CFD Optimization of Small Gas Ejectors Used in Navy Diving Systems

by

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Department of Mechanical Engineering and Materials Science
Duke University

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Edward J. Shaughnessy

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the Department of Mechanical Engineering and Materials Science in the Graduate School of Duke University

2012
ABSTRACT

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Abstract

Optimization of small gas ejectors is typically completed by selecting a single set of operating conditions and optimizing the geometry for the specified conditions. The U.S. Navy is interested in utilizing a small gas ejector design in multiple diving systems with varying operational conditions. This thesis is directed at developing a Quasi Newton-Raphson Multivariate Optimization method using Computational Fluid Dynamics (CFD) to evaluate finite difference approximations. These approximations are then used as inputs to the gradient vector and the Hessian matrix of the standard Newton-Raphson multivariate optimization method. This optimization method was shown to be timely enough for use in the design phase of a multiple parameter system. CFD investigation of the level curves of the simulation cost function hypersurface verified the success of the method presented at optimizing each independent parameter. Additional CFD simulations were used to investigate the ejector performance for operational conditions deviating from the operational conditions used during optimization. A correlation was developed for selecting the optimum throat diameter, and corresponding maximum efficiency, as functions of the input conditions only. Experimental models were manufactured using fused deposition modeling and evaluated with good agreement to the CFD simulation results.
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### Definition of Key Terms

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<th>Description</th>
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<tr>
<td>$P$</td>
<td>Pressure</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$V$</td>
<td>Velocity</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach Number</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds Number</td>
</tr>
<tr>
<td>$D$</td>
<td>Diameter</td>
</tr>
<tr>
<td>$L$</td>
<td>Length</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance, identified by subscript</td>
</tr>
<tr>
<td>$A$</td>
<td>Area, identified by subscript</td>
</tr>
<tr>
<td>$R$</td>
<td>Gas constant</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>Mass flow rate</td>
</tr>
<tr>
<td>$\dot{M}$</td>
<td>Mass flow rate total, primary + secondary</td>
</tr>
<tr>
<td>$f(\bar{x})$</td>
<td>Multivariate cost function for mass flow rate</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>Vector position of independent parameters</td>
</tr>
<tr>
<td>$\nabla f(\bar{x})$</td>
<td>Gradient of cost function</td>
</tr>
<tr>
<td>$H(\bar{x})$</td>
<td>Hessian matrix of 2nd partial derivatives</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Mass flow rate efficiency</td>
</tr>
<tr>
<td>psia</td>
<td>Absolute pressure</td>
</tr>
<tr>
<td>psid</td>
<td>Differential total pressure across the nozzle</td>
</tr>
<tr>
<td>psig</td>
<td>Gauge pressure</td>
</tr>
<tr>
<td>fsw</td>
<td>Feet of seawater</td>
</tr>
<tr>
<td>acfm</td>
<td>Actual cubic feet per minute</td>
</tr>
<tr>
<td>Subscript</td>
<td>Description</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
</tr>
<tr>
<td>N</td>
<td>Nozzle exit plane</td>
</tr>
<tr>
<td>a</td>
<td>Ambient conditions</td>
</tr>
<tr>
<td>P</td>
<td>Primary fluid or flow</td>
</tr>
<tr>
<td>S</td>
<td>Secondary fluid or flow</td>
</tr>
<tr>
<td>o</td>
<td>Stagnation properties upstream of the nozzle</td>
</tr>
<tr>
<td>T</td>
<td>Ejector throat</td>
</tr>
<tr>
<td>E</td>
<td>Ejector diffuser exit</td>
</tr>
<tr>
<td>EFF</td>
<td>Effective position of fully expanded primary jet</td>
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Acknowledgements

I would like to thank Dr. Lew Nuckols for his encouragement to undertake graduate studies at Duke University. I would also like to thank my advisor, Dr. Laurens Howle, for the guidance and wisdom he shared throughout my work toward completing this thesis. I am thankful to Dr. Edward Shaughnessy, and Dr. Donald Bliss, for their conversations and dedication to being members of my thesis committee.

My attendance in graduate school was made possible through the SMART Scholarship for Service Program and the support from my supervisor, Mike Palmer, at the Naval Surface Warfare Center (NSWC) in Panama City, FL. I am also thankful to Greg Holbrook and Richard Brantley at the NSWC Hydrospace Laboratory for facilitating the experiments completed for my thesis. I am especially thankful to Kirk VanZandt for his endless knowledge on gas ejectors and dedication to helping me complete my experimentation.

I want to extend the warmest thanks for the love and support from Melissa Iandoli, my fiancée and future wife, who supported me unconditionally through many long hours working on this thesis. Ultimately, I am thankful to my Lord, Jesus Christ, for through Him all things are possible.
1. **Introduction**

1.1 **Motivation**

In December 1978, The United States Navy released a technical evaluation of the MK 12 Surface Supported Diving System (MK 12 SSDS) for mixed-gas diving operations. The MK 12 SSDS was developed for use during extended duration dives with an extreme exposure depth limit of 450 fsw requiring helium-oxygen breathing gas mixtures (Coulombe, 1978).

![Image of a U.S. Navy Diver wearing the MK 12 SSDS](image)

**Figure 1:** A U.S. Navy Diver wearing the MK 12 SSDS (Coulombe, 1978).
Figure 1 shows a Navy Diver wearing the MK 12 SSDS in the mixed-gas configuration. The backpack required both a high pressure primary gas source supplied from an umbilical tethered to the surface vessel, and a secondary emergency air source stored locally with the diver for use if the primary source was interrupted. During both primary and emergency modes, the breathing air is added by constant mass injection through the ejector and re-circulated internally to minimize gas usage, making the MK 12 SSDS a semi-closed circuit rebreather. The flow path of breathing gas inside the re-circulator assembly can be seen in Figure 2.

The gas source, whether from an umbilical tethered to a ship or an emergency back-up bottle, is input at the ejector nozzle. It enters the ejector throat where it entrains and mixes with the secondary gas re-circulating from the canister. The mixed-gas stream then enters the helmet through a one-way valve. When the diver inhales, oxygen is consumed and CO₂ is produced and exhaled back into the helmet. Now CO₂ is in the breathing gas and needs to be removed. CO₂ rich gas exits the helmet through another one-way valve into the canister where the CO₂ is absorbed, which reduces the volume of gas. The re-circulated secondary gas then exits the canister into the plenum surrounding the ejector nozzle. When additional gas is added through the nozzle, it induces the secondary gas surrounding the nozzle to be pulled into the ejector throat, completing the re-circulating cycle.
Figure 2: Rear view of the internal flow path inside the re-circulator assembly of the MK 12 SSDS (Coulombe, 1978).
In the MK 12 SSDS, the U. S. Navy is utilizing the performance of the gas ejector to provide a pumping pressure to induce a re-circulation of secondary gas through the canister. There are two processes in the design of a rebreather which are related to the ejector performance: the input gas at the ejector nozzle must replace approximately the same amount of oxygen (by mass) that the diver is consuming during breathing, and the CO₂ produced during breathing needs to be circulated (by volume) through the canister to be removed from the breathing gas. To ensure a sufficient amount of CO₂ is removed, a minimum re-circulating volumetric flow rate must be achieved. The ejector is positioned such that it induces flow through the canister in order to circulate the flow, reducing the work required by the diver’s breathing.

Major issues involved in the proper design of a semi-closed circuit rebreather are replacing the oxygen that the diver is consuming and maintaining a minimum volumetric flow rate through the canister to prevent build-up of toxic CO₂ in the diver’s helmet. Additional issues that are significant are the amount of gas storage required for the diver, both on the ship and in the emergency bottle, and the duration of the emergency gas supply. The gas ejector design is one of the important design considerations for improving semi-closed circuit rebreather performance.

Ejectors can be used with incompressible fluids as well as compressible fluids and are referred to as gas ejectors throughout this paper to specify the use of compressible fluids. In many applications, gas ejectors use a converging-diverging
nozzle to supply the primary gas to take advantage of the increased energy and momentum transfer that supersonic nozzles can provide. The optimization during this study uses a converging nozzle only. The operational conditions used for optimization are air for both primary and secondary gases, standard ambient temperature and pressure, and a differential pressure of 250 psid across the primary nozzle. These operational conditions result in a choked, under-expanded nozzle during optimization. In Chapters 3 and 4, the operational conditions deviate from the optimization conditions in order to evaluate the effect of changing input conditions on the optimum geometry. The input conditions in those two chapters mostly result in a choked, under-expanded nozzle; however, some input conditions during CFD evaluation and experimentation did result in a subsonic nozzle.

1.2 Concept, Problem and Purpose

Gas ejectors are sometimes referred to as injectors, ejector pumps, or jet pumps. This is in reference to the difference in pressure across to the ejector body providing a pumping force which acts to drive the fluid through a system. Initially, the operation of an ejector in a diving system can be described in three sections: the Entrainment Section, the Mixing Chamber Section, and the Diffuser Section, seen in Figure 3. In the Entrainment Section 1 of the ejector in Figure 3, a small high pressure stream of gas is injected into a larger area of a low pressure gas. The high pressure gas is referred to as
the primary gas, or sometimes as the motive gas. The low pressure gas in the surrounding area is referred to as the secondary, or ambient gas.

In reference to the system of Figure 2, the secondary gas is the gas that has been circulated through the diver’s helmet, through the canister, and back to the plenum surrounding the ejector nozzle. The re-circulated secondary gas is entrained by the primary gas and “pulled” into the Mixing Chamber. Inside the Mixing Chamber Section 2 of Figure 3, the two streams are mixed with momentum from the primary stream being transferred to the secondary stream. At the exit of the mixing chamber, the mixed stream enters the Diffuser Section 3 of Figure 3, where it expands, slows down, and recovers pressure from the flow. This pressure is what provides the pumping action sometimes mentioned for performance measures of an ejector, and is what helps circulate the gas through the MK 12 SSDS breathing apparatus.

![Diagram of the operation of a small gas ejector](image)

**Figure 3: Diagram of the operation of a small gas ejector with: 1) The Entrainment Section 2) The Mixing Chamber Section and 3) The Diffuser Section.**

Gas Ejectors are purely mechanical devices in that they do not have any moving parts. The only contributing variables to the ejector performance can be tied to the input
conditions, the properties of the primary and secondary gases, and the physical geometry of the ejector itself.

When an ejector is designed for a production unit, there are limitations to the geometry that can be used; for example there are no ideal corners, rather they must have a defined radius. For this reason, there are a large number of geometric variables seen in Figure 4 that could potentially have an effect on the overall ejector performance.

![Diagram of ejector geometry](image)

**Figure 4: The geometry definitions for the small gas ejector.**

There are the classic ejector parameters like the throat diameter (D_T), the throat length (L_T), the nozzle diameter (D_N), the diffuser angle (θ), the exit diameter (D_E), and the nozzle placement relative to the throat entrance (d_N). Additional parameters for this particular system include the entrance radius of the ejector throat (R_{T,IN}), the exit radius of the ejector throat (R_{T,OUT}), the diffuser exit radius (R_{E,OUT}), and potentially the exterior geometry of the nozzle. Each of these geometric parameters has been identified as potential candidates for optimization.
Designing an ejector for use in diving requires the knowledge of the operational conditions of the particular dive. The depth of the dive will change the ambient pressure of the secondary gas, while it may also change the required gas mixture and the primary nozzle pressure in order to supply a specific mass of oxygen to the diver. This presents a problem because ejectors are typically designed for operation at one set of operational conditions and gas properties.

The U.S. Navy is interested in designing ejectors for use in multiple new diving systems. The first system is a new re-breather similar in operation to the MK 12 SSDS system pictured in Figure 1 and Figure 2. The new system would have a different operating depth with a maximum depth of around 350 fsw, while the MK 12 SSDS had a maximum operating depth of 450 fsw (Coulombe, 1978). A second system the U.S. Navy is interested in is an incapacitated diver rescue system, where an ejector would be used to re-circulate air in an open helmet during a shallow water rescue mission of an incapacitated diver. A third potential use identified by the Navy is in a portable recompression chamber, where an ejector could be used to pump and circulate air inside a portable chamber without requiring any additional power.

These three examples of ejector use in diving present the problem of optimizing an ejector for use in multiple diving systems where the operational depths, gas type, and supply pressures may vary widely between systems. The purpose of this research is to determine a timely process for which a commercial Computational Fluid Dynamics
(CFD) software package can be used during a design phase to optimize a gas ejector for use in multiple diving applications. In addition to optimizing an ejector for a specific application (referred to as the design point), it is important to determine if any similarity exists for the optimum geometry and maximum performance when deviating from the optimization design point.

1.3 Literature Review

1.3.1 History and Application

The injector was originally invented in 1858, by a Parisian inventor named Henri Giffard, for use in pumping water into steam boilers for steam locomotives (Kranakis, 1982). Today, injectors are referred to by many different names including jet pumps, jet ejectors, and gas ejectors. The original analysis of the injector did not take place until 1908 by Henri Poincare in his book *Thermodynamique* (Kranakis, 1982). In his book, Poincare approached the explanation of the performance of the injector with the laws of thermodynamics (Kranakis, 1982). Thermodynamics plays an important role in the approach used throughout much of the literature presented.

The reasonably simple operation of an ejector is completely opposite of the complex processes that take place inside the ejector itself. Multiple processes may be taking place simultaneously, which can include any or all of the following: acceleration of the surrounding particles by impact from the primary fluid, viscous friction at the boundary of the primary jet, pressure gradients during primary jet expansion, and
change of state of the fluids involved (Kroll, 1947). This is a preview of the many problems that ejector designs encounter. Although many systems can eliminate one or two of these processes, like a change of state (with gas only systems) or pressure gradients (using perfectly expanded supersonic nozzles), there may also be additional important processes not identified by Kroll; such as chemical reactions between gases, or additional fluid property gradients (density, viscosity, temperature etc.).

The ability to be used with liquids, gases, and combinations of the two make the ejector device versatile in application. Ejector jet pumps are being considered for use in refrigeration cycles because they can use water as the refrigerant fluid, and are powered by waste heat (Eames, et al., 1999 and El-Dessouky, et al., 2002). Ejectors can also be used for thrust augmentation in high entrainment ratio aircraft (Dutton & Carroll, 1986 and Presz Jr. & Werle, July 2002). The corrosive resistance properties of ejectors make them useful in many chemical engineering processes where they are used to pump corrosive fluids, slurries, fumes, and dust filled gases (Yadav & Patwardhan, 2008).

Steam-water ejectors are two phase ejectors that can be used both as a feeding pump or as a condensor (Butrymowicz, et al., 2009). Microapplications for ejectors are beginning to be studied for uses like micro steam generators (Eid, et al., 2010) or potentially as a substitute for a compressor in a micro turbine (Gardner, et al., 2010).

A generalized modeling of ejectors is very difficult because of the range of input conditions, fluid interactions, and design controls that are involved. Initially, most
investigations begin with a basic 1D analysis involving conservation equations. Many investigations have been completed for determining the optimum design geometry for an ejector; although the majority of these investigations are focused on a specific set of operating conditions, or “special cases”, of ejector optimization (Dutton & Carroll, 1986). In other words, there is a lot of literature on optimization and sensitivity analysis for ejectors systems; however, each focuses on a specific set of operational conditions and fluids with the sensitivity analysis specific to that design point. Therefore it is required that an optimization or sensitivity analysis be conducted on each set of operational conditions of interest.

1.3.2 Geometry Design for Ejector Performance

Ejector designs are typically based around a performance parameter that is important to the particular application. There are mainly two approaches to optimizing an ejector. The first approach is minimizing primary mass flow rate when you have known operating pressures for the ejector inlet and outlet. For this optimization, the secondary flow has a constant pressure and mass flow rate, with a known pumping pressure at the ejector exit required (Dutton & Carroll, 1986). The optimum ejector uses the minimum primary flow that will result in the required increase in pressure. The second approach is maximizing the ejector exit to inlet compression ratio when the primary and secondary mass flow rates and stagnation pressures are known (Dutton & Carroll, 1986).
Therefore a good understanding of the performance parameter for a given system is important before attempting an optimization or sensitivity analysis. One drawback to these approaches is that either the mass flow ratio or the compression ratio must be known. In the current design, the maximum flow ratio is required; however, in internal flow through a packed bed, like the CO₂ canister in the MK12 SSDS, the backpressure, and hence the required compression ratio, increases with increasing volumetric flow rate. Therefore, in the current design, neither combination of mass flow ratio or compression ratio is constant. For example, the optimization method presented by Emanuel (1976) assumes constant pressure mixing with a low inlet Mach Number. In his analytical method, there is a known primary and secondary mass flow rate that is constant. This contrasts with the current study where the secondary mass flow rate is variable with both the geometry and the backpressure created by the ejector itself. Using the optimization method presented by Emanuel (1976) would require that the assumptions and design constraints are consistent with the study of interest, and in this case they are not due to the constant secondary mass flow rate assumption.

The first geometric parameter to consider is the nozzle in order to determine the fluid properties of the primary flow exiting the nozzle. The nozzle is typically a controlled variable within the system. For simplicity, isentropic flow equations are used for both sonic and subsonic conditions at the primary nozzle. If more precise analysis is required for the actual performance of a nozzle, then experimentally determined
discharge coefficients can be used (Shapiro A. H., 1953). For large Reynolds Number flows, Re \( \sim 10^6 \) or more, the discharge coefficient is on the order of 0.99, while in low Reynolds Number flows the coefficient may be much less than 0.99 (Shapiro A. H., 1953).

Interestingly, the size and shape of the nozzle are rarely optimized themselves, but are typically a given value for the system, with the ejector geometry optimized around this particular nozzle. However, it has been shown that for supersonic ejectors, if a nozzle is designed for minimum obstruction of the secondary flow and positioned well inside the ejector throat, the efficiency can be increased (Watanawanavet, 2005). In the current design problem, the nozzle for the system cannot be designed for this type of positioning and the use of a supersonic ejector is not applicable due to the transient nature of diving. Nozzle placement in subsonic ejectors has also been studied (Zhang, Jin, Huang, & Tian, 2009). The knowledge of a potential increase in efficiency due to nozzle position is noted. A study of micro ejectors shows the power density of a micro ejector can be optimized by selecting an appropriate nozzle diameter and Reynolds number at the nozzle exit (Gardner, et al., 2010). This may be an important design consideration for the future, but the current study uses a fixed nozzle diameter specific to the system currently in use.

Throughout the literature, the most important design parameter is the ratio of the throat diameter, or the smallest diameter of the mixing chamber, to the diameter of
the primary nozzle. It is also important to determine if you are going to use a constant area mixing chamber or a constant pressure mixing chamber. For a comparison of analytically modeling a constant pressure vs. constant area mixing chamber see the dissertation written by Liao (2008) where he used conservation equations to derive generalized 1D equations for ejector designs with both styles of mixing chambers.

A constant area mixing chamber is used in the current study which has a constant entrance radius with variable throat diameter and chamber length. Many studies have been done to determine a relationship between the maximum mass flow ratio and the diameter ratio that corresponds to it; although, typically they are bound by specific conditions. For example, the maximum efficiency for an ejector as the pressure ratio across the primary nozzle is increased is presented by Kroll (1947). Kroll also presented the diameter ratio specific to the maximum flow ratio again as a function of the pressure ratio across the primary nozzle. However, he presents this information for two separate ejectors: one as a steam-air ejector, and the other as an air-air ejector. Kroll also maintains a limit to the application of these relationships for a discharge pressure to be near atmospheric ambient pressure. There is clearly a significance of the fluid properties and input conditions to both the mass flow ratio and the appropriate diameter ratio for optimum design; however, by this method experiments would necessarily need to be completed for every gas combination intended to be used for each set of input conditions on the ejector.
The length to diameter ratio of the mixing chamber is also normally reported for specified conditions of ejector operation. It is important to design the ejector throat long enough for complete mixing to occur. One assumption typically made for 1D analysis is that there is complete mixing in the mixing chamber. This assumption is not always correct and the throat length design does affect the overall efficiency of the ejector. The investigation by Keenan, et al. (1948) led to the conclusion that the throat length necessary for complete mixing depends on both the mixing process and any shock process. The viscosity was determined to be a major factor of the best throat length for ejectors in the study of two phase ejector flows (Yanzhong & Wang, 2011). This study concluded that the proper throat length depended on a pseudo-shock length, defined as the position of fully recovered pressure in the mixing chamber, which was highly dependent on viscosity ratios of the primary to secondary flows. Clearly there are multiple parameters that may affect the true best throat length including compressible flow effects, turbulence, pressure gradients, and the dominating mixing process.

Keenan, et al. (1948) presents the optimum distance from the nozzle exit plane to the mixing chamber exit plane as a function of the stagnation pressure ratio of the primary to secondary fluids. This is an interesting combination as it includes the nozzle placement relative to the throat entrance added to the throat length for a total distance. Additionally, it provides a corellation between the mismatch in stagnation pressures to a total mixing length. What is interesting about this study is that it shows a dependency of
the optimum nozzle position on the input conditions of the ejector. The study by
Keenan, et al. (1948) used a converging-diverging nozzle in a supersonic ejector where
the secondary flow was also compressible and shocks were present in the ejector throat.
This differs from the current study where a converging nozzle is used and the secondary
and mixed flows are assumed to be subsonic.

The diffusing section is used to recover pressure from the mixed flow. Again,
during 1D analysis, one typically assumes the flow is completely mixed with a uniform
momentum at the exit of the throat corresponding to the entrance of the diffuser. If this
assumption is not true, the pressure recovery would be reduced and the effectiveness of
the diffuser limited. The exit diameter of the diffuser section is typically defined by an
overall ejector length and a diverging angle for the diffuser. Many times in literature, the
experimental models are tabulated in order to compare geometry, similar to the table
presented by Kroll (1947).

The external geometry of the nozzle also seems to be an important design issue,
especially when designing a constant area mixing chamber. A theory for high-efficiency
supersonic ejectors showed that moving the nozzle tip well into the throat of the ejector
would promote an initial velocity of the secondary fluid; therefore, reducing the velocity
gradient between the two mixing streams and ultimately enhancing the efficiency of the
momentum transfer (Watanawanavet, 2005). This would lead one to assume that
optimizing the nozzle geometry may lead to the ability to adjust the position of the
nozzle to more efficiently transfer momentum from the primary flow to the secondary
flow; however, in the current system, the nozzle geometry is limited to practical
manufacturing methods.

1.3.3 Ejector Optimization

Optimization of ejectors has been attempted for many configurations. Generally,
the systems being optimized are special cases or specific ranges of flow conditions and
have limited applicability of the method or results to a more general ejector design.
Selection of the appropriate performance parameter is important for optimizing an
ejector system. In general, there are two performance parameters that are used to
describe the overall efficiency of an ejector: the entrainment ratio, or mass flow ratio of
the primary and secondary fluids, and the compression ratio, or the pressure increase
across the ejector.

For example, a pressure ratio is used for the ejector design by Kroll (1947) where
the compression ratio is known, or required, and the mass flow rate is being optimized
for the minimum possible primary mass flow rate to produce the compression required.
This study by Kroll (1947) seems to provide one with an appropriate diameter ratio and
an estimation of the maximum overall efficiency given that the inlet and exit pressures
are known. The issue with using this data is that one is confined to the use of either an
air-air or air-steam ejector at ambient pressures not much different than atmospheric
conditions and that the pressure difference between the secondary fluid and the discharge pressure are both known.

A list of optimization methods was presented by Dutton & Carroll (1986) where a few different approaches from literature are discussed along with their limitations and design scope. The method by Dutton & Carroll (1986) determines the primary nozzle Mach Number and the appropriate area ratio for optimizing a gas ejector. This method was based on a 1D analytical solution for a constant area mixing ejector. Dutton & Carroll (1986) showed that this method works for optimizing one of the following: mass flow ratio, compression ratio, or the stagnation pressure ratio, assuming both of the other two values are known. The limit of the method is the use of diatomic fluids for both the primary and secondary flows with approximately equal molecular weight and stagnation temperatures. Again for this method to be successful at least two of the three flow conditions must be constant or assumed, which is different than the current study where only one flow condition is completely known.

An additional informative analysis was completed by Wacholder & Dayan (1984) in which an adjoint sensitivity method was used to analyze a supersonic ejector. With this method, a single computation produced a set of sensitivity coefficients for a large number of independent variables. This analysis was shown to be very valuable in a system with a large number of independent parameters (Wacholder & Dayan, 1984). However, the sensitivity is based around a specific design point for both the fluid
properties and the geometry. As is typically seen in ejector analysis where the primary mass flow rate is being optimized, this sensitivity study assumes that the secondary mass flow rate, stagnation pressure, and ambient pressure are all known.

A subsonic ejector is one that usually employs a converging nozzle below or at the critical pressure ratio for sonic flow in the nozzle throat. This means that the primary flow exiting the ejector is at the same pressure as the secondary fluid and the mixed stream is subsonic in the mixing chamber. Numerical simulations were done to optimize the nozzle position in a subsonic ejector (Zhang, et al., 2009). In this paper, all flow parameters were set and only the nozzle position was altered. It was determined that there was an optimal nozzle position, however, it varied depending on the flow parameters. It was also discovered that the subsonic jet behaved closely with the free jet theory but is limited to the empirical nature of free jet theory and requires coefficients to be determined experimentally (Zhang, et al., 2009).

Again, analysis of the type mentioned above is limited to optimizing a single variable at a time. Additionally, in diving applications, it is also important to know and control the mass flow rate of the primary flow. This constraint requires that the nozzle be operated at or above choked flow conditions. Therefore, subsonic ejector optimization is not necessarily applicable to the current research.

CFD has also been used for ejector performance evaluation as well as optimization (Liao, 2008 and Watanawanavet, 2005). In his dissertation Liao (2008)
developed a numerical model to predict ejector performance. CFD was then used as an analysis tool and compared to the numerical results with good comparison. A sensitivity analysis was completed using CFD for individual parameters around a single design point. In the thesis written by Watanawanavet (2005), CFD was used during an optimization process where the geometry parameters were varied independently until an optimum condition was found. The optimization was also completed with steam as the motive fluid and with the mass flow ratio and a Reynold ratio between the primary and secondary fluids used as a design parameter. In the current study, the secondary mass flow rate or velocity is not assumed at any local position. This proves the potential power for CFD as an analysis tool for ejectors, but again represents the necessity to optimize an ejector specific to the operational conditions of the system being analyzed.

The small gas ejector in a re-breather operates under changing operating conditions with varying gases depending on the dive profile. Additionally, neither the secondary mass flow rate nor the compression ratios are controlled. Maximizing the mass flow ratio increases the circulation and maximizing the compression ratio facilitates the flow of the gases through the CO2 canister. Counter to a traditional mass flow optimization where the secondary flow rate is known and the primary is minimized, in the ejector used for diving it is important to maintain a constant primary flow rate and maximize the secondary flow rate. When the mass flow rate increases, the backpressure will also increase (due to increased actual volume flow rate), but there is a
negative feedback where an increasing backpressure acts to reduce the secondary flow rate which can change the optimal conditions. While each of the methods reviewed give valuable insight into many different aspects of ejector design the small gas ejector used in diving applications will operate outside the constraints and assumptions of the current literature.

In reality, most optimization methods or sensitivity studies are confined to specific operating conditions. The optimization method used in this thesis is similar in that the actual optimization is specific to one set of input conditions and fluid properties. Additionally, building similarity relationships between input properties and the optimum geometry will be studied in order to expand the application of the optimization at the initial, single of input conditions.

1.3.4 Flow Considerations

The majority of 1D analysis of ejectors is completed without consideration of the complex internal flow inside the ejector especially in the mixing chamber. Supersonic ejectors tend to develop normal shock fronts inside the ejector throat prior to the flow exiting into the diffuser. Two phase steam-water ejectors deal with phase changes and interaction of fluids with highly differing properties. Turbulent mixing between shear layers presents many problems for analytically modeling ejectors systems. Subsonic ejectors still may have compressible flow interactions, chemical reactions, as well as thermodynamic and heat transfer processes occurring inside the flow. With this many
variations and potential flow interactions it is impossible to consider any solution to a single ejector optimization as a general solution.

Schlieren photography was used by Keenan, et al. (1948) in order to visualize the shock waves inside the supersonic ejector throat. Compressible flow ejectors display shock waves during experimentation which make visualization techniques useful for understanding the supersonic flow inside an ejector and the potential effects of the shock positions and transition from supersonic to subsonic flow. In their shock visualization experiments, Keenan, et al. (1948) used a converging nozzle and changed the backpressure to shift the position of the shock within the mixing chamber. They concluded that if the mixed flow is supersonic, the mixing length from the nozzle exit to the mixing chamber exit is dominated by the shock placement and nearly constant. They also concluded that if the mixed flow is subsonic, the mixing processes dominate and the mixing chamber length may vary with pressure ratios and nozzle placement.

Ejectors are considered supersonic when the mixed flow is supersonic, and they are considered subsonic when the mixed flow is subsonic. Supersonic ejectors use converging-diverging nozzle while subsonic ejectors use converging nozzles only. Although they are called subsonic ejectors, this is slightly misleading for the fact that the flow at the nozzle exit can be sonic and if that flow is also underexpanded, the flow outside of the nozzle can accelerate and become supersonic; however, the mixed flow remains subsonic in the mixing chamber. The fluid exits the nozzle as a stream and is
usually referred to as a jet. Underexpanded jets have been studied in order to determine the spreading rate and fluid properties as a function of the distance traveled from the nozzle exit plane (Yuceil & Otugen, 2002 and Mate, et al., 2001 and Kashimura, et al., 2011). With the assumptions that no mass from the secondary flow penetrates the boundary of the primary jet and that the jet expands adiabatically, the flow conditions for the equivalent “fully expanded” jet, where the primary flow pressure equals the secondary flow pressure, can be analytically determined (Yuceil & Otugen, 2002). The study by Yuceil & Otugen (2002) continues by defining a Convective Mach Number for underexpanded free jets where a range of Convective Mach Number between 0.5 and 0.9 shows significant turbulent effects on mixing, while values above and below this range the turbulent mixing is insignificant. Convective Mach Number is used in turbulent shear flows to describe the relative motion between the convection speed of the largest eddies and the external flow velocity (Smits & Dussauge, 2006). Convective Mach Number may be an important indicator of similarity for ejector optimization with changing input conditions and properties.

Kashimura, et al., (2011) studied the shock cell development and flow properties in turbulent sonic jets from converging nozzles. They presented computer generated schlieren photographs for density distribution, temperature distribution, pressure distribution, and shock cell structures. Kashimura, et al., (2011) show a connection between the axial distribution of flow properties and the shock cell development to the
pressure ratio of the nozzle. This is encouraging information for using only input properties, namely the pressure ratio, as a similarity parameter.

In the study by Liepmann & Gharib (1992) they used flow visualization techniques to study the vorticity in the near field of jets exiting converging nozzle. Their conclusion was a connection between the Reynolds number and the number, placement, and size of vortex pairs near the nozzle exit plane (Liepmann & Gharib, 1992). These vortex pairs were directly related to the entrainment of surrounding fluid and ultimately mixing of the two fluids. The jet structure and impact on entrainment ratio is not typically inspected due to optimization occurring at a set design point for input conditions.

1.3.5 Underexpanded Sonic Nozzles

This section reviews the work by Yuceil & Otugen (2002) in more detail. It is important to discuss the range and assumptions of the scaling presented. The study by Yuceil & Otugen (2002) was conducted in order to characterize the flow properties of an underexpanded free jet discharging into a sufficiently large plenum. It is assumed that the jet expansion is adiabatic and there is not a significant mass flux of secondary fluid across the boundary of the jet. Some of the intermediate steps of the following derivation have been ommitted in favor of presenting only the equations relevant to this thesis. See Yuceil & Otugen (2002) for the complete derivation.
Figure 5: Illustration of the expansion of the primary jet when the nozzle is underexpanded. When \( P_N > P_A \) the primary flow will expand to an effective expansion plane defined when \( P_{EFF} = P_A \).

The current thesis uses a converging nozzle; therefore, for an underpanded scenario to exist, the Mach Number at the nozzle exit must be unity, so \( M_N = 1 \). When the stagnation pressure and temperature are known for a gas exiting at Mach 1, all of the properties of the primary flow at the nozzle exit plane can estimated with isentropic flow equations. In Figure 5, the primary flow is exiting the nozzle at a pressure greater than the surrounding ambient fluid and expands as it travels downstream due to the pressure gradient. At some position downstream, the primary flow will have expanded until the pressure is equal to the surrounding ambient fluid. That position can be
considered the effective expansion plane where the primary pressure has become fully expanded. The flow properties at the effective expansion plane are termed the effective flow properties for a fully expanded jet. The intent of the scaling by Yuceil & Otugen (2002) was to establish similarity between the known input flow conditions and the flow conditions at a position downstream where the pressure has equalized to the surrounding ambient pressure. For simplicity, the ambient gas in Figure 5 is assumed to be stagnant and the primary jet behaves as if it were a free jet.

The underexpansion pressure ratio is defined as $u$ in Equation (1). This is the ratio of the static pressure of the primary flow at the nozzle exit plane to the static pressure of the ambient surrounding fluid. Using conservation of momentum for the primary jet and manipulating the definition of Mach Number, Yuceil & Otugen (2002) were able to express the effective velocity as a function of $u$, the Mach Number at the nozzle exit, $M_N$, and the specific heat ratio, $\gamma$, shown in Equation (2).

$$u = \frac{P_N}{P_a}$$

$$\frac{V_{\text{EFF}}}{V_N} = 1 + \frac{u - 1}{\gamma M_N^2 u}$$
With the effective velocity known, the energy equation for adiabatic flow can be used to solve for the effective temperature and is rearranged to be a function of \( u, M_N, \) and \( \gamma \) in Equation (3).

\[
\frac{T_{\text{EFF}}}{T_N} = \frac{(\gamma + u - 1)}{\gamma u} - \frac{(\gamma - 1)}{2} \left( \frac{u - 1}{\gamma M_N u} \right)^2
\]  \hspace{1cm} (3)

The gas is assumed to behave like an ideal gas so substituting the equation of state for an ideal gas into Equation (3), the effective density can also be written as a function of \( u, M_N, \) and \( \gamma \) shown in Equation (4).

\[
\frac{\rho_{\text{EFF}}}{\rho_N} = \frac{2\gamma^2 M_N^2 u}{2\gamma M_N^2 u(\gamma + u - 1) - (\gamma - 1)(u - 1)^2}
\]  \hspace{1cm} (4)

Continuity is rearranged to the form of Equation (5) and with the effective density and effective velocity known, Equation (2) and Equation (4) can be substituted into the continuity equation therefore expressing the effective area to the actual nozzle area as a function of \( u, M, \) and \( \gamma \) only. Equation (6) has been simplified to the effective diameter ratio versus the area ratio that is originally solved for from the continuity.
equation. The importance of this scaling is two-fold. First, each of the flow parameters at the fully expanded position have been written in terms of $u$, $M_N$, and $\gamma$, which are known from the inputs to the system. Secondly, it provides information about how input conditions alter the effective parameters of the primary flow when it has become fully expanded. The assumption that there is a negligible mass flux across the jet boundary would lead to an additional assumption that the physical mixing of the two flows does not begin until the primary flow has become fully expanded.

The last important parameter identified by Yuceil & Otugen (2002) is a Convective Mach Number in Equation (7). The Convective Mach Number is described by Yuceil & Otugen (2002) to be a measure of the effects of flow compressibility on turbulence. The Convective Mach Number can also be thought of as a relative velocity between the convective speed of the largest eddies and the velocity of the surrounding fluid (Smits & Dussauge, 2006). It is presented by Yuceil & Otugen (2002) that a flow with Convective Mach Number in the range of $0.5 < M_C < 0.9$ will show significant compressibility effects. Outside of this range, on both the lower and upper ends, it was shown that compressibility was insignificant (Yuceil & Otugen, 2002).

$$M_C = \frac{V_{EFF}}{\sqrt{\gamma R T_{EFF}}} \left[ 1 - \frac{1}{1 + \left( \frac{\rho_{EFF}}{\rho_a} \right)^{\gamma - 1}} \right]$$

(7)
Because it is considered a relative Mach Number, it would seem necessary to know the relative velocities of the primary and secondary flows. This makes the quantification of this effect fairly difficult, and in the case of optimizing an ejector, almost impossible. The Convective Mach Number calculated by Yuceil & Otugen (2002) has assumed the surrounding fluid is stationary in the vicinity of the effective expansion plane. This allows one to neglect the velocity of the secondary flow to simplify the definition of the Convective Mach Number to include only the velocity of the primary flow. Using this simplification in the current study, it may be possible to quantitatively compare similarity between input conditions where turbulent mixing is expected to be a contributor to the performance of the ejector.

1.3.6 Ejector Designs in Diving

As mentioned in the Motivation section, the U. S. Navy has previously included a small gas ejector in the MK 12 SSDS semi-closed circuit rebreather. It was originally designed for a maximum depth of 450 fsw. The gas ejector used in the MK 12 SSDS was optimized for use at greater depths, to increase the duration of the emergency backup system, or for use in a future breathing apparatus. The analytical optimization completed by Nuckols & Sexton (1987) had an increase in efficiency of around 100%. The efficiency parameter was defined as diving duration time for a given supply of primary gas. Nuckols & Sexton (1987) concluded that the maximum efficiency gain could be realized by optimizing the ratio of the nozzle exit area to the mixing chamber area.
Following the analytical optimization methods completed by Nuckols & Sexton (1987), the ejector system used by the U.S. Navy underwent further manual optimization at the Naval Surface Warfare Center Panama City Division. In the work completed by VanZandt et al. (1998), the throat diameter, mixing chamber length to diameter ratio, nozzle placement relative to the ejector throat entrance, and entrance effects were manually evaluated to determine the optimum non-dimensional ratios for each of these parameters. The ejector efficiency, defined as the total mass flow to primary mass flow ratio, was increased by an additional 10%. One additional important phenomena realized during experimental evaluation of the new ejector was a large increase in ejector performance with depth of operation. This leads to a hypothesis that the ambient pressure contributes to the calculation of the secondary mass flow rate and may have an influence on the ejector optimization at different operating ambient pressures.

1.3.7 Literature Summary

In summary, the optimization of a small gas ejector involves many parameters. In addition to having multiple independent design parameters, it is apparent that the optimal design changes for different operational conditions. This leads to the point of why there are multiple attempts to optimize ejectors, and while they are all validated by experimentation and hold true near the design point of optimization, they cannot be generalized to ejector designs far from the optimization or analyzed design point.
1.4 Limitations and Design Controls

To make clear the limitations that are placed on an ejector design, the designer must specify the thermodynamic state and properties of both fluids, the compressible properties of the primary flow exiting the nozzle, pressures for the primary, secondary and outlet boundaries, and all of the internal geometry associated with ejector design. With this many variables, it is absolutely imperative that the constraints on the system in question and the performance parameters as well as independent variables are clearly defined. The conditions for this research are outlined below.

![Ejector geometry](image)

**Figure 6**: Ejector geometry displayed here (also Figure 4) used to define geometric constraints and input conditions.

The nozzle has a diameter of 0.0157 in and is a constant during both optimization and experimentation. The placement of the nozzle is considered an independent variable for optimization. The nozzle outside geometry, the external angle, and outside diameter at the nozzle exit have been identified as potential parameters for optimization; however, they are considered less important and are held constant in this thesis. The
primary flow properties at the exit of the nozzle are estimated by isentropic relationships for a known upstream thermodynamic state. The primary fluid properties are held constant during optimization but are varied during the CFD Evaluation and Experimentation. The primary stagnation fluid properties during optimization were air at $P = 264.7$ psia and $T = 70^\circ F$.

The secondary fluid is considered to be at the stagnation pressure of the ambient gas surrounding the entire system (i.e. the internal pressure of the hyperbaric chamber during experimentation). In many theoretical approaches, this fluid is typically assumed known or given a specified velocity. In this work, the secondary thermodynamic state far upstream is defined and the mass flow rate and velocity are determined by the CFD simulation. Ultimately, the secondary mass flow rate is what is being maximized for a specified primary mass flow rate. The secondary fluid properties are held constant during optimization but are varied during the CFD evaluation and experimentation. The secondary stagnation fluid properties during optimization are air at $P = 14.7$ psia and $T = 70^\circ F$.

The geometry of the ejector has multiple parts that could potentially influence the ejector performance. The secondary suction chamber is assumed to be sufficiently large to not restrict the secondary flow. The ejector mixing chamber has entrance and exit radii, the throat diameter, and the throat length that all contribute to the mixing of the primary and secondary flows. The radii are held constant while the throat diameter
and throat length are independent parameters being optimized. The ejector diffuser section has an entrance radius, a diffuser angle, a length, and an exit radius culminating at the ejector exit diameter. The diffuser angle and both radii are held constant while the exit diameter is an independent parameter being optimized and the total length is a floating dependent variable. In multivariate optimization techniques the complexity increases greatly with increased number of parameters. The number of independent parameters in this thesis was limited to four and were chosen based on ejector performance sensitivity reported in much of the literature and previous experimentation by the U.S. Navy.

It was mentioned previously that the primary and secondary fluids are air during optimization of the small gas ejector. During CFD evaluation and experimentation the gas properties will be varied. However, the Laminar Flow Element (LFE) used to measure the flow rates is based on viscosity; therefore, the evaluation of gas ejectors in this paper is only representing single gas ejector systems. The exact values of the geometry, fluid properties, and operating conditions will be presented and discussed in more detail in Chapter 2.

1.5 Preview of Chapters

The literature review presented some of the historical approaches to gas ejector design and optimization techniques. Presently, there is not an exact analytical solution to an ejector design and most optimization or sensitivity analyses are limited in application
far from the design point of each individual study. The literature review focused on the importance of different design considerations seen in the literature for using a gas ejector in a dive setting and where the design for a diving application branches off from the historical optimization techniques and experimental data.

Chapter 2 discusses the use of a Quasi Newton-Raphson method for optimizing a multivariate function. The Quasi Newton-Raphson method used involves using CFD software to provide inputs to finite difference approximations used to populate the gradient vector and the Hessian matrix used in the Newton-Raphson optimization method. This chapter describes the theory of the optimization method, the setup of the CFD software, the resulting optimized geometry for the small gas ejector, and selection process for manufacturing experimental models.

Chapter 3 involves using the CFD with the selected experimental model geometry from the previous chapter to evaluate the performance of the small gas ejector. Initially evaluating the level curves in the computational cost function hypersurface will verify that the small gas ejector was optimized for each independent parameter at the specified design point. Additionally the performance of the small gas ejector will be evaluated in conditions deviating from the optimization design point. A similarity analysis will be presented which establishes a correlation between the optimum geometry and cost function with changes to any of the input properties.
Chapter 4 will first describe the experimental design and materials required. The data collection method and analysis to determine the uncertainty of the data collected will then be presented. Following the experimental design, the chapter will present the data for the level curves of the experimental cost function hypersurface at the design point to verify the ejector was optimized for each independent parameter at the design point. The experimental results will then be presented using the similarity developed in the CFD Evaluation chapter.

Finally, Chapter 5 will tie the three main parts of this study together: CFD Optimization, CFD Evaluation, and Experimentation. The discussion will focus on the use of the optimization method as a design phase tool for engineering and the design considerations of the results presented throughout the paper. The recommendations section will discuss future expansion of this gas ejector study.
2. **Optimization of the Small Gas Ejector**

The purpose of this section is to determine a timely process for which CFD can be used to optimize a gas ejector during the design phase of a new system. CFD can provide valuable information to the design engineer and greatly diminish the need for experimental work. In addition to optimizing the ejector for a specific design point, it is important to study the effects of changing boundary conditions and fluid properties on the optimization of the independent geometric parameters. After the ejector has been optimized, CFD will be used to evaluate the cost function sensitivity of changes to each independent parameter in order to select specific ejector geometry for experimentation.

The optimization method uses CFD simulations as inputs to finite difference approximation equations which are then used as inputs for a Newton-Raphson multivariate optimization. When numerical approximations of the derivatives are used as inputs to the Newton-Raphson method, it is sometimes termed a Quasi Newton-Raphson method; therefore, the method presented for optimization in this paper is considered a Quasi Newton-Raphson method.

### 2.1 The Quasi Newton-Raphson Multivariate Optimization Method

A multivariate Newton-Raphson method of optimization begins with a second order Taylor series approximation for a function of \( n \) independent parameters.
\[ f(\tilde{x} + \Delta x) = f(\tilde{x}) + \Delta x^T \nabla f(\tilde{x}) + \frac{1}{2} \Delta x^T H(\tilde{x}) \Delta x \] (8)

In Equation (8), \( f(\tilde{x}) \) is a real function about the vector position \( \tilde{x} \) with \( \nabla f(\tilde{x}) \) being the gradient of \( f(\tilde{x}) \) and \( H(\tilde{x}) \) being the Hessian matrix of second partial derivatives of the function \( f(\tilde{x}) \) about the position \( \tilde{x} \). Equation (8) is valid for a real function \( f(\tilde{x}) \) that is continuous and at least twice differentiable with the gradient vector and Hessian matrix also being continuous. The vector \( \Delta x \) is the same order as the vector \( \tilde{x} \) and is assumed to have a norm close to zero. That is, \( \Delta x \) is a small step in each independent parameter close to their original position.

The condition for the function \( f(\tilde{x}) \) to be maximized is that the gradient be a vector of zeros and with the Hessian being negative definite. Differentiation of Equation (8) with respect to \( \Delta x \) gives:

\[ f'(\tilde{x} + \Delta x) = \nabla f(\tilde{x}) + H(\tilde{x}) \Delta x \] (9)

Setting the derivative to zero leads to:

\[ \nabla f(\tilde{x}) + H(\tilde{x}) \Delta x = 0 \] (10)

Solving Equation (10) for \( \Delta x \) gives:

\[ \Delta x = -H^{-1}(\tilde{x}) \nabla f(\tilde{x}) \] (11)
The vector $\Delta \mathbf{x}$ represents the vector change in position which would maximize the function $f(\mathbf{x})$ when the first and second derivatives are known and the Hessian is negative definite.

Analytically, the Newton-Raphson method would assume an optimized position of a known function where the gradient and Hessian could be calculated. It would then solve the equation successively until the solution is below some critical value and the function is maximized. When an exact analytical function is not known, any number of numerical methods can be utilized to approximate both the gradient vector and the Hessian matrix. The finite difference approximations in Equations (12)-(14) will be used to provide the approximations for the derivatives used in the Newton-Raphson method (Tannehill, Anderson, & Pletcher, 1997).

\[
f_x(x, y) = \frac{f(x + \Delta x, y) - f(x - \Delta x, y)}{2\Delta x}
\]  
(12)

\[
f_{xx}(x, y) = \frac{f(x + \Delta x, y) - 2f(x, y) + f(x - \Delta x, y)}{\Delta x^2}
\]  
(13)

\[
f_{xy}(x, y) = \frac{f(x + \Delta x, y + \Delta y) - f(x + \Delta x, y - \Delta y) - f(x - \Delta x, y + \Delta y) + f(x - \Delta x, y - \Delta y)}{4\Delta x \Delta y}
\]  
(14)

The subscripts of the functions denote which variable is being differentiated. For example, $f_x(x, y)$ in Equation (12) represents the derivative of a function of multiple
variables, x and y, with respect to the first variable, x. It is also important to note here that the Hessian matrix is symmetric or that it is given $f_{xy}(x, y) = f_{yx}(x, y)$.

To take the finite difference approximations one step further, CFD simulations will provide the inputs to Equations (12) - (14) in order to evaluate the finite difference approximations. Each term in a finite difference approximation would be evaluated with an individual CFD simulation. For example, there are two terms in Equation (12) $f(x + \Delta x, y)$ and $f(x - \Delta x, y)$ that would be evaluated with CFD by changing the first variable x by a selected value of $\pm \Delta x$, respectively. When these two simulations are completed, the gradient of $f(\bar{x})$ with respect to the first variable x could be calculated with Equation (12) and inserted as the first term of the gradient vector $\nabla f(\bar{x})$ in Equation (11).

In this way, two CFD simulations are required to evaluate the gradient of the cost function with respect to each independent parameter, three CFD simulations are required to evaluate the partial second derivatives of the cost function with respect to a single variable, and four CFD simulations are required to evaluate the partial second derivatives of the cost function with respect to any two independent parameters. When every term of both the gradient vector and Hessian matrix has been determined, the vector $\Delta x$ can be calculated by evaluating Equation (11) and the new position estimated to maximize $f(\bar{x})$ can be found by Equation (15).
\[ \tilde{x}_{\text{new}} = \tilde{x}_{\text{old}} + \Delta \tilde{x} \quad (15) \]

This solution has assumed that the Hessian matrix is indeed negative definite and the approximations are very accurate. When dealing with a real function and making so many approximations, \( \Delta \tilde{x} \) can be assumed to represent the direction of which the step should be taken but not necessarily the exact magnitude of the step. In order to determine the best step magnitude in the direction of \( \Delta \tilde{x} \), a line search should be completed using Equation (16).

\[ \tilde{x}_{\text{new}} = \tilde{x}_{\text{old}} + \gamma \Delta \tilde{x} \quad (16) \]

Equation (16) is a line search technique called the Wolfe condition where the coefficient \( \gamma \) typically has values between 0 and 1 and is called a damping factor (Wolfe, 1969). A set of CFD simulations would be evaluated for the series of \( \tilde{x}_{\text{new}} \) positions and the maximum position would be selected as the actual \( \tilde{x}_{\text{new}} \). The CFD simulations required to evaluate of Equations (12) through (16) culminating with a single new position would constitute a single iteration of the optimization method. This means that the more variables included in the optimization, the more CFD simulations must be run; thus, increasing the overall time per iteration. Using the Wolfe condition theoretically helps reduce the total number of iterations of the Newton-Raphson method by converging more quickly and was, therefore, used between calculations of each of the iterations.
2.2 **CFD in the Optimization Method**

The small gas ejector of interest has previously gone through multiple design iterations to increase its efficiency mentioned in the literature review (Nuckols & Sexton, 1987 and VanZandt, et al., 1998). Therefore, the current ejector geometry is used as a starting point for the CFD Optimization. It was decided to use commercial CFD software to streamline the optimization process as well and provide an easy transition to real world application for the engineers at NSWC PCD using this approach in the future.

The CFD setup included a mesh density study to determine the optimum mesh size which would provide reliable results while maintaining as minimum a run time as possible. The CFD model was configured using only thermodynamic states for the fluid properties on the boundaries and maintained as much of the default Flow Simulation settings as possible.

The CFD is used to evaluate each term in the finite difference approximations which lead to the complete Hessian matrix and gradient vector for the cost function. Once the Hessian matrix and gradient vector are completed, the system can be solved for the vector representing the appropriate changes of each parameter. Those changes were then applied to the geometry of the ejector for the first optimization iteration. This iterative process was continued until the resulting changes in geometry were negligible either in ejector efficiency or below manufacturing tolerances for which the system is considered converged.
2.2.1 CFD Software

SolidWorks Flow Simulation 2011 was used during both the CFD Optimization and the CFD Evaluation portions of this work. The following information can be found in more detail in the SolidWorks technical manual (SolidWorks, 2011). Flow Simulation uses a Finite Volume (FV) method applied to a spatially rectangular computational mesh to solve the governing equations with the computational mesh designed in a Cartesian Coordinate System. With the FV method, the governing equations are discretized and are in a conservative form. The spatial derivatives are approximated by implicit differences operators and have a second order accuracy while the time derivatives are approximated using an implicit first order Euler scheme. The models in this work were simulated under steady-state conditions.

The computational mesh is constructed in the rectangular computational domain. The computational mesh is refined locally at the solid-fluid interface, in any specified fluid regions, at any solid-solid surfaces, and in the fluid region during calculations. The locally refined mesh remains orthogonal to the computational mesh and continuously splits cells into child cells until a specified cell size or curvature of a solid-liquid interface within a single cell has reached a threshold value.

Flow Simulation uses the Favre-averaged Navier-Stokes equations to predict turbulent flows. This procedure requires additional information and therefore the $k - \varepsilon$ model of transport equations is used for the turbulent kinetic energy and corresponding
dissipation rate. Flow Simulation employs a single system of equations to describe both laminar and turbulent flows while solving the Navier-Stokes equations. This allows for a transition between laminar and turbulent states to be possible, and more importantly, the simulation predicts laminar or turbulent flow locally and solves the appropriate case.

Flow Simulation uses a no-slip condition on solid-liquid interfaces. Pressure boundaries can be specified with total pressure, static pressure, or environmental pressure where the flow direction determines whether it is total (in-flow) or static (out-flow). Pressure boundaries require temperature, fluid properties, and turbulent parameters to be specified. The CFD configurations for this thesis used pressure boundaries only, and allow the simulation to determine the actual flow properties such as the nozzle, exit flow, etc.

2.2.2 CFD Model Configuration

SolidWorks Flow Simulation 2011 provided a very user friendly environment and repeatable simulation results. The Flow Simulation project was applied to the ejector model with the current geometry. In order to minimize the user contributions to the results, the SolidWorks default settings (turbulence parameters, wall conditions, etc.) were all maintained. In addition, the boundary conditions only included total pressure properties, or ambient pressure properties, which are all known from given operating conditions. Standard room temperature was defined for all open boundaries and air was
used for both the primary and secondary fluids. The conditions chosen for optimization were an ejector using air as the primary gas with a gauge pressure of 250 psig (264.67 psia) and air as the secondary gas at 1 atm (14.7 psia), or also referred to as surface conditions.

Figure 7: An axial cross sectional view of the CFD setup. The external black box represents the complete computational domain.

A cross sectional view of the ejector used for CFD modeling is shown in Figure 7. The computational domain is the black box on the outside of the ejector body and is the smallest cube that encapsulates all of the fluid portions of the internal geometry. The internal geometry was simplified as much as possible without losing important features necessary for ejector design. For example, the nozzle would not realistically have a thin wall for the entire feature, but simplifying the internal nozzle geometry also simplifies the mesh and decreases simulation time.
The entire flow setup includes three pressure boundaries and a porous medium restriction that simulates a variable flow restriction similar to a CO₂ canister or the flow meter used during experimentation. The setup can be seen in Figure 8 where the boundary condition being set is highlighted in the command window.

Figure 8: The pressure boundary conditions and the porous medium restriction defined in Flow Simulation to simulate ejector performance.

Flow Simulation allows the user to establish localized meshes. This was an important feature as it allowed the mesh density to be concentrated in positions of the greatest gradients while minimizing the overall mesh for parts of the fluid domain that
have small gradients decreasing the final mesh, and therefore, decreasing simulation times. Two local mesh definitions were used. The first was at the face of the nozzle where the primary gas is exiting the nozzle and interacting with the surrounding gas.

![Figure 9: The local mesh definitions: at the face of the nozzle (upper left) with resulting mesh upper right) and a cylinder encompassing the primary jet (lower left) with resulting mesh (lower right).](image)

The second local mesh was a cylindrical volume defined as an empty or “invisible” volume that encompassed the primary jet as it entered the ejector throat. These local mesh definitions were positioned to concentrate computational cells where local
gradients were expected to be the largest which resulted in increased stability of the simulation during the mesh convergence study.

2.2.3 Mesh Convergence

A mesh convergence analysis was completed as a first step in the optimization process. The local mesh definitions presented previously were defined initially. Then, the default minimum mesh density in Solidworks Flow Simulation was used as a starting point. The ejector body was aligned in such a way as to have the axis of the nozzle and ejector body both aligned with the y-axis of the computational domain. The beginning mesh used was the one that Flow Simulation automatically generated for the lowest refinement setting. The overall mesh density was increased by a set number for subsequent simulations. The results were then compared to determine when the mesh was dense enough to provide an appropriately accurate solution while also providing a minimum run time per simulation. A decision was made at this point that a slight reduction in accuracy would be appropriate for a much greater decrease in simulation time. The reason this was deemed appropriate was that the nature of an iterative optimization problem would allow for the simulations to self-correct to a certain extent.

The first step in the optimization process was to develop a mesh generation that could be repeated for multiple models with changing geometry. In addition, the mesh needs to be dense enough to provide relatively accurate results, while being sparse
enough to provide relatively fast computation results in order to compute each iteration step in a timely manner.

Figure 10: Mesh convergence of the Efficiency, $\eta$, with increasing mesh density.

It can be seen in the mesh convergence of Figure 10 that the solution did not converge to one particular value; instead, it sharply rose to a specific magnitude and oscillated around that value continuously for the increasing mesh size. This is fairly unusual behavior as one would expect to see that the solution would converge to one specific value and become asymptotic. For an optimization problem this can be considered a systematic error, and due to the nature of optimization being a relative comparison of outputs for different physical geometry, the oscillating convergence does not present a problem during this process.
The oscillations seen in Figure 10 do not seem to be converging to, nor diverging from, a unique solution. Theoretically, if the mesh density is increased sufficiently it can be assumed that the solutions would converge to a single value. It is not advantageous to determine what that value is when relativity is all that is important in the Newton-Raphson method; especially when increasing the mesh density significantly increases the computational time required to complete each iteration. Therefore, it can be estimated that any mesh within these oscillations will provide a reliable solution to be used with the Newton-Raphson optimization method, and the mesh selection was determined by the densest mesh that still provided a reasonable computational time. The mesh definition defined in Flow Simulation is shown on the left side of Figure 11 with the mesh generation information shown on the right side.

![Figure 11: The initial mesh settings from the mesh optimization. The mesh has 37, 33, and 144 cells in the X, Z, and Y-directions, respectively. After refinement, there were roughly 100,000 fluid or partial fluid/solid cells.](image)
2.2.4 CFD Convergence

Iterations of Newton’s method were completed to provide points for the CFD convergence criteria. The first criterion for convergence was that the overall efficiency of the ejector had reached a maximum plateau. In other words, the change in overall efficiency would go to zero for subsequent iterations completed. The second criterion for convergence was that each individual parameter should also remain at or near a constant value when compared to previous iterations. This criterion is more subjective, but when a parameter is at or below a reasonable manufacturing tolerance, that parameter’s value is considered converged.

The optimization configurations were set up in Flow Simulation where an identical flow simulation project could be applied to each individual geometric configuration. The configurations were specific to the thirty-three separate configurations needed in order to completely populate the Hessian matrix and gradient vector. Computation of all of the thirty-three configurations and subsequent solving of Equation (9) would provide the data point for a single iteration of the optimization method.

There are two signs of a completely converged system. The first indication can be seen in Figure 12 with the convergence of the mass flow efficiency with the number of iterations of the optimization method. Additional to the graphical convergence of the mass flow efficiency, $\eta$, the Hessian matrix can provide information with limited
mathematical analysis. When the Hessian matrix is negative definite, the mass ratio function has been maximized. The concavity of the Hessian matrix was determined at the end of each iteration.

Figure 12: The convergence of the cost function of the Mass Flow Ratio with iterations of the Quasi Newton-Raphson method.

It was seen that during the earlier iterations that the concavity of the Hessian matrix varied and sometimes included saddle points. This was important when doing the Wolfe line search between iterations mentioned previously in order to determine the true direction and step toward the maximized function. The iterations toward the end of the optimization process did, however, all have negative definiteness of the Hessian matrix. This was just a secondary sign that provided confidence in knowing that the system had truly been optimized at the end of iteration 11 in Figure 12.
The second indication that the cost function has converged deals with the convergence of the individual parameters. It is seen in Figure 13 that each individual parameter converged to a specific value as the optimization process continued until at the end the percent change of each parameter for a new model was relatively zero. The “percent change” in $d_n$ is deceiving because it is an assembly measurement and not a dimension of the physical model. Therefore, this value depends on where the measurement is taken from and the fact that it is the only parameter that can actively be changed during experimentation made it less important to be perfectly optimized. It can also be considered converged due to the fact that the absolute change is on the order of one thousandth of an inch. It is also interesting to note here that the throat length

**Figure 13: Convergence of independent parameters. Each parameter approaches an asymptote with increased iterations.**
parameter initially was changed in the opposite direction of being optimized. During these early iterations, the Hessian matrix was a saddle point which would indicate that there would be two separate step directions pointing toward optimization. Each direction would be optimizing in one or more variables while doing the opposite for the other variables. What this means is that when there was a saddle point, the line search provided the direction to step in which the dominant parameter was being optimized. This led to one or more less dominant variables, in this case \( L_T \), moving in the wrong direction. Once the throat diameter was optimized, the Hessian matrix became negative definite and the throat length was then optimized in the correct direction.

### 2.3 Optimization Results

The optimization results for the changes in geometry and the resulting increase in efficiency are shown in Table 1. The optimization of the small gas ejector predicts that an increase of 8.87% in the mass flow efficiency can be realized by geometric changes. The changes in geometry are not extreme as they all fall within the ranges of assumed optimized values seen in the literature. In terms of normalized dimensions, the throat diameter to the nozzle diameter, as well as the exit diameter to the throat diameter, should be increased while the throat length to the throat diameter, and the nozzle placement distance relative to the throat entrance, should be reduced.
Table 1: Optimization results for the independent parameters along with the normalized ratios and the overall increase in efficiency.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th></th>
<th>Optimized</th>
<th></th>
<th>Change [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value [in]</td>
<td>Ratio</td>
<td>Value [in]</td>
<td>Ratio</td>
<td></td>
</tr>
<tr>
<td>$D_T$</td>
<td>0.375</td>
<td>23.9</td>
<td>0.429</td>
<td>27.4</td>
<td>14.5%</td>
</tr>
<tr>
<td>$L_T$</td>
<td>3.000</td>
<td>8.0</td>
<td>2.842</td>
<td>6.6</td>
<td>-5.3%</td>
</tr>
<tr>
<td>$D_E$</td>
<td>1.250</td>
<td>3.3</td>
<td>1.579</td>
<td>3.7</td>
<td>26.3%</td>
</tr>
<tr>
<td>$d_N$</td>
<td>-0.250</td>
<td>-15.9</td>
<td>-0.196</td>
<td>-12.5</td>
<td>21.5%</td>
</tr>
<tr>
<td>$\eta$</td>
<td>11.6</td>
<td>N/A</td>
<td>12.6</td>
<td>N/A</td>
<td>8.9%</td>
</tr>
</tbody>
</table>

2.3.1 Level Curves of the Simulation Cost Function Hypersurface

After the Newton-Raphson method converged to an optimum model, the next step was to verify that each geometric parameter was optimized independently around the optimized geometry. Additional CFD simulations were run with the geometric parameters being changed independently to determine the sensitivity of the mass flow ratio to each parameter. This provided confidence in the results of the optimization. The sensitivity analysis for each parameter was also used to provide a range of values to select the geometry of the physic models for experimentation.

Figure 14 shows the CFD results for the dependence of $\eta$ on each of the independent parameter beings optimized. Figure 14 has significance for two different
observations. The first is that the optimized geometry that was determined from the optimization was indeed optimized for each individual parameter. Additionally, it provides enough information to strategically select the geometry for the physical models that will be manufactured for experimentation.

Figure 14: Level curves of the cost function hypersurface. Solid triangles represent the optimized geometry. These results verify the success of the optimization method.

2.3.2 Experimental Model Selection

It is important for experimental verification that the change in geometry for each parameter produces a large enough change in mass flow rate to be distinguishable with the limitations of the flow meter that will be used during data collection. The CFD
simulations used to develop Figure 14 were set up for each parameter individually in order to span geometries that were appropriate to that parameter. With the throat diameter, for example, an area ratio was used instead of the diameter ratio where the maximum throat diameter was chosen such that the area was twice as large as the area of the optimum throat diameter area. Three additional configurations were then chosen between the maximum sized model and the optimum model with equal change in throat diameter. The minimum throat diameter was chosen such that the area was half as large as the area of the optimized model throat diameter. Again, three additional configurations were then chosen between the minimum sized model and the optimum model with equal change in throat diameter.

The exit diameter was done in the same fashion as the throat diameter, although for the physical models being produced there are limits on the minimum and maximum dimensions of the exit diameter due to system constraints. The minimum exit diameter must still be greater than the throat diameter, while the maximum exit diameter must be smaller than the 2 in tube required to connect to the laminar flow element (LFE).

For the throat length, a range of 3 in was chosen (over 100% of the actual length) where four configurations were created with a throat length larger and smaller than the optimized geometry, each with equidistant change in throat length of 0.375 in. For the Nozzle Distance, a range of 0.9 in was chosen which included four configurations with a negative distance (away from the throat entrance) and five configurations with a
positive distance (into the entrance of the throat). Each Nozzle Distance configuration
had a change in distance of 0.1 in.

Table 2: The physical model geometry and identification are shown. The columns of
the ejector ID, separated by a hyphen, identify the throat diameter, the throat length
and the exit diameter from left to right, respectively.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Ejector ID</th>
<th>$D_t$</th>
<th>$L_t$</th>
<th>$D_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>D - D - D</td>
<td>0.375</td>
<td>3.00</td>
<td>1.25</td>
</tr>
<tr>
<td>Optimized</td>
<td>B - B - B</td>
<td>0.429</td>
<td>2.84</td>
<td>1.58</td>
</tr>
<tr>
<td>Throat Diameter, $D_t$</td>
<td>A - B - B</td>
<td>0.313</td>
<td>2.84</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>AA - B - B</td>
<td>0.371</td>
<td>2.84</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>CC - B - B</td>
<td>0.496</td>
<td>2.84</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>C - B - B</td>
<td>0.563</td>
<td>2.84</td>
<td>1.58</td>
</tr>
<tr>
<td>Throat Length, $L_t$</td>
<td>B - A - B</td>
<td>0.429</td>
<td>1.50</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>B - C - B</td>
<td>0.429</td>
<td>4.50</td>
<td>1.58</td>
</tr>
<tr>
<td>Exit Diameter, $D_e$</td>
<td>B - B - A</td>
<td>0.429</td>
<td>2.84</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>B - B - C</td>
<td>0.429</td>
<td>2.84</td>
<td>1.80</td>
</tr>
</tbody>
</table>

*A denotes the parameter has been decreased
*B denotes the parameter has the optimized geometry
*C denotes the parameter has been increased
*D denotes the parameter has the original geometry

Table 2 shows a list of the chosen geometry for the experimental models and the
identification system developed to distinguish them. It was decided to manufacture four
models for evaluating the throat diameter because the throat diameter was the
parameter in which the cost function is most sensitive. The throat length and exit
diameter have two models each built for experimentation. The nozzle distance is an
adjustable parameter within the experimental design; therefore, it is able to be evaluated at more design points for sensitivity.

The geometry of the physical models was selected from the CFD simulations shown in Figure 14. Based on the measuring tolerance of the LFE, it was important for any two models to have a minimum of 0.5 acfm difference in flow rate in order to confidently distinguish between the two models during experimentation. This requirement was not a problem for the throat length, as the length could be increased or decreased by over 100%. The exit diameter did present a problem for the actual geometry selection. In order to realize a large enough change in flow rate, the largest exit diameter would need to be increased to over the size of the entrance to the LFE. For that reason, the exit diameter was selected as large as possible to maintain a realistic wall thickness and connect to a 2 in flexible hose. Additionally, the smaller exit diameter needed to be reduced below the diameter of the throat for a large enough change in flow rate. It was decided to make the exit diameter slightly larger than the throat diameter and still maintain the same radius and diffuser angle.
3. Computational Fluid Dynamics Evaluation

The ejector design presented in this thesis deviates from traditional ejector design and optimization because of the changing operational conditions that occur during a diving mission. Previous ejectors were designed for a specific operational condition or manually optimized at one set of operational conditions and subsequently evaluated or operated at one or many other conditions (Nuckols, 1987). Therefore, in order to expand the knowledge of ejector optimization, it is important to evaluate how the operational conditions being changed will dictate the geometry that will provide the highest efficiency.

The approach of this chapter is to change a single operational condition at a time through a specific range of conditions that could be expected during a diving profile. For example, the ejectors would be tested while inside a hyperbaric chamber in order to increase the ambient pressure. The ambient pressure would be increased while maintaining a constant differential pressure across the nozzle. At each ambient pressure, the differential pressure can be varied across the nozzle through a range of predetermined pressures.

The final variable is the input gas properties. The gas properties of both the primary and secondary gases can be changed, but they will necessarily be changed together so that the primary and secondary gases are always the same. In order to determine if different gas mixtures would require additional ejector optimization, the
experimentation will be expanded to using He and CO₂ for both the primary and secondary gases. The primary-secondary combinations will be air-air, He-He, and CO₂-CO₂.

The main goal is to develop trends for the optimized ejector geometry based on changing ambient pressures, changing primary mass flow rates, and changing the operating gas mixture. The second goal is to predict the changes in the optimized ejector geometry in a way that can be experimentally evaluated with changing one or multiple operational conditions and gas properties.

Experimental model geometry was selected in Section 2.3.2 where the geometry of ten physical models was identified to be manufactured for experimental evaluation. The CFD simulations during this chapter were evaluating the performance of the same ejector model geometry that was selected in Section 2.3.2. This reduces the number of simulations at each boundary condition while providing a direct comparison for the experimentation to follow.

3.1 Simulation Plan

The models will each be subjected to multiple boundary conditions. Table 3 shows the range of simulation boundary conditions specific to each gas. These boundary conditions were selected with the purpose of evaluating conditions potentially expected in diving and for reasonable experimental testing. It is important to notice that the ambient depths are different for each gas. This was done intentionally to match the
ambient density at as many test points as possible while knowing that the primary nozzle pressure and gas properties were different. For example, the ambient density of CO$_2$ at 1.32 ata is approximately the same as air at 2 ata and He at 13.99 ata.

Table 3: Range of simulation boundary conditions specific to each gas.

<table>
<thead>
<tr>
<th>Gas</th>
<th>Ambient Boundary</th>
<th>Nozzle Total Pressure Boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1 15</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>2 29</td>
<td>79</td>
</tr>
<tr>
<td></td>
<td>3 44</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>4 59</td>
<td>109</td>
</tr>
<tr>
<td></td>
<td>5 74</td>
<td>124</td>
</tr>
<tr>
<td></td>
<td>6 88</td>
<td>138</td>
</tr>
<tr>
<td>CO2</td>
<td>1 15</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>1.32 19</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>1.97 29</td>
<td>79</td>
</tr>
<tr>
<td></td>
<td>2.63 39</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>3.29 48</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td>3.95 58</td>
<td>108</td>
</tr>
<tr>
<td>He</td>
<td>1 15</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>4 59</td>
<td>109</td>
</tr>
<tr>
<td></td>
<td>6.99 103</td>
<td>153</td>
</tr>
<tr>
<td></td>
<td>13.99 206</td>
<td>256</td>
</tr>
<tr>
<td></td>
<td>20.98 308</td>
<td>358</td>
</tr>
<tr>
<td></td>
<td>27.97 411</td>
<td>461</td>
</tr>
</tbody>
</table>

Table 3 lists every nozzle pressure for each ambient depth that has been identified as an important boundary condition. Each of the ten experimental model geometries could be simulated at every boundary condition identified in Table 3. This presented a problem of time required to run the CFD. For that reason, a priority matrix was created to allow for the most important boundary conditions to be considered first.
The basic idea for simulations was to develop connections between each set of boundary conditions. For example, it was important to evaluate the optimum geometry for air with primary nozzle pressure of 250 psid at each of the depths identified for air, for the entire range of differential pressures for air at 1 ata ambient pressure, and for the different gases with 250 psid differential pressure at 1 ata ambient pressure. In this way, a similarity reduction could be attempted for each boundary condition independently. Following independent boundary condition changes, the simulations would move to changing multiple boundary conditions simultaneously (i.e. changing differential pressures at ambient pressures other than 1 ata, etc.).
3.2 Results

3.2.1 Verification of Optimized Geometry

The simulations of the physical models were created with a primary pressure of 250 psid, an ambient pressure of 14.7 psia, and air as the primary and secondary fluids. These simulations will be used to verify that the theoretical optimization was correct, as well as to predict the actual performance of the experimental models. A quadratic curve fit was used to fit the data between the five models with different throat diameters. The predicted maximum diameters from the quadratic curve fits were used to plot the changing optimum throat diameter.

The CFD simulations for predicting the performance of the physical models, seen in Figure 15, verify that the geometry determined during optimization is indeed the optimum geometry. Additionally, one can see that the throat diameter seems to be the dominant geometric parameter influencing the performance of the small gas ejector. The exit diameter shows very little change between the minimum exit diameter (just larger than the throat diameter) and the maximum exit diameter (just smaller than the LFE entrance diameter). The throat length shows a slightly larger influence than the exit diameter; however, the range of which the length is varied is large (± 50%) compared to the change in flow it produces. The nozzle placement can significantly affect the performance if the nozzle is either too far into the throat or too far outside the throat. Again, the range of nozzle positions is large, and while near the optimum position
(within ± 0.5 in) the performance remains relatively constant.

Figure 15: CFD evaluation of the level curves of the simulation cost function hypersurface. This figure is similar to Figure 14 but is showing the results of simulating the actual physical geometry of the experimental models.

The throat diameter, on the other hand, has a very significant influence on the efficiency. The throat diameter has a very distinct maximum and falls off sharply on both sides of that maximum. It also displays the largest change in efficiency with the smallest percent change in nominal value.
3.2.2 Changing Boundary Conditions and Gas Properties

It will also be important to evaluate the ejector physical models for varying the boundary conditions and gas properties. Each boundary condition or gas property is varied independently from the others and then dependently in order to determine if any trends exist.

![Figure 16: The results for model BBB with an air-air ejector at 250 psid nozzle pressure. There is a trend for the optimum throat diameter increasing with increasing ambient pressure.](image)

It is seen in Figure 16 that the optimized geometry can change with changing ambient pressures. The trend that developed was an increase in the optimum throat diameter as the physical model was subject to increasing ambient pressures.

Additionally, as the ambient pressure is increased, the overall efficiency of the system...
also increases. This increase in efficiency is fortunate for CO$_2$ scrubbing capabilities as a minimum volume flow rate must be maintained.

Figure 17: The results for model BBB at 250 psid nozzle pressure with different gases identify a trend for shifting the optimum throat diameter when changing the gas type. In Figure 17 the optimum throat diameter shifts to larger diameters with depth. It is also noted that the optimum throat diameter for He at surface conditions is smaller than the optimum throat diameter for air at surface conditions, and the optimum throat diameter for CO$_2$ at surface conditions is higher. This shows a dependency on gas type for shifting the optimum throat diameter. Figure 17 also shows each gas follows similar trends for increasing optimum throat diameters and increased efficiency with increasing ambient pressures.
The ambient pressures were chosen in order to match the gas density of the ambient fluid inside the system. For example, the air at 3.00 ata in Figure 16 has the same density as the CO$_2$ at 1.97 ata and the He at 20.98 ata of Figure 17. Therefore, the He ejectors were subjected to a much larger range of ambient pressures and correspondingly had a much larger range in efficiency, whereas CO$_2$ was subject to a smaller range of ambient pressures and correspondingly showed a much smaller range of efficiency.

![Graph](image)

**Figure 18:** The results for model BBB with an air-air ejector at 1 ata ambient pressure identify a trend for the shifting of the optimum throat diameter with gas type and nozzle differential pressure.

The left plot of Figure 18 shows the dependence of optimum throat diameter with changing gas properties. The optimized throat diameter decreases with He and
increases with CO2. Additionally, the efficiency has increased with CO2 and decreased with He. The plot on the right side of Figure 18 is for air at 1 ata and changing the differential pressure across the primary nozzle. It can be seen that the optimized throat diameter decreases with lower differential pressures and increases with higher differential pressures. Additionally, the efficiency increases when the differential pressure decreases.

It is important here to summarize the effects that can be expected with changing boundary conditions and gas properties. Table 5 displays the trends identified for the optimum throat diameter changing relative to the optimized boundary conditions of an air-air ejector at 1 ata ambient pressure and 250 psid.

Table 5: The trends associated with changing the boundary conditions and gas properties when referenced to the values at the optimized conditions.

<table>
<thead>
<tr>
<th>Gas Type</th>
<th>Optimized Conditions</th>
<th>Increasing Ambient Pressure</th>
<th>Increasing Differential</th>
<th>Decreasing Differential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_T$</td>
<td>$\eta$</td>
<td>$D_T$</td>
<td>$\eta$</td>
</tr>
</tbody>
</table>

*References Air at Optimized conditions  
**All other trends reference the same gas at optimized conditions

The summarized information in Table 5 shows that the trends are consistent for how the throat diameter should be increasing or decreasing based on a change of boundary conditions. For example, when the differential pressure was increased, the
throat diameter of the ejector should increase as well to maintain optimum performance regardless of which gas is being used. Additionally, the gas properties are important at any particular combination of boundary conditions. Therefore, if the optimized system uses air for the primary and secondary gases, and the gas is changed to helium, the throat diameter should be decreased appropriately.

These trends are important to understand as they give the design engineer an intuitive guide as to how an ejector should be designed for any combination of boundary conditions and gas properties. It does not, however, provide a magnitude of the change in physical dimensions, only the trend as to whether the geometry should be increased or decreased.

3.3 Discussion

The results presented in Section 3.2 show that all of the input conditions do, in fact, change the optimum geometry. The results for simulating changing boundary conditions seem to have similarity throughout each of the plots. To evaluate these effects, it is important to look at non-dimensional parameters in order to potentially establish a link between optimum geometry and maximum efficiency when deviating from the optimization design point.
3.3.1 Non-dimensional Parameters

The development of the non-dimensional parameters discussed can be seen in Appendix A. The results of the Buckingham Pi Theorem are shown in Equation (17).

\[
\frac{\dot{M}_s}{M_p} = f \left( \text{Re}_N, \frac{D_T}{D_N}, \frac{L_T}{D_N}, \frac{P_N}{P_a} \right)
\]  

(17)

Where the Pi groups are then labeled:

\[ \eta = 1 + \frac{\dot{M}_s}{M_p} = \Pi_1 \quad \Pi_2 = \text{Re}_N \quad \Pi_3 = \frac{D_T}{D_N} \quad \Pi_4 = \frac{L_T}{D_N} \quad \Pi_5 = \frac{P_N}{P_a} \]

The cost function is the entrainment ratio which is a function of four Pi groups. The Reynolds Number \( \Pi_2 \) is defined at the nozzle exit plane. The next two, \( \Pi_3 \) and \( \Pi_4 \), are both physical geometry parameters being optimized. The last Pi group \( \Pi_5 \) is the pressure ratio of the nozzle exit plane to ambient conditions. It was shown in Section 1.3.5 of the Literature Review that all of the flow conditions for the fully expanded primary jet can be expressed as functions of this pressure ratio. Therefore, a relationship between the cost function and any optimized parameters with \( \Pi_5 \) can be expressed in terms flow properties derived in Section 1.3.5. In other words, the flow parameters of the fully expanded primary jet can be used for scaling and still satisfy the non-dimensional analysis of Equation (17) because they are all functions of the pressure ratio. The first non-dimensional parameter is the Reynolds Number of the primary flow exiting the nozzle in Equation (18):
\[ \text{Re}_N = \frac{\rho_N V_N D_N}{\mu_N} \]  

where \( V_N, D_N, \rho_N \) and \( \mu_N \) are the velocity, diameter, density, and dynamic viscosity local to the nozzle exit plane. The nozzle diameter is a constant during simulation and experimentation. The velocity and viscosity are functions of the local temperature and gas properties. In the case of air as the primary and secondary gas, the fluid properties are initially the same. Because the flow is sonic (for the majority of the time) and the stagnation temperature is held constant at room temperature, the only parameter that is varying is the density, which is a function of temperature (invariant) and pressure. In Equation (18), the Reynolds number increases proportionally with increasing primary nozzle pressure and ambient pressure. This increase also corresponds with an increase in the diameter ratio presented in Table 5 for increasing primary and ambient pressures.

It was shown in Section 1.3.5 that the pressure ratio at the nozzle exit plane can influence the properties of the final fully expanded primary jet. The equations in Section 1.3.5 were used to develop functional relationships between the flow parameters and both the optimum throat diameter and the maximum efficiency. The effective diameter ratio of Equation (6) is re-written in terms of \( \Pi_z \) in Equation (19).

\[ \frac{D_{\text{EFF}}}{D_N} = \sqrt{\frac{2\gamma M_N^2 \Pi_z (\gamma + u - 1) - (\gamma - 1)(\Pi_z - 1)^2}{2\gamma M_N^2 + 2\gamma (\Pi_z - 1)}} \]  

(19)
The ejector performance in this thesis can be thought of as a momentum transfer from the primary flow to the secondary flow. When discussing momentum, the important parameters are density, area, and velocity. The area ratio is simply a function of the effective diameter ratio of Equation (19). It is also relevant to include a density relationship in the scaling. The velocity of the secondary fluid is unknown, and because of the importance velocity plays in momentum transfer, it was determined relevant to include the Convective Mach Number as a measure of relative impact of the primary flow turbulence on the secondary fluid. The effective density ratio of Equation (4) is re-written as a function of $\Pi_5$ in Equation (20), and the Convective Mach Number of Equation (7) is re-written in terms $\Pi_5$ and the effective density ratio in Equation (21).

$$\frac{\rho_{\text{EFF}}}{\rho_N} = \frac{2\gamma M_N^2 \Pi_5}{2\gamma M_N^2 \Pi_5 (\gamma + \Pi_5 - 1) - (\gamma - 1)(\Pi_5 - 1)^2}$$ (20)

$$M_C = \frac{1}{\sqrt{\gamma R}} \left[ \frac{(\gamma + \Pi_5 - 1) - (\gamma - 1) \left( \frac{\Pi_5 - 1}{\gamma M_N \Pi_5} \right)^2}{1 + \left( \frac{\rho_{\text{EFF}}}{\rho_u} \right) \frac{1}{2}} \right]$$ (21)

These non-dimensional groups are used in the following sections to develop correlations between the boundary conditions and the optimum geometry and corresponding maximum efficiency.
3.3.2 Similarity for Optimum Throat Diameter

The plots in this section present the geometry that corresponds with the maximum efficiency at the given boundary conditions. The maximum point was determined by maximizing the second order polynomial best fit between the five throat diameters at any given set of boundary conditions. As an example, a quadratic curve was fit to the data for air at 1 ata and 250 psid in Figure 18, and the maximum of that quadratic function determines the optimum diameter which is the first data point (lowest) of the 250 psid data set in Figure 19.

![Figure 19: Similarity between ambient boundary conditions with constant nozzle differential pressure for an air-air ejector.](image)

Figure 19 presents data for an air-air ejector operating at three different nozzle differential pressures, while increasing the ambient pressure from one atmosphere.
absolute to six atmospheres absolute. It can be seen that the optimum geometry between ambient boundary conditions is a fairly strong function of the Reynolds Number with constant differential pressure. However, when the differential pressure is changed, there is a break in the similarity. The range of diameter ratios for the physical models is roughly between 20 and 36. Any data for the nozzle diameter outside this range was extrapolated, although kept for comparison within the plots.

When the primary pressure is the only boundary condition changed, the effective change in the primary flow parameters can be expressed as a function of the changing underexpansion ratio at the nozzle exit plane. Figure 20 shows the relationship between the optimum diameter ratio and the Reynolds Number combined with the effective diameter ratio.
Figure 20: Correlation between optimum throat diameters for an air-air ejector while changing both the differential pressure and ambient pressure.

The three differential pressures at all six different ambient pressures collapse with a semi-log relationship. Looking back to Equation (15), if the gas is changed, it will change both gas properties which are the density and the viscosity; therefore, it depends on the gas selected whether the Reynolds Number will increase or decrease for a change in gas. The Reynolds Number is used again in Figure 21 to establish a correlation between optimum throat diameters with changing gases and increasing ambient pressures.
There is a distinct similarity between the optimum throat diameters for each gas while increasing the ambient pressure. The flow properties, and hence the underexpansion ratio, are governed by the Mach Number and specific heat ratio of the gas in question. Additionally, the expansion outside of the nozzle is affected by changing the specific heat of a gas. Therefore, the effective diameter ratio was again selected to develop a correlation between separate gases.

Figure 21: Similarity between optimum throat diameters with ambient pressure changes for individual gases at 250 psid differential pressure.
Figure 22: The correlation between optimum throat diameters for changing gases and ambient pressures while maintaining 250 psid differential pressure.

Figure 22 shows a strong logarithmic relationship between the optimum throat diameters when both the gas properties and ambient pressure are varied. Thus far, ambient pressure and primary pressure have been connected, and the gas properties and ambient pressure have been connected. In order to complete the correlation, additional points were added for changing multiple boundary conditions simultaneously. The same non-dimensional Pi groups from previous plots were used with the correlation in Figure 23 with good results.
Figure 23: Correlation between optimum throat diameter ratios with all input conditions. The optimum diameter ratio displays an exponential approach relationship with Reynolds Number and the effective diameter ratio.

3.3.3 Similarity for Maximum Efficiency

It is not only important to understand how the optimum geometry changes with gas properties and boundary conditions, but it is also important to be able to estimate the performance of the ejector at the optimum point. The maximum efficiency was determined from the same quadratic curve fits that were used to determine the optimum throat diameter. Initially, the same Pi groups from the correlation with the diameter ratio were used in order to determine any similarity for the maximum efficiency.
There exists similarity between maximum efficiency for the differential pressure curves seen in Figure 24. Now the momentum transfer becomes important for efficiency, and the effective density ratio and Convective Mach Number are introduced. Additionally, the area ratio is only a function of the primary flow, and therefore, the exponents for each non-dimensional group were determined manually for a best fit.
Figure 25: Correlation between the maximum efficiency and changing differential and ambient pressures. The exponents of the Pi groups were manually optimized.

The gas properties were also seen to influence the overall performance. Using the same non-dimensional groups and exponents as in Figure 25, the correlation with changing gas properties and ambient pressures are shown in Figure 26.
Figure 26: The correlation between the maximum efficiency and gas properties at changing ambient pressures. The optimized exponents were used from Figure 25.

Again there exists a strong logarithmic relationship for the maximum efficiency and the non-dimensional Pi groups selected. The same conditions are present here as they were with the diameter ratio correlation to combine all of the maximum efficiency plots into a single correlation plot. Although the exponents were optimized for Figure 25, it was decided to complete an additional, manual exponent optimization for the correlation shown in Figure 27.
Figure 27: Correlation for the maximum efficiency for all operational conditions with manually optimized exponents showing a strong linear relationship.

The significance of the correlation presented in Figure 27 is easier to grasp when the range of conditions in which the correlation is valid are explicitly displayed. Table 6 below lists the operational conditions in which the ejector was evaluated. For diving systems, there are mainly four controlled inputs to the ejector system that can be varied. These are the primary fluid properties, the primary fluid thermodynamic state, the ambient fluid properties, and the ambient fluid thermodynamic properties. For most systems, the operational conditions are selected and the ejector geometry is optimized to operate at those particular conditions. But it can be seen, both in this work and
literature, the geometry, and inherently the performance, are largely dependent on these input conditions. Therefore it would seem that engineering for diving systems would require optimization of every individual system that included an ejector.

**Table 6: Range of boundary conditions for the development of the correlation plots of Figure 23 and Figure 27.**

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_a$</td>
<td>$P_{o,\text{min}}$</td>
<td>$P_{o,\text{max}}$</td>
<td>$P_a$</td>
<td>$P_{o,\text{min}}$</td>
</tr>
<tr>
<td>1</td>
<td>14.7</td>
<td>65</td>
<td>465</td>
<td>1.00</td>
<td>14.7</td>
</tr>
<tr>
<td>2</td>
<td>29.4</td>
<td>80</td>
<td>480</td>
<td>1.32</td>
<td>19.4</td>
</tr>
<tr>
<td>3</td>
<td>44.1</td>
<td>95</td>
<td>495</td>
<td>1.97</td>
<td>29.0</td>
</tr>
<tr>
<td>4</td>
<td>58.8</td>
<td>110</td>
<td>510</td>
<td>2.63</td>
<td>38.7</td>
</tr>
<tr>
<td>5</td>
<td>73.5</td>
<td>125</td>
<td>525</td>
<td>3.29</td>
<td>48.4</td>
</tr>
<tr>
<td>6</td>
<td>88.2</td>
<td>140</td>
<td>540</td>
<td>3.95</td>
<td>58.1</td>
</tr>
</tbody>
</table>

The greatest achievement of this correlation is combining each of these possible inputs into a single plot for selecting the optimum diameter of an ejector. Additionally, a correlation was established to estimate the maximum performance, which also depended only on these possible inputs. These correlations require no information to be known or assumed from the secondary flow inside the ejector. Therefore, it would give the engineer a quick method to determine if the performance required under certain conditions could be achieved, and if so, what the corresponding optimum throat diameter would be for that given condition. These correlations are good, but they are not exact; however, the idea is that test and evaluation of a system can be reduced when an approximate optimum geometry can be selected from a known correlation.

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4. **Experimentation**

The primary goal for the experimentation is to determine if the numerical optimization technique used provided the most optimized model for the given conditions for which it was optimized. This is important in order to trust that the CFD software coupled with the optimization method could be used during a design phase in the future in order to reduce the amount of experimental testing necessary during the design process. The secondary goal for the experimentation is to evaluate the trends for the shifting of the optimized geometry with changing input conditions developed in Chapter 3.

4.1 **Materials**

1. Small Gas Ejector Models (x10)
2. Laminar Flow Element (LFE)
3. Hyperbaric Chamber
4. XY Rotary Table
5. Ejector Mount
6. Nozzle Mount
7. Dial Indicator
8. Pressure Sensors (x3)
9. Temperature Thermistors (x2)
10. Data Acquisition Control Unit (DACU)
11. Computer Recording System
12. Coupler, Ejector to Tube Connection
13. Clear Plastic Tubing with 2 in Inside Diameter
14. O-rings
15. Pressure Gauges
16. High Pressure (HP) Gas Sources (air, CO₂, He)
17. Pressure Reducing Regulator
4.2 Method

The mass flow rates exiting the small gas ejector can be measured using a Laminar Flow Element (LFE). A LFE measures the pressure drop across a flow restriction, and based on a calibration curve and reference viscosity, one can calculate the flow rate exiting the LFE. The primary mass flow rate entering the ejector can be calculated using compressible flow equations for flow through a nozzle and will be verified using a small size LFE. These two mass flow rates provide the data necessary for plotting the efficiency for the overall ejector performance.

Each of the ten experimental models that were manufactured was evaluated through a range of experimental configurations in order to develop similar trends to those determined in Chapter 3. Each model was tested first at the optimization conditions, followed by changing the gas type and input conditions.

4.2.1 Experimental Setup

The experimental setup can be seen in Figure 28. A high pressure (primary) gas source supplies the gas to a manual pressure reducing regulator. The gauge downstream of the regulator indicates the gauge pressure of the primary gas that is injected into the system through a converging nozzle and into the ejector.
Figure 28: Experimental setup for testing the performance of the small gas ejector geometry selected from the CFD optimization.

The primary gas entrains the ambient (secondary) gas around the entrance of the ejector to flow into the ejector. The primary flow transfers momentum to the secondary flow inside the ejector, and the mixed total flow exits the ejector and into the straight tube. There is a coupling with O-rings on both ends that prevents gas leakage from the exit of the ejector or the entrance of the straight tube.

The straight tube is a requirement of the LFE which helps the flow become laminar through the LFE restriction. The flow enters the LFE where the pressure drop from the entrance to the exit of the element is measured. This pressure drop corresponds to a volume flow rate when the gas properties are known. The flow then exits the LFE through a straight pipe and back into the ambient gas.
It is important to note all of the measurements being taken during this process. First, the primary pressure gauge must be monitored and recorded in order to maintain the appropriate differential pressure between the primary gas and secondary gas. The ambient temperature is monitored for equilibrium, especially after pressurizing the hyperbaric chamber where the ambient temperature may rise significantly. The ambient pressure is continuously monitored and vented to account for the increase in mass from the primary gas source to maintain a constant ambient pressure. The temperature is also measured at the entrance to the LFE to be used for a temperature correction factor for the viscosity of the gas inside the LFE for a more accurate flow rate output. The differential pressure across the LFE is measured in order to maintain the flow rate information for the gas flowing through the LFE.

Each of the measurements have wires that are routed through the Bulkhead Coupling to the DACU and then to the computer for data collection. The DACU reads voltage outputs from the thermistors and the pressure gauges, and then transfers those readings to the computer. The computer uses a DOS based computer program to convert the voltages to the appropriate temperatures and pressures, respectively.

### 4.2.2 Ejector Models

The ejector models were rapid-prototyped by a process called Fused Deposition Modeling (FDM) by a company named Stratasys. FDM is a rapid prototyping process in which a 3D printing machine creates the part directly from a digital stereolithography
file. The Ejector models were created from ABS thermoplastic material with a tolerance of ±0.0015 inches per inch stated by the manufacturer.

Figure 29: An ejector model with the individual features labeled. Functional features include the threading, knurling, and O-ring groove. Identification features include the Ejector ID and the color of the model.

The ejector models similar to the one pictured in Figure 29 were all manufactured with specific features used for identification and assembly into the test fixture. The functional features include the threading, knurling, and O-ring groove. The threading on the inlet of the ejector models is what allows the ejector to be switched in and out of the text fixture rapidly. The knurling was included in order to facilitate easier assembly by hand. An O-ring groove was included in the ejector design in order to create a gas tight seal between the ejector body and the Coupler to the straight tube.
The identification features included the Ejector ID (see Table 2 in Section 2.3.2) which was built into each model from the part file during the FDM process. Additionally, the models were color-coded based on the specific geometry that was being evaluated with that model. For example, the blue models were the four models with the throat diameter being varied. The green models were the two models with the throat length being varied. The yellow models were the two models with the exit diameter being varied. The Optimized model was a plain white color.

4.2.3 Laminar Flow Element

The main measuring device used in this setup was the Meriam Laminar Flow Element (LFE). An LFE is a flow restriction that uses a differential pressure across an element of capillary passages to measure flow of gases. Meriam defines these passages as “capillary” because the effective diameter of each passage is on the order of thousandths of an inch while the length is several inches. The reduced dimensions of the capillary passages also work to the advantage of forcing the flow to be laminar through the LFE, which provides a linear relationship between pressure drop and volume flow rate. It was important to design enough straight tube before and after the LFE in order for the laminar assumption and calibration curves to be accurate. Meriam recommends at least ten diameters upstream of the LFE and five diameters downstream of the LFE.
Figure 30: The experimental setup of the LFE used to measure the flow rate exiting the ejector. The pressure drop and temperature entering the LFE are the measurements needed to calculate the flow rate by the calibration curve.

Figure 30 shows the LFE in the experimental configuration. The pressure drop across the LFE and the temperature at the LFE entrance are measured as seen in Figure 30. The LFE calibration curves were generated for using air at standard temperature and pressures. However, this did not present a problem since the calibration for measuring ACFM with the LFE was dependent on viscosity only. Therefore, it was imperative that the temperature was measured with the thermistor just upstream of the LFE entrance seen in Figure 30. With the gas properties known, easy temperature and viscosity correction factors could be applied in order to generate the correct calibration curve during experimentation. The standard calibration curve for the LFE with air at standard temperature and pressure is shown in Figure 31.
Figure 31: The calibration curve of air at standard temperature and pressure for the LFE is linear with 23.28 acfm for 8 inH₂O.

4.2.4 Hyperbaric Chamber

A 30 cubic foot Hyperbaric Chamber was used in order to change the ambient pressure during experimentation. The Hyperbaric Chamber was pressurized with the same gas that is used for the primary nozzle. After each pressure change, time was given for the temperature to become steady inside the chamber before data collection began. Mass was continuously added to the hyperbaric chamber via the primary nozzle so a vent was needed in order to maintain a constant ambient pressure.

During the He and CO₂ gas experiments, the hyperbaric chamber needed to be purged in order to ensure the purity of the gas encompassing the internal volume was
the same as the purity of the primary gas being injected. This was important for the calibration of the LFE, as well as to have a known density for both the primary and secondary gases. The Hyperbaric Chamber was pressurized to 15 psig followed by depressurizing for a total of eight cycles. These cycles added enough mass of the gas being used for the secondary gas to be above 99% purity. The ultimate purity could only be as high as the gas in the HP storage tanks; therefore, it was assumed that the HP gas sources were 100% pure for the calculations of the purge procedure.

4.2.5 Design of Ejector Experiment

The experimentation itself was accomplished by regulating the pressure upstream of a converging nozzle. This nozzle was discharged directly into the opening of the ejector body with the secondary gas surrounding the nozzle being entrained into the ejector body. The ejector body setup was built on an XY Rotary Table with an Ejector Attachment Block where the ejectors could be threaded into place as seen in Figure 32.
Figure 32: Top view of the ejector test setup built on top of an XY Rotary Table.

This setup made it possible to exchange ejectors rapidly and without any adjustments between experiments. Additionally, it allowed for translation of the entire ejector axially so that it was possible to adjust the nozzle position. The dial indicator was zeroed at the position selected by the CFD Optimization and was used as an indicator for positioning the nozzle both into and away from the ejector inlet.

The exit of the ejector was connected to a 2 in inner diameter clear plastic tube via the coupler with an O-ring positioned on either side to prevent any gas leakage. The flow then continued down the plastic tubing and into the LFE. The pressure drop is measured across the LFE, and with a known gas viscosity, the mass flow rate can be calculated for the flow exiting the LFE.
The entire ejector evaluation system was placed inside of a 30 cuft hyperbaric chamber in order to increase the ambient pressure for the secondary gas and outlet input conditions. The entire time, the temperature inside the hyperbaric chamber and the temperature just before entering the LFE were monitored. The pressure inside the hyperbaric chamber was monitored and controlled. The pressure loss across the ejector body and tubing, as well as the pressure loss across the LFE, were monitored.

![Image of hyperbaric chamber and ejector setup](image)

**Figure 33: The ejector setup inside the hyperbaric chamber.**

The DACU was connected to each of the monitoring probes. This raw data was transferred to the computer in voltages, and using the appropriate conversion factors, presented in typically expected values. The DACU is simply a voltage reader which transfers the raw data to the computer to be collected. The Volume Filler Tank in Figure
33 was filled with water in order to decrease the amount of gas needed to pressurize the Hyperbaric Chamber.

### 4.3 Data Collection and Error Analysis

The raw data collected for each ejector was a fluctuating value highly dependent on the ejector and input conditions. For example, the fluctuations of the output data were much different for any particular model when testing He at 1 ata and 50 psid than it was for testing CO₂ at 4 ata and 450 psid.

![Example raw data output during experiment with Model BBB with air-air at 1 ata ambient pressure and 250 psid nozzle pressure.](image)

**Figure 34:** Example raw data output during experiment with Model BBB with air-air at 1 ata ambient pressure and 250 psid nozzle pressure.
Each data point in the raw data output of Figure 34 was taken every ten seconds during the experiment. The raw data is accumulated continuously during the ten seconds and the average is then reported to the computer as a single data point. It is clearly visible that the raw data is unsteady for individual readings; however, the time average of the raw data does converge after a sufficiently long period of time. The question then becomes - what is a long enough period of time of taking data in order to consider an experimental value converged?

Figure 35: Example of the same raw data output of Figure 34 for collecting 4 minutes of data for the experiment with model BBB with air-air at 1 ata ambient pressure and 250 psid nozzle pressure.

The time averaging of the data converges fairly quickly around the 4 minute mark. There is a loss in some accuracy for shortening the experiments, but there is a
much greater gain in the time required to complete the experiments. A minimum of 4 minutes was determined to be the threshold for collecting a good experimental data point.

The raw data files were analyzed to determine the uncertainty for the data collected. A Shapiro-Wilk normality test was completed for every experimental run, and it was determined that the raw data sets were normally distributed (Shapiro & Wilk, 1965). An example of a Normal Probability plot generated during a Shapiro-Wilk normality test is shown in Figure 36. This allowed for a Monte-Carlo simulation to be used in order to establish an uncertainty for the averaging of the raw data.

![Figure 36: Example of a probability plot for a two sided Shapiro-Wilk Normality test with a standard significance of 0.05. A linear normal probability occurs when the population is normally distributed.](image-url)
Monte-Carlo simulations were completed by analyzing each individual raw data file. The mean and range of each data file were calculated and used as inputs to the Monte-Carlo simulation. A standard normal distribution was generated and a sample set of data with the same number of data points was created using the raw data mean and range. The sample sets were created assuming the range of the raw data file was within 2-sigma of the standard normal distribution. The mean of the Monte-Carlo simulation was then computed and compared to the original mean giving an approximate error for that particular simulation. There were 100,000 Monte-Carlo simulations run for each raw data file with the overall maximum error presented in Table 7.

Table 7: The overall maximum error seen during Monte-Carlo simulations for each raw data set. The experimental data is assumed to be within 6.11% of the actual value with 99% confidence.

<table>
<thead>
<tr>
<th>Monte-Carlo Simulation</th>
<th>Confidence Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>99%</td>
</tr>
<tr>
<td>4 min run time</td>
<td>6.11%</td>
</tr>
<tr>
<td>50 min run time</td>
<td>2.11%</td>
</tr>
</tbody>
</table>

The time averaging of the experiments for a minimum of 4 minutes proved to be a reliable length of time for data collection. Although a significantly higher degree of accuracy could have been achieved by collecting data for 50 minutes, the 4 minute average is accurate enough and the time saved in reducing each experiment length was
valuable in completing the desired range of configurations. For simplicity, each data point was assumed to have the maximum percent error from the Monte Carlo simulations presented in Table 7.

### 4.4 Experimental Approach

The approach to evaluating the ejectors experimentally almost mirrored the approach for the CFD simulations for the ejectors with changing input conditions. The results for the CFD simulations were used as a guide to determine which experimental tests would be the most conclusive and provide the best data. The same ranges of pressures were tested as shown in Table 3. A new priority matrix shown in Table 8 was developed as a guide to the order of experimentally evaluating the ejector models.

#### Table 8: Priority matrix for determining the order of experimentation.

<table>
<thead>
<tr>
<th>Model ID</th>
<th>Air Surface</th>
<th>Air Depth</th>
<th>CO2 Surface</th>
<th>CO2 Depth</th>
<th>He Surface</th>
<th>He Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>D - D - D</td>
<td>High</td>
<td>Low</td>
<td>Med</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>B - B - B</td>
<td>High</td>
<td>High</td>
<td>High</td>
<td>High</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>AA - B - B</td>
<td>High</td>
<td>Low</td>
<td>Med</td>
<td>Low</td>
<td>Med</td>
<td>Med</td>
</tr>
<tr>
<td>A - B - B</td>
<td>High</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>C - B - B</td>
<td>High</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>CC - B - B</td>
<td>High</td>
<td>Low</td>
<td>Med</td>
<td>Low</td>
<td>High</td>
<td>Med</td>
</tr>
<tr>
<td>B - A - B</td>
<td>High</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
<td>Med</td>
<td>Med</td>
</tr>
<tr>
<td>B - C - B</td>
<td>High</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
<td>Med</td>
<td>Med</td>
</tr>
<tr>
<td>B - B - A</td>
<td>High</td>
<td>Low</td>
<td>Med</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>B - B - C</td>
<td>High</td>
<td>Low</td>
<td>Med</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
</tr>
</tbody>
</table>
The goal for prioritizing experiments was to minimize the time by evaluating both the most important input conditions (i.e. the optimized model geometry for all conditions) as well as the least time intensive (i.e. a full range of primary pressures was quick once at a given ambient pressure) while eliminating tests with less importance (i.e. nozzle placement at depth would have required chamber pressurization for each data point and was neglected).

4.5 Results

To prove that the optimization technique was successful, the physical ejector models were experimentally evaluated with the input conditions used during the optimization process.

Figure 37: Level curves of the experimental cost function hypersurface.
These results provide a reasonable expectation of using CFD software for optimization of a small gas ejector. The optimized geometries are identified by solid triangles in each plot of Figure 37 with quadratic curve fits represented with solid lines to illustrate the estimated maximum.

Figure 38: A combination of the first plot in Figure 15 and Figure 37 for comparing the experimental results to the CFD simulations. The experimental results agree well with the CFD simulations in both magnitude and maxima.

The experimental data for the level curve of the cost function dependency on throat diameter pictured in Figure 38 is in good agreement with the CFD simulations for the same conditions. In fact, the experimental results were very reliably in agreement with the CFD simulations throughout experimentation.
4.5.1 Perfectly Expanded Sonic Flow and Maximum Efficiency

The following results were from an experiment that was attempting to determine for what pressure ratio across the nozzle would provide the maximum mass flow efficiency. The CBB model was tested with an air-air ejector for additional primary pressures at an ambient pressure of 5 ata in order to provide a complete range of subsonic, to sonic, to underexpanded sonic conditions at the nozzle exit.

For air at 5 ata, the ambient pressure inside the Hyperbaric Chamber was approximately 73.5 psia. The critical total pressure for the nozzle to be sonic would be approximately 140 psia with a pressure ratio of total primary to total secondary pressure of ~1.9. The majority of the experiments used a range from 50 psid to 450 psid. While this experiment used those same primary pressures, additionally, it was evaluated at approximately 21 psid, 36 psid, and 101 psid. These additional pressures were used in order to develop a curve around the point where the nozzle becomes sonic at 140 psia (66.5 psid).
Figure 39: The ejector efficiency versus the total pressure ratio across the nozzle.

Starting with subsonic pressure ratios in Figure 39 (left), as the primary pressure approaches perfectly expanded sonic flow, or when the pressure ratio ≈ 1.9, the efficiency is increasing. It can be seen that the efficiency continues to increase as the nozzle becomes underexpanded until it reaches a maximum efficiency around when pressure ratio ≈ 3.0. As the nozzle becomes more underexpanded the efficiency begins to decrease again, but does so at a much slower rate than on the subsonic side.

4.5.2 Optimal Nozzle Placement

The four independent parameters that were evaluated independently were the throat diameter, throat length, exit diameter, and nozzle distance to the throat entrance. The three independent parameters that were not discussed much throughout this work
were the throat length, exit diameter, and nozzle distance to the throat entrance. The cost function was much less sensitive to each one of these parameters independently as was shown in Figure 14 and Figure 15. However, the mass flow efficiency does depend on these parameters to a certain extent which was discussed in the literature review.

The optimum throat length and exit diameter did not change with input conditions, while the nozzle placement was dependent on the input conditions. Figure 40 shows the optimum nozzle placement normalized to the throat diameter for changes in the primary nozzle pressure.

![Figure 40: Optimum nozzle distance for changing primary pressure.](image)

The zero mark on the x-axis of Figure 40 represents the ejector throat entrance plane where positive values indicate nozzle placement inside the ejector throat and
negative values indicate nozzle placement away from the throat entrance. Figure 40 shows that the optimal nozzle placement has a dependence on initial conditions. The solid lines are rational curve fits with confidence intervals shown only for the data with greatest estimated error for clarity. The nozzle placement shows a dependency on the primary nozzle pressure with a much larger sensitivity when the primary pressure is decreased.

4.5.3 Similarity

The experimental data for all input conditions is plotted with the same non-dimensional parameters that were used to develop the correlation for the optimum diameter ratio and maximum ejector efficiency for Figure 23 and Figure 27.
Figure 41: Correlation for the optimum diameter ratio for maximum efficiency as an exponential approach function of the nozzle Reynolds Number and the effective diameter ratio.

The optimum throat diameter corresponding to the maximum efficiency for any given input condition is presented in Figure 41 as a function of the Reynolds Number and the effective diameter ratio. The maximum error of all data points is represented by the error bar. The optimum throat diameter increases with Reynolds Number and the effective diameter of the primary jet and approaches an asymptotic value. Again, any data points outside the diameter ratio range of 20 to 36 were extrapolated from the quadratic curve fit of the level curve.
Figure 42: The correlation for the maximum efficiency has a linear dependence with Pi groups for the experimental data.

The maximum efficiency for any given input condition is shown in Figure 42 as a linear function of Reynolds Number, the effective diameter ratio, the effective density ratio, and the Convective Mach Number.

4.6 Discussion

Figure 37 is a clear indicator that the optimization method used was successful at selecting the optimum geometry for each of the four independent parameters. It is also clear that the throat diameter is the most dominant parameter while the efficiency is least sensitive to the exit diameter. The throat length and nozzle placement have some
significant sensitivity, although they were evaluated through much larger ranges than either the throat diameter or exit diameter.

The CFD simulations in Figure 38 estimated the mass flow efficiency to be less than the experimental data shown. This is an interesting result because typically during CFD simulations, losses due to minor effects, such as surface roughness or scale effects, are minimal, which would lead the CFD to provide an overestimate of the system performance. However, underestimation by the CFD simulations was repeated throughout the experimentation with consistent agreement between experiment and CFD, and therefore, this systematic error was considered unimportant to the overall ejector evaluation.

As the primary pressure approaches perfectly expanded sonic flow from subsonic conditions, the efficiency increases. This is an expected trend as the mass flow rate and velocity continue to increase as the primary pressure is increased until choked flow conditions with Mach 1 at the nozzle exit plane. It is also seen that as the pressure is increased and the nozzle becomes underexpanded, the efficiency continues to increase until it reaches a maximum at a value well above the pressure ratio for perfectly expanded flow. This is an indication that the turbulence created by an underexpanded nozzle promotes mixing and momentum transfer and enhances the efficiency. It is not known if this specific limit related to the pressure ratio corresponds to additional ambient pressures than five atmospheres absolute evaluated in Figure 39. There is
clearly a balance between the turbulent enhancement of the mixing and energy losses due to dissipation of energy (i.e. energy lost through a shock event).

The optimal throat length and exit diameter did not show any dependence on input conditions and were not displayed during the results. This does not mean that for any change in optimum throat diameter these values would still remain the same. In other words, it cannot be determined by this experimentation whether these two parameters have dependencies on another parameter, such as the throat diameter; however, evaluating any dependency would require additional physical models to be manufactured, and therefore, was not explored in this work. The nozzle placement did show a dependency on the input conditions seen in Figure 40. What is important here is that the experiments were evaluating independent parameters only, and while the greatest gains may be realized by optimizing the dominant parameter in the throat diameter, there may be additional increases in performance for optimizing the remaining parameters around the optimum throat diameter selected.

4.6.1 Similarity

To expand on the CFD optimization, it was important to test the physical models under various changes in input conditions and gas properties. The results of these experiments followed the CFD simulations. This information is critical in understanding the limitations to optimizing a small gas ejector for diving missions, as well as
determining that the CFD simulations could be used for expanding the optimization of the small gas ejectors for use in multiple diving systems.

The optimum throat diameter in Figure 41 approaches an asymptote as $x \to \infty$. The data follows an exponential approach function of the form of Equation (22).

$$y = A \times (1 - e^{-Bx}) + C \quad (22)$$

The constants $A$ and $C$ represent the horizontal asymptote and y-intercept, respectively. The constant $B$ is used to control the rate of response of the function. The correlation for optimum throat diameter and non-dimensional parameters was consistent between the CFD simulations and experimentation, although the experimental data showed a stronger approach to an asymptote. The coefficients of Equation (22) for both the CFD simulations and experimental data are presented in Table 9.

Table 9: This table is comparing the coefficients of the correlation curves for the optimum diameter and maximum efficiency for simulation and experimental data.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Exponential Approach Curve For Optimum Diameter</th>
<th>Linear Curve For Maximum Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A$</td>
<td>$B$</td>
</tr>
<tr>
<td>Simulation</td>
<td>26.73</td>
<td>2.56E-05</td>
</tr>
<tr>
<td>Experiment</td>
<td>22.77</td>
<td>3.32E-05</td>
</tr>
</tbody>
</table>
The correlation in Figure 42 for the maximum efficiency follows a linear relationship of the form in Equation (23).

\[ y = M \times x + B_2 \]  \hspace{1cm} (23)

The constants \( M \) and \( B_2 \) represent the slope and y-intercept, respectively. The linear relationship for the experimentation was consistent with the CFD simulations. The coefficients of Equation (23) for both the CFD and experimentation are presented in Table 9.

![Figure 43: Comparison of the exponential approach curves of the simulation and experimental data for the correlation developed for the optimum diameter ratio.](image)

Figure 43: Comparison of the exponential approach curves of the simulation and experimental data for the correlation developed for the optimum diameter ratio.
Figure 43 is a plot of the two exponential approach curves estimating the optimum throat diameter as a function of the Reynolds Number and the effective diameter ratio. Figure 44 is a plot of the two linear curves estimating the maximum efficiency as a function of the Reynolds Number, the effective diameter ratio, the effective density ratio, and the Convective Mach Number. The correlation curves for the experimental data is a good match with the correlation curves predicted during CFD evaluation.

\[
\eta_{\text{max}} = (Re_N)^{0.25} \left( \frac{\rho_{\text{EFF}}}{\rho_a} \right)^{-0.25} \left( \frac{D_{\text{EFF}}}{D_N} \right)^{-0.75} (M_C)^{-0.125}
\]

Figure 44: Comparison of the linear curves of the simulation and experimental data for the correlation developed for maximum efficiency.
5. Conclusions

5.1 Optimization

The Quasi-Newton Raphson Multivariate Optimization method presented in this thesis was successful at optimizing the small gas ejector for a specific operational condition. When analyzing the level curves of the cost function hypersurface, it was determined that the throat diameter is the dominant geometry for which the cost function is the most sensitive, as this level curve has the greatest curvature. Although it was successful, there are still limitations to this method. In the current study, eleven geometric parameters were identified as possible candidates for optimization. Due to the substantial increase in computing cost, the optimization was limited to the use of four independent parameters.

The use of correlation curves developed in Chapters 3 and 4 provide the ability to select a throat diameter based on input conditions only. A multivariate optimization could then be completed around this approximately optimized throat diameter where a larger number of parameters could be optimized with the expectation of converging with fewer iterations.

5.2 Similarity and Design Considerations

A correlation was developed where the optimum throat diameter can be selected specific to input conditions only; namely, the primary nozzle pressure, ambient
pressure, and gas properties. This result is unique in that it deviates far from the original optimization conditions yet can be predicted fairly well with changes to any of the input values. The optimum throat diameter has an exponential approach relationship with the nozzle exit Reynolds Number and the effective diameter for the expansion of the primary nozzle.

The performance of the ejector is also self-similar across all of the input conditions being changed. The entrainment ratio increases linearly with the nozzle exit Reynolds Number, the effective diameter ratio, the effective density ratio, and the Convective Mach Number. The combination of these two correlations provides valuable information for a quick estimate of optimum throat diameter for given operational conditions and subsequent evaluation of the maximum performance.

The optimum throat diameter and optimum nozzle placement were seen in Section 4.5 to be dependent on input conditions, while the throat length and exit diameter were not dependent on the input conditions evaluated in this work. The limitation is that the correlations were developed around independent evaluation of the throat diameter only. It was discussed in the literature review that the throat length, nozzle placement, and exit diameter are dependent on the throat diameter and would necessarily scale with the throat diameter; however, the exact scaling was not investigated during this work. That is to say, the correlation provides an estimate of the optimum throat diameter, but there may be additional increases in performance for
optimizing the rest of the geometry to the optimum throat diameter at any particular set of input conditions.

5.3 Recommendations

The approach taken by this thesis was to use CFD coupled with a Newton-Raphson Optimization method in order to develop an optimization process that is feasible for use during the design phase for a diving system using a small gas ejector. Additionally, CFD simulation and experimentation were used to evaluate the optimal geometry of the small gas ejector for input conditions deviating from the original optimization conditions. This section will discuss recommendations for improvement on the optimization and similarity studies presented, as well as discuss additional research areas for gas ejector optimization and design.

5.3.1 Optimization and Design Considerations

This thesis presented a multivariate optimization method in Chapter 2 that was verified as being capable at predicting the optimum geometry for a specific operational condition. The CFD setup used for the simulations was a 3D computational domain with a significantly reduced mesh density. Local mesh definitions allowed for a much higher mesh density to be concentrated in areas where the gradients were known to be large. Future use of this optimization method could utilize the axisymmetric nature of a gas ejector in order to solve the CFD simulations in a 2D computational domain. This may
be able to reduce the overall time for optimization or allow for additional parameters to be optimized in the same amount of time. There is also a potential for reducing the overall optimization time by selecting the optimal throat diameter geometry from the correlations presented in Chapters 3 and 4, followed by optimizing additional parameters around the optimal throat diameter.

The correlations presented in Chapters 3 and 4 were determined based on independent optimization of the throat diameter only. The engineer should first focus on selecting the most dominant geometry for the performance of the small gas ejector. Once the optimal throat diameter is determined, the performance for the given input conditions can be estimated. If an increase in performance is necessary, additional optimization could then take place for the parameters that are dependent on the throat diameter.

5.4 Future Research

The method of optimization presented in this thesis proved to be a useful technique for small numbers of independent parameters. The small gas ejector used in previous U.S. Navy diving systems has eleven potential geometric parameters that were identified as possible contributors to the ejector performance. The time involved with this method becomes very costly with increased numbers of independent parameters. Additionally, the input conditions and gas properties must necessarily be set during the optimization. This means for a transient system the physical design can only be
optimized for a single set of operational conditions. It is therefore important to expand
the knowledge of ejector optimization to include more of the geometric parameters as
well as additional gases and gas mixtures.

5.4.1 Additional Parameter Optimization

The experimental evaluation of the small gas ejector in this thesis was limited to
independently changing the optimization parameters. The CFD Optimization method
included dependencies between the independent parameters. The value in using the
CFD Optimization method over experimental evaluation is that for each of the optimal
throat diameters in the correlation of Figure 41 there may be a separate set of optimal
throat length, exit diameter, and nozzle placements. It would be useful to develop
correlations for optimizing the remaining geometric parameters. Selecting input
conditions that are not the same in pressures or gas types, but are predicted to have the
same optimal throat diameter using the correlation from Section 4.5.3, and optimizing
the remaining independent parameters at those input conditions, would provide insight
on whether the remaining geometry have similarity for their optimal values related to
input conditions, the throat diameter, or both.

There are still seven identified geometric parameters in Figure 4 that could be
optimized. Therefore, combining these additional parameters into future optimization
methods would be ideal. Additionally, it is not known how the correlation curves
presented in this thesis would translate to optimizing ejectors with different nozzle
diameters. It would be just as important to optimize an ejector with the same input conditions as this thesis, but using a larger or smaller nozzle, and comparing the optimal diameter ratio of the ejector throat to the new nozzle diameters.

5.4.2 Multiple Gas Systems

In reality, diving applications use mixed-gas systems. These systems may include two different pure gases as the primary and secondary fluids, as well as gas mixtures for both the primary and secondary fluids. The experimental evaluation of the current ejector used an LFE to measure the flow rate where the viscosity of the gas must be known. When using gas mixtures, this value is much harder to predict, and therefore, a substitute for the LFE as the flow measuring device may be necessary. SolidWorks Flow Simulation allows for mixed gases to be defined at the inlet boundaries of the computational domain, and therefore, could be used to optimize a mixed-gas system in the same manner as presented in Chapter 2.

5.5 Summary

Using SolidWorks Flow Simulation to evaluate the finite difference approximations for inputs to the Quasi Newton-Raphson Multivariate Optimization method proved to be a feasible optimization technique for a small system with limited independent parameters. The Flow Simulation was also shown to be a valuable analysis
tool for predicting ejector performance. Some of the important discoveries in this work are as follows:

1. The Quasi-Newton-Raphson optimization technique used in conjunction with SolidWorks flow simulation can be used as a design phase tool for small systems with a limited number of independent parameters.

2. A correlation was developed for selecting the optimum throat diameter vs. Reynolds Number at the nozzle exit plane and the effective diameter ratio.

3. A correlation was developed for predicting the ejector performance vs. the Reynolds number at the nozzle exit plane, the effective density to ambient density ratio, the effective diameter to nozzle diameter ratio, and the Convective Mach Number.

4. The throat diameter and nozzle placement had dependencies on input conditions while the throat length and exit diameter did not.

5. An estimated optimum diameter ratio and projected maximum performance can be determined with knowledge of input conditions only.
A. Dimensional Analysis

Dimensional analysis can provide the functional relationship between parameters groups and can be deduced even if the governing differential equations are unknown (Lin, et al., 1988). Without knowing the governing equations, the non-dimensional groups provide the knowledge of the existence of a relationship between the unknown cost function and non-dimensional groups on independent parameters without providing the proper scaling for an exact relationship to be developed. The important idea is that the number of independent parameters that govern the flow can be reduced and will provide a basis for analyzing similarity.

One of the difficulties of analyzing gas ejectors is the abundance of parameters involved in the potential solutions. Many times, the analysis takes on a simplified model in order to determine the parameters with the largest effects on performance. The dimensional analysis completed in this section takes a similar approach but includes as many of the input flow parameters as possible in order to establish potential relationships between physical quantities. It is important, especially when the governing equations are not known, to include any parameters that may appear in an exact mathematical problem (Lin, Segel, & Handelman, 1988).
Figure 45: The control volume considered for dimensional analysis.

The control volume depicted in Figure 45 is used in order to select the appropriate parameters used for dimensional analysis. The complete state of the primary flow at the nozzle exit can be determined with the properties $P_N$, $T_N$, $\mu_N$, $\gamma_N$, $R_N$, $\rho_N$, $V_N$, and $D_N$. The known geometry of the nozzle is $D_N$; the gas fluid properties are $\mu_N$, $\gamma_N$, $R_N$; and the flow properties are $P_N$, $T_N$, $\rho_N$, and $V_N$. These flow properties can be determined using isentropic flow equations given the fluid total pressure and total temperature upstream of the nozzle.

The secondary flow properties at the entrance to the ejector throat cannot be known exactly; however, it is assumed the secondary flow is subsonic in the ejector and fluid properties can initially be estimated as the upstream, ambient, stagnation conditions. The complete state of the secondary fluid far upstream of the throat entrance
is known by the fluid properties $P_S$, $T_S$, $\mu_S$, $\gamma_S$, $R_S$, $\rho_S$, and $V_S$. The diameter for the secondary flow can be determined by using the known geometry of the nozzle and ejector throat. The secondary gas fluid properties are $\mu_S$, $\gamma_S$, $R_S$, $P_S$ and $T_S$ and are assumed to be the known secondary stagnation properties, with $\rho_S$ calculated using the Ideal Gas Law. The initial velocity, $V_S$, for the secondary gas is assumed to be zero.

Table 10 lists the dimensions of each of the identified parameters. There are four primary dimensions in this system: length, time, mass, and temperature. Using the Buckingham Pi Theorem, there should be 11 dimensionless Pi groups.

**Table 10: List of the dimensional parameters of the non-dimensional analysis**

<table>
<thead>
<tr>
<th>Dimensional Parameters</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_N$, $D_T$, $L_T$</td>
<td>$L$</td>
</tr>
<tr>
<td>$T_N$, $T_A$</td>
<td>$T$</td>
</tr>
<tr>
<td>$R$</td>
<td>$L^2 t^{-2} T^{-1}$</td>
</tr>
<tr>
<td>$P_N$, $P_A$</td>
<td>$ML^{-1} t^{-2}$</td>
</tr>
<tr>
<td>$\mu_N$, $\mu_A$</td>
<td>$ML^{-1} t^{-1}$</td>
</tr>
<tr>
<td>$\rho_N$, $\rho_A$</td>
<td>$ML^{-3}$</td>
</tr>
<tr>
<td>$V_N$</td>
<td>$Lt^{-1}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$1$</td>
</tr>
<tr>
<td>$M_S$</td>
<td>$M t^{-1}$</td>
</tr>
</tbody>
</table>
It is automatically seen that the specific heat ratio is a dimensionless parameter itself; therefore, it is its own Pi group. Also, the specific gas constant, \( R \), and specific heat ratio, \( \gamma \), are constant gas properties, and since this thesis was interested in single gas systems, the primary and secondary specific heat ratio and specific gas constants are equal, respectively. Table 11 lists the dimensional parameters in matrix form, while Table 12 is showing the reduced row echelon form of Table 11. The non-dimensional Pi groups are determined using Table 12.

**Table 11: List of dimensional parameters in matrix form. There are four primary dimensions. The first four parameters are the repeating parameters.**

<table>
<thead>
<tr>
<th>( \rho_N )</th>
<th>( V_N )</th>
<th>( D_N )</th>
<th>( T_N )</th>
<th>( \dot{M}_S )</th>
<th>( D_T )</th>
<th>( L_T )</th>
<th>( T_A )</th>
<th>( R )</th>
<th>( P_N )</th>
<th>( P_A )</th>
<th>( \rho_A )</th>
<th>( \mu_N )</th>
<th>( \mu_A )</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>-3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>-1</td>
<td>-1</td>
<td>-3</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>M</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>t</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>T</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

123
Table 12: Reduced Row Echelon Form of Table 11. The Pi groups of the parameters on the right are established in terms of the repeating parameters on the left.

<table>
<thead>
<tr>
<th>$\rho_N$</th>
<th>$V_N$</th>
<th>$D_N$</th>
<th>$T_N$</th>
<th>$\dot{M}_S$</th>
<th>$D_T$</th>
<th>$L_T$</th>
<th>$T_A$</th>
<th>$R$</th>
<th>$P_N$</th>
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<th>$\rho_A$</th>
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The first Pi group turns out to be the mass flow ratio of the primary flow to the secondary flow in Equation (24). This Pi group is the cost function during the optimization technique and the measurable variable during experimentation.

$$\Pi_1 = \frac{\dot{M}_S}{\rho_N V_N D_N} = \frac{\dot{M}_S}{\dot{M}_P} \quad (24)$$

The second and third Pi groups, in Equations (25) and (26), happen to be related to the geometry of the ejector itself. These two were expected and come up often in the literature as parameters that can be optimized to increase ejector performance.

Here the throat length is non-dimensionalized by the nozzle diameter, while typically in literature it is non-dimensionalized by the throat diameter.

$$\Pi_2 = \frac{D_T}{D_N} \quad (25)$$
The next Pi group is for the dynamic viscosity term of the primary flow that results in a Reynolds Number, seen in Equation (27), defined for the primary flow exiting the nozzle. The significance of the Reynolds Number may be valuable due to the fact that it has the fluid properties as part of its definition. This leads to an idea that a correlation could potentially be identified, regardless of primary fluid type.

\[
\Pi_3 = \frac{L_T}{D_N} \tag{26}
\]

The next Pi group for the dynamic viscosity term of the primary flow resulting in a Reynolds Number, seen in Equation (27), defined for the primary flow exiting the nozzle. The significance of the Reynolds Number may be valuable due to the fact that it has the fluid properties as part of its definition. This leads to an idea that a correlation could potentially be identified, regardless of primary fluid type.

\[
\Pi_4 = \frac{\rho_N V_N D_N}{\mu_N} \tag{27}
\]

The Pi group resulting from the gas constant is considered where the equation for Mach Number is manipulated into the form of Equation (5). Substituting the right hand side of Equation (5) into Equation (28), and with the knowledge that the system being investigated will be operated around \(M_N = 1\), \(\Pi_5\) reduces to a value on the order of 1 and this Pi group can be neglected.

\[
\Pi_5 = \frac{R}{V_N} = \frac{R}{\gamma R M_N^2} \sim 1 \tag{28}
\]

The Next two Pi groups are dealing with the pressure terms where the pressure at the nozzle is scaling with the kinetic energy of the nozzle, and they can be combined in order to get a ratio of the primary pressure at the nozzle exit to the ambient total pressure.
\[ \Pi_6 = \frac{P_N}{\rho_N V_N^2} \]  

(29)

\[ \Pi_7 = \frac{P_N}{P_a} \]  

(30)

The next three Pi groups can be manipulated to give ratios of primary to secondary flow parameters.

\[ \Pi_8 = \frac{T_N}{T_a} \]  

(31)

\[ \Pi_9 = \frac{\mu_N}{\mu_a} \]  

(32)

\[ \Pi_{10} = \frac{\rho_N}{\rho_a} \]  

(33)

Again, the knowledge of the system in question allows for simplification. In Equation (31), it is noted that the nozzle temperature is purely a function of the gas specific heat ratio and the Mach Number at the nozzle exit. However, this system is being analyzed close to Mach Number = 1, and the specific heat ratios are all of the same order of magnitude; therefore, the temperature at the nozzle exit is relatively invariant and the temperature ratio is approximately constant. The fluid dynamic viscosity is purely a function of temperature, and because the temperature ratio is approximately constant, it follows that the viscosity ratio must also be approximately constant.
Assuming the fluids are perfect gases, the density ratio can be represented as a function of the temperature and pressure ratios. It has been established that the temperature ratio is approximately constant and therefore the density ratio reduces to a function of the pressure ratio, \( \Pi_7 \), which is shown in Equation (34).

\[
\Pi_{10} = \frac{\rho_N}{\rho_a} = f\left(\frac{P_N T_a}{P_a T_N}\right) = f(\Pi_7)
\]  

(34)

The final Pi group is the gas specific heat which was identified as being dimensionless, and therefore, its own Pi group from Table 10. The specific heat ratio for gases is on the order of 1, and therefore, further reduces the complexity of the system.

\[
\Pi_{11} = \gamma \sim 1
\]  

(35)

Each Pi group that has been identified as being on the order of 1 or approximately constant can be neglected as being a minor contributing factor to the ejector performance. Writing the Buckingham Pi theorem in the form of Equation (36), and neglecting the insignificant Pi groups, results in Equation (37) with a reduced number of dimensionless groups.

\[
\Pi_1 = f(\Pi_2, \ldots, \Pi_{11})
\]  

(36)

\[
\frac{\dot{M}_S}{\dot{M}_P} = f\left(\text{Re}_N, \frac{D_T}{D_N}, \frac{L_T}{D_N}, P_N, P_a\right)
\]  

(37)
It has been shown in previous literature that the pressure ratio can be used to establish a curve for predicting an ejector’s performance (Kroll, 1947); however, it was limited to a specific gas with the discharge pressure close to atmospheric pressure. It has also been shown that for increasing Reynolds Number ejector efficiency (defined differently than in the current work) will also increase (Gardner, et al., 2010); although, the effect of Reynolds Number on the entrainment ratio specifically, or how it may influence the optimum geometry, has not been presented. The second and third Pi groups listed are physical geometry values of the ejector and the primary nozzle. Once the system is optimized, these values can be varied independently in order to determine if the optimum value changes with input conditions. Therefore, these two Pi groups necessarily become additional cost functions and can be considered initially as functions of the Reynolds Number and pressure ratio also.

The limitations of this dimensional analysis are that there were potentially important Pi groups that have been neglected for this analysis, like a temperature ratio, based on assumed conditions. The analysis does provide Pi groups that can be considered important to the current ejector’s performance. Without governing equations, it is difficult to establish the true scaling and influence of each Pi group. To further investigate the last Pi group, it is necessary to look to literature specifically related to underexpanded jets and turbulent mixing of fluids. The paper on Scaling Parameters of Underexpanded Supersonic Jets by Yuceil, et al., (2002) provides some
very useful scaling for the current study and is reviewed in detail in Section 1.3.5. The study by Yuceil, et al., (2002) included a converging nozzle with the exit Mach number equal to 1. One of the assumptions is that as an underexpanded flow begins to expand into the surrounding fluid, the mass flux of the secondary fluid across the jet boundary is negligible.

What this means is that the diameter of the primary jet will grow until the pressure is equal to the surrounding static pressure while reducing the available area for the secondary flow entering the ejector throat. It also leads to an assumption that mixing of the two flows does not begin until the pressure has equalized between the two flows. The second assumption of Yuceil, et al., (2002) is that when a high pressure flow is expanding into a low pressure flow, the high pressure flow has oblique shock waves that are adiabatic but *not* isentropic. With these assumptions considered valid, the equations of Section 1.3.5 can be used with the Dimensional Analysis presented here.
References


