satisfying, as it largely under-

² For example, Giannone, Lenza, and Reichlin [2008] show that a simple stylized model with few variables is subject to misspecification, which leads to an overestimate of the contribution of exogenous shocks to the overall behavior of macroeconomic variables.
estimates the standard deviations of real variables. Nevertheless, the model correctly predicts a large redistribution of the variance from high to lower frequencies during the Great Moderation, as observed in the data. Performing a counterfactual exercise in the spirit of Stock and Watson [2002, 2003], Ahmed, Levin, and Wilson [2004], Primiceri [2005], Boivin and Giannoni [2006], we obtain several interesting results. First, if the statistical properties of the exogenous shocks are held fixed at their pre-1983 values, the change in monetary policy does not have any effect on the variance of the real variables. Second, when the persistence and the variance of exogenous shocks are as estimated in the post-1983 period, the change in monetary policy has a large effect on the reduction of high-frequency volatilities. In fact, approximately 40 percent of the overall decline of the high-frequency volatilities of consumption, output, and investment is due to the altered monetary policy rule. The rest of the high-frequency volatilities’ decline is due to the changed parameters of the exogenous processes. Third, the main cause of the redistribution of the spectral density of the macroeconomic variables from high to medium frequencies is the increased persistence of the TFP shock and investment-specific technology shock. However, the changed monetary policy partially contributes to the redistribution of the spectral density, since its effect on the volatilities is much smaller at medium frequencies than at high frequencies.

The rest of the paper is organized as follows. In Section 2.2 we provide an intuition about the role of a monetary policy change on the spectral density of the variables using a simple New Keynesian model. In Section 2.3, we describe a medium-scale DSGE model. In Section 2.4 we present the estimation and calibration procedures. In Section 2.5 we describe the main findings of the paper, and Section 2.6 concludes with several final remarks.
2.2 Monetary Policy and Spectral Density

In order to provide some intuition about the effects of the change in monetary policy on the level and on the redistribution of the spectral density of real variables, we first consider a basic New Keynesian Model, as in Galí [2008]. This model is characterized by two rigidities. First, the perfect competition assumption is abandoned by assuming that each firm produces a differentiated good and sets its price. Second, firms set their prices a lá Calvo [1983], i.e. in any given period, only a fraction of randomly picked firms is allowed to reset their prices.

The non-policy block of the model is composed of the New Keynesian Phillips Curve

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t, \]

and the dynamic IS equation, given by

\[ \tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r^a_t) + E_t (\tilde{y}_{t+1}). \]

Here, \( E_t \) denotes expectation conditional on the information at time \( t \), \( \pi_t \) denotes the inflation rate at time \( t \), \( r^a_t \) is the natural real interest rate, \( \tilde{y}_t \) is the output gap defined as the deviation of output from its flexible price counterpart, \( \beta \) is the discount factor, \( \kappa = \lambda \left( \sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \) with \( \lambda = \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta(1-\alpha+\alpha\varepsilon)} \), \( \sigma \) is the inverse of intertemporal elasticity of substitution, \( 1-\alpha \) is the labor share in the production function, \( \varphi \) is the inverse of Frisch elasticity of labor supply, \( \theta \) is the price stickiness parameter, and \( \varepsilon \) is the elasticity of substitution among the differentiated goods. The dynamics of the model are governed by two exogenous processes. First, the level of technology, which we denote as \( a_t \), follows a first order autoregressive (AR(1)) process:

\[ a_t = \rho_a a_{t-1} + \varepsilon^a_t, \text{ where } \varepsilon^a_t \sim N (0, 1). \]

Second, the monetary policy shock, denoted as \( \nu_t \), follows a similar first order au-
toregressive process:

\[ v_t = \rho_v v_{t-1} + \varepsilon^v_t, \quad \text{where} \ \varepsilon^v_t \sim N(0,1). \]

The monetary policy shock is considered to be an exogenous component of the nominal interest rate rule:

\[ i_t = \rho + \phi_x \pi_t + \phi_y \bar{y}_t + v_t, \]

where \( i_t \) is the nominal interest rate at time \( t \), and \( \rho \) is the household’s discount rate, with \( \rho = -\log(\beta) \). Up to a first order approximation, total output can be written as the following function of the two exogenous processes,

\[ y_t = \Lambda_v (\phi_x, \phi_y, \rho_v, \Theta) v_t + \Lambda_a (\phi_x, \phi_y, \rho_a, \Theta) a_t, \tag{2.1} \]

where \( \Lambda_a \) and \( \Lambda_v \) are functions of the Taylor rule parameters \((\phi_x, \phi_y)\), the persistence parameters of the exogenous processes \((\rho_a \text{ or } \rho_v)\), and all the other structural parameters of the model gathered in the \( \Theta \). In particular, Galí [2008] shows that

\[ \Lambda_v (\phi_x, \phi_y, \rho_v, \Theta) = -\frac{(1 - \beta \rho_v)}{(1 - \beta \rho_v)(\sigma (1 - \rho_v) + \phi_y) + \kappa (\phi_x - \rho_v)} \]  

\[ \Lambda_a (\phi_x, \phi_y, \rho_a, \Theta) = \psi \left( 1 - \frac{\sigma (1 - \rho_a) (1 - \beta \rho_a)}{(1 - \beta \rho_a)(\sigma (1 - \rho_a) + \phi_y) + \kappa (\phi_x - \rho_a)} \right), \tag{2.3} \]

where \( \psi = \frac{1 + \phi}{\sigma (1 - \alpha) + \phi + \alpha} \) and \( \kappa \) is defined as above. These expressions imply that the relationship between the persistence of the exogenous shocks and the level of output is non-linear in the monetary policy parameters. Assuming that \( \varepsilon^a_t \) and \( \varepsilon^v_t \) are independent, it is trivial to obtain the variance of output:

\[ Var(y_t) = \left[ \Lambda_v (\phi_x, \phi_y, \rho_v, \Theta) \right]^2 \frac{1}{1 - \rho_v^2} + \left[ \Lambda_a (\phi_x, \phi_y, \rho_a, \Theta) \right]^2 \frac{1}{1 - \rho_a^2}, \tag{2.4} \]

and its spectrum:

\[ S^y(\omega) = \left[ \Lambda_v (\phi_x, \phi_y, \rho_v, \Theta) \right]^2 S^v(\omega) + \left[ \Lambda_a (\phi_x, \phi_y, \rho_a, \Theta) \right]^2 S^a(\omega), \tag{2.5} \]
where \( S^\nu (\omega) \) denotes the spectrum of output at frequency \( \omega \). Following Galí [2008], we can derive similar expressions defining the relationship between inflation and the two exogenous processes, as well as the variance and spectrum of inflation.\(^3\) Now assume that the parameters of the Taylor rule change from \((\phi_\pi, \phi_y)\) to \((\phi'_\pi, \phi'_y)\), as a result of a change in monetary policy. Moreover, assume that all the parameters in \( \Theta \), and the persistence parameters of the exogenous process, \( \rho_a \) and \( \rho_v \), remain unchanged. In this case, since \( \Lambda_a \) and \( \Lambda_v \) depend on the parameters of the Taylor rule, the unconditional variance of output changes. In addition, provided that \( \rho_a \neq \rho_v \), the relative contributions of the two shocks to the variance of output also change, thus implying a different shape of the output spectrum.

To illustrate the magnitude of the effects of a change in monetary policy to the spectral density and spectral distribution of the economic variables, we perform a numerical exercise. First, we calibrate preference and technology parameters following Galí’s baseline calibration: \( \beta = 0.99, \sigma = 1, \alpha = 1/3, \varepsilon = 6, \) and \( \theta = 2/3 \). The values of the autoregressive coefficients of the two shocks and the coefficients of the Taylor rule are the following: \( \rho_a = 0.8, \rho_v = 0.5, \phi_\pi = 1.5, \) and \( \phi_y = 0.125 \). Given this parameterization, using (2.4) and (2.5), we can compute several statistics of interest. In particular, we consider the standard deviation of output, the standard deviations of inflation, and the spectral distribution of the two variables. To obtain information about the shape of the spectrum of the two variables, following Pancrazi [2009] we consider two intervals of frequencies: the high frequencies, defined as the fluctuations with periodicity between 2 and 32 quarters, and the medium frequencies, defined as the fluctuations with periodicity between 32 and 80 quarters. The standard deviation

\[^3\] The expressions are:

\[ \pi_t = \Lambda_v^\pi (\phi_\pi, \phi_y, \rho_v, \Theta) \nu_t + \Lambda_a^\pi (\phi_\pi, \phi_y, \rho_a, \Theta) a_t \]

with

\[ \Lambda_v^\pi (\phi_\pi, \phi_y, \rho_v, \Theta) = \frac{\kappa}{(1-\beta \rho_v) \sigma (1-\rho_v) + \phi_y + \kappa (\phi_\pi - \rho_v)} \]

\[ \Lambda_a^\pi (\phi_\pi, \phi_y, \rho_a, \Theta) = -\psi \left( \frac{\kappa}{(1-\beta \rho_a) \sigma (1-\rho_a) + \phi_y + \kappa (\phi_\pi - \rho_a)} \right) \]
of output and inflation at these intervals of frequencies are reported in Table 2.1.

Now assume that the monetary authority decides to respond to inflation more aggressively, which implies a larger $\phi_\pi$. Therefore, keeping all other parameters of the model constant, we set $\phi_\pi$ to be 6. The resulting standard deviations implied by the model are shown in Table 2.2. To illustrate the change from a different angle, in Table 2.3 we compute the percentage change of the variances driven by the new-monetary policy. These tables present some interesting findings. First, a change of the response of monetary authority to inflation has effects both on the stabilization of inflation itself and on the stabilization of output. In our exercise, the effect on inflation is larger: the variance of inflation declines by 87 percent, whereas the variance of output declines by 10 percent. Second, the decline of the volatility of output differs across the different frequencies. In fact, while at high frequencies the decline of the variance of output is 24 percent, the new monetary policy induces a slightly larger variance of output at medium frequencies. In contrast, the stabilization effect on inflation appears to be fairly uniform at all frequencies. This result can be visualized in Figures 2.1 and 2.2, where we plot the normalized spectrum of output and inflation under the two different monetary policies. As the figures show, the change in monetary policy largely affects the shape of the spectrum of output and inflation.

Why does the shape of the output spectrum change with a change of $\phi_\pi$? As equations (2.1), (2.2), and (2.3) suggest, a different $\phi_\pi$ leads to a change of the relative weight of the two shocks in output. In other words, $\Lambda_v$ and $\Lambda_a$ do not change proportionally, since they depend on the autocorrelations of the two different exogenous processes. To explore the effects of the change in $\phi_\pi$ we compute the variance decompositions of output and inflation, displayed in Table 2.4. The variance decompositions

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4 The normalized spectrum is a useful tool for exploring the relative contribution of different frequencies to the total variance of a variable. Since output has different total variance in the two scenarios, we rescale the spectra so that those variables have variance equal to unity.
decomposition indicates the fraction of the variance attributable to the monetary shock and to the technology shock. Since in this experiment we assume that the exogenous processes do not change, the variance decomposition is affected only by the change in the Λ functions. In particular, the increase in $\phi_\pi$ implies that the dynamics of output are driven to a larger degree by the technology shocks, $a_t$, than by the monetary shocks, $v_t$ with respect to the model with a lower $\phi_\pi$. Since the technology shock is more persistent than the monetary shock, there is a redistribution of the volatility of output toward lower frequencies.

The purpose of this simple example was to illustrate that different monetary policy could have large effects on the level of the spectral density of the real macroeconomic variables and on its distribution. However, the model considered above is relatively unrealistic, since it abstracts from the investment sector, it features few rigidities, and it is driven only by two shocks. Therefore, in the following sections we consider a richer theoretical framework. This framework allows us to address the question whether a change in the monetary policy after the early 1980s affected the variances of the real variables and their temporal distributions.

2.3 Medium-Scale DSGE Model

We use a fairly standard DSGE model, in the spirit of CEE and Smets and Wouters (2003). The model is driven by four shocks: a neutral technology shock, an investment-specific shock, a fiscal policy shock, and a monetary policy shock. The model is augmented by a number of real and nominal rigidities. The nominal rigidities include price and wage stickiness, and indexation to the past inflation. The real rigidities evolve from the habit formation in consumption, monopolistic competition in factor and product markets, and investment adjustment costs.
2.3.1 Households

The economy is populated by a continuum of households indexed by \( j \in [0, 1] \). House-
hold’s preferences are defined over consumption, \( c_{jt} \), and labor, \( l_{jt} \). Each household \( j \) maximizes lifetime utility that takes the following form:

\[
U = E_t \sum_{t=0}^{\infty} \beta^{t} \left\{ \log (c_{jt} - bc_{jt-1}) - \psi \frac{l_{jt}^{1+\gamma}}{1 + \gamma} \right\},
\]

(2.6)

where \( \beta \) denotes the subjective discount factor, \( b \) is the habit persistence parameter and \( \gamma \) is the inverse of Frisch labor supply elasticity.\(^5\)

Households own physical capital. The capital stock, \( k_t \), is assumed to evolve over time according to the following law of motion

\[
k_{t+1} = (1 - \delta) k_t + \mu_t \left( 1 - S \left( \frac{x_t}{x_{t-1}} \right) \right) x_t,
\]

(2.7)

where \( \delta \) is the depreciation rate of capital stock, \( x_t \) represents the gross investment, and \( \mu_t \) is the investment-specific technology shock that follows an autoregressive process, given by

\[
\log (\mu_t) = \rho_{\mu} \log (\mu_{t-1}) + \sigma_{\mu} \varepsilon_{\mu,t}, \text{ where } \varepsilon_{\mu,t} \sim N(0, 1).
\]

The function \( S(\cdot) \) is an investment adjustment cost function, as introduced by CEE. We assume that in the steady state \( S = S' = 0 \) and \( S'' > 0 \), which implies no adjustment costs in the vicinity of the steady state. We assume the following function form:

\[
S \left( \frac{x_t}{x_{t-1}} \right) = \frac{\kappa}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right)^2.
\]

The first-order conditions with respect to consumption, capital, capacity utiliz-
ation, and investment are fairly standard, whereas the first-order conditions with

\(^5\) We can omit the subscript \( j \) with consumption, because households are assumed to have access to a complete set of Arrow-Debreu securities, and can fully ensure against the idiosyncratic risks.
respect to labor and wages are more complex. We follow the set-up of Erceg, Henderson, and Levin [2000] and assume that each household supplies differentiated labor services to the production sector. In order to avoid this heterogeneity spilling over into consumption heterogeneity, they assume that utility is separable in consumption and labor, and that, because of the existence of complete markets, households can fully ensure against the employment risks. In addition, we assume that a representative labor aggregator combines households’ labor in the same proportion as firms would choose. This ensures that her demand for the $j$-th household’s labor is the same as the sum of the firms’ demands for this type of labor.

Specifically, the labor aggregator uses the following production technology:

$$l_t^d = \left( \int_0^1 l_{jt}^{\eta-1} \, dj \right)^{\frac{\eta}{\eta-1}}, \quad (2.8)$$

where $\eta \in [0, \infty)$ is the elasticity of substitution among different types of labor, and $l_t^d$ is the aggregate labor demand. She maximizes profits subject to (2.8), taking as given all differentiated labor wages $w_{jt}$ and the aggregate wage index $w_t$. Her demand for the labor of household $j$ is given by

$$l_{jt} = \left( \frac{w_{jt}}{w_t} \right)^{-\eta} l_t^d \quad \forall j. \quad (2.9)$$

Households set their wages following Calvo setting, i.e. in any given period, a fraction $\theta_w \in [0, 1)$ of randomly picked households is not allowed to optimally set their wages. Instead, they partially index their wages to the past inflation, $\Pi_{t-1}$, which is controlled by the indexation parameter $\chi_w \in [0, 1]$. The remaining fraction of households who are allowed to reset their wages, choose the same optimal wage,
i.e. \( w^*_i = w_{jt} \forall j \), that maximizes (3.1). The first-order condition to this problem is:

\[
\frac{\eta - 1}{\eta} w_t^* E_t \sum_{k=0}^{\infty} (\beta \theta w)^k \lambda_{t+k} \left( \frac{k}{s=1} \frac{\Pi^{w_{t+s-1}}}{\Pi_{t+s}} \right)^{1-\eta} \left( \frac{w_t^{*}}{w_{t+k}} \right)^{-\eta} l_{t+k}^d
\]

\[
= E_t \sum_{k=0}^{\infty} (\beta \theta w)^k \psi \left( \frac{k}{s=1} \frac{\Pi^{w_{t+s-1}}}{\Pi_{t+s}} \frac{w_t^{*}}{w_{t+k}} \right)^{-\eta(1+\gamma)} \left( l_{t+k}^d \right)^{1+\gamma}
\]

where \( \lambda_t \) is the Lagrangian multiplier associated with the household’s budget constraint.

2.3.2 The Final Good Producer

The final good producer aggregates intermediate goods, \( y_{it} \), into the homogenous final good, \( y_t^d \), using a Dixit and Stiglitz [1977] production function:

\[
y_t^d = \left( \int_{0}^{1} y_{it}^\varepsilon \, di \right)^{\frac{1}{\varepsilon - 1}},
\]

(2.10)

where \( \varepsilon \) is the elasticity of substitution among the intermediate goods. The final good producer chooses the bundle of goods that minimizes the cost of producing \( y_t^d \), taking all intermediate goods prices \( p_{it} \), final domestic good price \( p_t \), and the quantity of intermediate goods \( y_{it} \) as given. The unit price of the output unit is equal to its unit cost \( p_t \) :

\[
p_t = \left( \int_{0}^{1} p_{it}^{1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}}.
\]

The input demand function \( y_{it} \) for each intermediate good \( i \) is then given by:

\[
y_{it} = \left( \frac{p_{it}}{p_t} \right)^{-\varepsilon} y_t^d \quad \forall i,
\]

where \( y_t^d \) is the aggregate demand.
2.3.3 Intermediate Goods Producers

There is a continuum of intermediate goods producers indexed by $i$ on the unit interval. Each differentiated good is produced by a single intermediate firm $i$ that rents capital services $k_{it}$, and labor services $l^d_{it}$, using the production function:

$$y_{it} = A_t k_{it}^\alpha (l^d_{it})^{1-\alpha},$$

where $\alpha$ is the capital share in the production function and $A_t$ represents the neutral technology process, given by the following autoregressive process:

$$\log (A_t) = \rho \log (A_{t-1}) + \sigma \varepsilon_{A,t},$$

where $\varepsilon_{A,t} \sim N(0,1)$.

Each intermediate goods firm chooses amount of $k_{it}$ and $l^d_{it}$ to rent, taking the input prices $r_t$ and $w_t$ as given. The standard static first-order conditions for cost minimization imply that real marginal cost is the same for all firms. Therefore it does not have a subscript $i$ associated with it. The real marginal cost is given by

$$mc_t = \left( \frac{1}{1-\alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right) \frac{w_t^{1-\alpha} r_t^\alpha}{A_t}.$$

We assume that the intermediate goods firms set their prices a la Calvo [1983] and Yun [1996]. That is, in each period, a fraction $\theta_p \in [0,1)$ of firms is not allowed to change their prices, and can only index them by the past inflation, which is controlled by the indexation parameter $\chi_p \in [0,1]$. The remaining $1-\theta_p$ firms that are allowed to reset their prices in period $t$, choose optimal price $p^*_t$, which is the solution to the following maximization problem:

$$\max_{p_{it}} E_t \sum_{k=0}^{\infty} (\beta \theta_p)^k \frac{\lambda_{t+k}}{\lambda_t} \left\{ \left( \sum_{s=1}^{k} \prod_{t+s-1}^{t+s-1} \frac{\prod_{t+s-1}^{t+s-1} \prod_{t+s-1}^{t+s-1} p_{it}}{p_{t+k}} - mc_{t+k} \right) y_{it+k} \right\}$$

subject to:

$$y_{it+k} = \left( \sum_{s=1}^{k} \prod_{t+s-1}^{t+s-1} \frac{\prod_{t+s-1}^{t+s-1} p_{it}}{p_{t+k}} \right)^{1-\varepsilon} y^d_{it+k}.$$
If we define recursively:

\[
g_1^t = \lambda_t mc_t y_t^d + \beta \theta_p E_t \left( \frac{\Pi_t}{\Pi_{t+1}} \right)^{-\varepsilon} g_{t+1}^1
\]

\[
g_2^t = \lambda_t \Pi_t^* y_t^d + \beta \theta_p E_t \left( \frac{\Pi_t}{\Pi_{t+1}} \right)^{1-\varepsilon} \left( \frac{\Pi_t^*}{\Pi_{t+1}^*} \right) g_{t+1}^2,
\]

where \( \Pi_t^* = \frac{p_t^*}{p_t} \), the first-order condition to this problem can be written as \( \varepsilon g_1^t = (\varepsilon - 1) g_2^t \).

Finally, considering the price setting, the aggregate price index is:

\[
p_t^{1-\varepsilon} = \theta_p \left( \frac{\Pi_t}{\Pi_{t-1}} \right)^{1-\varepsilon} p_{t-1}^{1-\varepsilon} + (1 - \theta_p) \left( p_t^* \right)^{1-\varepsilon}.
\]

2.3.4 The Government Problem

The monetary authority follows the interest rate rule given by:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left( \frac{\Pi_t}{\Pi} \right)^{\gamma_n} \left( \frac{y_t^d}{y_t^d} \right)^{\gamma_y} \exp(m_t), \tag{2.11}
\]

where \( R_t \) is the nominal gross return on capital in period \( t \), \( \Pi \) represents the target level of inflation which is equal to the inflation in the steady state, \( R \) is the steady state nominal gross return on capital, \( y_t^d \) is the steady-state level of output, and \( m_t \) represents the shock to monetary policy with the following law of motion:

\[
m_t = \sigma_m \varepsilon_{mt}, \text{ where } \varepsilon_{mt} \sim N(0, 1).
\]

Interest rate smoothing, i.e. the presence of \( R_{t-1} \) in the Taylor rule, is justified because we want to match the smooth profile of the interest rate, observed in the U.S. data.

The fiscal authority, or government, runs a balanced budget. Government spending, \( g_t \), is modeled as an exogenous autoregressive process, given by

\[
\log(\frac{g_t}{g}) = \rho_g \log(\frac{g_{t-1}}{g}) + \sigma_g \varepsilon_{g,t}, \text{ where } \varepsilon_{g,t} \sim N(0, 1).
\]
Here $g$ represents the steady-state level of government spending, defined as a constant portion, $S_g$, of the steady-state level of output.

2.3.5 Aggregation

The aggregate demand is given by

$$y^d_t = y_t + x_t + g_t + \mu_t^{-1} a(u_t) k_t,$$

where $u_t$ is the variable capacity utilization and $\mu_t^{-1} a(u_t)$ is the physical cost of use of capital in resource terms. Following Altig et al. [2005] and CEE, we assume that $u_t = 1$ in the steady state and $a(1) = 0$, and that the value of the curvature of $a$ in the steady state, $a'(1)/a''(1)$ $\geq$ 0. We assume the functional form that satisfies these properties, given by

$$a(u_t) = \gamma_1 (u_t - 1) + \frac{\gamma_2}{2} (u_t - 1)^2.$$

After some manipulations, the goods market clearing condition is:

$$c_t + x_t + g_t + \mu_t^{-1} a(u_t) k_t = \frac{A_t (k_{t-1})^\alpha (l^d_t)^{1-\alpha}}{v_t^p},$$

where $v_t^p = \int_0^1 \left( \frac{p_t}{p_t} \right)^{-\varepsilon} d\iota$ is the price dispersion term that is, considering the Calvo price setting, given by

$$v_t^p = \theta_p \left( \frac{\Pi_{t-1}}{\Pi_t} \right)^{-\varepsilon} v_{t-1}^p + (1 - \theta_p) (\Pi_t^\varepsilon)^{-\varepsilon}.$$

Finally, the labor market clearing condition is obtained by integrating (3.9) over all households $j$

$$l^d_t = \frac{1}{v_t^w} l_t.$$
where \( v_t^w = \int_0^1 \left( \frac{w_{j,t}}{w_t} \right)^{-\eta} dj \) is the wage dispersion term that is, considering the Calvo wage setting, given by

\[
v_t^w = \theta_w \left( \frac{w_{t-1} \Pi_{t-1}^x}{w_t \Pi_t} \right)^{-\eta} v_{t-1}^w + (1 - \theta_w) (\Pi_t^{\pi^*})^{-\eta},
\]

where \( \Pi_t^{\pi^*} = \frac{w_t^*}{w_t} \).

### 2.4 Estimation and Calibration

#### 2.4.1 Estimation

The goal of this paper is to assess whether changes in the monetary policy after the early 1980s contributed to the decline of the variance of the real variables and to its temporal redistribution. To address this question, we first split the sample into two subsamples. The first subsample covers the period 1947:I-1978:IV, whereas the second subsample covers the period 1983:I-2007:IV. We eliminate the four years from 1978 to 1982 from the sample, since it is generally believed that the monetary policy rule being followed in that period was very different from the other sub-periods.\(^6\) We then estimate the processes for the investment-specific technology, TFP, exogenous component of the monetary policy rule, and the exogenous process of government spending. We use their observable counterparts in the estimation process.

First, let us consider the monetary policy rule as in (2.11). We obtain ordinary least squares estimates of the Taylor rule parameters in the two subsamples and estimate the monetary policy shock, \( m_t \), as the residual from this regression. This allows us to estimate the variance of the monetary policy shock. After taking logs,

equation (2.11) becomes:

\[
\log \left( \frac{R_t}{R} \right) = \gamma_R \log \left( \frac{R_{t-1}}{R} \right) + (1 - \gamma_R) \gamma_R \log \left( \frac{\Pi_t}{\Pi} \right) + (1 - \gamma_R) \gamma_R \log \left( \frac{y_{d,t}}{y} \right) + m_t,
\]

where \( \Pi \) is the average inflation in each subsample, and \( R = \Pi / \beta \) is the steady state interest rate. Inflation, \( \Pi_t \), is measured as the percentage change in the consumption deflator from NIPA. The real interest rate, \( R_t \), is measured as three-month T-bills rate obtained from the International Financial Statistics, and \( y_{d,t}/y \) is the output gap, defined as the cyclical component of the real per capita gross domestic product. To take into account the role of the medium frequencies, we use a bandpass filter as implemented by Christiano and Fitzgerald [2003] to isolate the fluctuation between 2 and 80 quarters.

To obtain the parameters of the investment-specific technology process, we use the relative price of investment with respect to consumption as the observable. In fact, equation (2.7) and equation (2.12) imply that the relative price of the investment good with respect to the consumption good is \( 1/\mu_t \). Therefore, the level of the investment-specific technology, \( \mu_t \), can be estimated as the inverse of this relative price. National Income and Product Accounts (NIPA) provide data for both the investment deflator and the consumption deflator. The consumption deflator is computed as the real consumption price index of nondurables and services. The investment deflator is computed as the real price of private investment. The issue of the quality improvement of capital goods over time and its effects on the measured relative price of investment is well-explored in the macroeconomics literature. Gordon [1989] provides estimates of the quality adjusted price of several types of durable equipment. However, Gordon’s time series covers only the postwar period
until 1983. Cummins and Violante [2002] and Pakko [2002] extended Gordon’s procedure using forward extrapolation to obtain updated quality-adjusted price series. However, Moulton [2001] revealed that NIPA currently takes into account the quality adjustment for electronic equipment, the component of investment intuitively more subject to quality changes. Since the two procedures deliver the same qualitative results, as illustrated by Pakko [2002], and since the forward extrapolation relies on some questionable assumption, e.g. that the quality bias in the price indexes has not changed since 1983, in this paper we use the price series from NIPA. We then estimate the parameters of an AR(1) process on the relative price of investment to obtain point estimates of $\rho_\mu$ and $\sigma_\mu$ in each of the two subsamples.

We follow a similar approach to estimate the parameters of the TFP process. We account for the variable capacity utilization by constructing a measure of TFP as:

$$ TFP_t = \left( \frac{Y_t}{L_t^{1-\alpha} (U_t K_t)^{\alpha}} \right). $$

We set the labor share, $1 - \alpha$, equal to 0.64, which is obtained as the average value of the labor share series recovered from the BLS. From the same source we recover annual data on capital services, $K_t$. We interpolate the capital services series to obtain quarterly series, assuming constant growth within the quarters of the same year. Non-farm business measures of hours, $H_t$, and output, $Y_t$, are also retrieved from the BLS. Finally, the series of capacity utilization, $U_t$, is retrieved from the Federal Reserve Board. This measure is based on the manufacturing data.

Finally, we estimate the parameters governing the government spending process using data on government consumption expenditure from NIPA, and obtain the point estimates of $\rho_g$ and $\sigma_g$ in the two subsamples.

Table 2.5 shows the estimates of the parameters governing the four exogenous processes. First, the persistence of both the TFP and investment-specific technology processes increased, with a larger increase of the TFP persistence. In contrast, the
persistence of the government spending shock did not change. Second, the standard deviation of the innovations decreased for all the shocks, except for the monetary policy. The decrease of the standard deviation is more remarkable for the government spending innovation. This result is somewhat expected, since the first subsample includes the Korean War and since the government spending was more stable in the second subsample.

Since our goal is to explore the role of the different monetary policies and the different technology processes before and during the Great Moderation, we gather the estimated parameters of the Taylor rule in vectors $\Gamma_i$, and the parameters of the exogenous processes in $\Theta_i$, with $i = 1, 2$, where $i$ indicates the subsample used for the estimation.

2.4.2 Calibration

Since the goal of this paper is to assess the role of the different shocks in the change of variance of the real variables during the Great Moderation, we assume that the structural parameters of the model are constant during the entire post-war period. We calibrate the structural parameters of the model, using the corresponding data statistics or the conventional wisdom. We choose the subjective discount factor $\beta$ to be $1.03^{-1/4}$, which corresponds to the annualized real interest rate of 3 percent. Following CEE, we set the habit persistence parameter, $\theta$, to 0.65. The preference parameter associated with labor, $\psi$, is chosen so that the agents allocate one-third of their time endowment to work. The depreciation rate parameter, $\delta$, is set equal to 0.025, which implies an annual capital depreciation rate of 10 percent. We assign a value of 0.36 to the capital share in production function to match the steady state share of labor of 64 percent. Following Altig et al. [2005], we set the elasticity of substitution between different types of labor equal to 21, and the elasticity of substitution between differentiated intermediate goods equal to 6. The price stickiness
parameter is set at 0.6, which implies price contracts lasting 2.5 quarters, whereas the wage stickiness parameter is set at 0.64, implying wage contracts lasting 2.8 quarters. Both values are taken from CEE. We assume no price indexation, following Cogley and Sbordone [2004] and Levin, Onatski, Williams, and Williams [2005], who find that there is no indexation in product prices. Finally, the wage indexation is very close to unity, following Levin et al. [2005] who find a high degree of wage indexation. Values of the calibrated parameters are summarized in Table 2.6.

2.5 Results

2.5.1 Model Performance

After we estimate the exogenous processes of the model in both subsamples and feed them into the model, we assess if the model is able to generate reasonable predictions for the behavior of the macroeconomic variables. Table 2.7 displays the model predictions for the high-frequency standard deviations of output, consumption, and investment in two subsamples. In particular, we define as $M(\Gamma_i, \Theta_j), i, j = 1, 2$, the model in which the Taylor rule parameters, $\Gamma_i$, are estimates of (2.13) using data from subsample $i$, and the parameters of the exogenous processes, $\Theta_j$, are estimated using data from subsample $j$. We also report the estimates of the corresponding data moments for comparison.\(^7\)

The model performs remarkably well in replicating the behavior of the standard deviations of the variables in the two subsamples. Although the model slightly underestimates the volatility of output and investment in the first subsample, it is able to match the ratios of the standard deviations of output, consumption, and

---

\(^7\) The data on consumption, output, and investment are retrieved from the NIPA. The consumption series is given by real per capita personal consumption expenditures on nondurables and services series. Output is measured by real per capita gross domestic product series, and investment by real per capita private investment. To obtain estimates of the high- and medium-frequency standard deviations as defined in Section 2, all data are filtered using the band pass filter implemented by Christiano and Fitzgerald [2003].
investment. Moreover, the model matches almost exactly the standard deviations of the variables in the second subsample, and also predicts the decline of the high-frequency volatility during the Great Moderation period. This result confirms that our model, driven by the estimated exogenous processes, generates realistic high-frequency dynamics of the macroeconomic variables.

However, since we want to explore the changes in the spectral shapes of the macroeconomic variables during the Great Moderation, we are also interested in the model predictions of the medium-frequency volatilities. Table 2.8 displays the model medium-frequency standard deviations of output, consumption, and investment, as well as their data counterparts. The model largely underestimates the standard deviations of the variables at medium frequencies. Therefore, the propagation mechanism governing the intertemporal dynamics of the model appears to be weak, since it cannot generate fluctuations at medium frequencies similar in magnitude to those observed in the data. Although the model fails to quantitatively capture the medium-frequency behavior of the macroeconomic variables, it produces rather interesting qualitative implications. The model correctly predicts the absence of moderation at medium frequencies, as observed in the data. Whereas the standard deviation at high frequencies largely declines from the first to the second subsample, the standard deviation at medium frequencies exhibits a different behavior. In fact, it largely increases for investment, slightly increases for output, and marginally declines for consumption. Hence, the model is able to qualitatively predict the spectral redistribution of the variance from high frequencies to low frequencies during the Great Moderation.

To assert this implication, in Table 2.9 we present the percentage contribution of the high-frequency variance to the total variance of the variables in the two subsamples, implied both by the data and by the model. Two important results emerge. First, the data show a redistribution of the variance from high to medium frequencies
during the Great Moderation. In fact, whereas the high frequencies account for approximately 40 percent of the total variance of the output and consumption and for 66 percent of the total variance of investment in the pre-1978 period, the contribution of the high frequencies for all the variables drops to about 20 percent during the Great Moderation. This result is a consequence of the specific nature of the Great Moderation, which is characterized by a sharp decline of the volatility only at high frequencies of the macroeconomic variables.\footnote{See Pancrazi [2009].} Since the high-frequency volatilities declined remarkably, and the medium-frequency volatilities did not, the medium frequencies capture a larger fraction of the total variance during than before the Great Moderation. Second, as already pointed out, the model is not well suited to explain the medium-frequency fluctuations of the variables, since the largest fraction of the total variability of output, consumption, and investment is captured only by the high frequencies. Nevertheless, the model correctly predicts a large redistribution of the variance from high frequencies to low frequencies during the Great Moderation. Our primary goal in this paper is to explore what the main driving force is behind this redistribution of the variance from high to medium frequencies.

2.5.2 Counterfactuals: Role of Monetary Policy

In 2.2, by using a simple model we showed that a change in the monetary policy parameters could potentially imply a redistribution of the spectral density of real variables. In this section, we evaluate the role of a change in the monetary policy during the Great Moderation period in explaining both the decline in the variance and its redistribution from high to medium frequencies, as observed in the data, performing two counterfactual exercises.

First, we compute the standard deviations implied by the model $M(\Gamma_2, \Theta_1)$, where the exogenous processes are kept as estimated in the first subsample, $\Theta_1$, but...
we allow the monetary policy to adopt the rule estimated in the second subsample, \( \Gamma_2 \). As Table 2.10 shows, the role of the monetary policy change is negligible. The model moments at both intervals of frequencies are essentially unaffected when we allow only the parameters of the Taylor rule to change. Therefore, we conclude that a different monetary policy during the post-1983 period alone could not have played a significant role in the decline of the high-frequency volatilities of the real macroeconomic variables, nor in the redistribution of their volatilities from high to medium frequencies.

Then, in the second counterfactual exercise, we consider the model \( M(\Gamma_1, \Theta_2) \). We fix the coefficients of the Taylor rule as estimated in the first subsample, \( \Gamma_1 \), but we now feed into the model the exogenous processes estimated in the second subsample, \( \Theta_2 \). Table 2.11 shows the implied model moments. In this scenario, the standard deviations of the real macroeconomic variables are strongly affected at both high and medium frequencies. However, the change in the estimates of the exogenous processes alone does not reproduce the same moments as in the case in which both the exogenous processes and the coefficients of the Taylor rule change, \( M(\Gamma_2, \Theta_2) \). In particular, the decline of the high-frequency volatilities when the Taylor rule coefficients are fixed to their first subsample values, \( \Gamma_1 \), is smaller than when the Taylor rule coefficients are estimated in the Great Moderation period, \( \Gamma_2 \). The difference between these scenarios is much more pronounced at high frequencies than at medium frequencies. Therefore, once we assume that the exogenous processes have changed from the Pre-Great Moderation to the Great Moderation period, the role of monetary policy is not anymore negligible. In particular, when both exogenous processes and the Taylor rule coefficients change, the contribution of the monetary policy to the total reduction of the high-frequency volatilities of the macroeconomic variables is evident; 38 percent of the overall decline of the high-frequency variance of output and investment, and 43 percent for consumption, is due to the different
monetary policy.

The change in the exogenous processes and the change in the monetary policy have opposite effects at medium frequencies. In fact, the larger persistence of the TFP and investment-specific technology causes an increase of the medium-frequency volatility of output and investment. In contrast, the change in monetary policy mitigates this increase, since it drives down the medium-frequency volatilities. However, the effect of the monetary policy on the medium-frequency volatilities is significantly smaller than on the high-frequency volatilities.

This finding is supported when we compute the percentage contribution of the high frequencies to the total variance of the macroeconomic variables in this counterfactual scenario, as reported in Table 2.12. The change in the exogenous processes alone implies a redistribution of the spectral density of output, investment, and consumption from high to medium frequencies, as suggested by the decline in the percentage contribution of high frequencies. This redistribution is mainly caused by the increase in persistence of the TFP, and of the investment-specific technology, since they are the major contributors to the overall variance of the real variables, as we show in the next section. However, the change in monetary policy amplifies this effect, since the different monetary policy causes a large decline in the high-frequency volatility and it has only a marginal effect on the medium-frequency volatility. Therefore, the monetary policy in the Great Moderation period contributed to the change of the spectral shape of the real macroeconomic variables, since it unequally altered the volatilities at different frequencies.

2.5.3 Variance Decomposition

Another interesting question we address is, what fraction of the high-frequency and medium-frequency variances is attributable to each of the four exogenous shocks that drive the dynamics of the model? Tables 2.13 and 2.14 display the variance decompo-
ositions of output, consumption, and investment in the two subsamples, respectively. If we consider the model fitted to the pre-Great Moderation period, \( M(\Gamma_1, \Theta_1) \), the largest fraction of the high-frequency variance of output depends on the TFP shock. Monetary shocks explain only about 2 percent of the total variance of the real variables, whereas the investment-specific and government spending shocks together are the source of less than 10 percent of the total variance for each of the three variables.

However, the percentage contributions of the shocks change significantly when we consider medium-frequency fluctuations. First, notice that although the TFP shock explains most of the medium-frequency variances of both output and investment, its share drops sharply with respect to the high-frequency contribution. The sharpest decline of the TFP shock share is for consumption; it drops from 83.6 percent at high frequencies to 30.8 percent at medium frequencies. Therefore, the other shocks become relatively more important when lower frequency fluctuations are considered. In fact, the government spending shock is the main source of medium-frequency movements in consumption. The importance of the investment-specific technology shock for all three macroeconomic variables increases notably when medium frequencies are considered. Finally, the effect of the monetary policy shock on the variances also remains negligible at medium frequencies.

Table 2.14 displays the variance decompositions of the three macroeconomic variables implied by the model driven by the exogenous processes estimated during the Great Moderation period, \( M(\Gamma_2, \Theta_2) \). At high frequencies, the TFP shock drives the largest fraction of the fluctuations of all three real variables. The contribution of the monetary policy shock in the total variance drops essentially to zero for all variables. The investment-specific technology shock explains approximately 5 percent of the total variance, as in the model estimated in the pre-Great Moderation period, whereas the contribution of the government spending shock at high frequencies significantly declines in the second subsample. At medium frequencies, the TFP
remains the largest driving force for the fluctuations of the real variables. The role of the investment-specific shock increases, especially for consumption (from 4.8 percent to 24.2 percent) and investment (from 8 percent to 21.4 percent), which makes it the second most important shock. The government spending shock and the monetary policy shock play a minor role in explaining the medium-frequency fluctuations of the variables.

Using a simpler model, we showed that different monetary policy can potentially alter the variance decomposition of real variables. In order to explore whether that was the case during the Great Moderation period, we compute the variance decomposition of the counterfactual process \( M (\Gamma_1, \Theta_2) \), shown in Table 2.15. Since in this model only the parameters of the exogenous processes are fixed to their second sub-sample values, the differences between the decompositions in Tables 2.14 and 2.15 are driven solely by the change in monetary policy. Therefore, by comparing the results, we infer the effects that the monetary policy adopted during the Great Moderation had on variance decompositions of output, consumption, and investment. The differences are negligible, not greater than 3 percent for any of the four shocks. Therefore, we conclude that the monetary policy did not alter the variance decompositions of the real macroeconomic variables.

2.6 Conclusions

In this paper, we focus on the two main characteristics of the Great Moderation: the significant reduction of the high-frequency volatility of real macroeconomic variables, which has been largely explored in the literature, and the absence of moderation of their medium-frequency volatility, as recently observed by Pancrazi [2009]. In particular, using a medium scale DSGE model as in CEE, we explore whether the more aggressive monetary policy in the post-1983 period accounts for the reduction of the variance and its different temporal distribution in the last three decades. We
show that the “good luck” hypothesis, the notion that the nature of the exogenous processes has changed during the Great Moderation period, mainly accounts for both facts.

As a theoretical framework, we consider a model driven by four shocks: a neutral technology shock, an investment-specific technology shock, a government spending shock, and a monetary policy shock. The structural parameters of the model are calibrated and kept constant throughout the whole sample, whereas the parameters governing the exogenous processes are estimated in the two subsamples, pre-Great Moderation and the Great Moderation period, and fed into the model.

Using the predictions of this model when performing several counterfactual exercises, we conclude that a change in monetary policy during the post-1983 period alone did not play a significant role in accounting for the two facts characterizing the Great Moderation: the decline of the high-frequency volatilities of the real macroeconomic variables and the redistribution of their volatilities from high to medium frequencies. It is only with a simultaneous change in both the exogenous processes and monetary policy, that the contribution of the monetary policy to the total reduction of the high-frequency volatilities of the macroeconomic variables becomes evident; it accounts for 38 percent of the overall decline of the high-frequency variance of output and investment, and 43 percent of the decline in the case of consumption. Moreover, we document that the effects of the monetary policy are much larger at high frequencies than at medium frequencies.

The change in the exogenous processes alone largely accounts for the redistribution of the spectral density of output, investment, and consumption from high to medium frequencies. This redistribution is mainly caused by the increase in persistence of the TFP and the investment-specific technology processes, as two major contributors to the overall variance of the real variables. However, the change in monetary policy amplifies this effect, since it has a much larger effect on the high-
frequency volatility. This leads us to conclude that monetary policy in the Great Moderation period contributed to the transformation of the spectral shape of the real macroeconomic variables, even though much less than the change of the exogenous processes.

We also perform a variance decomposition exercise, and show that in both subsamples the TFP shock and the investment-specific technology shock are the two most important driving forces of the variances at both high and medium frequencies. In both subsamples, the role of the investment-specific shock increases at medium frequencies. The role of the monetary policy shock is negligible in both subsamples at all frequencies. The fiscal policy shock is relatively more important in the first subsample, and at medium frequencies. However, its role is only relevant in explaining medium-frequency movements of consumption. Finally, we show that these results are not affected by the change in monetary policy.

2.7 Tables and Figures

Table 2.1: Standard Deviations of Output and Inflation when $\phi_\pi = 1.5$

<table>
<thead>
<tr>
<th></th>
<th>Percentage</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma$</td>
<td>$\sigma_{HF}$</td>
<td>$\sigma_{MF}$</td>
</tr>
<tr>
<td>Output</td>
<td>2.02</td>
<td>1.40</td>
<td>0.91</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.43</td>
<td>0.32</td>
<td>0.19</td>
</tr>
</tbody>
</table>

*Note:* The table reports the standard deviation of output and inflation implied by the New Keynesian model when the inflation parameter in the Taylor rule is $\phi_\pi = 1.5$. The first column reports the total standard deviations, the second column reports the high-frequency standard deviations, defined as the fluctuations between 2 and 32 quarters, and the third column reports the medium-frequency standard deviations, defined as the fluctuations between 32 and 80 quarters.
Table 2.2: Standard Deviations of Output and Inflation when $\phi_\pi = 6$

<table>
<thead>
<tr>
<th>Percentage</th>
<th>$\sigma$</th>
<th>$\sigma^{HF}$</th>
<th>$\sigma^{MF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.92</td>
<td>1.22</td>
<td>0.97</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.15</td>
<td>0.12</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Note: The table reports the standard deviation of output and inflation implied by the New Keynesian model when the inflation parameter of the Taylor rule is $\phi_\pi = 6$. The first column reports the total standard deviations, the second column reports the high-frequency standard deviations, defined as the fluctuations between 2 and 32 quarters, and the third column reports the medium-frequency standard deviations, defined as the fluctuations between 32 and 80 quarters.

Table 2.3: Change of Variances Driven by Different Monetary Policies

<table>
<thead>
<tr>
<th>Percentage</th>
<th>$\sigma$</th>
<th>$\sigma^{HF}$</th>
<th>$\sigma^{MF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>-10</td>
<td>-24</td>
<td>14</td>
</tr>
<tr>
<td>Inflation</td>
<td>-87</td>
<td>-86</td>
<td>-91</td>
</tr>
</tbody>
</table>

Note: The table reports the percentage change in the variance of output and inflation from an increase of the inflation parameter of the Taylor rule from 1.5 to 6. The first column reports the percentage change in the total variances, the second column reports the percentage change in the high-frequency variances, defined as the fluctuations between 2 and 32 quarters, and the third column reports percentage change in the medium-frequency variances, defined as the fluctuations between 32 and 80 quarters.
Table 2.4: Variance Decomposition with Two Different Monetary Rules

<table>
<thead>
<tr>
<th></th>
<th>$\phi_\pi = 1.5$</th>
<th></th>
<th>$\phi_\pi = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Technology Shock</td>
<td>Monetary Shock</td>
<td>Technology Shock</td>
</tr>
<tr>
<td>Output</td>
<td>58</td>
<td>42</td>
<td>91</td>
</tr>
<tr>
<td>Inflation</td>
<td>43</td>
<td>57</td>
<td>12</td>
</tr>
</tbody>
</table>

*Note:* The table reports the percentage contributions of the technology shock and monetary shock on the total variance of output and inflation, for the two cases when the inflation parameter of the Taylor rule is 1.5, and 6.
Table 2.5: Estimated Parameters of the Taylor Rule and Exogenous Processes

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Taylor Rule</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest Rate Smoothing Coefficients</td>
<td>( \gamma_R )</td>
<td>0.98 [0.03]</td>
</tr>
<tr>
<td>Inflation Coefficients</td>
<td>( \gamma_\pi )</td>
<td>0.20 [0.02]</td>
</tr>
<tr>
<td>Output Coefficients</td>
<td>( \gamma_y )</td>
<td>0.50 [0.00]</td>
</tr>
<tr>
<td>Monetary Policy Shock</td>
<td>( \sigma_m )</td>
<td>0.12 [0.02]</td>
</tr>
<tr>
<td>Investment-Specific Technology Shock</td>
<td>( \rho_\mu )</td>
<td>0.88 [0.03]</td>
</tr>
<tr>
<td>Standard Deviation Innovation</td>
<td>( \sigma_\mu )</td>
<td>0.54 [0.06]</td>
</tr>
<tr>
<td>TFP Shock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Persistence</td>
<td>( \rho_A )</td>
<td>0.71 [0.06]</td>
</tr>
<tr>
<td>Standard Deviation Innovation</td>
<td>( \sigma_A )</td>
<td>0.82 [0.05]</td>
</tr>
<tr>
<td>Government Spending Shock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Persistence</td>
<td>( \rho_g )</td>
<td>0.95 [0.02]</td>
</tr>
<tr>
<td>Standard Deviation Innovation</td>
<td>( \sigma_g )</td>
<td>2.24 [0.51]</td>
</tr>
</tbody>
</table>

**Note:** The table displays the estimates of the parameters of the Taylor rule and of the exogenous shocks of the model. The first column reports the estimates using data in the first subsample (1947:1-1978:4). The second column reports the estimates using data in the second subsample (1983:1-2007:4). Standard deviations are reported in percent. Standard errors are displayed in brackets.
Table 2.6: Calibration of the Structural Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ – Subjective discount factor</td>
<td>0.9926</td>
</tr>
<tr>
<td>$b$ – Habit persistence in consumption</td>
<td>0.65</td>
</tr>
<tr>
<td>$\psi$ – Preference parameter with labor</td>
<td>5</td>
</tr>
<tr>
<td>$\gamma$ – Inverse of Frisch labor supply elasticity</td>
<td>1</td>
</tr>
<tr>
<td>$\delta$ – Depreciation rate of capital</td>
<td>0.025</td>
</tr>
<tr>
<td>$\alpha$ – Capital share in the production function</td>
<td>0.36</td>
</tr>
<tr>
<td>$\varepsilon$ – EOS among differentiated intermediate goods</td>
<td>6</td>
</tr>
<tr>
<td>$\eta$ – EOS among different types of labor</td>
<td>21</td>
</tr>
<tr>
<td>$\kappa$ – Investment adjustment cost parameter</td>
<td>1.5</td>
</tr>
<tr>
<td>$S_g$ – Share of Government spending in GDP</td>
<td>0.17</td>
</tr>
<tr>
<td>$\gamma_2$ – Coefficient of the capital utilization function</td>
<td>0.0655</td>
</tr>
<tr>
<td>$\theta_w$ – Wage stickiness</td>
<td>0.64</td>
</tr>
<tr>
<td>$\chi_w$ – Wage indexation</td>
<td>0.98</td>
</tr>
<tr>
<td>$\theta_p$ – Price stickiness</td>
<td>0.6</td>
</tr>
<tr>
<td>$\chi_p$ – Price indexation</td>
<td>0</td>
</tr>
</tbody>
</table>

*Note:* The table reports the values of the calibrated structural parameter of the DSGE model.
Table 2.7: Model and Data High-Frequency Standard Deviations

<table>
<thead>
<tr>
<th></th>
<th>$M(\Gamma_1, \Theta_1)$ Data: 1947-1978</th>
<th>$M(\Gamma_2, \Theta_2)$ Data: 1983:2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.64 (1.94) [0.20]</td>
<td>1.07 (0.97) [0.09]</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.81 (0.88) [0.11]</td>
<td>0.51 (0.60) [0.05]</td>
</tr>
<tr>
<td>Investment</td>
<td>4.59 (5.44) [0.67]</td>
<td>2.97 (2.96) [0.23]</td>
</tr>
</tbody>
</table>

Note: The table reports the high-frequency standard deviations of output, consumption, and investment implied by the model and estimated in the data. The high-frequencies correspond to fluctuations between 2 and 32 quarters. The first column reports the moments implied by the model $M(\Gamma_1, \Theta_1)$, where $\Gamma_1$ is the set of parameters of the Taylor rule estimated using data in the first subsample (1947:1-1978:4), and $\Theta_1$ is the set of parameters of the exogenous processes estimated using also data in the first subsample. The second column reports the data moments in the first subsample. The third column reports the moments implied by the model $M(\Gamma_2, \Theta_2)$, where $\Gamma_2$ is the set of parameters of the Taylor rule estimated using data in the second subsample (1983:1-2007:4), and $\Theta_2$ is the set of parameters of the exogenous processes estimated using also data in the second subsample. The fourth column reports the data moments in the second subsample.
Table 2.8: Model and Data Medium-Frequency Standard Deviations

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.80 2.44 [0.24]</td>
<td>0.95 1.94 [0.35]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.53 1.10 [0.09]</td>
<td>0.47 1.55 [0.23]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>0.81 3.56 [0.37]</td>
<td>3.01 5.96 [1.10]</td>
</tr>
</tbody>
</table>

Note: The table reports the medium-frequency standard deviations of output, consumption, and investment implied by the model and estimated in the data. The medium-frequencies correspond to fluctuations between 32 and 80 quarters. The first column reports the moments implied by the model M(Γ₁, Θ₁), where Γ₁ is the set of parameters of the Taylor rule estimated using data in the first subsample (1947:1-1978:4), and Θ₁ is the set of parameters of the exogenous processes estimated using also data in the first subsample. The second column reports the data moments in the first subsample. The third column reports the moments implied by the model M(Γ₂, Θ₂), where Γ₂ is the set of parameters of the Taylor rule estimated using data in the second subsample (1983:1-2007:4), and Θ₂ is the set of parameters of the exogenous processes estimated using also data in the second subsample. The fourth column reports the data moments in the second subsample.

Table 2.9: Contributions of the High Frequencies to the Total variance

<table>
<thead>
<tr>
<th></th>
<th>Data 1947-1978</th>
<th>Data 1983:2007</th>
<th>Model M(Γ₁, Θ₁)</th>
<th>Model M(Γ₂, Θ₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>38</td>
<td>20</td>
<td>80</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>38</td>
<td>14</td>
<td>69</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>66</td>
<td>20</td>
<td>72</td>
<td>48</td>
</tr>
</tbody>
</table>

Note: The table reports the percentage contributions of the high frequency variance to the total variance for output, consumption, and investment. The first and second columns report these statistics estimated from the data in the two subsamples, respectively. The third and fourth columns report the statistics implied by the model.
Table 2.10: Model Standard Deviations: Counterfactual in Which Only the Taylor Rule Parameters Change

<table>
<thead>
<tr>
<th></th>
<th>High Frequencies</th>
<th>Medium Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M(\Gamma_1, \Theta_1)$</td>
<td>$M(\Gamma_2, \Theta_2)$</td>
</tr>
<tr>
<td>Output</td>
<td>1.64</td>
<td>1.07</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.81</td>
<td>0.51</td>
</tr>
<tr>
<td>Investment</td>
<td>4.59</td>
<td>2.97</td>
</tr>
</tbody>
</table>

Note: The table displays the high-frequency and medium-frequency standard deviations of output, consumption, and investment implied by the model. The model $M(\Gamma_2, \Theta_1)$ represents the counterfactual model in which the parameters of the Taylor rule, $\Gamma_2$, are as estimated using data in the second subsample (1983:1-2007:4) and the parameters of the exogenous processes, $\Theta_1$, are as estimated using data in the first subsample (1947:1-1978:4).
Table 2.11: Model Standard Deviations: Counterfactual in Which Only the Parameters of the Exogenous Processes Change

<table>
<thead>
<tr>
<th></th>
<th>High Frequencies</th>
<th></th>
<th>Medium Frequencies</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( M(\Gamma_1, \Theta_1) )</td>
<td>( M(\Gamma_2, \Theta_2) )</td>
<td>( M(\Gamma_1, \Theta_2) )</td>
<td>( M(\Gamma_1, \Theta_1) )</td>
</tr>
<tr>
<td>Output</td>
<td>1.64</td>
<td>1.07</td>
<td>1.29</td>
<td>0.80</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.81</td>
<td>0.51</td>
<td>0.60</td>
<td>0.53</td>
</tr>
<tr>
<td>Investment</td>
<td>4.59</td>
<td>2.97</td>
<td>3.59</td>
<td>2.82</td>
</tr>
</tbody>
</table>

Note: The table displays the high-frequency and medium-frequency standard deviations of output, consumption, and investment implied by the model. The model \( M(\Gamma_1, \Theta_2) \) represents the counterfactual model in which the parameters of the Taylor rule, \( \Gamma_1 \), are as estimated using data in the first subsample (1947:1-1978:4) and the parameters of the exogenous processes, \( \Theta_2 \), are as estimated using data in the first subsample (1983:1-2007:4).
### Table 2.12: Contributions of the High Frequencies to the Total Variance

<table>
<thead>
<tr>
<th>Percentage</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M(\Gamma_1, \Theta_1)$</td>
</tr>
<tr>
<td>Output</td>
<td>80</td>
</tr>
<tr>
<td>Consumption</td>
<td>69</td>
</tr>
<tr>
<td>Investment</td>
<td>72</td>
</tr>
</tbody>
</table>

*Note:* The table reports the model implied percentage contributions of the high frequency variance to the total variance for output, consumption, and investment.
Table 2.13: Variance Decomposition in $M(\Gamma_1, \Theta_1)$

<table>
<thead>
<tr>
<th>Percentage</th>
<th>Monetary</th>
<th>Investment-specific</th>
<th>TFP</th>
<th>Government Spending</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High Fr.</td>
<td>Medium Fr.</td>
<td>High Fr.</td>
<td>Medium Fr.</td>
</tr>
<tr>
<td>Output</td>
<td>2.2</td>
<td>0.7</td>
<td>3.0</td>
<td>20.0</td>
</tr>
<tr>
<td>Consumption</td>
<td>2.0</td>
<td>0.2</td>
<td>4.6</td>
<td>28.7</td>
</tr>
<tr>
<td>Investment</td>
<td>2.0</td>
<td>0.6</td>
<td>7.6</td>
<td>37.7</td>
</tr>
</tbody>
</table>

*Note:* The table represents the percentage contribution of the four shocks to the high-frequency, which corresponds to fluctuations between 2 and 32 quarters, and medium-frequency, which corresponds to fluctuations between 32 and 80 quarters, variance of output, consumption, and investment in the model $M(\Gamma_1, \Theta_1)$, where $\Gamma_1$ is the set of parameters of the Taylor rule estimated using data in the first subsample (1947:1-1978:4), and $\Theta_1$ is the set of parameters of the exogenous processes estimated using also data in the first subsample.
Table 2.14: Variance Decomposition in $M(\Gamma_2, \Theta_2)$

<table>
<thead>
<tr>
<th></th>
<th>Monetary</th>
<th>Investment-specific</th>
<th>TFP</th>
<th>Government Spending</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High Fr.</td>
<td>Medium Fr.</td>
<td>High Fr.</td>
<td>Medium Fr.</td>
</tr>
<tr>
<td>Output</td>
<td>0.1</td>
<td>0.0</td>
<td>3.1</td>
<td>9.7</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.1</td>
<td>0.0</td>
<td>4.8</td>
<td>24.2</td>
</tr>
<tr>
<td>Investment</td>
<td>0.1</td>
<td>0.0</td>
<td>8.0</td>
<td>21.4</td>
</tr>
</tbody>
</table>

*Note:* The table represents the percentage contribution of the four shocks to the high-frequency, which corresponds to fluctuations between 2 and 32 quarters, and medium-frequency, which corresponds to fluctuations between 32 and 80 quarters, variance of output, consumption, and investment in the model $M(\Gamma_2, \Theta_2)$, where $\Gamma_2$ is the set of parameters of the Taylor rule estimated using data in the second subsample (1983:1-2007:4), and $\Theta_2$ is the set of parameters of the exogenous processes estimated using also data in the second subsample.
Table 2.15: Variance Decomposition in $M(\Gamma_1, \Theta_2)$

<table>
<thead>
<tr>
<th></th>
<th>Monetary</th>
<th>Investment-specific</th>
<th>TFP</th>
<th>Government Spending</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High Fr.</td>
<td>Medium Fr.</td>
<td>High Fr.</td>
<td>Medium Fr.</td>
</tr>
<tr>
<td>Output</td>
<td>2.52</td>
<td>0.32</td>
<td>2.2</td>
<td>8.5</td>
</tr>
<tr>
<td>Consumption</td>
<td>2.66</td>
<td>0.20</td>
<td>3.5</td>
<td>22.2</td>
</tr>
<tr>
<td>Investment</td>
<td>2.33</td>
<td>0.35</td>
<td>5.5</td>
<td>19.0</td>
</tr>
</tbody>
</table>

Note: The table represents the percentage contribution of the four shocks to the high-frequency, which corresponds to fluctuations between 2 and 32 quarters, and medium-frequency, which corresponds to fluctuations between 32 and 80 quarters, variance of output, consumption, and investment in the model $M(\Gamma_1, \Theta_2)$, where $\Gamma_1$ is the set of parameters of the Taylor rule estimated using data in the first subsample (1947:1-1978:4), and $\Theta_2$ is the set of parameters of the exogenous processes estimated using data in the second subsample (1983:1-2007:4).
Figure 2.1: Spectrum of Output in the New Keynesian Model with Two Alternative Monetary Policies

![Figure 2.1](image1.png)

*Note*: The figure plots the normalized spectrum of output implied by the New Keynesian model when the inflation parameter of the Taylor rule is 1.5, solid line, and 6, dashed line.

Figure 2.2: Spectrum of Inflation in the New Keynesian Model with Two Alternative Monetary Policies

![Figure 2.2](image2.png)

*Note*: The figure plots the normalized spectrum of inflation implied by the New Keynesian model when the inflation parameter of the Taylor rule is 1.5, solid line, and 6, dashed line.
3

Exchange Rate Dynamics in an Estimated Small Open Economy DSGE Model

3.1 Introduction

Understanding exchange rates dynamics is one of central questions in international macroeconomics. Over the last decade there have been numerous studies trying to understand what are necessary aspects of a model that would reproduce real exchange rate movements present in the data (for example, Bouakez [2007], Chari, Kehoe, and McGrattan [2002], Corsetti, Dedola, and Leduc [2008], Kollmann [2001], Obstfeld and Rogoff [2000]). Since most of these papers use calibrated models, this paper contributes to this literature by estimating a full-fledged small open economy DSGE model that features various real and nominal rigidities. This framework allows me to evaluate the importance of different features of the model in explaining observed exchange rate dynamics. Furthermore, I also investigate which are the main shocks that contribute to the volatility of main macroeconomic variables, with the emphasis on the volatility of real exchange rate.

In particular, I estimate a small open economy DSGE model, which builds on the
model of CEE, by incorporating an open economy component into it. In particular, the model features exporting and importing firms, which face price stickiness a l’a Calvo [1983] and Yun [1996], implying a low exchange rate pass through.\(^1\) The model incorporates various nominal and real frictions: variable capacity utilization, habit persistence in consumption, adjustment cost to investment, wage and price stickiness, and wage and price indexation. This allows me to assess the importance of different frictions in different dimensions, especially in replicating exchange rate movements. To do so, I compare implied real exchange rate persistence of the model that excludes a particular rigidity or rigidities with the one implied by the benchmark model.

My results are as follows. The benchmark model performs rather well in replicating persistence of the real exchange rate in all three countries, while exchange rate volatility is explained relatively better in the case of Australia and Canada than in the case of UK. Furthermore, the most important model frictions in the replication of the real exchange rate persistence are the price stickiness parameters, in particular the domestic price stickiness parameter, importing price stickiness, and indexation parameter. Relevance of the importing price stickiness is increasing in the share of imports in total consumption basket. One possible explanation for this result might be as follows. As described in my model, the real exchange rate is defined as the nominal exchange rate corrected by the relative price of the domestic and world economy. Since my model is a small open economy model, world price cannot be altered by the economic decisions of the agents in a small domestic economy and can be considered constant. Therefore, all movements in the relative price come from the movements in the domestic CPI level. Higher degree of domestic price stickiness implies that on average domestic firms are allowed to change prices less often, suggesting higher persistence of inflation (measured by the change in domestic CPI). This higher inflation

\(^1\) Conventional wisdom suggests that the higher value of these parameters, the lower exchange rate pass through.
persistence implies higher persistence of the real exchange rate. The same intuition follows when I consider the price stickiness of the importing goods, which are the part of the aggregate price index in the domestic economy. Since I allow for the local currency pricing, i.e. domestic importers set prices of imported goods in their own currency, the frequency with which they change their prices will undoubtedly influence the real exchange rate persistence through its effect on the persistence of domestic inflation. In addition, wage stickiness and wage indexation parameters also play an important role for the replication of the exchange rate dynamics. Moreover, all these frictions are important for explaining volatility of the real exchange rate as well. Finally, I find that among the domestic shocks, the most important shocks for explaining the volatility of the exchange rate are the investment-specific technology shock and the monetary policy shock, while among the so-called world shocks the most important is the foreign interest rate shock.

Despite the burgeoning theoretical literature, not much work has been done on the estimation side. For example, Adolfson, Laseen, Linde, and Villani [2007] estimate a small open economy DSGE model using the Euro Area data and employing Bayesian techniques. However, their primary interest is to evaluate how well this model fits the European data. In addition, their model features much larger number of shocks than my model. My model features nine shocks, six of which coming from the domestic economy, and three from the foreign economy. Specifically, domestic shocks are: preference shock, labor supply shock, neutral technology shock, investment specific shock, monetary policy shock and asymmetric technology shock, whereas the foreign economy shocks are: shock to the interest rate, and shocks to foreign inflation and output.

I estimate the model using the data for the following three countries: Australia, Canada, and UK. I choose these countries as examples of small open developed economies, whereas the rest of the world is approximated by the US data. I use
the post Bretton-Woods data, which leads to a sample period 1972:I - 2006:IV. The
variables used as observables are: inflation, interest rate, output, consumption, ex-
ports, and real exchange rate for each of the three countries, together with inflation,
interest rate, and output of the US economy, which is chosen as the approximation
of the rest of the world. I estimate the model using Bayesian methods. I use the
Kalman filter to evaluate the likelihood of the model, under the assumption that
all structural shocks are normally distributed. Then, by combining prior distribu-
tions of the structural parameters and the likelihood function, I recover posterior
distributions of the parameters. I use a random walk Metropolis-Hastings algorithm
to sample from the proposal posterior distribution. For each estimated structural
parameter, I obtain the chain of the draws from the posterior distribution. Finally,
I take the mean of the chain to be the point estimate of the parameter. A subset
of the structural parameters is calibrated in a standard fashion.

This paper is related to several papers in the existing literature. For instance,
Chari, Kehoe, and McGrattan [2002] show that in the two-country model adding
price stickiness is not enough to match the real exchange rate volatility. In fact, only
after adding preferences separable in leisure their model can reproduce observed
exchange rate volatility. Furthermore, Kollmann [2001] shows that sticky nominal
wages and prices can help in matching exchange rate volatility. Also, Bergin and
Feenstra [2001] show that translog-linear preference forms can reproduce high degree
of volatility of the real exchange rate. Finally, Devereux and Engel [2002] show
that allowing for local currency pricing is a key element for matching the exchange
rate volatility. None of these papers is successful in replicating the exchange rate
persistence very well.

The rest of the paper is organized as follows. Section 3.2 provides relevant empiri-

\footnote{Different moments of the posterior distribution can be chosen as point estimates. The most
commonly used moments are mean and the mode of the posterior distribution. Results are not
significantly influenced if mode is used as a point estimate.}
cal evidence using the data from three mentioned small open economies. Theoretical model is described in Section 3.3. Section 3.4 describes the data, estimation procedure, calibration, prior distributions of the estimated parameters, and estimation results. In Section 3.5, the model exchange rate dynamics is confronted with the empirical exchange rate dynamics. Section 3.6 concludes.

3.2 Empirical Evidence

This section presents empirical evidence regarding the nominal and real exchange rate dynamics. Figures 3.1 to 3.3 show exchange rate series for three countries that serve as examples of developed small open economies, in order to demonstrate that high exchange rate persistence and volatility is a common developed small open economies phenomenon. Specifically, I consider the following three countries: Australia, Canada and UK. The figures display log-levels of nominal and real exchange rates, expressed in domestic currencies to the US dollar, for the post Bretton-Woods sample period 1972:I - 2006:IV. The dashed line represents nominal exchange rate, which is expressed as the number of home currency units needed to buy one foreign currency unit. This implies that decrease of the nominal exchange rate corresponds to the appreciation of the home-country currency, whereas increase of the nominal exchange rate corresponds to the depreciation of the home-country currency.

Real exchange rate, represented by the solid line, is constructed as the consumer price index (CPI) based exchange rates, i.e. as the product of the nominal exchange rate and relative price levels between two countries, where the measure of price level is the CPI. Specifically,

\[ s_t^r = \frac{s_t \text{CPI}_{t}^{\text{foreign}}}{\text{CPI}_{t}^{\text{domestic}}}, \]

where \( s_t^r \) represents the real exchange rate, \( s_t \) is the nominal exchange rate, \( \text{CPI}_{t}^{\text{foreign}} \) is the foreign price index, and \( \text{CPI}_{t}^{\text{domestic}} \) is the domestic price index.
Figures 3.4 to 3.6 display quarterly growth rates of nominal and real exchange rates. Exchange rates are highly volatile in all three countries. The volatility of the series is approximated by its standard deviation, while the persistence of the series is approximated by its first-order autocorrelation coefficient. Table 3.1 shows the values of these statistics for the three countries. The exchange rate is the most volatile and the least persistent in the UK economy. Exchange rates in Canadian and Australian economy show very similar dynamics, being twice less volatile than exchange rate in the UK economy, and more persistent than the same indicator in the UK economy.

3.3 The Model

In this section I describe the model. Specifically, I use a small open economy DSGE model that builds on the model proposed by CEE, by incorporating small open economy components into it. In addition to a standard closed DSGE model setting, exporting and importing sectors are added into the model. The model features various types of rigidities, such as price and wage stickiness and indexation parameters, variable capacity utilization, investment adjustment costs, and habit persistence in consumption.

3.3.1 Households

The economy is populated by a continuum of households indexed by $j \in [0, 1]$. Household’s preferences are defined over consumption, $c_{jt}$, and labor, $l_{jt}$. Each household maximizes lifetime utility that takes the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left\{ \log (c_{jt} - hc_{jt-1}) - \varphi_t \psi \frac{l_{jt}^{1+\gamma}}{1 + \gamma} \right\}, \quad (3.1)$$

where $\beta \in (0, 1)$ is the discount factor, $h$ is the habit persistence parameter, $\gamma$ is the inverse of Frisch labor supply elasticity, while $\xi_t$ and $\varphi_t$ represent preference shock
and labor supply shock, which follow autoregressive processes:

\[
\log \xi_t = \rho \log \xi_{t-1} + \sigma \varepsilon_{\xi,t} \\
\log \varphi_t = \rho \varphi_{t-1} + \sigma \varepsilon_{\varphi,t},
\]

where \( \varepsilon_{\xi,t} \sim \mathcal{N}(0,1) \) and \( \varepsilon_{\varphi,t} \sim \mathcal{N}(0,1) \). Households consume both domestically produced goods and imported goods.

Aggregate consumption of the household, \( c_{jt} \), is given by the constant elasticity of substitution (CES) index of domestically produced and imported goods:

\[
c_{jt} = \left\{ (1 - \alpha_c)\frac{1}{\eta_c} (c_{jt}^d)_{\eta_c}^{-1} + \alpha_c \frac{1}{\eta_c} (c_{jt}^m)_{\eta_c}^{-1} \right\}^{\frac{\eta_c}{\eta_c - 1}},
\]

where \( c_{jt}^d \) represents the consumption of domestically produced goods, \( c_{jt}^m \) is the consumption of imported goods, \( (1 - \alpha_c) \) is the home bias in consumption, and \( \eta_c \) is the elasticity of substitution between domestic and imported consumption goods.

Household chooses the best allocation of its resources between domestically produced and imported consumption goods, by maximizing total consumption subject to the following budget constraint:

\[
p_t c_{jt}^d + p_{it} c_{jt}^m = p_t c_{jt}, \tag{3.2}
\]

where \( p_t \) is the price of domestically produced goods, \( p_{it} \) is the price of imported goods in domestic currency, and \( p_t \) is the aggregate CPI. After some manipulations of the first-order conditions of this problem, demands for domestically produced consumption goods and imported consumption goods are:

\[
c_{jt}^d = (1 - \alpha_c) \left( \frac{p_t}{p_t^c} \right)^{-\eta_c} c_{jt}, \tag{3.3}
\]

\[
c_{jt}^m = \alpha_c \left( \frac{p_{it}}{p_t^c} \right)^{-\eta_c} c_{jt}. \tag{3.4}
\]
The expression for the aggregate CPI, \( p_c^t \), can be obtained by plugging (3.3) and (3.4) back into (3.2):

\[
p_c^t = \left( 1 - \alpha_c \right) p_t^{1-\eta_c} + \alpha_c (p_t^m)^{1-\eta_c} \right)^{\frac{1}{1-\eta_c}}.
\]

Households are assumed to own physical capital \( k_{jt} \), which accumulates according to the following law of motion:

\[
k_{jt+1} = (1 - \delta) k_{jt} + \mu_t \left( 1 - S \left[ \frac{i_{jt}}{i_{jt-1}} \right] \right) i_{jt},
\]

where \( i_{jt} \) denotes gross investment, \( \delta \) is the parameter denoting the depreciation rate of capital, and \( \mu_t \) is the investment-specific technological shock that follows an autoregressive process, given by

\[
\log \mu_t = \Upsilon \mu + \log \mu_{t-1} + \sigma \mu \varepsilon_{\mu,t},
\]

where \( \varepsilon_{\mu,t} \sim \mathcal{N}(0, 1) \). The function \( S[\cdot] \) introduces investment adjustment costs and satisfies following properties in the steady state: \( S[\Upsilon \mu] = S'[\Upsilon \mu] = 0 \), and \( S''[\Upsilon \mu] > 0 \), where \( \Upsilon \mu \) represents steady state growth rate. These assumptions imply no adjustment cost up to the first-order in the vicinity of a steady state. Household chooses optimal level of investment in each period, as well as the optimal allocation of investment resources. As in the case of total consumption index, I analogously define total investment index as the CES aggregate of domestic and imported investment goods given as:

\[
i_{jt} = \left\{ (1 - \alpha_i) \frac{1}{\eta_i} (i_{jt}^d)^{\frac{\eta_i-1}{\eta_i}} + \alpha_i \frac{1}{\eta_i} (i_{jt}^m)^{\frac{\eta_i-1}{\eta_i}} \right\}^{\frac{\eta_i}{\eta_i-1}}, \tag{3.5}
\]

where \( i_{jt}^d \) denotes domestic investment goods, \( i_{jt}^m \) denotes imported investment goods, \( \eta_i \) is the elasticity of substitution between domestic and imported investment goods,
and \((1 - \alpha_i)\) is the home bias in investment goods. Then, household’s demands for these two types of investment goods are the following:

\[
\begin{align*}
\bar{i}_{jt}^d &= (1 - \alpha_i) \left( \frac{p_t}{p_t^i} \right)^{-\eta_i} i_{jt}, \\
\bar{i}_{jt}^m &= \alpha_i \left( \frac{p_t^{m}}{p_t} \right)^{-\eta_i} i_{jt},
\end{align*}
\]

(3.6) (3.7)

where \(p_t^i\) is the aggregate investment price index obtained by plugging (3.6) and (3.7) into (3.5):

\[
p_t^i = \left\{ (1 - \alpha_i)p_t^{1-\eta_i} + \alpha_i(p_t^{m})^{1-\eta_i} \right\}^{\frac{1}{1-\eta_i}}.
\]

Households can trade on the whole set of possible Arrow-Debreu securities, which are indexed both by household \(j\) and by time \(t\) in order to capture both idiosyncratic and aggregate risk. In addition, households hold an amount \(b_{jt+1}\) of domestic government bonds that pay a nominal gross interest rate of \(R_t\) between periods \(t\) and \(t + 1\), and amount \(b_{jt+1}^*\) of foreign government bonds that pay a nominal gross interest rate \(R_t^*\). Furthermore, to ensure a well-defined steady state of the model, I assume that the foreign interest rate is increasing in the level of the country debt.\(^3\) To capture this fact I introduce a function \(\Phi(\cdot)\), which is assumed to be a decreasing function of foreign asset holdings, \(a_t^f\)\(^4\), where \(a_t^f = \frac{\kappa b_t}{p_t}\). This formulation implies that if the country is a net borrower, it will be charged a premium on the foreign interest rate, whereas if the country is a net lender it will receive remuneration on its savings. The nominal exchange rate is denoted by \(s_t\) and is given in terms of domestic currency needed to buy a unit of foreign currency, i.e. increase in \(s_t\) implies exchange rate depreciation, whereas decrease in \(s_t\) implies exchange rate appreciation. Considering

\(^3\) This is a standard approach in this literature. See Schmitt-Grohé and Uribe [2003], Benigno [2009], Adolfson et al. [2007].

\(^4\) I use the functional form proposed by Schmitt-Grohé and Uribe [2003], given by \(\Phi(a) = -\psi_2 \left(e^{a} - \bar{a}^f - 1\right)\), where \(\psi_2\) and \(\bar{a}^f\) are constant parameters.
all stated above, the budget constraint of the \( j \)-th household, expressed in the domestic currency terms, is the following:

\[
\begin{align*}
& \frac{p_t^c}{p_t} c_{jt} + \frac{p_t^i}{p_t} i_{jt} + \frac{b_{jt+1}}{p_t} + \frac{s_t b_{jt+1}^*}{p_t} + \int q_{jt+1,t} a_{jt+1} d\omega_{jt+1,t} \\
= & \quad w_{jt} l_{jt} + (r_t u_{jt} - \mu_t^{-1} a [u_{jt}]) k_{jt} \\
+ & R_{t-1} \frac{b_{jt}}{p_t} + R_t^* \Phi \left( \frac{s_t-1 b_t^*}{p_t} \right) \frac{s_t b_{jt}^*}{p_t} + a_{jt} + T_t + \Gamma_t
\end{align*}
\]

where \( w_{jt} \) is the real wage, \( r_t \) is the real rental price of capital, \( u_{jt} \) is the capital utilization, \( \mu_t^{-1} a [u_{jt}] \) is the physical cost of capital utilization in resource terms, \( T_t \) is a lump-sum transfer, and \( \Gamma_t \) are the firms’ profits. In addition, I assume that \( a [1] = 0 \), \( a' \) and \( a'' > 0 \). The Lagrangian associated with this problem is the following:

\[
\begin{align*}
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix}
\xi_t \left\{ \log (c_{jt} - h c_{jt-1}) - \varphi_t q_t^{i_{jt}} \right\} \\
\frac{p_t^c}{p_t} c_{jt} + \frac{p_t^i}{p_t} i_{jt} + \frac{b_{jt+1}}{p_t} + \frac{s_t b_{jt+1}^*}{p_t} + \int q_{jt+1,t} a_{jt+1} d\omega_{jt+1,t} - w_{jt} l_{jt} \\
- (r_t u_{jt} - \mu_t^{-1} a [u_{jt}]) k_{jt} - R_{t-1} \frac{b_{jt}}{p_t} - R_t^* \Phi \left( \frac{s_t-1 b_t^*}{p_t} \right) \frac{s_t b_{jt}^*}{p_t} \\
- a_{jt} - T_t - F_t \\
-Q_{jt} \left\{ k_{jt+1} - (1 - \delta) k_{jt} - \mu_t \left( 1 - S \left[ \frac{i_{jt}}{b_{jt-1}} \right] \right) i_{jt} \right\}
\end{bmatrix}
\end{align*}
\]

Household chooses \( c_{jt}, b_{jt}, b_{jt}^*, u_{jt}, k_{jt+1}, i_{jt}, w_{jt}, l_{jt} \) and \( a_{jt+1} \); \( \lambda_{jt} \) represents the Lagrangian multiplier associated with the budget constraint and \( Q_{jt} \) represents the Lagrangian multiplier associated with installed capital. If I define the marginal Tobin’s Q as the ratio of these two multipliers, i.e. \( q_{jt} = \frac{Q_{jt}}{\lambda_{jt}} \), then first-order conditions
with respect to $c_{jt}$, $b_{jt}$, $b_{jt}^*$, $u_{jt}$, $k_{jt+1}$, and $i_{jt}$ are respectively:

$$
\xi_t \left( c_{jt} - hc_{jt-1} \right)^{-1} - h_t \beta E_t \xi_{t+1} \left( c_{jt+1} - hc_{jt} \right)^{-1} = \lambda_{jt} \frac{p_t^e}{p_t}
$$

$$
\lambda_{jt} = \beta E_t \left\{ \lambda_{jt+1} \frac{R_t}{\Pi_{t+1}} \right\}
$$

$$
\lambda_{jt} s_t = \beta E_t \left\{ \lambda_{jt+1} \frac{R_t^* \Phi(a_t^s)_{s+1}}{\Pi_{t+1}} \right\}
$$

$$
r_t = \mu_t^{-1} a'[u_{jt}]
$$

$$
\lambda_{jt} q_{jt} = \beta E_t \lambda_{jt+1} \left\{ (1 - \delta) q_{jt+1} + \left( r_{jt+1} u_{jt+1} - \mu_{t+1}^{-1} a [u_{jt+1}] \right) \right\}
$$

$$
- \lambda_{jt} \frac{p^i_t}{p_t} + \lambda_{jt} q_{jt} \mu_t \left( 1 - S \left[ \frac{i_{jt}}{i_{jt-1}} \right] - S' \left[ \frac{i_{jt}}{i_{jt-1}} \right] \frac{i_{jt}}{i_{jt-1}} \right)
$$

$$
+ \beta E_t \lambda_{jt+1} q_{jt+1} \mu_{t+1} S' \left[ \frac{i_{jt+1}}{i_{jt}} \right] \left( \frac{i_{jt+1}}{i_{jt}} \right)^2 = 0.
$$

When taking the first-order conditions with respect to wages and labor, I follow the set-up of [Erceg, Henderson, and Levin (2000)] and assume that each household supplies differentiated labor services to the production sector. In order to avoid this heterogeneity spilling over into consumption heterogeneity, they assume that utility is separable in consumption and labor, and that, because of the existence of complete markets, households can fully ensure against the employment risks. Furthermore, in this environment, the equilibrium price of Arrow-Debreu securities ensures that the consumption does not depend on idiosyncratic shocks. In addition, I assume that a representative labor aggregator combines households’ labor in the same proportion as firms would choose, which ensures that her demand for $j$–th household’s labor is the same as the sum of the firms’ demands for this type of labor. Specifically, the labor aggregator uses the following production technology:

$$
l^d_t = \left( \int_0^1 \int_0^1 l^o_{i_{jt+1}} dj \right)^{\frac{\sigma}{\gamma-1}}, \quad (3.8)
$$
where $\eta \in [0, \infty)$ is the elasticity of substitution among different types of labor, and $l^d_t$ is the aggregate labor demand. She maximizes profits subject to (3.8), taking as given all differentiated labor wages $w_{jt}$ and the aggregate wage index $w_t$. Her demand for the labor of household $j$ is given by,

$$l_{jt} = \left( \frac{w_{jt}}{w_t} \right)^{-\eta} l^d_t \quad \forall j.$$  \hfill (3.9)

Households set their wages following Calvo setting, i.e. in each period, a fraction $\theta_w \in [0, 1)$ of randomly picked households is not allowed to optimally set their wages. Instead, they partially index their wages to the past inflation, which is controlled by the indexation parameter $\chi_w \in [0, 1]$. The remaining fraction of households reset their wages $w_{jt}$ to maximize (3.1), which leads to the following first order condition:

$$\frac{\eta - 1}{\eta} w^{opt}_t \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \lambda_{t+k} \left( \frac{\Pi_{s=1}^{\chi_w t} \Pi_{t+s}^{t+1}}{\Pi_{s=1}^{\chi_w t} \Pi_{t+s}^{t+1}} \right)^{1-\eta} \left( \frac{w^{opt}_t}{w_{t+k}} \right)^{-\eta} \psi_{t+k} =$$

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \left( \xi_{t+k} \phi_{t+k} \psi_{t+k} \right) \left( \frac{\Pi_{s=1}^{\chi_w t} \Pi_{t+s}^{t+1}}{\Pi_{s=1}^{\chi_w t} \Pi_{t+s}^{t+1}} \right)^{1-\gamma} \left( \frac{w^{opt}_t}{w_{t+k}} \right)^{-\eta} \left( \lambda_{t+k} \right)^{1-\gamma},$$

which can be expressed recursively as $f^1_t = f^2_t$, where $f^1_t$ and $f^2_t$ are defined as

$$f^1_t = \frac{\eta - 1}{\eta} (w^{opt}_t)^{1-\eta} \lambda_t w_t^{\eta l^d_t} + \beta \theta_w \mathbb{E}_t \left( \frac{\Pi_{s=1}^{\chi_w t} \Pi_{t+s}^{t+1}}{\Pi_{s=1}^{\chi_w t} \Pi_{t+s}^{t+1}} \right)^{1-\eta} \left( \frac{w^{opt}_{t+1}}{w^{opt}_t} \right)^{1-\eta} f^1_{t+1},$$

$$f^2_t = \psi d_t \phi_t \left( \frac{w_t}{w^{opt}_t} \right)^{\eta} \left( l^d_t \right)^{1+\gamma} + \beta \theta_w \mathbb{E}_t \left( \frac{\Pi_{s=1}^{\chi_w t} \Pi_{t+s}^{t+1}}{\Pi_{s=1}^{\chi_w t} \Pi_{t+s}^{t+1}} \right)^{-\eta} \left( \frac{w^{opt}_{t+1}}{w^{opt}_t} \right)^{\eta} f^2_{t+1}.$$  \hfill (3.10)

### 3.3.2 Firms and Price Setting

There are five types of the firms in the model: a final domestic good producer, intermediate goods producers, importing goods producers, and exporting goods producers. In this section, I describe the problems each of these producers faces.
**Final Good Producer**

The final domestic good producer aggregates intermediate goods into the homogeneous final good, using the following production function of the Dixit and Stiglitz (1997):

\[
y^d_t = \left( \int_0^1 y^d_{it} \frac{1}{\epsilon} \, di \right)^{\frac{1}{\epsilon - 1}},
\]

(3.10)

where \( \epsilon \) is the elasticity of substitution between intermediate goods. The final good producer chooses the bundle of goods that minimizes the cost of producing \( y^d_t \), taking all intermediate goods prices \( p_{it} \), final domestic good price \( p_t \), and the quantity of intermediate goods \( y_{it} \) as given. The unit price of the output unit is equal to its unit cost \( p_t \):

\[
p_t = \left( \int_0^1 p_{it}^{1-\epsilon} \, di \right)^{\frac{1}{1-\epsilon}}.
\]

The input demand function \( y_{it} \) for each intermediate good \( i \) is then given by:

\[
y_{it} = \left( \frac{p_{it}}{p_t} \right)^{-\epsilon} y^d_t \quad \forall i,
\]

where \( y^d_t \) is the aggregate demand.

**Intermediate Good Producers**

Each differentiated good is produced by a single intermediate firm \( i \in [0,1] \) that rents capital services \( k_{it} \), and labor services \( l^d_{it} \), using the production function:

\[
y_{it} = A_t k_{it}^\alpha (l^d_{it})^{1-\alpha} - \phi z_t,
\]

where \( \phi \) is the parameter that corresponds to the fixed cost of production, and \( A_t \) is the neutral technology process, given by:

\[
\log A_t = Y_A + \log A_{t-1} + \sigma_A \varepsilon_{A,t},
\]
where $\varepsilon_{A,t} \sim \mathcal{N}(0,1)$. Following Altig et al. [2005], I assume that fixed costs are subject to the permanent shock $z_t$, which ensures that along the balanced-growth path fixed costs do not vanish, and that profits are approximately zero in the steady state.

Each intermediate goods firm chooses amount of $k_{it}$ and $l_{it}^d$ to rent, taking the input prices $r_t$ and $w_t$ as given. The standard static first-order conditions for cost minimization imply that real marginal cost is the same for all firms, i.e. does not have a subscript $i$ associated with it, since all intermediate goods firms face the same aggregate technology shocks as well as the same input prices. Real marginal cost is given by

$$mc_t = \left( \frac{1}{1 - \alpha} \right)^{1-\alpha} \frac{w_t^{1-\alpha} r_t^\alpha}{A_t},$$

where

$$w_t = \varrho (1 - \alpha) A_t k_{it}^\alpha (l_{it}^d)^{-\alpha},$$

$$r_t = \varrho \alpha A_t k_{it}^{\alpha-1} (l_{it}^d)^{1-\alpha}.$$

I assume that the intermediate goods firms set their process a l’a Calvo [1983] and Yun [1996]. That is, in each period, a fraction $\theta_p \in [0,1)$ of firms is not allowed to change their prices, and can only index them by the past inflation, which is controlled by the indexation parameter $\chi_p \in [0,1]$. The remaining $1 - \theta_p$ firms that are allowed to reset their prices in period $t$, solve the following maximization problem:

$$\max_{p_{it}} \mathbb{E}_t \sum_{k=0}^{\infty} \left( \beta \theta_p \right)^k \frac{\lambda_{t+k}}{\lambda_t} \left\{ \left( \sum_{s=1}^{k} \Pi_{t+s-1}^{\chi_p} \frac{p_{it}}{p_{t+k}} - mc_{t+k} \right) y_{it+k} \right\}$$

s.t. $y_{it+k} = \left( \sum_{s=1}^{k} \Pi_{t+s-1}^{\chi_p} \frac{p_{it}}{p_{t+k}} \right)^{-\varepsilon} y_{t+k}^d.$
If I define recursively:

\[ g_1^t = \lambda_t m c_t y^d_t + \beta \theta_p \mathbb{E}_t \left( \frac{\Pi_t^{\chi_p}}{\Pi_{t+1}} \right)^{-\varepsilon} g_1^{t+1} \]

\[ g_2^t = \lambda_t \Pi_{t,\text{opt}} y^d_t + \beta \theta_p \mathbb{E}_t \left( \frac{\Pi_t^{\chi_p}}{\Pi_{t+1}} \right)^{1-\varepsilon} \left( \frac{\Pi_{t,\text{opt}}}{\Pi_{t+1,\text{opt}}} \right) g_2^{t+1}, \]

where \( \Pi_{t,\text{opt}} = \frac{p_{t,\text{opt}}}{p_t} \), the first-order condition to this problem can be written as

\[ \varepsilon g_1^t = (\varepsilon - 1) g_2^t. \]

Finally, considering the price setting, the aggregate price index is:

\[ p_t^{1-\varepsilon} = \theta_p \left( \frac{\Pi_{t-1}^{\chi_p}}{\Pi_{t-1}} \right)^{1-\varepsilon} p_{t-1}^{1-\varepsilon} + (1 - \theta_p) p_{t,\text{opt}}^{1-\varepsilon}. \]

**Importing Firms**

There is a continuum of importing firms indexed by \( i \) on the unit interval. The problem that these firms solve can be described as follows. First, importing firm \( i \) buys a homogenous good in the world market at the price \( p_t^* \) and turns it into a differentiated imported good through a differentiating technology and brand naming. Then, imported goods "packer" mixes these differentiated imported goods \( y_{it} \), using the production technology:

\[ y_{it}^{m} = \left[ \int_0^1 (y_{it}^{m})^{\varepsilon_m-1} \frac{d\varepsilon_m}{\varepsilon_m} \right]^{\frac{\varepsilon_m}{\varepsilon_m-1}}, \quad (3.11) \]

to produce the final imported good \( y_{it}^{m} \). Finally, he sells the final imported good to the households, who decide to consume it or to invest it. The parameter \( \varepsilon_m \) is the elasticity of substitution across differentiated importing goods.

The imported goods packer chooses the bundle of goods that minimizes the cost of producing \( y_{it}^{m} \), taking all imported goods prices \( p_{it}^{m} \), final imported goods basket price \( p_t^{m} \), and the quantity of imported goods \( y_{it}^{m} \) as given. The unit price of the
output unit is equal to its unit cost \( p_t^m \):

\[
p_t^m = \left( \int_0^1 (p_{it}^m)^{1-\varepsilon_m} \, di \right)^{\frac{1}{1-\varepsilon_m}},
\]

whereas the demand function for each differentiated imported good \( i \) is given by:

\[
y_{it}^m = \left( \frac{p_{it}^m}{p_t^m} \right)^{-\varepsilon_m} y_t^m \quad \forall i.
\]

Finally, total amount of imported goods is obtained by integrating over all differentiated imported goods:

\[
Y_t^m = \int_0^1 y_{it}^m \, di.
\]

In order to allow for the incomplete exchange rate pass-through to import prices, I assume that also these firms are subject to price stickiness a\` la Calvo (1983) and Yun (1996). That is, in each period, a fraction \( \theta_m \in [0, 1) \) of firms is not allowed to change their prices, and can only index them by the past inflation, which is controlled by the indexation parameter \( \chi_m \in [0, 1] \). The remaining \( 1 - \theta_m \) that are allowed to reset their prices in period \( t \), solve the following maximization problem:

\[
\max_{p_{it}^m} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_m)^k \frac{\lambda_{t+k}}{\lambda_t} \left\{ \left( \sum_{s=1}^{k} (\Pi_{t+s-1}^m)^{\chi_m} \frac{p_{it}^m}{p_{t+k}^m} - \frac{s_{t+k} p_{t+k}^m}{s_{t+k} p_{t+k}^m} \right) y_{it+k}^m \right\}
\]

\[
y_{it+k}^m = \left( \sum_{s=1}^{k} (\Pi_{t+s-1}^m)^{\chi_m} \frac{p_{it}^m}{p_{t+k}^m} \right)^{-\varepsilon_m} y_{t+k}^m,
\]

where the marginal value of a dollar to the household is treated as exogenous by the firm, and where \( \frac{s_{t+k} p_{t+k}^m}{p_{t+k}^m} \) represents the real marginal cost that is equal to the nominal marginal cost (the price of homogenous foreign good that they buy on the world market) divided by the imported goods price index.
Following the same strategy as in the problem of intermediate good firms, first order condition of importing firms can be written as \( \varepsilon_m (g_t^m)^1 = (\varepsilon_m - 1)(g_t^m)^2 \), with

\[
(g_t^m)^1 = \lambda_t s_i p_i^m y_t^m + \beta \theta_m \mathbb{E}_t \left( \frac{(\Pi_{-1}^m)\chi_m}{\Pi_{-1}^m} \right)^{-\varepsilon_m} (g_{t+1}^m)^1,
\]

\[
(g_t^m)^2 = \lambda_t \Pi_{t, opt}^m y_t^m + \beta \theta_m \mathbb{E}_t \left( \frac{(\Pi_{-1}^m)\chi_m}{\Pi_{-1}^m} \right)^{1-\varepsilon_m} \left( \frac{\Pi_{t, opt}^m}{\Pi_{t+1, opt}^m} \right) (g_{t+1}^m)^2,
\]

where \( \Pi_{t, opt}^m = \frac{p_{t, opt}^m}{p_t^m} \). Given the price setup these firms face, the price index of import goods is:

\[
(p_t^m)^{1-\varepsilon_m} = \theta_m \left( \frac{\Pi_{t-1}^m}{\Pi_{t-1}^m} \chi_m \right)^{1-\varepsilon_m} (p_{t-1}^m)^{1-\varepsilon_m} + (1 - \theta_m) (p_{t, opt}^m)^{1-\varepsilon_m}.
\]

**Exporting Firms**

There is a continuum of exporting firms indexed by \( i \) on the unit interval. Each firm \( i \) buys a homogenous final domestic good in the domestic market and differentiates it by differentiating technology or brand naming. Then, they sell these differentiated goods to the rest of the world. The demand for each variety of these goods comes from the households in the foreign economy, who decide to consume it or invest to it, and can be written as follows:

\[
e_{i,t} = \left( \frac{p_{it}^e}{p_t^e} \right)^{-\varepsilon_e} e_t,
\]

where \( \varepsilon_e \) is the elasticity of substitution between differentiated exporting goods. Total amount of exported goods, \( E_t \), is thus obtained by integrating over all differentiated exporting goods:

\[
E_t = \int_0^1 e_{it} di,
\]

with the exported goods price index:

\[
p_t^e = \left( \int_0^1 (p_{it}^e)^{1-\varepsilon_e} di \right)^{\frac{1}{1-\varepsilon_e}}.
\]
Assuming that the domestic economy is small relative to the foreign economy, and thus plays a negligible part in the aggregate foreign consumption, the demand for the final export good in the foreign economy is:

\[ e_t = \left( \frac{p_t^e}{p_t^*} \right)^{-\eta_f} y_t^*, \]

where \( \eta_f \) is the elasticity of substitution between domestic and foreign goods in the foreign economy, and \( y_t^* \) is the output of the rest of the world.

I assume that exporters exhibit local-currency pricing, i.e. they take into account the conditions of the foreign market when setting prices. In order to allow for incomplete exchange rate pass-through in the export market, I assume that export prices are sticky à la Calvo [1983] and Yun [1996]. In particular, each period, fraction \( 1 - \theta_e \) of the firms is allowed to change the price, while the outstanding \( \theta_e \) firms can only index their prices by the past inflation \( \left( \Pi_t^* = \frac{p_t^*}{p_{t-1}^*} \right) \). Indexation parameter is denoted by \( \chi_e \in [0, 1] \). The nominal marginal cost of exporting firms is the price of domestic final good expressed in the foreign currency \( \left( \frac{p_t^c}{p_t^*} \right) \), which divided by the export price index gives real marginal cost \( \left( \frac{p_t^c}{\Pi_t^*} \right) \). Hence, exporting firm \( i \) that is allowed to reset its price solves:

\[
\max_{p_{i,t}} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_e)^k \frac{\lambda_{t+k}}{\lambda_t} \left\{ \left( k \sum_{s=1}^{k} \left( \Pi_{t+s-1}^* \right)^{\chi_e} \frac{p_{it}^e}{p_{t+k}^e} - \frac{p_{t+k}^e}{s_{t+k}p_{t+k}^e} \right) e_{it+k} \right\}
\]

\[
\text{s.t. } e_{i,t+k} = \left( k \sum_{s=1}^{k} \left( \Pi_{t+s-1}^* \right)^{\chi_e} \frac{p_{it}^e}{p_{t+k}^e} \right)^{-\varepsilon_e} e_{t+k}.
\]

Similar to the importing goods case, the recursive representation of the problem is

\[ \varepsilon_e (g_t^e)^1 = (1 - \varepsilon_e) (g_t^e)^2, \]

where

\[ (g_t^e)^1 = \lambda tmc^e e_t + \beta \theta_e \mathbb{E}_t \left( \frac{(\Pi_t^*)^{\chi_e}}{\Pi_{t+1}^*} \right)^{-\varepsilon_e} (g_{t+1}^e)^1 \]
\[(g_t^e)^2 = \lambda_t \Pi_{opt,t}^e e_t + \beta \theta_t E_t \left( \frac{(\Pi_t^e)_{\chi e}}{\Pi_{t+1}^e} \right)^{1-\varepsilon_e} \left( \frac{\Pi_{opt,t}^e}{\Pi_{opt,t+1}^e} \right) (g_{t+1})^2.\]

Finally, given Calvo pricing,

\[1 = \theta_e \left( \frac{(\Pi_{t-1}^e)_{\chi e}}{\Pi_t^e} \right)^{1-\varepsilon_e} + (1 - \theta_e) (\Pi_{opt,t}^e)^{1-\varepsilon_e}.\]

### 3.3.3 Government

The monetary authority uses the following interest rate rule:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left( \frac{\Pi_t^e}{\Pi^c} \right)^{\gamma_\Pi} \left( \frac{s_t}{s} \right)^{\gamma_s} \left( \frac{\gamma_y}{\gamma_{yd}} \right)^{1-\gamma_R} \exp(m_t),
\]

where \(\Pi^c\) is the target level of inflation, \(R\) is the steady state nominal gross return to capital, \(\gamma_{yd}\) is the steady state gross growth rate of \(y_{td}\), \(m_t\) is a random shock to monetary policy given as \(m_t = \sigma_m \varepsilon_{mt}\), where \(\varepsilon_{mt} \sim N(0,1)\). Finally, \(s_t\) denotes the nominal interest rate.\(^5\)

The role of the Government is to set up lump sum transfers in each period so that the Government budget constraint given by,

\[
T_t = \int_0^1 \frac{m_{jd} dj}{p_t} - \int_0^1 \frac{m_{jd-1} dj}{p_t} + \int_0^1 \frac{b_{jd+1} dj}{p_t} - R_{t-1} \int_0^1 \frac{b_{jd} dj}{p_t}
\]

holds in each period.

\(^5\) Lubik and Schorfheide [2005, 2006] show that in the case of Canada and UK Central Banks systematically take into account level of the nominal exchange rates in setting the interest rate, while the Central Bank of Australia does not. This implies that Central Banks of Canada and UK recognize nominal exchange rate as the part of the Taylor rule, whereas this variable is not in the Taylor rule that Australian Central Bank follows. However, I will allow for the exchange rate to be part of the Taylor rule in all three cases.
3.3.4 Foreign Assets

As described above, I define net foreign assets as \( a^f_t = \frac{s_{t-1}b^e_t}{p_t} \). The evolution of net foreign assets is:

\[
\frac{s_{t-1}b^e_t}{p_t} - \frac{R^*_t}{p_t} s^*_t = \frac{s_{t-1}b^e_t}{p_t} s^*_t E_t - \frac{s_{t-1}b^e_t}{p_t} Y^m_t,
\]

which could be, after substituting the previously mentioned definition of the foreign assets, written as follows:

\[
a^f_t = \frac{R^*_t}{p_t} \left( a^f_{t-1} \right) s^*_t = \frac{s_{t-1}b^e_t}{p_t} s^*_t E_t - \frac{s_{t-1}b^e_t}{p_t} Y^m_t.
\]

3.3.5 Aggregation

The aggregate demand for the final domestic good is

\[
y^d_t = c^d_t + i^d_t + e_t + \mu^{-1}a[u_t]k_t.
\]

After plugging in the expressions for \( c^d_t, i^d_t, e_t \), the demand of intermediate good \( i_t \), and the final good producer production function, and integrating over all firms, the aggregate demand for the final domestic good becomes:

\[
\left( 1 - \alpha_i \right) \left( \frac{p_t}{p_s} \right)^{-\eta_i} c_t + \left( 1 - \alpha_c \right) \left( \frac{p_t}{p_s} \right)^{-\eta_c} i_t + e_t + \mu^{-1} a[u_t]k_t = \frac{A_t \left( u_t k_{t-1} \right)^{\alpha} \left( \frac{v^p_t}{v^p_{t-1}} \right)^{1 - \alpha} - \phi z_t}{v^p_t},
\]

where \( v^p_t = \int_0^1 \left( \frac{p_{a}}{p_{m}} \right)^{-\epsilon} \) is the price dispersion term that is, considering the Calvo price setting, given by

\[
v^p_t = \theta_p \left( \frac{\Pi^\chi_t - 1}{\Pi^\chi_t} \right)^{-\epsilon} v^p_{t-1} + (1 - \theta_p) \Pi^{-\epsilon}_{t, \text{opt}}.
\]

The labor market clearing condition is obtained by integrating (3.9) over all households \( j \):

\[
l^d_t = \frac{1}{v^w_t} l_t,
\]

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where \( v^w_t = \int_0^1 \left( \frac{w_{t|t-1}}{w_t} \right)^{-\eta} dj \) is the wage dispersion term, that, because of the presence of sticky wages, is given by,

\[
v^w_t = \theta_w \left( \frac{w_{t-1}}{w_t} \prod_{t-1}^{\infty} \right)^{-\eta} v^w_{t-1} + (1 - \theta_w) \left( \prod_{t}^{w*} \right)^{-\eta}.
\]

Using the definition of total imports, the market clearing condition at the imported goods market can be written as:

\[
Y^m_t = \int_0^1 y^m_{i,t} di = y^m_{t} v^m_{t} = (\xi^m_{t} + \eta^m_{t}) v^m_{t},
\]

where \( v^m = \int_0^1 \left( \frac{\Pi^m_{t-1}}{\Pi^m_t} \right)^{-\varepsilon^m} di \), given Calvo price setting the evolution of this term is

\[
v^m_t = \theta_m \left( \frac{\Pi^m_{t-1}}{\Pi^m_t} \right)^{-\varepsilon^m} v^m_{t-1} + (1 - \theta_m) \left( \Pi^m_{t,opt} \right)^{-\varepsilon^m}.
\]

Analogously, using the definition of the total exports, market clearing of the exports market can be written as:

\[
E_t = \int_0^1 e^i_{i,t} di = e_t \int_0^1 \left( \frac{p^e_{i,t}}{p^e_t} \right)^{-\varepsilon_e} di = v^e_t \left( \frac{p^e_t}{p^e_t} \right)^{-\eta} y^*_t,
\]

where \( v^e_t = \int_0^1 \left( \frac{p^e_{t}}{p^e_t} \right)^{-\varepsilon_e} di \), with the evolution:

\[
v^e_t = \theta_e \left( \frac{\Pi^e_{t-1}}{\Pi^e_t} \right)^{-\varepsilon_e} v^e_{t-1} + (1 - \theta_m) \left( \Pi^e_{t,opt} \right)^{-\varepsilon_e}.
\]

### 3.3.6 Relative Prices and Marginal Costs

Since I introduce foreign sector to the closed economy setting of the model, it is convenient to define the following relative price expressions:

\[
\kappa^{c,d}_t = \frac{p^c_t}{p_t}, \quad \kappa^{c,m}_t = \frac{p^c_t}{p^m_t}, \quad \kappa^{m,d}_t = \frac{p^m_t}{p_t},
\]

\[
\kappa^{i,d}_t = \frac{p^i_t}{p_t}, \quad \kappa^{i,m}_t = \frac{p^i_t}{p^m_t}, \quad \kappa^{e,*}_t = \frac{p^e_t}{p_t}.
\]
Furthermore, since nominal and real marginal costs of the exporting and importing firms are given in terms of relative prices, it is convenient to define them as follows:

$$m_{c}^{m,m} = \frac{s_{t}p_{t}^{e}}{p_{t}^{m}} = \frac{s_{t}p_{t}^{e} p_{t}^{e} p_{t}}{p_{t}^{m,e} p_{t}^{e} p_{t}} = \frac{1}{m_{c}^{e}} \frac{1}{\kappa_{e}^{m,d}} \frac{1}{\kappa_{e}^{e}}$$

and

$$m_{c}^{e} = \frac{p_{t} p_{t-1} p_{t}^{e} s_{t-1}}{s_{t} p_{t}^{e} p_{t-1} p_{t}^{e} s_{t-1}} = m_{c}^{e} \frac{\Pi_{t} s_{t-1}}{\Pi_{t} s_{t}}.$$

### 3.3.7 The Foreign Economy

The foreign economy is considered as exogenously given. That is, I consider foreign inflation, foreign interest rate, and foreign output to be exogenous. Following Adolfson et al. [2007] and Justiniano and Preston, I model the foreign economy as the vector autoregressive (VAR) model. Denote a vector of foreign variables as $F^{e} = [\Pi_{t}^{e}, R_{t}^{e}, y_{t}^{e}]$. Then, the data generating process is assumed to take the following form:

$$F_{t}^{e} = A * F_{t-1}^{e} + \varepsilon_{t}^{e}.$$

Parameters of the matrix $A$ are then introduced into the model and estimated.

### 3.4 Estimation

A subset of the deep structural parameters of the model is estimated using Bayesian techniques, whereas the other subset of the parameters are calibrated in a standard fashion.

#### 3.4.1 Estimation Procedure

The structural parameters of the model are denoted by $\theta \in \Theta$, and are estimated using Bayesian techniques. After writing the competitive equilibrium conditions of
the model in the log-linearized form, the state space representation of the model can be written as:

\[ S_t = AS_{t-1} + B\varepsilon_t \]  
\[ \text{obs}_t = CS_{t-1} + D\varepsilon_t, \] 

where (3.12) represents a state equation, and (3.13) represents a measurement equation that connects model variables to the vector of observables, \( \text{obs}_t \). Since it is not possible to write the likelihood function of this model in the closed form, I use the Kalman filter to evaluate the likelihood of the model, under the assumption that all structural shocks are normally distributed. Let us denote the data as \( Y^T = \{y_t\}_{t=1}^T \), and the likelihood of the model given the set of structural parameters as \( (Y^T|\theta) \). The likelihood function is then combined with the prior density \( \pi(\theta) \) to form the posterior density \( \pi(\theta|Y) \) as

\[ \pi(\theta|Y) \propto (Y^T|\theta) \pi(\theta). \]  

I use a random walk Metropolis-Hastings algorithm to sample from the proposal posterior distribution, which is a multivariate normal \( N(0,c) \), where \( c \) is chosen to guarantee the acceptance rate between 35 and 45 percent. I generate 1 million draws and use first 30 percent of the draws as so-called burn-up period. Once I obtain Markov chains, I use their means as point estimates of the parameters.

To perform the variance decomposition, I start from the state space representation and obtain \( MA(\infty) \) form of the model, to evaluate the relevance of the shocks in different time horizons. In particular, I can rewrite (3.12) as

\[ (I - AL)S_t = B\varepsilon_t \]  
\[ S_t = (I - AL)^{-1} B\varepsilon_t. \]

Plugging (3.15) into (3.13) to obtain the form that connects observables to the shocks,
gives:

$$\text{obs}_t = C (I - AL)^{-1} B \varepsilon_{t-1} + D \varepsilon_t$$

$$= \sum_{i=0}^{\infty} C (AL)^i B \varepsilon_{t-i} + D \varepsilon_t.$$ 

Using this $MA(\infty)$ form I can perform the variance decomposition and evaluate the contribution of each shock to the variance of the macroeconomic variables.

### 3.4.2 Data

I estimate the model using quarterly data for Australia, Canada, and UK, commonly used as examples of developed small open economies in the literature. The rest of the world is approximated by the US economy. I use post Bretton-Woods data, in that during this period exchange rates were held roughly constant. This leads to the sample period 1972:I-2006:IV. The vector of the observables is the following:

$$Y^T = (\log \Pi_t, \log R_t, \Delta \log y_t, \Delta \log c_t, \Delta \log E_t, \log s^*_t, \log \Pi^*_t, \log R^*_t, \Delta \log y^*_t)^\top,$$

where the first six series are associated with the domestic economy, and the remaining three with the US economy. Specifically, $\Pi_t$ denotes percentage change in the CPI, $R_t$ is the real interest rate measured by T-bills, $y_t$ is real per capita gross domestic product (GDP), $c_t$ is real per capita aggregate consumption, $E_t$ is real per capita total exports denominated in domestic currency, and $s^*_t$ is the real exchange rate.\(^6\)

Exchange rate series are obtained from the International Financial Statistics.\(^7\) Data on consumption and all other components of the Canadian GDP are recovered from the Canada’s National Statistical Agency.\(^8\) The rest of the data are downloaded from

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\(^6\) Real exchange rate is the CPI measured exchange rate. That is $\log s^*_t = \log s_t + \log(CPI^{US}) - \log(CPI^{DomesticCountry}).$

\(^7\) Available at http://ifs.apdi.net/imf.

\(^8\) Available at http://www.statcan.gc.ca.
the Data Stream International.\footnote{In case of Canada, to be consistent with the model, I construct nominal consumption excluding the consumption for durable goods. For the same reason I also exclude government spending from the output series. Furthermore, CPI is constructed using the consumption deflators for nondurable goods and services. However, in case of UK and Australia, since detailed data is not available, I use consumption of all goods, gross output that includes also government spending, and consumer price index that includes prices of all goods.}

Variables with superscript $*$ indicate rest of the world variables, approximated by the US data. In particular, $\Pi_\ast_t$ is a percentage change in the US CPI, $R_\ast_t$ is the US Treasury Bill Rate, and $y_\ast_t$ is real per capita US GDP. Output and CPI data are obtained from the National Income and Product Accounts, whereas Treasury Bill Rate data are from the Federal Reserve Board.

### 3.4.3 Calibration and Priors

I calibrate a subset of the structural parameters of the model. This procedure is preferred to estimating all parameters for several reasons. For example, the likelihood cannot provide additional information for some parameters; some parameters are better identified using micro data. Therefore, I choose to calibrate these parameters in a standard fashion. In particular, the discount factor $\beta$, which is difficult to identify, is set to 0.99 in order to match the annual interest rate of 4 percent. Following Altig et al. [2005] I calibrate the depreciation rate to be 0.025, and the share of capital to be 0.36. I choose a value of 7.5 for the preference parameter $\psi$, which implies that in the steady state households work one third of their time. Following CEE and Adolfson et al. [2007], the labor supply elasticity is set to 1, so that markup in wage setting is 1.05. I set the elasticity of labor supply parameter to 0.21. Finally, I choose the same calibration as well as prior distributions for all three countries.

All remaining structural parameters of the model are estimated. I first assign prior distributions to these parameters, summarized in Table 3.2. The choice of the
distribution families is as follows. First, as fairly standard in the literature, I impose beta distribution on all parameters with feasible values in the unit interval.\textsuperscript{10} These parameters include the autoregressive coefficients of the shocks, Calvo parameters, indexation parameters, habit persistence parameter, interest rate smoothing parameter, and the home bias in consumption and investment parameters. Second, for the parameters with positive values I impose a gamma distribution since it is defined on the $[0, \infty)$ space. These parameters include the standard deviations of the shocks, and the investment adjustment cost. Third, I impose a normal distribution on the unrestricted parameters, i.e. parameters that can take any value, such as parameters in the Taylor rule.\textsuperscript{11}

The choice of the moments of the prior distributions is as follows. For the habit persistence parameter I impose a beta distribution with the mean 0.65, which is equal to the estimate of this parameter obtained by Altig et al. [2005] for the US economy, whereas the standard deviation of this parameter is set to 0.1 percent. For the domestic Calvo parameter I use a beta distribution with the mean 0.675, which implies average price and wage stickiness of 3 quarters. Following Adolfson et al. [2007], I use smaller value for the foreign sector, in order to account for observed lower exchange-rate pass through. However, I use a higher standard deviation, 0.15 percent, to allow for higher uncertainty. I choose rather uninformative priors for the parameters that determine the elasticity of substitution between different types of goods. I use a gamma distribution with mean 2 and standard deviation 0.5, except for the elasticity of substitution between domestic and foreign goods, which I choose to be 1.5 following the results of Chari, Kehoe, and McGrattan [2002]. I set a normal

\textsuperscript{10} See An and Schorfheide [2007] for general discussion about the Bayesian techniques, including the choice of the priors, and Del Negro and Schorfheide [2008] for the discussion about forming priors.

\textsuperscript{11} This is consistent with the existing literature. I use the same priors as Rabanal and Rubio-Ramirez [2005], who take the same original estimates of Taylor adjusted for the annual data.
distribution with mean 2.76 and standard deviation 0.75 as the prior distribution for the investment adjustment cost, considering the estimate of Altig et al. [2005].

I choose a gamma distribution with mean 0.3 and standard deviation 0.2 percent as the prior distribution for all the shocks in the model. Following Adolfson et al. [2007], I impose a beta distribution with mean 0.85 and standard deviation 0.1 percent for all autoregressive coefficients. Finally, I impose normal distributions on the coefficients in the foreign VAR, following Justiniano and Preston.

3.4.4 Estimation Results

In Table 3.3 I report mean and standard deviations of the posterior distributions for all three countries in the benchmark model, i.e. a model that features all aforementioned rigidities. I choose the mean of the Markov chain of the estimated parameter as a point estimate. In Figures 3.7 and 3.8 I present prior and posterior distributions of the parameters whose implications on the real exchange dynamics will be discussed in the next section. The posterior distributions are obtained as the approximation of the distribution from the same family as the prior distributions, with mean and standard deviation equal to the mean and standard deviation of the Markov chain. In fact, in most of the cases the posterior distributions do differ from the prior distribution, suggesting that the data are very informative about the posterior distribution of the parameters.

I now turn to the discussion of the point estimates of the parameters and their comparison among the three countries. The habit formation parameter is very high in Canada and UK, whereas it is twice smaller in case of Australia. This result is in line with the estimates of Justiniano and Preston (2005), who estimate simpler model using the data for Canada, Australia and New Zealand.\footnote{Thier framework is very similar to the framework of Galí and Monacelli [2005].} The parameter that controls the share of imports in consumption, $\alpha_c$, is approximately 0.1 for Canada and

\footnote{Thier framework is very similar to the framework of Galí and Monacelli [2005].}
Australia, whereas for the UK it is higher, 0.3. Domestic price stickiness in Canada is rather high (0.751), which would imply that domestic producers can change their prices every four quarters. In case of the UK this parameter is slightly lower (0.726), whereas it is the lowest for Australia (0.632), which implies that the producers can change their prices on average every 2.7 quarters. This estimate is similar to the one of Justiniano and Preston who estimate this parameter to be 0.61. Also Adolfson et al. [2007] estimate that in the Euro area producers can change prices every 3.5 quarters. However, this result is not directly comparable with my estimation for the UK, even though the UK economy is one of the four biggest economies, output wise, in the Euro area. The estimates of other stickiness parameters (exporting goods and importing goods stickiness parameters) in all three economies are lower than the domestic one, which implies that producers in other sectors can change their prices more frequently. However, there are no existing estimates that can be used to compare my results, since the models that have been estimated are much simpler and do not include these parameters. The elasticity of substitution between domestic and imported goods is very high in all three countries. The lowest among EOS parameters is the one among the differentiated exported goods. This is true for all three countries.

The point estimates of the indexation parameters are quite low. The highest among indexation parameters is the wage indexation parameter, which is approximately 0.5. However, one should be careful in the interpretation of this result, since in the posterior distributions of these parameters are very close to their prior distributions. This result implies that the prior distributions for these parameters are very informative. One possible explanation is that in the estimation procedure I do not use the data on employment, i.e. I do not use series of hours worked, that would possibly provide more information for the estimation of this parameter.

The point estimates of the coefficients in the Taylor rule are rather similar among
the three countries. In particular, all Central Banks aggressively target inflation, since the inflation parameters in Canada, Australia, and UK are 1.523, 1.725 and 1.836 respectively. The coefficients that determine the responsiveness of the Central Bank to the exchange rate movements are very low in case of all three countries, being the lowest in the case of Canada. In Australia this coefficient is 0.102, whereas in case of UK it is 0.127.

The autoregressive coefficients is very high for the technology shock and asymmetric technology shock. The preference shock and labor supply shock are less persistent, especially for Canada.

In order to evaluate the importance of particular frictions in the explanation of the exchange rate dynamics, I estimate different versions of the model. In fact, I will shut down one by one different frictions, which naturally changes the point estimates from the benchmark model.

3.5 Exchange Rate Dynamics

After calibrating and estimating all the structural parameters of the model, I analyze the performance of an estimated model in terms of replicating the real exchange rate dynamics. I perform several exercises. First, I calculate the real exchange rate standard deviation and autocorrelation implied by the estimated model. Second, I assess the performance of the benchmark model, which includes number of nominal and real rigidities. Third, I evaluate the importance of each friction in the model for the replication of the real exchange rate dynamics. To do so, I exclude one friction at a time and reestimate the model. Fourth, I also evaluate the importance of particular type of frictions, such as price stickiness or indexation, by excluding all price stickiness parameters or all indexation parameters at a time and reestimating the model. The results are reported in Tables 3.4 and 3.5.
3.5.1 Benchmark Model

In this section I investigate the performance of the benchmark model in terms of its ability to explain the exchange rate dynamics. I use point estimates reported in Table 3.3 to simulate the model and calculate the implied model moments.\(^\text{13}\)

The performance of the model in replicating the exchange rate persistence in all three countries is remarkably good. In case of Canada, the model implied persistence is 0.92 compared to the persistence of 0.98 observed in the data. The performance of the model is even better for Australia and UK: 0.94 compared to 0.96 in the data for Australia, and 0.92 compared to 0.915 in the data for UK.

The model volatility of the real exchange rate in case of Canada is very similar to the data (3.09 in the model compared to 3.12 in the data), and slightly lower in case of Australia (3.03 in the model compared to 3.22 in the data). The model can reproduce a high real exchange rate volatility for UK (3.80), but it is approximately 1 percent lower than in the data.

Overall, the performance of the model along both these dimensions is pretty satisfying. However, the primary purpose of my analysis is to understand which of the channels are crucial for this successful replication of the exchange rate dynamics. I turn to this investigation in what follows.

3.5.2 Sensitivity Analysis

Persistence

To evaluate the importance of various rigidities of the model in explaining the exchange rate persistence, I compare the real exchange rate persistence of a model that excludes a particular rigidity or rigidities with the one of the benchmark model. The smaller is the difference, the less important the particular rigidity is.

My results suggest that the most important parameters for the replication of

\(^{13}\) I simulate the process 10000 times and report the mean values.
exchange rate persistence are the price and wage stickiness parameters. Specifically, the performance of the model significantly worsens when the domestic price stickiness parameter, $\theta_p$, is excluded from the model. This pattern is observed across all three countries. In fact, the real exchange rate persistence decreases from 0.92 to 0.73 in Canada, from 0.93 to 0.78 in Australia, and from 0.92 to 0.81 in the UK. This result suggests that the domestic price stickiness largely contributes to the high persistence of the real exchange rate in the model. A possible explanation for this result is as follows. As described in my model, the real exchange rate is defined as the nominal exchange rate corrected by the relative price of the domestic and world economy. Since my model is a small open economy model, the world price cannot be altered by the economic decisions of the agents in a small domestic economy and can be considered constant. Therefore, all movements in the relative price come from the movements in the domestic CPI level. A higher degree of domestic price stickiness implies that on average domestic firms are allowed to change prices less often, suggesting higher persistence of inflation (measured by the change in domestic CPI). This higher inflation persistence implies higher persistence of the real exchange rate.

A similar intuition follows when I consider the price stickiness parameter of the importing goods producers, $\theta_m$, which is a part of the domestic economy CPI. Since I allow for the local currency pricing, i.e. domestic importers set prices of imported goods in their own currency, the frequency with which they change their prices will undoubtedly influence the real exchange rate persistence through its effect on the persistence of domestic inflation. As Table 3.4 shows, if importers faced flexible prices the persistence of the real exchange rate would decrease, but less than in the case when domestic price stickiness is excluded from the model. The decrease in the persistence is the highest in the UK. This result can be explained by the fact that the share of imports in the consumption basket is the highest in the UK, thus implying
the biggest relative importance of this channel. Furthermore, my results suggest that the price stickiness in the exporting sector, $\theta_e$, is not very important in case of Canada and Australia (implied exchange rate persistence does not significantly change), whereas in UK it is as important as the price stickiness in the importing sector.

Finally, the wage stickiness parameter is also shown to play an important role in the replication of the exchange rate persistence. In fact, when I exclude this parameter, the model persistence falls by more than 0.12. A possible explanation is that, following results of Chari, Kehoe, and McGrattan [2002] who suggest that the stickier are the wages the less likely producers are to change their prices, and therefore inflation and real exchange rate will be more persistent. However, Chari, Kehoe, and McGrattan [2002] claim that importance of this parameter is not very big, somewhat contrary to my results.

I also show that the indexation parameters also have an important effect on the persistence of the real exchange rate. In particular, the domestic price indexation, $\chi_p$, and wage indexation, $\chi_w$, seem to be the most important among the indexation parameters. The presence of the indexation implies a higher persistence of the inflation, since prices will be indexed by the prices in the previous period. Therefore, the intuition described above follows also in this case. Again, the indexation parameter in the exporting sector is, as expected, less important than its counterpart in the importing sector.

I conclude this discussion by evaluating the effect of the elasticity of substitution parameters on the persistence of the real exchange rate. The procedure here is slightly different than in the previous cases. Specifically, if any of these parameters is set to zero the solution of the model will be undetermined. Therefore, for the solution of the model to be determined, the EOS parameters need to be positive. Therefore, I calibrate elasticity parameters to be equal to 0.9, which is a very small
value compared to the benchmark estimates, and then reestimate other parameters of the model. However, I conclude that imposing high values of these parameters is not crucial for the high exchange rate persistence. The most important among EOS parameters is $\varepsilon$, which defines the EOS among differentiated intermediate goods.

Therefore, the rigidities that have the highest effect on the high exchange rate persistence are the price stickiness, domestic price stickiness being the most important one, and the indexation, with the domestic goods indexation parameter as the most significant among them.

**Volatility**

Table 3.5 displays the implied real exchange rate volatilities when the rigidities are excluded from the model. Price and wage stickiness parameters turn out to play an important role in the replication of the real exchange rate volatility as well. The intuition is as follows. The model features local currency pricing, which implies that importing and exporting firms set their prices in local currency. Therefore, if a shock hits the economy and some producers are not allowed to adjust their prices, large fluctuations in the exchange rate will be needed for the model to return to the equilibrium.

The importance of the domestic price stickiness, as well as the price stickiness in the importing sector, is crucial in the replication of the exchange rate volatility. When these parameters are excluded from the model, the volatility of the real exchange rate series declines significantly. In particular, if I exclude the domestic price stickiness parameter, the volatility falls by approximately 1 percentage point in Canada, 0.6 percentage points in Australia and 0.7 percentage points in the UK.

The importance of indexation parameters is large also in this case. Once all indexation parameters are excluded, the volatility of the real exchange rate decreases by approximately more than 1 percentage point in all three countries. In this case,
the wage indexation seems to be relatively most important among all indexation parameters.

Finally, the performance of the model in this dimension is relatively better in case of Australia and Canada. A possible explanation might be that using a small open economy model is not suited for the analysis of the UK economy. Using a two-country model would induce the interaction between the domestic and world economy, and therefore, most likely have different implications on the dynamics of the real exchange rate.

3.5.3 Variance Decomposition

The model features nine shocks: intratemporal preference shock \((\varepsilon_{\xi,t})\), labor supply shock \((\varepsilon_{\varphi,t})\), aggregate technology shock \((\varepsilon_{A,t})\), investment specific shock \((\varepsilon_{\mu,t})\), monetary policy shock \((\varepsilon_{\mu,t})\), asymmetric technology shock \((\varepsilon_{z^*,t})\) and three foreign economy shocks \((\varepsilon_{\pi^*,t}, \varepsilon_{R^*,t}, \varepsilon_{y^*,t})\). Therefore, this model represents a desirable setting for the investigation of the main driving forces of the exchange rate volatility.

The model has the same number of shocks as observables, ruling out stochastic singularity. In Tables 3.6, 3.7 and 3.8 I report the contribution of each shock to the variance of the observables in the model at four different horizons: 4 quarters, 8 quarters, 12 quarters and 20 quarters, for three different countries. The result is derived using the point estimates of the parameters in the benchmark model. Among domestic shocks, the most important shocks are the monetary policy shock, labor supply shock, and investment specific shock. Monetary policy shock can explain the biggest part of the volatility of the real exchange rate in all three countries, and is about 45 percent.

Among the world shocks only the effect of the shock to the interest rate is important, and is about 8 percent in Canada, while in Australia and UK it contributes in explaining approximately 7 percent of the total volatility of the exchange rate.
3.6 Conclusions

In this paper I estimate a small open economy DSGE model using the data for three countries: Australia, Canada, and the UK. The model builds on a closed economy DSGE model of CEE and Smets and Wouters [2003], by incorporating an open economy component into it. In particular, I add two sectors: importing and exporting sector; in both sectors producers face price stickiness. I estimate the model using Bayesian estimation techniques.

One of the purposes of this paper was to assess how good is this model in replicating the real exchange rate dynamics, its persistence and volatility. I show that the benchmark model performs rather well along both dimensions in all of the three considered economies. Furthermore, I evaluate the importance of various rigidities of the model for the replication of exchange rate persistence and volatility. I find that the most important frictions for the replication of the real exchange rate persistence are the domestic price stickiness parameter, domestic indexation, wage stickiness and wage indexation. This result is because higher price or wage stickiness implies higher persistence of inflation, and therefore higher persistence of the real exchange rate. All of these parameters are also important in explaining the real exchange rate volatility as well.

Finally, I investigate the importance of the nine shocks of the model in explaining the volatility of the real exchange rate. I show that among the domestic shocks, the most important are the investment specific technology shock, monetary policy shock, and labor supply shock, whereas the shock to the foreign interest rate dominates among the world shocks.

3.7 Tables and Figures
Table 3.1: Dynamics of Nominal and Real Exchange Rate in the Data

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>Australia</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>St.dev</td>
<td>AR</td>
<td>St. dev</td>
</tr>
<tr>
<td>Nominal exchange rate</td>
<td>2.97</td>
<td>0.97</td>
<td>2.97</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>3.09</td>
<td>0.97</td>
<td>3.22</td>
</tr>
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</table>
Table 3.2: Prior Distributions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Form</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Habit persistence ($h$)</td>
<td>beta</td>
<td>0.65</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Share of imports in consumption ($\alpha_c$)</td>
<td>beta</td>
<td>0.4</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Share of imports in investment ($\alpha_i$)</td>
<td>beta</td>
<td>0.4</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>EOS - ($\eta_c$)</td>
<td>gamma</td>
<td>2</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>EOS - ($\eta_m$)</td>
<td>gamma</td>
<td>2</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>EOS - ($\eta_e$)</td>
<td>gamma</td>
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<td>0.5</td>
<td></td>
</tr>
<tr>
<td>EOS - ($\eta_w$)</td>
<td>gamma</td>
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<td>0.5</td>
<td></td>
</tr>
<tr>
<td>EOS - ($\varepsilon$)</td>
<td>gamma</td>
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<td>0.267</td>
<td></td>
</tr>
<tr>
<td>EOS - ($\eta_f$)</td>
<td>gamma</td>
<td>1.5</td>
<td>0.267</td>
<td></td>
</tr>
<tr>
<td>Calvo - domestic goods ($\theta_p$)</td>
<td>beta</td>
<td>0.675</td>
<td>0.1</td>
<td></td>
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<tr>
<td>Calvo - imported goods ($\theta_m,c$)</td>
<td>beta</td>
<td>0.5</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Calvo - exported goods ($\theta_e$)</td>
<td>beta</td>
<td>0.5</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Calvo - wages ($\theta_w$)</td>
<td>beta</td>
<td>0.675</td>
<td>0.1</td>
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<tr>
<td>Investment adjustment cost ($\kappa$)</td>
<td>normal</td>
<td>2.76</td>
<td>0.75</td>
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<tr>
<td>Indexation - domestic prices ($\chi_p$)</td>
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<td>0.5</td>
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<tr>
<td>Indexation - import goods ($\chi_m$)</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Indexation - export goods ($\chi_e$)</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Indexation - wages ($\chi_w$)</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Taylor rule: response to output</td>
<td>normal</td>
<td>1.5</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Taylor rule: response to inflation</td>
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<td>0.125</td>
<td>0.125</td>
<td></td>
</tr>
<tr>
<td>Taylor rule: response to interest rate</td>
<td>beta</td>
<td>0.8</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Taylor rule: response to the real interest rate</td>
<td>normal</td>
<td>0</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>AR(1) Technology shock</td>
<td>beta</td>
<td>0.85</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>AR(1) Preference shock</td>
<td>beta</td>
<td>0.85</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>AR(1) Labor supply shock</td>
<td>beta</td>
<td>0.85</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>AR(1) Asymmetric technology shock</td>
<td>beta</td>
<td>0.85</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Std of the preference shock</td>
<td>gamma</td>
<td>0.3</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Std of the labor supply shock</td>
<td>gamma</td>
<td>0.3</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Std of the technology shock</td>
<td>gamma</td>
<td>0.3</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Std of the investment specific shock</td>
<td>gamma</td>
<td>0.3</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Std of the monetary policy shock</td>
<td>gamma</td>
<td>0.3</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Std of the asymmetric technology shock</td>
<td>gamma</td>
<td>0.3</td>
<td>0.2</td>
<td></td>
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<tr>
<td>Std of the foreign output shock</td>
<td>gamma</td>
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<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Std of the foreign inflation shock</td>
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<td>0.3</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Std of the foreign interest rate shock</td>
<td>gamma</td>
<td>0.3</td>
<td>0.2</td>
<td></td>
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</tbody>
</table>
Table 3.3: Posterior Distributions: Benchmark Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Canada Mean</th>
<th>Canada St. dev</th>
<th>Australia Mean</th>
<th>Australia St. dev</th>
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<td>0.02</td>
<td>0.79</td>
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<tr>
<td>Imports share in cons. ( (\alpha_c) )</td>
<td>0.16</td>
<td>0.13</td>
<td>0.10</td>
<td>0.12</td>
<td>0.33</td>
<td>0.09</td>
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<td>Imports share in inv. ( (\alpha_i) )</td>
<td>0.31</td>
<td>0.10</td>
<td>0.15</td>
<td>0.11</td>
<td>0.32</td>
<td>0.08</td>
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<tr>
<td>EOS - ( (\eta_c) )</td>
<td>7.69</td>
<td>0.13</td>
<td>6.02</td>
<td>0.09</td>
<td>6.53</td>
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<tr>
<td>EOS - ( (\eta_m) )</td>
<td>4.32</td>
<td>0.14</td>
<td>2.34</td>
<td>0.14</td>
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<td>0.04</td>
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<td>Calvo - wages ( (\theta_w) )</td>
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<td>0.03</td>
<td>0.87</td>
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<td>0.08</td>
<td>0.34</td>
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<td>0.14</td>
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<td>0.07</td>
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<td>0.14</td>
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<td>0.18</td>
<td>0.23</td>
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<td>0.39</td>
<td>0.19</td>
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<tr>
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<td>0.15</td>
<td>1.84</td>
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<td>AR(1) Technology shock</td>
<td>0.94</td>
<td>0.14</td>
<td>0.95</td>
<td>0.01</td>
<td>0.85</td>
<td>0.03</td>
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<tr>
<td>AR(1) Preference shock</td>
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<td>0.76</td>
<td>0.02</td>
<td>0.73</td>
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<td>AR(1) Labor supply shock</td>
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<td>0.13</td>
<td>0.82</td>
<td>0.13</td>
<td>0.84</td>
<td>0.03</td>
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<tr>
<td>AR(1) Asymm. tech. shock</td>
<td>0.89</td>
<td>0.08</td>
<td>0.94</td>
<td>0.13</td>
<td>0.96</td>
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<tr>
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<td>0.02</td>
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<td>0.20</td>
<td>0.03</td>
<td>0.17</td>
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<td>Std - mon. policy shock</td>
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<td>0.01</td>
<td>0.21</td>
<td>0.17</td>
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<tr>
<td>Std - asymm. tech. shock</td>
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<td>0.12</td>
<td>0.21</td>
<td>0.15</td>
<td>0.32</td>
<td>0.11</td>
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<tr>
<td>Std - foreign output shock</td>
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<td>0.31</td>
<td>0.12</td>
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Table 3.4: Real Exchange Rate Persistence Under Different Model Specifications

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<td></td>
<td>$AR$</td>
<td>$AR$</td>
<td>$AR$</td>
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<tr>
<td>Benchmark Model</td>
<td>0.92</td>
<td>0.94</td>
<td>0.92</td>
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<tr>
<td>No $\theta_p$</td>
<td>0.73</td>
<td>0.78</td>
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<td>No $\theta_m$</td>
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<td>0.89</td>
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<td>0.89</td>
<td>0.85</td>
</tr>
<tr>
<td>No $\theta_w$</td>
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<td>0.82</td>
<td>0.83</td>
</tr>
<tr>
<td>Flexible prices</td>
<td>0.72</td>
<td>0.77</td>
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<tr>
<td>No $\epsilon$</td>
<td>0.88</td>
<td>0.88</td>
<td>0.79</td>
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<tr>
<td>No $\chi_p$</td>
<td>0.82</td>
<td>0.83</td>
<td>0.80</td>
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<tr>
<td>No $\chi_m$</td>
<td>0.89</td>
<td>0.89</td>
<td>0.83</td>
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<tr>
<td>No $\chi_e$</td>
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<td>0.88</td>
<td>0.85</td>
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<tr>
<td>No $\chi_w$</td>
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<td>0.85</td>
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<tr>
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<td>0.80</td>
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<td>EOS: $\eta_m$</td>
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<td>0.89</td>
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<td>0.89</td>
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<td>EOS: $\varepsilon$</td>
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Table 3.5: Real Exchange Rate Volatility Under Different Model Specifications

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<td>2.76</td>
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<tr>
<td>No $\theta_w$</td>
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<td>2.87</td>
<td>2.34</td>
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<td>2.31</td>
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</tr>
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<td>EOS: $\eta_e$</td>
<td>2.97</td>
<td>2.84</td>
<td>2.76</td>
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<td>3.13</td>
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Table 3.6: Variance Decomposition: Canada

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<th>Shock/Model</th>
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<th>8 quarters</th>
<th>12 quarters</th>
<th>20 quarters</th>
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<tr>
<td>Labor supply shock</td>
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<td>15.34</td>
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<td>17.85</td>
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<td>Technology shock</td>
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<tr>
<td>Monetary policy shock</td>
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<td>42.76</td>
<td>41.09</td>
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<td>2.99</td>
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<tr>
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<td>8.75</td>
<td>8.63</td>
<td>8.39</td>
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<td>0.02</td>
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Table 3.7: Variance Decomposition: Australia

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<th>12 quarters</th>
<th>20 quarters</th>
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<td>4.67</td>
<td>4.64</td>
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Table 3.8: Variance Decomposition: UK

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<th>12 quarters</th>
<th>20 quarters</th>
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<td>7.78</td>
<td>7.63</td>
<td>7.51</td>
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<tr>
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<td>10.98</td>
<td>11.01</td>
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<td>7.00</td>
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<td>3.24</td>
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Figure 3.1: Log-Level of the Nominal and Real Exchange rate: Canada, period 1972:IV-2006:IV

Figure 3.2: Log-Level of the Nominal and Real Exchange rate: Australia, period 1972:IV-2006:IV
FIGURE 3.3: Log-Level of the Nominal and Real Exchange rate: UK, period 1972:IV-2006:IV

![Graph showing Log-Level of the Nominal and Real Exchange rate for UK (1972:IV - 2006:IV)](image)

FIGURE 3.4: Growth Rates of Nominal and Real Exchange Rates: Canada, period 1972:I - 2006:IV

![Graph showing Quarterly Growth rate of the Nominal and Real Exchange Rate for Canada (1972:I - 2006:IV)](image)
Figure 3.5: Growth Rates of Nominal and Real Exchange Rates: Australia, period 1972:I - 2006:IV

Figure 3.6: Growth Rates of Nominal and Real Exchange Rates: UK, period 1972:I - 2006:IV
Figure 3.7: Prior and posterior distributions for some parameters for Canada.
Figure 3.8: Prior and Posterior Distributions of Stickiness and EOS Parameters for Canada.
Appendix A

Appendix to Chapter 1

A.1 Data Sources

Here I list all the data used in the paper, as well as their sources. I divide data into three categories: aggregate data, sectoral data (manufacturing, durable and nondurable sectors) and industry data (nineteen two-digit SIC level industries).

Aggregate data

The aggregate data are available at quarterly frequency over the sample period 1949:I-2006:IV; aggregate series used are:

- As the aggregate consumption measure I use Real Personal Consumption Expenditures on Nondurable Goods and Services, BEA NIPA tables.
- As the aggregate output measure I use seasonally adjusted total non farm Output from the BLS website (series ID: PRS85006033).
- As the labor input I use seasonally adjusted total non farm Hours from the BLS website (series ID: PRS85006033).
• As the capital measure I use private business Capital Services data from the Net Multifactor Productivity and Cost Tables at the U.S. BLS; this series measures the services derived from the stock of physical assets and software.

• Compensation of employees is obtained from the BEA NIPA tables.

• Aggregate inventories data are recovered from the BEA NIPA tables.

• As a CPI measure I use the GDP deflator that is calculated as the ratio of Gross Domestic Product (BEA, NIPA table 1.1.5) and Real Gross Domestic Product (BEA, NIPA table 1.1.6).

• As an aggregate stock price index I use S&P500 series, that is downloaded from professor Robert Shiller’s website.

**Sectoral and Industry Data** The sectoral data used in the empirical work are available at quarterly frequency for the period 1972:I-2005:III. The reason why the sample starts later than the aggregate sample is the availability of the data on Electric power index and Industrial production index. Federal Reserve Board provides these series only from 1972:I, which limits my sectoral TFP samples. All series are obtained for the manufacturing, durables, and nondurables sectors; Industry-level series correspond to sample period 1972:I-1997:III. Since in 1997 SIC was replaced by NAICS, I am not able to extend the industry-level data further than 1997. Sectoral and industry-level data are the following:

• Following Burnside, Eichenbaum, and Rebelo [1995] I use electricity consumption as a proxy for capital services and index of industrial production as the output measure. The former is measured as kilowatts of electricity used, and is given by the Electric power index series. Both series are obtained from the Federal Reserve Board website at the level of manufacturing, durable and
nondurable sectors, as well as at the level of two-digit SIC manufacturing industries.\footnote{Available at: https://www.federalreserve.gov/datadownload/}

- As the labor input for the three sectors and all two-digit SIC industries I use quarterly averages of monthly production workers, which is constructed as the product of the two time series: average weekly hours of production workers and number of production workers. Both series are retrieved from the BLS.

- Data on compensation of employees for the three sectors and two-digit SIC industries are obtained from the BEA website. They are used to compute the share of labor income in the total income. In particular, labor share is calculated as the ratio of nominal labor compensation of all employees and the nominal value added income. However, these data are not available in quarterly frequencies. In order to obtain the quarterly frequencies, I calculate the annual labor shares and assume constant growth of the labor share over the year.

- As the measure of Investment I use chain-type quantity indices for investment in private fixed assets by industry, obtained from the BEA, NIPA Tables. Since the investment data are available only in annual frequencies at the sectoral level, I interpolate them assuming constant growth within the quarters of the same year.

- Rates of capacity utilization are recovered from the Federal Reserve Board website. These data are available for the manufacturing, durables, nondurables level, as well as for the two-digit SIC level.

- As the measure of inventories for the manufacturing, durable and nondurable sectors I use the Real Private Inventories and Real Domestic Final Sales of
Business by Industry, BEA table 5.7.6A, billions of chained 2000 dollars, seasonally adjusted quarterly totals. Inventories data at the two-digit SIC level are not available from this source; instead, they are recovered from the Census Bureau website.²

- As the consumption measure for the level of manufacturing, durables and non-durables I use Real Personal Consumption Expenditures by major type, BEA NIPA tables.

- Data on sectoral stock returns and two-digit SIC industries stock returns are obtained from Kenneth French’s website.

- I use BEA NIPA tables to recover sectoral CPIs.

- Finally, all the data are converted to per capita terms using the civilian non-institutional population aged 16 and over, obtained from the BLS (series code: LNU00000000Q).

A.2 Identification Scheme

A.2.1 Inverting VECM

To impose structural restrictions described in the paper, it is necessary to obtain the Wold representation of the system. In order to do so, I first estimate the reduced form two (and three) variables vector error correction model (VECM) of the system using Johansen’s approach to full-information maximum likelihood estimation of a system characterized by the number of cointegrating relations, determined by the Likelihood Ratio Test.³ However, because of the presence of the cointegrating relation(s), it is not straightforward to invert this system.

² Available at: http://www.census.gov/indicator/www/m3/hist/m3bendoc.htm.
³ [Hamilton, 1994, pg 635-637] describes this procedure in detail.
In order to obtain closed-form expressions for the coefficients of \( C(L) \) I use the approach of Hansen [2005]. In particular, obtaining these expressions is crucial for the impulse response analysis (see for example Lutkepohl and Reimers [1992], Warne [1993], and Phillips [1998]).

Let the cointegrated VAR process be given by

\[
\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t,
\]

and assume that the assumptions of the Johansen’s representation theorem hold. Then the process has the following representation:

\[
X_t = C \sum_{i=1}^{t} \varepsilon_i + C(L) \varepsilon_t + C(X_0 - \Gamma_1 X_{-1} - \ldots - \Gamma_{k-1} X_{-k+1}),
\]

where \( C = \beta \perp (\alpha' \perp \Gamma \beta \perp)^{-1} \alpha' \perp \), and \( \alpha \perp \) and \( \beta \perp \) are orthogonal complements to \( \alpha \) and \( \beta \).

Hansen [2005] shows that the coefficients of \( C(L) \) can be obtained using the following formula:

\[
C_i = GQ^i E_{1,2},
\]

with

\[
G = (I - CT), -CT_1, \ldots, -CT_{k-1},
\]

\[
Q = \begin{bmatrix}
I + \Pi & \Gamma_1 & \cdots & \Gamma_{k-2} & \Gamma_{k-1} \\
\Pi & \Gamma_1 & \cdots & \Gamma_{k-2} & \Gamma_{k-1} \\
0 & \Pi & \Gamma_1 & \cdots & \Gamma_{k-2} \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & \Pi & \Gamma_1 \\
\end{bmatrix},
\]

\[
E_{1,2} = (I_p, I_p, 0, \ldots, 0)'.
\]

Once the \( C_i \) matrices are recovered, it is straightforward to write down \( MA(\infty) \) representation. The next step is to identify the structural shocks by imposing the
meaningful structural restrictions. The short-run and long-run restrictions imposed are described below.

A.2.2 Two-Variable System

Short-Run Identification  The element $1, 2$ of the short-run matrix $\Gamma_0$ is restricted to zero. In the case of two-variable system, after the $MA(\infty)$ representation is recovered, it is straightforward to impose the short-run restriction described in Section 2. This restriction is imposed using Cholesky decomposition.

Long-Run Identification  If the two series are found to be cointegrated all elements in the second column of the long-run matrix equal zero, i.e. $\tilde{\epsilon}_1$ is the only permanent shock. Hence, in this case one cannot use a simple Cholesky decomposition as in Blanchard and Quah [1989] to recuperate the structural news shock. In fact, in order to recover disturbance $\tilde{\epsilon}_1$ I follow the procedure proposed by King, Plosser, Stock, and Watson [1991]. This procedure allows one to impose the long-run restrictions using the fact of the existence of the cointegrating relations.

A.2.3 Three-Variable System

Short-Run Identification  The element $1, 2$ of the short-run matrix $\Gamma_0$ is restricted to be equal to zero. This assumption imposes that the shock to stock prices, $\varepsilon_2$, does not have a short-run impact on TFP. In addition, the shock $\varepsilon_3$ is restricted to be only a transitory shock, which implies that the third column of the long-run matrix $\Gamma (1)$ will be a zero-column. No restrictions are imposed on the shock $\varepsilon_1$, so as to let it potentially represent a surprise shock to technology. When combining these restrictions, the shock $\varepsilon_2$ can be uniquely identified.
**Long-Run Identification**  The long-run matrix is restricted to be lower triangular, and the shock $\tilde{\varepsilon}_1$ is recuperated. However, since the existence of two cointegrating vectors implies two zero columns in the long-run matrix, one more restriction needs to be imposed in the short-run matrix in order to uniquely identify all three shocks. I choose the 3, 2 element of the short-run matrix to be equal to zero. This additional restriction does not have any impact on the retrieved $\tilde{\varepsilon}_1$ shock. It is only needed to separate $\tilde{\varepsilon}_2$ and $\tilde{\varepsilon}_3$ shocks, and, therefore, does not influence the shock $\tilde{\varepsilon}_1$, the effects of which I am interested in.
A.3 Equations of the Model

Households maximize the following lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left( C_t^* - hC_{t-1}^*, N_{1t}, N_{2t} \right)$$

subject to:

$$C_{1t} \leq A_{1t} N_{1t}^{1-\alpha_1} (u_{1t} K_{1t})^{\alpha_1}$$

$$C_{2t} + X_{1t} + X_{2t} + \Delta I_t \leq A_{2t} N_{2t}^{1-\alpha_2} \left[ (1 - \rho) (u_{2t} K_{2t})^{-\nu} + \rho I_{t-\nu} \right]^{-\frac{\alpha_2}{\nu}}$$

$$C_t^* = \left[ \chi_1 C_{1t}^{\nu_1} + \chi_2 C_{2t}^{\nu_2} \right]^\frac{1}{\nu}$$

$$K_{1,t+1} = (1 - \delta^1 (u_{1t})) K_{1t} + X_{1t} \left( 1 - \frac{\kappa_1}{2} \left( \frac{X_{1t}}{X_{1t-1}} - 1 \right)^2 \right)$$

$$K_{2,t+1} = (1 - \delta^2 (u_{2t})) K_{2t} + X_{2t} \left( 1 - \frac{\kappa_2}{2} \left( \frac{X_{2t}}{X_{2t-1}} - 1 \right)^2 \right)$$

$$S_{t+1} = (1 - \delta_S) S_t + D_t \left( 1 - \frac{\kappa_S}{2} \left( \frac{D_t}{D_{t-1}} - 1 \right)^2 \right)$$

Lagrangian of this problem is the following:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l}
U \left( C_t^* - hC_{t-1}^*, N_{1t}, N_{2t} \right)
+ \lambda_{1t} \left[ A_{1t} (N_{1t})^{1-\alpha_1} (u_{1t} K_{1t})^{\alpha_1} - C_{1t} \right]
+ \lambda_{2t} \left[ A_{2t} N_{2t}^{1-\alpha_2} \left[ (1 - \rho) (u_{2t} K_{2t})^{-\nu} + \rho I_{t-\nu} \right]^{-\frac{\alpha_2}{\nu}} - D_t - X_{1t} - X_{2t} - \Delta I_t \right]
+ Q_{1t} \left[ (1 - \delta^1 (u_{1t})) K_{1t} + X_{1t} \left( 1 - \frac{\kappa_1}{2} \left( \frac{X_{1t}}{X_{1t-1}} - 1 \right)^2 \right) - K_{1,t+1} \right]
+ Q_{2t} \left[ (1 - \delta^2 (u_{2t})) K_{2t} + X_{2t} \left( 1 - \frac{\kappa_2}{2} \left( \frac{X_{2t}}{X_{2t-1}} - 1 \right)^2 \right) - K_{2,t+1} \right]
+ Q_{St} \left[ (1 - \delta_S) S_t + D_t \left( 1 - \frac{\kappa_S}{2} \left( \frac{D_t}{D_{t-1}} - 1 \right)^2 \right) - S_{t+1} \right]
\end{array} \right\}$$

First order conditions of this problem are the following:
\[ C_{1t} : \]

\[ U_{c1t} = \lambda_{1t}, \quad (A.1) \]

where

\[ U_{C_{1t}}^{KPR} = \left( \left( C_t - h C_{t-1} \right) (1 - \psi N_t^\theta) \right)^{-1-\sigma} \frac{\chi_1 C_{1t}^{\mu-1}}{[\chi_1 C_{1t}^{\mu} + \chi_2 (\gamma S_t)^\mu]} - h \beta E_t \left[ \left( \left( C_t - h C_t \right) (1 - \psi N_t^\theta) \right)^{-1-\sigma} \frac{C_t^\epsilon \chi_1 C_{1t}^{\mu-1}}{[\chi_1 C_{1t}^{\mu} + \chi_2 (\gamma S_t)^\mu]} \right] \]

\[ U_{C_{1t}}^{GHH} = \left( \left( C_t - h C_{t-1} \right) (1 - \psi N_t^\theta) \right)^{-1-\sigma} \frac{C_t^\epsilon \chi_1 C_{1t}^{\mu-1}}{[\chi_1 C_{1t}^{\mu} + \chi_2 (\gamma S_t)^\mu]} - h \beta E_t \left[ \left( \left( C_t - h C_{t-1} \right) (1 - \psi N_t^\theta) \right)^{-1-\sigma} \frac{C_{t+1}^\epsilon \chi_1 C_{1t+1}^{\mu-1}}{[\chi_1 C_{1t+1}^{\mu} + \chi_2 (\gamma S_{t+1})^\mu]} \right] \]

\[ [N_{1t}] : \]

\[ U_{N_{1t}} + \lambda_{1t} F_{N_{1t}} = 0, \quad (A.2) \]

where

\[ U_{N_{1t}}^{KPR} = \left( \left( C_t - h C_{t-1} \right) (1 - \psi N_t^\theta) \right)^{-1-\sigma} \left( C_t - h C_{t-1} \right) (-\psi \theta N_t^{\theta-1}) \]

\[ U_{N_{1t}}^{GHH} = \left( \left( C_t - h C_{t-1} \right) (1 - \psi N_t^\theta) \right)^{-1-\sigma} (-\psi \theta N_t^{\theta-1}) \]

\[ F_{N_{1t}} = A_{1t} \left( 1 - \alpha_1 \right) N_{1t}^{1-\alpha_1} \left( u_{1t} K_{1t} \right)^{\alpha_1} \]

\[ [N_{2t}] : \]

\[ U_{N_{2t}} + \lambda_{2t} F_{N_{2t}} = 0, \quad (A.3) \]

where

\[ U_{N_{2t}}^{KPR} = U_{N_{1t}}^{KPR} \]

\[ U_{N_{2t}}^{GHH} = U_{N_{1t}}^{GHH} \]

\[ F_{N_{2t}} = A_{2t} \left( 1 - \alpha_2 \right) N_{2t}^{1-\alpha_2} \left[ (1 - \rho) \left( u_{2t} K_{2t} \right)^{-\nu} + \rho I_t^{-\nu} \right]^{\frac{\alpha_2}{\nu}} \]
\[ [S_t+1] : \]
\[ -Q_{St} + \beta \left[ U_{St+1} + Q_{S,t+1} (1 - \delta_S) \right] = 0. \]

Define \( q_{St} = \frac{Q_{St}}{\lambda_{2t}} \) and rewrite the previous equation as:
\[ q_{St} = \beta \left[ \frac{U_{St+1}}{\lambda_{2t}} + \frac{Q_{S,t+1}}{\lambda_{2t}} (1 - \delta_S) \right], \quad (A.4) \]

where
\[
U_{St}^{KPR} = \left( (C_t^* - hC_{t-1}^*) (1 - \psi N_t^\theta) \right)^{1-\sigma} \frac{C_t^* \chi_2 \gamma (\gamma S_t)^{\mu-1}}{[\chi_1 C_{1t}^\mu + \chi_2 (\gamma S_t)^\mu]} - h\beta E_t \left[ \left( (C_{t+1}^* - hC_t^*) (1 - \psi N_{t+1}^\theta) \right)^{1-\sigma} \frac{C_{t+1}^* \chi_2 \gamma (\gamma S_{t+1})^{\mu-1}}{[\chi_1 C_{1t+1}^\mu + \chi_2 (\gamma S_{t+1})^\mu]} \right]
\]
\[
U_{St}^{GHH} = \left( (C_t^* - hC_{t-1}^*) - \psi N_t^\theta \right)^{-\sigma} \frac{C_t^* \chi_2 \gamma (\gamma S_t)^{\mu-1}}{[\chi_1 C_{1t}^\mu + \chi_2 (\gamma S_t)^\mu]} - h\beta E_t \left[ \left( (C_{t+1}^* - hC_t^*) - \psi N_{t+1}^\theta \right)^{-\sigma} \frac{C_{t+1}^* \chi_2 \gamma (\gamma S_{t+1})^{\mu-1}}{[\chi_1 C_{1t+1}^\mu + \chi_2 (\gamma S_{t+1})^\mu]} \right]
\]

\[ [K_{1,t+1}] : \]
\[ -Q_{1t} + \beta E_t \left[ \lambda_{1,t+1} F_{K_{1,t+1}} + Q_{1,t+1} (1 - \delta_1) \right] = 0, \]

where
\[ F_{K_{1,t+1}} = \alpha_1 A_{1,t+1} (N_{1,t+1})^{1-\alpha_1} (u_{1,t+1} K_{1,t+1})^{\alpha_1-1} u_{1,t+1}. \]

Define \( q_{1t} = \frac{Q_{1t}}{\lambda_{1t}} \), and rewrite the previous equation as:
\[ -q_{1t} + \beta E_t \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \left[ F_{K_{1,t+1}} + q_{1,t+1} (1 - \delta_1) \right] = 0 \quad (A.5) \]

\[ [K_{2,t+1}] : \]
\[ -Q_{2t} + \beta \left[ \lambda_{2t+1} F_{K_{2,t+1}} + Q_{2,t+1} (1 - \delta_2) \right] = 0, \]

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where
\[
F_{K2,t+1} = A_{2,t+1} (N_{2,t+1})^{1-\alpha_2} [(1 - \rho) K_{2,t+1}^{-\nu} + \rho I_{t+1}^{-\nu}]^{-\alpha_2 - 1} \\
\left( -\frac{\alpha_2}{\nu} \right) (1 - \rho) (-\nu) (u_{2,t+1} K_{2,t+1}^{-\nu})^{-1} u_{2,t+1}
\]

Define \( q_{2t} = \frac{Q_{2t}}{\lambda_{2t}} \), and rewrite the equation as:
\[
-q_{2t} + \beta \frac{\lambda_{2,t+1}}{\lambda_{2t}} [F_{K2,t+1} + q_{2,t+1} (1 - \delta_2)] = 0 \quad (A.6)
\]

[I_{t+1}):
\[
\lambda_{2t} = \beta E_t \lambda_{2t+1} [F_{I_{t+1}} + 1], \quad (A.7)
\]

where
\[
F_{I_{t+1}} = -\frac{\alpha_2}{\nu} (A_{2,t+1} N_{2,t+1})^{1-\alpha_2} [(1 - \rho) (K_{t+1})^{-\nu} + \sigma I_{t+1}^{-\nu}]^{-\alpha_2 - 1} \rho (-\nu) (I_{t+1})^{-\nu - 1}
\]

[X_1t] :
\[
1 = q_{1t} \left[ 1 - \frac{\kappa_1}{2} \left( \frac{X_{1t}}{X_{1,t-1}} - 1 \right)^2 - \frac{\kappa_1}{2} \left( \frac{X_{1t}}{X_{1,t-1}} - 1 \right) \frac{X_{1t}}{X_{1,t-1}} \right] + \\
\beta E_t q_{1,t+1} \frac{\lambda_{2,t+1}}{\lambda_{2t}} \left[ \kappa_1 \left( \frac{X_{1,t+1}}{X_{1t}} - 1 \right) \left( \frac{X_{1t+1}}{X_{1t}} \right)^2 \right] \quad (A.8)
\]

[X_2t] :
\[
1 = q_{2t} \left[ 1 - \frac{\kappa_2}{2} \left( \frac{X_{2t}}{X_{2,t-1}} - 1 \right)^2 - \frac{\kappa_2}{2} \left( \frac{X_{2t}}{X_{2,t-1}} - 1 \right) \frac{X_{2t}}{X_{2,t-1}} \right] + \\
\beta E_t q_{2,t+1} \frac{\lambda_{2,t+1}}{\lambda_{2t}} \left[ \kappa_2 \left( \frac{X_{2,t+1}}{X_{2t}} - 1 \right) \left( \frac{X_{2,t+1}}{X_{2t}} \right)^2 \right] \quad (A.9)
\]
\[ [D_t] : \]

\[
1 = q_{St} \left[ 1 - \frac{\kappa_S}{2} \left( \frac{D_t}{D_{t-1}} - 1 \right)^2 - \kappa_S \left( \frac{D_t}{D_{t-1}} - 1 \right) \frac{D_t}{D_{t-1}} \right] \\
+ \beta E_t q_{S,t+1} \frac{\lambda_{2,t+1}}{\lambda_{2t}} \left[ \kappa_S \left( \frac{D_{t+1}}{D_t} - 1 \right) \left( \frac{D_{t+1}}{D_t} \right)^2 \right] \]  

(A.10)

\[ [u_{1t}] : \]

\[
q_{1t} \left( \delta_1^1 + \delta_2^1 (u_{1t} - 1) \right) = A_{1t} N_{1t}^{1-\alpha_1} (u_{1t} K_{1t})^{\alpha_1-1} \]  

(A.11)

\[ [u_{2t}] : \]

\[
q_{2t} \left( \delta_1^2 + \delta_2^2 (u_{2t} - 1) \right) \\
= A_{2t} N_{2t}^{1-\alpha_2} \left[ (1 - \rho) (u_{2t} K_{2t})^{-\nu} + \rho I_t^{-\nu} \right]^{-\alpha_2^{-1}} \alpha_2 (1 - \rho) (u_{2t} K_{2t})^{-\nu-1} \]  

(A.12)

\[
C_{1t} = A_{1t} N_{1t}^{1-\alpha_1} K_{1t}^{\alpha_1} \]  

(A.13)

\[
C_{2t} + X_{1t} + X_{2t} + \Delta I_t = A_{2t} N_{2t}^{1-\alpha_2} \left[ (1 - \rho) K_{2t}^{-\nu} + \rho I_t^{-\nu} \right]^{-\alpha_2^{-1}} \]  

(A.14)

\[
C_t^* = \left[ \chi_1 C_{1t}^{\mu} + \chi_2 C_{2t}^{\mu} \right]^\frac{1}{\mu} \]  

(A.15)

\[
N_{1t} + N_{2t} = N_t = 1 - L_t \]  

(A.16)


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Biography

Marija Vukoti´c was born in Podgorica, Montenegro on March 8th, 1980. She earned her B.A. in Economics, *cum laude*, from the University of Belgrade in December 2002. Before joining the graduate program at Duke University in August 2004, Marija held a research position as a junior economist in the G-17 Institute in Belgrade, Serbia. She was awarded M.A. in Economics from Duke University in 2005. She is planning on graduating from Duke University with doctorate degree in Economics in spring of 2010.