A Modeling Tool for Fuel Price Risk Management in Power Generation Portfolio Planning

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May 2012

Master’s project submitted in partial fulfillment of the requirements for the Master of Environmental Management degree in the Nicholas School of the Environment of Duke University

2012
Abstract

Power plants are significant capital investments whose returns are largely determined by such uncertain factors as future fuel prices and environmental regulations. Traditionally regulated utilities in the United States initiate power plant investment decisions through a process called Integrated Resource Planning (IRP). The aim of this centralized planning approach is to comprehensively assess both supply-side and demand-side options for satisfying load in the utility’s service territory over the next 20-30 years, and to establish a road map of actions that will serve load at least cost while adhering to reliability standards. Given the current level of uncertainty associated with many types of power plant investments, industry experts have identified a need for innovative planning practices within the IRP framework that can highlight resource tradeoffs, consider a variety of outcome metrics, and test a broader range of resource portfolios.

This document describes the development and evaluation of a decision tool that is capable of capturing the tradeoff between upfront capital costs and fuel price risk in power plant investment decisions. An adaptation of mean-variance portfolio theory (MVP), a financial risk management framework, serves as the foundation of the model. As an extension of previous applications of MVP to power sector planning, this project explores the potential for implementing a parametric approach to fuel price inputs as well as a multi-period decision structure. Additionally, the model is designed as an open-source tool that operates on a widely accessible software platform so that a range of IRP stakeholders—including utilities, public utilities commissions, and environment-focused non-governmental organizations—can openly discuss data inputs, assumptions, and results.

The decision tool is evaluated under five input scenarios that represent different possible future fuel price trajectories and greenhouse gas emissions concerns. Results suggest that the parametric fuel price modeling methodology demonstrated here may be a valuable approach to future IRP applications of MVP. The multi-period structure is problematic in that outputs cannot be compiled across periods. Additionally, use of a computationally limited software platform precludes sufficient representation of economies of scale in power plant construction. Although these drawbacks may limit MVP applications to the exploratory analysis phase of IRP, with further development, the decision tool developed here could be useful in addressing some of the limitations of current IRP practices with respect to decision-making under uncertainty.
**List of Tables**

Table 1: Example user input for electricity demand.
Table 2: Engineering and financial assumptions for three power plant options.
Table 3: Plant type assignments.
Table 4: Coal price model components.
Table 5: LCOE calculation components: indices, parameters, and variables.
Table 6: Expected LCOE and standard deviation values.
Table 7: LCOE covariance matrix.
Table 8: Nonlinear program components: indices, parameters, decision variables, and intermediate decision variables.

**List of Figures**

Figure 1: Major US ISO/RTOs, January 2011.
Figure 2: The efficient frontier concept from mean-variance portfolio theory.
Figure 3: Port-Optim model schematic.
Figure 4: Fuel price drift and volatility.
Figure 5: Fuel price module schematic.
Figure 6. Coal and natural gas price projections.
Figure 7: LCOE module schematic.
Figure 8: Portfolio optimization module schematic.
Figure 9: Period 1 capacity additions and portfolio costs for the Base Case scenario.
Figure 10: Cumulative capacity additions through 2032 under Base Case scenario.
Figure 11: Period 1 efficient frontiers for three scenarios of natural gas price volatility.
Figure 12: Period 1 efficient frontiers for three scenarios of greenhouse gas emissions concern.
Table of Contents

I. Introduction ............................................................................................................................................. 1
   1.1 Big Decisions, Little Certainty ............................................................................................................ 1
   1.2 Decision-Making in Traditionally Regulated Territories ................................................................. 2
   1.3 Integrated Resource Planning and Risk Management ..................................................................... 4
   1.4 Objectives .......................................................................................................................................... 8

II. Methods ................................................................................................................................................. 10
   2.1 Model Overview ............................................................................................................................... 10
   2.2 User Inputs ........................................................................................................................................ 11
   2.3 Plant Assumptions ............................................................................................................................ 12
   2.4 Fuel Price Module ............................................................................................................................. 15
   2.5 Levelized Cost of Electricity Module .............................................................................................. 19
   2.6 Portfolio Optimization Module ......................................................................................................... 24
   2.7 Model Outputs .................................................................................................................................. 30

III. Application .......................................................................................................................................... 32
   3.1 The Base Case .................................................................................................................................. 32
   3.2 Natural Gas Volatility Scenarios ...................................................................................................... 35
   3.3 Greenhouse Gas Concern Scenarios ............................................................................................... 37

IV. Conclusions ......................................................................................................................................... 40

References .................................................................................................................................................. 43

Appendix: Model Code ................................................................................................................................ 46
I. Introduction

1.1 Big Decisions, Little Certainty

The electric power industry in the United States faces many challenges today. Chief among these is the need to meet growing demand while also replacing aging infrastructure over the next several decades (EIA, 2011a; EIA, 2011b). Although efficiency gains will be achieved by residential, commercial, and industrial power users, significant power plant capacity expansions are expected. For example, the US Energy Information Administration (EIA) projects that over 200 GW of new capacity will be needed between now and 2035 (EIA, 2011a).

However, new power plant investments are fraught with market, technology, and regulatory uncertainties. Despite the failure of recent efforts to pass climate-related legislation, regulation of greenhouse gas emissions is an ongoing possibility that would greatly impact the capital and/or operating costs of new coal plants (Electric Power Research Institute (EPRI), 2011). Availability and prices for natural gas will be determined in part by unknown future supplies derived from horizontal drilling and hydrofracturing of shale deposits (Costello, 2010); this extraction process is subject to continuing inquiries into environmental and human health effects (Majumdar et al., 2012). New nuclear units face significant financing hurdles because of their high upfront costs, and may also be subject to new rules surrounding (1) disaster preparedness and liability in the wake of Japan’s recent Fukushima Daiichi nuclear accident, and (2) waste disposal given the current political impasse on long-term storage of high-level nuclear waste (Holt, 2011). Finally, renewable technologies such as wind and solar hinge substantially on sometimes transient government incentives, and their cost-competitiveness moving forward will also be determined by industry learning, technological development, and power storage advances (EPRI, 2011).

These are just a few of the risks associated with currently available power generating technologies. Given that power plants with capacity ratings on the order of 1,000 MW cost billions of dollars to build and millions more each year to operate, investors and other stakeholders are understandably wary of industry uncertainties.
1.2 Decision-Making in Traditionally Regulated Territories

During the early 20th century, state legislatures across the US granted power producers monopoly status within specified service territories, and for many decades state public utilities commissions (PUCs) regulated electricity prices, capital investments, and allowed rates of return for each electric utility operating within each state (Bosselman, 2010). Despite a wave of electric power industry deregulation in the 1990s, competitive markets for electricity still do not exist in many states. This is true for much of the Southeast and part of the Western US, as evidenced by the lack of Regional Transmission Organizations (RTOs) and Independent System Operators (ISOs) in these regions (Figure 1; FERC, 2011). In these traditionally regulated areas of the country, power is produced and sold as it was for much of the 20th century: predominantly by vertically integrated investor-owned utilities that serve millions of customers and own thousands of megawatts of power-producing capacity. For example, North Carolina’s two major vertically integrated utilities, Duke Energy Carolinas and Progress Energy Carolinas, together were responsible for about 70% of the 136 million MWh of retail power sales in the state in 2010 (EIA, 2012).

Figure 1: Major US ISO/RTOs, January 2011. Source: Federal Energy Regulatory Commission, 2011. ISOs and RTOs are organizations that manage power transmission networks and operate
competitive wholesale electricity markets within their service territories. The white space on the map highlights areas of the country that lack an ISO/RTO and in most cases are served predominantly by traditionally regulated monopoly utilities.

Decisions to build new power plants in traditionally regulated territories start with utility Integrated Resource Plans (IRPs). An IRP considers the electricity demand forecast for the utility’s service territory over the next 2-3 decades and lays out a multi-year plan for meeting demand through efficiency gains, existing power plant capacity, and new power plant builds. The analyses underlying these plans are complex. They consist of detailed power system reliability studies, production simulations, and investment cost models that are carried out using powerful software packages (Stoll, 1989; Logan et al., 1994). Because these analyses are time-consuming and resource-intensive, utilities often use relatively simple and quick screening analyses to identify candidate resource portfolios that will then be subject to the detailed analysis noted above (Stoll, 1989; Duke Energy Carolinas, 2010). Although power systems economics manuals, peer-reviewed literature, and the IRPs themselves can offer broad sketches of the technical analyses typically employed in the planning process, the data and specific planning approaches and algorithms used by individual utilities are considered confidential, proprietary information. As such, scrutiny by independent analysts and even state or federal regulators is generally prohibited.

The final IRP must be approved by the state PUC, which assesses the plan based on a set of rules established by the PUC or congressionally. The rules vary by state but are generally aimed at ensuring that utilities conduct and present technical analysis that serves as the basis of long-term resource decisions, which have been made pursuant to the goals of safety, reliability, and cost-effectiveness in electricity production. Cost-effectiveness is judged based primarily on expected infrastructure capital and operating costs over the relevant 2-3 decade planning period. Because fuel prices, environmental regulations, electricity demand, and other key factors are subject to uncertainty as discussed above, PUC rules sometimes include requirements for risk minimization. The North Carolina Utilities Commission, for example, states in its Rules and Regulations, Chapter 8, Rule R8-60(g):

“The utility shall analyze potential resource options and combinations of resource options to serve its system needs, taking into account the sensitivity of its analysis to variations in future estimates of peak load, energy requirements, and other significant assumptions,
including, but not limited to, the risks associated with wholesale markets, fuel costs, construction/implementation costs, transmission and distribution costs, and costs of complying with environmental regulation.”

However, even where short- and long-term risk minimization is an explicit objective, numerical risk benchmarks and acceptable risk management tools are rarely specified (as is the case for North Carolina). Thus the utility is given the burden of developing risk assessment and minimization techniques, and of demonstrating that its IRP reflects low-cost outcomes regardless of future circumstances. It is also important to note that IRP dockets at PUCs are generally non-binding; that is, the plans are meant as general road maps that will be updated regularly and supplemented with additional analyses each time power plant contracting and construction is imminent.

In areas of the country where competitive power markets exist (see Figure 1 above), traditional IRP approaches have mostly been replaced by market-based selection of long-term resource supply (Chupka et al., 2008). This is a reflection of the assumption that competitive markets will select resources in a way that maximizes economic efficiency. Given that (1) markets also set electricity prices in these regions, (2) investment risk is mainly shouldered by producers, and (3) power producers are prevented from wielding market power, overall there is little need for centralized review of any one producer’s future resource plans.

1.3 Integrated Resource Planning and Risk Management

As noted above, the details of utility IRP methodologies are not publicly available due to proprietary information rights, so careful scrutiny of the technical analyses is ordinarily not possible. However, industry experts and PUC initiatives can offer insights into the state of IRP practices. A 1994 report commissioned by the US National Renewable Energy Laboratory notes a “perception that the capabilities of current utility planning models are inadequate with regard to renewable resources” (Logan et al., 1994). More recently, some analysts have suggested that given current industry uncertainty, utility IRP approaches could benefit from (1) exploring a broader scope of potential resource mixes, (2) evaluating potential resource mixes using a range of outcome metrics, and (3) considering the tradeoffs associated with different resource mixes (Chupka et al., 2008). A briefing paper from the National Regulatory Research Institute highlights potential benefits of fuel diversification, resource planning at the portfolio level in
addition to stand-alone technology analysis, and giving due consideration to risk and option value (Costello, 2005). Some PUCs (for example the California Public Utilities Commission) have begun to take a more active role in utility planning, by conducting and soliciting independent planning analyses and by setting acceptable risk evaluation frameworks and metrics (California Energy Commission, 2007). Thus, experts have identified opportunities for developing modeling and planning approaches that are capable of capturing important attributes of nontraditional options for meeting demand (such as renewable resources) and that quantify risks in innovative ways.

A number of innovative planning methodologies focused on risk management have been demonstrated. A few of the more prominent approaches include value-at-risk analysis (California Energy Commission, 2007), real options theory (Dixit and Pindyck, 1994; Patiño-Echeverri et al., 2007), and mean-variance portfolio theory (Bazilian and Roques, 2008), which have all been adapted from the field of finance to power sector planning. Mean-variance portfolio theory (MVP) in particular is noteworthy for its potential to highlight the value of risk-hedging strategies early in the planning process, and to capture tradeoffs between resource portfolio cost and risk.

MVP was originally developed in the 1950s as a way of assembling portfolios of financial securities (Bazilian and Roques, 2008). The overall expected return of a portfolio of securities $R_p$ is simply the weighted average of the individual expected returns of each security (Van Horne and Wachowicz, 2008):

$$R_p = \sum_{j=1}^{m} W_j R_j$$

where

$W_j$ = proportion (weight) of funds invested in security $j$;

$R_j$ = expected return for security $j$; and

$m$ = total number of different securities in the portfolio.
The overall portfolio risk $\sigma_p$ (taken here as the standard deviation of possible portfolio returns) is a function of not just the individual risks of each security, but also the assumed correlation of returns between each security (Van Horne and Wachowicz, 2008):

$$\sigma_p = \left( \sum_{j=1}^{m} \sum_{k=1}^{m} W_j W_k \sigma_{j,k} \right)^{1/2}$$

where

$W_j =$ proportion (weight) of funds invested in security $j$;

$W_k =$ proportion (weight) of funds invested in security $k$;

$m =$ total number of different securities in the portfolio; and

$\sigma_{j,k} =$ covariance between possible returns for securities $j$ and $k$.

Thus, an investor should assemble a portfolio of assets whose returns are not well-correlated with one another, so that if the value of any one asset begins to decline, the value of other assets is most likely stable or increasing. Using this strategy, an investor can ensure that the value of the portfolio as a whole remains relatively stable over time. An important concept within MVP is the ‘efficient frontier’ that is observed on plots of portfolio returns versus portfolio risk (Figure 2).
Figure 2: The efficient frontier concept from mean-variance portfolio theory. Image adapted from California Energy Commission, 2007.

For each possible level of portfolio risk, optimization procedures can be used to identify the mix of assets with the highest expected portfolio return. A rational investor would only hold a portfolio that lies on the efficient frontier, because every portfolio below the frontier has a higher level of risk than necessary to achieve a given return (Bazilian and Roques, 2008). The concept of the efficient frontier also illustrates the capability of MVP to capture the tradeoff between risk and return, and to present this tradeoff in a simple way that allows investors to choose portfolios of assets that fit their attitudes toward risk.

MVP was first applied to electric power sector planning in the 1970s but has been used most frequently for electric sector analysis in the last decade (Bazilian and Roques, 2008). In this application, the assets are power plants rather than financial securities, and the relevant return metric most frequently used is the levelized cost of generating electricity (LCOE)\(^1\). The overall LCOE of a portfolio of resources can be minimized for each possible level of portfolio risk, using optimization techniques as described above. The investor can then consult the efficient frontier results and plan to assemble the mix of generating resources identified as providing the

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\(^1\) LCOE is a metric, given in $/MWh, that allows simple comparisons of power plants that have different upfront capital costs, operating costs, and lifetimes (LCOE computations will be discussed in detail in the Methods section below).
minimum expected portfolio LCOE for the level of risk he or she prefers. Most MVP applications from the early 2000s focus on fuel price uncertainty as the primary risk factor under consideration, and evaluate resource portfolios on a country-wide basis (Bazilian and Roques, 2008).

In recent years, many theoretical extensions of MVP analysis for power sector planning have been developed. For example, De Laquil, Awerbuch & Stroup (2005) considered not only fuel price uncertainty, but also construction period risks and uncertainties in fixed and variable O&M cost streams. Roques et al. (2008) applied MVP to generating companies in competitive markets rather than traditionally regulated territories. A load-factor-based variant of MVP has also been developed to account for the capabilities of different generating technologies to serve different load segments (Gotham et al., 2009).

There are a number of drawbacks to power sector applications of MVP theory to date. One drawback is that the approach is static in nature and has not been developed as a multi-period application that adapts to continually changing circumstances (Roques et al., 2008). Another major shortcoming is the use of historical prices and cost data to compute risk metrics and correlation factors. These drawbacks significantly limit the applicability of MVP to real-world planning frameworks used by utilities, PUCs, and other power sector stakeholders.

1.4 Objectives

The goal of this Master’s project is to develop a decision tool to aid planners in thinking about portfolios of capacity additions. Although there are a number of financial, engineering, and regulatory risks that affect the return of electricity generating assets over time, the tool developed here focuses on fuel price risk, as a natural first step towards an open-source model to design investment portfolios. MVP theory is applied to power sector resource portfolio assembly, and fuel price risk is quantified via a widely used securities price modeling framework that allows the user to test different assumptions about future fuel price trajectories. A multi-period framework is also developed that builds and dispatches plants dynamically in response to changing demand.

The remainder of the report proceeds as follows. Section 2 describes the Port-Optim model, including user inputs, the fuel price module, LCOE module, portfolio optimization module, and the model outputs. Section 3 presents an application of the Port-Optim model to the
regulated utility territory served by Duke Energy Carolinas, illustrating typical model results. Section 4 discusses the insights offered by the model, its major limitations, areas for further development, and potential uses for electric utility planning in traditionally regulated areas of the United States.
II. Methods

2.1 Model Overview

The Port-Optim Model is a Mathworks Matlab-based decision tool that is designed to inform portfolio-level electric capacity planning on the timescale of decades. The user inputs demand forecasts for the utility service area of interest, as well as fuel price drift, volatility, and correlation factors that reflect the utility’s expectations about future coal and natural gas prices. The model uses the input data to project future fuel prices, calculate expected per-unit electricity costs from several generation technology options, and assemble a set of minimum-cost resource portfolios having a range of price risk. Generally, a lower expected portfolio cost is a tradeoff for greater potential portfolio variability, or risk. The model results allow the user to choose a resource portfolio plan that fits the risk preferences of the utility service territory.

As illustrated below in Figure 3, the user inputs are fed into the Port-Optim model, which is comprised of plant assumption data plus three main computational components: the fuel price module, the LCOE module, and the portfolio optimization module. The model outputs are exported to an Excel spreadsheet for review and analysis.

User Inputs
- Demand
- Fuel price factors

Port-Optim Model
- Plant assumptions
- Fuel price module
- LCOE module
- Portfolio optimization

Outputs
- Future resource mixes
- Portfolio costs + variance

Figure 3: Port-Optim model schematic.

The model inputs are described below in Section 2.2. The computational machinery of the model is described in Sections 2.3, 2.4, 2.5, and 2.6, including detailed schematic diagrams and algorithm lists for each module. Finally, the outputs are described in Section 2.7.
2.2 User Inputs

The user inputs required to start the model include forecast electricity demand for the utility service territory of interest and expected fuel price drift, volatility, and correlation factors over the model planning horizon.

**Demand.** Expected electricity demand is represented both by total annual power consumption (in MWh) and by annual peak capacity requirement (in MW). Values are input at 5-year intervals for the full 60-year planning horizon (see Table 1 below; values are based on Duke Energy Carolinas 2010 Integrated Resource Plan and will be used to demonstrate the model below in Section 3). Electricity demand is a key constraint in the model’s portfolio optimization routine. Annual MWh demand over the long term is also part of the model’s indirect representation of economies of scale; hence the longer time horizon of MWh demand values than MW values seen in Table 1. Scale economy representation will be discussed in detail in Sections 2.3 and 2.6.

<table>
<thead>
<tr>
<th>Year</th>
<th>Annual Generation (MWh)</th>
<th>Peak Capacity (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>91,100,000</td>
<td>17,800</td>
</tr>
<tr>
<td>2017</td>
<td>98,700,000</td>
<td>19,400</td>
</tr>
<tr>
<td>2022</td>
<td>108,500,000</td>
<td>21,100</td>
</tr>
<tr>
<td>2027</td>
<td>118,700,000</td>
<td>22,700</td>
</tr>
<tr>
<td>2032</td>
<td>125,300,000</td>
<td>23,800</td>
</tr>
<tr>
<td>2037</td>
<td>125,300,000</td>
<td>-</td>
</tr>
<tr>
<td>2042</td>
<td>125,300,000</td>
<td>-</td>
</tr>
<tr>
<td>2047</td>
<td>125,300,000</td>
<td>-</td>
</tr>
<tr>
<td>2052</td>
<td>125,300,000</td>
<td>-</td>
</tr>
<tr>
<td>2057</td>
<td>125,300,000</td>
<td>-</td>
</tr>
<tr>
<td>2062</td>
<td>125,300,000</td>
<td>-</td>
</tr>
<tr>
<td>2067</td>
<td>125,300,000</td>
<td>-</td>
</tr>
</tbody>
</table>

**Fuel price factors.** Expected fuel price drift, volatility, and correlation factors are the second group of user inputs to the Port-Optim model. The fuel price module contains algorithms that project future coal and natural gas prices. The algorithms will be described in detail below in Section 2.4, but two of the key factors determining future fuel price trajectories are annual drift
and volatility. Both factors are percentages measured in relation to the initial fuel price. Drift is the expected annual growth rate in fuel prices, and volatility is the potential annual variability in this growth rate. For example, if natural gas prices are assumed to have a drift of 2% and a volatility of 0%, the price would be expected to grow smoothly by 2% each year (Figure 4a). If gas prices are 4.90 $/tcf today and assumed to have a drift of 0% and a volatility of 2%, the expected price at any future year would be 4.90 $/tcf, but prices would fluctuate by 2% annually around this expected price (Figure 4b).

![Figure 4: Fuel price drift and volatility. (a) At 2% drift and 0% volatility, natural gas prices begin at 4.90 $/tcf and grow smoothly by 2% annually. (b) At 0% drift and 2% volatility, five separate price trajectories begin at 4.90 $/tcf and fluctuate by 2% annually around a constant expected price of 4.90 $/tcf (black dotted line).](image)

Fuel price correlation is the tendency of natural gas and coal prices to track each other year to year. A correlation of 0 would indicate that natural gas and coal prices move completely independently of one another from one year to the next, while a correlation of 1 would indicate that their year-to-year movements are linearly dependent on one another or linearly dependent on a third common factor.

### 2.3 Plant Assumptions

In addition to the demand and fuel price inputs described above, the Port-Optim model relies on a set of engineering and financial assumptions about power plant operation and costs. These plant assumptions, along with the user inputs described in Section 2.2, together comprise the raw data on which the major model computations are based. Fuel price projections are created via a set of algorithms described below in Section 2.4, and these price projections are
further used to compute LCOE values for three power generation technologies: coal steam, natural gas combined cycle (NGCC), and wind (Section 2.5). The LCOE values are inputs to the final portfolio optimization routine, which selects minimum-cost resource mixes for a range of portfolio risk levels, over several planning periods (Section 2.6). The current section presents a description of the plant assumptions used within the model.

Table 2 shows the model’s numerical assumptions about the plant types available to the portfolio optimization routine for use in meeting demand over time.

Table 2: Engineering and financial assumptions for three power plant options.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Units</th>
<th>Value</th>
<th>Additional Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nameplate Capacity</td>
<td>Rated power output of plant.</td>
<td>MW</td>
<td>700</td>
<td>700</td>
</tr>
<tr>
<td>Heat Rate</td>
<td>Average heat input required per unit of energy production.</td>
<td>btu/kWh</td>
<td>8,800</td>
<td>7,000</td>
</tr>
<tr>
<td>Heat Content of Fuel</td>
<td>Average heat content per unit of fuel.</td>
<td>Fuel-specific</td>
<td>19.5 mmbtu/short ton</td>
<td>1 mmbtu/tcf</td>
</tr>
<tr>
<td>Plant Lifetime</td>
<td>Useful life of plant.</td>
<td>Years</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>Capacity Factor</td>
<td>Annual energy production as a percentage of maximum production potential.</td>
<td>None</td>
<td>30%, 60%, 90%</td>
<td>30%, 60%, 90%</td>
</tr>
<tr>
<td>Annual Capital + Financing Cost</td>
<td>Annual cost of physical plant plus annual financing charge accounting for WACC and taxes†.</td>
<td>$</td>
<td>140,000,000</td>
<td>42,000,000</td>
</tr>
</tbody>
</table>

†WACC, weighted average cost of capital. IECM inputs for coal and NGCC plants: 55% debt at 6% interest rate, 45% equity at 12% required return, 34% federal taxes, 4.15% state taxes, 2% property taxes, discount rate of 8.7%. Coal plant is assumed to have NOx, SO2, PM, and mercury controls installed. Wind capital + financing costs computed using Excel’s PMT function with IECM financing and discount assumptions, and a capital cost of $2,400,000 per MW (from EIA 2010a).
Notice that the dispatchable generation technologies, coal steam and NGCC plants, have three capacity factors listed in Table 2. The portfolio optimization routine has the option of building and operating dispatchable technologies at any of these three capacity factors. Because wind is non-dispatchable, the model can only choose to build and run a wind farm at a 30% capacity factor. In effect this means that the model has seven plant types to choose from in meeting demand over time (Table 3).

Table 3: Plant type assignments.

<table>
<thead>
<tr>
<th>Plant Type</th>
<th>Technology Description</th>
<th>Capacity Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Coal Steam</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>Coal Steam</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>Coal Steam</td>
<td>0.9</td>
</tr>
<tr>
<td>4</td>
<td>NGCC</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>NGCC</td>
<td>0.6</td>
</tr>
<tr>
<td>6</td>
<td>NGCC</td>
<td>0.9</td>
</tr>
<tr>
<td>7</td>
<td>Wind</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The capacity factor options available to the model serve as an indirect mechanism for encouraging economies of scale in power plant construction, and they also allow for a dynamic modeling approach that optimizes plant output over multiple time periods, as overviewed below.

Direct representation of economies of scale in the portfolio optimization module would require formulation as a mixed-integer nonlinear program, which is computationally difficult. Because the Port-Optim model is designed to run in seconds using widely accessible software, a less computationally intensive nonlinear program formulation is used. In order to simulate scale-economy planning behavior, the model may choose to build a larger plant than needed to meet demand in the near term, if demand is expected to continue to grow in the long term. In this case, the plant is operated at a lower capacity factor initially and then ramped up as demand grows over time. This mechanism incentivizes larger builds and thereby simulates scale-economy behavior. Simulating such behavior is important because as indicated in Section 2.5, the model assumes that plant capital costs vary linearly at the $/MW cost of a 700 MW plant (see Table 2). In reality, the capital costs of smaller plants are not simple linear reductions of the total cost of a 700-MW-scale plant. Instead, the $/MW costs of power plant construction and financing vary
with power plant size (EIA, 2010a). Thus, a mechanism for simulating scale economy behavior is needed to improve the odds that use of scale-economy costs is reflective of the actual size of power plants built. Section 2.6 below describes the mechanism for simulating scale economy behavior in detail.

Including several capacity factor options is also a basis for operating plants dynamically as fuel prices and demand levels change over time. As described above, a power plant can be built and operated at one capacity factor in one period, and then ramped up or down in subsequent periods based on future circumstances. Sections 2.7 and 4 discuss the implications of this approach to incorporating a dynamic element into the MVP framework for resource portfolio planning.

2.4 Fuel Price Module

The fuel price factors input by the user are incorporated into fuel price algorithms that project expected coal and natural gas prices and price variances at specified future years. For each five-year planning period, the portfolio optimization module will assemble resource portfolios based on the expected LCOE and variance values of the different plant options at year 20. Thus, the fuel price module must provide year-20 price projections for each planning period (beginning in 2017, 2022, 2027, and 2032). A schematic of the fuel price module is presented below (Figure 5).

Figure 5: Fuel price module schematic. Blue diamonds are user inputs, green diamonds are assumptions based on external data (here, EIA, 2011a), and yellow circles are calculated values used further in the LCOE module.
Prices are assumed to change from year to year according to a geometric Brownian motion (GBM) model. GBM is a stochastic process model that is commonly used to forecast future securities prices (Dixit and Pindyck, 1994). Although GBM is capable of representing several key dimensions of future coal and natural gas price fluctuation, it is important to note that commodity price modeling is a nontrivial task that occupies significant amounts of time and expertise within industry, government, and academia. The GBM price models used here are not meant to capture complex commodity market dynamics. Instead, they are intended to simulate assumptions about the overall trends in fuel price movement over time, as well as the nature of year-to-year price fluctuations anticipated to occur.

Given its widespread use for securities modeling, GBM is a natural starting point for projecting fuel prices. Simple fuel price models such as these give the model the capability of incorporating user-defined values reflecting utility planning department and/or PUC expectations about future prices. These values, the drift and volatility factors, also allow scenario analysis with respect to fuel price variations—an exercise that will be demonstrated below in Section 3. The use of price models to forecast coal and natural gas prices eliminates the need to restrictively assume that future price movements will reflect historical price trends, which is a significant limitation of MVP applications to electric power sector planning to date.

Equations (1) and (2) below represent the price models for coal and natural gas, respectively, and Table 4 lists and describes the coal price model components (the natural gas price model components are analogous).

\[
C_y = C_{y-1} + \alpha_C*(C_{y-1})*dt + \omega_C*(C_{y-1})*dz \tag{1}
\]

\[
N_y = N_{y-1} + \alpha_N*(N_{y-1})*dt + \omega_N*(N_{y-1})*dz \tag{2}
\]
Table 4: Coal price model components.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_y)</td>
<td>Coal price in year (y) ($/short ton).</td>
</tr>
<tr>
<td>(C_{y-1})</td>
<td>Coal price in year (y-1) ($/short ton).</td>
</tr>
<tr>
<td>(\alpha_C)</td>
<td>Coal price annual drift (unitless).</td>
</tr>
<tr>
<td>(\omega_C)</td>
<td>Coal price annual volatility (unitless).</td>
</tr>
<tr>
<td>(dt)</td>
<td>Period length (years).</td>
</tr>
<tr>
<td>(dz)</td>
<td>Increment of a Wiener process: (dz = \epsilon\sqrt{dt}), where (\epsilon \sim N(0,1)).</td>
</tr>
</tbody>
</table>

GBM model components as described in Dixit and Pindyck (1994).

The default drift factors in Equations (1) and (2) are set so that the expected fuel prices through time approximately track the Reference case fuel price projections from the US Energy Information Administration’s Annual Energy Outlook 2011 (EIA, 2011a). Additionally, the default volatility factors are set so that the fuel price standard deviations approximately track the high and low fuel price projections from the High Coal Cost, Low Coal Cost, High Shale Recovery Per Well, and Low Shale Recovery Per Well side cases of the AEO2011. Figure 6 illustrates the expected fuel prices and standard deviations for period 1 under the default drift and volatility settings.

Figure 6. Coal and natural gas price projections. Projections were obtained from fuel price models given in Equations (1) and (2). Solid lines are expected prices; dotted lines are ± one s.d.
As noted above in Section 1.3, fuel price correlation is a critical assumption of MVP applied within the electric power sector. The default coal and natural gas price correlations were set to reflect the projected coal and natural gas price correlation represented in the AEO2011 Reference Case, which is 0.99 over the relevant time horizon.

Another advantage of using a GBM model for coal and natural gas price forecasts is that analytical solutions can be obtained for expected prices and variance at time points of interest (Dixit and Pindyck, 1994). For example, the expected coal price at year $t$ is given by

$$E[C(t)] = C_0 e^{\alpha_C t}$$

(3)

where

- $C_0$ = initial coal price
- $\alpha_C$ = coal price annual drift
- $t$ = year of interest

The variance in the expected coal price at year $t$ is given by

$$\text{Var}[C(t)] = C_0^2 e^{2\omega_C^2 t} (e^{\omega_C^2 t} - 1)$$

(4)

where

- $C_0$ = initial coal price
- $\alpha_C$ = coal price annual drift
- $t$ = year of interest
- $\omega_C$ = coal price annual volatility

The natural gas price expectation and variance are computed analogously. Because the fuel price module is intended to provide year-20 price projections for each planning period, here $t$ is set at 20 and initial fuel prices ($C_0$ and $N_0$) are set at the AEO2011 Reference Case coal and natural gas price projections for 2017 (period 1), 2022 (period 2), 2027 (period 3), and 2032 (period 4).
Because the drift and volatility factors in these equations are known quantities (defined by user inputs), numerical values can be obtained for the expectation and variance of coal and natural gas prices at year 20 for each planning period. These values can then be used to obtain year-20 expected LCOE and variance values for coal– and natural gas–fired power plants, as described in the next section.

2.5 Levelized Cost of Electricity Module

As noted above, LCOE is a $/MWh metric that allows simple cost comparisons of power plants that have different upfront capital costs, operating costs, and lifetimes. LCOE values typically encompass capital and financing costs, fixed O&M, variable O&M (including fuel and other variable components), and sometimes associated transmission investments (Tidball et al., 2010; EIA, 2010b; EPRI, 2011). Here, a simplified model of power plant LCOE is used that excludes fixed O&M, variable O&M besides fuel costs, and transmission investment. An LCOE module schematic is presented in Figure 7, and the LCOE algorithms are given below as Equations (5), (6), and (7). Table 5 describes the equation components.
Figure 7: LCOE module schematic. Blue diamonds are user inputs, green diamonds are plant assumptions (Table 2), yellow circles are intermediate outputs from the fuel price module, purple circles are intermediate values calculated within the LCOE module, and pink circles are calculated values from the LCOE module that will be further used within the portfolio optimization module.

\[
\text{LCOE}_p = \left( \frac{\text{AnnCap}_p + \text{Fuel}_p}{\text{AnnElec}_p} \right)
\]

\[
\text{where}
\]

\[
\text{Fuel}_p = \frac{\text{Fuelprice}_p}{\text{HeatCon}_p} \times \text{HeatRate}_p \times \text{Nameplate}_p \times \text{CapFac}_p \cdots \\
\times \text{kWhperMWh} \times \text{HrperYr}
\]

(5)
\[ AnnElec_p = Nameplate_p \times CapFac_p \times HrperYr \] (7)

Table 5: LCOE calculation components: indices, parameters, and variables.

<table>
<thead>
<tr>
<th>Indices</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>Plant type</td>
<td>1-7 (see Table 3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Units; Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( AnnCap_p )</td>
<td>Annual capital + financing cost for plant type ( p )</td>
<td>$/; Listed in Table 2</td>
</tr>
<tr>
<td>( HeatCon_p )</td>
<td>Heat content of fuel for plant type ( p )</td>
<td>Listed in Table 2</td>
</tr>
<tr>
<td>( HeatRate_p )</td>
<td>Average heat rate of plant type ( p )</td>
<td>Btu/kWh; Listed in Table 2</td>
</tr>
<tr>
<td>( Nameplate_p )</td>
<td>Nameplate capacity of plant type ( p )</td>
<td>700 MW (see Table 2)</td>
</tr>
<tr>
<td>( CapFac_p )</td>
<td>Capacity factor of plant type ( p )</td>
<td>Listed in Table 3</td>
</tr>
<tr>
<td>( kWhperMWh )</td>
<td>Conversion factor from kWh to MWh</td>
<td>1,000</td>
</tr>
<tr>
<td>( HrperYr )</td>
<td>Number of hours in a year</td>
<td>8,760</td>
</tr>
<tr>
<td>( Fuel_p )</td>
<td>Total annual cost of fuel for plant type ( p )</td>
<td>$/; Computed according to Equation (6)</td>
</tr>
<tr>
<td>( AnnElec_p )</td>
<td>Annual energy production of plant type ( p )</td>
<td>MWh; Computed according to Equation (7)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Units; Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Fuelprice_p )</td>
<td>Fuel price for plant type ( p )</td>
<td>$/short ton or $/tcf; Computed according to Equation (3) and its natural gas analog</td>
</tr>
</tbody>
</table>

As shown in Table 5, all of the terms in Equations (5), (6), and (7) are known parameters, other than the random variable \( Fuelprice_p \). However, as described above in Section 2.4, the fuel price module has analytically obtained expectation and variance values for \( Fuelprice_p \). This means that the expectation and variance of the LCOE for plant type \( p \) can be obtained for year 20 of each planning period as follows:

\[ E[LCOE_p] = E[a \times Fuelprice_p + b] = a \times E[Fuelprice_p] + b \]

and

\[ \text{Var}[LCOE_p] = \text{Var}[a \times Fuelprice_p + b] = a^2 \times \text{Var}[Fuelprice_p] \]
where \( a \) and \( b \) are constants:

\[
\begin{align*}
a &= \left(\frac{1}{HeatCon_p}\right) \left(\frac{1}{AnnElec_p}\right) \cdot HeatRate_p \cdot Nameplate_p \cdot CapFac_p \cdot k\text{W}h\text{per}MWh \cdot Hr\text{per}Yr \\
b &= \frac{AnnCap_p}{AnnElec_p}
\end{align*}
\]

Because \( a \) and \( b \) are constants defined by known parameters, and numerical values have been analytically obtained for \( E[Fuelprice_p] \) and \( \text{Var}[Fuelprice_p] \), as demonstrated for \( C(t) \) via Equations (3) and (4) above, numerical values can also be obtained for \( E[LCOE_p] \) and \( \text{Var}[LCOE_p] \).

Using the parameters listed in Tables 2 and 3, the default fuel price drifts and volatilities, and the equations described above, the following values are calculated for LCOE expectation and variance (shown below as standard deviation) for period 1:

**Table 6: Expected LCOE and standard deviation values.**

<table>
<thead>
<tr>
<th>Plant Type</th>
<th>Technology Description</th>
<th>Capacity Factor</th>
<th>Exp. LCOE ($/MWh)</th>
<th>s.d. ($/MWh)</th>
<th>EIA LCOE ($/MWh)</th>
<th>EPRI LCOE ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Coal Steam</td>
<td>0.3</td>
<td>97.59</td>
<td>9.01</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Coal Steam</td>
<td>0.6</td>
<td>59.54</td>
<td>9.01</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Coal Steam</td>
<td>0.9</td>
<td>46.86</td>
<td>9.01</td>
<td>85.5-110.8</td>
<td>54-60</td>
</tr>
<tr>
<td>4</td>
<td>NGCC</td>
<td>0.3</td>
<td>74.00</td>
<td>15.19</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>NGCC</td>
<td>0.6</td>
<td>62.59</td>
<td>15.19</td>
<td>60.0-74.1</td>
<td>49-79</td>
</tr>
<tr>
<td>6</td>
<td>NGCC</td>
<td>0.9</td>
<td>58.78</td>
<td>15.19</td>
<td>61.9-115.0</td>
<td>75-138</td>
</tr>
<tr>
<td>7</td>
<td>Wind</td>
<td>0.3</td>
<td>69.04</td>
<td>0</td>
<td>81.9-115.0</td>
<td>-</td>
</tr>
</tbody>
</table>

†EIA and EPRI values are shown for comparison, from EIA (2010b) and EPRI (2011). The expected LCOE values calculated by the Port-Optim model are comparable to those from EIA and EPRI, but are lower across the board in part due to the model’s exclusion of fixed and variable O&M costs. Additionally, the EIA values for coal steam plants are higher than those from EPRI in part due to an assumed 3% increase in the cost of capital that is meant to serve as a proxy for the impact of a $15 per metric ton CO₂ emissions fee. EIA also includes transmission investment charges in all levelized cost estimates.

In addition to LCOE expectation and standard deviation for each individual plant type, a covariance matrix for the LCOE values is required for the portfolio optimization module, as indicated in Section 1.3. By default, a correlation factor of \( r = 0.99 \) for coal and natural gas prices is assumed based on the AEO2011 price forecast correlation. Because LCOE is assumed
here to be a single-variable function of fuel price, the plant-by-plant LCOE correlation factors are equivalent to the fuel price correlation factor. A correlation of \( r = 0 \) is assumed for the LCOE of wind versus coal steam and NGCC plants.

The quantity \( \sigma_{j,k} \), introduced in Section 1.3 as the covariance between possible returns for securities \( j \) and \( k \), is obtained as follows (Van Horne and Wachowicz, 2008):

\[
\sigma_{j,k} = r_{j,k} \sigma_j \sigma_k
\]

where

- \( r_{j,k} \) = the correlation coefficient for securities \( j \) and \( k \)
- \( \sigma_j \) = the standard deviation of security \( j \)
- \( \sigma_k \) = the standard deviation of security \( k \)

Using this equation, along with the plant-specific LCOE standard deviations computed above (see Table 6 for example) and the correlation factor assumptions just described, an LCOE covariance matrix is obtained:

**Table 7: LCOE covariance matrix.**

<table>
<thead>
<tr>
<th>Plant Type</th>
<th>Coal Steam</th>
<th>NGCC</th>
<th>Wind</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Coal Steam</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>81.22</td>
<td>81.22</td>
<td>81.22</td>
</tr>
<tr>
<td>2</td>
<td>81.22</td>
<td>81.22</td>
<td>81.22</td>
</tr>
<tr>
<td>3</td>
<td>81.22</td>
<td>81.22</td>
<td>81.22</td>
</tr>
<tr>
<td>NGCC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>135.57</td>
<td>135.57</td>
<td>135.57</td>
</tr>
<tr>
<td>5</td>
<td>135.57</td>
<td>135.57</td>
<td>135.57</td>
</tr>
<tr>
<td>6</td>
<td>135.57</td>
<td>135.57</td>
<td>135.57</td>
</tr>
<tr>
<td>Wind</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
The key outputs of the fuel price and LCOE modules, then, are year-20 expected LCOE values for each plant type, plus the 7 x 7 covariance matrix that represents the correlations between the LCOE values of pairs of plant types through time (see Tables 6 and 7). These outputs are used as inputs to the final component of the model: the portfolio optimization module, described in the next section.

Note that the planning horizon is set at 20 years based on the plant lifetime of the shortest-lived plant (that is, a wind farm). This restriction is necessary because the LCOE values calculated here will be used to make decisions about optimal resource portfolio composition. Thus, the planning horizon can be no longer than the shortest-lived plant type, because beyond that time point the originally planned portfolio will have changed due to retirement of the shortest-lived plant.

2.6 Portfolio Optimization Module

The purpose of the portfolio optimization module is to assemble the least-cost set of resources for a range of possible portfolio standard deviation values, using the mathematical framework of MVP discussed above in Section 1.3. The user is then able to select the portfolio that meets the utility’s or PUC’s risk preferences. The optimization routine is carried out sequentially four times (once for each planning period, or every 5 years beginning in 2017). For each of the four planning periods, the model selects resource portfolios using year-20 expected plant LCOE and standard deviation values computed as discussed above, along with a plant-to-plant LCOE covariance matrix. A module schematic is presented in Figure 8, and the parameters, variables, objective function, and constraints of the optimization program are described below.
Figure 8: Portfolio optimization module schematic. Blue diamonds are user inputs, green diamonds are plant assumptions (Table 2), pink circles are intermediate outputs from the LCOE module, orange circles are intermediate values calculated within the portfolio optimization module, light green rectangles are nonlinear program constraints, the bright green rectangle is the nonlinear program objective function, the gray triangle is the set of nonlinear program decision variables, and the red star represents the nonlinear program outputs.

1. Indices, Parameters, Decision Variables, and Intermediate Decision Variables. Table 8 lists and describes these key components of the nonlinear program formulation.
Table 8: Nonlinear program components: indices, parameters, decision variables, and intermediate decision variables.

<table>
<thead>
<tr>
<th>Indices</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p,q$</td>
<td>Plant type</td>
<td>1-7 (see Table 3)</td>
</tr>
<tr>
<td>$t$</td>
<td>Planning period</td>
<td>1-4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Units; Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CapFac_p$</td>
<td>Capacity factor of plant type $p$</td>
<td>Listed in Table 3</td>
</tr>
<tr>
<td>$LCOE_{p,t}$</td>
<td>Expected LCOE for plant type $p$ for planning period $t$</td>
<td>$$/MWh; Computed in LCOE module according to Equations (5), (6), and (7)</td>
</tr>
<tr>
<td>$Covar_{p,q,t}$</td>
<td>Covariance through time of plant type $p$ LCOE with respect to plant type $q$ LCOE, for planning period $t$</td>
<td>Unitless; Computed in LCOE module</td>
</tr>
<tr>
<td>$CP_t$</td>
<td>Capacity requirement for planning period $t$</td>
<td>MW; User-defined (for example see Table 1)</td>
</tr>
<tr>
<td>$G_t$</td>
<td>Generation requirement for planning period $t$</td>
<td>MWh; User-defined (for example see Table 1)</td>
</tr>
<tr>
<td>$D35_t$</td>
<td>Generation requirement 35 years after the start of planning period $t$</td>
<td>MWh; User-defined (for example see Table 1)</td>
</tr>
<tr>
<td>$D25_t$</td>
<td>Generation requirement 25 years after the start of planning period $t$</td>
<td>MWh; User-defined (for example see Table 1)</td>
</tr>
<tr>
<td>$SlackLim35_t$</td>
<td>Upper limit for allowance of slack that will still be available 35 years after the start of planning period $t$</td>
<td>MWh; $SlackLim35_t = D35_t - G_t$</td>
</tr>
<tr>
<td>$SlackLim25_t$</td>
<td>Upper limit for allowance of slack that will still be available 25 years after the start of planning period $t$</td>
<td>MWh; $SlackLim25_t = D25_t - G_t$</td>
</tr>
<tr>
<td>$HrperYr$</td>
<td>Number of hours in a year</td>
<td>8,760</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision Variables</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{p,t}$</td>
<td>Capacity of plant type $p$ to build and dispatch for planning period $t$</td>
<td>MW</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Intermediate Decision Variables</th>
<th>Description</th>
<th>Units; Computation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{p,t}$</td>
<td>Energy production of plant type $p$ for planning period $t$</td>
<td>MWh; $y_{p,t} = x_{p,t} \times CapFac_p \times HrperYr$</td>
</tr>
<tr>
<td>$GenTot_t$</td>
<td>Total energy production of all plants for planning period $t$</td>
<td>MWh; $GenTot_t = \sum_p y_{p,t}$</td>
</tr>
<tr>
<td>( FracGen_{p,t} )</td>
<td>Fraction of energy production from plant type ( p ) as a share of total energy production from all plant types, for planning period ( t )</td>
<td>Unitless; ( FracGen_{p,t} = \frac{y_{p,t}}{GenTott} )</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>( SlackTott )</td>
<td>Total slack available 25 and 35 years after the start of planning period ( t )</td>
<td>( \text{MWh}; \quad SlackTott = \left( \sum_{p=1}^{3} (0.9 - CapFac_p) \times x_{p,t} + \sum_{p=4}^{6} (0.9 - CapFac_p) \times x_{p,t} \right) \times HrperYr )</td>
</tr>
</tbody>
</table>

\[ \sum_{p} = \text{summation over} \ p. \]

\[ \sum_{p=1}^{3} = \text{summation over} \ p \text{ from 1 to 3}. \]

2. **Objective function.** Because minimizing portfolio LCOE can bias the results in favor of building out excess amounts of the lowest-cost per MWh (or lowest risk) resources in order to lower overall portfolio LCOE or variance, here the objective function minimizes total portfolio cost (in $; that is, portfolio $/MWh * total MWh portfolio generation). Simultaneously, the objective function maximizes the intermediate decision variable \( SlackTott \). ‘Slack’ is the total amount of spare generation potential (in MWh) available from all dispatchable plant types in a given planning period (that is, the extra generation potential available if all dispatchable plants ramped up to a maximum capacity factor of 90%). This variable is important for indirectly representing the producer’s incentive to take advantage of economies of scale in power plant construction; it will be discussed in more detail below. The objective function is as follows (refer to Table 8 for components):

For each sequential planning period \( t \),

\[
\text{Min} \left( \sum_{p} LCOE_{p,t} \times y_{p,t} \right) / SlackTott
\]

3. **Constraints.** The objective function is to be minimized subject to the following constraints (refer to Table 8 for components):
Portfolio standard deviation constraint:

\[ \sigma = \sqrt{\left( \sum_p \sum_q \text{FracGen}_{p,t} \ast \text{FracGen}_{q,t} \ast \text{Covar}_{p,q,t} \right)} \]

For each planning period, the objective function is minimized 10 different times, each time using a different value for the portfolio standard deviation equality constraint. The 10 portfolio s.d. values are selected by identifying the maximum and minimum non-zero plant-specific year-20 LCOE s.d. values. 10 equally spaced values between this max and min (inclusive) are used as s.d. equality constraints. This number of portfolio standard deviation values was chosen to minimize model running time while still yielding an illustrative range of portfolio compositions.

Capacity requirements must be satisfied:

\[ \left( \sum_{p=1}^{6} x_{p,t} \right) + 0.15 \gamma_{t} \geq CP_t \quad \text{for all } t \]

Dispatchable plant types are assumed as 100% firm capacity; wind farms are assumed as 15% firm capacity, based on practice used in Duke Energy Carolinas 2010 Integrated Resource Plan (DEC, 2010).

Generation requirements must be satisfied:

\[ \text{GenTot}_t \geq G_t \quad \text{for all } t \]

Coal steam capacity existing since previous planning period or earlier must continue to be dispatched:

\[ \sum_{p=1}^{3} x_{p,t} \geq \sum_{p=1}^{3} x_{p,t-1} \quad \text{for } t=2,3,4 \]

NGCC capacity existing since previous planning period or earlier must continue to be dispatched:

\[ \sum_{p=4}^{6} x_{p,t} \geq \sum_{p=4}^{6} x_{p,t-1} \quad \text{for } t=2,3,4 \]
Wind capacity existing since previous planning period or earlier must continue to be dispatched:

\[ x_{7,t} \geq x_{7,t-1} \text{ for } t=2,3,4 \]

Coal steam slack for any one planning period cannot exceed forecast of incremental MWh demand 35 years after the start of the planning period:

\[ 3 \text{HrperYr}\sum_{p=1} (0.9 - \text{CapFac}_{p})x_{p,t} \leq \text{SlackLim}_{35t} \text{ for all } t \]

Coal steam plus NGCC slack for any one planning period cannot exceed forecast of incremental MWh demand 25 years after the start of the planning period:

\[ 6 \text{HrperYr}\sum_{p=1} (0.9 - \text{CapFac}_{p})x_{p,t} \leq \text{SlackLim}_{25t} \text{ for all } t \]

Non-negativity constraints:

\[ x_{p,t} \geq 0 \text{ for all } p,t \]

Thus, the portfolio optimization module takes the demand forecasts originally input by the user and the expected LCOE and covariance values computed via the fuel price and LCOE modules, and finds the lowest total cost portfolio of generating resources that meets the MWh and MW demand constraints, for a range of possible portfolio standard deviation values. The portfolio optimization routine is run for 4 sequential 5-year planning periods. In the first period, demand and non-negativity are the only constraints on the system. In periods 2-4, constraints are added to reflect capacity built in previous periods. For example, if 1000 MW of coal capacity are built in period 1, a constraint is added for period 2 that sets a minimum of 1000 MW of coal capacity to be dispatched. This does not duplicate plant costs given that they are amortized to annual payments (thus a coal plant put online at year 1 of period 1 will be in its sixth year of operation at year 1 of period 2 and so far will have been paid off to 5/40 or 1/8 of its total cost; hence the constraint added at period 2 simply forces the user to continue paying the fixed capital costs of the plant the model selected in period 1). Furthermore, in addition to continuous dispatch of existing units from period to period, more capacity can be built as well.
As introduced briefly above, slack generation is defined as any unused generation potential of the dispatchable resources selected by the model. For example, suppose the model selects 1000 MW of coal capacity at a capacity factor of 0.3 in planning period 1. Maximum plant usage is defined as a capacity factor of 0.9; therefore, the 1000 MW coal plant at capacity factor 0.3 would yield slack generation of \((0.9 - 0.3) \times 1000 MW \times 8760\) hr/yr, or 5,256,000 MWh/year. The concept of slack generation is used within the model to artificially encourage exploitation of economies of scale through building larger plants than needed in the near term in preparation for growing demand in the future. As shown above in the nonlinear program formulation, MWh of slack generation is dividing the objective function to be minimized, thus encouraging as much slack generation as possible. Buildup of slack generation potential is limited, however, according to how much incremental demand is expected over the lifetime of the plants in question (see Table 8, SlackLim35, and SlackLim25). If a plant’s initial annual slack generation cannot be completely utilized (due to growing demand) by the end of the plant’s useful life, the plant would violate the module’s slack limit constraints and would not be selected. A noteworthy shortcoming of this approach to encouraging economies of scale is that it does not apply to wind plants because they are non-dispatchable. Demand forecasts 30 and 40 years into the future are also challenging, so accurate slack limits are likely to be difficult to obtain in practice.

Finally, the Port-Optim model does not consider construction time; that is, plants are assumed to be built and go online at the beginning of the planning period in question; they are further assumed to run at their designated capacity factor for the full 5 years of the planning period.

2.7 Model Outputs

At the end of the fourth planning period, the model has constructed a 20-year recommendation of plant builds and use, for each level of portfolio standard deviation. The final model outputs are the year-20 expected portfolio LCOE and standard deviation values for planning period 1, as well as a forecast of the capacity mixes observed at year 15 under each risk preference scenario.

There is no simple way to combine the portfolio cost and s.d. data from one planning period to the next, given that the portfolios change from period to period. Thus, a composite
portfolio cost and standard deviation for the entire planning horizon (encompassing all four planning periods) unfortunately cannot be reported. Though any wind farms built in planning period 1 would be heading toward the end of their useful lives by planning period 3 (and thus portfolio decisions at period 3 based on an assumed 20-year continued portfolio existence are not strictly valid), the resource mixes standing after planning period 3 are nonetheless somewhat indicative of the end results of planning over the long term using this approach. In Section 3 below, an illustrative application of the model is presented, complete with graphical displays of the final model outputs just described.
III. Application

This section illustrates how the model can be used to find 10 portfolios that meet an annual MWh demand that is assumed to grow as presented in Table 1, Section 2.2. That is, total annual energy use is forecast to grow from its current 2012 level of 91,100,000 MWh to 125,300,000 MWh by 2032. In addition, peak capacity needs are forecast to grow from 17,800 MW in 2012 to 23,800 MW in 2032. Beginning in 2032, demand is assumed to stay flat because of uncertainty in future demand levels beyond 2030. These demand inputs were chosen to reflect the current demand forecast of Duke Energy Carolinas (DEC, 2010), a major vertically integrated utility operating in North and South Carolina.

3.1 The Base Case

In the Base Case model run, default fuel price drifts and volatilities are assumed; that is, fuel price trajectories are approximately reflective of EIA’s AEO2011 Reference Case and Side Cases projecting prices under high and low fuel supply scenarios. Figures 9 and 10 below display the final outputs described in Section 2.7: resource mixes and year-20 portfolio LCOE and s.d. values for period 1, and the resource mixes expected to be in place after period 3 decisions are made.
Figure 9: Period 1 capacity additions and portfolio costs for the Base Case scenario.
Several key points are evident from Figure 9. First, the Port-Optim model is capable of capturing the cost-risk tradeoff discussed above in Section 1.3. The leftmost portfolios, for example Portfolios 1-3, have the highest expected portfolio LCOE values but the lowest s.d. values. Conversely, the rightmost portfolios, for example Portfolios 8-10, have the lowest expected portfolio LCOEs but the highest s.d. values. A planner could consider these portfolio options, observe that Portfolio 1 has an expected LCOE of 95 ± 8 $/MWh and Portfolio 10 has an expected LCOE of 68 ± 15 $/MWh, and decide whether to choose one of these extremes or a portfolio in between. Thus the model offers a nuanced perspective on portfolio costs that presents the planner with a straightforward representation of the cost-risk tradeoff inherent in portfolio decision-making.

Second, a comparison of Portfolios 1 and 2 suggests that the use of the slack capacity mechanism as a proxy for scale economy incentives may distort the model results in favor of dispatchable technologies. This is evident from the fact that Portfolio 1 is both lower cost and lower risk than Portfolio 2, simply by virtue of diversifying slightly with a small wind addition to
an otherwise pure-coal resource mix. The model selected this mix for Portfolio 1 because the portfolio s.d. constraint necessitated inclusion of low-risk wind capacity to reduce overall portfolio risk. In Portfolio 2, an all-coal mix is sufficient to meet the s.d. constraint, and because coal yields slack capacity, the objective function (which minimizes total portfolio cost, not per-unit cost, and is also impacted by slack) is optimized in a way that likely does not reflect the lowest possible portfolio LCOE for this risk level. An obvious solution is to optimize the per-unit portfolio cost, but as noted above in Section 2.6, this can result in extreme over-building in order to satisfy s.d. constraints at minimum per-unit cost. This shortcoming introduced by the slack-based mechanism for encouraging scale economy building will be explored further in Sections 3.3 and 4.

Figure 10 indicates the model’s ability to over-build dispatchable resources in the near term, operate at a low capacity factor initially, and ramp up output over time as demand continues to grow. Recall that the incremental capacity needs for period 1 are less than 2,000 MW. Yet as seen in Figure 9, at all risk levels, capacity has been over-built in anticipation of future demand increases. For example, of the approximately 4,600 MW of capacity built for Portfolio 6, all are operating at a capacity factor of 0.3 during period 1 (data not shown). Figure 10 shows that very little additional capacity has been built for Portfolio 6 after period 3; however, by period 3 most of the Portfolio 6 capacity is operating at capacity factors of 0.9 or 0.6 (data not shown). Thus, the model is capable of building new capacity in one period and dynamically ramping up output as needed in future periods.

3.2 Natural Gas Volatility Scenarios

Two scenarios of natural gas volatility were run and compared to the Base Case scenario described in Section 3.1. In the Volatile NG scenario, natural gas volatility was set as 30% higher than the default volatility in the Base Case. In the Stable NG scenario, natural gas volatility was set as 30% lower than the default value. Demand inputs remained the same as for the Base Case. Figure 11 below displays the period 1 efficient frontiers for these three scenarios of natural gas price volatility.
Figure 11 shows that different assumptions about fuel price volatility can lead to very different sets of cost-risk combinations that the planner must choose from. These different cost-risk combinations across scenarios can also mean very different menus of resource mixes available to choose from. For example, in contrast to the coal and natural gas portfolios seen in the Base Case results (Figure 9), under the Stable NG scenario the model constructs portfolios composed of only natural gas and wind (data not shown), with 100 to 1300 MW wind capacity added to natural gas–based portfolios to further reduce price risk. These results are most likely due to lower natural gas price risk than coal price risk; thus, while natural gas plants are least-cost and are used to meet most of the demand, wind capacity is the only way to further reduce price risk in order to present a menu of cost-risk combinations. In the Volatile NG scenario, portfolio composition is similar to that seen in the Base Case, except there is substantially less overbuilding to prepare for future demand increases (most portfolios are limited to ~3000 MW.
capacity; data not shown). Given that per-unit portfolio costs are higher in this scenario than in the Base Case, the strategy of overbuilding to exploit slack capacity benefits is less advantageous.

One strategy for making planning choices using results such as these is to compare the resource mixes for the portfolios represented on each of the efficient frontiers produced by the model. The planner’s goal would be to identify resource mixes that yield a favorable cost-risk balance for any of the given fuel price volatility scenarios.

3.3 Greenhouse Gas Concern Scenarios

Two scenarios of greenhouse gas concern were also run and compared to the Base Case scenario. In the GHG Concern scenario, interest rates and equity returns on coal steam investments were increased to 7% and 14%, respectively, which had an overall effect of increasing the WACC for coal plants by about 1.5%, similar to EIA’s assumption of 3% higher cost of capital for coal plants due to greenhouse gas emissions concerns. The increased WACC results in a higher annual capital cost for coal plants under this scenario. In the GHG, No Wind scenario, these same GHG concern inputs are used, but wind farms are not available as a resource choice in the portfolio optimization module. Demand inputs remained the same in both scenarios as for the Base Case. These scenarios were designed to test the benefits of wind additions to resource mixes in a future where carbon emissions are perceived to have negative impacts on investment value.
Figure 12 demonstrates that different valuations of carbon emissions lead the model to select different sets of portfolios. Again, a planner could identify resource mixes that perform favorably across all scenarios.

In addition, note that in the GHG Concern scenario, relative to the Base Case, wind replaces coal as the resource of choice used to supplement NGCC capacity and reduce portfolio risk in the higher-risk portfolios (Portfolios 6-9; resource mix data not shown). Coal is still a significant portion of the lower-risk resource mixes, however, and these two observations explain why the GHG Concern efficient frontier rises above that of the Base Case for the low-risk portfolios, but dips below the Base Case frontier for the high-risk portfolios. The benefit of wind capacity to reduce per-unit cost relative to coal is again highlighted here, and in the Base Case coal is the risk-mitigating resource of choice most likely because of its contribution to slack capacity. Indeed, in contrast to the GHG Concern scenario, in the GHG, No Wind scenario the
efficient frontier is simply a shifting of the Base Case frontier upwards to higher costs, because coal and natural gas are the only resource options used. Much like the results observed in Section 3.1, this latent wind benefit suggests that the slack capacity mechanism of encouraging scale economy builds has significant drawbacks in arbitrarily disfavoring non-dispatchable technology options.
IV. Conclusions

This Master’s project has sought to use the framework of a well-tested financial risk management tool, mean-variance portfolio theory, to represent fuel price uncertainty and minimize power plant investment risks due to this uncertainty. The model created allows the user to test different scenarios of future demand and fuel price trajectories, and to select resource portfolios by accounting for both the expected cost and variance of system-wide electricity production.

*GBM fuel price modeling.* In using a geometric Brownian motion model for fuel prices coupled with user-defined price trajectory factors, the Port-Optim model demonstrates an alternative approach for estimating future price risk and correlations. Rather than relying on empirical data, the GBM approach allows many possible future price trajectories to be tested so that resource portfolios may be selected on the basis of robustness across a range of possible futures. Results from model runs suggest that different assumptions about future fuel prices can lead to very different sets of portfolio choices that are not always intuitive based on individual plant LCOE values. Given the model’s capability of highlighting a range of portfolio choices under different possible future circumstances, the unconstrained approach demonstrated here to modeling fuel prices within an MVP framework may prove beneficial in real-world planning settings where future fuel prices are seen as highly uncertain.

*Multi-period MVP framework.* The dynamic MVP approach implemented in the Port-Optim model is workable in that power plants can be built in one period and re-dispatched at a higher or lower level in subsequent periods based on changing cost, risk constraints, and demand. However, the current model is not capable of tracking plant retirements over time and dynamically optimizing portfolios accordingly. A further consequence of this limitation is that the resource portfolio additions are not co-optimized with existing plant capacity. Additionally, portfolio cost and variance outputs for one period cannot be combined with those for future periods, so the model cannot give a final ‘total’ cost of the candidate portfolio plans over the time horizon of interest.

*Representing economies of scale.* The slack capacity approach to encouraging economies of scale is a component of the model that is closely related to the dynamic framework just described. Again, this approach is workable in that larger plants are indeed built early and
ramped up over time, which does simulate a tendency of project developers to capture economies of scale. However, smaller plants are still built, and when this occurs, the true construction and financing costs have not been properly represented because the assumed costs are linear functions of the total cost of large coal, NGCC, and wind installations, which ignores the effects of economies of scale. In addition, model results suggest a second significant drawback to this approach: the inclusion of slack capacity as a divisor in the objective function incentivizes selection of dispatchable plants over non-dispatchable technologies, sometimes in a way that leads to assembly of suboptimal portfolios.

**Future research and applications.** Further development of the modeling framework presented here should focus on addressing the challenges of (1) representing a dynamic planning process that incorporates existing units and plant retirement options, and (2) encouraging economies of scale without arbitrarily discouraging non-dispatchable generation options. The latter goal could easily be achieved given access to optimization software that can run mixed-integer nonlinear programs. However, a central goal of this project was to develop an open-source model that can be run quickly, and not only by utilities with significant computational resources and planning staff, but also by PUCs and NGOs interested in power sector planning approaches. Still, even if the two challenges noted here are addressed, MVP approaches to power sector planning do not appear to be well-suited to provide detailed recommendations for building and operating power plants. Given that multi-period results cannot be meaningfully combined, this risk management framework appears to be most appropriate for broad, exploratory analysis of supply options over multi-decade time horizons.

MVP-based analysis may therefore be informative at the screening stage of the utility IRP process, in identifying additional candidate resource portfolios for further analysis based on their cost-risk tradeoffs. This methodology is one approach for addressing IRP issues that some industry observers have cited: the need to capture risk-hedging benefits of renewable resources, consider a broader range of possible resource mixes, and highlight resource tradeoffs as part of planning practices. The Port-Optim model developed here is a starting point for an open-source decision tool for utilities, PUCs, and other power sector stakeholders. With further development to include a more representative range of resource choices, engineering constraints, and more detailed financial considerations, a static version of the Port-Optim model could ultimately be
used collaboratively or separately by parties to IRP proceedings to aid in decision-making under uncertainty.
References


<http://205.254.135.24/oiaf/aeo/electricity_generation.html>

<http://www.eia.gov/todayinenergy/detail.cfm?id=1830>


Appendix: Model Code

Matlab utilizes two types of files: script files and function files. Script files provide overall programming instructions and may call function files to perform subtasks. The Port-Optim model uses a primary script that calls 8 function files in the course of executing the code. Below is the Port-Optim script:

```matlab
%*********************************************************************
%*********************************************************************
%       PORT-OPTIM MODEL by Kenneth Sercy
%*********************************************************************
%*********************************************************************

clear
%**********************
% USER INPUTS
%**********************

%prompt user for MWh generation inputs
prompt={'Enter current demand (million MWh):',...
    'Enter year 2017 demand (million MWh):',...
    'Enter year 2022 demand (million MWh):',...
    'Enter year 2027 demand (million MWh):',...
    'Enter year 2032 demand (million MWh):',...
    'Enter year 2037 demand (million MWh):',...
    'Enter year 2042 demand (million MWh):',...
    'Enter year 2047 demand (million MWh):',...
    'Enter year 2052 demand (million MWh):',...
    'Enter year 2057 demand (million MWh):',...
    'Enter year 2062 demand (million MWh):',...
    'Enter year 2067 demand (million MWh):'};
dlg_title='Demand Inputs';
um_lines=[1 50];
defaultanswer={'91.1', '98.7', '108.5', '118.7', '125.3','125.3',...
    '125.3','125.3','125.3','125.3','125.3','125.3'};
options.Resize='on';
options.WindowStyle='normal';
options.Interpreter='tex';
answer=inputdlg(prompt,dlg_title,num_lines,defaultanswer,options);
dem=[str2num(answer{1}), str2num(answer{2}), str2num(answer{3}),...
    str2num(answer{4}), str2num(answer{5}), str2num(answer{6}),...
    str2num(answer{7}), str2num(answer{8}), str2num(answer{9}),...
    str2num(answer{10}),str2num(answer{11}),str2num(answer{12})];
demadj=dem*1000000;

%prompt user for MW peak capacity inputs
prompt={'Enter current peak capacity (thousand MW):',...
    'Enter year 2017 peak capacity (thousand MW):',...
    'Enter year 2022 peak capacity (thousand MW):',...
    'Enter year 2027 peak capacity (thousand MW):',...
    'Enter year 2032 peak capacity (thousand MW):',...
    'Enter year 2037 peak capacity (thousand MW):',...
    'Enter year 2042 peak capacity (thousand MW):',...
    'Enter year 2047 peak capacity (thousand MW):',...
    'Enter year 2052 peak capacity (thousand MW):',...
    'Enter year 2057 peak capacity (thousand MW):',...
    'Enter year 2062 peak capacity (thousand MW):',...
    'Enter year 2067 peak capacity (thousand MW):'};
dlg_title='Peak Capacity Inputs';
um_lines=[1 50];
defaultanswer={'115.1', '130.7', '147.5', '162.5', '178.9','178.9',...
    '178.9','178.9','178.9','178.9','178.9','178.9'};
options.Resize='on';
options.WindowStyle='normal';
options.Interpreter='tex';
answer=inputdlg(prompt,dlg_title,num_lines,defaultanswer,options);
dem=[str2num(answer{1}), str2num(answer{2}), str2num(answer{3}),...
    str2num(answer{4}), str2num(answer{5}), str2num(answer{6}),...
    str2num(answer{7}), str2num(answer{8}), str2num(answer{9}),...
    str2num(answer{10}),str2num(answer{11}),str2num(answer{12})];
demadj=dem*1000000;
```
'Enter year 2022 peak capacity (thousand MW):', ...
'Enter year 2027 peak capacity (thousand MW):', ...
'Enter year 2032 peak capacity (thousand MW):');
dlg_title='Peak Capacity Inputs';
num_lines=[1 50];
defaultanswer={'17.8', '19.4', '21.1', '22.7', '23.8'};
options.Resize='on';
options.WindowStyle='normal';
options.Interpreter='tex';
answer=inputdlg(prompt,dlg_title,num_lines,defaultanswer,options);
cap=[str2num(answer{1}), str2num(answer{2}), str2num(answer{3}),...
    str2num(answer{4}), str2num(answer{5})];
capadj=cap*1000;

%prompt user for fuel price drift,volatility,correlation factors
prompt={'Enter coal price drift factor:',...
    'Enter natural gas price drift factor:',...
    'Enter coal price volatility factor:',...
    'Enter natural gas price volatility factor:',...
    'Enter correlation factor for coal and natural gas prices:'};
dlg_title='Fuel Price Module Inputs';
num_lines=[1 50];
defaultanswer={'0.007','0.02','0.09','0.065','0.99'};
options.Resize='on';
options.WindowStyle='normal';
options.Interpreter='tex';
answer=inputdlg(prompt,dlg_title,num_lines,defaultanswer,options);
fuelmodin=[str2num(answer{1}),str2num(answer{2}),...
    str2num(answer{3}),str2num(answer{4}),str2num(answer{5})];

kWhperMWh=1000; %kWh to MWh conversion
kmvec=kWhperMWh*ones(1,7); %conversion vector
hrperyr=8760; %no. hrs per yr
hyvec=hrperyr*ones(1,7); %hr per yr vector

coalprice1=41.4; %initial coal price, dollars per short ton, period 1.
coalprice6=42.2; %initial coal price, period 2
coalprice11=43.9;
coalprice16=45.4;
ngprice1=4.9; %initial ng price, dollars per thousand cubic feet, per 1.
ngprice6=5.4;
ngprice11=6.2;
ngprice16=6.6;
codrift=fuelmodin(1); %annual price drift factors
ngdrift=fuelmodin(2);
covol=fuelmodin(3); %annual price volatility factors
ngvol=fuelmodin(4);
fuelcorr=fuelmodin(5); %coal-ng price correlation factor

%%%%%%%%%%%%%%%%%%%%%
% PLANT OPERATION ASSUMPTIONS
%%%%%%%%%%%%%%%%%%%%%

nameplate=700; %nameplate capacity assumption, MW
npall=nameplate*ones(1,7); %1 x 7 nameplate vector
heatreteco=8800; %coal plant heatrate assumption, btu/kWh
heatrateng=7000; % NGCC plant heatrate assumption, btu/kWh
hrall=[heatrateco*ones(1,3),heatrateng*ones(1,3),0]; % 1 x 7 heatrate vector
enconco=19500000; % coal plant energy content of fuel, btu/short ton
enconng=1000000; % NGCC energy content of fuel, btu/thousand cubic feet
enall=[enconco*ones(1,3),enconng*ones(1,3),1]; % 1 x 7 energy content vector
lifetimeco=40; % plant lifetime, years
lifetimeng=30;
lifetimewn=20;
lifetimeall=[lifetimeco,lifetimeco,lifetimeco,lifetimeeng,lifetimeeng,...
            lifetimeeng,lifetimewn]; % plant lifetime vector
horizon=min(lifetimeall);
cfco=[0.3,0.6,0.9]; % coal plant capacity factor options
cfng=[0.3,0.6,0.9]; % NGCC plant capacity factor options
cfwn=[0.3]; % wind farm capacity factor options
cfall=[cfco,cfng,cfwn]; % capacity factor vector
elecannco=cfco*nameplate*8760; % annual electric production coal plant, MWh
elecannng=cfng*nameplate*8760; % annual electric production NGCC, MWh
elecannwn=cfwn*nameplate*8760; % annual electric production wind farms MWh
elecall=[elecannco,elecannng,elecannwn]; % 1 x 7 annual electric vector
plantannco=140000000; % annual capital + financing cost coal plant, dollars
plantannng=42000000; % annual capital + financing cost NGCC, dollars
plantannwn=127000000; % annual capital + financing cost wind farm, dollars
plantall=[plantannco*ones(1,3),plantannng*ones(1,3),plantannwn*ones(1,3)]; % 1 x 7 annual cap vector

%****************************************
% FUEL PRICE MODELS & LCOE COMPUTATIONS
%****************************************
% period 1
ecol=coalprice1*exp(codrift*horizon); % expected coal price at horizon year
engl=ngprice1*exp(ngdrift*horizon); % expected ng price at horizon year
eall1=[ecol*ones(1,3),engl*ones(1,3),0]; % expected fuel price vector

% fuel price variances at horizon year
varcol=coalprice1^2*exp(2*codrift*horizon)*(exp(covol^2*horizon)-1);
varngl=ngprice1^2*exp(2*ngdrift*horizon)*(exp(ngvol^2*horizon)-1);
varall1=[varcol*ones(1,3),varngl*ones(1,3),0]; % fuel price variance vector

% coefficients for LCOE computation
coa_1=ones(1,7)./elecall./enall.*hrall.*npall.*cfall.*kmvec.*hyvec;
cob_1=plantall./elecall;

% period 2
eco2=coalprice6*exp(codrift*horizon);
eng2=ngprice6*exp(ngdrift*horizon);
eall2=[eco2*ones(1,3),eng2*ones(1,3),0];

varco2=coalprice6^2*exp(2*codrift*horizon)*(exp(covol^2*horizon)-1);
varng2=ngprice6^2*exp(2*ngdrift*horizon)*(exp(ngvol^2*horizon)-1);
varall2=[varco2*ones(1,3),varng2*ones(1,3),0];

coa_2=ones(1,7).*elecall./enall.*hrall.*npall.*cfall.*kmvec.*hyvec;
cob_2=plantall./elecall;
\text{%period 3}
\begin{align*}
\text{eco3} &= \text{coalprice11} \* \exp(\text{codrift} \* \text{horizon}); \\
\text{eng3} &= \text{ngprice11} \* \exp(\text{ngdrift} \* \text{horizon}); \\
\text{eall3} &= [\text{eco3} \* \text{ones}(1,3), \text{eng3} \* \text{ones}(1,3), 0]; \\
\text{varco3} &= \text{coalprice11}^2 \* \exp(2 \* \text{codrift} \* \text{horizon}) \* (\exp(\text{covol}^2 \* \text{horizon}) - 1); \\
\text{varng3} &= \text{ngprice11}^2 \* \exp(2 \* \text{ngdrift} \* \text{horizon}) \* (\exp(\text{ngvol}^2 \* \text{horizon}) - 1); \\
\text{varall3} &= [\text{varco3} \* \text{ones}(1,3), \text{varng3} \* \text{ones}(1,3), 0]; \\
\text{coa}_3 &= \text{ones}(1,7)./\text{elecall}./\text{enall}.*/\text{hrall}.*/\text{npall}.*/\text{cfall}.*/\text{kmvec}.*/\text{hyvec}; \\
\text{cob}_3 &= \text{plantall}./\text{elecall}; \\
\end{align*}

\text{%period 4}
\begin{align*}
\text{eco4} &= \text{coalprice16} \* \exp(\text{codrift} \* \text{horizon}); \\
\text{eng4} &= \text{ngprice16} \* \exp(\text{ngdrift} \* \text{horizon}); \\
\text{eall4} &= [\text{eco4} \* \text{ones}(1,3), \text{eng4} \* \text{ones}(1,3), 0]; \\
\text{varco4} &= \text{coalprice16}^2 \* \exp(2 \* \text{codrift} \* \text{horizon}) \* (\exp(\text{covol}^2 \* \text{horizon}) - 1); \\
\text{varng4} &= \text{ngprice16}^2 \* \exp(2 \* \text{ngdrift} \* \text{horizon}) \* (\exp(\text{ngvol}^2 \* \text{horizon}) - 1); \\
\text{varall4} &= [\text{varco4} \* \text{ones}(1,3), \text{varng4} \* \text{ones}(1,3), 0]; \\
\text{coa}_4 &= \text{ones}(1,7)./\text{elecall}./\text{enall}.*/\text{hrall}.*/\text{npall}.*/\text{cfall}.*/\text{kmvec}.*/\text{hyvec}; \\
\text{cob}_4 &= \text{plantall}./\text{elecall}; \\
\end{align*}

\text{%expected LCOE per tech at horizon year, periods 1-4}
\begin{align*}
\text{lcoe_objfunA} &= \text{coa}_1.\*\text{eall1}+\text{cob}_1; \text{ 1x7, $/MWh} \\
\text{lcoe_objfunB} &= \text{coa}_2.\*\text{eall2}+\text{cob}_2; \\
\text{lcoe_objfunC} &= \text{coa}_3.\*\text{eall3}+\text{cob}_3; \\
\text{lcoe_objfunD} &= \text{coa}_4.\*\text{eall4}+\text{cob}_4; \\
\end{align*}

\text{%LCOE s.d. per tech at horizon year, periods 1-4}
\begin{align*}
\text{stdp1} &= (\text{coa}_1.\^2.\*\text{varall1})^{.5}; \text{ 1x7, $/MWh} \\
\text{stdp2} &= (\text{coa}_2.\^2.\*\text{varall2})^{.5}; \\
\text{stdp3} &= (\text{coa}_3.\^2.\*\text{varall3})^{.5}; \\
\text{stdp4} &= (\text{coa}_4.\^2.\*\text{varall4})^{.5}; \\
\text{stdin} &= [\text{stdp1}; \text{stdp2}; \text{stdp3}; \text{stdp4}]; \\
\end{align*}

\text{%LCOE covariances}
\begin{align*}
\text{coco1} &= 1\*\text{stdp1}(1)^2; \text{ coco2} = 1\*\text{stdp2}(1)^2; \text{ coco3} = 1\*\text{stdp3}(1)^2; \\
\text{coco4} &= 1\*\text{stdp4}(1)^2; \\
\text{ngng1} &= 1\*\text{stdp1}(4)^2; \text{ ngng2} = 1\*\text{stdp2}(4)^2; \text{ ngng3} = 1\*\text{stdp3}(4)^2; \\
\text{ngng4} &= 1\*\text{stdp4}(4)^2; \\
\text{cong1} &= \text{fuelcorr}\*\text{stdp1}(1)\*\text{stdp1}(4); \text{ cong2} = \text{fuelcorr}\*\text{stdp2}(1)\*\text{stdp2}(4); \\
\text{cong3} &= \text{fuelcorr}\*\text{stdp3}(1)\*\text{stdp3}(4); \text{ cong4} = \text{fuelcorr}\*\text{stdp4}(1)\*\text{stdp4}(4); \\
\text{covhalf1} &= [\text{coco1} \* \text{ones}(1,3), \text{cong1} \* \text{ones}(1,3), 0]; \\
\text{covhalf2} &= [\text{coco2} \* \text{ones}(1,3), \text{cong2} \* \text{ones}(1,3), 0]; \\
\text{covhalf3} &= [\text{coco3} \* \text{ones}(1,3), \text{cong3} \* \text{ones}(1,3), 0]; \\
\text{covhalf4} &= [\text{coco4} \* \text{ones}(1,3), \text{cong4} \* \text{ones}(1,3), 0]; \\
\end{align*}
% LCOE covariance matrices, periods 1-4
cov_sdconA=[covhalf1_1;covhalf1_1;covhalf1_1;covhalf1_2;...
covhalf1_2;covhalf1_2;zeros(1,7)];%7x7
cov_sdconB=[covhalf2_1;covhalf2_1;covhalf2_1;covhalf2_2;...
covhalf2_2;covhalf2_2;zeros(1,7)];
cov_sdconC=[covhalf3_1;covhalf3_1;covhalf3_1;covhalf3_2;...
covhalf3_2;covhalf3_2;zeros(1,7)];
cov_sdconD=[covhalf4_1;covhalf4_1;covhalf4_1;covhalf4_2;...
covhalf4_2;covhalf4_2;zeros(1,7)];

% Portfolio optimization

% set up nonlinear program (NLP) inputs

% covariance matrix
covar=cov_sdconA;

% slack limits, MWh (for RHS slack constraints)
slklim(:,1)=[(demadj(9)-demadj(2));(demadj(7)-demadj(2))];
slkcoeffco=(0.9-cfco)*8760; % coeffs for LHS slack constraints
slkcoeffng=(0.9-cfng)*8760;

% solver options
options=optimset('Display','notify-detailed','Algorithm','interior-point');
start=[0 0 0 0 0 0 0]; % dec var starting values
A=[-1 -1 -1 -1 -1 -1 -0.15; -2628 -5256 -7884 -2628 -5256 -7884 -2628;...
slkcoeffco 0 0 0; slkcoeffco slkcoeffng 0]; % LHS ineq constraints
b=[-(capadj(2)-capadj(1)); -(demadj(2)-demadj(1)); slklim]; % RHS ineq constraints
lb=[0 0 0 0 0 0 0]; % non-negativity (lower bounds)

% set portfolio standard deviation values for NLP constraints:
sdvec=linspace(min(stdin(1,1:6))/1.1,max(stdin(1,:)),nsd);

% loop through s.d. values and record NLP output for each (for period 1)
for i=1:nsd
    param(i)=sdvec(i); % input s.d. constraint
    [x(:,i,1),fval(1,i)]=fmincon(@(x) objfunA(x,lcoe_objfunA),...
        start,A,b,[],[],lb,[],@(x) sdconA(x,cov_sdconA,param(i)),options);

    gen(:,i)=cfall'*8760.*x(:,i,1); % record generation per tech
    gentot1(i)=sum(gen(:,i)); % record total gen per s.d. run
    fracgen(:,i)=gen(:,i)./gentot1(i); % record fracgen per tech
    y(i)=sum(fracgen(:,i).*covar(:,1))+...
        sum(fracgen(2,i).*covar(:,1))+
        sum(fracgen(3,i).*covar(:,1))+
        sum(fracgen(4,i).*covar(:,1))+
        sum(fracgen(5,i).*covar(:,1))+
        sum(fracgen(6,i).*covar(:,1))+
        sum(fracgen(7,i).*covar(:,1));

end
z(1,i)=sqrt(y(i)); %record portfolio s.d.
captot1(i)=sum(x(1:6,i,1)); %record dispatchable capacity
capact1(i)=sum(x(:,i,1)); %record total capacity (incl wind)
slk(:,i)=x(:,i,1).*[slkcoeffco'; slkcoeffng'; 0]; %record slack gen
slktot5(i)=sum(slk(:,i)); %record total slack gen per s.d. for period
1
port(1,i)=sum(fracgen(:,i).*lcoe_objfunA');
porttot(1,i)=port(1,i)*gentot1(i);
sdtot(1,i)=z(1,i)*gentot1(i);
end

%period 2 code
sizex=size(x);
covar=cov_sdconB;
slklim(:,2)=[(demadj(10)-demadj(3));(demadj(8)-demadj(3))]; %update slack limits
sdvec=linspace(min(stdin(2,1:6))/1.1,max(stdin(2,:)),nsd);
for j=1:sizex(2)
    param(j)=sdvec(j);
    %update constraints:
    A=[-1 -1 -1 -1 -1 -1 -0.15; -2628 -5256 -7884 -2628 -5256 -7884 -2628;... slkcoeffco 0 0 0 0; slkcoeffco slkcoeffng 0; -1 -1 0 0 0 0 0;... 0 0 -1 -1 0 0 0 0 0 -1];
b=[-(capadj(3)-capadj(1)); -(demadj(3)-demadj(1));... slklim(:,2);-sum(x(1:3,j,1)); -sum(x(4:6,j,1)); -x(7,j,1)];
[x(:,j,2),fval(2,j)]=fmincon(@(x) objfunB(x, lcoe_objfunB),... start,A,b,[],[],lb,[],@(x) sdconB(x,cov_sdconB,param(j)),options);
gen(:,j)=cfall'*8760.*x(:,j,2);
gentot2(j)=sum(gen(:,j));
fracgen(:,j)=gen(:,j).gentot2(j);
y(j)=sum(fracgen(1,j).*fracgen(:,j).*covar(1,:))+... sum(fracgen(2,j).*fracgen(:,j).*covar(2,:))+... sum(fracgen(3,j).*fracgen(:,j).*covar(3,:))+... sum(fracgen(4,j).*fracgen(:,j).*covar(4,:))+... sum(fracgen(5,j).*fracgen(:,j).*covar(5,:))+... sum(fracgen(6,j).*fracgen(:,j).*covar(6,:))+... sum(fracgen(7,j).*fracgen(:,j).*covar(7,:));
z(2,j)=sqrt(y(j));
captot2(j)=sum(x(1:6,j,2));
capact2(j)=sum(x(:,j,2));
slk(:,j)=x(:,j).*[slkcoeffco'; slkcoeffng'; 0];
sltot9(j)=sum(sltk(:,j));
port(2,j)=sum(fracgen(:,j).*lcoe_objfunB');
porttot(2,j)=port(2,j)*gentot2(j);
sdtot(2,j)=z(2,j)*gentot2(j);
end

%period 3 code
sizex=size(x);
covar=cov_sdconC;
slklim(:,3)=[(demadj(11)-demadj(4));(demadj(9)-demadj(4))];
sdvec=linspace(min(stdin(3,1:6))/1.1,max(stdin(3,:)),nsd);
for k=1:sizex(2)
    param(k)=sdvec(k);
    A=[-1 -1 -1 -1 -1 -1 -0.15; -2628 -5256 -7884 -2628 -5256 -7884 -2628;...
        slkcoeffco 0 0 0 0 0 0 0; slkcoeffco slkcoeffng 0;...
        -1 -1 0 0 0 0 -1 -1 0 0 0 0 0 -1];
    b=[-(capadj(4)-capadj(1)); -(demadj(4)-demadj(1));...
        slklimit(:,3); -sum(x(1:3,k,2)); -sum(x(4:6,k,2)); -x(7,k,2)];
    [x(:,k,3),fval(3,k)]=fmincon(@(x) objfunC(x, lcoe_objfunC),...
        start,A,b,[],[],lb,[],@(x) sdconC(x, cov_sdconC, param(k)),options);
    gen(:,k)=cfall'*.8760.*x(:,k,3);
    gentot3(k)=sum(gen(:,k));
    fracgen(:,k)=gen(:,k)./gentot3(k);
    y(k)=sum(fracgen(1,k).*fracgen(:,k).*covar(:,1))+...
        sum(fracgen(2,k).*fracgen(:,k).*covar(:,2))+...
        sum(fracgen(3,k).*fracgen(:,k).*covar(:,3))+...
        sum(fracgen(4,k).*fracgen(:,k).*covar(:,4))+...
        sum(fracgen(5,k).*fracgen(:,k).*covar(:,5))+...
        sum(fracgen(6,k).*fracgen(:,k).*covar(:,6))+...
        sum(fracgen(7,k).*fracgen(:,k).*covar(:,7));
    z(3,k)=sqrt(y(k));
    captot3(k)=sum(x(1:6,k,3));
    capact3(k)=sum(x(:,k,3));
    slk(:,k)=x(:,k).*[slkcoeffco'; slkcoeffng'; 0];
    slktot14(k)=sum(slk(:,k));
    port(3,k)=sum(fracgen(:,k).*lcoe_objfunC');
    porttot(3,k)=port(3,k)*gentot3(k);
    sdtot(3,k)=z(3,k)*gentot3(k);
end

%period 4 code
sizex=size(x);
covar=cov_sdconD;
slklim(:,4)=[(demadj(12)-demadj(5));(demadj(10)-demadj(5))];
sdvec=linspace(min(stdin(4,1:6))/1.1,max(stdin(4,:)),nsd);
for m=1:sizex(2)
    param(m)=sdvec(m);
    A=[-1 -1 -1 -1 -1 -1 -0.15; -2628 -5256 -7884 -2628 -5256 -7884 -2628;...
        slkcoeffco 0 0 0 0 0 0 0; slkcoeffco slkcoeffng 0;...
        -1 -1 0 0 0 0 -1 -1 0 0 0 0 0 -1];
    b=[-(capadj(5)-capadj(1)); -(demadj(5)-demadj(1));...
        slklimit(:,4); -sum(x(1:3,m,3)); -sum(x(4:6,m,3)); -x(7,m,3)];
    [x(:,m,4),fval(4,m)]=fmincon(@(x) objfunD(x, lcoe_objfunD),...
        start,A,b,[],[],lb,[],@(x) sdconD(x, cov_sdconD, param(m)),options);
    gen(:,m)=cfall'*.8760.*x(:,m,4);
    gentot4(m)=sum(gen(:,m));
    fracgen(:,m)=gen(:,m)./gentot4(m);
    y(m)=sum(fracgen(1,m).*fracgen(:,m).*covar(:,1))+...
        sum(fracgen(2,m).*fracgen(:,m).*covar(:,2))+...
Example function files are provided below. Two types of function files were used. The first type specifies the objective function to be minimized as part of the portfolio optimization routine. One objective function file is needed for each planning period represented in the model:

```matlab
function f = objfunA(x, lcoe_objfunA)

decvar=[x(1),x(2),x(3),x(4),x(5),x(6),x(7)];

cfco=[0.3,0.6,0.9];
cfng=[0.3,0.6,0.9];
cfwn=[0.3];
cfall=[cfco,cfng,cfwn];

slkcoeffco=(0.9-cfco)*8760;
slkcoeffng=(0.9-cfng)*8760;
slkcoeff39=[slkcoeffco 0 0 0 0];
slkcoeff29=[0 0 0 slkcoeffng 0];
slktot=sum(slkcoeff39.*decvar)+sum(slkcoeff29.*decvar);

gen=cfall*8760.*decvar;
gentot=sum(gen);
fracgen=gen./gentot;
lcoe=lcoe_objfunA;
f = (sum(lcoe.*fracgen)*gentot)/slktot; %obj fun for period 1
```

The second type of function file defines the nonlinear portfolio standard deviation constraint. Again, one standard deviation function file is needed for each of the four planning periods:

```matlab
function [c, ceq] = sdconA(x, cov_sdconA, param)
decvar=[x(1),x(2),x(3),x(4),x(5),x(6),x(7)];
```
\[
\begin{align*}
\text{cfco} &= [0.3, 0.6, 0.9]; \\
\text{cfng} &= [0.3, 0.6, 0.9]; \\
\text{cfwn} &= [0.3]; \\
\text{cfall} &= [\text{cfco}, \text{cfng}, \text{cfwn}]; \\
\text{covar} &= \text{cov}_\text{sdconA}; \\
\text{gen} &= \text{cfall} \times 8760.0 \times \text{decvar}; \\
\text{gentot} &= \text{sum}(\text{gen}); \\
\text{fracgen} &= \text{gen} / \text{gentot}; \\
\text{y} &= \text{sum}(\text{fracgen}(1) \times \text{fracgen} \times \text{covar}(1,:), \ldots) + \\
& \quad \text{sum}(\text{fracgen}(2) \times \text{fracgen} \times \text{covar}(2,:), \ldots) + \\
& \quad \text{sum}(\text{fracgen}(3) \times \text{fracgen} \times \text{covar}(3,:), \ldots) + \\
& \quad \text{sum}(\text{fracgen}(4) \times \text{fracgen} \times \text{covar}(4,:), \ldots) + \\
& \quad \text{sum}(\text{fracgen}(5) \times \text{fracgen} \times \text{covar}(5,:), \ldots) + \\
& \quad \text{sum}(\text{fracgen}(6) \times \text{fracgen} \times \text{covar}(6,:), \ldots) + \\
& \quad \text{sum}(\text{fracgen}(7) \times \text{fracgen} \times \text{covar}(7,:), \ldots); \\
\text{z} &= \sqrt{\text{y}}; \\
\text{c} &= []; \\
\text{ceq} &= \text{z} - \text{param};
\end{align*}
\]