

# Essays in Macroeconomics

by

Béla Személy

Department of Economics  
Duke University

Date: \_\_\_\_\_

Approved:

\_\_\_\_\_  
Adriano A. Rampini (co-chair), Supervisor

\_\_\_\_\_  
Juan F. Rubio-Ramírez (co-chair)

\_\_\_\_\_  
A. Craig Burnside

\_\_\_\_\_  
S. Viswanathan

Dissertation submitted in partial fulfillment of the requirements for the degree of  
Doctor of Philosophy in the Department of Economics  
in the Graduate School of Duke University  
2011

ABSTRACT  
(Economics)

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# Abstract

During 2007-2009 the U.S. economy experienced the most severe financial crisis since the Great Depression. Why did the financial crisis turn into such a severe recession? And what were the causes of the Great Recession? This dissertation consists of two essays examining these questions. In the first essay I study the extent to which the increase in uncertainty might have contributed to the severity of the crisis. The second essay examines the reasons behind the fall in the personal saving rate as measured in the National Income and Product Accounts.

In the first essay I study the effects of changes in uncertainty on optimal financing and investment in a dynamic firm financing model in which firms have access to complete markets subject to collateral constraints. Entrepreneurs finance projects with their net worth and by issuing state-contingent securities, which have to be collateralized with physical capital. An increase in uncertainty leads to deleveraging, as entrepreneurs reduce their demand for external financing and fund a larger share of their investment from net worth. Upon an increase in uncertainty, investment initially falls as entrepreneurs decrease the scale of their projects. Investment recovers as entrepreneurs build up net worth and transition into an environment with high uncertainty. Quantitatively, changes in uncertainty have large effects on optimal leverage and investment dynamics.

The spendthrift nature of U.S. households leading up to the financial crisis has been cited as a major contributing factor for the Great Recession. Indeed, the personal saving rate has been falling since the end of the 1970s, dropping to as low as nearly 1 percent before the financial crisis. The reasons behind the decline in the personal saving rate have yet to be understood, and thus constituting an important puzzle for economic research. In the second essay, joint work with Maurizio Mazzocco, we provide a potential explanation for the decline in the personal saving rate. Specifically, we show that a single variable can potentially explain the decline in the U.S. personal saving rate from 10 percent in the early eighties to nearly 1 percent in 2007. This variable is medical expenditure by health institutions net of the employers' contributions to pension and insurance funds. Furthermore, if we differentiate between contributions to pension funds and to health plans, we find that the main reason behind the dramatic reduction in the U.S. personal saving rate is the stagnation of employers' contributions to pension funds that started in the early eighties combined with the sharp rise in expenditure by health institutions.

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# 1

## Introduction

During 2007-2009 the U.S. economy experienced the most severe financial crisis since the Great Depression. Why did the financial crisis turn into such a severe recession? And what were the causes of the Great Recession? This dissertation consists of two essays examining these questions. In the first essay I study the extent to which the increase in uncertainty might have contributed to the severity of the crisis. The second essay examines the reasons behind the fall in the personal saving rate as measured in the National Income and Product Accounts.

In the first essay I study the effects of changes in uncertainty on optimal financing and investment in a dynamic firm financing model in which firms have access to complete markets subject to collateral constraints. The main contribution of this essay is the study of the effects of changes in uncertainty in an environment where financing is based on an optimal long-term contract. Specifically, I derive comparative statics and using calibrated parameter values, I then provide quantitative results.

Dynamic firm financing is modeled as in Rampini and Viswanathan (2010a), who derive collateral constraints endogenously from limited enforcement constraints. In this setting, uncertainty jointly determines firms' optimal capital structure and investment decisions. Specifically, collateral constraints impose limits on borrowing, which forces entrepreneurs to finance their projects with both net worth and external funds. In turn, limits to borrowing affect firms' investment choices, as available net worth determines the investment that entrepreneurs can afford.

The main result of this essay is that an increase in uncertainty leads to deleveraging, as entrepreneurs reduce their demand for external financing and fund a larger share of their investment from net worth. Upon an increase in uncertainty, investment initially falls as entrepreneurs decrease the scale of their projects. Investment recovers as entrepreneurs build up net worth and transition into an environment with high uncertainty. Quantitatively, changes in uncertainty have large effects on optimal leverage and investment dynamics.

The spendthrift nature of U.S. households leading up to the financial crisis has been cited as a major contributing factor to the Great Recession. Indeed, the personal saving rate has been falling since the end of the 1970s, dropping to as low as nearly 1 percent before the financial crisis. The reasons behind the decline in the personal saving rate have yet to be understood, and thus constituting an important puzzle for economic research.

In the second essay, joint work with Maurizio Mazzocco, we provide a potential explanation for the decline in the personal saving rate. Specifically, we show that a single variable can potentially explain the decline in the U.S. personal saving rate from 10 percent in the early eighties to nearly 1 percent in 2007. This variable is

medical expenditure by health institutions net of the employers' contributions to pension and insurance funds. Furthermore, if we differentiate between contributions to pension funds and to health plans, we find that the main reason behind the dramatic reduction in the U.S. personal saving rate is the stagnation of employers' contributions to pension funds that started in the early eighties combined with the sharp rise in expenditure by health institutions.

Our findings have important implications for policy analysis. They indicate that households were partially responsible for the decline in the U.S. saving rate. Had they increased their contributions to pension plans to compensate for the reduction in the employers', the saving rate would not have declined or would have declined by less. Thus, any policy designed to increase the contributions to pension plans by households will have a positive effect on the saving rate. However, our results also suggest that the main reason behind the decline in household savings is the dramatic rise in health expenditure, which was possible only because an increasing amount of resources were diverted from other uses. As a consequence, a policy designed to increase the pension contributions of households would merely be a short-term solution. A policy that can have long-term effects on the saving rate is one aimed at structurally changing the health sector and at reducing its expenditure.

# Optimal Leverage and Investment under Uncertainty

## 2.1 Introduction

During the recent financial crisis the U.S. economy has experienced a significant increase in measured uncertainty as documented in Bloom et al. (2009). At the same time the economy suffered a severe recession with sharp contractions in investment and credit, among other indicators. Figure 2.1 plots the evolution since 2006 of the implied volatility index, VIX, as a measure of uncertainty, real investments, and real commercial loans advanced to U.S corporations. Furthermore, as can be seen from Figure 2.1, in the aftermath of the crisis non-financial corporations sharply increased their holdings of liquid assets, prompting the question of why firms are not investing more given that they have access to so much liquidity?<sup>1</sup> Motivated

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<sup>1</sup> See “Show us the money,” The Economist, July 1, 2010 and “The cost of repair,” The Economist, October 7th, 2010.

by these observations, in this paper I study the effects of changes in uncertainty on financing and investment decisions in a dynamic firm financing model, where firms have access to complete markets subject to collateral constraints.

The main contribution of this paper is the study of the effects of changes in uncertainty in an environment where financing is based on an optimal long-term contract. Specifically, I derive comparative statics results and using calibrated parameter values, I then provide quantitative results.

Dynamic firm financing is modeled as in Rampini and Viswanathan (2010a), who derive collateral constraints endogenously from limited enforcement constraints. In this setting, uncertainty jointly determines firms' optimal capital structure and investment decisions. Specifically, collateral constraints impose limits on borrowing, which forces entrepreneurs to finance their projects with both net worth and external funds. In turn, limits to borrowing affect firms' investment choices, as available net worth determine the investment that entrepreneurs can afford.

Entrepreneurs use physical capital as their only factor of production. Investment in physical capital is funded from the existing net worth and external financing. External financing has benefits, but comes with a risk for entrepreneurs. On the one hand, ex-ante higher leverage allows entrepreneurs to increase their investment and achieve faster growth if ex-post returns on investments are high. On the other hand, external financing carries a risk for entrepreneurs, as the repayment of debt in periods of low returns reduces entrepreneurs' net worth. As reductions in net worth constrain future investment decisions, the scale of the project determines the trade-off between faster growth when realized returns are high and the risk of losing net worth in states with low returns. Thus, entrepreneurs have to choose not only

their investment policy but also their financing policy.

In this paper I show that increases in uncertainty amplify the risk of borrowing. With an increase in uncertainty, the variance of the returns that entrepreneurs face on their investment increases. As a result, in periods when returns are low, repaying debt leads to larger reductions of net worth. Consequently, upon an increase in uncertainty entrepreneurs will decrease the scale of their projects and will delever; that is, entrepreneurs will reduce their demand for external financing and fund a larger share of investments from their net worth. Thus, an increase in uncertainty initially leads to a fall in optimal investment. Investment recovers as entrepreneurs build up their net worth and transition into an environment with higher uncertainty and lower leverage.

It is instructive to relate this result to the standard result on the effect of uncertainty on investment when firms face convex adjustment costs. As shown in Abel (1983), an increase in uncertainty induces a precautionary savings behavior, and since capital is the only vehicle through which firms can save, increased uncertainty leads to an increase in investment. In this paper, in addition to investment, entrepreneurs also choose their financing policy. Consequently, the precautionary savings motive can manifest either through an increased investment or through a decrease in external financing. This paper shows that when all collateral constraints bind before and after an increase in uncertainty, entrepreneurs can only save by increasing their investment, just as in Abel (1983). However, when some collateral constraints are slack, entrepreneurs can also save by borrowing less at the margin, which may reduce their investment.

In the long run, the change in uncertainty will be reflected in firms' capital

structure. Upon an increase in uncertainty, firms decrease their demand for external financing and will finance a larger share of their investment from their net worth. In the new environment with high uncertainty, firms will have larger net worth and lower leverage and will be able to operate the firm at the initial scale. Thus this paper highlights the importance of capital structure as the main mechanism through which uncertainty affects firm dynamics.

The model has several important implications. First, the paper has implications for corporate risk management practices. The main prediction of the model is that upon an increase in uncertainty, risk management concerns override firms' financing needs, and as a result investment decreases. The need to hedge fluctuations in net worth implies that entrepreneurs issue fewer claims against lower states; however this comes at the expense of their financing needs, resulting in reduced investments.

Furthermore, the predictions of this paper are in line with the observed increase in liquid assets held by non-financial corporations. The model features complete markets, subject to collateral constraints, which allow firms to engage in risk management. Firms can hedge idiosyncratic risk by issuing fewer claims against lower states, but also by conserving net worth in all states to take advantage of future investment opportunities. Conserving net worth against all states in this context can be thought of as hoarding cash. The results in this paper show that upon an increase in uncertainty firms will increase their cash holdings, thus providing a potential explanation for the observed increase in liquid assets holdings.

Additionally, it is important to note that leverage and collateral are determined in equilibrium. This is so despite the fact that collateral constraints are derived endoge-

nously from limited enforcement constraints,<sup>2</sup> where the tightness of the constraint is governed by one parameter. Models that feature collateral constraints typically assume that collateral constraints are always binding and thus the leverage ratio is exogenously fixed.<sup>3</sup> Indeed, the occasionally binding nature of collateral constraints is crucial to the results in this paper.

Finally, it is instructive to compare an economy with complete markets, subject to collateral constraints, to an economy with incomplete markets that is subject to the same constraints. When markets are incomplete, entrepreneurs insure against fluctuations in productivity by conserving net worth. Thus, under incomplete markets firms tend to have higher capitalization. While entrepreneurs are less able to insure against risk in the economy, higher capitalization allows entrepreneurs to weather unexpected changes in uncertainty. In economies with complete markets, entrepreneurs can hedge states with low returns. Since hedging improves risk sharing, entrepreneurs need not conserve as much of their net worth. But this also implies that entrepreneurs will be thinly capitalized in the face of unexpected shocks. As a result in economies with complete markets, subject to collateral constraints, shocks tend to be amplified, while economies with incomplete markets, also subject to collateral constraints, tend to dampen the effects of uncertainty shocks.<sup>4</sup>

This paper builds on Rampini and Viswanathan (2010a), who study risk-neutral entrepreneurs subject to limited liability, whereas this paper assumes that entrepre-

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<sup>2</sup> See Rampini and Viswanathan (2010a) and Kehoe and Levine (1993). For an alternative environment with endogenously incomplete markets, see Geanakoplos (1997), Geanakoplos (2003), Dubey et al. (2005), Geanakoplos (2009).

<sup>3</sup> See Kiyotaki and Moore (1997), Iacoviello (2005), with the notable exception of Mendoza (2010) and Khan and Thomas (2010).

<sup>4</sup> Cooley et al. (2004) also find that complete markets tend to amplify the shocks in the economy.

neurs are risk-averse. The main implication of the assumption of risk-averse entrepreneurs can be found in the optimal firm size. Specifically, in the model with collateral constraints, well-capitalized (high net worth) entrepreneurs will operate at the same optimal size as in the frictionless economy. Crucially, this result allows for the analytical derivation of comparative statics results. Furthermore, by using calibrated values, I compute the effects of uncertainty shocks and show that the mechanism presented in the model is quantitatively significant.

The paper is related to several lines of research. First, I follow the literature that considers dynamic incentive problems as the main determinant of firm financing and capital structure. Specifically, I consider limited enforcement problems between financiers and investors as in Albuquerque and Hopenhayn (2004), Lorenzoni and Walentin (2007), Rampini and Viswanathan (2010a), and Rampini and Viswanathan (2010b). Albuquerque and Hopenhayn (2004) consider the case of a firm which needs financing for a project with an initial non-divisible investment, whereas here I consider a standard neoclassical investment problem; moreover the limits of enforcement differ in the two specifications. Lorenzoni and Walentin (2007) are the first to derive endogenously collateral constraints from limited enforcement constraints and study its implications on investment and Tobin's  $q$ . Because of constant returns to scale, in their setup firm-level net worth does not matter; moreover, they assume that all collateral constraints bind. The focus in this paper is on the effect of uncertainty on the capital structure, and the interaction between net worth and demand for external financing. Aggregate implications of limited enforcements are further studied in Cooley et al. (2004) and Jermann and Quadrini (2007). None of these papers analyze the effect of changes in uncertainty on capital structure and investment dynamics.

Implications of incentive problems due to private information about cash flows or moral hazard on capital structure and investment dynamics are studied in Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007), DeMarzo et al. (2009). However they do not consider the effect of changes in the level of uncertainty on leverage and investment dynamics.

Second, there is a recent literature that looks at the aggregate effect of uncertainty shocks. In the presence of adjustment costs,<sup>5</sup> Bloom (2009) and Bloom et al. (2009) argue that uncertainty shocks represent important driving forces for business cycles,<sup>6</sup> although the quantitative importance of this mechanism is debated in the literature, see Bachmann and Bayer (2009). As in the above-mentioned papers, my focus is also on the effect of uncertainty on capital accumulation; however I do not consider the presence of adjustment costs.

Third, uncertainty shocks also have been considered in models with financing frictions; this literature considers models in which entrepreneurs have private information about their cash flows and obtain financing through optimal, one-period debt contracts as in Townsend (1979) and Bernanke et al. (1999). In this setup, changes in uncertainty affect credit spreads, thus influencing the cost of financing. The first paper that formalized this insight is Williamson (1987); recently it received further attention in Christiano et al. (2009) and Gilchrist et al. (2009). In contrast to the above literature, this paper considers optimal long-term contracts where the agency friction is limited enforcement, and furthermore assumes that the cost of financing

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<sup>5</sup> There is a large literature on the role of uncertainty on capital accumulation, but the focus is on the long run effects of uncertainty, see Abel (1983), Dixit (1989), Caballero (1991), Dixit and Pindyck (1994), Bertola and Cabellero (1994), Abel and Eberly (1996), Abel and Eberly (1999).

<sup>6</sup> This idea initially was developed in Bernanke (1983).

is constant, so the mechanism described above is absent.

Finally, Arellano et al. (2010) study fluctuations in uncertainty in an economy without capital, whereas Fernández-Villaverde et al. (2010) consider the effect of fluctuations in uncertainty on small, open economies.

The outline of this paper is as follows. The next section presents the model and characterizes the solution. Section 3 contains the main analytical results of the paper, while Section 4 contains the quantitative results. The last section concludes.

## 2.2 The Model

This section presents a neoclassical investment model where entrepreneurs have access to a complete set of state-contingent securities, subject to collateral constraints, as in Rampini and Viswanathan (2010a). Due to the collateral constraints, entrepreneurs have to finance their investment from their net worth and external funds. External funds are provided by lenders who have access to a limitless supply of capital. Credit markets are subject to limited enforcement; that is, entrepreneurs can default on their loan obligations and divert cash flows and a fraction of their capital holdings. Lenders discount the future at the rate  $\tilde{\beta} \equiv R^{-1}$ , and are willing to supply funds as long as, in net present value terms, the loans are repaid.

There is a measure one of risk-averse, relatively impatient entrepreneurs, who discount the future at the rate,  $\beta < \tilde{\beta}$ . Entrepreneurs have access to a production technology with decreasing returns to scale.<sup>7</sup> Capital,  $k$ , is the only factor of production, which depreciates at a constant rate  $\delta \in (0, 1)$ .

**Assumption 1.** *The production function,  $f$ , is strictly increasing, strictly concave*

<sup>7</sup> This assumption can alternatively be motivated by a decreasing industry demand function.

and differentiable,  $f'(k) > 0$ ,  $\lim_{k \rightarrow 0} f'(k) = \infty$ ,  $\lim_{k \rightarrow \infty} f'(k) = 0$ .

The return on capital,  $k'$ , is subject to stochastic shocks  $A(s')f(k')$ , where  $A(s')$  is the realization of the total factor productivity in state  $s' \in S$ . Let the history of events up to time  $t$  be denoted by  $s^t = [s_0, \dots, s_{t-1}, s_t]$ , where  $s_t \in S^t$ ,  $\forall t$ . Furthermore, assume that the state  $s$  follows a Markov chain process with transition matrix,  $\pi(s_t, s_{t+1})$ , with  $s_t \in S, \forall t$ .

**Assumption 2.** For all  $s, \hat{s} \in S$ , where  $\hat{s} > s$ ,  $A(\hat{s}) > A(s)$  and  $A(s) > 0$ ,  $\forall s \in S$

Entrepreneurs have the possibility to default. Upon default they can divert the cash flow and  $(1 - \theta) \in (0, 1)$  fraction of available capital, whereas creditors can seize the remaining  $\theta$  fraction of the resale value of capital. A crucial assumption of the model is that defaulting entrepreneurs are not excluded from either capital nor physical goods markets.

### 2.2.1 Limited Enforcement

Entrepreneurs enter into long-term contracts with risk-neutral lenders who have unlimited capital. The contract specifies payments,  $p_t(s^t)$  between entrepreneurs and lenders. These payments can be negative or positive, depending on whether entrepreneurs need financing or pay back their loans. In order for lenders to participate in this contract, the present value of net payments must be non-negative:

$$\sum_{t=0}^{\infty} \sum_{s^t} R^{-t} \pi(s_0, s^t) p_t(s^t) \geq 0 \quad (2.1)$$

where  $\pi(s_0, s^t) = \pi(s_0, s_1) \times \dots \times \pi(s_{t-1}, s_t)$ .

Additionally, since entrepreneurs might default in any future period or state, lenders must ensure that in an eventual case of default, the value of the assets that they recoup will cover the present value of their net payments. Since in the present context, the value that lenders can recoup equals  $\theta$  fraction of the resale value of the capital stock, the enforcement constraint is:

$$\theta k_{t+1}(s^t)(1 - \delta) \geq \sum_{j=t}^{\infty} \sum_{s^j} R^{t-j} \pi(s_t, s^j) p_j(s^j), \quad \forall s^j \in S \quad (2.2)$$

Notice that the enforcement constraint takes a very simple form; the present value of capital holdings serves as collateral for the entrepreneurs' future promised payments. Because of the possibility of default, entrepreneurs can credibly issue promises against state  $s^{t+j}$ , of up to the  $\theta$  fraction of the resale value of undepreciated capital in that state.

Denote  $Rb_1(s_0, s_1)$  as the present value of all future payments from the entrepreneur to the lender in state  $s_1$ . Then (2.1) can then be written as follows

$$\begin{aligned} Rb_1(s_0, s_1) &= \sum_{t=0}^{\infty} \sum_{s^t} R^{-t} \pi(s_0, s^t) p_t(s^t) \\ &= p_0(s_0) + R \sum_{s_1|s_0} R^{-1} \pi(s_1, s_2) b_2(s_1, s_2) \\ &= p_0(s_0) + \sum_{s_1|s_0} \pi(s_1, s_2) b_1(s_1, s_2) \end{aligned} \quad (2.3)$$

Notice that (2.3) implies that entrepreneurs issue state-contingent, one-period securities; however these securities are priced by the lenders with the probability that particular states occur. This is intuitive; since lenders are risk-neutral, they

price state-contingent assets only with the probability of that state occurring; that is, without correcting for any risk factor. With this notation, the enforcement constraint (2.2) can be written:

$$\theta k_{t+1}(s^t)(1 - \delta) \geq Rb_{t+1}(s_t, s_{t+1}), \forall s_{t+1} \in S \quad (2.4)$$

Furthermore, conditions (2.4) makes it clear that the long-term contract can be implemented by a sequence of one-period contracts, where entrepreneurs issue state-contingent claims that are subject to state-contingent collateral constraints. Notice that, in general enforcement constraints depend on the value of default, and these enforcement constraints have to hold in all future periods. This implies that for entrepreneurs to make credible promises to repay the loan, lenders need to know all preference parameters and to keep track of the whole history of repayments. In the present context, lenders only have to observe the current level of entrepreneurs' physical capital, and thus the informational requirements on the lenders' knowledge is greatly reduced. Specifically, an important implication of the present model is that lenders need to know only the per-period publicly available asset holdings of entrepreneurs. Thus, the value of the default, in general a value function itself, now depends only on the level of physical assets. This simplifies the problem considerably<sup>8</sup>.

We now turn to the entrepreneurs' problem.

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<sup>8</sup> See for example the treatment in Marcet and Marimon (1992), Kehoe and Levine (1993), Alvarez and Jermann (2000), and Marcet and Marimon (2009)

### 2.2.2 Entrepreneurs' Problem

Entrepreneurs choose dividends, investment and financing to maximize the expected utility of their future dividend consumption. I assume entrepreneurs are risk-averse over their dividend payments.

**Assumption 3.** *The utility function,  $u$ , is strictly increasing, strictly concave, and differentiable:  $u'(d) > 0$ ,  $\lim_{d \rightarrow 0} u'(d) = \infty$ ,  $\lim_{d \rightarrow \infty} u'(d) = 0$ .*

Using the collateral constraints (2.4), the entrepreneurs' problem can be written in recursive form. Furthermore, the problem can be substantially simplified with the introduction of an additional variable, net worth. Define net worth in state  $s'$  as  $w(s') = z'f(k') + k'(1 - \delta) - Rb(s')$ , the return on investment and resale value of capital less the state-contingent debt to be repaid. The introduction of net worth allows the reduction of the number of potential state variables from at least three  $(k', b(s'), s')$  (where, notice, debt in every state of the economy is part of the state variables), to only two  $(w(s'), s')$ , significantly simplifying the problem. I suppress notation by assuming that every variable depends implicitly on  $(w, s)$ . The entrepreneurs' problem can then be written as:

$$V(w, s) = \max_{\{d, k', b(s'), w(s')\}} \left\{ u(d) + \beta \sum_{s' \in S} \pi(s, s') V(w(s'), s') \right\} \quad (2.5)$$

subject to

$$w + \sum_{s' \in S} \pi(s, s') b(s') \geq d + k' \quad (2.6)$$

$$A(s')f(k') + k'(1 - \delta) \geq w(s') + Rb(s'), \quad \forall s' \in S \quad (2.7)$$

$$\theta k'(1 - \delta) \geq Rb(s'), \quad \forall s' \in S \quad (2.8)$$

and

$$d \geq 0, \quad k' \geq 0.$$

Entrepreneurs in each period use their net worth and potential borrowing to fund gross investments,  $k'$ , and pay out dividends,  $d$ , as can be seen from the budget constraint (2.6). To obtain funding, entrepreneurs issue state-contingent securities that they promise to buy back in the next period. Next period's net worth depends on the amount of investment, the realized state of the economy and the cost of financing, as can be seen in equation (2.7).

Given the possibility of default, entrepreneurs' promises to repay their debt are not credible and they need to secure their borrowing with their physical capital. Lenders are willing to provide financing only if, in case of default, entrepreneurs' assets can cover the provided funds. This is encoded in the collateral constraints (2.8), which need to hold in every state of the world.

Entrepreneurs issue state-contingent claims, secured with their capital holdings. Obtained financing must be repayed at a cost  $Rb(s')$ . Entrepreneurs have to trade off their need for investment with the cost of financing. Borrowing against state  $s'$  reduces next period's net worth in that state. This implies that borrowing against state  $s'$  carries a risk for entrepreneurs, as investment in state  $s'$  will be constrained by the available net worth. Notice also that the above collateral constraints are similar to the one used in Kiyotaki and Moore (1997), with the exception that the collateral constraints here are derived endogenously from a limited enforcement problem, and that borrowing is state-contingent.

Next, I turn to the characterization of the recursive problem. The below proposition states that the entrepreneurs' problem is well-defined and there exists a unique value function  $V$  satisfying (2.5) - (2.8).

**Proposition 1.** *(i) There is a unique  $V$ , satisfying (2.5) - (2.8). (ii)  $V$  is continuous, strictly increasing, and strictly concave in  $w$ . (iii)  $\forall \hat{s}, s \in S$  such that  $\hat{s} > s$ ,  $\pi(\hat{s}, s')$  strictly first order stochastically dominates  $\pi(s, s')$ ,  $V$  is increasing in  $s$ .*

The proofs for Parts (i) - (iii) are relatively standard. The concavity of the production function, and the risk-aversion of entrepreneurs guarantee that  $V$  is a unique, strictly increasing and strictly concave function of net worth.

Denote the Lagrange multipliers on the constraints (2.6), (2.7), (2.8) as  $\lambda, \beta\pi(s, s')\lambda(s'), \beta\pi(s, s')\lambda(s')\mu(s')$ . The first-order conditions for the entrepreneur are:

$$\lambda = u'(d) \tag{2.9}$$

$$\lambda = \beta \sum_{s' \in S} \pi(s, s')\lambda(s')(A(s')f'(k') + 1 - \delta + \mu(s')\theta(1 - \delta)) \tag{2.10}$$

$$\lambda = \beta R\lambda(s')(1 + \mu(s')), \quad \forall s' \in S \tag{2.11}$$

$$\mu(s')(\theta k'(1 - \delta) - Rb(s')) = 0, \mu(s') \geq 0 \quad \forall s' \in S \tag{2.12}$$

The envelope condition is  $V_w(w, s) = \lambda$ . Due to the assumptions on the production and utility functions, capital and dividends is always positive; thus I do not include that constraint in the above Kuhn-Tucker conditions.

Condition (2.9) governs the dividend payout policy of entrepreneurs, and the envelope condition makes it clear that dividend payout depends on the marginal

valuation of net worth. Condition (2.10) governs the optimal investment of entrepreneurs. Notice that in states when the collateral constraint (2.8) does not bind for any state  $s' \in S$  next period,  $\mu(s') = 0$ , (2.10) reduces to the standard Euler equation, where optimal investment is governed by the marginal revenue of capital weighted by entrepreneurs' stochastic discount factor. A binding collateral constraint (2.8),  $\mu(s') > 0$ , drives a wedge between the marginal product of capital and the relative marginal utilities of wealth. Specifically, binding collateral constraints imply that entrepreneurs use physical capital both as a factor of production and as an asset that can be used for collateral. Thus, internal funds require a premium in the presence of binding collateral constraints.

Equation (2.11) governs the evolution of entrepreneurial net worth. Optimal next period net worth depends on the financing need of entrepreneurs. In states where the collateral constraint does not bind,  $\mu(s') = 0$ . In states however when the collateral constraint binds, the use of capital for collateral purposes is encoded in the value of  $\mu(s')$ .

The next proposition shows that the problem (2.5) - (2.8) has a unique solution.

**Proposition 2.** *Denote  $x_0 \equiv [d_0, k'_0, b_0(s'), w_0(s')]$ . The optimal policy  $x_0$  is unique.*

Next, I discuss the solution of the frictionless problem, when entrepreneurs are not relatively impatient and borrowing is not subject to collateral constraints.

### 2.2.3 Frictionless Case

In this section I consider the frictionless case, when there are no collateral constraints and entrepreneurs have the same discount factor as lenders. In that case, entrepreneurs can perfectly insure against idiosyncratic productivity shocks,  $\lambda = \lambda(s')$

for all  $s' \in S$ ,<sup>9</sup> and operate on the optimal scale. Indeed, the optimal capital stock then is given by:

$$1 = \beta \sum_{s' \in S} \pi(s, s') (A(s') f'(\bar{k}') + 1 - \delta)$$

Since markets are complete, firms' capital structure is indeterminate, and firms operate at the optimal scale,  $\bar{k}'$ , at all levels of net worth. Next, I turn to the case when entrepreneurs are relatively impatient and face collateral constraints.

#### 2.2.4 Characterization of the Optimal Policy

In this section I characterize the optimal financing and investment policies of entrepreneurs. Due to the presence of collateral constraints, entrepreneurs must finance part of their investments with their internal funds. Depending on their level of net worth, entrepreneurs must accumulate enough internal funds to be able to afford levels of investment that maximize the return on their project. Entrepreneurs' optimal policies determine their financing demand, investment choice and their optimal accumulation of internal funds. Throughout this section I derive the results under the assumption of constant investment opportunities,  $\pi(s, s') = \pi(s')$ .

The optimal financing policy can be best characterized by analyzing the shadow value of collateral,  $\mu(s')$ .

**Proposition 3** (Optimal Financing Policy). *(i) There exists  $\underline{w} > 0$  such that if  $w < \underline{w}$ , then the collateral constraint in all states bind  $\mu(s') > 0 \forall s' \in S$ . (ii) The marginal value of net worth is (weakly) decreasing in the state  $s'$ , whereas the multipliers on collateral constraints are (weakly) increasing in the state  $s'$ ;  $\forall s', s'_+ \in$*

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<sup>9</sup> See Chapter 8, in Ljungqvist and Sargent (1994).

$S$ , such that  $s'_+ > s'$ ,  $\lambda(s'_+) < \lambda(s')$  and  $\mu(s'_+) > \mu(s')$ . **(iii)** There exist  $\bar{w} > 0$  such that if  $w \geq \bar{w}$  then  $\mu(s') = 0$ ,  $\forall s' \in S$

To understand the implications of the model for financing demand, recall that entrepreneurs must finance part of their investment with their own net worth. Thus entrepreneurs' investment decisions are constrained by their available net worth. The first part of the proposition states that when the net worth of entrepreneurs is low enough, entrepreneurs will exhaust their debt capacity against all future states. Of course, entrepreneurs do this because, in all future states, the marginal return on their investment will be higher than their cost of financing, which in the current model is  $R$ .

The second part of the proposition characterizes the shadow value of collateral in different states of nature. The proposition says that in states when returns are low the shadow value of collateral is lower. Intuitively, entrepreneurs would always like to borrow less against states with low returns, as in such states repaying the debt leads to losses of net worth. Entrepreneurs will always want to issue more claims against states with high returns, but the collateral constraints restrict the amount of funds that can be borrowed. Thus entrepreneurs' shadow value of collateral is higher against states with high returns.

The last part of the proposition states that when entrepreneurs have accumulated enough net worth, they will choose to issue fewer claims than the value of their collateral, against all future states. The intuition for this result is that when the level of net worth is high enough, entrepreneurs will be able to perfectly hedge the idiosyncratic fluctuations in productivity, and they will no longer value physical

assets for the purpose of collateral.

Turning now to the optimal investment policy,

**Proposition 4** (Optimal Investment Policy). *There exists  $\bar{w} > 0$  such that (i) if  $w, w_+ < \bar{w}$ , for  $w < w_+$  such that  $w < w_+$  then  $k'(w) < k'(w_+)$ . (ii) If  $w \geq \bar{w}$ , then  $k' = \bar{k}$ .*

Entrepreneurs with low levels of net worth will be constrained in their investment opportunities, as part of their investments need to be financed by net worth. As entrepreneurs increase their net worth, their investment decisions become less constrained. Entrepreneurs keep accumulating net worth until the return on their investment is greater or equal to their cost of financing  $R$ . Thus, as long as long as net worth is low enough, in that entrepreneurs are constrained in their investment choice, entrepreneurs' optimal investment policies are increasing in their net worth. However, once optimal investment reaches the level at which the marginal return on investment equals the opportunity cost of investment,  $R$ , entrepreneurs stop accumulating further capital.

Notice that the maximal level of investment,  $\bar{k}$ , is also the solution to the neoclassical investment problem with complete markets, same discount factor, and no collateral constraints. Since all risk is idiosyncratic, entrepreneurs can perfectly insure against this risk, and at all levels of net worth they will invest  $\bar{k}$ . In the presence of collateral constraints, when net worth is low, investment will be constrained by the available net worth. Thus entrepreneurs will have to build up their net worth in order to afford the same level of investment as in the problem without limits to borrowing.

To understand the optimal dividend policy recall the optimality conditions (2.9) and the envelope condition:  $\lambda = u'(d) = V_w(w, s)$ . Intuitively risk aversion ensures that entrepreneurs increase their dividend payout in line with accumulation of internal funds.

The evolution of optimal net worth is presented in the proposition below.

**Proposition 5** (Net worth transition dynamics). *Suppose  $\pi(s, s') = \pi(s')$ ,  $\forall s, s' \in S$ . (i)  $\forall s', s'_+ \in S$ , such that  $s'_+ > s'$ ,  $w(s'_+) \geq w(s')$ , with equality if  $\mu(s'_+) = \mu(s') = 0$ . (ii)  $w(s')$  is increasing in  $w$ ,  $\forall s' \in S$ ; for  $w$  sufficiently small,  $w(s') > w$ ,  $\forall s' \in S$ ; and for  $w$  sufficiently large,  $w(s') < w$ ,  $\forall s' \in S$ . (iii)  $\forall s' \in S$ ,  $\exists w$  dependent on  $s'$  such that  $w(s') = w$ .*

Part (i) of Proposition 5 states that the higher the returns on the projects are in a state, the larger will net worth be in the next period. To understand Part (ii), recall the optimality condition (2.11). If the level of initial net worth is low enough, entrepreneurs will be able to grow by leveraging up against future states. That is, at levels of initial net worth at which  $\beta R(1 + \mu(s')) > 1$ , entrepreneurial net worth increases. However when net worth is high enough, entrepreneurs, being relatively impatient, have no incentive to save and thus they will pay out net worth as dividends. Therefore, next periods' net worth decreases. Part (iii) states that there exists a unique level of net worth in each state at which net worth stays constant.

The equilibrium outcome will be a stationary distribution of firms, in terms of their net worth. The next proposition shows the existence of a stationary distribution and characterizes its support.

**Proposition 6** (Existence of a Stationary Distribution). *There exists a unique sta-*

tionary distribution of net worth. Define  $w_l$ ,  $\underline{s}$ ,  $w_u$ , and  $\bar{s}$ , where  $s \geq \underline{s}$ , and  $s \leq \bar{s}$ ,  $\forall s \in S$ , such that  $\mu(w_l, \underline{s}') = 1/(\beta R) - 1$  and  $\mu(w_u, \bar{s}') = 1/(\beta R) - 1$ . Then the support of the stationary distribution is  $w \in [w_l, w_u]$ .

The partial equilibrium framework allows for the characterization of the stationary distribution and to provide sharp bounds on its support. From (2.11), notice that whenever  $\mu(\underline{s}') < 1/(\beta R) - 1$ ,  $\lambda < \lambda(\underline{s}')$  which implies that  $w > w(\underline{s}')$ . Thus, entrepreneurs in state  $\underline{s}'$  choose to have lower net worth. However, since entrepreneurs were already constrained in their investment choices, a further decline in capitalization will further constrain their investment possibilities. This implies that with a decline in net worth, the Lagrange multiplier on the collateral constraint,  $\mu$ , next period will have to rise. When  $\mu(\underline{s}')$  increases such that  $\mu(\underline{s}') > 1/(\beta R) - 1$ , from (2.11) we have that  $\lambda > \lambda(\underline{s}')$ ; thus entrepreneurs will increase their net worth. The symmetric argument applies when  $\beta R(1 + \mu(\bar{s}')) > 1$ .

Levels of net worth, at which  $\mu(\underline{s}') > 1/(\beta R) - 1$  and  $\mu(\bar{s}') < 1/(\beta R) - 1$  are transient. Whenever, net worth is low enough,  $w < w_l$ , regardless of the realization of the shocks entrepreneurs' net worth in next period increases. Similarly, when net worth is high enough,  $w > w_u$ , entrepreneurs prefer to pay out net worth as dividends and thus next period's net worth decreases. As a result levels of net worth outside of the support  $w \in [w_l, w_u]$  are transient.

In the more general case, with autocorrelated shocks, investment and financing policies will depend both on the current state and net worth. In Section 4, I study the quantitative implications when investment opportunities are stochastic. There the properties of the technology shocks will be calibrated to empirically plausible

measures of autocorrelation and volatility.

### *2.2.5 Risk-Averse Entrepreneurs*

Let me now turn to the discussion of the importance of risk-averse entrepreneurs. As I have shown above, the assumption of risk aversion implies that investment equals frictionless investment when net worth is sufficiently high. In contrast, with risk-neutral entrepreneurs this is not the case. This result has several implications.

Above a threshold level of net worth, entrepreneurs will hedge all future states; that is, entrepreneurs will conserve net worth against all states to be able to take advantage of future investment opportunities. This happens despite the fact that conserving net worth is costly, as entrepreneurs are relatively impatient. Accumulating net worth against all states can also be interpreted as firms holding onto cash or liquid assets, which I will discuss in the next section. If entrepreneurs are risk-neutral, as shown in Rampini and Viswanathan (2010b) entrepreneurs will not hedge states with high returns when investment opportunities are constant.

The assumption of risk aversion makes also it also convenient to derive comparative statics results. When entrepreneurs' net worth is high enough, then the project is operated at the same scale as in the frictionless economy. And since this scale of operation does not depend on the level of uncertainty, the bounds for the stationary distribution can be exactly pinned down. This significantly simplifies the derivation of comparative static results.

Finally, the results in this paper crucially depend on whether the collateral constraints bind. When agents are risk averse using calibrated parameters, I find that collateral constraints will bind in some regions of the state variable, while not in

others. In a model with risk-neutral agents, under the current parameterization all collateral constraints bind and thus from a quantitative point of view, the effects discussed in this paper will not be present.

### *2.2.6 Collateral Constraints and Borrowing Constrained States*

In this section I discuss the nature of entrepreneurs' collateral constraints and how financing depends on them. Since investment is constrained by the available net worth, entrepreneurs want to accumulate net worth as fast as possible. However entrepreneurs also want to insure against states with low realization of shocks. That is, they want to transfer net worth from high states to low states. Collateral constraints (2.8) imply that entrepreneurs cannot promise to pay more than the value of their collateral in the subsequent period. As a result, collateral constraints impose a limit on how much insurance entrepreneurs can achieve. Consequently, collateral constraints tend to bind against states with high realizations of shocks, and be slack against states with low returns. However, this does not mean that entrepreneurs are borrowing constrained in states with high returns. After all, both their investment and financing policies are choice variables. It simply means that in the absence of collateral they cannot shift enough wealth from states with high realizations of productivity to states with low realizations of productivity.

In fact, the lower their net worth is, the more constrained entrepreneurs become. To understand this, notice that collateral constraints imply that part of investment must to be funded from entrepreneurs' available net worth. The lower the net worth, the lower the down payment entrepreneurs can afford, and the more constrained their investment choices becomes. And since collateral constraints impose a limit on

firms' maximum leverage, they tend to be borrowing constrained precisely at lower levels of net worth, which can happen after a series of realizations of low productivity shocks.

### 2.3 Uncertainty, Financing, and Investment Decisions

In this section I analyze the effects of an increase in uncertainty on investment and financing decisions in the presence of collateral constraints. All results are derived under the assumption of constant investment opportunities; that is  $\pi(s, s') = \pi(s')$ .

The effect of uncertainty can be modeled in two equivalent ways. On the one hand one can assume two stochastic probability distributions, in which case one probability distribution second order stochastically dominates the other probability distribution. The two probability measures will impact the optimal investment and financing decision through their impact on the prices of state-contingent securities. Since lenders are risk neutral, the state-contingent securities are priced according to their probability measures.

Changes in uncertainty are modeled as a mean preserving spread over the magnitude of productivity shocks. As such, the price of state-contingent securities remains the same, however entrepreneurs' demand for state-contingent debt will change as the magnitude of shocks change. The effect of uncertainty is modeled by comparing the stationary distribution of net worth, and the resulting investment and financing decision under two total factor productivity processes,  $A_L, A_H$ . The two productivity processes have the same mean, but differ only in their variance. Denote the mean productivity level as  $\bar{A}_i = \sum_{s \in S} \pi(s) A_i(s), \forall i \in \{L, H\}$ . Then

**Assumption 4.** *Define the mean of the two productivity levels as  $\bar{A}_i = \sum_{s \in S} \pi(s) A_i(s)$ ,*

and define the variances as  $\sigma_i^2 = \sum_{s \in S} \pi(s)(A_i(s) - \bar{A}_i)^2$ ,  $\forall i \in \{L, H\}$ . Assume that  $\bar{A}_L = \bar{A}_H$ , but  $\sigma_L < \sigma_H$ .

Let us turn to the effect of uncertainty on firm investment and financing decisions. The next proposition shows that dividend payout decreases with uncertainty.

**Proposition 7** (Uncertainty and Dividend Payout). *Dividend payout decreases with an increase in uncertainty; that is  $d_L > d_H$  for all  $w$ .*

The intuition behind this result can be understood as follows. Since the value function is strictly concave, an increase in uncertainty induces a precautionary savings behavior for entrepreneurs; as a result entrepreneurs want to save more. Thus, at any given level of net worth, entrepreneurs decrease their dividends payout in order to be able to save more.

The next proposition states that when uncertainty is high, firms decide to hedge at lower levels of net worth, whereas firms reach the level of net worth that allows their investment choice to be unconstrained at higher levels of net worth.

**Proposition 8** (Uncertainty and Financing Demand). *Denote  $\underline{s}, \bar{s} \in S$  such that  $s \geq \underline{s}$ , and  $s \leq \bar{s}$ ,  $\forall s \in S$ . (i) Denote  $\underline{w}_L$  and  $\underline{w}_H$  such that  $\mu_L(\underline{w}_L, \underline{s}') = 0$  and  $\mu_H(\underline{w}_H, \underline{s}') = 0$ . Then  $\underline{w}_H < \underline{w}_L$ . (ii) Denote  $\bar{w}_L$  and  $\bar{w}_H$  such that  $\mu_L(\bar{w}_L, \bar{s}') = 0$  and  $\mu_H(\bar{w}_H, \bar{s}') = 0$ . Then  $\bar{w}_H > \bar{w}_L$ .*

Intuitively, uncertainty affects the risk of external financing. On the one hand, when uncertainty is high, entrepreneurs may want to borrow more against states with high returns; however the collateral constraints limit the amount of external financing provided by lenders. Thus, entrepreneurs cannot hedge the larger risks by

borrowing more against states with high returns. On the other hand, entrepreneurs' in low states now face lower returns on their project. As a consequence in periods with low returns, servicing the debt leads to larger losses of net worth, thus rendering entrepreneurs more constrained in next periods' investment decisions. As a result, entrepreneurs' incentive is to hedge more states with low returns, by borrowing less against those states.

The next proposition states the effect of uncertainty on investment decisions.

**Proposition 9** (Uncertainty and Investment). *(i) Assume  $\underline{w}_H$  the level of net worth such that  $\mu_H(\underline{w}, \underline{s}') = 0$ . If  $w < \underline{w}$  then  $k'_L < k'_H$ . (ii) Assume  $\bar{w}_H$  the level of net worth such that  $\mu_H(\bar{w}_H, \bar{s}') = 0$ . If  $w \geq \bar{w}_H$  then  $\bar{k}'_L = \bar{k}'_H$ . (iii) There exists  $\underline{w}_H < \hat{w} < \bar{w}_H$ , such that if  $w \leq \hat{w}$  then  $k'_L \leq k'_H$ . If  $w \geq \hat{w}$  then  $k'_L \geq k'_H$ .*

To understand the above result, remember that the value function is concave in net worth. The concavity of the value function induces a precautionary motive for entrepreneurial savings. When all collateral constraints bind entrepreneurial savings can happen only through an increase in capital accumulation.

Moreover, the maximum level of investment does not depend on the level of uncertainty. This is intuitive, since investors would never invest such that the marginal return on capital would be lower than the cost of financing,  $R$ . Or put it differently, since entrepreneurs can save using state-contingent securities with return  $R$  as well, state-contingent securities represent an opportunity cost for entrepreneurs. Thus, they will never accumulate levels of capital at which the marginal return on capital is less than the opportunity cost,  $R$ . Alternatively, when net worth is high enough, entrepreneurs can perfectly insure against idiosyncratic shocks and thus the level of

risk does not matter for their optimal decision.

The last part of the proposition states that there is a threshold level of net worth  $\hat{w}$ , below which entrepreneurs invest more when uncertainty is high, and above which, entrepreneurs decrease their investment with an increase in uncertainty. The intuition behind this result is the following. When net worth is low enough entrepreneurs will borrow to the maximum extent of their collateral. With an increase in uncertainty, as long as collateral constraints bind, entrepreneurs choose to invest more due to precautionary reasons. Thus there is a region of net worth where entrepreneurs' investment increases with uncertainty. However with an increase in uncertainty, firms start hedging at lower levels of net worth, invest less and thus lower their capital growth. But when uncertainty is high, entrepreneurs reach the maximum scale of their project at higher levels of net worth. However with lower growth, there must be a threshold level of net worth,  $\hat{w}$ , above which entrepreneurs will operate on a lower scale, as compared to when uncertainty was low. Thus, when the level of net worth is high enough, entrepreneurs' investment decreases with uncertainty.

It is instructive to relate these findings to the result on the effect of uncertainty on investment in the presence of convex adjustment costs, as in Abel (1983). In that model, firms' value function is concave because of the assumed constant returns to scale production functions and convex adjustment costs. Upon an increase in uncertainty, entrepreneurs' precautionary motive for savings increases, but the only vehicle through which entrepreneurs can save is capital, so they invest more. In this paper, when all collateral constraints bind, savings can only increase through more investment in physical capital. However, when some collateral constraints are slack, entrepreneurs can also save by borrowing less at the margin, which may reduce their

investment. Indeed, with an increase in uncertainty, entrepreneurs with high enough net worth chose to save more by decreasing their demand of external financing. With lower funds entrepreneurs invest less and operate at a lower scale.

Let us now turn to the quantitative results.

## 2.4 Quantitative Results

In this section I show that quantitatively the effects presented in the previous section are significant. First, I look at comparative statics; that is, how levels of uncertainty and the collateral constraints affect the stationary distribution, especially the leverage ratio. Then, I compute the effects of uncertainty shocks in a calibrated economy.

The idiosyncratic shock process is modeled as a two-state Markov Chain process, with a symmetric transition matrix. Specifically, assume that the productivity level can be written as

$$A(s_t) = \begin{cases} A_L = \bar{A} - \sigma \\ A_H = \bar{A} + \sigma \end{cases} \quad (2.13)$$

where  $\bar{A}$  is the unconditional value of the productivity process,  $\bar{A} = (A_L + A_H)/2$  and the variance is such that  $\sigma = (A_H - A_L)/2$ .

The literature on estimating the properties of firm level total factor productivity processes does not provide uniformly accepted values for the autocorrelation and unconditional variance. In fact, estimates for both parameters differ widely across studies. For example Veracierto (2002) finds the unconditional volatility of the technology shock to be  $\sigma = 0.056$ , while Cooper and Haltiwanger (2006) finds that the unconditional volatility,  $\sigma = 0.30$ . Conditional volatility estimates are even harder to find in the literature. The exception is Bloom (2009), who derives the estimates

from stock market data. In this paper I follow Bloom et al. (2009), who calibrate the volatility process of idiosyncratic and aggregate productivity to match moments of the cross-sectional dispersion of the inter-quartile sales growth and moments based on a GARCH(1,1) estimated conditional heteroscedasticity of GDP growth.<sup>10</sup> I consider, however, only idiosyncratic productivity, and will follow the parametrization in Bloom et al. (2009). For the specific values, I assume that when uncertainty is low, the standard deviation of productivity is  $\sigma_L = 0.067$ , while when uncertainty is high  $\sigma_H = 0.13$ , thus uncertainty increases twofold.

As with the volatility of the idiosyncratic productivity process, there is no consensus on the estimate for the autocorrelation parameter either. For example Veracierto (2002) estimates the autocorrelation of the idiosyncratic shock to be 0.83, while Cooper and Haltiwanger (2006) estimate a higher autocorrelation parameter of 0.885. Gomes (2001) and Khan and Thomas (2010) calibrate the autocorrelation parameter to be 0.65, to match the persistence of the investment process. Here too I follow Bloom et al. (2009), and assume the autocorrelation to be 0.86, resulting in the following transition matrix for the stochastic process:

$$\pi(s, s') = \begin{bmatrix} 0.93 & 0.07 \\ 0.07 & 0.93 \end{bmatrix}$$

I take relatively standard values for the remaining parameters. The values of the parameters are summarized in Table 1. Specifically, following Bernanke et al. (1999), I assume a yearly discount factor for lenders to be equal to  $\tilde{\beta} = 0.95$ , which implies a yearly gross interest rate of  $R = 1/0.95 = 1.0524$ . Entrepreneurs are assumed to

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<sup>10</sup> These moments in both cases are: mean, standard deviation, skewness, and serial correlation of the annual IQR sales growth rates and GDP growth.

have a CRRA utility function; that is  $u(d) = d^{(1-\gamma)}/(1 - \gamma)$  with the coefficient of risk aversion,  $\gamma = 1$ . Turning to the production function, I assume that the capital share in the production function is  $\alpha = 0.33$ , while the yearly depreciation is 10%.

The two relatively unconventional values are the magnitude of the relative discount factor and the collateral constraint parameter  $\theta$ . For the relative impatience parameter I assume  $\beta = 0.93$ , which implies a yearly premium on internal funds of 2.2%. In comparison, Iacoviello (2005) assume the quarterly premium on internal funds to be 1.1%, which gives a yearly premium of 4.4%. Finally, I assume that the share of physical capital that can be pledged as collateral is 70%; that is  $\theta = 0.7$ . Depending on the level of uncertainty, this results in a book leverage ratio between 0.53 and 0.59 in line with the book leverage of 0.587, as found in Covas and Den Haan (2010) and Covas and Den Haan (2011) using Compustat data. Using Flow of Funds data Jermann and Quadrini (2010) report a somewhat lower book leverage ratio; they find that the ratio of debt to capital over the period of 1984-2009:1 for the Nonfinancial Business Sector is 0.46.

Define  $Z = W \times S$  and  $\phi(Z)$  as the cross sectional distribution of firms over net worth and idiosyncratic shocks. Now define the leverage ratio as total liabilities over net worth:

$$L = \int_Z \frac{\sum_{s' \in S} \pi(s, s') b(s')}{w} \phi(z) \tag{2.14}$$

Table 1 summarizes the parameter values.

Table 2.1: Parameter Calibration

$\beta$	$\gamma$	$R$	$\alpha$	$\delta$	$\theta$	$\bar{A}$	$\rho$	$\sigma_L$	$\sigma_H$
0.93	1	1/0.95	0.33	0.1	0.7	1	0.86	0.067	0.13

The next section presents the comparative statics.

#### 2.4.1 Comparative Statics

In this section I present the descriptive statistics of the stationary distributions for two levels of uncertainty. Table 2 contains the results.

Table 2.2: Stationary Distribution

Variables	$\sigma = 0.067$	$\sigma = 0.13$
Output, $y$	1.42	1.42
Capital stock, $k'$	2.90	2.90
Dividend, $d$	1.04	1.05
External Financing, $\sum_{s' \in S} \pi(s, s')b(s')$	1.70	1.56
Net Worth, $w$	2.23	2.38
Book Leverage	0.58	0.53
Leverage Ratio, $L$	0.76	0.67

The primary difference across levels of uncertainty is the difference between the equilibrium levels of net worth, debt, and leverage ratio. Notice that the real variables remain equal across different levels of uncertainty, however the capital structure changes dramatically. In the stationary distribution when uncertainty is high, firms choose to increase their internal funds by more than 7.1% just to be able to afford the same level of investment and reach the same output. The consequence of this change in the capital structure is that the leverage ratio in the economy decreases by 12%.

#### 2.4.2 Transitional Dynamics

This section contains the results of how firms respond to unanticipated changes in uncertainty. Studying the effects of unexpected shocks is important in light of the

recent financial crisis of 2007-2009 that was widely thought of as not having been anticipated by most market participants or economists. Indeed, there was a widespread belief that house prices could not fall and developments in the financial sector allowed risks to be spread widely. Even the current Chairman of the Federal Reserve Bank, Ben Bernanke, declared on March 28, 2007: “At this juncture, however, the impact on the broader economy and financial markets of the problems in the sub-prime market seems likely to be contained.”<sup>11</sup> Thus, it seems warranted as a baseline scenario to investigate the effects of unanticipated changes in uncertainty. However, in the next section I also present results for the stochastic volatility case.

Basically, by allowing for an unanticipated, permanent uncertainty shock, I trace the transitional dynamics from a low to a high volatility environment. In order to compute the transitional dynamics, I first solve the entrepreneurs’ problem (2.5) - (2.8) for the optimal decisions. To compute the stationary distribution, I simulate a large number of firms and trace them over time, until based on some metric, the stationary distribution converges. I choose 100000 firms and track them over time until the mean of net worth converges.

Having computed the stationary distribution, I increase the volatility of shocks. Then I trace the evolution of firms for 20 periods. I repeat this last step 500 times and chose the mean value of the variables. This last step is required to rule out the dependence of transitional dynamics on a particular draw of idiosyncratic shocks.

In order to understand the effects of uncertainty shocks, first, I describe the optimal policies in a constant investment opportunity environment. Figures 2.2

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<sup>11</sup> See “The Economic Outlook”, March 2007, Report before the joint economic committee, U.S. Congress.

and 2.3 show the optimal policies for investment ( $k'$ ), Lagrange multipliers on the collateral constraints ( $\mu(s')$ ), optimal dividend policy, and optimal net worth in next period ( $w(s')$ ), in both cases when uncertainty is low and high.

Notice that when uncertainty is high, entrepreneurs start to hedge at lower levels of net worth as compared to when uncertainty is lower. This is so because as the ex-post risk of losing net worth in the economy increases, entrepreneurs issue fewer claims against the state with lower returns. Moreover, entrepreneurs reach the unconstrained scale of their projects at higher levels of net worth. However, costly hedging crowds out investment, and as such entrepreneurs increase the size of their project at a lower rate. This can be seen by comparing the optimal investment policies ( $k'$ ) of firms, and noticing that the slope is lower in the case of high uncertainty, under the region where entrepreneurs hedge. Finally, notice that the stationary distribution widens with an increase in uncertainty, as can be seen from the bottom right panel in the two figure. The bounds of the stationary distribution increase, as the bounds are at levels of net worth at which the optimal policy crosses the 45 degree line.

Figure 2.4 presents the transitional dynamics for investment, debt, net worth, and output for the case of autocorrelated productivity shocks; that is, when investment opportunities are stochastic. The parameters are listed in Table 2. Notice that the effect of an unexpected, permanent increase in uncertainty is large; net investment drops by more than 40%. Furthermore, upon an increase in uncertainty dividend payout is reduced by more than 2% as entrepreneurs use their net worth for investment rather than pay it out as dividends. In order to hedge the increased risk, entrepreneurs reduce their demand for external financing, which decreases the

leverage ratio.

The main difference, however, resides in the different behavior of real and financial variables. Notice that after 5 periods (years), entrepreneurs manage to build up enough net worth to essentially operate at the same scale at which they operated when uncertainty was low. However, the balance sheet undergoes significant change for 15 more periods, as entrepreneurs conserve enough net worth to hedge the higher risk in the economy. With constant investment opportunities, entrepreneurs will never hedge the highest state, as they are able to insure against risks just by hedging states with low realizations of returns. This is not the case anymore with stochastic investment opportunities. Now, with an increase in uncertainty, entrepreneurs will hedge all states; that is, they will conserve net worth against all states in order to be able to take advantage of future investment opportunities. Thus with autocorrelated shocks, an increase in uncertainty will increase entrepreneurs' holdings of cash.

### *2.4.3 Stochastic Volatility*

In this section I present the results for the case of stochastic volatility. With stochastic volatility, entrepreneurs are aware that uncertainty in the economy can change and will hedge accordingly. To model stochastic volatility, I assume, just as before, that productivity shocks follow a Markov process. However, now the magnitude of the variance changes as well. Thus, I assume that

$$A(s_t) = \begin{cases} A_L = \bar{A} - \sigma_{t-1} \\ A_H = \bar{A} + \sigma_{t-1} \end{cases} \quad (2.15)$$

where  $\sigma_t \in \{\sigma_L, \sigma_H\}$ . I assume that  $\sigma_t$  follows a Markov process with the transition probabilities

$$\pi_\sigma(s, s') = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

This implies that the yearly autocorrelation in the stochastic volatility process is 0.4.

To study the effects of an uncertainty shock in this environment, I follow Arellano et al. (2010). First, I compute the stationary distribution in this economy when uncertainty changes as well. Next, I force uncertainty to be low for 10 periods, and then I increase the level of uncertainty from then on for 25 periods. Figure 2.5 presents the results from this experiment.

As a result of the low uncertainty environment, leverage in the economy is increased by more than 4%. This implies that relative to steady state, entrepreneurs decrease their net worth by 1.5%. Upon an increase in uncertainty entrepreneurs delever in order to hedge the larger shocks in the economy. Hedging, however, comes at the expense of financing needs and thus results in an initial reduction in investments. As entrepreneurs build up net worth, investment recovers and dividend payout increases. Furthermore, notice that now the leverage ratio falls below the steady state level; thus, as before, changes in uncertainty in the long run will be reflected in entrepreneurs' capital structure.

The main message of this section however is that even in the presence of stochastic volatility, changes in uncertainty have large effects on optimal financing and investment dynamics. Let me now turn to some related implications of this model and a discussion of the importance of complete markets for these results.

#### 2.4.4 *Why Do Firms Save So Much?*

As Figure 2.1 documents, by the third quarter of 2010, US nonfinancial corporations amassed approximately \$1.95 trillion in liquid assets.<sup>12</sup> Furthermore Bates et al. (2009) provide evidence that firms' cash to assets ratio more than doubled between 1980 and 2006, while recent reports<sup>13</sup> suggest that firm level cash holdings have increased significantly after the recent recession. In this section I show that an increase in uncertainty induces firms to conserve more net worth against all states. Conserving net worth against all states can be interpreted as cash, and thus this paper potentially can explain why, in the aftermath of the financial crisis, US nonfinancial firms appear to have increased their holdings of cash and liquid assets on their balance sheets.

With higher uncertainty, firms start hedging states with low returns at lower levels of net worth. Moreover, firms can fully hedge fluctuations in productivity at larger levels of net worth, as compared to an environment with low uncertainty. Thus, with increases in uncertainty, firms engage in more risk management. Furthermore, if one interprets as cash the net worth that entrepreneurs conserve against all future states, then the results show that entrepreneurs save more cash when uncertainty in the economy is high.

Table 3 presents the ratio of cash and net worth under the two uncertainty regimes.

Notice that when uncertainty is low firms do not hold cash. However when

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<sup>12</sup> See the Flow of Funds data from the Federal Reserve Board, Table L102.

<sup>13</sup> On October 26 Moody's Investor Service estimated that nonfinancial U.S companies are hoarding \$943 billion of cash, an increase from \$775 billion since the end of 2008. See <http://www.reuters.com/article/idUSN2614487020101027>.

Table 2.3: Holdings of Liquid Assets

	$\sigma = 0.067$	$\sigma = 0.13$
Liquid assets, $\int_Z (w - \bar{w})/w \phi(z)$	0%	0.93%

uncertainty in the economy increases, entrepreneurs choose to hold cash worth almost 1% of their net worth. As such, these results are qualitatively in line with the documented increase in liquid assets after the financial crisis of 2007-2009. As documented in Bloom et al. (2009), measured uncertainty increased during this period, and thus increasing holdings of cash for risk management purposes might be a rational response from firms.

The predictions of this model are also in line with empirical results documenting the gradual increase in cash holding by U.S firms.<sup>14</sup> Campbell et al. (2001) and Comin and Philippon (2005) document a secular increase in various measures of firm level volatility. According to the results in this paper, firms experiencing an increase in uncertainty hedge more, and thus will increase their holdings of liquid assets in order to take advantage of future investment opportunities.

#### 2.4.5 Collateral Constraints and Asymmetric Responses

The presence of collateral constraints implies that shocks can have asymmetric effects, depending on whether collateral constraints bind.

To understand the embedded asymmetry, take for example the case of a permanent increase in uncertainty. With increased risk, entrepreneurs want to hedge more states with lower realizations of shocks. However, collateral constraints restrict the issuance of state-contingent securities against states with higher realizations of

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<sup>14</sup> See, for example Bates et al. (2009).

shocks, thus making it more difficult for entrepreneurs to hedge. Moreover, since entrepreneurs are risk-averse, upon an increase in uncertainty firms do not cut their dividend payout enough to meet the financing needs required to hedge the larger shocks. Instead entrepreneurs cut their investments and external financing. Thus, an increase in uncertainty will lead entrepreneurs to decrease the scale of their project, which leads to a recession.

The response of the economy to an unexpected decrease in uncertainty, however, is very different, as can be seen in Figure 2.6. Upon an increase in uncertainty, entrepreneurs will have too much net worth accumulated relative to the risks in the economy, and as such they have an incentive to pay out the extra net worth as dividends. Risk aversion, however, implies that it is not optimal to pay out all the extra net worth in one payment, and as such they will gradually pay out the extra net worth until their balance sheet reflects the risks in the economy. Notice, however, that when entrepreneurs need to pay out their net worth, collateral constraints will be less binding, as entrepreneurs' hedging and financing needs will be reduced. As a result investment will react less to a decrease in uncertainty.

#### *2.4.6 Financial Innovation*

What is the effect of financial innovation on capital structure and investment decision? Financial innovation in the context of the present model can be thought of as an increase in the lenders' ability to collateralize loans. That is, financial innovation is modeled as an increase in the value of the parameter  $\theta$ .

Table 4 below presents how the stationary distributions are affected by financial innovation. First, notice that under the stationary distribution, the capital stock

increases. To understand this result, recall that well capitalized entrepreneurs operate on the same scale as in the frictionless case. As a result it is not that financial innovation affect the maximal scale at which entrepreneurs run their projects. Instead financial innovation increases the aggregate capital stock through its affect on the stationary distribution. Intuitively, financial innovation allows entrepreneurs to accumulate capital at a faster, since it enables them to have more access to external financing. Thus entrepreneurs with lower levels of net worth can operate at a higher scale. This is the reason why capital stock increases.

Table 2.4: Stationary Distribution with Financial Innovation,  $\theta = 0.8$

Variables	$\sigma = 0.067$	$\sigma = 0.13$
Output, $y$	1.43	1.43
Capital stock, $k'$	2.95	2.95
Dividend, $d$	1.03	1.04
External Financing, $\sum_{s' \in S} \pi(s, s')b(s')$	1.98	1.84
Net Worth, $w$	1.99	2.15
Book Leverage	0.67	0.62
Leverage Ratio, $L$	0.99	0.88

Second, notice that under financial innovation the leverage ratio,  $L$ , as defined in (14), increases. The reason is that greater access to external financing enables entrepreneurs with lower levels of net worth can operate at a higher scale. But since financing investment from net worth is costlier than financing from internal funds, entrepreneurs choose to finance investment more from external funds rather than net worths. As a result financial innovation leads to an increase in leverage. Increased leverage however implies that entrepreneurs will now be less capitalized in the face of changes in uncertainty.

Higher leverage also means that entrepreneurs potentially may be more affected

by changes in the level of uncertainty in the economy. Figure 2.7 presents the effect of an increase in uncertainty in the case of a financially more developed economy. Notice that on impact of uncertainty shocks on investment is larger, as the decrease in investment increases from 40.22 percent to 45.48 percent.

## 2.5 Modeling Financing Frictions: Complete or Incomplete Markets with Collateral Constraints

To better understand the importance of complete markets, subject to collateral constraints, in this section I compute a model with exogenously incomplete markets where financing is subject to collateral constraints. The key question is how do results change if one assumes instead of complete markets, markets that are exogenously incomplete?

The entrepreneurs' problem now becomes:

$$V(w, s) = \max_{\{d, k', b', w(s')\}} \left\{ u(d) + \beta \sum_{s' \in S} \pi(s, s') V(w(s'), s') \right\}$$

subject to

$$w + b' \geq d + k'$$

$$A(s')f(k') + k'(1 - \delta) \geq w(s') + Rb', \quad \forall s' \in S$$

$$\theta k'(1 - \delta) \geq Rb'.$$

When markets are exogenously incomplete, entrepreneurs can insure against fluctuations in productivity only by conserving net worth. This is the only way for entrepreneurs to be able to take advantage of future investment opportunities. This happens despite the fact that entrepreneurs are relatively impatient; thus, it is costly

for them to save net worth. Relative impatience, however, also means that entrepreneurs will not save enough to become forever unconstrained. Upon a series of low realizations of shocks, entrepreneurs' net worth may be reduced so much that their investment choices become constrained and the collateral constraint will bind again.

The precautionary savings motive implies that the collateral constraint will bind as long as entrepreneurs have net worth below a threshold level. Above this threshold level, entrepreneurs can choose the unconstrained level of investment. Importantly, however, the unconstrained level of investment will be below the investment level achieved under complete markets. The reason is that, as shown in Angeletos and Calvet (2006), when markets are incomplete, entrepreneurs cannot insure against the idiosyncratic production risk and thus will charge a risk premium on net worth.

Upon an increase in uncertainty, in the region of net worth where the collateral constraint binds in both regimes of uncertainty, entrepreneurs will choose to accumulate capital at a faster rate. This is due to the precautionary savings motive, just as in Abel (1983). The maximum level of investment, however, now will be lower, since an increase in uncertainty leads to an increase in the risk premium on net worth. Figure 2.8 plots the optimal investment policy in both cases of constant and stochastic investment opportunities.

What do these results imply for the overall effect of a permanent increase in uncertainty? For the numerical results, I use the same parameter value as in Table 1. The next table presents descriptive statistics of the stationary distributions under different levels of uncertainty.

From the table above, one can see that in economies with incomplete markets

Table 2.5: Stationary Distribution under Incomplete Markets

Variables	$\sigma = 0.067$	$\sigma = 0.13$
Output, $y$	1.42	1.42
Capital stock, $k'$	2.91	2.93
Dividend, $d$	1.04	1.04
External Financing, $\sum_{s' \in \mathcal{S}} \pi(s, s')b(s')$	1.74	1.74
Net Worth, $w$	2.21	2.23
Book Leverage	0.59	0.59
Leverage Ratio, $L$	0.78	0.77

the long run effect of an increase in uncertainty is to increase the capital stock. The intuition behind this result can be found in the shape of the stationary distribution. In the case of stochastic investment opportunity the precautionary savings effect overcomes the risk premium effect and a larger mass of entrepreneurs will have higher investments. This is in stark contrast to the comparative statics results under complete markets.

Additionally, results from this section also imply that incomplete markets tend to dampen the effects of uncertainty shocks. Intuitively, under incomplete markets conserving net worth is the only way to insure against risks in the economy. But this implies that entrepreneurs become overly capitalized. And since changes in uncertainty effect capital structure, uncertainty shocks have a small impact on investment decisions.

Under complete markets subject to collateral constraints, results are both qualitatively and quantitatively different. With complete markets, entrepreneurs can hedge risks by issuing state-contingent securities subject to collateral constraints. Thus entrepreneurs are now better able to hedge risk and are not required to conserve as much net worth. But this implies that in the face of an increase in uncertainty firms

find themselves undercapitalized and thus their reaction to the shock is larger. As a result, complete markets subject to collateral constraints tend to amplify shocks in the economy.

These results highlight the importance of how different choices of modeling of financial frictions can have profoundly different implications.

## 2.6 Conclusion

In this paper I studied the effect of uncertainty on financing demand and investment in the context of a model where firms face collateral constraints. The main innovation of this paper is to study the effect of uncertainty in a dynamic model of firm financing, where financing is advanced based on an optimal long-term contract.

Collateral constraints limit the amount that firms can borrow. Thus, investment needs to be financed both with internal funds and external funds. In this setting, uncertainty affects the capital structure of firms, specifically the optimal mix of internal and external financing that firms use to fund their investment projects. Uncertainty implies that external financing is risky, in the sense that if the return on investment is low, servicing the debt leads to a loss of internal funds, which constrains future investment choices.

Upon an increase in uncertainty, entrepreneurs reduce their demand for external financing and consequently the scale of their production. Investment and output rebound as entrepreneurs build up internal funds and transition to an equilibrium with high uncertainty. Quantitatively, an unexpected increase in uncertainty has large effects on optimal leverage and investment dynamics.

In this paper I assume that the price of capital does not change. An important ex-

tensions would be to endogenize the price of capital, since then collateral constraints then would depend on the resale value of capital. Fluctuation in the price of capital could potentially amplify the mechanism presented in this paper. Furthermore, it would be important to consider the effect of uncertainty on credit spreads, as fluctuations in the cost of financing will affect the maximum scale of investment and thus could further amplify the mechanism presented above. These lines of research are left for future work.

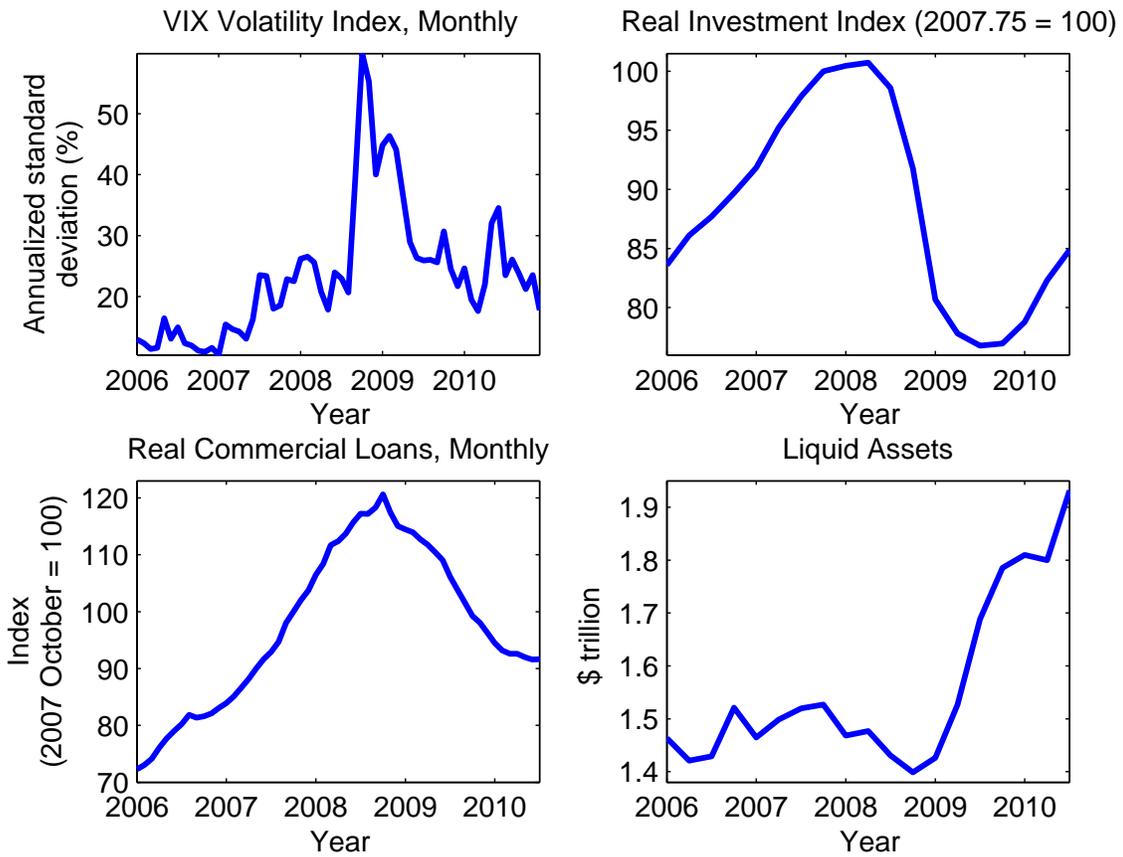
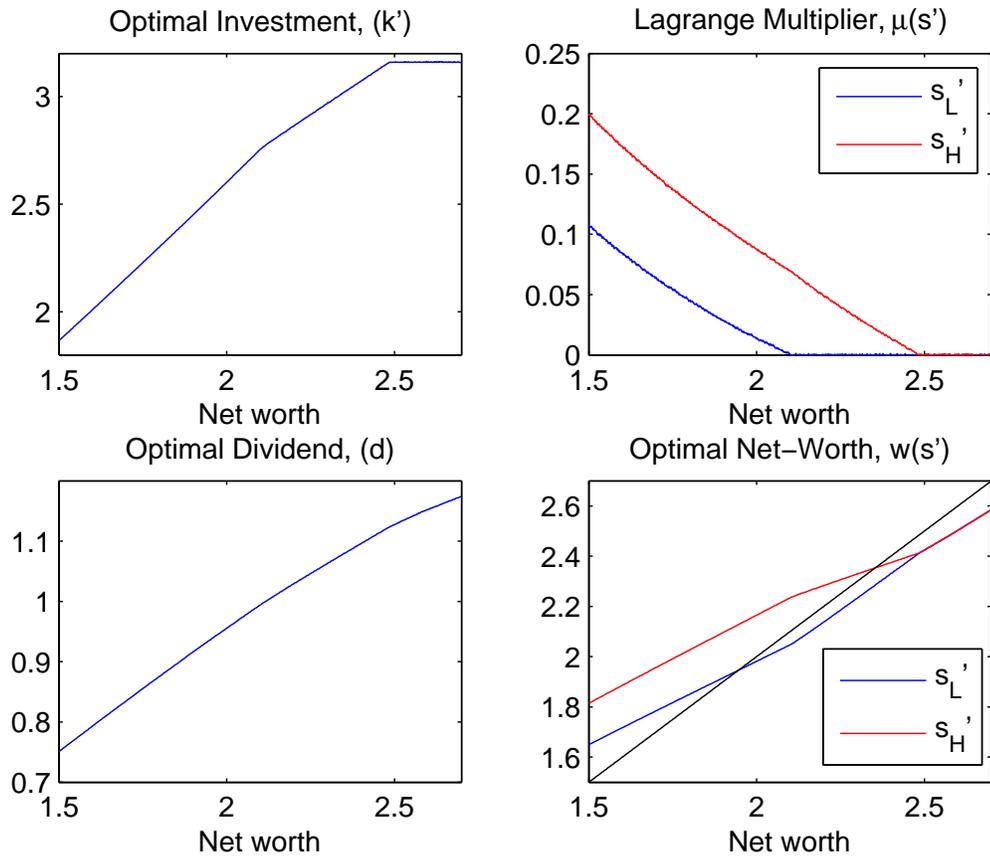
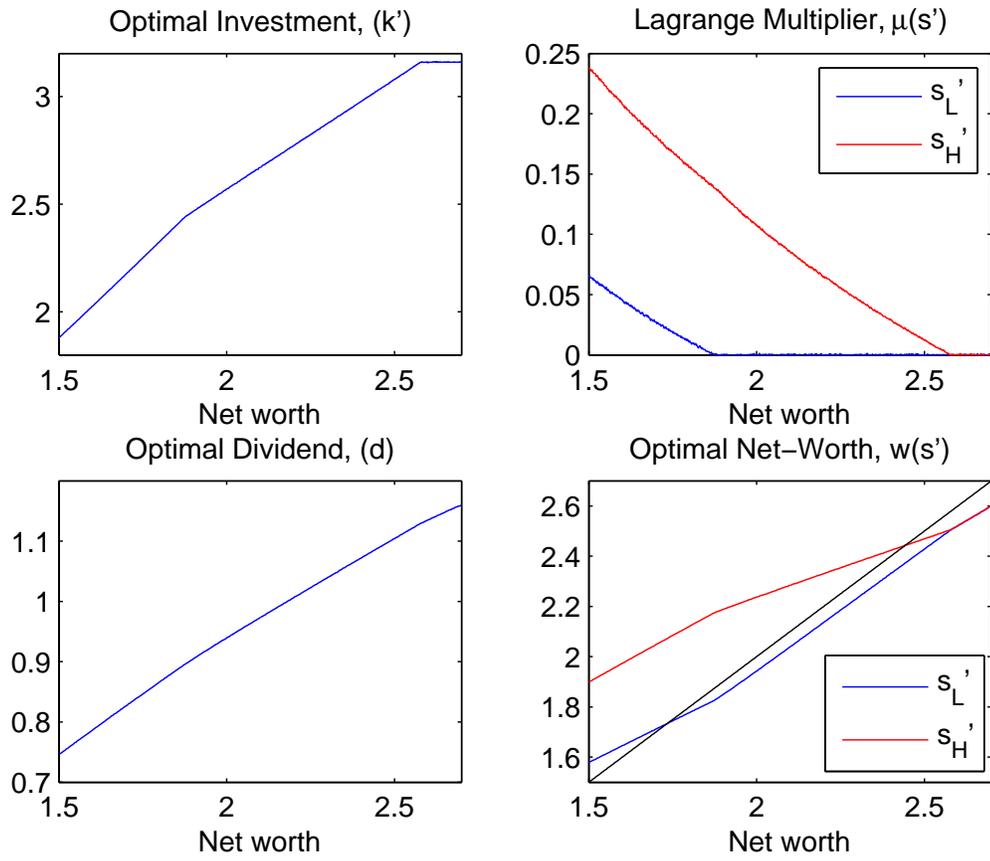


FIGURE 2.1: Aggregate Economic Indicators



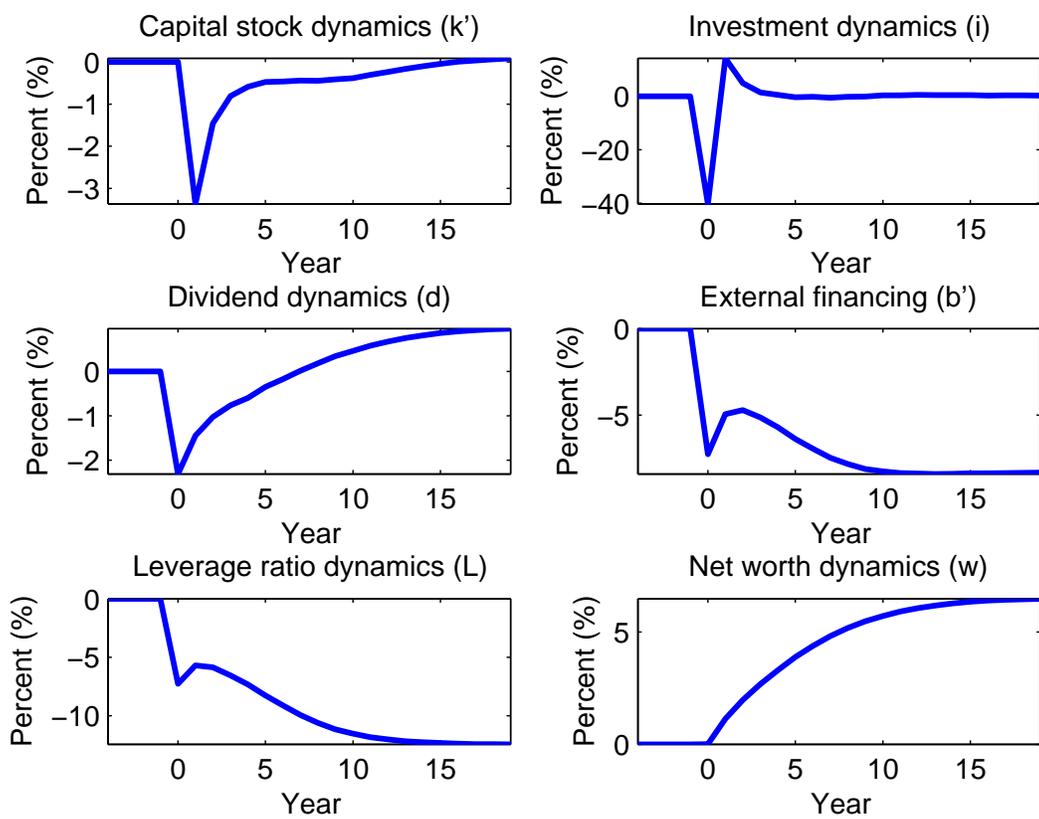
Parameter values are:  
 $\beta = 0.93, R = 1/0.95, \gamma = 1, \alpha = 0.33, \delta = 0.1, \theta = 0.7, \sigma_H = 0.067$

FIGURE 2.2: Optimal Policy - Low Volatility,  $\pi(s, s') = \pi(s')$



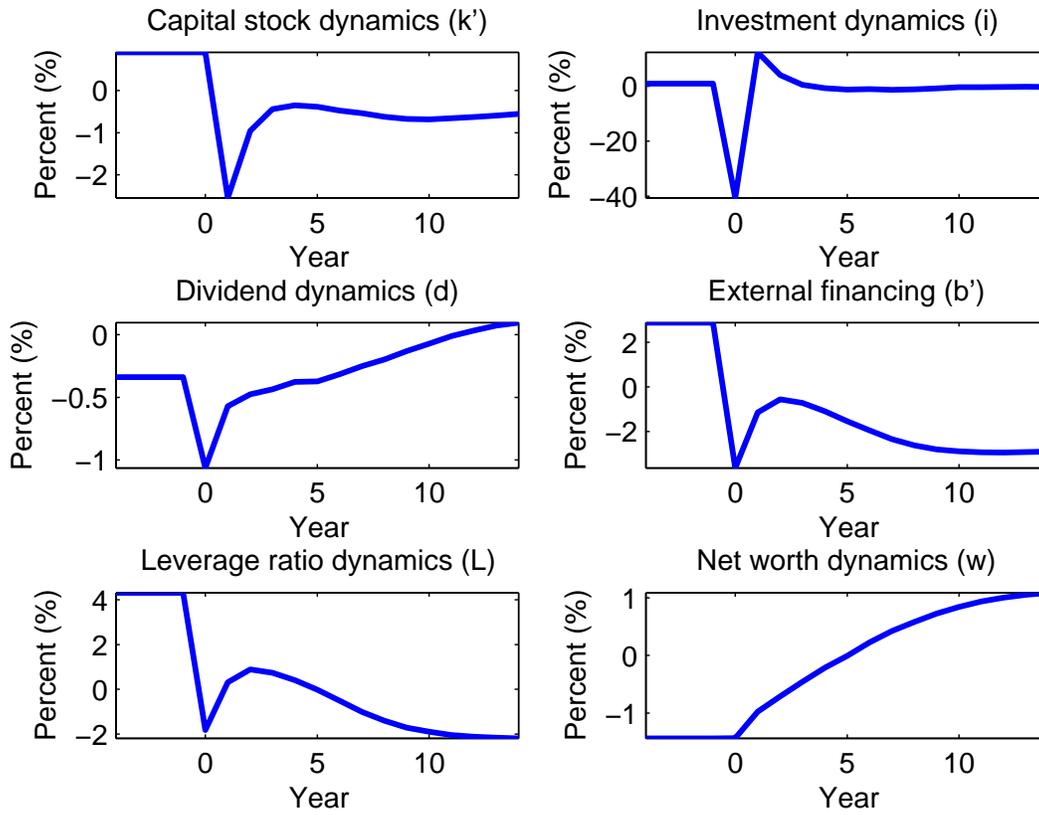
Parameter values are:  
 $\beta = 0.93, R = 1/0.95, \gamma = 1, \alpha = 0.33, \delta = 0.1, \theta = 0.7, \sigma_H = 0.13$

FIGURE 2.3: Optimal Policy - High Volatility,  $\pi(s, s') = \pi(s')$



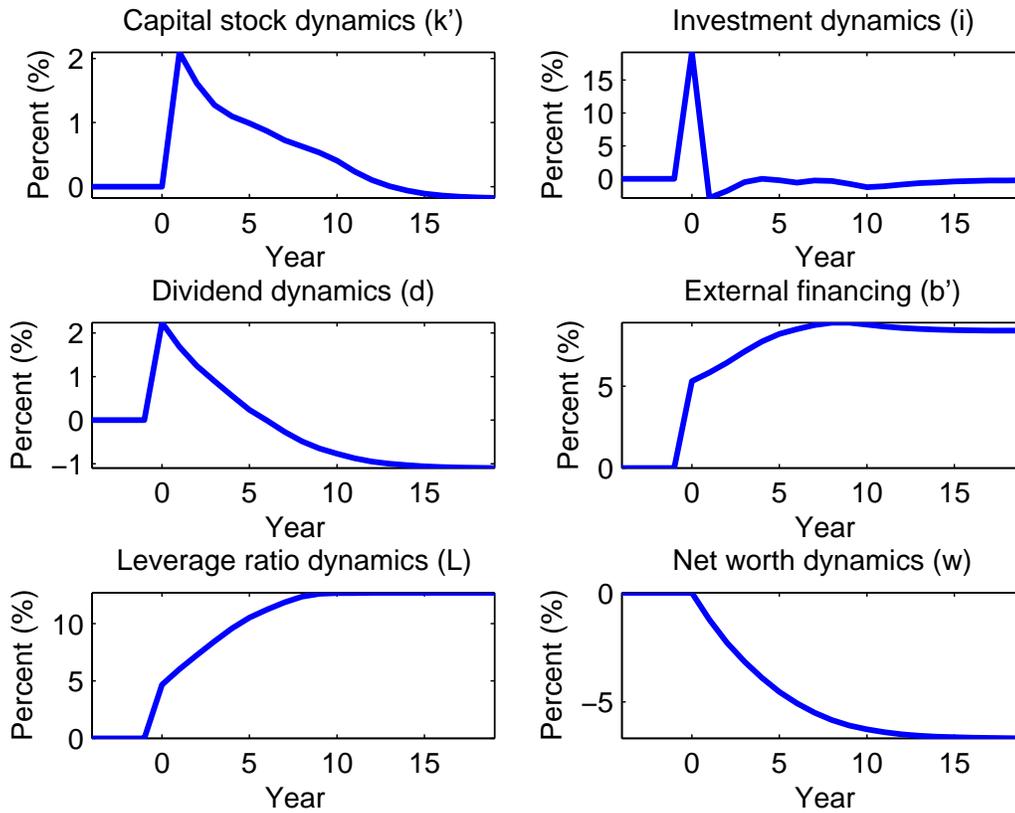
The figure depicts the impact of an unexpected increase in uncertainty that occurs in period 0. All variables are relative to their steady-state values under low uncertainty. Parameter values are:  $\beta = 0.93$ ,  $R = 1/0.95$ ,  $\gamma = 1$ ,  $\alpha = 0.33$ ,  $\delta = 0.1$ ,  $\theta = 0.7$ ,  $\rho = 0.86$ ,  $\sigma_L = 0.067$ ,  $\sigma_H = 0.13$ .

FIGURE 2.4: Increase in Uncertainty, Transitional Dynamics



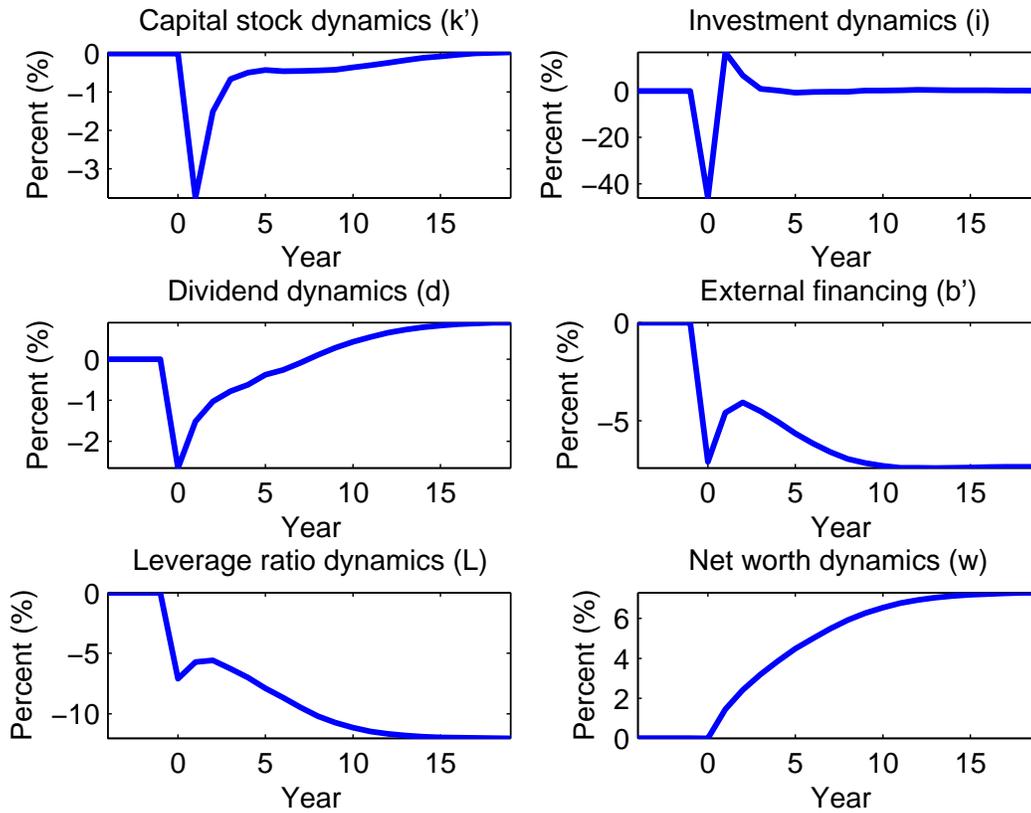
All variables are relative to their steady-state values. Parameter values are:  $\beta = 0.93$ ,  $\gamma = 1$ ,  $R = 1/0.95$ ,  $\alpha = 0.33$ ,  $\delta = 0.1$ ,  $\theta = 0.7$ ,  $\rho_A = 0.86$ ,  $\rho_\sigma = 0.4$ ,  $\sigma_L = 0.067$ ,  $\sigma_H = 0.13$ .

FIGURE 2.5: Uncertainty Shock, Stochastic Volatility



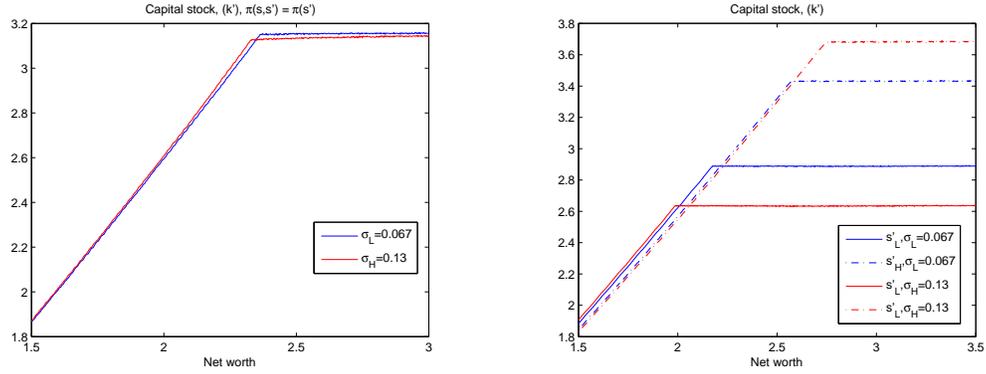
The figure depicts the impact of an unexpected increase in uncertainty that occurs in period 0. All variables are relative to their steady-state values under high uncertainty. Parameter values are:  $\beta = 0.93$ ,  $\gamma = 1$ ,  $R = 1/0.95$ ,  $\alpha = 0.33$ ,  $\delta = 0.1$ ,  $\theta = 0.7$ ,  $\rho = 0.86$ ,  $\sigma_L = 0.067$ ,  $\sigma_H = 0.13$ .

FIGURE 2.6: Decrease in Uncertainty, Transitional Dynamics



The figure depicts the impact of an unexpected increase in uncertainty that occurs in period 0. All variables are relative to their steady-state values under high uncertainty. Parameter values are:  $\beta = 0.93$ ,  $\gamma = 1$ ,  $R = 1/0.95$ ,  $\alpha = 0.33$ ,  $\delta = 0.1$ ,  $\theta = 0.8$ ,  $\rho = 0.86$ ,  $\sigma_L = 0.067$ ,  $\sigma_H = 0.13$ .

FIGURE 2.7: Increase in Uncertainty and Financial Innovation, Transitional Dynamics



(a) Constant investment opportunity (b) Stochastic investment opportunity

Panel (a) shows the optimal scale with iid shocks, and (b) shows the optimal scale with correlated shocks. Parameter values are:  $\beta = 0.93$ ,  $\gamma = 1$ ,  $R = 1/0.95$ ,  $\alpha = 0.33$ ,  $\delta = 0.1$ ,  $\theta = 0.7$ ,  $\sigma_L = 0.067$ ,  $\sigma_H = 0.13$ .

FIGURE 2.8: Optimal Scale of Production in Incomplete Markets

## The Decline of the Personal Saving Rate and the Rise of Health Expenditures

\* joint work with Maurizio Mazzocco

### 3.1 Introduction

It is a well-known fact that the U.S. personal saving rate has declined from around 10 percent in the early eighties to nearly 1 percent in 2007. This drop may be of concern to economists and policy-makers because it may signal an increased dependence on foreign investment and a future reduction in capital stock with negative consequences for labor productivity, wages, and national output. In the past twenty years economists have attempted to explain this dramatic drop. An examination of the related literature indicates that the sharp decline is still a puzzle. Parker (1999), in his seminal paper, states that “Each of the major current theories of the decline in the U.S. saving rate fails on its own to match significant aspects of the

macroeconomic or household data.” Guidolin and Jeuness (2007) review a number of arguments and theories that have been proposed and conclude that, “the recent decline of the U.S. private saving rate remain a puzzle.”

The main contribution of this paper is to show that the entire decline in the saving rate can be explained by one variable: medical expenditure by health institutions net of the employers’ contributions to pension and insurance funds. Parker (1999) has correctly argued that health expenditure on its own cannot explain the drop in household savings. The reason behind this conclusion is that health expenditure has been growing since the early sixties. Therefore, if it were the main explanation, the saving rate should have started its decline in the sixties. However, when the employers’ contributions are subtracted from the medical expenditure, the resulting variable has the features required to explain the evolution of household savings. This variable is constant until the early eighties and it increases at a steep rate for the rest of the period. To test whether this variable can explain on its own the decline in the saving rate, we consider a hypothetical situation in which health expenditure by institutions net of the employers’ contributions is equal to zero for the entire period; we then compute the corresponding saving rate. In this hypothetical case, the saving rate is constant at approximately 9 percent from the sixties to today. This result confirms the hypothesis that this variable on its own can explain the decline.

With the goal of better understanding the decline in the saving rate, we decompose the employers’ contributions in its two main components: contributions to pension funds and to health plans. We find that the major force behind the decline is the evolution of the contributions to pension plans. As a percentage of income, this variable grew at approximately the same rate as health expenditure until 1980,

but from that year on it started to decline. Thus, health expenditure kept rising at a sharp rate without the counteracting effect of the pension contributions. The main observable outcome was a reduction in the saving rate.

Our findings have important implications for policy analysis. They indicate that households were partially responsible for the decline in the U.S. saving rate. Had they increased their contributions to pension plans to compensate for the reduction in the employers' share, the saving rate would not have declined or would have declined by less. Thus, any policy designed to increase the contributions to pension plans by households will have a positive effect on the saving rate. However, our results also suggest that the main reason behind the decline in household savings is the dramatic rise in health expenditure, which was possible only because an increasing amount of resources was diverted from other uses. As a consequence, a policy designed to increase the pension contributions of households would merely be a short-term solution. A policy that can have long-term effects on the saving rate is one aimed at structurally changing the health sector and at reducing its expenditure.

A strength of this paper is that the findings presented here are obtained using simple summary statistics and do not depend on the use of a particular model. One can therefore interpret our results as non-parametric in the sense that they do not rely on particular modeling assumptions. There is a price to pay, however, for the absence of a model. Without a model, we cannot decompose the effect of the increase in net health expenditures into demographic changes, health price changes, and demand changes. Performing this decomposition is extremely valuable, but it is left for future research. In addition, with no model, we cannot make statements about the optimal saving rate for the U.S. economy. We cannot determine whether

a saving rate of 2 percent is low or whether a saving rate of 8 percent is optimal. We can only provide the explanation for its decline from 8 to 2 percent. For a discussion on the optimality of the current saving rate, one can read Scholz et al. (2006) and Lusardi et al. (2001).

The rest of the paper proceeds as follows. The next section provides a brief discussion of related papers. In section 2, we describe the data sets and define the variables used in this paper. Section 3 develops a simple model of health expenditure and saving decisions that is used to interpret the empirical analysis. Section 4 presents the main result and section 5 discusses how the increase in medical expenditure was financed. Section 6 concludes.

## 3.2 Related Papers

A large number of studies have analyzed the decline in the U.S. saving rate. In this section, we will discuss the papers with findings that are related to ours. For a thorough review of the literature see Browning and Lusardi (1996), Parker (1999), and Guidolin and Jeuness (2007).

Gokhale et al. (1996) is the first paper to discuss the steep increase in health expenditure in the past 50 years and to suggest a possible relationship with the drop in the saving rate. In table 2, they report that medical consumption as a percentage of disposable income was 3.9 in the fifties, 5.2 in the sixties, 7.3 in the seventies, 10.1 in the eighties, and 12.8 in the early nineties. This pattern suggests that medical expenditure on its own cannot explain the decline in the saving rate. Health expenditure was already growing at a steep rate in the sixties and seventies, whereas the saving rate started its decline in the eighties. Probably for this reason, Gokhale et al.

(1996) do not directly explore the effect of the increase in medical consumption on household saving. Instead, they use it mainly as a motivation for decomposing the changes in the saving rate in four components: the changes generated by variations in the intergenerational distribution of resources; the effects produced by changes in the cohort-specific consumption propensities; the changes produced by modifications in the rate of government spending; the effects of changes in demographics. The decomposition of the changes in the saving rate is based on a simple life-cycle model and relies on the assumptions implicit in that model. Their results suggest that the decline in saving rate can be traced to two factors. The government redistribution of resources from young generations with low propensities to consume to old generations with higher propensities. Our results differ from the one reported by Gokhale et al. (1996) in several respects. First, we show that the sharp increase in health expenditure can explain on its own the decline in U.S. saving rate if considered jointly with the evolution of the employers' contributions to insurance funds. Second, the increase in medical consumption, while being more pronounced for the older generations, affected all age groups. Finally, we show that the expansion in health expenditure funded by private health insurance is of the same order of magnitude as the increase experienced by Medicare and Medicaid.

In a related paper, Parker (1999) considers the main explanations given in the literature for the decline in the saving rate, namely the effect of asset value appreciations, durable goods, changes in the intergenerational distribution of resources, financial innovations, and changes in the discount factors. He then evaluates whether these hypotheses are consistent with patterns observed in micro and macro data. He concludes that each of the main theories fails on its own to explain the decline

in saving rate since it cannot match significant aspects of aggregate and household data. In particular, he rejects the hypothesis of changes in the intergenerational distribution of resources, which include the increase in medical expenditure, for two reasons. First, the trends in the government redistribution of resources and in the increase in health expenditure predate the drop in the saving rate. Second, the ratio of consumption to income increases for all generations and not only the old ones. Both conclusions are correct. In particular, without subtracting the evolution of the employers' contributions, health expenditure cannot explain the decline in saving rate.

In a paper that is contemporaneous to Parker's, Gale and Sabelhaus (1999) start from the observation that one may require different measures of aggregate savings to answer different economic questions. They then study the evolution of the U.S. saving rate using several measures of this aggregate variable. They first use the measure employed by NIPA. In this case, the pattern displayed by the saving rate is consistent with the findings of previous papers, which document a steep decline that starts in the early eighties. They then consider an alternative measure that adjusts personal savings obtained using the NIPA definition for durable goods, retirement accounts, inflation, and tax accruals. Using this measure, the authors still report a decline in the saving rate but of a smaller magnitude. Gale and Sabelhaus consider also a third measure that adds capital gains. In this case, as one may expect, the saving rate at the end of the nineties was at the highest level in the last forty years. In our paper, we only consider the standard measure of aggregate savings used in NIPA. Thus, we have nothing to say about the evolution of saving rates obtained using alternative measures.

The most recent survey we could find is the paper by Guidolin and Jeuness (2007). The first part of the paper examines whether the decline in the U.S. saving rate is real or a simple statistical artifact generated by measurement issues. Since the decline is evident in all the standard measures considered in their paper, they conclude that it cannot be easily explained by measurement issues. In the second part of the paper, Guidolin and La Jeunesse review many of the theories that have been proposed and conclude that the drop in saving rate that started in the early eighties is still a puzzle.

### 3.3 Data Description and Variable Definition

We use data from the National Income and Product Accounts (NIPA), the National Health Expenditure Accounts (NHEA). Our main variable of interest is the evolution of the Personal Saving Rate (NIPA, Table 2.1, line 34).

The National Income and Product Accounts are published by the Bureau of Economic Analysis and constitutes the basis for the majority of macroeconomic research in the U.S. We use annual data for the years 1960-2008, for it to be comparable to the period for which NHEA has available data. All files were downloaded on 1/20/2010 from the BEA website (<http://bea.gov/national/index.htm>).

We construct our variables using data from Table 2.1, Personal Income and Its Disposition; Table 2.5.5, Personal Consumption Expenditures by Function; Table 7.8, Supplements to Wages and Salaries by Type. Furthermore, we use two additional tables, Table 6.11(A-D), Employer Contributions for Employee Pension and Insurance Funds by Industry and by Type, and Table 3.6, Contributions to Government Social Insurance, to understand the evolution of employers' contributions to

both private and public pension and health insurance funds. We measure Personal Health Expenditures as the sum of items from Table 2.4.5, as listed in Table 3.1. The variables used to compute our different savings rate measures are defined in Table 3.2.

Table 3.1: Components of Personal Health Expenditures

Item	Line in NIPA Table 2.4.5
Therapeutic appliances and equipment	line 21
Pharmaceutical and other medical products	line 40
Health care	line 60
Net health insurance	line 93

Table 3.2: Components of Personal Saving Rate

Variable	Source
Personal Saving Rate	Table 2.1, line 34
Employer Contributions for Employee Pension and Insurance Funds	Table 2.1, line 7
Employer Contributions for Employee Pension Funds	Table 7.8, lines 6,7,10,11
Employer Contributions for Employee Health Insurance Funds	Table 7.8, line 16

Data for current personal taxes is taken from Table 2.1, line 25. From Table 3.6 we obtain data for employers' payroll tax for Medicare line 6, while Medicare payroll tax paid by employees is taken from line 25.

The National Health Expenditure Accounts are published since 1964 by the Department of Health and Human Services. The NHEA accounts provide not only a comprehensive measure of total spending on health care goods and services, but also a breakdown of the sources of funds that finance these expenditures. This is impor-

tant, since looking at the funding sources one can have a better understanding of the decision making level at which medical expenditures are made. Moreover, funding sources also reveal the relative level of expenditure that are financed from public and private sources. All NHEA files were downloaded on 1/20/2010 from the Centers for Medicare and Medicaid Services website (<http://www.cms.hhs.gov>).

The NHEA are generally compatible with the NIPA, as can also be seen from the reconciliation project in Sensenig and Wilcox (2001). Since we are interested in the evolution in the NIPA of households' health expenditures, we use the comparable Personal Health Care data from the NHEA. The Personal Health Care data from NHEA tracks the Personal Health Expenditures measures in the NIPA remarkably well, the ratio of the two personal health expenditure measures throughout the period 1960-2008 is between 0.94 - 1.06. The variables that we use from the National Health Expenditure Accounts are defined in Table 3.3.

Table 3.3: Definitions of Variables from NHEA

Variable	Source
Personal Health Care	line 104
Out of pocket expenditure	line 107
Health Institutions	line 104 less line 107
Private Funds	line 106
Private Health Insurance	line 108
Public Funds	line 113
Gov't programs	line 146
Medicare	line 115
Medicare Part A	line 159 + 335 + 476
Medicare Part B, C, D	line 115 less line above
Medicaid	line 118 and 135
Health insurance premiums paid by employers	NIPA Table 7.8, line 16
Health insurance premiums paid by households	line 9 less line above

### 3.4 A Model of Savings and Health Expenditure

In this section we develop a simple model of saving, health expenditure decisions, and employers' contributions. The model will be used to provide some insight on which components of health expenditure and employers' contributions may have affected the evolution of household savings in the past 50 years. We will then use this insight to derive different hypothetical measures of the saving rate that can be employed to understand the decline in household savings.

Consider an economy characterized by a representative household which lives for  $T$  periods. The household has a discount factor  $\beta$  and preferences over a standard non-durable consumption good  $c$  and its health status  $h$ . The corresponding utility function takes the form  $u(c, h)$ . In each period  $t$ , the health status is a function of two variables: the amount of resources spent to maintain and improve its health  $c_t^h$  and a health shock  $\epsilon_t$ ; that is

$$h_t = f(c_t^h, \epsilon_t).$$

The only source of uncertainty in the model is the health shock. The history of the shocks up to period  $t$  will be denoted by  $\sigma^t$  and the corresponding probability by  $P(\sigma^t)$ .

The amount of resources spent on the household health depends on two variables: the health plan used by the household  $b_t$  and the expenditure of health institutions (hospitals, nursing homes, dentist and physician practices)  $I_t^h$ . As a consequence,

$$c_t^h = g(b_t, I_t^h).$$

The health plan used by the household depends on two variables: the health insurance premium paid by the household  $P_t^i$ , which is under its direct control; the

employer's contributions to the its health plan  $P_t^e$ , which the household takes as given. This implies that

$$b_t = g(P_t^i, P_t^e).$$

In each period, the individual is endowed with an amount of income  $y_t$  and incurs out-of-pocket health expenses  $c_t^{out}$ , which are fully determined by the health plan and the health shock. For simplicity, it is assumed that the evolution of the income process is deterministic. The household can save an amount  $s_t$  using a risk-free asset with a gross interest rate  $R_t$ . The employer makes contributions to the household pension fund  $P_t^p$ . To simplify the model we will assume that the household can use these contributions in each period. This is a strong assumption since there are limitations on how these resources can be used before retirement. But it does not affect the insight we will provide using the model. The consumption of the non-durable good is the numeraire and the expenditure on all other goods is described in units of this numeraire.

In each period, the individual chooses the consumption of the non-durable good, how much to pay in health insurance premium, and how much to save as the solution of the following problem:

$$\begin{aligned} \max_{c_t, P_t^i, s_t} \sum_{t=1}^T \beta^t \sum_{\sigma_t} P(\sigma_t) u^i(c_t, h_t) & \quad (3.1) \\ \text{s.t. } c_t + P_t^i + c_t^{out} = y_t + P_t^p + R_t s_{t-1} - s_t & \quad \text{for any } t, \sigma_t \\ h_t = f(c_t^h, \epsilon_t), \quad c_t^h = g(b_t, I_t^h), \quad b_t = g(P_t^i, P_t^e). & \end{aligned}$$

Using this stylized model we can describe which components of the saving rate can explain its decline. We can then discuss modifications in the definition of the

saving rate that allow us to evaluate the effect of these components. The standard definition of the saving rate takes the following form:

$$r_t = \frac{\text{disposable income} - \text{total expenditure}}{\text{disposable income}}.$$

Previous papers that have studied the evolution of the saving rate using NIPA data have defined disposable income as the income generated by the household plus the employer's and government's contributions to health plans and pension funds. Since our model does not consider the government's contributions, disposable income can be defined as follows:

$$\text{disposable income} = y_t + P_t^e + P_t^p,$$

In those same papers, total expenditure is defined as the sum of expenditure that is under the direct control of the individual,  $c_t + P_t^i + c_t^{out}$ , plus the expenditure of health institutions  $I_t^h$ , i.e.

$$\text{total expenditure} = c_t + P_t^i + c_t^{out} + I_t^h.$$

As a consequence, the definition of saving rate used in previous papers takes the following form:

$$r_t^1 = \frac{y_t + P_t^e + P_t^p - (c_t + P_t^i + c_t^{out} + I_t^h)}{y_t + P_t^e + P_t^p}.$$

Gokahle, Kotlikoff, and Sabelhous (1996) have provided evidence that health expenditure has increased at a steep rate during the period under consideration. A simple way to test whether medical expenditure can explain on its own the decline in the saving rate is to analyze the evolution of the saving rate under the assumption that health expenditure as a fraction of disposable income was equal to some

constant during this period. To simplify the exposition we will assume without loss of generality that the constant is zero. This is equivalent to replacing the saving rate  $r_t^1$  with the following hypothetical one:

$$r_t^2 = \frac{y_t + P_t^e + P_t^p - (c_t + P_t^i)}{y_t + P_t^e + P_t^p}.$$

One may argue, however, that it is incorrect to include in disposable income the employer's contributions to the household health plan  $P_t^e$ , since larger contributions generally imply a higher level of expenditure by health institutions. To take into consideration this potential problem, we will also consider an alternative definition of the saving rate, in which  $\frac{P_t^e}{y_t + P_t^e + P_t^p}$  is constant during the period. As for health expenditure, we will assume, without loss of generality, that this constant is zero by setting  $P_t^e$  equal to zero. The corresponding saving rate can be written as follows:

$$r_t^3 = \frac{y_t + P_t^p - (c_t + P_t^i)}{y_t + P_t^p}.$$

In 1974 and 1978 two laws passed that affected the size of the employers' contributions to pension funds  $P_t^p$ . The Employee Retirement Income Security Act (ERISA) of 1974 is a federal law that sets minimum standards for most voluntarily established pension and health plans in private industry. It was designed to provide additional protection for individuals enrolled in these plans. There is mixed evidence on the effect of ERISA on the employers' contributions to pension funds. For instance, Ledolter and Power (1984) provide evidence that ERISA significantly increased the termination and significantly reduced the formation of private retirement plans by

employers. But E. Rejda and Schmidt (1984) find that ERISA did not have a significant effect on insured pension contributions. The Revenue Act of 1978 included a provision that became Internal Revenue Code Sec. 401(k) under which employees are not taxed on the portion of income they elect to receive as deferred compensation rather than as direct cash payments. The law went into effect on Jan. 1, 1980. The general belief is that employers started to replace the defined-benefit pension plans with the defined-contribution (DC) plans with the Revenue Act of 1978. For instance, Even and MacPherson (1994), and Papke (1999) find evidence that the introduction of a 401(k) or other DC plans reduced the participation in and funding to defined benefit plans. Since with DC plans the employer is not required to make pension contributions, this event resulted in a reduction in  $P_t^p$ .<sup>1</sup> To evaluate the effect of these two laws on  $P_t^p$  and on the U.S. saving rate, we will consider a saving rate in which we set to zero the employers' contributions from pension plans. We therefore analyze the following saving rate:

$$r_t^4 = \frac{y_t - (c_t + P_t^i)}{y_t}.$$

Finally, we will evaluate whether the medical expenditure by institutions is the only component that is relevant to explain the decline in the saving rate. To that

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<sup>1</sup> A number of papers have studied the effect of Individual Retirement Accounts (IRA) and 401(k) plans on national and personal savings. The evidence is mixed. Gale and Scholz (1994) find that IRAs have a small impact on saving. Venti and Wise (1986), Venti and Wise (1990), and Poterba et al. (1996) reach different conclusions. Their findings suggest that IRAs and 401(k) have significant and positive effects on national and personal savings. These papers are only marginally related to our findings. They do not relate the rise in popularity of these retirement programs to the NIPA saving rate. In addition, they do not provide evidence on the effect of the Revenue Act of 1978 on  $P_t^p$ .

end, we will consider a saving rate that includes out-of-pocket expenses, i.e.

$$r_t^5 = \frac{y_t - (c_t + P_t^i + c_t^{out})}{y_t}.$$

In the next section we will use the measures of the saving rate introduced in this section, NIPA and NHEA data to show that the dramatic drop in the private saving rate that occurred over the past 25 years can be explained by the steep increase in health expenditure by institutions which was not matched by a corresponding increase in the employers' contributions to health and pension plans.

### 3.5 Main Result

The saving rate in the household sector has experienced a significant drop in the past 25 years. This well-known fact is illustrated in Figure 3.1, which depicts this decline using the NIPA data and the standard definition of saving rate  $r_t^1$  discussed in the previous section. It shows that during the sixties and seventies the saving rate in the household sector was approximately constant at around 9-10%. But in the early eighties the private saving rate started to decline at a fast pace until it reached the alarmingly low level of 2% in 2005. Several papers have attempted to explain this pattern, but there is no agreement on the actual cause. In this section, we will show that the steep drop in saving rate can be fully explained by a single variable: health expenditure by institutions net of employers' contributions to pension and insurance funds.

To provide evidence on the effect of health expenditure on savings, we start with a simple empirical exercise. We consider the evolution of the personal saving rate for an hypothetical situation in which health expenditure is always equal to zero,

i.e. we use  $r_t^2$  as a measure of the saving rate. Figure 3.2 describes the evolution of  $r_t^2$  using data on total health expenditure from NIPA and NHEA. Two features of Figure 3.2 are worth discussing. First, the evolution of  $r_t^2$  obtained using NIPA data on health expenditure is very much alike the evolution obtained using NHEA data. We will therefore interchangeably use the NIPA and NHEA data to study the effect of health expenditure. Second, when health expenditure is excluded from the computation of the saving rate, the decline that started in the early eighties vanishes. However, the saving rate increases at a steep rate in the first twenty years of the period under investigation. Had health expenditure been equal to zero for the entire period, household savings would have grown from 14% in 1960 to 20% in 1980. Then, they would have remained around this level for the rest of the period.

To understand why  $r_t^2$  increases in the sixties and seventies, in Figure 3.3 we report the evolution of health expenditure as a percentage of disposable income. The data indicate that health expenditure has been growing as a percentage of income at a steady rate from 1960 to today, the only exception being the period from 1993 to 2000. This finding explains the pattern observed in Figure 3.2. The actual saving rate is constant during the sixties and seventies. Hence, when a steadily growing variable is added to it, the resulting saving rate will display an increasing pattern in the first twenty years. The evolution of health expenditure reported in Figure 3.3 also suggests that health expenditure cannot explain on its own the drastic decline in household savings. For this hypothesis to be correct this variable has to remain constant as a percentage of disposable income until the early eighties and then it has to increase.

In Figure 3.4 we add two new variables to health expenditure: the employers'

contributions for employee pension and insurance funds and total health expenditure minus these contributions, both as percentage of disposable income. It is important to remark that in the computation of the saving rate the employers' contributions are included in disposable income. They should therefore be subtracted from medical expenses to determine the net effect of the variable of interest. Figure 3.4 shows that from 1960 to 1980 the rate of increase in health expenditure was offset by a similar rate of increase in the employers' contributions. As a consequence, health expenditure net of these contributions was constant at around 2% of disposable income. Starting from 1980, however, health expenditure kept rising as a percentage of income, whereas the employers' contributions remained constant at the 1980 level. In addition, from 1993 to 1997 the contributions experienced a decline which compensated for the fact that health expenditure was flat during that period. Thus, net health expenditure grew also in those years.

All this implies that health expenditure net of employers' contributions displays the pattern required to explain the decline in saving rate. It was constant until the early eighties and it grew at a steep rate since then. To confirm this hypothesis, we report in Figure 3.5 net health expenditure, the actual saving rate, and the hypothetical saving rate obtained by setting to zero health expenditure net of employers' contributions from its computation  $r_t^4$ . After a small increase from 1960 to 1966, the saving rate  $r^4$  remained constant at around 12% for the entire period. This finding confirms the hypothesis that the sharp increase in net health expenditure explains on its own the decline in the household saving rate.

We will now evaluate whether the large increase in health expenditure was paid by households as out-of-pocket expenses. The NIPA data do not distinguish between

out-of-pocket and the rest of health expenditure, but the NHEA collects data on both variables. Figure 3.6 describes out-of-pocket health expenses, health expenditure by institutions, and health expenditure by institutions net of contributions using NHEA data. All variables are reported as a percentage of disposable income. The figure shows that out-of-pocket expenses declined by approximately 1% during this period. This finding implies that the large increase in health expenditure is fully explained by the growth in the medical expenses of institutions. Figure 3.6 also indicates that health expenditure by institutions net of employers' contributions displays the features required to be the main reason behind the decline in household savings. It shows that the difference between the health expenditure of institutions and the employers' contributions was negative and constant until the early eighties. But starting from that period the medical expenses by institutions started to increase without being matched by a corresponding increase in the employers' contributions. This also corresponds to the period in which the saving rate began its decline. To confirm this hypothesis, we report the evolution of the hypothetical saving rate  $r_t^5$ , which was defined in the previous section by setting to zero net medical expenditure of institutions. In figure 3.7, this hypothetical saving rate is approximately constant at around 9%.

The employers' contributions are composed of two main components: contributions to pension plans and contributions to health plans. We conclude this section by analyzing their evolution. Figure 3.8 indicates that the changes in the contributions to pension plans as well as to health plans are responsible for the flattening of total contributions in the early eighties. As a percentage of income, prior to 1979 the contributions to pension plans grew at the same rate as health expenditure. But

from that year, these contributions started to decline and in 2000 they were about two percentage points lower than in 1979. As discussed in the previous section, the likely explanation for this drop is the shift to DC pension plans and possibly the passing of ERISA. The contributions to health plans display a different pattern. As a percentage of income, they grew until 1993. However, even in those years the rate of growth was smaller than the one experienced by health expenditure and in the early eighties it declined even further. Then, in the nineties it flattened at around 6%.

We will now evaluate the importance of the contributions to pension funds in explaining the decline in the saving rate relative to the contributions to health plans. To achieve this goal, we first estimate the rate at which the contributions to pension plans grew as a percentage of disposable income in the sixties and seventies. This rate is computed by regressing these contributions from 1960 to 1979 on a time trend. The rate is estimated to be 0.16 percentage points. We then compute the hypothetical level of pension contributions that would have prevailed if they had grown at the constant rate of 0.16 as a percentage of disposable income. Finally, we calculate the standard saving rate  $r_t^1$  by using the hypothetical employers' pension contributions in place of the actual ones. Figure 3.9 describes the evolution of this variables, of the hypothetical and actual employers' contributions, and of medical expenses of institutions. From 1960 to 1990, the hypothetical contributions grew at the same rate as medical expenditure by institutions. It is only after 1990 that health expenditure started growing at a slightly larger rate than the hypothetical contributions. This implies that the corresponding hypothetical saving rate is constant for most of the period and declines slightly only after 1990. This result suggests that the reduction

in employers' contributions to pension funds are the main reason for the decline in the U.S. saving rate. The change in contributions to health plans had only a small effect after 1990.

We can therefore conclude that households are partially to blame for the events that explain the sharp decline in saving rate. Had households increased their personal contributions to pension funds to compensate for the reduction in the employers' contributions, the saving rate would not have experienced the decline observed in the data. Our analysis indicates, however, that the main reason behind the drop in household savings is the steep rate at which medical expenses by institutions grew. If health expenditure keeps rising at the rate observed in the past fifty years, even a large increase in household contributions to pension plans would not be enough to avoid a further decline in the U.S. saving rate.

### 3.6 How Is the Increase In Health Expenditure Paid For?

The objective of this section is to understand how the increase in household health expenditure observed between 1960 and 2008 was financed. To that end, it helps to divide it in private health expenses, which are defined as expenses paid by consumers or health insurance companies, and public health expenses, which are composed of all the expenditures that are paid using public funds. Public health expenditure will be further divided in its two main components: Medicare, which covers people older than 65 and disables, and Medicaid, which assists low-income families. Figure 3.10 documents the evolution of total, private, and public health expenditure as a percentage of household disposable income. It shows that public as well as private health expenses grew during the period under consideration. Private health expen-

diture almost doubled as a fraction of income, experiencing an increase from 4.9% in 1960 to 9% in 2008. Public health expenditure grew at a faster rate, increasing from 1.3% in 1960 to 8.4% in 2008. In Figure 3.11 we report the dynamics of public health expenditure, Medicare and Medicaid expenses.<sup>2</sup> According to the data, from their inception in 1966 both programs increased at a similar rate, with Medicare and Medicaid reaching, respectively, 4% and 3% of disposable income in 2008.

We will start by describing how the rise in private health expenses were financed. There are three possible groups that may have funded it. First, health insurance companies may have paid for the increase through a reduction in their underwriting gains which are defined as the difference between the premium received and the benefit paid. Second, employers may have funded a portion of the rise by way of an increase in the health insurance premiums they pay for their employees. Finally, consumers may have financed part of the rise through an increase in their health insurance premiums and through an increase in their out-of-pocket expenses. With regard to the health insurance companies, a well-known fact is that in the past several decades private insurers have generally experienced the health insurance underwriting cycle, three consecutive years of underwriting gains followed by three consecutive years of losses.<sup>3</sup> As a consequence, a reduction in underwriting gains cannot have financed the increase in private health expenditure. Figure 3.12 reports the amount of private health expenditure that was paid by consumers and employers. As a frac-

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<sup>2</sup> Medicare and Medicaid account for a large fraction of public health expenditure. For instance, in 2008 they added up to 84% of public health expense

<sup>3</sup> This cycle has been documented for instance in J. Reed, S. Roberts, and R. Maule (1989), J. Gabel, R. Formisano, B. Lohr, S. DiCarlo (1991), and in the executive report prepared in 2003 by the consulting firm Milliman USA ([www.aha.org/aha/content/2003/pdf/MillimanReport030410.pdf](http://www.aha.org/aha/content/2003/pdf/MillimanReport030410.pdf))

tion of disposable income, the insurance premiums paid by employers increased at a steep rate, going from 0.9% in 1960 to 5% in 2008. In contrast, the contribution of consumers increased only slightly during this period from 4.2% in 1960 to 4.8% in 2008. In Figure 3.13 we decompose the consumer's payments in out-of-pocket expenses and premiums. From the early eighties the premiums paid by consumers grew at a rate that is similar to the one experienced by employers. During the same period, however, the consumers enjoyed a reduction in out-of-pocket expenses that compensated for the increase in premiums. These results suggest that the rise in private health expenditure was paid almost entirely by employers and that the budget constraint of consumers was barely affected by it.

We will now discuss the financing of the health expenses related to Medicare. To that end, it is useful to divide Medicare in its four components, Part A, B, C, and D, since they are funded in different ways. The original Medicare program consisted only of Part A and Part B. Part A covers only inpatient hospital stays. Part B helps pay for some of the services and products administered on an outpatient basis and not covered by Part A. In 1997, with the passage of the Balanced Budget Act, Medicare beneficiaries were given the option to receive their Medicare benefits through private health insurance plans, instead of through the original Medicare plans A and B. These plans are known as Part C. Medicare Part D was enacted as part of the Medicare Prescription Drug, Improvement, and Modernization Act of 2003 (MMA) and went into effect on January 1, 2006. The objective of this program is to subsidize the costs of prescription drugs for Medicare beneficiaries. Figure 3.14 shows that Part A is the component of Medicare that grew at the faster rate, increasing from 0.24% of disposable income in 1966, when it was introduced, to 2.4% in 2008. The rest of

Medicare expenses increased at a more moderate rate, going from 0.1% in 1966 to 1.2% in 2005. In 2006 the introduction of Part D generated a jump to 1.5%. In 2008, Part B, C, and D amount to 1.7% of disposable income.

Given the sharp increase in Part A expenditure, we will start by documenting how this component of Medicare was financed. These health expenses are paid using the Hospital Insurance (HI) trust fund, which is financed for the most part by a Medicare payroll tax. For instance, in 2006 86% of the funds came from it. The second source of funds in order of importance was interest payments on the trust fund, which accounted for just 7% of the total.<sup>4</sup> The evolution of the Medicare payroll tax, which is paid half by the employee and half by the employer, is displayed in Figure 3.15. One can see that the expansion of Medicare Part A was funded with an increase in the corresponding payroll tax. The tax receipts increased from 0.4% of disposable income in 1966 to 1.7% in 2008. To understand the effect of this change in payroll taxes on the household budget constraint, however, one has to analyze the evolution of the total amount of taxes paid by households. The dynamics of this variable is documented in Figure 3.15, which shows that throughout this period total taxes paid by the household sector as a fraction of income remained approximately constant around 14%. This result suggests that the increase in Medicare payroll tax was compensated by a reduction in other personal taxes. As a consequence, the household budget constraint was not affected by the rise in Medicare Part A expenses.

Medicare expenses related to Parts B, C, and D are paid using the Supplementary

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<sup>4</sup> The percentages for 2006 have been computed using data from the 2007 Annual Report of the Boards of Trustees of the Federal Hospital Insurance and Federal Supplementary Medical Insurance Trust Funds.

Medical Insurance (SMI) Trust Fund. This fund is financed primarily through the monthly premiums that all beneficiaries of Parts B, C, D must pay and general government revenues. For instance, in 2006 general revenues accounted for 76% of the SMI Trust Fund inflow, while beneficiary premiums accounted for 21% of the Trust Fund inflow. Figure 3.16 describes the evolution of the general revenues allocated to the SMI Trust Fund and of the monthly premiums, both as a fraction of income. At the beginning of the program, general revenues and premiums represented an identical and small fraction of disposable income. They were both equal to 0.12%. With time, however, the two series started to diverge with the general revenues accounting for a larger fraction of the inflow to the Trust Fund. In 2008, general revenues amounted to 1.74% of income, while the beneficiary premiums were one third of them and only 0.66% of income. These results suggest that a larger fraction of Parts B, C, and D expenditures are paid with general government revenues, even if the personal taxes have not increased in this period, as documented in Figure 3.15. As a result this source of funding should have had no effect on the household budget constraint. The beneficiary premiums are the only variable we have found whose small increase seems to have affected the household budget constraint.

We will now turn our attention to Medicaid. Financing for the Medicaid program is shared by the federal government and the states. Unlike Medicare, federal funding for Medicaid comes entirely from general revenues. Funding for the state share of Medicaid costs comes from a variety of sources, but at least 40% must be financed by the state, and up to 60% may come from local governments. The main sources of funds are personal income, sales, and corporate income taxes. For instance, the National Association of State Budget Officers reports that in 2006 about 80% of the

state share of their Medicaid costs was financed by state general funds, most of which are raised from personal income, sales, and corporate income taxes. The remaining 20% was financed by other state funds. As shown in Figure 3.15, personal taxes did not increase during this period. Consequently, the households budget constraint was not directly affected by the increase in Medicaid expenditure.

The evidence presented in this section suggests that the increase in private health expenditure was paid almost exclusively by employers. It also indicates that nearly all of the rise in the public health expenses were paid using general government revenues, payroll taxes, and premiums for the beneficiaries of Parts B, C, and D. We have provided evidence that personal taxes did not increase during this period. There are, therefore, two possible ways in which the government could have collected the revenues used to pay for public health expenditure. One possibility is that the government increased taxes on production and imports, taxes on corporate income, and/or taxes from the rest of the world. The second possibility is that the government issued debt. Figure 3.17 describes the evolution of the government deficit and of the sum of personal taxes, taxes on production and imports, taxes on corporate income, and taxes from the rest of the world. It shows that the total amount of taxes collected during this period stayed constant as a fraction of income at around 30%. It also shows that the per-period federal budget was mainly in deficit since the early 1980s. These two findings suggest that the general revenues used to pay for public health expenditure were collected either through savings in other parts of the general expenditures or through an increase in debt.

All these results indicate that the only change that may have had a direct impact on the household budget constraint is the increase in premiums for the beneficiaries

of Parts B, C, and D. The increase, however, was minor and only affected people that were 65 or older.

### 3.7 Conclusions

In the paper we present the following results:

1. Health expenditure by institutions net of employers' contributions to insurance and pension funds explains on its own the entire decline that the U.S. saving rate experienced from the early eighties to today.

2. To explain the decline it is crucial to subtract the employers' contributions from medical expenses. As a percentage of income, the employers' contributions grew at the same rate as health expenditure until the early eighties. But from that period onward, they remained constant whereas medical expenses kept growing at the same rate. As a consequence, the saving rate started to decline only in the early eighties even if health expenditure has been growing since the sixties.

3. If one divides the employers' contributions in contributions to pension funds and to health plans, the contributions to pension funds had the largest effect on the decline in saving rate.

These results suggest that the policy that is more likely to have long run effects on the U.S. saving rate is one designed to curb the medical expenses by health institutions. In the last section of this paper, we have shown that the medical expenditure funded by private health insurance increased as a percentage of income at the same rate as the one funded by Medicare and Medicaid. We have also shown that all the main components of health expenditure have increased. Medical expenses for hospital care, physician and clinical services, prescription drugs, and dental care

have all risen. As a consequence, a reform of the health sector can be successful only if it leads to structural changes in all of its areas.

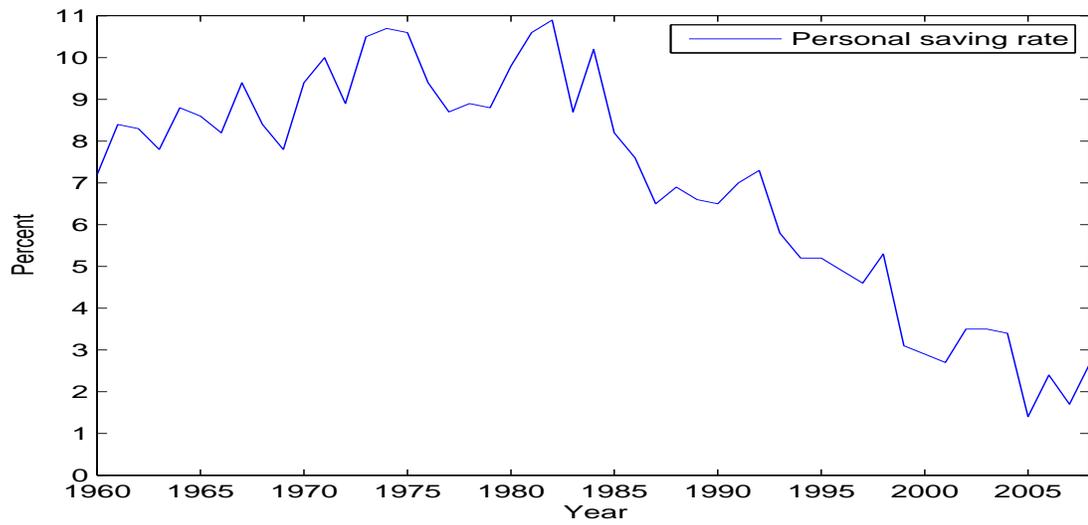


FIGURE 3.1: Household Saving Rate from NIPA

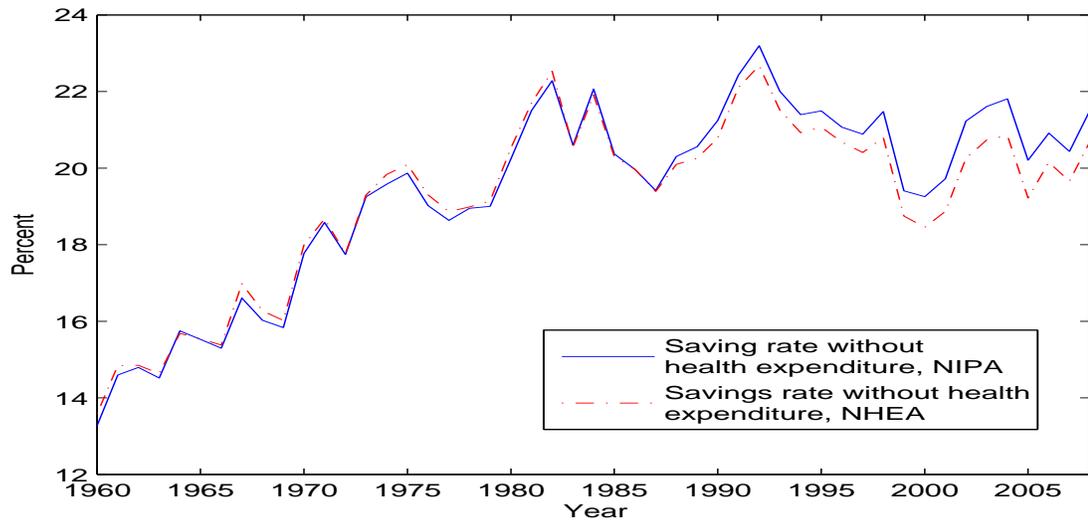


FIGURE 3.2: Household Saving Rate Without Health Expenditure from NIPA and NHEA

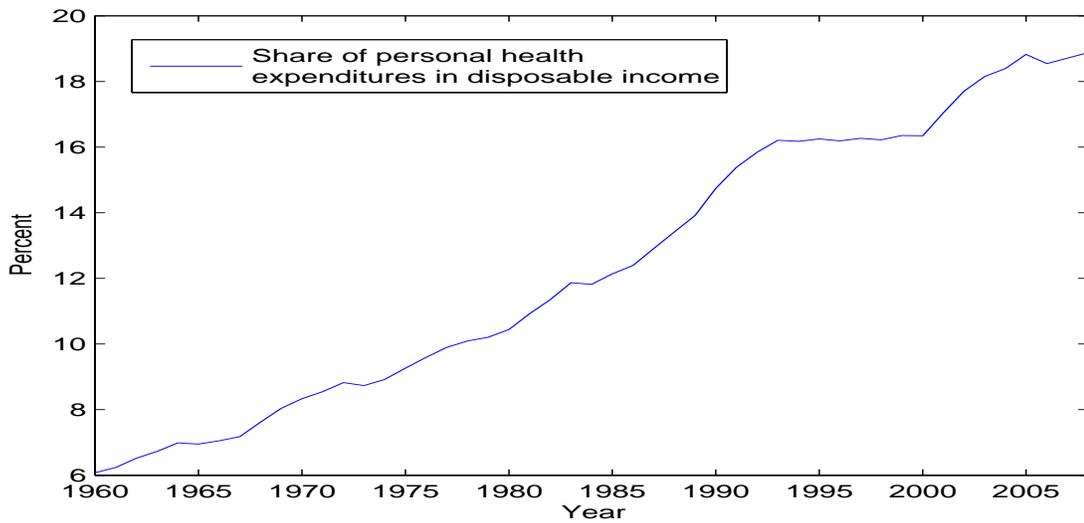


FIGURE 3.3: Total Health Expenditure as a Percentage of Disposable Income, NIPA

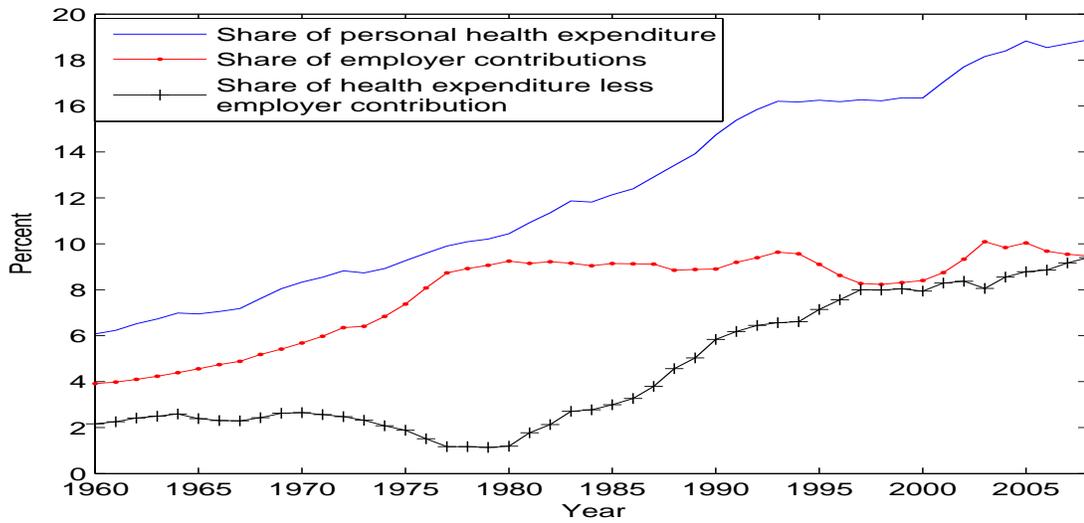


FIGURE 3.4: Health Expenditure and Employers' Contributions as a Percentage of Income, NIPA

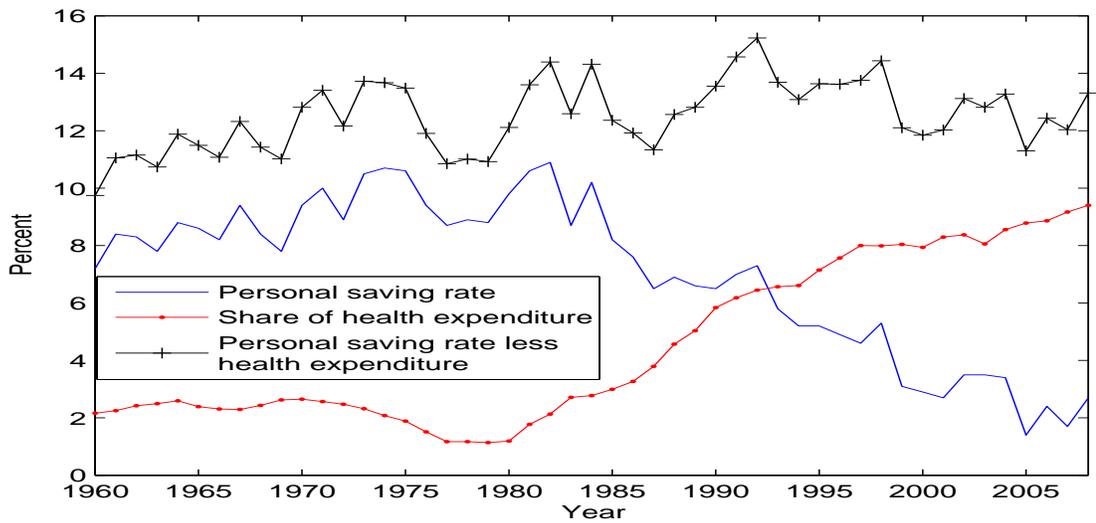


FIGURE 3.5: Saving Rates and Health Expenditure Net of Employers' Contributions, NIPA

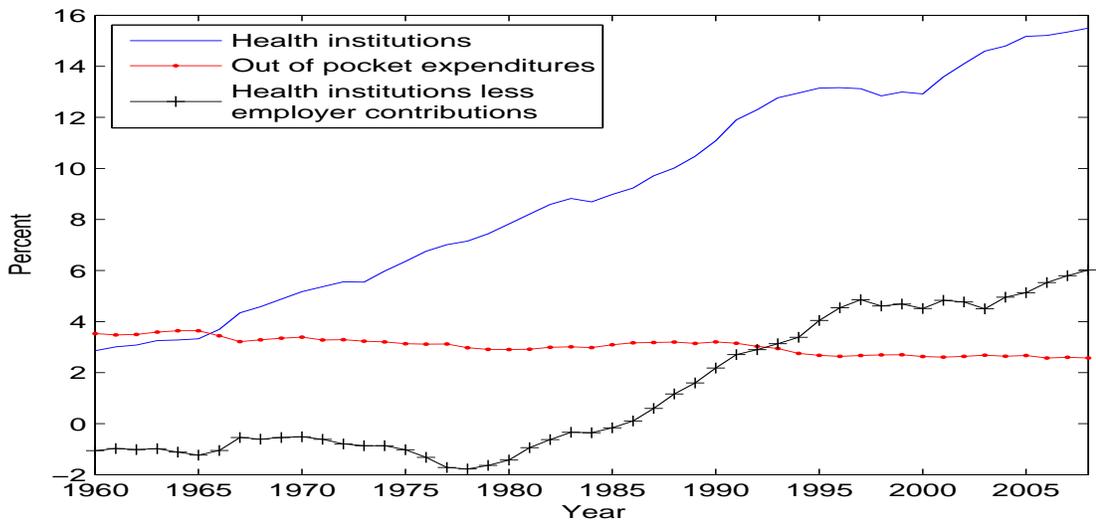


FIGURE 3.6: Different Measures of Health Expenditure as a Percentage of Income, NHEA

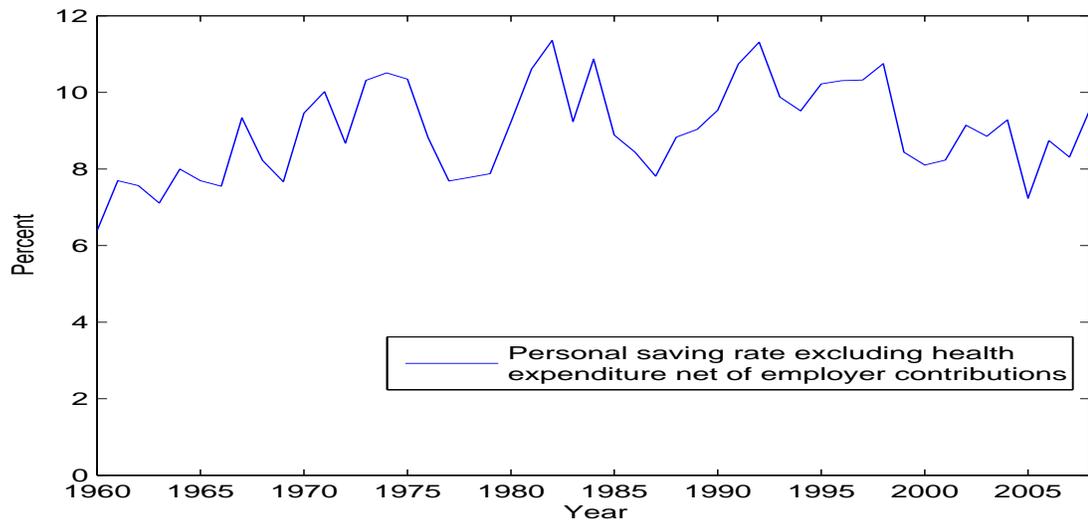


FIGURE 3.7: Saving Rate without Net Health Expenditure by Institutions

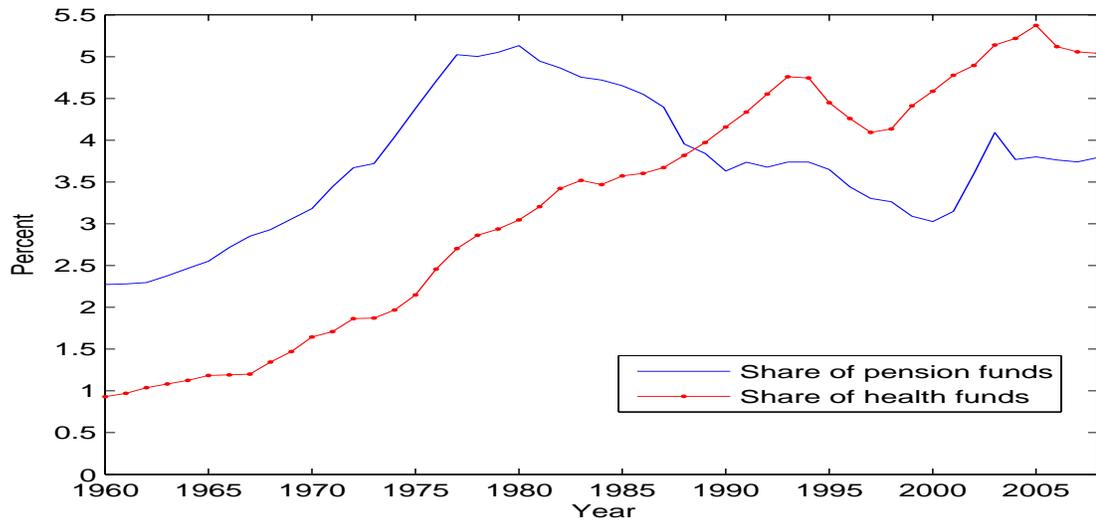


FIGURE 3.8: Employers' Contributions to Pension and Health Plans as a Percentage of Income

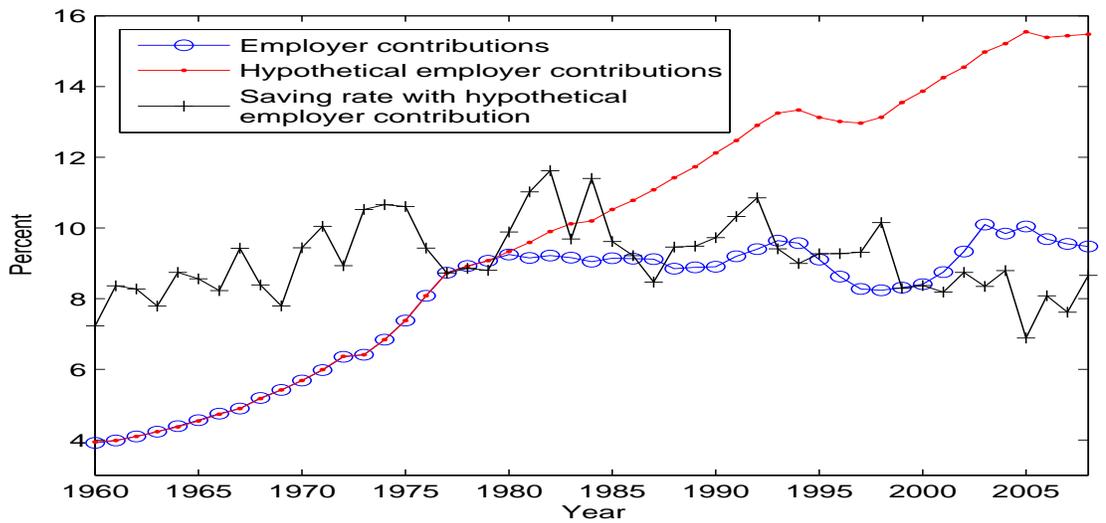


FIGURE 3.9: Saving Rate with Projected Pension Contributions

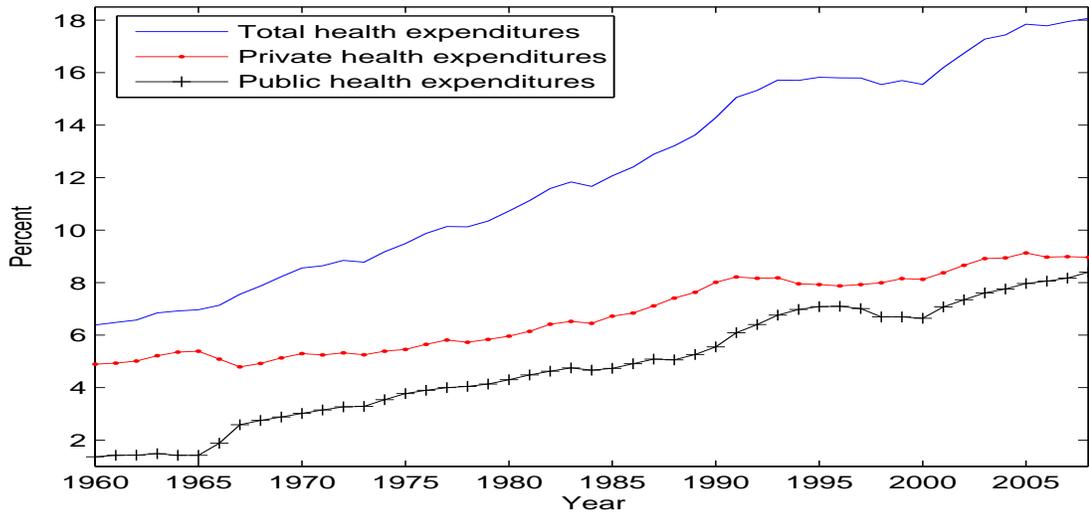


FIGURE 3.10: Components of Health Expenditure as Percentage of Disposable Income

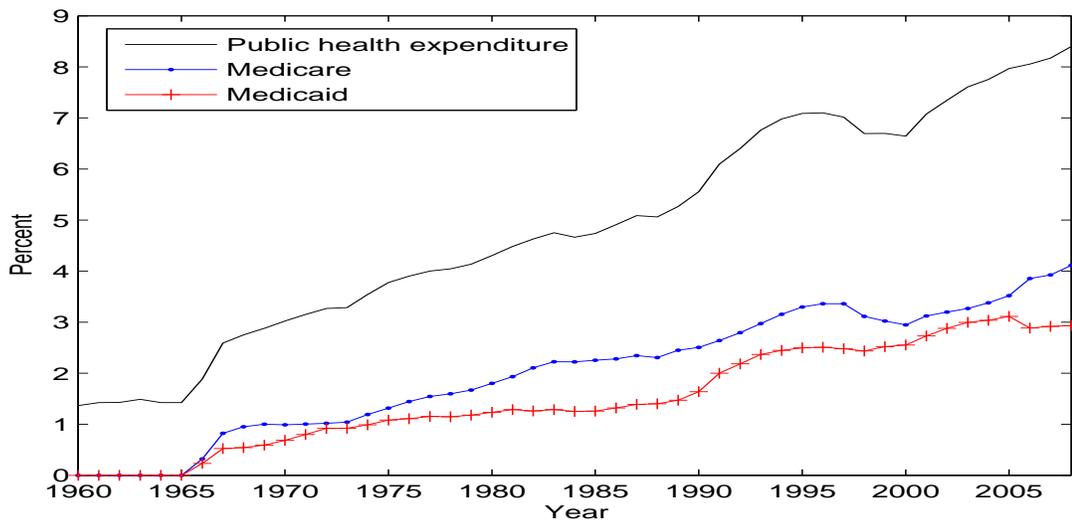


FIGURE 3.11: Components of Public Health Expenditures as Percentage of Disposable Income

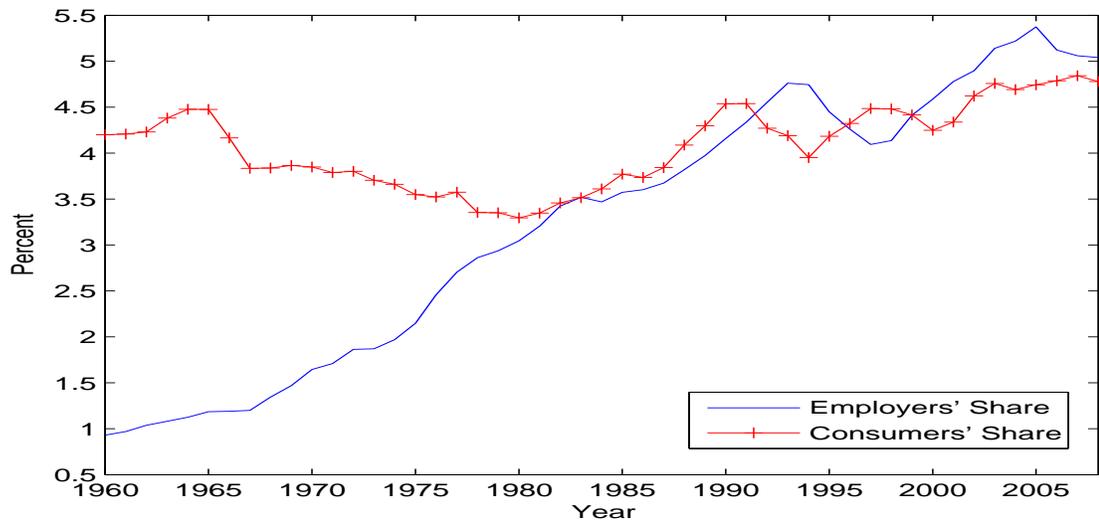


FIGURE 3.12: Contributions by Consumers and Employers to Health Expenditure as Percentage of Disposable Income

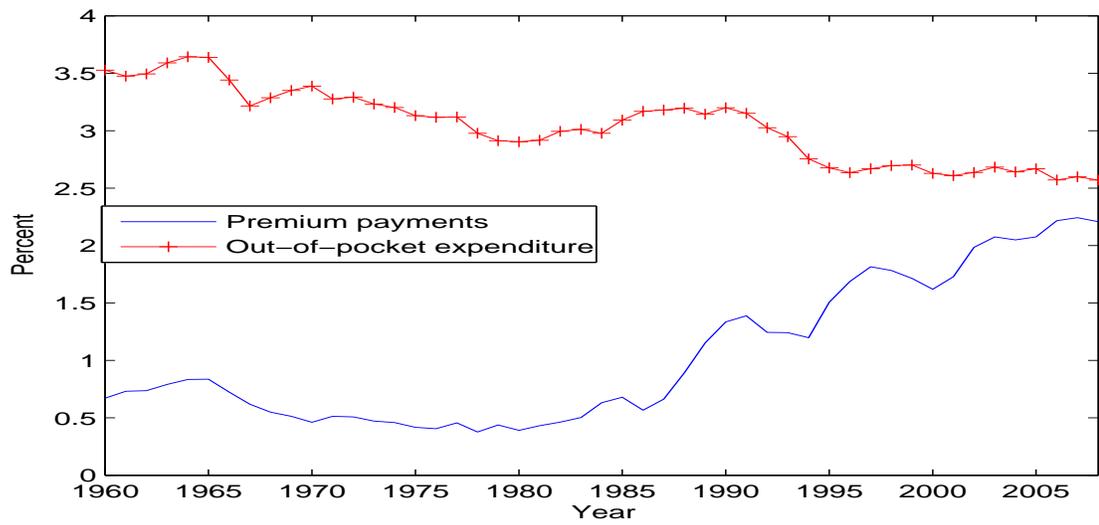


FIGURE 3.13: Decomposing Consumer Health Expenditure as Percentage of Disposable Income

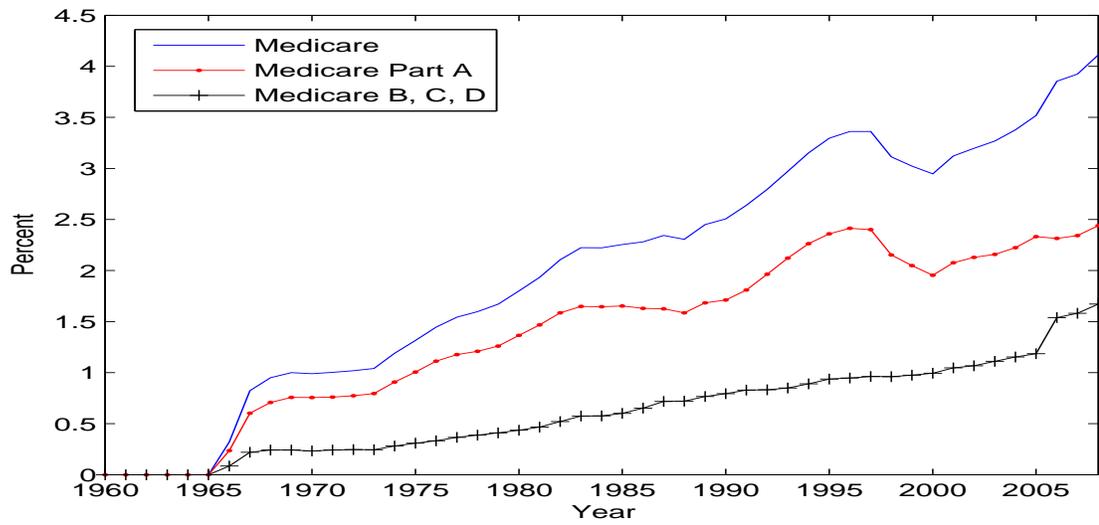


FIGURE 3.14: Medicare Components as Percentage of Disposable Income

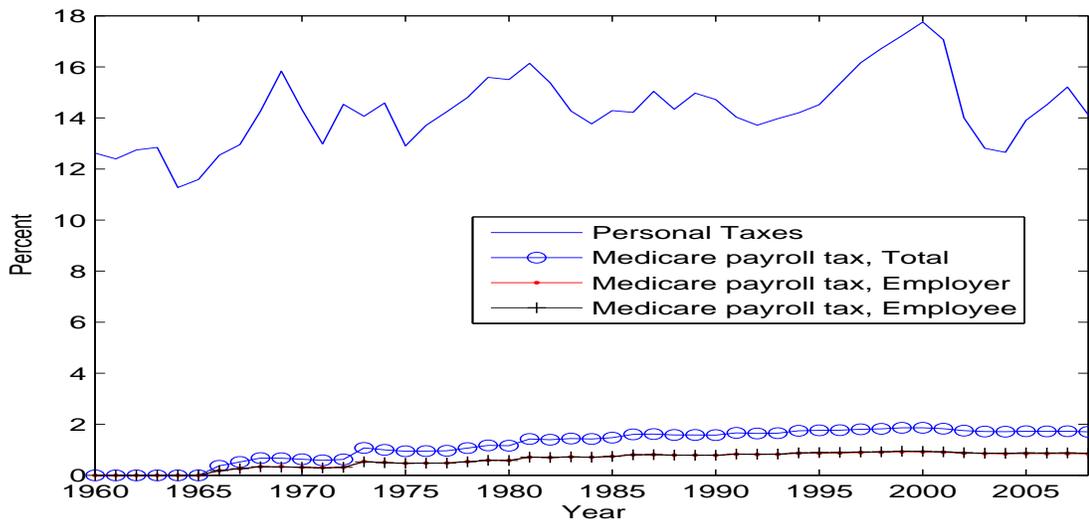


FIGURE 3.15: Personal Taxes and Medicare Payroll Tax as Percentage of Disposable Income

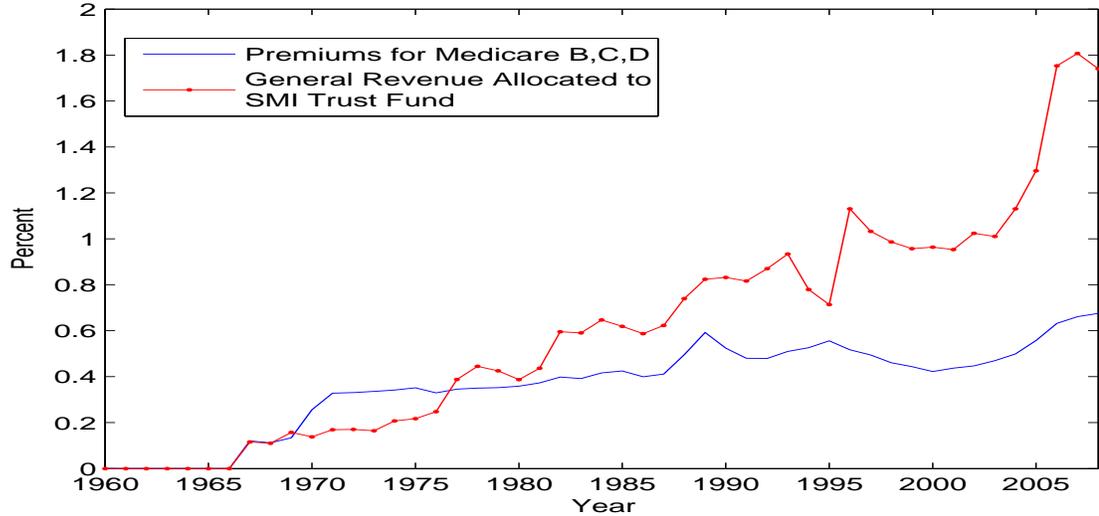


FIGURE 3.16: Revenue Allocated to SMI Trust Funds and Premiums as Percentage of Disposable Income

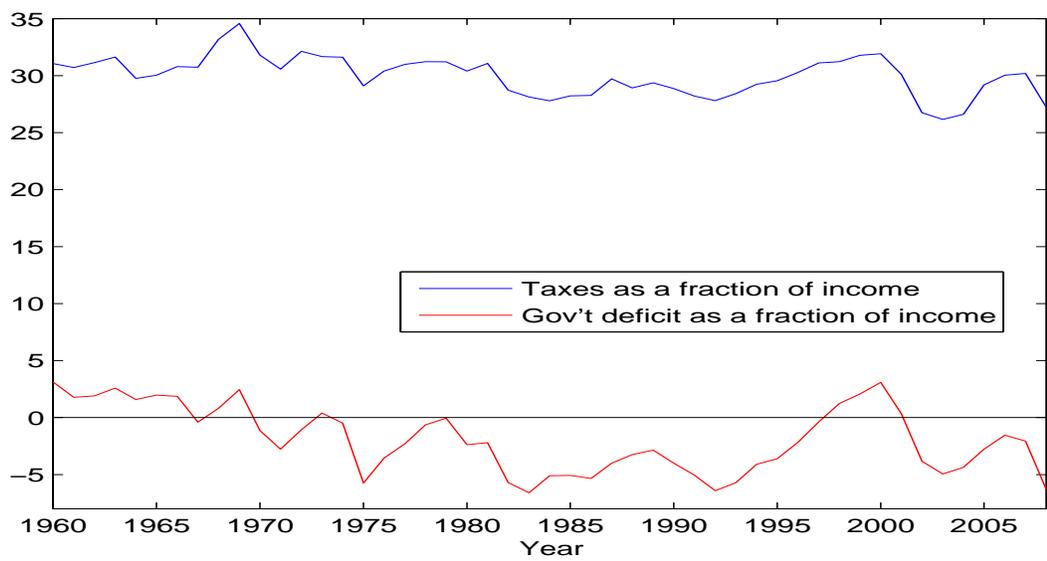


FIGURE 3.17: Total Taxes and Budget Deficits as Percentage of Disposable Income

# Appendix A

## Appendix to Chapter 1

To prove Propositions (1)-(6), I adapt the proofs from Rampini and Viswanathan (2010a), to the case of risk averse entrepreneurs. Define  $x$  as the set of choice variables  $x \equiv \{d, k', b(s'), w(s')\}$ , where  $x \in \mathbb{R}_+^{2+S} \times R^S$ , and the constraint set  $\Gamma(w, s)$  is given by the state variables such that (2.6) - (2.8) is satisfied. Lemma 10 shows that the choice set is convex

**Lemma 10.** *(i)  $\Gamma(w, s)$  is convex, given  $(w, s)$ , and convex in  $w$  and monotone in the sense that  $w \leq \hat{w}$  implies  $\Gamma(w, s) \subseteq \Gamma(\hat{w}, s)$ .*

*Proof of Lemma 10.* Suppose  $x, \hat{x} \in \Gamma(w, s)$ . For  $\phi \in (0, 1)$ , let  $x_\phi \equiv \phi x + (1 - \phi)\hat{x}$ . Then  $x_\phi \in \Gamma(w, s)$  since equations (2.6), (2.8) are linear, and from (2.7) as  $f$  is concave, we have  $f(k'_\phi) \geq \phi f(k') + (1 - \phi)f(\hat{k}')$ .

Let  $x \in \Gamma(w, s)$  and  $\hat{x} \in \Gamma(\hat{w}, s)$ . For  $\phi \in (0, 1)$ , let  $x_\phi \equiv \phi x + (1 - \phi)\hat{x}$ . Since (2.7), (2.8) does not include  $w$ , and  $\hat{w}$ , and  $\Gamma(w, s)$  is convex given  $w$ ,  $x_\phi$

satisfies these equations. Moreover, since  $x$  and  $\hat{x}$  satisfy equation (2.6) at  $w$ , and  $\hat{w}$ , respectively, and equation (2.6) is linear in  $x$  and  $w$ ,  $x_\phi$  satisfies the equation at  $w_\phi$ . Thus,  $x_\phi \in \Gamma(\phi w + (1 - \phi)\hat{w}, s)$ . In this sense,  $\Gamma(w, s)$  is convex in  $w$ .

If  $w < \hat{w}$ , then  $\Gamma(w, s) \subseteq \Gamma(\hat{w}, s)$  □

*Proof of Proposition 1. Part (i).* Define the operator  $T$  as follows:

$$(Tg)(w, s) = \max_{x \in \Gamma(w, s)} u(d) + \beta \sum_{s' \in S} \pi(s, s') g(w(s'), s')$$

where  $x$  and  $\Gamma(w, s)$  is defined above. To show that the problem (2.5) has a unique solution  $V$ , it is enough to show that the operator  $T$  satisfies Blackwell's sufficient conditions for a contraction.

Suppose  $g(w, s) \geq f(w, s)$ , for all  $(w, s) \in \mathbf{R}_+ \times S$ . Then, for any  $x \in \Gamma(w, s)$

$$(Tg)(w, s) \geq u(d) + \beta \sum_{s' \in S} \pi(s, s') g(w(s'), s') \geq u(d) + \beta \sum_{s' \in S} \pi(s, s') f(w(s'), s').$$

Thus,

$$(Tg)(w, s) \geq \max_{x \in \Gamma(w, s)} u(d) + \beta \sum_{s' \in S} \pi(s, s') f(w(s'), s') = (Tf)(w, s)$$

for all  $(w, s) \in \mathbf{R}_+ \times S$ . Thus,  $T$  satisfies monotonicity.

Next, we show that  $T$  satisfied discounting

$$T(f + a)(w, s) \leq \max_{x \in \Gamma(w, s)} u(d) + \beta \sum_{s' \in S} \pi(s, s') (f + a)(w(s'), s') = (Tf)(w, s) + \beta a$$

Thus  $T$  is a contraction and by the contraction mapping theorem, has a unique fixed point  $V$ .

Part **(ii)**. Let  $x_0 \in \Gamma(w, s)$  and  $\hat{x}_0 \in \Gamma(\hat{w}, s)$  attain  $(Tf)(w, s)$  and  $(Tf)(\hat{w}, s)$ , respectively. Suppose  $f$  is increasing in  $w$  and suppose  $w \leq \hat{w}$ . Then,

$$\begin{aligned} (Tf)(\hat{w}, s) &= u(\hat{d}_0) + \beta \sum_{s' \in S} \pi(s, s') f(\hat{w}_0(s'), s') \geq u(d) + \beta \sum_{s' \in S} \pi(s, s') f(w(s'), s') \\ &\geq \max_{x \in \Gamma(w, s)} u(d) + \beta \sum_{s' \in S} \pi(s, s') f(w(s'), s') = (Tf)(w, s) \end{aligned}$$

Hence,  $Tf$  is increasing in  $w$ . Moreover, suppose  $w < \hat{w}$ , then

$$\begin{aligned} (Tf)(\hat{w}, s) &\geq u((\hat{w} - w) + d_0) + \beta \sum_{s' \in S} \pi(s, s') f(w_0(s'), s') \\ &\geq u(d_0) + \beta \sum_{s' \in S} \pi(s, s') f(w_0(s'), s') > (Tf)(w, s) \end{aligned}$$

implying that  $Tf$  is strictly increasing. Hence,  $T$  maps increasing functions into strictly increasing functions, which implies that  $V$  is strictly increasing.

Suppose  $f$  is concave. Then, for  $\phi \in (0, 1)$ , let  $x_{0,\phi} \equiv \phi x_0 + (1 - \phi)\hat{x}_0$  and  $w_\phi \equiv \phi w + (1 - \phi)\hat{w}$ , we have

$$\begin{aligned} (Tf)(w, s) &\geq u(d_{0,\phi}) + \beta \sum_{s' \in S} \pi(s, s') f(w_{0,\phi}(s'), s') \\ &> \phi u(d_0) + (1 - \phi)u(\hat{d}_0) + \beta \sum_{s' \in S} \pi(s, s') f(w_{0,\phi}(s'), s') \\ &= \phi(Tf)(w, s) + (1 - \phi)(Tf)(\hat{w}, s) \end{aligned}$$

Thus, the concavity of  $u$  ensures that  $Tf$  is strictly concave, and  $T$  maps concave functions into strictly concave functions, which implies that  $V$  is strictly concave. Since  $V$  is increasing and strictly concave in  $w$ , it must be continuous in  $w$ .

Part **(iii)**. Let  $S = s_1, \dots, s_n$ , with  $s_{i-1} < s_i$ , for all  $i = 2, \dots, n$  and  $N = 1, \dots, n$ . Define the step function on the unit interval  $b : [0, 1] \rightarrow \mathbf{R}$  as  $b(\nu) =$

$\sum_{i=1}^n b(s'_i) \mathbf{1}_{B_i}(\nu)$ ,  $\forall \nu \in [0, 1]$ , where  $\mathbf{1}$  is the indicator function,  $B_1 = [0, \pi(s, s'_1)]$ , and

$$B_i = \left[ \sum_{j=1}^{i-1} \pi(s, s'_j), \sum_{j=1}^i \pi(s, s'_j) \right], \quad i = 2, \dots, n.$$

For  $\hat{s}$ , define  $\hat{B}_i$ ,  $\forall i \in N$ , analogously. Let  $B_{ij} = B_i \cap \hat{B}_j$ ,  $\forall i, j \in N$ , of which at most  $2n - 1$  are non-empty. Then, we can define the step function  $\hat{b} : [0, 1] \rightarrow \mathbf{R}$  as

$$\hat{b}(\nu) = \sum_{j=1}^n \sum_{i=1}^n b(s'_i) \mathbf{1}_{B_{ij}}(\nu), \quad \nu \in [0, 1]$$

We can then define the stochastic debt policy for  $\hat{B}_j$ ,  $\forall j \in N$ , with positive Lebesgue measure ( $\lambda \hat{B}_j > 0$ ), as  $\hat{b}(s'_i | s'_j) = b(s'_i)$  with conditional probability  $\chi(s'_i | s'_j) = \lambda(B_{ij}) / \lambda(\hat{B}_j)$ .

Moreover, this implies a stochastic net worth

$$\begin{aligned} \hat{w}(s'_i | s'_j) &= A(s'_j) f(k') + k'(1 - \delta) - R \hat{b}(s'_i | s'_j) \\ &\geq A(s'_i) f(k') + k'(1 - \delta) - R b(s'_i) = w(s'_i), \quad a.e. \end{aligned}$$

with strict inequality when  $i < j$ , since  $\lambda(B_{ij}) = 0$ , whenever  $i > j$ . Moreover,  $\hat{w}(s'_i | s'_j) > w(s'_i)$  with positive probability given our assumption stated in the Proposition.

Now suppose  $\hat{s} > s$  and  $f(w, \hat{s}) \geq f(w, s)$ ,  $\forall w \in \mathbf{R}_+$ . Let  $x_0$  attains  $(Tf)(w, s)$ .

Then

$$\begin{aligned} (Tf)(w, \hat{s}) &\geq u(d_0) + \beta \sum_{\hat{s}' \in S} \pi(\hat{s}, \hat{s}') \sum_{s' \in S} \chi(s' | \hat{s}'), \hat{s}') \\ &> u(d_0) + \beta \sum_{s' \in S} \pi(s, s') f(w_0(s'), s') = (Tf)(w, s) \end{aligned}$$

Thus,  $T$  maps increasing functions into strictly increasing functions, implying that  $V$  is strictly increasing in  $s$ .  $\square$

*Proof of Proposition 2.* We now show that  $x_0$  that attains  $V(w, s)$  is unique. To see this, we first show that  $w_0(s')$  is unique  $\forall s' \in S$ . Suppose that there exist  $\tilde{x}_0$  with  $\tilde{w}_0(s') \neq w_0(s')$  for some  $s' \in S$  that also attains  $V(w, s)$ . Then a convex combination  $x_{0,\phi}$  is feasible and attains a strictly higher value due to strict concavity of  $V(w, s)$ , a contradiction. Thus  $x_{0,\phi}$  is unique in terms of  $w_0(s')$ ,  $\forall w$  and  $s$ .

To see that the choice of the optimal capital stock  $k'_0$  is unique, suppose that  $x_0$  and  $\tilde{x}_0$  both attain  $V(w, s)$ , but  $k'_0 \neq \tilde{k}'_0$ . Since  $\Gamma(w, s)$  is convex, by taking the convex combination of  $x_0$  and  $\tilde{x}_0$ , note that

$$\begin{aligned} A(s')f(k'_{0,\phi}) + k_{0,\phi}(1 - \delta) &> \phi[A(s')f(k'_{0,\phi}) + k_{0,\phi}(1 - \delta)] \\ &+ (1 - \phi)[A(s')f(\tilde{k}'_{0,\phi}) + \tilde{k}_{0,\phi}(1 - \delta)] \end{aligned}$$

However this implies that at  $x_{0,\phi}$ , (2.7) is slack, and hence there exists a feasible choice that attains a strictly higher value, a contradiction. Thus  $x_0(w, s)$  is unique in terms of  $k'_0$ , for all  $w$  and  $s$ .

Given  $k'_0$  and  $w'_0(s')$ , for all  $s' \in S$ ,  $b_0(s')$  is uniquely determined by (2.7), and the payout policy,  $d_0$ , by (2.6).  $\square$

*Proof of Proposition 3. Part (i):* Using (2.6)

$$\begin{aligned} w &= d + k' - \sum_{s' \in S} \pi(s, s')b(s, s') > k' - \sum_{s' \in S} \pi(s, s')b(s, s') \\ &> k' - \theta k'(1 - \delta) = (1 - \theta)(1 - \delta)k' \end{aligned}$$

where, for the first inequality we use that  $\lim_{d \rightarrow 0} u(d) = \infty$ , which implies that  $d > 0$ , and the second inequality uses the collateral constraint (2.8). Notice however, that the above results implies that as  $w \rightarrow 0$ , investment  $k' \rightarrow 0$ . Now using (2.10) and substituting out  $\mu(s')$  from (2.11)

$$1 \geq \beta \sum_{s' \in S} \pi(s, s') \frac{\lambda(s')}{\lambda} \left( \frac{A(s')f'(k') + (1 - \delta)(1 - \theta)}{1 - \theta(1 - \delta)/R} \right)$$

As  $k \rightarrow 0$  implies that  $f'(k') \rightarrow \infty$ , but from the above equation then it must be that  $\lambda(s')/\lambda \rightarrow 0$ . But then using the collateral constraint (2.8):  $\mu(s')\lambda(s')/\lambda \rightarrow (\beta R)^{-1}$ , resulting that  $\mu(s') > 0 \quad \forall s' \in S$ . Thus, by continuity,  $\exists \underline{w} > 0$ , such that  $\forall w < \underline{w} : \mu(s') > 0 \quad \forall s' \in S$ .

Part **(ii)**: If  $w(s') \leq w(s'_+)$ , then  $\lambda(s') \geq \lambda(s'_+)$  by concavity. Moreover, from (2.11),  $\lambda(s')(1 + \mu(s')) = \lambda(s'_+)(1 + \mu(s'_+))$ , thus  $\mu(s') \leq \mu(s'_+)$ . Suppose instead that  $w(s') > w(s'_+)$ . Then, from (2.7),  $\mu(s') = 0$ , but then  $\lambda(s') = \lambda(s'_+)(1 + \mu(s'_+))$ , resulting that  $\lambda(s') \geq \lambda(s'_+)$ . But this implies that  $w(s') \leq w(s'_+)$ , which is a contradiction.

Part **(iii)**: From part **(ii)**, it is enough to show that  $\mu(\bar{s}') = 0$  if  $w > \bar{w} > 0$ , where  $\bar{s}' \geq s' \quad \forall s' \in S$ . Since the marginal product of capital in states when the collateral constraint binds is  $A(s')f'(k') + (1 - \delta)(1 - \theta)$ , while the marginal product of capital in states when the collateral constraint doesn't bind is  $A(s')f'(k') + (1 - \delta)$ , and noting that the opportunity cost of investment is  $R$  it has to be that  $\exists \bar{k}$  such that  $k' < \bar{k} \quad \forall w > 0$ .

Assume that  $w$  is high enough, in the sense that  $\bar{k}'$  is the optimally chosen level of investment. I prove the statement by contradiction. First, assume that  $w(\bar{s}')$  is

chosen optimally such that  $\mu(\bar{s}') = 0$ . But this implies that  $w(\bar{s}') = A(\bar{s}')f(\bar{k}') + \bar{k}'(1 - \delta) - Rb(\bar{s}')$ . Then (2.11) implies that  $\lambda = R\beta\lambda(\bar{s}')$ .

Now take  $w_1(\bar{s}')$  such that  $\mu_1(\bar{s}') > 0$ . Then

$$\begin{aligned} w_1(\bar{s}') &= A(\bar{s}')f(\bar{k}') + \bar{k}'(1 - \delta)(1 - \theta) \\ &< A(\bar{s}')f(\bar{k}') + \bar{k}'(1 - \delta) - Rb(\bar{s}') = w(\bar{s}') \end{aligned}$$

Thus  $w(\bar{s}') > w_1(\bar{s}')$ . However, from (2.11),  $\lambda = R\beta\lambda_1(\bar{s}')(1 + \mu_1(\bar{s}'))$  and  $\lambda = R\beta\lambda(\bar{s}')$  implying that  $\lambda_1(\bar{s}') < \lambda(\bar{s}')$ , resulting that  $w_1(\bar{s}') > w(\bar{s}')$ . But this is a contradiction, and thus it cannot be that at the optimum  $\mu(\bar{s}') > 0$ . Then  $\exists \bar{w} > 0$  such that  $\mu(s') > 0 \forall s' \in S$ .  $\square$

*Proof of Proposition 4.* Part (i). Suppose that  $w < \bar{w}$ . If  $\mu(s') > 0 \forall s' \in S$ , then  $k' = (w - d)/(1 - \theta(1 - \delta)/R)$ . Take any  $w^+, w$  such that  $\mu(s') > 0 \forall s' \in S$ . Assuming that  $w^+ > w$ , implies that  $\lambda^+ < \lambda$ . Now proceed by assuming that  $k'^+ < k'$ . From (2.7)  $w^+(s') < w(s') \forall s' \in S$ . Then from (2.10)

$$\begin{aligned} \lambda \left( 1 - \frac{\theta(1 - \delta)}{R} \right) &= \beta \sum_{s' \in S} \pi(s, s') \lambda(s') (A(s')f'(k') + (1 - \delta)(1 - \theta)) \\ &< \beta \sum_{s' \in S} \pi(s, s') \lambda^+(s') (A(s')f'(k'^+) + (1 - \delta)(1 - \theta)) \\ &< \lambda^+ \left( 1 - \frac{\theta(1 - \delta)}{R} \right) \end{aligned}$$

However this implies that  $\lambda < \lambda^+$ , which is a contradiction. As a result  $k'^+ > k'$  whenever  $\mu(s') > 0 \forall s' \in S$ .

Suppose now that  $\mu(s') = 0$ , for some  $s' \in S$ . Then from (2.10) and (2.11) we get

$$\begin{aligned}
1 &= \beta \sum_{s' \in S} \pi(s') \frac{\lambda(s')}{\lambda} (A(s')f'(k') + (1 - \delta) + \theta\mu(s')(1 - \delta)) \\
&= \beta \sum_{s'|\mu(s')>0} \pi(s') \frac{\lambda(s')}{\lambda} (A(s')f'(k') + (1 - \delta)(1 - \theta)) + \sum_{s'|\mu(s')>0} \pi(s')\theta(1 - \delta)/R \\
&\quad + \beta \sum_{s'|\mu(s')=0} \pi(s') \frac{\lambda(s')}{\lambda} (A(s')f'(k') + (1 - \delta))
\end{aligned}$$

Take as before  $w^+ > w$  and  $k'^+ < k'$ . Then  $f'(k'^+) \geq f'(k')$ . Moreover, for  $\{s'|\mu(s') = 0\}$ ,  $\lambda(s')/\lambda = (\beta R)^{-1}$ . Since  $\lambda^+ < \lambda$ , the above equation implies that  $\exists s'$  such that  $\lambda^+(s') < \lambda(s')$ . But  $k'^+ < k'$  implies that for  $\{s'|\mu(s') > 0\}$ ,  $w^+(s') < w(s')$  and hence  $\lambda^+(s') > \lambda(s')$ , a contradiction. Hence,  $k'$  and  $w(s')$  are strictly increasing in  $w$  for  $w \leq \bar{w}$ .

Part **(ii)**. If  $w > \bar{w}$  then  $\mu(s') = 0 \forall s' \in S$ . Combining (2.10) and (2.11) we get

$$R = \sum_{s' \in S} \pi(s') (A(s')f'(\bar{k}) + 1 - \delta)$$

Thus the maximum level of capital,  $\bar{k} = f'^{-1}((R - 1 + \delta)/\sum_{s' \in S} \pi(s')A(s'))$ .  $\square$

*Proof of Proposition 5.* Part **(i)** follows directly from Proposition 2 **(ii)**.

Part **(ii)**. To show that  $\exists w$  such that  $w(s') > w$ ,  $\forall s' \in S$  note that Proposition 2 implies that for  $w$  sufficiently small  $\mu(s') > 0$ ,  $\forall s' \in S$ . Since  $w(s') = A(s')f'(k') + k'(1 - \delta)(1 - \theta)$ , we have that:

$$\frac{\partial w(s')}{\partial w} = A(s')f'(k') \frac{\partial k'}{\partial w} + (1 - \delta)(1 - \theta) \frac{\partial k'}{\partial w} > 0$$

where we use that the production function is increasing in its argument,  $f' > 0$ , and Proposition 3 (i). To show that  $\exists w$  such that  $w(s') < w, \forall s' \in S$  note that Propositions 2 also implies that for  $w$  sufficiently large  $\mu(s') = 0, \forall s' \in S$ . From (2.11) then  $\lambda = \beta R \lambda(s') > \lambda(s'), \forall s' \in S$ , which implies that if  $w > \bar{w}$  then  $w(s') < w, \forall s' \in S$ .

Part (iii): By the theorem of maximum  $w(s')$  is continuous in  $w$ , and the intermediate value theorem and part (ii) hence imply the result.  $\square$

*Proof of Proposition 6.* Define the induced state space  $W = [w_l, w_u] \in \mathbf{R}$  with its Borel subsets  $\mathcal{W}$ . Take  $P$  to be the induced transition function on  $(W, \mathcal{W})$ , with the associated operator on bounded continuous functions  $T : B(W, \mathcal{W}) \rightarrow B(W, \mathcal{W})$  and the associated operator on probability measures  $T^* : P(W, \mathcal{W}) \rightarrow P(W, \mathcal{W})$ .

First, I show that  $P$  is monotone (that is, for any bounded, increasing function  $f$ , the function  $Tf$  defined by  $(Tf)(w) = \int f(w')P(w, dw'), \forall w$ , is also increasing) and has the Feller property (that is, for any bounded, continuous function  $f$ , the function  $Tf$  is also continuous). Take any bounded, increasing function  $f$ . Then  $(Tf)(w) = \sum_{s' \in S} \pi(s')f(w(s')(w))$  is increasing since  $w(s')(w)$  is increasing by part (ii) of Lemma 2. For any bounded, continuous function  $f$ ,  $(Tf)(w)$  is moreover continuous as  $w(s')(w)$  is continuous by the theorem of maximum.

Next, I show that  $\exists w^0 \in \mathcal{W}, \varepsilon > 0$ , and  $N \geq 1$ , such that  $P^N(w_l, [w^0, w_h]) \geq \varepsilon$  and  $P^N(w_h, [w_l, w^0]) \geq \varepsilon$ .

First, notice that all levels of net worth outside of  $[w_l, w_h]$  are transient. To see this, suppose that  $w < w_l$ . Then  $\mu(w, s') > \mu(w_l, s') = 1/(\beta R) - 1, \forall s' \in S$ . But then from (2.11) if  $\mu(w, s') > 1/(\beta R) - 1, \forall s' \in S$ , then  $w < w(s'), \forall s' \in S$ .

Similarly, assume that  $w > w_h$ . Then  $\mu(w, s') < 1/(\beta R) - 1$ ,  $\forall s' \in S$ , and from (2.11) if  $\mu(w, s') < 1/(\beta R) - 1$ ,  $\forall s' \in S$ , results that  $w > w(s')$ ,  $\forall s' \in S$ .

Since  $\mu(w_l, \bar{s}') > 1/(\beta R) - 1$ , take  $w^0$  such that  $w_0 = w(\bar{s}')(w_l)$ . Then  $P(w_l, [w^0, w_h]) \geq \pi(\bar{s}')$  and  $N_1 = 1$  and  $\exists \varepsilon_1 > 0$  such that  $\pi(\bar{s}') > \varepsilon_1 > 0$ .

Now given  $w^0 = w(\bar{s}')(w_l)$ , I show that  $\exists N_2 \geq 1$ , such that  $P^{N_2}(w_h, [w_l, w^0]) \geq \varepsilon_2$ . The idea is that given a sufficiently long sequence of the lowest productivity realization results in a net worth lower than  $w^0$ . Notice that  $\mu(w, \underline{s}') < 1/(\beta R) - 1$ ,  $\forall w > w_l$ . This implies that  $w(\underline{s}')(w) < w$ ,  $\forall w > w_l$ . Thus  $\exists N_2 < \infty$  and  $\varepsilon_2 > 0$  such that  $P^{N_2}(w_h, [w_l, w^0]) \geq \varepsilon_2$ , where  $\pi^{N_2}(\underline{s}') > \varepsilon_2$ .  $\square$

*Proof of Proposition 7.* First, we show that when in both regimes of uncertainty all collateral constraints bind,  $d_L > d_H$ . Assume that the the optimal choice variables are constant across regimes of uncertainty. That is, assume that  $w$  is low enough such that  $\mu_i(s') > 0$ , for all  $i \in \{L, H\}$  and  $s' \in S$ . Then  $d_L = d_H$ ,  $k'_L = k'_H$ ,  $\mu_L(s') = \mu_H(s')$ ,  $b_L(s') = b_H(s')$ , and  $V_L = V_H$ . But then from (2.7) and since (2.8) holds with equality:  $w_i = A_i(s')f(k'_i) + k'_i(1 - \delta)(1 - \theta)$ ,  $\forall i \in \{L, H\}$ . But Assumption 4 implies that  $Var(w_L(s')) < Var(w_H(s'))$ . And since  $V$  is concave,  $\sum_{s' \in S} \lambda_H(s') > \sum_{s' \in S} \lambda_L(s')$ , which implies that (2.10) cannot hold for  $A_H(s')$ , which is a contradiction. Thus, then  $d_L \neq d_H$ ,  $k_L \neq k_H$ , and  $V_L \neq V_H$  when  $w$  is low enough such that  $\mu_i(s') > 0$  for all  $i \in \{L, H\}$ ,  $s' \in S$ .

Assume instead that  $d_L < d_H$ , and  $\mu_i(s') > 0$  for all  $i \in \{L, H\}$ ,  $s' \in S$  still holds. Then  $k'_L > k'_H$ . But this implies that  $\lambda_L > \lambda_H$ . But  $k'_H < k'_L$  implies that  $A_H(s')f'(k'_H) + (1 - \delta)(1 - \theta)$  increases for all  $s' \in S$  and such,  $\lambda_H(s')$  must decrease for all  $s' \in S$ . To see that this cannot happen, take  $w$  such that  $0 < \mu(w, \underline{s}') < 1/(\beta R) -$

1, for all  $i \in \{L, H\}$ . But from (2.11) results that  $\lambda_H = \beta R \lambda_H(\underline{s}')(1 + \mu_H(\underline{s}'))$ . Now using the assumption  $\mu(w, \underline{s}') < 1/(\beta R) - 1$ , results that  $\lambda_H < \lambda_H(\underline{s}')$ ; that is  $w > w_H(\underline{s}')$ . But this is a contradiction, as from the concavity of the value function  $w_H(\underline{s}') < w$  implies that  $d_H(\underline{s}') < d_H$  and  $\lambda_H(\underline{s}') > \lambda_H$ . As a result it must be that  $d_L > d_H$  and  $k'_L < k'_H$  whenever  $\mu_i(s') > 0$  for all  $i \in \{L, H\}$ ,  $s' \in S$ .

Next, we show that when all collateral constraints are slack,  $d_L < d_H$ . Assume that  $w$  is high enough such that  $\mu_i(s') = 0, \forall i \in \{L, H\}$ ,  $s' \in S$ . First, note that optimal investment is given by  $R = \sum_{s' \in S} \pi(s') (A(s') f'(\bar{k}') + 1 - \delta)$ , thus the optimal capital stock does not change with uncertainty,  $\bar{k}'_L = \bar{k}'_H = \bar{k}'$ . Now assume that  $d_L = d_H$ . Note, that when  $\mu(s') = 0$ , for all  $s' \in S$ , then  $\lambda_i = \beta R \lambda_i(s')$  has to hold for all  $i \in \{L, H\}$ , and  $s' \in S$ . From (2.10) however since  $Var(A_L(s')) < Var(A_H(s'))$  it must be that, states with more extreme realizations of the shocks have to be weighed less. For example it has to be that  $\lambda_H(\underline{s}') < \lambda_L(\underline{s}')$ . But  $\lambda_L(s') = \lambda_H(s')$  for all  $s' \in S$ , thus a contradiction. As a result it cannot be that  $d_L = d_H$  when  $w$  is high enough such that  $\mu_i(s') = 0, \forall i \in \{L, H\}$ .

Now take  $d_L < d_H$ . This implies that  $\lambda_L > \lambda_H$ . From (2.11) it has to be that  $\lambda_H = \beta R \lambda_H(s')$  for all  $s' \in S$ . However this implies that  $\lambda_H < \lambda_H(s')$ , for all  $s' \in S$ . But then (2.10) cannot hold, contradiction. Thus it must be that  $d_L > d_H$  when  $w$  is such that  $\mu_i(s') = 0$  for all  $i \in \{L, H\}$  and  $s' \in S$ .

Finally, since  $V$  is strictly concave function it then must be that  $d_L > d_H$  for all  $w$ . □

First, I show that the collateral constraint binds against the lowest state when the marginal return on capital in that state is greater than the cost of financing.

Then, I prove Proposition 8.

**Lemma 11.**  $\mu(s') > 0$  if  $A(s')f'(k') + (1 - \delta) > R$ , for all  $s' \in S$ .

*Proof of Lemma 11.* Take  $\underline{s}' \in S$  such that  $\mu(\underline{s}') > 0$ . Now I show that  $\mu(\underline{s}') > 0$  if  $A(\underline{s}')f'(k') + (1 - \delta) > R$ . Suppose otherwise; assume that  $\mu(\underline{s}') = 0$  and  $A(\underline{s}')f'(k') + (1 - \delta) > R$ . Denote the optimal debt against state  $\underline{s}'$  as  $b(\underline{s}')$ . Then increase borrowing against state  $\underline{s}'$  by  $\varepsilon > 0$ , furthermore increase investment by  $\hat{k}' = k' + \pi(\underline{s}')\varepsilon$ , such that  $A(\underline{s}')f'(\hat{k}') + (1 - \delta) > R$  still holds and  $d$  stays the same. But then (2.6) still holds,  $\hat{w}(s') > w(s')$  for all  $s' \in S$ . Thus there exist a allocation that achieves higher utility and satisfies the budget constraints, contradiction. As such it cannot be that  $\mu(\underline{s}') = 0$  and  $A(\underline{s}')f'(k') + (1 - \delta) > R$ .  $\square$

*Proof of Proposition 8.* Part (i). From Proposition 7 we know that for all  $w$  such that if  $\mu_i(s') > 0$  for all  $i \in S$  then  $k'_L < k'_H$ . But then  $A_H(\underline{s}')f'(k'_H) + 1 - \delta < A_L(\underline{s}')f'(k'_L) + 1 - \delta$ . Moreover  $A_H(\underline{s}')f'(k'_H) + 1 - \delta$  is decreasing in  $k'_H$ . And since, from Proposition 4,  $k'_H$  is increasing in  $w_H$  then it must be that there is  $0 < \underline{w}_H < \bar{w}_H$  such that  $A_H(\underline{s}')f'(k'_H(\underline{w}_H)) + 1 - \delta = R$ ,  $A_L(\underline{s}')f'(k'_L(\underline{w}_H)) + 1 - \delta > R$ , and  $A_L(\underline{s}')f'(k'_L(\bar{w}_H)) + 1 - \delta = R$ . But then from Lemma 2, at  $\underline{w}_H$ ,  $\mu_H(\underline{w}_H, \underline{s}') = 0$ , whereas  $\mu_L(\underline{w}_H, \underline{s}') > 0$ . Thus  $\underline{w}_H < \underline{w}_L$ .

Part (ii). Now I show that if  $\mu_H(\bar{w}_H, \bar{s}') = 0$  and  $\mu_L(\bar{w}_L, \bar{s}') = 0$  then  $\bar{w}_H > \bar{w}_L$ . First take the case when the level of net worth is high enough that entrepreneurs can fully insure in both regimes of uncertainty. Then from (2.6) and since  $d_L(w) > d_H(w)$  results that  $\sum_{s' \in S} \pi(s')b_L(s') < \sum_{s' \in S} \pi(s')b_H(s')$ . Moreover, since  $Var(A_L(s')) < Var(A_H(s'))$  and from (2.11) results that  $b_H(\bar{s}') > b_L(\bar{s}')$ . But recall that from (2.8),

$\theta k'(1 - \delta) \geq Rb(s')$ . Thus, take  $\bar{w}_L$  such that at  $w = \bar{w}_L - \varepsilon$ , for  $\varepsilon > 0$  very small,  $\theta k'_L(w)(1 - \delta) = Rb_L(w, \bar{s}')$ , but  $\theta \bar{k}'_L(1 - \delta) > Rb_L(\bar{w}_L, \bar{s}')$ . But then entrepreneurs in the high uncertainty regime cannot afford to borrow  $b_H(\bar{w}_L, \bar{s}')$  such that  $\theta \bar{k}'_H(1 - \delta) > Rb_H(\bar{w}_L, \bar{s}')$ , as  $b_H(\bar{w}_L, \bar{s}') > b_L(\bar{w}_L, \bar{s}')$ . As such entrepreneurs can not be perfectly insured at  $\bar{w}_L$ . As a result at  $\bar{w}_L$ , when  $\mu_L(\bar{w}_L, \bar{s}') = 0$ ,  $\mu_H(\bar{w}_L, \bar{s}') > 0$ .  $\square$

*Proof of Proposition 9. Part (i).* See Proposition 7 above.

Part (ii) See Proposition 7 above.

Part (iii) Using the results from Proposition 8, we have that  $\mu_H(\underline{s}') < \mu_L(\underline{s}')$ , whenever  $\mu_L(\underline{s}') > 0$ . Thus  $\exists \omega > 0$  such that  $\mu_H(\underline{s}') = 0$  and  $\mu_L(\underline{s}') > 0$ . Moreover, since  $\mu_H(\bar{s}') > \mu_L(\bar{s}')$ , if  $\mu_H(\bar{s}') > 0$ , then there is  $\hat{\omega}_L < \hat{\omega}_H$ . This implies that at  $\hat{\omega}_L$ ,  $k'_H < k'_L$ . But then since  $\exists \underline{\omega} > 0$  such that  $k'_H > k'_L$ , and  $\bar{\omega}$  such that  $k'_H \leq k'_L$ , and that  $k'$  is monotone and continuous, it results that  $\exists \hat{\omega} > 0$  such that if  $\omega < \hat{\omega}$  then  $k'_H > k'_L$ , whereas if  $\omega > \hat{\omega}$  then  $k'_H < k'_L$ .  $\square$

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# Biography

Béla Személy was born in Targu Mures, Romania on July 10, 1980. He earned his B.A. in Finance at the Academy of Economic Studies, Bucharest, Romania in 2003. He continued his studies at Central European University, Budapest, Hungary, where he graduated with Honors and earned an M.A. in Economics. Béla completed his M.A. thesis under the supervision of Professor John S. Earle. Béla began his studies at Duke University in August 2005, where during his first year was awarded the Distinguished Economics Department Graduate Fellowship. He is planning on graduating from Duke University with a Ph.D. degree in Economics in May 2011.