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Scattering of an ultrasoft pion and the $X(3872)$

Eric Braaten,[1,2] H.-W. Hammer,[2] and Thomas Mehen[3]

[1]*Department of Physics, The Ohio State University, Columbus, Ohio 43210, USA*

[2]*Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, 53115 Bonn, Germany*

[3]*Physics Department, Duke University, Durham, North Carolina 27708, USA*

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The identification of the $X(3872)$ as a loosely-bound charm-meson molecule allows it to be described by an effective field theory, called XEFT, for the $D^*\bar{D}$, $D\bar{D}^*$, and $D\bar{D}\pi$ sector of QCD at energies small compared to the pion mass. We point out that this effective field theory can be extended to the sector that includes an additional pion and used to calculate cross sections for the scattering of a pion and the $X(3872)$. If the collision energy is much smaller than the pion mass, the cross sections are completely calculable at leading order in terms of the masses and widths of the charm mesons, pion masses, and the binding energy of the $X(3872)$. We carry out an explicit calculation of the cross section for the breakup of the $X(3872)$ into $D^{*+}\bar{D}^{*0}$ by the scattering of a very low energy π^+.

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I. INTRODUCTION

The $X(3872)$ is a remarkable hadron that was discovered by the Belle Collaboration in 2003 in the exclusive decay $B^\pm \to K^\pm X$ [1] and then quickly confirmed by the CDF Collaboration through inclusive production in $p\bar{p}$ collisions [2]. The most compelling interpretation of this particle is a loosely-bound charm-meson molecule whose quantum numbers are $J^{PC} = 1^{++}$ and whose particle content is [3–8]

$$X = \frac{1}{\sqrt{2}}(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}). \quad (1)$$

Its binding energy, E_X, relative to the $D^{*0}\bar{D}^0$ threshold is extremely small, less than 1 MeV. As a consequence, it has universal properties that are determined by E_X or, equivalently, by the large $D^{*0}\bar{D}^0$ scattering length in the even charge-conjugation channel [9].

Fleming, Kusunoki, Mehen, and van Kolck have constructed an effective field theory called XEFT that can be used to calculate corrections to the universal properties of the $X(3872)$ systematically [10]. In this effective field theory, the elementary constituents are the neutral charm mesons D^0, D^{*0}, $\bar{D}^0$, and $\bar{D}^{*0}$ and the neutral pion π^0. There are two types of interactions: contact interactions between the pairs of charm mesons $D^{*0}\bar{D}^0$ and $D^0\bar{D}^{*0}$ and the pion transitions $D^{*0} \leftrightarrow D^0\pi^0$ and $\bar{D}^{*0} \leftrightarrow \bar{D}^0\pi^0$. XEFT was designed to describe systems consisting of $D^{*0}\bar{D}^0$, $D^0\bar{D}^{*0}$, and $D^0\bar{D}^0\pi^0$ with total energy very close to the $D^{*0}\bar{D}^0$ threshold, including the $X(3872)$. In Ref. [10], the authors presented power-counting arguments that guarantee that the pion transitions can be treated perturbatively. They calculated the decay rate of the $X(3872)$ into $D^0\bar{D}^0\pi^0$ to next-to-leading order in XEFT. Fleming and Mehen have applied XEFT at leading order to decays of the $X(3872)$ into the P-wave charmonium states χ_{cJ} and one or two pions [11]. They factored the amplitude for the decay into a long-distance XEFT matrix element and a short-distance factor that can be calculated using heavy-hadron chiral perturbation theory (HHχPT).

The original formulation of XEFT has a rather limited domain of validity. One limitation is that it does not describe charged charm mesons. This limits its domain of applicability to the energy region within a few MeV of the $D^{*0}\bar{D}^0$ threshold, because the $D^{*+}D^-$ threshold is higher only by about 8 MeV. This limitation is easily removed by generalizing XEFT to include the charged charm mesons D^+, D^{*+}, D^-, and D^{*-} and the charged pions π^+ and π^-. This generalization extends the domain of applicability of XEFT to all energies relative to the $D^{*0}\bar{D}^0$ threshold that are small compared to the $D^* - D$ mass difference, which is approximately equal to the pion mass $m_\pi \approx 135$ MeV. If the P-wave charmonium state $\chi_{c1}(2P)$, whose quantum numbers are also 1^{++}, lies in this region, it may be necessary to also include it as an explicit degree of freedom in order to carry out accurate calculations beyond leading order.

Canham, Hammer, and Springer have obtained universal results for scattering of charm mesons with the $X(3872)$ that depend only on the $D^{*0}\bar{D}^0$ scattering length [12]. They calculated the S-wave phase shifts for D^0X scattering and for $D^{*0}X$ scattering by solving the three-body problem for the charmed mesons. The D^0X scattering length and the $D^{*0}X$ scattering length are both about an order of magnitude larger than the $D^{*0}\bar{D}^0$ scattering length. These universal results can be regarded as the predictions of XEFT at zeroth order in the pion transitions. Thus XEFT can be applied to systems consisting of three charm mesons with energy close to the appropriate threshold.

In this paper, we point out that XEFT can also be applied to systems consisting of $D^*\bar{D}^*$, $D^*\bar{D}\pi$, $D\bar{D}^*\pi$, and $D\bar{D}\pi\pi$ with total energy close to the $D^*\bar{D}^*$ threshold. The scattering processes whose cross sections are calculable using XEFT include πX elastic scattering and $\pi X \to D^*\bar{D}^*$ at collision energy much smaller than m_π. In Sec. II, we summarize the case for the $X(3872)$ as a loosely-bound charm-meson molecule and we describe some of its universal properties. In Sec. III, we calculate the cross section for $\pi^+ X \to D^{*+}\bar{D}^{*0}$. In Sec. IV, we discuss elastic $\pi^+ X$ scattering. Our results are summarized in Sec. V.

II. THE *X*(3872)

We begin by summarizing the case for the $X(3872)$ as a loosely-bound charm-meson molecule whose particle content is given in Eq. (1). The only experimental information that is necessary to make this identification is the determination of its quantum numbers and the measurements of its mass. The quantum numbers of the $X(3872)$ can be identified as 1^{++} by combining the following information:

(i) the observation of its decay into $J/\psi\gamma$, which implies that it is even under charge conjugation [13],
(ii) analyses of the momentum distributions from its decay into $J/\psi\pi^+\pi^-$, which imply that its spin and parity are 1^+ or 2^- [14],
(iii) either the observation of its decays into $D^0\bar{D}^0\pi^0$ [15], which disfavors 2^- because of angular momentum suppression, or the observation of its decay into $\psi(2S)\gamma$ [16], which disfavors 2^- because of multipole suppression.

A recent analysis of decays into $J/\psi\pi^+\pi^-\pi^0$ by the *BABAR* Collaboration favors the quantum numbers 2^{-+}, but does not exclude the assignment 1^{++} [17]. In the absence of compelling experimental evidence to the contrary, we will assume that the quantum numbers of the $X(3872)$ are 1^{++}.

There are claims in the literature that, even if the quantum numbers of the $X(3872)$ are 1^{++}, there are difficulties in interpreting it as a molecule. The *BABAR* Collaboration has observed a branching fraction for the decay mode $\psi(2S)\gamma$ that is larger than that for $J/\psi\gamma$ [16]. They claimed that this result is generally inconsistent with a purely molecular interpretation. This conclusion seems to be based on a specific model by Swanson in which the $X(3872)$ is a molecule with explicit $J/\psi\rho$ and $J/\psi\omega$ components [8]. Swanson's model predicts that the branching fraction for $\psi(2S)\gamma$ is more than 2 orders of magnitude smaller than that for $J/\psi\gamma$ [18]. However, the decay to $\psi(2S)\gamma$ is sensitive to short-distance components of the $X(3872)$ wave function. The calculated rate for the decay into $\psi(2S)\gamma$ in Ref. [18] is not a universal prediction for all molecular models. Universal predictions (such as the calculations of this paper) rely only on the long-distance behavior of the $X(3872)$ wave function. The proper conclusion from the observation of the decay into $\psi(2S)\gamma$ in Ref. [16] is that Swanson's model is excluded, not the molecular hypothesis. There is also a claim that the prompt production rate of the $X(3872)$ observed at the Tevatron is larger than a theoretical upper bound on the production rate for a molecule by orders of magnitude [19]. However, the derivation of that upper bound was based on an incorrect assumption about the region of support for the integral over the relative momentum in the production amplitude [20]. If that assumption is relaxed, the upper bound is compatible with the observed production rate at the Tevatron.

Measurements of the mass of the $X(3872)$ in the $J/\psi\pi^+\pi^-$ decay channel [21] imply that its energy relative to the $D^{*0}\bar{D}^0$ threshold is

$$M_X - (M_{D^{*0}} + M_{D^0}) = -0.42 \pm 0.39\ \text{MeV}. \qquad (2)$$

The mass of the $X(3872)$ has also been measured in the $D^0\bar{D}^0\pi^0$ decay channel [15] and in the $D^{*0}\bar{D}^0$ decay channel [22]. Measurements in the $D^0\bar{D}^0\pi^0$ channel are biased towards larger values by a contribution from a threshold enhancement of $D^{*0}\bar{D}^0$ above the threshold [23]. Measurements in the $D^{*0}\bar{D}^0$ channel are further biased towards larger values by the analysis procedure that assigns an energy above threshold to $D^0\bar{D}^0\pi^0$ or $D^0\bar{D}^0\gamma$ events below the threshold [24]. The quantum numbers 1^{++} imply that the $X(3872)$ has an S-wave coupling to $D^{*0}\bar{D}^0$. Its tiny energy relative to the $D^{*0}\bar{D}^0$ threshold implies that it is a resonant coupling. This system is therefore governed by the universal behavior of particles with short-range interactions and an S-wave threshold resonance that is predicted by nonrelativistic quantum mechanics [9]. The universal properties of the system are determined by the pair scattering length a only. If $a > 0$, one of the universal properties is the binding energy: $E_X = 1/(2\mu a^2)$, where μ is the reduced mass of the pair. Another universal property is the root-mean-square separation of the constituents: $r_X = a/\sqrt{2}$. Identifying the mass difference in Eq. (2) as $-E_X$, we find that the charm mesons in the $X(3872)$ have an astonishingly large rms separation: $r_X = 4.9^{+13.4}_{-1.4}$ fm.

The universal properties of an S-wave threshold resonance in the 2-body sector can be derived from the universal transition amplitude for scattering of the constituents, which is a function of the total energy E of the pair relative to the scattering threshold in their center-of-momentum frame:

$$\mathcal{A}(E) = \frac{2\pi/\mu}{-1/a + \sqrt{-2\mu E - i\epsilon}}. \qquad (3)$$

If a is positive, this amplitude has a pole at the energy of the bound state: $E = -E_X$, where $E_X = 1/(2\mu a^2)$. The rate for a process involving the bound state can be calculated diagrammatically by introducing a vertex for the coupling of the bound state to its constituents. Up to an irrelevant phase, the coupling constant G for the vertex is the square root of the residue of the pole in the amplitude in Eq. (3) at $E = -E_X$:

$$G = (8\pi^2 E_X/\mu^3)^{1/4}. \qquad (4)$$

Taking into account the amplitude $1/\sqrt{2}$ for the constituents of X to be $D^{*0}\bar{D}^0$ or $D^0\bar{D}^{*0}$ from Eq. (1), the vertex factor for the coupling of X to either pair of charm mesons is $G/\sqrt{2}$.

III. CHARM MESON SCATTERING INTO $X\pi$

The masses of many different particles enter into the cross section that we will calculate. It is therefore useful to introduce a compact notation for these masses. We denote the masses of D^{*0} and D^{*+} by M_{*0} and M_{*+}, or by M_* if the difference can be neglected. We denote the masses of D^0 and D^+ by M_0 and M_+, or by M if the difference can be neglected. We denote the masses of π^0 and π^+ by m_0 and m_+, or by m_π if the difference can be neglected. The mass of the $X(3872)$ is $M_X = M_{*0} + M_0 - E_X$, where E_X is its binding energy. The differences between the D^* mass and the sum of the D and π masses are denoted by

$$\delta_{00} = M_{*0} - M_0 - m_0 = 7.14 \pm 0.07 \text{ MeV}, \qquad (5)$$

$$\delta_{0+} = M_{*+} - M_0 - m_+ = 5.85 \pm 0.01 \text{ MeV}. \qquad (6)$$

We will refer to the energy scale set by these mass differences as δ.

In XEFT, there is an extremely small energy scale: the binding energy of the $X(3872)$, which is less than 1 MeV. In the generalization of XEFT that includes charged charm mesons and charged pions, there is also the relatively small energy scale δ. The reason the D^* mesons have relatively long lifetimes is because δ is small compared to m_π, which makes their hadronic decay rates comparable to their electromagnetic decay rates. A pion with momentum comparable to m_π is generally referred to as a *soft pion*. We will refer to a pion with kinetic energy comparable to or smaller than δ as an *ultrasoft pion*. The momentum scale for an ultrasoft pion is $\sqrt{m_\pi \delta}$, which is about 30 MeV.

Two types of small mass ratios that appear in XEFT calculations are the ratio of a pion mass to a charm meson mass, such as $m_0/M_0 = 0.072$ and $m_0/M_{*0} = 0.067$, and the ratio of a mass difference δ to a pion mass, such as $\delta_{0+}/m_0 = 0.043$ and $\delta_{00}/m_0 = 0.053$. The numerical values of these ratios are all about 5%. We therefore treat all the ratios $m_\pi/M_{(*)}$ and δ/m_π as order ϵ. The ratio of the binding energy of the $X(3872)$ to its mass is much smaller: $E_X/M_X = 0.0001$ for $E_X = 0.42$ MeV. We will treat this ratio as $O(\epsilon^3)$. We will organize our calculations to include all terms suppressed by $O(\epsilon)$, but we will feel free to drop corrections of $O(\epsilon^2)$. Thus a multiplicative factor of the mass of $X(3872)$ can be approximated as $M_X \approx M_{*0} + M_0$. The reduced mass of D^{*0} and $\bar{D}^0$ can be expressed as $\mu \approx MM_*/M_X$, since the corrections are $O(E_X/M_X)$ and hence $O(\epsilon^3)$ in our power counting. Likewise, the reduced mass of π^+ and X is $\mu_{\pi^+X} \approx M_X m_+/(2M_*)$ and the reduced mass of π^+ and D^0 is $\mu_{\pi^+D} \approx Mm_+/M_*$. The corrections to these approximations are suppressed by $\delta/M_{(*)}$ and hence $O(\epsilon^2)$.

We would like to calculate the cross section for the scattering of an ultrasoft pion and the $X(3872)$ into a pair of spin-1 charm mesons $D^*\bar{D}^*$. We find it convenient to first calculate the cross section for the time-reversed process: the formation of $X(3872)$ by the reaction $D^*\bar{D}^* \to X\pi$. At leading order in the pion transitions, this reaction proceeds only through the Feynman diagram in Fig. 1. Our notation for the lines representing the X, the charm mesons, and the pion is such that the number of solid lines and dashed lines is conserved at every vertex. The pion is ultrasoft if the *collision energy* of the D^* and $\bar{D}^*$, which is their kinetic energy in the center-of-mass frame, is small compared to m_π. In the diagram in Fig. 1, the virtual D is off its energy shell by an amount that is small compared to m_π. Thus this reaction is within the region of validity of XEFT. The reactions $D^*\bar{D} \to X\pi$ and $D\bar{D} \to X\pi$ can proceed through diagrams similar to the one in Fig. 1 except that the number of dashed lines is not conserved at every vertex. The virtual charm meson that is exchanged must therefore be off its energy shell by an amount of order m_π. Thus these reactions can not be described by XEFT.

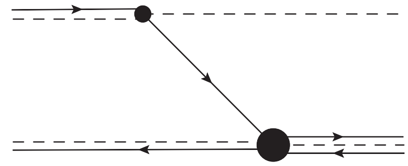

FIG. 1. Feynman diagram for the reaction $D^{*+}\bar{D}^{*0} \to X\pi^+$. Spin-1 and spin-0 charm mesons are represented by solid-dashed and solid lines, respectively. Pions are represented by dashed lines. The $X(3872)$ is represented by a solid + dashed + solid line.

We proceed to calculate the cross section for the reaction $D^{*+}\bar{D}^{*0} \to X\pi^+$. We take the incoming momenta of the D^{*+} and $\bar{D}^{*0}$ to be $\pm\boldsymbol{p}$, and we take the outgoing momenta of the π^+ and X to be $\pm\boldsymbol{k}$. We take the collision energy $E_{\rm cm} = p^2/M_*$ to be small compared to m_π, so the π^+ is ultrasoft. The T-matrix element is

$$\mathcal{T} = \frac{(G/\sqrt{2})(g/f_\pi)(\boldsymbol{k}\cdot\boldsymbol{\epsilon}_+)(\boldsymbol{\epsilon}_X^*\cdot\boldsymbol{\epsilon}_0)}{E_X + [\boldsymbol{p} - (M_*/M_X)\boldsymbol{k}]^2/(2\mu)}, \qquad (7)$$

where $\boldsymbol{\epsilon}_X^*$, $\boldsymbol{\epsilon}_+$, and $\boldsymbol{\epsilon}_0$ are the polarization vectors of the X, D^{*+}, and $\bar{D}^{*0}$, respectively. The $D^*D\pi$ coupling constant g/f_π can be determined from the measured partial width for $D^{*+} \to D^0\pi^+$:

$$g/f_\pi = 2.8 \times 10^{-4} \text{ MeV}^{-3/2}. \qquad (8)$$

The coupling constant g defined here differs from the dimensionless coupling constant $g = 0.6$ used in Refs. [10,11] by a factor of $\sqrt{2m_\pi}$. The simple expression in the denominator of Eq. (7) for the propagator of the virtual D^0 can be obtained most easily by exploiting the approximate Galilean invariance of this reaction. Galilean invariance holds when the sum of the masses is exactly equal in the initial and final states. For the reaction $D^{*+}\bar{D}^{*0} \to X\pi^+$, the sum of the masses decreases between the initial and final states by only about $\delta_{0+} \approx 5.85$ MeV, which is about 1 part in 700. We can take advantage of the approximate Galilean invariance by calculating the D propagator in the rest frame of the $X(3872)$. The momentum $\boldsymbol{p} - (M_*/M_X)\boldsymbol{k}$ in the denominator of Eq. (7) is just the momentum transferred through the virtual D meson in that frame. The propagator calculated in the center-of-mass frame involves the sum or difference of three kinetic energies. Those three terms can be combined to give the single kinetic energy in the denominator of Eq. (7) plus terms that have been neglected because they are suppressed by a factor of $E_X/M_X = O(\epsilon^3)$.

Squaring the T-matrix element in Eq. (7) and averaging and summing over the spins of the D^{*+}, $\bar{D}^{*0}$, and X, we get

$$\frac{1}{9}\sum_{\text{spins}}|\mathcal{T}|^2 = \frac{2G^2(g/f_\pi)^2\mu^2k^2}{3[2\mu E_X + p^2 - 2(M_*/M_X)(\boldsymbol{p}\cdot\boldsymbol{k}) + (M_*/M_X)^2k^2]^2}. \tag{9}$$

After integrating over the momenta of the outgoing X and π, our final result for the reaction rate is

$$v_{\text{rel}}\sigma[D^{*+}\bar{D}^{*0} \to X\pi^+] = \frac{2G^2(g/f_\pi)^2\mu^2\mu_{\pi X}k^3}{3\pi\Delta(p^2, (M_*^2/M_X^2)k^2, -2\mu E_X)}, \tag{10}$$

where Δ is the triangle function: $\Delta(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$. The pion momentum k is determined by conservation of energy:

$$\frac{k^2}{2\mu_{\pi X}} = \delta_{0+} + E_X + \frac{p^2}{M_*}. \tag{11}$$

By completing the square in the variable p^2 in the denominator of Eq. (10) and dividing by the relative velocity $v_{\text{rel}} = 2p/M_*$, we obtain our final expression for the cross section:

$$\sigma[D^{*+}\bar{D}^{*0} \to X\pi^+] = \frac{G^2(g/f_\pi)^2M_XM_*^2m_\pi k^3/(24\pi p)}{[p^2 - (M_*/2M)(m_\pi\delta_{0+} - M_XE_X)]^2 + M_*^2m_\pi\delta_{0+}M_XE_X/M^2}. \tag{12}$$

At small momentum p, the cross section diverges as $1/p$ but the reaction rate $v_{\text{rel}}\sigma[D^{*+}\bar{D}^{*0} \to X\pi^+]$ is well behaved. The asymptotic behavior of the cross section for large momentum p is

$$\sigma[D^{*+}\bar{D}^{*0} \to X\pi^+] \to \frac{(g/f_\pi)^2M_XM(m_\pi/\mu)^{5/2}(2M_XE_X)^{1/2}}{12p^2}. \tag{13}$$

In Fig. 2, we show the reaction rate $v_{\text{rel}}\sigma[D^{*+}\bar{D}^{*0} \to X\pi^+]$ as a function of the $D^{*+}\bar{D}^{*0}$ collision energy $E_{\text{cm}} = p^2/M_*$ for several values of the binding energy E_X of the $X(3872)$. From the expression in Eq. (12), we can see that there are two competing momentum scales that govern the behavior of the reaction rate at small p: $(m_\pi\delta_{0+})^{1/2} \approx 30$ MeV and $(M_XE_X)^{1/2}$, which is approximately 60 MeV if $E_X = 1$ MeV. If the binding energy E_X decreases to less than about $m_\pi\delta_{0+}/M_X \approx 0.2$ MeV, the peak in the reaction rate shifts from zero collision energy to a positive value near $(m_\pi\delta_{0+} - M_XE_X)/(2\mu)$. This is the collision energy for which there is no momentum transferred through the virtual D^0. The peak at nonzero collision energy is evident in the curve for $E_X = 0.125$ MeV in Fig. 2.

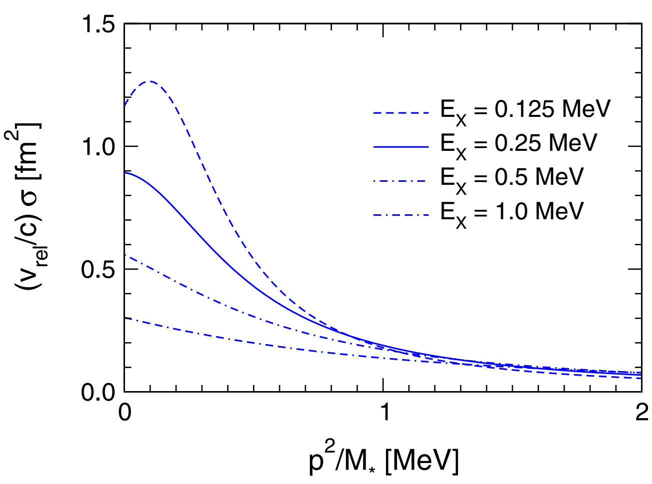

FIG. 2 (color online). Reaction rate $v_{\text{rel}}\sigma$ for $D^{*+}\bar{D}^{*0} \to X\pi^+$ as a function of the collision energy $E_{\text{cm}} = p^2/M_*$ for several values of the binding energy E_X.

The reaction rate for $D^{*0}\bar{D}^{*0} \to X\pi^0$ can be calculated in a similar way. In addition to the Feynman diagram in Fig. 1, which involves exchange of a virtual D^0, there is a second diagram that involves exchange of a virtual $\bar{D}^0$. The T-matrix element is therefore the sum of two terms. The term corresponding to the Feynman diagram in Fig. 1 differs from the expression in Eq. (7) by an isospin Clebsch-Gordan factor of $1/\sqrt{2}$. In the energy conservation condition in Eq. (11), δ_{0+} must be replaced by δ_{00}.

We now consider the reaction $\pi^+X \to D^{*+}\bar{D}^{*0}$. We take the incoming momenta of the π^+ and X to be k and we take the outgoing momenta of the D^{*+} and $\bar{D}^{*0}$ to be p. They are related by the conservation of energy condition in

Eq. (11). The collision energy is $E_{\rm cm} = k^2/(2\mu_{\pi X})$. The energy threshold for the breakup process is $\delta_{0+} + E_X$. The cross section can be obtained from the production cross section in Eq. (12) by changing the flux factor from $M_*/(2p)$ to $\mu_{\pi X}/k$ and by changing the phase space factor from $\mu_{\pi X}k/\pi$ to $M_* p/(2\pi)$:

$$\sigma[X\pi^+ \to D^{*+}\bar{D}^{*0}] = \frac{G^2(g/f_\pi)^2 M_X M_*^2 m_\pi kp/(24\pi)}{[p^2 - (M_*/2M)(m_\pi\delta_{0+} - M_X E_X)]^2 + M_*^2 m_\pi \delta_{0+} M_X E_X/M^2}. \tag{14}$$

IV. πX ELASTIC SCATTERING

Another process in the $D\bar{D}\pi\pi$ sector that is calculable using XEFT is $\pi^+ X$ elastic scattering. At leading order in the pion transitions, $\pi^+ X$ elastic scattering proceeds through six one-loop diagrams. The two diagrams in Fig. 3 involve virtual D^{*+} and $\bar{D}^{*0}$ mesons and virtual D^0 and D^- mesons, respectively. There are also four additional diagrams with Weinberg-Tomozawa vertices. One of them is the first diagram in Fig. 3 with the virtual D^{*+} propagator shrunk to a point to obtain a $\pi^+ D^0 - \pi^+ D^0$ vertex. Another one is obtained from the second diagram by shrinking the virtual D^- propagator to a point to obtain a $\pi^+\bar{D}^{*0} - \pi^+\bar{D}^{*0}$ vertex. The other two diagrams involve a $\pi^+ D^{*0} - \pi^+ D^{*0}$ vertex and a $\pi^+\bar{D}^0 - \pi^+\bar{D}^0$ vertex, respectively.

In Ref. [10], the authors carried out a power-counting analysis for XEFT that determined that the $D^* - D\pi$ vertices can be treated perturbatively. They did not consider the Weinberg-Tomozawa (WT) vertices for $D\pi - D\pi$ and $D^*\pi - D^*\pi$. These come from the kinetic term in the HHχPT Lagrangian,

$$\mathcal{L} = {\rm Tr}[H_a^\dagger (iD_0)_{ba} H_b], \tag{15}$$

where the chiral covariant derivative is

$$(D_0)_{ba} = \partial_0 \delta_{ba} - \frac{1}{2f^2}[\pi, \partial_0 \pi]_{ba} + O(\pi^4), \tag{16}$$

and our definitions for the fields H_b, $H_a^\dagger$, and π can be found in Appendix A of Ref. [10]. The WT coupling for D^0 mesons is obtained by plugging the second term in Eq. (16) into Eq. (15), yielding

$$\mathcal{L}_{\rm WT} = \frac{1}{2f^2} P_0^\dagger \pi^- i\overleftrightarrow{\partial}_0 \pi^+ P_0, \tag{17}$$

where P_0 is the field for the D^0. There is a similar coupling for the D^{*0}. Next we rewrite Eq. (17) in terms of nonrelativistic pion fields, as is appropriate for the XEFT Lagrangian. The relativistic fields $\pi^\pm$ when written in terms of nonrelativistic fields are

$$\pi^\pm = \frac{1}{\sqrt{2m_\pi}}(e^{-im_\pi t}\hat{\pi}^\pm + e^{im_\pi t}\hat{\pi}^{\mp\dagger}), \tag{18}$$

where the $\hat{\pi}^\pm (\hat{\pi}^{\pm\dagger})$ denotes a nonrelativistic field that annihilates (creates) $\pi^\pm$ mesons. The XEFT Lagrangian for the WT vertex is

$$\mathcal{L}_{\rm WT} = \frac{1}{2f^2} P_0^\dagger (\hat{\pi}^{+\dagger}\hat{\pi}^+ - \hat{\pi}^{-\dagger}\hat{\pi}^-) P_0, \tag{19}$$

with a similar term for D^{0*} mesons. We have kept only the terms in which the phase factors $e^{\pm i m_\pi t}$ cancel. Other terms describe processes that are outside the range of validity of XEFT. The WT vertex is $O(Q^0)$ in the power counting of Ref. [10]. This means a diagram with a WT vertex is the same order as a diagram obtained by replacing the WT vertex with a virtual D or D^* propagator and two $D^* - D\pi$ couplings, since $D^* - D\pi$ vertices are $O(Q)$ and D and D^* propagators are $O(1/Q^2)$. The one-loop diagrams for $\pi^+ X(3872)$ elastic scattering with a WT vertex are then the same order in Q as the diagrams in Fig. 3. However, applying charge conjugation to Eq. (19), we see that the Feynman rule for $\pi^+ D^{0(*)} \to \pi^+ D^{0(*)}$ has the opposite sign as the Feynman rule for $\pi^+\bar{D}^{0(*)} \to \pi^+\bar{D}^{0(*)}$, so the four diagrams with a WT vertex add up to zero. The only nonvanishing one-loop contribution to $\pi^+ X(3872)$ elastic scattering comes from the diagrams in Fig. 3.

An explicit one-loop calculation of $\pi^+ X$ elastic scattering would be worthwhile as a test of the calculational technology of XEFT. For collision energies below the breakup threshold $\delta_{0+} + E_X$, the scattering amplitude will be real-valued. As the binding energy E_X decreases to 0, the constituents of the $X(3872)$ have a larger and larger probability of being well separated. The cross section near threshold should therefore reduce in this limit to the sum of the cross sections for scattering off the individual constituents.

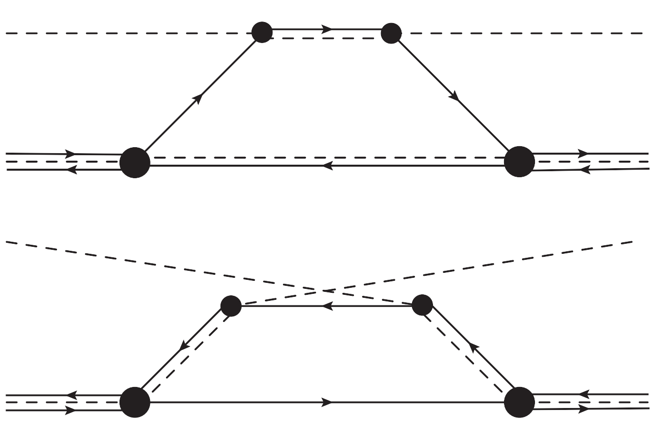

FIG. 3. Two of the six one-loop Feynman diagrams for $\pi^+ X$ elastic scattering. The other four diagrams involve a Weinberg-Tomazawa vertex and add up to zero.

V. SUMMARY

XEFT is an effective field theory that was originally designed to describe systems consisting of $D^{*0}\bar{D}^0$, $D^0\bar{D}^{*0}$, and $D^0\bar{D}^0\pi^0$ with total energy relative to the $D^{*0}\bar{D}^0$ threshold that is small compared to the 8 MeV $D^{*+}D^-$ threshold. The only important energy scale in this effective field theory is the binding energy E_X of the $X(3872)$. XEFT can be generalized to an effective field theory that includes charged charm mesons and charged pions and describes systems consisting of $D^*\bar{D}$, $D\bar{D}^*$, and $D\bar{D}\pi$ with total energy relative to the $D^*\bar{D}$ threshold that is small compared to m_π. In addition to the tiny energy scale E_X, this effective field theory also describes the ultrasoft energy scale δ set by the difference between $D^* - D$ mass splittings and m_π. We have pointed out that XEFT can also be applied to systems consisting of $D^*\bar{D}^*$, $D^*\bar{D}\pi$, $D\bar{D}^*\pi$, and $D\bar{D}\pi\pi$ with total energy relative to the $D^*\bar{D}^*$ threshold that is small compared to m_π. We have used XEFT to calculate the cross section for $\pi^+ X \to D^{*+}\bar{D}^{*0}$ for ultrasoft collision energy. This cross section is completely determined by the masses and widths of the charm mesons, the pion masses, and the binding energy of the $X(3872)$.

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