3D Microwave Imaging through Full Wave Methods
for Heterogenous Media

by

Mengqing Yuan

Department of Electrical and Computer Engineering
Duke University

Date: ______________________

Approved:

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Qing H. Liu, Supervisor

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Rhett George

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William T. Joines

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Martin Brooke

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Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Electrical and Computer Engineering in the Graduate School of Duke University
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Abstract

In this thesis, a 3D microwave imaging method is developed for a microwave imaging system with an arbitrary background medium. In the previous study on the breast cancer detection of our research group, a full wave inverse method, the Diagonal Tensor approximation combined with Born Iterative Method (DTA-BIM), was proposed to reconstruct the electrical profile of the inversion domain in a homogenous background medium and a layered background medium. In order to evaluate the performance of the DTA-BIM method in a realistic microwave imaging system, an experimental prototype of an active 3D microwave imaging system with scanning antennas is constructed. For the objects immersed in a homogenous background medium or a layered background medium, the inversion results based on the experimental data show that the resolution of the DTA-BIM method can reach finely to a quarter of wavelength of the background medium, and the system’s signal-noise-ratio (SNR) requirement is 10 dB. However, the defects of this system make it difficult to be implemented in a realistic application. Thus, another active 3D microwave imaging system is proposed to overcome the problems of the previous system. The new system employs a fixed patch antenna array with electric switch to record the data. The antenna array introduces a non-canonical inhomogeneous background in the inversion system. The analytical Greens’ functions employed in the original DTA-BIM method become unavailable. Thus, a modified DTA-BIM method, which use the numerical Green’s functions combined with measured voltage, is proposed.
This modified DTA-BIM method can be used to the inversion in a non-canonical inhomogeneous background with the measured voltages (or $S_{21}$ parameters). In order to verify the performance of this proposed inversion method, we investigate a prototype 3D microwave imaging system with a fixed antenna array. The inversion results from the synthetic data show that this method works well with a fixed antenna array, and the resolution of reconstructed images can reach to a quarter wavelength even in the presence of a strongly inhomogeneous background medium and antenna couplings. In addition, a time-reversal method is introduced as a pre-processing step to reduce the region of interest (ROI) in our inversion. Furthermore, a Multi-Domain DTA-BIM method is proposed to fit the discontinued inversion regions. With these improvements, the size of the inversion domain and the computational cost can be significantly reduced, making the DTA-BIM method more feasible for rapid response applications.
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In recent years, object detection with electromagnetic (EM) waves has become a fast growing research topic due to the nondestructive property of the EM method. A number of EM inversion methods, for example, the Born-type iterative methods and contrast-source inversion [68]-[50], have been developed for microwave imaging technology in different applications, such as breast cancer diagnosis and screening, subsurface sensing and through-wall imaging [53]-[46].

Diverse inversion applications have different requirements for inversion results. Some applications need to find out targets’ shapes and positions only, for example, Ground-penetrating radar (GPR) and through-wall imaging. More sophisticated applications prefer to reconstruct the electrical profile of the inversion domain, such as, breast cancer detection. My research belongs to the second category. The electrical profile of the target region need to be reconstructed.

In general, an EM inversion method in the second category consists of a forward solver and an inverse solver. The forward solver is used to calculate the scattered fields at probes and gradient information. The inverse solver is used to reconstruct the electrical profile of the imaging domain. The performance of the inversion de-
pends on the performance of the forward solver and the inverse solver. For the for-
ward solver, in the last decades, various methods, such as the finite-element method
(FEM), method of moments (MOM) [30], and finite-difference time-domain (FDTD)
method have been employed as full-wave solution techniques. And the Born approx-
imation [5] has been proposed and implemented as an approximate method. But
these methods have their limitations, especially for objects embedded in a complex
background medium. Although the full-wave methods can provide accurate results,
they need to solve a large system matrix (FEM and MOM) or require large num-
ber of time steps. It is therefore usually too expensive for a realistic 3D biomedical
imaging system. For example, in order to obtain the scattered field in the MOM,
we need to solve the current distribution in the computation domain, which nor-
mally needs $O(N^3)$ CPU time ($N$ is the number of unknowns) if a direct matrix
inversion method is used. The Born approximation is a fast method to solve the
scattering without the inversion of a system matrix, but it is valid for weak scat-
tering only. The accuracy of the Born approximation decreases rapidly with the
increasing target contrast and size. Several improved Born approximation methods,
including the extended Born approximation (EBA) [64], [65], the quasi-linear (QL) or
quasi-analytical (QA) approximation [71]-[72] and the Diagonal Tensor Approxima-
tion (DTA) [61, 62] have been proposed to overcome the weak scattering limitations.
Normally in these methods, only $O(N^2)$ CPU time is needed to obtain the current
distribution in the computation domain. Furthermore, $O(N \log N)$ algorithms, for
example, Stabilized Bi-Conjugate Gradient Fast Fourier Transform (BCGS-FFT),
have been developed for homogeneous and layered background media [73]. The per-
formance of these improved approximations under higher contrasts has been verified
by [29]-[63] with synthetical data.

In a realistic EM inversion application, for example, breast cancer diagnosis and
screening, a fast approximate method, such as the Born types approximation: the
Figure 1.1: The experimental setup for the 3D MWI system of Semenov et al. (a) The system layout. (b) The geometrical configuration of a 2D inverse problem. ●: the transmitter positions in the object coordinate system, ○: the receiver positions for a given transmission direction. [57]

EBA, QL and DTA methods, may be favored due to the computational time requirement. In this research, we choose the DTA method as our forward solver.

1.1 Review of Existing Experimental 3D Microwave Imaging (MWI) Systems

For the applications which need to reconstruct the electrical profile of the target region, several experimental 3D MWI prototype systems [57], [59], [53], [54] has been proposed and fabricated in the past years.

Semenov et al. [57], [59] have developed two similar prototype experimental 3D MWI systems basing on 2D vector Born reconstruction method. They operate at 2.36 GHz and 0.9 GHz respectively. These systems consisted of a stationary cluster of 32 transmitters, a mechanically positioned receiver, a cylinder chamber filled with matching fluid, and a computer controlled data acquisition system. The accuracy of this data acquisition system could reach $-120$ dB, while the SNR was $30$ dB. The arrangement of antenna and the receiver scanning scheme were showed in Fig.
1.1. In these systems, the signal in the inversion volume was determined by the vertical polarized TM wave. The experiment data were very close to the TM wave from vertical magnetic dipole. Thus, 2D vector Born method could be applied to inversion. The receiver rotated around the axis of system by 180° and collected data (amplitude and phase). The receiver repeated this collection 48 times in different vertical levels. The total scanning time will take about 8 hours. After the data were obtained, 2D vector Born method was used to reconstruct stacking 2D images and build a 3D image subsequently. Results of these systems demonstrated an ellipsoid with complex electrical permittivity $\bar{\varepsilon} = 70 + i17$ of dimension about $5.5 \times 5.5 \times 6.5$ cm$^3$ with 2 semispherical holes inside distinguishable in the water ($\bar{\varepsilon} = 77.9 + i9.7$). The reconstructed target shape and the real part of the complex permittivity showed a reasonable agreement with the original target, but the imaginary part of the complex permittivity could not be reconstructed correctly.

Basing on [14], [2], [52], Meaney et al. built another prototype 3D MWI system operating at 0.3-1 GHz, as shown in Fig. 1.2 (a) and (c). The system employed monopole antennas to excite 2D wave, reconstructing 2D images by the Newton-Raphson method in conjunction with hybrid finite-element boundary-element method. The antenna array is shown in the Fig. 1.2 (b). Each antenna worked as transmitter and receiver, respectively. When a transmitter excited signals, other antennas were switched to the receiver mode sequentially. In order to eliminate the coupling between antennas, all non-active antennas were connected to a load through a switch. Similar to Semenov et al.’s system, 2D slices were built at different levels by moving the antenna array vertically. Subsequently, 3D images were built by stacking 2D images. By this approach, the data acquisition time had been significantly reduced to 10-15 minutes for scanning one breast, and the total inversion time was 7 – 20 minutes. The inversion results basing on the data from exams on 43 patients was promising, but the resolution of the image was coarse, in the range of 1 cm. Their
Figure 1.2: The experimental setup for the 3D MWI system of Meaney et al. (a) The system layout [52]. (b) The antenna array layout [52]. (c) Experimental system in clinic [54].

report is the first clinical experience on converting the MW signal into 3D breast images.

Although the above systems show impressive progress in MWI, all are not strictly 3D inversion systems since they employ 2D inversion methods only. Generally, the signal captured on the receiver is from all objects in whole domain due to the diffraction and refraction, but the 2D inversion method assumes the signal only comes from the objects in a plane, resulting in a not so accurate separated 2D inversion. In order to overcome these shortcomings, Liu et al. develop a 3D inversion method (DTA-BIM) [72] to reconstruct the object immersed in a homogenous or a layered
background medium. The performance of the DTA-BIM method with synthetic data has been verified [61, 62].

My PH.D work is to implement the DTA-BIM method in a realistic physical 3D microwave imaging system. Firstly, we build an experimental prototype active 3D MWI system to evaluate the performance of the DTA-BIM method and the signal noise ratio (SNR) requirement under different kinds of background media setup. Experimental results indicate these methods work well in a homogenous background or a layered background medium, and the resolution of inversion image can reach a quarter wavelength [7]-[9]. However, this system exposes some implementation limitations. One problem is the long data acquisition time makes it difficult to be implemented in the rapid-response applications. Another problem is the difficulty in antenna fabrication and maintenance. The third is that the linear voltage-field conversion is not strictly correct and introduces noise in the DTA method, compromising the inversion result. Therefore, a new active 3D microwave imaging system is proposed. This system employs a fixed patch antenna array combining with a electric switch to record the data. The data acquisition time can be reduced to second range and makes it more feasible for a rapid-response application. However, the antenna array influences the inversion system by introducing a non-canonical inhomogeneous background. Here, we define a homogeneous or layered medium background as a canonical background as its Green’s function can be found analytically. On the other hand, if the background is arbitrarily inhomogeneous (other than planar, spherical, or cylindrical layered medium), we define it a non-canonical background medium as its Green’s function cannot be found analytically. An example of such a very practical situation is an array of antenna elements with significant mutual couplings being used in a microwave imaging chamber. In addition, the linear voltage-field conversion does not work any more due to the complicated polarization of the near-field distribution on antenna. In order to implement DTA-BIM method in this system,
we propose a modified DTA-BIM scheme to extend the 3D EM inversion to a non-
canonical inhomogeneous background. With the usage of the numerical Green’s
function, the DTA-BIM method can work for arbitrary background medium. The
method also includes a field-voltage conversion to ensure that the DTA-BIM method
can work with measured voltage directly.

1.2 Contributions of this Dissertation

(1) An experimental active 3D microwave imaging system is constructed to eval-
uate the performance of the general DTA-BIM method and the SNR requirement
under different background in a realistic environment. In a homogenous background
medium or a layered background medium, the inversion based on the experimental
results show that the resolution of the DTA-BIM method can reach a quarter of
wavelength of the background with 10 dB SNR.

(2) In order to overcome the disadvantages exposed in our experimental active 3D
microwave imaging system, another active 3D microwave imaging system is proposed,
which employs a fixed patch antenna array combining with a electric switch to record
the data.

(3) In the proposed active 3D microwave imaging system, the whole inversion prob-
lem become a inversion in a non-canonical inhomogeneous background medium due
to the structure of antenna. In order to make the DTA-BIM method still workable
in this system, a modified DTA-BIM method using numerical Green’s functions is
proposed.

(4) To utilize the measured voltage directly in the modified DTA-BIM method, we
develop a method to convert the scattered field on the probe (the antenna) to the
measured wave port voltage.

(5) The inversion results through the modified DTA-BIM method shows that it works well with synthetic data.

(6) we introduce a time-reversal method to reduce the region of interest (ROI) in our inversion, which significantly reduces the size of inversion domain and the inversion time, resulting in a more feasible system for the applications that require rapid response.

1.3 Dissertation Outline

- In Chapter 2, we present the general DTA-BIM method to reconstruct the objects immersed in a homogenous or layered background medium.

- In Chapter 3, the design of an experimental active 3D microwave imaging system with moving antenna is discussed. The inversion results based on the experimental data and general DTA-BIM method are presented. The disadvantages of the system are discussed.

- In Chapter 4, the design of another experimental active 3D microwave imaging system with static antenna array is discussed. A comparison between the experimental data and the simulation data is shown.

- In Chapter 5, we present the modified DTA-BIM method. Approaches that taking advantage of the symmetry of the system to reduce the simulation cases to obtain the Green’s function is discussed. The field-voltage conversion method is presented too. The verification of all the methods employed in the modified DTA-BIM method
is demonstrated.

• In Chapter 6, the inversion results based on the modified DTA-BIM method and the synthetical data are given.

• In Chapter 7, we introduce a time reversal method to reduce the size of inversion domain. In order to cooperate with the reduced size inversion region, a Multi-Domain DTA-BIM method is proposed. The inversion results based on the Multi-Domain DTA-BIM method are presented.

• In Chapter 8, we present conclusions of the study and some proposed future work are discussed.
2.1 The Scattering Problem

In a homogenous or layered medium 3D problems shown in Fig. 2.1, the time-harmonic scattered electric field (with $e^{j\omega t}$ time convention) at an observer can be expressed by the volume equivalence principle.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2.1.png}
\caption{General scattering problem with 3D objects in a layered medium. Each layer has different complex dielectric constant. The transmitters and receivers are located in layer $p$ and $m$, respectively.}
\end{figure}
\[ E^{sct}(r) = -j\omega\tilde{\mu}_b \int_D G(r,r') \cdot J(r')dr' = \tilde{k}_b^2 \int_D G(r,r') \cdot \chi(r') E(r')dr' \] (2.1)

where \( \tilde{\mu}_b \) is the complex permeability of the background. \( J = j\omega(\tilde{\epsilon} - \tilde{\epsilon}_b)E \) is the induced current density in the object, \( \tilde{\epsilon} = \epsilon_0\epsilon_r - j\sigma/\omega \) is the object complex permittivity (\( \epsilon_r \) is the relative dielectric constant of object, \( \epsilon_0 \) is the permittivity of free space, \( \sigma \) is the conductivity of object), and \( \tilde{\epsilon}_b \) is the complex permittivity of the background. \( E \) is the total field in the computation domain, \( \chi(r') \) is a contrast value defined as \( \chi(r') = \tilde{\epsilon}(r')/\tilde{\epsilon}_b(r') - 1 \), and \( G(r,r') \) is a dyadic Green’s function from a source point at \( r' \) to the observer at \( r \) in a homogeneous or layered medium background. \( D \) is the computation domain, and \( \tilde{k}_b^2 = \omega^2\epsilon_b\mu_b \).

According to (2.1), to obtain the scattered field, the dyadic Green’s function \( G(r,r') \) and the total field \( E \) in the computation domain must be solved first. Generally, this is done with the MOM by solving the volume integral equation in the computation domain. However, this is an expensive method to use in terms of both memory and CPU time requirements. In our work, we use the DTA method to calculate \( E \).

2.2 The General Diagonal Tensor Approximation (DTA)

To reduce the computational costs from the MOM, here we apply the diagonal tensor approximation [61, 62] to solve the scattering problem for objects in a homogeneous or layered background medium. For this purpose, the total field \( E \) in equation (2.1) inside the computation domain can be expressed as

\[ E(r) = E^{inc}(r) + E^{sct}(r) = E^{inc}(r) + \tilde{k}_b^2 \int_D G(r,r') \cdot \chi(r') E(r')dr' \] (2.2)
where $\mathbf{E}^{inc}(\mathbf{r})$ is the incident field, which can be obtained analytically for a homogeneous or layered background medium.

Generally, for a weak cross-polarization system, the scattered field can be approximately related to the incident field by a diagonal scattering tensor $\mathbf{\Gamma}$. The scattered field can be expressed as

$$\mathbf{E}^{sct}(\mathbf{r}) \approx \mathbf{\Gamma}(\mathbf{r}, \mathbf{r}_s) \cdot \mathbf{E}^{inc}(\mathbf{r})$$  \hspace{1cm} (2.3)

where $\mathbf{r}_s$ is the source position, and $\mathbf{\Gamma}(\mathbf{r}, \mathbf{r}_s)$ is expressed as

$$\mathbf{\Gamma} \approx \begin{bmatrix} \gamma_x & 0 & 0 \\ 0 & \gamma_y & 0 \\ 0 & 0 & \gamma_z \end{bmatrix}$$  \hspace{1cm} (2.4)

Therefore, the $\mathbf{E}(\mathbf{r'})$ inside the computation domain can be approximated by

$$\mathbf{E}(\mathbf{r'}) \approx [\mathbf{I} + \mathbf{\Gamma}(\mathbf{r'}, \mathbf{r}_s)] \cdot \mathbf{E}^{inc}(\mathbf{r'})$$  \hspace{1cm} (2.5)

(2.1) can be re-written as

$$\mathbf{\Gamma}(\mathbf{r}, \mathbf{r}_s) \cdot \mathbf{E}^{inc}(\mathbf{r}) \approx \mathbf{\tilde{k}}^2 \int_{D} \mathbf{G}(\mathbf{r}, \mathbf{r'}) \cdot \chi(\mathbf{r'}) [\mathbf{I} + \mathbf{\Gamma}(\mathbf{r'}, \mathbf{r}_s)] \mathbf{E}^{inc}(\mathbf{r'}) d\mathbf{r'}$$  \hspace{1cm} (2.6)

In order to solve (2.6), we use the localized nonlinear approximation and expand $\mathbf{\Gamma}(\mathbf{r'}, \mathbf{r}_s)$ in terms of Taylor series of $\mathbf{r}$,

$$\mathbf{\Gamma}(\mathbf{r'}, \mathbf{r}_s) = \mathbf{\Gamma}(\mathbf{r}, \mathbf{r}_s) + \nabla \mathbf{\Gamma}(\mathbf{r}, \mathbf{r}_s) \cdot (\mathbf{r'} - \mathbf{r}) + \cdots$$  \hspace{1cm} (2.7)

Retaining the zeroth-order term in (2.7), we have

$$\mathbf{\Gamma}(\mathbf{r'}, \mathbf{r}_s) = \mathbf{\Gamma}(\mathbf{r}, \mathbf{r}_s)$$  \hspace{1cm} (2.8)
Therefore, (2.6) can be written as

\[ \Gamma(r, r_s) \cdot E^{inc}(r) \approx \kappa_0^2 \int_D G(r, r') \cdot \chi(r') \left[ I + \Gamma(r, r_s) \right] E^{inc}(r') \, dr' \]  

(2.9)

and the approximate tensor \( \Gamma(r, r_s) \) can be calculated by [61]

\[ \begin{bmatrix} \gamma_x & \gamma_y & \gamma_z \end{bmatrix}^T = \{ \text{diag}[E^{inc}(r')] - G^b(r') \}^{-1} \cdot E^b(r') \]  

(2.10)

where

\[ \text{diag}[E^{inc}(r')] = \begin{bmatrix} E_x^{inc}(r') & 0 & 0 \\ 0 & E_y^{inc}(r') & 0 \\ 0 & 0 & E_z^{inc}(r') \end{bmatrix} \]  

(2.11)

\[ E^b(r') = \kappa_0^2 \int_D G(r', r'') \cdot \chi(r'') E^{inc}(r'') \, dr'' \]  

(2.12)

\[ G^b(r') = \begin{bmatrix} g_{xx}(r') & g_{xy}(r') & g_{xz}(r') \\ g_{yx}(r') & g_{yy}(r') & g_{yz}(r') \\ g_{zx}(r') & g_{zy}(r') & g_{zz}(r') \end{bmatrix} \]  

(2.13)

In the above,

\[ g_{ij}(r') = \kappa_0^2 \int_D G_{ij}(r', r'') \cdot \chi(r'') E_j^{inc}(r'') \, dr'' \]  

(2.14)

where \( i, j = x, y, z \), \( G_{ij}(r', r'') \) is the \( ij \)th component of the dyadic Green’s function for an ideal electric dipole source in a homogeneous or layered medium background, which can be obtained analytically also. For convenience, in the later part of this dissertation, we refer \( G_{ij}(r', r'') \) as the Green’s function from cell to cell in the DTA method.
After the total field inside the object is approximated by the DTA method, a numerical integration of the dyadic Green’s function operating on the induced electric current density can be performed to arrive at the electric field at an observer. For simplicity, we will discretize (2.2) to obtain the electric field as

\[
E(r_i) = E^{inc}(r_i) + \tilde{k}_0^2 \sum_{j=1}^{N} G(r_i, r_j) \cdot \chi(r_j) E(r_j) \Delta V_j
\]  

(2.15)

where \(r_j\) is the \(j\)th cell center position, \(G(r_i, r_j)\) is the dyadic Green’s function from a current at \(j\)th cell to the observer at \(i\)th cell, and \(\Delta V_j\) is the volume of the \(j\)-th cell. For convenience, in the later part of this dissertation, we refer \(G(r_i, r_j)\) as the Green’s function from cell to receiver in the DTA method.

2.3 The Inverse Solver

Discretize (2.1), we get

\[
\tilde{k}_0^2 \Delta V \sum_{n=1}^{N} G(r_m, r_n) \cdot E(r_n) \chi(r_n) = E^{sct}_m
\]  

(2.16)

where \(\Delta V\) is the uniform cell volume, \(\chi(r) = \tilde{\epsilon}(r) / \tilde{\epsilon}_b(r) - 1\) is the complex contrast. \(m\) is the receiver index. \(r_m\) is the receiver position.

The matrix form of (2.16) is

\[
A \chi = E^{sct}
\]  

(2.17)

In (2.17), \(A\) and \(\chi\) are both unknown and \(A\) depends on \(\chi\), therefore, it is a nonlinear equation. In our work, we employ the Born Iterative Method (BIM) [42] to solve (2.17). For each iteration of the BIM, we have known \(\chi\) from the previous
iteration, and can solve $E$ by the DTA method. Thus $A = A_n$ is known, and (2.17) become a linear equation for $\chi$.

For this linear equation, we define a cost function $F(\chi)$ at the $(n+1)$-th iteration

$$F(\chi_{n+1}) = \frac{\|E^{sct} - A_n\chi_{n+1}\|^2}{\|E^{sct}\|^2} + \gamma^2 \frac{\|\chi_{n+1}\|^2}{\|\chi_n\|^2}$$

(2.18)

where $\gamma^2$ is a regularization parameter. According to our experience, we set $\gamma = 0.1$ in our inversion.

The minimization of the cost function leads to

$$F'(\chi_{n+1}) = 0 \implies \left( \frac{A_n^\dagger A_n}{\|E^{sct}\|^2} + \frac{\gamma^2}{\|\chi_n\|^2} I \right) \chi_{n+1} = \frac{A_n^\dagger E^{sct}}{\|E^{sct}\|^2}$$

(2.19)

Therefore, the contrast at the $(n + 1)$-th iteration ($\chi_{n+1}$) can be solved by the Conjugate Gradients method (CG). The iteration process continues until the contrast function converges.
An Experimental MWI Prototype System with Dipole Antennas

In order to evaluate the performance of the general DTA-BIM method with measured data, an experimental MWI prototype system employing moving dipole antennas and scanning data acquisition method is constructed. We evaluate following items:

(1) the signal level and the accuracy.
(2) the data calibration method.
(3) the highest tolerable noise level in the inversion method.
(4) the methods to eliminate the antennas coupling.
(5) the performance of the data acquisition method.
(6) the performance of the DTA-BIM method method with a homogenous background medium.
(7) the performance of the DTA-BIM method method with a layered background medium.
3.1 Antenna design

We employ electric dipole antennas in this experimental system. For the general DTA-BIM method, as shown in equations in chapter 2, we use the scattered field in the inversion. Actually, the measured data is the scattered voltage (or S parameters) on the antenna feeding structure. Therefore, in order to implement the DTA-BIM method with measured data, it is necessary to convert the measured voltage to the scattered field. Here, we employ a linear voltage-E field conversion method.

In this system, we use the characterized tap water ($\epsilon_r = 80$, $\sigma = 0.3$ S/m) as the background medium, and use $\lambda/2$ dipole antenna as the transmitter and receiver. The antenna design frequency is 1.7 GHz. Fig. 3.1 shows the structure of the antenna. The total length of two antenna arms is 15 mm. The matching balun (length 7.4 mm) on the front end is used for reducing the radiation from the outer conductor of coaxial cable.

3.2 System Setup for Objects in a Homogenous Medium

The first experiment is to evaluate the performance of the DTA-BIM method in a homogeneous background medium. We set up a rectangular chamber filled with tap water working as a infinite homogeneous background. Theoretically, in order
to make a limited size space working as a infinite homogeneous background, an absorbing material is required on the inner boundary of the space. In our system, the conductivity of the tap water can decay the microwave signal significantly in a short distance. Thus, a thick layer of water can be considered as an absorbing material. Our tests show, for a microwave signal with power +20 dBm at the antenna design frequency 1.7 GHz, the maximum penetration thickness is 15 cm. Therefore, if a receiving antenna has a distance more than 15 cm from the chamber boundary, no signal from the chamber boundary can be detected. It proves that the chamber with taped water perfectly match a homogeneous background medium. The system setup is shown in Fig. 3.2.

To avoid the coupling between antennas, only 2 antennas are utilized in the system. One is transmitter, another is receiver. The transmitter is mounted on a 3D positioner (0.01 mm accuracy), the receiver is mounted on the 2D positioner (2 um accuracy). The data acquisition system consists of a HP 8753E network analyzer, a desktop computer and two computer-controlled positioners. The transmitter travels to 9 or 15 positions according to different experiment setups. For each transmitter
3.2.1 Data Acquisition

In order to obtain the scattered field from the target, two sets of data are recorded. First, we measure the incident field $E^{inc}$ (without the targets in the inversion domain) by scanning the positions discussed in the previous section. Subsequently, the targets are placed into the inversion domain at a specified position. The total fields $E$ are measured with the same scanning pattern. The scattered field can be obtained by $E^{scat} = E - E^{inc}$. The data acquisition time for each sampling position, including the network analyzer sampling time and the receiver moving time, is 6 seconds. Therefore, for a case with 15 transmitter positions and 105 receiver sampling positions for each transmitter position, the measurement of recording two sets of data takes about 5 hours.
3.2.2 The Data Calibration Method

The long data acquisition time and the long-lasting movement of the antenna introduce noises during the measurement. These noises include the thermal noise shift of the vector network analyzer, the cable noise, the antenna positioning error, etc. We employ the scattered field in the inversion, which is the difference between the $E$ and $E^{inc}$. A weak noise in the $E^{inc}$ or $E$ could be a relative large error for the scattered field. Therefore, these noises may degrade the inversion result. It is necessary to calibrate the measured data before the inversion. We obtain these noises by measurement. For each data set, we do two extra measurements at the beginning and at the end of the antenna scanning respectively. These two measurements measure the field with the same transmitter and receiver positions. The difference between these two measurements is the summation of these noises in the data set.

3.3 Inversion Results for Objects in a Homogenous Background Medium

For all inversion cases with a homogenous background medium, the inversion domain is $8 \times 8 \times 8$ cm$^3$ and divided into $16 \times 16 \times 16$ uniform cells.

3.3.1 Case 1: Two Clay Balls in a Homogeneous Water Background

Fig. 3.4 shows the inversion results for two clay balls ($\epsilon_r = 5$, $\sigma = 0.01$ S/m, 5 mm in diameter) in the homogeneous water background. The distance between the opposite transmitters and receivers (variable d in Fig. 3.3) is 9.2 cm. The result shows two balls are located correctly, and the size is correct also. However, the value of permittivity can not be recovered properly. It is caused by the large contrast between the background and the objects, and the small size of the objects.

Fig. 3.5 shows the amplitude of incident and scattered fields at receiver positions for the two clay balls case. The average scattered field is $-75$ dB. The noise in the system is -85 dB in average, resulting a 10 dB SNR.
Figure 3.4: (a) The layout of two clay balls in the inversion domain. The distance between two balls is 3 cm. (b) Reconstructed relative permittivity. (c) Reconstructed conductivity.

The inversion time is eight hours on a computer with an Intel Q6600 CPU.

3.3.2 Case 2: One Clay Ball and One Metal Ball in a Homogeneous Water Background

Fig. 3.6 shows the result for the combination of one clay ball and one metal ball placed in the homogeneous water background. The positions of transmitters and receivers are the same as the two clay balls case. This experiment is utilized to test whether a weak scattered field can be detected from a strong scattered background. According to the inverted conductivity, the clay ball can be still detected. But the conductivity of metal ball can not be reconstructed correctly, it is caused by the induced current distribution on the metal ball is not uniform. Therefore, only the part of metal ball with strongest induced current can be reconstructed. The inversion
3.4 System Setup for Objects Placed in a Layered Background Medium

In this setup, we invert the targets buried in a layered medium to evaluate the performance of the DTA-BIM method for layered medium. The layout is similar to the homogenous background system except two plastic slabs are placed parallel in the water to form a five-layer medium. The system setup and the layout of five-layer medium are shown in Fig. 3.7. The thickness of plastic slabs is 2.2 mm, distance between two plastic slabs is 6.2 cm. Due to the high contrast of electric permittivity between plastic slabs and water, most of the incident field are reflected on the interface of plastic slabs and water. In order to keep the average scattered field at $-75$ dB, the distance between opposite transmitters and receivers is decreased to 6.2 cm. Multiple source excitations are produced by placing the dipole transmitting antenna at nine locations on the plane $z = 3.1$ cm with $x = ih$, $y = jh$, where $i, j = -1, 0, 1$, and $h = 3.5$ cm. Experimental data for $E_x$ are collected for each transmitter location by scanning the receiving antenna automatically at 99 locations on the plane of with $z = 3.1$ cm, where $y = id$ and $x = je$, with $i = -5, \cdots, 5$, $d = 1$.

![Figure 3.5: Incident and scattered field pattern at all receiver positions.](image-url)
cm, and \( j = -5, \ldots, 5 \), \( e = 1.25 \) cm.

In the DTA method, we employ the analytical layered medium Green’s function for an ideal electric dipole.

### 3.5 Inversion Results for Objects in a Layered Background Medium

For all inversion cases with a homogenous background medium, the inversion domain is \( 6 \times 6 \times 6 \) cm\(^3\) and divided into \( 16 \times 16 \times 16 \) uniform cells.

#### 3.5.1 Case 3: Three Dielectric Spheres in a Layered Background Medium

Fig. 3.8 shows the experiment setup and reconstructed result for three clay balls (1 cm in diameter) placed in two vertical plastic slabs (\( \epsilon_r = 2.4 \), \( \sigma = 0.01 \) S/m). The
horizontal distance between the top two balls is 3 cm, the vertical distance between two rows of balls is 2.5 cm.

The inversion result shows that the location and the size of three balls are properly reconstructed. Compared to the case of two 0.5 mm clay balls in the homogeneous water background, the reconstructed permittivity is more closer to the real value. It is due to the bigger sizes of target and the higher sampling density of inversion. The inversion time is eight hours and 30 minutes on a computer with an Intel Q6600 CPU.

3.5.2 Case 4: Two Metallic Spheres in a Layered Background Medium

In the above experiments, dielectric spheres with permittivity value smaller than the background have been imaged. To demonstrate the performance of the system and algorithms for objects having a higher complex permittivity value than the background, we show here an example for two metallic spheres in Fig. 3.9 (a) and (b). The setup is the same as in the previous experiment in Fig. 3.8, except that the two metallic spheres with 0.8 cm diameter are separated by 3 cm center to center in the Y direction of the $Z=0$ plane.

The reconstruction results for the permittivity and conductivity are shown in Fig. 3.9 (c)-(f). The results clearly demonstrate the presence of the two metallic objects either from the permittivity image or from the conductivity image. As expected, Fig. 3.9 also shows that the resolution in the $Z$ direction is lower than in the other two directions because the scattered field information is collected only on the $XY$ plane. The inversion time is similar to the case 3.

These experimental examples shows that the DTA-BIM method can also be applied to image objects in a layered medium.
3.6 Conclusion

We have developed a microwave tomographic imaging system prototype to evaluate the performance of the 3D microwave imaging with experimental data when objects are buried in a homogenous or multilayered medium. Such a system and data sets for 3D objects in a layered medium are not known to exist previously.

Our inversion results show the general DTA-BIM method works well with 10 dB SNR and the analytical Green’s function. The targets can be localized correctly and the image resolution can reach a quarter wavelength of the background medium, which is much higher than any other reported experimental MWI systems [54, 34, 10, 58, 57, 59].

However, some defects exist in the system. The long data acquisition time and the time-consuming inversion make the system infeasible for applications which require rapid response. Meanwhile, The total length of two antenna arms is 15 mm only. There are implementation difficulties with this tiny structure of the dipole antenna. First, the size of the dipole antenna is too small to fabricate identical antennas. Second, the antenna arms are not robust enough. It is difficult to maintenance the antenna and keep the performance. Therefore, an easy-fabricated and robust antenna is required.

In addition, though the linear conversion from antenna voltage to electric field works in this system, it introduces unnecessary noise in the inversion and degrade the result. A inversion method which can directly use the measured voltage is required.
Figure 3.7: (a) 3D view for the system setup for spheres placed in layered medium. (b) An example of the geometry of the layered medium: the transmitting dipole antenna moves on the plane of $z_T = -4.6$ cm and the receiving dipole antenna scans on the plane of $z_R = 4.6$ cm to form multiple transmitter-receiver locations.
Figure 3.8: (a) Experiment setup for three clay balls placed in two vertical plastic slabs. (b) Reconstructed relative permittivity. (c) Reconstructed conductivity.
Figure 3.9: Experiment setup and results for the microwave imaging of two metal spheres in a layered medium. (a) The two spheres of 0.8 cm diameter are contained in an imaging domain of $6 \times 6 \times 3$ cm$^3$ and located on the plane of $z = 0$. The interfaces of the five-layer medium are located at $z_1 = -1.78$ cm, $z_2 = -1.55$ cm, $z_3 = 1.55$ cm, and $z_4 = 1.78$ cm. Experimental data are collected using 9 transmitter locations on the plane of $z_t = -3.1$ cm and 99 receiving antenna locations on the plane of $z_r = 3.1$ cm. (b) The distance between two spheres is 3 cm in the $y$ direction. (c) Reconstructed permittivity (angle view). (d) Reconstructed conductivity (angle view). (e) 2D cross-section of the reconstructed permittivity at $z = 0$ plane. (f) 2D cross-section of the reconstructed conductivity at $z = 0$ plane.
An MWI System with a Fixed Patch Antenna Array

In order to overcome the defects in the 3D MWI system shown in chapter 3, a new active 3D MWI system with a fixed patch antenna array is proposed. The system employs a fixed bowtie shaped patch antenna array and a high speed data acquisition sub-system. The patch antenna array is fabricated on a imaging chamber, as shown in Fig. 4.1 (c). This structure can simplify the fabrication and maintenance of the imaging system. The data acquisition sub-system is a fixed high speed electrical switch. The total data acquisition time on this MWI system is expected to be several minutes. These two advantages make the inversion system more feasible for a realistic application. In this dissertation, we will only discuss the imaging system based on the fixed patch antenna and the imaging chamber.

The antenna array work both as transmitters and receivers. The center region of the chamber is the inversion domain. The structure of the patch antenna array introduces an inhomogeneous background medium in the inversion system, and produces difficulties in solving the Greens’ functions in the DTA-BIM method. With
the fast development of numerical EM methods and the computer technology, many EM solvers are available and can solve the forward problem with high accuracy, for example, the CST Microwave studio and the HFSS. In this work, a strategy is proposed to implement the DTA method in an inhomogeneous background medium: pre-calculating the Green’s function numerically. However, due to the complexity of our inversion system and the high accuracy requirement for our simulation, it is necessary to verify whether these commercial EM softwares can meet our requirements.

In this chapter, an imaging chamber composed of a bowtie shaped patch antenna arrays is fabricated to provide the measured data, as shown in Fig. 4.1 (d).

In this chapter, we will show the simulation results on the chamber and the comparison of the simulation results and the experimental results.

4.1 Patch Antenna and Chamber

The structure of the bowtie shaped patch antenna is shown in Fig. 4.1(a), which has a size of $9 \times 14$ mm$^2$. The substrate is FR4 ($\varepsilon_r = 4.9$, 1.6 mm in thickness). The medium beyond the patch is a matching medium, acetone. The electric profile of acetone is shown in Fig. 4.2(a) and (b). Because the conductivity of acetone will decay the RF signal propagating in the chamber, in order to make a balance between system resolution and the scattered signal level [41], the antenna operation frequency is 2.7 GHz. In order to simplify the simulation, we use bulk electric profile for the acetone, with $\varepsilon_r = 21, \sigma = 0.01$ S/m.

There are eight interlacing antennas on each side panel. All the eight antennas share a ground (the ground covers the whole panel). The layout of the antenna array is shown in Fig. 4.1(b). The chamber size is $10 \times 10 \times 10$ cm$^3$. The four side walls are fabricated with identical antenna array, totally 32 antennas. The bottom is sealed by a single layer copper PCB board, while the top is open. The chamber is filled with a matching medium. The purpose of sealing the five walls by copper ground is
Figure 4.1: (a) The structure of the bowtie shaped patch antenna. The size of patch is 0.9 × 2.5 cm. (b) The layout of eight antennas on a panel. The size of panel is 10 × 10 cm². (c) The system model in the numerical method. The number in the picture is the antenna index in the simulation. (d) The photo of the physical chamber. The size of chamber is 10 × 10 × 10 cm³.

to isolate the noise coming from the environment. According to the experiment, the fringing field comes from the antenna connector is about −50 dB. If this field can go through the substrate and reach antenna, it will cover the scattered field from inversion targets.

Due to the implementation limitations (poisonous, flammable and evaporable) of acetone in clinic, another matching medium, glycerin. With the glycerin, the antenna working frequency shifts to 4.4 GHz. The electric profile of glycerin is shown in .4.2(c) and (d). Similar to the simulation case using acetone, we use bulk electric profile for
Figure 4.2: (a) The relative permittivity of acetone. (b) The conductivity of acetone. (c) The relative permittivity of glycerin. (d) The conductivity of glycerin.

the glycerin in our simulation, with $\epsilon_r = 4.7, \sigma = 0.46$ S/m.

4.2 The Comparison between the Simulation Results and the Experimental Results

4.2.1 Case 1: The Chamber Filled with Acetone

For the matching medium, acetone, the simulated $S_{21}$ in the CST Microwave Studio agrees well with the measured $S_{21}$, as shown in Fig. 4.3. For the magnitude, the simulated results almost overlap the measurement. The difference mainly happened when $|S_{21}|$ is less than -30 dB. For the phase, there is only a constant shift in $S_{21}$ because of the physical connector length different from the one of the simulation model.
4.2.2 Case 2: The Chamber Filled with Glycerin

For the matching medium, glycerin, the simulated $S_{21}$ in the CST Microwave Studio still agrees the measured $S_{21}$ (shown in Fig. 4.4). Similar to the acetone case, the simulated $|S_{21}|$ almost overlap the measured $|S_{21}|$. There is only a constant phase shift between simulated and measured $S_{21}$.

4.3 Discussion

The measured $S_{11}$ on different antennas in the chamber show that the coupling among the antennas do not affect the antenna radiation performance too much. All
Figure 4.4: (a) The simulated and measured $S_{11}$ in glycerin for antennas 1, 4 and 7. (b) The magnitude of $S_{28,14}$ versus frequency. (c) The magnitude of $S_{21}$ for eleven antennas at 4.4 GHz (the antenna indices listed in fig. 4.1 (c)). The index of eleven antennas are 1, 5, 15, 26, 7, 17, 28, 30, 20, 10 and 14. The transmitter is antenna 14. (d) The phase of $S_{21}$ for eleven antennas at 4.4 GHz.

antennas still have the same working frequency range. Therefore, the layout of the antenna array and the chamber structure is feasible for a realistic MWI application.

From above comparisons, we verify that the simulation tool, the CST Microwave Studio, can simulate the $S_{21}$ with $-30$ dB accuracy for the antenna array fabricated in a chamber. Because the 32 antennas are distributed on the four side walls of the chamber, the simulation results show that the 3D field distribution in the chamber can be solved with high accuracy also. It means we can use this simulation tool to obtain the Green’s functions required by the DTA-BIM method with $-30$ dB
accuracy. If other numerical tools can provide the same accurate solution, we also can use them in our simulation.

Thus, we confirm our proposal for the numerical incident field and the numerical Green’s function in the DTA-BIM method is feasible.
The Modified DTA-BIM Method

In order to solve the contrast $\chi$ in equation (2.16), we need to firstly solve the Green’s function from cell to receiver and the total electrical field in the inversion domain. The total electrical field is calculated by equation (2.5). It depends on the Green’s function from cell to cell and the incident fields in the inversion domain.

Generally, it is impossible to obtain the Green’s function and the incident fields required in the DTA method analytically for a non-canonical inhomogeneous background. But the investigations in chapter 4 show that we can use a numerical method to solve all Green’s functions required in the DTA method. However, due to the meshing requirement of the DTA method, a large number of Green’s functions and incident fields need to be solved.

For example, if the inversion domain is discretized into $N$ cells, the direct simulation method to solve the Green’s function from cell to cell is to place an ideal electric dipole at a cell center with $\hat{x}$, $\hat{y}$ and $\hat{z}$ polarization respectively and put receivers at all the cell centers to get the response. Thus, in order to solve all the Green’s functions from cell to cell, $3N$ simulations is required. Same number of simulations are required for the solution of the Green’s function from cell to probe. To solve
the incident fields at all the cell centers from every transmitter, $N_t$ simulations are required ($N_t$ is the number of transmitters in the system). For example, we define the center region of the chamber fabricated in chapter 4 as the inversion domain. The two diagonal corners of the inversion domain are at $(-2, -2, -2)$ and $(2, 2, 2)$ cm, respectively. The size of the inversion domain is $4 \times 4 \times 4$ cm$^3$, $1.6\lambda \times 1.6\lambda \times 1.6\lambda$ at the working frequency. The background material is acetone. In order to acquire a sufficient accuracy in the DTA method, the sampling density of the mesh is at least 10 cells per wavelength. Therefore, we need to discretize the inversion region by at least $16 \times 16 \times 16 = 4096$ cells. The number of simulations for this inversion domain is $6 \times 4096 + 32 = 24608$. Due to the complexity of the simulation model, the CST Microwave studio requires about 10 hours to finish one simulation. Thus, the total simulation workload to solve all Green’s functions required in the DTA method is 246080 hours, more than 28 years if only one core computer is used. It is a huge workload and infeasible for realistic applications. Therefore, we need to find better methods to reduce the number of simulations.

Considering the symmetry structure of the inversion system, it is obviously we can take advantage of this symmetry property combining with the reciprocity theorem to reduce the workload. In this chapter, we will show our methods to reduce the number of simulation cases. For the Green’s function from cell to cell, we can reduce the number of simulations by factor of about 8. For the Green’s function from cell to probe, we can reduce the number of simulations from $3N$ to $N_r$ ($N_r$ is the number of probes in the system). Thus, for the inversion example in the last paragraph, the number of simulation cases can be reduced to about 1550.

Though this number of simulations is still a big workload, but we focus our efforts on a fixed biomedical imaging system whose sensors (antennas) are in one fixed configuration, thus the system response can be repeatedly used in the forward and inverse simulations. For such a fixed scattering measurement system, in an
approximation method, we can assume the Green’s function for the background medium in this system is fixed. Although this pre-calculation of the Green’s function might be expensive, it is a one-time simulation; thus the method is effective for a fixed measurement system such as a microwave imager. If the scattering measurement system is fixed, the cost on pre-calculation becomes negligible for a long term usage.

Meanwhile, considering the realistic measured data are usually the wave port voltage values (or the $S$ parameters if voltages are normalized) at the probe (antenna) feeding ports. Here, we derive the relationship between the wave port voltage and the scattered field from the object on the wave port. We also develop a vector wave port Green’s function to let the DTA method use measured voltage directly. With these modifications, the inversion method is modified to fit the new input.

5.1 Reciprocity for the Green’s Function of an ideal electric dipole source with a point observer

As mention above, we would like to use the reciprocity theorem to reduce the number of simulation cases. Here, we show how the reciprocity theorem works for an ideal electric dipole source with a point observer.

If the fields produced by electric current sources $\mathbf{J}_i(\mathbf{r})$ and $\mathbf{J}_j(\mathbf{r})$ are denoted by $\mathbf{E}_i(\mathbf{r})$ and $\mathbf{E}_j(\mathbf{r})$ respectively, then according to the reciprocity theorem we have

$$\int_D \mathbf{J}_i(\mathbf{r}) \cdot \mathbf{E}_j(\mathbf{r}) d\mathbf{r} = \int_D \mathbf{J}_j(\mathbf{r}) \cdot \mathbf{E}_i(\mathbf{r}) d\mathbf{r} \quad (5.1)$$

Choosing these sources as point dipoles $\mathbf{J}_i = \hat{a}_i \delta(\mathbf{r} - \mathbf{r}_i)$, and $\mathbf{J}_j = \hat{a}_j \delta(\mathbf{r} - \mathbf{r}_j)$ where $\hat{a}_i$ and $\hat{a}_j$ are the dipole directions, we obtain

$$\int_D [\hat{a}_i \delta(\mathbf{r} - \mathbf{r}_i)] \cdot \mathbf{E}_j(\mathbf{r}) d\mathbf{r} = \int_D [\hat{a}_j \delta(\mathbf{r} - \mathbf{r}_j)] \cdot \mathbf{E}_i(\mathbf{r}) d\mathbf{r} \quad (5.2)$$
or
\[
\hat{a}_i \cdot G(r_i, r_j) \cdot \hat{a}_j = \hat{a}_j \cdot G(r_j, r_i) \cdot \hat{a}_i
\]

(5.3)

Therefore,
\[
G(r_i, r_j) = [G(r_j, r_i)]^T
\]

(5.4)

according to the reciprocity theorem for an ideal electric dipole source with a point observer in an arbitrary inhomogeneous background medium.

5.1.1 Use of Symmetry to Reduce the Number of Simulations for the Green’s function from Cell to Cell

To approximate the total electric field inside the object in the DTA method by equation (2.14), the dyadic Green’s function is needed for sources located at every cell. This precalculation can be time consuming. However, the symmetry of the inversion system can be explored to reduce the number of numerical simulations needed for this dyadic Green’s function.

Theoretically, in order to solve (2.14), we need to run \(N\) simulations (\(N\) is the total number of cells) to obtain the Green’s functions from \(N\) sources to \(N\) receivers. For each simulation, there is an ideal electric dipole source at the \(i\)th cell center and \(N\) point probes at all cell centers. But if the measurement system is symmetric as in typical microwave imaging systems (such as that in [33] and the system shown in chapter 4), more specified, as the rectangular chamber used in Fig. 4.1, which is symmetric along \(x = y, x = 0, y = 0\) planes respectively, we only need to simulate about \(N/8\) cases, then use the symmetry to obtain \(N^2\) Green’s functions.

Here, we use a 2D case to show our scheme of using the symmetry and reciprocity together. As shown in Fig. 5.1, there are 16 cells in the inversion domain, and the system is symmetric along the \(x = y, x = 0\) and \(y = 0\) planes respectively. In the first
Figure 5.1: (a) Simulate the cases with source at cell 11, 12, 16, respectively. (b) Obtain the Green’s function for source at cell 15 by mirroring along \( x = y \) plane. (c) Obtain the Green’s function for source at cell 9, 10, 13 and 14 by mirroring along \( x = 0 \) plane. (d) Obtain the Green’s function for source at cell 1-8 by mirroring along \( y = 0 \) plane.

step, we need to place a point dipole source at cells 11, 12 and 16, respectively to get the Green’s function from source cells 11, 12 and 16 to all cells. Because the system is symmetric along \( x = y \) plane, we can obtain the Green’s function from cell 15 to all cells by mirroring the results for source at cell 12 about the \( x = y \) plane. Then, we can obtain the Green’s function from source cells 9, 10, 13 and 14 by mirroring the results for source cells 11, 12, 15 and 16 about the \( x = 0 \) plane. Finally, in the same way we obtain the Green’s function for source cells 1-8. Through this scheme, we need only 3 simulations, instead of 16 simulations, to obtain the \( 16 \times 16 \) Green’s
functions. Similarly, for a 3D system with 2D symmetry with respect to \( x = y, x = 0 \) and \( y = 0 \) planes, the total simulation requirement will be significantly reduced by a factor about 8 (through above steps, \( y = 0 \) symmetry can reduce the number of simulation cases by half; \( x = 0 \) symmetry can further half the number of simulation cases; finally, \( x = y \) symmetry can reduce the number of simulation cases by about half; therefore, the factor is approximately \( 2 \times 2 \times 2 = 8 \)).

5.2 The Green’s Function from Inversion Domain to a Receiver

Once the electric field has been found by the DTA inside the object domain discretized by \( N \) cells, the electric field at a receiver at \( \mathbf{r} \) can be found from (2.1). To do this, we need to know Green’s function \( G(\mathbf{r}, \mathbf{r}') \) for sources at the \( N \) cells inside the inversion domain to the receiver. This requires \( N \) numerical simulations. But according to the reciprocity theorem in equation (5.4), we can obtain this by the transpose of \( G(\mathbf{r}', \mathbf{r}) \), i.e., the Green’s function from a source at the receiver location to all cells in the inversion domain. Thus, the total simulation requirement will be significantly reduced by a factor of \( N \) for one receiver outside the object. If there are \( N_R \) receivers, the saving factor will be \( N/N_R \).

5.3 The Wave Port Green’s Function \( G_u \)

In a realistic system, the receiving probes are usually not ideal point electric dipoles as described in the section 5.2, but wave ports to characterize the antenna feeding ports. The measured data are usually the wave port voltage values (or the \( S \) parameters if voltages are normalized) at these antenna feeding ports. Here, we derive the relationship between the wave port voltage and the scattered field from the object on the wave port. With this conversion, we can use the measured voltage (or the \( S \) parameters) in the DTA method directly.
Assume that an antenna is fed by a wave port with a port area \( S \), and and the object is located at \( \mathbf{r} \in D \) outside the antenna. We setup two cases: In case 1, we have an induced electric current density \( \mathbf{J} \) inside a volume \( D \) in the inhomogeneous background medium, and the scattered voltage response \( V \) on wave port is calculated through \( \mathbf{E}_{\text{sc}} \) and \( \mathbf{H}_{\text{sc}} \) radiated by this induced source; in case 2, we excite the antenna with a waveport and obtain the electric field response at \( \mathbf{r} \in D \). We will derive the reciprocity relation for the voltage and electric and magnetic fields.

We define the electric and magnetic fields for the guided mode in the wave port as \( \mathbf{e}_m \) and \( \mathbf{h}_m \). Here we assumed that \( \mathbf{e}_m \) and \( \mathbf{h}_m \) have been already normalized. Then in Case 1 where an induced electric current density \( \mathbf{J}(\mathbf{r}) \) for \( \mathbf{r} \in D \), according to [27], the voltage on a wave port as a receiver can be obtained by

\[
\int_S \{ \mathbf{E}_{\text{sc}}(\mathbf{r}) \times [-\mathbf{h}_m(\mathbf{r})] \} \cdot \mathbf{n} ds = V_{\text{sc}}
\]  

(5.5)

where \( \mathbf{E}_{\text{sc}} \) is the scattered electric field response on the wave port due to the induced electric current source in \( D \), and \( \mathbf{n} \) is the outward normal of the wave port surface. Equation (5.5) can be rewritten as

\[
V_{k_{\text{sc}}} = \int_S [-\mathbf{h}_m(\mathbf{r}) \times \mathbf{n}] \cdot \mathbf{E}_{\text{sc}}(\mathbf{r}) ds
\]

\[
= \int_D \int_S \mathbf{n} \times \mathbf{h}_m(\mathbf{r}) \cdot \mathbf{G}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') d\mathbf{r}'
\]

\[
= \int_D \mathbf{G}_u(\mathbf{r}_k, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') d\mathbf{r}'
\]

(5.6)

where the vector Green’s function

\[
\mathbf{G}_u(\mathbf{r}_k, \mathbf{r}') = \int_S \mathbf{n} \times \mathbf{h}_m(\mathbf{r}) \cdot \mathbf{G}(\mathbf{r}, \mathbf{r}') ds
\]

(5.7)

can be obtained by a numerical method. \( \mathbf{r}_k \) is the center of the \( k \)-th antenna port.
5.4 Reciprocity for the Wave Port Green’s Function

As mentioned above, we would like to use the reciprocity theorem to reduce the number of simulations from $3N$ to $Nr$. But in the modified DTA-BIM method, the dyadic Green’s function from cell to probe becomes a vector Green’s function, and the response of a ideal electric dipole is voltage for this vector Green’s function. Therefore, the relationship shown in the section 5.1 cannot work anymore. But the vector Green’s function $G_u$ comes from the integration of the field, therefore, $G_u$ should be reciprocal in some form. Here, we show the new relationship of the reciprocity theorem for the wave port vector Green’s function.

Now we are ready to discuss the two cases and their reciprocity relationship: In case 1, we have an induced electric current density $J_1$ inside a volume $D$ in the inhomogeneous background medium, and the scattered voltage response $V$ on wave port is calculated through $E^{sct}$ and $H^{sct}$ radiated by this induced source; in case 2, we excite the antenna with a wave port mode and obtain the electromagnetic field response $E_2$ and $H_2$ at $r \in D$. We will derive the reciprocity relation for the voltage and electric and magnetic fields.

In Case 1, the induced electric current source $J_1(r)$ for $r \in D$ produces electric field $E_1$ and magnetic field $H_1$ in wave port $S$.

In Case 2, for a wave port source at feeding position $S$, the equivalent currents corresponding to the $m$-th mode of the incident field in the port can be expressed as

$$J_2(r) = -\hat{n} \times h_m(r); \quad M_2(r) = \hat{n} \times e_m(r), \quad r \in S \quad (5.8)$$

where $h_m(r)$ and $e_m(r)$ are the incident electric and magnetic fields in the wave port for an inward propagating mode respectively. This wave port excitation produces an electric field $E_2$ at $r \in D$, which is outside the wave port surface $S$. 

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According to the reciprocity theorem, we have
\[
\int_S [J_2(r) \cdot E_1(r) - M_2(r) \cdot H_1(r)] ds = \int_D J_1(r) \cdot E_2(r) dr \quad (5.9)
\]
and
\[
\int_S [-\hat{n} \times h_m(r) \cdot E_1(r) - \hat{n} \times e_m(r) \cdot H_1(r)] ds = \int_D J_1(r) \cdot E_2(r) dr \quad (5.10)
\]
As the normalized voltage and current on a wave port can be defined as [27]
\[
V = \int_S \{ E_1(r) \times [-h_{tm}(r)] \} \cdot \hat{n} ds; \quad I = \int_S [e_{tm}(r) \times H_1(r)] \cdot \hat{n} ds, \quad (5.11)
\]
where \(e_{tm}\) and \(h_{tm}\) are the transverse field distributions of the \(m\)-th waveguide mode propagating into the wave port, equation (5.9) can be rewritten as
\[
V + I = \int_D J_1(r) \cdot E_2(r) dr \quad (5.12)
\]
In the special case where the current density \(J_1\) is a point dipole, i.e., \(J_1 = \hat{a} \delta(r - r_1)\), we have
\[
V + I = \hat{a} \cdot E_2(r_1) \quad (5.13)
\]
Therefore, the electric response of a wave port source is equal to the summation of the normalized voltage and current responses on the port due to an electric dipole source.

### 5.5 The Modified Inversion Method

Because we have already developed the electric field-voltage conversion method in the section 5.3, here we would like to use measured voltage on the probe wave port to
do the inversion directly, the inversion method shown in chapter 2 must be modified to fit this new input.

For a general 3D microwave imaging system shown in Fig. 5.2, the scattered voltage on the probe wave port can be calculated as

\[
V_{sct}^m = \int_D G_u(r_m, r') \cdot J(r') dr' = j \omega \varepsilon_b \int_D G_u(r_m, r') \cdot \chi(r') E(r') dr'
\]  
(5.14)

where \(V_{sct}^m\) is the measured scattered voltage on the \(m\)-th antenna port centered at \(r_m\), \(J\) is the induced electric current density inside a volume \(D\) in the inhomogeneous background medium, and \(G_u\) is defined in 5.7.

Discretizing (5.14), we obtain

\[
j \omega \varepsilon_b \triangle V \sum_{n=1}^N G_u(r_m, r_n) \cdot E(r_n) \chi(r_n) = V_{sct}^m
\]  
(5.15)

where \(\triangle V\) is the uniform cell volume, \(\chi(r) = \varepsilon(r)/\varepsilon_b(r) - 1\) is the complex contrast.

The matrix form of (5.15) is

\[
A \chi = V_{sct}
\]  
(5.16)
We can use the same method described in chapter 2 to obtain final inversion equation

\[
\left( \frac{\mathbf{A}_n^\dagger \mathbf{A}_n}{\| \mathbf{V} \|^2} + \frac{\gamma^2}{\| \chi_n \|^2} \mathbf{I} \right) \chi_{n+1} = \frac{\mathbf{A}_n^\dagger \mathbf{V}}{\| \mathbf{V} \|^2}
\]  

(5.17)

Here, we still use the CG method to solve the \( \chi_{n+1} \).

5.6 The Verification of the Modified DTA Method

Here, we will verify the results of the methods proposed in the previous sections by comparing with synthetic data.

5.6.1 Verification of the Reciprocity of an Ideal Electric Dipole Source to a Point Probe

In order to verify the reciprocity theorem in an actual numerical simulation, we set up two kinds of simulations, as shown in Fig. 5.3(a). For setup 1, we place an electric dipole with \( \hat{x} \), \( \hat{y} \) and \( \hat{z} \) polarization respectively at position \( \mathbf{r}_1 = (0,0,1) \) cm and a point probe at \( \mathbf{r}_2 = (2.5,3.0,7.5) \) cm. For setup 2, we put an electric dipole with \( \hat{z} \), \( \hat{x} \) and \( \hat{y} \) polarization respectively at position \( \mathbf{r}_2 \) and a point probe at \( \mathbf{r}_1 \). The background is a homogenous material with relative permittivity of 5 and conductivity of 0.01 S/m; the inner (blue) cuboid is a rectangular object with size of \( 3 \times 4 \times 5 \) cm\(^3\) with the relative permittivity 10 and the conductivity 0.1 S/m. This setup form an inhomogeneous environment. Fig. 5.3(b) shows that the \( E_z \) response of the \( \hat{x} \) dipole in setup 1 is the same as the \( E_x \) response of the \( \hat{z} \) dipole in setup 2. Fig. 5.3(c) shows that the \( E_y \) response of the \( \hat{z} \) dipole in setup 1 overlaps the \( E_z \) response of the \( \hat{y} \) dipole in setup 2. Fig. 5.3(d) shows that the \( E_x \) response of the \( \hat{y} \) dipole in setup 1 exactly matches the \( E_y \) response of the \( \hat{x} \) dipole in setup 2. Through these simulations, the reciprocity of the ideal electric dipole source to a point probe is verified in simulation.

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Figure 5.3: Verification of the reciprocity of an ideal electric dipole source to a point probe in an inhomogeneous medium with a dielectric cube of dimensions $3 \times 4 \times 5$ cm$^3$ centered at $(1.0,1.3,3.2)$ m, $\varepsilon_r = 10$ and $\sigma = 0.1$ S/m in a homogeneous background with $\varepsilon_{rb} = 5$ and $\sigma_b = 0.01$ S/m. (a) Simulation setup with a $x$-oriented electric dipole at $\mathbf{r}_1 = (0,0,1)$ cm and a point probe at $\mathbf{r}_2 = (2.5,3.0,7.5)$ cm. (b) Transient $E_z$ response of setup 1 versus transient $E_x$ response of setup 2. (c) Transient $E_y$ response of setup 1 versus transient $E_z$ response of setup 2. (d) Transient $E_x$ response of setup 1 versus transient $E_y$ response of setup 2.

5.6.2 Verification of the Reciprocity of an Ideal Electric Dipole Source to a Wave Port

Next, we verify the reciprocity between an ideal electric dipole source and a wave port in a Planar Inverted F Antenna (PIFA) placed in medium with $\varepsilon_{rb} = 5$ and $\sigma_b = 0.01$ S/m. The antenna wave port is extended to air. The configuration is shown in Fig. 5.4(a), while the PIFA antenna geometry is give in Fig. 5.4(b). We set up two kinds of simulations to verify that the voltage response on a wave port
is reciprocal to the electric field response at a point probe in simulation. For setup 1, we place an \( \hat{x} \), \( \hat{y} \) and \( \hat{z} \) polarized electric dipole at position \( \mathbf{r}_1 = (0.5, 0.7, 1.0) \) cm respectively and a receiving mode wave port (the red rectangle) on the antenna feeding coax. For setup 2, we put a point probe at \( \mathbf{r}_1 \) to record the electrical field, and change the wave port as source with the TEM mode in the coax. Except for the wave port of a coaxial cable, the background is a homogenous material with relative permittivity of 5 and conductivity of 0.01 S/m. Fig. 5.4(c) shows the voltage response on the wave port from \( \hat{x} \) dipole in setup 1 and the \( E_x \) response of setup 2. Fig. 5.4(d) shows the voltage response on the wave port from \( \hat{y} \) dipole in setup 1 and the \( E_y \) response of setup 2. Fig. 5.4(e) shows the voltage response on the wave port from \( \hat{z} \) dipole in setup 1 and the \( E_z \) response of setup 2. Every pair of curves in Fig. 5.4 (c)-(e) agree very well.

Through these simulations, we know the voltage response on a wave port is reciprocal to the electric field response at a point probe in a numerical simulation.

5.6.3 The Accuracy of the Green’s Function obtained from Mapping

In the last synthetic inversion case in chapter 6, the antenna fabricated on the chamber is no symmetry, as shown in Fig. 6.1(a). Due to this non symmetric structure, the antenna radiation pattern is non symmetric in \( xoy \) plane, as shown in Fig. 5.6 (c).

Although the antenna is not symmetric, the chamber setup and antenna array layout in the synthetic inversion system is symmetric along \( x = y, x = 0, y = 0 \) planes, respectively. We employ the mapping method discussed in chapter 5 to solve the Green’s function from cell to cell. Obviously the non symmetric structure of antenna introduces error in the mapping, it is necessary to investigate the accuracy of the Green’s function obtained from this method.

Fig. 5.6 shows the top view of the chamber. The black dots are cell centers. In
we divide the inversion domain into 4 regions, indexed from 1 to 4, as shown in Fig. 5.6. In our mapping scheme, we only need to simulate the Green’s function from all cells to a source cell in the region 1. Then for the Green’s function from all cells to a source cell in the region 2, we can obtain them from mapping along \( x = y \) plane. For the Green’s function from all cells to a source cell in the region 3, we solve them by mapping along \( x = 0 \) plane. Finally we use mapping along \( y = 0 \) plane to solve the Green’s function from all cells to a source cell in the region 4.

In the verification, we randomly pickup some source positions in region 2, 3 and 4, respectively. For the Green’s function from other cells to this source, we compare the difference between the one obtained by mapping and the one from direct simulation. The error is calculated by

\[
E = \frac{\sum_{j=1}^{N} ||G_{mn}^{\text{map}}(r_i, r_j) - G_{mn}^{\text{sim}}(r_i, r_j)||^2}{\sum_{j=1}^{N} ||G_{mn}^{\text{sim}}(r_i, r_j)||^2}
\]

where \( i \neq j \) and \( m, n = x, y, z \). \( r_j \) is the position of the \( j \)th cell. \( G_{mn}^{\text{map}} \) is the Green’s function solved by mapping. \( G_{mn}^{\text{sim}} \) is the Green’s function obtained by simulation. \( N \) is the total cell number. In our inversion case shown in chapter 5, \( N = 1728 \). \( r_i \) is the position of the \( i \)th source cell. The errors are shown in Tab. 5.1.

Table 5.1: Errors between the mapped Green’s function and the simulated Green’s function

<table>
<thead>
<tr>
<th>Region Id</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

As can be seen, due to the non symmetric antenna structure, the error increment is proportional with the mapping numbers. Every mapping produce about 3% error. But the maximum error is still less or equal to 8%. Because the DTA method is an approximation method, these Green’s functions can be used in our inversion. In following sections, we will verify the accuracy of the DTA method with these mapped
Green’s functions.

5.6.4 Verification of the Wave Port Green’s Function $G_u$

Here, we setup an imaging chamber to test the accuracy of the Green’s function for a wave port scattered voltage in a non-canonical inhomogeneous background medium. The chamber is sealed by five grounded PCB panels, and only open at the $+y$ direction. There are 8 PIFA fabricated on each side panel, thus totally 32 antennas in the chamber, as shown in Fig. 5.7(a). Each antenna is fed by a coax as shown in Fig. 5.4; there is a wave port on the cross-section of each coax. The chamber size is $10 \times 10 \times 10$ cm$^3$, filled with a fluid with relative permittivity of 7.5 and conductivity of 0.01 S/m. A cuboid (size is $4 \times 1.6 \times 2$ cm$^3$) with relative permittivity of 13 and conductivity of 0.1 S/m is placed close to the bottom of the chamber, as shown in Fig. 5.7(b). The cuboid is meshed by $20 \times 8 \times 10$ cells in forward scattering calculation in equation (5.14). In the simulation, there is a point probe at each cell center to record the electric field.

Firstly, we remove the cuboid from the chamber. We record the electric field at each cell center as $E^{inc}$. According to the reciprocity theorem, this field is also the vector Green’s function $G_u(r_2, r_1)$ from cell center to the wave port. Then we put the cuboid into the chamber and record the electric field at each cell center as the total field $E$. Because we know the contrast of the cuboid, we can obtain the induced current at each cell center by $J = j \omega \varepsilon_0 \chi E$, then the scattered voltage at each wave port can be obtained by (5.14).

We calculate the scattered wave port voltage by operating the vector wave port Green’s function on the induced current density given calculated by the FDTD method through Wavenology EM. We then compare this voltage with the FDTD calculation by Wavenology EM. Fig. 5.8 shows the comparison of the simulated scattered voltage and the reference result by Wavenolgy EM at each wave port in
frequency domain. The result from (5.14) matches the reference result very well. The small mismatch at the high frequency part is due to the lower meshing density of (2.15) for (5.14) in high frequency range (10 sampling points per wavelength at 2.75 GHz). From this test, we know that (5.14) works well in this complicated antenna array. This verifies the vector wave port Green’s function in this realistic microwave imaging chamber.

5.6.5 The Scattered Field by the DTA Method in a System with Five PEC Panels

The above examples verify the reciprocity and Green’s functions for inhomogeneous media. Next, we set up a non-canonical inhomogeneous background case to test the accuracy of scattered field calculated by the proposed DTA method combined with the numerical Green’s functions. This case is designed to test a measurement system which employs impedance-matched point dipole probes to measure the field directly. As shown in Fig. 5.9(a), the background medium contains five PEC panels and two cuboids. The distance between facing PEC panels is 10 cm. The size of each cuboid is $8 \times 8 \times 4$ cm$^3$. The electric properties of all materials are shown in Fig. 5.9(b). There are 44 probes in the simulation, as shown in Fig. 5.9(c), distributed along 4 straight lines, each line having 11 probes. The distance between two adjacent probes along the same line is 8 mm.

The target is a sphere placed at the center of the system (the red sphere shown in Fig. 5.9(c)), which has a radius of 10 mm, and with $\varepsilon_r = 4$ and $\sigma = 0.1$ S/m. Fig. 5.10 shows the scattered electric field at the probes at 2.75 GHz. It shows the scattered field through the DTA method is much better than the Born approximation, even for a sphere with size of $0.34 \lambda_{bg}$ with contrast $\chi = 1$; the scattered field calculated by the DTA method has 10% relative RMS error compared with the reference full-wave results.
5.6.6 The Scattered Voltage from Two Cubes by the DTA Method in a Chamber

In this example, we study a microwave imaging chamber to test the accuracy of the scattered voltage at wave ports by the DTA method combined with the numerical Green’s function. The chamber is sealed by 5 PCB panels, only open at the +z direction. Similar to the case in Fig. 5.8, with 8 PIFA antennas fabricated on each side panel, thus totally 32 antennas. Each antenna is fed by a wave port on the cross-section of the coax. The chamber size is \(10 \times 10 \times 10\, \text{cm}^3\), filled with a fluid having relative permittivity of 5 and conductivity of 0.01 S/m. Two cuboids of dimensions \(4 \times 4 \times 4\, \text{cm}^3\) (equal to \(0.6\lambda \times 0.6\lambda \times 0.6\lambda\)) are placed in the chamber, as shown in Fig. 5.11. The top cuboid has relative permittivity of 8 and conductivity of 0.2 S/m. The bottom cuboid has relative permittivity of 6 and conductivity of 0.02 S/m. The forward computation domain is the bounding box of the two cuboids, meshed by \(12 \times 12 \times 12\) cells.

Fig. 5.12 shows the electric field distribution at \(z = 5\, \text{mm}\) plane in the microwave imaging chamber in Figure 5.11. In this setup, there is not object in the chamber. The transmitter is antenna #14.

Fig. 5.13 shows the scattered voltage on 32 probes at frequency 2.75 GHz when the source port is port 17. The scattered voltage results calculated by the DTA method agree well with the reference full-wave results, with 15% relative RMS error.

5.6.7 The Scattered Voltage from Eight Small Cubes by the DTA Method in a Chamber

Finally, we examine the DTA accuracy for the scattering from multiple small objects. In this case, 8 cubes (each with dimensions \(12 \times 12 \times 12\, \text{mm}^3\)) are placed in two layers in the computation domain. The positions of these 8 cubes are shown in Fig. 6.4, with the distance between any adjacent cells being \(\lambda/4\) in the background fluid at 2.7 GHz; and the size of cube is \(\lambda/4\) also. All cubes have the same electric properties.
of a relative permittivity 10 and conductivity 0.2 S/m.

Fig. 5.15 shows the scattered voltage at the 32 wave ports at frequency 2.75 GHz (source port is port 17). The scattered voltage through the DTA method has 18% relative RMS error compared with the reference full-wave results.

For the last two cases, due to the Green’s function used in the DTA method has a maximum error of 8%, we can estimate the DTA method will obtain better performance with more accurate Green’s function. Therefore, it is shown that our proposed method works well in a complicated non-canonical inhomogeneous background. The DTA method combined with the electric field-voltage conversion can provide acceptable scattering field results for such complicated configurations. Therefore, we expect that this approximate DTA solver will be useful for both the forward and inverse scattering computation in microwave imaging.

5.7 Conclusions

We propose a modified DTA method to work with numerical Green’s function and the measured wave port voltage. We also propose a scheme which takes advantage of the symmetrical structure of the inversion system to reduce simulation workload. The verification through the synthetic data shows the proposed methods works very well. Therefore, the modified DTA method can be used to calculate the scattered fields from arbitrary objects in a non-canonical inhomogeneous background with acceptable accuracy. It is the first application of the Diagonal Tensor Approximation to calculate the scattered fields or the scattered voltage on probes from arbitrary objects in a non-canonical inhomogeneous background. The promising results shows that modified DTA method can significantly extend the application domain of the original DTA method from cannonical background media to arbitrary non-canonical inhomogeneous background.
Figure 5.4: Verification of the reciprocity of an ideal electric dipole source to a wave port in a PIFA antenna placed in a medium with $\varepsilon_r b = 5$ and $\sigma_b = 0.01$ S/m. (a) Antenna geometry. (b) Simulation setup. (c) Transient $E_x$ response of setup 1 versus transient wave port voltage response of $\hat{x}$ polarized electric dipole. (d) Transient $E_y$ response of setup 1 versus transient wave port voltage response of $\hat{y}$ polarized electric dipole. (e) Transient $E_x$ response of setup 1 versus transient wave port voltage response of $\hat{z}$ polarized electric dipole.
Figure 5.5: (a) Simulation setup for a PIFA antenna placed in a medium with $\varepsilon_r = 5$ and $\sigma_b = 0.01$ S/m. (b) 3D radiation pattern. (c) 2D radiation pattern at $\theta = 90^\circ$. (d) 2D radiation pattern at $\phi = 0^\circ$. 
Figure 5.6: Region definition for the Green’s function mapping.
Figure 5.7: Verification of the vector wave port Green’s function $G_u(r_m, r_1)$ for a realistic microwave imaging chamber with 32 PIFA antennas. (a) Simulation setup of the chamber, with each of the four side panels having 8 PIFA antennas given in Fig. 5.4. The bottom face is PEC, and the top face is open to air. (b) A cuboid in the chamber (size view). (c) A cuboid in the chamber (front view).
Figure 5.8: Comparison of scattered voltage at wave ports calculated by the vector wave port Green’s function and by the FDTD method with Wavenology EM for the problem in Figure 5.7. (a) Wave ports 17 is used as a source port to calculate the scattered wave port voltage. (b) $|U^{scat}|$ on port 17. (c) $|U^{scat}|$ on port 14. (d) $|U^{scat}|$ on port 25.
Figure 5.9: The scattered field by DTA in a system with five PEC panels. (a) Angle view of the simulation setup, where the red dot is the ideal electric dipole source. (b) Cross-section of the case, where for $\varepsilon_r = 2$ and $\sigma = 0.001$ S/m for the homogenous material except for the PEC and cuboids; for the top cuboid, $\varepsilon_r = 3$ and $\sigma = 0.15$ S/m; for bottom cuboid, $\varepsilon_r = 1$ and $\sigma = 0.001$ S/m. (c) The point probes in the simulation (dark dots), the red sphere ($r = 10$ mm) is the target.
Figure 5.10: Comparison of the scattered electric field for the system in Figure 5.9 calculated by DTA, Born approximation, and Wavenolgy EM for (a) $|E^sct_x|$, (b) $|E^sct_y|$, and (c) $|E^sct_z|$.
Figure 5.11: Two cubes in an imaging chamber. (a) Angle view of the configuration. (b) Top view of the configuration, with the dashed line indicating the forward simulation domain. (c) Side view of the configuration.
Figure 5.12: The electric field distribution at $z = 5$ mm plane in a microwave imaging chamber in Figure 5.11. In this figure, there is not object in the chamber; the antenna #14 is the transmitter. (a) $|E_x|$ distribution. (b) $|E_y|$ distribution. (c) $|E_z|$ distribution.
Figure 5.13: The calculated scattered voltage from the two cubes in a microwave imaging chamber in Figure 5.11. Comparison of the (a) real part, (b) imaginary part and (c) magnitude of the scattered voltage with the reference full-wave results obtained by Wavenology EM.
Figure 5.14: Scattered voltages from eight small cubes in the microwave imaging chamber. (a) Angle view of the simulation setup. (b) Top view of the configuration with the red dashed line indicating the computational domain. (b) The side view of the configuration.
Figure 5.15: Comparison of the DTA calculated scattered voltage with Wavenology EM for the (a) real part, (b) imaginary part and (c) magnitude of the scattered voltage at the 32 ports.
Inversion with the Modified DTA-BIM Method

6.1 The Model for a 3D Microwave Imaging System

In order to test the performance of the new proposed DTA-BIM inversion method, a 3D microwave imaging chamber with an antenna array is set up. The antenna array is built on a rectangular chamber with 5 PCB panels (the material of PCB board is FR4 with relative permittivity of 4.4), and only open at the +Z direction. The chamber size is $10 \times 10 \times 10 \text{ cm}^3$, filled by a fluid with relative permittivity of 5 and conductivity of 0.01 S/m. There are 8 Planar Inverted F Antennas (PIFA) fabricated on each panel, hence totally 32 antennas in the chamber. Fig. 6.1(a) shows the structure of the PIFA antenna with its $S_{11}$ magnitude shown in Fig. 6.1(b). Fig. 6.1(c) shows the layout of the antennas on one panel. All antennas share the same ground to isolate the noise from the environment. Each antenna is fed by a wave port on the coax. The wave ports are indexed from 1 to 32. The antenna operating frequency is 2.8 GHz, where $|S_{11}|$ is less than -10 dB. The structure of the chamber is shown in Fig. 6.1(d). The antenna in the system is different from the one shown in chapter 4. The reason is the system in chapter 4 employs acetone as the
background fluid. With the high permittivity of acetone (the relative permittivity is about 21), the antenna size can be very small at the working frequency 2.8 GHz. It is possible to place eight antennas in a $10 \times 10 \text{ cm}^2$ panel. For the system used in this chapter, we utilize a fluid with relative permittivity of 5.0 as the background fluid. Meanwhile, we keep the work frequency at 2.8 GHz. Under this situation, the bowtie shaped patch antenna become very large. If the chamber size is kept as $10 \times 10 \times 10 \text{ cm}^3$, the distances among the antennas in the antenna array become very small, producing large coupling among antennas in the array. Therefore, the antenna is redesigned by the PIFA structure to fit this background fluid.

The inversion domain is the center region of the chamber in Fig. 6.2(a). The two corners of inversion domain are $(-24, -24, -24), (24, 24, 24)$ mm respectively. The inversion domain is shown as the red dash line in Figs. 6.2(b) and (c). The inversion domain is divided into $12 \times 12 \times 12$ uniform cells. Therefore, there are 1728 unknowns to be reconstructed.

In order to perform imaging, we need to collect the data of the scattered voltage $V^{\text{sc}}$; furthermore, we need to calculate the incident electric field $E^{\text{inc}}$ at the center of each voxel (cell), cell-to-cell dyadic Green’s function $G$ and cell-to-antenna Green’s function $G_u$. We obtain these data by three steps. Firstly, we fill the chamber with a fluid only, without any object inside; and there is a point observer at each cell center. We excite 32 wave ports sequentially; the voltage recorded on each wave port is the incident voltage $V^{\text{inc}}$, the electric field at each observer is $E^{\text{inc}}$, and the relationship between the wave port voltage and the electric field at each observer is $G_u$. Secondly, we place targets into the chamber, and excite 32 wave ports sequentially again. The voltage recorded on a wave port is the total voltage $V$. The scattered voltage is $V^{\text{sc}} = V - V^{\text{inc}}$. Based on this scheme, we can obtain $32 \times 32 = 1024$ scattered voltages. The third step is to calculate the Green’s function $G$. The detail method is discussed in chapter 5 for the forward problem.
6.2 Numerical Examples

With the above imaging chamber, we can calculate the synthetic data for the scattered voltages at the antenna array for various objects placed inside the chamber. Shown below are several examples to illustrate the performance of the DTA-BIM inversion method for this inhomogeneous background medium.

6.2.1 Case 1: Two $\lambda/2$ Cubes

In this first case, there are two cubes (each with a size of $24 \times 24 \times 24$ mm$^3$) placed at the two corners of inversion domain. The positions of the cubes are shown in Fig. 6.2. The one close to the chamber bottom (corner positions are $(-24, -24, -24)$ and $(0, 0, 0)$ mm) has a relative permittivity value of 6 and conductivity of 0.02 S/m. The one close to the chamber top (corner positions at $(0, 0, 0)$ and $(24, 24, 24)$ mm) has a relative permittivity value of 8 and conductivity of 0.2 S/m.

Due to the fact that $\omega \varepsilon_0 \Delta \varepsilon_r >> \Delta \sigma$ for our inversion frequency, only the reconstructed permittivity is evaluated. Fig. 6.3 shows the inverted electric profile for Case 1. As can be seen, the relative permittivity of two cubes can be reconstructed correctly, and the shape and the size of the cubes are correct also. This case successfully demonstrates that the proposed method can reconstruct the objects with size up to $\lambda/2$ and the contrast of $\chi \approx 0.6$ in a non-canonical inhomogeneous background.

6.2.2 Case 2: Eight $\lambda/4$ Cubes

Next we study a case with multiple small cubes to examine the resolution of this inversion method. In this case, 8 cubes (each has size of $12 \times 12 \times 12$ mm$^3$) are placed as two layers in the inversion domain. The positions of 8 cubes are shown in Fig. 6.4, where the $x$, $y$ and $z$ distance between any two adjacent cubes is $\lambda/4$ in the background fluid at 2.8 GHz, and the size of each cube is $\lambda/4$ also. All cubes have the same electric profile with a relative permittivity value of 10 and conductivity of
0.2 S/m.

Fig. 6.5 shows the reconstructed electric profile for Case 2. As can be seen, the reconstructed relative permittivity values of eight cubes are as high as 7.1, close to the real value of 8. For the positions and the sizes, in X-Y cross-section, the same layer 4 cubes are reconstructed correctly also; meanwhile, 4 cubes can be clearly distinguished. Because there are not any antennas on the chamber top and bottom surfaces, the resolution of the image is worse in the Z direction. For this inversion result, it is observed that the resolution of reconstructed image can reach $\lambda/4$.

Fig. 6.6(a) shows that the inversion data error converges rapidly within only 6 iterations. Fig. 6.6(b) shows the comparison between the scattered voltage from synthetic data and the scattered voltage from the reconstructed electrical profile. They match well.

From Case 2, we show that our method works well for reconstructing multiple, small and closely arranged objects in a noncanonical inhomogeneous background. The resolution of reconstructed image can reach $\lambda/4$ in the lateral direction.

6.2.3 Case 3: A Layered Cube

In this case, we place a layered cube at the center of the inversion domain as shown in Fig. 6.7. The inner layer of the cube [blue part in Fig. 6.7(b)] is filled with a small cube. The outer cube [two opposite corners are $(-16, -16, -16)$ and $(16, 16, 16)$ mm] has a relative permittivity of 8 and conductivity of 0.2 S/m. The inner small cube [corner positions are $(-8, -8, -8)$ and $(8, 8, 8)$ mm] has relative permittivity of 2.08 and zero conductivity.

Fig. 6.8 shows the reconstructed dielectric profile for Case 3. As can be seen, the size and the thickness of the outer cube can be distinguished clearly. Due to the fact that there is not any antenna on the chamber top and bottom surfaces, the top face of the outer cube is not fully closed. Meanwhile, part of the inner cube can be
reconstructed correctly, and the relative permittivity of the center of the inner cube \( \epsilon_r = 2 \) is the same as the ground truth.

From Case 3, we observe that our method works well for a complicated layered cube in a non-canonical inhomogeneous background.

6.3 Conclusions

It is the first application of a 3D inversion solver, the modified DTA-BIM method, to reconstruct targets in a non-canonical inhomogeneous background with measured probe voltages. The inversion result shows this 3D inversion solver can obtain a supper resolution of \( \lambda/4 \) in a non-canonical inhomogeneous background. In addition, our inversion cases for the targets in a non-canonical inhomogeneous background shows the modified DTA-BIM method can work in any non-canonical inhomogeneous background, this significantly widen the application domain of the inversion method.

But this method still has some rooms to improve. One consideration is the high computational cost of the DTA method (\( O(N^2) \) CPU time) can be significantly reduced if the inversion domain can be reduced to a limit range. But it requires to shrink the region of interest (ROI) firstly. Hence, we introduce time-reversal method to reduce our inversion domain.
Figure 6.1: The model for a 3D microwave imaging system [43]. (a) The structure of the PIFA antenna (unit: mm). (b) The $S_{11}$ of PIFA antenna in the chamber. (c) The antenna layout on one panel (unit: mm). (d) The structure of chamber.
Figure 6.2: Case 1: Inversion of two cubes centered at \((-12, -12, -12)\) mm and \((12, 12, 12)\) mm. (a) 45-degree angle view, (b) top view, and (c) side view with the red dash line indicating the inversion domain.
Figure 6.3: Inversion result of Fig. 6.2 (Case 1). (a) Cross sections of the reconstructed relative permittivity profile for the bottom cube. (b) Cross sections of the relative permittivity profile for the top cube. (c) Iso surface of the reconstructed relative permittivity ($\epsilon_r = 5.8$ for the bottom cube, $\epsilon_r = 8.0$ for the top cube).
Figure 6.4: Case 2 setup with eight cubes. (a) 45-degree angle view, (b) top view, and (c) side view with the red dash line indicating the inversion domain.
Figure 6.5: Reconstruction of Case 2. (a) Cross section of the relative permittivity profile for case 2 (side view). (b) Cross section of the relative permittivity profile for case 2 (top view). (c) Iso surface of the relative permittivity for case 2 with $\epsilon_r = 7$. 
Figure 6.6: (a) Convergence curve of inversion case 2. (b) Inverted scattered voltage vs. simulated scattered voltage.

Figure 6.7: Case 3: A layered cube inside a chamber. (a) 45-degree angle view. (b) Side view.
Figure 6.8: Reconstruction of the layered cube in Case 3. (a) Cross section of the relative permittivity profile (top view). (b) Cross section of the relative permittivity profile (side view). (c) Iso-surface of the the relative permittivity (Iso value is 5.5).
7

The Time Reversal Method and the Implementation in Microwave Imaging

7.1 Introduction

The time-reversal method [21, 18] has attracted growing attention in recent years. This method utilizes the reciprocity of wave propagation in a time-invariant medium to find the shape and the location of a source. The basic principle of the time-reversal method is shown in Fig. 7.1. In this system, an array of receivers record the scattered transient signal from a source. Then the signal on the receivers can be reversed in time sequence and propagated back. There will be a focus on the source location. Based on this capability, the time-reversal method can be used to localize significant energy in a complex (cluttered) environment space. In recent years, time-reversal theory has been applied successfully in the acoustics regime. The applications include the target imaging and detection [38]-[16] and the underwater communication [39]. The time reversal technique has been also introduced to the electromagnetics regime recently. The topics include forest communications [17], [35], underground object imaging and detection [32]-[17], breast cancer detection [25], [36], [37] and hardware
realization of the time-reversal mirror (the transmitter-receiver array, also called as TRM).

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig7_1.png}
\caption{Schematic of time-reversal process. A transient source radiate a signal to a transmitter-receiver array. The signals are reversed in time and radiated back to the inversion domain. In a domain with significant multipath, a focusing is introduced at the original source.}
\end{figure}

The focusing quality in the time-reversal method is decided by the size of the effective aperture of transmitter-receiver array. This effective aperture is not only the physical size of the transmitter-receiver array. It also includes the effect of the environment. A complicated background will create so-called multipath effect and can significantly increase the aperture of transmitter-receiver array. Therefore, in a highly cluttered, multipath environment, the effective aperture may become very large, yielding so-called super-resolution focusing [38]. In other words, the resolution of time reversal in a highly cluttered medium may be better than suggested by the actual physical aperture alone [38], [51].

In this research, the numerical Green’s function and the incident field must be obtained by a numerical method, i.e., we must build a simulation modal for the inversion system. Thus, we can easily reverse the time of the recorded signal and send it back to the inversion domain by a numerical method. If the focus can be found, we can limit our inversion domain to the focus region only, which can significantly reduce our inversion domain and computational cost.
In this chapter, we will discuss the implementation of the time reversal method in our simulation and how to transform the single domain DTA-BIM method to fit multiple discontinue inversion domains. In the final section of this chapter, numerical results are given.

7.2 Review of the Time Reversal Method with FDTD Simulation

In recent years, several research on the time-reversal method with near field measurement are reported [23], [36], [37]. Especially, [36] and [37] employ the time-reversal method with Finite-difference time-domain (FDTD) simulation to locate the breast cancer, which is similar to our research (our simulation tool Wavenogy EM. uses FDTD method). Therefore, their experience will give some help.

The 3D simulation modal in [37] is shown in Fig. 7.2. The virtual TRM array has five lines of receivers along the $x$ direction, with 21 receivers in each line (along the $y$ axis). The spacing between each receiver in the $x$ or $y$ direction is 0.057 cm. The center element of the array (which is placed on the same axial slice as the target) is the $x$ polarized ideal electrical dipole source with a differentiated Gaussian pulse of 50 ps width. The breast model comes from a dielectric profile mapping of Magnetic resonance imaging (MRI) data. Obviously, the breast tissue makes the simulation volume become a very complicated environment and can create significant multipath effect.

The simulation procedure is firstly recording the transient signals (3 electric components and 3 magnetic components) on the TRM without the tumor inside breast and call them the incident fields. Then an artificial sphere (3 mm in diameter, $\epsilon_r = 39$ and $\sigma = 8$ S/m) is place in the breast, the position is shown in Fig. 7.2 (b) and (c). Re-simulate the modal and recording the six components again, the data in this simulation is the total fields. The scattered fields are obtained by subtracting the incident fields from the total fields. Finally, reverse the time sequence of the
scattered fields and excite them at TRM simultaneously. Fig. 7.3 shows the $yz$ slices with the strongest focus for the normalized time-reversed $E_x$ field scattered from the tumor. As can be seen, the position and the size of the focus almost overlap with the tumor. Therefore, we know the time-reversal method combining with FDTD method works very well for a complicated breast model.

7.3 Simulation Setup for the Time Reversal Method

The reported time-reversal methods need clutters to create multipath effect, but there is a not clear statement on how to setup the shape, layout and the position of the clutters. Due to the structure of our inversion chamber, it is a non-canonical inhomogeneous background. Theoretically, this inhomogeneous property can be considered as a clutter. However, this layout of the clutter will make the transmitter-receiver array stay before the clutter. This setup did not appear in any existing time-reversal reports. It is necessary to investigate whether this kind of clutter layout can work.

7.3.1 Test of Clutter Layout Requirement in the Time-Reversal Method

Here, we try two kinds of clutter layouts and check the performance of the time-reversal method in these two setups respectively. The simulation tool is Wavenology EM. Package.

Fig. 7.4 shows the simulations setup. All the simulations have the same geometry layout. The source is an ideal electric dipole with $\hat{z}$ polarization at $(2200, 0, 0)$ mm. The target is a PEC sphere with radius 150 mm. Its center is located at $(-1000, 0, 0)$ mm. There is a wall ($\epsilon_r = 2.5$) working as clutter with two diagonal corners at $(-100, -1000, -400)$ and $(100, 1000, 400)$ mm. The difference of the two setups is the position of the receiver array. One has all receivers and source staying at the same side of the clutter, as shown in Fig. 7.4(c); while the other puts the receivers and source at the opposite side of the clutter, as shown in Fig. 7.4 (d). All receivers
have the same distance of 20 mm to the surface of the wall. We excite a 1st order Blackman-Harris Window (BHW) pulse on the source. The total simulation time window is 30 ns. The simulation background medium is air.

For the time-reversal simulation, Three steps are proceeded. First, we simulate the case with setup as shown in Fig. 7.4 (a). The $E_z$ component on the receivers are recorded. The results are regarded as the total field ($E_{\text{tot}}^i(t)$) (here, $i$ is the receiver index). Then we change the material of the target to the background medium and re-simulate the case with the same mesh grid, time step and time window. The $E_z$ component recorded in this case are regarded as the incident field ($E_{\text{inc}}^i(t)$). Thus, the scattered field from the target can be calculated as $E_{\text{sct}}^i(t) = E_{\text{tot}}^i(t) - E_{\text{inc}}^i(t)$.

Finally, we remove the original source, reverse the time of the $E_{\text{sct}}^i(t)$; put $\hat{z}$ polarized ideal electric dipole source at the original corresponding receiver positions and excite them.

Fig. 7.5 (a) shows the $E_{\text{inc}}^z(t)$ and $E_{\text{tot}}^z(t)$ received by receiver 6 for the setup Fig. 7.4 (c). Here, receiver 6 is the center receiver in the receiver-array. Fig. 7.5 (b) shows the scattered $E_z(t)$ on receiver 6 and the excitation pulse on source 6. Here, source 6 is has the same position of receiver 6. The excitation pulse is time-reversed $E_z(t)$.

Fig. 7.6 shows the results for two different setups. Obviously, when receivers and source staying at the same side of the clutter case, clear focus on the target is observed. On the contrary, no focus on the target is observed when receivers and source staying at the different sides.

7.3.2 Chamber Simulation Setup in the Time Reversal Method

For our chamber simulation, the PIFA antennas work as both transmitter and receiver. Thus, from the above simulations, there should be clutters before the antennas. We design a setup for the time-reversal simulation on the chamber. As shown in
Fig. 7.7, there is a dielectric wall \((\epsilon_r = 10)\) before each antenna array. The thickness of the wall is 2 mm. The distance of the dielectric wall to the antenna array is 3 mm.

The dielectric wall before the antenna is equivalent to changing the wave impedance before the antenna, we need to verify whether the antenna can keep the similar performance as the original design. Fig. 7.8 shows the clutter do not affect the \(|S_{11}|\) too much, still work around frequency 2.8 GHz.

Another consideration is that the PIFA antenna used in this inversion system is a narrow band antenna. For the time-reversal method, it is suggested to use wide-band signal. Therefore, we still need to verify whether the time-reversal method can be implemented in our inversion chamber with narrow band antenna. Our antennas work in a narrow band around 2.8 GHz. To make the antenna radiate a maximum energy around 2.8 GHz. We setup a transient excitation pulse on the source antenna which has a maximum energy around 2.8 GHz, i.e., the maximum frequency range of the transient excitation pulse should be 8.8 GHz for the 1* order BHW pulse. This will significantly increase the computational cost. However, the time-reversal method only need three simulations and is still acceptable.

We setup a single target case to verify the performance of the time-reversal method in our chamber environment. The target is a cube with size \(12 \times 12 \times 12 \text{ mm}^3\) and relative permittivity 60. The cube center is at \((-20, -20, 20)\) mm, as shown in Fig. 7.9 (a). For the original source in the total field and incident field cases is antenna 5. The sources in the time-reversal simulation are all other antennas except antenna 5. Fig. 7.9 (b) shows the strongest focusing in the simulation. Fig. 7.9 (c) shows this strongest focusing is exactly at the cube. Through this simulation, we know the time-reversal method can be used to find the target position in our inversion chamber.
7.4 The Multi-Domain DTA-BIM Method

The time-reversal method can help to find focus on the targets. If we can locate the approximate position and the size of the inversion target, we can inverse the focus region only. Therefore, the computational cost of the inversion can be reduced. Here, we propose a Multi-Domain DTA-BIM method to fit this situation. We use a 2D case to illustrate the scheme of this method.

For example, the inversion domain is split into multiple sub-domains, index them from 1 to $K$, as shown in Fig. 7.10. Assuming there are only two non-uniform size focuses at the left-top and right-bottom corners respectively. Because there are no focuses in the sub-domain $2$ to $K - 1$, we can think there are no induced currents in these regions. Thus the electric profile of these regions can be considered as the same as the background material and do not need to solve. We only need to inverse the unknown in the sub-domain 1 and $K$.

Recalling the discretized DTA method in the (5.15), we can re-organize it according to the sub-domain definition,

$$
j \omega \varepsilon_b \Delta V \sum_{k=1}^{K} \sum_{n=1}^{N_k} G_u(r_m, r_{nk}) \cdot E(r_{nk}) \chi(r_{nk}) = V_{m}^{sct} \tag{7.1}
$$

where $\Delta V$ and $\chi(r)$ has the same definition as that in (5.15). $k$ is the sub-domain index, $K$ is the total number of the sub-domains. $r_{nk}$ is the position of the $n$th cell in the sub-domain $k$. $N_k$ is the number of cell in the sub-domain $k$. Due to the electrical profile in the non-focusing sub-domains is known, we only need to solve the unknowns in the focused sub-domains. (7.1) can be re-written as

$$
j \omega \varepsilon_b \Delta V \sum_{k=s,t, \ldots}^{N_k} \sum_{n=1}^{N_k} G_u(r_m, r_{nk}) \cdot E(r_{nk}) \chi(r_{nk}) = V_{m}^{sct} \tag{7.2}
$$
where $s, t, ...$ is the index of the focused sub-domain. Therefore, the matrix $A$ and $\chi$ (used in $A\chi = V^{sct}$) coming from (7.2) must be smaller than that from (5.16). Similarly, because the cells in the non-focusing sub-domains do not contribute any induced currents on the whole inversion domain, the computation cost of each $E(\mathbf{r})$ can be reduced from $N$ to $N_s$ (here, $N_s$ is the total number of the unknown in the focused sub-domains). Thus, the total computation cost for the DTA-BIM method is reduced.

### 7.5 Numerical Example of the Multi-Domain DTA-BIM Method

Here, we will use the two $\lambda/2$ cubes case shown in chapter 6 to test the performance of the multiple domains DTA-BIM method. In our simulation, the antenna array is fed by lumped port, and the recorded data is the lumped port voltage.

The first step is to locate the focus region in the chamber. We feed antenna 5 by a lumped port (shown in Fig. 7.7 (a)) with a $1^{st}$ order BHW transient pulse (bandwidth is $0.1 - 9$ GHz), and record the transient voltage on all the lumped ports. The data from without cubes case is the incident voltage ($V^{inc}(t)$); The data from with cubes case is the total voltage ($V^{tot}(t)$); the scattered voltage is calculated by $V^{sct}(t) = V^{tot}(t) - V^{inc}(t)$. Then we reverse the time sequence of the $V^{sct}(t)$ and send these $V^{sct}(t)$ to each lumped port. In the simulation, we use snapshot to find the focus. Fig. 7.11 shows the simulation results. There are two focuses in the chamber.

With these two focused regions, we define two sub-domains, the size of the sub-domains are $(-24, -24, -24) - (4, 0, 24)$ and $(-4, 0, -24) - (24, 24, 24)$ mm$^2$ respectively. The layout of the the sub-domains is shown in Fig. 7.12.

Then we use 7.2 to implement the inversion. The inversion result is shown in Fig. 7.13. As can be seen, the inversion result through the multiple domains DTA-BIM method is almost the same as by the single domain DTA-BIM method. Therefore, we know the multi-domain DTA-BIM method can work in our inversion
7.6 Conclusion

We introduce the time-reversal method as preprocessing method to estimate the target position and size in our inversion method. With these known focus information, a multi-domain DTA-BIM method is proposed to reduce computational cost. The numerical result shows the multiple domains DTA-BIM method works in our chamber situation.
Figure 7.2: (a) Iso-surface plot of the breast skin in the 3D FDTD simulation. The dash lines is the positions of the planar 2D receiver array for the virtual TRM. A single transmitter (center element of the TRM) is at \((x, y) = (2.96, 0)\) cm. (b) Top view of the breast model \((yz\) slice at \(x = 2.96\) cm), the white dot in the breast is the tumor. (c) Coronal view of the breast model \((xy\) slice at \(z = 2.3\) cm), the white dot in the breast is the tumor. The darker regions are denser fibroglandular tissues. To increase the contrast with the background and improve visibility, the artificial skin layer and tumor are shown in white [37].
Figure 7.3: $yz$ slices with the strongest focus for the normalized time-reversed $E_x$ field scattered from the target located at $(2.96, 0.8, 2.3)$ cm [37].
Figure 7.4: Time-reversal simulation setup for clutter’s layout testing. (a) The source, clutter (wall) and target (angle view). (b) The transient excitation pulse. (c) The receivers are located in the $-x$ region (before the clutter). (d) The receivers are located in the $+x$ region (behind the clutter).
Figure 7.5: Signals in the time-reversal simulations. (a) The received $E_z$ on receiver 6 (the center receiver in the receiver-array). (b) The scattered $E_z$ on receiver 6 and the excitation pulse on source 6.

Figure 7.6: Focus in the time-reversal simulations. (a) Strong focus in case Fig. 7.4(c). (b) No focus in case Fig. 7.4(d).
Figure 7.7: (a) Inversion chamber with clutter (angle view). The number aside the antenna is the antenna index in the simulation. (b) Inversion chamber with clutter (top view).

Figure 7.8: $|S_{11}|$ for antenna 5. The position of antenna 5 is shown in Fig. 7.7 (a).
Figure 7.9: (a) The time-reversal simulation setup for a single dielectric cube in the chamber with clutter. (b) The electrical field snapshot for the strongest focus in the simulation. (c) The position and the size of the focus in the chamber.
Figure 7.10: The sub-domain definition (indexed from 1 to $K$) for the original inversion domain. The sub-domain with shadow means there is focus in this sub-domain.
Figure 7.11:  (a) The time-reversal simulation setup for two $\lambda/2$ cubes in the chamber with clutter. (b) A very strong focus at the top region of chamber. (c) A weak focus at the bottom region of chamber. (d) The overlap of the chamber and the strong focus.

Figure 7.12:  The two target sub-domains (the regions with the green and blue color) in the inversion for the two $\lambda/2$ cubes case (the case 2 in chapter 6.)
Figure 7.13: (a) Convergence curve of the inversion for the two $\lambda/2$ cubes in the chamber through the Multi-Domain DTA-BIM method. (b) Cross sections of the reconstructed relative permittivity profile for the bottom cube. (c) Cross sections of the relative permittivity profile for the top cube. (d) Iso surface of the reconstructed relative permittivity ($\varepsilon_r = 5.8$ for the bottom cube, $\varepsilon_r = 8.0$ for the top cube).
8

Conclusion and Future Work

8.1 Conclusion

We have developed a microwave tomographic imaging system prototype to evaluate the performance of 3D microwave imaging through the general DTA-BIM method with experimental data. We also extend this system to a layered-medium setup to test the performance of 3D microwave imaging through the general DTA-BIM method when objects are buried in a multilayered medium. Such a system and data sets are firstly introduced for 3D microwave imaging in a layered background medium. The inversion results show the general DTA-BIM method works well and can obtain a resolution of a quarter wavelength of the background medium, which is much higher than any other reported experimental MWI systems [54, 34, 10, 58, 57, 59].

We also demonstrate the first application of the Diagonal Tensor Approximation to calculate the scattered fields from arbitrary objects in a non-canonical inhomogeneous background. This extends the application domain of the DTA method from previous cannonical background media, such as homogeneous and layered-medium background, to arbitrary inhomogeneous background media. This method relies
on the numerical computation of the Green’s functions. We take advantage of the symmetry of the inversion configuration and the reciprocal property of the Green’s function to reduce the number of the simulation cases. Furthermore, we develop a necessary formulation to relate the wave port voltage at an antenna to the fields in the computation domain. Extensive numerical results show that this method can accurately obtain the scattered fields from arbitrary objects in a non-canonical inhomogeneous background.

The promising inversion results based on this modified DTA-BIM method show that a nonlinear inversion can be implemented in an arbitrary non-canonical inhomogeneous background, significantly extending the EM inversion applications.

We introduce a time-reversal method as pre-processing step to reduce the inversion region. A Multi-Domain DTA-BIM method is proposed to cooperate with the reduced size inversion region, which reduces the computational cost of the inversion method and makes it more applicable for rapid response applications. Numerical results based on this Multi-Domain DTA-BIM method show this idea is workable with our inversion setup.

8.2 Future Work

First, though our modified DTA-BIM method provides promising inversion results for an arbitrary non-canonical inhomogeneous background, all current data sources are synthetic. It is desirable to test this method with experimental data in the future.

Second, we introduce a time-reversal method in our inversion to reduce the inversion region. We employ wall-like clutter in our simulation to get focus. However, this type of clutter does not fit some applications, such as the breast cancer detection. Therefore, more investigations should be carried out to find more friendly clutter, including the material, shape and layout in the system. Another interesting topic in the time-reversal method is the focus localization. For the snapshots shown in
Fig. 7.6 (a), Fig. 7.9 (b) and Fig. 7.11 (b) and (c), we may detect fake focuses. The fake focuses produce unnecessary computation domains in the Multi-Domain DTA-BIM method. Thus, a more effective and robust focus-localization method is needed in the future.

Third, though we show the DTA-BIM method can work in our inversion cases with the experimental data or the synthetic data. There are not tests for the cases in which the number of unknown in the inversion domain is smaller than that of measured data. With the introduction of the time-reversal method and the Multi-Domain DTA-BIM method, the inversion method may become an over-determined problem. Hence, more tests should be carried out on this topic.
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Biography

Personal Information

Name: Mengqing Yuan
Place of birth: Guangxi, China.
Date of birth: Nov. 2, 1973

Education

Ph.D., Electrical and Computer Engineering, Duke University, USA, April 2011.
B.S., Electrical Power Engineering, Huazhong University of Science and Technology, China, July 1995.

Peer-Reviewed Publications


12. C. Yu, M. Yuan, Q. H. Liu, J. Stang, Y. Zhang, R. T. George, G. A. Ybarra,
