The Hidden Costs of Central Bank Borrowing

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Abstract

This paper explores a previously overlooked unintended consequence of a private bank accepting Central Bank loans as a lender of last resort. Applying the basic Markowitz Security Model, I explore the potential effect of a private bank accepting a Central Bank loan as a signal of increased risk of investment in that private bank to the private markets. Finding a possibility that private investors will charge a penalty risk premium for having sought Central Bank financing, I consider the effects of this premium in three different game theoretic scenarios, each with a different set of assumptions that could apply in different Economic settings. Depending on the specific environment, possible effects include dependence on Central Bank financing, bankruptcy, or an eventual return to the private financial markets for future funding.

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Table of Contents:

I. Introduction
II. Review of Literature
III. The Central Bank: Monetary Policy Watchdog vs. Lender of Last Resort
IV. Risk Signals: The Effect of Central Bank Borrowing on Investor Portfolio Allocation
V. Private Bank Funding and Source Choices and Conditions
VI. Implications and Limitations
VII. Conclusions and Questions for Further Research

I. Introduction

As financial stability becomes an increasing concern in the public eye, concerns over liquidity risk have driven much of the discussion over the appropriate regulatory response. Banks lend on a much longer time scale than they borrow, and if their ability to borrow is somehow curtailed, a serious risk of default results. In the recent European sovereign debt crisis we have seen US money market investors increasingly avoid commercial paper and securities issued by continental banks. One way to understand this investor pullout is as a type of bank-run by investors. Diamond and Dybvig’s standard model of bank runs implies that bank runs can be avoided through the availability of a Central Bank as a lender of last resort. However, if investors react negatively to receiving information that banks are borrowing from a Central Bank, this solution could fail to materialize in the case of money market funding. In this paper, I consider the effect a private bank borrowing from the Central Bank may have on the availability of investor funds for the bank. I explore in a game theoretic context the effect on a bank’s capital structure equilibrium when they receive Central Bank funding and how banks can avoid a negative feedback loop of Central Bank borrowing, losing private funding, and having to depend on Central Bank funding or lose financial viability.

II. Review of Literature

Diamond and Dybvig (1983) wrote the foundational model of bank runs. They see bank runs as a cooperation game with two separate equilibria: bank run and no bank run. The different outcomes are completely dependent on the expectation of other depositors’ actions. Both federal deposit insurance and Central Banks (as lenders of last resort) can act as a deterrent to runs and to make sure depositors choose to keep their deposits in a bank. Chari and Jagannathan (1988) combine this paper and findings in Bryant (1980) which depend on information-induced shocks, to demonstrate that both new information or something as simple as long queues to withdraw funds from a bank, are sufficient to trigger a bank run. Sleet and Smith (2000) create a highly formal system to describe the safety net of deposit insurance and lenders of last resort, but their model stays restricted to four players: borrowers, lenders, bankers, and government. All of these models remain silent on the question of the role of money market funds in bank financing, as opposed to financing through the Central Bank.
The above literature, while helpful in understanding depositor behavior, does not necessarily apply to money market investors’ behavior, as the literature primary focuses on policy recommendations meant to preclude bank runs. The bank securitization literature started by Myers and Majluf (1984) uses the concept of lemons and information asymmetry to explain how banks would be largely underfunded if they depended solely on deposit-based financing, and James (1988) expands this discussion into how banks can use securities to bridge this gap. The literature on maturity transformation—the process by which longer term assets are financed with shorter term liabilities—adds to this with Thakor (1992), showing how banks create liquidity and are able to gain returns through maturity mismatching. While this literature admits the existence of money market funding, it never directly considers any connection between Central Bank funding to the availability of money market funding.

Despite the wealth of literature on private banking, and numerous papers recommending Central Bank funding as a solution to some market failures or other problems, there are very few papers that address the effects of that Central Bank funding on private banks. Kahane (1977), Koehn and Santomero (1980) and Besank and Kanatas (1993), to name just a sample, all explore the effect of capital requirements on bank asset allocation. Kahn and Santos (2005) model bank regulation, but focus almost purely on decisions made by the regulatory bodies, not the effects regulatory bodies have on bank decisions.

As this review of this literature demonstrates, Central Banks are often discussed in the literature, but the direct effect of using Central Banks as a lender of last resort remains, so far as I can tell, unexplored. This paper attempts to fill this gap in the literature and connect private investing in money market funds and Central Bank activity.

III. The Central Bank: Monetary Policy Watchdog vs. Lender of Last Resort

In order to understand the interactions between money market funds and Central Bank activity, it is necessary to understand the basic incentives of Central Banks. Central banks tend to have five goals for monetary policy:

1. High employment
2. Economic growth
3. Stability of financial markets
4. Interest-rate stability
5. Stability in foreign exchange markets (Mishkin 317).

Price stability, or low and stable inflation, has been increasingly viewed as the most important goal of monetary policy (315). Price stability leads to lower amounts of uncertainty in markets, and so banks use the idea of a nominal anchor—a nominal variable such as the nominal exchange rate, wage and price controls, or the money supply.

It can be seen, then, that Central Banks primarily understand themselves to be players in the macroeconomy, and so all their interventions in the microeconomic world of private banks shall have to keep this motivation in mind.
Generally speaking, Central Banks have three instruments with which to achieve their goals:

1. Open market operations
2. Discount policies
3. Reserve requirements

This paper focuses on the effects of 2: Discount policies. A more in-depth look at how the discount policy is generally used is thus necessary. This paper shall focus on the type of discount window used by the US Central Bank, the Federal Reserve. The Federal Reserve has three types of discount loans available for banks:

1. Primary credit
2. Secondary credit
3. Seasonal credit

Primary credit is the most important tool from a monetary policy perspective. Healthy banks are allowed to borrow all the funds they want in the short term (generally overnight). This interest rate is generally about 100 basis points (1 percentage point) above the federal funds target rate, and is referred to as the discount rate. It is also important to note that the discount rate imposes a price ceiling on the federal funds rate, because were the federal funds rate ever to rise above the discount rate, then banks would start borrowing solely from the discount window until demand decreased for interbank loans enough that the federal funds rate dropped back below the discount window rate.

Secondary credit is the primary focus of this paper. Secondary credit is given banks that are in financial trouble and are experiencing liquidity problems. The interest rate on secondary credit is 50 basis points above the discount rate. However, in times of extreme crisis, central banks have been known to set up special lender-of-last resort facilities that offered securities without the 50 basis point premium (such as through an auction).

Discount loans also affect the monetary base. When a central bank makes a discount loan of N to a private bank, that bank is credited with N on its balance sheet. The monetary liabilities of the Central Bank have increased by N and the monetary base, too, increases by this amount. However, if a bank pays off the loan from the Central Bank, it reduces its borrowing by N, and pays back the interest on the loan, \( r_c \). Once one takes into account the interest paid in the loan, the total effect is a slight shrinking of the monetary base.

One potential pitfall of this secondary credit is that it introduces a certain amount of moral hazard: if a bank expects to be bailed out by the Central Loans with discount loans when it gets into trouble, it will be willing to take on more risk. Banks thus take on more risk, exposing the government’s deposit insurance agency—hence taxpayers—to more risk (Mishkin 385). This becomes especially problematic for especially large banks, which can consider themselves too big to fail.

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1 Seasonal credit is not applicable to this paper and shall not be discussed.
IV. Risk Signals: The Effect of Central Bank Borrowing on Investor Portfolio Allocation

Consider the situation where it becomes known to an investor that a private bank has taken on a Central Bank loan in a negative economic climate. The investor can be relatively confident the private bank is using the Central Bank as a lender of last resort. This loan acts as a signal to the investor that no other private investor is willing to invest in the bank, which lets him know that no one else in the market is willing to loan to the private bank. The private investor updates his previous risk assessment to take into account this new information.

Let us explore this intuition using the Markowitz model of portfolio investment. Assign the investor the standard utility function

\[ u(x) = ax - \frac{b}{2}x^2, \]  

where \( x \) is the return rate. Note that this equation defines a risk averse investor, where

\[ u''(x) = -b < 0. \]

Now let us take a random Normal variable, \( R_p \), to be the portfolio return rate. Let

\[ E[R_p] = \mu, \text{ and} \]
\[ Var[R_p] = \sigma^2. \]

Substituting \( R_p \) for \( x \) into 4.1, we see

\[ E[u(R_p)] = a\mu - \frac{b}{2}\mu^2 - \frac{b}{2}\sigma^2. \]

It follows that

\[ \frac{\partial E[u(R_p)]}{\partial \sigma^2} = -\frac{b}{2} < 0. \]

Accordingly, if we hold \( \mu \) constant, we see that as variation in \( R_p \) increases, the expected utility of the investor decreases. Thus, upon hearing that a private bank has taken out a Central Bank loan, the investor will want to take action.

We assume that investment in the bank takes the form of a risky security with return rate of the Normally distributed random variable \( R_b \) with some previously arrived at expected value \( \mu_b \) and variation, \( \sigma_b^2 \). We can, for the sake of simplicity, separate this out from the rest of the investor’s portfolio and treat as a two-security portfolio,
\[ R_p = \alpha_B R_B + \alpha_S R_S. \quad 4.7 \]

\( R_S \) is the return rate on the investor’s portfolio minus the bank security, and \( \alpha_B \) and \( \alpha_S \) are the relative weights of the bank security and the rest of the portfolio minus the bank investment, with \( \alpha_B + \alpha_S = 1 \). Treating this under the standard Markowitz model of portfolio investment for two securities, the standard minimum variance weighting is

\[ \alpha_B = \frac{\sigma_S^2 - \rho_{SB} \sigma_S \sigma_B}{(\sigma_B^2 + \sigma_S^2 - 2 \rho_{SB} \sigma_S \sigma_B)} \quad 4.8 \]

Where \( \sigma_S^2 = Var(R_S) \), \( \sigma_B^2 = Var(R_B) \), and \( \rho_{SB} = Corr(R_S, R_B) \).

From 4.8, we can take the first derivative with respect to \( \sigma_B \) to derive

\[ \frac{\partial \alpha_B}{\partial \sigma_B} = \frac{\sigma_S \sigma_B \rho_{SB} \sigma_B - 2 \sigma_S}{(\sigma_B^2 + \sigma_S^2 - 2 \rho_{SB} \sigma_S \sigma_B)^2}. \quad 4.9 \]

Because the denominator is always positive, as is \( \sigma_S \sigma_B \), \( \frac{\partial \alpha_B}{\partial \sigma_B} < 0 \) when \( \rho_{SB} \sigma_B < 2 \sigma_S \), or

\[ \rho_{SB} \sigma_B < 2 \sigma_S. \quad 4.10 \]

Since risk is traditionally measured by the standard deviation of an asset, we see that when the risk of the bank asset increases, the relative weighting of it in a given investor’s portfolio decreases, so long as 4.10 holds. According to 4.10, there could counter-intuitively be the opposite effect and the relative weighting of \( \alpha_B \) increase when the correlation between the bank security and the rest of the market multiplied by the mean of the portfolio and individual bank variances is greater than twice the variance of the rest of the portfolio. This can be rewritten as

\[ \rho_{SB} < 2 \frac{\sigma_S}{\sigma_B}. \quad 4.11 \]

which relates the correlation between the rest of the portfolio and the bank security to two times the ratio of the two.

The question of how often we can expect 4.11 to hold clearly arises. Because \( \sigma_S \) and \( \sigma_B \) are necessarily positive, this can only possibly happen when the correlation between the rest of the portfolio and the bank is positive. If the investor’s portfolio is well-diversified, we would expect the variance of the portfolio, or even the portfolio minus a single security, to generally be less than that of the variance of the individual bank security. Thus, we would expect \( \frac{\sigma_S}{\sigma_B} \) to be less than unity. However, it seems a rather unlikely scenario for this ratio to drop beneath the \( \frac{1}{2} \) which would be the minimum
possible value that would allow for 4.11 to be false, and even that would be when $\rho_{SB}$ approached unity—another unlikely occurrence. All in all, this seems rather unlikely in most circumstances. It would, however, be perhaps plausible if a general economic downturn were caused by increased volatility in the expected ability for a bank to repay its liabilities, such as in the recent financial crisis.

However, since the above scenario would not seem to hold in most cases, the rational investor response to a bank taking on a Central Bank loan should be to decrease his exposure to that bank and to decrease the weight of that asset in his portfolio. This seems to place the private bank in a bind. If the response of the average private investor to a Central Bank loan is to decrease his exposure, then this would mean that the capital infusion provided by the Central Bank loan would be offset to some degree by private investors withdrawing their investments. Since the financial instrument we are discussing is most likely a loan, it may be that the terms of the loan prohibit any such withdrawal of funds from the bank. However, we would still expect that private investors would not be willing to extend financing to these banks in the future. The private bank would have to somehow make up for this capital decrease either through another central bank loan or through somehow bypassing the above results and still acquiring financing through the private markets.

One possible way to acquire financing through private markets despite the results above could be to pay a risk premium as a penalty on having borrowed from a Central Bank. The above calculations assume that the return from the bank security stays constant. If the return on the bank security were to increase for the investor, then he may be willing to take on the increased risk signaled to the market by the Central Bank. The simplest way for this increased return to be accomplished would be for private bank to pay some risk premium as a penalty. The following sections shall explore this case.

V. Private Bank Funding Source Choices and Conditions

V.1 A Base Equation

If we understand taking a central bank loan as a signal of weakness to the markets, it would seem to follow that the greater the number of loans from the Central bank, the greater the effect on the penalty rate. Let us restrict the market rate to a single market rate, $r_m$, and single market penalty rate for going to the central bank, $p_m$, and $k$ representing the number of times a bank has recently acquired a central bank loan. Then we can define the interest rate for the private bank seeking financing as

$$r_n = I(r_m,k,...)r_c + [1 - I(p_m,k,...)][r_m + F(p_m,k)].$$

$I(r_m,k,...)$ is some rule function with range $\{0,1\}$, where $I = 1$ when the bank chooses central bank financing, and $I=0$ when the bank raises capital on private markets. It will depend on the market interest rate $r_m$, the number of times a bank has recently acquired a central bank loan $k$, and a set of other factors that go beyond the scope of this paper (such as liquidity, capital ratios, etc.).
If we consider the case where there is only a single market lending and penalty rate, and if the Central Bank is only available for financing a single loan (or the private bank is only interested in a single loan from the central bank), then we can disregard $k$ as trivial, which is explored in section V.2 below.

If there are, however, multiple loans the bank is attempting to receive and we relax the restriction that the central bank will only fund a single loan, then we must consider the interaction between $k$ and $p_m$. If we assume the simplest case, where this is a strictly linear interaction and

$$F(p_m, k) = kp_m,$$  \hspace{1cm} 5.2

then

$$r_n = I(r_m, k, ..., r_c) + [1 - I(p_m, k, ...)] [r_m + kp_m].$$  \hspace{1cm} 5.3

To explore more about the function $I$, we consider three cases in the subsections that follow. Each subsection explores different cases where a bank has gone to the Central Bank because, following section IV’s conclusions, it had no other available funding. Subsection V.2 considers a simple 2-stage game restricting the availability of Central Bank financing to period 1. Subsection V.3 lifts this restriction to begin to explore the case where a private bank does not perceive funding from the Central Bank to be limited in the near future. Subsection V.4 considers the case where money market imperfections allow for multiple private investors, each with separate base lending and risk premium rates.

\textbf{V.2 A 2-Stage Sequential Game with Restricted Central Bank Loan Availability}

Consider a base case where there is a private bank seeking funding for two stages. Because Central Banks are not in the business of acting as a financial backer of private banks indefinitely, we assume that Central Bank financing is available only in the first stage. This scenario can be modeled with a simple 2 stage sequential game with 3 players.

The Private Bank $K$
Private Investor $A$
Central Bank $C$

In stage 1, $K$, seeking funding, goes to $A$. $A$ examines $K$’s balance sheets, and is willing to lend at some market rate $r_m$. The Central bank is willing to lend at $r_C$.

The choice facing $K$ in stage 1 can be represented as follows
In stage 2, we assume that Central Bank financing has been exhausted. We further assume that rate $r_m$ remains unchanged for borrowing from $A$ if $K$ borrowed from $A$ in stage 1. However, if $K$ borrowed from $C$ in stage 1, $A$ perceives this as an increase of risk and thus charges a market risk premium of $p_m$. Thus, the cost of financing for stage 2 is as follows:

<table>
<thead>
<tr>
<th>Stage 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Private financing in Stage 1</td>
<td>Central Bank financing in Stage 1</td>
<td></td>
</tr>
<tr>
<td>$r_m$</td>
<td>$r_m + p_m$</td>
<td></td>
</tr>
</tbody>
</table>

The game can be reduced to the following two options:

<table>
<thead>
<tr>
<th>Private financing</th>
<th>Central Bank financing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_m + \frac{r_m + p_m}{1 + r_D}$</td>
<td>$r_C + \frac{r_m + p_m}{1 + r_D}$</td>
</tr>
</tbody>
</table>

Where $r_D$ is $K$’s discount rate. Thus, $K$ will choose central bank financing when

$$r_C + \frac{r_m + p_m}{1 + r_D} < r_m + \frac{r_m}{1 + r_D},$$

which can be reduced to the following condition for $K$ choosing central bank financing:

$$r_C < r_m - \frac{p_m}{1 + r_D}.$$  \hspace{1cm} 5.5

From this equation, we know that if a bank only needs to access Central Bank financing for a single loan, or the Central Bank is only willing to lend for a single loan, we can define $I$ as follows:

$$I = 1 \text{ if } r_C < r_m - \frac{p_m}{1 + r_D},$$  \hspace{1cm} 5.6

$$I = 0 \text{ if } r_C > r_m - \frac{p_m}{1 + r_D}.$$  \hspace{1cm} 5.7
This, however, would seem problematic. The Federal Reserve, as a rule, lends above the market rate, so that $r_C > r_m$. Because we assume that $p_m > 0$, we thus find that 5.5 will never be satisfied and K will never seek Central Bank financing, unless private funds are completely unavailable, in which case K is forced to seek Central Bank financing. This same intuition would seem to hold even when we expand past 2 stages.

V.3 A Multi-period Game with Unlimited Central Bank Loan Availability

However temporary in nature secondary discount window lending may be in general, there may be cases where the private bank may (correctly or incorrectly) perceive that the discount window loan availability may not be restricted in the near future. To explore this possibility, let us now consider a case similar to subsection V.2’s, but where we allow Central Bank financing to continue being an option past a single stage.

In this case, we still have the same three players:

The Private Bank K  
Private Investor A  
Central Bank C

For simplicity’s sake, assume that K has already received a loan from the Central Bank at rate $r_C$ in some Stage 0. It now faces the following decision:

<table>
<thead>
<tr>
<th>Stage 1</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Private</td>
<td>Central</td>
</tr>
<tr>
<td>financing</td>
<td>$r_m + p_m$</td>
<td>$r_C$</td>
</tr>
</tbody>
</table>

In stage 2, we assume that the private markets have imposed some penalty rate as defined by the function $F(p_m, k)$. For simplicity, assume that $F(p_m, k) = kp_m$ as in 5.2 above.²

Stage 2 can be described as follows:

<table>
<thead>
<tr>
<th>Stage 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stage 1 Private financing</td>
<td>Stage 1 Central Bank financing</td>
</tr>
<tr>
<td></td>
<td>$r_m + p_m$</td>
<td>$r_m + 2p_m$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Stage 2 Private Financing</th>
<th>Stage 2 Central Bank Financing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r_C$</td>
<td>$r_C$</td>
</tr>
</tbody>
</table>

² So long as $\frac{\partial F(p_m, k)}{\partial k} > 0$, the specifics of $F$ will not change the intuition contained in this paper.
In the above chart, we see that the cost of a loan in stage 2 is dependent both on whether \( K \) received Central Bank or private financing in stage 1. If \( K \) opted for private financing in stage 1 and decides to take private financing in stage 2, then it pays the base rate \( r_m \) plus the risk premium rate \( p_m \) for having borrowed from the Central Bank in stage 0. If \( K \) opted for Central Bank financing in stage 1, but decides to go back to the private markets in stage 2, it pays the base rate \( r_m \) plus the risk premium rate \( 2p_m \) for having borrowed twice from the Central Bank (\( k=2 \)). If \( K \) opts for Central Bank financing, it pays \( r_C \) no matter whether it sought private funds or Central Bank financing in the previous period.

The game reduces to the following for the two stages:

<table>
<thead>
<tr>
<th>Stage 2 Private Financing</th>
<th>Stage 1 Private financing</th>
<th>Stage 1 Central Bank financing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 2 Central Bank Financing</td>
<td>( r_m + p_m + \frac{r_m + p_m}{1 + r_D} )</td>
<td>( r_C + \frac{r_m + 2p_m}{1 + r_D} )</td>
</tr>
<tr>
<td>Stage 2 Central Bank Financing</td>
<td>( r_m + p_m + \frac{r_C}{1 + r_D} )</td>
<td>( r_C + \frac{r_C}{1 + r_D} )</td>
</tr>
</tbody>
</table>

Either \( r_m + p_m > r_C \), \( r_m + p_m < r_C \), or \( r_m + p_m = r_C \). If \( r_m + p_m > r_C \), then there is an equilibrium at \(<\text{Central Bank, Central Bank} >\). If \( r_m + p_m < r_C \), then there is a standard Nash equilibrium at \(<\text{Private, Private} >\). If \( r_m + p_m = r_C \), then there is no stable equilibrium, and \( K \) is strictly indifferent between private and central bank financing in the final round. However, in this case private financing in stage 1 strictly dominates central bank financing.

\[ V.4 \text{ Expanding the Investment Pool} \]

Let us consider what happens when we relax the assumption that all private investors are the same and merely take a single external market rate. The game is expanded to a sequential game in which there are four players:

- The Bank \( K \)
- Private Investor \( A \)
- Private Investor \( B \)
- Central Bank \( C \).

In stage 1, \( K \) goes to \( A \). \( A \) examines \( K \)’s balance sheets, and is willing to lend at \( r_A \). Next, \( K \) turns to \( B \). Similarly, \( B \) examines \( K \)’s balance sheets, and is willing to lend at \( r_B \). \( K \) has another option of financing: the Central Bank \( C \) acting as a lender of last resort. The Central Bank is willing to lend at \( r_C \).
Stage 1

<table>
<thead>
<tr>
<th>Private financing</th>
<th>Central Bank financing</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \min[r_A, r_B] )</td>
<td>( r_C )</td>
</tr>
</tbody>
</table>

In stage 2, \( C \) is not longer willing to lend to \( K \). As in the first game, if private financing was used in stage 1, \( A \) and \( B \) remain willing to lend \( r_A \) and \( r_B \), respectively. However, if Central Bank financing was used, then private financiers view the risk of the bank as having increased, and they charge a risk premium—\( p_A \) and \( p_B \), respectively.

Stage 2

<table>
<thead>
<tr>
<th>Private financing in Stage 1</th>
<th>Central Bank financing in Stage 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \min[r_A, r_B] )</td>
<td>( \min[r_A + p_A, r_B + p_B] )</td>
</tr>
</tbody>
</table>

This game can be reduced to:

<table>
<thead>
<tr>
<th>Private financing</th>
<th>Central Bank financing</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \min[r_A + \frac{r_A + p_A}{1 + r_D}, r_B + \frac{r_B + p_B}{1 + r_D}] )</td>
<td>( \min[r_C + \frac{r_A + p_A}{1 + r_D}, r_C + \frac{r_B + p_B}{1 + r_D}] )</td>
</tr>
</tbody>
</table>

\( K \) chooses to go with Central bank financing when

\[
\min[r_C + \frac{r_A + p_A}{1 + r_D}, r_C + \frac{r_B + p_B}{1 + r_D}] < \min[r_A + \frac{r_A + p_A}{1 + r_D}, r_B + \frac{r_B}{1 + r_D}] \tag{5.8}
\]

with \( r_D \) as the discount rate. \( K \) chooses to bypass central bank financing when

\[
\min[r_C + \frac{r_A + p_A}{1 + r_D}, r_C + \frac{r_B + p_B}{1 + r_D}] > \min[r_A + \frac{r_A + p_A}{1 + r_D}, r_B + \frac{r_B + p_B}{1 + r_D}] \tag{5.9}
\]

There are four cases to consider:

<table>
<thead>
<tr>
<th>Minimum 1</th>
<th>Minimum 2</th>
<th>Condition for Central Bank financing</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_C + \frac{r_A + p_A}{1 + r_D} )</td>
<td>( r_A + \frac{r_A}{1 + r_D} )</td>
<td>( r_C + \frac{r_A + p_A}{1 + r_D} &lt; r_A + \frac{r_A}{1 + r_D} )</td>
</tr>
<tr>
<td>( r_C + \frac{r_A + p_A}{1 + r_D} )</td>
<td>( r_B + \frac{r_B}{1 + r_D} )</td>
<td>( r_C + \frac{r_A + p_A}{1 + r_D} &lt; r_B + \frac{r_B}{1 + r_D} )</td>
</tr>
<tr>
<td>( r_C + \frac{r_B + p_B}{1 + r_D} )</td>
<td>( r_A + \frac{r_A}{1 + r_D} )</td>
<td>( r_C + \frac{r_B + p_B}{1 + r_D} &lt; r_A + \frac{r_A}{1 + r_D} )</td>
</tr>
<tr>
<td>( r_C + \frac{r_B + p_B}{1 + r_D} )</td>
<td>( r_B + \frac{r_B}{1 + r_D} )</td>
<td>( r_C + \frac{r_B + p_B}{1 + r_D} &lt; r_B + \frac{r_B}{1 + r_D} )</td>
</tr>
</tbody>
</table>
Case 1 reduces to the same as 3.2, where the simplified decision rule is

$$r_C < r_A - \frac{p_A}{1 + r_D}. \quad 5.10$$

The condition for K seeking Central Bank financing in case 2 can be rewritten as

$$r_C < r_B + \frac{r_B - r_A - p_A}{1 + r_D}. \quad 5.11$$

Case 2 assumes the following conditions:

A) $$r_C + \frac{r_A - p_A}{1 + r_D} < r_C + \frac{r_B + p_B}{1 + r_D}. \quad 5.12$$

B) $$r_A + \frac{r_A}{1 + r_D} < r_B + \frac{r_B}{1 + r_D}. \quad 5.13$$

Condition A is equivalent to

$$r_A + p_A < r_B + p_B. \quad 5.14$$

We can rewrite equation 5.14 as

$$r_A - r_B < p_B - p_A, \quad 5.15$$

and from condition B, we can deduce that

$$r_A < r_B. \quad 5.16$$

From 5.16, we know that $$r_B - r_A > 0.$$ 

Equation 5.11 can be rewritten as

$$r_C - r_B < \frac{r_B - (r_A + p_A)}{1 + r_D}. \quad 5.17$$

As such, when the conditions in case 2 hold, Central Bank financing is only sought when the difference between the lending rate of a Central Bank loan and a loan from investor B is less than the discounted value of the difference between the lending rate of investor B and the sum of the rate of lending for investor A plus investor A’s risk premium for seeking a Central Bank loan.

Case 3 is a mirror image to case 2, and so, following the same reasoning from 5.12 to 5.17 the simplified decision rule can be written as
\[ r_C - r_A < \frac{r_A - (r_B + p_B)}{1 + r_D} , \]

where Central Bank financing is sought when the difference between the lending rate of a central bank loan and a loan from investor A is less than the discounted value of the difference between the lending rate of investor A and the sum of the rate of lending for investor B plus investor B’s risk premium for seeking a central bank loan.

Case 4 mirrors case 1, and the simplified decision rule is

\[ r_C < r_B - \frac{p_B}{1 + r_D} . \]

As has been discussed previously, the Federal Reserve sets its discount window lending rate above that of the market. As such, we should expect that

\[ r_C > r_A , \text{ and} \]
\[ r_C > r_B . \]

However, as with equation 5.5 in the single-investor case, 5.11 and 5.19 in cases 1 and 4, respectively, will never satisfy equations 5.20 and 5.21 unless investors A and B are completely unwilling to lend. On the other hand, cases 3 and 4 do give a very specific case in which central bank financing may be sought—specifically when either the gap between interest rates between investors A and B are high such as when markets are highly inefficient, or when the penalty rate is extremely low. While this does not seem extremely likely to happen very often, it also cannot be simply dismissed.

**VI. Implications and Limitations**

In order to understand the implications of this model, let us first reconsider the incentives faced by the Central Bank. Central Banks are in a unique position with respect to their debt acquisition. In most cases, creditors merely desire a timely payback of their investment. The goals of a Central Bank, however, are considerably wider. As has been previously discussed, the primary goal of Central Bank policy is price stability, or a low and stable inflation rate. However, as mentioned in section III, central bankers commonly list five other goals: high employment, economic growth, financial market stability, interest rate stability, and stability in foreign exchange markets. These goals have a macroeconomic focus which are not necessarily the same as the goals of the bank. There is the possibility for a Central Bank to use its lending authority to attempt to exercise control over the governance of the bank to which it is lending, especially in times of economic distress when the volume of these loans increases.

As previously mentioned in section III, every Central Bank loan has the effect of temporarily increasing the monetary base in the economy. If the primary goal of the Central Bank is to stabilize the financial market and control the inflation rate, then so long as the market is not doing particularly well—and the economy is not experiencing
anything like stagflation—the Central Bank will not have any problem with extending many of these loans. However, as soon as inflationary pressures pick up, the Central Bank will want to unwind these commitments.

It follows from the above paragraph that during times of economic hardship when a Central Bank is pursuing expansionary monetary policy, a private bank would be likely to consider itself to be relatively unthreatened by a Central Bank cutting off its supply of discount window loans. This perceived availability of funds would only be increased if the Central Bank saw the closing of a bank as a threat to the financial system. This concern could be increased if there are many banks at risk of closing and that are seeking Central Bank funding. This case approaches the model in V.3 and allows for the possibility of private banks to enter back into the private market for financing after taking a Central Bank loan (though indefinite dependence on Central Bank funding is also possible, depending on the specifics of the scenario). On the other hand, if the economy is doing well and the Central Bank is not pursuing an expansionary monetary policy, then the private bank would understand the Central Bank to be less enthusiastic about increasing the monetary base in the economy. This situation more closely approaches the model in V.2, where the private bank considered the availability of Central Bank discount window loans to be very limited, and we found that the only case in which a private bank would go to the Central Bank would be one in which no private lenders were willing to lend to the private bank at all. The considerations imply that we would expect discount window loans to be more widely used by struggling private banks in economic downturns than by struggling private banks during economic upswings. Additionally, the Central Bank could increase or decrease the cost of these loans merely by signaling to the markets that it would be lenient in extending them.

A further implication of the above modeling is that the Brander-Lewis (1986) model implies that in a non-competitive market, firm risk-taking decreases as debt-equity ratios increase. In the above model, private banks are seeking debt and thus would be increasing their debt-equity ratio and it would seem that banks in this position should decrease their risk. If Central Banks are intervening, this effect may increase, because Central Banks are interested in stability of financial markets. Banks going bankrupt is not particularly good for stability in financial markets, and so Central Banks are most likely going to discourage such behavior. This discouragement can have teeth, especially if the Central Bank is able to threaten not funding another round of investing and force the private bank into the private market before it is ready. This set of incentives conveniently lines up so that once the private bank feels it is ready to enter the private market, it will no longer feel as constrained by the Central Bank’s threat of removing funding, and thus will be able to increase its risk even before it has returned to the private market.

As was seen in the models above, there are several cases where private banks can avoid dependency on the Central Bank even after receiving a loan from it. In its simplest form, if a private bank is able to acquire sufficient Central Bank financing before it would feel pressure that the Central Bank will cut off financing (such as in a single financing round simply to shore up unexpected short-term liquidity issues), then according to the
equilibrium found in game we considered in section V.3, so long as the sum of the base rate and penalty rate for going to the Central Bank is less than or equal to the rate the Central Bank loans at (, or \( r_m + p_m \leq r_C \)) then the bank will either be incentivized to go back to the private market or will be indifferent between private market and Central Bank funding. From this, we can see that the larger the gap between \( r_C \) and \( r_m \), the larger premium an investor can charge and still expect to see private banks willing to pay it.

Another interesting implication comes from the discussion of section V.4 when market imperfections are considered and investors may set their own base lending rates and penalties for having gone to the Central Bank. If the investor with the less expensive base lending rate does not have a sufficiently high penalty rate for having borrowed from a Central Bank, then, per equations 5.10 and 5.19, the only case in which a private bank will go to a Central Bank is when private investors are completely unwilling to lend to it. In this case, there would be no way for a bank to escape becoming dependent on Central Bank financing short of bankruptcy. However, if the investor with the less expensive base rate has a sufficiently large penalty, then there would seem to be a specific equation that can be used to determine whether the private bank should continue to seek Central Bank financing. In this case, the larger the gap between the rate of a Central Bank loan and the rate of the loan of the more expensive investor, the larger the gap has to be between the more and less expensive private investor in order for the private bank to consider continuing to borrow from the Central Bank. This would have interesting implications if, say, the Fed were to dramatically decrease the market fed funds rate, theoretically increasing the gap between the private lenders’ rate and the Federal Reserve’s.

Finally, there are a few cases where my assumptions do not hold that could also provide alternative results. First, if the market is unaware that lending was done by the Central Bank, then it would show up on balance sheets just like any other loan and investors would be none the wiser, thus making any signal impossible. Second, if, for some other reason, markets do not impose any penalty for acquiring Central Bank funding, then this model’s central assumption would fail and the above modeling would not apply.

According to the discussion in section IV, this latter case could happen when \( \rho_{SB} < 2 \frac{\sigma_S}{\sigma_B} \).

As previously discussed, this seems generally unlikely to happen. However, it would seem to be perhaps plausible if a general economic downturn were caused by increased volatility in the expected ability for a bank to repay its liabilities, such as in the recent financial crisis. Assuming the above condition fails in this case, this would suggest that we might not expect to see investors penalizing banks that accepted Federal Reserve funding in the most recent financial crisis. Assuming that the intuition is similar between accepting bailout funds as to accepting lender-of-last-resort funds, this would also suggest that the Treasury department’s insistence that all the major banks accept TARP funding was perhaps unnecessary.
VII. Conclusion and Questions for Further Research

This paper explored a previously unexplored cost to central bank financing in which accepting a loan of last resort was interpreted as a signal of undercapitalization. It first started with considering the result of a private bank taking out a loan from the Central Bank as a lender of last resort on investors’ portfolios and, using the Markowitz model, found that accepting the loan would, in equilibrium, result in a decrease of exposure to the bank. As such, in any period after this loan, the private bank would have to pay a premium to return to the primary markets. Using basic game theory, it then considered numerous different possible cases and restrictions to see the effect this premium could have on decision making for the private bank. In some cases, the equilibrium was to become dependent on the Central Bank, or else pay increasingly high premiums to return to the private market.

Clearly, many questions remain unanswered and are left for further research. This is to be expected—this is a first foray into interactions between fields that seems to have been largely unexplored previously. A first further research question is to consider the signaling mechanism in more detail. It is not always known, and so a signaling model taking into account asymmetric information between the investor and the private bank could be of great benefit.

Second, the function I call I could use some exploration. In the literature, there seems to be a lack of exploration as to when banks would go to the Central Bank. The above model would suggest that it is not necessarily only when private funds are strictly unavailable. I would be very useful to perform econometric analysis on, but at this time, data not aggregating beyond use regarding this is unavailable.

Ultimately, this paper explored the relationship between three actors: the central bank, the private investor, and the private bank. While all three players are all influential in money markets and affect each other, the way specialization happens to occur in the field of economics leaves a gap in exploring the complex relations among these three players. It is the hope of this author that this paper will increase interest in exploring further consequences of these interactions. This can only lead to a much-needed greater understanding of financial markets.
Works Cited:


