Essays in Corporate Finance

by

Ryan D. Pratt

Business Administration
Duke University

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Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in Business Administration
in the Graduate School of Duke University
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Abstract

I study the effect of human capital on firms’ leverage decisions in a structural dynamic model. Firms produce using physical capital and labor. They pay a cost per employee they hire, thus investing in human capital. In default a portion of this human capital investment is lost. The loss of human capital constitutes a significant cost of financial distress. Labor intensive firms are more heavily exposed to this cost and respond by using less leverage. Thus the model predicts a decreasing relationship between leverage and labor intensity. Consistent with this prediction, I show in the data that high labor intensity leads to significantly less use of debt. In the model a move from the lowest to the highest decile of labor intensity is accompanied by a drop in leverage of 21 percentage points, very close to the 27 percentage point drop in the data. Overall, I argue that human capital has an important effect on firm leverage and should receive more attention from capital structure researchers.

Furthermore, I study a two-period contracting problem in which entrepreneurs need financing but have limited commitment. If an entrepreneur chooses to default, he can divert a proportion of the project’s output. Entrepreneurs are heterogeneous with respect to their ability to divert output. In particular, I focus on the special case with only two types of entrepreneurs. “Opportunistic” entrepreneurs can divert output, but “dependable” entrepreneurs cannot. I find that, if the proportion of dependable entrepreneurs is sufficiently high, it is optimal to write contracts that induce second period default by the opportunistic entrepreneurs. This critical pro-
portion generally decreases with the severity of the agency problem. The model delivers both cross-sectional and time-series predictions about default, investment, and output.
For Jessica, Payton and Luke
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Acknowledgements

I would like to thank Alon Brav, Simon Gervais, John Graham, Adriano Rampini, Lukas Schmid and Qi Chen for their support and encouragement. I would also like to thank Hyunseob Kim, David Robinson, Vish Viswanathan, Toni Whited and seminar participants at Brigham Young University, Duke University, University of Rochester, Vanderbilt University and The Board of Governors of the Federal Reserve for helpful comments. I am also grateful to Michael Albert, Jawad Addoum, Howard Kung and Sam Melessa for helpful conversations.
A Structural Model of Human Capital and Leverage

1.1 Introduction

Quantitatively, human capital is the most important factor of production. As shown in Figure 1.1, on average roughly 60% of firm output is used to compensate its labor force. Yet despite its quantitative significance, labor has received relatively little attention in the corporate finance literature. When explicitly modeled, labor typically takes the form of a manager who controls a project and must be properly incentivized by the project’s owners. The study of the problem of the separation of ownership and control is central in corporate finance and has led to, among other things, a large literature on executive compensation. This focus on key managerial employees, however, ignores a much larger group of employees who have neither significant ownership nor control individually, but who collectively are tremendously important to the success of the firm.

Typically, labor is implicitly treated as a purely variable factor of production. When labor markets are frictionless employees can be hired and fired costlessly and
always earn their marginal product. In such a setting labor has no effect on firm value or firm policies. In the presence of labor market frictions, however, labor becomes a quasi-fixed factor of production and contributes to firm value. Such frictions may include the inability of workers to insure their labor income, costly search and matching, or the acquisition of firm-specific human capital. With frictional labor markets there is a match-specific surplus associated with the employment relationship, which must be divided between workers and employees. Workers are better off if they can keep their current jobs, and firms are better off if they can keep their current employees. When part of the value of the firm comes from rents earned from labor, a value maximizing firm will consider the effect of its policies on labor. Corporate policies may affect the firm's labor expenses and its ability to retain workers, which in turn affect firm value. Such policies may include capital structure, dividend policy, cash management, corporate governance, investment, mergers and acquisitions, etc.

Recently a literature has emerged that studies the interaction between labor markets and corporate finance. This literature presents mounting evidence that human capital has an important impact on corporate policies. Despite this evidence, to the best of my knowledge no dynamic model of labor and corporate finance currently exists. The purpose of this paper is to fill this gap, formalizing the way in which we think about the effect of labor markets on corporate finance and providing a lens through which to interpret recent contributions to this literature. I focus on the interaction between human capital and capital structure. I augment a standard neoclassical model of the firm to include a simple model of job search. Since workers who are laid off cannot find alternative employment immediately, unemployment is costly to workers. Leverage exposes workers to unemployment risk, for which they are compensated \textit{ex ante} through higher wages. Additionally, I assume that labor adjustment is costly to the firm. The firm incurs hiring and training costs per employee hired, and also pays a severance to any employees dismissed outside of default.
In the spirit of the tradeoff models of capital structure, firms in my model can be
financed either with debt or equity, with debt enjoying a tax advantage. Default is
endogenous, with firms choosing to default whenever their equity value falls to zero.
The model features deadweight costs of default that are proportional to the firm’s
capital stock. This direct cost of financial distress has been studied in the previous
literature, with many concluding that the costs appear too low to rationalize debt
levels observed in the data.

My model adds an important cost of financial distress that has mostly been
ignored in the literature. In default, a proportion of the firm’s employees lose their
jobs, and the human capital investment embodied in these employees is lost. I argue
that this cost can be quantitatively large and should not be ignored by models of
capital structure. Firms who rely more heavily on human capital in the production
process will have lower leverage ratios because they face higher costs of financial
distress.

I calibrate the model and perform simulations to assess the quantitative signif-
icance of human capital for capital structure. The model produces leverage levels
consistent with those in the data. Furthermore, the model is able to match the cross-
sectional pattern of leverage and labor intensity. Leverage in the model decreases
monotonically in labor intensity, closely matching the pattern in the data. A move
from the lowest to the highest decile of labor intensity is associated with a drop in
leverage of 21 percentage points in the model, very close to the 27 percentage point
drop in the data. In addition to matching the cross-section of labor intensity, the
model is able to match the cross-sectional relationship between leverage and several
other previously recognized predictors of capital structure, including the market to
book ratio, profitability, and cash flow volatility. Overall, I conclude that human
capital is quantitatively significant and should not be ignored by capital structure
researchers.
1.2 Related Literature

My paper is primarily related to a new and growing literature that studies the interaction between labor markets and corporate finance. This literature provides mounting evidence that labor markets are important factors in determining corporate policies. Yet few models exist which help to formalize the analysis of the interaction between labor markets and corporate finance. I help fill this gap by deriving a dynamic model in which labor, investment and leverage can be studied jointly. To the best of my knowledge, this is the first paper that allows the joint study of these three important corporate policies. Such a model will help with the interpretation of the results in recent empirical papers on labor and corporate finance as well as open up avenues for future research in this area.

In a closely related theoretical paper, Berk, Stanton, and Zechner (2010) write a model in which firms sign long term wage contracts with employees. Bankruptcy limits the ability of the firm to commit to the long term contract, thus introducing a cost of debt. The firm trades off this cost against the tax advantages of debt to determine the optimal capital structure. While the mechanism in my model is similar, there are a number of important differences. In Berk, Stanton, and Zechner (2010) the focus is on solving for the optimal dynamic wage contract. In order to do so a number of simplifying assumptions are made. In their model, the capital structure decision is static, there is no hiring or firing outside of bankruptcy, there is no investment decision, and debt is risk-free. In my model, I focus on the dynamics of capital structure, taking the structure of the wage contract as given. I endogenize investment and hiring, which allows me to study their interactions with financing and to assess their quantitative importance. The previous structural corporate finance literature has demonstrated that investment is a key driver of leverage (Hennessy and Whited 2005, 2007, Gomes and Schmid 2010). Since investment and hiring policies
interact, it is important to allow for both to be endogenous in a model that analyzes the importance of human capital.

The idea of bankruptcy limiting the firm’s ability to commit to long-term contracts is key in Butt-Jaggia and Thakor (1994) as well. They write a model in which workers choose to allocate their time between firm-specific and general investment activities. Their focus is on the moral hazard problem of workers under-investing in firm-specific activities because these activities lack value in the outside labor market. Long-term contracts help to mitigate this moral hazard, but such contracts do not survive bankruptcy. Thus, debt is costly in that it makes it more difficult for the firm to induce firm-specific investments. A similar cost of debt is emphasized in the model of Qian (2003). She argues, as I do, that these costs of debt are more significant for labor intensive firms and provides empirical evidence to support the theoretical claim that leverage and human capital intensity should be negatively related. In these models the source of relationship-specific surplus is firm-specific human capital, whereas in my model it derives from costly job search. Furthermore, in addition to predicting the direction of the relationship between human capital and leverage, I evaluate its quantitative significance.

In a recent empirical paper motivated by the costs of financial distress borne by employees, Agrawal and Matsa (2010) use variation in state unemployment benefits to estimate the effect of unemployment risk on corporate financing policies. When unemployment benefits become more generous, the cost to workers of becoming unemployed drops, so it becomes less costly for firms to compensate workers for bearing unemployment risk. Agrawal and Matsa (2010) show that firms increase their leverage when their workers receive improvements in state unemployment benefits. I provide evidence from a structural model that complements their empirical evidence.

In one of the early papers in this literature Hanka (1998) presents some of the first
evidence on the relationship between corporate debt and the terms of employment. He finds that firms with higher debt have larger and more frequent reductions in employment, a higher propensity to use part time employees, and pay lower wages. Interestingly, he finds that these relationships appear to have emerged around 1970 and strengthened over time, a period during which the importance of human capital for firm production increased dramatically (Acemoglu 2002). Thus, as human capital has become more important for firms, its effect on firm policies has increased. Given this evidence, it is reasonable to expect the effects emphasized in my paper to be of increasing importance going forward.

Matsa (2010) studies the interaction between labor and capital structure in a setting with collective bargaining. He uses changes in states’ collective bargaining laws to show that an increase in union bargaining power leads to significant increases in leverage. The interpretation is that capital structure is used by the firm as a strategic variable to influence wage negotiation. By pledging cash flows to debt holders, the firm decreases the size of the pie over which unionized workers will bargain. Hennessy and Livdan (2009) formalize this intuition in a model of firm-supplier bargaining. In their paper, the relationship between a firm and its workers is governed by implicit relational contracts. By selling debt, firms increase their share of the relationship surplus. However, with a smaller remaining surplus, there is a smaller range of credible payments with which to supply incentives to the workers. Reduced incentives offset the firm’s increased share, leading to an interior debt solution. Simintzi, Vig, and Volpin (2010) argue that the theoretical prediction that an increase in employee bargaining power leads to an increase in leverage relies on the assumption that debt is not renegotiable. When debt is renegotiable, an increase in employee bargaining power leads to a decrease in leverage. The intuition given is that more employee bargaining power leads to higher operating leverage in the form of wages, which crowds out financial leverage. They test their prediction using cross-country data.
and find support for a negative relationship between employee bargaining power and leverage. In contrast to Matsa (2010), Simintzi, Vig, and Volpin (2010) do not focus on settings with collective bargaining.

Benmelech, Bergman, and Seru (2011) study the interaction between financial constraints and hiring using an approach similar to that used in the literature on investment-cash flow sensitivity. They find that hiring is sensitive to cash flow and that this sensitivity is higher for firms that are more financially constrained. Financial constraints matter for hiring when labor is not a purely variable factor of production.

The study of labor as a fixed or quasi-fixed factor of production dates back to Oi (1962). Fixed costs associated with labor include investments in recruiting, hiring and training. Labor adjustment costs have been used in many settings in the literature and play an important role in my paper. Hamermesh (1989) uses a plant-level data set to examine the nature of labor adjustment costs at the micro level. He finds that adjustment tends to be lumpy, indicating that labor adjustment costs contain a significant fixed component rather than purely convex costs that would imply smoother adjustment. Further, Hamermesh (1995) studies the source of adjustment costs using a small set of establishments. He finds that both gross costs (of hiring and firing) and net costs (of labor adjustment) affect dynamic labor demand, but that gross costs and the turnover that helps generate them are most important. See also Hamermesh and Pfann (1996) for a review of the literature on factor adjustment costs.¹

¹ A recent literature in asset pricing investigates the role of labor in determining prices (Merz and Yashiv 2007, Bazdresch, Belo, and Lin 2009, Kuehn, Petrosky-Nadeau, and Zhang 2010, Donangelo 2011, Zanetti 2010). These papers generalize standard neoclassical models to allow for frictions in the labor market, either in the form of exogenous adjustment costs to labor or through search and matching frictions. They find that labor adjustment costs matter for returns in both the cross-section and in the time series, and that accounting for labor market factors significantly improves the fit of asset pricing models.
minants of capital structure. Starting from the idea that firms choose their capital structures based on an optimal tradeoff between tax advantages and distress costs, many researchers have concluded that observed leverage ratios appear puzzlingly low (Graham 2000). This has spurred a large literature seeking to better understand the costs associated with distress. Almeida and Philippon (2007) note that defaults cluster in bad times, so that the present value of distress costs depends on risk premia. They find that accounting for these risk premia helps explain why firms otherwise appear to use debt conservatively. Glover (2011) argues that there is selection bias in the observed costs of distress, since firms with high distress costs will endogenously avoid these costs by using less debt. Using a structural model, he estimates the unobserved distribution of distress costs, finding that unconditional average distress costs are nearly twice as large as average costs conditional on distress. My paper is consistent with the argument in Glover. I focus on the loss of human capital as a specific cost of default and argue that firms with large amounts of human capital will choose debt conservatively to avoid losing their human capital. This is similar to the argument made in Titman (1984). He argues that stakeholders other than bondholders and shareholders are affected by financial distress and that optimal capital structure will take account of the costs borne by these parties. While his model focuses on customers’ relationship with a durable goods producer, Titman (1984) is clear that the same argument can be applied to suppliers and workers.

Rampini and Viswanathan (2010) develop a dynamic model of firm financing based on the need to collateralize promises to pay with tangible assets. Their model considers intangible capital generally, whereas I focus specifically on human capital, an important component of intangible capital. Moreover, Rampini and Viswanathan (2010) focus on the inability of lenders to repossess intangible capital, assuming that in case of default the firm retains all intangible capital. In my model the fact that the firm loses its human capital in default is key.
My paper relies on insights from the optimal contracting literature. Harris and Holmstrom (1982) solve for the optimal long-term wage contract between a firm and an employee of unknown ability. Michelacci and Quadrini (2009) study the dynamics of long-term wage contracts in a setting where firms are financially constrained. He (2011) studies a dynamic agency problem where agents are allowed to privately save. In each of these papers the optimal employment contract insures employees against idiosyncratic shocks, a feature shared by Berk, Stanton, and Zechner (2010). Furthermore, He (2011) finds that dismissals are accompanied by sizable severance payments in the optimal contract. While my model does not solve for the optimal wage contract, I incorporate these results into my model. Employees demand higher wages as compensation for unemployment risk, and employees who are laid off outside of bankruptcy are given a compensating severance payment.

Finally, my paper is motivated by the literature on the theory of the firm. Zingales (2000) argues that existing theories of the firm work well for old economy firms that are characterized by relatively sharp firm boundaries and heavy reliance on specialized physical capital. He argues that in the modern service and technology oriented economy, human capital has emerged as the most important asset. Representing a firm by its physical capital is no longer a reasonable approximation. Thus, in order to further our understanding of corporate finance, it is necessary to think about other sources of firm value and how capital structure and governance decisions should adapt to the new economy. By focusing on human capital as a major source of firm value and on the loss of human capital as a significant cost of financial distress, I am aiming to narrow the gap between the realities of modern firms and financial modeling.
1.3 Empirical Evidence

In this section I present empirical evidence to support the claim that human capital has a quantitatively important effect on capital structure. As argued above, a firm’s employees have a significant stake in the firm and have much to lose in the case of unemployment. Several papers have studied the earnings losses of displaced workers, with the consensus being that they are significant and long-lasting (Couch and Placzek 2010, Jacobson, LaLonde, and Sullivan 1993, Ruhm 1991, Stevens 1997, Kletzer and Fairlie 2003). Figure 1.2 is taken from Couch and Placzek (2010), which contains an excellent summary of the studies in this area, and shows earnings losses of a sample of employees in Connecticut who were separated in mass layoffs starting in 1999. The magnitude and duration of the losses in this study are consistent with the findings of similar studies using different data sets, sample periods, and geographic regions. The average quarterly earnings in their sample is $13,288. The figure shows a spike at the time of separation which is driven by lump-sum severance payments to a subset of the displaced workers. Following separation, workers suffer an immediate loss of around 35%. Two years later earnings remain 15% below pre-separation levels, and they remain depressed through six years, when the sample ends. As Couch and Placzek (2010) argue, the losses presented in Figure 1.2 is representative of the conclusions drawn from the other papers that have studied earnings losses of displaced workers.

Figure 1.2 presents evidence that workers who are laid off experience significant economic consequences. This constitutes a significant cost of financial distress. Firms whose production technology is relatively labor intensive are more heavily exposed to this cost of financial distress, and will therefore respond by using less leverage. Figure 1.3 plots the mean market leverage of Compustat firms sorted into deciles of labor intensity. Labor intensity is measured as the ratio of total wage bill to total
assets.\textsuperscript{2} The figure shows a striking univariate relationship between leverage and labor intensity. Firms in the lowest decile of labor intensity have an average of 42% market leverage versus 16% for firms in the highest decile, a difference of a factor of 2.6, and leverage is monotonically decreasing in labor intensity. Figure 1.4 performs the same exercise using book leverage in place of market leverage. The pattern for book leverage is nearly identical to that for market leverage.

Figures 1.3 and 1.4 show that the univariate relationship between leverage and labor intensity is significant. However, labor intensity is likely to be correlated with a number of other determinants of capital structure. Table 1.1 reports multivariate results for regressions of market leverage on deciles of labor intensity and other well-known predictors of capital structure (Rajan and Zingales 1995). All regressions include year fixed effects and cluster standard errors at the firm level. Each dependent variable is measured in deciles to account for nonlinearities in the relationships and to facilitate the comparison of coefficients. Table 1.1 shows that labor intensity is a significant predictor of leverage, on par with previously recognized predictors in terms of both the magnitude of its coefficient and its explanatory power. Moving up one decile of labor intensity results in a drop in market leverage of 2.8 percentage points. For comparison, the largest coefficient in the regression is on the market to book ratio. Moving up one decile in terms of the market to book ratio results in a drop in leverage of 3.7 percentage points. Deciles of labor intensity explain 27.9 percent of the variation in market leverage, compared to a high of 36 percent for the market to book ratio.

When all variables are included in the regression, the coefficient on each variable is attenuated, suggesting the importance of controlling for each determinant. Size

\textsuperscript{2} Measuring labor intensity using the wage bill puts severe restrictions on the data. Reporting labor expenses is voluntary, and is reported consistently by roughly 10\% of Compustat firms. Ballester, Livnat, and Sinha (2001) find that disclosure is more common for large firms, labor-intensive firms, and firms in regulated industries. I also find that firms who report labor expenses have slightly higher leverage on average.
and cash flow volatility lose their significance, while labor intensity remains one of
four significant predictors of leverage, along with tangibility, market to book, and
profitability.

For completeness, Table 1.2 presents the same regressions using book leverage as
the dependent variable. The results are very similar, with the most notable difference
being that market to book and profitability are less reliably related to book leverage.
This is not surprising because of how these variables feed directly into the firm’s
market value. In these regressions labor intensity is second only to tangibility in terms
of the significance of its relationship to leverage, as measured both by the magnitude
of its coefficient and the $R^2$ of the regression. Moving up one decile in labor intensity
results in a drop of 2.7 percentage points in leverage, versus a rise in leverage of 2.9
percentage points for a corresponding change in tangibility. Labor intensity explains
24.7 percent of the variation in book leverage, versus the 28.7 percent explained by
tangibility. As is the case for market leverage, labor intensity survives as one of four
significant predictors of capital structure in a multiple regression.

This evidence suggests that labor intensity is an important factor in a firm’s
capital structure decision. Tangibility, size, market to book, profitability, and cash
flow volatility have long been recognized as important and reliable predictors of
capital structure. The evidence presented in this section suggests that labor intensity
should be included in this list.

1.4 Model

In this section I write a model to formalize and quantify the dependence of capital
structure on labor intensity. The model is most closely related to the models of
Hennessy and Whited (2007) and Cooley and Quadrini (2001). The model consists
of a large number of firms which are described by three state variables: net worth
($\omega$), labor ($n$) and a persistent, idiosyncratic productivity shock ($z$). In what follows
I solve a single firm’s optimization problem, using primes to denote future values and subscripts on functions to denote partial derivatives. I assume that the shock $z$ has Markov transition function $\Gamma(z'|z)$ and takes on values in a compact set $[\underline{z}, \overline{z}]$ with $\overline{z} > 0$. Each period the firm chooses next period’s capital ($k'$), labor ($n'$), and debt ($b'$). Investment, hiring, and financing decisions are made simultaneously.

Voluntary labor turnover occurs at a constant rate $\delta_n$, so that gross hiring is given by

$$h = n' - (1 - \delta_n)n. \quad (1.1)$$

Gross adjustments to the labor stock are costly to the firm. Specifically, the firm pays a cost $\eta$ for each worker it hires. This cost can represent recruiting, screening, and training costs. In order to lay off workers the firm must pay a severance payment of $g^-$ per worker, which is determined endogenously below. The total costs of labor adjustment are then given by

$$g = g^+ h \cdot 1_{(h \geq 0)} - g^- h \cdot 1_{(h < 0)}. \quad (1.2)$$

These costs are consistent with the literature on labor adjustment costs, which has found that gross adjustment costs play a very important role in the dynamics of labor demand (Hamermesh 1989, 1995).

Debt in the model consists of a standard one-period debt contract. The market value of debt is denoted by $b'$, where negative values of $b'$ are interpreted as cash holdings. Since there is no mechanism in the model for firms to have both cash and debt, $b'$ is best interpreted as net corporate debt. In the model, firms endogenously default whenever it is in shareholders’ best interest. The interest rate on debt will therefore reflect default risk and will be an endogenous function of all information available at the time of financing. Let $r(k', n', b', z)$ denote the endogenous interest rate, with any corporate saving taking place at the risk free rate $r_0$. Interest income
is taxed at rate \( \tau_i \), so the discount factor in the model is \( \beta = \frac{1}{1+\tau_0(1-\tau_i)} \). I solve for the endogenous interest rate below.

The firm’s budget constraint can be written as

\[
\omega + b' = k' + g + d_g,
\]

where \( d_g \) denotes the gross dividends paid by the firm. This equation states that the firm uses its net worth plus new issues of debt to purchase capital, hire (or lay off) workers, and pay dividends. Negative values of \( d_g \) are interpreted as equity issuance, which is subject to flotation costs. Following the previous literature (Gomes 2001, Hennessy and Whited 2005, 2007, Gomes and Schmid 2010), flotation costs are modeled in reduced form. Here flotation costs consist of fixed, linear, and quadratic terms

\[
\Lambda(d_g) = \lambda_0 + \lambda_1|d_g| + \lambda_2d_g^2.
\]

Positive dividends are subject to taxation at a rate \( \tau_d \). Letting \( \phi_i \) be an indicator variable for equity issuance, net dividends received by shareholders are given by

\[
d = d_g - (1 - \phi_i)\tau_dd_g - \phi_i\Lambda(d_g).
\]

The firm uses capital and labor to produce output

\[
f' = z'A \cdot F(k', n')^\nu.
\]

The function \( F \) is strictly increasing and concave in both arguments and is homogeneous of degree one. I assume that \( \nu < 1 \) so that the output function exhibits decreasing returns to scale. This is consistent with empirical observations on the growth dynamics of firms, namely that small firms grow faster than large firms. \( A \) is simply a scaling constant that is used to set the steady state value of capital to one in each simulation below.
Operating profits are equal to output less wages and a fixed cost of production that must be paid each period

\[ \pi' = f' - w(k', n', b', z)n' - f_0. \]  

(1.7)

Wages are agreed upon at the beginning of the period and, like the interest rate, are an endogenous function of all information that is available at the time they are agreed upon. Wages are subject to two sources of risk. First, since wages are a fixed claim it is possible that the firm will not be able to pay the promised wages in some states, a situation that I refer to as wage default. Second, when the firm defaults on its debt obligations, a proportion of employees lose their jobs. As will be shown below, unemployment is costly to the workers, so wages will reflect both the possibility of wage default and of unemployment due to financial default. Accounting for labor in the production process introduces operating leverage in the profit function in the form of wages which are fixed in the short run. This operating leverage can crowd out financial leverage. The amount of labor-induced operating leverage varies in the cross-section with the wage bill \( w \cdot n \), or labor intensity, of the firm. Thus, the more a firm relies on labor for production, the less financial leverage it will use. The function \( w \) will be derived in the following section.

Capital depreciates at a constant rate \( \delta_k \), and depreciation and interest expense are tax-deductible, implying that the firm’s taxable income is

\[ y' = \pi' - \delta_k k' - r b'. \]  

(1.8)

Following Hennessy and Whited (2007), I specify a corporate tax schedule with a kink at zero taxable income to approximate loss limitations. Firms with positive taxable income pay a marginal rate \( \tau_c^+ \), while the tax rate for firms with negative taxable income is \( \tau_c^- \), where \( 0 < \tau_c^- < \tau_c^+ < 1 \). The firm’s effective marginal tax rate is then given by

\[ \tau_c = \tau_c^+ \cdot 1(y' \geq 0) + \tau_c^- \cdot 1(y' < 0). \]  

(1.9)
The kink in the tax schedule implies that debt is less attractive to firms that have a higher probability of incurring negative taxable income. For firms which are sufficiently likely to have negative taxable income, debt becomes tax-disadvantaged.

Corporate earnings are equal to operating profits less the tax bill, which can be written as

\[ e' = (1 - \tau_c) p' + \tau_c \delta_k k' + \tau_c r' b'. \] (1.10)

This way of writing earnings emphasizes the tax benefits of depreciation and interest. The firm’s net worth each period consists of earnings plus depreciated capital minus debt obligations

\[ \omega' = e' + (1 - \delta_k) k' - (1 + r) b'. \] (1.11)

The objective of the firm is to maximize the expected value of future dividends. Investors are risk neutral and discount the future at a rate \( \beta = \frac{1}{1 + r_d (1 - \tau_i)} \), where \( \tau_i \) is the tax rate charged on interest income. The Bellman equation that describes this problem can be written

\[
V(\omega, n, z) = \max \left\{ 0, \max_{k', n', b'} d + \beta E[V(\omega', n', z') | z] \right\}
\] (1.12)

The outer “max” reflects the shareholders’ limited liability. Any time that the realization of the productivity shock is bad enough that the equity value would be negative, the firm will choose to default on its obligations. It is clear that the value function will be increasing in both its first and third arguments over the domain in which the value function is positive. This fact will be useful in deriving both the labor market and the debt market equilibria.

1.4.1 Labor Market Equilibrium

In this section I solve for the wage \( w(k', n', b', z) \) that is required by employees. Firms and employees agree on wages at the same time as investment, hiring, and
financing decisions are made. As the notation makes clear, wages depend on all of the information that is available at that time. I assume that employment agreements are long term, while wages are determined each period according to the state of the firm. The worker will continue to work for the firm until he is laid off or leaves voluntarily. The terms of the employment agreement are thus determined to meet a required level of lifetime (rather than single period) utility for the employees. I assume that outside of default the firm fully insures its employees’ human capital, in the sense that the employees’ lifetime utilities do not depend on the stochastic shocks. Several papers in the optimal contracting literature have studied the optimal wage contract in a variety of economic settings (Harris and Holmstrom 1982, Michelacci and Quadrini 2009, Berk, Stanton, and Zechner 2010, He 2011). In each setting the optimal contract provides insurance to employees against adverse shocks to their human capital. This result follows naturally from the inability of employees to diversify this risk away. While solving for the optimal wage contract is outside the scope of this paper, I rely on these previous studies to motivate the structure of the contract.

Employees are compensated for bearing risk through two channels. First, the contracted wage will respond endogenously to the probability that a worker will be laid off in bankruptcy, thus compensating him \textit{ex ante} for bearing this risk. Workers may also be laid off outside of bankruptcy when the firm receives a negative shock and needs to downsize. In this case, the firm pays each separated worker a severance payment that compensates him \textit{ex post} for the costs associated with unemployment. He (2011) finds such severance payments to be optimal in a setting where employees can privately save. The severance payment is endogenous and will be determined below.

Determination of the wage proceeds in two steps. Because firms cannot costlessly replace workers and because workers cannot immediately find another job, there is a match-specific surplus associated with an employment relationship. As is common
in the labor literature, in the first step I use generalized Nash bargaining to divide this surplus. The outcome of the Nash bargaining game is a market wage $w_0$ which provides the necessary level of lifetime utility to the employees. Since the firm cannot commit to paying the market wage each period in perpetuity, the second step is to add a risk premium to the market wage to account for the possibility that the worker is laid off. This second step produces the state-dependent wage $w(k', n', u', z)$. This solution process is outlined graphically in Figure 1.6.

**Job Search and Nash Bargaining**

Unemployed workers receive an exogenous unemployment benefit of $w_u$ net of costs incurred while searching for employment. The probability of a successful employment search is denoted by $p_e$, which I treat as exogenous. Let $U_e$ denote the lifetime expected utility for a worker who is currently employed, and let $w_0$ denote the constant wage that delivers this lifetime utility if received with certainty forever, that is

$$U_e \equiv \sum_{t=0}^{\infty} \beta^t u(w_0) = \frac{u(w_0)}{1-\beta}.$$  

Similarly, let $U_u$ denote the lifetime expected utility for a worker who is currently unemployed, which is equal to the unemployment benefit plus the expected discounted continuation value

$$U_u = u(w_u) + \beta (p_e U_e + (1-p_e)U_u). \quad (1.13)$$

Combining these two equations gives the surplus to an employee from the employment relationship

$$S_e(w_0) = U_e - U_u = \frac{u(w_0) - u(w_u)}{1-\beta(1-p_e)}. \quad (1.14)$$

The firm’s surplus from the employment relationship is simply given by $g^+$, since the firm can replace an employee at any point by paying the hiring cost.

Letting $\theta$ represent the employees’ absolute bargaining power, the market wage is given by the solution to the Nash bargaining equation $\theta g^+ = (1-\theta)S_e(w_0)$. Solving
this equation for \( w_0 \) gives the wage that divides the total surplus associated with the employment relationship according to each party’s bargaining power

\[
\begin{align*}
  u(w_0) &= \frac{\theta}{1 - \theta} g^+ (1 - \beta (1 - p_e)) + u(w_u).
\end{align*}
\] (1.15)

The market wage is increasing in the workers’ relative bargaining power, in the hiring costs, and in the unemployment benefit. Equation (1.15) is a standard wage equation with Nash bargaining.\(^3\)

**Default and the Wage Risk Premium**

There are two sources of risk for which the employees in the model must be compensated *ex ante*. First, a fraction \( \xi_n \) of employees are laid off when the firm defaults. Employees who are laid off in default receive no severance payment, and thus they incur all of the costs of unemployment. Letting \( p_d(k', n', l', z) \) represent the endogenous default probability, the probability of a worker becoming unemployed due to default is \( p_u = p_d \cdot \xi_n \).

Second, since wages are promised before output is realized, there is the possibility that the firm will not be able to deliver the promised wages, a case that I call wage default. This occurs if the promised wage bill is sufficiently large relative to output, which is most likely to occur when the productivity shock is low and employment is high. As this possibility makes clear, labor is a fixed claim in the short run which exposes the firm to negative shocks in very much the same way that leverage does. Firms that are more exposed to leverage through their labor contracts will optimally choose less financial leverage all else equal.

It is worth noting that wage default does not play a prominent role in the model. In fact, in the simulations reported below it never occurs. It is, however, a theoretical

\(^3\) See Blanchard and Gali (2010) and Zanetti (2010) for applications of this wage-setting equation in a general equilibrium framework.
possibility that must be accounted for in the model. Firms could, in theory, choose a very high level of labor and a very low level of capital, such that for bad realizations of productivity they would be unable to pay their employees. Though this turns out to never be optimal, in order for the model to be complete I must specify what would happen if the firm were to choose such policies. Thus, wage default can be thought of as being off equilibrium.\(^4\)

Formally, wage default occurs any time that the firm has negative operating income and zero continuation value. In this case the firm has insufficient output to pay its promised wages and lacks the ability to borrow to finance payroll. Because zero continuation value is a necessary condition for wage default, firms that cannot pay their wages also default on their debt obligations.\(^5\)

I assume that employees are given priority over creditors in default. There are two reasons for this assumption. First, wages are generally paid throughout the production process, so that the accumulated wages due at the time of a default would be relatively small. Second, Section 507 paragraph (a)(4) of the U.S. Bankruptcy Code allows for $11,725 of wages earned by each employee within 180 days preceding the bankruptcy filing to be given priority over creditors’ claims.\(^6\) Thus, when default occurs, employees first collect all available resources up to their full wages, then creditors collect any remaining resources and the firm shuts down. When wages cannot be paid in full, creditors get nothing.

Since \(V\) is nondecreasing in both net worth and the productivity shock, there

\(^4\) Another way to handle this would be to specify a constraint requiring that employees are always paid their promised wages. Such a constraint would place a state-contingent limit on the number of employees the firm could hire.

\(^5\) Note that if the fixed costs of production are large enough, the firm could theoretically default on its promised wages even if holds positive net cash. In the calibrations below, however, the fixed cost parameter is always set low enough that this is not a possibility.

\(^6\) This dollar amount is adjusted for inflation every three years, with the next adjustment scheduled for April 1, 2013. Also, anecdotal evidence suggests that in cases where labor petitions the bankruptcy court for larger amounts than that to which they are entitled by the U.S. Bankruptcy code, the petition is often granted.
exists a unique critical value $z_n(k', n', b', z)$ such that the firm is unable to pay the promised wages if and only if $z' < z_n$. I let $p_n$ denote the probability that the firm defaults on its promised wages

$$p_n(k', n', b', z) = \int_{z_n}^{z_n} \Gamma(z, dz'). \quad (1.16)$$

If the firm cannot pay its employees and is forced to shut down, default costs equal to a proportion $\xi_k$ of the capital stock are incurred. Available resources to be recovered by employees are then given by

$$R_n = f' - f_0 + (1 - \xi_k)(1 - \delta_k)k'. \quad (1.17)$$

Employees recover all the output less fixed costs of production, plus depreciated capital less deadweight costs.

The lifetime expected utility of an employed worker can be written as

$$U_e = \frac{u(w_0)}{1 - \beta} = (1 - p_n)u(w(k', n', b', z)) + \int_{z_n}^{z_n} u(R_n/n')\Gamma(z, dz')$$

$$+ \beta (p_uU_u + (1 - p_u)U_e). \quad (1.18)$$

The first equality follows from the definition of $w_0$. The second equality requires that the utility derived from this period’s wages plus the discounted continuation utility from being either employed or unemployed must equal the utility that would be derived from receiving $w_0$ forever. The first line is the employees’ expected period utility. With probability $(1 - p_n)$, employees receive the promised wages. Otherwise, employees receive their expected recovery value conditional on wage default. Finally, the last line represents the expected continuation utility.

Combining equations (1.18) and (1.13) gives the following equation for the risk-
adjusted wage:

$$u(w(k', n', y', z)) = \frac{1}{1 - p_n} \left[ u(w_0) + \beta p_u (u(w_0) - u(w_a)) \frac{1}{1 - \beta(1 - p_e)} - \int_z^{z_n} u(R_{n'/n'}) \Gamma(z, dz') \right].$$

(1.19)

The second term in brackets on the right hand side is the required compensation for unemployment risk. It is increasing in the probability of unemployment and also in the costliness of unemployment. The third term represents the compensation for the risk of wage default. The wage premium is increasing in the probability of wage default and decreasing in the expected recovery conditional on wage default.

**Severance**

When the firm has a negative productivity shock, it may choose to layoff workers. I assume that workers who are laid off outside of default receive a severance payment that fully compensates them for the costs of being unemployed. There are several motivations for this assumption. When labor markets are competitive, employees will demand compensation for unemployment risk. This compensation can be made *ex ante* through higher wages or *ex post* through severance payments. In the case of layoffs that occur in default, severance payments cannot be promised, so employees are compensated for default risk with higher wages *ex ante*. Taking this same approach to compensate employees who are laid off outside of bankruptcy would complicate things significantly, however, since calculating the wage premium would require knowing the firm’s policy functions beforehand. Thus, assuming that workers who are laid off outside of bankruptcy receive a compensating severance payment greatly simplifies the problem. Qualitatively, at least, the difference is immaterial, since in either case the firm would be required to bear the cost of the risk it imposes on its employees. In a dynamic contracting model, He (2011) finds that large severance payments that sustain the employees’ consumption at pre-layoff levels are
optimal.

The endogenous severance payment $g^\ast$ satisfies

$$u(w_u + g^\ast) + \beta(p_e U_e + (1 - p_e) U_u) = U_e.$$  \hspace{1cm} (1.20)

This equation states that the utility that workers get from the unemployment benefit and the severance payment plus the expected discounted continuation utility is equal to the utility from remaining employed. Solving this equation for $g^\ast$ yields the severance payment

$$g^\ast = u^{-1} \left[ \frac{u(w_0) - \beta(1 - p_e)u(w_u)}{1 - \beta(1 - p_e)} \right] - w_u. \hspace{1cm} (1.21)$$

Intuitively, the severance payment is increasing in the disutility of being unemployed and decreasing in the unemployment benefit.

It is worth noting that the severance payment as specified assumes that the worker consumes the entire severance payment in the period he is laid off. This assumption is made for tractability, since alternatively I would have to solve a dynamic consumption-savings problem with stochastic labor income. The severance payment as specified provides an upper bound on what the severance payment would have to be if the agent were to dynamically smooth his consumption. Rather than complicate the model by embedding a dynamic consumption-savings problem in the current framework, I perform sensitivity analysis by exogenously varying the severance payment to ensure that this assumption does not drive my results.

1.4.2 Debt Market Equilibrium

In this section I characterize the endogenous default condition for the firm as well as the corresponding interest rate $r(k', n', U, z)$. Default in the model is endogenous, with firms choosing to default whenever is is in shareholders’ best interest. In default
firms are liquidated, with bondholders receiving all of the proceeds after employees are made whole, and equity holders are left with nothing.\footnote{The assumption that firms are liquidated in default is made for simplicity and is not crucial. In practice, default is more likely to result in renegotiation or reorganization than liquidation. What matters in the model is that there are costs associated with default. The parameters that determine default costs are set in the calibration to reflect actual costs observed in default consistent with the prior literature, rather than the potentially larger costs associated with liquidation.}

As is common in the previous literature, I assume that when a firm defaults it incurs a deadweight cost that is a proportion \( \xi_k \) of its capital stock. Bondholders’ recovery can then be written as

\[
R = (1 - \tau_c) \pi' + \tau_c \delta_k k' + (1 - \xi_k)(1 - \delta_k)k'.
\]

Consistent with the previous literature, I assume that the tax benefit of debt is lost when the firm defaults.

In addition to the standard default costs, a proportion \( \xi_n \) of employees lose their jobs in default. As is well documented in the labor literature, job loss is costly to employees. The loss of human capital associated with the unemployment caused by default augments the costs of default. Firms that rely more heavily on human capital in the production process face higher overall default costs and will therefore optimally choose to use less leverage than firms who rely less on human capital.

Default in the model occurs any time that the equity value function equals zero. Since the value function is increasing in the productivity shock and in net worth, there exists an endogenous critical value \( z_d(k', n', n', z) \) such that the firm defaults if and only if \( z' < z_d \). Given this definition, the firm’s conditional default probability can be written as

\[
p_d(k', n', n', z) = \int_{z}^{z_d} \Gamma(z, dz').
\]
that wage default necessarily implies debt default, while the reverse is not true. If there are not sufficient resources to make employees whole, then bondholders are left with nothing. Thus, for any $z' < z_n$ the bondholder’s recovery is zero. The critical regions of interest are illustrated graphically in Figure ??.

Assuming competitive lending, the bank’s zero profit condition can be written as

$$b_1 \beta (1 - p_d) (1 + r_0 (1 - \tau_i)) y' + \int_{z_n}^{z_n} R \Gamma(z, d' z') \right].$$

(1.24)

The left hand side of (1.24) equals the funds provided by the bank, and the right hand side is the discounted expected payoff. When there is no default, the lender receives the entire principal plus interest. Otherwise, the lender gets his expected recovery value conditional on default. Rearranging this equation gives the endogenous interest rate

$$r(k', n', y', z) = \frac{1}{1 - \tau_i} \left[ \frac{1 + r_0 (1 - \tau_i) - \int_{z_n}^{z_n} R/y \Gamma(z, d' z')}{1 - p_d} - 1 \right].$$

(1.25)

As expected, the interest rate is increasing in the probability of default and decreasing in the lender’s recovery.

1.4.3 Optimal Firm Behavior

The dynamic programming problem formalized in (1.12) has no analytical solution. I can, however, analytically characterize the optimal firm policies and use numerical techniques to solve for the equity value function. In this section I assume differentiability of the value function for expositional purposes.\(^8\) The numerical solution makes no assumptions about the differentiability of the value function.

Let $\phi_d$ be an indicator variable that takes a value of one if the firm is paying dividends and $\phi_i$ be an indicator variable for equity issuance. The firm’s objective is

\(^8\) Due to limited liability, the value function is convex near values of net worth at which equity value becomes zero. Without concavity, I cannot prove differentiability. Fixed costs pose an additional problem for differentiability.
to maximize the right-hand side of (1.12). The firm’s optimal capital stock is given by the solution to
\[ \beta E [V'_k \omega_k] = 1 - \phi_d \tau_d + \phi_i \Lambda_{d_i}. \] (1.26)

Substituting in \( \omega_k \), equation (1.26) becomes
\[ \beta E \left\{ V'_k \left[ (1 - \tau_c)(f'_k - w_k n' - r_k b') + \tau_c \delta_k + (1 - \delta_k) \right] \right\} = 1 - \phi_d \tau_d + \phi_i \Lambda_{d_i}. \] (1.27)

The left-hand side of equation (1.27) is the discounted shadow value of an additional unit of capital. The right-hand side is the marginal financing cost. For firms which are paying dividends at the margin, an additional unit of capital decreases the net dividend by \( 1 - \tau_d \). For firms which are issuing equity at the margin, an additional unit of capital increases equity issuance by \( 1 + \Lambda_{d_i} \), where \( \Lambda_{d_i} \) is the marginal cost of equity issuance. For many firms the benefits of investment will not be sufficient to overcome the fixed costs of equity issuance. Such firms will neither issue equity nor pay dividends, instead using debt as the marginal source of financing. In this case, the marginal cost of financing is simply 1.

Let \( \phi_h \) be an indicator variable for positive gross hiring, and let \( \phi_{lo} \) be an indicator variable for layoffs. The firm’s optimal hiring policy is given by the solution to
\[ \beta E [V'_n \omega_n + V'_n] = (1 - \phi_d \tau_d + \phi_i \Lambda_{d_i})(\phi_h g^+ - \phi_{lo} g^-). \] (1.28)

Solving for \( \omega_n \), the optimal condition for dynamic labor demand is given by
\[ \beta E \left\{ V'_n \left[ (1 - \tau_c)(f'_n - (w_n n' + w) - r_n b') + V'_n \right] \right\} = (1 - \phi_d \tau_d + \phi_i \Lambda_{d_i})(\phi_h g^+ - \phi_{lo} g^-). \] (1.29)

Equation (1.29) begins to make explicit the relationship between labor and financing. The left-hand side is the discounted shadow value of an additional employee, while the right-hand side is the cost of hiring an additional employee. The first term in parentheses on the right-hand side reflects the firm’s endogenous financing regime.
The effective cost of hiring an additional employee depends on whether the firm is paying dividends, issuing equity, or issuing new debt at the margin. In particular, the effective cost of hiring is highest for firms which are issuing equity at the margin and lowest for firms which are paying dividends. The final term represents the direct costs of hiring or firing. Firms that are hiring must pay $g^+$ to hire an additional employee, whereas firms who are laying off workers have a negative marginal hiring cost, since laying off one less worker means the firm has one less severance payment to make.

The optimal financing policy is given by

$$\beta E[V_\omega \omega_b] = 1 - \phi_d r_d + \phi_i \Lambda_1.$$  \hspace{1cm} (1.30)

Substituting in the solution for $\omega_b$ gives the following equation for optimal financing:

$$\beta E\{V_\omega' [(1 - \tau_c)(w_b n' + (r_b b' + r)) + 1]\} = 1 - \phi_d r_d + \phi_i \Lambda_1.$$  \hspace{1cm} (1.31)

As with the other optimal policies, the optimal financing policy depends on the financing regime as is emphasized in Hennessy and Whited (2005). The right-hand side of equation (1.31) is the marginal cost of an extra dollar of equity financing or, alternatively, the marginal benefit of an extra dollar of debt financing. In equilibrium, this is set equal to the marginal cost of debt financing, which appears on the left-hand side.

The relationship between labor and financing is made more explicit in (1.31). There are two terms on the left-hand side that contribute to the marginal cost of debt. The term $r_b b' + r$ is the marginal interest expense. When an additional dollar of debt is raised, interest must be paid on that dollar. Additionally, in the case of risky debt, an additional dollar of debt causes the debt to be riskier, thus increasing the interest rate on the entire debt outstanding. These two terms capture the marginal interest expense and would be present in any model of optimal debt financing.
In addition to the marginal interest expense, risky debt leads to higher wages to compensate employees for the unemployment risk that they bear. This effect, captured by the term \( w_h n' \), is consistent with the model of Titman (1984) and emphasized in the model of Berk, Stanton, and Zechner (2010). This term is an important driving force in the model and will vary significantly in the cross-section of labor intensity. A model which fails to account for the distress costs borne by employees implicitly sets \( w_h n' = 0 \), which understates the marginal cost of debt.

1.5 Quantitative Results

1.5.1 Baseline Calibration

In this section I calibrate the model to match certain features of the data and to assess the quantitative significance of human capital for capital structure. Table 1.3 shows the parametrization of the benchmark model. Wherever possible, parameters are chosen using guidance provided by previous research and to match moments from the data.

I first need to choose functional forms for the production function and the technology shocks. As is standard in the literature, I use a Cobb-Douglas production function, \( F(k, n) = k^{\alpha_k} n^{\alpha_n} \). Here the returns to scale parameter \( \nu = \alpha_k + \alpha_n \). I set \( \nu = .85 \) as in Jermann and Quadrini (2003) and Atkeson and Kehoe (2001). Atkeson, Khan, and Ohanian (1996) provide discussion justifying this parameter choice. I set \( \alpha_n = .6 \) to match the average ratio of labor expense to assets in my sample. This value is also consistent with labor’s share of output in the aggregate.

It is also standard in the literature to specify the technology shocks to be an AR(1) process in logs. As such, I set

\[
log(z') = \rho_z log(z) + \sigma_z \epsilon',
\] (1.32)

where \( \epsilon' \) is distributed standard normal. I set \( \rho_z = .684 \) as in Hennessy and Whited
(2007). The conditional volatility of productivity varies significantly in the literature and is not directly comparable across different production functions. As a baseline I set $\sigma_z = 0.092$.

The fixed costs of production deduct directly from the firm’s operating profits and are therefore important for matching firm value. As in Zhang (2005), for each simulation I set fixed costs to target the average Tobin’s Q in the data, which leads to a baseline level of fixed costs $f_0 = 0.1$. I set $\delta_k = 0.15$ to match the average rate of depreciation and $\delta_n = 0.15$ to match the average rate of voluntary turnover in the manufacturing sector from the Bureau of Labor Statistics Job Opening and Labor Turnover Survey (JOLTS).

The calibration of the tax environment and equity issuance costs follows Hennessy and Whited (2007) closely. As in their paper, the maximum corporate tax rate is set equal to 40%, while the minimum rate is 20%, and the tax rate on interest income is set to 29%. I set the tax rate on dividend income to 12%. This differs somewhat from Hennessy and Whited (2007) in that they specify a convex dividend tax schedule with a maximum rate of 12%. Fixed costs of equity issuance are set to 0.02 to target the average frequency of equity issuance as reported in Hennessy and Whited (2005) and Hennessy and Whited (2007). Similarly, quadratic costs of equity issuance are set to 0.36 targeting the average equity issuance to assets ratio. Proportional costs of issuance are set equal to 0.1, very close to the value of 0.091 estimated by Hennessy and Whited (2007).

Proportional default costs are set equal to 0.05. This value is lower than those typically used in the literature, but is consistent with the low estimates of direct default costs measured in the empirical literature (Weiss 1990, Andrade and Kaplan 1998). There is not a lot of guidance in selecting the proportion of jobs lost in default. As a baseline I choose to set $\xi = 0.3$.9

9 This parameter, as well as the proportional default costs, are chosen to approximate costs that
The probability of a successful job search is set to approximate the present value wage loss of employees who are laid off. Jacobson, LaLonde, and Sullivan (1993) study a panel of employees who are laid off from distressed firms and finds long-term wage losses equal to 25%. Their paper focuses on long-tenured employees who likely suffer more significant losses from displacement. Couch and Placzek (2010) find more modest results that they argue are more consistent with results from different data sources, time periods, and geographic regions. They find that six years after separation displaced workers are still suffering 15% wage loss. Since all jobs are identical in my model, I need to choose a low probability of successful search to approximate the magnitude of earnings losses from these studies. I thus set $p_e = .3$, which implies that earnings on average are 8.4% lower six years after separation. This parameter should not be interpreted literally, since the probability of finding a job during a year of unemployment is clearly greater than 30%. Rather, it should be interpreted as a parameter that controls how costly an unemployment spell is in expectation.

Notice from equation (1.15) that hiring costs $g^+$, the unemployment benefit $w_u$, and the market wage $w_0$ are determined only up to a scaling constant. I therefore set these parameters to deliver a market wage of 1. Hiring costs $g^+$ are set equal to .75, which implies that hiring costs are three-quarters of a worker’s annual payroll. (Hamermesh and Pfann 1996) review the research on estimates of direct costs of labor adjustment and report that some studies find external costs up to one year’s payroll for a worker. They also report that these costs increase rapidly with the skill of a worker, being low for high-turnover, low-skilled workers and high for low-turnover, high-skilled jobs. My model is motivated by the importance of human capital in the production process, so I choose the hiring costs to wage ratio to be in line with what it would be for relatively high-skilled workers. The unemployment benefit $w_u$ is set would occur in default on average, not conditional on liquidation, as mentioned above.
equal to .5, which approximates the stated goal of the unemployment insurance system to replace half of a worker’s wages. Given these values, the employees bargaining power $\theta$ is set equal to .68, which sets the market wage $w_0$ to 1. Finally, I choose log utility to represent the workers’ preferences, which represents a very conservative level of relative risk aversion.

1.5.2 Numerical Solution and Baseline Results

Since the model has no closed-form solution, I solve the model using value function iteration. Specification of the state space requires grids for the control variables $\{k', n', b'\}$ as well as a grid for net worth $\omega$ and a grid for the productivity shocks $z$. The dimensionality of the state-space is rather large for this problem, restricting the number of grid points that can be used in each dimension. The state space for $z$ is especially expensive computationally since both $z$ and its lagged value are needed to calculate the firm’s realized net worth. As such, I use 9 grid points for $z$ in the interval $[-3\sigma_z, 3\sigma_z]$. The grids for $k'$ and $n'$ can be bounded above by solving the equations

$$f_k(\bar{k}, \bar{n}, \bar{z}) = \delta_k$$

$$f_n(\bar{k}, \bar{n}, \bar{z}) = w_0 + g^+ \delta_n$$

jointly for $\bar{k}$ and $\bar{n}$. Intuitively, (1.33) says that the maximum value of capital sets the marginal productivity of capital when labor and productivity are at their highest values equal to the marginal economic cost of capital, which is the depreciation rate. Similarly, (1.34) says that the maximum value of labor sets the marginal productivity of labor when capital and productivity are at their highest values equal to the marginal economic cost of labor, which is the wage rate plus the cost of replacing workers who quit voluntarily. It would never be profitable for a firm to choose any value of capital larger than $\bar{k}$ nor any value of labor larger than $\bar{n}$. The
lower bounds for \( k' \) and \( n' \) can similarly be determined by solving

\[
\begin{align*}
    f_k(k, n, z) &= \delta_k \\
    f_n(k, n, z) &= w_0 + g^+ \delta_n
\end{align*}
\]

jointly for \( k \) and \( n \). Thus, \( k' \) and \( n' \) can be confined to a compact set. The state space for \( k \) is then determined by choosing 40 points in the interval \([k, \bar{k}]\), while the state space for \( n \) contains 30 points in the interval \([\bar{n}, \bar{n}]\).

Given my assumptions on tax rates, the state space for \( b' \) can be confined to a compact set \([\bar{b}, \bar{b}]\). To see this, note that when a firm’s cash holdings are sufficiently large the probability of negative taxable income becomes negligible. Since the corporate tax rate on positive taxable income is higher than the tax rate on interest income, it is more efficient for saving to take place outside of the firm. Thus, \( b' \) will be bounded below. Conversely, given that capital is bounded, the firm’s value is bounded from above. If debt were to exceed that value, the firm would clearly prefer to default. Thus debt must be bound above. I set \( \bar{b} = (1 - \tau_c^+ (1 - \tau_d) (f(k, \bar{n}, \bar{z}) - w_0 \bar{n} - f_0) / r_0 \), which is an approximation of the maximum possible firm value, and I set \( b = -\bar{b} \).

The state space for \( b' \) consists of 30 points in the interval \([\bar{b}, \bar{b}]\).

Finally, the state space for net worth contains 30 points in the interval \([\bar{\omega}, \bar{\omega}]\). Through some experimentation I choose \( \omega \) low enough so that \( V(\omega, \cdot, \cdot) = 0 \) and \( \bar{\omega} \) large enough to exceed the maximum value of realized net worth. The state space is sufficient in that the optimal policy never occurs at any endpoint of the grid.

Figure 1.7 plots the firm’s value function. In each panel the dashed line holds productivity fixed at its long-run mean, the dotted line fixes productivity 1.5 standard deviations above the mean, and the solid line fixes productivity 1.5 standard deviations below the mean. Panel A plots the firm’s value as a function of net worth. For each level of productivity there exists a critical level of net worth such that the firm will default if realized net worth is below that threshold. Firms with high pro-
ductivity can sustain lower net worth values than can firms with lower productivity.

Panel B of Figure 1.7 plots the firm’s value as a function of its employment stock. The shape of the value function reflects the treatment of labor as a quasi-fixed factor. For low values of labor, the value function is increasing, reflecting the fact that human capital investment is valuable to the firm. It is possible, however, for the firm to overinvest in human capital. Because the firm cannot freely dispose of employees, if the firm accumulates too much labor it can be detrimental to firm value. In contrast, if labor were treated as a purely variable factor it would have no effect on firm value at all, since in that case labor could be instantaneously and costlessly adjusted to its optimal level. In such a model, the plot in Panel B would simply be a flat line at 0. Labor would play no role in firm value and have no effect on firm policies.

Table 1.4 shows the results of the baseline parametrization. I simulate 1000 firms for 200 years, keeping only the last 30 years to ensure that firms have reached their steady state distribution. Importantly, the model is able to produce leverage values in line with those found in the data even with very low proportional costs of distress. The model produces an average level of market leverage of 18%, compared to the average in the data of 21%. Firms in the model rely on equity issuance more heavily than do firms in the data, issuing equity 24% of the time compared to 18% in the data. Equity issuance is also larger in the model than in the data, averaging 17% of assets in the model compared to 9% in the data. The model matches the payout ratio very closely. On average 24% of earnings are paid out as dividends in the model, compared to 22% in the data. The average ratio of labor expense to assets is 36% in the model versus 37% in the data, once again a very close match. The model produces Tobin’s Q values that are a little higher than in the data (1.77 versus 1.49), but not higher than those produced in other studies in the literature (Zhang 2005, Hennessy and Whited 2007, Gomes and Schmid 2010). Average investment is 18%
in the model versus 15% in the data, and the model-implied default frequency of 4% is also somewhat higher than the target rate of 2% found in the data. Overall the model seems to fit the data quite well.

In addition to delivering the value and policy function, the numerical solution delivers the endogenous functions for the wage premium and the interest rate. As is highlighted by equation (1.31), these functions are crucial for determining the marginal cost of debt, and hence the optimal debt policy. Figures 1.8 and 1.9 illustrate graphically the effect from equation (1.31). The dashed line in Figure 1.8 plots total wage expense as a function of debt, while the solid line plots total interest expense as a function of debt. As debt increases both labor expenses and interest expense increase in a convex fashion. This reflects the increasing risk of default as debt increases.

Figure 1.9 plots the firm’s marginal cost of debt, the derivative of the plots in Figure 1.8. The solid line plots the marginal interest expense, the dashed line plots the marginal labor expense, and the dotted line plots the sum of the two, or the total marginal cost of debt. In a model that ignores labor the optimal debt policy would be determined by the solid line. Accounting for labor, the dotted line determines the optimal level of debt. The absolute magnitude of labor related distress costs, and hence the difference between the dotted and solid lines, will depend on how heavily the firm relies on human capital to produce. Thus, the figure illustrates that ignoring labor related costs of distress leads to understatement of the costs of debt and, therefore, overstatement of optimal debt levels.

An important feature of the model is that wages increase as firms approach distress, following the pattern of interest rates. At first glance, this result seems to be at odds with the data. For example, Figure 1.2 shows that on average wages dip very modestly preceding a separation event. The workers represented in the figure were part of mass layoffs, so the firms involved were likely in distress. There
are, however, several factors to consider here. First, my model abstracts from any incentive effects of distress. Workers are assumed to always work while they are employed. In reality, workers for distressed companies may not be as productive as those employed by healthy companies. Second, all workers are homogeneous in my model. In reality, distressed firms are likely to lose their best employees, those who can most easily find a different job, while retaining those who would have a more difficult time finding alternative employment. The more relevant comparison, then, is what happens to productivity-normalized wages for distressed firms. If worker productivity in distressed firms drops more significantly than do wages, then these firms still are more burdened by labor related costs in distress. Finally, in the case that wages do actually decline in distress even when properly measured, this would amount to an additional cost of distress borne by employees, for which they would demand compensation \textit{ex ante}. With competitive labor markets, the firm will end up paying employees for bearing this risk one way or another.

1.5.3 Cross-section of Labor Intensity

As was shown above, firm leverage decreases significantly with labor intensity. In this section I evaluate the model’s ability to match this empirical fact. In the model, labor intensity is controlled by the parameter $\alpha_n$. Table 1.5 shows the leverage generated by the model when this parameter is varied over a range of values. The bottom half of the panel shows the labor intensity percentile in the data that is matched by each value of $\alpha_n$, and the corresponding average market leverage.\footnote{To avoid noisy leverage calculations at the percentile level, first the average leverage for each decile is computed, then the average leverage for the percentile is computed by interpolating between adjacent deciles.} Overall the model does a good job at matching leverage in the cross-section of labor intensity. The model predicts a jump in market leverage from 13% to 34% as labor intensity drops from the 92nd to the 6th percentile, a factor of 2.6. The corresponding change in
leverage in the data is from 12% to 39%, a factor of 3.3. Thus, the model accounts for 79% of the difference in leverage between the lowest and the highest deciles of labor intensity.

To further investigate the relationship between leverage and labor intensity, I run regressions similar to those run in Table 1.1 on simulated data. I construct variables in the model that correspond to each of the empirical variables from Table 1.1 with the exception of tangibility. Since firms use homogeneous capital to produce, there is no notion of tangibility in the model. The measure of size in the model is $k$, which is also the book value of the firm and corresponds to total assets in the data. As in the data, labor intensity in the model is measured as the total wage bill normalized by total assets, $(wn)/k$. The market to book ratio is measured as $(V+b)/k$, profitability is measured as $\pi/k$, and cash flow volatility is measured as the standard deviation of $\pi/k$ over the previous five periods.

Table 1.6 shows the results of running the regressions on simulated data. For each of the seven different levels of labor intensity in Table 1.5, I simulate 1000 firms for 200 years, keeping only the last 30 years. Since measurement of cash flow volatility requires five previous periods, only firms which have been alive at least five years are included. Each of the independent variables enters the regression in deciles, allowing me to directly compare coefficients between Tables 1.1 and 1.6. In general, the model does a good job matching the regressions from the data. The coefficient on labor intensity is -0.030 in the simulated data versus a coefficient of -0.028 in the data. This is expected since the model is designed to match the relationship between labor intensity and leverage. The coefficients on profitability and the market to book ratio in the simulated regressions have the same sign as their counterparts in the actual data, though their magnitudes are smaller. Cash flow volatility also enters the regression with the correct sign, though the magnitude is slightly larger in the simulated regressions (-0.025 versus -0.019). The model does not match the sign on
the size coefficient, though in both simulated and actual regressions size is the least informative predictor of leverage, as measured by $R^2$. While the coefficient on size in Table 1.1 is positive, there is some disagreement about this relationship in the literature. Rampini and Viswanathan (2010) argue that when leverage is measured properly it does not depend on size.

In the multiple regression all variables remain significant, owing largely to the very large sample size. As is the case in Table 1.1, the coefficients on the market to book ratio, profitability, and cash flow volatility are all attenuated when all independent variables are included. Size becomes more negatively related to leverage in the simulated multiple regressions, a pattern that is not consistent with the data. It is worth noting that the coefficient on labor intensity is unchanged in the multiple regression. This is because most of the variation in labor intensity is due to variations in the exogenous parameter $\alpha_n$, whereas the variation in the other variables is due to endogenous responses to technology shocks. Thus there are correlations among the other determinants of capital structure, whereas labor intensity is largely uncorrelated with each of them. This could be addressed by specifying a model where labor intensity responds endogenously to some sort of technology shock. It is not clear that this approach is preferable, however, since one of the criticisms of this class of models is that they do not allow for enough heterogeneity across firms.

1.6 Conclusion

I write a dynamic quantitative model of corporate finance that incorporates labor market frictions. Labor market frictions induce match-specific rents associated with the employment relationship. Termination of this relationship is costly to both firms and employees. The loss of human capital that accompanies unemployment represents a significant cost of financial distress, large enough to have a quantitatively important affect on capital structure. I calibrate the model and find that it is able
to match informative moments from the data. Average leverage in the data is 21% compared to 18% in the model, despite using default costs that are only 5% of the capital stock. The model is also able to match the cross-sectional pattern of leverage and labor intensity. The model predicts a monotonically decreasing relationship between leverage and labor intensity, with a move from the lowest to the highest decile of labor intensity resulting in a drop in leverage of 21 percentage points, compared to a drop of 27 percentage points in the data. Furthermore, the model is able to match the cross-sectional relationships between leverage and the market to book ratio, profitability, and cash flow volatility. Overall the model provides the first quantitative evidence on the importance of human capital for the firm’s capital structure decision.
Table 1.1: **Market Leverage and Labor Intensity.** The table shows the results from regressing market leverage on labor intensity as well as a number of previously recognized predictors of capital structure. The data is from Compustat with financial firms dropped. Labor intensity is measured as total labor expense divided by total assets. Each independent variable is measured in deciles to help control for nonlinearities and to facilitate comparison of coefficients.

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<td>(0.002)</td>
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<tr>
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<td>(0.001)</td>
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<td></td>
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<td>0.309</td>
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*** p<0.01, ** p<0.05, * p<0.1
Robust standard errors in parentheses
Table 1.2: **Book Leverage and Labor Intensity.** The table shows the results from regressing book leverage on labor intensity as well as a number of previously recognized predictors of capital structure. The data is from Compustat with financial firms dropped. Labor intensity is measured as total labor expense divided by total assets. Each independent variable is measured in deciles to help control for nonlinearities and to facilitate comparison of coefficients.

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<td></td>
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<tr>
<td>Cash Flow Vol.</td>
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<td>-0.000</td>
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<td>0.364***</td>
<td>0.294***</td>
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<td>(0.010)</td>
<td>(0.009)</td>
<td>(0.010)</td>
<td>(0.009)</td>
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*** p<0.01, ** p<0.05, * p<0.1

Robust standard errors in parentheses
Table 1.3: Calibration of Benchmark Model.

### Panel A: Production and Taxes

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<thead>
<tr>
<th>Parameter</th>
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<td>$r_0$</td>
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<td>$\nu$</td>
<td>Returns to Scale</td>
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<tr>
<td>$\alpha_n$</td>
<td>Labor's Share of Output</td>
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<td>$f_0$</td>
<td>Fixed Costs of Production</td>
<td>0.1</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Persistence of Productivity Shock</td>
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</tr>
<tr>
<td>$\sigma_z$</td>
<td>Conditional Volatility of Productivity Shock</td>
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</tr>
<tr>
<td>$\delta_k$</td>
<td>Depreciation Rate of Capital Stock</td>
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<tr>
<td>$\delta_n$</td>
<td>Rate of Voluntary Labor Turnover</td>
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<tr>
<td>$\tau_c^+$</td>
<td>Maximum Corporate Tax Rate</td>
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</tr>
<tr>
<td>$\tau_c^-$</td>
<td>Minimum Corporate Tax Rate</td>
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<td>$\tau_i$</td>
<td>Tax Rate on Interest Income</td>
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<td>$\tau_d$</td>
<td>Tax Rate on Dividends</td>
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### Panel B: Equity Issuance and Default Costs

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<td>$\lambda_0$</td>
<td>Fixed Costs of Equity Issuance</td>
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<td>Proportional Costs of Equity Issuance</td>
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<td>Quadratic Costs of Equity Issuance</td>
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<td>$\xi_k$</td>
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### Panel C: Labor Market

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<td>$g^+$</td>
<td>Hiring Costs</td>
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<td>$w_u$</td>
<td>Unemployment Benefit</td>
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<td>$\theta$</td>
<td>Workers’ Bargaining Power</td>
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<td>$p_e$</td>
<td>Probability of Successful Job Search</td>
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Table 1.4: Results of Benchmark Calibration

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<td>Average Net Market Leverage</td>
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<td>0.18</td>
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<tr>
<td>Average Wages/Assets</td>
<td>0.37</td>
<td>0.36</td>
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<tr>
<td>Equity Issuance Frequency</td>
<td>0.18</td>
<td>0.24</td>
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<tr>
<td>Average Equity Issuance/Assets</td>
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<tr>
<td>Average Payout Ratio</td>
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<td>Average Investment/Assets</td>
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<td>Average Tobin’s Q</td>
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<td>Default Frequency</td>
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Table 1.5: **Leverage and the Cross-Section of Labor Intensity.** The table shows leverage as a function of labor intensity both in the model and in the data. The labor intensity parameter, $\alpha_n$, is varied across simulations, producing a model implied labor intensity and leverage. The labor intensity from the model is mapped into a percentile of labor intensity in the data for which the corresponding leverage is calculated. To avoid noisy calculations at the percentile level, leverage is calculated at the decile level of labor intensity, then leverage at the percentile is interpolated between adjacent deciles.

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<th>$\alpha_n$</th>
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<th>0.6</th>
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<tr>
<td>Wages/Assets</td>
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<td>0.48</td>
<td>0.36</td>
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<td>Labor Intensity Percentile</td>
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<tr>
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Table 1.6: **Leverage and Labor Intensity in the Model.** The table shows the results from regressing market leverage on labor intensity as well as a number of previously recognized predictors of capital structure using data simulated from the model. For comparison with Table 1.1, the independent variables are all measured in deciles.

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</tr>
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<td>-0.012</td>
<td>-0.012</td>
<td>-0.012</td>
<td>-0.012</td>
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<tr>
<td>Market/Book</td>
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<td>-0.007</td>
<td>-0.007</td>
<td>-0.007</td>
<td>-0.007</td>
<td>-0.007</td>
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<tr>
<td>Profitability</td>
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<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>Cash Flow Vol.</td>
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<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
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</tr>
<tr>
<td>Constant</td>
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<td>0.256</td>
<td>0.272</td>
<td>0.317</td>
<td>0.368</td>
<td>0.489</td>
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<tr>
<td>$R^2$</td>
<td>0.315</td>
<td>0.007</td>
<td>0.018</td>
<td>0.081</td>
<td>0.210</td>
<td>0.343</td>
</tr>
</tbody>
</table>
Figure 1.1: Labor’s Share of Income. The figure is taken from the National Income and Product Accounts (NIPA) tables for the period 1959-2010. In aggregate, 58% of income goes to employee compensation, 23% to operating surplus (payments to financial claimholders), 11% to depreciation, and 8% to taxes on production.
do not exhibit the sharp dip reported by JLS. In the period prior to job loss, the workers experiencing mass layoff receive, on average, about $950 in their final pay. This observation is due to the receipt of lump sum severance payments by a limited number of workers.

In the period immediately following job loss, using the fixed-effects and time trend models, the estimated reductions in earnings are $4,254 (32 percent) and $4,341 (33 percent), respectively. Six years later, the average quarterly earnings losses for that year are $1,699 (13 percent) and $1,923 (15 percent), respectively. The initial losses reported here for the mass layoff sample are similar to those of other separators; however, their earnings recover more slowly and they plateau at a lower level. The result that earnings losses of the mass layoff sample using administrative data for Connecticut are similar in magnitude to the range of estimates obtained by other researchers for prime-age workers using data from the DWS and PSID is important in establishing that, in economic conditions less severe than those experienced in Pennsylvania at the time of the JLS study, different data types yield similar results.

Using propensity score methods to match those who experience mass layoff to continuously employed workers who have identical probabilities of displacement, differences in earnings paths of these individual pairs are taken in each pre- and postdisplacement period and averaged.

Three working papers using administrative data to track earnings of workers who experience mass layoff also do not find a substantial dip in earnings in predisplacement periods (Lengermann and Vilhuber 2002; Shoeni and Dardia 2003; and Andrew K. G. Hildreth, Till M. von Wachter, and Elizabeth W. Handwerker 2007). Hildreth et al. additionally find upward spikes in earnings in the year prior to separation, as reported here.

The earnings of the group of displaced workers in 1998:IV averaged $13,228. This amount was used in calculating the percentage earnings losses in the text. The graphed parameter estimates and associated standard errors are contained in Web Appendix K.

JLS (1993b, figure 5.10) examine earnings losses among all separators in high and low unemployment regions of Pennsylvania. The comparison includes both those who separate as part of a mass layoff and other job changers. They report earnings losses among all separators 47 percent smaller in low relative to high unemployment regions. Thus, they also find significant variation in earnings losses associated with business cycle conditions.

**Figure 1.2: Earnings Losses of Displaced Workers.** The figure is taken from Figure 2 in Couch and Placzek (2010) and shows the earnings losses of workers who were laid off in mass layoffs in Connecticut starting in 1999. Estimates from two different models are plotted. The solid line represents estimates where a time trend is included in the model for wages. The dashed line includes individual fixed effects. The average quarterly salary in the sample was $13,288.
Figure 1.3: Market Leverage and Labor Intensity. The figure plots mean market leverage against deciles of labor intensity. Firms are sorted into deciles based on labor intensity, then within-decile means of market leverage are calculated. The data is taken from Compustat after dropping financial firms. Only firms who report labor expenses are included in the sample.
Figure 1.4: Book Leverage and Labor Intensity. The figure plots mean book leverage against deciles of labor intensity. Firms are sorted into deciles based on labor intensity, then within-decile means of book leverage are calculated. The data is taken from Compustat after dropping financial firms. Only firms who report labor expenses are included in the sample.
Figure 1.5: Solution for Endogenous Wages. The figure illustrates the solution method to arrive at the firms endogenous wage function $w(k', n', b', z)$. In the first step, Nash Bargaining is used to divide up the relationship surplus between the firm and the employees. This gives a certainty equivalent wage that is constant over time. In the next step, the wages are determined as a function of the endogenous state of the firm such that the employees are exactly compensated for the risk that they bear.

Figure 1.6: Critical Regions of Productivity Shocks. The figure illustrates the critical regions of the state of technology, $z$. For the lowest values of the shock, creditors get nothing while employees recover a portion of their promised wages. For slightly better productivity states, employees are paid their promised wages, but the firm chooses to default on its debt obligations. Finally, for productivity states better than $z_d$, the firm pays its creditors and keeps operating.
Figure 1.7: Value Function. Panel A plots the firm’s value as a function of net worth, while Panel B plots the firm’s value as a function of its labor stock. In each panel, the dashed line fixes productivity at the long-run average level, the dotted line fixes \( z \) 1.5 standard deviations above the mean, and the solid line fixes \( z \) 1.5 standard deviations below the mean.
Figure 1.8: **Wages and Interest Expense vs. Debt.** The dashed line plots the wage risk premium as a function of debt in the model, while the solid line plots the interest rate risk premium. As debt exposes both employees and bondholders to risk, each must be compensated for bearing that risk.
Figure 1.9: Marginal Cost of Debt. The top panel plots the marginal costs and benefit of debt for a firm with high labor intensity, while the bottom panel plots the same for a firm with low labor intensity. In each panel the solid line plots the marginal interest expense, the dashed line the marginal wage expense, and the dotted line the total marginal cost associated with debt (the sum of the first two lines), each as a function of debt in the model. Additionally, the solid horizontal line plots a hypothetical marginal benefit of debt curve. In a model which ignores labor, the optimal debt policy would be determined by the intersection of solid lines, whereas in my model the optimal debt policy is determined by the dotted line’s intersection with the marginal benefit curve. While all firms’ optimal leverage is affected by the presence of labor, the effect is much greater for labor intensive firms.
Optimal Financing Contracts with Heterogeneous Entrepreneurs

2.1 Introduction

It has long been accepted that agency problems are significant determinants of firm investment, size, survival, and capital structure. Jensen and Meckling (1976) laid the groundwork for a vast literature aimed at understanding the agency conflicts that arise between a firm’s shareholders and its managers and how best to alleviate them. Recently the dynamic agency literature has sought to characterize how the evolution of agency problems over time influences firm dynamics. These models generally feature an entrepreneur who needs financing to run a project. The relationship is subject to an agency problem, and the evolution of this problem over time determines firm characteristics. The intuition behind dynamic agency is the following: in order to provide incentives to do the right thing, the principal needs to make the agent’s continuation value as high as possible. The continuation value determines the agent’s access to capital today, but is itself determined by the agent’s future access to capital. Albuquerque and Hopenhayn (2004) consider a general model of limited
commitment in which the entrepreneur can choose to default and accept an outside option. Clementi and Hopenhayn (2006) consider the case of unobservable shocks to output. One limitation of these models is that there is no default in equilibrium. To address this counterfactual implication, Hopenhayn and Werning (2008) consider a model in which the agent’s outside option is random and privately observed. The asymmetric information regarding the outside option delivers default in equilibrium and allows the dynamics of debt, default, and capital to be jointly studied.

I study a two-period model with a set of entrepreneurs who each have a project that needs financing. Each entrepreneur must finance an initial fixed cost and capital input in each period. Each entrepreneur has the option to default and to divert a fraction of his project’s output. However, entrepreneurs are heterogeneous with respect to this ability. That is, some entrepreneurs are able to divert a larger proportion of the output and some a smaller proportion. Each entrepreneur’s type is private information, which induces asymmetric information regarding the entrepreneur’s outside option. This information asymmetry implies that default may occur in equilibrium. The purpose of this paper is to analyze the effect of the presence of heterogeneous agents on the optimal dynamic financing contract. My model is similar to Hopenhayn and Werning (2008) in that I study equilibrium default that arises from asymmetric information regarding the entrepreneur’s outside option. The key difference is that I study the optimal financing contract under *ex ante* private information. As far as I know, this is the first paper to study optimal dynamic financing contracts with *ex ante* hidden information. Importantly, while Hopenhayn and Werning (2008) study an infinite-horizon project, at present my model has only two periods.

The entrepreneur heterogeneity in my model has several potential interpretations. It may be that the heterogeneity actually represents some unobservable characteristic of the project rather than the entrepreneur. For example, the organizational structure of the project may help determine how easily the entrepreneur can divert
output. If the entrepreneur alone exercises executive power, then it may be easier for him to divert output. A structure wherein several individuals together exercise executive power would then make it more difficult for the entrepreneur to divert output. Alternatively, it may simply be a matter of skill or talent. That is, some entrepreneurs may be more skilled at diverting output than others. Finally, the heterogeneity could be interpreted as differences in the entrepreneurs’ ethical standards or social networks. If the entrepreneur considers it unethical to divert output or if he would pay a high social cost within his social network for choosing to divert, then effectively his ability to divert output would be low. While it is left to each reader to determine his interpretation of the model, I have in mind the last one. As such, I consider only two types of entrepreneurs: some who can divert a positive proportion of output and others who cannot divert any output. I call entrepreneurs who can divert output “opportunistic,” and I call those who cannot divert “dependable.”

I choose to model differences in entrepreneur ethics as a constraint on the choice set, what Rabin (1995) calls “moral constraints.” Note that it need not be the case that there are people who actually cannot steal. It may be that people derive utility from not stealing (“moral preferences”) or that they pay a social cost when they choose to steal. These different possibilities turn out to be observationally equivalent. In choosing this modeling approach, I am simply trying to capture the idea that some people are more averse to certain behaviors than others. For example, Fisman and Miguel (2007) find that there are significant and persistent cross-country differences in the number of parking tickets received by United Nations diplomats in New York City, even though diplomatic immunity protected each diplomat from any legal enforcement of the tickets. Diplomats from some countries did not receive any parking tickets during the sample period, while diplomats from other countries received hundreds. Presumably each diplomat would stand to gain roughly equally from parking illegally, yet something prevented the diplomats who received no park-
ing tickets from taking advantage of their immunity. Effectively, these diplomats behaved as if they had a constraint on their choice set. It is outside the scope of this paper to investigate why such heterogeneity arises. Instead, I take entrepreneur heterogeneity as given and investigate the implications.

The presence of a proportion of the population who will voluntarily adhere to social norms may significantly alter the predictions of agency theory. Carlin and Gervais (2009) examine a principle-agent model where not all agents need incentives to exert effort. They derive optimal contracts that differ meaningfully from the standard principle-agent contract.

There are several possible outcomes for the optimal dynamic financing contract. First, the lender could potentially offer a menu of contracts to screen entrepreneurs. I show that this outcome is not possible. It turns out that any contract that could potentially screen entrepreneurs allows the opportunistic entrepreneurs to profit at the expense of the dependable entrepreneurs. If a lender offers this menu of contracts, she will lose money if she happens to lend to an opportunistic entrepreneur and will earn a profit if she happens to lend to a dependable entrepreneur. Another lender could offer an alternative contract that dependable entrepreneurs would prefer, thus skimming them away and leaving the original lender with only opportunistic entrepreneurs.

Alternatively, it may be optimal to induce the opportunistic entrepreneurs to reveal their type by defaulting in the first period. Then the dependable entrepreneur’s project could potentially be unconstrained in the second period. However, this strategy leaves the dependable entrepreneur with the burden of making the lender whole after the first period loss. Also, the project scale in the second period would still be constrained by the need to induce first period default. The entrepreneurs’ continuation utility is determined by the project scale in the second period. If that scale is too large, the opportunistic entrepreneur will not be willing to give up his continuation
utility by defaulting in the first period.

If, instead, the opportunistic entrepreneur were induced to default in the second period, then a smaller burden would be passed to the dependable entrepreneur because the opportunistic entrepreneur would already have made the first period debt payment. Second period default has an additional benefit. The opportunistic entrepreneur receives a greater payoff when he chooses to default, thus his continuation utility is higher with second period default. The prospect of receiving this larger payoff in the second period disciplines his behavior in the first period. The debt payment that he will be willing to pay in the first period is therefore larger than it would be if he were not to default in the second period.

Finally, it may simply be optimal for the lender to offer a contract that prevents default. This is the same contract that would be optimal if every entrepreneur were opportunistic. If this is the case, then the presence of dependable agents has no effect on the optimal contract.

I use numerical methods to compare the different contracts. I find that the contract that induces second period default dominates the contract that induces first period default. Generally, there is a critical proportion of dependable agents for which it becomes optimal to induce second period default. If there are too few dependable entrepreneurs, the cost of default is too high, and the optimal contract provides incentives against default. If there are enough dependable entrepreneurs, then the benefits of running the project at a larger scale outweigh the costs, and second period default becomes optimal.

The model provides empirical predictions about the types of contracts likely to be seen. For example, if citizens of some countries adhere more closely to social norms than in other countries, then the model predicts that defaults will be lower while investment and output will be higher in those countries. Moreover, the model predicts that entrepreneurs with a large amount of social capital are less likely to default. For
example, the model would predict that family firms would have lower default rates and higher investment and output, all else equal. Also, if adherence to social norms varies over time, then the model yields time series predictions. Suppose, for example, it is less socially costly to strategically default in bad times; then the model would predict that contracts written in bad times would provide less capital and stronger incentives against default than contracts written in good times. Furthermore, the model predicts that, all else equal, firms that produce output that is easier to divert will have higher rates of default, lower investment, and lower output. For example, if it is easier to divert output from a service firm than a firm that produces tangible goods, then the model would predict that service firms would have higher rates of default, lower investment, and lower output.

The paper proceeds as follows. In section 2, I describe the model and solve for the benchmark, no-default contract that would be optimal if every entrepreneur were opportunistic. In section 3, I characterize the optimal contracts in the presence of heterogeneous entrepreneurs. In section 4, I compare contracts using numerical evidence. Section 5 concludes.

2.2 Model

The model consists of entrepreneurs and lenders, all of whom are risk-neutral and have the same discount factor. The gross risk-free interest rate is $R$. Each entrepreneur has a project that only he can run. The project lasts two periods and requires capital input $k_t$, delivering deterministic output $y(k_t), t = 1, 2$. $y$ is twice continuously differentiable, strictly concave, and satisfies the Inada conditions. Capital depreciates fully each period. In addition to capital input, the project requires an initial fixed cost $F$. The entrepreneur has no initial wealth, and so he must borrow to finance the initial cost and each period’s capital input.

A set of competitive lenders offers financing contracts to the entrepreneurs. A
contract specifies first period capital allocation and debt payment, \( \{k_1, D_1\} \), second period capital allocation and debt payment contingent on first period repayment, \( \{k_2, D_2\} \), and second period capital allocation and debt payment contingent on first period default, \( \{k^d_2, D^d_2\} \). Lenders have the ability to commit to a contract.

A proportion \( 1 - \phi \) of entrepreneurs are “opportunistic”, and the remaining proportion \( \phi \) are “dependable.” Opportunistic entrepreneurs have limited commitment. If an opportunistic entrepreneur chooses to default rather than pay \( D_t \), he can divert a proportion \( \alpha \) of the output and the lender can recover a proportion \( \beta \). Thus, \( \alpha \) parameterizes the severity of the agency problem. I assume that \( \alpha + \beta < 1 \) so that default destroys value. If a dependable entrepreneur chooses to default, he cannot divert any output. Thus, a dependable entrepreneur will never choose to default. Each entrepreneur’s type is private information.

2.2.1 A static example

There are potentially both static and dynamic benefits to allowing default to occur in equilibrium. To try to separate the two, I begin by presenting a simple static example. The model is the same as that presented in the previous section except that the project lasts only one period. If the entrepreneur could commit to repay the loan, then the optimal contract would consist of capital input of \( k^* \), where \( y'(k^*) = R \).

At this level the marginal product is equal to the cost of capital. The entrepreneur would repay \( R(k^* + F) \) and would be left with the rest.

Suppose instead that the entrepreneur cannot commit to repay. If he chooses to default he can divert \( \alpha y(k) \), so the most the lender can expect to be repaid is \( (1 - \alpha) y(k) \). If the fixed cost is large enough, this limited commitment constraint restricts the amount of capital that can be deployed, as shown in Figure 2.1. In the figure, the blue dashed line represents the contracted debt payment, and the green dashed curve represents the pledgable amount, \( (1 - \alpha) y(k) \). The maximum loanable
amount of capital, $\bar{k}$, is determined by the intersection.

Figure 2.1: Static Example: Limited Commitment Constrains Lending. With full commitment lending would take place up to capital level $k^*$, whereas with limited commitment lending is constrained such that the amount of capital deployed is $\bar{k}$.

Finally, suppose that a proportion $\phi$ of the entrepreneurs are dependable while the remaining entrepreneurs are opportunistic. If the lender is willing to allow default, then the capital will not be constrained as illustrated in Figure 2.1. Instead, the lender can lend more than $\bar{k}$ and allow the opportunistic entrepreneurs to default. If the gains from running the project at a larger scale are significant enough, then the dependable types will be willing to pay more than $R(k + F)$ so as to satisfy the lender’s participation constraint. The dependable entrepreneur solves

$$\max_{k,D} y(k) - D \quad (2.1)$$
\begin{equation}
\phi D + (1 - \phi) \beta y(k) \geq R(k + F) \tag{2.2}
\end{equation}
\begin{equation}
y(k) \geq D \tag{2.3}
\end{equation}

Equation (2.2) is the lender’s participation constraint, which will bind in equilibrium. Equation (2.3) is a feasibility constraint on the debt payment. Since the entrepreneur has no wealth, the debt payment cannot be larger than the project output. Ignoring (2.3) (since if it binds, the dependable entrepreneur gets zero), the first order condition for this program is

\[
y'(k) = \frac{R}{\phi + (1 - \phi) \beta}.
\tag{2.4}
\]

It is easy to see from this condition that, as \( \phi \) approaches one, \( k \) approaches \( k^* \). Intuitively, the equilibrium outcome converges to first best as the limited commitment constraint disappears. This means that if there are enough dependable entrepreneurs, then there exists a unique equilibrium where the amount of capital borrowed is strictly greater than \( \bar{k} \). This illustrates the static benefit from allowing default. Note that in this equilibrium, the opportunistic entrepreneurs will be strictly better off than the dependable entrepreneurs.

Note here that I have used the dependable entrepreneur’s payoff as the objective function. This is because, given default, the lender loses money from lending to an opportunistic entrepreneur and makes a profit from lending to a dependable entrepreneur. In order to prevent another lender from skimming away only the dependable entrepreneurs, the lender must offer a contract which maximizes the dependable entrepreneurs’ utility. This result holds generally and will be discussed in more detail below.
2.2.2 First Best Two Period Contract: $\phi = 1$

Before analyzing the constrained optimal contract, I characterize the first best contract. The first best contract obtains when all entrepreneurs have full power to commit to repay. In terms of the parameters of the model, this corresponds to the special case $\phi = 1$. In this case, there will be no default and no need to provide repayment incentives. The contract maximizes the entrepreneur’s payoff subject to the lender’s participation constraint and feasibility constraints on the debt payments. It is easy to see from the assumptions on the production function that the project has a unique optimal scale, $k^*$, defined by $y'(k^*) = R$. At this level the cost of capital is equal to the marginal product. Since the entrepreneur can commit to repay, the project is run at the optimal scale both periods, $k_1 = k_2 = k^*$. The lender’s participation constraint will bind, $RD_1 + D_2 = R^2(k_1 + F) + Rk_2$, but specific debt payments are indeterminate. This is another general feature of the model: when the project can be run at the optimal scale the timing of debt payments is indeterminate. Throughout this paper I focus on unique optimal contracts, and hence on contracts where the project is constrained away from the optimal scale. A more formal characterization of the first best contract is given in the appendix.

2.2.3 Benchmark Contract: $\phi = 0$

In a typical model of dynamic financing constraints with limited commitment, all entrepreneurs would be homogeneous and would be subject to the limited commitment constraint. Thus, I characterize the optimal contract in the special case in which every agent is opportunistic, $\phi = 0$. I will use this contract as a benchmark against which to compare the contracts that do not prevent default.

The optimal contract maximizes the entrepreneur’s payoff subject to the lender’s participation constraint, the entrepreneur’s incentive constraints that arise from limited commitment, and feasibility constraints on the debt payments. Competitive
lending implies that the lender’s participation constraint will bind at the optimum, so the contract will be the same whether the objective is to maximize total output or to maximize the entrepreneur’s payoff. Formally, I solve:

\[
\max_{k_1, k_2, k_2^d, D_1, D_2, D_2^d} Ry(k_1) + y(k_2) \tag{2.5}
\]

s.t.

\[
RD_1 + D_2 \geq R^2(k_1 + F) + Rk_2 \tag{2.6}
\]

\[
R(y(k_1) - D_1) + y(k_2) - D_2 \geq R\alpha y(k_1) + y(k_2^d) - D_2^d \tag{2.7}
\]

\[
R(y(k_1) - D_1) + y(k_2) - D_2 \geq R(y(k_1) - D_1) + \alpha y(k_2) \tag{2.8}
\]

\[
R(y(k_1) - D_1) + y(k_2) - D_2 \geq R\alpha y(k_1) + \alpha y(k_2^d) \tag{2.9}
\]

\[
y(k_1) \geq D_1 \tag{2.10}
\]

\[
y(k_2) \geq D_2 \tag{2.11}
\]

\[
y(k_2^d) \geq D_2^d \tag{2.12}
\]

Equation (2.6) is the lender’s participation constraint. Equations (2.10)-(2.12) are feasibility constraints on the debt payments. The debt payments cannot be larger than the output since the entrepreneur has no wealth. I assume that the entrepreneur consumes anything that he does not pay out as a debt payment in the first period. The incentive constraints (2.7) - (2.9) ensure that there will be no default in equilibrium. Equation (2.7) ensures that the entrepreneur prefers to make the contracted payments in each period rather than default in the first period; equation (2.8) ensures that the entrepreneur will not default in the second period; and equation (2.9) ensures that the entrepreneur will not default in both periods. Since default destroys value, the optimal contract will prescribe payments which satisfy these constraints. To see this, note that there is no information asymmetry between the lender and the entrepreneur. Thus, any default by the entrepreneur
would be anticipated by the lender and would not prevent the lender from breaking even. Since the lender will always break even, the entrepreneur would bear the cost of any default, implying that default will not occur in the optimal contract. Note that this feature is a common yet counterfactual implication of many models of dynamic financing constraints. This is one of the questions that my model seeks to address: under what conditions will default occur in an optimal dynamic contract?

Proposition 1 summarizes the properties of the optimal contract. For a proof, see the appendix.

**Proposition 1.** Suppose that \( F \) is small enough that the project can feasibly be financed but large enough that the first best capital allocation is not possible in the second period, \( k_2 < k^* \). Then there is a unique optimal contract for \( \phi = 0 \) with the following properties:

(i) There is no default in equilibrium.

(ii) \( k_1 \) is determined implicitly by the following equation:

\[
R(k_1 + F) + y^{-1}(Ry(k_1)) = (2 - \alpha) y(k_1)
\]  \( (2.13) \)

(iii) \( k_2 \) is given by:

\[
y(k_2) = Ry(k_1)
\]  \( (2.14) \)

(iv) \( D_1 = y(k_1) \) and \( D_2 = (1 - \alpha) y(k_2) \)

(v) \( k_2^d = D_2^d = 0 \)

(vi) \( k_1 \) and \( k_2 \) are strictly decreasing in \( F \).

Proposition 1 holds as long as \( F \) is neither “too big” nor “too small.” To understand what is meant by “too big” and “too small,” consider Figure 2.2. The solid blue upper curve in Figure 2.2 represents \( y(k) \), while the dashed blue lower curve
represents \((1 - \alpha)y(k)\). The solid red upper line has slope \(R\) and is tangent to \(y(k)\) at \(k = k^*\). The dashed green lower line represents the second period debt payment, \(D_2\), as a function of second period capital, \(k_2\) and is defined by the lender’s participation constraint \((2.6)\), which binds in equilibrium. This line has slope \(R\) and intercept

\[
R^2(k_1 + F) - Ry(k_1),
\]

where I have substituted in the equilibrium condition \(D_1 = y(k_1)\). Equation \((2.15)\) is essentially the negative of the entrepreneur’s net worth after the first period. Since the entrepreneur can divert \(\alpha y(k_2)\), the most that the lender can require is \((1 - \alpha)y(k_2)\), so \(D_2 \leq (1 - \alpha)y(k_2)\). The largest possible value of \(k_2\) is therefore given by the point of intersection of the dashed curve and the dotted line. Beyond that the entrepreneur will find it optimal to default. An increase in \(F\) raises the dashed line both directly and through its effect on \(k_1\). The largest value of \(F\) such that the project can feasibly be financed would occur where the dashed line would be tangent to the dashed curve. Note that this does not mean that the project is not potentially profitable for larger \(F\), only that the limited commitment constraints make financing impossible.

On the other hand, a decrease in \(F\) pushes the dashed line downward, pushing \(k_2\) out toward \(k^*\). \(F\) is too small when the intersection occurs at a value of \(k\) larger than \(k^*\). In this case, \(k_2 = k^*\), and the debt payments become indeterminate. This is because, though the limited commitment constraints exist, the fixed cost is small enough that they do not bind. There is nothing “wrong” with this case, except that it is less interesting than the case in which constraints bind.

Figure 2.2 also helps illustrate the dynamic dependence between \(k_1\) and \(k_2\). The above discussion highlights the fact that the amount of capital the entrepreneur can deploy in the second period depends on the amount deployed in the first period. Likewise, the amount of capital deployed in the second period determines the amount
Figure 2.2: Dynamic Dependence Between $k_1$ and $k_2$. The figure shows the dependence between capital levels $k_1$ and $k_2$. $k_2$ is determined by the intersection of the dashed line and the dashed curve, while the dashed line is determined by $k_1$.

that can be deployed in the first period. This is because the prospect of a payoff tomorrow provides the bonding necessary to induce the entrepreneur to make his debt payment today. A larger second period payoff generates a greater incentive to repay in the first period, and hence a larger amount of first period capital can be financed. This dynamic dependence is a key feature of dynamic models of financing constraints, and it will be exploited to produce optimal contracts featuring default in the following sections.

2.3 Optimal Contract with Heterogeneous Entrepreneurs

In this section I will analyze the optimal contract in the presence of heterogeneous entrepreneurs. There are several possibilities to consider when searching for the
optimal contract with heterogeneous entrepreneurs. First, as with any problem of \textit{ex ante} private information, it is natural to ask whether it is possible for the lender to screen the entrepreneurs with a menu of contracts. I show below that this is not possible. Thus, in equilibrium, each entrepreneur will accept the same contract.

Another possibility is that the optimal contract with heterogeneous entrepreneurs induces default by opportunistic entrepreneurs. Notice that when every entrepreneur is opportunistic, there is no default in the optimal contract by construction. Default imposes a positive cost and yields no benefit, so in the absence of asymmetric information default will not occur. In contrast, the optimal contract with heterogeneous entrepreneurs does not necessarily preclude default. Avoiding default forces the project to be run at a suboptimal scale. As I showed in the static example, when default is allowed the project can potentially be run at a larger scale.

Additionally, there are dynamic benefits from default that arise in the two-period setting. If opportunistic entrepreneurs can be induced to default in the first period, then the dependable entrepreneurs could signal their type by making the first period payment. Then, in the second period, the lender would lend the dependable type as much as he wanted. This does not mean, however, that the dependable entrepreneur would be completely unconstrained in the second period. If the lender offered this contract, she would lose money if she happened to lend to an opportunistic entrepreneur. In order to satisfy the lender’s \textit{ex ante} participation constraint, the dependable type would have to pay more in the second period to cover the lender’s possibility of loss. Here the contract relies on the lender’s (and the dependable entrepreneur’s) ability to commit. By the time the second period arrives the lender would know what type of entrepreneur she lent to, but she must still require the dependable entrepreneurs to compensate her for the (unrealized) risk of loss. The dependable entrepreneur’s second period payoff would be further constrained by the need to provide incentive for first period default to the opportunistic entrepreneurs.
If the project were run with too much capital in the second period, then the opportunistic entrepreneur would prefer to wait to default. A contract in which there is first period default will dominate a contract without default if the benefits of running the project at a larger scale and type revelation are enough to offset the costs of default imposed on the dependable entrepreneurs.

Alternatively, an optimal contract might induce the opportunistic entrepreneurs to default in the second period. Remember from the static example above that opportunistic entrepreneurs are strictly better off than dependable entrepreneurs when they choose to default. This fact is especially useful when default occurs in the second period. The opportunistic entrepreneur’s higher payoff in the second period when he will default provides the bonding necessary to induce him to make a larger first period debt payment. Thus, second period default will allow the project to be run at a larger scale in both the first and the second periods. A contract in which the opportunistic entrepreneurs default in the second period will dominate a contract with no default if these benefits outweigh the costs of default.

Note that it would never be optimal to allow the opportunistic entrepreneur to default in both periods. Once the opportunistic entrepreneur has revealed his type by defaulting in the first period, the lender knows with certainty that she can only extract \((1 - \alpha)y(k_2)\) from him in the second period. If she asked for more than this he would default, and she would be left with \(\beta y(k_2) < (1 - \alpha)y(k_2)\). There is no reason to incur the cost of second period default after types have been revealed.

2.3.1 Some Intermediate Results

Before I can characterize the optimal contract with first period default, it is necessary to establish some intermediate results. First, it is not obvious what the objective function should be in the presence of heterogeneous entrepreneurs. I argued in the case of homogeneous entrepreneurs that the same contract would maximize both total
output and the entrepreneur’s payoff given competitive lending. This is not true in the case of heterogeneous entrepreneurs. Consider, for example, the objective to maximize total expected output. If this were the objective, then the optimal contract would exploit the dependable types. To see this, note that the output is constrained by the opportunistic entrepreneurs’ limited commitment constraints and will be maximized when these constraints are relaxed as much as possible. The lender can relax these constraints by demanding all of the output in both periods as payment, and the dependable entrepreneur would comply. The opportunistic entrepreneur’s constraints would be relaxed as much as possible by exploiting the dependable entrepreneur in this way, enabling total output to be maximized.

This could not be an equilibrium contract, however. Under this contract the lender would profit from the dependable entrepreneurs and lose money from lending the opportunistic entrepreneurs, breaking even in expectation. Another lender could then offer an alternative contract that left a small surplus for the dependable entrepreneurs and effectively skim them away from the first lender. The first lender would be left with only the opportunistic entrepreneurs, from whom she would lose money.

This suggests a general result. Suppose under some contract that the lenders lose money from lending to an opportunistic entrepreneur. This could be because the opportunistic entrepreneur will default under the contract or because the contract specifies payments by the opportunistic entrepreneur that are too low to break even (so as to avoid the deadweight cost of default). Then, in order to break even in expectation, it must be that the lenders earn a profit from lending to dependable entrepreneurs. Suppose further that the contract does not maximize the payoff to dependable entrepreneurs. Then any other contract which offered the dependable entrepreneurs a higher payoff could skim them away. If the alternative contract also offered a higher payoff to opportunistic entrepreneurs, then clearly it would
dominate the original contract. If the alternative contract offered a lower payoff to opportunistic entrepreneurs, then they would accept the original contract, and the lender who offered it would lose money. Thus I have shown

**Lemma 2.** *Any equilibrium contract under which the lender loses money from doing business with opportunistic entrepreneurs must maximize the payoff to the dependable entrepreneurs.*

The second intermediate result that will be useful in characterizing the optimal contracts with heterogeneous entrepreneurs has to do with the incentive constraints. With heterogeneous entrepreneurs, the constrained optimization problem that defines each optimal contract will have multiple incentive constraints. Since only the opportunistic entrepreneurs require incentive constraints, each constraint is in place to prevent them from doing something. For example, if the contract is designed to induce first period default, there will be three incentive constraints that guarantee that the opportunistic entrepreneurs prefer to default in the first period rather than repay each period, default in the second period, or default in both periods.

One of the challenges in analytically characterizing the optimal contracts is that it is difficult to tell which incentive constraints bind in general. As I will show below, it turns out that different constraints bind in different regions of the parameter space. However, there is one general statement that can be made. With the exception of the no-default, pooling equilibrium contract, each contract calls for the opportunistic entrepreneurs to behave differently than the dependable entrepreneurs, so each contract will have a constraint which prevents the opportunistic entrepreneurs from mimicking the dependable entrepreneurs. In a separating equilibrium this constraint will prevent the opportunistic entrepreneurs from accepting the contract designed for the dependable entrepreneurs and making both specified debt payments. In a pooling equilibrium, it will simply prevent the opportunistic entrepreneurs from making
both debt payments.

Suppose that this constraint binds, so that both types of entrepreneurs are equally well off under a given contract. Then the opportunistic entrepreneur would be just as well off to mimic the dependable entrepreneur by accepting the same contract and making the specified debt payments. In fact, by revealed preference, this deviation would leave the opportunistic entrepreneur as well off as any of his possible choices given the offered contract(s). That is, the opportunistic entrepreneurs can do no better than behaving exactly the way the dependable entrepreneurs do. Thus, the dependable entrepreneur’s contract must be incentive compatible. Furthermore, the dependable entrepreneur’s contract must always satisfy the lender’s participation constraint and the debt payment feasibility constraints. Thus, his contract is subject to all of the constraints of the benchmark, no-default contract, which has a unique solution by Proposition 1.

Suppose further that, lenders lose money from lending to opportunistic entrepreneurs. Then lenders must profit from lending to dependable entrepreneurs. The dependable entrepreneur’s contract is subject to incentive compatibility constraints, feasibility constraints, and the lender’s participation constraint, just like the benchmark contract. Since the lender earns a profit from the dependable entrepreneurs, it must be that they would be strictly better off under the benchmark contract. Then, by Lemma 2, this cannot be an equilibrium contract. I have shown

**Lemma 3.** Consider a contract under which the opportunistic entrepreneurs and the dependable entrepreneurs behave differently but get the same payoff. Then,

(i) neither entrepreneur can be better off than he would be with the benchmark, no-default contract.

(ii) Moreover, if the lender loses money from lending to the opportunistic entrepreneurs, then both types would be strictly better off with the benchmark
Lemma 3 does not imply that the relevant incentive constraint will be slack at the solution to the constrained optimizations below, only that it must be for there to be hope that the contract under consideration will dominate the benchmark contract.

2.3.2 Non-existence of a Separating Equilibrium

In this section I will show that it is not possible for the lender to offer a menu of contracts to screen entrepreneurs. There are several cases to consider. First, suppose that the lenders try to screen the entrepreneurs with a menu of contracts in which neither type defaults. In this case, the payoffs to each type of entrepreneur must be the same, otherwise the one with the lower payoff would change his choice of contract. Then, by Lemma 3, neither entrepreneur can be better off than he would be with the benchmark contract. But, by Proposition 1, the benchmark contract is a unique maximum of the solution to the optimization without default. So, in equilibrium, both types of entrepreneurs will be offered the same contract, namely the benchmark contract. Thus, it is not possible to implement a separating equilibrium without default.

Suppose, on the other hand, that the lenders try to implement a separating equilibrium in which the opportunistic entrepreneurs default. In this case, the lender loses money if she lends to an opportunistic entrepreneur, but profits if she lends to a dependable entrepreneur. This menu of contracts relies on the lender’s ability to commit, since she would prefer not to lend to anyone who would accept the opportunistic entrepreneur’s contract.

Here there are two subcases to consider. First, suppose that under the original menu of contracts the dependable entrepreneurs are just as well off as the opportunistic entrepreneurs. Then Lemma 3 implies that both entrepreneurs would be strictly better off under the benchmark contract. Instead, suppose that the oppor-
tunistic entrepreneur’s contract leaves him strictly better off than the dependable
entrepreneurs. This means that if the opportunistic entrepreneur switched contracts
and made the debt payments, he would be strictly worse off. If, instead, he switched
contracts and defaulted, he would be weakly worse off, by revealed preference. Then
another lender could offer an alternative contract with a slightly lower debt payment,
$D_2$. This contract would make the dependable entrepreneur better off. Furthermore,
lowering $D_2$ would not affect the payoff that the opportunistic entrepreneur would
receive if he switched contracts and defaulted. Thus, the other lender could skim
away the dependable entrepreneurs with this alternative contract, leaving the original
lender to lose money lending to only opportunistic entrepreneurs. I have shown

**Proposition 4.** *There is no separating equilibrium in which opportunistic entrepreneurs
choose one contract and dependable entrepreneurs choose another.*

### 2.3.3 Contract without Default

From this point forward, I consider only pooling equilibria. In a pooling equilib-
rium, both types of entrepreneurs accept the same contract. If the lender wants to
prevent default, the contract must satisfy the opportunistic entrepreneur’s incentive
constraints. Thus, the contract will maximize output subject to the lender’s par-
ticipation constraint, the opportunistic entrepreneur’s incentive constraints, and the
payment feasibility constraints. This is exactly the benchmark contract solved above,
and is the same as would be implemented if the lender knew that every entrepreneur
was opportunistic. In a more general setup with heterogeneous entrepreneurs, the
no-default contract would have incentive constraints that were determined by the en-
trepreneur who was capable of diverting the most output. Note that without default
the economy cannot benefit from the presence of dependable entrepreneurs.

**Remark 1.** *The optimal no-default contract with heterogeneous entrepreneurs is the
same as the benchmark contract of Proposition 1.*
2.3.4 First Period Default

It is natural to think that if default is to occur optimally, that the dependable entrepreneurs would prefer that it occur in the first period. This way they would have the opportunity to signal their type by their repayment, and the lender should be willing to lend them more in the second period. On the other hand, dependable entrepreneurs will have to compensate the lender for the revenue lost to default. Due to the fixed cost required to start the project, the debt outstanding is generally greater in the first period than in the second, so first period default could potentially leave the dependable entrepreneurs with a sizeable burden. Furthermore, the amount that the lender will lend to the dependable entrepreneurs in the second period will be constrained by the need to induce first period default. By defaulting in the first period, the opportunistic entrepreneurs gives up their continuation value. This continuation value is increasing in the second period capital. If the contract assigned too much second period capital to the dependable entrepreneurs, then the opportunistic entrepreneurs would not be willing to give up their continuation value by defaulting in the first period.

The optimal contract that induces first period default is given by the solution to:

\[
\max_{k_1, k_2, D_1, D_2} R(y(k_1) - D_1) + y(k_2) - D_2
\] (2.16)
\[ \phi(RD_1 + D_2) + (1 - \phi)(R\beta y(k_1) + D_2^d) \geq R^2(k_1 + F) + R(\phi k_2 + (1 - \phi)k_2^d) \]  
(2.17)

\[ Ray(k_1) + y(k_2^d) - D_2^d \geq R(y(k_1) - D_1) + y(k_2) - D_2 \]  
(2.18)

\[ Ray(k_1) + y(k_2^d) - D_2^d \geq R(y(k_1) - D_1) + \alpha y(k_2) \]  
(2.19)

\[ (1 - \alpha) y(k_2^d) \geq D_2^d \]  
(2.20)

\[ y(k_1) \geq D_1 \]  
(2.21)

\[ y(k_2) \geq D_2 \]  
(2.22)

Notice that, because the opportunistic entrepreneurs default, Lemma 2 implies that the objective function must be the dependable entrepreneur’s payoff. Equation (2.17) requires that the lender at least break even in expectation. The dependable entrepreneurs will make the first period payment, \( D_1 \), while the opportunistic entrepreneurs will default, leaving the lender to recover \( \beta y(k_1) \). In the second period, the dependable entrepreneurs will make the payment \( D_2 \), and the opportunistic entrepreneurs will make the payment \( D_2^d \). Equation (2.18) ensures that the opportunistic entrepreneurs prefer to default in the first period rather than make the contracted debt payments; equation (2.19) ensures that the opportunistic entrepreneurs prefer to default in the first period rather than in the second period; and equation (2.20) ensures that they will not default in both periods. Equations (2.21) and (2.22) are feasibility constraints on the debt payments.

The contract relies critically on the lender’s ability to commit. If she could not commit, then in the second period, after types have been revealed, she would lend \( k^* \) to the dependable types and recover as much as possible from the opportunistic types. The lender would do this by choosing \( k_2^d \) to maximize \((1 - \alpha) y(k_2^d) - Rk_2^d \). The unique solution to this program is given by \( k_2^* \) defined by \( y'(k_2^*) = R/(1 - \alpha) \). Of course, if the lender could not commit to not behave this way in the second period,
it would be more difficult for her to induce first period default.

Proposition 5 summarizes the properties of the optimal contract with first period default. For a proof, see the appendix.

**Proposition 5.** Suppose in the optimal contract with first period default that \( k_2 < k^* \) and \( k^*_d < k^* \). Then,

(i) \( D_1 = y(k_1) \),

(ii) \( D^d_2 = (1 - \alpha) y(k^*_d) \), and

(iii) the incentive compatibility constraint (2.19) binds.

Because I focus on contracts that are constrained away from the optimal scale, I require that \( k_2 \) and \( k^*_d \) be less than \( k^* \). Part (i) of Proposition 5 indicates that the optimal contract requires all of the first period output as a first period debt payment. This is intuitive, since raising \( D_1 \) is consistent with inducing first period default and also helps to relax constraints for the second period. Part (ii) says that the lender will require the maximum obtainable second period payment from the opportunistic entrepreneur. While not surprising, this condition is not entirely obvious, since punishing the opportunistic entrepreneur in the second period makes it more difficult to induce first period default. Part (iii) says that the incentive constraint which prevents the opportunistic entrepreneur from defaulting in the second period rather than the first binds. Notice that Lemma 3 implies that (2.18) must be slack if this contract is to outperform the benchmark contract.

### 2.3.5 Second Period Default

A contract that induces second period default has the potential to improve on the benchmark contract. If the opportunistic entrepreneurs are induced to default in the second period, they will receive a higher second period payoff. This provides
incentives for them to make a larger debt payment in the first period, allowing the project to be run at a larger scale.

The optimal contract that induces default in the second period is given by the solution to

$$\max_{k_1, k_2, k_2^d, D_1, D_2, D_2^d} R(y(k_1) - D_1) + y(k_2) - D_2$$  \hspace{1cm} (2.23)$$

s.t.

$$\phi D_2 + (1 - \phi) \beta y(k_2) \geq R^2(k_1 + F) + R(k_2 - D_1)$$  \hspace{1cm} (2.24)$$

$$R(y(k_1) - D_1) + \alpha y(k_2) \geq R(y(k_1) - D_1) + y(k_2) - D_2$$  \hspace{1cm} (2.25)$$

$$R(y(k_1) - D_1) + \alpha y(k_2) \geq R\alpha y(k_1) + y(k_2^d) - D_2^d$$  \hspace{1cm} (2.26)$$

$$R(y(k_1) - D_1) + \alpha y(k_2) \geq R\alpha y(k_1) + \alpha y(k_2^d)$$  \hspace{1cm} (2.27)$$

$$y(k_1) \geq D_1$$  \hspace{1cm} (2.28)$$

$$y(k_2) \geq D_2$$  \hspace{1cm} (2.29)$$

$$y(k_2^d) \geq D_2^d$$  \hspace{1cm} (2.30)$$

Equation (2.24) is the lender’s participation constraint given second period default. In the second period the lender will be paid $D_2$ by the dependable entrepreneur, while the opportunistic entrepreneur will default, leaving the lender to recapture $\beta y(k_2)$. Equations (2.25) - (2.27) are the opportunistic entrepreneur’s incentive constraints. They ensure that the opportunistic entrepreneurs prefer to default in the second period rather than make the contracted payments, default in the first period, or default in both periods. Equations (2.28) - (2.30) are feasibility constraints on debt payments. Proposition 6 summarizes the properties of the optimal second period default contract.

**Proposition 6.** There is a critical value of $k_2$ defined by $y(k_2^c) = \frac{R}{\alpha + \beta + \phi(1-\alpha-\beta)}$ such that, if $k_1 < k^*$ and $k_2 < k_2^c$ then
(i) \( D_1 = y(k_1) \),

(ii) \( k_2^d = D_2^d = 0 \), and

(iii) the incentive compatibility constraint (2.26) binds.

Notice that Proposition 1 and Proposition 5 characterize the benchmark and first period default contracts given that \( k_2 < k^* \) where the contract is constrained away from the optimal scale. With second period default, \( k_2 \) will never reach \( k^* \), since there are no subsequent periods to regain the loss due to default. The critical value \( k_2^c \) is the largest value of \( k \) such that default “pays for itself” by allowing the project to be run at a higher scale. That is, absent any wealth constraints, the optimal second period capital deployment would be \( k_2^c \). Thus, \( k_2^c \) here plays the role that \( k^* \) played in the first period default and no default contracts. Requiring \( k_2 < k_2^c \) means that I am requiring the contract to be constrained away from the optimal scale.

Part (i) states that the entire first period output is required as a debt payment. This is intuitive, since it extracts as much as possible from the opportunistic entrepreneur. It is not obvious, however, since raising the first period debt payment makes it more difficult to prevent first period default. Part (ii) is as expected: to more easily prevent first period default it should be punished as much as possible. Part (iii) indicates that the incentive constraint which prevents the opportunistic entrepreneur from defaulting in the first period binds. Notice that Lemma 3 implies that (2.25) must be slack if the contract is to outperform the benchmark.

2.4 Comparison of Contracts

In this section, I provide evidence from numerical optimizations that facilitate a better understanding of the optimal contracts. Throughout this section I use \( y(k) = 10x^{.75} \) as the production function; I set the loss due to default \( 1 - \alpha - \beta = .03 \); and I set the interest rate to \( R = 1.05 \).
2.4.1 Fixed Cost

Figure 2.3 shows the payoff to the dependable entrepreneur as a function of the fixed cost. The four panels show the same plot for different combinations of $\phi$ and $\alpha$. The blue dotted line shows the payoff from the benchmark contract; the green dashed line shows the payoff from the contract with first period default; and the red solid line shows the payoff from the contract with second period default.

Figure 2.3 suggests that, while there are regions where first period default dominates the benchmark contract, first period default is dominated by second period default. In the contract with first period default, the dependable entrepreneur must make a large second period payment to offset the expected loss from first period default.
default. In return, the dependable entrepreneur is able to run the project at a larger scale. Evidently, this tradeoff is worthwhile for large $F$ when $\phi$ is large, but it is not better than the second period default contract. Because of this evidence, from here on I will focus on comparing the contract with second period default against the benchmark contract.

Not surprisingly, Figure 2.3 indicates that inducing default is more helpful when the proportion of dependable entrepreneurs is high. This is very straightforward. Not so obvious is the fact that inducing default is more helpful when the opportunistic entrepreneurs are able to steal a large portion of the output. One might think that, since the lender’s recovery is lower when the opportunistic entrepreneur can steal a lot, it would be better to disallow default. The intuition behind the result illustrated by the figure is that, when $\alpha$ is high, the limited commitment constraint more severely restricts the project scale when there is no default. When the project scale is low, marginal productivity is high. Since inducing default increases the project scale, default is particularly useful when marginal productivity is high.

Figure 2.3 illustrates that, for a wide range of parameter values, inducing default will be optimal if the fixed cost is large enough. In each panel, inducing default in the second period allows the project to be run in fixed cost regions where it would not be feasible without default. In the fourth panel, inducing default dominates the benchmark contract even when there is no fixed cost. For the remainder of this section I will use base case values of $\phi = 0.5$, $\alpha = 0.5$, and $F = 500$ unless otherwise stated.

Figure 2.4 and Figure 2.5 graphically illustrate the optimal contract with second period default and the benchmark contract as functions of the fixed cost. Figure 2.4 shows the payoffs to each type of entrepreneur in the top panel and the capital deployment in periods one and two in the lower two panels. Figure 2.5 show the payoffs to each type of entrepreneur in the top panel and the first and second period
debt payments in the lower two panels. In terms of the behavior of the optimal contract, there are several distinct regions of \( F \), labeled Regions I-V in the figures.

Region I occurs at the smallest values of fixed cost. In fact, given the base case parameterization, \( F \) would have to be negative in order for the contract to be in Region I. In this region, \( k_1 = k^* \) and \( k_2 = k_2^c \), and the contract is free of all constraints except that the opportunistic entrepreneur must prefer to default than to repay in the second period. Thus \( D_2 = (1 - \alpha) y(k_2) \), incentive constraint (2.25) binds, and both types of entrepreneur fair equally well. In this region the no default contract is equal to the first best. As \( F \) increases, the first period debt payment increases correspondingly.

As \( F \) continues to increase, \( k_1 \) decreases until the first period debt payment reaches a maximum, \( y(k_1) = D_1 \). At this point, the contract transitions into Region III, which begins around \( F = 100 \) in Figures 2.4 and 2.5. Since \( y(k_1) = D_1 \), the dependable entrepreneurs are extracting everything they can from the opportunistic entrepreneurs in the first period. Both incentive constraints bind, so as \( F \) increases the contract must decrease both to satisfy the contract’s constraints. Again, in this region the contract without default dominates the contract with second period default.

As \( F \) increases the concavity of \( y \) implies that it is increasingly expensive to decrease \( k_1 \) and \( k_2 \). A point is reached at which it is cheaper for the dependable
entrepreneur to pay a higher debt payment in the second period than it is for him to lower \( k_1 \) and \( k_2 \). At this point, Region IV begins, which occurs around \( F = 400 \) in Figures 2.4 and 2.5. In this region \( k_1 \) and \( k_2 \) remain flat and \( D_2 > (1 - \alpha) y(k_2) \) and increases with a slope of \( R^2 \) as \( F \) increases. The opportunistic entrepreneur’s payoff remains flat, while the dependable entrepreneur’s payoff decreases with a slope of \( R^2 \). This means that the opportunistic entrepreneurs are strictly better off than the dependable entrepreneurs. Note that the opportunistic entrepreneur’s incentive constraint (2.26) that prevents him from defaulting in the first period still binds. Proposition 6 applies in Regions III and IV, and the contract with second period default dominates the no-default contract throughout most of Region IV. Notice also that the base case parameterization lies in Region IV.

Region V begins around \( F = 950 \) in Figures 2.4 and 2.5. In this region, the project is simply too expensive to run. Note that without default, projects would become too expensive around \( F = 550 \). For reference, the first best contract would permit projects to be run until around \( F = 1300 \) in this parameterization.

2.4.2 Proportion of Dependable Entrepreneurs: \( \phi \)

Figure 2.6 shows the payoff to each type of entrepreneur under both the no default and second period default contracts as functions of the proportion of dependable entrepreneurs, \( \phi \). Also, the figure shows the social surplus for the optimal contract. Note that everything shown in Figure 2.6 lies in Region IV as described above. The blue dotted line in Figure 2.6 plots the payoffs to either type of entrepreneur under the no default contract and is coincident with the light blue dash-dotted line in the left half of the figure. Note that this contract makes no distinction between dependable and opportunistic entrepreneurs, so the payoff is a constant. The solid red line shows the payoff to the opportunistic entrepreneur. Note that it is monotonically increasing, so that the opportunistic entrepreneurs are better off in a population with more
dependable entrepreneurs. Also, since the plot lies in Region IV, the opportunistic entrepreneurs are everywhere strictly better off than the dependable entrepreneurs. The green dashed line shows the payoff to the dependable entrepreneurs under the second period default contract. Not surprisingly it increases monotonically with $\phi$. At a value of about $\phi = 0.55$ the dependable entrepreneurs switch from preferring the no default contract to the second period default contract. The dash-dotted light blue line shows the total social surplus under the optimal contract. For $\phi < 0.55$, both types of entrepreneurs receive the same, no default payoff. At $\phi = 0.55$ there is a discontinuous jump in social surplus due to the fact that opportunistic entrepreneurs become much better off when default is induced. For $\phi > 0.55$, social surplus is monotonically increasing, approaching the first best value around 1370.

One interesting feature of Figure 2.6 is that the difference between the opportunistic and the dependable entrepreneur’s payoffs is increasing in $\phi$. The intuition behind this is the following. Dependable entrepreneurs absorb the cost of default. Because that cost is high when there are many opportunistic entrepreneurs who default and few dependable entrepreneurs to absorb the cost, the contract in this region is very distorted from first best. A relatively small amount of capital is used in the second period, limiting the difference between the entrepreneurs’ payoffs. On the other hand, when there are very few opportunistic entrepreneurs, the per capita cost of default on the dependable entrepreneurs becomes negligible, and the contract approaches first best. In this case, the second period capital is very high, and so the opportunistic entrepreneur’s payoff for defaulting is also very high.

Figure 2.6 allows us to make both cross-sectional and time series predictions about what types of contracts we are likely to observe in different environments. For example, suppose that citizens in some countries adhere more strictly to social norms and customs than in other countries. It may be that the social cost incurred when one breaks the custom is different across countries. Insofar as repayment of debt
can be considered a social norm, we would expect higher investment, higher output, and lower levels of default in countries with strong adherence to social norms. The same line of reasoning could allow us to make cross-sectional predictions at the entrepreneur level. We would predict that entrepreneurs with strong social networks are more likely to adhere to social norms. This may imply, say, that lending contracts for family firms provide higher investment and have lower default rates. Additionally, the model would predict that contracts where the lender and the borrower have a pre-existing relationship would also provide higher investment and have lower default rates than contracts between strangers, all else equal.

This line of reasoning also allows us to make time series predictions. Suppose that the social cost associated with default varies over time. For example, it may be less socially costly to strategically default in bad times when one can blend in with many people who are defaulting out of necessity. If this is true, we would expect to see contracts that call for lower investment, produce lower output, and have lower default rates written during bad times.

2.4.3 Proportion that can be Diverted: $\alpha$

Figure 2.7 shows the payoff to each type of entrepreneur under both contracts, as well as the social surplus under the optimal contract, plotted as functions of $\alpha$, the proportion of output that the opportunistic entrepreneurs can steal. The blue dotted line plots the payoff to either type of entrepreneur under the no default contract and is coincident with the light blue, dash-dotted line in the left half of the figure. This payoff decreases until the no default contract becomes infeasible around $\alpha = 0.53$. The solid red line plots the payoff to the opportunistic entrepreneur. It is strictly decreasing in $\alpha$ and has kinks when shifting across regions as described in the previous section. The green dashed line shows the payoff to the dependable entrepreneur given a second period default contract. Note that the dependable entrepreneur be-
comes better off under the second period default contract at a value of approximately \( \alpha = 0.51 \). The light blue dash-dotted line shows the total social surplus from the optimal contract. It is equal to the payoff to each entrepreneur when the no default contract is optimal. When the second period default contract becomes optimal there is a discontinuous jump in social surplus, since the opportunistic entrepreneurs are strictly better off under this contract.

We can use Figure 2.7 to help guide empirical predictions. The main feature of the figure is that inducing default has more potential benefit when the agency problem is particularly significant. Thus, holding constant the proportion of dependable entrepreneurs, we would expect to see higher default rates, lower output and lower investment in industries where diverting output is particularly easy. If it is easier, for example, to divert output for a service firm than for a firm that produces tangible goods, then the model would predict that service firms would have lower investment and output and higher default rates.

### 2.5 Conclusion

I study a two-period contracting problem in which borrowers differ in their willingness to strategically default. “Opportunistic” borrowers default whenever it is in their best interest, whereas “dependable” borrowers always pay their debts. I find that the optimal contracts depend on the average willingness to default in the population. When a sufficient proportion of the population is dependable, the optimal contract is structured such that opportunistic borrowers will choose to default in the second period. This is because the gains from running the projects at a larger scale are sufficient to offset the losses that occur when a subset of firms default. Delaying type revelation is optimal because it allows some of the rents extracted by opportunistic borrowers to come in the form of returns from a positive NPV project. In contrast, when the proportion of opportunistic borrowers surpasses a critical thresh-
old, lending to all types is constrained by the limited commitment problem. In this case there is no default in equilibrium, but firms are constrained to run at a smaller scale. Furthermore, I find that default is more likely to be optimal when the agency problems are particularly severe. The model has implications for contracting in an environment with adverse selection and delivers both time series and cross sectional predictions about equilibrium default, investment, and output.
Figure 2.4: Entrepreneur Payoffs and Capital Deployment. The figure shows entrepreneur payoffs and capital deployments as a function of the fixed cost for parameters $\phi = 0.5$ and $\alpha = 0.5$. Payoffs are shown for each type of entrepreneur and capital deployments are shown for each possible contract.
Figure 2.5: Entrepreneur Payoffs and Debt Payments. The figure shows entrepreneur payoffs and debt payments as a function of the fixed cost for parameters $\phi = 0.5$ and $\alpha = 0.5$. Payoffs are shown for each type of entrepreneur and debt payments are shown for each possible contract.
Figure 2.6: Entrepreneur Payoffs and Social Surplus as a Function of φ. The figure shows the payoffs received by entrepreneurs as well as social surplus as a function of φ, the proportion of dependable entrepreneurs in the population. Payoffs are shown for both dependable and opportunistic entrepreneurs, and social surplus is shown for both the no default and optimal contracts. Other parameters are set to α = 0.5 and F = 500.
Figure 2.7: Entrepreneur Payoffs and Social Surplus as a Function of $\alpha$.
The figure plots entrepreneur payoffs and social surplus for the optimal contract as a function of $\alpha$, the proportion of output that opportunistic entrepreneurs can divert. Other parameters are set to $\phi = 0.5$ and $F = 500$. 
Appendix A

Proofs

Characterization of the First Best Contract. The first best contract is given by the solution to

$$\max_{k_1,k_2,D_1,D_2} R(y(k_1) - D_1) + y(k_2) - D_2$$  \hspace{1cm} (A.1)

s.t.

$$RD_1 + D_2 \geq R^2(k_1 + F) + Rk_2$$  \hspace{1cm} (A.2)

$$y(k_1) \geq D_1$$  \hspace{1cm} (A.3)

$$y(k_2) \geq D_2$$  \hspace{1cm} (A.4)

Competitive lending implies that the lender’s participation constraint (A.2) will bind at the optimum. Substituting this into (A.1) yields the objective

$$R(y(k_1) - R(k_1 + F)) + y(k_2) - Rk_2$$  \hspace{1cm} (A.5)

Assuming that the project has positive net present value, the feasibility constraints on debt payments will not bind. The first order conditions for $k_1$ and $k_2$ are given by $y'(k_1) = R$ and $y'(k_2) = R$, which implies $k_1 = k_2 = k^*$. \hfill \qed
Proof of Proposition 1. The optimal contract without default is given by the solution to (2.5)-(2.12). Notice that (2.7) and (2.9) are relaxed by making \( y(k_2^d) \) as small as possible, so without loss of generality we can set \( k_2^d = 0 \). This is expected, since punishing default makes it easier to discourage. (2.7) is further relaxed by making \( D_2^d \) as large as possible, so (2.12) then implies that \( D_2^d = 0 \). I will show below that (2.7) will bind at the optimum, so \( k_2^d = D_2^d = 0 \) is necessary. This causes (2.7) to equal (2.9), so we are down to two limited commitment constraints. Furthermore, (2.8) reduces to \( D_2 \leq (1 - \alpha)y(k_2) \), which then replaces (2.11).

Now, suppose that \( D_1 < y(k_1) \). Raise \( D_1 \) and \( k_2 \) by the same differential amount. This increases the objective function and does not violate any constraints as long as \( y'(k_2) \geq R \), which we know must hold. Thus, without loss of generality \( D_1 = y(k_1) \).

Now, suppose that \( D_2 < (1 - \alpha)y(k_2) \). Then raise \( k_2 \) by \( dk_2 \) and \( D_2 \) by \( Rdk_2 \). We can increase the objective without violating any constraints by doing this repeatedly until \( D_2 = (1 - \alpha)y(k_2) \) or \( k_2 = k^* \), whichever comes first. If \( k_2 = k^* \), then the fixed cost is small enough that the limited commitment constraints do not bind, and the contract exhibits indeterminacy in the debt payments. This is not the case I focus on. So, \( k_2 < k^* \) implies \( D_2 = (1 - \alpha)y(k_2) \).

We have reduced the program to

\[
\max_{k_1, k_2} Ry(k_1) + y(k_2) \tag{A.6}
\]

s.t.

\[
Ry(k_1) + (1 - \alpha)y(k_2) \geq R^2(k_1 + F) + Rk_2 \tag{A.7}
\]

\[
y(k_2) \geq Ry(k_1) \tag{A.8}
\]

(A.7) is the lender’s participation constraint, which must bind by competitive lending. Suppose that (A.8) does not bind. Then we could increase \( k_1 \), which raises the objective and does not violate (A.8). Thus, both constraints must bind, and together they determine \( k_1 \) and \( k_2 \).
It remains only to prove part (vi) of Proposition 1. Note that (2.13) implicitly defines $k_1(F)$. Implicit differentiation gives

$$R(k'_1(F) + 1)(y^{-1})'(Ry(k_1)) \ast Ry'(k_1) \ast k'_1(F) = (2 - \alpha)y'(k_1) \ast k'_1(F). \quad (A.9)$$

Rearranging and substituting (A.8) into (A.9) gives

$$k'_1(F)(R + y'(k_1)(\frac{R}{y'(k_2)} - (2 - \alpha))) = -R, \quad (A.10)$$

which implies that $k'_1(F) < 0$ if and only if

$$\frac{R}{y'(k_1)} > \frac{R}{y'(k_2)} - (2 - \alpha). \quad (A.11)$$

Note that the right-hand side of (A.11) must be negative, while the left-hand side is positive. Thus, $k'_1(F) < 0$. \hfill \Box

**Proof of Proposition 5.** The optimal contract subject to first-period default by opportunistic entrepreneurs is given by the solution to (2.16) - (2.22). Competitive lending implies that (2.17) will bind.

Suppose that $D_1 < y(k_1)$ at the solution. Then raise $D_1$ by $dD_1$, $k_2$ by $dk_2$, and $k^d_2$ by $dk^d_2$. The objective function is increased as long as

$$y'(k_2)dk_2 > RdD_1. \quad (A.12)$$

(2.17) will still hold as long as

$$\phi(dk_2 - dD_1) + (1 - \phi)dk^d_2 \leq 0. \quad (A.13)$$

(2.18) is still satisfied as long as

$$y'(k^d_2)dk^d_2 \geq y'(k_2)dk_2 - RdD_1, \quad (A.14)$$

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and (2.26) is still satisfied as long as \( y'(k_2^d)dk_2^d \geq \alpha y'(k_2)dk_2 - RdD_1 \), which is clearly implied by (A.14). Now, it is possible to choose \( dk_2^d \) to satisfy (A.13) and (A.14) as long as

\[
\frac{y'(k_2)dk_2 - RdD_1}{y'(k_2)} \leq \frac{\phi}{1 - \phi}(dD_1 - dk_2). \tag{A.15}
\]

It is possible to choose \( dk_2 \) to satisfy (A.15) and (A.12) as long as

\[
\frac{RdD_1}{y'(k_2)} < \frac{(1 - \phi)R + \phi y'(k_2^d)}{(1 - \phi)y'(k_2) + \phi y'(k_2^d)}dD_1. \tag{A.16}
\]

(A.33) can be shown to hold if and only if \( k_2 < k^* \), which is assumed. Thus \( D_1 = y(k_1) \).

Now, suppose that \( D_2^d < (1 - \alpha)y(k_2^d) \) at the solution. Then raise \( D_2^d \) by \( dD_2^d \), \( k_2 \) by \( dk_2 \), and \( k_2^d \) by \( dk_2^d \). The objective function is increased as long as

\[
dk_2 > 0. \tag{A.17}
\]

(2.17) will still be satisfied as long as

\[
R(\phi dk_2 + (1 - \phi)dk_2^d) \leq (1 - \phi)dD_2^d. \tag{A.18}
\]

(2.25) will still be satisfied as long as

\[
y'(k_2^d)dk_2^d - dD_2^d \geq y'(k_2)dk_2. \tag{A.19}
\]

Finally, (2.26) will still be satisfied if \( y'(k_2^d)dk_2^d - dD_2^d \geq y'(k_2)dk_2 \), which is implied by (A.19). Now, it is possible to choose \( dk_2^d \) to satisfy (A.18) and (A.19) as long as

\[
\frac{y'(k_2)dk_2 + dD_2}{y'(k_2^d)} \leq dD_2/R - \frac{\phi}{1 - \phi}dk_2. \tag{A.20}
\]

It is possible to choose \( dk_2 \) to satisfy both (A.17) and (A.20) as long as

\[
\frac{(1 - \phi)(y'(k_2^d) - R)}{R((1 - \phi)y'(k_2) + \phi y'(k_2^d))}dD_2 > 0. \tag{A.21}
\]

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Condition (A.21) holds if and only if $k_2^d < k^*$, which is assumed. Thus, $D^d_2 = (1 - \alpha)y(k^d_2)$.

Now, suppose that (2.19) is slack at the solution. Raise $k_2$ by $dk_2$, $k^d_2$ by $dk^d_2$, and $D_2$ by $dD_2$. The objective is increased as long as

$$y'(k_2) dk_2 > dD_2.$$  \hfill (A.22)

(2.17) is still satisfied as long as

$$R(\phi dk_2 + (1 - \phi)dk^d_2) \leq \phi dD_2.$$  \hfill (A.23)

(2.18) is still satisfied as long as

$$y'(k^d_2) dk^d_2 \geq y'(k_2) dk_2 - dD_2.$$  \hfill (A.24)

$dk^d_2$ can be chosen to satisfy (A.23) and (A.24) as long as

$$\frac{y'(k_2) dk_2 - dD_2}{y'(k^d_2)} \leq \frac{\phi}{1 - \phi} (dD_2/R - dk_2).$$ \hfill (A.25)

Finally, $dD_2$ can be chosen to satisfy (A.22) and (A.25) as long as

$$\frac{R((1 - \phi)y'(k_2) + \phi y'(k^d_2))}{R(1 - \phi) + \phi y'(k^d_2)} dk_2 < y'(k_2) dk_2.$$ \hfill (A.26)

Condition (A.26) can be shown to hold if and only if $k_2 < k^*$, which is assumed. \qed

**Proof of Proposition 6.** The optimal contract with second period default is given by the solution to (2.23) - (2.30). First, notice that setting $k^d_2 = D^d_2 = 0$ makes (2.26) and (2.27) easier to satisfy, so we can set these accordingly without loss of generality. If (2.26) turns out to bind at the optimum, as I will show below, then $k^d_2 = D^d_2 = 0$ is a necessary condition for an optimum.

Suppose, to the contrary, that (2.26) is slack at the optimum. Then raise $k_1$ by $dk_1$ and $D_1$ by $Rdk_1$. All constraints will still be satisfied, and the objective function
will be increased as long as \( k_1 < k^* \), which is assumed. Thus, (2.26) binds at the optimum.

Now, suppose that \( D_1 < y(k_1) \) at the optimum. Then raise \( D_1 \) by \( dD_1 \), \( D_2 \) by \( dD_2 \), and \( k_2 \) by \( dk_2 \). The objective function is increased as long as

\[
y'(k_2)dk_2 > RdD_1 + dD_2.
\]  

(2.27)

(2.24) will still be satisfied as long as

\[
(R - (1 - \phi)\beta y'(k_2))dk_2 \leq RdD_1 + \phi dD_2.
\]  

(A.28)

(2.26) will still be satisfied as long as

\[
\alpha y'(k_2)dk_2 \geq RdD_1.
\]  

(A.29)

(2.25) and (2.29) will still be satisfied as long as

\[
(1 - \alpha)y'(k_2)dk_2 \leq dD_2 \leq y'(k_2)dk_2.
\]  

(A.30)

Suppose we make \( dD_2 \) as small as possible given (A.30), \( dD_2 = (1 - \alpha)y'(k_2)dk_2 \).

Making this substitution we are left with the following inequalities:

\[
\alpha y'(k_2)dk_2 > RdD_1 \]

\[
(R - (\phi(1 - \alpha) + (1 - \phi)\beta)y'(k_2))dk_2 \leq RdD_1.
\]  

(A.31)

(A.32)

\( dD_1 \) can be chosen to satisfy this set of inequalities as long as

\[
y'(k_2) > \frac{R}{\alpha + \beta + \phi(1 - \alpha - \beta)}.
\]  

(A.33)

which is assumed.
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Biography

Ryan David Pratt was born on August 30, 1980 in Salt Lake City, Utah. He earned Bachelor’s degrees in Computer Science and Mathematics from the University of Utah in 2004 and a Ph.D. in Finance from Duke University in 2012. After graduating he will be Assistant Professor of Finance at Brigham Young University.