ELECTROMECHANICAL ANALYSIS OF A COMPLETE ARM PROSTHESIS (EMAS)

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Abstract: A characterisation of a complete arm prosthesis is necessary to develop effective control. This is a description of the use of Lagrange methodology to describe the system and to optimise for motion control.

The Lagrange equations of motion are derived from the Newtonian equations of motion. Lagrange analysis describes the system in terms of Kinetic (T) and Potential energies (V). The Kinetic energy (T) is found through a generalised co-ordinate system, where T is a function of the co-ordinates and time derivates. In the non-conservative prosthetic arm, potential energy (V) is found from the generalised forces. These descriptions encompass both electrical and mechanical energies, which are then used to provide the optimum control settings.

This analysis method allows multiple terminal analysis points to be combined, allowing an electrical network with losses, and a mechanical network with losses, combined by a coupling network. Thus the analysis allows for *n* mechanical and electrical terminals in the network. This network approach lends itself to a complete prosthetic arms system, where terminals in the network can range from individual fingers to shoulder joints.

Introduction:

An alternative to the Newton-Euler formulation is the Lagrangian formulation. Whereas the Newton-Euler formulation might be said to be a "force balance" approach to dynamics, the Lagrangian formulation is an "energy-based" approach to dynamics [1].

Generalised Co-ordinates:

The generalised co-ordinates $q_1...q_n$, completely locates the dynamic system. Let T and V be the total kinetic energy and potential energy stored in the dynamic system. The Lagrangian L, can be defined by

$$L(q_i, \dot{q}_i) = T - V \tag{1}$$

Since the Potential and Kinetic energies are functions of q_i and $\dot{q}_i, (i = 1,......,n)$, so therefore is the Lagrangian L.

Mechanical:

The Lagrangian equations of motion of dynamic systems are given by

$$\frac{d}{dt}\frac{\partial L}{\partial q_i} - \frac{\partial L}{\partial q_i} = Q_i \qquad i = 1,...,n \tag{2}$$

Where Q_i is the generalized force corresponding to the generalised co-ordinate q_i. Considering the virtual work done by non-conservative forces acting on the system can identify the generalised force.

The kinetic energy of a link is given by

$$T_i = \frac{1}{2} m_i \vec{v}_{ci}{}^T \vec{v}_{ci} + \frac{1}{2} \vec{\omega}_i{}^T I_i \vec{\omega}_i \tag{3}$$

Where v_i is the velocity vector of the centre of the linkage and ω_i is the angular velocity vector with reference to the base co-ordinate frame. m_i is the mass of the link and I_i is the inertia tensor at the centre expressed in the base co-ordinates.

The total kinetic energy stored in the whole arm linkage is then given by

$$T = \sum_{i=1}^{n} T_i \tag{4}$$

The expression for the kinetic energy is written in terms of the velocity and angular velocity of each link member, which are not independent variables. The above equation can be re-written in terms of an independent and complete set of generalised co-ordinates, specifically joint displacements $q = [q_1, q_2, ..., q_n]^T$.

The Jacobian matrix of nDOF serial manipulator is given by

$$J = \begin{matrix} J_{L1} & J_{L2} & ... & J_{Ln} \\ J_{A1} & J_{A2} & .... & J_{An} \end{matrix} \tag{5}$$

J_{Li} and J_{Ai} are column vectors of the Jacobian matrix respectively associated with the linear and angular velocities. Using vector J_{Li}, the linear velocity of the end-effector can be written as:

$$v_e = J_{L1}\dot{q}_1 + ... + J_{Ln}\dot{q}_n \tag{6}$$

Similarly, the angular velocity of the end-effector was expressed as a linear combination of the column vectors J_{Ai}

$$\omega_e = J_{A1}\dot{q}_1 + ... + J_{An}\dot{q}_n \tag{7}$$

The same method can be applied to v_{ci} and ω_i by regarding the link as an end-effector.

$$v_{ci} = J_{L1}^{(i)}\dot{q}_1 + ... + J_{Li}^{(i)}\dot{q}i = J_L^{(i)}\dot{q} \tag{8}$$

$$\omega_{(i)} = J_{A1}^{(i)}\dot{q}_1 + ... + J_{Ai}^{(i)}\dot{q}_i = J_A^{(i)}\dot{q} \tag{9}$$

The motion of link i depends on only joints 1 through i. Therefore,

$$J_L^{(i)} = [J_{Li}^{(i)}....J_{Li}^{(i)}0...0] \tag{10}$$

$$J_A^{(i)} = [J_{A1}^{(i)}....J_{Ai}^{(i)}0....0] \tag{11}$$

Substituting expressions for v_{ci} and ω_i into the equation for kinetic energy gives,

$$T = \frac{1}{2}\sum_{i=1}^{n}(m_i \dot{q}^T J_L^{(i)T} J_L^{(i)} \dot{q} + \dot{q}^T J_A^{(i)T} I_i J_A^{(i)} \dot{q}) = \frac{1}{2}\dot{q}^T H \dot{q} \tag{12}$$

H is given as an nXn matrix.

$$H = \sum_{i=l}^{n}(m_i J_L^{(i)T} J_L^{(i)} + J_A^{(i)T} I_i J_A^{(i)}) \tag{13}$$

The matrix H incorporates all the mass properties of the whole arm linkage, as reflected to the joint axes, and is referred to as the manipulator inertia tensor. The manipulator inertia tensor has properties similar to those of individual inertia tensors.

The quadratic form associated with the manipulator inertia tensor represents kinetic energy, and the manipulator inertia tensor is positive definite since kinetic energy is always strictly positive unless the system is at rest.

However, the manipulator inertia tensor involves Jacobian matrices that vary with arm configuration. Hence the manipulator inertia tensor is configuration-dependent. Let H_{ij} be the [i, j] components of the manipulator inertia tensor H, then the total kinetic energy can be rewritten in a scalar form so that

$$T = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}H_{ij}\dot{q}_i\dot{q}_j \tag{14}$$

Where H_{ij} is a function of $q_1,....q_n$.

In addition to the computation of the kinetic energy, the potential energy V and generalized forces are required to be found in order to derive Lagrange's equation of motion.

Let g be the vector representing the acceleration of gravity with reference to the base co-ordinate frame. Then the potential stored in the whole arm linkage is given by

$$V = \sum_{i=1}^{n}m_i \vec{g}^T {}_0^{ci}\vec{r} \tag{15}$$

Where ${}_0^{ci}\vec{r}$ is the position vector of the centre of the link, which is dependent on the arm configuration. Thus the potential function is a function of $q1,...., qn$ [2].

By substituting gravity torque, inertia torques, and coriolis and centrifuge effects into the original Lagrange equations gives the mechanical definition for a complete arm system.

$$\sum_{j=1}^{n} H_{ij}\ddot{q}_j + \sum_{j=1}^{n}\sum_{k=1}^{n} h_{ijk}\dot{q}j\dot{q}_k + G_i = q_i \qquad\qquad i = 1,2,....,n \qquad (16)$$

Where:

$$G_i = \sum_{j=1}^{n} m_j g^T J_{Li}^{(j)} \qquad (17) \quad \text{and} \qquad h_{ijk} = \frac{\partial H_{ij}}{\partial q_k} - \frac{1}{2}\frac{\partial H_{jk}}{\partial q_i} \qquad (18)$$

Electrical:

The first step in analysing a complicated electromechanical system by a conservation of energy approach is to reduce the system containing electromechanical coupling terms to a minimum. To do this, separate out all purely electrical parts and all purely mechanical parts of the system including losses. This separation procedure is carried out to the extent that each electrical terminal pair is coupled to one energy store, either magnetic or electrical. Any internal interconnections between circuits that are coupled to different energy storages are included in the external electrical network. The mechanical variables represented by the mechanical terminal pairs are those, which affect energy storage in the electric and magnetic fields. The separation procedure results in the general conservative electromechanical coupling network in Figure 1 in which there are n electrical terminals and m mechanical terminals pairs. Each electrical terminal pair will be coupled to either magnetic field energy storage or electric energy field storage. The total stored energy W in the coupling network is
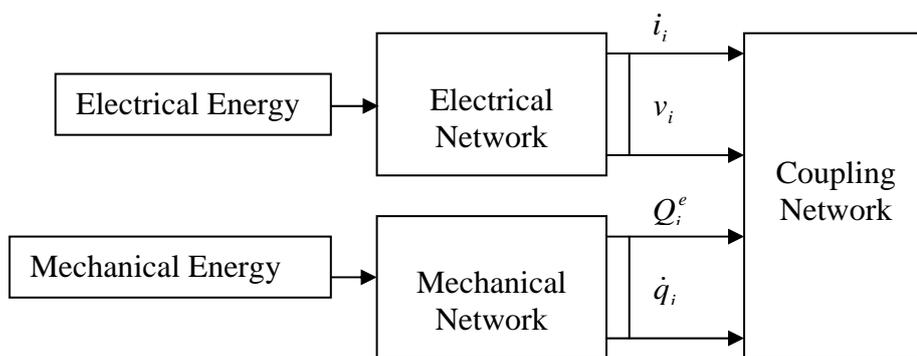

Figure 1. Electromechanical System with Coupling Network

Represented by $W^{el} = W^{\bar{q}} + W^{\psi}$ $\qquad\qquad (19)$

Where $W^{\bar{q}}$ is the energy stored in electric fields and W^{Ψ} is energy stored in magnetic fields. It is assumed that W is an instantaneous configuration of the system. Consider an electrical terminal pair coupled to the electrical field storage. When the $\bar{q}_i$ and q_i are specified independently, the current in the *ith* terminal is $i_i = d\bar{q}_i/dt$ and the voltage v_i at the *ith* terminal is given by the internal constraints. Next, consider an

electrical terminal pair that is coupled to magnetic field storage. When the ψ_i and x_i are specified independently, the voltage in the *ith* terminal is $v_i = d\psi_i/dt$ and the current i_i at the *ith* terminal is given by the internal constraints. It should be mentioned that instead of specifying the $\bar{q}_i$ and ψ_i the voltages v_i and the currents i_i could have been considered as independent.

The next problem is to find the generalized force due to the electromechanical coupling. Since the m mechanical terminal pairs are characterised by independent variables, it is possible to consider each mechanical terminal pair individually to find the force. Let us define the generalized force Q_k^e as the force applied to the *kth* mechanical co-ordinate by the coupling network. Q_k^e can be found by considering that an arbitrary placement dq_k of the *kth* mechanical co-ordinate during the time dt takes place. All other mechanical co-ordinates are fixed and the electrical variables may change in accordance to the internal constraints due to the electrical network. This means that only one electrical variable at each electrical terminal can be changed arbitrarily. All losses are said to be either part of the mechanical network or the electrical network.

Step by Step implementation and Conclusions:

Mechanical Network:

Select a suitable co-ordinate system to represent the mechanical configuration if the system. Obtain the Kinetic energy (T) Obtain the potential energy (V), as a function of the co-ordinates, if a conservative system. If the system is non-conservative, find the generalised forces in the form of Q_i .

Electrical Network:

Use the currents to form the generalised co-ordinates. Obtain the total electric energy co-efficient, as a function of the mechanical and electrical co-ordinates. Calculate the power quantities. Define the extended Lagrangian.

$$L^{el} = T + W^{el} - V \tag{20}$$

The Lagrangian method of modelling allows for a complete analysis method in terms of energy. Understanding the energies and losses of a system allows for a more complete implementation for control system design. Lagrange offers one method for modelling a system such as a prosthetic arm. By combining the electrical and mechanical segments for analysis, allows for a more in-depth understanding of a prosthetic arm, than has previously been the case.

References:

1. Sciavicco, L, Siciliano, B, "Modelling and Control of Robot Manipulators", Springer, ISBN 1 85233 221 2
2. Atia, K,R,E, "Dynamic Analysis, Design and Control of an Industrial Parallel Robot" University of Glasgow, Thesis 2000.