Essays in Financial Economics

by

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Business Administration
Duke University

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Pietro Peretto

Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in Business Administration
in the Graduate School of Duke University

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Abstract

I study the link between economic growth and asset prices using stochastic endogenous growth models. In these settings, long-term growth prospects are endogenously determined by innovation and R&D. In equilibrium, R&D endogenously drives a small, persistent component in productivity which generates long-run uncertainty about economic growth. With recursive preferences, this growth propagation mechanism helps reconcile a broad spectrum of equity and bond market facts jointly with macroeconomic fluctuations.
I dedicate my dissertation to my parents and my sister.
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Acknowledgements

I am indebted to my advisor, Ravi Bansal and my committee members, Hengjie Ai, Pietro Peretto and Lukas Schmid for invaluable guidance and support. I also thank Jawad Addoum, Hao Chen, Jonas Arias, Hyunseob Kim, Ryan Pratt, and Nikhil Sharma for helpful conversations that extend well beyond research.
I study the link between economic growth and asset prices using stochastic endogenous growth models. In these models, long-term growth prospects are endogenously determined by innovation and R&D. In equilibrium, R&D endogenously drives a small, persistent component in productivity which generates long-run uncertainty about economic growth. With recursive preferences, this growth propagation mechanism helps reconcile a broad spectrum of equity and bond market facts jointly with macroeconomic fluctuations. The two papers included in my dissertation are (i) “Innovation, Growth, and Asset Prices,” which is joint work with Lukas Schmid and “Equilibrium Growth, Inflation, and Bond Yields.” The first paper focuses on the equity market implications while the second paper focuses on the bond market implications.
2

Innovation, Growth, and Asset Prices

2.1 Introduction

Asset prices reflect anticipations of future growth. Likewise, long-term growth prospects mirror an economy’s innovative potential. At the aggregate level, such innovation is reflected in the sustained growth of productivity. Empirical measures of innovation, such as R&D expenditures, are typically quite volatile and fairly persistent. Such movements in the driving forces of growth prospects should naturally be reflected in the dynamics of growth rates themselves. Indeed, in US post-war data productivity growth has undergone long and persistent swings.\footnote{See e.g. Gordon (2010), and Jermann and Quadrini (2007)} Similarly, innovation-driven growth waves associated with the arrival of new technologies such as television, computers, the internet, to name a few, are well documented.\footnote{See e.g. Helpman (1998) and Jovanovic and Rousseau (2005)} Asset prices naturally reflect this low-frequency variation in growth prospects. In particular, if agents fear that a persistent slowdown in economic growth will lower asset prices, such movements will give rise to high risk premia in asset markets.

In this paper, we use a tractable model of innovation and R&D in order to link as-
set prices and aggregate risk premia to endogenous movements in long-term growth prospects. More specifically, our setup has two distinguishing features. First, we embed a stochastic model of endogenous growth based on industrial innovation\(^3\) into an otherwise standard real business cycle model. Here technological progress and sustained growth are determined endogenously by the creation of new patents and technologies through R&D. New patents facilitate the production of a final consumption good and can be thought of as intangible capital. Second, we assume that households have recursive preferences,\(^4\) so that they care about uncertainty regarding long-term growth prospects.

Our results suggest that extending macroeconomic models to account for the endogeneity of innovation and long-term growth goes some way towards an environment that jointly captures the dynamics of quantities and asset prices. When calibrated to match the empirical evidence on productivity and long-run economic growth, our model can quantitatively rationalize key features of asset returns in the data. In particular, it generates a realistic equity premium and a low and stable risk-free interest rate without relying on excessively high risk aversion. Moreover, it generates a sizeable spread between the returns on physical capital and intangible capital, which is commonly related to the value premium in the data.

Our model supports the notion that movements in long-term growth prospects are a significant source of risk priced in asset markets. Such ‘long-run risks’ (in the sense of Bansal and Yaron (2004)) arise endogenously in our production economy suggesting that stochastic models of endogenous growth are a useful framework for general equilibrium asset pricing. At the center of this framework is a strong propagation and amplification mechanism for shocks which is tightly linked to the joint dynamics of innovation and asset prices. High equilibrium returns provide strong incentives for

\(^3\) Following Romer (1990) and Grossman and Helpman (1991a)

\(^4\) Epstein and Zin (1989), Backus et al. (2004)
agents to engage in innovation and investing in R&D. This pricing effect reinforces the impact of exogenous shocks, thus providing an amplification mechanism. On the other hand, R&D leads to the development of new technologies which will persistently boost aggregate growth, so that aggregate growth appears in long waves, thus providing a propagation mechanism. Such endogenous persistence feeds back into asset prices with recursive preferences. When prolonged slumps in economic growth coincide with low asset valuations, households will require high risk premia in asset markets.

Formally, we first show that in the model innovation and R&D endogenously drive a small, but persistent component in the growth rate of measured aggregate productivity. More specifically, we decompose productivity growth into a high-frequency component driven by an exogenous shock, as well as an endogenous component driven by R&D. While the shock induces fluctuations at business cycle frequency comparable to standard macroeconomic models, the innovation process in the model translates this disturbance into an additional, slow-moving component generating macroeconomic movements at lower frequencies. Naturally, these productivity dynamics induce persistent uncertainty about the economy’s long-term growth prospects that will be reflected in the dynamics of aggregate quantities.

Persistent variation in consumption and cash flow growth is reflected in risk premia in asset markets given our preference specification. With recursive Epstein-Zin utility with a preference for early resolution of uncertainty not only are innovations to realized consumption and dividend growth priced, but also innovations to expected consumption and dividend growth. The propagation mechanism in the model translates shocks to the level of technology into (i) innovations to expected consumption growth, generating endogenous long-run risks in consumption, and (ii) innovations to expected dividend growth, generating realistic low-frequency movements in price-dividend ratios. Furthermore, in the model, physical capital is endogenously more
exposed to predictable variation in growth than intangible capital, which generates a sizeable value spread. More broadly, our paper suggests that equilibrium growth is risky.

Our model thus allows us to identify economic sources of long-run risks in the data. In particular, it identifies R&D and innovation as economic sources of a predictable component in productivity growth, sometimes referred to as long-run productivity risk (as in Croce (2008), Gomes et al. (2008), Backus et al. (2007), Backus et al. (2010), Favilukis and Lin (2010)). Indeed, in line with the predictions of the model, we provide novel empirical evidence that measures of innovation have significant predictive power for aggregate growth rates including productivity, consumption, output and cash flows at longer horizons.

Our paper is related to a number of different strands of literature in asset pricing, economic growth and macroeconomics. The economic mechanisms driving the asset pricing implications are closely related to Bansal and Yaron (2004). In a consumption-based model, Bansal and Yaron directly specify both consumption and dividend growth to contain a small, persistent component. This specification along with the assumption of Epstein-Zin recursive utility with a preference for early resolution of uncertainty, allows them to generate high equity premia as compensation for these long-run risks. While the empirical evidence for the long-run risk channel is still somewhat controversial, the ensuing literature on long-run risk quantitatively explains a wide range of patterns in asset markets, such as those in equity, government, corporate bond, foreign exchange and derivatives markets. We contribute to this literature by showing that predictable movements in growth prospects are an equilibrium outcome of stochastic models of endogenous growth and by providing novel empirical evidence identifying economic sources of long-run risks in the data.

A number of recent papers have examined the link between technological growth and asset prices. Garleanu et al. (2009) model technological progress as the arrival
of large, infrequent technological innovations and show that the endogenous adoption of these innovations leads to predictable movements in consumption growth and expected excess returns. Garleanu et al. (2011) examine the implications of the arrival of new technologies for existing firms and their workers, and show that in an overlapping-generations model innovation creates a systematic risk factor labeled displacement risk. Pastor and Veronesi (2009) explain bubble-like behavior of stock markets in the 1990s by the arrival of new technologies.

While our model has implications for consumption dynamics and asset returns that are related to these models, our approach is quite different but complementary. In these models of technology adoption, the arrival of new technologies is assumed to be exogenous, while we examine the asset pricing and growth dynamics implications of the endogenous creation of new technologies by means of R&D, which leads to a distinct set of empirical predictions. Moreover, by embedding a model of endogenous technological progress into a real business cycle model, our paper provides a straightforward and tractable extension of the workhorse model of modern macroeconomics. In this respect, the paper is closer to recent attempts to address asset pricing puzzles within versions of the canonical real business cycle model. Starting from Jermann (1998), Boldrin et al. (2001) and Kogan (2004) recent examples include Campanale et al. (2008), Kaltenbrunner and Lochstoer (2008), Ai (2008) and Kuehn (2008), who explore endogenous long-run consumption risks in real business cycle models with recursive preferences, Gourio (2009) and Gourio (2010) who examines disaster risks. Particularly closely related are recent papers by Croce (2008), Backus et al. (2007), Backus et al. (2010), Gomes et al. (2008) and Favilukis and Lin (2010) who examine the implications of long-run productivity risk with recursive preferences for the equity premium, and the cross-section of stock returns, respectively. While they specify long-run productivity risk exogenously, our model shows how such risk arises endogenously and can be linked to innovation. Tallarini (2000) considers the sepa-

Methodologically, our paper builds on and is closely related to recent work by Comin and Gertler (2006) and Comin et al. (2009). Building on the seminal work by Romer (1990) and Grossman and Helpman (1991a), these authors integrate innovation and adoption of new technologies into a real business cycle model and show that the resulting stochastic endogenous growth model features rich movements at lower than business cycle frequencies, which they label medium term business cycles. We contribute to this literature by linking medium term cycles to long run risks and aggregate risk premia, and examining its asset pricing implications with recursive preferences. Moreover, while they consider low-frequency movements around a trend, we focus on the low frequency movements of the trend growth rate. This is an important distinction from an asset pricing perspective.

The paper is structured as follows. In section 2 we describe our benchmark model. In section 3 we qualitatively explore the growth and productivity processes arising in equilibrium and detail their links with the real business cycle model. We examine its quantitative implications for productivity, macroeconomic quantities and asset prices in section 4, along with a number of empirical predictions. Section 5 concludes.

2.2 Model

We start by describing our benchmark endogenous growth model. We embed a model of industrial innovation in the tradition of Romer (1990) into a fairly stan-
standard macroeconomic model with convex adjustment costs and recursive Epstein-Zin preferences. In the model, rather than assuming exogenous technological progress, growth instead arises through research and development (R&D) investment. R&D investment leads to the creation of intermediate goods or new patents used in the production of a final consumption good. An increasing number of intermediate goods is the ultimate source of sustained growth, hence the model is a version of an expanding-variety model of endogenous growth.

**Household**  The representative household has Epstein-Zin preferences defined over consumption:

$$U_t = \left\{ (1 - \beta)C_t^{1-\gamma} + \beta(E_t[U_{t+1}^{1-\gamma}])^{\frac{\psi}{\theta}} \right\}^{\frac{1}{1-\gamma}},$$  \hspace{1cm} (2.1)

where $\gamma$ is the coefficient of relative risk aversion, $\psi$ is the elasticity of intertemporal substitution, and $\theta \equiv \frac{1-\gamma}{1-1/\psi}$. When $\psi \neq \frac{1}{\gamma}$, the agent cares about news regarding long-run growth prospects. We will assume that $\psi > \frac{1}{\gamma}$ so that the agent has a preference for early resolution of uncertainty and dislikes shocks to long-run expected growth rates.

The household maximizes utility by participating in financial markets and by supplying labor. Specifically, the household can take positions $Z_t$ in the stock market, which pays an aggregate dividend $D_t$, and in the bond market, $B_t$. Accordingly, the budget constraint of the household becomes

$$C_t + Q_t Z_{t+1} + B_{t+1} = W_t L_t + (Q_t + D_t) Z_t + R_t B_t$$  \hspace{1cm} (2.2)

where $Q_t$ is the stock price, $R_t$ is the gross risk free rate, $W_t$ is the wage and $L_t$ denotes hours worked.

As described above, the production side of the economy consists of several sectors, so that the aggregate dividend can be further decomposed into the individual payouts
of these sectors, in a way to be described below.

As usual, the setup implies that the stochastic discount factor in the economy is given by

\[ M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\psi} \left[ \mathbb{E}_t(U_{t+1}^{1-\gamma}) \right]^{\gamma-1} \frac{U_{t+1}^{1-\gamma}}{U_t^{\gamma-1}} \]  \quad (2.3)

where the second term, involving continuation utilities, captures preferences concerning uncertainty about long-run growth prospects. Furthermore, since the agent has no disutility for labor, she will supply her entire endowment, which we normalized to unity.

**Final Goods Sector** There is a representative firm that uses capital \( K_t \), labor \( L_t \) and a composite of intermediate goods \( G_t \) to produce the final (consumption) good according to the production technology

\[ Y_t = (K_t^\alpha \Omega_t^{1-\alpha}L_t^{1-\xi})^{1-\xi}G_t^\xi \]  \quad (2.4)

where the composite \( G_t \) is defined as

\[ G_t \equiv \left[ \int_0^{N_t} X_{i,t}^{\frac{1}{\nu}} \, di \right]^{\nu}. \]  \quad (2.5)

\( X_{i,t} \) is intermediate good \( i \in [0, N_t] \), where \( N_t \) is the measure of intermediate goods in use at date \( t \), and \( \alpha \) is the capital share, \( \xi \) is the intermediate goods share, and \( \nu \) is the elasticity of substitution between the intermediate goods. Note that \( \nu > 1 \) is assumed so that increasing the variety of intermediate goods raises the level of productivity in the final goods sector. This property is crucial for sustained growth. In our quantitative work, we will think of intermediate goods as new patents or intangible capital.

The productivity shock \( \Omega_t \) is assumed to follow a stationary Markov process. Because
of the stationarity of the forcing process, sustained growth will arise endogenously from the development of new intermediate goods. We will describe the R&D policy below.

The firm’s objective is to maximize shareholder value. Taking the stochastic discount factor $M_t$ as given, this can be formally stated as

$$
\max_{\{I_t,L_t,K_{t+1},X_{i,t}\}_{t \geq 0}} E_0 \left[ \sum_{t=0}^{\infty} M_t D_t \right] \quad (2.6)
$$

The firm’s dividends are

$$
D_t = Y_t - I_t - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t} \, di \quad (2.7)
$$

where $I_t$ is capital investment, $W_t$ is the wage rate, and $P_{i,t}$ is the price per unit of intermediate good $i$, which the final goods firm takes as given. The last term captures the costs of buying intermediate goods at time $t$.

In line with the literature on production-based asset pricing, we assume that investment is subject to capital adjustment costs, so that the capital stock evolves as

$$
K_{t+1} = (1 - \delta) K_t + \Lambda \left( \frac{I_t}{K_t} \right) K_t \quad (2.8)
$$

where $\delta$ is the depreciation rate of capital and $\Lambda(\cdot)$ the capital adjustment cost function\(^5\).

---

\(^5\) $\Lambda(\cdot)$ is specified as in Jermann (1998)

$$
\Lambda \left( \frac{I_t}{K_t} \right) = \frac{\alpha_1}{1 - \xi} \left( \frac{I_t}{K_t} \right)^{1 - \frac{1}{\xi}} + \alpha_2
$$

The parameter $\xi$ represents the elasticity of the investment rate. The parameters $\alpha_1$ and $\alpha_2$ are set so that there are no adjustment costs in the deterministic steady state.
Denoting by $q_t$ the shadow value of capital, the firm’s optimality conditions are

$$q_t = \frac{1}{N_t}$$

$$W_t = (1 - \alpha)(1 - \xi)\frac{Y_t}{L_t}$$

$$1 = E_t \left[ M_{t+1} \left\{ \frac{1}{q_t} \left( \alpha(1 - \xi)\frac{Y_{t+1}}{K_{t+1}} + q_{t+1}(1 - \delta) - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}A_{t+1} \right) \right\} \right]$$

$$P_{i,t} = (K_t^\alpha(\Omega_tL_t)^{1-\alpha})^{1-\xi} \nu \xi \left[ \int_0^{N_t} X_{i,t}^{\frac{1}{\nu}} di \right]^{\nu \xi - 1} \frac{1}{\nu} X_{i,t}^{\frac{1}{\nu} - 1}$$

where $\Lambda_t \equiv \Lambda \left( \frac{h}{K_t} \right)$ and $\Lambda_t' \equiv N' \left( \frac{h}{K_t} \right)$. The last equation determines the final good producer’s demand for intermediate input. Importantly, that demand is procyclical, as it depends positively on $\Omega_t$.

Intermediate Goods Sector  Intermediate goods producers have monopoly power. Given the demand schedules set by the final good firm, monopolists producing the intermediate goods set the prices in order to maximize their profits. Intermediate goods producers transform one unit of the final good in one unit of their respective intermediate good. In this sense production is “roundabout” in that monopolists take final good production as given as they are tiny themselves. This fixes the marginal cost of producing one intermediate good at unity.

Focusing on symmetric equilibria, the monopolistically competitive characterization of the intermediate goods sector implies

$$P_{i,t} = P_t = \nu$$

That is, each intermediate goods producer charges a markup $\nu > 1$ over marginal cost. Hence, intermediate profits are

$$\Pi_{i,t} = \Pi_t = (\nu - 1)X_t$$
where $X_{i,t} = X_t = \left( \frac{\xi}{\nu} (K_t^\alpha (\Omega_t L_t)^{1-\alpha})^{1-\xi} N_t^{\nu^{\xi-1}} \right)^{\frac{1}{1-\xi}}$. Consequently, the intermediate good input and hence monopoly profits are procyclical. The value of owning exclusive rights to produce intermediate good $i$ is equal to the present discounted value of the current and future monopoly profits

$$V_{i,t} = V_t = \Pi_t + \phi E_t[M_{t+1}V_{t+1}]$$

(2.11)

where $1 - \phi$ is the probability that an intermediate good becomes obsolete. Again, given the procyclicality of profits, values of patents are procyclical as well. Since the value of patents are the payoff to innovation, as described below, this implies that the returns to innovation are procyclical and risky.

**R&D Sector** Innovators develop new patents for intermediate goods used in the production of final output. They do so by conducting research and development, using the final good as input at unit cost. Patents of newly developed products can be sold to intermediate goods producers. Assuming that this market is competitive, the price of a new patent will equal its value to the new intermediate goods producer. For simplicity, we assume that households can directly invest in research and development.

We link the evolution of the measure of intermediate goods or patents $N_t$ to innovation as

$$N_{t+1} = \vartheta_t S_t + \phi N_t$$

(2.12)

where $S_t$ denotes R&D expenditures (in terms of the final good) and $\vartheta_t$ represents the productivity of the R&D sector that is taken as exogenous by the R&D sector. In a similar spirit as Comin and Gertler (2006), we assume that this technology coefficient involves a congestion externality effect capturing decreasing returns to scale in the
innovation sector

\[
\vartheta_t = \frac{\chi \cdot N_t}{S_t^{1-\eta} N_t^\eta}
\]  

(2.13)

where \( \chi > 0 \) is a scale parameter and \( \eta \in [0, 1] \) is the elasticity of new intermediate goods with respect to R&D. Since there is free entry into the R&D sector, the following break-even condition must hold:

\[
E_t[M_{t+1}V_{t+1}](N_{t+1} - \phi N_t) = S_t
\]

(2.14)

which says that the expected sales revenues equals costs, or equivalently, at the margin, \( \frac{1}{\vartheta_t} = E_t[M_{t+1}V_{t+1}] \).

Resource Constraint  Final output is used for consumption, investment in physical capital, factor input used in the production of intermediate goods, and R&D:

\[
Y_t = C_t + I_t + N_tX_t + S_t
\]

(2.15)

\[
= C_t + I_t + N_t^{1-\nu}G_t + S_t
\]

(2.16)

where the last equality exploits the optimality conditions and the term \( N_t^{1-\nu}G_t \) captures the costs of intermediate goods production. Given that \( \nu > 1 \) reflecting monopolistic competition, it follows that increasing product variety increases the efficiency of intermediate goods production, as the costs fall as \( N_t \) grows.

Stock Market  We assume that the stock market value includes all the production sectors, namely the final good sector, the intermediate goods sector, as well as the research and development sector. The aggregate dividend then becomes

\[
D_t = D_t + \Pi_tN_t - S_t
\]

(2.17)
Defining the stock market value to be the discounted sum of future aggregate dividends, exploiting the optimality conditions, this value can be rewritten as

\[ Q_t = q_t K_{t+1} + N_t (V_t - \Pi_t) + E_t \left[ \sum_{i=0} M_{t+1+i} V_{t+i+1} (N_{t+i+1} - \phi N_{t+i}) \right] \] (2.18)

as in Comin et al. (2009). The stock return is defined accordingly. Therefore, the stock market value comprises the current market value of the installed capital stock, reflected in the first term, the market value of currently used intermediate goods interpreted as patents or blueprints, reflected in the second term, as well as the market value of intermediate goods to be developed in the future, as reflected in the third term. Therefore, in addition to the tangible capital stock, the stock market values intangible capital as well as the option value of future intangibles.

**Forcing Process** We introduce uncertainty into the model by means of an exogenous shock \( \Omega_t \) to the level of technology. We assume that \( \Omega_t = e^{a_t} \), and \( a_t = \rho a_{t-1} + \epsilon_t \), with \( \epsilon_t \sim N(0, \sigma^2) \) and \( \rho < 1 \). Note first that this process is strictly stationary, so that sustained growth in the model will not arise through exogenous trend growth in exogenous productivity, but endogenously. Second, while formally, \( \Omega_t \) resembles labor augmenting technology, it does not represent measured TFP in our setting. Rather, measured TFP in the model can be decomposed in an exogenous component, driven by \( \Omega_t \), and an endogenous component which is driven by the accumulation of intermediate goods and hence innovation, which is also the source of sustained growth. We discuss the dynamics of productivity in detail in section 3.

2.3 Equilibrium Growth and Productivity

In our benchmark model, sustained growth is an equilibrium phenomenon resulting from agents’ decisions. Moreover, these decisions generate growth rate and productivity dynamics contrasting with those implied by more standard macroeconomic
frameworks. In this section we describe these patterns qualitatively, while we will provide supportive empirical evidence and a quantitative analysis in the next section. First, it is convenient to represent the aggregate production function in our benchmark model in a form that permits straightforward comparison with specifications used commonly in macroeconomic models where growth is given exogenously. To that end, note that using the equilibrium conditions derived above, final output can be rewritten as follows:

\[ Y_t = \left( \frac{\xi}{\nu} \right)^{\frac{\epsilon}{\nu - \epsilon}} K_t^\alpha (\Omega_t L_t)^{1-\alpha} N_t^{\frac{\nu - \epsilon}{\nu - \epsilon}} \]  

(2.19)

For sustained growth to obtain in this setting we need to impose a parametric restriction. Technically, to ensure balanced growth, we need the aggregate production function to be homogeneous of degree one in the accumulating factors \( K_t \) and \( N_t \). We will thus impose the parameter restriction that \( \alpha + \frac{\nu - \epsilon}{1 - \epsilon} = 1 \). In this case, we have a production function that resembles the standard neoclassical one with labor augmenting technology \( Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha} \) where total factor productivity (TFP) is

\[ Z_t = A\Omega_t N_t \]  

(2.20)

and \( A \equiv \left( \frac{\xi}{\nu} \right)^{\frac{\epsilon}{1 - \epsilon}} > 0 \) is a constant. The equilibrium productivity process thus contains a component driven by the exogenous forcing process, \( \Omega_t \), and an endogenous trend component reflecting the accumulation of intermediate goods, \( N_t \).

In our quantitative work, we will contrast the implications of the benchmark with those of a nested standard real business cycle model with exogenous growth. We can achieve this by specifying the aggregate stock of R&D exogenously. More specifically, in the latter model, we specify TFP as \( \tilde{Z}_t = A\Omega_t \tilde{N}_t \) and a deterministic trend \( \tilde{N}_t = e^{\mu t} \).

Hence, the fundamental difference between our model and the canonical real business
cycle framework is that the trend component of the TFP process, \( N_t \), is endogenous and fluctuates in our setup but exogenous and deterministic in the RBC model. Our benchmark model thus endogenously generates a stochastic trend, which is consistent with the evidence for OECD countries in Cogley (1990).

This stochastic trend is naturally reflected in the dynamics of productivity growth rates. Clearly, given a realistically persistent process for \( a_t \), we have

\[
\Delta z_t = \Delta n_t + \Delta a_t \quad (2.21)
\]

\[
\approx \Delta n_t + \epsilon_t \quad (2.22)
\]

where lowercase letters denote logs. In contrast, with a deterministic trend, we have \( \Delta z_t \approx \mu + \epsilon_t \). Accordingly, while in the counterpart with exogenous growth, productivity growth will be roughly i.i.d., it will inherit a second component in the benchmark model which depends on the accumulation of patents. Therefore, qualitatively and quantitatively, the dynamics of productivity growth reflect the dynamics of innovation.

To see this more explicitly, rewrite the growth rate of productivity, \( \Delta Z_t \), as \( \Delta Z_t = \Delta N_t \cdot \Delta \Omega_t \). Given a realistically persistent calibration of \( \{ \Omega_t \} \) in logs, we have \( \Delta \Omega_t \approx e^{\epsilon_t} \). On the other hand, given the accumulation of \( N_t \) as \( N_t = \vartheta_t \cdot S_{t-1} + \phi N_{t-1} \), the growth rate of patents becomes \( \Delta N_t = \vartheta_t \cdot \hat{S}_{t-1} + \phi \), where we set

\[
\hat{S}_t \equiv \frac{S_t}{N_t}
\]

We will refer to \( \hat{S}_t \) as the R&D intensity. Accordingly, we find \( \Delta Z_t \approx (\vartheta_t \cdot \hat{S}_{t-1} + \phi)(e^{\epsilon_t}) \). Thus,

\[
E_t[\Delta Z_{t+1}] \approx E_t \left[ (\vartheta_t \cdot \hat{S}_t + \phi)(e^{\epsilon_{t+1}}) \right] \quad (2.23)
\]

\[
\approx \vartheta_t \cdot \hat{S}_t + \phi \quad (2.24)
\]
Our model thus exhibits variation in expected growth driven by the R&D intensity. Empirically, the R&D intensity is a fairly persistent and volatile process. In a realistic calibration of the model, we therefore expect productivity growth to exhibit substantial low-frequency variation and persistent uncertainty about growth prospects. Favorable economic conditions, as captured by a positive shock to $a_t$, also affect productivity and growth through their equilibrium effect on innovation and hence $N_t$, thus propagating shocks further. This is quite in contrast to the counterpart with exogenous growth, where expected productivity growth is approximately constant.

The equilibrium productivity growth dynamics implied by the model resemble closely those specified by Croce (2008). Croce specifies productivity to contain an i.i.d. component as well as a small, but persistent component. He refers to that latter component as long-run productivity risk and shows that this specification allows to generate substantial risk premia in a production economy. While he exogenously specifies these dynamics, we show that such long-run productivity risk arises naturally in a setting with endogenous growth and that it is linked to innovation. Our model thus allows to empirically identify economic sources of long-run risk.

We can get further insights into the determinants of the stochastic trend by exploiting the specification of the innovation sector. From the law of motion for patents and the optimality condition for R&D it follows that the growth rate of the measure of intermediate goods satisfies

\[
\frac{N_{t+1}}{N_t} = \phi + E_t \left[ (X_{t+1}V_{t+1})^{\frac{\eta}{1-\eta}} \right] 
= \phi + E_t \left[ \chi \sum_{j=1}^{\infty} M_{t+j} \frac{1}{\phi^j} \Pi_{t+j} \right]^{\frac{\eta}{1-\eta}}
\]

(2.25)
where $M_{t+j} = \prod_{s} M_{t+s}^j$ is the $j$-step ahead stochastic discount factor and $M_{qt}$ ≡ 1. This implies that growth is directly related to the discounted value of future profits in the intermediate goods sector. This observation has two important implications. First, growth rates will naturally inherit the procyclicality of profits. Second, the average growth rate is endogenously related to the discount rate. Quantity dynamics therefore reflect risk premia. With recursive preferences, equilibrium growth will also depend on the endogenous amount of persistent long-term uncertainty. This is quite in contrast to the real business cycle model, where, as shown by Tallarini (2000), risk premia do not affect quantity dynamics.

2.4 Quantitative Implications

In this section we calibrate our model and explore its ability to replicate key moments of both macroeconomic quantities and asset returns. Rather than matching standard business cycle moments, we calibrate our model of endogenous growth to be quantitatively consistent with long-run dynamics of the aggregate economy, by which we mean isolating appropriate frequency bands in growth rates using a band-pass filter. On the other hand, we find it instructive to compare our benchmark model with a version in which trend growth is given exogenously. In the following, we refer to the benchmark endogenous growth model as ENDO, and the exogenous growth counterpart as EXO. The models are calibrated at a quarterly frequency.

2.4.1 Parameter Choices

Our benchmark model has three main components: Recursive preferences, a technology to produce final consumption goods, and an innovation technology. Recursive preferences have been used extensively in recent work in asset pricing. We follow this literature and set preference parameters to standard values that are also sup-

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6 See Bansal and Yaron (2004).
ported empirically. The parameters related to the final goods sector are set to match long-run dynamics in the aggregate economy. We identify the long-run components of growth rates with movements at frequencies between 100 and 200 quarters, that we isolate using a bandpass filter. We follow Comin and Gertler (2006) in calibrating the parameters related to the intermediate goods and R&D sectors. These choices are also consistent with empirical evidence in the growth literature. Critically, satisfying balanced growth helps provide further restrictions on parameter values. Table 2.1 summarizes our parameter choices.

We begin with a description of the calibration of the preference parameters. The elasticity of intertemporal substitution $\psi$ is set to value of 1.85 and the coefficient of relative risk aversion $\gamma$ is set to a value of 10, which are standard values in the long-run risks literature. An intertemporal elasticity of substitution larger than one is consistent with the notion that an increase in uncertainty lowers the price-dividend ratio. Note that in this parametrization, $\psi > \frac{1}{\gamma}$, which implies that the agent dislikes shocks to expected growth rates and is particularly important for generating a sizeable risk premium in this setting. The subjective discount factor $\beta$ is set to an annualized value of 0.984 so as to be consistent with the level of the riskfree rate.

In the final goods sector, the capital share $\alpha$ is set to .35, the intermediate goods (materials) share $\xi$ is set to 0.5, and the depreciation rate of capital $\delta$ is set to 0.02. These three standard parameters are calibrated to match steady-state evidence. The capital adjustment cost function is standard in the production-based asset pricing literature. The adjustment cost parameter $\zeta$ is set at 0.70 to match the relative volatility of long run consumption growth to output growth.

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7 See Bansal et al. (2007) uses Euler conditions and a GMM estimator to provide empirical support for the parameter values.

8 This choice is also consistent with the estimation evidence in van Binsbergen et al. (2011).

9 See, for example, Jermann (1998), Croce (2008), Kaltenbrunner and Lochstoer (2008) or van Binsbergen et al. (2011) for estimation evidence.
We now turn to the calibration of the parameters relating to the stationary productivity shock \( a_t \equiv \log(\Omega_t) \). Note that this shock is different than the Solow residual since the final goods production technology includes a composite input consisting of an expanding variety of intermediate goods, as detailed in the previous section. The persistence parameter \( \rho \) is set to an annualized value of 0.95 and is calibrated to match the first autocorrelation of R&D intensity. Furthermore, this value for \( \rho \) allows us to be consistent with the first autocorrelations of the key quantity growth rates and productivity growth.\(^{10}\) The volatility parameter \( \sigma \) is set at 1.75% to match long-run output growth volatility.

For the remaining parameters, the markup in the intermediate goods sector \( \nu \) is set to a value of 1.65 and the elasticity of new intermediate goods with respect to R&D \( \eta \) is set to a value of 0.85. While the markup of intermediate inputs is difficult to measure, varying the parameter around a reasonable range does not change our key quantitative results. The parameter \( \eta \) is within the range of panel and cross-sectional estimates from Grichiles (1990). Since the variety of intermediate goods can be interpreted as the stock of R&D (a directly observable quantity), we can then interpret one minus the survival rate \( \phi \) as the depreciation rate of the R&D stock. Hence, we set \( \phi \) to 0.9625 which corresponds to an annualized depreciation rate \( 1 - \phi \) of 15% which is a standard value and that assumed by the BLS in the R&D stock calculations. The scale parameter \( \chi \) is used to help match balanced growth evidence and set at a value of 0.332.

We calibrate the exogenous growth model (EXO) to facilitate direct comparison with our benchmark model. To do so, we set a trend growth parameter \( \mu \) equal to 1.90%\(^{10}\). To provide further discipline on the calibration of \( \rho \), note that since the ENDO model implies the TFP decomposition, \( \Delta z_t = \Delta a_t + \Delta n_t \), we can project log TFP growth on log growth of the R&D stock to back out the residual \( \Delta a_t \). The autocorrelations of the extracted residual \( \Delta \hat{a}_t \) show that we cannot reject that it is white noise. Hence, in levels, it must be the case that \( a_t \) is a persistent process to be consistent with this empirical evidence. In our benchmark calibration, the annualized value of \( \rho \) is .95.
to match average output growth and adjust the volatility of the forcing process to match the volatility of consumption growth of the benchmark model\textsuperscript{11}.

Long-Run Dynamics

Table 2.2 reports quantitative implications of the model for long-run economic performance. The benchmark model is quantitatively in line with the average growth rate of the economy and the long-run components $\sigma^{LR}$ of output, consumption and investment volatility, as targeted by our calibration. Two observations are nevertheless noteworthy.

First, the exogenous growth counterpart, while similarly calibrated, generates counterfactually little long-run movements in quantities. This finding reflects the lack of a strong propagation mechanism or, equivalently, the lack of endogenous persistence that workhorse models of macroeconomics in the real business cycle tradition are well known to exhibit and will be discussed in more detail below, in section 4.4.

Second, while at business cycle frequencies investment growth is much more volatile than both consumption and output growth, in the long run, it is actually smoother. This suggests that movements at higher frequencies are driven by a different set of shocks. Our model of endogenous growth is most readily thought of as theory of long-run movements and therefore we focus on innovations to productivity, to which we turn now.

2.4.2 Productivity Dynamics

Many of the key implications of the benchmark model can be understood by looking at the endogenous dynamics of total factor productivity (TFP) growth, $\Delta Z_t$, which we outlined in section 3:

$$E_t[\Delta Z_{t+1}] \approx \vartheta_t \cdot \tilde{S}_t + \phi$$

\textsuperscript{11} Extensive robustness checks with other model specifications are available in a separate appendix on request.
where \( \hat{S}_t \equiv \frac{S_t}{N_{t}} \) is the R&D intensity. Therefore, qualitatively, the dynamics of TFP are driven by endogenous movements in R&D.

Quantitatively, the implications of the model will thus depend on the ability of our calibration to match basic stylized facts about R&D activity and innovation. As table 2.2 documents, the model is broadly consistent with volatilities and autocorrelations of R&D investment, the stock of R&D and R&D intensities in the data. Crucially, as in the data, the R&D intensity is a fairly volatile and persistent process and we match its annual autocorrelation of 0.93.

The above decomposition of the expected growth rate of TFP therefore suggests a highly persistent component in TFP growth. Table 2.4 confirms this prediction, both in the data as well as in the model. While uncovering the expected growth rate of productivity as a latent variable in the data (as in Croce (2008)) suggests an annual persistence coefficient of 0.93, our model closely matches this number with a persistence coefficient of 0.95. Moreover, the volatilities of expected TFP growth rates in the data and in the model roughly match. Note that in contrast to our benchmark model, the EXO specification implies that TFP growth is roughly i.i.d., inconsistent with the empirical evidence.

Qualitatively, the above decomposition and the persistence of R&D intensity suggests that R&D intensity should track productivity growth rather well. Figures 2.1 and 2.2 visualize these patterns in the model, using a simulated sample path, as well as in the data. The plots visualize the small, but persistent component in TFP growth induced by equilibrium R&D activity.

From an empirical point of view, these results suggest that R&D activity, and especially the R&D intensity should forecast productivity growth rather effectively. We confirm this prediction in table 2.6, which documents results from projecting productivity growth on R&D intensity or R&D growth, respectively, over several horizons. In the data, R&D intensity and growth forecasts productivity growth over
several years, significantly, and with $R^2$’s increasing with the horizon. Qualitatively, the model replicates this pattern rather well.

The intuition for these results comes from the endogenous R&D dynamics that the model generates. This can be readily gleaned from the impulse responses to an exogenous shock displayed in figure 2.4. It exhibits responses of quantities in the patents sector. Crucially, after a shock profits rise persistently. Intuitively, a positive shock in the final goods sector raises the demand $X_t$ for intermediate goods, and with $\Pi_t = (\nu - 1)X_t$, this translates directly into higher profits. Naturally, given persistently higher profits, the value of a patent goes up, as shown in the third panel. Then, in turn, as the payoff to innovation is the value of patents, this triggers a persistent increase in the R&D intensity. This yields the persistent endogenous component in productivity growth displayed above. Crucially, the exogenous shock has two effects. It immediately raises productivity of the final output firm temporarily (due to the mean-reverting nature of the shock), leading to standard fluctuations at business cycle frequency. In addition, it also induces more R&D which will be reflected in the creation of more patents which has a permanent effect on the level of productivity. Moreover, the increases in R&D are persistent, leading to fluctuations at lower frequencies. Intuitively, in this setting, an exogenous shock to the level of productivity endogenously generates a persistent shock to the growth rate of the economy, or in other words, it generates growth waves.

2.4.3 Consumption Dynamics and Endogenous Long-Run Risk

In the previous section, we documented that the benchmark model has rich implications for the dynamics of measured TFP, which will naturally be reflected in the quantity dynamics of our production economy. With a view towards asset pricing, we focus on the implications for consumption dynamics in this section. In particular, we examine the dynamics of expected consumption growth that the model generates.
While Bansal and Yaron (2004) have shown in an endowment economy that persistent variation in expected consumption growth coupled with recursive preferences can generate substantial risk premia in asset markets, the empirical evidence regarding this channel is still controversial. In this light, providing theoretical evidence in production economies supporting the mechanism would be reassuring.\footnote{While several papers have considered how such long run risks can arise endogenously in production economies (Croce (2008), Kaltenbrunner and Lochstoer (2008), Campanale et al. (2008)), these studies operate in versions of the real business cycle model (proxied by the EXO specification here) and typically do not generate sufficient endogenous risks to match asset market statistics.}

Table 2.7 documents basic properties of consumption growth in the model. While the model matches the volatility of consumption growth, it also roughly replicates its annual autocorrelation. This is in sharp contrast to the EXO specification, where consumption growth is barely autocorrelated. More importantly, the table also documents that the benchmark model produces substantial variation in expected consumption growth, considerably more than the EXO specification. Similarly, this is also reflected in the substantial long-term volatilities that consumption growth exhibits in the model. In line with the asset pricing literature, we will refer to the volatility of consumption growth as business cycle or short-run risk and persistent variation in expected consumption growth as long-run risk. This suggests that the benchmark model generates quantitatively significant long-run risks in consumption growth.

Note that while Bansal and Yaron (2004) (in an endowment economy setting) and Croce (2008) (in a production economy setting) generate long-run risks by introducing independent, persistent shocks to consumption and productivity growth respectively, in our model fluctuations in realized consumption growth and persistent variation in expected consumption growth are driven by one source of exogenous uncertainty only. Hence, the model translates this disturbance in substantial low-frequency movements in consumption growth, or, in other words, provides a strong...
mechanism to propagate this shock. Accordingly, shocks to realized consumption growth also act as shocks to expected consumption growth.

Table 2.8 reports long-horizon autocorrelations of consumption growth, in the data and in the model. We restrict the empirical sample to 1953 to 2008, to ensure consistency with the availability with R&D data. While our model matches the first autocorrelation of consumption growth almost exactly, the second and third autocorrelation are negative in the data and positive in the model, and more importantly, outside the 95% confidence interval. On the other hand, all longer horizon autocorrelations are within that confidence interval.

In order to quantify the persistence in consumption growth in the model, we now compute the expected consumption growth process. We do so in two ways. In the first method, we take the consumption growth policy function from the numerical solution and directly take conditional expectations to obtain the expected consumption growth policy. Then we can directly simulate the process using this function. In the second method, we first simulate log consumption growth $\Delta c_t$ as well as the state variables log capital-to-R&D-capital ratio $\hat{k}_t$, and log productivity shock $a_t$ from the model and proceed by running the following regression $\Delta c_{t+1} = \beta_0 + \beta_1 \hat{k}_t + \beta_2 a_t + \epsilon_{c,t+1}$ so that the fitted values from this regression give the expected consumption growth process. Table 2.9 reports the results. The two methods yield practically the same process. This is not surprising as consumption growth in the model is approximately log-linear in the state variables. Also the endogenous expected consumption growth dynamics generated from our model are roughly similar to the exogenous specification ($x_t$) from Bansal and Yaron (2004). In particular, our process is slightly more persistent but slightly less volatile than the one in Bansal and Yaron.

Naturally, persistence in expected consumption growth is just a reflection of persistent dynamics in productivity growth. Empirically, this suggests that measures related to innovation, and the R&D intensity and R&D growth in particular, should
have forecasting for consumption growth. We verify this in table 2.10, which reports results from projecting future consumption growth over various horizons on the R&D intensity and growth. Empirically, these innovation measures predict future consumption growth over horizons up to 5 years, with significant point estimates, and $R^2$'s are increasing with the horizon. Qualitatively, the model reflects this pattern reasonably well. This gives empirical support to the notion of innovation-driven low-frequency variation in consumption growth.

2.4.4 Fluctuations and Propagation

While consumption dynamics are important for asset pricing, endogenous persistent variation in expected productivity growth suggests a propagation mechanism for quantities more generally, which, as alluded to above, standard macro models typically lack. We therefore now turn to a more systematic discussion of the macroeconomic implications of the model.

Table 2.11 reports standard business cycle statistics implied by the model. While the model is calibrated to replicate long-run dynamics of the aggregate economy, the table shows that it is also reasonably consistent with basic business cycle statistics. In particular, our benchmark model does just as well as the EXO model, which is essentially a version of a standard real business cycle model, meaning that the ENDO model generates high-frequency dynamics in line with the canonical real business cycle model. On the other hand, all specifications predict investment to be too smooth. This is because the model is calibrated to generate realistically smooth long-run investment dynamics, suggesting that different shocks drive investment volatility at business cycle frequencies.\(^{13}\)

\(^{13}\) Similarly, we abstract from endogenous movements in labor supply, as those mostly drive fluctuations at business cycle frequencies.
Looking beyond the standard business cycle statistics, the macroeconomic implications of our benchmark model and the exogenous growth counterpart are quite different, as we now explore.

Table 2.12 reports autocorrelations of basic growth rates, in the data, as well as in the ENDO and the EXO models. Note first that while all growth rates exhibit considerable positive autocorrelation at annual frequencies, the corresponding persistence implied by the EXO models is virtually zero, and sometimes even negative. This is one of the main weaknesses of the real business cycle model (as pointed out e.g. in Cogley and Nason (1995)). In stark contrast, our ENDO model generates substantial positive autocorrelation in all quantities, and in general are quantitatively close to their data counterparts. Note that the exogenous component of productivity is the same in both model. Accordingly, the ENDO model possesses a strong propagation mechanism induced by the endogenous component of productivity, e.g. by R&D.

The intuition for this endogenous propagation is of course simple, and tightly linked to the dynamics of TFP documented in the previous section. To the extent that innovation induces a persistent component in productivity, this will be reflected in quantity dynamics. Recall however, that the TFP dynamics implied by the model are consistent with the empirical evidence. As for consumption growth, this suggests that the drivers of expected productivity growth, namely the R&D intensity and R&D growth, should forecast quantity growth. This is verified in table 2.13 for output growth.

The propagation mechanism implies that macroeconomic quantities display markedly different behavior at different frequencies, in other words, it implies a rich intertemporal distribution of growth rates. The results in table 2.12 also suggest that the implied volatilities of growth rates of the EXO and ENDO models are basically undistinguishable at short horizons, in the ENDO model they grow fairly quickly over longer horizons. Basically, the ENDO model generates significant dynamics at
lower frequencies, while the EXO model does not.

Another implication of the model is that it generates cash flow dynamics in line with the empirical evidence. First of all, it generates strongly procyclical profits. This can be seen from figure 2.4. In our setting, this is driven by the procyclical demand for intermediate goods. Second, the model generates a persistent component in dividend growth. This can be seen in table 2.15, which documents considerable volatility in conditional expected dividend growth, which implies substantial variation in the conditional mean of cash flow growth. This is visualized in figure 2.6. Again, this is in stark contrast to the exogenous growth specification. This will be important from an asset pricing perspective, as only the benchmark model generates sufficient long-term uncertainty about dividend growth.

2.4.5 Asset Pricing Implications

The productivity dynamics in the model and the resulting endogenous persistence in consumption and cash flows generate sizeable risk premia in asset markets, as we now document. Endogenous persistence in growth rates affects asset prices in our model, because when agents have Epstein-Zin utility with a preference for an early resolution of uncertainty, not only are innovations to realized consumption and dividend growth priced, but also innovations to expected consumption and dividend growth.

Consistent with the multi-sector structure of our model, the stock market is a claim to the net payout from production; equation (18) provides a decomposition of the value of this claim into the value of physical capital and patents, hence intangible capital. Accordingly, we can separately define the returns on physical capital, the return on intangible capital, and the spread between the two. We will suggestively relate that spread to the value premium, the return spread between high book-to-market stocks (value stocks) and low book-to-market stocks (growth stocks). The
link is more suggestive as growth firms in the data likely are intangibles intensive but also hold physical capital, while in our model they do not, and likewise for value firms.

Table 2.14 reports asset market statistics, for the benchmark model and alternative specifications. Quantitatively, the benchmark model generates a sizeable excess return on stocks of close to 3%, an premium on physical capital in excess of 4%, a value spread close to the excess return on the aggregate stock market, plus a low and smooth risk free rate. The volatility of the aggregate stock market returns is close to 5%, with considerable volatilities of the return on physical capital and the value spread as well.

While sizeable, the premia and volatilities of returns in the model do not rationalize their empirical counterparts entirely. In line with our interpretation of the benchmark endogenous growth model as a model of long-run dynamics, we view the model implied premia and volatilities as those components reflecting uncertainty about long-term growth prospects and productivity. As documented earlier, the benchmark model is calibrated to match such long-run risks in the language of Bansal and Yaron, while it does not generate realistic business cycle or short-run risks, such as investment volatility. Indeed, Ai et al. (2010) report that empirically the productivity-driven fraction of return volatility is just around 6%, which is roughly consistent with our quantitative finding. On the other hand, table 2.14 also reports the asset pricing implications of a version of the endogenous growth model which is calibrated to match short-run consumption risk in a long-sample starting from the great depression. This calibration produces an overall equity premium of close to 6%, and a value premium of a similar magnitude.

In order to understand these results, it is instructive to compare the asset pricing implications of the benchmark model with those of the exogenous growth specification. To facilitate comparison, we focus on the returns on physical capital in the following.
While, as discussed previously, the quantity implications of the models are similar at high-frequencies, the pricing implications are radically different. As can be seen from the table, the risk free rate is counterfactually high in the exogenous growth specifications, and the equity premium is close to zero and only a tiny fraction of what obtains in the benchmark model. These differences are intimately connected to differences in low-frequency dynamics that the two models generate. Intuitively, in the settings with exogenous growth, expected growth rates are roughly constant (as in the real business cycle model), therefore diminishing households’ precautionary savings motive. In such a setting, households want to borrow against their future income, which in equilibrium can only be prevented by a prohibitively high interest rate. In the endogenous growth setting, however, taking advantage of profit opportunities in the intermediate goods sector leads to long and persistent swings in aggregate growth rates, and higher volatility over longer horizons. In this context, households optimally save for low growth episodes, leading to a lower interest rate in equilibrium.

Moreover, in contrast to intangible capital, the claim to physical capital is very risky in the model. This suggests that physical capital is endogenously more exposed to long-run uncertainty. The reason is twofold. First, as discussed above, the model generates endogenous long-run risks in consumption growth reflected in the stochastic discount factor. Second, the level of the risk premium also implies that in equilibrium, dividends on physical capital are risky. The reason is that these dividends naturally inherit a persistent component from the endogenous component of productivity. These cash flow dynamics not only affect risk premia, but naturally, also asset market valuations, as documented in figure 2.6. Specifically, the figure documents that following a productivity shock expected growth rates respond strongly in a persistent fashion in the ENDO model whereas in the EXO model expected growth rates are virtually unresponsive to the shock. In particular, expected div-
idend growth rates endogenously exhibit substantial persistent variation consistent with the setup in Barsky and DeLong (1993), who show that such a process can explain long swings in stock markets, and in Bansal and Yaron (2004).

The impulse responses also show that innovations to realized consumption and dividend growth are tightly linked to innovations to expected growth. Both of these innovations are priced when agents have Epstein-Zin utility with a preference for early resolution of uncertainty. In this case, agents fear that persistent slowdowns in growth coincide with a fall in asset prices. Therefore bad shocks are simultaneously bad shocks for the long run, which renders equity claims very risky.

Figure 2.7 illustrates this. In the benchmark model the response of the stochastic discount factor is substantially larger on impact than in the exogenous growth counterpart as a shock to realized consumption growth leads to a revision in growth expectations which is picked up in the stochastic discount factor as a revision to expected continuation utility. This is in contrast to the exogenous growth specification, where consumption growth is essentially iid. Moreover, the benchmark model displays stronger co-movements of returns and discount factor, leading to a higher risk premium as the latter is $E[r_d - r_f] \approx -\text{cov}(m, r_d)$.

2.4.6 Asset Prices and Growth

While the endogenous growth rate dynamics in the model in conjunction with recursive preferences help explain large risk premia in the data, asset prices also have important feedback effects on the macroeconomy. In particular, realistic risk premia in the model foster growth and amplify long-run movements in growth rates, a phenomenon we label long-run amplification. This is in contrast to real business cycle models, in which risk and risk premia do not affect quantity dynamics, a point which was forcefully made by Tallarini (2000). Formally, these feedback effects can be traced to equation (26) which relates the growth rate of the economy to discount
rates and profit opportunities in the intermediate goods sector. Our model suggests that such a feedback can be quantitatively significant.

Table 2.15 provides quantitative evidence on long-run amplification. It reports the volatilities of conditional means, long-run risks in other words, of various quantities. It does so for the benchmark model, the exogenous growth model, and a version of the endogenous growth model solved with CRRA preferences by setting the IES to the inverse of risk aversion. Not surprisingly, movements in conditional means are much more pronounced in the benchmark model relative to the exogenous growth model. Notably, however, the CRRA case of the endogenous growth model barely generates movements in conditional means. Thus, in our benchmark model, realistic asset price implications provide long-run amplification. Recursive preferences in conjunction with endogenous persistent fluctuations in growth rates increase the volatility of asset prices. Incentives to innovate reflect prices however, which renders innovation more volatile and amplifies long-run movements in growth.

We provide quantitative evidence on average growth rate effects in table 2.16, where we report sensitivity of model implications with respect to the key preference parameters, risk aversion and intertemporal elasticity of substitution. Consider first varying risk aversion, in the first 2 columns. Consistent with the results in Tallarini (2000), varying risk aversion barely affects standard business cycle statistics, that is, second moments. In other words, while varying risk aversion does not affect the amount of risk in the economy, it affects the price of risk and risk compensation, reflected in substantial differences in risk premia. Therefore, relative to Tallarini, the benchmark endogenous growth model exhibits a new effect, namely sensitivity of the average growth rate relative to the risk aversion. Specifically, raising risk aversion fosters growth. This has a simple intuition: Higher compensation for the same risks, or similarly, higher price for the same magnitude of risks reflected by in a higher Sharpe ratio, makes investment in risky assets more attractive and therefore
channels resources towards innovation and R&D. This is reflected in higher R&D investment, as measured by the R&D intensity, and hence higher growth.

In the last two columns, we keep risk aversion fixed at the benchmark level, but vary the intertemporal elasticity of substitution. Note that for all specifications we have $\psi > \frac{1}{\gamma}$, so that irrespective of the specification, agents have a preference for early resolution of uncertainty. Varying the IES changes the amount of risk in the economy, and its intertemporal distribution. Raising the IES is akin to increasing the propensity to substitute over time, which increases the response of investment to productivity and expected productivity growth and accordingly its volatility. In turn this smoothes consumption growth and increases its persistence. This raises the volatility of the conditional mean of consumption growth. Raising the IES therefore reduces short-run risk and increases long-run risk, while lowering the IES increases short-run risk and reduces long-run risk. With a high price of long-run risk, the net effect is an increase of the risk premium in the first case, and a fall in the latter case. As above, the average growth rate of the economy is increasing in the Sharpe ratio.

2.4.7 Long-Term Comovement

Our model also has realistic implications for comovement between prices and quantities at lower frequencies. In the following we identify low frequency movements in growth rates using a bandpass filter which isolates movements at frequencies between 32 and 100 quarters.

Figure 2.8 reveals that the model replicates the low-frequency comovements between productivity and quantities in the data. This is noteworthy because it reveals the significant variation macro data exhibit at lower frequencies and the significant comovement between productivity and quantities, which is mirrored by the ENDO model.

Figure 2.9 shows the close match between the price-dividend ratio and productivity
growth in the data and the benchmark model at low-frequencies. This strongly sug-
gests productivity-driven slow movements in asset market valuations in the data. In
the model, these movements are driven by variation in expected cash flows, induced
by time variation in R&D intensity. The long swings in price-dividend ratios are
consistent with the evidence in Barsky and DeLong (1993).
At lower frequencies we also find strong cross-correlations between stock returns and
consumption growth. This is displayed in figure 2.10, indicating the lag-lead struc-
ture between returns and consumption growth. In the data and at low frequencies,
returns lead consumption growth by several quarters and the lead correlations die
away more slowly (relative to the lag correlations). In other words, lower-frequency
movements in returns contain important information regarding long-run movements
in future growth. The ENDO model replicates this feature whereas the EXO model
does not. This important divergence between the two models is due to the fact that
in the ENDO model, growth rates contain a predictable component, which is absent
in the EXO models, that is a key determinant of asset prices. In sum, the benchmark
model is able reconcile the long-term relationship between returns and growth that
the neoclassical growth model fails to produce.

2.5 Conclusion

Starting from the notion that asset prices reflect expectations about future growth,
we provide a quantitative analysis of a production economy whose long-term growth
prospects are endogenously determined by innovation and R&D. By integrating in-
novation and R&D into a real business cycle model with recursive preferences, our
model constitutes a straightforward and highly tractable extension of the workhorse
model of modern macroeconomics. In sharp contrast to the latter, however, our
baseline model jointly rationalizes key features of asset returns and long-run macroe-
conomic performance in the data.
In the model, favorable economic conditions boost innovation and the development of new technologies. Since technological progress fosters long-run economic growth, endogenous innovation generates a powerful propagation mechanism for shocks reflected in persistent variation in long-term growth prospects. With recursive preferences, innovations to expected growth are priced and lead to high risk premia in asset markets, as agents fear that persistent slowdowns in growth coincide with low asset valuations. Formally, we show that R&D drives an endogenous predictable component in measured productivity, which gives an innovation-based explanation of long-run productivity risk in the data.

Our model thus allows to empirically identify economic sources of long-run risks. Indeed, we document novel empirical evidence that measures of innovation have significant predictive power for aggregate growth rates at longer horizons.
Table 2.1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ENDO</th>
<th>EXO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^4$</td>
<td>0.984</td>
<td>0.984</td>
</tr>
<tr>
<td>$\psi$</td>
<td>1.85</td>
<td>1.85</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.5</td>
<td>-</td>
</tr>
<tr>
<td>$\nu$</td>
<td>1.65</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>$\rho^4$</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.332</td>
<td>-</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.9625</td>
<td>-</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.83</td>
<td>-</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.75%</td>
<td>0.97%</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>$\mu * 4$</td>
<td>-</td>
<td>1.90%</td>
</tr>
</tbody>
</table>

Table 2.2: Long-Run Dynamics

<table>
<thead>
<tr>
<th>Data</th>
<th>ENDO</th>
<th>EXO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\Delta y]$</td>
<td>1.90%</td>
<td>1.90%</td>
</tr>
<tr>
<td>$\sigma_{\Delta y}^{LR}$</td>
<td>0.24%</td>
<td>0.22%</td>
</tr>
<tr>
<td>$\sigma_{\Delta c}^{LR}$</td>
<td>0.28%</td>
<td>0.24%</td>
</tr>
<tr>
<td>$\sigma_{\Delta i}^{LR}$</td>
<td>0.18%</td>
<td>0.17%</td>
</tr>
</tbody>
</table>

Table 2.3: Innovation Dynamics

<table>
<thead>
<tr>
<th>Data</th>
<th>ENDO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\Delta s}$</td>
<td>4.89%</td>
</tr>
<tr>
<td>$AC1(\Delta s)$</td>
<td>0.21</td>
</tr>
<tr>
<td>$AC1(\Delta n)$</td>
<td>0.90</td>
</tr>
<tr>
<td>$AC1(S/N)$</td>
<td>0.93</td>
</tr>
</tbody>
</table>
Table 2.4: Expected Productivity Growth Dynamics

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>ENDO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_\tilde{x}$</td>
<td>0.93</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma(\tilde{x})$</td>
<td>1.10%</td>
<td>1.20%</td>
</tr>
</tbody>
</table>

Table 2.5: Productivity Growth Dynamics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>ENDO</th>
<th>EXO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AC_1(\Delta z)$</td>
<td>0.09</td>
<td>0.11</td>
<td>-0.02</td>
</tr>
<tr>
<td>$\sigma(E_t[\Delta z_{t+1}])$</td>
<td>0.38%</td>
<td>0.15%</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\Delta z(5)}$</td>
<td>9.29%</td>
<td>4.15%</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\Delta z(10)}$</td>
<td>15.79%</td>
<td>5.55%</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\Delta z(20)}$</td>
<td>25.24%</td>
<td>6.86%</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.6: Productivity Growth Forecasts

Forecasts with R&D Intensity

<table>
<thead>
<tr>
<th>Horizon (k)</th>
<th>Data</th>
<th>ENDO</th>
<th>S.E.</th>
<th>R²</th>
<th>$\beta$</th>
<th>S.E.</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.014</td>
<td>0.009</td>
<td>0.031</td>
<td>0.075</td>
<td>0.075</td>
<td>0.039</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.031</td>
<td>0.015</td>
<td>0.080</td>
<td>0.142</td>
<td>0.142</td>
<td>0.062</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.049</td>
<td>0.024</td>
<td>0.120</td>
<td>0.204</td>
<td>0.204</td>
<td>0.077</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.069</td>
<td>0.032</td>
<td>0.174</td>
<td>0.261</td>
<td>0.261</td>
<td>0.088</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.091</td>
<td>0.041</td>
<td>0.232</td>
<td>0.314</td>
<td>0.314</td>
<td>0.095</td>
<td></td>
</tr>
</tbody>
</table>

Forecasts with R&D Growth

<table>
<thead>
<tr>
<th>Horizon (k)</th>
<th>Data</th>
<th>ENDO</th>
<th>S.E.</th>
<th>R²</th>
<th>$\beta$</th>
<th>S.E.</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.431</td>
<td>0.190</td>
<td>0.113</td>
<td>0.560</td>
<td>0.560</td>
<td>0.037</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.820</td>
<td>0.315</td>
<td>0.192</td>
<td>1.070</td>
<td>1.070</td>
<td>0.060</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.230</td>
<td>0.452</td>
<td>0.262</td>
<td>1.533</td>
<td>1.533</td>
<td>0.076</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.707</td>
<td>0.522</td>
<td>0.376</td>
<td>1.948</td>
<td>1.948</td>
<td>0.084</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.092</td>
<td>0.599</td>
<td>0.444</td>
<td>2.322</td>
<td>2.322</td>
<td>0.090</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.7: Consumption Dynamics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>ENDO</th>
<th>EXO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\Delta c}$</td>
<td>1.42%</td>
<td>1.42%</td>
<td>1.42%</td>
</tr>
<tr>
<td>$AC1(\Delta c)$</td>
<td>0.40</td>
<td>0.39</td>
<td>-0.002</td>
</tr>
<tr>
<td>$\sigma(E_t[\Delta c_{t+1}])$</td>
<td>0.51%</td>
<td>0.09%</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\Delta c(5)}$</td>
<td>6.63%</td>
<td>3.14%</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\Delta c(10)}$</td>
<td>11.97%</td>
<td>4.30%</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\Delta c(20)}$</td>
<td>21.18%</td>
<td>5.58%</td>
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</tr>
</tbody>
</table>

Table 2.8: Consumption Autocorrelations

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>ENDO</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>lower</td>
</tr>
<tr>
<td>$AC1(\Delta c)$</td>
<td>0.40</td>
<td>0.39</td>
<td>0.05</td>
</tr>
<tr>
<td>$AC2(\Delta c)$</td>
<td>-0.09</td>
<td>0.26</td>
<td>-0.01</td>
</tr>
<tr>
<td>$AC3(\Delta c)$</td>
<td>-0.17</td>
<td>0.21</td>
<td>-0.06</td>
</tr>
<tr>
<td>$AC4(\Delta c)$</td>
<td>-0.11</td>
<td>0.17</td>
<td>-0.11</td>
</tr>
<tr>
<td>$AC5(\Delta c)$</td>
<td>0.06</td>
<td>0.13</td>
<td>-0.15</td>
</tr>
<tr>
<td>$AC6(\Delta c)$</td>
<td>0.10</td>
<td>0.11</td>
<td>-0.17</td>
</tr>
<tr>
<td>$AC7(\Delta c)$</td>
<td>-0.02</td>
<td>0.09</td>
<td>-0.20</td>
</tr>
<tr>
<td>$AC8(\Delta c)$</td>
<td>-0.16</td>
<td>0.05</td>
<td>-0.24</td>
</tr>
<tr>
<td>$AC9(\Delta c)$</td>
<td>-0.17</td>
<td>0.03</td>
<td>-0.25</td>
</tr>
<tr>
<td>$AC10(\Delta c)$</td>
<td>-0.01</td>
<td>0.02</td>
<td>-0.27</td>
</tr>
</tbody>
</table>

Table 2.9: Expected Consumption Growth Dynamics

<table>
<thead>
<tr>
<th></th>
<th>BY</th>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.979</td>
<td>0.981</td>
<td>0.981</td>
</tr>
<tr>
<td>$\hat{\sigma}_x$</td>
<td>0.12%</td>
<td>0.10%</td>
<td>0.10%</td>
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</tbody>
</table>
Table 2.10: Consumption Growth Forecasts

<table>
<thead>
<tr>
<th>Horizon (k)</th>
<th>Data</th>
<th>ENDO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β</td>
<td>S.E.</td>
</tr>
<tr>
<td>1</td>
<td>0.217</td>
<td>0.084</td>
</tr>
<tr>
<td>2</td>
<td>0.395</td>
<td>0.178</td>
</tr>
<tr>
<td>3</td>
<td>0.540</td>
<td>0.276</td>
</tr>
<tr>
<td>4</td>
<td>0.703</td>
<td>0.347</td>
</tr>
<tr>
<td>5</td>
<td>0.842</td>
<td>0.401</td>
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</table>

Table 2.11: Business Cycle Statistics

<table>
<thead>
<tr>
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<th>Data</th>
<th>ENDO</th>
<th>EXO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\Delta c}/\sigma_{\Delta y}$</td>
<td>0.61</td>
<td>0.61</td>
<td>1.13</td>
</tr>
<tr>
<td>$\sigma_{\Delta i}/\sigma_{\Delta c}$</td>
<td>4.38</td>
<td>2.23</td>
<td>0.79</td>
</tr>
<tr>
<td>$\sigma_{\Delta s}/\sigma_{\Delta y}$</td>
<td>2.10</td>
<td>1.64</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{\Delta z}/\sigma_{\Delta y}$</td>
<td>1.22</td>
<td>1.52</td>
<td>1.54</td>
</tr>
</tbody>
</table>
Table 2.12: First Autocorrelations

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>ENDO</th>
<th>EXO</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC1(Δz)</td>
<td>0.09</td>
<td>0.11</td>
<td>-0.020</td>
</tr>
<tr>
<td>AC1(Δc)</td>
<td>0.40</td>
<td>0.46</td>
<td>-0.002</td>
</tr>
<tr>
<td>AC1(Δy)</td>
<td>0.37</td>
<td>0.21</td>
<td>0.001</td>
</tr>
<tr>
<td>AC1(Δi)</td>
<td>0.25</td>
<td>0.14</td>
<td>0.012</td>
</tr>
<tr>
<td>AC1(Q)</td>
<td>0.95</td>
<td>0.96</td>
<td>0.89</td>
</tr>
</tbody>
</table>
Table 2.13: Output Growth Forecasts

<table>
<thead>
<tr>
<th>Horizon (k)</th>
<th>Data</th>
<th>ENDO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\beta}$</td>
<td>S.E.</td>
</tr>
<tr>
<td>1</td>
<td>0.020</td>
<td>0.013</td>
</tr>
<tr>
<td>2</td>
<td>0.046</td>
<td>0.022</td>
</tr>
<tr>
<td>3</td>
<td>0.068</td>
<td>0.029</td>
</tr>
<tr>
<td>4</td>
<td>0.089</td>
<td>0.041</td>
</tr>
<tr>
<td>5</td>
<td>0.114</td>
<td>0.051</td>
</tr>
</tbody>
</table>

Forecasts with R&D Growth

<table>
<thead>
<tr>
<th>Horizon (k)</th>
<th>Data</th>
<th>ENDO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta$</td>
<td>S.E.</td>
</tr>
<tr>
<td>1</td>
<td>0.267</td>
<td>0.130</td>
</tr>
<tr>
<td>2</td>
<td>0.453</td>
<td>0.261</td>
</tr>
<tr>
<td>3</td>
<td>0.572</td>
<td>0.387</td>
</tr>
<tr>
<td>4</td>
<td>0.763</td>
<td>0.457</td>
</tr>
<tr>
<td>5</td>
<td>0.940</td>
<td>0.499</td>
</tr>
</tbody>
</table>

Table 2.14: Asset Pricing Implications

<table>
<thead>
<tr>
<th>ENDO</th>
<th>ENDO-HV</th>
<th>EXO</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Moments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[r_f]$</td>
<td>1.21%</td>
<td>1.21%</td>
</tr>
<tr>
<td>$E[r_{mf}^* - r_f]$</td>
<td>2.92%</td>
<td>5.76%</td>
</tr>
<tr>
<td>$E[r_m^* - r_f]$</td>
<td>4.10%</td>
<td>8.33%</td>
</tr>
<tr>
<td>$E[r_{m}^* - r_{ic}^*]$</td>
<td>3.27%</td>
<td>6.89%</td>
</tr>
<tr>
<td><strong>Second Moments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\Delta c}$</td>
<td>1.42%</td>
<td>2.72%</td>
</tr>
<tr>
<td>$\sigma_{r_f}$</td>
<td>0.30%</td>
<td>0.38%</td>
</tr>
<tr>
<td>$\sigma_{r_{mf}^* - r_f}$</td>
<td>4.86%</td>
<td>6.73%</td>
</tr>
<tr>
<td>$\sigma_{r_m^* - r_f}$</td>
<td>7.08%</td>
<td>9.49%</td>
</tr>
<tr>
<td>$\sigma_{r_{m}^* - r_{ic}^*}$</td>
<td>5.13%</td>
<td>7.81%</td>
</tr>
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</table>
Table 2.15: Volatility of Expected Growth Rates

<table>
<thead>
<tr>
<th></th>
<th>ENDO</th>
<th>EXO</th>
<th>ENDO-CRRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(E_t[\Delta z_{t+1}])$</td>
<td>0.38%</td>
<td>0.15%</td>
<td>0.06%</td>
</tr>
<tr>
<td>$\sigma(E_t[\Delta y_{t+1}])$</td>
<td>0.42%</td>
<td>0.08%</td>
<td>0.09%</td>
</tr>
<tr>
<td>$\sigma(E_t[\Delta i_{t+1}])$</td>
<td>0.37%</td>
<td>0.05%</td>
<td>0.21%</td>
</tr>
<tr>
<td>$\sigma(E_t[\Delta d_{t+1}])$</td>
<td>0.92%</td>
<td>0.18%</td>
<td>0.10%</td>
</tr>
</tbody>
</table>

Table 2.16: Sensitivity Analysis: Preference Parameters

First Moments

<table>
<thead>
<tr>
<th></th>
<th>$\gamma = 2$</th>
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<th>$\psi = 0.5$</th>
<th>$\psi = 2.2$</th>
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<tr>
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<td>$E[r_m^* - r_f]$</td>
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<td>6.27%</td>
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<tr>
<td>$E[S/N]$</td>
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<td>0.084</td>
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Other Moments

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<th>$\psi = 0.5$</th>
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Figure 2.1: Growth Rates and R&D Intensity

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Figure 2.4: Expected Growth Rates
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3

Equilibrium Growth, Inflation and Bond Yields

3.1 Introduction

Explaining the nominal yield curve is a challenge for standard economic models. Notably, Dunn and Singleton (1986) and Backus et al. (1989) highlight the difficulty of consumption-based models with standard preferences in explaining the sign, magnitude, and volatility of the term spread. More recently, consumption-based models with richer preference specifications and model dynamics, such as Wachter (2006), Piazzesi and Schneider (2006), Gallmeyer et al. (2007), Bansal and Shaliastovich (2009), have found success in reconciling bond prices. However, Donaldson et al. (1990), den Haan (1995), Rudebusch and Swanson (2008), Rudebusch and Swanson (2012), van Binsbergen et al. (2011) show that extending these endowment economy models to environments with endogenous production still have difficulty in explaining the term premium jointly with key macroeconomic aggregates. This paper shows that endogenizing long-run growth prospects in a production-based model and assuming agents have recursive preferences can help rationalize the slope of the yield

\footnote{Standard preferences in this context refers to a power utility specification.}
curve jointly with a broad spectrum of macroeconomic facts.

Specifically, I link nominal bond prices to firm decisions using a stochastic endogenous growth model with imperfect price adjustment. Inflation is determined by the price-setting behavior of monopolistic firms. Due to the assumption of sticky prices, in equilibrium, inflation is equal to the present discounted value of current and future marginal costs of the firm. Notably, marginal costs are negatively related to productivity, which is crucial for understanding the link between inflation and growth in the model. Long-run productivity growth is driven by R&D investments and leads to sustained growth. In a stochastic setting, this mechanism generates an endogenous stochastic trend and leads to substantial long-run uncertainty about future growth. This framework has two distinguishing features. First, I embed an endogenous growth model of vertical innovations\(^2\) into a standard New Keynesian DSGE model,\(^3\) which in contrast to the latter type of models, trend growth is endogenously determined by firm investments. Second, households are assumed to have recursive preferences\(^4\) so that they are sensitive towards uncertainty about long-term growth prospects.

While New Keynesian DSGE models have been successful in quantitatively explaining a wide array of empirical macroeconomic features and become the standard framework for modern monetary policy analysis, these models have typically found difficulty in replicating salient features in asset markets.\(^5\) The results of this paper show that incorporating the endogenous growth margin, in conjunction with recursive preferences, allows these models to reconcile a broad set of asset market

\(^{2}\) See e.g. ?, Aghion and Howitt (1992), and Peretto (1999).

\(^{3}\) See Woodford (2003) and Gali (2008) for textbook treatments on this class of models.


\(^{5}\) See Smets and Wouters (2007) and Christiano et al. (2005) are examples of these models that have been able to match the impulse responses of key macroeconomic variables to nominal and real shocks. Rudebusch and Swanson (2008), Kurmann and Otrok (2011) are examples that document the failures of such models in replicating asset price facts.
facts, including the average nominal yield curve and the equity premium, jointly with macroeconomic facts.

The endogenous growth channel generates persistent low-frequency movements in aggregate growth and inflation rates that are negatively correlated. The intuition for these dynamics is as follows. In good times, innovation rates increase, which lead to a persistent rise in productivity growth and consequently, a persistent increase in consumption and dividend growth. Furthermore, a prolonged boom in productivity growth lowers marginal cost of firms for an extended period, which leads firms to lower the price of their goods persistently; in the aggregate, this implies a prolonged decline in inflation. When agents have Epstein-Zin recursive utility with a preference for an early resolution of uncertainty, they are very averse to persistent changes in long-run growth prospects. Hence, the innovation-driven low-frequency cycles in consumption and dividend growth imply a high equity premium. In addition, the long-run negative correlation between consumption growth and inflation implies that long nominal bonds have low payoffs when long-run growth is expected to be low. Thus, long bonds are particularly risky which lead to an upward sloping yield curve and sizeable bond risk premium.

When monetary policy follows a Taylor rule and is aggressive in stabilizing inflation, the negative long-run relationship between growth and inflation also implies that an increase in the slope of the yield curve predicts higher future growth. In particular, a persistent decline in inflation leads to a sharp and persistent drop in the short-term nominal rate. Consequently, the slope of the nominal yield spread

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6 There is strong empirical support for these dynamics. For example, Barksy and Sims (2009) and Kurmann and Otrok (2011) document that anticipations of future increases in productivity growth are associated with sharp and persistent declines in inflation.

7 The basic mechanism that long-run movements in productivity growth drive term spreads is empirically supported. Indeed, Kurmann and Otrok (2010) use a VAR to show that news about long-run productivity growth explain over 60% of the variation in the slope of the term structure.
becomes steeper while expected growth increases, as in the data.\footnote{See, for example, Ang et al. (2006) for empirical evidence of this relationship.}

Given that a calibrated version of the model can rationalize a broad set of asset pricing and macroeconomic facts, this framework serves as an ideal laboratory to quantitatively evaluate the effect of changes in monetary policy on asset prices in counterfactual policy experiments. Specifically, in the model, monetary policy is characterized by a short-term nominal interest rate rule that responds to current inflation and output deviations. Due to the presence of nominal rigidities, changes in the short rate affect the real rate which alters real decisions, including R&D. Thus, monetary policy can influence trend growth dynamics. Moreover, by varying the policy parameters, even such short-run stabilization policies can have substantial effects on the level and dynamics of long-run growth, which in turn have important implications for real and nominal risks. For example, more aggressive output stabilization implies that the short rate, and thus the real rate, will increase more in response to an increase in output. Since R&D rates are procyclical, a larger rise in the real rate will dampen the increase in R&D more and lower the volatility of expected growth rates. With less uncertainty about long-run growth prospects, risk premia decline. On the other hand, more aggressive output stabilization amplifies the volatility of expected inflation. Since expected inflation is countercyclical, a sharper rise in the real rate depresses expected inflation even further. Greater uncertainty about expected inflation makes long bonds even riskier which increases the slope of the nominal yield curve.

In another example, more aggressive inflation stabilization implies that the short rate, and thus the real rate, will increase more in response to an increase in inflation. Since inflation and R&D rates are negatively correlated, a larger rise in the real rate will further depress R&D and thus, amplify R&D rates. Furthermore, more volatile R&D rates imply that expected growth rates are more volatile. Higher uncertainty
about long-term growth prospects therefore increases risk premia. Additionally, more aggressive inflation stabilization will naturally smooth expected inflation which lowers the slope of the nominal yield curve. Thus, inflation and output stabilization have opposite effects on asset markets. In short, these results suggest that monetary policy, even when targeting short-run deviations, can have a substantial impact on asset markets by distorting long-run growth and inflation dynamics.

My paper is related to a number of different strands of literature in asset pricing, economic growth and macroeconomics. The basic economic mechanisms driving the equity markets are closely related to Bansal and Yaron (2004) (henceforth, BY). In a consumption-based model, BY specify both consumption and dividend growth to contain a small, persistent component, which exogenously leads to long and persistent swings in the dynamics of these quantities. This specification along with the assumption of Epstein-Zin recursive utility allows them to generate high equity premia as compensation for these long-run risks.

The economic mechanisms driving the nominal bond prices are related to the endowment economy models of Piazzesi and Schneider (2006) and Bansal and Shaliasstovich (2009). Both of these papers extend the BY framework to a nominal setting by specifying the evolution of inflation exogenously. Critically, in order to match the upward sloping nominal yield curve, both papers require that the long-run correlation between consumption growth and inflation is negative, which they find empirical support for. These joint consumption and inflation dynamics imply that long bonds are particularly risky, as they have low payoffs when expected consumption growth is low. My paper shows that these joint consumption and inflation processes are a natural implication of a stochastic endogenous growth model.

Methodologically, my paper is most closely related to Kung and Schmid (2011) (henceforth, KS) who show that in a standard stochastic endogenous growth model with expanding variety, equilibrium R&D decisions generate persistent low-frequency
movements in measured productivity growth. Naturally, these productivity dynamics are then reflected in consumption and dividend growth. With recursive preferences, these endogenous low-frequency cycles help to reconcile equity market data. Further, KS documents that the model creates a strong feedback effect between asset prices and growth, which amplifies low-frequency movements in aggregate growth rates, which further increases risk premia. My paper relies on similar economic mechanisms for generating a sizeable equity premium and low riskfree rate. On the other hand, my paper differs from KS by extending these ideas to a nominal economy with imperfect price adjustment. Specifically, I embed an endogenous growth model of vertical innovations into a standard New Keynesian setup. These extensions allow me to study the determination of nominal bond prices jointly with equity prices and also, the implications of monetary policy for both growth and asset prices.

More broadly, my paper relates to a number of recent papers that study how long-run risks arise endogenously in general equilibrium production economies. Some examples include Tallarini (2000), Uhlig (2010), Backus et al. (2010), Croce (2008), Campanale et al. (2008), Kaltenbrunner and Lochstoer (2008), Kuehn (2008), Ai (2009), and Gourio (2009). These papers typically work in versions of the standard real business cycle model, where growth is given exogenously. One conclusion from calibrated versions of these important contributions is that while long-run risks do arise endogenously in such settings, they are typically not quantitatively sufficient to rationalize key asset market statistics. In contrast, the endogenous growth paradigm does deliver quantitatively significant endogenous long-run risks through the R&D and innovation decisions of firms.

My paper also relates to the literature examining the term structure of interest rates in general equilibrium production-based models. Donaldson, Johnsen and Mehra (1990) and den Haan (1995) document that variants of standard business cycle models with additively separable preferences have trouble reproducing the size-
able positive nominal term premium observed in the data. These shortcomings are inherently linked to the fact that additively separable preferences cannot generate sufficient risk premia. Indeed, Backus et al. (1989) also highlight similar issues in an endowment economy setting.

Wachter (2006) shows that a consumption-based model with habit preferences can explain the nominal term structure of interest rates. However, Rudebusch and Swanson (2008) show that in production-based model with habit preferences, nominal bond prices can only be reconciled at the expense of distorting the fit of macroeconomic variables, such as real wages and inflation. In contrast, my model can explain the nominal structure of interest rates, jointly with equity prices, in a production setting while still maintaining a good fit to a broad set of macroeconomic variables.

van Binsbergen et al. (2011) and Rudebusch and Swanson (2012) consider standard production-based models with recursive preferences and highlight the difficulty these models have in quantitatively explaining the nominal term structure of interest rates with macro aggregates. In particular, these papers demonstrate that a very large coefficient of relative risk aversion is required to match to the slope of the nominal yield curve. In contrast, in my model, I explain the slope of the yield curve with a standard calibration. This difference is again attributed to the fact that standard neoclassical models with exogenous growth lack the strong propagation mechanism of endogenous growth models and therefore do not generate enough long-run consumption growth volatility. So, while the neoclassical models generate negative correlation between consumption growth and inflation to give a positively sloping yield curve, the quantity of real long-run risks is too small, and thus, the bond risk premium is too small.

Furthermore, Rudebusch and Swanson (2012) also document that incorporate expected growth shocks to the productivity process actually makes the nominal yield curve downward sloping. A positive expected productivity growth shock leads to a
very large wealth effect that reduces the incentives to work and leads to a sharp rise in wages. The increase in wages raises marginal costs and thus, inflation increases. In addition, expected consumption growth naturally inherits the long-run dynamics of productivity growth. Thus, this shock leads to a positive correlation between expected consumption growth and inflation, which leads to a downward sloping average nominal yield curve. In contrast, the movements in expected productivity growth in my model are endogenous and affected by labor decisions. Namely, an increase in the labor input raises the marginal productivity of R&D and therefore raises the incentives to innovate. An increase in labor hours will raise both the level of output and the expected growth rate of output, *ceteris paribus*. Put differently, the labor input has a significantly higher marginal value in the endogenous growth model than in the neoclassical setting, where productivity is exogenous. Consequently, in the endogenous growth setting, agents have higher incentives to supply labor in good times to boost expected growth prospects. Importantly, this dampens the sharp rise in wages from the wealth effect of persistently higher future growth. Consequently, the endogenous growth channel allows my model to maintain the strong negative correlation between expected consumption growth and inflation that is critical for explaining the nominal yield curve.

Finally, my paper is closely related to a few recent papers exploring how various policy instruments can distort the intertemporal distribution of consumption risk. In a companion paper, Kung (2011) studies monetary policy design in a similar New Keynesian endogenous growth model. In particular, Kung (2011) examines the welfare tradeoffs between short-run risks, long-run risks and the level of trend growth of various interest rate policies. Croce et al. (2012) (henceforth, CKNS) demonstrate how tax smoothing fiscal policies can amplify low-frequency movements in growth rate in a real business cycle model with financial frictions, which can increase risk premia significantly. Croce et al. (2011) (henceforth, CNS) study fiscal policy design
in a stochastic endogenous growth model with expanding variety. Similarly, they find that fiscal policies aimed at short-run stabilization significantly amplify long-run consumption volatility and decreases welfare. In contrast, in my paper and Kung (2011), I find that interest rate rules targeting short-run output stabilization decreases long-run consumption volatility while inflation stabilization increases long-run consumption volatility. Thus, while CKNS and CNS study the role of fiscal policy on growth dynamics, my paper and Kung (2011) study the role of monetary policy. Hence, I view these papers as complementary.

The paper is structured as follows. Section 2 outlines the benchmark endogenous growth model and the exogenous growth counterparts. Section 3 qualitatively illustrates the growth and inflation dynamics of the model. Section 4 explores the quantitative implications of the model. Section 5 concludes.

3.2 Model

The benchmark model embeds a endogenous growth model of vertical innovations into a fairly standard New Keynesian model. The representative household is assumed to have recursive preferences defined over consumption and leisure. These preferences imply that the household is sensitive towards fluctuations to expected growth rates, which is a key margin in this model. The production side is comprised of a final goods sector and an intermediate goods sector. The final goods sector is characterized by a representative firm that produces the consumption goods using a bundle of intermediate goods inputs that are purchased from intermediate goods producers. The intermediate goods sector is comprised of a continuum of monopolists of unit measure. Each monopolist sets prices subject to quadratic price adjustment costs and uses firm-specific labor, physical capital, and R&D capital inputs to produce a particular intermediate goods. Also, each monopolist accumulates the physical and R&D capital stocks subject to convex adjustment costs. The monetary
authority is assumed to follow a modified Taylor rule.

Under a certain parameter configuration and exogenous R&D policy, the benchmark endogenous growth model collapses to a standard New Keynesian setup with exogenous growth. Moreover, to highlight the implications of the endogenous growth mechanism, I compare the benchmark growth model to two paradigms of exogenous growth, one with a deterministic trend and the other with a stochastic trend.

3.2.1 Endogenous Growth

Representative Household Assume that the household has recursive utility over streams of consumption $C_t$ and leisure $L - L_t$:

$$U_t = \left\{ (1 - \beta) (C_t^\gamma) \frac{1 - \gamma}{1 - \psi} + \beta (E_t [U_{t+1}^{1-\gamma}]) \right\} \frac{\theta}{1 - \gamma}$$

$$C_t^\star = C_t (L - L_t)^\tau$$

where $\gamma$ is the coefficient of risk aversion, $\psi$ is the elasticity of intertemporal substitution, $\theta = \frac{1 - \gamma}{1 - 1/\psi}$ is a parameter defined for convenience, $\beta$ is the subjective discount rate, and $L$ is the agent’s time endowment. The time $t$ budget constraint of the household is

$$p_t C_t + \frac{B_{t+1}}{P_{t+1}} = D_t + W_t L_t + B_t$$

where $p_t$ is the nominal price of the final goods, $B_{t+1}$ are nominal one-period bonds, $R_{t+1}$ is the gross nominal interest rate set at time $t$ by the monetary authority, $D_t$ is nominal dividend income received from the intermediate firms, $W_t$ is the nominal wage rate, and $L_t$ is labor supplied by the household. The household’s intertemporal condition is

$$1 = E_t \left[ M_{t+1} \frac{P_t}{P_{t+1}} \right] R_{t+1}$$
where
\[ M_{t+1} = \beta \left( \frac{C_t^{s+1}}{C_t^*} \right)^{\frac{1}{1-\gamma}} \left( \frac{C_t^{s+1}}{C_t^*} \right)^{-1} \left( \frac{U_{t+1}^{1-\gamma}}{E_t[U_{t+1}^{1-\gamma}]} \right)^{1-\frac{1}{\theta}} \]
is the stochastic discount factor. The intratemporal condition is
\[ \frac{W_t}{P_t} = \frac{\tau C_t}{L - L_t} \]

**Final Goods**  A representative firm produces the final (consumption) goods in a perfectly competitive market. The firm uses a continuum of differentiated intermediate goods \( X_{i,t} \) as input in the CES production technology
\[ Y_t = \left( \int_0^1 X_{i,t}^{\nu+1} \, dt \right)^{\frac{1}{\nu+1}} \]
where \( \nu \) is the elasticity of substitution between intermediate goods. The profit maximization problem of the firm yields the following isoelastic demand schedule with price elasticity \( \nu \)
\[ X_{i,t} = Y_t \left( \frac{P_{t,i}}{P_t} \right)^{-\nu} \]
where \( P_t \) is the nominal price of the final goods and \( P_{t,i} \) is the nominal price of intermediate goods \( i \). The inverse demand schedule is
\[ P_{t,i} = P_t Y_t^\frac{1}{\nu} X_{i,t}^{-\frac{1}{\nu}} \]

**Intermediate Goods**  The intermediate goods sector will be characterized by a continuum of monopolistic firms. Each intermediate goods firm produces \( X_{i,t} \) with physical capital \( K_{i,t} \), R&D capital \( N_{i,t} \), and labor \( L_{i,t} \) inputs using the following technology, similar to Peretto (1999),
\[ X_{i,t} = K_{i,t}^\alpha (Z_{i,t} L_{i,t})^{1-\alpha} \]
where total factor productivity (TFP) is

\[ Z_{i,t} = A_t N_{i,t}^\eta N_t^{1-\eta} \]

where \( A_t \) represents a stationary aggregate productivity shock, \( N_t \equiv \int_0^1 N_j dj \) is the aggregate stock of R&D and the parameter \( \eta \in [0, 1] \) captures the degree of technological appropriability. Thus, firm-level TFP is comprised of two aggregate components, \( A_t \) and \( N_t \), and a firm-specific component \( N_{i,t} \). In contrast to the neoclassical production function with labor augmenting technology, TFP contains an endogenous component determined by firm decisions. In particular, the firm can upgrade its technology directly by investing in R&D. Furthermore, there are spillover effects from innovating; namely, firm-level investments in R&D will also improve aggregate technology. These spillover effects are crucial for generating sustained growth in the economy and a standard feature in modern endogenous growth models.\(^9\)

The law of motion for \( A_t \), in logs, is

\[ a_t = (1 - \rho)a^* + \rho a_{t-1} + \sigma \epsilon_t \]

where \( a_t \equiv \log(A_t) \), \( \epsilon_t \sim N(0, 1) \) is i.i.d., and \( a^* > 0 \) is the unconditional mean of \( a_t \).

The law of motion for \( K_{i,t} \) is

\[ K_{i,t+1} = (1 - \delta_k) K_{i,t} + \Phi_k \left( \frac{I_{i,t}}{K_{i,t}} \right) K_{i,t} \]

\[ \Phi_k \left( \frac{I_{i,t}}{K_{i,t}} \right) = \frac{\alpha_{1,k}}{1 - \frac{1}{\zeta_k}} \left( \frac{I_{i,t}}{K_{i,t}} \right)^{1-\frac{1}{\zeta_k}} + \alpha_{2,k} \]

where \( I_{i,t} \) is capital investment (using the final goods) and the function \( \Phi_k(\cdot) \) captures capital adjustment costs. The parameter \( \zeta_k \) represents the elasticity of new capital

\(^9\) See, for example, Romer (1990), Grossman and Helpman (1991b), and Aghion and Howitt (1992).
investments relative to the existing stock of capital. The parameters $\alpha_{1,k}$ and $\alpha_{2,k}$ are set to values so that there are no adjustment costs in the deterministic steady state.\footnote{\textsuperscript{10} Specifically, $\alpha_{1,k} = (\Delta Z_{ss} - 1 + \delta_k)^{\frac{1}{\xi_k}}$ and $\alpha_{2,k} = \frac{1}{\xi_k - 1} (1 - \delta_k - \Delta Z_{ss})$.}

The law of motion for $N_{i,t}$ is

$$N_{i,t+1} = (1 - \delta_n)N_{i,t} + \Phi_n \left( \frac{S_{i,t}}{N_{i,t}} \right) N_{i,t}$$

$$\Phi_n \left( \frac{S_{i,t}}{N_{i,t}} \right) = \frac{\alpha_{1,n}}{1 - \frac{1}{\zeta_n}} \left( \frac{S_{i,t}}{N_{i,t}} \right)^{1 - \frac{1}{\zeta_n}} + \alpha_{2,n}$$

where $S_{i,t}$ is R&D investment (using the final goods) and the function $\Phi_n(\cdot)$ captures adjustment costs in R&D investments and has the same functional form as the capital adjustment cost equation of Jermann (1998). The parameter $\zeta_n$ represents the elasticity of new R&D investments relative to the existing stock of R&D. The parameters $\alpha_{1,n}$ and $\alpha_{2,n}$ are set to values so that there are no adjustment costs in the deterministic steady state.\footnote{Specifically, $\alpha_{1,n} = (\Delta N_{ss} - 1 + \delta_n)^{\frac{1}{\xi_n}}$ and $\alpha_{2,n} = \frac{1}{\xi_n - 1} (1 - \delta_n - \Delta N_{ss})$.}

Substituting the production technology into the inverse demand function yields the following expression for the nominal price for intermediate goods $i$

$$P_{i,t} = P_t Y_t^\beta \left[ K_{i,t}^{\alpha_{i,t}} (A_t N_{i,t}^\eta N_{i,t}^{1-\eta} L_{i,t})^{1-\alpha} \right]^{-\frac{1}{\beta}}$$

$$\equiv P_t J(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t)$$

Further, nominal revenues for intermediate firm $i$ can be expressed as

$$P_{i,t} X_{i,t} = P_t Y_t^\beta \left[ K_{i,t}^{\alpha_{i,t}} (A_t N_{i,t}^\eta N_{i,t}^{1-\eta} L_{i,t})^{1-\alpha} \right]^{-\frac{1}{\beta}}$$

$$\equiv P_t F(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t)$$

\textsuperscript{10} Specifically, $\alpha_{1,k} = (\Delta Z_{ss} - 1 + \delta_k)^{\frac{1}{\xi_k}}$ and $\alpha_{2,k} = \frac{1}{\xi_k - 1} (1 - \delta_k - \Delta Z_{ss})$.

\textsuperscript{11} Specifically, $\alpha_{1,n} = (\Delta N_{ss} - 1 + \delta_n)^{\frac{1}{\xi_n}}$ and $\alpha_{2,n} = \frac{1}{\xi_n - 1} (1 - \delta_n - \Delta N_{ss})$.\footnote{Specifically, $\alpha_{1,n} = (\Delta N_{ss} - 1 + \delta_n)^{\frac{1}{\xi_n}}$ and $\alpha_{2,n} = \frac{1}{\xi_n - 1} (1 - \delta_n - \Delta N_{ss})$.}
For the real revenue function $F(\cdot)$ to exhibit diminishing returns to scale in the factors $K_{i,t}$, $L_{i,t}$, and $N_{i,t}$ requires the following parameter restriction:

$$[\alpha + (\eta + 1)(1 - \alpha)] \left(1 - \frac{1}{\nu}\right) < 1$$

or

$$\eta(1 - \alpha)(\nu - 1) < 1$$

The intermediate firms face a cost of adjusting the nominal price a lá Rotemberg (1982), measured in terms of the final goods as

$$G(P_{i,t}, P_{i,t-1}; P_t, Y_t) = \frac{\phi_R}{2} \left(\frac{P_{i,t}}{\Pi_{ss}P_{i,t-1}} - 1\right)^2 Y_t$$

where $\Pi_{ss} \geq 1$ is the steady-state inflation rate and $\phi_R$ is the magnitude of the costs.

The source of funds constraint is

$$D_{i,t} = P_t F(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t) - W_{i,t} L_{i,t} - P_t I_{i,t} - P_t S_{i,t} - P_t G(P_{i,t}, P_{i,t-1}; P_t, Y_t)$$

where $D_{i,t}$ and $W_{i,t}$ are the nominal dividend and wage rate, respectively, for intermediate firm $i$. Firm $i$ takes the pricing kernel $M_t$ and the vector of aggregate states $\Upsilon_t = [P_t, K_t, N_t, Y_t, A_t]$ as exogenous and solves the following recursive program to maximize shareholder value $V_{i,t} = V(i)(\cdot)$

$$V(i)(P_{i,t-1}, K_{i,t}, N_{i,t}; \Upsilon_t) = \max_{P_{i,t}, I_{i,t}, S_{i,t}, K_{i,t+1}, N_{i,t+1}, L_{i,t}} \frac{D_{i,t}}{P_t} + E_t \left[M_{t+1} V(i)(P_{i,t}, K_{i,t+1}, N_{i,t+1}; \Upsilon_{t+1})\right]$$
subject to

\[
\frac{P_{i,t}}{P_t} = J(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t)
\]

\[
K_{i,t+1} = (1 - \delta_k) K_{i,t} + \Phi_k \left( \frac{I_{i,t}}{K_{i,t}} \right) K_{i,t}
\]

\[
N_{i,t+1} = (1 - \delta_n) N_{i,t} + \Phi_n \left( \frac{S_{i,t}}{N_{i,t}} \right) N_{i,t}
\]

\[
D_{i,t} = \mathcal{P}_t F(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t) - W_{i,t} L_{i,t} - \mathcal{P}_t I_{i,t} - \mathcal{P}_t S_{i,t} - \mathcal{P}_t G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}; \mathcal{P}_t, Y_t)
\]

The corresponding first order conditions are

\[
\Lambda_{i,t} = \phi_R \left( \frac{P_{i,t}}{\Pi_{ss} P_{i,t-1}} - 1 \right) \frac{Y_t}{\Pi_{ss} P_{i,t-1}} - E_t \left[ M_{t+1} \phi_R \left( \frac{P_{i,t+1}}{\Pi_{ss} P_{i,t}} - 1 \right) \frac{Y_{i+1} P_{i,t+1}}{\Pi_{ss} P_{i,t}^2} \right]
\]

\[
Q_{i,k,t} = \frac{1}{\phi'_{i,k,t}}
\]

\[
Q_{i,n,t} = \frac{1}{\phi'_{i,n,t}}
\]

\[
Q_{i,k,t} = E_t \left[ M_{t+1} \left\{ \frac{\alpha (1 - \frac{1}{\nu}) Y_{t+1}^{\frac{1}{\nu}} X_{i,t+1}^{1 - \frac{1}{\nu}}}{K_{i,t+1}} + \frac{\Lambda_{i,t+1} \left( \frac{\eta}{\nu} \right) Y_{t+1}^{\frac{1}{\nu}} X_{i,t+1}^{1 - \frac{1}{\nu}}}{K_{i,t+1}} \right\} \right]
\]

\[
+ E_t \left[ M_{t+1} Q_{i,k,t+1} \left( 1 - \delta_k - \frac{\phi'_{i,k,t+1} I_{i,t+1}}{K_{i,t+1}} + \Phi_{i,k,t+1} \right) \right]
\]

\[
Q_{i,n,t} = E_t \left[ M_{t+1} \left\{ \frac{\eta (1 - \alpha) (1 - \frac{1}{\nu}) Y_{t+1}^{\frac{1}{\nu}} X_{i,t+1}^{1 - \frac{1}{\nu}}}{N_{i,t+1}} + \frac{\Lambda_{i,t+1} \left( \frac{\eta (1 - \alpha)}{\nu} \right) Y_{t+1}^{\frac{1}{\nu}} X_{i,t+1}^{1 - \frac{1}{\nu}}}{N_{i,t+1}} \right\} \right]
\]

\[
+ E_t \left[ M_{t+1} Q_{i,n,t+1} \left( 1 - \delta_n - \frac{\phi'_{i,n,t+1} S_{i,t+1}}{N_{i,t+1}} + \Phi_{i,n,t+1} \right) \right]
\]

\[
\frac{W_{i,t}}{P_t} = \frac{(1 - \alpha) (1 - \frac{1}{\nu}) Y_{t}^{\frac{1}{\nu}} X_{i,t}^{1 - \frac{1}{\nu}}}{L_{i,t}} + \frac{\Lambda_{i,t} \left( \frac{1 - \alpha}{\nu} \right) Y_{t}^{\frac{1}{\nu}} X_{i,t}^{1 - \frac{1}{\nu}}}{L_{i,t}}
\]

where \( Q_{i,k,t}, Q_{i,n,t}, \) and \( \Lambda_{i,t} \) are the shadow values of physical capital, R&D capital
and price of intermediate goods, respectively.\(^\text{12}\)

**Central Bank** The central bank follows a modified Taylor rule specification that depends on the lagged interest rate and output and inflation deviations:

\[
\ln \left( \frac{R_{t+1}}{R_{ss}} \right) = \rho_r \ln \left( \frac{R_t}{R_{ss}} \right) + \rho_\pi \ln \left( \frac{\Pi_t}{\Pi_{ss}} \right) + \rho_y \ln \left( \frac{\bar{Y}_t}{Y_{ss}} \right) + \sigma_\xi \xi_t
\]

where \(R_{t+1}\) is the gross nominal short rate, \(\bar{Y}_t \equiv \frac{Y_t}{N_t}\) is detrended output, and \(\xi_t \sim N(0, 1)\) is a monetary policy shock. Variables with a \(ss\)-subscript denote steady-state values. Given this rule, the central bank chooses \(\rho_r, \rho_\pi, \rho_y, \text{ and } \Pi_{ss}\).

**Symmetric Equilibrium** In the symmetric equilibrium, all intermediate firms make identical decisions: \(P_{i,t} = P_t, X_{i,t} = X_t, K_{i,t} = K_t, L_{i,t} = L_t, N_{i,t} = N_t, I_{i,t} = I_t, S_{i,t} = S_t, D_{i,t} = D_t, V_{i,t} = V_t\). Also, \(B_t = 0\). The aggregate resource constraint is

\[
Y_t = C_t + S_t + I_t + \frac{\phi_R}{2} \left( \frac{\Pi_t}{\Pi_{ss}} - 1 \right)^2 Y_t
\]

where \(\Pi_t \equiv \frac{P_t}{P_{R-1}}\) is the gross inflation rate.

**Nominal Yields** The price of a \(n\)-period nominal bond \(P_t^{(n)}\) can be written recursively as:

\[
P_t^{(n)} = E_t \left[ M_{t+1} \frac{1}{\Pi_{t+1}} P_{t+1}^{(n-1)} \right]
\]

where \(P_t^{(0)} = 1\) and \(P_t^{(1)} = \frac{1}{R_{t+1}}\). The yield-to-maturity on the \(n\)-period nominal bond is defined as:

\[
y_t^{(n)} = -\frac{1}{n} \log \left( P_t^{(n)} \right)
\]

\(^{12}\) \(\Phi_{i,k,t} = \Phi_k \left( \frac{I_{i,t}}{N_{i,t}} \right), \Phi_{i,n,t} = \Phi_n \left( \frac{S_{i,t}}{N_{i,t}} \right), \Phi'_{i,k,t} = \alpha_{1,k} \left( \frac{I_{i,t}}{N_{i,t}} \right)^{-\frac{1}{\gamma_k}}, \Phi'_{i,n,t} = \alpha_{1,n} \left( \frac{S_{i,t}}{N_{i,t}} \right)^{-\frac{1}{\gamma_n}}\) are defined for notational convenience.
3.2.2 Exogenous Growth

This setup also nests a fairly standard New Keynesian setup with exogenous growth when the technological appropriability parameter $\eta$ is set to 0 and the aggregate stock of R&D $N_t$ is exogenously specified. Under these conditions the production function of the intermediate firm can be expressed as

$$X_{i,t} = K_{i,t}^{\alpha}(Z_{i,t}L_{i,t})^{1-\alpha}$$

$$Z_t = A_t N_t$$

where $N_t$ follows a stochastic process. Note that TFP is now exogenous and comprised of a stationary component $A_t$ and a trend component $N_t$. I consider two versions of the exogenous growth model, one with a deterministic trend and the other with a stochastic trend in productivity, to compare with the benchmark endogenous growth model.

Deterministic Trend  The law of motion for $N_t$ is

$$N_t = e^{\mu t}$$

where $\mu$ is parameter governing the average growth rate of the economy. Equivalently, this expression can be rewritten in log first differences as

$$\Delta n_t = \mu$$

where $\Delta n_t \equiv \ln(N_t) - \ln(N_{t-1})$.

Stochastic Trend  The log growth rate of $N_t$ is specified as

$$\Delta n_t = \mu + x_{t-1}$$

$$x_t = \rho_x x_{t-1} + \sigma_x \epsilon_{x,t}$$

where $\epsilon_t, \epsilon_{x,t} \sim iid \ N(0, 1)$, $corr(\epsilon_t, \epsilon_{x,t}) = 0$, and $\rho_x$ is the persistence parameter of the autoregressive process $x_t$. 

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3.3 Equilibrium Growth and Inflation

This section will provide a qualitative description of the growth and inflation dynamics. Sustained growth in the benchmark endogenous growth model is attributed to the R&D decisions of intermediate firms. In a stochastic setting, this framework generates (i) low-frequency movements in the growth rate of real and nominal variables and (ii) a negative long-run correlation between aggregate growth rates and inflation. These long-run dynamics are essential for explaining asset market data. In contrast, a standard model with exogenous growth lacks a strong propagation mechanism that can generate low-frequency movements in growth rates. While incorporating exogenous low-frequency shocks into productivity growth in the neoclassical model will lead to low-frequency cycles in aggregate growth rates, this shock will consequently drive a strong positive relation between expected consumption growth and expected inflation. Furthermore, the endogenous growth paradigm allows monetary policy to play an important role in influencing long-run growth dynamics, and hence risk. This section describes the equilibrium growth and inflation dynamics qualitatively, and contrast them with the exogenous growth setups described above. The next section presents empirical evidence supporting these patterns and provides a quantitative assessment of the model.

**Growth Dynamics** First, I will characterize the equilibrium growth dynamics. In the benchmark endogenous growth model, substituting in the symmetric equilibrium conditions yields the following the aggregate production function

\[
Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha}
\]

\[
Z_t = A_t N_t
\]

Recall that the trend component, \(N_t\), is the equilibrium stock of aggregate R&D capital in this setting and endogenously determined by the intermediate firms. In
other words, the benchmark model generates an endogenous stochastic trend. Furthermore, log productivity growth is

$$\Delta z_t = \Delta a_t + \Delta n_t$$

Since, $a_t$ is typically calibrated to be persistent shock, then $\Delta a_t \approx \epsilon_t$. Using this approximation we can rewrite the expression above as

$$\Delta z_t = \Delta n_t + \epsilon_t$$

where Also, since $\Delta n_t$ is determined at $t - 1$, the expected growth rate is equal to the stock of R&D growth:

$$E_{t-1}[\Delta z_t] = \Delta n_t$$

Thus, low-frequency movements are driven by endogenous R&D rates, which are fairly persistent and volatile processes in the data. Furthermore, the propagation mechanism of endogenous growth framework implies that the single shock $a_t$ will generate both high-frequency movements and low-frequency movements. In contrast, in a standard neoclassical paradigm with a deterministic trend, log productivity growth is

$$\Delta z_t = \mu + \epsilon_t$$

where again, the approximation $\Delta a_t \approx \epsilon_t$ is used. Hence, the expected growth rate is constant:

$$E_{t-1}[\Delta z_t] = \mu$$

Hence, the exogenous growth model will only exhibit high-frequency movements around the trend. In the exogenous growth model with a stochastic trend, productivity growth is

$$\Delta z_t = \mu + x_{t-1} + \epsilon_t$$
where again, the approximation $\Delta a_t \approx \epsilon_t$ is used. Thus the expected growth rate is equal to the drift plus a persistent shock $x_{t-1}$, often referred to in the literature as "long-run productivity risk."\(^\text{13}\)

$$E_{t-1}[\Delta z_t] = \mu + x_{t-1}$$

Thus, the endogenous growth paradigm provides a structural interpretation for this low-frequency component by linking it to innovation rates.\(^\text{14}\) Importantly, in the endogenous growth framework, the monetary authority can affect this low-frequency component, or in other words, alter the distribution of long-run productivity risk. As will be shown in the next section, monetary policy can significantly alter risk premia through this channel.

**Inflation Dynamics** In the models above, the log-linearized inflation dynamics depend on real marginal costs and expected inflation:

$$\tilde{\pi}_t = \gamma_1 \tilde{mc}_t + \gamma_2 E_t[\tilde{\pi}_{t+1}]$$

where $\gamma_1 = \frac{\nu - 1}{\phi_R} > 0$, $\gamma_2 = \beta \Delta Y_{ss}^{1 - \frac{1}{\psi}} > 0$, and lowercase tilde variables denote log deviations from the steady-state.\(^\text{15}\) Recursively substituting out future $\tilde{\pi}$ terms implies that current and expected future real marginal costs drive inflation dynamics. Moreover, real marginal cost can be expressed as the ratio between real wages and the marginal product of labor. Furthermore, real marginal costs, in log-linearized form, can be expressed as:

$$\tilde{mc}_t = \tilde{w}_t + \tilde{\alpha}t - (1 - \alpha)\tilde{\alpha}_t - (1 - \alpha)\tilde{n}_t$$

\(^\text{13}\) See Croce (2010).

\(^\text{14}\) Kung and Schmid (2011) also highlight this mechanism in explaining the equity premium.

\(^\text{15}\) See appendix for details.
where lowercase tilde variables denote log deviations from the steady-state. Thus, inflation is driven by the relative dynamics of these aggregate variables. In the endogenous growth model, after a good productivity shock, $\tilde{w}_t$, $\tilde{l}_t$, and $\tilde{n}_t$ all increase after an increase in $\tilde{a}_t$. Notably, in a calibrated version of the benchmark model, the magnitude of the responses of last two terms in the equation are larger than that of the first two terms. Consequently, marginal costs and inflation decrease persistently after a positive productivity shock. On the other hand, as discussed above, expected growth rates increase persistently after a positive productivity shock. Thus, expected inflation and expected growth rates have a strong negative relationship in the benchmark model, as in the data. These dynamics will be examined further in the section below.

### 3.4 Quantitative Results

This section explores the quantitative implications of the model using simulations. Perturbation methods are used to solve the model. To account for risk premia and potential time variation, a higher-order approximation around the stochastic steady state is used. Furthermore, ENDO 1 will refer to the benchmark endogenous growth model, ENDO 2 is the same as ENDO 1 but with no policy uncertainty, EXO 1 will refer to the exogenous growth model with a deterministic trend, and EXO 2 will refer to the exogenous growth model with a stochastic trend.

#### 3.4.1 Calibration

This part presents the quarterly calibration used to assess the quantitative implications of the benchmark growth model (ENDO 1). Table 3.1 reports the calibration of the benchmark model along with the other three models that are used for comparison purposes. Worth emphasizing, the core set of results are robust to reasonable deviations around the benchmark calibration. Recursive preferences have been used
extensively in recent work in asset pricing.\textsuperscript{16} I follow this literature and set preference parameters to standard values that are also supported empirically.\textsuperscript{17} Standard parameters in the final goods and intermediate goods sector are set to standard values in the New Keynesian DSGE literature. Non-standard parameters in the intermediate goods sector are used to match R&D dynamics. Critically, satisfying balanced growth helps provide further restrictions on parameter values.

I begin with a description of the calibration of the preference parameters. The parameter $\tau$ is set to match the steady-state household hours worked. The elasticity of intertemporal substitution $\psi$ is set to value of 2 and the coefficient of relative risk aversion $\gamma$ is set to a value of 10, which are standard values in the long-run risks literature. An intertemporal elasticity of substitution larger than one is consistent with the notion that an increase in uncertainty lowers the price-dividend ratio. Note that in this parametrization, $\psi > \frac{1}{\gamma}$, which implies that the agent dislikes shocks to expected growth rates and is particularly important for generating a sizeable risk premium in this setting. The subjective discount factor $\beta$ is set to a value of 0.9963 so as to be broadly consistent with the level of the riskfree rate. In the endogenous growth setting $\beta$ also has important effect on the level of the growth rate. In particular, increasing $\beta$ (the agent is more patient) increases the steady-state growth rate. Holding all else constant (including $\beta$), the direct effect of an increase in growth is an increase in the level of the riskfree rate. On the other hand, the direct effect of increasing $\beta$ and holding the level of the growth rate fixed is a decrease in the level of the riskfree rate.

I now move to the calibration of the standard parameters from the production-side. In the final goods sector, the price elasticity of demand is set at 6, which

\textsuperscript{16} See Bansal and Yaron (2004).

\textsuperscript{17} See Bansal et al. (2007) uses Euler conditions and a GMM estimator to provide empirical support for the parameter values.
corresponds to a markup of 0.2. In the intermediate goods sector, the capital share \( \alpha \) is set to 0.33 and the depreciation rate of capital \( \delta \) is set to 0.02, which are calibrated to match steady-state evidence. The quadratic price adjustment function was first proposed in Rotemberg (1982) and is standard in the literature. The price adjustment cost parameter \( \phi_R \) is set to 70 and is calibrated to match the impulse response of output to a monetary policy shock.\(^{18}\) Table 3.11 provides sensitivity analysis for \( \phi_R \) and shows that the core results hold for a wide range of values used in the literature. The capital adjustment cost function is standard in the production-based asset pricing literature.\(^{19}\) The capital adjustment cost parameter \( \zeta_k \) is set at 7.8 to match the relative volatility of investment growth to consumption growth.

The nonstandard parameters are now discussed. The depreciation rate of the R&D capital stock \( \delta_n \) is calibrated to a value of 0.0375 which corresponds to an annualized depreciation rate of 15% which is a standard value and that assumed by the BLS in the R&D stock calculations. The R&D capital adjustment cost parameter \( \zeta_n \) is set at 4.6 to match the relative volatility of R&D investment growth to consumption growth. The degree of technological appropriability \( \eta \) is set at 0.1 to match the steady-state value of the R&D investment rate.

I now turn to the calibration of the parameters relating to the stationary productivity shock \( a_t \). Note that this shock is different than the Solow residual since measured productivity includes an endogenous component that is related to the equilibrium stock of R&D. A decomposition of total factor productivity in our benchmark model is provided below, which provides a mapping between the exogenous growth model and the endogenous growth model. The persistence parameter \( \rho \) is set to

\(^{18}\) This value is also consistent with structural estimation evidence from Ireland (2001). In a log-linear approximation, this parameter corresponds to an average price duration of 4.3 quarters in the Calvo-pricing framework.

\(^{19}\) See, for example, Jermann (1998), Croce (2008), Kaltenbrunner and Lochstoer (2008) or van Binsbergen et al. (2011) for estimation evidence.
0.985 and is calibrated to match the first autocorrelation of R&D intensity, which determines the growth rate of the R&D stock (expected growth component) and in turn, is a critical determinant of asset prices.\(^{20}\) The volatility parameter \(\sigma\) is set at 1.36\% to match consumption growth volatility. The constant determining the mean of the process \(a^*\) is set to match balanced growth evidence.

The monetary policy rule parameters are within the standard range of estimated values in the literature.\(^{21}\) The parameter governing the sensitivity of the interest rate to inflation deviations \(\rho_\pi\) is set to 1.5. The parameter governing the sensitivity of the interest rate to output deviations is set to 0.16. The volatility parameter \(\sigma_{xz}\) is set to 0.3\%.\(^{22}\) The parameter that determines the steady-state value of inflation \(\Pi_{ss}\) is set to match the average level of inflation in the data.

I now turn to the calibration of the other three models for which the benchmark model is compared to. ENDO 2 is the same as ENDO 1 but with no policy uncertainty \(\sigma_\xi = 0\). Thus in ENDO 2, there is only one exogenous shock. In the two exogenous growth models, the common parameters with the growth model are kept the same to facilitate a direct comparison. However, since the trend component is exogenous in those models, this will entail additional parameters governing the exogenous dynamics of the trend. In the the exogenous growth model with a stochastic trend EXO 2, the process for the low-frequency productivity growth shock is calibrated so that the expected productivity growth component matches key features of the endogenously expected growth component in EXO 1. In particular, the persistence parameter \(\rho_x\) is set to match the first autocorrelation of \(E[\Delta z_t]\) with that of

\(^{20}\) To provide further discipline on the calibration of \(\rho\), note that since the ENDO models imply the TFP decomposition, \(\Delta z_t = \Delta a_t + \Delta n_t\), we can project log TFP growth on log growth of the R&D stock to back out the residual \(\Delta a_t\). The autocorrelations of the extracted residual \(\Delta \hat{a}_t\) show that we cannot reject that it is white noise. Hence, in levels, it must be the case that \(a_t\) is a persistent process to be consistent with this empirical evidence. In our benchmark calibration, the annualized value of \(\rho\) is .94.

\(^{21}\) See, for example, Clarida et al. (2000) and Rudebusch (2002).

\(^{22}\) See, for example, Smets and Wouters (2007).
ENDO 1. The volatility parameter \( \sigma_x \) is set to match the volatility of \( E[\Delta z_t] \) with that of ENDO 1. The parameter governing the average level of the trend \( \mu \) is set at 0.55% to match the average level of output growth. EXO 1 is the same as EXO 2 except that \( \sigma_x \) is set to 0 so that the trend is deterministic, as typically assumed in the literature.

### 3.4.2 Macroeconomic Dynamics

Table 3.2 reports the key macroeconomic moments. Note that the benchmark growth model (ENDO 1) closely fits key business cycle moments. Worth noting, the model is able to match investment volatilities, which is a challenging feature in the data to explain jointly with high risk premia in standard production-based models with recursive preferences. In particular, when IES is high enough, small amounts of capital adjustment costs discourage investment volatility. However, for the same reason, small nominal interest rate shocks that induce movements in the real rate due to nominal rigidities, lead to strong incentives to adjust labor and capital inputs. Indeed, comparing ENDO 1 with the benchmark model without interest rate shocks (ENDO 2), one can see that the volatility of log hours growth, physical investment growth, and R&D investment growth are substantially larger in ENDO 1. Thus, incorporating standard New Keynesian features, nominal rigidities and policy uncertainty, help alleviate a long-standing problem in standard real business cycle (RBC) models with recursive preferences.

At business cycle frequencies, the benchmark growth model performs at least as well as the exogenous growth counterparts, EXO 1 and EXO 2. This can be seen by comparing ENDO 1 with the last columns of Table 3.2. At low-frequencies, the benchmark growth model generates substantial endogenous long-run uncertainty in aggregate growth rates, which is reflected in the sizeable volatility of expected

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23 See, for example, Croce (2008) and Kaltenbrunner and Lochstoer (2008).
productivity growth. This is reported in Table 3.3. When firms receive a positive productivity shock, they increase the levels of inputs persistently, including R&D. As with standard business cycle models, increasing the level of inputs will lead to persistent cyclical movements around the trend. However, the persistent increase in the R&D input will also generate persistence in productivity growth, as discussed in Section 3. Naturally, other aggregate growth rates, such as consumption, dividends, and output, will inherit these long-run productivity dynamics. Hence, the model generates both high- and low-frequency movements in quantities with a single productivity shock. Empirically, this mechanism suggests that measures related to innovation should have forecasting ability for aggregate growth rates. This is verified in tables 3.6 and 3.7, which report results from projecting future consumption growth and productivity growth over various horizons on the growth rate of the R&D capital stock. In the data, the growth rate of the R&D stock predicts future consumption and productivity growth over horizons up to 4 years, with significant point estimates, and $R^2$’s are increasing with the horizon. Qualitatively, the model reflects these patterns reasonably well. This gives empirical support to the notion of innovation-driven low-frequency variation in growth rates. Furthermore, these low-frequency growth dynamics are crucial for asset prices, which are discussed in detail in the section below.

In contrast, in the exogenous growth model with a deterministic trend (EXO 1), expected productivity growth is approximately constant, as shown in section 3. Figure 3.3 plots the dynamic response of expected growth rates to a one standard deviation shock to productivity for both the endogenous and exogenous growth models. Table 3.9 further emphasizes this point by reporting the low-frequency volatilities of consumption, output, investment, and labor hours growth for ENDO 1 and EXO 1. The low-frequency component is identified using a bandpass filter with a bandwidth
of 32 to 200 quarters.\textsuperscript{24} Thus, incorporating expected growth shocks is needed to generate significant low-frequency growth dynamics in the exogenous growth paradigm.

EXO 2 adds a persistent expected productivity growth shock.\textsuperscript{25} While adding these shocks do allow for significant low-frequency movements in aggregate growth rates, the dynamic responses of aggregate quantities to these expected growth shocks imply a very different relationship between trend and cycle components than in the endogenous growth framework. Notably, this difference will imply very different implications for asset prices, which will be discussed below. In the endogenous growth models, the endogenous trend and cycle components share a strong positive relationship: Increases in the level of inputs translate to both increases in the level and growth rate of output. Increases in factor inputs will directly raise the level of output. In addition, increases in labor and physical capital inputs will raise the marginal product of R&D while an increase in R&D will directly raise productivity growth. Thus, labor and physical capital will have a long-run impact on growth by affecting the incentives to innovate. To contrast, in the exogenous growth model, factor inputs only affect the cycle but not the trend. Consequently, in response to a good productivity shock, firms in the endogenous growth model will increase factor inputs more aggressively than in the exogenous growth model, which is illustrated in Figure 3.2. In the exogenous growth model with a stochastic trend (EXO 2), a long-run growth shock will generate a very large wealth effect that increases consumption of goods and leisure initially. The fall in labor supply leads to an initial drop in labor hours and thus, the level of output. An increase in consumption and a fall in output then require investment in physical capital to fall. Given the strong desire for leisure, there is an initial rise in wages to maintain the equilibrium levels

\textsuperscript{24} Specifically, I use the bandpass filter from Christiano and Fitzgerald (2003).

\textsuperscript{25} These shocks are often referred to as “news shocks” in the macro literature. See, for example, Cochrane (1994), Beaudry and Portier (2006), and Jaimovich and Rebelo (2009). In essence, the endogenous growth model links these “news shocks” to equilibrium innovation decisions.
of output and consumption. These responses are depicted in Figure 3.4. Thus, in EXO 2, the trend and cycle components are negatively related; in other words, good news about future productivity leads to bad news today, which goes against empirical evidence.\textsuperscript{26} In contrast, the economic forces of the endogenous growth model naturally generate a positive link between low-frequency movements in productivity growth and short-run macroeconomic aggregates.

### 3.4.3 Asset Prices

This section discusses the asset pricing implications of the model, which critically hinges on the low-frequency dynamics of consumption growth and inflation that was outlined in the previous section. Since it is assumed that the agent has Epstein-Zin utility with a preference for an early resolution of uncertainty, this implies that not only are innovations to realized consumption and dividend growth priced, but also innovations to expected consumption and dividend growth. Table 3.4 reports the first and second moments of the risk-free rate, equity returns, and nominal yields. Quantitatively, the benchmark growth model is broadly consistent with the financial moments from the data. Also, note that comparing ENDO 1 with ENDO 2 demonstrates that the policy shock has a negligible impact on asset prices in this framework, because, as documented above, these shocks do not have a significant effect on the intertemporal distribution of risk, and primarily effect short-run investment and labor fluctuations. This highlights the importance of productivity shocks in driving low-frequency dynamics (through the endogenous growth mechanism), which in turn is reflected in asset prices. Remarkably, the core results and rich dynamics of the benchmark growth model are generated with a single shock.

I first begin with a discussion of the model implications for real risks. Notably,

\textsuperscript{26} See, for example, Beaudry and Portier (2006), Beaudry and Lucke (2010), and Kurmann and Otrok (2011).
the benchmark growth model generates a low and smooth risk-free rate and a sizeable equity premium. The equity premium is just under 4%. The volatility of the equity premium is 5.4%, which is a little over one-third of the historical volatility of the market excess return. Since the model is productivity-based, this number can be thought of as the productivity-driven fraction of historical excess return volatility. On the other hand, it is well known that dividend-specific shocks explain a good portion of stock return volatility. In particular, Ai, Croce and Li (2010) report that empirically the productivity-driven fraction of return volatility is around 6%, which is consistent with this quantitative finding. To understand these results, it is useful to compare the benchmark endogenous growth model (ENDO 1) with the exogenous growth model with a deterministic trend (EXO 1). First note that while the two models have similar business cycle statistics, in EXO 1 the equity premium is close to zero and the risk-free rate is counterfactually high. This stark contrast between the two models is due to the fact that these two paradigms generate very different low-frequency growth dynamics, as described above. In particular, the strong propagation mechanism of the endogenous growth model generates substantial long-run uncertainty in aggregate growth rates while the EXO 1 model does not. As households with recursive preferences are very averse to uncertainty about long-run growth prospects, this implies that households have a much higher precautionary savings motive in the endogenous growth setting. In equilibrium, this leads to lower real interest rates in ENDO 1 than in EXO 1. Moreover, ENDO 1 also generates a substantial equity premium, which is due to aggregate growth rates, including consumption and dividends, naturally inheriting the innovation-driven low-frequency dynamics of productivity growth. Thus, the dividend claim is very risky, which is reflected in the sizeable equity premium.

EXO 2 incorporates exogenous long-run uncertainty through productivity growth, where the calibration of this shock is set so that it replicates the volatility and
persistence of the expected productivity growth component in ENDO 1, which is reported in Table 3.3. In essence, incorporating this shock is a reduced-form way of capturing long-run uncertainty in productivity growth. Evidently, incorporating this shock helps the exogenous growth paradigm generate a larger equity premium and lower risk-free rate than in EXO 1. However, even though the expected productivity dynamics share very similar properties, note that the equity premium in EXO 2 is smaller than in ENDO 1 and the risk-free rate is larger than ENDO 1. This difference is due to the observation above that the relationship between cycle and trend is very different between ENDO 1 and EXO 2. In ENDO 1, recall that short-run and long-run fluctuations are strongly positively correlated so that these shocks reinforce each other. In EXO 2, recall that short-run and long-run fluctuations are negatively related, and therefore hedge each other. Furthermore, the ENDO 1 and EXO 2 will have drastically different implications for nominal yields, which is discussed below.

Now I turn to the model implications for nominal yields, which are also reported in Table 3.4. Note that the benchmark growth model (ENDO 1) closely matches both the means and volatilities of nominal yields for maturities of 4, 8, 12, 16, and 20 quarters in the data. Also, the average 20 quarter minus 1 quarter yield spread is a little over 1% as in the data. In contrast, the yield spread is close to zero in the exogenous growth model with a deterministic trend (EXO 1) and negative in the exogenous growth model with a stochastic trend (EXO 2). These results are intimately linked to the long-run co-movement between inflation and consumption growth. In particular, long bonds are riskier than short bonds if they have lower expected real payoffs (expected inflation is high) in states where marginal utility is high (expected consumption growth is low). Thus, in this setting where the agent has recursive preferences, expected inflation and expected consumption growth need to be negatively related for the models to produce an upward-sloping average nominal
yield curve and sizeable term spread.\footnote{Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2009) highlight this point in endowment economy setups where inflation and consumption growth are exogenously specified and assumed to be negatively correlated. Furthermore, they find strong empirical evidence for this negative relationship.}

I first begin with a discussion of the equilibrium inflation and growth dynamics. Note that in the bottom row of Figure 3.1 the benchmark model (ENDO 1) replicates the negative low-frequency patterns in consumption growth and inflation from the data. As before, the low-frequency component is identified using a bandpass filter, where the bandwidth is from 32 to 200 quarters. Indeed, Table 3.10 shows that the ENDO 1 closely matches low-frequency correlations between macro growth rates and inflation. Furthermore, expected consumption growth and expected inflation are strongly negatively correlated. To understand the mechanics behind these endogenous long-run dynamics, it is instructive to look at the impulse response functions from Figure 3.6. First, I will focus on the IRFs for ENDO 1 which correspond to the solid lines. When a good productivity shock is realized, expected growth rates, including consumption growth, increase persistently as discussed above. Now, I will also show that expected inflation declines persistently in response to a good productivity shock. Recall from Section 3, inflation depends on positively on the wage to capital ratio and labor hours worked, and negatively on the productivity shock and the R&D stock to physical capital stock ratio:

\[
\tilde{\pi}_t = \gamma_1 \tilde{mc}_t + \gamma_2 E_t[\tilde{\pi}_{t+1}]
\]
\[
\tilde{mc}_t = \tilde{w}_t + \alpha \tilde{l}_t - (1 - \alpha) \tilde{a}_t - (1 - \alpha) \tilde{n}_t
\]

where \(\gamma_1 = \frac{\nu - 1}{\phi R} > 0\), \(\gamma_2 = \beta \Delta Y_{ss}^{1-\frac{1}{\psi}} > 0\), and lowercase tilde variables denote log deviations from the steady-state. As discussed above, \(\tilde{w}_t\) and \(\tilde{l}_t\) increase in response to an increase in \(\tilde{a}_t\). In addition, because the firm has very high incentives to take advantage of the good shock to increase long-run growth prospects, the stock of R&D
increases more relative to the increase in physical capital. Thus, \( \tilde{n}_t \) also increases. Quantitatively, the increase in the terms \((1 - \alpha)\tilde{a}_t\) and \((1 - \alpha)\tilde{n}_t\) dominates the increase in \( \tilde{w}_t \) and \( \alpha\tilde{n}_t \), so that marginal costs, and therefore inflation declines. In sum, the productivity shock drives a strong negative relationship between expected consumption growth and expected inflation. Moreover, these dynamics are reflected in asset markets by an upward sloping yield curve and sizeable term spread.

Importantly, there is strong empirical support for these model-implied inflation-growth dynamics. Barksy and Sims (2009) and Kurmann and Otrok (2011) show in a VAR that a positive shock to expected productivity growth (“news shock”) leads to large and persistent decline in inflation. In the benchmark growth model, fluctuations to expected productivity growth are driven by R&D rates. Specifically, in the benchmark growth model, the expected productivity growth component is the growth rate of the stock of R&D. As discussed above, a persistent increase in R&D during good times lowers marginal costs and inflation persistently. Indeed, in the data innovation rates and inflation share a strong negative relationship, as predicted by the model. The top left plot of Figure 3.1 provides visual evidence of the negative long-run relationship between R&D and inflation. The benchmark model exhibits similar patterns which can be seen in the top right plot of Figure 3.1. As before, the low-frequency component is identified using a bandpass filter, where the bandwidth is from 32 to 200 quarters. Table 3.10 corroborates the visual evidence by showing that the long-run correlations between inflation and macro growth rates, including the R&D stock and measured productivity, are indeed strongly negative in the data and the benchmark model. Furthermore, the model predicts that measures related to innovation should forecast inflation rates with negative loadings on the innovation variable. This is verified in Table 3.8, which reports the results from projecting future inflation rates on the growth rate of the R&D stock for horizons of one to four
years. In the data, the $R^2$ values are sizeable and the point estimates are negative and statistically significant. The forecasting regressions from the model correspond to population values. Qualitatively, the model reflects these features reasonably well.

The negative long-run relationship between expected growth and inflation rates is also crucial for reconciling the empirical observation that increases in the term spread predict higher future economic growth.\footnote{See, for example, Ang et al. (2006) for empirical evidence that find that term spread forecasts future growth.} In the benchmark model, a positive productivity shock leads to a persistent increase in expected growth and a persistent decline in inflation. Given that the monetary authority is assumed to follow a Taylor rule and aggressively responds to inflation deviations, a persistent fall in inflation leads to sharp and persistent drop in the short-term nominal rate. Consequently, the slope of the nominal yield spread becomes steeper.\footnote{Kurmann and Otrok (2011) provide empirical support for this mechanism.} Figure 3.5 verifies this intuition by showing that a positive productivity shock leads to a persistent increase in the yield spread, where the solid line corresponds to the benchmark growth model. In sum, the model predicts that a rise in the slope of the yield curve is associated with an increase in future growth rates. This is verified in table 3.5, which reports consumption growth forecasts with the 20 quarter yield spread. In the data, the $R^2$ values are sizeable and the point estimates are positive and statistically significant. The forecasting regressions from the model correspond to population values. In particular, the regressions from ENDO 1 produces $R^2$ that are of similar magnitude as the ones from the data and positive point estimates.

To highlight the importance of the endogenous growth mechanism for explaining the term structure, it is useful to compare the benchmark model (ENDO 1) with the exogenous growth models EXO 1 (deterministic trend) and EXO 2 (stochastic trend). In EXO 1, the average nominal yield curve is upward sloping, however, the slope is
counterfactually small, as reported in Table 3.4. This shortcoming inherently related to the inability of the model to generate a sizeable equity premium. Namely, the model lacks an strong propagation mechanism that generates quantitatively sufficient long-run consumption uncertainty. Figure 3.6 highlights this point. Note that the dashed lines in Figure 3.6 correspond to the impulse response functions for EXO 1 in response to a productivity shock. Expected inflation and expected consumption growth are negatively related, which drive the upward sloping yield curve. While expected productivity growth is close to constant, consumption smoothing will drive persistence in consumption growth. However, this channel is quantitatively small, which can be readily seen in the impulse response for expected consumption growth and comparing it with the the response from ENDO 1.

Interestingly, incorporating exogenous long-run uncertainty by adding a persistent expected productivity growth shock, as in EXO 2, makes the slope of the nominal yield curve negative. This counterfactual implication implies that incorporating this shock to the exogenous growth framework leads to a positive relationship between expected consumption growth and expected inflation.\textsuperscript{30} In particular, a positive growth shock has a large wealth effect that reduces the incentives to work. Thus, wages need to rise sharply and persistently to induce households to supply labor in order to maintain the equilibrium level of consumption and output. Quantitatively, the large and persistent rise in wages along with the eventual increase in labor hours eventually dominates the increase in the trend-capital ratio. Thus, marginal costs and inflation eventually increase. These dynamics are depicted in Figure 3.7.

The positive relationship between expected consumption growth and inflation in EXO 2 also implies that a decline in the slope of the yield curve forecasts higher future economic growth, which is counterfactual. A positive long-run growth shock increases expected consumption growth and inflation. A persistent increase in inflation leads

\textsuperscript{30} This is also documented in Rudebusch and Swanson (2012).
to a sharp and persistent increase in the short-term nominal rate. Consequently, the
slope of the nominal yield spread decreases. This intuition is verified in table 3.5,
which reports consumption growth forecasts with the 20 quarter yield spread. In the
data, the $R^2$ values are sizeable and the point estimates are positive and statistically
significant. The forecasting regressions from the model correspond to population
values. In particular, the regressions from EXO produce negative point estimates.
In sum, the endogenous growth margin is critical for reconciling nominal bond data.

3.4.4 Policy Experiments

While monetary policy shocks have a negligible impact on long-run growth dynamics
and asset prices, this section demonstrates that changing the policy parameters can
have a large quantitative impact on the intertemporal distribution of risk in the
benchmark growth model. Specifically, changing the policy parameters alters the
transmission of the productivity shock. This section explores the effects of varying the
intensity of inflation and output stabilization on asset prices. In the model, monetary
policy is characterized by a short-term nominal interest rate rule that responds to
current inflation and output deviations. Due to the presence of nominal rigidities,
changes in the short rate affect the real rate which alters real decisions, including
R&D. Thus, monetary policy can influence trend growth dynamics. Moreover, by
varying the policy parameters, even such short-run stabilization policies can have
substantial effects on the level and dynamics of long-run growth, which in turn have
important implications for real and nominal risks.

Figure 3.8 reports the effects of varying the policy parameter $\rho_y$, where a larger
value means a more aggressive stance on output stabilization. In particular, more
aggressive output stabilization implies that the short rate, and thus the real rate, will
increase more in response to an increase in output. Since R&D rates are procyclical,
a larger rise in the real rate will dampen the increase in R&D more and lower the
volatility of expected growth rates. With less uncertainty about long-run growth prospects, risk premia would decline. On the other hand, more aggressive output stabilization amplifies the volatility of expected inflation. Since expected inflation is countercyclical, a sharper rise in the real rate depresses expected inflation even further. Greater uncertainty about expected inflation makes long bonds even riskier which increases slope of the nominal yield curve.

Figure 3.9 reports the effects of varying the policy parameter $\rho_\pi$, where a larger value means a more aggressive stance on inflation stabilization. In particular, more aggressive inflation stabilization implies that the short rate, and thus the real rate, will increase more in response to an increase in inflation. Since inflation and R&D rates are negatively correlated, a larger rise in the real rate will further depress R&D and thus, amplify R&D rates. Furthermore, more volatile R&D rates imply that expected growth rates are more volatile. Higher uncertainty about long-term growth prospects therefore increases risk premia. Additionally, more aggressive inflation stabilization will naturally smooth expected inflation which lowers the slope of the nominal yield curve. Thus, inflation and output stabilization have opposite effects on asset markets. In short, these results suggest that monetary policy, even when targeting short-run deviations, can have a substantial impact on asset markets by distorting long-run growth and inflation dynamics.

3.5 Conclusion

This paper examines the nominal yield curve implied by a stochastic endogenous growth model with imperfect price adjustment. In good times when productivity is high, firms increase R&D, which raises expected growth rates. In addition, the increase in productivity and R&D lowers marginal costs persistently. As firms face downward sloping demand curves, a fall in marginal costs leads firms to lower prices, which in the aggregate, leads to a persistent decline in inflation. Thus, the model en-
dogenously generates low-frequency movements in productivity growth and inflation that are negatively related. From the perspective of a bondholder, these dynamics imply that long bonds have lower expected payoffs than short bonds when long-run growth prospects are expected to be grim. When households have recursive preferences, this implies that long bonds have particularly low payoffs when marginal utility is high. As a result, the model generates an upward sloping average nominal yield curve and sizeable term spread. In addition, when the monetary authority follows a Taylor rule, the negative relationship between expected growth and inflation implies that a rise in the slope of the yield curve predicts higher future growth.

More broadly, this paper offers a unified framework to study macroeconomics and asset pricing. Notably, incorporating the endogenous growth margin with assumption of recursive preferences into a standard New Keynesian DSGE framework allows this class of models to explain a wide array of stylized facts in asset pricing. From a macroeconomic perspective, the endogenous growth mechanism allows these models rationalize both high- and low-frequency dynamics of aggregate variables. From a production-based asset pricing perspective, incorporating sticky prices and nominal interest rate shocks allow these models to explain the observed high investment volatility.
Table 3.1: Quarterly Calibration

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<th>EXO 1</th>
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Table 3.2: Macroeconomic Moments

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<th>EXO 1</th>
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<td>0.60</td>
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Table 3.3: Expected Productivity Growth Dynamics

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Table 3.4: Asset Pricing Moments

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<td>$E[r^{*}_d - r_f]$</td>
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<td>5.33%</td>
<td>6.24%</td>
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Table 3.5: Consumption Growth Forecasts with 20Q Yield Spread

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85
Table 3.6: Productivity Growth Forecasts with R&D Growth

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<tr>
<td></td>
<td>β</td>
<td>S.E.</td>
</tr>
<tr>
<td>1</td>
<td>0.431</td>
<td>0.190</td>
</tr>
<tr>
<td>2</td>
<td>0.820</td>
<td>0.315</td>
</tr>
<tr>
<td>3</td>
<td>1.230</td>
<td>0.452</td>
</tr>
<tr>
<td>4</td>
<td>1.707</td>
<td>0.522</td>
</tr>
</tbody>
</table>

Table 3.7: Consumption Growth Forecasts with R&D Growth

<table>
<thead>
<tr>
<th>Horizon (k)</th>
<th>Data</th>
<th>ENDO 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β</td>
<td>S.E.</td>
</tr>
<tr>
<td>1</td>
<td>0.217</td>
<td>0.084</td>
</tr>
<tr>
<td>2</td>
<td>0.395</td>
<td>0.178</td>
</tr>
<tr>
<td>3</td>
<td>0.540</td>
<td>0.276</td>
</tr>
<tr>
<td>4</td>
<td>0.703</td>
<td>0.347</td>
</tr>
</tbody>
</table>

Table 3.8: Inflation Forecasts with R&D Growth

<table>
<thead>
<tr>
<th>Horizon (k)</th>
<th>Data</th>
<th>ENDO 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β</td>
<td>S.E.</td>
</tr>
<tr>
<td>1</td>
<td>-0.543</td>
<td>0.168</td>
</tr>
<tr>
<td>2</td>
<td>-1.065</td>
<td>0.409</td>
</tr>
<tr>
<td>3</td>
<td>-1.560</td>
<td>0.671</td>
</tr>
<tr>
<td>4</td>
<td>-2.015</td>
<td>0.927</td>
</tr>
</tbody>
</table>
Table 3.9: Volatility of Low-Frequency Components

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>ENDO 1</th>
<th>EXO 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta c)$</td>
<td>0.75%</td>
<td>0.88%</td>
<td>0.65%</td>
</tr>
<tr>
<td>$\sigma(\Delta y)$</td>
<td>1.16%</td>
<td>1.29%</td>
<td>0.99%</td>
</tr>
<tr>
<td>$\sigma(\Delta i)$</td>
<td>3.06%</td>
<td>2.78%</td>
<td>2.09%</td>
</tr>
<tr>
<td>$\sigma(\Delta l)$</td>
<td>1.29%</td>
<td>0.36%</td>
<td>0.32%</td>
</tr>
</tbody>
</table>

Table 3.10: Low-Frequency Correlations

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>ENDO 1</th>
<th>EXO 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$corr(\Delta c, \pi)$</td>
<td>-0.77</td>
<td>-0.87</td>
<td>-0.25</td>
</tr>
<tr>
<td>$corr(\Delta y, \pi)$</td>
<td>-0.74</td>
<td>-0.72</td>
<td>-0.02</td>
</tr>
<tr>
<td>$corr(\Delta z, \pi)$</td>
<td>-0.52</td>
<td>-0.65</td>
<td>0.04</td>
</tr>
<tr>
<td>$corr(\Delta n, \pi)$</td>
<td>-0.63</td>
<td>-0.82</td>
<td>-</td>
</tr>
</tbody>
</table>

$corr(E[\Delta c], E[\pi])$: -0.96 0.23

Table 3.11: Sensitivity Analysis: Price Adjustment Costs

<table>
<thead>
<tr>
<th>1st Moments</th>
<th>$\phi_R = 10$</th>
<th>$\phi_R = 40$</th>
<th>$\phi_R = 70$</th>
<th>$\phi_R = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\Delta y]$</td>
<td>2.76%</td>
<td>2.53%</td>
<td>2.20%</td>
<td>1.97%</td>
</tr>
<tr>
<td>$E[r^*_f - r_f]$</td>
<td>4.32%</td>
<td>4.07%</td>
<td>3.74%</td>
<td>2.16%</td>
</tr>
<tr>
<td>$E[y^{(20)} - y^{(1)}]$</td>
<td>1.25%</td>
<td>1.20%</td>
<td>1.13%</td>
<td>1.10%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2nd Moments</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\Delta c}/\sigma_{\Delta y}$</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>$\sigma_{\Delta l}/\sigma_{\Delta y}$</td>
<td>0.42</td>
<td>0.64</td>
<td>0.77</td>
<td>0.80</td>
</tr>
<tr>
<td>$\sigma_{\Delta i}/\sigma_{\Delta c}$</td>
<td>3.86</td>
<td>4.16</td>
<td>4.38</td>
<td>4.50</td>
</tr>
<tr>
<td>$\sigma_{\Delta l}/\sigma_{\Delta c}$</td>
<td>3.41</td>
<td>3.42</td>
<td>3.44</td>
<td>3.44</td>
</tr>
<tr>
<td>$\sigma_{\Delta c}$</td>
<td>1.42%</td>
<td>1.42%</td>
<td>1.42%</td>
<td>1.42%</td>
</tr>
<tr>
<td>$\sigma_{\pi}$</td>
<td>1.57%</td>
<td>1.55%</td>
<td>1.52%</td>
<td>1.49%</td>
</tr>
<tr>
<td>$\sigma_{E[\Delta z]}$</td>
<td>0.64%</td>
<td>0.63%</td>
<td>0.61%</td>
<td>0.59%</td>
</tr>
<tr>
<td>$AC1(E[\Delta z])$</td>
<td>0.98</td>
<td>0.98</td>
<td>0.97</td>
<td>0.97</td>
</tr>
</tbody>
</table>
Figure 3.1: Long-Run Co-Movement
Figure 3.2: Level Dynamics
Figure 3.3: Expected Growth Rates
Figure 3.4: Level Dynamics (Expected Growth Shock)
Figure 3.5: Asset Prices
**Figure 3.6:** Expected Inflation and Growth Mechanisms (Productivity Shock)
FIGURE 3.7: Expected Inflation and Growth Mechanisms (Expected Growth Shock)
Figure 3.8: Varying Intensity of Inflation Stabilization
Figure 3.9: Varying Intensity of Output Stabilization
Bibliography


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Biography

Howard Kung was born on February 8, 1982 in New York. He did his undergraduate studies in Economics and Mathematics at the University of Virginia and his doctorate in Business Administration at Duke University. After graduating from Duke he will be an assistant professor in Finance at the University of British Columbia.