

Empirical Analysis on the Effects of Jumps on Realized Beta and the Disentanglement of Jump Beta¹

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Abstract

This paper constructs jump-robust estimators for the beta in Capital Asset Pricing Model (CAPM) in order to test the robustness of the recently developed Realized Beta in the presence of large discontinuous movements, or jumps, in stock prices. To complete the analysis on effect of jump on Realized Beta, this paper also disentangles jump beta and diffusive beta from the Realized Beta measurement in order to examine whether stocks react differently to jumps under the CAPM. Then, the results are compared to recent literatures tackling the same problem from different approaches.²

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1. Introduction

With the availability of high-frequency data in recent years, new lights are shed on half-a-century-old models, such as the well-known Capital Asset Pricing Model (CAPM), and new estimators are developed with implications for high-frequency analysis. This paper aims to test the robustness of one of those estimators, specifically the “Realized Beta” from Andersen, Bollerslev, Diebold, and Wu (2005). This paper and Andersen, Bollerslev, Diebold, and Wu (2005) both take an interest in CAPM because of its simplicity and compatibility with high-frequency analysis.

In the past, many models have been developed to measure and minimize risk. For example, Markowitz (1952) put forth the Markowitz model that yields an efficient frontier consisting of (efficient) portfolios with risks, represented by variances, minimized through diversification. One downside, however, of the Markowitz model is the large number of estimates required. The Capital Asset Pricing Model (CAPM), independently developed in Treynor (1961, 62), Sharpe (1964), Lintner (1965a,b), and Mossin (1966), simplifies the problem with the key insight that only non-diversifiable market (or systematic) risk should be priced. As a result, the number of estimates required is greatly reduced. Specifically, since firm-specific (or idiosyncratic) risks are assumed to be diversifiable and uncorrelated with each other, covariance between assets can be derived solely from systematic risk and would not, therefore, require additional estimations as in Markowitz model. Though more advanced multi-factor models have been developed since Sharpe (1964) and others formulated the CAPM, the key insight of CAPM to price only the non-diversifiable risk is impeccable. Though with addition non-diversifiable risk factors, the multi-factor models, nevertheless, employ the same concept. Thus is the simplicity embedded in the CAPM.

To understand the compatibility of CAPM with high-frequency analysis, one important component of CAPM, the beta, must be introduced. According to CAPM, beta measures the systematic risk, or the exposure of each individual asset to the fluctuations in the returns of the market portfolio, a properly weighted portfolio containing all available assets. The greater the magnitude of the beta, the more the return on the asset fluctuates with the return on the market portfolio. Thus, assets with greater magnitude of beta are considered more “risky.” In real

markets most of the betas are positive. However, it is possible for assets to have negative beta.³ When beta is negative, the return on the corresponding asset simply moves in the opposite direction with the return on the market portfolio.

With beta introduced, the extension of beta to high-frequency analysis can be explained without delay. Since true betas are not directly observable, the CAPM betas are traditionally estimated by regressing asset return on the market return, i.e. the return on a market index used as a proxy for the market portfolio, with, typically, 5 years of monthly data as in Fama and MacBeth (1973). It is also mathematically equivalent to estimate the betas by dividing the covariance between the asset return and market return by the variance of the market return, since the idiosyncratic risk is not correlated with the market return⁴. This methodology can be extended to high frequency, because the covariance and variance measurements used to calculate the CAPM beta could easily be replaced with their recently developed high-frequency counterparts. For example, Andersen, Bollerslev, Diebold, and Wu (2005) calculated high-frequency beta, named Realized Beta, with Pearson Covariance and Realized Variance⁵ and concluded that betas are time varying, as oppose to the traditional model of betas that are constant throughout time. Realized Beta introduced in Andersen, Bollerslev, Diebold, and Wu (2005) is a more accurate measurement of CAPM beta because it employs more information than the traditional regression on monthly returns.

This paper, motivated by Andersen, Bollerslev, Diebold, and Wu (2005), seeks to test the robustness of their time-varying Realized Beta in the presence of discontinuities in stock prices (or “jumps”). To perform the test, this paper proposes to construct a jump-robust beta employing the jump-robust covariance measurement introduced in Barndorff-Nielsen Shephard (2004a). Then, with the jump-robust beta, it is possible to isolate the effects of jumps on Realized Beta by taking the difference between the Realized Beta and jump-robust beta and access whether the jumps, as observed in real markets, pose any risk in addition to systematic risk faced by investors. In addition, this paper disentangles the beta in time periods with jumps, or jump beta, from the Realized Beta measurement by using the jump-robust beta. The results will be compared to

³ For example, Chicago Board Options Exchange Market Volatility Index, or VIX, has a negative beta when S&P500 is used as the market portfolio.

⁴ See section 2.1 for specifics.

⁵ See section 3.1 and 3.2.

Todorov and Bollerslev (2010), which also disentangled jump beta, though with a different method.

In the next section, the theoretical framework underlying this research will be explained; in particular, the Capital Asset Pricing Model (CAPM) and the underlying pricing model. Then, in section 3, statistical methods employed in the study shall be described. In particular, a jump robust estimator of the CAPM beta will be constructed to study the effects of jumps. Next, in section 4, data source and data treatment will be clarified. Finally, in section 5, the empirical results will be related and, in section 6, the conclusions.

2. Theoretical Framework

2.1 Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) can be used to determine the required return of individual securities. Under the CAPM model, return on stock i is as follows:

$$R_i = \beta_i R_m + \epsilon_i, \quad (1)$$

where $R_i = r_i - r_f$ is the excess return on stock i (i.e. return on stock i minus the risk-free rate), R_m is the excess return on a market index m , and ϵ_i is the idiosyncratic risk of stock i , which is uncorrelated with R_m or the idiosyncratic risk of any other stock under CAPM assumptions⁶.

Note β_i , the regression coefficient between R_i and R_m , is often referred to as the systematic or market exposure of stock i , because it measures the sensitivity of excess return on stock i to changes in excess returns on market. This paper takes an interest in β_i because it represents not only the exposure to systematic variations but also the key to predict returns on individual assets as in equation (1). However, since the variation in the risk free rate is small relative to that of the individual stock returns over the sample time horizon, r_f would not contribute much to the change in r_i . Therefore, r_f can be safely disregarded and the model can be simplified to

$$r_i = \beta_i r_m + \epsilon_i. \quad (2)$$

This paper employs CAPM model not only for its simplicity but also its compatibility with high-frequency analysis. Traditionally, β_i in equation (2), referred to as CAPM beta onward, has been measured by regressing monthly returns of stock i on monthly returns of a market index m over the 5 years. However, given minute-by-minute trading prices, it is possible to compute returns and beta of higher frequencies (e.g. 1-minute returns and daily beta) from the CAPM model. With high-frequency beta, various tests can be performed, including testing the robustness of beta measurements in the presence of “jumps” (i.e. discontinuities in stock prices as discussed in next subsection).

⁶ I.e. $0 = Cov(\epsilon_i, R_m) = Cov(\epsilon_i, r_m - r_f) = Cov(\epsilon_i, r_m)$; $0 = Cov(\epsilon_i, \epsilon_j)$, where $i \neq j$.

Under CAPM assumption that there is no correlation between idiosyncratic risk ϵ_i and systematic risk as reflected by the return on market r_m , CAPM beta can be derived using the following method. First, covariance of stock i and market m in the CAPM setting can be computed as follows:

$$\begin{aligned} Cov(r_i, r_m) &= Cov(\beta_i r_m + \epsilon_i, r_m) \\ &= \beta_i Cov(r_m, r_m) + Cov(\epsilon_i, r_m) \\ &= \beta_i Var(r_m). \end{aligned}$$

Finally, dividing both sides by variance of the market return gives a formula for beta:

$$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)}. \quad (3)$$

As equation (3) indicates, β_i depends on the variance of returns on market m and covariance between returns of stock i and market m . Thus, to provide an estimation of β_i , variance and covariance estimation methodology must be explained. First, the underlying model for stock prices must be defined to set the foundation for the variance and covariance estimations.

2.2 Stochastic Model of Stock Prices

The CAPM depends on returns, which, in turn, depends on underlying prices of the stocks. Therefore, before proceeding to variance and covariance estimation methods, it is necessary to define the underlying pricing model. This paper models the prices with a simple continuous-time stochastic process, which states that log-prices of stocks are moved by a deterministic and a random drift, as follows:

$$dp(t) = \mu(t)dt + \sigma(t)dw(t), \quad (4)$$

where $\mu(t)$ is the time-varying (deterministic) drift of log-price $p(t)$ and $\sigma(t)dw(t)$ is the time-varying volatility component, representing the random drift. In particular, $\sigma(t)$ is the instantaneous volatility of the stock and $w(t)$ is the Wiener process, or Standard Brownian motion, often used to model random movements. In modern literatures, it is often assumed that $\sigma(t)$ depends on the Wiener increment $dw(t)$, while $\mu(t)$ does not. In addition, since the log-price $p(t)$ satisfies the stochastic differential equation above, the efficient price of a security is

said to follow a Geometric Brownian motion, a widely used model for derivative analysis. The famous Black-Scholes' formula, for example, derives pricing for call and put options based on the assumption that prices follow a Geometric Brownian motion, or that the log-prices can be represented by equation (4). Despite the numerous advantages of the continuous-time (diffusive) stochastic process, it is important to note that equation (4) produces a continuous sample path of stock prices, which might not reflect the movements in real stock prices.

The continuous-time stochastic process remained a dominant assumption for stock prices until Merton (1976) points out that discontinuities, or “jumps,” would distort results from the well-known Black-Scholes equation, which assumes stock prices to perform under equation (4), to produce a systematic bias for option pricing. In addition, Anderson, Bollerslev, and Diebold (2007) shows that many volatility estimation models produce biased results because the underlying assumption of continuous sample path is violated in practice. Andersen, Benzoni, and Lund (2002) also argues to include a jump term in the standard stochastic differential equations to better model real stock prices. As this paper aims to discuss the importance of jumps in measuring the CAPM beta, it is necessary to include a (Poisson-driven) jump term to incorporate these discontinuities:

$$dp(t) = \mu(t)dt + \sigma(t)dw(t) + \kappa(t)dq(t), \quad (5)$$

where $\kappa(t)$ is the magnitude of the jumps and $q(t)$ is the Poisson counting of jumps until time t . Together, the term $\kappa(t)dq(t)$ represents jump.

2.3 Modified CAPM

With the presence of jumps in stock prices, the CAPM model needs to be revised, as Todorov and Bollerslev (2010) suggests, to incorporate the cumulative return from intervals with jumps, or jump return. First, the return on market m can be decomposed into cumulative return from intervals without jumps, or diffusive return, r^C and jump return r^J as follows:

$$r_m = r^C + r^J.$$

Then, we define the modified return on stock i as follows:

$$r = \beta_C r^C + \beta_J r^J + \epsilon, \quad (6)$$

where r is the return on stock i^7 , β_C is the sensitivity of r to diffusive return r^C and β_J is the sensitivity of r to jump return r^J . If r_i is affected by the diffusive return and jump return alike, i.e. $\beta_C = \beta_J = \beta_i$, equation (6) would be equivalent to equation (2). Then, the jump return of the market m would pose the same risk on r , and $\beta_J = \beta$ can be computed from equation (3). However, if $\beta_C \neq \beta_J$, the beta computed from equation (3) would be an average of diffusive beta, β_C , and jump beta, β_J , weighted by the variance of r^C and r^J respectively, as follows⁸:

$$\beta = \beta_C \left(\frac{\sigma_C^2}{\sigma_C^2 + \sigma_J^2} \right) + \beta_J \left(\frac{\sigma_J^2}{\sigma_C^2 + \sigma_J^2} \right) \quad (7)$$

Since neither the diffusive beta nor the jump beta is observed, it is necessary to disentangle each to study how stocks reward differently or similarly in time intervals with and without jumps.

3 Statistical Methods

3.1 Variance Estimations

Realized, or historical, Variance of a sequence of prices $P(t)$ can be derived from the returns. Let log-stock prices be $p(t) = \ln(P(t))$. The return of j^{th} interval on day t can be calculated as follows:

$$r_{t,j} = p(t - 1 + j) - p(t - 1 + j), \quad (8)$$

where M is the number of intervals in day t and $0 < j \leq M$. Then, Realized Variance can be defined as:

$$RV_t \equiv \sum_{j=1}^M r_{t,j}^2, \quad (9)$$

⁷ From here onward, the subscript i will be dropped for clarity and simplicity in equations, except when multiple assets are discussed as in section 3.2. But it is understood r always refers to return on a specific stock i , and all β 's are specific to stock i . Also, note that the variances in equation (7) refer to variances on the market returns.

⁸ *Proof:* $\beta_i = \frac{\text{Cov}(r_m, r_i)}{\text{Var}(r_m)} = \frac{\text{Cov}(\beta_C r^C + \beta_J r^J + \epsilon_i, r_m)}{\text{Var}(r^C) + \text{Var}(r^J)} = \frac{\beta_C \sigma_C^2 + \beta_J \sigma_J^2}{\sigma_C^2 + \sigma_J^2} = \beta_C \left(\frac{\sigma_C^2}{\sigma_C^2 + \sigma_J^2} \right) + \beta_J \left(\frac{\sigma_J^2}{\sigma_C^2 + \sigma_J^2} \right) \blacksquare$

where M is the number of samples in a day. Note that equation (9) uses only returns from within each trading day (intraday returns), discarding any overnight returns (inter-day returns). As a result, any jumps resulted from overnight returns are excluded from Realized Variance. When delta goes to 0, Realized Variance converges to integrated variance plus the jumps:

$$\lim_{\Delta \rightarrow 0} RV_t = \int_{t-1}^t \sigma^2(s) ds + \sum_{j=1}^M \kappa^2(t_j),$$

where $\Delta = \frac{1}{M}$ is the sampling frequency⁹, $\sigma^2(s)$ is the time-continuous integrable variance function and $\kappa^2(t_j)$ is the squared discrete jump term (as introduced in equation (5)). It is clear that Realized Variance is not a robust measure of the variance σ^2 in the presence of jumps.

Therefore, to improve the robustness of variance estimation in the presence of jumps, Barndorff-Nielsen and Shephard (2004b) introduces Bi-power Variance, a jump-robust measure of variance, as follows:

$$BV_t \equiv \mu_1^{-2} \frac{M}{M-1} \sum_{j=2}^M |r_{t,j-1}| |r_{t,j}|, \quad (10)$$

where $\mu_1^{-2} = \frac{\pi}{2}$ and M is the number of samples in each day. Bi-power Variance is a jump robust estimator of the integrated variance, since Barndorff-Nielsen and Shephard (2004b) has shown

$$\lim_{\Delta \rightarrow 0} BV_t = \int_{t-1}^t \sigma^2(s) ds.$$

The Bi-power Variance converges to continuous true, or integrated (i.e. jump-free), variance function as Δ goes to zero. Intuitively, in the presence of any jump, one of the two consecutive returns is bound to be larger. The product of the smaller return and the larger return, however, will be small and thus neutralize the effects of the jump. Therefore, instead of squaring the returns, Barndorff-Nielsen and Shephard (2004b) multiplies the absolute value of the previous return by the absolute value of the current return to mitigate the effects of any potential jumps.

⁹ Smaller the Δ means higher sampling frequency.

3.2 Covariance Estimation

Ever since the development of Realized and Bi-power Variances, modern literatures, such as Barndorff-Nielsen and Shephard (2004a), have expanded the concepts into multivariate settings. For example, Pearson Covariance,

$$PCov_t(r_i, r_m) = \sum_{j=1}^M r_{i,j} r_{m,j}, \quad (11)$$

is commonly used to calculate the covariance between two assets for the unique property that Pearson Covariance is only sensitive to linear relations between the returns of the two assets. Barndorff-Nielsen and Shephard (2004a) develops the distribution theory for Pearson Covariance, deriving a limit for the Realized Correlation,

$$R\beta_t = \frac{PCov_t(r_i, r_m)}{RV_m}, \quad (12)$$

calculated from Pearson Covariance and Realized Variance. Barndorff-Nielsen and Shephard (2004a) also shows empirically that the proposed limit theory provides a good model for predicting behavior of finite samples. Built on these theories and others, Andersen, Bollerslev, Diebold, and Wu (2005) employs Pearson Covariance in the calculation of Realized Beta as in equation (12). However, the Realized Beta in equation (12) is not jump robust, because Pearson Covariance is susceptible to jumps in underlying prices, as Barndorff-Nielsen and Shephard (2004a) points out.

To solve the problem, Barndorff-Nielsen and Shephard (2004a) introduces a jump-robust estimator of covariance. First, a portfolio approach to compute Realized Covariance is developed. This approach decomposes the Realized Covariance into Realized Variances, which can easily be computed from equation (9). A later subsection shall show that Realized Covariance developed with the portfolio approach is equivalent to Pearson Covariance. One advantage, however, of the portfolio approach is that the Realized Variances used to compute the Realized Covariance can be replaced by jump-robust Bi-power Variances to result in a jump-robust estimator of covariance, denoted Bi-power Covariance. With Bi-power Covariance and Bi-power Variance of the market, it is possible to obtain a jump-robust measure of CAPM beta, denoted

Bi-power Beta. And thus, the jump beta can be disentangled from equation (7). First, portfolio theory that underlies the proposed portfolio approach much be introduced.

3.2.1 Portfolio Method

First introduced in Barndorff-Nielsen and Shephard (2004a), a polarization identity allows the decomposition of covariance into variances, estimated through either Realized Variance or Bi-power Variance. First, a portfolio with equal weights of stock i and S&P 500 Futures Index (SPFU) m can be constructed as follows¹⁰:

$$r_p = \frac{1}{2}r_i + \frac{1}{2}r_m, \quad (13)$$

where r_i is the return on stock i and r_m is the return on the SPFU. Then,

$$\begin{aligned} Var(r_p) &= Var\left(\frac{1}{2}r_i + \frac{1}{2}r_m\right) \\ &= \left(\frac{1}{2}\right)^2 Var(r_i) + \left(\frac{1}{2}\right)^2 Var(r_m) + 2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) Cov(r_i, r_m) \\ &= \frac{1}{4}Var(r_i) + \frac{1}{4}Var(r_m) + \frac{1}{2}Cov(r_i, r_m). \end{aligned}$$

The expansion of the Variance function can be easily proved by the applying the definitions of Variance and Covariance in terms of Expectation¹¹. Rearranging the terms, the covariance can be represented as follows:

$$Cov(r_i, r_m) = 2 \times \left[Var(r_p) - \frac{1}{4}Var(r_i) - \frac{1}{4}Var(r_m) \right]. \quad (14)$$

Thus, the covariance is decomposed into different variance measures.

3.2.2 Realized Covariance

Using Realized Variance for the variance measures in equation (14), Realized, or historical, Covariance can be defined as follows:

¹⁰ We use S&P 500 Futures Index as a proxy for the market portfolio. Since the risk free rate is relatively low compared with returns on the market, S&P 500 Futures Index is essentially identical to S&P 500 Index. Thus we shall refer to it as SPFU below.

¹¹ $Var(r) = E[r^2] - E[r]^2$; $Cov(r, s) = E[rs] - E[r]E[s]$.

$$RCov_t(r_i, r_m) \equiv 2 \times \left[RV_{p,t} - \frac{1}{4} RV_{i,t} - \frac{1}{4} RV_{m,t} \right], \quad (15)$$

where $RCov_t(r_i, r_m)$ is the Realized Covariance between r_i and r_m , and $RV_{p,t}$, $RV_{i,t}$, and $RV_{m,t}$ are the Realized Variance of portfolio p , stock i , and index m , respectively, as defined below.

With known r_i and r_m , r_p can be easily computed for each time interval in equation (13). Then using equation (9), Realized Variance for portfolio p is as follows:

$$RV_{p,t} = \sum_{j=1}^M r_{p,t,j}^2,$$

where M is the number of samples on day t , when both stock i and market index m has price data available. Note that for any weight w given to stock i and $(1 - w)$ to market m , the Realized Covariance calculated by equation (15) is exactly the same as Pearson Covariance in equation (11). To simplify notations, assume $w = \frac{1}{2}$.¹² The proof is as follows:

$$\begin{aligned} RCov_t(r_i, r_m) &= 2 \times \left[RV_{p,t} - \frac{1}{4} RV_{i,t} - \frac{1}{4} RV_{m,t} \right] \\ &= 2 \times \left[\sum_{j=1}^M r_{p,t,j}^2 - \frac{1}{4} \sum_{j=1}^M r_{i,t,j}^2 - \frac{1}{4} \sum_{j=1}^M r_{m,t,j}^2 \right] \\ &= 2 \times \left[\sum_{j=1}^M \left(\frac{1}{2} r_{i,t,j} + \frac{1}{2} r_{m,t,j} \right)^2 - \frac{1}{4} \sum_{j=1}^M r_{i,t,j}^2 - \frac{1}{4} \sum_{j=1}^M r_{m,t,j}^2 \right] \\ &= 2 \times \left[\sum_{j=1}^M \left(\frac{1}{4} r_{i,t,j}^2 + \frac{1}{4} r_{m,t,j}^2 + \frac{1}{2} r_{i,t,j} r_{m,t,j} \right) - \frac{1}{4} \sum_{j=1}^M r_{i,t,j}^2 - \frac{1}{4} \sum_{j=1}^M r_{m,t,j}^2 \right] \\ &= 2 \times \left[\sum_{j=1}^M \frac{1}{2} r_{i,t,j} r_{m,t,j} \right] \\ &= \sum_{j=1}^M r_{i,t,j} r_{m,t,j} = PCov_t(r_i, r_m) \end{aligned}$$

¹² Similar proof can be done with any weight w .

Though there is no obvious jump-robust analogue of Pearson Covariance, it is easy to derive a jump-robust covariance measurement from equation (14). By analogue of equation (15), Bi-power Covariance can be defined as follows:

$$BCov_t(r_i, r_m) \equiv 2 \times \left[BV_{p,t} - \frac{1}{4} BV_{i,t} - \frac{1}{4} BV_{m,t} \right], \quad (16)$$

where $BV_{p,t}$, $BV_{i,t}$, and $BV_{m,t}$ are the Bi-power Variance of portfolio p , stock i , and index m , respectively. Note that since Bi-power Variance is jump-robust, Bi-power Covariance is also jump-robust because it is solely computed from Bi-power Variances as in equation (10).

3.3 Betas

The jump beta was only briefly introduced in Equation (7) as the sensitivity on jump return r^J . A more intuitive explanation, however, would be that jump beta describes how stock returns behave in the presence of systematic price jumps, or jumps in the price of the market index. If jump beta were the same as its continuous counterpart, stock would behave the same way in time periods with and without jumps. In this case, investors need not worry because the risk and return on the stock would not be any different during systematic price jumps. Yet, if jump beta were higher, the underlying stock would be inherently riskier in time intervals with jumps, though compensated in its returns for the increased riskiness. For example, Todorov and Bollerslev (2010) finds that their version of jump beta is quite different from their diffusive beta and suggested that portfolios intended to hedge for large systematic movements should be constructed differently from portfolios hedging the regular day-to-day market movements. Thus, it is important to know whether jump beta is different from its continuous counterpart. And so, to unravel the jump beta, the following subsections will define three different estimators of beta and derive an estimator for the jump beta.

3.3.1 Realized and Bi-power Betas

As derived in the CAPM model, beta for stock i can be computed by dividing covariance between stock i and market m by the variance of returns on market m . Realized Covariance and Bi-power Covariance from section 3.2.2 and 3.2.3 can be readily substituted into equation (3) to calculate the Realized Beta and Bi-power Beta. However, the choice of the Variance of returns

on the market m is still a matter of interest. There are two possible options: first is to use Realized Variance or Bi-power Variance for both; and second is to use Realized Variance for Realized Beta and Bi-power Variance for the Bi-power Beta. This paper shall employ both methodologies in order to disentangle jump beta from equation (7). The first method will be implemented for its additive property. The second will be applied to estimate the diffusive beta.

So now after substituting equation (15) and (16) into equation (3), we get Realized Beta,

$$R\beta_{i,t} = \frac{RCov_t(r_i, r_m)}{RV_{m,t}} = \frac{2 \times \left[RV_{p,t} - \frac{1}{4}RV_{i,t} - \frac{1}{4}RV_{m,t} \right]}{RV_{m,t}}, \quad (17)$$

Bi-power Beta,

$$B\beta_{i,t} = \frac{BCov_t(r_i, r_m)}{RV_{m,t}} = \frac{2 \times \left[BV_{p,t} - \frac{1}{4}BV_{i,t} - \frac{1}{4}BV_{m,t} \right]}{RV_{m,t}}, \quad (18)$$

and Diffusive Bi-power Beta,

$$DB\beta_{i,t} = \frac{BCov_t(r_i, r_m)}{BV_{m,t}} = \frac{2 \times \left[BV_{p,t} - \frac{1}{4}BV_{i,t} - \frac{1}{4}BV_{m,t} \right]}{BV_{m,t}}, \quad (19)$$

Note that the Realized, Bi-power, and Diffusive Bi-power Beta, constructed in equation (17), (18), and (19), respectively, is a daily measure rather than the traditional approach assuming only one beta for each stock throughout time. This fact will become important later on, when betas are examined throughout different periods.

3.3.2 Jump Beta and Effect of Jumps

It might be difficult to see how these betas interact to disentangle the jump beta without studying their limit behaviors. Thus, to clarify, the limits for the betas defined in equation (17), (18), and (19) must be examined. First, since Realized Beta is not jump robust, it can be decomposed into jump beta and diffusive beta as in equation (7) as follows:

$$R\beta_{i,t} \rightarrow \beta_C \left(\frac{\sigma_C^2}{\sigma_C^2 + \sigma_J^2} \right) + \beta_J \left(\frac{\sigma_J^2}{\sigma_C^2 + \sigma_J^2} \right),$$

where the arrow represents asymptotic convergence in probability. Next, note that Diffusive Bi-power Beta is jump-robust because it can be computed solely from the jump-robust Bi-power Variances. So, the Diffusive Bi-power Beta converges in probability to the diffusive beta:

$$DB\beta_{i,t} \rightarrow \beta_C.$$

Then, because Bi-power Beta uses Realized Variance of the market index as denominator, Bi-power Beta converges in probability to diffusive beta times the weight as follows¹³:

$$B\beta_{i,t} \rightarrow \beta_C \left(\frac{\sigma_C^2}{\sigma_C^2 + \sigma_J^2} \right).$$

Finally, it is possible to proceed and disentangle jump beta as follows¹⁴:

$$\hat{\beta}_J = \frac{R\beta_{i,t} - B\beta_{i,t}}{1 - B\beta_{i,t}/DB\beta_{i,t}} \quad (20)$$

Thus is the estimator for jump beta.

Another measurement of interest is the effect of jumps on the Realized Beta. As mentioned before, Realized Beta and Bi-power Beta are additive because both have Realized

¹³ Since the jump-robust Bi-power Variance of the market converges in probability to the diffusive variance, or variance of returns on days without jumps,

$$BV_{m,t} \rightarrow \sigma_C^2,$$

Realized Variance of the market converges in probability to the variances of the returns on days with and without jumps,

$$RV_{m,t} \rightarrow \sigma_C^2 + \sigma_J^2,$$

and the Diffusive Bi-power Beta converges in probability to the diffusive beta, the proof is as follows:

$$\begin{aligned} B\beta_{i,t} &= \frac{BCov_t(r_i, r_m)}{RV_{m,t}} \\ &= \frac{BCov_t(r_i, r_m)}{BV_{m,t}} \left(\frac{BV_{m,t}}{RV_{m,t}} \right) \\ &= DBP_{i,t} \left(\frac{BV_{m,t}}{RV_{m,t}} \right) \rightarrow \beta_C \left(\frac{\sigma_C^2}{\sigma_C^2 + \sigma_J^2} \right) \blacksquare \end{aligned}$$

¹⁴ Proof: $\hat{\beta}_J = \frac{R\beta_{i,t} - B\beta_{i,t}}{1 - B\beta_{i,t}/DB\beta_{i,t}} \rightarrow \frac{\beta_C \left(\frac{\sigma_C^2}{\sigma_C^2 + \sigma_J^2} \right) + \beta_J \left(\frac{\sigma_J^2}{\sigma_C^2 + \sigma_J^2} \right) - \beta_C \left(\frac{\sigma_C^2}{\sigma_C^2 + \sigma_J^2} \right)}{1 - \beta_C \left(\frac{\sigma_C^2}{\sigma_C^2 + \sigma_J^2} \right) / \beta_C} = \frac{\beta_J \left(\frac{\sigma_J^2}{\sigma_C^2 + \sigma_J^2} \right)}{1 - \left(\frac{\sigma_C^2}{\sigma_C^2 + \sigma_J^2} \right)} = \frac{\beta_J \left(\frac{\sigma_J^2}{\sigma_C^2 + \sigma_J^2} \right)}{\frac{\sigma_J^2}{\sigma_C^2 + \sigma_J^2}} = \beta_J$, where the \rightarrow represents convergence in probability.

Variance of the market index as the denominator. Therefore, by taking the difference, it is possible to define effects of jumps on Realized Betas, or EJ , as

$$EJ \equiv R\beta_{i,t} - B\beta_{i,t} \rightarrow \beta_J \left(\frac{\sigma_J^2}{\sigma_C^2 + \sigma_J^2} \right). \quad (21)$$

Following directly from the limits defined above, it's clear that EJ converges in probability to the jump beta multiplied by the weight of jump variance, or the variance for time intervals with jumps. Intuitively, EJ is the impact of jumps on Realized Beta measurements. Thus, it is necessary to take into consideration not only the change in beta in the presence of jumps but also the contribution of jumps to the total variance. Suppose jump beta is vastly different from diffusive beta, but if jumps were infrequent, the contribution of jump variance to the total variance would be insignificant and the effect of jumps on Realized Beta measurement would be negligible. So, EJ is also important in determining the robustness of the Realized Beta measurement. Thus concludes our theoretical framework. Section 5 will discuss the empirical results calculated based on equation (20) and (21).

4 Data

4.1 Data Source

This paper uses minute-by-minute price data, obtained from commercial data vendor price-data.com, for 14 commonly traded stocks in S&P 100 and the S&P500 Futures Index. As shown in Table 3, the time horizon for the price data varies, with the longest from April 1997 to December 2010, and shortest from May 2006 to Dec. 2010. However, all the graphs will be drawn on the same times scale for ease of cross-stock comparison. On each trading day included in the dataset, the price data are available for each minute from 9:35am to 4:00pm. Though New York Stock Exchange opens at 9:30am on trading days, the first five minutes of price data are excluded, since the price movements in the first five minutes contain much noise as the market tries to adjust to overnight announcements or aftermarket trading. The choice of sampling frequency will be discussed in the next subsection, as it is a little more technical. Furthermore, the stock splits are adjusted for so as not to be mistaken for extraordinarily large jumps, though stock splits that happen after the market closes are already accounted for as overnight returns are

excluded from the analysis in this paper. Thus are the data available for the research in this paper. Next, decisions must be made over which stocks to choose for the study.

Among the 14 stocks chosen, seven are from the Financial Services sector, four from Food, and three from Pharmaceutical, as shown in Table 3. Stocks are picked from different sectors in order to do a comprehensive study and to eliminate the potential bias arisen from a single stock or a single sector. Here we have a substantially high number of financial stocks because this paper wishes to study how the betas for financial stocks vary over times, especially during times of financial stress. Firms are also chosen from the Food and Pharmaceutical industry, because they tend to be relatively stable during financial stress as food and over-the-counter drugs (OTCs) are necessities. The food and pharmaceutical stocks are expected to provide contrast to the financial stocks and also provide support for the validity of our methods. Hence, by comparing and contrasting the Financial Stocks and Food/Pharmaceutical, some insights might emerge about the overall market during times of financial stress.

4.2 Microstructure Noise

This paper discusses results obtained from high-frequency sampling of stock prices. Thus, there is the question of how frequent should the prices be sampled. More information is obtained when sampling frequency is high. However, when sampling at high frequency, information like the bid-ask spread¹⁵ is also included in the dataset, introducing additional movements obscuring the actual price of the stock, or in general “microstructural noise.” To display the tradeoff between additional information and microstructural noise, Andersen, Bollerslev, Diebold and Labys (2001) calculates and plotted the average Realized Variance (a measurement of deviation from the mean as explained below) at different sampling frequencies, ranging from one to 60 minutes. The graph in Andersen, Bollerslev, Diebold and Labys (2001) is reproduced for JP Morgan & Chase Co. in Graph 1, plotting average Realized Volatility against sampling frequency. Microstructural noise contributes to the high average Realized Volatility at higher frequencies. As the sampling frequency decreases, so do microstructural noise, and thus a lower average Realized Volatility. Between 5- and 10-minute sampling frequencies, the average Realized

¹⁵ Bid-ask spread is the difference between the purchasing price and selling price of the same security. In order to make money, brokers sell securities to buyers (ask) at a price slightly higher than the price they buy (bid) from sellers. This phenomenon obscures the true price, because each transaction is either at the bid price or ask price, providing potential discontinuities.

Volatility starts to flatten out, meaning forgoing any more information would not eliminate much microstructural noise. Thus, it's a good tradeoff between information and microstructural noise around the 5-minute sampling frequency.

5 Results

5.1 Effects of Jumps on Realized Beta

Table 1 summarizes the effects of jumps on Realized Beta. The results are important because Realized Beta, calculated from Pearson Covariance and Realized Variance of the market, would be significantly biased if effects of jumps on beta prove to be significant. The biased beta estimation would generate biased estimation of expected returns and covariance of securities, leading to non-optimal investment decisions, and thus, posing additional risk for investors. As Table 1 indicates, the simple student t-test rejects null hypothesis at 0.1 percent significance level for more than half of the chosen sample stocks. In other words, for most cases, the effect of jumps, or EJ , as calculated from equation (21) is significantly different from 0. However, as shown in column five in Table 1, EJ rarely contributes to more than 10 percent of the average of the Realized betas. Specifically, the relative effect of jumps rarely exceeds 3 percent for the financial stocks that are listed in the top section of Table 1. Thus, EJ may not be economically significant, as it does not contribute much to the Realized betas.

The same observation can also be found in Graph 2, which plots Realized betas against Bi-power betas for JPM. The red line in the graph is the 45° line and the red circle is the mean of Realized betas plotted against the mean of Bi-power betas. As the cluster of points fall symmetrically around the 45° line and the red circle falls almost perfectly on the 45° line, it is clear that Realized betas and Bi-power betas have almost 1:1 ratio. In other words, the difference between Realized betas and Bi-power betas, i.e. EJ , is almost zero. This observation is consistent with the data in Table 1.

5.1 Diffusive and Jump Beta

Table 2 summarizes the results obtained from applying equation (19) and (20) to the 14 chosen stocks. The Realized and Bi-power betas are copied from Table 1 for comparison. Diffusive Bi-

power Beta is expected to be greater than Bi-power Beta, since Bi-power Beta is equivalent to Diffusive Bi-power Beta multiplied by the weight $\frac{\sigma_c^2}{\sigma_c^2 + \sigma_j^2}$, which is less than 1¹⁶. Looking through all 14 stocks in Table 2, it can be quickly verified that Diffusive Bi-power betas are indeed greater than Bi-power betas as expected. However, it is surprising, at first, to see that the Diffusive Bi-power betas are greater than Realized betas for all of the 14 chosen stocks. After closer examinations, results, nevertheless, make sense mathematically. Since food and pharmaceutical stocks all have negative EJ as indicated in Table 1, Realized betas are, on average, necessarily smaller than Bi-power betas, which are smaller than Diffusive Bi-power betas by construction as explained earlier and as shown in Table 2. With some work, it can also be shown that in general, Diffusive Bi-power Beta will be larger than Realized Beta if the weight $\frac{\sigma_c^2}{\sigma_c^2 + \sigma_j^2}$ is less than $1 - REJ$, where REJ is the Relative Effect of Jumps as indicated in column 5 of Table 1¹⁷.

In addition, Table 2 also reports Jump betas for the 14 stocks. The Jump betas are indeed quite different from the Diffusive Bi-power betas as suggested in Todorov and Bollerslev (2010). However, for a few stocks, such as BAC, JPM, HNZ, and WAG, the Jump beta are negative, contrary to findings in Todorov and Bollerslev (2010) that indicate jump betas are positive across stocks. Furthermore, the Jump betas are mostly smaller in magnitude than their diffusive counterparts, again deviating from results in Todorov and Bollerslev (2010) suggesting that jump betas are usually larger than their diffusive counterparts. These differences, nevertheless, does not conclude contrary findings, because Table 2 also shows that for each stock, the standard error of mean for Jump betas is, sometimes, almost as large as, and sometimes, even larger than the mean of Jump betas itself. Thus, definitive conclusions can hardly be drawn from such volatile

¹⁶Since squares of real numbers are greater than 0, $\sigma_c^2 < \sigma_c^2 + \sigma_j^2$. Dividing both sides by the value on the right hand side gives the desired result: $\frac{\sigma_c^2}{\sigma_c^2 + \sigma_j^2} < 1$.

¹⁷ Since $REJ = \frac{EJ}{R\beta} = \frac{R\beta - B\beta}{R\beta}$, $1 - REJ = 1 - \frac{R\beta - B\beta}{R\beta} = \frac{B\beta}{R\beta}$. Rearranging the terms gives $B\beta \left(\frac{1}{1 - REJ} \right) = R\beta$. Decomposing Bi-power Beta into Diffusive Bi-power Beta times the weight yields $DB\beta \left(\frac{\sigma_c^2}{\sigma_c^2 + \sigma_j^2} \right) \left(\frac{1}{1 - REJ} \right) = R\beta$. If $\frac{\sigma_c^2}{\sigma_c^2 + \sigma_j^2} < 1 - REJ$, $\left(\frac{\sigma_c^2}{\sigma_c^2 + \sigma_j^2} \right) \left(\frac{1}{1 - REJ} \right) < (1 - REJ) \left(\frac{1}{1 - REJ} \right) = 1$. Thus, $R\beta = DB\beta \left(\frac{\sigma_c^2}{\sigma_c^2 + \sigma_j^2} \right) \left(\frac{1}{1 - REJ} \right) < DB\beta$, given that $\frac{\sigma_c^2}{\sigma_c^2 + \sigma_j^2} < 1 - REJ$. ■

results. On the same note, Todorov and Bollerslev (2010) also points out that their jump betas are not persistent.

5.3 Financial Stocks vs. Food and Pharmaceutical Stocks

The relatively small EJs for financial stocks came to the center of attention when Table 1 was examined in detail. The financial stocks' betas, ranging from 0.79 to 0.94¹⁸, almost double some of the betas of food and pharmaceutical stocks. This observation is consistent with the expectation of higher systematic risks associated with financial firms. However, the magnitude of EJs for the financial stocks, ranging from 0 to 0.025, small relative to the food and pharmaceutical stocks' EJs , which all, save one, have magnitude greater than 0.025. One possible culprit would be the nature of jump-robust estimators. Given that financial stocks have betas that are almost twice the betas of food and pharmaceutical stocks, it would be more difficult for jump-robust Bi-power betas to “classify” and “remove” changes in Realized betas as EJ . However, further investigations are needed to determine the true cause of the observations. It is also interesting to note that most of the EJs for the chosen financial stocks are positive, while all of the EJs for the food and pharmaceutical stocks are negative. Table 2 shows that Jump betas are largely positive for food and pharmaceutical stocks, indicating that the weight $\frac{\sigma_j^2}{\sigma_c^2 + \sigma_j^2}$ is less than 0, which is impossible for real numbers. However, this result might not necessarily indicate fallacy in theory or computation. As discussed before, the Jump beta measurements have such a big variance, so the results must be considered with caution.

5.4 Time-Varying Beta

The second argument expands upon the fact that beta is time-varying, as opposed to being fixed as presented in traditional risk analysis. As both Realized beta and Bi-power beta are both derived from variances, which are time-varying, this paper implicitly assumes beta is time-varying. The top numbers reported in column two and three of Table 1 represent the mean of betas across time for each stock, while the numbers in parentheses below display the standard error of the betas across time. Even with our prejudice, beta should be uniform throughout time if

¹⁸ This range excludes results from GS and NYX, which does not have sufficient amount of data, as shown in Table 3, to be comparable with other stocks. If included, the results would be biased towards the period of the U.S. Financial crisis around 2007.

it is indeed fixed as claimed in traditional risk analysis. What first caught the attention is the higher average Realized Beta for MS and NYX, which has data availability limited to 2006-2010. However, as shown in Graph 3, the Bi-power beta for JP Morgan & Chase Co. is far from being uniform. The beta spiked around 2008 when the recent financial crisis happened. Then, other figures in Graph 3 tells similar story, the Bi-power beta for almost every Financial stock spiked around 2008. It is interesting to note that GS has a much more moderate spike, perhaps due to its reduced exposure to Collateralized Debt Obligations (CDO) market as it claims. Graph 4 and 5, which display the betas of stocks from the Food and Pharmaceutical industry, does not share the same story. Besides the fact that the mean of each stock in the two industry is much lower than 1, as indicated by the lower red lines in Graph 5 and 6, the betas seem to be relatively uniform throughout time, with no apparent spike around the recent financial crisis. Perhaps it's unique to stocks in the Financial sector that the betas peak (above 1) during financial crisis.

6 Conclusions

From the results, it is clear that the effects of the jumps on the Realized Beta are, in general, too small to have any significant impact. This is especially true for the financial stocks. The evidence indicates that the effects of jumps are not important enough so they do not pose increased risk for investors. Though scope of this paper is limited by the availability of high-frequency data, the results propose important applications in portfolio management. Since beta is not significantly different in the presence of jump, we suggest portfolio managers not devote any effort to hedge the large price movements, or jumps. This paper finds that jump betas are in general lower than their diffusive counterparts, contrary findings in Todorov and Bollerslev (2010). However, as the standard errors on the average Jump betas are sometimes significantly larger than average Jump betas themselves, serious conclusions can hardly be drawn without further research. Nevertheless, this observation coincides with results from Todorov and Bollerslev (2010) suggesting non-persistent jump beta measurements.

Following the footsteps of Andersen, Bollerslev, Diebold, and Wu (2005), we find that the time-varying feature of betas to be more apparent in financial stock. It is clear from that beta spikes up during financial crisis. With further testing, we may be able to conclude the increase of beta during financial crisis. However, no particular patterns are observed in Food and

Pharmaceutical stocks. One explanation may be that financial stocks are more liquid and volatile than Food and Pharmaceutical stocks, and they become more sensitive when the market is stressed. Thus concludes our research.

7. Tables

Stocks	Realized Beta	Bi-power Beta	EJ	REJ^{19}	$ t\text{-stat} ^{20}$	$t\text{-critical}^{21}$	Reject null?
Financial Services							
BAC	0.8715 (0.0083)	0.8681 (0.0089)	0.0034 (0.0043)	0.39%	0.7949	3.2934	No
BK	0.8482 (0.0065)	0.8722 (0.0072)	-0.0239 (0.0050)	-2.82%	4.8126	3.2934	Yes
GS	0.8807 (0.0072)	0.8575 (0.0079)	0.0232 (0.0050)	2.63%	4.6339	3.2939	Yes
JPM	0.9442 (0.0063)	0.9188 (0.0072)	0.0253 (0.0044)	2.68%	5.7729	3.2934	Yes
MS	1.2513 (0.0149)	1.2049 (0.0165)	0.0461 (0.0093)	3.68%	4.9483	3.2985	Yes
NYX	1.0328 (0.0149)	1.0001 (0.0178)	0.0328 (0.0113)	3.18%	2.8926	3.2987	No
WFC	0.7906 (0.0074)	0.7910 (0.0081)	$-3.9334e^{-4}$ (0.0043)	-0.05%	0.0907	3.2934	No
Food							
HNZ	0.4277 (0.0039)	0.4676 (0.0047)	-0.0400 (0.0035)	-9.35%	11.3402	3.2934	Yes
KFT	0.3173 (0.0057)	0.3515 (0.0067)	-0.0342 (0.0046)	-10.78%	7.4348	3.2955	Yes
KO	0.5645 (0.0047)	0.5715 (0.0056)	-0.0070 (0.0031)	-1.24%	2.2581	3.2934	No
PEP	0.4974 (0.0045)	0.5461 (0.0057)	-0.0487 (0.0040)	-9.79%	12.1097	3.2934	Yes
Pharmaceutical							
CVS	0.5783 (0.0058)	0.6289 (0.0066)	-0.0507 (0.0049)	-8.77%	10.3850	3.2934	Yes
PFE	0.6702 (0.0054)	0.6957 (0.0064)	-0.0254 (0.0041)	-3.79%	6.1375	3.2934	Yes
WAG	0.6241 (0.0054)	0.6809 (0.0064)	-0.0568 (0.0050)	-9.10%	11.3723	3.2935	Yes

Table 1 shows the main results from the calculations. In column 2-4, the top number in each row represents the mean and the bottom number in parentheses represents the standard error of the mean. It's clear that the null hypothesis is rejected for most stocks at 0.1% significance level. Thus, we conclude that the jump contributions are statistically significantly different from 0. However, the relative effect of jump is rarely above 10%. And for the supposedly riskier Financial stocks, the relative jumps rarely contributes to more than 3% of the Realized Beta. Thus, even though the results may be statistically significant, the effect of jump is economically insignificant.

¹⁹Relative Effect of Jump is denoted as $REJ = \frac{EJ}{R\beta}$.

²⁰ $H_0: EJ = 0; H_a: EJ \neq 0$.

²¹At 0.1% significance level

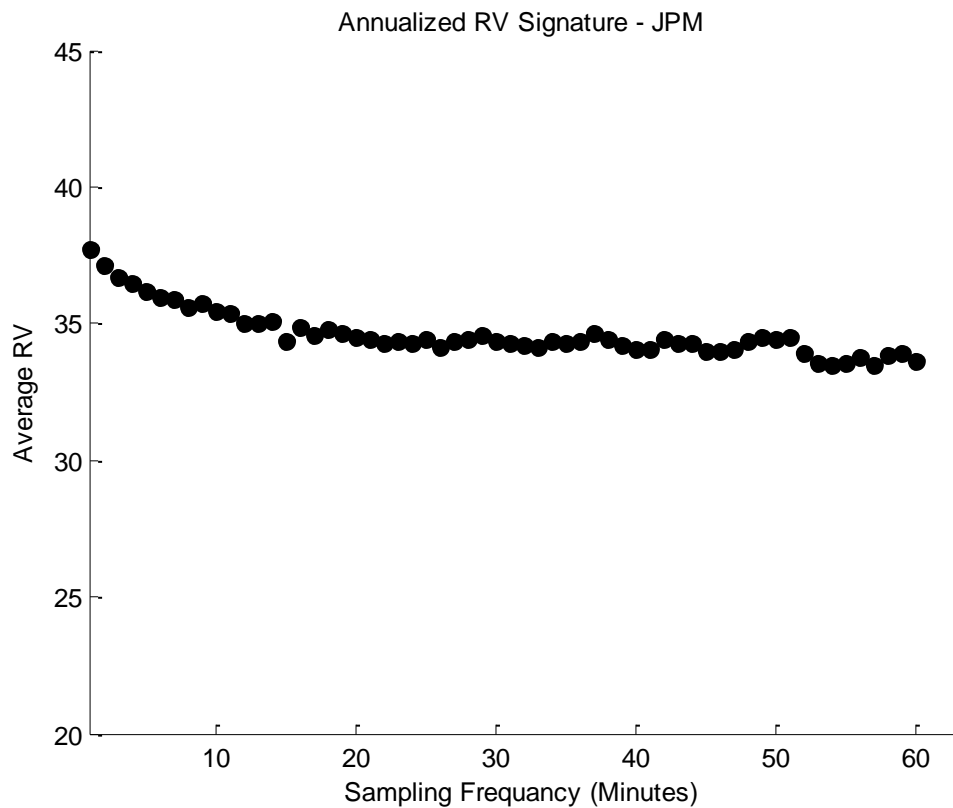
	Financial Services						
	BAC	BK	GS	JPM	MS	NYX	WFC
$R\beta$	0.8715 (0.008)	0.8482 (0.007)	0.8807 (0.007)	0.9442 (0.006)	1.2513 (0.015)	1.0328 (0.015)	0.7906 (0.007)
$B\beta$	0.8681 (0.009)	0.8722 (0.007)	0.8575 (0.008)	0.9188 (0.007)	1.2049 (0.017)	1.0001 (0.018)	0.791 (0.008)
$DB\beta$	0.935 (0.010)	0.941 (0.008)	0.930 (0.009)	0.990 (0.008)	1.308 (0.018)	1.087 (0.019)	0.855 (0.009)
$\hat{\beta}_J$	-0.069 (0.990)	0.847 (0.670)	2.072 (1.214)	-0.865 (1.023)	0.484 (0.958)	1.040 (1.446)	0.611 (1.081)
	Food				Pharmaceutical		
	HNZ	KFT	KO	PEP	CVS	PFE	WAG
$R\beta$	0.4277 (0.004)	0.3173 (0.006)	0.5645 (0.005)	0.4974 (0.005)	0.5783 (0.006)	0.6702 (0.005)	0.6241 (0.005)
$B\beta$	0.4676 (0.005)	0.3515 (0.007)	0.5715 (0.006)	0.5461 (0.006)	0.6289 (0.007)	0.6957 (0.006)	0.6809 (0.006)
$DB\beta$	0.506 (0.005)	0.385 (0.007)	0.615 (0.006)	0.589 (0.006)	0.680 (0.007)	0.750 (0.007)	0.735 (0.007)
$\hat{\beta}_J$	-0.377 (0.691)	0.253 (1.418)	0.155 (0.537)	0.067 (0.626)	0.122 (1.186)	0.479 (0.547)	-1.236 (1.434)

Table 2 shows the Diffusive Bi-power Betas and the Jump Betas calculated for the 14 chosen stocks. Similar to Table 1, the top number in each cell represents the mean while the bottom number in parentheses represents standard error of the mean. The Realized and Bi-power betas are copied from Table 1 for ease of comparison. It is interesting to note that most of the Jump betas have smaller magnitude than the Diffusive Bi-power beta, except for BAC, GS, and WAG. However, since the standard error of the mean for each Jump beta is so large relative to the mean itself, the results hardly hold any significance. Nevertheless, it is also interesting to note that the Realized betas are fairly close to the Diffusive Bi-power betas, reinforcing the observations from Table 1.

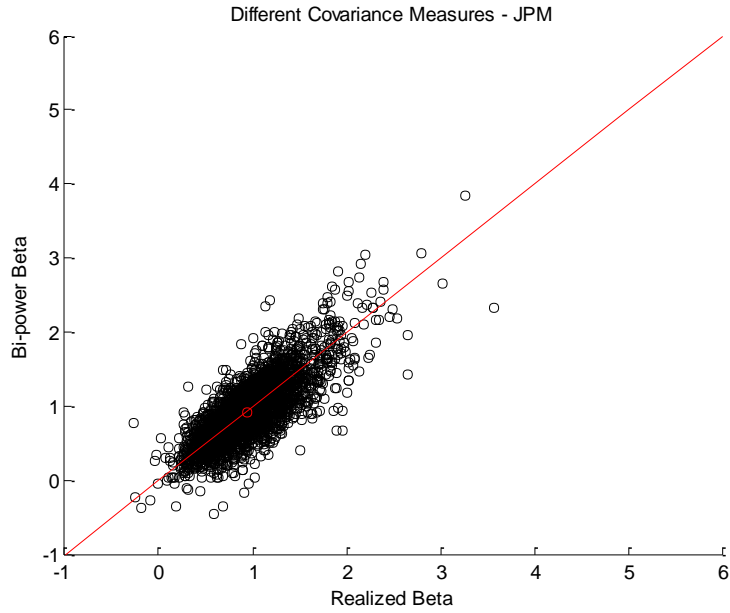
Ticker	Full Name	Date Range	Number of days in sample
Financial Services			
BAC	Bank of America Corp.	04/09/1997 to 12/30/2010	3411
BK	The Bank of New York Mellon Corp.	04/08/1997 to 12/30/2010	3410
GS	Goldman Sachs Group, Inc.	05/05/1999 to 12/30/2010	2896
JPM	JP Morgan Chase & Co.	04/09/1997 to 12/30/2010	3411
MS	Morgan Stanley	01/17/2006 to 12/30/2010	1230
NYX	NYSE Euronext	05/08/2006 to 12/30/2010	1195
WFC	Wells Fargo & Co.	04/09/1997 to 12/30/2010	3409
Food			
HNZ	H.J. Heinz Company	04/09/1997 to 12/30/2010	3411
KFT	Kraft Foods Inc.	06/13/2001 to 12/30/2010	1949
KO	The Coca-Cola Company	04/09/1997 to 12/30/2010	3412
PEP	PepsiCo, Inc.	04/09/1997 to 12/30/2010	3413
Pharmaceutical			
CVS	CVS Caremark Corp.	04/08/1997 to 12/30/2010	3410
PFE	Pfizer Inc.	04/09/1997 to 12/30/2010	3411
WAG	Walgreens Co.	08/01/1997 to 12/30/2010	3332
Market Index			
SPFU	Standard & Poor's 500 Futures Index	01/02/1997 to 12/30/2010	3482

Table 3 shows the Ticker, Full name, Date Range and Number of days in sample for the 14 stocks chosen for this study. The stocks are divided in to their respective industry. Note that the price data for some of the stocks have the same data range, but might have different days missing, resulting in different number of days in sample.

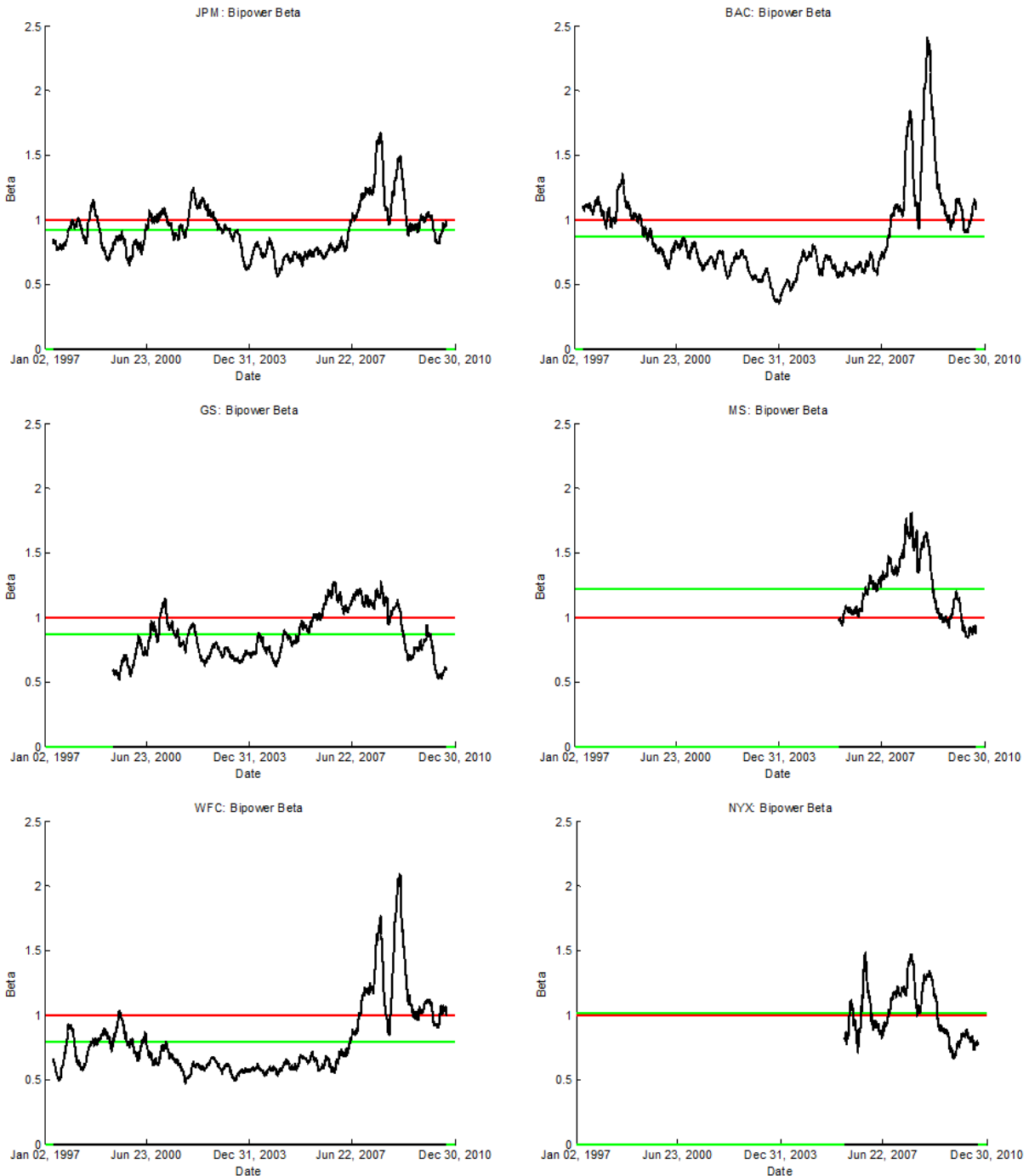
8. Figures



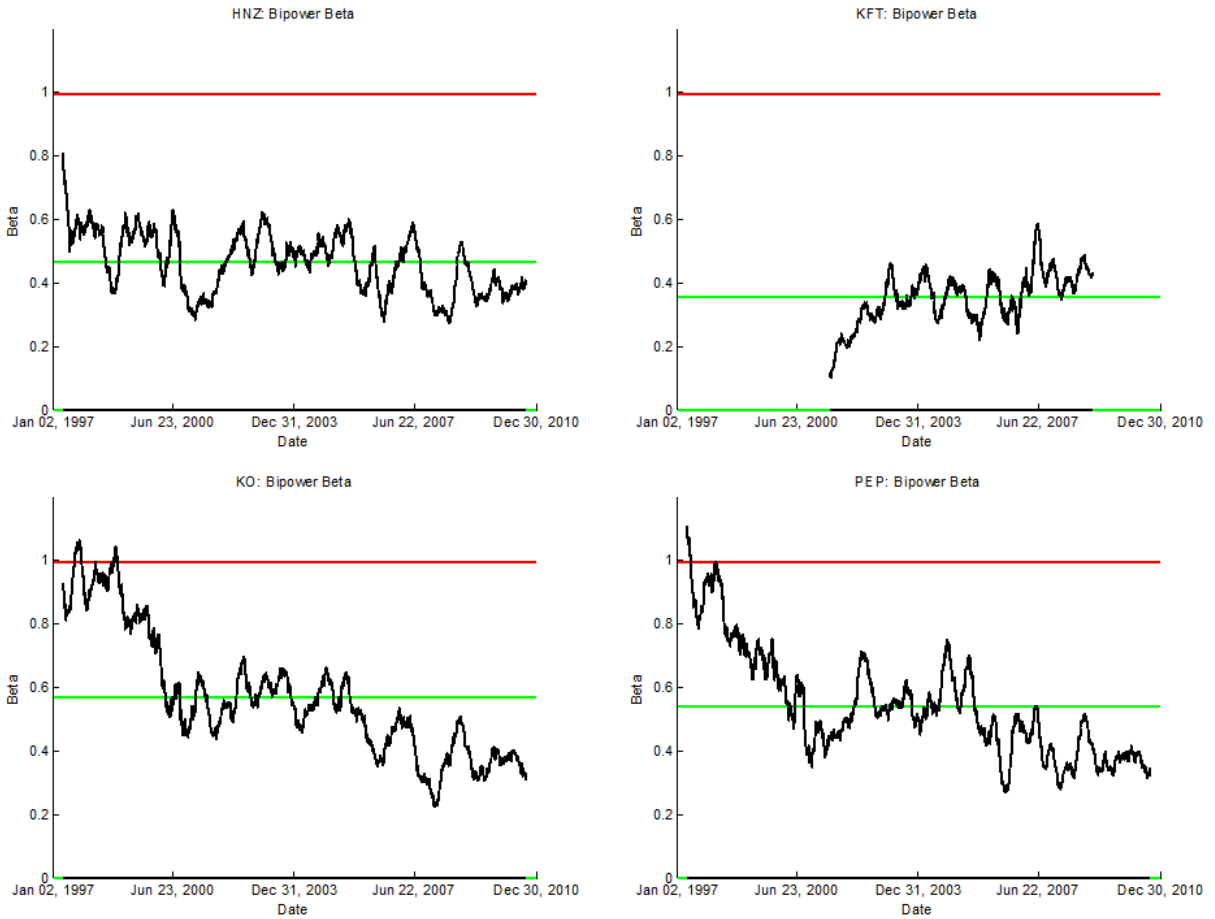
Graph 1, introduced in Andersen, Bollerslev, Diebold and Labys (2001), describes the relationship between average Realized Variance of the stock JPM from Apr. 1997 to Dec. 2010 and the sampling frequency in minutes on the x-axis. As the plot indicates, the higher the sampling frequency, the higher the average Realized Variance as more microstructural noise contributes to the Realized Variance measurement at higher sampling frequency. So an ideal sampling frequency, which captures enough variation but does not include so much microstructural noise, would be 5-minute (or 10-minute).



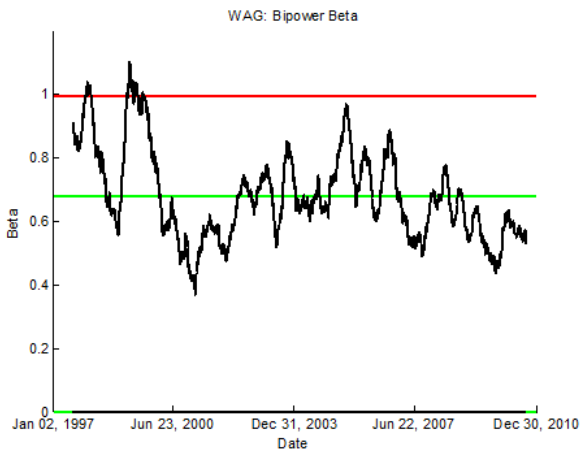
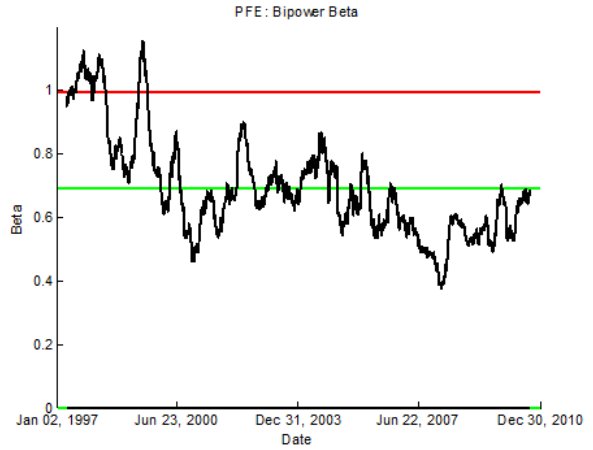
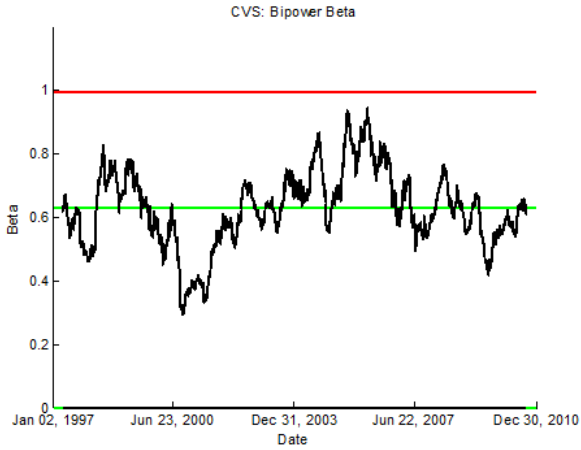
Graph 2 plots Realized beta against Bi-power beta for JPM. The red dot in the middle of the cluster is the mean of Realized Beta plotted against the mean of the Bi-power Beta. And the red line is the 45 degree line. Cluster of points seems to center around the 45 degree line and the red dot is right below the line. The contribution of jump is significantly different from 0 at even 0.1% significance level as indicated in Table 1. However, as we can see from the graph, the relative contribution of jump is insignificant, as the red dot falls almost perfectly on the 45 degree line, meaning Realized Beta and Bi-power Beta have almost a 1:1 ratio.



Graph 3 plots the jump-robust Bi-power betas for six financial stocks in order to examine the jump-free trends in betas. One financial stock, BK, is excluded from the graphs simply because of space constraints and aesthetics. Here, all the Bi-power betas plotted are averaged over 3 months, or 63 trading days, to wash out noise in daily Bi-power betas. The red line indicates a beta of 1, whereas the green line represents the mean of each stock's Bi-power beta. Notice not all stocks have the same time horizon, but all are plotted on the same time scale for ease of comparison. It is interesting to note that all the Bi-power betas in Graph 3 peaks above 1 around the U.S. Financial Crisis in 2008, and stocks with data from the first half of the decade have Bi-power beta peaking barely above 1 around 2000.



Graph 4 plots the Bi-power betas for randomly selected stocks from the Food Industry. A downward trend is observed, as opposed to the spikes observed in the financial stocks. The red and green lines represent the same as in Graph 3. But here, we can see the green line (mean) is much lower than the red line (1).



Graph 5 plots the Bi-power betas for Pharmaceutical stocks. It tells similar story as Graph 4. A downward trend can be observed from some of the stocks' Bi-power betas in Graph 5.

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