Fluctuation Effects in One-Dimensional Superconducting Nanowires

by

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Department of Physics
Duke University

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Robert Behringer

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Horst Meyer

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Berndt Mueller

Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Physics in the Graduate School of Duke University
2010
Abstract

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This thesis focuses on the fluctuation in the switching current $I_s$ of superconducting Al nanowires. We discovered that the maximum current which nanowires can support is limited by a single phase slip at low temperature.

Al superconducting nanowires less than 10 nm wide were fabricated based on a MBE grown InP ridge template in an edge-on geometry. The method utilizes a special substrate featuring a high standing 8nm-wide InP ridge. A thin layer of Al was evaporated on the substrate and Al on the ridge formed nanowires.

The fluctuation effects starts to dominate in the nanowire due to reduced energy barrier. One of such effects is the phase slip. The phase slip is a topological event, during which the superconducting phase between two superconducting electrodes changes by $2\pi$. The phase slip broadens the normal-superconducting transition. Part of the nanowire becomes normal during the phase slip and forms a normal core. The normal core generates heat and causes the premature switching in superconducting nanowires.

The nanowire becomes superconducting below the critical temperature $T_c$. The superconducting-normal transition was studied in the thesis. The transition of nanowires with superconducting leads qualitatively fits the thermally activated phase slip (TAPS) theory. On the other hand, the transition of the nanowires with normal leads showed a resistive tail due to the inverse-proximity effect.

The nanowire switches from the superconducting state to the normal state as
the current is increased. Ideally, the maximum current is set by a pair-breaking mechanism, by which the kinetic energy of quasi-particles exceeds the bonding energy of Cooper pairs. This is called the critical current, $I_c$. In practice, the measured maximum current, called the switching current $I_s$, cannot reach $I_c$ because of the phase slip.

$I_s$ shows stochasticity due to the phase slip. For the nanowires with superconducting leads, the average $I_s$ approximately follows but falls below $I_c$. The fluctuation in $I_s$ shows non-monotonic behavior, in contrast to other studies. The fluctuation first increases and then decreases rapidly with increasing temperature. The fluctuation behavior is consistent with a scenario where the switch is triggered by a single phase slip at low temperature while by multiple phase slips at higher temperature. Thermal activation of phase slips appears dominant at most temperatures. However, in the thinnest nanowire, the saturation of the fluctuation at low temperature indicates that the phase slips by macroscopic quantum tunneling.

The superconducting nanowires with normal leads were also studied. One of the distinctive properties of our nanowire (the critical field of 1D nanowire is 10 times larger than that of a 2D superconducting film) allowed us to study the same nanowire with different leads (superconducting or normal). Both the average $I_s$ and the fluctuation in $I_s$ differed qualitatively depending on whether the leads were superconducting or normal. The temperature dependence of the average $I_s$ followed the $I_c$ of the Josephson junction instead of the phenomenological pair-breaking $I_c$. The difference was found to depend on both the temperature (close to $T_c$ or 0) and the length (shorter or longer than the charge imbalance length). Our study also showed that nonlinear current-voltage (IV) curves were observed due to the inverse-proximity effect.
To my loving and supportive family: father, mother, sister, wife and son
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List of Abbreviations and Symbols

Symbols

- $e$: Electron charge
- $c$: Speed of light
- $K$: Kelvin
- $A$: Ampere
- $\Omega$: Ohm
- $V$: Voltage
- $\xi$: Superconducting coherence length
- $T$: Temperature
- $T_c$: Critical temperature
- $I_c$: Pair-breaking critical current
- $I_s$: Switching current
- $R_q$: Quantum resistance $= h/(2e)^2 = 6.5K\Omega$

Abbreviations

- 1D: One-dimensional
- 2D: Two-dimensional
- 3D: Three-dimensional
- PMMA: Poly(methyl methacrylate)
- MIBK:IPA: Methyl isobutyl ketone: isopropanol
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>MBE</td>
<td>Molecular Beam Epitaxy</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning Electron Microscope</td>
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<tr>
<td>TEM</td>
<td>Transmission Electron Microscope</td>
</tr>
<tr>
<td>AFM</td>
<td>Atomic Force Microscope</td>
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<tr>
<td>E-beam</td>
<td>Electron beam</td>
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<tr>
<td>PS</td>
<td>Phase slip</td>
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<tr>
<td>TAPS</td>
<td>Thermally activated phase slip</td>
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<td>QPS</td>
<td>Quantum phase slip</td>
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<tr>
<td>IV</td>
<td>Current-voltage</td>
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<tr>
<td>SIS</td>
<td>Superconductor-insulator-superconductor</td>
</tr>
<tr>
<td>SNS</td>
<td>Superconductor-normal metal-superconductor</td>
</tr>
<tr>
<td>TTL</td>
<td>Transistor-transistor Logic</td>
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The low dimensional systems have been studied extensively in past several decades, due to the interests from both the fundamental science and the potential application. One of the significant driving forces in the field is the demands of ever decreasing size of the integrated circuits (IC). This requires scientists to find ways to make the things ever smaller and explore the properties for potential applications.

At the time, it was clear that if a conducting wire is narrow and short enough, called one-dimensional (1D) wire, the conductance becomes quantized, which is different from a regular three-dimensional (3D) conductor obeying Ohm’s law. In 1D wire, rather than moving diffusively, electrons move ballistically governed by the Landauer-Buttiker formalism. The question we wanted to ask was how other properties of 1D systems, such as superconductivity, would differ from a 3D counterpart.

The first question was how to define a 1D superconductor. In the reality, strict low dimensional systems do not exist. They are called 1D or 2D when the corresponding dimensions are smaller than a characteristic length. For example, in an electronic system, a wire is called a 1D quantum wire when the lateral dimension of the wire
is smaller than the Fermi wavelength, $\lambda_F$

$$
\lambda_F = \frac{2\pi \hbar}{\sqrt{2mE_F}}
$$

(1.1)

where $m$ is effective mass of the electron and $E_F$ is the Fermi energy. It shows quantized conductance at low temperature at which the thermal excitation to a higher level is exponentially suppressed.

Similarly, the superconducting coherence length $\xi$ is the characteristic length for a superconductor. The definition of $\xi$ will be introduced in Chapter 2. But simply speaking, the superconducting electrons can be described by a complex wavefunction. The wavefunction keeps the coherence over a distance ($\xi$). The superconductivity does not change much over this distance. When the lateral dimension of a superconductor is smaller than $\xi$, it is called a 1D superconductor.
Broadening of superconducting-normal transition

It may not be a total surprise that the research on 1D superconductors was started 40 years ago, earlier than the study of low dimensional electronic systems, which were made possible only by the advance in the micro-fabrication. The reason is simple. The superconducting coherence length $\xi$ can be up to a few micrometers in some materials. More importantly, $\xi$ approaches infinity when the temperature is close to $T_c$. Thus, the low dimension can be achieved without requiring advanced fabrication techniques.

One of the major findings was the fluctuation effect, namely the phase slip. The exact definition of the phase slip is introduced in Chapter 2. But roughly speaking, the phase slip is a spacial-temporal event, during which the superconducting order parameter phase between two bulk superconductors changes by $2\pi$. The phase slip also forms a normal core with the size about $\xi$.

The phase slip caused by the thermal activation (thermally activated phase slip, TAPS) is responsible to the broadening of the normal-superconducting transition of a 1D superconductor. The theory and experiments showed good agreement, shown in Fig. 1.1 [1–5]. The phase slip was also used to explain a range of interesting phenomena including so called the phase slip center [6], which was featured by voltage steps in current-voltage (IV) curves. The phase slip center is still one of the commonly observed phenomena in 1D superconductors [7–10]. However, we will show that such phenomena can be possibly due to spurious factors.

Resistive tail in narrower 1D superconductor

Benefiting from the advance in microfabrication, the narrower 1D superconducting wires could be produced in the eighties. Giordano first measured the resistive transition of such wires and found a resistance tail in addition to the broadened
Figure 1.2: The resistance tail measured by Giordano. The curve shows the resistance transition of narrow superconducting wires. The dots are the experimental value. The solid curves are the fitting by the TAPS theory and the dash curves are the fitting including QPS [11].

transition induced by TAPS [11–13], as shown in Fig. 1.2. He attributed this to the phase slips caused by macroscopic quantum tunneling (quantum phase slip, QPS). The experiment inspired a number of theorists working on the quantum macroscopic tunneling and possible quantum phase transition in the 1D superconductor [14–16]. However, the results were criticized because of possible weak links, inhomogeneity and granularity.

Superconducting-insulating transition

In the year 2000, the field was reignited by the finding of superconducting-insulation transition in 1D superconductors. The samples were fabricated by coating carbon nanotubes with superconducting materials. The experiment showed that an extremely narrow superconducting nanowire may become insulating if the normal resistance of the nanowire is larger than quantum resistance for Cooper pairs $R_q = h/(2e)^2 = 6.5\Omega$ [17], as shown in Fig. 1.3. The authors of reference [17] also found
the resistive transition can be roughly fitted by the QPS theory [18].

This observation may not be a complete surprise for people who are familiar with the superconducting-insulating transition in disordered two-dimensional (2D) superconducting films (for a review, see [20]). Experiments showed that the film turned from the normal-superconducting transition to normal-insulating transition with decreasing film thickness, as shown in Fig. 1.4 [19]. The disorder is believed to play an important role in such behavior. Although the exact type of disorder is unclear, it is introduced by evaporating the superconductor onto a cold substrate (typically at liquid Helium temperature). The quantum phase transition is one of the possible explanations. However, up to date, unambiguous evidence is absent, and the existence of QPS is still under debate.

Most recent developments in 2D superconducting film indicated that the superconductivity existed in the 2D film as thin as a single-atomic-layer. The experiments were made possible by epitaxially growing film and subsequently measuring the superconducting gap using scan tunneling microscope (STM) [21–23]. Fig. 1.5 showed that the superconducting gap existed in a single-atomic-layer thick In film. The result
Figure 1.4: Superconducting-insulating transition in a 2D superconducting Bi film. The film turned from the normal-superconducting transition to the normal-insulating transition with decreasing film thickness. [19]

indicates that the superconducting-insulating transition observed in 2D disordered film was likely due to inhomogeneity. A possible scenario was that the metal atoms aggregated on the film and formed isolate islands connected by tunneling junctions.
Recent developments in 1D superconductor

After the superconducting-insulating transition in nanowires [17] and the following QPS papers [18] were published, a consistent picture had emerged. Wider superconducting wires show the effect of TAPS. Narrower wires show the effect of TAPS near $T_c$ and that of QPS at lower temperature. When the wires become extremely narrow, they become insulating. The insulating phase was tentatively explained in several theoretical works, which claim that the QPS coherently proliferated so that the Cooper pairs localized. Thus, the wire became insulating [24,25].

The only question is how universal the property is across different systems. This turns out to be a difficult task. To our best knowledge, up to date, no other superconducting nanowires have shown the superconducting-insulating transition. Our Al
superconducting nanowires did not show any signs of insulating even though their size is comparable. All these indicate that the superconducting-insulating transition is not as universal as previously thought. It may only happen under certain condition with particular systems.

Recent developments further complicated the issue. Originally thought to be QPS, the nanotube based 1D superconductors were found to follow the TAPS theory all the way down to lowest temperature, through nonlinear current-voltage (IV) measurement [26]. On the other hand, Altomare in our group measured nonlinear IV curves in Al nanowires and found the signature of QPS [27]. The newest development is that the QPS was rediscovered in the some 1D superconductors by measuring the switching current fluctuation [28].

In the thesis, I show the result on the switching current fluctuation of our superconducting nanowires. The fluctuation shows non-monotonic temperature dependence, contrary to previous study [28]. The fluctuation increases and then decreases with increasing temperature. The fluctuation behavior is consistent with a scenario where the switch is triggered by a single phase slip at low temperature while by multiple phase slips at higher temperature. At lower T, quantitative estimation shows the heat generated by a single PS likely causes a thermal runaway triggering the switching transition. The behavior presented in this work may be relevant to superconducting nanowires of different materials, due to the commonality of a restricted geometry. Thus it is likely that single TAPS and QPS limited switching current is a generic feature of superconducting nanowire systems.

Along with the studies on the phase slips, a number of other interesting phenomena were also observed in 1D superconductors, including the so called anti-proximity effect, S-shaped IV curves, Current-Field-introduced superconductivity [29–31]. On the theory side, using superconducting nanowires as the current standard and the superconducting quantum-bit for quantum computation were proposed [32,33].
The thesis is organized as followings. The theoretical background is introduced in the second chapter. The third chapter is the fabrication and experimental setup. The fourth chapter is the result and the last chapter is the conclusion and describes future research.

Along the way, I will illustrate the difficulty and trickiness of such measurements, including the influence of external noise and environmental interference, influence of connecting leads (superconducting leads vs. normal leads), and heating issues, etc.
2
Basic Theories

2.1 A brief introduction to superconductivity

Next year will be the 100th year anniversary of the discovery of superconductivity by Onnes in 1911 [34]. Looking back at the history, many problems have been solved, but yet new phenomena continue to emerge and the superconductivity is still one of the frontiers of physics research.

The hallmark of the superconductivity is its zero resistivity. It was first discovered that the electrical current could pass the superconductor without dissipation below certain temperature, called the critical temperature $T_c$. In other words, the superconductor is a perfect conductor. Soon, Meissner and Ochsenfeld found the superconductor exhibited the perfect diamagnetism, which means no magnetic field can be found inside the superconductor below $T_c$. This phenomenon was later called Meissner effect and cannot be simply explained as a perfect conductor.

Early theoretical attempts to address the superconductivity phenomenon were made by F. London and H. London [35]. They proposed two equations to govern the
electrical and magnetic field in a superconductor

\[
\mathbf{E} = \frac{\partial}{\partial t} (\Lambda \mathbf{J}_s)
\]

\[
\mathbf{H} = -\frac{c}{\nabla} \times (\Lambda \mathbf{J}_s)
\]

(2.1)

\[
\Lambda = \frac{4\pi \lambda^2}{c^2} = \frac{m}{n_s e^2}
\]

These were called London functions, where \( \lambda \) is London penetration length, \( \mathbf{J}_s \) is the superconducting current (supercurrent) density and \( n_s \) is the superconducting electron density.

In 1950, Ginzburg and Landau introduced a complex pseudo-wavefunction \( \psi \), called an order parameter to describe the behavior of superconducting electrons near \( T_c \) [36]. The superconducting electron density is given by the modulus of the wavefunction,

\[
n_s = |\psi(x)|^2
\]

(2.2)

The free energy of the superconductor can be written as a Taylor expansion of the order parameter

\[
f = f_n + \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \frac{1}{2m^*} \left| \left( \frac{\hbar}{i} \nabla - \frac{e^*}{c} \mathbf{A} \right) \psi \right|^2 + \frac{\hbar^2}{8\pi}
\]

(2.3)

In the absence of magnetic field and the gradient of the order parameter, the free energy can be written as

\[
f_s - f_n = \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4
\]

(2.4)

where \( f_n \) is the free energy at the normal state , \( f_s \) is the free energy at the superconducting state, \( \alpha \) and \( \beta \) are the coefficients of the second order and the fourth order expansion and \( \hbar^2/8\pi \) is the energy density of the magnetic field.

As shown in Fig. 2.1, the phase transition happens when \( \alpha \) changes from positive to negative. When \( \alpha \) and \( \beta \) are both positive, the normal state has lowest energy.
Figure 2.1: G-L free energy at normal and superconducting state. The superconductor is at normal state when $\alpha > 0$ (left curve); it is at superconducting state when $\alpha < 0$ (right curve).

The superconductor is at normal state and $\psi = 0$. When $\alpha$ is negative and $\beta$ is positive, the superconducting state has lowest energy. The superconductor is at superconducting state and $\psi$ has non-zero value.

The G-L theory is a triumph of physical intuition. It used a general phase transition theory to describe the behavior of a superconductor near $T_c$ without providing a microscopic mechanism. Although the Ginzburg-Landau (GL) theory was not generally appreciated initially, it is now accepted as a great physics intuition which reveals the quantum-mechanics nature of the superconducting state.

The London equations and G-L theory successfully described the electrodynamic property of a superconductor. However, neither of them gave a microscopic mechanism for superconductivity, essentially, why a current can flow without resistance. This question was answered by Bardeen, Cooper and Shrieffer in their famous BCS theory [37]. The details of the theory is complicated. But simply put, they showed that even a weak attractive potential between two electrons near Fermi sea could cause these two electrons form a bound state to lower the energy. The two electrons forming the bound state had the equal but opposite momentum and spin and were called Cooper pairs. The total momentum of the pair was resilient against individ-
ual electron scattering. As a result, the Cooper pair moved in the superconductor without resistance. The origin of the attractive potential between electrons was from the second order interaction by electron-phonon scattering.

One of the key prediction of the BCS theory was the equation for the energy gap

\[ E_g(0) = 2\Delta(0) = 3.528k_B T_c \]

The measurement agreed with the calculation very well. This is one of the most decisive verification of the BCS theory [38].

The latest chapter in the superconductivity history is the discovery of high temperature superconductors by Bednorz and Muller in 1986. This was a surprise and challenge to BCS theory, which had been very successful. Since then, remarkable progresses have been made in searching for new materials to achieve higher temperature. However, the underlying mechanism is still not understood very well and it continues to be one of the difficult problems in physics.

My research focuses on the fluctuation effect of the 1D conventional superconductor, in particular Aluminum (Al). Most phenomena can be understood in the frame work of the BCS theory. But for convenience’ sake, G-L theory is often used. This chapter starts with the introduction of Josephson Junctions (JJs) and phase slips (PSs). Then the phase slip in 1D superconductor is introduced and the phase slip rate is calculated. The last section is a brief description of the proximity effect and inverse-proximity effect.

2.2 Josephson effect and phase slip

2.2.1 Josephson effect and Josephson junction

The Josephson effect was first predicted in 1962 [39]. It was found that a supercurrent

\[ I_s = I_c \sin(\delta\phi) \]
can flow between two superconductors separated by a thin tunneling barrier. Here the critical current $I_c$ is the maximum current that the junction can support and $\delta \phi$ is the phase difference between two superconducting electrodes. The change of $\delta \phi$ satisfies

$$\frac{d(\delta \phi)}{dt} = \frac{2V}{\hbar}$$

where $V$ the voltage difference between 2 electrodes.

The above relations are valid not only for the superconductors separated by the insulating layer (called SIS), but also for them separated by a normal metal (SNS) and a constriction (ScS), as shown in Fig. 2.2. The device having this kind of geometry is called the Josephson junction (JJ).

The free energy stored in the junction can be calculated by evaluated time average of $IV$. The result shows

$$F = -E_J \cos(\delta \phi) - \text{const}$$

where $E_J = (hI_c/2e)$ is Josephson energy.

Ambegaokar and Baratoff [40] calculated the temperature dependence of $I_c$, which follows the equation

$$I_cR_n = \frac{\pi \Delta}{2e} \tanh \frac{\Delta}{2k_B T}$$

where $R_n$ is the resistance of the junction in the normal state and $\Delta$ is the BCS gap.
2.2.2 RCSJ model

A more complete description of the Josephson junction includes the quasiparticle tunneling and capacitive effect. This is called Resistive and Capacitance Shunted Junction (RCSJ) model [38]. In the model, the current can be written as

\[ I = I_c \sin(\delta\phi) + \frac{V}{R} + CdV/dt \]  (2.10)

The total current is the sum of the Josephson current, quasiparticle tunneling current and the current on capacitor. Since \( V \) is a function of \( \delta\phi \), the equation can be rewritten as a second order differential equation

\[ \frac{d^2(\delta\phi)}{d\tau^2} + \frac{1}{Q} \frac{d(\delta\phi)}{d\tau} + \sin(\delta\phi) = \frac{I}{I_c} \]  (2.11)

where \( \tau = \omega_p t \) with \( \omega_p = \sqrt{2eI_c/hC} \) being called the plasma frequency, and \( Q = \omega_p RC \) is called the quality factor.

The differential equation is analogous to a mechanical system, where a particle of mass \( (h/2e)^2C \) is moving along the \( \delta\phi \) axis in the effective potential

\[ U(\delta\phi) = -E_J \cos(\delta\phi) - (hI/2e)\delta\phi \]  (2.12)
subjected to a viscous drag force \( (\hbar/2e)^2(1/R)d(\delta\phi)/dt \). This is called the tilted washboard potential model (Fig. 2.3), which can be treated as a periodical potential tilted by the applied current.

2.2.3 Overdamped and underdamped Josephson junction

The junction can be classified as either overdamped or underdamped depending on whether the quality factor \( Q \) is much less or greater than 1. Different damping results in different behavior of current-voltage (IV) curves.

For an overdamped Josephson junction \( (Q \ll 1) \), which is typically the result of a small \( C \), the phase \( \delta\phi \) moves in the potential with a large drag force. Therefore, when \( I \) is increased during IV scan, it is possible that the voltage cannot catch up to the change of the current even when the current exceed \( I_c \). As a result, the voltage asymptotically approaches its full value (Fig. 2.6, \( u = \infty \)). The voltage is

\[
V = R(I^2 - I_c^2)^{1/2}
\]  

(2.13)

For an underdamped Josephson junction \( (Q \gg 1) \), the phase move relatively easily due to a small viscous force. As soon as the current exceeds \( I_c \), the phase keeps running and a voltage arises between two electrodes. The phase will do so even if the current is reduced to below \( I_c \). Speaking of IV curves, a hysteresis occurs when the current is ramped up and down, as shown in Fig. 2.4.

The overdamping and underdamping of Josephson junction can be illustrated by a particle moving in a tilted washboard potential. In overdamping situation, the friction in the potential is so huge that the particle is always trapped in the local minimum. On the other hand, in underdamping situation, the particle may keep running downhill given an initial momentum to overcome the first energy barrier.
2.2.4 Phase slip in Josephson junction

The above discussion is based on the analysis in absence of the fluctuation, including both thermal and quantum. Taking the simpler thermal fluctuation as an example, the phase oscillates in the energy minimum because of the thermal fluctuation at a finite temperature. Thus, the phase has a small probability $\sim e^{\Delta U(I)/k_BT}$ to overcome the energy barrier and go to the next energy minimum. During the event, the phase changes by $2\pi$ and this is called the phase slip. The attempt frequency $\omega_A$ is

$$\omega_A = \omega_p (1 - (I/I_c)^2)^{1/4}$$  \hspace{1cm} (2.14)

The current dependence of the energy barrier can be approximated quite well as

$$\Delta U(I) = 2E_J(1 - I/I_c)^{3/2}$$  \hspace{1cm} (2.15)

The $3/2$ power is the result of the cubic term in the Taylor expansion of the sinusoidal potential.

For the underdamped Josephson junction, one phase slip is enough to switch the junction into the normal state before the current reaches $I_c$, due to the hysteresis. To distinguish it from $I_c$, we call it the switching current $I_s$, at which the junction

---

**Figure 2.4:** Hysteretic IV curves of an underdamped Josephson junction [38]
Figure 2.5: Fluctuation in $I_s$ caused by the phase slip in Josephson junction. The escape temperature is a parameter independent phase slip rate. It is close the real temperature at thermal regime but saturates to a constant value at quantum regime [41].

switches from the superconducting state to the normal state. This one-to-one relationship between the phase slip and the switching provides a method to measure the phase slip rate, which is hard to measure by other means. Because of the stochasticity of the phase slip, the switching is also a stochastic process. The $I_s$ can be found through IV measurement. $I_s$ distribution can be obtained by repeatedly measuring $I_s$ and plotting the histogram [42]. The width of $I_s$ distribution (standard deviation) shows the magnitude of the fluctuation. The temperature dependence of the fluctuation in $I_s$ reveals the origin of the fluctuation. In Josephson junctions, the thermal activation causes the standard deviation to be proportional to $T^{3/2}$ at high temperature, while the macroscopic quantum tunneling (called quantum phase slip, QPS) causes the fluctuation saturate to a finite value at low temperature [41].

The phase slip also affects IV curves of overdamped Josephson junctions. As indicated by the Josephson relation (Eq. 2.7), each phase slip generates a voltage pulse. The sum of the voltage pulses gives a measurable voltage and causes the nonlinearity in IV curves before $I_c$ is reached. The nonlinearity in IV increases with
increasing temperature, as shown in Fig. 2.6.

2.3 Phase slip in 1D superconductor

2.3.1 Free energy of 1D superconductor

1D superconductor is defined as a superconducting wire, the lateral dimension of which is much less than the superconducting coherence length $\xi$. In other words, the variation of the order parameter $\psi$ is negligible in lateral dimension. In the absence of the magnetic field, the free energy of a superconductor is

$$F(\psi(r)) = \int \left( |\nabla \psi|^2 - \alpha |\psi|^2 + \frac{1}{2} \beta |\psi|^4 \right) d\mathbf{r} \quad (2.16)$$

according to G-L theory [3]. In 1D, the solution satisfying minimized energy based on the variation principle is

$$-\nabla^2 \psi - \alpha \psi + \beta |\psi|^2 \psi = 0 \quad (2.17)$$

The usual constant current solution is

$$\psi_k = f_k \exp(ikx) \quad (2.18)$$

$$f_k^2 = (\alpha - k^2)/\beta \quad (2.19)$$
where $x$ is the coordinate and $k$ is the wavevector. The current density (in reduced unit) is

$$J = (1/2i)(\psi^* \nabla \psi - \psi \nabla \psi^*) = kf^2 = k(\alpha - k^2)/\beta \quad (2.20)$$

The current density has maximum at $k = \sqrt{\alpha/3}$. This gives the mean-field critical current:

$$J_c = 2\alpha^{3/2}/3\sqrt{3}\beta \sim (T_c - T)^{3/2} \quad (2.21)$$

$k$ is denoted as $k_n$ since it has multiple values. Each $k_n$ gives an unique solution, which can be visualized as a helix spiraling about the $x$-axis (Fig. 2.7A). In the absence of the current, the free energy has minimum and degenerates at $k_n$ corresponding to different $\psi_k$. The system needs overcome an energy barrier in order to pass from one configuration to another. According to the Josephson relation (Eq. 2.7,

$$(2e/\hbar)\Delta V = (\partial/\partial t)\Delta(\arg \psi)$$

where $\Delta(\arg \psi)$ is the phase difference between two ends. A small voltage across the 1D superconductor will tighten the loop and eventually change $k_n$. In term of the visualized solution (Fig. 2.7), changing $k_n$ means adding or subtracting turns from the spirals. As a result, the calculated free energy is a periodic function of the phase difference $\arg \psi$ between two electrodes.

In the presence of the current, this free energy diagram tilts, similar to the tilted
washboard potential shown in the Josephson junction (Fig. 2.3B). Many concepts derived from Josephson junction are also valid for the 1D superconductor, for example, the phase slip. Note that the ”phase” in the phase slip of 1D superconductor does not refer to the phase of the helix in the real space, but rather the index $n$ in $k_n$, or allowed wave-numbers.

Unlike the Josephson junction, where the discussion on phase slip only focuses on the phase space because the physical distance between two superconductors is small, the phase slip in the 1D superconductor has the meaning in both the phase space and the real space. In the phase space, the phase slips by $2\pi$ similar to a Josephson junction. In the real space, the phase slipping happens at a random position along the wire. The superconducting coherence length $\xi$ determines the spacial extension of the phase slip, illustrated in Fig. 2.7B. The order parameter in that particular position temporarily becomes zero and a normal core is formed. The length of the normal core cannot be shorter than $\xi$. The superconductivity recovers and the helix solution reestablished (with one more or one less turn) after the phase slip.

The phase slip generates a voltage pulse between the superconducting electrodes according to Eq. 2.7. When no current flows through the wire, the washboard is horizontal. $\delta \phi$ has equal chance to slip either $+2\pi$ or $-2\pi$. The positive and negative voltage pulses cancel each other. However, when a current flows, $\delta \phi$ prefers one side than the other. As a result, a net voltage is built between the electrodes and a zero bias resistance is observed in a superconducting wire.

The rigorous calculation [2–4] shows that the energy needed to overcome (the free energy barrier) during the phase slip is

$$\Delta F = \frac{\sqrt{2}}{3\pi} H_c^2 \xi \sigma$$

(2.22)

where $\sigma$ is the area of the cross section, $H_c$ is the critical magnetic field of the bulk material. The order of magnitude estimation can be made as followings. The energy
needed to overcome is close to the condensation energy of the normal core from the superconducting state to the normal state. Restricted by the coherence length, the volume of normal core is the cross-section area $\sigma$ times the coherence length $\xi$. The condensation energy density is $H^2_c$, so the energy barrier will be $H^2_c \xi \sigma$, which is close to the rigorous calculation.

Near $T_c$, the free energy barrier can be transformed to experimentally accessible quantities [18]

$$\Delta F = 0.83 \frac{L}{\xi_0} \frac{R_n}{R_n} k_B T_c \left(1 - \frac{T}{T_c}\right)^{3/2}$$

(2.23)

where $L$ is the length of the wire, $R_n$ is the normal resistance and $\xi_0$ is the superconducting coherence length at zero temperature [43]. This is equivalent to

$$\Delta F = \left(\sqrt{6} \hbar / 2e\right) I_c$$

(2.24)

where

$$I_c = \frac{\pi}{3\sqrt{3} e R_n \xi_0} \left(1 - \frac{T}{T_c}\right)^{3/2}$$

(2.25)

and $\Delta_0$ is the superconducting gap at zero temperature. The above derivation is based on the G-L theory near $T_c$. Experimentally, $\Delta F$ at all temperature is needed. Here, we assume Eq. 2.24 to be correct at all temperature. However, instead of using $I_c \sim (1 - T/T_c)^{3/2}$ near $T_c$, we use a phenomenological

$$I_c(T) = I_c(0) \left(1 - \left(\frac{T}{T_c}\right)^2\right)^{3/2}$$

(2.26)

valid at all temperature, where $I_c(0)$ is the critical current at zero temperature [44]. $I_c(0)$ is a fitting parameter. Note that Eq. 2.24 is similar to the Josephson energy $E_J$. Such treatment was used in other experimental studies [28].

The above calculation is based on zero current. As shown in the washboard potential (Fig. 2.3), the applied current lowers the free energy barrier. The current
dependence of $\Delta F$ is quite cumbersome in the original calculation [3], but it can be approximated as

$$\Delta F(I) = \Delta F(0)(1 - I/I_c(T))^{5/4} \quad (2.27)$$

Note that the 5/4 power is based on G-L theory near $T_c$. No direct measurement or calculation are reported at low temperature. Bezryadin et al. used 5/4 power to fit their fluctuation data. But, the problem is that too many parameters and assumptions were made. Moreover, their latest result inclines to a 3/2 power (not published). The 3/2 power is consistent with our measurement in the fluctuation measurement. It is the same as the power in a Josephson junction. The exactly reason is unclear. But 3/2 power is a result of a cubic potential. This term is a natural approximation of such potential from Taylor expansion.

The free energy barrier is a function of both the current and the temperature

$$\Delta F(T, I) = \frac{\sqrt{6\hbar}}{2e} I_c(T) \left(1 - \frac{I}{I_c(T)}\right)^{3/2} \quad (2.28)$$

This is the equation used in our analysis.

2.3.2 Phase slip rate and resistance caused by phase slip

In the following, we show that how the phase slip results a measurable resistance. Starting from the phase slip due to thermal activation (TAPS), the phase slip rate $\Gamma_{\text{TAPS}}$ is proportional to $e^{\Delta F/k_BT}$. As we mentioned before, the positive and negative voltage pulses from $2\pi$ and $-2\pi$ cancel each other. Therefore, the net voltage is proportional to the difference between these two phase slips. The net phase slip rate is

$$\Gamma_{\text{TAPS}} \propto \exp \frac{-\Delta F_0 - \delta F_I/2}{k_BT} - \exp \frac{-\Delta F_0 + \delta F_I/2}{k_BT} \quad (2.29)$$

$$\propto 2 \exp \frac{-\Delta F_0}{k_BT} \sinh \frac{\delta F_I}{k_BT}$$
where $\delta F = (\pi \hbar/e) I$ is the free energy difference caused by the current $I$, and $\Delta F_0$ is the zero bias free energy. The attempt frequency is

$$\Omega_{TAPS} = \frac{L}{\xi_T} \sqrt{\frac{\Delta F_0}{k_B T \tau_{GL}}}$$

where

$$\tau_{GL} = \frac{\pi \hbar}{8 k_B (T_c - T)}$$

is G-L relaxation time. The total voltage generated by phase slips is

$$V = \frac{\hbar \Omega_{TAPS}}{e} \exp \frac{-\Delta F_0}{k_B T} \sinh \frac{\pi \hbar I}{2e k_B T}$$

The zero bias resistance $R = dV/dI$ is

$$R_{TAPS} = \frac{\pi \hbar^2}{2e^2 k_B T} \Omega_{TAPS} \exp \frac{-\Delta F_0}{k_B T}$$

A small correction to include normal electrons has to be made when compared with the measured resistance. The total resistance can be modeled as two resistor with the resistance $R_{TAPS}$ and $R_n$ in parallel. The measured resistance is

$$\frac{1}{R} = \frac{1}{R_{TAPS}} + \frac{1}{R_n}$$

Unlike the well-established TAPS rate, there is no unanimous conclusion on the phase slip rate due to macroscopic quantum tunneling (quantum phase slip, QPS) to date. The earliest attempt was made by Giordano [11, 12], who tried to explain the resistance tail he observed in thin superconducting strip ($\sim$ 40 nm). His key argument is that instead of using $k_B T$ as energy scale, $\hbar/\tau_{GL}$ should be used in the quantum tunneling. Following the same argument for TAPS, the QPS rate can be derived as followings. First,

$$\Gamma_{QPS} \propto \exp \frac{\Delta F_0}{\hbar/\tau_{GL}}$$
The attempting frequency can be rewritten as

\[ \Omega_{QPS} = \frac{L}{\xi_T} \sqrt{\frac{\Delta F_0}{\hbar/\tau_{GL}}} \frac{1}{\tau_{GL}} \]  \hspace{1cm} (2.36)

The average voltage is

\[ V = \frac{\hbar \Omega_{QPS}}{e} \exp \left( -a_{QPS} \Delta F_0 \right) \sinh \left( \frac{\pi \hbar I}{2e \hbar/\tau_{GL}} \right) \]  \hspace{1cm} (2.37)

The resistance becomes

\[ R_{QPS} = B_{QPS} \frac{\pi \hbar^2}{2e^2 \hbar/\tau_{GL}} \Omega_{QPS} \exp \left( -a_{QPS} \Delta F_0 \right) \frac{1}{\hbar/\tau_{GL}} \]  \hspace{1cm} (2.38)

where \( B_{QPS} \) and \( a_{QPS} \) are two fitting parameters, due to the limited understanding on the macroscopic quantum tunneling. Similar to the approach for TAPS, the measured resistance, including \( R_n \), \( R_{TAPS} \) and \( R_{QPS} \), is

\[ \frac{1}{R} = \frac{1}{R_{TAPS} + R_{QPS}} + \frac{1}{R_n} \]  \hspace{1cm} (2.39)

2.3.3 Current dependence of phase slip rate

As shown in Eq. 2.32, the voltage is a nonlinear function of the current. So only the local resistance (differential resistance) can be defined. It is

\[ R = \frac{dV}{dI} = \frac{\hbar \Omega_{TAPS}}{e} \exp \left( -\frac{\Delta F_0}{k_B T} \right) \cosh \left( \frac{\pi \hbar I}{2e k_B T} \right) \]  \hspace{1cm} (2.40)

In QPS case, \( k_B T \) is substituted by \( \hbar/\tau_{GL} \).

A number of experiments measure the phase slip rate based on above equation [26, 27]. However, we show this is somewhat flawed. There are two reasons. First, the above equations are based on the approximation for small \( I \). The full expression of the phase slip with a current is

\[ \Gamma_{TAPS} \propto \exp \left( -\frac{\Delta F_0 (1 - I/I_c)^{5/4}}{k_B T} \right) - \exp \left( -\frac{\Delta F_0 (1 - I/I_c)^{5/4} - \pi \hbar/eI}{k_B T} \right) \]  \hspace{1cm} (2.41)
As can be seen, Eq. 2.40 is only an approximation when $I$ is small. At a large bias, the first exponential term dominates so that the phase slip rate becomes

$$\Gamma_{TAPS} \propto \exp \left( -\Delta F_0 \left( 1 - \frac{I}{I_c} \right)^{5/4} \right) \frac{5}{4} k_B T$$

(2.42)

The equation can be further written as

$$\Gamma_{TAPS} \propto \exp \left( -\Delta F_0 \left( 1 - \frac{I}{I_c} \right)^{3/2} \right) \frac{3}{2} k_B T$$

(2.43)

using $3/2$ power deduced from the measurement.

As a result, the differential resistance can differ from Eq. 2.40 significantly.

Second, as we show in Chapter 4, a few phase slip can trigger the switching transition. It means that the 1D superconductor can become normal before nonlinear IV can be observed. Nonlinear IV is not a reliable way to measure the phase slip rate.

2.4 Proximity and inverse proximity effect

The proximity effect is widely used for the superconductivity. Surprisingly, however, the proximity effect is not clearly defined. In the famous textbook by Tinkham, *Introduction to Superconductivity*, it is defined in one sentence: *in which Cooper pairs from a superconducting metal in close proximity diffuse into the normal metal* [38]. It hardly mentions how and why the Cooper pairs ”diffuse” into normal metal. In this section, we cannot answer all these questions because of our limited understanding. However, we will introduce importantly concepts relevant to our current study.

The proximity effect is often used to describe a relatively transparent interface between a superconductor and a normal metal, primarily on the normal-metal side. The basic experimental facts in early experiments include:
(1) When two superconducting films are stacked together, the superconducting gap of the film varies between the gaps of each individual superconductor depending on the relative thickness of the films.

(2) The gap of a superconducting film on top of a thick normal metal increases with increasing film thickness.

(3) A superconducting gap can develop in a thin normal metal film in contact with a thick superconducting film.

Clearly, the superconductor and the normal metal influence each other. Because mutual influence only happens in the region close to the interface, the overall phenomena is called proximity effect.

In fact, the proximity effect consists of two important physics processes, Andreev reflection and the diffusion of the reflected quasiparticles. When a normal metal is in contact with a superconductor, normal electrons near the Fermi surface cannot enter the superconductor directly because of the superconducting gap. Thus, an electron-hole pair is generated at the interface. The generated electron joins with the incoming electron to form a Cooper pair in the superconductor. The retroreflected hole can be seen as an annihilation of an electron. Thus, a net effect is that two electrons in the normal metal enter the superconductor forming a Cooper pair. The above process is ballistic and happens exactly at the interface. In the normal metal, the retroreflected hole moves diffusively. The diffusive process is coherent. This is why the process at the interface can affect a larger region on the order of $\xi$.

The proximity effect lower the resistance of the proximity region. However, it is incorrect to think that the superconductivity is introduced in the normal metal [45]. This appears to contradict to a SNS junction, which can become totally superconducting. The reason for superconducting SNS junction is because the two normal-superconducting interfaces. The reflected holes from one interface are reflected to
virtual electrons at the other interface. This process moves Cooper pairs from one superconductor to the other. Thus, the superconductivity appears in a SNS junction.

Extensive studies have focused on the normal side of the interface. For example, the pair correlations, reentrant resistance and Andreev interferometry etc. have been demonstrated [45–47]. However, what happens inside the superconductor was poorly understood. The experiments showed that a measurable resistance could be observed in a superconducting wire sandwiched by two normal leads. The resistance is proportional to the resistance of the wire at normal state with the length about twice of $\xi$. Klapwijk et al. first studied the temperature dependence of the extra resistance [48]. Unfortunately, it is difficult to compare the solution with our result quantitatively because the numerical solution of the Usadel equation was used.

Phenomenologically, our understanding is that the normal electrons enter the superconductor and are needed to be converted to superconducting electrons. The conversion process causes the potential to drop and thus, a resistance appears. To distinguish this from the proximity effect, we call it inverse proximity effect.

The inverse proximity effect seems to be inconsistent with the proximity effect. If the extra resistance appears wherever there is a NS interface, it should be measured in a SNS junction too. This is definitely not what researches have seen. Unfortunately, not many studies focused on this subject, possibly due to the lack of practical interests. So I just present my understanding. At the interface, the superconducting gap retains its full value for a "strong" superconductor. Therefore, the normal electrons enter the superconductor by Andreev reflections. This lowers the resistance of the normal metal. On the other hand, when a weak superconductor is at the interface, such as a narrow superconducting wire, the gap becomes smaller at the position closer to the interface. Therefore, quasiparticles enter the superconductor by thermal excitation. These quasiparticles need to be converted to the superconducting electrons. The conversion process gives rise to the resistance.
3

Sample fabrication and experimental setup

3.1 Sample fabrication

This section is organized as followings. A brief introduction of the fabrication process is described in the next paragraph. This information is enough for a general knowledge. However, for the people who want to follow the procedure, the second part describes the detailed procedure and highlights the key challenges during the fabrication. At the end of the section, the microscopic structure of the nanowires is shown.

The procedure was based on the method developed by Altomare [49]. The quantum-well-template method was used to fabricate ultra-narrow Al nanowires. The procedure is as follows:

(a) It starts from a MBE-grown (Molecular Beam Epitaxy) quantum well structure. The substrate was an 8nm InP (Indium Phosphate) layer sandwiched between two thick InGaAs (Indium-Gallium-Arsenide) layers. The whole structure is on the top of a GaAs (Gallium Arsenide) substrate.

(b) The unusual part of fabrication was that the pattern was on the sidewall of the
Figure 3.1: Nanowire fabrication process. The GaAs and InGaAs were represented by the same color since they have close properties.

...crystal instead of the usual top surface.

(c) Metal films as supporting pads were deposited by Electron-Beam (E-beam) lithography. The distance between two pads varied from 1.5 µm to 20 µm.

(d) A selective etch, which attacked InGaAs and GaAs without affecting InP, was used.

(e) A thin layer of Al (8-10 nm) was evaporated onto the structure. The Al on the top of InP ridge connected the two electrodes and formed 1D superconducting nanowire studied in the experiment.

Before evaporating electrodes and selective etch, oxygen plasma etch (Technics PE II-A Plasma System) was used to clean the residue of E-beam resist (PMMA, Poly(methyl methacrylate)) on the surface.

The substrate was provided by Professor Charles W. Tu at University of California at San Diego. The nanowire fabrication was done in our lab at Duke.
Two different supporting pads were fabricated: one is 10 nm Ti and the other is bilayer Au/Ti (6/5 nm). Combined with 8-10 nm Al from the final evaporation, two kinds of leads were formed: superconducting leads (bilayer Al/Ti) and normal leads (tri-layer Al/Au/Ti). The behavior of the nanowires was different for different leads, which is shown in Chapter 4.

The E-beam lithography is a standard process (Fig. 3.2):

(a) Clean the substrate in Acetone for 10 minutes in the ultrasonic bath

(b) Spin PMMA (4% 450K in anisole) on the substrate (5000 RPM for 20 seconds), bake the sample on the hot plate at 180°C for 10 minutes

(c) Expose certain area to form the pattern by E-beam lithography

(d) Dissolve the exposed area by developer (MIBK:IPA, 1:3)

(e) Thermally evaporate a thin layer of metal film
Figure 3.3: Edge profile of metal films at various condition.

(f) Lift-off and remove PMMA using acetone

O₂ plasma etching (1 minute at 300 mbar) following the lift-off was used to remove PMMA residue.

3.1.1 Key challenges in fabrication

The actual procedure is not as simple as described. This is indeed a difficult process. It took us years to develop and is still not fully under control. In this section, I listed key steps during the fabrication.

Spin PMMA

This step is trivial in a regular lithography. However, this turns out to be the most challenging step during the nanowire fabrication.

As can be seen in Fig. 3.2e, the undercut of PMMA is crucial in the liftoff process. Fig. 3.3 shows different edge profile of metal films under different condition. Fig. 3.3a shows an optimum undercut. PMMA isolates the metal on the substrate from the metal on PMMA. When the E-beam writing condition is not optimal, for example,
Figure 3.4: Optical microscope images of PMMA on substrate. (a) PMMA on a single substrate. The horizontal stripes due to interference indicate different thickness of PMMA. The edge of the substrate is not covered by PMMA. (b) PMMA on a back-to-back substrates. The coverage of PMMA on substrates is different from one place to the other. This shows the difficulty of the spin coating process.

exposure time or developing time is not correct, the edge can be torn like in Fig. 3.3b. Because the metal pads serve as leads for the measurement, the torn edges cut off the connection between the leads and the nanowires.

PMMA should neither be too thin nor too thick. When it is too thin, the metal joins together (Fig. 3.3c). It forms a torn edge similar to Fig. 3.3b. When PMMA is too thick (Fig. 3.3d), the edge becomes very thin at the end. The metal pads may not be able to support themselves after the chemical etching, because the etching dissolves a small amount of the substrate under metal pads. As a result, the leads are shorted through the Al in the trench.

The undercut can be easily controlled if the thickness of PMMA is under control. The thickness of PMMA is easy to control in a normal lithography where the pattern is in the center area of the substrate. However, this is very difficult in our case because the pattern is only 2 \( \mu \text{m} \) away from the edge of the substrate. The thickness changes toward the edge. Close to the edge, the thickness is reduced rapidly and the substrate is barely covered by PMMA (Fig. 3.4a).
The problem was alleviated by placing two substrates back to back. This improves the PMMA coverage, as shown in Fig. 3.4b. However, it is not a foolproof solution. Numerous trials have to be made before a successful coating was realized.

The detailed procedure described is as following:

(a) Place two substrates back to back together
(b) Align the two top surfaces at the same level
(c) Drop an extremely small amount of PMMA between two substrates to glue them together and bake the sample to harden PMMA.
(d) Place the substrates in the special vice (Fig. 3.5)
(e) Put the vice on the spin coater, the vice prevents two substrates to separate at high rotating speed
(f) Bake the sample again to harden PMMA on the top surface

The step (b) was done by looking both surfaces under a microscope in a direction almost in parallel with the surface and perpendicular to the interface line. The light was scattered at the interface if there was a mismatch. Two substrates were carefully adjusted in position until the light reflection was gone.

Chemical etching

The solution used for chemical etching is $H_3PO_4 : H_2O_2 : H_2O$ (1:1:138 by volume). The etching depth depends on the thickness of evaporated Al. Usually, for 10 nm Al, the etching depth is 30 nm. The etching rate is around 1 nm/s. The etching rate changes over time due to the natural decomposition $H_2O_2$. In principle, longer etching time is better because Al in the trench has to be separated from Al on and ridge and supporting pads. However, the time cannot be too long because of
Figure 3.5: Special vice to hold the sample together. (a) The schematic of the vice. (b) A photo of the vice.

Following reasons. First, although the ideal selectivity (the etching rate ratio between InGaAs and InP) is high, the solution unavoidably attacks several mono-layers of InP. This creates a height difference between supporting pads and the InP ridge. The Al on the ridge will have trouble to connect to the metal pads. Second, the etching is isotropic, which means that the substrate under supporting pads is also etched. The thin supporting pads are not rigid enough to support itself, if too much substrate underneath is gone. Overall, this process requires a fine balance between two competing factors.

Before chemical etching and after liftoff process, the sample was cleaned with $O_2$ plasma etching (Technics PE II-A Plasma System). The process is as followings:

(a) Pump the chamber

(b) Flush the chamber with pure $O_2$ for 10 min at the pressure of 300mbar

(c) Start etching for 1 min
Figure 3.6: Shadow evaporation. (a) is the schematic of the device and (b) is a photo of the device. The blade can move back and forth.

Shadow evaporation

The pattern on the substrate serves as a bridge between the nanowire and the external circuit. On the nanowire side, the metal film must be very thin (around 10 nm). On the other side where Au wire is bonded, it requires a thick Au layer (around 60 nm).

The final Al evaporation also requires the bonding area of the substrate to be blocked during evaporation, as shown in Fig. 3.1e. The blocking minimizes the chance that Al in the trench shorts electrodes. This step cannot be done by another E-beam lithography, because PMMA may damage the ridge during spin coating.

To simplify the process, a shadow evaporating device is made (Fig. 3.6). The device was used to block the bonding area during the evaporation. The position of the blocker (a blade) can be adjusted. The procedure is as followings. First, after lithography, the supporting pads were evaporated while the bonding area was blocked by the blade. Two different kinds of supporting pads were used: one was Ti (10 nm) and the other was Au/Ti (5/6 nm). Next, the bonding pads (Au/Cr 60/5 nm) were evaporated while part of supporting pads was blocked. The final step was to evaporate Al again while blocking the bonding area.

Fig. 3.7 shows the schematic of the nanowire and pads.
3.1.2 Microscopic structure of the nanowire

Fig. 3.8 shows the nanowires on the InP ridge imaged by Transmission Electron Microscope (TEM). When imaging, the electron beam passed through nanowires and InP ridge in the direction perpendicular to the substrate surface. Continuous wires were observed on the top of InP, as can be seen in Fig. 3.8a. Nanowires consist of many crystallized Al nuclei (Fig. 3.8a) scattered along the longitudinal direction. Between nuclei, there exists either amorphous Al or tiny Al crystal, which cannot be distinguished clearly. The TEM images also indicate the wire cannot be arbitrarily thin. Al tended to aggregate and many weak links were created, particularly when less Al was evaporated (Fig. 3.8c).

3.2 Experimental setup

The importance of the measurement for the experimental works cannot be overemphasized. Without proper measurements, the experimental results can be misleading or sometimes plain wrong. This is why I feel it is important to list a thorough experimental setup in a separate section.

This section includes measurement methods and the equipment. The methods
Figure 3.8: TEM images of a nanowire. (a) a nanowire on the InP ridge, (b) a defective InP ridge, (c) a granular nanowire, (d) a high-resolution TEM image.

include the resistance measurement, current-voltage (IV) measurement, differential resistance \((dV/dI)\) measurement, magneto-resistance measurement. The noise characteristic of various equipments, including the voltage meter, the IV meter, the function generator and various filters, is introduced next. In the end, I show the temperature control method in \(^3\)He system and the dilution refrigerator.

Note that our measurement methods changed and improved over time. Not all data was obtained from the following setups. In the result section, we will specify the condition if it is differ from following setups.

3.2.1 Measurement method

Resistance measurement

Fig. 3.9 shows the setup to measure the resistance of the sample. This is a four-terminal measurement using a lock-in amplifier (Princeton Applied Research 124). The typical value for the excitation AC voltage is 0.1 V (RMS value) and the value for
the resistance of $R$ is $10\ \text{M}\Omega$ unless specified otherwise. The frequency was chosen to be prime numbers, for example 11, 13, 17 or 23 Hz, to reduce line frequency interference (60 Hz). The filters can be used in the circuit but it is not necessary because the lock-in has a very low noise.

The four terminals were connected to the leads next to the nanowire by the methods shown in Fig. 3.10.

*IV measurement*

The constant current mode was used to measure IV curves, because the samples were superconductors. The IV measurement was done by two methods (Fig. 3.11).
Figure 3.11: Experimental setup to measure IV curves. (a) Measured by the home-made low noise power supply controlled by the computer. The voltmeter is a HP3478A multimeter. (b) Measured by a Keithley 6430 source meter.

One was to use a home-made power supply. The power supply provides a voltage. A large resistor (1 MΩ) was used as current limiting resistor to convert the source to the constant current source. The other was to use Keithley 6430 sourcemeter. The sourcemeter has the option to operate in either the constant voltage mode or constant current mode. It measures the voltage or the current directly.

Generally speaking, the home-made power supply is preferred since it has less noise than the Keithley 6430.

Figure 3.12: Experimental setup to measure differential resistance.
Figure 3.13: Experimental setup to measure Magneto-Resistance.

**Differential resistance measurement**

The setup is shown in Fig. 3.12. It measures the local slope of IV curve and detects a small nonlinearity in IV curve. The basic principle is that a small AC modulation was added onto the main signal. In our case, the main signal is a constant current provided by Keithley 6430. A small AC current was added through a transformer. The AC current (70 nA, RMS) was generated by an AC voltage excitation (0.07 V) and a 1 MΩ current limiting resistor. The AC voltage on the sample was measured by the lock-in amplifier through a home-made voltage amplifier.

**Magneto-resistance measurement**

The setup was designed based on the principle of a resistance bridge (Fig. 3.13). This circuit can detect a very small change in the sample resistance. The AC voltage was 0.07 V (RMS) and the AC current was 70 nA (RMS). During the measurement, R1 and R2 were tuned first so that the potential at B was the same as it at A. Then
Figure 3.14: Experimental setup to measure $I_s$ fluctuation. The function generator (Tektronix 3000 series) outputs a sawtooth wave. The voltage drop on the sample is fed into the peak detection circuit. The circuit controls a switch to bypass the current and simultaneously outputs a squarewave to the oscilloscope.

A magnetic field was applied and the change in the resistance was recorded.

**Switching current fluctuation measurement**

The switching current $I_s$ is defined as the current at which the nanowire switches to normal state from superconducting state. The value can be found on a IV curve. The basics of the fluctuation measurement are the IV measurement. In principle, this can be done by repeatedly measuring IV curves using above methods. However, to get a better statistics, more than 10,000 traces are needed. The above method is impracticable because the time needed to perform 10,000 IV curves is enormous (several minutes for a single IV trace). Therefore, great engineering efforts were made to measure the fluctuation successfully.

The measurement was achieved by using a function generator coupled with an oscilloscope with deep memory (Fig. 3.14). The function generator outputs a sawtooth wave. A current limiting resistor was used to convert the voltage source to a
current source. An oscilloscope synchronized with the function generator recorded the voltage drop across the sample. The recorded data from the oscilloscope (voltage vs. time) could be converted to IV curve since the current was linked to the time. The data were analyzed by a program to determine $I_s$. To reduce the heat generated when the nanowire turned to normal, a circuit was designed to bypass the current when a voltage jump was detected.

Fig. 3.15 shows the simplified schematic of the bypass circuit. The signal from the amplifier is first compared with a reference voltage. The reference is set slightly above the zero by a potentiometer. The output of the comparator is fed to a flip-flop (74 series). The flip-flop does a logic calculation. It raises the output when a voltage jump is detected and resets everything when a TTL (Transistor-Transistor Logic) signal is received. The TTL signal is the trigger signal from the function generator. The falling edge of TTL signal open the switch at the beginning of each cycle. The output of the flip-flip controls two opto-couplers: one bypasses the current; the other goes to the oscilloscope.

To better understand this process, we plot Fig. 3.16.

As shown in Fig. 3.16, the bypass circuit reduced the heating when the nanowire became normal. The heating time is around 100 $\mu$s. This is 100 times shorter than
Figure 3.16: Waveform comparison. From the top to the bottom: (1) the voltage outputted by the function generator, (2) TTL signal from the trigger of the function generator, (3) the actual current flowing through the sample, (4) the voltage drop across the sample, (5) the signal recorded by the oscilloscope.

3.2.2 Noise characteristic of equipments and filters

Proper filtering in electrical transport measurement is always important, especially for low noise measurement at low temperature. The noise level from external circuit must be small enough to avoid unwanted excitation. It is more so in measuring a superconductor because many approaches and principles are different.

We use a simple example to show why and how the approach is different. RC filters are widely used in our lab to reduce the noise. A regular RC filter is shown in Fig. 3.17a. The voltage noise \( \delta V_s \) on the sample is

\[
\delta V_s = \frac{1}{R + \frac{1}{j\omega C}} \frac{R_s}{\frac{1}{j\omega C} + \frac{1}{j\omega C} R_s} \delta V \tag{3.1}
\]

where \( \omega \) is the frequency and \( \delta V \) is the voltage noise from the power supply. At high frequency, voltage noise on the sample is shorted by the capacitor so \( \delta V_s \) becomes
very small. What happens if the sample is a superconductor? It is true that the voltage across the sample drops to zero. But it does not mean that the sample is not affected. The noise should be analyzed in terms of the current noise. In a regular RC filter, all current goes through the superconductor. Therefore, the current noise \( \delta I \) is \( \delta V/R \) and it is not affected by the capacitor \( C \). One way to solve the problem is to add another resistor in series with the sample after \( C \). \( \delta I \) becomes \( \delta V_s/R \) for high frequency noise. This greatly reduces the current noise.

Above example shows how tricky the measurement might be when dealing with superconductors. We tried minimized the experimental artifacts through two approaches. One is to choose or make low noise electronics, including using the homemade power supply, battery-powered amplifiers etc. The other is to heavily filter the signal, including low temperature RC filters, thermo-coax filters, Farad beads filters etc.

In this section, the noise characteristic of various equipments we used and the performance of filters were shown. One of my biggest mistakes was to take many things for granted in terms of the equipments. Each new equipments or method should have been carefully examined before being used.
**Figure 3.18:** Noise characteristic of Keithley 6430 sub-Femtoamp SourceMeter. The noise was measured when the output was set to 0 V.

*Noise characteristic of Keithley 6430 sub-Femtoamp SourceMeter*

Modern digital electronics, especially with a catchy name and a high price tag, can be very misleading in the field of the low noise measurement. The equipment may have user-friendly interface and easy computer access. The fact that the equipment shows the 6-digit precision of the voltage seems to suggest the output to be equally stable. As shown in Fig. 3.18, however, the equipment is far from ideal to measure the sample. If all the noise is integrated, it gives us 1 mV (RMS value) voltage fluctuation. Suppose a typical 10 KΩ resistance in series with the sample, it gives us the current fluctuation up to 100 nA. This equipment should be used only when convenience is the priority or the noise can be averaged out.
Figure 3.19: Noise characteristic of the function generator.

**Tektronix AFG 3215 Function generator**

The noise spectrum of the function generator is shown in Fig. 3.19. The voltage fluctuation is 0.4 mV in RMS integrated up to 1 GHz. However, the voltage noise from the function generator has a less impact on the measurement because of the big current limiting resistor (1 MΩ). For example, 0.4 mV voltage fluctuation gives 0.4 nA current fluctuation, which is negligible compared with 20 nA fluctuation observed in the experiment. In addition, there were RC filters and ferrite-bead filter to further filter out the noise.

**HP3478A multimeter**

The device is used to measure the voltage drop on the sample. The voltage fluctuation is 0.7 mV integrating all frequency. This device also generates noise affecting sample
if it is not filtered properly (Fig. 3.20). Later, we used a homemade amplifier to isolate the multimeter from the sample.

Other devices

All other devices, including the homemade power supply, homemade voltage amplifier (INA 110) and PAR 124 Lock-in amplifier have negligible noise.

Performance of RC filter

The importance of filtering cannot be stressed enough. Our main result, $I_s$ fluctuation, is on the order of 20 nA. Any current noise estimated in previous section can significantly exaggerate the fluctuation. Therefore, the noise should be reduced as much as possible.

Since all measurements were quasi-static measurement, low-pass filters were used
Figure 3.21: Performance of RC filter near cutoff frequency. The output of the Keithley sourcemeter with and without filter. The output of the source-meter is set to 0 V. The cutoff frequency of the filter is 1.6 KHz (10 KΩ and 10 nF)

to cutoff unwanted high frequency noise. Many kinds of filters were used, including low temperature RC filters, Ferrite-bead filters, room temperature copper powder filters and Thermocoax filters.

Low pass RC filters are one of commonly used low pass filters. RC filters are described in Fig. 3.17. Except the problem mentioned earlier, RC filters have two other problems. One is that the filters cannot remove the noise right above cut-off frequency. The other is that they do not work well at RF frequency due to the stray capacitance.

The attenuation of RC filters decreases exponentially with the frequency. This may be sufficient in ordinary situation, but it still can jeopardize the measurement. For instance, Fig. 3.21 compares the noise characteristic of Keithley 6430 with and
without the RC filter. There is a -45 dBm spike (1.2 mV in voltage) at 60 KHz. With a RC filter at the cutoff frequency of 1.6 KHz, the spike decreases by a factor of 1,000 to -75 dBm (0.04 mV in voltage). Though the voltage is small, it still generates 40 nA current fluctuation with 1 KΩ series resistance.

Two different designs were used: Aluminum-box RC filter and PCB-board RC filters. Box RC filters were usually placed at room temperature and PCB-board RC filters were placed at low temperature near the sample. PCB-board filters were installed at 1K pot (around 1.4 K) in ³He system and at mixing chamber (20 mK) in the dilution refrigerator. Placing the filters at low temperature and close to the sample not only reduces thermal noise (∼ √T) but also filters out the noise picked up from adjacent cables.

RC filters work poorly at high frequency, as shown in Fig. 3.22. It also shows that better design improves the performance. An Al-box filter has many resonant
frequencies. By making the filter on a PCB board and choosing better components, the performance of the filter in the hundreds of MHz range is greatly improved.

**Performance of Ferrite-bead filter**

In order to remedy the poor performance of RC filters above 100 MHz, Ferrite-bead filters were used. As can be seen from Fig. 3.23, the cutoff frequency is around 10 MHz. The signal is suppressed to an undetectable value above 100 MHz.

**Performance of metal-powder filter**

To further suppress RF noise above the GHz range, metal-powder filters were fabricated and used. Fig. metal powder filter shows the attenuation of the filter.
Performance of Thermocoax filter

Like metal-powder filters, Thermocoax filters were used to further suppress high frequency RF noise. Thermocoax cable (manufactured by Philips) is a coax cable designed for high temperature usage in the vacuum. However, it turns out to have an excellent filtering effect at cryogenic temperature. The cable was installed in the dilution refrigerator. The performance is shown in Fig. 3.24.

Position of the filters

In the $^3$He system (introduced in the next section), Ferrite-bead filters were used at room temperature and further away from the sample. Metal-powder filters at room temperature were installed at the cable feed-through on the cryostat. PCB RC filters were installed at low temperature close to the sample.

In the dilution refrigerator (introduce in the next section), Ferrite-bead filters
were also at room temperature and further away from the sample. Thermocoax
cables were used for wiring inside the dilution refrigerator. PCB RC filters were
installed and thermalized at the mixing chamber.

3.2.3 Temperature control

Temperature control is crucial in the fluctuation measurement. As shown in Chapter
4, the mean value of \( I_s \) changes with temperature. If the temperature is poorly
regulated, the distribution of \( I_s \) includes not only intrinsic fluctuation, but also the
temperature change. Actually, a great deal of effort in early stage focuses on how to
minimize the temperature fluctuation and drift.

The controlling principles and methods are different for \(^3\)He cryostat and the
dilution refrigerator. We will introduce them separately as follows.

Temperature control in the \(^3\)He cryostat

In the \(^3\)He cryostat (Oxford), the latent heat of evaporation is used to cool the liquid
\(^3\)He down to 0.3 K. The boiling temperature of liquid \(^3\)He is 3.19 K at 1 atmosphere.
The temperature is lowered when the pressure is reduced. A sorption pump is used
to reduce the pressure. The pump absorbs \(^3\)He gas when the temperature of the
pump drops. The absorption rate depends on temperature. Thus, the temperature
of the \(^3\)He can be controlled through tuning the temperature of the sorption pump.

Temperature control in dilution refrigerator

The dilution refrigerator (Leiden minikelvin 126-TOF) lowers the temperature using
the property \(^3\)He/\(^4\)He mixture. Generally speaking, the \(^3\)He/\(^4\)He mixture is separated
into \(^3\)He rich phase and \(^4\)He rich phase at low temperature. The binding energy of
\(^3\)He atoms in these two phases is usually different. Under certain condition, \(^3\)He
atoms tend to escape from \(^3\)He rich phase to \(^4\)He rich phase. This process lowers the
Figure 3.25: Temperature control in $^3$He system. The sample and thermometer sharing the same substrate were submerged in the liquid $^3$He. The temperature controller (Picowatt, TS-530A) read the thermometer and adjusted the power to heat the heater through PID control.

temperature of the mixture. It is similar to lowering the temperature by pumping on liquid. But in this case, $^3$He is pumped by the $^4$He rich phase instead of the vacuum.

The base temperature of the dilution refrigerator is 10 mK. A major difference between the dilution refrigerator and $^3$He system in terms of the design is the position of the sample. The sample in the dilution refrigerator is outside of the liquid while the sample in $^3$He system is inside. This determines a different cooling method between the dilution refrigerator and the $^3$He system.

In the dilution refrigerator, the sample can be cooled through both substrate (phonon) or measuring cables (electron). The thermal conductance of substrate becomes extremely poor at low temperature (the thermal conductance by phonons is proportional to $T^3$). Therefore, the sample was best cooled through the measuring cables (the thermal conductance of the electron gas is proportional to $T$).

The design of the sample holder is shown in Fig. 3.26. The mixing chamber stays
Figure 3.26: The schematic of sample holder in the dilution refrigerator.

at base temperature. A copper rod used as a thermal sink is attached onto the mixing chamber through a stainless steel rod (thermal isolator). A copper cable connecting the copper rod and the mixing chamber serves as a thermal link. It ensures that the temperature of the thermal sink can be elevated by the heater while the mixing chamber remains at around base temperature. The heater is made of a resistive wire (1 KΩ) thermally anchored to a copper spool. The measuring cables are thermally anchored to the copper rod. The sample is cooled through the measuring wires and so is the temperature of the sample controlled.

In principle, the temperature can be controlled by a PID controller as is done in 3He system. But there is an inherent problem. The temperature controller responds to the reading of a resistance bridge. The resistance bridge averages the signal from the thermometer over a time span of seconds. On the other hand, we apply a 10 Hz sawtooth wave. Apparently, the temperature controller is too slow to regulate the
Figure 3.27: Increased temperature vs. applied power in dilution refrigerator. The squares are the experimental data and the red curve is the fitting by the power law. The inset shows the fitting equation and parameters.

Figure 3.27: Increased temperature vs. applied power in dilution refrigerator. The squares are the experimental data and the red curve is the fitting by the power law. The inset shows the fitting equation and parameters.

temperature during ramping. It can only correct long term temperature drift.

The temperature control in dilution refrigerator turns out to be much easier than in the $^3$He system. The temperature of the sample is controlled by two competing processes: the cooling from the mixing chamber and the heating from the heater. The temperature is stable as long as two above processes are stable. This is different from the $^3$He system, where the cooling power from the sorption pump changes with time.

The only potential problem comes from the heat generated by the sample. The maximum power generated by the sample is 50 nW when the nanowire turns normal. Fig. 3.27 shows that $3 \times 10^{-7}$ W is needed to increase 1 mK at 100 mK. Therefore, the temperature change due to the heating from the sample is negligible.
4 Experimental result

4.1 Superconducting-normal transition of superconducting nanowires

Before I start this chapter, I would like to list all samples here for the future reference. where L, W, H and R are length, width, height and resistance (at normal state)

<table>
<thead>
<tr>
<th>Sample</th>
<th>L(µm)</th>
<th>W(nm)</th>
<th>H(nm)</th>
<th>R(KΩ)</th>
<th>Leads</th>
<th>Residue R(KΩ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>1.5</td>
<td>8</td>
<td>10</td>
<td>0.5</td>
<td>Al/Ti (10/10 nm)</td>
<td>0</td>
</tr>
<tr>
<td>S2</td>
<td>10</td>
<td>8</td>
<td>10</td>
<td>3.8</td>
<td>Al/Ti (10/10 nm)</td>
<td>0</td>
</tr>
<tr>
<td>S3</td>
<td>10</td>
<td>8</td>
<td>10</td>
<td>5</td>
<td>Al/Ti (10/10 nm)</td>
<td>0</td>
</tr>
<tr>
<td>S4</td>
<td>10</td>
<td>8</td>
<td>8</td>
<td>13</td>
<td>Al/Au/Ti (8/5/6 nm)</td>
<td>0.5</td>
</tr>
<tr>
<td>S5</td>
<td>3</td>
<td>8</td>
<td>8</td>
<td>3</td>
<td>Al/Au/Ti (8/5/6 nm)</td>
<td>0.5</td>
</tr>
<tr>
<td>S6</td>
<td>20</td>
<td>8</td>
<td>8</td>
<td>18</td>
<td>Al/Au/Ti (8/5/6 nm)</td>
<td>0.5</td>
</tr>
</tbody>
</table>

respectively. Note that the actual cross section area of nanowires is smaller than the product of W and H because of the surface oxidation.

The superconducting-normal (S-N) transition measurement (resistance vs. temperature) plays an important role in the development of phase slip theory. Indeed, the transition curve was the very first experimental evidence for the Thermal Activated Phase Slip (TAPS) theory [1]. Resistance vs. Temperature (R-T) measure-
ments were also used to claim phase slips by quantum tunneling (quantum phase slip, QPS) [11,18].

In this section, we show R-T measurement of superconducting nanowires with both normal leads and superconducting leads. We also compare the nanowires with different leads. We found that the nanowires with normal leads showed a resistance tail, similar to Giordano’s QPS measurement [11]. The resistance tail disappeared in the nanowires with superconducting leads. The R-T curves roughly fit TAPS theory close to $T_c$.

The experiments were first done on the nanowires with permanent normal leads, as shown in Fig. 4.1. The nanowires showed a residue resistance down to 300 mK. To highlight the change, the resistance (subtracting the residue resistance at 300 mK) was plotted in the log scale. As can be seen from Fig. 4.1b, R-T curve (after subtracting the residue resistance) fitted QPS theory very well (Eq. 2.39). The fitting was even better in several other samples. The impact of the magnetic field was shown in Fig. 4.1c. Surprisingly, R-T curves (subtracting residue resistance) of two nanowires with significantly different length but the same cross section showed almost identical behavior at low temperature, as shown in Fig. 4.1d.

Our early belief was that the residue resistance was due to the inverse proximity effect (Chapter 2). The value of the resistance was close to a normal section of the nanowire with the length of superconducting coherence length $\xi$. The resistance caused by phase slips can be calculated by subtracting the residue resistance from the total resistance, because the residue resistance saturates at low temperature. The curve can be fitted by QPS equation, as shown in Fig. 4.1b. Since the fitting showed the good agreement, we concluded that QPS was observed in our sample.

However, Fig. 4.1d indicated otherwise. It implied that the resistance tail was possibly due to the interface, because the resistance caused by phase slips should be proportional to the length. Indeed, it turns out that the R-T measurement was
greatly affected by whether the leads were normal or superconducting, as shown in Fig. 4.2. It shows the nanowire became completely superconducting below 1.3 K. The broadened transition was fitted by TAPS theory. The fitted $\xi_0=30$ nm is smaller than estimated coherence length 100 nm. This might be because the transition was complicated by the superconducting transition of the leads, which caused the inverse proximity effect.

The result is quite significant. First, we showed there was no resistance tail
The experiment was done on S2. The nanowire with superconducting leads was fitted by TAPS theory (Eq. 2.34). The fitting parameters were $\xi_0 = 30$ nm and $T_c = 1.47$ K. The inset shows the fitting result. The superconducting leads were turned normal by 0.1 T magnetic field. The resistance of the nanowire with normal leads was obtained by fitting the linear component of IV curves of the nanowire.

when the leads were superconducting. The result is directly in contradiction with Giordanos experiment, which was one of the pillars supporting the theoretical articles on QPS appearing later [11]. Our nanowire is several times thinner than his. By any means, the resistance tail should be more pronounced rather than diminishing if it is due to QPS. Second, we showed the importance of the superconductor-normal metal interface. The behavior was dominated by the interface instead of intrinsic properties of nanowires.
Figure 4.3: IV curves of the nanowire with different filters. (A) The influence of filters. The filters were regular RC filters with cutoff frequency of 1.6 KHz (R=10 KΩ, C=10 nF). The measurement was done at 300 mK under zero magnetic field. (B) The influence of filters at B=2.5 T (near the critical field) at 300 mK. The filters were also regular RC filters with cutoff frequency of 1.6 KHz (R=10 KΩ, C=10 nF), 160 Hz (R=10 KΩ, C=100 nF) and 16 Hz (R=10 KΩ, C=1 µF). The sample was S3.

4.2 IV measurement

A superconductor switches from the superconducting state to the normal state when the current is increased. In the IV measurement, a voltage jump was shown when the current exceeds a critical value. Ideally, the largest current a superconductor can sustain is called the depairing critical current ($I_c$), at which the kinetic energy of the quasi-particles exceed the superconductor gap. Experimentally, however, the superconductor will switch from the superconducting state to the normal state at a lower current. To distinguish it from $I_c$, we call it the switch current $I_s$. In this section, we show IV curves and the temperature dependence of $I_s$.

4.2.1 IV measurement affected by electrical noise

Before we start, we would like to show the importance of the filtering and how the understanding can be completely wrong due to inadequate filtering. As seen
in Fig. 4.3A, the characteristics changed qualitatively before and after filters were used. Without the filters, the voltage jumped up and down indicating the sample was oscillating between the superconducting and normal state. Adding more filters resulted a larger $I_s$. Fig. 4.3B shows the impact of filters with different cutoff frequency near the transition. The IV curves changed from a step-like feature to a single jump. Although the nanowire was measured near the critical field, the step-like feature was also observed near $T_c$. The feature is similar to the so called phase slip center [38]. Although it was discovered long time ago, researchers continue to observe similar behavior and attribute it to the phase slip center [7–10]. We demonstrated that such behavior was potentially due to imperfect filtering.

4.2.2 Differential resistance of nanowires with normal leads

As can be seen from previous section, heavy filtering reduced the nonlinearity of IV curves of nanowires with normal leads. At certain point, the nonlinearity is too small to see from IV curve. But further differential resistance ($dV/dI$) measurement revealed a detectable nonlinearity. The measurement measures the local resistance
and is sensitive to the change of IV.

Fig. 4.4 shows $dV/dI$ curves of a nanowire with normal leads at different temperature. Experimentally, we found the curves could be fitted very well by an exponential function plus a constant.

Further investigation showed the nonlinearity was mainly due to the normal superconducting interface, more specifically, the inverse proximity region. This is indicated from the similarity $dV/dI$ between S5 and S6, which have similar cross section area but very different length (3 $\mu$m vs. 20 $\mu$m). Therefore, the current dependence of the nanowire in the inverse proximity region contribute the observed nonlinear IV in my samples. Unfortunately, no theoretical guideline was found about the quantatively behavior.

Note that this does not contradict with our previous result [27], which was on a much longer nanowire (100 $\mu$m). Not only nonlinearity is more pronounced in that case, but also the criteria determining short and long nanowire can be different.

4.2.3 IV curves of nanowires with superconducting leads

The nonlinearity below $I_s$ disappeared in nanowires with superconducting leads, as shown in Fig. 4.5A. This was confirmed by $dV/dI$ measurements, which also did not show any nonlinearity. The only exception was when the temperature was very close to $T_c$ (Fig. 4.5B). The nonlinearity could be due to phase slips.

Lacking nonlinearity at low temperature appears to be in contradiction with the theory mentioned Chapter 2, in which we show a nonlinear IV is developed in an overdamped Josephson junction. The inconsistency is caused by the fact that the calculation is based on the assumption that the heat from phase slips can be removed efficiently. Goldbart et al. proposed that the heat from phase slips may raise the local temperature and causes the premature switching [51]. Thus, the nanowire turns to normal before it shows nonlinearity. A thorough discussion and detailed
Figure 4.5: IV measurement of nanowires with superconducting leads. (A) IV curves at different temperatures. The arrows show the current sweeping direction. (B) IV curves near \( T_c \). The sample is S3. The sample was measured in the dilution refrigerator.

calculation are postponed to the fluctuation measurement section. However, the result clearly indicates that using nonlinear IV to estimate phase slip rate may be questionable [26,27].

4.2.4 Temperature dependence of \( I_s \) with normal and superconducting leads

It turns out that the difference between nanowires with normal and superconducting leads is more than the occurrence of the nonlinearity. \( I_s \) can be reduced by as much as 50% when leads become normal from superconducting state. This was observed on the same wire when the leads were turned to normal from superconducting state by a small magnetic field (0.1 T). The critical field of the leads (0.1 T) is much smaller than that of the nanowire (\(~1\) T). The sharp decrease of \( I_s \) was observed immediately after the magnetic field exceeded the critical field of the leads. This demonstrated that the reduction of \( I_s \) was not because of the field dependence of \( I_s \) of the nanowire itself.

Fig. 4.6 shows the qualitative difference between nanowires with normal and
Figure 4.6: Temperature dependence of $I_s$ with superconducting and normal leads. $I_s$ was average over 10,000 IV measurement. The sample was S2. The leads were turned to normal by 0.1 T magnetic field. The fitting parameters for superconducting leads were $I_{c0}=4.45 \, \mu A$ and $T_c=1.36 \, K$ and those for normal leads were $I_{c0}=1.89 \, \mu A$ and $T_c=1.32 \, K$. The inset shows the temperature dependence of $I_s$ of S5 fitted by $I_c$ of the Josephson junction. The fitting parameters were $I_{c0}=0.83 \, \mu A$ and $T_c=1.4 \, K$.

superconducting leads. The temperature dependence of nanowires with superconducting leads was fitted by the phenomenological depairing $I_c$ equation (Eq. 2.26). There are no theoretical guidelines available for the temperature dependence of the nanowires with normal leads. Experimentally, however, nanowire with normal leads can be fitted very well by the $I_c$ equation for a Josephson junction (Eq. 2.9). The temperature dependence is not due to the magnetic field, as demonstrated by the nanowire with permanent normal leads (inset in Fig. 4.6).

This is indeed a surprise. Typically, it is believed that in a long superconducting nanowire, the measurement should reflect the intrinsic properties of the nanowire.
In our case, the length of the longest nanowires (20 µm) is 200 times longer than the superconducting coherence length ξ (100 nm). Thus, one expects that the boundary condition could be neglected. Clearly, the experiment showed the opposite.

The exact reason for the difference is unclear. Several factors may have been involved, including the inverse proximity effect, intrinsic property, Joule heating etc. First, there is the inverse proximity effect at the normal-superconducting interface. The superconducting gap changes gradually near the interface. It is possible that the superconductivity of nanowire near the interface is destroyed first. Then the current-turned normal section affects the section next to it. This section further affects the nanowire inside and soon the whole nanowire becomes normal. The problem of this explanation is that the resistance should increase gradually with increasing current. However, this was not what we seen in the experiment.

Second, Klapwijk et al. proposed that there exists a critical voltage to turn the superconducting nanowire to normal [52]. The non-equilibrium energy distribution in the nanowire due to the injection of quasiparticles from the normal metal may alter the superconducting gap. As a result, the superconducting nanowire may turn to normal before depairing \( I_c \) is reached. The influence of quasiparticles to the superconductivity has been demonstrated previously [53]. Unfortunately, we do not have the access of the theoretical model so it is difficult for us to compare.

Third, the Joule heating from the residue resistance may also cause the early switching. Due to the inverse proximity effect, the resistance can be measured in a superconducting nanowire. The resistance may generate a significant amount of Joule heating. In such confined geometry, it is difficult to dissipate the heat by phonons duo to the electron-phonon decoupling and the Kapitza resistance. Therefore, the dominant dissipation is by qualsiparticles to the normal leads. The residue resistance can be modeled as a normal nanowire with a length of ξ. One end of the nanowire is attached to a normal lead, so the temperature is fixed at the ambient temperature.
The other end connects to the superconducting nanowire and there is no heat flow. Setting a proper boundary condition, the temperature of the hot side is

\[ T_f = \sqrt{\frac{I^2R^2}{L_0} + T_0^2} \]  

(4.1)

according to the Wiedemann-Franz (W-F) law, where \( I \) is the current, \( R \) is the residue resistance at each side, \( L_0 \) is the Lorenz number and \( T_0 \) is the temperature of the leads. As can be seen from Fig. 4.7, the temperature rises about 0.3 K at base temperature according to the model. However, we should be aware of that because this subject is rarely studied, not all the assumptions are necessarily correct. For example, the heat transfer in the inverse proximity region may not follow the W-F law. A recent work showed the thermal conductance of an inverse proximity region has better thermal conductance than a superconductor, but a direct comparison to the W-F law is still lacking [54].

In the end, the reduction in \( I_s \) may be a combination of above factors. Nonetheless, the behavior can still be universal to all superconducting nanowires, due to
commonly restricted geometry.

More interestingly, the reduction of $I_s$ was influenced by both the length and temperature. $I_s$ of a short nanowire (1.5 $\mu$m) with normal leads exceeded its value with superconducting leads close to $T_c$, as shown in Fig. 4.8. The behavior is similar to a so-called anti-proximity effect [29]. In their article, Tian et al. found that the superconductivity of the nanowires could be suppressed by normal leads. More experiments showed the superconductivity could depend both on the magnetic field and on the applied current [31]. They found that the superconductivity reappeared when the current exceeded a critical value in a magnetic field.

The characteristic length to determine above phenomena is the charge imbalance length $\Lambda$ instead of $\xi$. Charge imbalance is a non-equilibrium effect caused by the conversion of quasi-particles from a normal metal to a superconductor. In a normal metal-superconductor interface, the quasi-particles from the normal metal are converted to the Copper pairs through the electron-phonon interaction. The charge imbalance length is the length scale for the interaction to happen. The typical value of $\Lambda$ is about 1 $\mu$m in such system. This is 10 times longer than $\xi$. Our result is consistent with a theoretical calculation [55], in which the authors predicted a higher $I_s$ when the length of the nanowire is close to $\Lambda$.

Finally, we want to emphasize that experiments on the normal and superconducting leads, and short and long nanowires were performed consecutively after each temperature was fixed. Therefore, the result was not caused by spurious factors such as the change of the temperature due to the external magnetic field.

4.2.5 Retrapping current

IV curves of superconducting nanowires show hysteresis. The origin of the hysteresis was believed to be due to the intrinsic underdamping, similar to an underdamped Josephson junction (Chapter 2). However, recent experiments indicated the hystere-
Figure 4.8: Temperature dependence of $I_s$ with superconducting and normal leads. The nanowire is 1.5 $\mu$m long with $R_n=500$ $\Omega$. The inset shows the behavior near $T_c$.

Hysteresis might be caused by self-heating. Tinkham et al. showed that the heating might be the origin of the hysteresis in superconducting nanowires based on the shape of the nonlinear IV curves near $T_c$ [56]. Another experiment showed heating in a SNS junction by directly measuring the electron temperature [57]. Here I measured the retrapping current, $I_r$, of superconducting nanowires. The result shows the heating is the cause for the hysteresis in the IV curves. $I_r$ is defined as the current at which the nanowire reenters the superconducting state in a down-sweep.

$I_r$ reveals the highest temperature point on the wire, which is the signature of hysteresis caused by heating and a thermal runaway. Two samples, S1 and S2 were studied. S1 is 10 nm wide, 1.5 $\mu$m long and S2 is 10 nm wide, 10 $\mu$m long. In the short superconducting nanowire (S1), heat is conducted electronically (W-F law),
yielding

\[ I_r = \sqrt{4L_0(T_c^2 - T_0^2)/R} \]  \hspace{1cm} (4.2)

where \( L_0 \) is the Lorentz number and \( R \) is the normal resistance of the nanowire, and \( T_0 \) is the ambient temperature. In the long S2, \( I_r \) follows

\[ I_r = \sqrt{\sigma_K w L(T_c^4 - T_0^4)/R} \]  \hspace{1cm} (4.3)

due to the Kapitza boundary resistance between the nanowire and the surroundings, where \( \sigma_K \) is Kapitza conductance, \( w \) is the width and \( L \) is the length. Note that \( T^4 \) is expected for 3D phonons, while a lower exponent (\( T^3 \)) is the expected for phonons of reduced dimensionality. In the experiments, we observed a qualitatively difference between short and long nanowire. The temperature dependence of \( I_r \) fits the functions very well respectively. The retrapping data shown was based on the normal leads. \( I_r \) data of long wire does not depend on the states of the leads, while \( I_r \) of the short wire was higher when the leads were in the SC state. This further suggests the dominant heat dissipation of the long wire was through the substrate and that of the short wire was through the leads.

The result strongly suggests the origin of the hysteresis in IV measurement of a superconducting nanowire is the heating.
4.3 $I_s$ fluctuation measurement

4.3.1 $I_s$ fluctuation of nanowires with superconducting leads

Ideally, a superconductor can sustain a current up to a so-called de-pairing critical current ($I_c$), at which the kinetic energy of quasiparticles exceeds the bounding energy of Cooper pairs. In practice, however, the measured maximum current (switching current $I_s$) is often below $I_c$. In bulk superconducting wire, for example, the current flows and concentrates on the surface. This current also generates a magnetic field on the surface. The magnetic field lowers $I_c$ and destroys the superconductivity. In a 2D film, the generation and the movement of the vortex may switch the film to a resistive state.

These can be avoided in a 1D superconducting nanowires, but it has a new challenge to reach $I_c$ due to energetically more economical fluctuation events – phase slips (PSs). How do phase slips affect $I_s$? We have discussed in Chapter 2 that Josephson junctions within RCSJ model can be classified as either underdamped or overdamped, depending on whether the quality factor, $Q = \sqrt{2eI_cC/\hbar R}$, is greater or less than 1. An under-damped Josephson junction is readily driven normal by a single phase slip event. Subsequent to overcoming the free-energy barrier, the phase keeps running downhill as there is insufficient damping to retrap the phase in a local minimum. Thus, the Josephson junction exhibits zero resistance up to $I_s$ and the voltage is hysteretic in an IV measurement. In contrast, in an overdamped Josephson junction, the phase moves diffusively; the IV is nonlinear and hysteresis is often not present.

Due to an extremely small capacitance, a long superconducting nanowire is believed to be heavily overdamped ($Q \ll 1$). However, most experiments to date have shown behavior at low temperature reminiscent of underdamping, such as a hysteretic IV and lacking of nonlinear IV. The apparent paradox can be solved if the
switching is due to other aspect of phase slips, such as heating. A recent model calculation showed that the heat generated by PSs can introduce a premature switching and result a non-monotonic change of fluctuation in $I_s$ with temperature [51]. This scenario was invoked to explain a counterintuitive monotonically-decreasing fluctuation with increasing temperature in MoGe superconducting nanowires [28], where it was necessary to suppose a specific combination of the number of phase slips needed to induce the switching and phase slips with different origins (TAPS and QPS).

In this section, we report on the measurement of the switching current $I_s$ fluctuation. We find clear evidence that the switching was induced by a single TAPS over a sizable temperature range (single TAPS regime), causing the fluctuation to increase as $T^{2/3}$. Moreover, evidence supportive of QPS was obtained in the narrowest nanowire at low $T$, indicated by the saturation of the fluctuation. As the temperature rises, the fluctuation collapses beyond $T \sim 0.8$ K. This initially increasing then rapidly decreasing fluctuation as temperature rises is reminiscent of the behavior in an under-damped Josephson junction. Moreover, it is consistent with the picture that a single phase slip triggers the switching at lower temperature and multiple phase slips become necessary at higher temperature. At lower temperature, quantitative estimation shows the heat generated by a single phase slip likely causes a thermal runaway, triggering the switching transition [51].

Three samples, S1, S2 and S3, were studied. S1 is 10 nm wide, 1.5 $\mu$m long; S2 is 10 nm wide, 10 $\mu$m long; S3 is 9 nm wide, 10 $\mu$m long. S3 was obtained by further oxidizing the surface of S2. S1 and S2 were measured in the $^3$He system and S3 was measured in the dilution refrigerator. The method is described in details in Chapter 3. The upsweep ramp rate was 50 $\mu$A/s at a repetition frequency of 10 Hz. Immediately after a voltage jump, the current was bypassed to reduce the resistive heating time to $\sim 100$ $\mu$s, thus reducing the overall heating and ensuring adequate time to thermalize the sample ($\sim 10^{-7}$s) before the next cycle.
To measure $I_s$ fluctuation, we performed $\sim 10,000$ IV sweeps at each temperature, recorded $I_s$ during the up-sweep and plotted the histogram. Fig. 4.10A shows the histograms at different temperature. The probability density function $P(I)$ obeys the expression:

$$P(I) = \Gamma(I) \left( \frac{dI}{dt} \right)^{-1} \left( 1 - \int_0^I P(u)du \right)$$  \hspace{1cm} (4.4)$$

where $dI/dt$ is the constant ramp rate, and $\Gamma(I)$ is the switching rate at the current $I$ [42]. If a single phase slip triggers the switching, the switching rate equals the phase slip rate, enabling to extract the PS rate from the measured distribution. For
each distribution, we deduce the mean value $\langle I_s \rangle$ and the standard deviation $\Delta I_s$.

The temperature dependence of $\Delta I_s$ can reveal the origin of the fluctuation. Qualitatively speaking, TAPS leads to a power law dependence with $T$, while QPS causes $\Delta I_s$ to saturate at low temperature.

The distribution can be approximated analytically if switching is induced by individual PSs. We use TAPS as an example. The phase slip rate $\Gamma(I)$ is proportional to $\exp(-\Delta F(T)(1 - I/I_c)^{3/2})$. After linearizing the current dependence of free energy in $\Gamma(I)$, $P(I)$ is solved and the solution is a Gumbel (Extreme Value, EV) distribution

$$P(x) = \frac{1}{\beta} \exp \left( \frac{x - \alpha}{\beta} - e^{\frac{x - \alpha}{\beta}} \right)$$

(4.5)

The order-of-magnitude agreement between the experimental data and the fitting (Fig. 4.10B) indicates the transition was induced by phase slips.

The fluctuation is proportional to $T^{2/3}$ in the single phase slip regime in all samples, as shown in Fig. 4.10C and D. The TAPS rate is proportional to $\exp(\Delta F(T, I)/k_B T)$, where $\Delta F(T, I)$ is given in Eq. 2.28. The $2/3$ power arises from the inverse of the exponent in the current dependence of the free energy barrier, $\Delta F(T, I) = \Delta F(T)(1 - I/I_c)^{3/2}$ [41].

When the nanowire diameter shrinks further, it becomes more probable for the phase to undergo a macroscopic quantum tunneling process through the barrier (QPS). The QPS rate is often approximated as $\exp(\Delta F(I)/k_B T^*)$, where $T^*$ is a constant. Accordingly, the phase slip rate should saturate for $T < T^*$. This is indeed what is observed in S3, as shown in Fig. 4.10D.

One thing I would like to point out is that the temperature in Fig. 4.10D was obtained by the heat transfer analysis rather than the direct measurement, due to the design of the sample and the cooling mechanism in the dilution refrigerator. At very low temperature, the sample can only be cooled through the electrical wirings.
Figure 4.11: Temperature dependence of $\Delta I_s$ and $\langle I_s \rangle$ of S3 (raw temperature). (A) The temperature shown is the temperature of the sample holder. The fitting parameter is $I_c(0) = 3.8 \mu A$ and $T_c=1.33$ K. (B) shows the calculated temperatures for the different value of $\sigma_K$. Each curves were shifted by 2 nA for better visibility. The result shown previously is based on $\sigma_K = 80 W/K^4 m^2$.

because of insufficient phonon cooling from the substrate. Part of the leads on the substrate is the normal metal (Au/Cr). When a large current was applied, the pads generated Joule heat and raised the local temperature. The $^3$He system does not have such problem because samples were submerged in the liquid $^3$He.

The pads were Au/Cr (60/5 nm) thin-film strip. The resistance was 60 $\Omega$ and the size of the strip was 375 $\mu$m long, 3 $\mu$m wide and 60 nm thick. At a large bias, the heat generation and dissipation can be modeled by the differential equation

$$-\frac{d}{dx}\left(\frac{L_0}{\rho} \frac{dT}{dx}\right) = I^2 \rho - \frac{w}{\sigma_K}(T^4 - T_0^4)$$

(4.6)

where $L_0$ is Lorenz number, $\rho$ is the resistance per unit length, $\sigma_K$ is the Kapitza thermal conductance, $w$ is the width of the strip and $T_0$ is ambient temperature. One end of the strip stays at ambient temperature $T_0$, while the other end is the temperature of the nanowire. Using proper boundary conduction, the equation was solved numerically (A finite element method software, COMSOL, was used.) The
Kapitza resistance was unknown and was used as a variable parameter. In the intermediate temperature range, mean switching current $\langle I_s \rangle$ was fitted by the depairing $I_c$ (Eq. 2.26) quite well. This indicated the temperature rise in this region was minimal. Using the above condition as upper bound, we calculated the temperature of the nanowire. As shown in Fig. 4.11B, using $\sigma_K = 80 W/K^4 m^2$ maintains $\langle I_s \rangle$ close to the depairing limit. However, the saturation was clearly visible even when $\sigma_K$ is reduced to $20 W/K^4 m^2$, where $\langle I_s \rangle$ deviates from the depairing limit significantly.

The above heat analysis set an upper bound for the actual temperature. The superconducting leads next to the nanowire do not generate heat and are good thermal blocker. This can lower the temperature. In any case, the saturation is visible and can be possibly due to QPS.

The $T^{2/3}$ dependence is approximate; there is a weak dependence of fluctuation on $I_c(T)$. At intermediate temperatures, a full simulation becomes necessary. The
thermal phase slip rate $\Gamma(T, I)$ in Eq. 4.4 is a function of both $T$ and $I$. It equals to

$$
\Gamma_{TAPS}(T, I) = \Omega_{TAPS} \exp \frac{\Delta F(T, I)}{k_B T}
$$

(4.7)

where

$$
\Omega_{TAPS} = \frac{L}{\xi_T} \sqrt{\frac{\Delta F(T, I)}{k_B T}} \frac{1}{\tau_{GL}}
$$

$$
\Delta F(T, I) = \frac{\sqrt{6} h}{2e} I_c(T) \left(1 - \frac{I}{I_c(T)}\right)^{3/2}
$$

(4.8)

$$
I_c(T) = I_c(0) \left(1 - \left(\frac{T}{T_c}\right)^2\right)^{3/2}
$$

(4.9)

as derived in Eq. 2.30 and Eq. 2.28.

The distribution $P(I)$ can be obtained through solving Eq. 4.4 by plugging $\Gamma(T, I)$ into the equation. Two quantities, $I_c(0)$ and $T_c$, are unknown. Since they can not be determined by other means, they are used as fitting parameters. The distribution is close to the Gumbel distribution.

There are several methods to calculate the standard deviation of $P(I)$. The most rigorous method is to generate random values based on $P(I)$ and then calculate the standard deviation. However, the above process is very time consuming. Since $P(I)$ is close to the Gumbel distribution. The standard deviation $\Delta x$ of the Gumbel distribution (Eq. 4.5) is $(\pi/\sqrt{6})\beta$. $\beta$ can be obtained through the derivative of $\log(P(x))$, at $x = \alpha$

$$
\Delta x = \frac{\pi}{\sqrt{6}}\beta = \frac{d(\log(P(x)))}{dx}
$$

(4.10)

The above method greatly reduced the time.

The whole process is as follows. First, choose proper values for $I_c(0)$ and $T_c$. Second, solve Eq. 4.4 numerically and obtain $P(I)$ at different temperature. Third,
find the peak values and calculate the derivatives of \( \log(P(I)) \) at the peak values.

Fourth, plot the theoretical \( \Delta I_s(T) \) and \( \langle I_s \rangle(T) \) and compare them with the experimental values. Fifth, adjust \( I_c(0) \) and \( T_c \) until a good fit between the experiment and theory is obtained.

Through the fitting, we noticed that the calculated \( \Delta I_s \) had to be multiplied by a constant scaling factor 1.2 to match the data. Good agreement is achieved up to \( \sim 0.8 \text{ K} \) in S2 as shown in Fig. 4.12. S2 fits equally well but with a slightly different scaling factor 1.3.

Below \( T \sim 0.4 \text{ K} \) \( \Delta I_s \) shows a tendency to flatten, deviating from the calculation, and indicating that QPSs are becoming important. Above \( 0.8 \text{ K} \), \( \Delta I_s \) falls below the simulated value, and collapses near \( T \sim 1.1 \text{ K} \). This behavior is associated with the need for more-than-one phase slips to heat up the wire as the current drops [51]. A similar collapse is familiar in Josephson junctions within the phase diffusion regime, where multiple phase slips are required to induce switching [58–60].

To achieve a consistent picture, we address the question whether heat generated by a single phase slip is sufficient to raise the local temperature and trigger switching. A single phase slip deposits an energy \( \phi_0 I \) in a time \( \phi_0 I/(I^2 R_{\text{core}}) \sim 50 \text{ ps} \), where \( \phi_0 = h/2e \) is the flux quantum, and \( R_{\text{core}} \) is the normal state resistance of the phase slip core. Heating spreads to a region surrounding the core determined by a charge imbalance length \( \Lambda \) on either side [51]. There is insufficient time for heat removal through the substrate or liquid \(^3\text{He} \) (Kapitza boundary resistance limited). After a PS event, this region either stays normal or returns to superconducting depending on whether the critical current \( I_c \) at the elevated temperature \( T_f \) is exceeded or not. \( T_f \) can be estimated as

\[
T_f = \sqrt{T_0^2 + \frac{\phi_0 I}{(\gamma A \Lambda)}}
\]  

(4.11)

where \( \gamma = C_v/T = 135J/(m^3K^2) \), \( C_v \) is the specific heat of normal Al [61], \( A=\)
Figure 4.13: Transition temperature from single PS regime to multiple PSs regime. The red curve is measured $\langle I_s \rangle$. The black curve is simulated depairing $I_c$. The black shaded region indicates the temperature rise due to single PS at current $I$ (in horizontal direction). The cross point where the black curve exits the shaded area is the transition point from single to multiple PS(s) regime. The ambient temperature $0.7$ K can be found by moving the cross point horizontally back to $\langle I_s \rangle$ (shown by green arrows).

100 nm$^2$, $\Lambda=0.8 \mu m$, and $T_0$ is the ambient temperature. If the current exceeds the $I_c(T_f)$, this region becomes normal and its resistance contributes further to heating, causing a thermal runaway. Setting $\Lambda \sim 0.8 \mu m$, Fig. 4.13 shows that a transition from the single phase slip regime to the multiple phase slip regime occurs around $0.7$ K, consistent with our experimental result. The value of $0.8 \mu m$ is close to the findings in several recent experiments [62, 63] on Al wires of sub-$\mu m$ diameter, and is expected to be temperature independent [64].

In summary, we have demonstrated that 1D Al superconducting nanowires can be switched into the normal state by a single phase slip over a sizable temperature range. At low temperature, QPS-induced switching was found in a narrow wire. In the single TAPS regime, $I_s$ fluctuation is proportional to $T^{2/3}$, reminiscent of a Josephson junction. The fluctuation collapses at higher temperature, where multiple phase slips are needed to trigger switching. Heating appears to play a major role in the switching process. Nevertheless, the remarkable similarity between the observed behaviors and
Figure 4.14: $I_s$ fluctuation measurement with normal and superconducting leads. (A) and (B) are S1 and S2 respectively.

recent findings in a variety of Josephson junctions [58–60], gives reason for caution in identifying heating as the sole mechanism responsible. The behavior presented in this work may be relevant to superconducting nanowires of different materials, due to the commonality of a restricted geometry. Thus it is likely that single TAPS and QPS limited switching current is a generic feature of superconducting nanowire systems.
4.3.2 \( I_s \) fluctuation of nanowires with normal leads

The \( I_s \) fluctuation in nanowires with normal leads differ from that with superconducting leads significantly. Unfortunately, we do not have a satisfactory explanation for the phenomena. Therefore, I just describe observed phenomena in this section.

As seen in Fig. 4.14A, in the short nanowire S1, the fluctuation is nearly zero above 1.1 K. Then the fluctuation increases slowly with decreasing temperature. Around 0.9 K, the fluctuation saturates at 5 nA. The fluctuation with normal leads is always less than that with superconducting leads.

The behavior changes qualitatively in the long nanowire S2. A sharp fluctuation peak occurs around 1.2 K. At the right side of the peak (higher temperature), \( I_s \) with normal leads follows that with superconducting leads. So does \( \langle I_s \rangle \). At the left side of the peak, \( I_s \) falls sharply to around 1 K and then gradually saturates to 5 nA. Below 1.2K, the fluctuation with normal leads is smaller than that with superconducting leads.

The behavior of \( I_s \) of the long nanowire with normal leads is consistent with the superconducting nanowire with permanent normal leads, as shown in Fig. 4.15.
These are very interesting phenomena. The derivation from phase slip indicates the fluctuation to be zero, since the leads can no longer "fix" the phase difference. So there should be no phase slip and the fluctuation is zero. Apparently, that model is too simple. The normal-superconducting interface certainly complicated issue here.
Conclusion and future works

In summary, I developed a process to fabricate superconducting nanowires as thin as a few nanometers. We finally have a complete and consistent picture of nanowires with superconducting leads. The nanowires still can become completely superconducting slightly below $T_c$. The fluctuation in $I_s$ was consistent with the scenario that the switching transition is caused by a single phase slip at low temperature and by multiple phase slips at high temperature. The result on the nanowires with normal leads show many interesting behaviors. Unfortunately, we are not able to understand them fully.

We were able to fabricate long, narrow and uniform nanowires. Our unique fabrication procedure, plus a series of other measures we took, helped us to minimize the surface oxidation. Benefiting from above measures, we observed that an extremely narrow superconducting Al nanowire ($< 10$ nm) still became completely superconducting, contrary to what many people had seen in the past.

We took great efforts to eliminate the electrical noise. The efforts include thoroughly characterizing the available equipment, designing/fabricating electronic devices and heavily filtering. They allow us to stay ahead of competing groups and
observe the true behavior of the sample. Unfortunately, sometimes this means that the result is less exciting and will be controversial. For example, we were able to show that some commonly observed phenomena, such as phase slip center, may be due to spurious factors. Better filtering also lowers nonlinear IV.

We changed the properties of the leads for the same nanowire and found that they significantly affect the behavior of the nanowire, which was never demonstrated by others. The behavior of nanowires with normal leads is interesting but has not been well understood. But we can point out that a number of recent studies trying to probe intrinsic properties of superconducting nanowires were affected by superconductor-normal metal interface.

In terms of fabrication, we believe our superconducting nanowires are the thinnest and longest of its kind. The width of the nanowire almost reaches the limit of the method by physical vapor deposition (thermal/e-beam evaporation). Any nanowires thinner than this will seriously suffer from continuity, granularity and surface oxidation. In terms of physics, however, it is not clear how much new physics can be obtained. In order to understand it completely, a breakthrough in fabrication, such as epitaxial growth or self assembly, is necessary. The ultimate question might be how narrow a superconductor can really be. This can be useful for probing the underlying mechanism of superconductivity.

This is an interesting an exciting field. Unfortunately, my research has to stop here due to graduation. We helped to answer a number of questions but plenty of them are still to be answered. If someone decide to continue the work, one of the immediate questions is the interface between a nanowire and the normal metal. To understand this issue, more theoretical work needs to be done. Another question is how different materials behave, particularly whether MoGe can be insulating if it is made by our method. There are still many interesting questions worth to study, but unfortunately, the project is not well funded.
In the end, I would like to share a few mistakes that I have made.

One of the biggest mistakes is not to measure the sample as soon as they were fabricated. I successfully made several samples at the early stage of the experiment. At the time, I decided to fabricate more samples and measure them altogether. Unfortunately, all the samples quickly deteriorated because of oxidation, which I did not know at the time. More devastatingly, because a crucial step was not fully understood then, I could not make new samples for a long time. The problem was not identified earlier because it was made complex by a number of factors, including evaporation speed control and the condition of the substrate.

If I could have done the measurement earlier, I might have realized the importance of the leads. Then I could directly fabricate the sample with superconducting leads, without wasting too much time on the sample with normal leads.

I should not blindly trust some measuring equipment. It never came to my mind that these equipments caused problems before I actually tested them, which was several years later. The test method is simple and it can be done much earlier. If I chose the most suitable method and equipment to do the measurement, then many earlier measurements will not be wasted.

I also wish I could have been more systematic while developing the fabrication method. At the beginning, I tended to skip steps to simplify the process. This appeared to be successful initially, as is demonstrated by several working samples. However, all of sudden, I could no longer make new samples. I ended up spend huge amount of time in checking every steps and all the possibilities. All the time I saved initially was spent later.

The reason I list my mistakes is not to feel regret or get sympathy. Looking forward, I hope to learn from lessons and be successful in the future.
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Biography

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Peng Li joined the department of physics in 2003 as a teaching assistance. Later, he entered Prof. Albert Chang’s research group and started to work on the superconducting nanowires. Prior to this, Peng Li obtained a master degree in physics department at Tsinghua in 2003, where he worked on the transport properties of carbon nanotubes. Peng Li obtained his B.S. in physics at Tsinghua in 2000.

Publications


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