Consumer privacy and the market for customer information

Curtis R. Taylor*

I investigate consumer privacy and the market for customer information in electronic retailing. The value of customer information derives from the ability of Internet firms to identify individual consumers and charge them personalized prices. I study two settings, a confidential regime in which the sale of customer information is not possible, and a disclosure regime in which one firm may compile and sell a customer list to another firm that uses it to price discriminate. Welfare comparisons depend critically on whether consumers anticipate sale of the list and on demand elasticity.

Dynamic pricing is the new version of an old practice, price discrimination. It uses a potential consumer's electronic fingerprint—his record of previous purchases, his address, maybe other sites he has visited—to size up how likely he is to balk if the price is high. If the customer looks price sensitive, he gets a bargain. If he doesn’t, he pays a premium.


1. Introduction

■ Motivation. In a recent survey, the Federal Trade Commission (FTC) found that 99% of online companies collect personal information from the individuals visiting their web sites (Seligman and Taylor, 2000). An article on one-to-one web marketing reports that “Most sites obtain [consumer] profile data by observing behavior on the site, tracking purchase behavior, asking questions with forms, or all three” (Allen, 1999, p. 2). In September 2000, Amazon.com conducted dynamic-pricing experiments in which DVD movies were sold to different customers at different prices (up to 40% different) based on their purchasing histories (Streitfeld, 2000). What is more, such tailor-made prices are not restricted to transactions on the Internet. Banks, airlines, long-distance companies, and even grocery stores use modern information technology to track individual customers and make them personalized offers.

Amazon was severely criticized by consumer privacy groups when news of its dynamic-pricing experiment came to light. The company publicly apologized and made refunds to 6,896 customers. Nevertheless, as Streitfeld (2000) observes, “With its detailed records on the buying

* Duke University; crtaylor@econ.duke.edu.

I thank Alessandro Acquisti, Tracy Lewis, Paul Klemperer, Preston McAfee, Canice Prendergast, Hal Varian, Huseyin Yildirim, and especially Margaret Meyer for helpful comments. I am grateful to my students Wolfgang Koehler and Oksana Loginova for their suggestions and assistance. Jacques Crémer and two anonymous referees provided very helpful editorial suggestions. Financial support was provided by the National Science Foundation (grant no. SES-0136817).

While the dynamic pricing of DVDs received more attention in the press, Amazon’s dynamic pricing was actually first detected in May 2000 when it was discovered to be offering discounts of over 20% on a popular MP3 player to some of its customers (Wolverton, 2000).

See, for example, Winnet (2000) and Khan (2000).
habits of 23 million consumers, Amazon.com is perfectly situated to employ dynamic pricing on a massive scale" (p. A1). Besides dynamic pricing, firms use consumer profile data to target ads and make product recommendations. Indeed, customized ads that use consumer profile data sell for ten times the price of untargeted advertisements (Schwartz, 2000).

It is not necessary for electronic retailers to rely only on their own consumer-profile data. There is an active market for personal consumer information served by such web-based marketing firms as Double Click and I-Behavior. These firms collect and sell customer data that typically include an individual’s: purchasing history, income, size of family, lifestyle interests, and motor vehicle ownership (Thibodeau, 2001). As Rendleman (2001) puts it, “Businesses are buying and selling customer data in a dizzying number of ways” (p. 1). Indeed, list brokers compile targeted mailing lists that sell for about $150 per 1,000 names, and a good mailing list reportedly can produce millions of dollars in sales all by itself (Rendleman, 2001).

Indeed, customer lists and consumer profile data are often among the most valuable assets owned by electronic retailers. For instance, when the web retailer Toysmart.com went bankrupt in June 2000, its creditors viewed Toysmart’s customer list as one of its most valuable assets. Only a legal challenge by the FTC prevented the sale of the list, which was collected under a company privacy policy that promised customers that they “can rest assured that your information will never be shared with a third party.” Similarly, Amazon’s privacy notice currently states, “Information about our customers is an important part of our business, and we are not in the business of selling it.” However, it then goes on to say, “As we continue to develop our business, we might sell or buy stores or business units. In such transactions, customer information generally is one of the transferred business assets.”

Consumers are becoming increasingly aware that their electronic purchases and other activities are being monitored, cataloged, and sold. Under pressure from consumer-privacy organizations, the FTC in March 2001 held a conference on consumer profiling and data exchange (Thibodeau, 2001). The commission’s own survey results indicate that 92% of respondents do not trust online companies to keep their personal information confidential, and 82% agreed that the government should regulate how online companies use personal information.

Indeed, consumers are already taking proactive measures to ensure their privacy. Slatalla (2000) reports that “A number of escrow services and online payment companies have begun to act as go-betweens to limit consumers’ exposure to sellers” (p. G4). Services such as PrivateBuy.com use disposable credit card numbers and phoney billing addresses to create an ostensibly untraceable online identity for the Internet shoppers who wish to protect their privacy. Also, McCullagh (2001) reports that “Consumers are able to rely on non-governmental rating and reputation systems to steer them toward desirable destinations... TRUSTe, BBBonline, and WebTrust offer ‘privacy seals’ to websites so consumers can take their business to only companies they trust. TRUSTe claims it has 2000 member companies, including many high-profile sites, and BBBonline has awarded its Privacy Seal to over 500 websites” (p. 3).

There are also other—albeit less sophisticated—strategies at the disposal of privacy-conscious electronic shoppers. Many consumers routinely refuse or remove cookies (electronic identifiers) from their computers; shop using several different computers; and pass up offers that they might otherwise be tempted to accept.

**Overview.** In this article I investigate consumer privacy and the market for customer information in the context of a simple strategic model. While most consumers probably have an inherent preference for privacy, the analysis presented here focuses on another potentially important reason for wishing to remain anonymous, discrimination in the form of dynamic pricing. Specifically, a model featuring a continuum of consumers who wish to purchase a distinct good from each

---

4 Emphasis added. The privacy notice is available at www.amazon.com.
5 In fact, it was TRUSTe that first brought Toysmart’s plan to sell its customer list to the attention of the FTC.
of two monopolists is explored. The consumers possess heterogeneous private demands for the goods, but they are initially indistinguishable by the firms. Each consumer's valuations for the two goods are positively correlated. This implies that a consumer's purchasing decision at firm 1 is valuable information for firm 2. In particular, firm 2 may wish to raise (or lower) its offer to a consumer if it learns that he did (or did not) purchase from firm 1.

I investigate two settings, a confidential regime in which firm 1 cannot sell or transfer customer information to firm 2, and a disclosure regime in which the sale of customer information is possible. Within the context of the disclosure regime, I explore two cases, one in which consumers are naive with regard to the sale of the customer list and one in which they fully anticipate it. In the case of naive consumers, I show that firm 1 often possesses incentives to charge high experimental prices in order to elicit information about its customers. If consumers are naive, then the firms prefer the disclosure regime to the confidential regime. Social surplus may be either lower or higher under the disclosure regime depending respectively on whether dynamic pricing leads to higher or lower average prices.

In the case when consumers anticipate sale of their information, some striking welfare reversals emerge. In particular, in equilibrium a fraction of consumers who have high valuations for both goods misrepresent their preferences by strategically refusing to buy from firm 1 if it sets a high price. This strategic-demand reduction has two important consequences. First, it undermines the market for customer information because it results in a worthless customer list. Second, it causes the effective demand facing firm 1 to be more elastic, often inducing it to post a lower price. Situations may occur, however, in which firm 1 nevertheless finds it optimal to post a high price. In this case, the deadweight loss associated with strategic-demand reduction adds to the inefficiency arising from monopoly pricing. When consumers fully anticipate sale of the customer list, the firms prefer the confidential regime to the disclosure regime. In particular, firm 1 would like to commit to a privacy policy under which it promises not to sell its customer list to firm 2. Of course, one need look no further than the landmark Toysmart case to see that such promises may be difficult to keep. Indeed, in the situation when firm 1 and firm 2 are actually a single entity selling a sequence of goods, it may be practically impossible to commit not to use customer information internally.

- **Related literature.** The strategic-demand reduction that occurs in the model presented in this article is a manifestation of the celebrated rachet effect. In this context, the most relevant article is Hart and Tirole (1988). These authors study a model of repeat purchases under conditions of private information by the consumer and imperfect commitment by the seller. In this setting, Hart and Tirole's findings are rather stark. Specifically, they find that under a long time horizon, the seller is generally compelled to charge a low price and to learn nothing until near the end of the game. This finding, however, has only limited connection with the model presented below, in which there are only two periods and where learning is incomplete because of the imperfect correlation in consumer valuations.

The economic literature on privacy—while quite young—is growing rapidly. Besides this one, there are several recent articles featuring theoretical treatment of privacy in electronic retailing and price discrimination: Calzolari and Pavan (2004), Acquisti and Varian (forthcoming), Dodds

---

6 The firms are modelled as pure monopolists in order to focus attention on information exchange. In fact, the firms could be oligopolists operating in distinct markets that feature differentiated goods, consumer search, or switching costs.

7 Segal (2003) studies an interesting (and somewhat related) model in which a monopolist does not know the distribution from which all of its customers' valuations are drawn. Because there is unconditional correlation in buyer types, Segal shows that it is generally optimal to use a contingent pricing mechanism rather than learning through sequential sales. This will not work in the model presented here because each consumer's valuations are drawn from a different distribution.

8 Rothschild (1974) was the first to study learning by a monopolist through experimental pricing.

9 This is similar to the finding of Baron and Besanko (1984), who show that a regulated monopolist can benefit from promising to forget about the past actions of its clients.

Calzolari and Pavan analyze a buyer whose tastes for the goods sold by each of two firms are perfectly correlated. Interestingly, in this context the authors show that it may be optimal for the first firm to commit to disclose customer information to the second firm if consumers view the two goods as complements. If consumer utility is separable in the two goods, however, then Calzolari and Pavan find that the first firm benefits from committing to keep customer information private.

Unlike the current article, Calzolari and Pavan do not consider the possibility of naive consumers. Also, most of their analysis is couched in the context of full commitment, where they consider the design of an abstract information transmission mechanism rather than explicit sale of a customer list. In essence, one can view Calzolari and Pavan as studying the question concerning the \textit{ex ante} optimal design of an information-sharing agreement between two firms, while the analysis presented here concerns the \textit{ex post} sale of customer information in the absence of any prior agreement.

Like Calzolari and Pavan, Acquisti and Varian (forthcoming) study consumer privacy in a setting where a buyer’s tastes for two goods are perfectly correlated. Acquisti and Varian, however, are primarily concerned with the design of an optimal pricing policy by a monopolist selling two goods in sequence under conditions of full commitment. Although they do not explicitly consider the sale of a customer list between firms, several of Acquisti and Varian’s findings are similar to results presented here. For instance, they find that dynamic pricing is optimal for the monopolist when consumers are naive but not when they are sophisticated. In particular, under full commitment, the revelation principle implies the optimality of eliciting a consumer’s private information up front by committing to a long-term price. There are, of course, also some important differences between the settings considered by Acquisti and Varian and this article. For example, under full commitment and in the absence of personalized service offerings, it is optimal for a monopolist to commit to charging either a high price for both goods or a low price for both goods when consumers are naive but not when they are sophisticated. As noted above, however, the absence of commitment analyzed here results in a more elastic effective demand for the first good, which may lead to a low price for the first good and a high price for the second one in equilibrium.

Dodds (2002) examines a model in which many agents contract sequentially with two principals and in which each agent’s “abilities” for performing the two tasks are positively correlated. In Dodd’s baseline model, the first principal cannot commit not to sell information about the contract signed by each agent. In this context, several of Dodd’s findings are similar in spirit to those presented here. Specifically, Dodds shows that commitment by the first principal to keep contract terms confidential generally benefits the principal but not the agents.

Wathieu (2002) shows that under certain circumstances, firms target consumers too finely, sacrificing overall efficiency for the sake of price discrimination. This creates a net opportunity for intermediaries who create value by maintaining coarse consumer access, which is Wathieu’s interpretation of the demand for privacy.

There are three other recent articles on privacy in electronic retailing that should be mentioned here. Deck and Wilson (2002) present an experimental investigation of price discrimination when customer tracking is and is not possible. Kleinberg, Papadimitriou, and Raghavan (2002) give several examples in which the value of information supplied by individuals to a transaction can be evaluated using concepts from cooperative game theory such as the core and Shapley value. Finally, Hann et al. (2002) present an interesting empirical analysis of the value of online privacy. Among other things, they find that benefits such as monetary reward and future convenience significantly affect individuals’ preferences over websites with differing privacy policies. Their findings also suggest that the value to an individual of protection against errors, improper access, and secondary use of personal information is worth between $30.49 and $44.62.

The rest of this article is organized as follows. Section 2 sets out the basic model. Section 3...
contains some key preliminary findings on the incentives for dynamic pricing. Section 4 analyzes the confidential regime in which the market for customer information does not exist. Section 5 investigates the value and pricing of the customer list. Section 6 contains the analysis of the disclosure regime when customers do not anticipate the sale of their information, and Section 7 analyzes the case in which they do anticipate the sale. Section 8 concludes. Proofs not presented in the text either appear in the Appendix or are available from the author upon request.

2. The model

- **The consumers.** There is a continuum of consumers with total mass of one. Consumers are risk neutral and do not discount the future. Each consumer is associated with a distinct index $i \in [0, 1]$, which can be thought of as his address. In each period $t = 1, 2$, each consumer demands one unit of a distinct nondurable product (good $t$). Specifically, consumer $i$’s valuation for good $t$ is $v_{it} \in \{v_L, v_H\}$, where $v_H > v_L \geq 0$. Both valuations $v_{i1}$ and $v_{i2}$ are privately known by consumer $i$ at the outset of the game. It is, however, common knowledge that $v_{i1}$ and $v_{i2}$ are determined by the outcome of two independent random variables $\bar{v}_{i1}$ and $\bar{v}_{i2}$, where

$$\Pr\{\bar{v}_{it} = v_H\} = \lambda_i, \quad t = 1, 2.$$  

The demand parameter, $\lambda_i \in [0, 1]$, can be thought of as a measure of income or intensity of taste for a particular class of goods. It is independently distributed throughout the population according to the nondegenerate distribution function $F(\lambda)$, and it is not observable.

- **The firms.** There are two risk-neutral firms (1 and 2) that have production costs of zero and that do not discount the future. Firm $t$ is the monopoly seller of good $t$.

Two privacy settings are considered, the confidential regime in which the market for customer information does not exist, and the disclosure regime in which firm 1 may sell customer information to firm 2. In particular, the customer list consists of the set of first-period purchasing decisions of each consumer, $q_{i1} \in \{0, 1\}$ for all $i \in [0, 1]$. In other words, the customer list merges a consumer’s address with his first-period purchasing decision. If firm 2 buys the customer list from firm 1, then it can use this information to engage in dynamic pricing (i.e., it may price discriminate based on whether a consumer did or did not buy good 1).

The game unfolds in several stages. First, each consumer observes his valuations, $v_{i1}$ and $v_{i2}$. The firms do not observe $v_{i1}$, $v_{i2}$, or $\lambda_i$ for any consumer, but the distribution $F(\lambda)$ is common knowledge. In the second stage, firm 1 posts price $p_1 \in \mathbb{R}_+$ for good 1. (Firm 2 is assumed to observe the offer, $p_1$.) Next, each consumer either accepts ($q_{i1} = 1$) or rejects ($q_{i1} = 0$) firm 1’s offer. Actions in the next two stages of the game depend on the privacy regime. Under the confidential regime, nothing happens in these stages. Under the disclosure regime, firm 1 offers to sell its customer list for $w \in \mathbb{R}_+$. Firm 2, then, either accepts ($x = 1$) or rejects ($x = 0$) this offer. Next, firm 2 makes offers to consumers. In particular, if firm 2 did not buy the customer list, then it posts the same price $p_{i2} = p_2$ to all consumers. Alternatively, if firm 2 did buy the list, then it offers $p_{i2} = p_{i1}^1$ to consumers who purchased good 1 and $p_{i2} = p_{i1}^0$ to consumers who did not. Finally, each consumer either accepts ($q_{i2} = 1$) or rejects ($q_{i2} = 0$) the offer made to him.

---

11 In this context, a consumer’s address is the means by which a firm identifies him. For simplicity, it is assumed that consumers cannot hide or change their addresses. See Tadelis (1999) for a model in which firms have reputations associated with their names, and in which names may be sold.

12 Including positive production costs and discounting would add notation without providing additional insights.

13 Since all consumers are stochastically equivalent and have independently distributed taste parameters, there is no loss in generality (and considerable notational savings) in assuming that they all receive the same offer in the first period.

14 Again, there is no loss of generality in assuming that consumers who are observationally equivalent receive the same offer. Also, Skreta (2002) verifies the optimality of take-it-or-leave-it price offers in a setting without commitment.

Consumer i’s payoff is

\[(v_{1i} - pl)q_{1i} + (v_{2i} - p_{2i})q_{2i}.\]

(Note that consumer i does not regard the goods as complements or substitutes, but his valuations are unconditionally correlated, as discussed in the next section.) The payoff to firm 1 under the confidential regime is \(p_1 Q_1\), where \(Q_1\) is the mass of consumers accepting its offer, and its payoff under the recognition regime is \(p_1 Q_1 + wx\). The payoff to firm 2 under the confidential regime is \(p_2 Q_2\), where \(Q_2\) is the mass of consumers accepting its offer. Its payoff under the recognition regime is

\[(1 - x)p_2 Q_2 + x(p_1^1 Q_1^1 + p_2^0 Q_2^0) - wx,\]

where \(Q_1^1\) and \(Q_2^0\) are the masses of consumers accepting the offers \(p_1^1\) and \(p_2^0\) respectively. The solution concept for the game is an efficient perfect Bayesian equilibrium (PBE) (i.e., a PBE in which payoff ties are resolved in favor of social efficiency).\(^\text{15}\)

3. Preliminary results

Although \(v_{1i}\) and \(v_{2i}\) are independent given \(\lambda_i\), their unconditional correlation is positive.\(^\text{16}\) A high (low) realization of \(v_{1i}\) is statistically associated with a high (low) value of \(\lambda_i\), which—in turn—is associated with a high (low) realization of \(v_{2i}\).

To see this formally, note that the population is composed of four types of consumers as distinguished by their valuations \((v_{1i}, v_{2i})\). A mass of \(E[X_2]\) of the consumers have valuations \((v_H, v_H)\); a mass of \(E[\lambda] - E[\lambda^2]\) have valuations \((v_H, v_L)\); a mass of \(E[\lambda] + E[\lambda^2]\) have valuations \((v_L, v_H)\); and a mass of \(1 - 2E[\lambda] + E[\lambda^2]\) have valuations \((v_L, v_L)\). Note that the mass of consumers with \(v_1 = v_H\) is \(E[\lambda]\) and the mass of consumers with \(v_1 = v_L\) is \(1 - E[\lambda]\). Using these observations, it is easy to see that the probability that an arbitrary consumer’s second-period valuation is \(v_2 = v_H\) given that his first-period valuation was \(v_1 = v_H\) is

\[E[\lambda \mid v_1 = v_H] = \frac{Pr\{v_2 = v_H \mid v_1 = v_H\}}{Pr\{v_1 = v_H\}} = \frac{E[\lambda^2]}{E[\lambda]},\]

and the probability that \(v_2 = v_H\) given \(v_1 = v_L\) is

\[E[\lambda \mid v_1 = v_L] = \frac{Pr\{v_2 = v_H \mid v_1 = v_L\}}{Pr\{v_1 = v_L\}} = \frac{E[\lambda] - E[\lambda^2]}{1 - E[\lambda]}.\]

Standard computations show that (1) and (2) imply the following basic relationship:

\[E[\lambda \mid v_1 = v_H] > E[\lambda] > E[\lambda \mid v_1 = v_L].\]

Next, let \(k_i\) denote the information firm 2 knows about consumer \(i\) when it makes him an offer. If firm 2 did not buy the customer list, then \(k_i = 0\), and if it did buy the list, then \(k_i = q_{1i}\). Firm 2’s belief is, then, denoted \(Pr\{v_{2i} = v_H \mid k_i\}.\(^\text{17}\)

\(^{15}\) There is generically a unique PBE outcome in each variant of the game considered. The efficiency criterion, therefore, seldom applies.

\(^{16}\) Simple algebra reveals

\[\frac{\text{Cov}[v_1, v_2]}{\sqrt{\text{Var}(v_1)\text{Var}(v_2)}} = \frac{E[\lambda^2] - (E[\lambda])^2}{E[\lambda] - (E[\lambda])^2} > 0.\]

Observe that the numerator of the fraction on the right side of this expression is \(\text{Var}[\lambda]\). Hence, if there were no uncertainty about \(\lambda\), then there would be zero correlation between \(v_1\) and \(v_2\). In other words, it is uncertainty about \(\lambda\) that generates the correlation in valuations.

\(^{17}\) Technically, firm 2’s beliefs about each individual may also depend on the mass of consumers who purchased good 1, \(Q_1\). It turns out, however, that in each version of the model considered below, there is a unique efficient PBE.
Finally, the equilibrium probability that consumer \( i \) accepts an offer of \( p_t \) by firm \( t \) is called the expected demand for good \( t \) by consumer \( i \) and is denoted \( D_{it}(p_t) \). It turns out that only two prices, \( p_t = v_L \) and \( p_t = v_H \), are ever charged by either firm in equilibrium. Hence, the following definition involves no loss of generality.\(^{18}\)

**Definition 1 (elasticity).** Consumer \( i \)'s expected demand for good \( t \) is called elastic, unit-elastic, or inelastic respectively as \( D_{it}(v_H)v_H \) is less than, equal to, or greater than \( D_{it}(v_L)v_L \).

The game is solved via backward induction. Note that consumer \( i \) will purchase good 2 if and only if \( v_{i2} \geq p_{i2} \).\(^{19}\) Hence, the expected demand for good 2 by consumer \( i \) is

\[
D_{i2}(p_{i2}) = \begin{cases} 
1, & \text{if } p_{i2} \leq v_L, \\
\Pr\{v_{i2} = v_H \mid k_i\}, & \text{if } p_{i2} \in (v_L, v_H], \\
0, & \text{if } p_{i2} > v_H.
\end{cases}
\]

Consumer \( i \)'s expected demand for good 2 is elastic, unit-elastic, or inelastic respectively as \( \Pr\{v_{i2} = v_H \mid k_i\} \) is less than, equal to, or greater than \( v_L/v_H \). This serves as proof of the following basic observation.

**Lemma 1 (second-period pricing).** In any PBE, firm 2's pricing strategy must satisfy

\[
p_{i2} = \begin{cases} 
v_L, & \text{if } \Pr\{v_{i2} = v_H \mid k_i\} < v_L/v_H, \\
v_H, & \text{if } \Pr\{v_{i2} = v_H \mid k_i\} > v_L/v_H,
\end{cases}
\]

for any \( i \in [0, 1] \).

If firm 2 purchases the customer list from firm 1 and if

\[
\Pr\{v_{i2} = v_H \mid q_{i1} = 0\} < \frac{v_L}{v_H} < \Pr\{v_{i2} = v_H \mid q_{i1} = 1\},
\]

then firm 2 will offer \( p_{i2}^0 = v_L \) to consumers who did not buy good 1 (because they have elastic expected demand for good 2), and \( p_{i2}^1 = v_H \) to consumers who bought good 1 (because they have inelastic expected demand for good 2).\(^{20}\) In other words, firm 2 will practice dynamic pricing.

It turns out that the most information firm 2 can ever infer from observing consumer \( i \)'s first-period purchasing decision is the value of \( v_{i1} \). Hence, from (1) and (2), a necessary condition for dynamic pricing to occur in equilibrium is

\[
E[\lambda \mid v_1 = v_L] < \frac{v_L}{v_H} < E[\lambda \mid v_1 = v_H]. \tag{4}
\]

Since an equilibrium in which dynamic pricing cannot occur is not very interesting, (4) is assumed to hold below.

\(^{18}\) The formula for arc elasticity in this context is

\[
\eta \equiv \frac{(D_{it}(v_H) - D_{it}(v_L))/(D_{it}(v_H) + D_{it}(v_L)))}{(v_H - v_L)/(v_H + v_L)}.
\]

It is straightforward to verify that \( \eta \) greater than, equal to, or less than one correspond to the respective revenue conditions given in the text.

\(^{19}\) As usual, equilibrium existence implies that almost every consumer \( i \in [0, 1] \) accepts with probability one if \( v_{i2} = p_{i2} \).

\(^{20}\) If expected demand is unit-elastic, then firm 2 is indifferent between charging \( v_L \) and \( v_H \), and equilibrium existence may require it to randomize in this case.
4. The confidential regime

Since there is no informational linkage between the markets in this benchmark setting, the equilibrium of the two-period game is a simple repetition of the one-shot equilibrium.

**Proposition 1 (the confidential regime).** There is a unique efficient PBE under the confidential regime. Equilibrium strategies are as follows:

\[
\begin{align*}
p_t &= \begin{cases} 
v_L, & \text{if } E[\lambda] \leq v_L/v_H, \\ v_H, & \text{if } E[\lambda] > v_L/v_H, \end{cases} \\
q_{ti} &= \begin{cases} 
1, & \text{if } p_t \leq v_{iti}, \\
0, & \text{if } p_t > v_{iti}, \end{cases}
\end{align*}
\]

for any \(i \in [0, 1]\) and \(t = 1, 2\).

This result follows from a simple application of mechanism design (see, for example, Bulow and Roberts, 1989). Note that if expected demand is elastic (or unit-elastic), then \(p_1 = p_2 = v_L\), and all consumers purchase both goods with probability one.\(^{21}\) In this case, the payoff to each firm is simply \(v_L\), and the expected payoff to a given consumer is \(2E[\lambda](v_H - v_L)\). This results in the maximal social surplus of \(2(E[\lambda]v_H + (1 - E[\lambda])v_L)\).

On the other hand, if expected demand is inelastic, then \(p_1 = p_2 = v_H\), and a given consumer buys good \(t\) with probability \(E[\lambda]\). In this case, the payoff to firm \(t\) is \(E[\lambda]v_H\), and every consumer receives a net payoff of zero. Hence, there is deadweight loss of \(2(1 - E[\lambda])v_L\) in this case. This is just the usual monopoly distortion. The firms find it optimal to forgo selling to low-value consumers in order to extract all the surplus from high-value ones. These welfare measures are useful for comparing the equilibrium outcome under the two variants of the disclosure regime studied below.

5. The market for customer information

The following simple result characterizes equilibrium play regarding sale of the customer list under the disclosure regime.

**Lemma 2 (full extraction).** Consider the continuation game beginning at the stage when firm 1 quotes a price for the customer list. If the value to firm 2 from observing the list is strictly positive, then firm 1 quotes a price equal to the value of the information and firm 2 purchases the list with probability one in any PBE. If the value to firm 2 from observing the list is zero, then the payoffs to the firms in any PBE are the same as their payoffs in the PBE in which firm 1 quotes a price of zero and firm 2 purchases the list.

This lemma indicates that there is no loss of generality in concentrating on equilibria in which firm 2 purchases the customer list with probability one. Note that firm 1 extracts the full value of any information embodied in the list because it has all the bargaining power.\(^{22}\) An implication of full extraction is that the equilibrium of the game coincides exactly with the situation in which firm 1 and firm 2 are actually a single entity. In other words, it is also appropriate to interpret the model in the context of a single monopolist that sells both goods and that keeps track of its customers’ purchasing patterns.

6. Naive consumers

Under the confidential regime, the sophistication of consumers was obviously not an issue. Under the disclosure regime, however, whether or not consumers anticipate the sale of their

\(^{21}\) There are multiple PBE if and only if expected demand is unit elastic. The efficiency criterion then dictates \(p_t = v_L\) for \(t = 1, 2\).

\(^{22}\) An arbitrary distribution of bargaining power between the firms can be easily incorporated. The less bargaining power firm 1 possesses, the weaker are its incentives for investing in information acquisition (i.e., experimental pricing) because this investment will be “held up” by firm 2.
information is an important distinction. Naivete is modelled here by supposing that consumers maximize their expected payoff in the first period without regard to how their purchasing decisions will influence the offers they receive in the second period.

**Lemma 3 (the value of information).** The equilibrium value of the customer list to firm 2 when consumers are naive is

\[ W(p_1) = E[V_2]V_H - (1 - E[V_1]) V_L - \max\{v_L, E[V]_v_H\}, \]

if \( p_1 \in (v_L, v_H) \); it is zero otherwise.

To understand this result, first notice that the customer list is valuable to firm 2 only to the extent that it permits it to discriminate among consumers. If \( p_1 > v_H \), then no consumers buy good 1, and if \( p_1 \leq v_L \), then they all do. In either case, the consumers are all observationally equivalent and the customer list is, therefore, worthless.

If, however, \( p_1 \in (v_L, v_H) \), then consumers with \( v_1 = v_H \) buy good 1 and those with \( v_1 = v_L \) do not. In this case, observing \( q_{i1} \) is equivalent to observing \( v_{i1} \). Hence, the probability that consumer \( i \) will pay \( p_2 = v_H \) for good 2 given \( q_{i1} = E[V_i | v_{i1}] \). Condition (4) and Lemma 1 then indicate that firm 2 should charge \( p_2 = v_H \) to consumer \( i \) if he purchased good 1 and \( p_2^0 = v_L \) if he did not. Under this dynamic-pricing scheme, only type-(\( v_H, v_L \)) consumers will not purchase good 2. The revenue accruing to firm 2 from using the customer list to price discriminate, therefore, is

\[ E[V_2]V_H + (1 - E[V_1]) V_L. \]

The value of the list to firm 2 is equal to the increase in its revenue from price discrimination. If \( v_L \geq E[V]_v_H \) and firm 2 did not have access to the list, then it would charge \( p_2 = v_L \) to all consumers. In this case, the value of the list derives from the ability to charge \( v_H \) to consumers who purchased good 1. Similarly, if \( E[V]_v_H > v_L \) and firm 2 did not have access to the list, then it would charge all consumers \( p_2 = v_H \). In this case, the value of the list derives from the ability to charge \( v_L \) to consumers who did not purchase good 1.

In order to characterize the equilibrium outcome of the game in this environment, define the constant

\[ \frac{1 + E[V | v = v_H]}{1 + E[V]} \]

Note that (3) implies \( \bar{V} > 1 \).

**Proposition 2 (naive consumers).** There is a unique efficient PBE outcome of the game under the disclosure regime when consumers are naive, and it is characterized as follows.

(i) If \( \bar{V}E[V] \leq v_L/v_H \), then firm 1 charges \( p_1 = v_H \); all consumers purchase good 1; the price of the customer list is zero; firm 2 charges \( p_2 = v_L \) to all consumers; all consumers purchase good 2.

(ii) If \( v_L/v_H < \bar{V}E[V] \), then firm 1 charges \( p_1 = v_H \); consumer \( i \) purchases good 1 if and only if \( v_{i1} = v_H \); the price of the customer list is positive; firm 2 charges \( p_2^i = v_H \) to consumers who purchased good 1 and \( p_2^0 = v_L \) to those who did not; only type-(\( v_H, v_L \)) consumers do not purchase good 2.

The most novel aspect of this result concerns firm 1’s pricing policy relative to the confidential regime. Even though consumer demand is the same under the confidential regime and the disclosure regime with naive consumers, firm 1 does not follow the same pricing strategy in equilibrium. When expected demand is elastic (i.e., \( E[V] \leq v_L/v_H \)), firm 1 faces a tradeoff under the disclosure regime with naive consumers. Its revenue from selling good 1 is maximized by charging \( p_1 = v_L \) (as it would do under the confidential regime), but this results in a worthless customer list (because all consumers buy). On the other hand, the value of the list is maximized...
by charging $p_1 = v_H$, but this generates less than optimal sales revenue, $E[\lambda]v_H < v_L$. So long as expected demand is not too elastic (i.e., $v_L/v_H < \frac{1}{\gamma}E[\lambda]$), firm 1 finds it optimal to sacrifice some revenue from selling good 1 in order to preserve the value of the list. That is, it experiments by charging a high price in order to generate valuable information.

The following welfare observations follow more or less directly from Proposition 2. (Note that firm 2 is always indifferent between the disclosure regime and the confidential regime because firm 1 extracts the full value of the customer list from it. Also, type-$(v_L, v_L)$ consumers are indifferent between the two regimes because they always receive zero surplus.)

**Corollary 1 (welfare with naive consumers).** When consumers are naive, the following equilibrium welfare comparisons hold.

(i) If $E[\lambda] < v_L/v_H$, then the confidential and disclosure regimes give rise to the same efficient outcome.

(ii) If $E[\lambda] < v_L/v_H < \frac{1}{\gamma}E[\lambda]$, then type-$(v_H, v_H)$ and -$(v_H, v_L)$ consumers are better off under the confidential regime; type-$(v_L, v_H)$ consumers are indifferent between the two regimes; firm 1 is better off under the disclosure regime; social surplus is higher under the confidential regime.

(iii) If $v_L/v_H < E[\lambda]$, then type-$(v_H, v_H)$ and -$(v_H, v_L)$ consumers are indifferent between the two regimes; type-$(v_L, v_H)$ consumers are better off under the disclosure regime; firm 1 is better off under the disclosure regime; social surplus is higher under the disclosure regime.

These welfare results are easily explained. First, if expected demand is sufficiently elastic, then firm 1 forsakes the market for information and prices at $v_L$. Moreover, since firm 2 learns nothing, it also prices at $v_L$ to all of the consumers. Hence, the outcome is the same as under the confidential regime. If, however, $v_L/v_H \in [E[\lambda], \frac{1}{\gamma}E[\lambda]]$, then firm 1 charges $v_H$ and firm 2 prices dynamically under the disclosure regime, while they both would have charged $v_L$ under the confidential regime. This results in higher producer surplus, lower consumer surplus, and lower total surplus overall. If, however, expected demand is inelastic, then the market for information creates a welfare improvement relative to the confidential regime. In particular, dynamic pricing results in lower prices and higher sales volume for good 2. Hence, the welfare impact of the market for information depends critically on what prices would be charged if sale of the customer list was not possible. It also depends critically on whether consumers anticipate sale of the list, as is demonstrated in the next section.

7. Sophisticated consumers

In this section, the unique efficient PBE outcome of the game under the disclosure regime is derived, assuming that consumers fully anticipate sale of the customer list. The key to the analysis is the determination of the expected demand function for good 1. This somewhat involved derivation appears in the Appendix as the proof of the following result.

**Proposition 3 (first-period demand).** Consider any efficient PBE of the game under the disclosure regime with sophisticated consumers. The mass of consumers accepting any offer $p_1 \in \mathbb{R}_+$ is

$$D_1(p_1) = \begin{cases} 1, & \text{if } p_1 \leq v_L, \\ E[\lambda] - p^* E[\lambda^2], & \text{if } p_1 \in (v_L, v_H), \\ 0, & \text{if } p_1 > v_H, \end{cases}$$

23 Under the disclosure regime, type-$(v_L, v_H)$ and -$(v_L, v_L)$ consumers do not buy good 1 and type-$(v_H, v_L)$ consumers do not buy good 2. This results in deadweight loss of $(1 - E[\lambda^2]) v_L$.

24 If expected demand is inelastic, then deadweight loss under the confidential regime is $2(1 - E[\lambda]) v_L$, which is easily seen to be greater than the deadweight loss under the disclosure regime of $(1 - E[\lambda^2]) v_L$. 

where
\[ \rho^* = \frac{E[\lambda^2]v_H + (1 - E[\lambda])v_L - \max\{v_L, E[\lambda]v_H\}}{E[\lambda^2](v_H - v_L)}. \]

The key difference between the naive- and sophisticated-consumer settings occurs over the price range \( p_1 \in (v_L, v_H) \). Three of the four types of consumers behave identically across the two settings when confronted with such prices: type-(\( v_H, v_L \)) consumers buy, and type-(\( v_L, v_H \)) and type-(\( v_L, v_L \)) consumers do not. Type-(\( v_H, v_H \)) consumers, however, do not behave the same across the two settings. Under the naive-consumer setting, all type-(\( v_H, v_H \)) individuals buy good 1 if \( p_1 \in (v_L, v_H) \), while under the sophisticated-consumer setting, a strictly positive fraction \( \rho^* \) of them reject such offers.\(^{25}\)

The intuition underlying this strategic-demand reduction is easily grasped. If—as in the naive-consumer setting—all type-(\( v_H, v_H \)) consumers accepted an offer of \( p_1 \in (v_L, v_H) \), then firm 2’s beliefs following an acceptance would be \( E[\lambda | v_1 = v_H] \), and its beliefs following a rejection would be \( E[\lambda | v_1 = v_L] \). In this case, condition (4) and Lemma 1 indicate that it would be optimal for firm 2 to set \( P_2 = v_H \) and \( p_2^0 = v_L \). This, however, cannot occur in equilibrium when consumers are sophisticated because
\[
v_H - p_1 + v_H - p_2^1 < v_H - p_2^0
\]
holds for \( p_1 \in (v_L, v_H) \). In other words, given firm 2’s beliefs and concomitant dynamic pricing, all type-(\( v_H, v_H \)) consumers would rather reject (not accept) the offer on good 1 in order to obtain a better offer on good 2. In fact, a similar—but slightly more involved—argument shows that no fraction less than \( \rho^* \) of type-(\( v_H, v_H \)) consumers can reject \( p_1 \in (v_L, v_H) \) in equilibrium.

On the other hand, no fraction higher than \( \rho^* \) of the type-(\( v_H, v_H \)) consumers can reject offers \( p_1 \in (v_L, v_H) \) either. For the intuition, suppose they all rejected such an offer. Then only type-(\( v_H, v_L \)) consumers would buy good 1. Given this, it would be optimal for firm 2 to set \( p_2^1 = v_H \) and \( p_2^0 = v_L \). This, however, cannot occur in equilibrium when consumers are sophisticated because
\[
v_H - p_2^0 < v_H - p_1 + v_H - p_2^1
\]
holds for \( p_2^0 \in \{v_L, v_H\} \) and \( p_1 \in (v_L, v_H) \).\(^{26}\) In other words, all type-(\( v_H, v_H \)) consumers would rather accept (not reject) the offer on good 1. Again, the general result follows from a similar—but slightly more involved—argument.

In order to characterize the equilibrium outcome of the entire game under the sophisticated-consumer setting, define the constant
\[ \gamma \equiv \frac{1}{E[\lambda]} \left(1 - \sqrt{E[(1-\lambda)^2]}\right). \]
Simple algebra and the fact that \( E[\lambda^2] - (E[\lambda])^2 > 0 \) establish that \( \gamma < 1 \).

Proposition 4 (sophisticated consumers). There is a unique efficient PBE outcome of the game under the disclosure regime when consumers are sophisticated, and it is characterized as follows.

(i) If \( E[\lambda] \leq v_L/v_H \), then firm 1 charges \( p_1 = v_L \); all consumers purchase good 1; the price of the customer list is zero; firm 2 charges \( p_2^1 = v_L \) to all consumers; all consumers purchase good 2.

(ii) If \( \gamma < E[\lambda] < v_L/v_H \), then firm 1 charges \( p_1 = v_L \); all consumers purchase

\(^{25}\) Simple algebra reveals \( \rho^* \in (0, 1) \).

\(^{26}\) The strict inequality is violated if \( p_2^0 = v_L \) and \( p_1 = v_H \). In this case, an “open-set” argument establishes the claim.
good 1; the price of the customer list is zero; firm 2 charges $p_2^1 = v_H$ to all consumers; consumer $i$ purchases good 2 if and only if $v_{i2} = v_H$. 

(iii) If $v_L / v_H < \gamma E[\lambda]$, then firm 1 charges $p_1 = v_H$; all type-$(v_H, v_L)$ consumers and $(1 - \rho^*)$ of the type-$(v_H, v_H)$ consumers buy good 1; the price of the customer list is zero; firm 2 charges $p_2^0 = p_2^2 = v_H$ to all consumers; consumer $i$ purchases good 2 if and only if $v_{i2} = v_H$.

This result exhibits some rather striking reversals from the naive-consumer setting. In particular, the strategic rejections by type-$(v_H, v_H)$ consumers lead to a more elastic expected demand function and a correspondingly larger range of parameter values over which firm 1 finds it optimal to set a low price. Hence, for $v_L / v_H \in [\gamma E[\lambda], \overline{\gamma E[\lambda]}]$, firm 1 sets $p_1 = v_L$ if consumers are sophisticated, whereas it sets $p_1 = v_H$ if they are naive. The reason for this difference is easy to understand. Recall that if consumers are naive and $v_L / v_H < \gamma E[\lambda]$, firm 1 sets $p_1 = v_H$ in order to maximize the sum of the revenue from selling good 1 and the revenue from selling the customer list. If consumers are sophisticated, however, then the “effective” expected demand for good 1 is lower and (perhaps most strikingly) the customer list is always worth zero because sophisticated consumers avoid revealing information that will hurt them. Thus, firm 1 finds it optimal to set $p_1 = v_L$ when $\gamma E[\lambda] \leq v_L / v_H$ (i.e., when effective expected demand is elastic) in order to maximize the revenue from selling good 1 alone. Note, in fact, that when $v_L / v_H \in [\gamma E[\lambda], E[\lambda]]$, expected demand for good 1 is elastic (because of the strategic-demand reduction), while expected demand for good 2 is inelastic. Hence, in this region of the parameter space, $p_1 = v_L$ and $p_2 = v_H$ for $i \in [0, 1]$. In other words, firm 1 receives a lower price and lower profit than firm 2 because of the strategic-demand reduction for good 1.

The customer list is worthless when consumers are sophisticated precisely because of the strategic rejections by type-$(v_H, v_H)$ consumers. In particular, if $E[\lambda] \leq v_L / v_H$, then $\rho^*$ is calibrated so that

$$\Pr\{v_{i2} = v_H \mid q_{i1} = 0\} < v_L / v_H = \Pr\{v_{i2} = v_H \mid q_{i1} = 1\}.$$ 

In this instance, however, Lemma 1 indicates that it is (weakly) optimal for firm 2 to set $p_{i2} = v_L$ even if $q_{i1} = 1$. In other words, purchase of good 1 does not provide a strong enough signal that $v_{i2} = v_H$ to justify dynamic pricing. Similarly, if $E[\lambda] > v_L / v_H$, then $\rho^*$ is calibrated so that

$$\Pr\{v_{i2} = v_H \mid q_{i1} = 0\} = v_L / v_H < \Pr\{v_{i2} = v_H \mid q_{i1} = 1\}.$$ 

In this case, Lemma 1 indicates that it is (weakly) optimal for firm 2 to set $p_{i2} = v_H$ even if $q_{i1} = 0$. In other words, refusal to purchase good 1 does not provide a strong enough signal that $v_{i2} = v_L$ to justify dynamic pricing.

The following welfare observations follow more or less directly from Proposition 4. (Note that firm 2 is always indifferent between the disclosure regime and the confidential regime because it learns no valuable information in either case and consumers behave identically in the second period under either regime. Also, type-$(v_L, v_L)$ consumers are indifferent between the two regimes because they always receive zero surplus.)

Corollary 2 (welfare with sophisticated consumers). When consumers are sophisticated, the following equilibrium welfare comparisons hold.

(i) If $E[\lambda] \leq v_L / v_H$, then the confidential and disclosure regimes give rise to the same efficient outcome.

(ii) If $\gamma E[\lambda] \leq v_L / v_H < E[\lambda]$, then type-$(v_H, v_H)$ and $(v_H, v_L)$ consumers are better off under the disclosure regime; type-$(v_L, v_H)$ consumers are indifferent between the two regimes; firm 1 is better off under the confidential regime; social surplus is higher under the disclosure regime.

(iii) If $v_L/v_H < \gamma E[\lambda]$, then all consumers are indifferent between the two regimes; firm 1 is better off under the confidential regime; social surplus is higher under the confidential regime.

These welfare results are easily explained. First, if $E[\lambda] \leq v_L/v_H$, then expected demand is elastic under both regimes and firm 1 optimally prices at $p_1 = v_L$. Moreover, since firm 2 learns nothing, it also prices at $p_2 = v_L$ to all of the consumers. If, however, $v_L/v_H \in [\gamma E[\lambda], E[\lambda])$, then firm 1 charges $p_1 = v_L$ under the disclosure regime (because effective expected demand is elastic) and $p_1 = v_H$ under the confidential regime (because expected demand is inelastic). Firm 2 learns no valuable information under either regime and, therefore, sets $p_2 = v_H$ to all consumers under both regimes. The lower price for good 1 under the disclosure regime results in higher consumer surplus, lower producer surplus, and higher total surplus over all. If, however, $v_L/v_H < \gamma E[\lambda]$, then expected demand is inelastic in both periods under both regimes and prices are always $v_H$. While the consumers are obviously, therefore, indifferent between the two settings, firm 1 prefers the confidential regime where it earns $E[\lambda]v_H$ rather than $(E[\lambda] - \rho^* E[\lambda^2])v_H$. Indeed, the deadweight loss created by strategic-demand reduction in this case exacerbates the inefficiency due to monopoly pricing.

Note that—in contrast to the case of naive consumers—when consumers are sophisticated, firm 1 always (weakly) prefers the confidential regime to the disclosure regime. In other words, firm 1 would like to publicly adopt a policy committing not to sell the customer list or support a public policy that disallows the resale of lists. Without such commitment, it faces strategic-demand reduction that both reduces its sales revenue and undermines the market for information. A commitment not to sell the customer list is, however, not always good for consumers or for social surplus. In particular, the fact that expected demand is more elastic under the disclosure regime can induce firm 1 to post a lower price that generates higher sales volume than under the confidential regime. When expected demand is quite inelastic, however, a commitment not to sell the customer list enhances welfare. While it does not solve the problem of monopoly pricing, it does eliminate the deadweight loss of $\rho^* E[\lambda^2]v_H$ from strategic-demand reduction.

8. Conclusion

- At its core, this article is concerned with property rights. Does a firm have the right to collect and sell valuable information about the identity and purchasing habits of its customers, or do consumers have the right to anonymity? Both settings were analyzed in the context of a simple strategic model without commitment.

It was shown that firms fare well under a customer disclosure regime when consumers do not anticipate the sale of their information. Indeed, in such a setting the opportunity to sell its customer list can provide a firm with incentives to charge high experimental prices. Such experimentation unambiguously lowers welfare because the loss in consumer surplus outweighs the value of the information obtained by the firms. When demand is very inelastic, however, welfare is actually higher under the disclosure regime when consumers are naive because firms offer lower prices to customers who did not previously purchase.

These welfare comparisons are modified sharply if consumers anticipate sale of the list. In this case, some consumers with high valuations engage in strategic-demand reduction when confronted with high prices. This has two important consequences. First, it undermines the market for customer information, since it results in a worthless customer list. Second, effective demand becomes more elastic, which can lead to lower equilibrium prices and higher welfare. Indeed, when consumers anticipate sale of the customer list, the firms would prefer to commit to not selling it, i.e., to adopt a binding privacy policy. Perhaps surprisingly, adoption of such a policy is not always good for welfare.

This article is an early exploration of a vein of research that is rich and relatively untapped. The growing ability of firms to store and recall customer information is reshaping markets and changing the landscape of competition. For instance, one often-proclaimed benefit of a regime with customer recognition is that it reduces consumer search by allowing firms to recommend
products and services in accordance with consumer profile data. This potential benefit was not
captured in the model presented above, and it would be interesting to see how it might modify the
findings. There are also interesting issues concerning the mode of competition in markets where
information about customers is fast becoming an essential ingredient for success. Finally, there
is a host of open policy questions surrounding privacy rights in electronic retailing. In short, it is
safe to say that economists and policy makers are only beginning to understand the social costs
and benefits of consumer privacy and the market for customer information.

Appendix

Proofs of Propositions 2, 3, and 4 follow.

Proof of Proposition 2. First, note from the specification of expected demand and from the formula for \( \tilde{W}(p_1) \) given in Lemma 3 that one of two prices, \( p_1 = v_L \) or \( p_1 = v_H \), must be optimal for firm 1.

Now, suppose \( E[\lambda] < v_L/v_H \). If firm 1 sets \( p_1 = v_H \), then it earns revenue from selling good 1 of \( E[\lambda] v_H \) and revenue from selling the customer list of \( E[\lambda] v_H - E[\lambda] v_L \). If it sets \( p_1 = v_L \), then it earns revenue from selling good 1 of \( v_L \) and revenue from selling the customer list of zero. Simple algebra then reveals

\[
E[\lambda] v_H + E[\lambda] v_H - E[\lambda] v_L \leq v_L \iff E[\lambda] \leq v_L/v_H.
\]

Next, suppose \( E[\lambda] > v_L/v_H \). If firm 1 sets \( p_1 = v_H \), then it earns revenue from selling good 1 of \( E[\lambda] v_H \) and revenue from selling the customer list of \( E[\lambda] v_H + (1 - E[\lambda]) v_L \). If it sets \( p_1 = v_L \), then it earns revenue from selling good 1 of \( v_L \) and revenue from selling the customer list of zero. Simple algebra and expression (1) then reveal

\[
E[\lambda] v_H + (E[\lambda] - E[\lambda]) v_H + (1 - E[\lambda]) v_L > v_L \iff E[\lambda | \lambda = v_H] > v_L/v_H.
\]

This holds by condition (4).

The rest of the claim follows directly from Lemmas 1 and 3. In particular, note that firm 2’s beliefs off the equilibrium path (e.g., if \( Q \neq D_1(p_1) \)) are irrelevant when consumers are naive. Q.E.D.

Proof of Proposition 3. The claim is established with a series of four lemmas. First, define \( \phi_0 (\phi_1) \) to be the probability that firm 2 charges consumer \( i \) \( p_{i2} = v_H \) if \( q_{i1} = 0 \) (respectively, \( q_{i1} = 1 \)). Similarly, \( 1 - \phi_0 \) (respectively, \( 1 - \phi_1 \)) is the probability that firm 2 charges consumer \( i \) \( p_{i2} = v_H \) if \( q_{i1} = 0 \) (respectively, \( q_{i1} = 1 \)). In this context, Lemma 1 says that

\[
\phi_{q_{i1}} = \begin{cases} 
1, & \text{if } Pr\{v_{i2} = v_H \mid q_{i1}\} < v_L/v_H, \\
\in [0, 1], & \text{if } Pr\{v_{i2} = v_H \mid q_{i1}\} = v_L/v_H, \\
0, & \text{if } Pr\{v_{i2} = v_H \mid q_{i1}\} > v_L/v_H.
\end{cases}
\]

The first basic lemma is used extensively in the proofs of the other three.

Lemma A1 (consumer incentives). In any PBE of the game under the disclosure regime with sophisticated consumers, consumer \( i \)'s first-period purchase decision must satisfy

\[
q_{i1} = \begin{cases} 
1, & \text{if } v_1 - p_1 > (\phi_0 - \phi_1)(v_{i2} - v_L), \\
0, & \text{if } v_1 - p_1 < (\phi_0 - \phi_1)(v_{i2} - v_L).
\end{cases}
\]

Proof. Consumer \( i \)'s expected payoff from purchasing good 1 for \( p_1 = v_1 - p_1 + \phi_1(v_{i2} - v_L) \), and his expected payoff from refusing to buy good 1 is \( \phi_0(v_{i2} - v_L) \). Q.E.D.

The next result shows that \( D_1(p_1) = 1 \) if \( p_1 < v_L \) and \( D_1(p_1) = 0 \) if \( p_1 > v_H \).

Lemma A2 (no signalling). Consider the continuation game that begins after firm 1 sets \( p_1 \) under the disclosure regime with sophisticated consumers.

(i) If \( p_1 < v_L \), then at least one PBE of the continuation game exists, and all consumers purchase good 1 in every PBE.

(ii) If \( p_1 > v_H \), then at least one PBE of the continuation game exists, and no consumer purchases good 1 in any PBE.

Proof. (i) Suppose \( p_1 < v_L \). First it is shown that all consumers buy good 1 in every PBE of the continuation game. Note that Lemma A1 indicates that all type-(\( v_H, v_H \)), -(\( v_H, v_L \)), and -(\( v_L, v_L \)) consumers will purchase good 1 because
\[ v_{11} - p_1 > (\phi_0 - \phi_1)(v_{12} - u_L), \quad \forall \phi_0 \text{ and } \phi_1 \in [0, 1]. \]

To see that all type-\((v_L, v_H)\) consumers must also buy, suppose to the contrary that there is a PBE in which a positive measure of type-\((v_L, v_H)\) consumers refuse to buy good 1. Then since all other types accept the offer, firm 2 knows which type of consumer it faces following a rejection. In particular, \(\Pr\{v_{12} = v_H \mid q_{11} = 0\} = 1\). Lemma 1 then gives \(\phi_0 = 0\). But then
\[ v_L - p_1 > (0 - \phi_1)(v_H - u_L), \quad \forall \phi_1 \in [0, 1]. \]

In other words, Lemma A1 indicates that all type-\((v_L, v_H)\) consumers strictly prefer to purchase good 1, contrary to supposition.

To establish existence, observe that the belief function,
\[
\Pr\{v_{12} = v_H \mid q_{11}\} = \begin{cases} E[\lambda], & \text{if } q_{11} = 1, \\ 1, & \text{if } q_{11} = 0, \end{cases}
\]
supports the outcome in which all consumers buy good 1 as a PBE of the continuation game.

(ii) Suppose \(p_1 > v_H\). First it is shown that no consumer buys good 1 in any PBE of the continuation game. Note that Lemma A1 indicates that all type-\((v_H, v_L), -(v_L, v_H), \) and \(-(v_L, v_L)\) consumers will refuse to purchase good 1 because
\[ v_{11} - p_1 < (\phi_0 - \phi_1)(v_{12} - u_L), \quad \forall \phi_0 \text{ and } \phi_1 \in [0, 1]. \]

To see that all type-\((v_H, v_H)\) consumers must also refuse to buy, suppose to the contrary that there is a PBE in which a positive measure of type-\((v_H, v_H)\) consumers buy good 1. Then since all other types reject such an offer, firm 2 knows which type of consumer it faces following an acceptance. In particular, \(\Pr\{v_{12} = v_H \mid q_{11} = 1\} = 1\). Lemma 1 then gives \(\phi_1 = 0\). But then
\[ v_H - p_1 < (\phi_0 - 0)(v_H - u_L), \quad \forall \phi_0 \in [0, 1]. \]

In other words, Lemma A1 indicates that all type-\((v_H, v_H)\) consumers strictly prefer not to purchase good 1, contrary to supposition.

To establish existence, observe that the belief function,
\[
\Pr\{v_{12} = v_H \mid q_{11}\} = \begin{cases} 1, & \text{if } q_{11} = 1, \\ E[\lambda], & \text{if } q_{11} = 0, \end{cases}
\]
supports the outcome in which no consumers buy good 1 as a PBE of the continuation game. Q.E.D.

The next result establishes that \(D_1(p_1) = E[\lambda] - \rho^*E[\lambda^2] \) if \(p_1 \in (v_L, v_H)\).

**Lemma A3 (strategic rejections).** Consider the continuation game that begins after firm 1 sets \(p_1\) under the disclosure regime with sophisticated consumers. If \(p_1 \in (v_L, v_H)\), then at least one PBE of the continuation game exists, and the following claims hold in every PBE.

(i) All type-\((v_L, v_L)\) consumers refuse to purchase good 1.

(ii) All type-\((v_H, v_L)\) consumers purchase good 1.

(iii) All type-\((v_L, v_H)\) consumers refuse to purchase good 1.

(iv) A fraction \(\rho^*\) of type-\((v_H, v_H)\) consumers refuse to purchase good 1.

**Proof.** Suppose \(p_1 \in (v_L, v_H)\). The first four steps of this proof demonstrate that only the behavior described in the lemma can occur in a PBE of the continuation game. The final step then verifies that a PBE involving this behavior exists.

(i) Note that
\[ v_L - p_1 < (\phi_0 - \phi_1)(v_L - u_L), \quad \forall \phi_0 \text{ and } \phi_1 \in [0, 1]. \]
Hence, Lemma A1 indicates that no type-\((v_L, v_L)\) consumers will purchase good 1 in any PBE of the continuation game.

(ii) Note that
\[ v_H - p_1 > (\phi_0 - \phi_1)(v_L - u_L), \quad \forall \phi_0 \text{ and } \phi_1 \in [0, 1]. \]
Hence, Lemma A1 indicates that all type-\((v_H, v_L)\) consumers will purchase good 1 in every PBE of the continuation game.

(iii) To see that no type-\((v_L, v_H)\) consumers will purchase good 1 in any PBE of the continuation game, suppose to the contrary that there is a PBE in which a fraction \(\alpha > 0\) of type-\((v_L, v_H)\) consumers purchase good 1. A necessary
condition for this is
\[ v_L - p_1 \geq (\phi_0 - \phi_1)(v_H - v_L). \]
Since the left side of this inequality is negative, it must be that \( \phi_0 < \phi_1 \). This being the case, note that
\[ v_H - p_1 > (\phi_0 - \phi_1)(v_H - v_L). \]

Hence, all type-\((v_H, v_H)\) consumers purchase good 1 in this PBE. Combining this with the previous two parts of the proof gives
\[ \Pr\{v_H = v_H \mid q_{i1} = 1\} = \frac{E[\lambda^2] + \alpha \left( E[\lambda] - E[\lambda^2] \right)}{E[\lambda] + \alpha \left( E[\lambda] - E[\lambda^2] \right)}. \]

and
\[ \Pr\{v_H = v_H \mid q_{i1} = 0\} = \frac{(1 - \alpha) \left( E[\lambda] - E[\lambda^2] \right)}{1 - 2E[\lambda] + E[\lambda^2] + (1 - \alpha) \left( E[\lambda] - E[\lambda^2] \right)}. \]
The first of these expressions is increasing and the second is decreasing in \( \alpha \). Moreover, for \( \alpha = 0 \), the first expression equals \( E[\lambda \mid v_1 = v_H] \) and the second equals \( E[\lambda \mid v_1 = v_L] \). Applying (4) then yields
\[ \Pr\{v_H = v_H \mid q_{i1} = 0\} < v_L/v_H < \Pr\{v_H = v_H \mid q_{i1} = 1\}. \]
Then, Lemma 1 then dictates that \( \phi_0 = 1 \) and \( \phi_1 = 0 \), in which case
\[ v_L - p_1 < (\phi_0 - \phi_1)(v_H - v_L). \]
But then, Lemma A1 indicates that no type-\((v_L, v_H)\) consumers buy good 1, contrary to supposition.

(iv) Showing that a fraction \( \rho^* \) of type-\((v_L, v_H)\) consumers reject \( p_1 \) in any PBE is somewhat involved. First, for \( \rho \in [0, 1] \), define the functions
\[ \pi_0(\rho) = \frac{E[\lambda] - E[\lambda^2] + \rho E[\lambda^2]}{1 - E[\lambda] + \rho E[\lambda^2]}, \]
and
\[ \pi_1(\rho) = \frac{E[\lambda^2] - \rho E[\lambda^2]}{E[\lambda] - \rho E[\lambda^2]} \]
Then, when a fraction \( \rho \) of type-\((v_L, v_H)\) consumers refuse to purchase good 1 and when all other types behave according to the claim, beliefs must satisfy
\[ \Pr\{v_H = v_H \mid q_{i1} \} = \pi_{q_{i1}}(\rho), \quad q_{i1} \in \{0, 1\}. \]
Next, note that \( \pi_0(\rho) \) is monotone increasing with \( \pi_0(0) = E[\lambda \mid v_1 = v_L] \) (review (2)) and \( \pi_1(\rho) \) is monotone decreasing with \( \pi_1(0) = E[\lambda \mid v_1 = v_H] \) (review (1)). Moreover, define
\[ \bar{\rho} = \frac{E[\lambda^2] - (E[\lambda])^2}{E[\lambda^2](1 - E[\lambda])}. \]
Then, simple algebra verifies that \( \bar{\rho} < 1 \) and
\[ \pi_0(\bar{\rho}) = E[\lambda] = \pi_1(\bar{\rho}) \]
In other words, the increasing function \( \pi_0(\rho) \) crosses the decreasing function \( \pi_1(\rho) \) at the point \((\bar{\rho}, E[\lambda])\). Simple algebra verifies the following implications:
\[ E[\lambda] < \frac{v_L}{v_H} \Rightarrow \pi_0(\rho^*) < \frac{v_L}{v_H} = \pi_1(\rho^*), \quad (A1) \]
\[ E[\lambda] > \frac{v_L}{v_H} \Rightarrow \pi_0(\rho^*) = \frac{v_L}{v_H} < \pi_1(\rho^*), \quad (A2) \]
and
\[ E[\lambda] = \frac{v_L}{v_H} \Rightarrow \rho^* = \bar{\rho}. \quad (A3) \]
With all this in hand, it is finally possible to prove that a fraction $\rho^*$ of type-$(v_H, v_H)$ consumers reject $p_1 \in (v_L, v_H)$ in any PBE.

First, by way of contradiction, suppose there is a PBE in which a fraction $\rho < \rho^*$ of type-$(v_H, v_H)$ consumers reject. Then

$$\pi_0(\rho) < \frac{v_L}{v_H} < \pi_1(\rho).$$

Lemma 1 then requires $\phi_0 = 1$ and $\phi_1 = 0$. But this implies

$$v_H - p_1 = (\phi_0 - \phi_1)(v_H - v_L).$$

Lemma A1 then implies that $\rho = 1$, contrary to supposition.

Next, by way of contradiction, suppose there is a PBE in which a fraction $\rho > \rho^*$ of type-$(v_H, v_H)$ consumers reject. There are two cases to consider.

(a) If $E[\lambda] \leq v_L/v_H$, then (A1) gives $\pi_1(\rho) < v_L/v_H$. Lemma 1 then requires $\phi_1 = 1$. But this implies

$$v_H - p_1 > (\phi_0 - \phi_1)(v_H - v_L).$$

Lemma A1 then implies that $\rho = 0$, contrary to supposition.

(b) If $E[\lambda] \geq v_L/v_H$, then (A2) and (A3) give $\pi_0(\rho) > v_L/v_H$. Lemma 1 then requires $\phi_0 = 0$. But this implies

$$v_H - p_1 > (\phi_0 - \phi_1)(v_H - v_L).$$

Lemma A1 then implies that $\rho = 0$, contrary to supposition.

(v) To establish existence, suppose that firm 2’s beliefs after observing the customer list are given by

$$\Pr\{\psi_2 = v_H \mid q_{1i}\} = \pi_{q_1}(\rho^*), \quad q_{1i} \in \{0, 1\}. \quad (A4)$$

A necessary condition for a fraction $\rho^*$ of type-$(v_H, v_H)$ consumers to refuse to buy good 1 is

$$v_H - p_1 = (\phi_0 - \phi_1)(v_H - v_L). \quad (A5)$$

Moreover, if this is satisfied, then Lemma A1 indicates that type-$(v_H, v_H)$ consumers prefer to buy good 1 and type-$(v_L, v_L)$ and $(v_L, v_H)$ consumers prefer not to buy it. Hence, if (A5) holds and a fraction $\rho^*$ of type-$(v_H, v_H)$ consumers refuse to buy good 1 (as they must in equilibrium), then the specified beliefs will be correct. The only question is whether values for $\phi_0$ and $\phi_1$ exist that satisfy (A5) and are consistent with equilibrium behavior by firm 2. There are three cases to consider.

(a) If $E[\lambda] < v_L/v_H$, then by (A1) and Lemma 1, $\phi_0 = 1$, and the value of $\phi_1$ is unrestricted. Hence, (A5) is satisfied in this case if and only if

$$\phi_1 = 1 - \frac{v_H - p_1}{v_H - v_L}, \quad (A6)$$

which is feasible.

(b) If $E[\lambda] > v_L/v_H$, then by (A2) and Lemma 1, $\phi_1 = 0$, and the value of $\phi_0$ is unrestricted. Hence, (A5) is satisfied in this case if and only if

$$\phi_0 = \frac{v_H - p_1}{v_H - v_L}, \quad (A7)$$

which is feasible.

(c) Finally, if $E[\lambda] = v_L/v_H$, then by (A3) and Lemma 1, the values of $\phi_0$ and $\phi_1$ are both unrestricted. Hence, (A5) is satisfied in this case if and only if

$$\phi_0 - \phi_1 = \frac{v_H - p_1}{v_H - v_L}, \quad (A8)$$

which is feasible. Q.E.D.

The following result completes the proof by showing that $D_1(p_1) = 1$ if $p_1 = v_L$ and $D_1(p_1) = E[\lambda] - \rho^*E[\lambda^2]$ if $p_1 = v_H$. © RAND 2004.
Lemma A4 (the critical prices). Consider the continuation game that begins after firm 1 sets \( p_1 \) under the disclosure regime with sophisticated consumers.

(i) If \( p_1 = v_L \), then there exists a PBE of the continuation game in which all consumers purchase good 1. Moreover, no other PBE of the continuation game yields a higher expected payoff to firm 1.

(ii) If \( p_1 = v_H \), then there exists a PBE of the continuation game in which the purchasing pattern of the consumers coincides with the one given in Lemma A3. Moreover, no other PBE of the continuation game yields a higher expected payoff to firm 1.

Proof. (i) Suppose \( p_1 = v_L \). To establish existence, consider the belief function

\[
\Pr\{v_2 = v_H \mid q_1\} = \begin{cases} 
E[\lambda], & \text{if } q_1 = 1, \\
1, & \text{if } q_1 = 0.
\end{cases}
\]

Given these beliefs, Lemma 1 requires \( \phi_0 = 0 \). But it is then a best response for all consumers to accept because

\[
v_{11} - v_L \geq (\phi_0 - \phi_1)(v_{12} - v_L).
\]

Also, when all consumers accept, beliefs are correct on the equilibrium path.

To see that no other PBE of the continuation game delivers a higher expected payoff to firm 1, note that no other PBE has higher sales volume. Hence, a more profitable PBE must involve lower sales volume and positive value for the customer list. If the customer list has positive value, then either

\[
(\text{A9}) \quad \Pr\{v_2 = v_H \mid q_1 = 0\} < \frac{v_L}{v_H} < \Pr\{v_2 = v_H \mid q_1 = 1\}.
\]

(a) First, there does not exist a PBE of the continuation game with sales volume less than one satisfying (A9). To see this, suppose otherwise. Note that Lemma 1 gives \( \phi_0 = 0 \) and \( \phi_1 = 1 \). Given this, Lemma A1 indicates that only type-(\( v_L, v_L \)) consumers reject because

\[
v_{11} - v_L > -(v_{12} - v_L)
\]

for all other types. But then,

\[
\Pr\{v_2 = v_H \mid q_1 = 0\} < v_L/v_H,
\]

contradicting (A9).

(b) Next, a PBE of the continuation game with sales volume less than one satisfying (A10) does exist, but all such equilibria deliver strictly lower payoffs than \( v_L \) to firm 1. To see this, note that Lemma 1 gives \( \phi_0 = 1 \) and \( \phi_1 = 0 \). Then, Lemma A1 implies that type-(\( v_H, v_L \)) consumers buy good 1 while type-(\( v_L, v_L \)) consumers do not. Both type-(\( v_H, v_H \)) and -(\( v_L, v_L \)) consumers are indifferent about buying good 1. Let \( \rho \) be the fraction of type-(\( v_H, v_H \)) consumers who do not buy good 1, and let \( \alpha \) be the fraction of type-(\( v_L, v_L \)) consumers who do buy it. Then it is straightforward to verify that so long as \( \rho \) and \( \alpha \) are not too large, there exists a PBE with the specified purchasing pattern in which \( \phi_0 = 1 \) and \( \phi_1 = 0 \). Moreover, firm 1’s payoff in such a PBE is

\[
\left( E[\lambda] - \rho E[\lambda^2] + \alpha \left( 1 - 2E[\lambda] + E[\lambda^2] \right) \right) v_L + (1 - \rho)E[\lambda^2]v_H + \\
\left( 1 - E[\lambda] \right) \rho E[\lambda^2] - \alpha \left( 1 - 2E[\lambda] + E[\lambda^2] \right) v_L - \max\{v_L, E[\lambda]v_H\},
\]

where the top line is the revenue from sale of good 1 and the bottom line is the revenue from sale of the customer list. Combining terms renders this as

\[
v_L + (1 - \rho)E[\lambda^2]v_H - \max\{v_L, E[\lambda]v_H\}.
\]

Simple algebra and the fact that \( E[\lambda^2] < E[\lambda] \) verify that this is less than \( v_L \).

(ii) Suppose \( p_1 = v_H \). To establish existence, suppose that \( \phi_0 = \phi_1 \) in accordance with (A6), (A7), and (A8). Then it is a strict best response for type-(\( v_L, v_L \)) and type-(\( v_L, v_H \)) consumers to reject and a weak best response for type-(\( v_H, v_L \))
and type-$(v_H, v_H)$ consumers to accept. Hence, there is a PBE of the continuation game in which the purchasing pattern of the consumers coincides with the one given in Lemma A3 and in which the beliefs are given in (A4).

Proving that no other PBE of the continuation game delivers a higher expected payoff to firm 1 takes two steps. Step (a) shows that the customer list is worth zero in every PBE of the continuation game. Then step (b) shows that the PBE in question involves the highest sales volume for good 1.

(a) To see that $W(v_H) = 0$ in every PBE of the continuation game, suppose to the contrary that $W(v_H) > 0$ in some PBE. In this case, either (A9) or (A10) must hold. If (A9) holds, then Lemma 1 gives $\phi_0 = 0$ and $\phi_1 = 1$. In this case, Lemma A1 indicates that all type-$(v_H, v_H)$ and no type-$(v_L, v_L)$ consumers purchase good 1. Given this, it is straightforward to verify that

$$\Pr\{v_2 = v_H \mid q_{11} = 0\} \leq E[\lambda_i \mid v_1 = v_L]$$

and

$$\Pr\{v_2 = v_H \mid q_{11} = 1\} \geq E[\lambda_i \mid v_1 = v_H].$$

Combining these with (4) results in a contradiction of (A9). If (A10) holds, then Lemma 1 gives $\phi_0 = 1$ and $\phi_1 = 0$. Given this, Lemma A1 indicates that no type-$(v_H, v_H)$, type-$(v_L, v_H)$, and type-$(v_L, v_U)$ consumers buy good 1. If no type-$(v_H, v_U)$ consumers buy it, then no updating occurs and $W(v_H) = 0$. If a positive measure of type-$(v_H, v_U)$ consumers buy good 1, then $\Pr\{v_2 = v_H \mid q_{11} = 1\} = 0$, contradicting (A10).

(b) Finally, to see that no PBE of the continuation game exists with higher sales volume, suppose to the contrary that there is a PBE with sales volume greater than $E[X] - p^*E[X^2]$. Now, Lemma A1 indicates that type-$(v_L, v_U)$ consumers never accept $p_1 = v_H$ in any PBE because

$$v_L - v_H < (\phi_0 - \phi_1)(v_L - v_U), \quad \forall \phi_0, \phi_1 \in [0, 1].$$

Hence, the PBE must involve acceptance by a fraction greater than $1 - \rho^*$ of type-$(v_H, v_H)$ consumers or by more than zero type-$(v_L, v_H)$ ones. This, however, implies

$$\Pr\{v_2 = v_H \mid q_{11} = 0\} < \frac{v_L}{v_H} < \Pr\{v_2 = v_H \mid q_{11} = 1\}.$$ 

Condition (4) and Lemma 1 then require $\phi_0 = 1$ and $\phi_1 = 0$. But then, Lemma A1 indicates that no type-$(v_H, v_H)$ or type-$(v_U, v_H)$ consumers will buy good 1, which is a contradiction. Q.E.D.

The equilibria identified in Lemma A4 are, in fact, the “correct” equilibria of the respective continuation games in the sense that they are the ones that must be played in order to ensure existence of a solution to firm 1’s pricing problem and, hence, to a PBE of the entire game. Q.E.D.

Proof of Proposition 4. First it is shown that firm 1 optimally posts one of two prices, $p_1 = v_L$ or $p_1 = v_H$, and that the customer list is worthless in either case. This takes three steps.

(i) If firm 1 posts $p_1 > v_H$, then no consumer purchases good 1, and the customer list is worth zero. Hence, its payoff from posting a price in this range is zero.

(ii) If firm 1 posts $p_1 < v_L$, then all consumers purchase good 1, and the customer list is worth zero. Hence, its payoff from posting a price in this range is $p_1$. The supremum payoff is $v_L$, which is attained in the equilibrium of the continuation game identified in the first part of Lemma A4. Moreover, Lemma A4 indicates that no other PBE of this continuation game delivers a higher expected payoff to firm 1.

(iii) If firm 1 posts $p_1 \in (v_L, v_H)$, then the purchasing pattern of the consumers coincides with the one given in Lemma A3. Moreover, (A1), (A2), and (A3) all indicate that if firm 2 purchases the customer list, then it is indifferent about practicing dynamic pricing. This implies that the customer list is worth zero. Hence, firm 1’s payoff from posting $p_1 \in (v_L, v_H)$ is $E[\lambda] - \rho^*E[\lambda^2]$. The supremum payoff is $E[\lambda] - \rho^*E[\lambda^2] v_H$, which is attained in the equilibrium of the continuation game identified in the second part of Lemma A4. Moreover, Lemma A4 indicates that no other PBE of this continuation game delivers a higher expected payoff to firm 1.

Next, simple algebra verifies

$$\left(E[\lambda] - \rho^*E[\lambda^2]\right) v_H \leq v_L \iff \gamma E[\lambda] \leq \frac{v_L}{v_H}.$$

The remainder of the claim then follows from Lemma 1. Q.E.D.
References


KHAN, S. “Travel Sites Aim Discounts at First-Timers: Tailor-Made Prices Expected to Deliver Repeat Business.” USA Today, October 9, 2000, p. 1B.


