Inventory Management and Supply Chain Finance: Theory and Empirics

by

Jordan D. Tong

Business Administration
Duke University

Date: ______________

Approved:

________________________
Jing-Sheng Song, Supervisor

________________________
Li Chen

________________________
A. Gürhan Kök

________________________
Richard P. Larrick

Dissertation submitted in partial fulfillment of
the requirements for the degree of Doctor of Philosophy
in Business Administration
in the Graduate School
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ABSTRACT

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Abstract

A payment scheme specifies when payments are made between firms in a supply chain. It has direct implications on how supply chain inventory is financed and managed. Longer supply chains due to globalization and the recent credit crisis have increased the pressure to make financing the supply chain more efficient. It was recently reported that 81% of UK firms say that market conditions have brought procurement and finance strategies in closer alignment. Meanwhile, information technology platform advancements provide opportunity for increased variety of payment schemes. It is therefore important to understand how different payment schemes should be captured in inventory decisions. This dissertation examines the impact of supply chain finance (the set of financial payment transactions that are triggered by supply chain events) on inventory management from both normative and behavioral perspectives.

We seek to address the following questions. From a normative perspective: How does the optimal inventory policy depend on the supply chain financing structure? What is the right inventory financing scheme for a supply chain? From a behavioral perspective: How do real managers psychologically process payments when making inventory decisions, and how are they affected by the supply chain financing scheme? The results are reported in three chapters, described below.

In the first chapter, “Payment schemes and the financed inventory,” we present a model of payment schemes in an echelon supply chain. A payment scheme specifies when payments are made between firms. Standard inventory decision models make strict assumptions about the payment scheme in order to avoid explicitly tracking financial flows. These assumptions, however, often do not hold in practice. We show that these assumptions can be relaxed. In particular, we introduce a model that allows us to track the financial flow of inventory models depending on the inventory policy and the payment scheme. We also define two new measures - financed inventory and margin backorders. These new measures allow us to leverage the structure of the payment scheme to define an equivalent problem that does
not have to explicitly track financial flows. We apply this method to the base stock model and economic order quantity model to demonstrate the sensitivity of the optimal inventory policy to the payment scheme. Our results provide simple closed-form formulas for inventory managers and also sheds light on what is the right payment scheme for a supply chain.

The second chapter, “The effect of payment schemes on inventory decisions: The role of mental accounting,” focuses on managerial behavior: how do manager’s mentally process and evaluate payments when making an inventory decision? Keeping the net profit structure constant, we study how the payment scheme affects inventory decisions in the newsvendor problem. Specifically, we examine three payment schemes which can be interpreted as the inventory order being financed 1) by the newsvendor herself, 2) by the supplier, and 3) by the customer. We find in laboratory experiments that the order quantities may be higher or lower than the expected profit-maximizing solution depending on the payment scheme. Specifically, the order quantity under newsvendor own financing is greater than that under supplier financing, which is, in turn, greater than the order quantity under customer financing. This observed behavior biases orders in the opposite direction as what a regular or hyperbolic time-discounted utility model would predict, and cannot be explained by loss aversion models. Instead, the findings are consistent with a model that underweights the order-time payments, which is consistent with the “prospective accounting” assumption in the mental accounting literature. A second study shows the results hold even if all actual payments are conducted at the same time, suggesting that the framing of the payment scheme is sufficient to induce mental accounting of payments at different times. We further validate the robustness of our model under different profit-margin conditions. Our findings contribute to the understanding of the psychological processes involved in newsvendor decisions and have implications for supply chain financing practices and supply chain contract design.

The third chapter, “Reference prices and transaction utility in inventory decisions,” studies another aspect of mental accounting in inventory decisions - the phenomenon that individuals often view a price as relative to other prices when making an evaluation. We
present a descriptive model of the effects of reference prices and transaction utility in a
newsvendor setting. The model predicts that an individual’s order is irrationally increasing
in past purchasing costs, decreasing in past selling prices, and decreasing in the proportion
of high profit margin to low profit margin products in the decision portfolio. Three labora-
atory experiments support the model’s predictions. These results suggest that managerial
supervision and/or intervention are most valuable after a sudden increase or decrease in the
cost or price of a product, or for a product that differs significantly in profit margin from
other products in the category. We further extend the study to a supply chain setting. We
show analytically that the supplier’s optimal wholesale price is lower when the newsvendor
is subject to reference effects compared to when the newsvendor is rational, and that the
supplier’s optimal retail price may be higher or lower depending on whether the reference
effect is stronger for the newsvendor or for customers. Finally, we show that supply chains
may suffer from a behavioral inefficiency we call a behavioral price whip: an increase in the
transfer price between two nodes may influence the upstream node to order more than is
rational while the downstream node demands less than is rational. These results suggest
that suppliers should carefully evaluate the reference effect on both customers and retailers,
and that everyday low pricing has a behavioral benefit over high-low pricing.
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1 Payment Schemes and the Financed Inventory

1.1 Introduction

Managing inventory costs is a high priority for firms, especially for manufacturing and merchandising companies. Total annual holding costs frequently exceed 30% of purchasing costs, and purchasing costs usually make up the largest category of the cost of goods sold (Horngreen et al. 2003). One of the most significant contributors to the holding cost is the cost of capital, or the return forgone by investing capital in inventory rather than elsewhere. The cost of capital depends on the payment scheme: the longer a firm must finance inventory, the greater the cost of capital. Moreover, the payment scheme affects the cash flow process, which determines the cash management costs for firms. There is a wide variety of payment schemes in practice, and the variety is increasing due to automatic electronic payments, the Internet, and information technology platforms. Unfortunately, however, most inventory control models make strict assumptions regarding the payment scheme. Thus, they cannot accommodate key accounting metrics for firms. Moreover, these models can subsequently make biased inventory policy recommendations when these assumptions are not met. In this paper, we show how variations in the payment scheme affect the way that inventory control models should incorporate financing related costs, and examine how the optimal inventory policy is affected.

It is well known that inventory holding costs consist of both physical and financial components. The holding costs that should be included in inventory decision models are what Horngreen et al. (2003) calls the relevant costs. The relevant physical holding cost consists of those costs that vary with the quantity of physical inventory on location (e.g., warehousing, obsolescence, breakage), and the relevant financial holding cost consists of those costs that vary with the amount of capital invested in inventory (e.g., price of units purchased). Also known, but perhaps less frequently mentioned, is that the backorder cost consists of physical and financing components. The relevant physical backorder cost consists of those costs that vary with the number of customers backordered (e.g., loss of goodwill,
bookkeeping, expediting), and the relevant financial backorder cost consists of those costs that vary with the amount of revenue delayed (e.g., price of units sold).

Most inventory models calculate the total inventory holding and backorder costs by simply multiplying the number of physical units on location by a single holding cost parameter and the number of physical backorders by a single backorder cost parameter. In this way, they implicitly assume that financial flows correspond perfectly to physical flows. That is, they assume that the amount of funds invested in inventory is always proportional to the number of physical units on hand, and that the amount of profit forgone due to a backorder is always proportional to the number of current physical backorders. We refer to these two proportionality assumptions together as the classic assumption. The classic assumption allows for simple analysis - only the physical flow of inventory needs to be tracked, and cash flows are assumed to correspond perfectly. For this reason it is both widely taught and used in research.

Unfortunately, the classic assumption frequently does not hold in practice. In reality, (particularly with the growing implementation of information systems, electronic payments, and supply chain financing solutions) financial flows rarely correspond perfectly to physical flows because payments are not restricted to being conducted at the same time as physical transfers. Over 50% of firms now have electronic purchase order creation and submission (with 24% more planning to implement within a year) and 50% also have ERP payment modules (with 16% more planning to implement within a year; Aberdeen Group 2011). In fact, one of the reasons CFOs are so interested in implementing a supply chain technology platform is specifically for the “ability to trigger financial activities of a much richer set of milestone events, such as arrival at a consolidator, vessel departure, customs clearance, or arrival at an inbound VMI hub” (Aberdeen Group 2006). Examples of variations in payment times between firms are numerous. For instance, a small seller may request payment from the buyer before will initiating a shipment. Additionally, under trade credit contracts, suppliers permit delayed payment. Under pay-on-pull contracts, a firm does not have to pay a supplier until pulling the inventory into assembly. And under pay-on-scan contracts, a retailer does
not pay a supplier until the final customer purchases or “scans” a good. Such variations in payment schemes are in reaction to the increased challenge of financing the supply chain in today’s economy. Longer supply chains due to globalization and the recent credit crisis have increased the pressure to make financing the supply chain more efficient. A 2009 Demica Report recently reported that 81% of UK firms say that “market conditions have brought procurement and finance strategies in closer alignment.” Therefore, it is important to understand how different payment schemes should be captured in inventory models.

In this paper we show that fortunately for many variations of payment schemes that violate the classic assumption, we can still evaluate inventory policies without explicitly tracking the cash flows. We accomplish this by first defining new measures we call the \textit{financed inventory} and the \textit{margin backorders}. The financed inventory consists of those units that have been paid for minus those units for which payments have been collected (i.e., those units of inventory that for which a firm is incurring financing costs). The margin backorders are those profit margins that could have been collected, but are on delay. Then, by leveraging the payment scheme, we express these financed measures in terms of physical measures, and transform the problem into one with only physical measures. In this way, we retain the simplicity of standard inventory models under the classic assumption, while accommodating a much wider range of payment schemes for which the classic assumption does not hold.

We now outline our main contributions.

1. We present a model of payment schemes, payment processes, and financed inventory/margin backorders which builds on existing classic inventory control models but can accommodate a variety of payment schemes in practice. We show how these models can connect inventory models with key financial accounting metrics such as the cash conversion cycle, the net working capital, and cash flow volatility.

2. We derive a transformation result to show how payment processes and financed inventory/margin backorders can be expressed in terms of physical measures. This result
allows us to relate classic inventory results to new financial accounting measures.

3. We apply the transformation result to single stage models, the classic economic order quantity and base stock models, to demonstrate how practitioners can easily calculate the optimal inventory policy under various payment scheme contracts. We also demonstrate how the true optimal solution may deviate significantly from the solution in which the manager incorrectly assumes that the classic assumption holds.

4. We apply the transformation result to multiechelon models to demonstrate how supply chain coordination is affected by the choice in payment scheme in a two stage echelon model. Finally, for echelon models of size $n$, we also express the cash flow processes (both the incoming and outgoing payment flows, and the net financed inventory process) according to an echelon base stock policy. In other words, we determine the necessary cash flows needed to support a base stock policy according to the payment scheme. These results should prove to be useful for future work on the interface of operations and finance.

We review related literature in Section 2. We define the payment scheme, financed inventory and margin backorders in Section 3. In Section 4 we relate these new measures to financial metrics and total costs. Section 5 contains our transformation results. In Sections 6-7, we discuss the impact of payment schemes on single stage and echelon models. Finally, we conclude in Section 8. All proofs are in the Appendix.

1.2 Literature Review

It has long been recognized that simple inventory models may not correctly capture the inventory financing costs of firms in practice. Among the first to comment on this problem was Beranek (1967), who showed that the standard EOQ model implicitly assumes that the funds invested in inventory are proportional to the physical inventory (the classic assumption), and demonstrated several financial arrangements that violate this assumption. Perhaps the most well-studied of these violations comes from the practice of suppliers pro-
viding trade credit (or permissible delay in payment). Subsequently, many researchers have examined how the EOQ model can be altered to accommodate various forms of trade credit (Haley and Higgins 1973; Goyal 1985; Rachamadugu 1989; Aggarwal and Jaggi 1995). Our results on the EOQ model serve to simplify and unify some of these results, as well as extend them to include backorders and backorder financing costs.

More recently, some scholars have studied the impact of trade credit in inventory models with stochastic demand (e.g., Gupta and Wang 2009; Maddah et al. 2004). These papers take the structure of trade credit as given and solve for the optimal inventory policy. Our work differs from these studies in that it analyzes the optimal inventory policy for a wide range of payment schemes, of which “net” trade credit terms is a special case (constant delays in payment).

Besides trade credit, researchers have investigated several other issues at the interface of operations and finance, such as asset-based financing (Buzacott and Zhang 2004), initial public offerings (Babich and Sobel 2004), investor dividends (Li et al. 2003) and bankruptcy costs (Xu and Birge 2004). Our paper differs in its main focus, which is on payment schemes, and considers only inventory decisions, treating the financial policy as independent.

From a behavioral perspective, Chen et al. (2011) study the psychological effect of different payment schemes on newsvendor decisions. They show that real managerial behavior may deviate from the optimal ordering behavior. Relatedly, several papers investigate how cost performance is sensitive to errors. For instance, Dobson (1988) studies the sensitivity of the total cost performance when there are errors in parameter inputs for the EOQ model. Additionally, Zipkin (2000) provides some discussion on cash flows in inventory models (for example, see Section 3.7 for a discussion on the comparison of the average cost and present-value models). Our paper provides normative solutions that are easily implementable in practice, and shows how errors due to applying incorrect payment scheme assumptions lead to biased inventory policies.
1.3 Payment Schemes and Financed Inventory

We first define our model setting and some fundamental supply chain activities. Then we introduce payment schemes, the financed inventory and the margin backorders. Throughout the paper, we use bold letters to denote vectors, usually of size $n$. For example, $x = (x_1, x_2, ..., x_n)$.

1.3.1 Preliminaries

Refer to Figure 1. We consider a serial supply chain of size $n$. The most upstream supplier at stage $n$ procures from an outside supplier and sells to stage $n - 1$ who, in turn, sells to stage $n - 2$. This process continues until stage 1 procures from stage 2 and sells to the final customer at stage 0.

For each stage $j$, $j = 0, 1, ..., n$ there are three fundamental supply chain activities: the order to stage $j + 1$, the shipment from stage $j + 1$, and the receipt from $j + 1$. Define the following associated time epochs:

$t_{j}^{o,k} = \text{the time of firm } j \text{'s } k \text{th order, } j = 1, 2, ..., n, k = 1, 2, ..$

$t_{j}^{s,k} = \text{the time of firm } j \text{'s } k \text{th shipment (from } j + 1), j = 1, 2, ..., n, k = 1, 2, ..$

$t_{j}^{r,k} = \text{the time of firm } j \text{'s } k \text{th receipt, } j = 1, 2, ..., n, k = 1, 2, ..$

where $t_{j}^{o,k}, t_{j}^{s,k}, t_{j}^{r,k} \in \mathbb{R}^+$, the positive real numbers. We assume these time epochs obey some simple properties.

Assumption 1. The system starts in equilibrium. We begin counting time epochs starting with the $n$th firm’s first order $t_{n}^{o,1}$. This unit subsequently triggers all of the other first
time epochs: $t_j^{o,1}, t_j^{s,1}, t_j^{o,1}; j = 1, 2, \ldots n$. Thus, the time epochs exhibit the following ordering $t_j^{o,k} \leq t_j^{s,k} \leq t_j^{r,k} \leq t_{j-1}^{s,k}$ for all $j = 1, 2, \ldots, n, k = 1, 2, \ldots$

In other words, although the system starts in equilibrium (there is inventory in the system), we only count time epochs starting from the most upstream firm’s first order. We call this first ordered unit the “trigger unit.” The trigger unit then subsequently starts the first time epoch for the rest of the system. For example, $t_1^{r,1}$ is the time that stage one receives the trigger unit. We have that firm $j$ must place an order before the shipment can be made from $j + 1$, which, in turn, must occur before the unit can be received. Moreover, the unit must be received before it can be shipped to $j - 1$. Define the following processes:

$$O_j(t) = \max \left\{ k : t_j^{o,k} \leq t \right\} = \text{cumulative order process for stage } j$$
$$S_j(t) = \max \left\{ k : t_j^{s,k} \leq t \right\} = \text{cumulative shipment process for stage } j$$
$$R_j(t) = \max \left\{ k : t_j^{r,k} \leq t \right\} = \text{cumulative receipt process for stage } j$$

In other words, $O_j(t)$ is the number of orders made by stage $j$ from time 0 until time $t$. Based on Assumption 1, we have the following lemma.

**Lemma 1.** $O_j(t) \geq S_j(t) \geq R_j(t) \geq S_{j-1}(t)$ for all $j = 1, 2, \ldots, n$

That is, for any time $t$, the cumulative number of stage $j$ orders is greater than the cumulative number of stage $j$ shipments, which, in turn, is greater than the cumulative number of stage $j$ receipts. Lastly, the cumulative number of stage $j$ receipts is greater than the cumulative number of stage $j - 1$ shipments. We also define the number of orders, shipments, and receipts in an interval as follows.

$$O_j(t, t + \xi) = O_j(t) - O_j(t + \xi) = \text{number of orders for stage } j \text{ in time } \xi$$
$$S_j(t, t + \xi) = S_j(t) - S_j(t + \xi) = \text{number of shipments for stage } j \text{ in time } \xi$$
$$R_j(t, t + \xi) = R_j(t) - R_j(t + \xi) = \text{number of receipts for stage } j \text{ in time } \xi$$
where $\xi \in \mathbb{R}^+$. 

Through these fundamental processes, we can describe the standard physical inventory measures. For instance, the physical inventory at stage $j$ is simply the number of units that have arrived, but not yet been shipped. Moreover, the physical backorders at stage $j$ are simply those orders that have been placed from stage $j - 1$, but not yet shipped by stage $j$. Finally, the physical in-transit inventory are those units that have been shipped but not yet received. Thus, we can express these measures as follows.

\[
P_j^p(t) = \text{local physical inventory at stage } j = R_j(t) - S_{j-1}(t)
\]

\[
B_j^p(t) = \text{local physical backorders at stage } j = O_{j-1}(t) - S_{j-1}(t)
\]

\[
IT_j^p(t) = \text{local physical in-transit inventory at stage } j = S_j(t) - R_j(t)
\]

Note that from Lemma 1, we have that the physical inventory, physical backorders and physical inventory in-transit are nonnegative.

In most inventory models, these three measures (physical inventory, physical backorders, and physical inventory in-transit) are used to describe the physical flow of the system. However, they do not capture the financial flow of the system. In order to describe the financial flow of the system, one must track the payment transfers between firms. The next section serves to address this issue.

1.3.2 Payment Schemes

We define a payment scheme for a supply chain of size $n$ by $n + 1$ payment terms.

\[
\tau = (\tau_n, \ldots, \tau_0) = \text{payment scheme}
\]
where each \( \tau_j \) represents the payment term between stage \( j \) and stage \( j + 1 \). We assume that the final customer, stage 0, collects no payment. Payment terms can take on many forms. In this paper, we consider payment terms in which payments for inventory between stages are always made in full (later, we discuss how our notation may be used to facilitate more complicated payments in which partial payments are permitted). We define each payment term \( \tau_j \) as having two components: a reference event \( \rho_j \), and a constant payment delay \( \delta_j \), so that we have

\[
\tau_j = (\rho_j, \delta_j) = \text{payment term}
\]

where \( \rho_j \in \mathcal{I} \) (defined below), and \( \delta_j \in \mathbb{R} \) (which can take on negative values in some cases).

We define two kinds of reference events: decision reference events and logistic reference events. Define:

\[
o_j = \text{firm } j\text{'s orders} \\
r_j = \text{firm } j\text{'s receipts} \\
s_j = \text{firm } j\text{'s shipments}
\]

We call the \( o_j \)s decision reference events because they track the information flow of the system. On the other hand, we call the \( r_j \)s and the \( s_j \)s logistic reference events because they track the downstream progress of the physical inventory. The collection of all decision and logistic reference events composes \( \mathcal{I} \).

\[
\mathcal{I} = \{o_j, s_j, r_j, j = 0, 1, \ldots, n\}
\]

Therefore, we have that the set of all possible payment terms is

\[
\mathcal{T} = \mathcal{I} \times \mathbb{R}
\]

However, for convenience, and because it is clear from context, we simply write \( \tau_j = \rho_j + \delta_j \)
instead of $\tau_j = (\rho_j, \delta_j)$. For example, $\tau_j = o_j + 30$ is interpreted as the payment term where firm $j$ must pay firm $j + 1$ 30 days after placing an order.

Finally, each payment term $\tau_j$ generates a payment process:

$$P_j(t|\tau_j) = \text{cumulative payment process for payment term } \tau_j$$

In other words, $P_j(t|\tau_j)$ is the number of payments that have occurred between stage $j$ and stage $j + 1$ by time $t$ under payment term $\tau_j$. We have

$$P_j(t|\tau_j) = \begin{cases} 
O_j(t - \delta_j) & \text{if } \rho_j = o_j \\
S_j(t - \delta_j) & \text{if } \rho_j = s_j \\
R_j(t - \delta_j) & \text{if } \rho_j = r_j 
\end{cases} \quad (2)$$

Example payment schemes Many payment schemes in practice can be accurately represented. Consider the simplest case: a retailer (stage 1) sells to the final customer (stage 0). Then the supply chain payment scheme is composed of two times, $(\tau_1, \tau_0)$, so that stage 1 pays her upstream supplier at $\tau_1$ and collects from her downstream customer at $\tau_0$. In the following examples we take the perspective of the retailer and refer to it as “the firm.”

- (Payments-on-receipts) Under payment scheme $\tau = (r_1, r_0)$, the firm pays the supplier upon receiving inventory, and does not collect payment until the customer receives the shipment.

- (Payments-on-shipments) Under payment scheme $\tau = (s_1, s_0)$, the firm pays the supplier upon the supplier’s shipment and collects payment upon shipping to the customer.

- (Payments-on-orders) Under payment scheme $\tau = (o_1, o_0)$, the firm pays the supplier upon ordering and collects payment from their customer upon the customer’s order (the firm’s demand arrival).

- (Classic assumption) Under payment scheme $\tau = (r_1, s_0)$, the firm pays the supplier
upon receiving inventory and collects payment from the customer upon shipping the inventory. We say that this payment scheme satisfies the *classic assumption* because the capital invested in inventory is always proportional to the physical inventory on hand.

- **(Pay-on-scan)** Under payment scheme $\tau = (s_0, s_0)$, the firm both pays the supplier and collects payment from the customer upon shipping. For example, retailers such as Wal-Mart have this type of agreement with many of their suppliers. They implement technology that automatically pays suppliers upon each customer scan at the register (Fahey 2003).

Moreover, payment schemes in practice often implement constant payment delays ($\delta \neq (0, 0)$). For instance,

- **(Net terms trade credit)** Under payment scheme $\tau = (r_1 + \hat{\delta}, s_0)$, the firm can delay their payment to the supplier for time $\hat{\delta}$. This is a common type of trade credit contract that suppliers frequently offer regular buyers (e.g., see Peterson and Rajan 1997).

- **(Pay-on-pull)** Under payment scheme $\tau = (s_0 + \hat{\delta}, o_0)$, the firm collects payment when they receive the order from the customer, but does not have to pay the supplier until $\hat{\delta}$ days after shipping. For example, Dell does not pay some suppliers until 36 days after the part is pulled into assembly (Holzner 2006), which they do just 4 days before shipping (it takes 4 days for assembly). Here we have also assumed that the customer pays upon ordering, which is consistent with Dell’s practices.

Though many contracts can be modeled closely in our framework, some contracts can not. For example, under “date” trade credit contracts, balances are settled at the end of pre-specified periods (e.g., on the last day of each month). Such a contract can not easily be captured by our payment scheme framework. Two other types of payment terms, though not directly captured by our modeling framework, are worth noting. Our payment scheme notation does not directly capture more complex contracts that allow for flexible payment
times at different rates, such as a trade credit contract that permits a 2% discount if payments are made within 10 days, and due by 30 days (so called “2/10 net 30” contracts). Moreover, some payment terms in practice allow for partial payment due at one time, with the rest of the payment due at a later time. Our model would serve as a nice starting point to model these types of payment terms, but would require an extension. We refer the reader to Peterson and Rajan (1997) for further discussion of trade credit contracts. In general, our framework works well with the increasing trend of “supply chain financing” under which payments are triggered based on decision and logistic “supply chain” reference times.

1.3.3 Financed inventory

Next, we define a new inventory measure, the financed inventory.

**Definition 1.** Firm $j$’s financed inventory are the number of units that firm $j$ has paid $j+1$ for minus the number of units for which payment has been collected from $j-1$.

The financed inventory measures the number of units for which the firm incurs opportunity cost of capital from holding inventory. Location 0 is the final customer, for whom we do not calculate financed inventory (although the customer’s payments affects the system through $\tau_0$). Firm $j$’s financed inventory is independent from $\tau_k, k \neq j, j-1$. Thus, we denote

$$I^f_j(t|(\tau_j, \tau_{j-1})) = \text{firm } j\text{'s financed inventory at time } t, j = 1, .., n$$

$$= P_j(t|\tau_j) - P_{j-1}(t|\tau_{j-1})$$

(3)

Note that although physical inventory is always nonnegative, the financed inventory $I^f_j(t)$ can be negative. When the limiting distribution exists, we drop the $t$ and denote $I^f_j(\tau_j, \tau_{j-1})$ the limiting distribution of the financed inventory for firm $j$.

Thus, from (3) in combination with (2), we can also express the financed inventory in
terms of fundamental activities. For instance, we have that

\[ I^f_j(t|(r_j + 10, s_j + 30)) = R_j(t - 10) - S_j(t - 30) \]

In general, we also have the following properties. We say \( \tau_j \leq \tau_{j-1} \) if \( P_j^f(t|\tau_j) \geq P_{j-1}^f(t|\tau_{j-1}) \) for all \( t \).

**Lemma 2.** The financed inventory exhibits the following properties:

1. \( I^f_j(t|\tau_j, \tau_{j-1}) \geq 0 \) for all \( t \) if \( \tau_j \leq \tau_{j-1} \), \( I^f_j(t|\tau_j, \tau_{j-1}) = 0 \) for all \( t \) if \( \tau_j = \tau_{j-1} \).
2. \( I^f_j(t|\tau_j, \tau_{j-2}) = I^f_j(t|\tau_j, \tau_{j-1}) + I^f_j(t|\tau_{j-1}, \tau_{j-2}) \) for all \( t \)
3. \( I^f_j(t|\tau_j, \tau_{j-1}) = -I^f_j(t|\tau_{j-1}, \tau_j) \) for all \( t \)
4. \( I^f_j(t|\tau_j, \tau_{j-1}) = I^f_j(t + \delta)(\tau_j + \delta, \tau_{j-1} + \delta) \), where \( \delta \in \mathbb{R} \) for all \( t \)

Part (1) states that the financed inventory is zero if payments are made and collected at the same time. For example, if payment is both made and received upon shipping (such as under pay-on-scan) we have \( I^f_j(t|s_j, s_j) = 0 \). However, unlike physical inventory, financed inventory can be negative. Part (2) demonstrates that the financed inventory can be expressed as the sum of its parts in the sense that financing inventory from times 1 to 2 and times 2 to 3 is the same as financing inventory from times 1 to 3. Part (3) says that exchanging the times payments are made and payments are collected is equivalent to switching the sign of the financed inventory. Finally, Part (4) observes that the financed inventory remains unchanged if one simultaneously looks forward in time and delays all payment times by the same amount of time. We define the entire supply chain’s financed inventory as the sum of each member’s financed inventory.

\[ \sum_{j=1}^{n} I^f_j(t|\tau_j, \tau_{j-1}) = \text{total supply chain financed inventory} \]

The following proposition is a direct result of Lemmas 1 and 2 parts (1) and (2).

**Proposition 1.** Fixing \( \tau_j \), we have \( I^f(t|\tau_j, o_j) \leq I^f(t|\tau_j, s_j) \leq I^f(t|\tau_j, r_j) \leq I^f(t|\tau_j, s_{j-1}) \)

for all \( j = 1, \ldots, n \) and \( t \). Similarly, fixing \( \tau_{j-1} \), we have \( I^f(t|o_j, \tau_{j-1}) \geq I^f(t|s_j, \tau_{j-1}) \geq \ldots \)
\[ I^f(t|(r_j, \tau_{j-1})) \geq I^f(t|(s_{j-1}, \tau_{j-1})) \text{ for all } j = 1, \ldots, n \text{ and } t. \]

### 1.3.4 Margin backorders

The financed inventory can be interpreted as the corresponding financial measure for the physical inventory. Similarly, we define firm \( j \)'s *margin backorders* below, which can be interpreted as the corresponding financial measure for the physical backorders.

**Definition 2.** Firm \( j \)'s margin backorders are those downstream demands that have realized, but for which payment has not yet been collected.

Note that for firm \( j \), the number of margin backorders depends only on the time of collection \( \tau_{j-1} \) from the downstream, and not on the time of payment to the upstream. Therefore, we denote

\[
B^f_j(t|\tau_{j-1}) = \text{firm } j \text{'s margin backorders at time } t, j = 1, \ldots, n
\]

\[
= P_{j-1}(t|o_{j-1}) - P_{j-1}(t|\tau_{j-1})
\]

Here, we have assumed that payment is “backordered” if it is received after the downstream’s demand. This assumption is made to be consistent with the definition of physical backorders, which assumes that a customer is “backordered” if it is satisfied after the downstream’s demand. (This assumption has implications for total cost accounting, which we will discuss later). The total supply chain margin backorders are defined as

\[
B^f(t|\tau) = \sum_{j=1}^{n} B^f_j(t|\tau_{j-1})
\]

**Lemma 3.** The margin backorders exhibit the following properties

1. \( B^f_j(t|\tau_{j-1}) \geq 0 \text{ for all } t \text{ if } o_{j-1} \leq \tau_{j-1}, B^f_j(t|\tau_{j-1}) = 0 \text{ for all } t \text{ if } o_{j-1} = \tau_{j-1}. \)

2. \( B^f_j(t|\tau_{j-1}) = B^f\left(t + \delta|\tau_{j-1} + \delta\right) \text{, where } \delta \in \mathbb{R} \text{ for all } t \)

The lemma observes that, as for the financed inventory, margin backorders are always positive if the payment collection is always after the downstream’s order. However, unlike
physical backorders, margin backorders can be negative. Moreover, the margin backorders remain unchanged if one simultaneously looks forward in time and delays the collection by the same amount of time.

1.4 Performance Metrics

The financed inventory can be used to express several frequently reported firm performance metrics.

1.4.1 CCC and net operating working capital

The cash-conversion cycle (CCC), or equivalently, the cash-to-cash time is the time it takes for a cash investment in inventory to mature. It is determined by the financed inventory as follows:

$$\frac{\bar{I}_{j}^{f}(\tau_{j}, \tau_{j-1})}{\lambda} = \text{average CCC for firm } j$$

$$\frac{1}{\lambda} \sum_{j=1}^{n} \bar{I}_{j}^{f}(\tau_{j}, \tau_{j-1}) = \text{average supply chain CCC}$$

where \( \lambda \) is the average demand rate. The net operating working capital is also related to the financed inventory. Let \( \theta_{j} \) be the average cost of goods sold for firm \( j \). Then we have the following good approximations

$$I_{j}^{f}(t|(\tau_{j}, \tau_{j-1})))\theta_{j} = \text{net operating working capital for firm } j$$

$$\sum_{j=1}^{n} I_{j}^{f}(t|(\tau_{j}, \tau_{j-1})))\theta_{j} = \text{total supply chain net operating working capital}$$

Clearly, all else equal, firms would like short cash-conversion cycle times and low (or even negative) working capital. Thus, all else being equal, firms would like to maintain low financed inventory.
1.4.2 Financing cost formulation

The financed inventory and the margin backorders have associated costs. Define the price paid between firms as follows

\[ c_j = \text{per unit cost paid by firm } j \text{ to firm } j+1 \]

where \( c_j < c_{j-1} \) for \( j = 1, 2, ..., n \). We also define each firm’s interest rate as

\[ \alpha_j = \text{firm } j \text{'s interest rate} \]

and \( \alpha = \min_j \{ \alpha_j \} \). We assume that the opportunity cost of capital from holding inventory is only due to the purchasing cost \( c_j \) (there is no cost from the value added, although such a cost it is not difficult to accommodate). Therefore, we can approximate the financed inventory holding and margin backorder costs as follows

\[ h_j^f = \alpha_j c_j = \text{unit financed inventory holding cost for firm } j \]
\[ b_j^f = \alpha_j (c_{j-1} - c_j) = \text{unit financed backorder margin cost for firm } j \]

In other words, the cost to finance one unit of inventory for one unit of time is the unit cost multiplied by the interest rate. On the other hand, the cost of a margin backorder is the profit margin that could have been earned multiplied by the interest rate. We express the total long run average financing costs for firm \( j \) as the sum of these two costs.

\[ \bar{C}_j^f = h_j^f \bar{I}_j^f(\tau_j, \tau_{j-1}) + b_j^f \bar{B}_j^f(\tau_{j-1}) = \text{average financing costs for firm } j \]

For the supply chain, we define the following decentralized cost

\[ \bar{C}_{dec}^f = h^f \cdot \bar{I}^f(\tau) + b^f \cdot \bar{B}^f(\tau) = \text{average decentralized supply chain financing cost} \]
For a centralized system, we define the following cost

$$\bar{C}_{cen}^f = h_{cen}^f \cdot \bar{I}(\tau) + b_{cen}^f \bar{B}_1^f(\tau_0) = \text{average centralized supply chain financing costs}$$

where

$$h_{cen}^f = \alpha c_n$$

$$b_{cen}^f = \alpha (c_1 - c_n)$$

The centralized system financing cost uses the smallest interest rate to finance all of its inventory. Margin backorders between firms do not incur costs, only the margin backorder to the outside final customer.

**Proposition 2.** \(\bar{C}_{cen}^f < \bar{C}_{dec}^f\) if there exists a \(t > 0\) and a \(j < n\) such that \(I_j^f(t|\tau_j, \tau_{j-1}) > 0\) and \(\alpha_j c_j > \alpha c_n\)

In other words, the supply chain suffers from inefficiency if any downstream stage finances inventory at a marked-up price or at a higher interest rate.

**Variances** Firms are also concerned with the uncertainty and volatility in their cash flows. For instance, firms are concerned with the variance of the incoming (related to \(\tau_j\)) or outgoing (related to \(\tau_{j-1}\)) cash flows. Define

$$\text{Var} \left( P_j((t, t+\xi)|\tau_j) \right) = \text{variance of payments associated with } \tau_j \text{between times } t \text{ and } t + \xi$$

Moreover, the variance of the financed inventory provides a good indicator of the variance of the net cash flows. Define

$$\text{Var} \left( I_j^f((t, t+\xi)|(\tau_j, \tau_{j-1})) \right) = \text{variance of the financed inventory between times } t \text{ and } t + \xi$$
In general, firms prefer low volatility and low uncertainty (all else equal) because volatility increases cash-management costs.

1.5 Transformations

In this section, we first review some standard physical inventory measures. We then show that payment processes and the financed inventory can be expressed through physical measures according to the payment scheme. Subsequently, we show how we can also express the average financed inventory through the averages of physical measures. Finally, we define a relaxed formulation of the average total inventory/backorder costs.

1.5.1 Review of physical inventory measures

We define the following physical measures in the standard way (we use the superscript $p$ to differentiate the physical inventory from the financed inventory).

$$L_j = \text{leadtime between stages } j+1 \text{and } j$$

$$I^p_j(t) = \text{local physical inventory at stage } j$$

$$IT^p_j(t) = \text{inventory in transit to stage } j$$

$$B^p_j(t) = \text{local physical backorders at stage } j$$

$$IO^p_j(t) = \text{inventory on order at stage } j$$

We also define the following combinations as follows

$$IN^p_j(t) = I^p_j(t) - B^p_j(t) = \text{local net inventory at stage } j$$

$$IOP^p_j(t) = IN^p_j(t) + IO^p_j(t) = \text{local inventory-order position at stage } j$$

1.5.2 Payment processes transformation

We seek to define some recursions in order to relate the financed inventory to physical inventory measures for any payment scheme.
Proposition 3. The payment process for any payment scheme can be expressed as follows

\[ P_j(t|\rho_j+\delta_j) = \begin{cases} 
P_j(t-\delta_j|o_n) - \sum_{i=j+1}^n [IT^p_i(t-\delta_j) + IP^p_i(t-\delta_j)] + B^p_{j+1}(t-\delta_j) & \text{if } \rho_j = o_j \\
\sum_{i=j+1}^n [IT^p_i(t-\delta_j) + IP^p_i(t-\delta_j)] & \text{if } \rho_j = s_j \\
P_j(t-\delta_j|o_n) - IT^p_n(t-\delta_j) - \sum_{i=j+1}^n [IP^p_i(t-\delta_j) + IT^p_{i-1}(t-\delta_j)] & \text{if } \rho_j = r_j 
\end{cases} \]

Thus, it is quite simple to express financed inventory in terms of physical inventory, physical backorders, and in-transit inventory for any possible payment scheme using this Proposition and (3). One simply needs to subtract the payments associated with \( \tau_{j-1} \) with the payments associated with \( \tau_j \).

1.5.3 Financed inventory transformation

The next lemma provides a convenient separation of the constant payment delays from the reference events for the financed inventory.

Lemma 4. The financed inventory has the following equivalent expressions

\[ \begin{align*}
I^f_j(t|\tau_j,\tau_{j-1}) &= I^f_j(t-\delta_j|\rho_j,\rho_{j-1}) + I^f_j(t|\rho_{j-1}+\delta_j,\rho_{j-1}+\delta_{j-1}) \\
&= I^f_j(t-\delta_{j-1}|\rho_j,\rho_{j-1}) + I^f_j(t|\rho_j+\delta_j,\rho_j+\delta_{j-1}) 
\end{align*} \tag{4} \]

and

\[ \begin{align*}
I^f_j(t|\rho_j+\delta_j,\rho_j+\delta_{j-1}) &= \begin{cases} 
O_j(t-\delta_{j-1},t-\delta_j) & \text{if } \rho_j = o_j \\
S_j(t-\delta_{j-1},t-\delta_j) & \text{if } \rho_j = s_j \\
R_j(t-\delta_{j-1},t-\delta_j) & \text{if } \rho_j = r_j 
\end{cases} 
\end{align*} \]

The lemma allows us to express the financed inventory under any payment scheme as the sum of two terms: for the first term there are no payment delays and for the second term the payment times differ only in the constant payment delays.
Proposition 4. There exists vectors $x^I_j, y^I_j, z^I_j \in \{-1, 0, 1\}$ such that the following relationships between the financed inventory and physical measures hold

\[
I^I_j(t|(\tau_j, \tau_{j-1})) = x^I_j \cdot IT^p(t - \delta_j) + y^I_j \cdot Ip(t - \delta_j) + z^I_j \cdot B^p(t - \delta_j) + I^I_j(t|(\rho_j, \rho_j - 1 + \delta_j - 1))
\]

Proposition 4 provides a way to express the financed inventory as a function of the in-transit inventory, the physical inventory, and the physical backorders for any payment scheme. The vectors $x^I_j, y^I_j, z^I_j$ can be interpreted as showing how physical measures “contribute” to the financed inventory. It also implies that if a firm is interested in reducing the financed inventory (and hence their CCC and working capital), one way is to reduce those physical measures that “contribute” to the financed inventory (decrease those measures associated with a 1 coefficient in $x_j, y_j, z_j$ or increase those associated with a $-1$). Another way to reduce the financed inventory is to change the payment scheme.

Corollary 1. $\bar{I}^I(\tau_j, \tau_{j-1}) = x^I_j \cdot \bar{IT}^p + y^I_j \cdot \bar{Ip} + z^I_j \cdot \bar{B}^p + \lambda(\delta_{j-1} - \delta_j)$

Similar to Proposition 4, Corollary 1 provides a way to express the average financed inventory as a function of the average pipeline inventory, the average physical inventory, and the average physical backorders for any payment scheme. It also shows that the average financed inventory is increasing linearly in $\delta_{j-1} - \delta_j$, so that all else being equal, the firm finances more inventory when payment collection is delayed by a constant amount of time or payments made are advanced by a constant amount of time.

For the supply chain, we also have the following useful conservation property

Proposition 5. If $\tau_n = s_n$ and $\tau_0 = s_0$ then $\sum_j x^I_j = 1, \sum_j y^I_j = 1, \sum_j z^I_j = 0$.

Proposition 5 implies that all of the in-transit inventory and physical inventory must translate to financed inventory. However, backorders need not translate to financed inventory because backorders do not correspond to real units of inventory in the system.
1.5.4 Margin back orders transformation

**Lemma 5.** *The margin back orders have the following equivalent expression*

\[ B^f_j(t|\tau_{j-1}) = B^f_j(t - \delta_{j-1}|\rho_{j-1}) + O_{j-1}(t - \delta_{j-1},t) \]

Lemma 5 allows us to express the margin back orders under any payment scheme in terms of only reference times, specifically the reference time for payment collection.

**Proposition 6.** *There exists vectors \( x^B_j, y^B_j, z^B_j \in \{-1, 0, 1\} \) such that the following relationship between the margin back orders and physical back orders holds*

\[ B^f_j(t|\tau_{j-1}) = x^B_j \cdot I^p(t - \delta_{j-1}) + y^B_j \cdot I^p(t - \delta_{j-1}) + z^B_j \cdot B^p(t - \delta_{j-1}) + O_{j-1}(t - \delta_{j-1},t) \]

Proposition 6 provides a way to express the margin back orders as a function of the physical back orders for any payment scheme. We also have the following corollary for the long run average.

**Corollary 2.** *\( \bar{B}^f_j(t|\tau_{j-1}) = x^B_j \cdot \bar{I}^p + y^B_j \cdot \bar{I}^p + z^B_j \cdot \bar{B}^p + \lambda \delta_{j-1} \)*

1.5.5 Total cost transformation

**Total cost formulation** In order to incorporate inventory holding and backorder costs, most standard inventory models use the following approach. Let

\[ C_j(t) = \text{total inventory holding and backorder costs at time } t \text{ for firm } j \]

**Definition.** (Classic Assumption) Let \( \hat{h}, \hat{b} > 0 \) be the total unit holding and backorder costs. We call the following cost formulation the *classic assumption*.

\[ C_j(t) = \hat{h}_j I^p_j(t) + \hat{b}_j B^p_j(t) \]

In other words, under the classic assumption the total holding cost is always proportional
to the physical inventory and the total backorder cost is always proportional to the physical backorders. In particular, if the holding cost is comprised of a physical unit holding cost \( h^p_j \) and a financial unit holding cost \( h^f_j \), then we must have \( \hat{h}_j = h^p_j + h^f_j \). Therefore, the classic assumption implicitly assumes that both the physical and the financial holding costs are always proportional to the physical inventory. Similarly, if \( b^p_j \) is the physical unit backorder cost and \( b^f_j \) is the margin unit backorder cost, then under the classic assumption we have \( \hat{b}_j = b^p_j + b^f_j \), and both the physical and the financial backorder costs are necessarily always proportional to the physical backorders.

Clearly, the classic assumption is quite strict, and does not hold under many payment schemes. We therefore make the following relaxation.

**Definition.** (Cost Relaxation) Let \( h^p_j, b^p_j \) be the unit physical holding and margin backorder costs and \( h^f_j, b^f_j \) the unit financial holding and backorder costs. We call the following relationships the *separation assumption*

\[
C_j(t|\tau_j, \tau_{j-1}) = h^p_j I^p(t) + h^f_j I^f_j(t|\tau_j, \tau_{j-1}) + b^p_j B^p_j(t) + b^f_j B^f_j(t|\tau_{j-1})
\]

Thus, we relax the classic assumption by allowing costs from financing the purchase to not be proportional to the physical inventory. Similarly, we allow the delayed profit margins to not be proportional to the physical backorders.

**Total cost transformation**

**Proposition 7.** The total inventory and backorder costs can be expressed as follows

\[
C_j(t|\tau_j, \tau_{j-1}) = h^p_j(t - \delta_j) + b^p_j(t - \delta_j) \\
+ \ell^p_j(t - \delta_j) \cdot IT^p(t - \delta_j) + \ell^f_j(t - \delta_j) \cdot IT^f_j(t|\rho_j - \delta_j, \rho_{j-1} + \delta_j + \delta_{j-1}) + b^p_j O_{j-1}(t - \delta_{j-1}, t) \\
= h^p_j(t - \delta_{j-1}) + b^p_j(t - \delta_{j-1}) \\
+ \ell^p_j(t - \delta_{j-1}) \cdot IT^p(t - \delta_{j-1}) + \ell^f_j(t - \delta_{j-1}) \cdot IT^f_j(t|\rho_j + \delta_j, \rho_{j-1} + \delta_{j-1}) + b^p_j O_{j-1}(t - \delta_{j-1}, t)
\]
where

\[ \ell_j^e(\tau_j, \tau_{j-1}) = h_j^e x_j^I + b_j^e x_j^B \]
\[ h_j^e(\tau_j, \tau_{j-1}) = h_j^e l_j + h_j^e y_j^I + b_j^e y_j^B \]
\[ b_j^e(\tau_j, \tau_{j-1}) = b_j^e l_j + b_j^e z_j^B + b_j^e z_j^B \]

and \( i_j \) is the vector of 0's with a 1 in the \( j \)th location.

Furthermore, the result can be extended to the average cost case.

**Corollary 3.** The total average inventory and backorder costs can be expressed as follows

\[ \bar{C}_j(\tau_j, \tau_{j-1}) = h_j^e(\tau_j, \tau_{j-1}) \cdot \bar{I} + b_j^e(\tau_j, \tau_{j-1}) \cdot \bar{B} + \ell_j^e(\tau_j, \tau_{j-1}) \cdot \lambda L + h_j^e \lambda (\delta_{j-1} - \delta_j) + b_j^e \lambda \delta_{j-1} \]

### 1.6 Applications to Inventory Models

#### 1.6.1 Single Stage EOQ Model

The EOQ model concerns inventory management with the presence of economies of scale. Demand arrives continuously at a known constant rate \( \lambda \). There is a fixed cost \( k \) to place an order, which arrives after a known leadtime \( L \). The objective is to determine an ordering policy that correctly balances the fixed cost of ordering with the costs of holding inventory and/or incurring backorders. The optimal inventory policy is of the following form: whenever the inventory position \( IP \) reaches \( \lambda L \), order the same batch size \( q \) so that the planned lowest net inventory \( IN \) is \( v \).

In a single stage inventory model, a payment scheme is fully defined by \( \tau = (\tau_1, \tau_0) \). For brevity, we only consider the possible set of reference times \( \rho_1, \rho_0 \in \{o_1, s_1, r_1, o_0, s_0\} \) (including additional reference times is not difficult.) The total average cost function for the policy \((q, v)\) under \( \tau \) is defined by the following cost function (where we drop the subscript
Table 1: Financed inventory according to selected payment schemes.

<table>
<thead>
<tr>
<th>( \rho_1 )</th>
<th>( \rho_0 )</th>
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1 when it is clear from context).

\[
C^{EOQ}(q,v,(\tau_1, \tau_0)) = (k + cq) \frac{\lambda}{q} + h^r(\tau_1, \tau_0) \bar{P}(q,v) + h^f(\tau_1, \tau_0) P^p(q,v) + b^e(\tau_1, \tau_0) \lambda L + h^f(\tau_1, \tau_0) \lambda_0 + b^f(\tau_1, \tau_0) \delta_0
\]

Table 1 shows how to write the financed inventory in terms of physical measures for all possible combinations of \( \tau_1, \tau_0 \). For this single stage model, we assume that there are no backorders at stage 2 (i.e., \( B_2^p = 0 \)). Then, based on Proposition 3, we can rewrite the cost function as follows (because of the abbreviated list of reference times, all vectors reduce to a scalar).

\[
C^{EOQ}(q,v,(\tau_1, \tau_0)) = (k + cq) \frac{\lambda}{q} + h^r(\tau_1, \tau_0) \bar{P}(q,v) + h^f(\tau_1, \tau_0) P^p(q,v) + \ell^c(\tau_1, \tau_0) \lambda L + h^f(\tau_1, \tau_0) \lambda_0 + b^f(\tau_1, \tau_0) \delta_0
\]

Table 2 shows the values of \( h^c, b^c, \) and \( w^c \) for all possible combinations of \( \tau_1, \tau_0 \).

Under a \((q,v)\) policy we have

\[
C^{EOQ}(q,v,(\tau_1, \tau_0)) = (k+cq) \frac{\lambda}{q} + h^c(\tau_1, \tau_0) (q+v^2) + b^c(\tau_1, \tau_0) \frac{v^2}{2q} + \ell^c(\tau_1, \tau_0) \lambda L + h^f(\tau_1, \tau_0) \lambda_0 + b^f(\tau_1, \tau_0) \delta_0 + b^f(\tau_1, \tau_0) \delta_0
\]

Define

\[
w^c(\tau_1, \tau_0) = \frac{b^c(\tau_1, \tau_0)}{b^c(\tau_1, \tau_0) + h^c(\tau_1, \tau_0)} \quad (5)
\]

as the effective critical fractile.

The optimal policy under each payment scheme is shown in the following proposition.
Table 2: From top to bottom: $h^e, b^e, \ell^e$ for selected payment schemes.

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<td>$s_0$</td>
<td>$-h^f - b^f$</td>
<td>$-h^f - b^f$</td>
</tr>
</tbody>
</table>
Proposition 8. The optimal batch size and lowest planned inventory level under $\tau$ is

$$q^*(\tau_1, \tau_0) = \sqrt{\frac{2k\lambda}{h^e(\tau_1, \tau_0)w^e(\tau_1, \tau_0)}}$$

$$v^*(\tau_1, \tau_0) = -(1 - w^e(\tau_1, \tau_0))q^*(\tau_1, \tau_0)$$

with optimal cost

$$C^{EOQ}(q^*(\tau_1, \tau_0), v^*(\tau_1, \tau_0)) = c\lambda + \sqrt{2k\lambda h^e(\tau_1, \tau_0)w^e(\tau_1, \tau_0)} + \ell^e(\tau_1, \tau_0)\lambda L + hf\lambda (\delta_0 - \delta_1) + b_j^f\lambda \delta_0$$

where $h^e(\tau_1, \tau_0), b^e(\tau_1, \tau_0)$ are defined in Proposition 2 and $w^e(\tau_1, \tau_0)$ in (5).

This proposition shows how the optimal batch size and minimum inventory levels are sensitive to the payment scheme. Interestingly, both the optimal policy and the optimal cost varies significantly depending on the payment scheme. One implication is that if managers incorrectly assume that the classic assumption holds, they will arrive at a suboptimal inventory policy leading to significant cost degradation.

Corollary 4. The optimal policy $(q^*(\tau_1, \tau_0), v^*(\tau_1, \tau_0))$ is independent of $\delta$. The total cost is increasing linearly in $\delta_0 - \delta_1$.

The corollary notes that the optimal policy does not depend on constant delay factors. Such independence is due to the average cost criteria, and is consistent with researchers who have found that the optimal EOQ policy is independent with “net” trade credit terms (e.g., Goyal 1985). Under this type of trade credit, payment to the supplier is due a certain number of days after inventory receipt. This is the special case where $\rho_m = r, \delta_m > 0$. Our results serve to extend this independence across constant delays in the payment scheme based on any reference time.

For most practical purposes, the average cost criteria provides good solutions when the inputs are stationary (see Hadley 1964). However, there may be some differences between a discounted cash flow approach and the average cost criteria. We refer the reader to Rachamadugu (1989) for an investigation of the difference between the average cost and the
discounted cash flow criteria in the EOQ trade credit problem.

1.6.2 Single Stage Base Stock Model

A similar procedure can be applied to the single stage base stock model. The base stock model concerns inventory management with randomness in the demand arrival process. Demand arrives randomly one unit at a time, and there is a positive leadtime $L$ for inventory. Assume the demand process is stationary and let $D$ be the demand in a leadtime, a random variable. $D$ has distribution $F$ and mean $\lambda L$. There is no fixed cost for placing an order. Thus, the objective is to determine an ordering policy that correctly balances the cost of holding inventory with the cost of incurring backorders. The optimal inventory policy is of the following form: if the inventory position $IP$ is greater than a base stock level, $y$, do nothing, but if $IP$ is less than $y$, place an order to bring $IP$ to $y$. Thus, once the inventory position reaches $x$, the base stock policy orders one unit of inventory each time a unit of demand arrives so that the inventory position remains constant.

In a single stage base stock model a payment scheme is again fully defined by $\tau = (\tau_1, \tau_0)$. The total average cost function for a base stock policy $y$ under $(\tau_1, \tau_0)$ is defined by the cost matrix

$$C^{BS}(y|(\tau_1, \tau_0)) = h^P \bar{P}(y) + h^I \bar{I}(x|(\tau_1, \tau_0)) + b^P \bar{B}^p(y) + b^f \bar{B}^f(y|(\tau_1, \tau_0))$$

which has the equivalent formulation according to Proposition 3.

$$C^{BS}(y|(\tau_1, \tau_0)) = h^c(\tau_1, \tau_0) \bar{P}(y) + b^c(\tau_1, \tau_0) \bar{B}^p(y) + \ell(\tau_1, \tau_0) + h^f \lambda (\delta_0 - \delta_1) + b^f \lambda \delta_0$$

Under a base stock policy, $S$, this simplifies to

$$C^{BS}(y|(\tau_1, \tau_0)) = h^c(\tau_1, \tau_0) \mathbb{E} [y - D]^+ + b^c(\tau_1, \tau_0) \mathbb{E} [y - D]^- + \ell(\tau_1, \tau_0) + h^f \lambda (\delta_0 - \delta_1) + b^f \lambda \delta_0$$

**Proposition 9.** The optimal base stock level under $(\tau_1, \tau_0)$ is $S^*(\tau_1, \tau_0) = F^{-1}(w^*(\tau_1, \tau_0))$.
The optimal cost is

\[ C^{BS}(y^*(\tau_1, \tau_0)) = \bar{C}(y^*(\tau_1, \tau_0)) + \ell^e(\tau_1, \tau_0) + h^f \lambda(\delta_0 - \delta_1) + b f \lambda \delta_0 \]

where

\[ \bar{C}(y^*(\tau_1, \tau_0)) = h^e(\tau_1, \tau_0) E[y^*(\tau_1, \tau_0) - D]^+ + b^e(\tau_1, \tau_0) E[y^*(\tau_1, \tau_0) - D]^- \]

and \( h^e(\tau_1, \tau_0), b^e(\tau_1, \tau_0) \) and \( \ell^e(\tau_1, \tau_0) \) are defined in Proposition 3.

The Proposition shows that the critical fractile, and hence the optimal in-stock probability, depends on the payment scheme. As before, however, the optimal policy is independent with the constant delay factors, as the next corollary shows.

Corollary 5. The optimal base stock policy \( y^*(\tau_1, \tau_0) \) is independent with \( \delta \). The total cost is increasing linearly in \( \delta_0 - \delta_1 \).

Similar implications from the EOQ model apply to the base stock model as well, so we will not repeat.

Lastly, as we did for the EOQ model, we comment on the choice of the average cost criteria which we have used to derive our results. Under the expected-present-value criterion, the derivation of the optimal policies under each payment scheme is much more involved. Fortunately, Zipkin (2000) derives the optimal base-stock levels under the expected value criterion for two of the above payment schemes, the classic assumption \( \tau = (r_1, s_0) \) (p. 389) and \( \tau = (r_1, o_0) \) (p. 233). The optimal solutions under the expected-present-value criterion are identical to our average cost criterion solutions in both cases. This is due to the fact that the optimal policy is myopic, and is consistent with Hadley (1964), who suggests that this phenomenon should hold “with equal force for all inventory models in which none of the parameters or distributions change with time.”
Table 3: The financed inventory for firms 1 and 2 for various values of $\tau_1$ given $\tau_2 = s_2$ and $\tau_0 = s_0$.

<table>
<thead>
<tr>
<th>$\tau_1$</th>
<th>$I_f^2(t)$</th>
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<td>$o_2$</td>
<td>0</td>
<td>$IT_f^2(t) + I_f^2(t) + IT_1(t) + I_1(t)$</td>
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<tr>
<td>$o_1$</td>
<td>$IT_f^2(t) + 2(t) - B_2(t)$</td>
<td>$B_2(t) + IT_1(t) + I_1(t)$</td>
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<tr>
<td>$o_0$</td>
<td>$IT_f^2(t) + I_f^2(t) + IT_1(t) + I_1(t) - B_1(t)$</td>
<td>$B_1(t)$</td>
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<tr>
<td>$s_2$</td>
<td>0</td>
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<tr>
<td>$s_1$</td>
<td>$IT_f^2(t) + I_f^2(t)$</td>
<td>$IT_f^2(t) + I_f^2(t)$</td>
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<tr>
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<td>$IT_f^2(t) + I_f^2(t) + IT_1(t) + I_1(t)$</td>
<td>0</td>
</tr>
<tr>
<td>$r_2$</td>
<td>$IT_f^2(t)$</td>
<td>$I_f^2(t) + IT_1(t) + I_1(t)$</td>
</tr>
<tr>
<td>$r_1$</td>
<td>$IT_f^2(t) + I_f^2(t) + IT_1(t)$</td>
<td>$I_1(t)$</td>
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### 1.6.3 Supply Chain Coordination: 2 Echelon Base-Stock Model

For a two-echelon base stock model, a payment scheme is defined by three payment times $\tau = (\tau_2, \tau_1, \tau_0)$. The supplier at stage 2 orders from an outside manufacturer with instantaneous supply and leadtime $L_2$. In order to focus specifically on the relationship between the two firms 1 and 2, we fix the payment times $\tau_2 = s_2 = o_2$, $\tau_0 = s_0$, and investigate the effect of the choice of $\tau_1$ on the system performance.

From Proposition 5 we have that the total supply chain financed inventory is independent of the choice of $\tau_1$. That is, we know that the total supply chain financed inventory is as follows.

$$ \sum_{j=1}^{2} I_f^j(t)(s_2, \tau_1, s_0) = IT_f^2(t) + I_f^2(t) + IT_1(t) + I_1(t) $$

Table 3 shows the division of financed inventory between stages 1 and 2 according to $\tau_1$. Table 4 shows the best response critical fractiles for each stage given $\tau_1$. Notice that each stage’s best response critical fractile will vary depending on the choice of $\tau_1$. Subsequently, both supply chain performance and division of costs will depend on the choice of $\tau_1$.

### 1.6.4 Evaluating Payment Flows in an Echelon Base Stock Model

In this section we investigate the payment flows that should exist in an echelon supply chain that operates on a base stock policy. Such metrics should be useful for providing a theoretical benchmark. For instance, one might compare the observed volatility in the working capital
Table 4: Critical fractiles for stages 1 and 2 for various values of $\tau_1$ given $\tau_2 = s_2$ and $\tau_0 = s_0$.

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<th>$\tau$</th>
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<th>c.f. stage 1</th>
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<td>$b_{21}' + b_{14}'$</td>
</tr>
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<td>$S_0$</td>
<td>$b_{21}' + b_{22}'$</td>
<td>$b_{21}' + b_{14}'$</td>
</tr>
<tr>
<td>$R_2$</td>
<td>$b_{21}' + b_{22}'$</td>
<td>$b_{21}' + b_{14}'$</td>
</tr>
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<td>$R_1$</td>
<td>$b_{21}' + b_{22}'$</td>
<td>$b_{21}' + b_{14}'$</td>
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with how much volatility there should be based only on the demand volatility and the payment scheme.

First, to be consistent with Assumption 1, we suppose that each stage starts with inventory $y_j$ and an empty supply stream so that each customer demand at stage 0 immediately triggers a demand at each subsequent upstream stage, all the way to the external source. However, recall that from Assumption 1 Part 2, we only begin counting time epochs following the first unit associated with the $n$th stage’s first order. Therefore, we have

**Lemma 6.** $O_j(t) = \left[D(t) - \sum_{i=j+1}^{n} y_i\right]^+$

The following proposition then characterizes all of the payment processes in terms of physical measures.

**Proposition 10.** In an echelon base stock model, the payment processes are

$$ P_j(t|\tau_j) = \begin{cases} 
D(t - \delta_j) - \sum_{i=j+1}^{n} y_i & \text{if } \rho_j = o_j \\
D(t - \delta_j) - \sum_{i=j+1}^{n} y_i - B_{j+1}(t - \delta_j) & \text{if } \rho_j = s_j \\
D(t - \delta_j) - \sum_{i=j}^{n} y_i + IN_j(t - \delta_j) & \text{if } \rho_j = r_j 
\end{cases} $$
Moreover, it is not difficult to calculate $IN^p_j(t)$ and $B^p_j(t)$ under an echelon base stock policy (see Zipkin 2000)

$$IN^p_j(t + \sum_{i=j}^{n} L_i) = y_j - B^p_{j+1}(t + \sum_{i=j+1}^{n} L_i) - D(t + \sum_{i=j+1}^{n} L_i, t + \sum_{i=j}^{n} L_i)$$

$$B^p_n(t) = 0$$

$$B^p_j(t + \sum_{i=j}^{n} L_i) = \left[ B^p_{j+1}(t + \sum_{i=j+1}^{n} L_i) + D(t + \sum_{i=j+1}^{n} L_i, t + \sum_{i=j}^{n} L_i) - y_j \right]^+$$

and in equilibrium, we have

$$IN^p_j = y_j - B^p_{j+1} - D_j$$

$$B^p_n = 0$$

$$B^p_j = \left[ B^p_{j+1} + D_j - y_j \right]^+$$

$$I^p_j = IN^p_j + B^p_j = y_j - B^p_{j+1} - D_j + B^p_j$$

Based on Proposition 10, we can evaluate the theoretical behavior of a firm’s incoming or outgoing payments. For instance, we can evaluate the variance of the incoming payments over time $\xi \in \mathbb{R}^+$ for firm $j$ by $Var \{ P_j((t, t + \xi)|\tau_j) \}$.

Besides the payment processes, it is also of interest to calculate the behavior of financed inventory because it is a representation of the net operating working capital for each firm $j$.

**Corollary 6.** In an echelon base stock model, we have

$$I^f_j(o_j + \delta_j, o_{j-1} + \delta_j) = y_j$$

$$I^f_j(s_j + \delta_j, s_{j-1} + \delta_j) = y_j + B_j(t - \delta_j) - B_{j+1}(t - \delta_j)$$

$$I^f_j(r_j + \delta_j, r_{j-1} + \delta_j) = y_j + B_j(t - \delta_j) - B_{j+1}(t - \delta_j) - IT^p_j(t - \delta_j) + IT^p_{j-1}(t - \delta_j)$$

Thus, we note that payments placed upon order times eliminate the variance in the financed inventory. That is, $Var \left( I^f_j((t, t + \xi)|(o_j, o_{j-1})) \right) = 0$. This is because such a
payment scheme corresponds exactly to the base stock policy. The payment policy can essentially be implemented as a one-for-one policy: each payment collection is immediately used to finance the next purchase.

1.7 Conclusion

This paper demonstrates how the payment scheme should affect the way financing costs are incorporated into inventory models. We provide a model of payment schemes and introduce new measures we call the financed inventory and margin backorders. The model allows us to relax some of the strict assumptions of classic inventory models. Our results provide managers with simple implementable solutions and also provide qualitative insights into how the payment scheme affects the optimal inventory policy. Lastly, we believe that this new approach to modeling financial flows in inventory problems will be useful in more complex problems at the interface of operations and finance.
2 The Effect of Payment Schemes on Inventory Decisions: The Role of Mental Accounting

2.1 Introduction

In the newsvendor problem, a decision maker chooses an inventory order quantity to meet a random future demand, with the goal to maximize the expected profit from selling the product. This model framework is commonly used for managing products with a short selling season and limited replenishment opportunities, such as fashion apparels and high-tech products. Due to lack of historical data and unexpected fluctuations in cost parameters, many newsvendor-type decisions in practice are made by humans based on subjective judgments (e.g., the fashion buying problem studied by Fisher and Raman 1996). Furthermore, even if the demand distribution and cost parameters are fully specified, human subjects are often observed deviating significantly from the optimal solution in experiments (Schweitzer and Cachon 2000). In this paper, we study how seemingly innocuous differences in the payment scheme can lead to significantly different inventory decisions in newsvendor experiments.

We define a payment scheme in the newsvendor problem as the amounts and times of payment transactions associated with the order quantity, the realized sales, and the leftover units. There are many payment schemes used in practice, designed for reasons including risk-sharing, cash constraints, and price discrimination. Clearly, a change in payment scheme may affect the order decision for perfectly rational reasons (if the net profit is altered). But will such change also induce certain behavioral effect? If so, to what extent? Finding answers to these questions can help inform the design of payment schemes in supply chain financing and contracting.

The literature on mental accounting (see Thaler 1999 for a review), most notably applied to consumer choice behavior, describes how individuals perceive multiple financial transactions by mentally aggregating or segregating them based on factors such as time or an uncertain event before making evaluations. In the newsvendor setting, the random demand event sets a natural boundary for payment transactions, both in terms of time and an uncer-
tain event. Thus, we posit that individuals mentally segregate payment transactions in the newsvendor problem into two time buckets before and after the demand realization, which we call “order-time payments” and “demand-time payments,” respectively. Because of this segregation, we reason that altering transactions before and after the demand realization in a payment scheme (while keeping the net profit constant) will lead to significantly different order decisions.

To examine this behavioral effect, we consider three payment schemes in our paper. The first payment scheme is similar to a standard wholesale price contract: the newsvendor pays for the order quantity at the order time and receives revenue after the demand realization. We call this payment scheme O (“Own-financing”) as the order is financed by the newsvendor’s own operating capital. In the second payment scheme, the newsvendor’s order payment is delayed until after the demand realization. As a result, there is no order-time payments; all transactions occur after the demand realization. We call this payment scheme S (“Supplier-financing”) because suppliers often offer this kind of cost-based loans to their customers (such as trade credits). However, the newsvendor can also obtain a similar cost-based loan from a third party (such as a bank). In the third payment scheme, newsvendor receives advanced revenue for the order quantity at the order time, but must refund the advanced payment for the leftover units after the demand realization. Thus, the order-time payments constitute a net profit for the inventory quantity ordered. We call this payment scheme C (“Customer-financing”) because large customers sometimes provide this kind of revenue-based loans to their suppliers (e.g., O’Sullivan 2007). Nevertheless, the newsvendor can also obtain a similar revenue-based loan from a third party (such as a bank) using the inventory as collateral. We eliminate differences in the interest rates and risk-premiums associated with these three payment schemes by setting them to zero. This allows us to keep the net profit structure constant and thus isolate the behavioral effect.

We present four descriptive models to predict the newsvendor ordering behavior. The first model assumes that individuals correctly aggregate the order-time and demand-time payments and choose the order quantity that maximizes the expected profit. The resulting
optimal order quantities are identical across all three payment schemes (this actually holds true for any utility models as long as it is based on the net profit). The second model assumes that individuals are loss averse with respect to the order-time and demand-time payments separately. The third model assumes that individuals discount the demand-time payments due to time-discounting preferences. Finally, the fourth model assumes that individuals underweight the order-time payments due to a mental accounting effect called “prospective accounting,” in which individuals fully account for transactions looking forward in time, but largely discount transactions looking backwards in time (Prelec and Loewenstein, 1998).

We conduct three experimental studies with human decision-makers to empirically test the predictions of the above models. In Study 1, we set the underage cost equal to the overdue cost. Thus, the expected profit-maximizing solution is to order the median demand under all three payment schemes, which allows us to neutralize the pull-to-center effect (i.e., the deviation from the optimal solution towards the center of the distribution; see Schweitzer and Cachon 2000). Our results show that order quantities exhibit a consistent decreasing pattern in the order of payment schemes O, S, and C, with the order quantities of scheme S being close to the expected profit-maximizing solution. This ordering behavior is inconsistent with the loss aversion model. It is also in the opposite direction with the prediction of the time discounting model (including hyperbolic discounting, Laibson 1997). Rather, the observed ordering behavior is consistent with the prospective accounting model that underweights the order-time payments. In Study 2, we further isolate the mental accounting effect from the physical timing of payments by having all physical payment transactions conducted after the demand realization. We find that the same ordering pattern in Study 1 is sustained. This result shows that the framing of the payment scheme is sufficient to induce the prospective accounting behavior. Finally, in Study 3, we demonstrate that the prospective accounting behavior is robust under high- and low-profit conditions, though the magnitude of the order differences across payment schemes is influenced by the profit condition.

Perhaps the most surprising finding from our study is that the behavioral effect of pay-
ment scheme works in the opposite direction against time discounting. When the interest rate is significant and/or the capital constraint is binding, the behavioral effect may be dominated by the time-discounting effect due to the tangible financial costs. However, when capital constraint is not an issue and the interest rate is negligible (as in our experiments), loans based on purchase cost (scheme S) or projected revenue (scheme C) might inadvertently lower the retailer’s order quantity relative to the case without the loan (scheme O). Therefore, to avoid any unintended consequences in practice, one should carefully evaluate the relative magnitude of these opposing effects when designing a payment scheme for supply chain financing and contracting.

Furthermore, our findings also provide a plausible explanation for the asymmetry of pull-to-center effect observed by Schweitzer and Cachon (2000) and Bolton and Katok (2008). These authors find that the pull-to-center effect is larger in the low-profit condition than in the high-profit condition. The framing of the payment scheme in their studies is similar to our scheme O. As a result, the asymmetry can be explained by the prospective accounting effect (see a discussion in Study 3). We further show that such asymmetry disappears under scheme S and is reversed under scheme C. Thus, the direction of the pull-to-center asymmetry is dependent on the framing of the payment scheme.

The rest of the paper is organized as follows. We provide a literature review in Section 2. We present the three stylized payment schemes and four decision models in Section 3. We present our experimental results in Section 4. We conclude with a discussion of the results and their managerial implications in Section 5.

### 2.2 Literature Review

There is a growing literature on behavioral operations management (see Bendoly et al. 2006 for a review). In this literature, researchers study how humans make operational decisions and how these decisions may differ from the rational decision. For example, Schweitzer and Cachon (2000) first find the pull-to-center effect in the newsvendor problem. Various influencing factors in newsvendor decisions are also investigated, such as decision heuristics...
(Bostian et al. 2008), the role of learning and feedback (Bolton and Katok 2008; Lurie and Swaminathan 2009), demand estimation biases (Feiler et al. 2011), psychological costs (Ho et al. 2010), and bounded rationality (Su 2008; Kremer et al. 2010). In a serial supply chain setting, Sterman (1989) and Croson and Donohue (2005, 2006) find that human subjects do not sufficiently account for the pipeline inventory and subsequently overreact to their inventory levels, contributing to the bullwhip effect. Loch and Wu (2008) examine the social preferences in supply chain contracts. Ho and Zhang (2008) and Katok and Wu (2009) further investigate the effectiveness of risk-sharing contracts, which are closely related to our paper as the behavioral effect of the payment scheme may also play a role in determining the effectiveness of risk-sharing contracts.

Mental accounting has long been used to help understand the psychology behind choice behavior (Kahneman and Tversky 1979; Tversky and Kahneman 1981; Thaler 1980). It provides an explanation for many phenomena in human behavior that seem irrational—most notably in consumer choice behavior (e.g., Thaler 1985; Heath and Soll 1996), and also in other functional areas, such as finance (Shefrin and Statman 1985) and accounting (Burgstahler and Dichev 1997). Our experimental findings provide an example of mental accounting in operations management. Consumers evaluate the transaction of a payment in return for a good. A consumer’s payment is mentally coupled with the consumption because the two are linked by the consumer good. Similarly, the newsvendor’s order-time payments are mentally coupled with the demand-time payments because the two are linked by the inventory ordered. Shafir and Thaler (2006) find that the typical wine connoisseur thinks of her initial purchase of a case of wine as an investment, later thinks of the wine as free when she drinks it, and so goes through the entire process never experiencing the pain of payment. Similarly, Prelec and Loewenstein (1998) find that people prefer to prepay for a vacation because they think that a prepaid vacation is more pleasurable than one that must be paid for after returning. This is because the payment is less painful if there is a future vacation to anticipate, while the vacation is more enjoyable if the payment has already been made. Gourville and Soman (1997) call the gradual reduction in relevance of
past payments “payment depreciation.” More generally, Prelec and Loewenstein (1998) call the mental accounting rule that fully recognizes future payments but largely writes off past payments “prospective accounting.” We contribute to the mental accounting literature by applying these concepts to the newsvendor problem.

Our paper is also related to the literature on the interface of operations and finance. The payment schemes we consider are actually stylized versions of real practices. For example, retailers often delay their payments to their supplier taking advantage of trade credits offered by the supplier (Peterson and Rajan 1997; Ng et al. 1999). The trade credit terms can certainly affect a firm’s optimal ordering policy (Haley and Higgins 1973; Gupta and Wang 2009; Song and Tong 2012). Similarly, small suppliers sometimes seek revenue-based loans by using their inventory as collateral. This has been gaining popularity in practice due to an increase in buyer-based supply chain financing solutions (see O’Sullivan 2007). Other financial considerations can also affect inventory decisions, such as asset-based financing (Buzacott and Zhang 2004) and capital constraints (Xu and Birge 2004; Babich and Sobel 2004; Xu and Zhang 2010). Our paper contributes to this literature by demonstrating that the choice of a financial payment scheme also has a behavioral effect on inventory decisions.

2.3 Models of Newsvendor Decision Making

In the newsvendor problem, a decision-maker chooses an order quantity \( q \) of a product to meet a future random demand \( D \). Let \( F(\cdot) \) denote the cumulative distribution function for the random demand. We assume that backlogs are not allowed (i.e., unmet customer demand is lost) and leftover inventory cannot be carried over to the subsequent period and has zero salvage value. The unit cost of the product is \( c \) and the selling price is \( p \) (with \( p > c \)).

Let us consider the following three payment schemes. 1) Payment scheme O (Own financing): the newsvendor pays the cost \( c \) per unit at order time and receives a revenue \( p \) per unit sold after the demand realization. 2) Payment scheme S (Supplier financing): the newsvendor pays nothing at order time; after the demand realization, she receives \( p - c \) per
unit sold and pays the external financing party (such as the supplier) $c$ per unit leftover. 3) Payment scheme C (Customer financing): the newsvendor receives $p - c$ per unit ordered at order time, but must refund $p$ per unit leftover back to the external financing party (such as the customer) after the demand realization. The payment schemes are summarized in Table 5.

<table>
<thead>
<tr>
<th>Payment Scheme</th>
<th>Payments at time of order</th>
<th>Payments after demand realization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Own Financing (O)</td>
<td>$-c$</td>
<td>$+p$</td>
</tr>
<tr>
<td>Supplier Financing (S)</td>
<td>0</td>
<td>$(p - c)$</td>
</tr>
<tr>
<td>Customer Financing (C)</td>
<td>$(p - c)$</td>
<td>0</td>
</tr>
</tbody>
</table>

In the newsvendor setting, the random demand event sets a natural separation point for payment transactions before and after it because it involves the resolution of an uncertain event. Thus, while there are three possible payment transactions (per unit ordered, per unit sold, and per unit leftover), we posit based on the mental accounting theory that individuals mentally segregate these transactions into two time buckets before and after the random demand event as shown in Table 5.

We comment that the above payment schemes also have practical interpretations. Payment scheme O corresponds to the standard wholesale price contract. Payment scheme S is similar in terms of the timing of the payments to trade-credit arrangements observed in practice (Peterson and Rajan 1997). However, it is a stylized scheme as it does not reflect the lower interest rate benefits that such arrangements typically offer relative to bank loans. Payment scheme C is similar to receiving revenue in advance from the customer (O’Sullivan 2007) or financing inventory from an external party using inventory as a collateral. This scheme is also stylized as the amount financed (or the revenue advanced) in practice may only be part of the total selling value of the inventory investment.

Next, we describe four models that differ in how the decision maker takes the payment scheme into account. We use the first normative model, which predicts the same order decision across payment schemes, as a benchmark. The other three models, inspired by
behavioral decision-making and consumer behavior literature, predict different ordering behaviors across payment schemes. In what follows, we use the term “reward function” to denote how the individual evaluates payment transactions under a given decision model.

### 2.3.1 Expected Profit-Maximizing Model

Let \( R^i(q, D) \) denote the reward function given the quantity \( q \) and demand realization \( D \) under the payment scheme \( i \in \{O, S, C\} \). If the decision-maker correctly aggregates the order-time and demand-time payments and chooses the optimal quantity to maximize the expected profit, then \( R^i(q, D) \) is simply the net profit given by

\[
R^i(q, D) = \begin{cases} 
-cq + p \min(q, D) & \text{if } i = O, \\
(p - c) \min(q, D) - c \max(q - D, 0) & \text{if } i = S, \\
(p - c)q - p \max(q - D, 0) & \text{if } i = C.
\end{cases}
\]

The optimal order quantity that maximizes the expected profit is \( q^i = \arg \max_q \mathbb{E}_D[R^i(q, D)] \).

**Proposition 11.** The expected profit-maximizing quantities under the three payment schemes are \( q^O = q^S = q^C = q^* \), where \( q^* = F^{-1}((p - c)/p) \).

It is easy to verify Proposition 1 by noting that \( R^O(q, D) = R^S(q, D) = R^C(q, D) \) for any \( q \) and \( D \). The term \((p - c)/p\) is known as the critical fractile. We note that \( q^O = q^S = q^C \) actually holds for any reward function as long as it is based on the net profit as shown above.

### 2.3.2 Loss Aversion Model

Prospect Theory, introduced by Kahneman and Tversky (1979), assumes that individuals are loss averse with respect to a reference wealth, such as the current wealth. In the newsvendor problem, it is reasonable to assume that individuals update their reference wealth after the order-time payments. Thus, individuals may be loss averse with respect to both the order-time and demand-time payments. Let \( \pi_i(q) \) denote the net payment at order time and \( \pi_i^2(q) \) denote the net payment after the demand realization under payment scheme \( i \in \{O, S, C\} \).
As in Schweitzer and Cachon (2000), we capture loss aversion using the utility function
\[ U^l(x) = \begin{cases} x & \text{if } x \geq 0; \\ \lambda x & \text{if } x < 0 \end{cases} \]
for \( \lambda > 1 \). Then the newsvendor’s total reward is:
\[ R_i(q) = U^l(\pi^1_i(q)) + U^l(\pi^2_i(q, D)) = \begin{cases} -\lambda q + p \min(q, D) & \text{if } i = O, \\ 0 + U^l((p - c) \min(q, D) - c \max(q - D, 0)) & \text{if } i = S, \\ (p - c)q - \lambda p \max(q - D, 0) & \text{if } i = C. \end{cases} \]

The following proposition compares the optimal order quantities \( q^i = \arg \max_q \mathbb{E}_D[R^i(q, D)] \) to the expected profit-maximizing solution \( q^* \).

**Proposition 12.** If the decision maker is loss averse and updates her reference wealth after order-time payments, then the optimal quantities under the three payment schemes are all less than the expected profit-maximizing solution, i.e., \( q^O < q^*, q^S < q^*, \) and \( q^C < q^* \).

**Proof:** The critical fractiles for payment schemes O and C are \((p - \lambda c)/p\) and \((p - c)/\lambda p\), respectively, which are both less than the expected-profit-maximizing critical fractile. Schweitzer and Cachon (2000) prove the result for payment scheme S, as they evaluate the loss aversion model assuming all payments are made at the same time. They show that the optimal solution is also less than the expected profit-maximizing solution. \( \square \)

### 2.3.3 Time-Discounting Model

The decision-maker may prefer to receive benefits earlier and delay costs until later, which is also known as the time-discounted utility model (see Frederick et al. 2002 for a discussion on time discounting). The discounting may be due to the real interest rate or due to behavioral preferences as in the present case. Under this model, the decision maker discounts the demand-time payments because they occur later. Let \( \delta \) \((0 < \delta < 1)\) denote this discount factor. The reward function \( R^i(q, D) \) can then be expressed as follows.

\[ R^i(q, D) = \begin{cases} -cq + \delta p \min(q, D) & \text{if } i = O, \\ \delta(p - c) \min(q, D) - \delta c \max(q - D, 0) & \text{if } i = S, \\ (p - c)q - \delta p \max(q - D, 0) & \text{if } i = C. \end{cases} \]
The following proposition compares the optimal order quantities \( q^i = \arg \max_q \mathbb{E}_D[R^i(q,D)] \) to the expected profit-maximizing solution \( q^* \).

**Proposition 13.** Under the time-discounting model, the optimal quantities under the three payment schemes have the following relationships: \( q^O < q^S = q^* < q^C \).

**Proof:** The critical fractiles for payment schemes O, S, and C are \((\delta p - c)/\delta p, (p - c)/p\), and \((p - c)/\delta p\), respectively. Because \((\delta p - c)/\delta p < (p - c)/p < (p - c)/\delta p\), we have \( q^O < q^S = q^* < q^C \). □

Note that because there are only two time points in the model set-up, there is no difference between standard time discounting and hyperbolic time discounting (Laibson 1997).

### 2.3.4 Prospective Accounting Model

Thaler (1985) suggests that for a consumer, the payment and consumption in a transaction are not seen as a separate loss and a gain, respectively. Rather, the payment is mentally linked or “coupled” with the thought of the associated consumption, and the consumption is “coupled” with the thought of the associated payment (Prelec and Loewenstein 1998).

However, the strength of these two couplings are not equal, and are strongly dependent on the sequence of events. Specifically, individuals use a mental accounting rule called “prospective accounting,” in which coupling is stronger when looking forward in time, but weaker when looking backward in time. The resulting phenomenon is consistent with underweighting whichever occurs first: the payment or the consumption. For example, consider how the prospective accounting rule applies to the case when payment precedes consumption, such as in a prepaid vacation. From the vantage point of the payment, the pain of payment is buffered because it is strongly coupled with the anticipated pleasure of the future vacation. From the vantage point of the vacation, the pleasure of the vacation is decoupled from the pain of the payment because it occurred in the past. Thus, in this case, the result of the prospective accounting is an overall underweighting of the pain of payment.

Instead of payment and consumption, the news vendor simply has outgoing payments and incoming payments, which occur either before or after the demand realization. Similar
to the consumer, we propose that, for the newsvendor, the order-time payments are coupled with the demand-time payments because they are connected by the number of units ordered. Thus, assuming outgoing payments are analogous to consumer payments (both are negative utilities) and incoming payments are analogous to consumption (both are positive utilities), we can implement the predictions of prospective accounting for our three payment schemes. Under payment scheme O, we assign an underweighting factor $\beta$ ($0 < \beta < 1$) to the order-time payments (which is the order cost). Under payment scheme C, we assign the underweighting factor $\alpha$ ($0 < \alpha < 1$) to the order-time payments (which is the net profit from the quantity ordered). Under payment scheme S, no transactions occur at the order time, so there is no underweighting. The reward function $R^i(q, D)$ under the prospective accounting model is given below.

$$R^i(q, D) = \begin{cases} 
-\beta cq + p \min(q, D) & \text{if } i = O, \\
(p - c) \min(q, D) - c \max(q - D, 0) & \text{if } i = S, \\
\alpha(p - c)q - p \max(q - D, 0) & \text{if } i = C.
\end{cases}$$

The following proposition compares the optimal order quantities $q^i = \arg \max_q \mathbb{E}_D[R^i(q, D)]$ to the expected profit-maximizing solution $q^*$.

**Proposition 14.** With prospective accounting, the optimal quantities under the three payment schemes have the following relationships: $q^O > q^S = q^* > q^C$.

**Proof:** The critical fractiles for payment schemes O, S, and C are $\frac{(p - \beta c)}{p}$, $\frac{(p - c)}{p}$, and $\frac{\alpha(p - c)}{p}$, respectively. Because $\frac{(p - \beta c)}{p} > \frac{(p - c)}{p} > \frac{\alpha(p - c)}{p}$, we have $q^O > q^S = q^* > q^C$. □

We refer the reader to Appendix A for a comparison of the above model with the consumer utility model of Prelec and Loewenstein (1998).
2.3.5 Summary

In this section, we have derived the predictions of four behavioral models that predict various ordering patterns under payment schemes O, S, and C. We provide a summary of the model predictions in Table 6. We note that the effects in each model need not be mutually exclusive. For example, in reality, time-discounting and prospective accounting may exist simultaneously. Because they bias orders in opposite directions, the resulting order would depend on which effect dominates (see Section 5 for further discussion).

Table 6: Summary of model predictions for orders under payment schemes O, S, and C.

<table>
<thead>
<tr>
<th>Newsvendor Model</th>
<th>Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected-Profit Maximization</td>
<td>( q^O = q^S = q^C = q^* )</td>
</tr>
<tr>
<td>Loss Aversion at the Times of Order and Demand</td>
<td>( q^O &lt; q^<em>, q^S &lt; q^</em>, q^C &lt; q^* )</td>
</tr>
<tr>
<td>Time Discounted Rewards</td>
<td>( q^O &lt; q^S = q^* &lt; q^C )</td>
</tr>
<tr>
<td>Prospective Accounting</td>
<td>( q^O &gt; q^S = q^* &gt; q^C )</td>
</tr>
</tbody>
</table>

2.4 Newsvendor Experiments

In this section, we present three newsvendor experiments to examine the behavioral effect of payment scheme on inventory decisions. To isolate the behavioral effect of the payment scheme, we eliminate factors such as capital constraints and interest rates in our experimental designs (see Section 5 for a discussion of the impact of these factors). In the first study, we test whether ordering behavior can be described by the models presented in the previous section. In the second study, we test whether the framing of the problem is sufficient to induce differences in order decisions even if all actual payments are made at the same time. In the third study, we test the robustness of the model predictions under high- and low-profit conditions.
2.4.1 Study 1: A Simple Payment Scheme Experiment

Experimental Design In Study 1, we test the three payment schemes O, S, and C (see Table 5) under parameters $c = $1, $p = $2 in a repeated newsvendor setting.

In each round, subjects roll three fair six-sided dice, the sum of which determines the demand for that round. Thus, demand is independent, identically distributed, and symmetric with a minimum value of 3, maximum value of 18, and mean value of 10.5. We choose to generate random numbers using three dice instead of a computer in order to facilitate participant understanding of the payment schemes through active demand generation and counting. The distribution of the sum of three dice is also well approximated by a normal distribution.

Recall that all payment schemes are equivalent in the sense that they produce identical total net profits or losses for any given ordering decision and demand realization. Furthermore, the actual average cost and underage costs are equal at $1 each and the expected-profit-maximizing solution under all payment schemes is to order either 10 or 11 units every period. The newsvendor pull-to-center effect suggests that participants are biased towards the mean of the demand distribution, or 10.5. Below we describe our experimental methods, present our results, and provide a discussion of the results for Study 1.

Methods We recruited 99 undergraduate and graduate students from Duke University. Bolton et al. (2010) find that qualitatively students and managers perform similarly in the newsvendor problem. Croson and Donohue (2006) also find managers’ and students’ inventory decisions are similar in a serial supply chain setting. Thus, we believe it is justifiable to use students as proxies for studying managerial behavior. The experimental conditions were assigned sequentially to the participants.\footnote{We conducted this study in two parts. In the first part, we randomly assigned one payment scheme O or S to each subject (57 subjects). In the second part, we randomly assigned one payment scheme S or C to each subject (42 subjects). Each subject completed the experiment under only one payment scheme. Although ideally we would have randomly assigned participants across all 3 payment schemes, we found no significant differences between the two repetitions of condition S, and therefore aggregated the data for analysis. This yielded 29 subjects for condition O, 49 for condition S, and 21 for condition C.} In exchange for their participation, participants received a minimum of $5 plus a $1 bonus for every 50 play dollars they had at the end.
of the game (each participant began with 100 play dollars). Participants earned anywhere from 7-13 dollars and took approximately 15 minutes to complete the experiment.

Participants were given an instruction sheet explaining the details of the game for the payment scheme to which they were assigned (see Appendix B). Instructions were also read out loud by a research assistant before beginning play. Participants were told that they would be selling “widgets” (represented by poker chips) and that customer demand for the widgets in a given round was represented by the sum of the rolling of three standard dice. Each participant interacted one-on-one with a research assistant, who facilitated payment transfers and recorded ordering decisions and dice rolls. A participant decided an order quantity vocally, placed that many poker chips into the “store” (represented by a square drawn on an index card), and made appropriate payment transfers. Then, the participant rolled the three dice, determined how many units were sold and/or leftover, and again made appropriate payment transfers. Finally, the participant removed all chips from the store to begin the next round.

Payment transfers were conducted in the form of play paper currency in denominations of 1, 5, and 10. All payments to the participant were conducted by the research assistant, while all payments from the participant were conducted by the participant. Appropriate payment transactions occurred immediately following the ordering decision and immediately following demand realization. The participant also moved the poker chips and rolled the dice themselves, which facilitated their understanding of the process. Game play was for 25 rounds, after which a follow-up question was administered: “If you could play the game again choosing only one order quantity, what number would you choose?” Finally, two written comprehension questions were administered at this time: “What is the minimum demand possible you can roll with three dice?” and “What is the maximum demand possible you can roll with three dice?”

Results All 99 participants completed the study. One participant in the S condition incorrectly answered both comprehension questions and also made multiple orders of more
than 18, and was therefore removed from the analysis (though all results hold when included). The resulting average ordering decisions in each round are shown in Figure 2, and a summary of our main results can be found in Table 7.

Table 7: Mean ordering quantities and differences in Study 1

<table>
<thead>
<tr>
<th>Payment Scheme</th>
<th>Average over 25 rounds</th>
<th>Round 1</th>
<th>Follow-up question</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Own Financing (O)</td>
<td>11.728 (1.392)</td>
<td>11.069 (3.390)</td>
<td>11.759 (1.766)</td>
<td>29</td>
</tr>
<tr>
<td>Supplier Financing (S)</td>
<td>10.573 (1.031)</td>
<td>10.271 (2.210)</td>
<td>10.448 (1.234)</td>
<td>48</td>
</tr>
<tr>
<td>Customer Financing (C)</td>
<td>9.749 (1.058)</td>
<td>9.238 (2.406)</td>
<td>9.571 (1.207)</td>
<td>21</td>
</tr>
</tbody>
</table>

Contrast tests

<table>
<thead>
<tr>
<th>Contrast tests</th>
<th>q^O − q^S</th>
<th>q^S − q^C</th>
<th>q^O − q^C</th>
</tr>
</thead>
<tbody>
<tr>
<td>q^O − q^S</td>
<td>1.155***</td>
<td>0.798</td>
<td>1.311***</td>
</tr>
<tr>
<td>q^S − q^C</td>
<td>0.824**</td>
<td>1.033</td>
<td>0.877*</td>
</tr>
<tr>
<td>q^O − q^C</td>
<td>1.979***</td>
<td>1.831*</td>
<td>2.188***</td>
</tr>
</tbody>
</table>

We conducted a repeated measures generalized linear model to analyze the 25 inventory order decisions under each payment scheme. We found that payment scheme significantly affected ordering behavior ($F(2, 95) = 18.88, p < 0.0001$). Specifically, we found that orders were highest under payment scheme O and lowest under payment scheme C. In order to test these differences, we conducted planned contrast tests. These tests showed that all three differences were significant: orders under O were significantly greater than orders
under S \((F(1, 95) = 18.10, p < 0.0001)\), orders under S were significantly greater than orders under C \((F(1, 95) = 7.46, p = 0.0075)\), and orders under O were significantly greater than orders under C \((F(1, 95) = 35.83, p < 0.0001)\). As Table 7 shows, these same trends are present in the first ordering decision (which is not confounded by experience or feedback), the average order quantity, and the follow-up question. However, not all differences were significant. Specifically, the differences appear to be more significant for the average orders and the follow-up question than for the first ordering decision (see a discussion on this in the summary below).

We also compared the average orders with the mean of the demand distribution, 10.5, because both the expected profit-maximizing criterion and the pull-to-center effect predicted orders near mean demand. We found that average orders under O were significantly greater than mean demand \((t(28) = 4.751, p < 0.001)\), average orders under C were significantly less than mean demand \((t(20) = -3.256, p = 0.004)\), while average orders under S were not significantly different from mean demand \((t(47) = 0.493, p = 0.624)\).

The actual demands generated by rolling the three dice were relatively consistent with the theoretical predictions. The means were 10.739, 10.557, and 10.764, under O, S, and C, respectively. Also, all participants (except the one eliminated participant in condition S) correctly answered 3 and 18 for the minimum and maximum possible demand that could be generated by rolling the three dice.

There was no significant difference in the overall ordering levels over time (Wilks’ Lambda = 0.701, \(F(24, 72) = 1.28, p = 0.212\)). In other words, there was no main effect for round. We also found no significant interaction between payment scheme and experience gained as more rounds were played (Wilks’ Lambda = 0.614, \(F(48, 144) = 0.83, p = 0.774\)).

**Summary** Study 1 establishes that payment schemes have a significant effect on ordering behavior in the newsvendor problem. We found that order decisions can be higher or lower than the expected profit-maximizing decision depending on the payment scheme, which is inconsistent with the expected profit-maximizing model. Specifically, we found that orders exhibit a consistent decreasing pattern in the order of schemes O, S, and C, with the order
quantities of scheme S being close to the expected profit-maximizing solution. These results are inconsistent with the loss aversion model and the time-discounting model. Rather, they are consistent with the prospective accounting model. Moreover, the differences appear to not only be robust over 25 rounds, but actually more significant over time. This suggests that the feedback individuals use to inform their future ordering quantity is also subject to the prospective accounting effect, making the order deviation robust over 25 rounds of experience (see Section 5 for a brief discussion of recency and anchoring and adjustment).

**Structural Parameter Estimates** In order to provide further validation of the prospective accounting model, and to obtain estimates for its parameters, we estimated $\beta$ and $\alpha$ using three structural estimation techniques. Note that the prospective accounting model reduces to the expected profit-maximizing model when $\beta = \alpha = 1$. Therefore, we can validate the model fit of the prospective accounting model against the expected profit-maximizing model by testing if $\beta$ and $\alpha$ are less than 1.

Our first two structural estimation approaches follow the N1 and N2 models of Olivares et al. (2008). The third approach we provide is a hybrid of N1 and N2, which we call NH. The differences between each approach lie in how they account for the variability in the observed order quantities. The N1 model attributes all of the order variability to a noisy underweighting factor, and uses an ordinary least squares regression to estimate the underlying underweighting factor. On the other hand, the N2 model attributes all of the order variability to errors in the order quantity (i.e., a “trembling hand”) and uses a nonlinear least squares regression to estimate the underweighting factor. Finally, we also provide a hybrid approach that we believe is reasonable in our problem setting. The NH model attributes some of the variability to a noisy underweighting factor and some of the variability to a “trembling hand.” It assumes that each participant has a unique underweighting factor (so that there is heterogeneity in underweighting factors across participants), but that the differences between order quantities across rounds for the same participant is due to a “trembling hand.” We approximate the demand distribution with a normal distribution with mean 10.5 and standard deviation 2.958, matching the mean and standard deviation.
Table 8: Mean ordering quantities and differences in Study 2.

<table>
<thead>
<tr>
<th>Payment Frame</th>
<th>Average over 25 rounds</th>
<th>Round 1</th>
<th>Follow-up question</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Own Financing (O)</td>
<td>11.648 (1.721)</td>
<td>10.800 (2.745)</td>
<td>11.900 (1.518)</td>
<td>20</td>
</tr>
<tr>
<td>Supplier Financing (S)</td>
<td>10.671 (1.020)</td>
<td>10.889 (2.055)</td>
<td>10.611 (0.850)</td>
<td>18</td>
</tr>
<tr>
<td>Customer Financing (C)</td>
<td>9.665 (1.159)</td>
<td>9.474 (2.144)</td>
<td>9.632 (1.165)</td>
<td>19</td>
</tr>
</tbody>
</table>

Contrast tests

- $q^O - q^S$: 0.977* -0.089 1.289**
- $q^S - q^C$: 1.006* 1.415 0.979*
- $q^O - q^C$: 1.983*** 1.326 2.268***

of the discrete demand distribution of the sum of three dice. The resulting estimates for $\beta$ and $\alpha$ (denoted with $\hat{\beta}$ and $\hat{\alpha}$) are reported in Table ??.

The results from the three estimate models are relatively consistent, with the estimates from the hybrid model falling in between those of the N1 and N2 models. In other words, in this experiment we found that on average individual’s orders are consistent with taking into account only about 70% of payments that occur at the order time when costs precede revenues (payment scheme O), and only about 80% of payments that occur at the order time when revenues precede costs (payment scheme C). For example, under payment scheme O, this suggests that an individual who orders 10 units at $1 each perceives the $10 cost as if it were only about $7.

Expected Profits. We also calculated the expected profits given each ordering decision of each participant. Rather than using actual profits (which is an outcome-based measure), we use the expected profit measure because it captures the participants’ decision efficiency. Expected profits were significantly affected by payment scheme ($F(2, 95) = 5.65, p = 0.005$). Average expected per-round profits (standard deviations in parentheses) by condition were 7.461(0.696), 7.805(0.240), and 7.746(0.348) for O, S, and C respectively. It is not surprising that the expected profits were highest under payment scheme S, since the average order quantity under S was closest to the expected-profit maximizing quantity. Contrast tests show that all differences between conditions are significant differences at the $p < 0.05$ level except the difference between S and C.
One might suggest that these differences in profit are not extremely large (the expected per-round profits are 4.61% greater under S than under O). This is due to the fact that the expected profit function is relatively flat near the optimal solution. However, other operational metrics are not as flat around the optimal solution. For example, the supplier’s revenue is the wholesale price times the newsvendor’s order. Thus, the supplier’s average per-round revenue is 20.29% greater under O than under C (on average the supplier sells 11.728 units to the newsvendor versus 9.749). The differences between payment schemes also impact customer service. We calculated the customer's expected in-stock rate for each ordering quantity. This analysis shows that the average expected per-round in-stock rates are 0.675(0.122), 0.564(0.116), and 0.462(0.120) for O, S, and C, respectively.

2.4.2 Study 2: A Payment Scheme Experiment with Same Payment Timing

Experimental Design The purpose of Study 2 is to test whether we can achieve similar results to Study 1 by manipulating only the framing of the payment scheme (i.e., when and how payments are determined), while eliminating the difference in the actual timing of payments transactions. According to Prelec and Loewenstein (1998), by knowing the size of a payment before an uncertain event, a consumer can mentally impute that payment even if the actual time the payment transaction occurs later. Study 2 investigates whether a similar phenomenon exists for the newsvendor. It tests whether the framing of payment scheme in the newsvendor problem can induce individuals to mentally set aside payments at the order time, even if all payment transactions are actually conducted after the demand realization.

Study 2 implements the same design as Study 1 with the following exception: all payments are postponed to the end of each round (i.e., conducted after the demand realization), even if some payments are determined at the time of the ordering decision. We will refer to these three payment schemes as payment frames, and denote our conditions with an overbar: Ō, Š, and Ĉ. We refer readers to Appendix B for the description of each payment frame in the instructions to the participants. Since all payments are delayed until after the demand
realization, there are no real differences between payment schemes in the actual financial position over time. However, the framing of the payment scheme may induce individuals to mentally impute the order-time payments even if the transactions are not conducted until after the demand realization. Specifically, payment frame $\bar{O}$ permits individuals to mentally set aside some cost at the order time, while payment frame $\bar{C}$ encourages individuals to mentally set aside some benefit at the order time. Thus, we expect results in Study 2 to be similar to Study 1. Below we describe our experimental methods, present our results, and provide a discussion of the results for Study 2.

**Methods and Results** We recruited 57 undergraduate and graduate students from Duke University. The methods were the same as Study 1 except for the following differences. First, as described above, all payments were conducted at end of each round, after the demand realization. Secondly, in addition to the comprehension questions asked in Study 1, at the end of the experiment we also asked each participant the question: “What do you think is the long-run average demand generated by rolling three dice?”

All 57 participants completed the study and were included in the following analyses. A
summary of our findings can be found in Table 9. As in Study 1, the repeated measures

generalized linear model showed that payment frame significantly affected ordering behavior

\((F(2, 54) = 10.94, p < 0.0001)\). Orders under \(\tilde{O}\) were significantly greater than orders under

\(\tilde{S} (F(1, 54) = 5.17, p = 0.0270)\). Orders under \(\tilde{S}\) were significantly greater than orders under

\(\tilde{C} (F(1, 54) = 5.34, p = 0.0246)\), and orders under \(\tilde{O}\) were significantly greater than orders

under \(\tilde{C} (F(1, 54) = 21.89, p < 0.0001)\). Comparing average orders to the mean demand

of 10.5, we again found that average orders under \(\tilde{O}\) were significantly greater than mean
demand \((t(19) = 2.98, p = 0.007)\), average orders under \(\tilde{C}\) were significantly less than mean
demand \((t(18) = -3.14, p = 0.006)\), while average orders under \(\tilde{S}\) were not significantly
different from mean demand \((t(17) = 0.80, p = 0.436)\).

Even though formal comparisons between Studies 1 and 2 would not be appropriate
because they are run at different times, we highlight the similarities and differences between
the results in the two studies. Overall, Table 9 demonstrates ordering behavior remarkably
similar to the results in Study 1, as given in Table 7. As in Study 1, orders did not
significantly change over time (Wilks’ Lambda = 0.472, \(F(24, 31) = 1.44, p = 0.1672\)). We
also found no significant interaction between payment frame and round (Wilks’ Lambda

\(= 0.309, F(48, 62) = 0.46, p = 0.4520\)). On the other hand, in Study 2 there are no

significant differences in the order quantities in Round 1. Also, the overall significance
of contrasts seem to be slightly less in Study 2 than in Study 1 for the average ordering
quantities (two less contrasts are significant at the \(p < 0.01\) level) and for the follow-up
question (one less contrast is significant at the \(p < 0.01\) level).

All participants correctly answered 3 and 18 for the minimum and maximum possible
demand that could be generated by rolling three dice. Participant estimates of the long-
run average demand with three dice revealed some variation, but were not significantly
affected by condition \((F(2, 54) = 0.34, p = 0.714)\). Their average estimates by condition

were 10.35 (0.96), 10.25 (0.65), and 10.45 (0.47) for \(\tilde{O}, \tilde{S}, \) and \(\tilde{C}\), respectively (standard
deviations in parentheses). T-tests show that these estimates are also not significantly
different from 10.5 (For \(\tilde{O}: t(19) = -0.7, p = 0.494\); for \(\tilde{S}: t(17) = -1.64, p = 0.1197\); for
<table>
<thead>
<tr>
<th>Payment Frame</th>
<th>Mean order quantity (standard deviation in parentheses)</th>
<th>Contrast tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average over 25 rounds</td>
<td>Round 1</td>
</tr>
<tr>
<td>O</td>
<td>11.648 (1.721)</td>
<td>10.800 (2.745)</td>
</tr>
<tr>
<td>S</td>
<td>10.671 (1.020)</td>
<td>10.889 (2.055)</td>
</tr>
<tr>
<td>C</td>
<td>9.665 (1.159)</td>
<td>9.474 (2.144)</td>
</tr>
</tbody>
</table>

Table 9: Mean and standard deviations of ordering quantities, and significance tests for differences between payment frames in Study 2. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model N1</th>
<th>Model N2</th>
<th>Model NH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.7555(0.0205)**</td>
<td>0.6979(0.0252)**</td>
<td>0.7389(0.0861)**</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.8019(0.0167)**</td>
<td>0.7778(0.0187)**</td>
<td>0.7942(0.0651)**</td>
</tr>
</tbody>
</table>

Table 10: Parameter estimates for Study 2. Standard errors are shown in parentheses. * $p < 0.05$, ** $p < 0.01$

\(\bar{C}: t(18) = -0.49, p = 0.630\). The actual mean demands were 10.787, 10.679, and 10.670, under $\bar{O}$, $\bar{S}$, and $\bar{C}$, respectively.

For Study 2, we followed the same procedure to estimate the $\beta$ and $\alpha$ parameters as we did in Study 1. The results are reported in Table 10. The estimates are almost identical to what we observed in Study 1, which again confirms that payment framing is sufficient to induce underweighting consistent with the prospective accounting model.

The expected per-round profits in Study 2 also closely resembled the results from Study 1. Expected profits were significantly affected by payment frame \(F(2, 54) = 4.50, p = 0.016\). Average expected profits by condition were 7.402 (0.657), 7.824 (0.262), and 7.735 (0.341) for $\bar{O}$, $\bar{S}$, and $\bar{C}$ respectively. Contrast tests show that all differences between conditions are significant differences at the $p < 0.05$ level except between $\bar{S}$ and $\bar{C}$.

**Summary** Study 2 establishes that the framing of the payment scheme can have a significant effect on order decisions even if the schemes have no differences in the actual timing.
of payments. This result provides further evidence for the mental accounting effect of payment schemes: by knowing the size of payments before the demand realization, individuals mentally set aside those payments and at the same time apply the prospective accounting rule to reach their order decision. In fact, we actually observed several participants who at the order time physically set aside or held in-hand the order-time payments, even though the actual transactions were not to be conducted until after the demand realization.

### 2.4.3 Study 3: Payment Scheme Experiments with High- and Low-Profit Conditions

#### Experimental Design and Hypotheses

In Study 3, we implement two repeated newsvendor experiments to test the effect of payment schemes for products with two different profit margins. The high-profit condition is conducted for a product with parameters $c = $1, $p = $4, which implies an actual average cost of $1 and an actual underage cost of $3. Under the expected profit-maximizing model, this yields a critical fractile of 75%. The low-profit condition is conducted for a product with parameters $c = $3, $p = $4, which implies an actual average cost of $3 and actual underage cost of $1. Under the expected profit-maximizing model, this yields a critical fractile of 25%. Within each high- and low-profit condition, we again test payment schemes O, S and C. Since in practice payments are usually made when they are determined, we use the payment schemes in Study 1. One can substitute the appropriate values of $c$ and $p$ into Table 5 to obtain a description of the payment schemes for the high- and low-profit conditions. As in the previous studies, demand is determined by the sum of three standard dice rolled by the subject in each round.

For all payment schemes O, S, and C, the expected profit-maximizing solution is 13 for the high-profit condition and 8 for the low-profit condition. The pull-to-center effect predicts that individuals are biased towards 10.5, the center of the distribution, causing actual orders to be somewhere between 13 and 10.5 for the high-profit condition, and somewhere between 8 and 10.5 for the low-profit condition. Nevertheless, the pull-to-center effect still predicts no difference between the payment schemes. Thus, although Study 3 does not allow us to
determine the relative magnitude of deviations from the expected profit-maximizing solution are whether due to the pull-to-center effect or due to payment schemes, it provides a test of robustness of the inequality predictions in Table 6 across high- and low-profit conditions.

Below we describe our experimental methods, present our results, and provide a discussion of the results for Study 3.

**Methods** We recruited 130 undergraduate and graduate students from Duke University—70 for the high-profit condition and 60 for the low-profit condition. The three payment schemes were assigned sequentially to the participants within each condition. In exchange for their participation, participants received a minimum of $5, with a bonus based on how much play money they earned in the game. In the high-profit condition, participants earned a $1 bonus for every 100 play dollars they had at the end of the game (each participant began with 100 play dollars). In the low-profit condition, participants earned a $1 bonus for every 50 play dollars they had at the end of the game (each participant began with 150 play dollars). For Study 3, each participant played the game for 20 rounds. In all respects except for the payment scheme parameter changes and the reduced number of rounds, the experimental design and methods were the same as in Study 1.

**Results** All 130 participants completed the study and were included in the analyses. The resulting average ordering decisions for each round are shown in Figure ??, and a summary of our findings can be found in Table 11.

**High-Profit Condition** For the high-profit condition, the repeated measures generalized linear model showed that payment scheme significantly affected ordering behavior ($F(2, 67) = 18.61, p < 0.0001$). We again found that average orders were highest under payment scheme O, and lowest under payment scheme C. Follow-up planned contrasts showed that some of the differences between conditions were significant, while others were not. Orders under O were not significantly greater than orders under S ($F(1, 67) = 3.34, p = 0.0719$). However, orders under S were significantly greater than orders under C ($F(1, 67) = 16.80, p = 0.0001$), and orders under O were significantly greater
than orders under C \((F(1, 67) = 35.75, p < 0.0001)\). Table 11 shows that this same pattern of significant differences appears to be present in the average order, round 1 order, and the follow-up question.

Though orders appear to be increasing over time in the high-profit condition, the effect was not significant (Wilks’ Lambda = 0.641, \(F(19, 49) = 1.44, p = 0.1502\)). We also found no significant interaction between payment scheme and round (Wilks’ Lambda = 0.719, \(F(38, 98) = 0.46, p = 0.9957\)). The actual mean demands were 10.787, 10.679, and 10.670, under O, S, and C, respectively.

**Low-Profit Condition** For the low-profit condition, the repeated measures generalized linear model showed that payment scheme significantly affected ordering behavior \((F(2, 57) = 7.15, p = 0.0017)\). We again found that average orders were highest under payment scheme O, and lowest under payment scheme C. However, the differences that were significant were not the same as in the high-profit condition. Order quantities under O were significantly greater than orders under S \((F(1, 57) = 9.22, p = 0.0036)\) but orders under S were not significantly greater than orders under C \((F(1, 57) = 0.19, p = 0.666)\). Orders under O were significantly greater than orders under C \((F(1, 57) = 12.04, p = 0.001)\). Again, Table 11 shows that these significance patterns appear in average order, round 1 order, and the follow-up question. Though from Figure ?? it appears that orders were decreasing over
Table 11: Mean ordering quantities and differences in Study 3.

<table>
<thead>
<tr>
<th></th>
<th>High-Profit Condition</th>
<th>Low-Profit Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean order quantity (standard deviation in parentheses)</td>
<td>Mean order quantity (standard deviation in parentheses)</td>
</tr>
<tr>
<td></td>
<td>Average over 20 rounds</td>
<td>Round 1</td>
</tr>
<tr>
<td><strong>Own Financing (O)</strong></td>
<td>11.821 (1.336)</td>
<td>11.250 (2.691)</td>
</tr>
<tr>
<td><strong>Supplier Financing (S)</strong></td>
<td>11.233 (1.020)</td>
<td>10.522 (1.344)</td>
</tr>
<tr>
<td><strong>Customer Financing (C)</strong></td>
<td>9.900 (0.892)</td>
<td>9.130 (1.359)</td>
</tr>
</tbody>
</table>

**Contrast tests**

<table>
<thead>
<tr>
<th></th>
<th>( q^o - q^s )</th>
<th>( q^s - q^c )</th>
<th>( q^o - q^c )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>High-Profit Condition</strong></td>
<td>0.588</td>
<td>1.333***</td>
<td>1.921***</td>
</tr>
<tr>
<td><strong>Low-Profit Condition</strong></td>
<td>1.173**</td>
<td>0.168</td>
<td>1.340***</td>
</tr>
</tbody>
</table>

We also found no significant interaction between payment scheme and round (Wilks’ Lambda = 0.440, \( F(38, 78) = 1.04, p = 0.428 \)). The actual mean demands were 10.523, 10.570, and 10.690, under O, S, and C, respectively.

**Summary** Study 3 examines the effect of payment schemes for high- and low-profit conditions. We find that in both conditions payment schemes significantly affect ordering decisions. Study 3 also provides a robustness check of the prospective accounting model. In support of the prospective accounting model, we find that in both conditions orders exhibit a decreasing pattern \( q^O > q^S > q^C \). Nevertheless, not all of these differences are significant. Specifically, for the high-profit condition, we find significant support for \( q^S > q^C \) and \( q^O > q^C \) but not for \( q^O > q^S \). On the other hand, for the low-profit condition, we find significant support for \( q^O > q^S \) and \( q^O > q^C \) but not for \( q^S > q^C \).

We offer the following explanation for this distortion. Because of the different profit parameters, the amount of order-time payments subject to underweighting under prospective...
accounting is different under schemes O, S, and C. Under the high-profit condition, the magnitudes of the order-time payments per unit under schemes O, S, and C are $1, $0, and $3, respectively. Thus, prospective accounting has a much greater impact on payment scheme C compared to schemes O and S. This is consistent with our observations that differences between O and C and between S and C are significant, but the difference between O and S is not. Similarly, for the low-profit condition, the magnitudes of order-time payments per unit are $3, $0, and $1, under O, S, and C, respectively. Thus, prospective accounting has a much greater impact on payment scheme O compared to schemes S and C, leading to significant differences between O and C and between O and S, but not between S and C.

To further understand this phenomenon, we calculate the optimal order quantities based on the prospective accounting model using the estimated factors $\beta$ and $\alpha$ obtained from Study 1 (specifically, the estimates from the NH method). The results are shown in Table 12, along with the observed differences in order quantities in Study 3. From this table, we see that the prospective accounting model is consistent with the findings of Study 3 in terms of the relative differences between order quantities under O, S, and C.

Table 12: Contrasts between payment schemes in the average observed order quantities in Study 3 and the theoretical order quantities according to the prospective accounting model.

<table>
<thead>
<tr>
<th>Payment Scheme Contrast</th>
<th>Predicted differences under Prospective Accounting model with $\beta = 0.7125, \alpha = 0.8137$</th>
<th>Average differences observed in Study 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High Profit</td>
<td>Low Profit</td>
</tr>
<tr>
<td>$q^O - q^S$</td>
<td>0.734</td>
<td>1.740</td>
</tr>
<tr>
<td>$q^O - q^C$</td>
<td>1.167</td>
<td>0.458</td>
</tr>
</tbody>
</table>

**Structural Parameter Estimates** In order to obtain an estimate for the parameters $\beta$ and $\alpha$ in the high- and low-profit conditions, we follow the same procedure as in Studies 1 and 2, but control for the pull-to-center effect. We assume the pull-to-center effect is of the same magnitude across each payment scheme and estimate it using maximum likelihood (the “TS” step in the two-step procedure proposed by Olivares et. al. 2008). The results
Table 13: Parameter estimates for Study 3. Standard errors are shown in parentheses.* $p<0.05$, ** $p<0.01$

<table>
<thead>
<tr>
<th>Profit Condition</th>
<th>Parameter</th>
<th>Model N1</th>
<th>Model N2</th>
<th>Model NH</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$\beta$</td>
<td>0.9638(0.0303)</td>
<td>0.7649(0.0353)**</td>
<td>0.8514(0.0829)</td>
</tr>
<tr>
<td>High</td>
<td>$\hat{\alpha}$</td>
<td>0.7757(0.0105)**</td>
<td>0.7848(0.0115)**</td>
<td>0.7801(0.0315)**</td>
</tr>
<tr>
<td>Low</td>
<td>$\beta$</td>
<td>0.7959(0.0138)**</td>
<td>0.8127(0.0174)**</td>
<td>0.8038(0.0459)**</td>
</tr>
<tr>
<td>Low</td>
<td>$\hat{\alpha}$</td>
<td>1.0845(0.0352)</td>
<td>0.9294(0.0390)*</td>
<td>1.0187(0.1179)</td>
</tr>
</tbody>
</table>

are reported in Table 13. These estimates again confirm our observations from the order quantity comparisons under schemes O, S, and C.

**Expected Profits** For the high-profit condition, expected profits were significantly affected by payment scheme ($F(2,67) = 11.98, p < 0.001$). The expected profits were highest under payment scheme O. Average expected per-round profits by condition were 26.690(0.500), 26.552(0.679), and 25.567(1.222) for O, S, and C respectively. Contrast tests show that all differences between conditions are significant differences at the $p<0.05$ level except between O and S. In other words, participants under O and S performed significantly better than those under C. For the low-profit condition, expected profits were significantly affected by payment scheme ($F(2,57) = 6.30, p = 0.003$). The average expected per-round profits were lowest under payment scheme O. Average expected profits by condition were 4.858(1.555), 5.881(0.743), and 5.847(0.496) for O, S, and C respectively. Contrast tests show that all differences between conditions are significant differences at the $p<0.05$ level except between S and C. In other words, participants under C and S performed significantly better than those under O. The expected profit analysis above also demonstrates how the effect of payment scheme interacts with the pull-to-center effect and the resulting effectiveness of ordering behavior. For the high-profit condition, the pull-to-center effect is mitigated by the effect of scheme O, but exacerbated by the effect of scheme C. Conversely, for the low-profit condition, the pull-to-center effect is mitigated by the effect of scheme C, but exacerbated by the effect of scheme O.

**On the Asymmetry of the Pull-to-Center Effect** Schweitzer and Cachon (2000) and
Bolton and Katok (2008) find that the pull-to-center effect is stronger, i.e., the deviation from the optimal order quantity towards the center of the demand distribution is larger, in the low-profit condition. They suggest stockout aversion, or that the high profit condition is more “intuitive” as possible explanations, but do not provide substantive evidence. The framing of the newsvendor problem in their papers is similar to the wholesale price contract (i.e., scheme O). Thus, to examine this issue, we compare the level of deviation from the optimal order quantity under high- and low-profit conditions for the three payment schemes.

We found that under payment scheme O, participants deviated farther from the optimal solution (toward the center of the distribution) under the low-profit condition compared to the high-profit condition ($F(5, 124) = 13.70, p = .0003$). Average orders under the low-profit condition were 2.478 above the optimal order 8, but average orders under the high-profit condition were only 1.179 below the optimal 13. This is the same asymmetry effect observed by Schweitzer and Cachon (2000) and Bolton and Katok (2008).

However, under payment scheme S, participants deviated approximately the same distance from the optimal solution in both the high- and low-profit conditions, and the deviations were not significantly different ($F(5, 124) = 1.70, p = .194$). Average orders under the low-profit condition were 1.305 above the optimal order 8, and average orders under the high-profit condition were 1.767 below the optimal 13. Thus, we observed that the asymmetry effect disappears under payment scheme S.

Finally, under payment scheme C, participants deviated farther from the optimal solution (toward the center of the distribution) under the high-profit condition compared to the low-profit condition ($F(5, 124) = 30.70, p < .0001$). Average orders under the high-profit condition were 3.100 below the optimal 13, but average orders under the low-profit condition were only 1.138 above the optimal order 8. Thus, the asymmetry effect in this case is in the opposite direction to that of payment scheme O.
2.5 Discussion

In this paper, we find that payment schemes have a significant behavioral effect on ordering decisions in the newsvendor problem. Our findings help us gain insights into how human subjects account for payments in the newsvendor problem. We provide evidence that order-time payments receive less weight than the demand-time payments, which is consistent with the prospective accounting model. As a result, we find that orders under payment scheme O, which is the same as a typical wholesale price contract, induces an order above the expected profit-maximizing solution. We also demonstrate that the framing of the payments is sufficient to induce differences in ordering behavior, even if the actual timing of payments are the same across payment schemes. Finally, we show that the prospective accounting effect of payment schemes is robust for high- and low-profit conditions. We also show that the differences between the payment schemes can explain the asymmetry of pull-to-center effect across profit conditions.

Our laboratory findings could also be partially explained by other behavioral theories. Schweitzer and Cachon (2000) apply the idea of “recency” to obtain a “chasing demand” heuristic, in which individuals use their previous order quantity as an anchor and adjusts towards the previous demand realization. One could also apply the same idea of recency to obtain a heuristic in which individuals also anchor on their previous order quantity and adjust by putting a greater weight on the most recent payment feedback (the payments after demand realization). This would also yield behavior similar to if the decision maker underweights the order-time payments, which is similar to what Gourville and Soman (1998) call “payment depreciation.” (Payment depreciation is essentially the second half of the prospective accounting rule that states that events are weakly coupled looking backwards in time.) However, such an adjustment heuristic does not explain why we find differences in ordering behavior between payment schemes in the first round of order decisions in our experiments. Another behavioral effect that is relevant to our setting is “debt aversion.” Because the newsvendor is in debt under payment scheme S, debt aversion would predict larger orders under scheme O than scheme S. However, it does not explain why payment
scheme O leads to orders larger than the expected profit-maximizing solution in Study 1. Furthermore, Prelec and Loewenstein (1998) suggest that “prospective accounting induces strong debt aversion,” which implies that debt aversion is a result of mental accounting rather than a stand-alone behavioral effect. Study 2 demonstrates that indeed our observations are not solely driven by the actual debt position over time, but how the individual mentally processes the payments over time.

The behavioral effect of payment schemes on newsvendor orders has direct implications on the newsvendor’s expected profit, the supplier’s revenue, and the customer’s service level. Therefore, from the newsvendor’s standpoint, one should strategically select the appropriate payment scheme to encourage a most efficient decision outcome. In our experiments, payment scheme S, which conducts all payments after the demand realization, encourages equal weighting of all payments and achieves the optimal order when the underage and underage costs are equal. However, payment schemes can also be used to mitigate other behavioral biases. For example, for the pull-to-center effect, the payment scheme that would lead to order quantities closest to the optimal solution is the payment scheme O under the high-profit condition and the payment scheme C under the low-profit condition. Furthermore, Study 2 demonstrates that even if the actual payment contract does not have a timing of the payments that induces an optimal ordering decision, one can simply rewrite the framing of payment scheme to encourage optimal behavior. On the other hand, the supplier and the customer would like to choose a payment scheme which induces the highest order, which, according to our experiments, is the wholesale price scheme (payment scheme O). In addition to practitioners, our results may inform future newsvendor experiments, as individuals weigh the payments correctly only under the framing of payment scheme S.

Our results also help us gain insight on the behavioral effect of financial contracts in practice. For instance, suppliers often offer retailers trade credit, allowing retailers to delay payment for goods until they make the sale, hoping that this will encourage higher orders. This intended effect of trade credit is captured by the time-discounting model in Section 2.3.3. When capital constraint is not an issue and the interest rate is negligible,
the practice of trade credit (corresponding to the payment scheme S) might inadvertently lower the retailer’s order quantity relative to that without trade credit (corresponding to the payment scheme O) as shown by our experiments. When the interest rate is significant or the capital constraint is binding, however, the prospective accounting effect may become second order. Similarly, though we show that providing the news-vendor revenue-based loans may decrease the inventory order, such a behavioral effect may also be dominated by the time discounting effect due to the tangible interest rate benefit. An interesting direction would be to determine the relative magnitudes of these opposing effects in practice and estimate the impact of mental accounting empirically.

Another application is in supply chain contract design and coordination (see Cachon 2003 for a review). A wholesale-price contract typically has payment transactions resembling the payment scheme O in this paper. Our results suggest that the retailer may place larger-than-optimal orders due to prospective accounting, reducing some of the supply chain inefficiency due to double marginalization (Lariviere and Porteus 2001). If the supplier can estimate the retailer’s underweighting factor as we did in Section 2.4, then she may coordinate the supply chain by setting the wholesale price equal to the unit production cost divided by the underweighting factor. Under a buy-back contract, the retailer receives a refund for leftover inventory after the demand realization. To the retailer, the refund payment is likely to be weighted more than the purchase cost incurred at the order time. Thus, the supplier may exploit this effect to achieve supply chain coordination by offering a smaller buy-back price for leftover inventory. For a similar reason, under a revenue-sharing contract, with prospective accounting, the supplier may be able to charge a higher wholesale price to the retailer and still achieve supply chain coordination. It would be interesting to empirically investigate these potential implications, although we acknowledge that there are many additional factors working simultaneously in real-world contract settings (see Zhang et al. 2012).

Finally, several extensions to our study merit further research. First, it would be interesting to test how individuals react to a payment scheme switch in the news-vendor
problem. This could enable the comparison of each individual’s decisions over different payment schemes. If this is to be carried out, we caution that some extra care should be taken to control for the recency and learning effects across the scheme switch. Second, it would also be interesting to test how individuals place inventory orders among multiple suppliers who offer different payment schemes. This could further shed light on the effect of payment schemes in a competitive environment.
3 Reference Prices and Transaction Utility in Inventory Decisions

3.1 Introduction

Transaction utility is a well-known concept in consumer behavior (Thaler 1980, 1985). When purchasing a good, consumers receive both acquisition utility (the value of the good if received as a gift minus its cost) and transaction utility (the “value of the deal”). Transaction utility explains why consumer behavior sometimes deviates from a rational economic model. Its magnitude depends on how the actual cost compares to a consumer’s reference cost. If the actual cost is lower than the reference cost, then the purchase is a good deal and the consumer’s transaction utility is positive. Otherwise, it is a bad deal and the consumer’s transaction utility is negative.

Because inventory decisions are also frequently made by individuals, it is likely that they are subject to similar psychological forces. That is, inventory decision makers may be affected by transaction utility. In this paper, we provide a descriptive model of reference effects and transaction utility in a newsvendor setting (henceforth referred to as the reference effects model). Our goal is to understand how the context of the inventory decision affects the decision maker: how do other prices (past or present) that are irrelevant for the rational decision maker influence the inventory decision?

Because the inventory manager makes two transactions (purchasing and selling), we assume that the inventory manager’s transaction utility has two components (the \textit{purchase transaction utility} and the \textit{sales transaction utility}) and depends on two reference parameters (the \textit{reference cost} and the \textit{reference price}). The inventory manager prefers to buy at a relatively low cost and sell at a relatively high price. Thus, the purchase transaction utility is increasing in the reference cost, but the sales transaction utility is decreasing in the reference price. The inventory manager’s order is increasing in the transaction utility, and greater than the expected profit-maximizing order if the transaction utility is positive.

There are several factors that may influence the inventory manager’s reference cost and
reference price. For consumers, the reference cost has been shown to be influenced by both past costs and the set of costs observed at the point of sale. We examine two corresponding factors for the inventory manager: past costs and prices, and the costs and prices of other products in the decision portfolio. The reference effects model assumes that the reference cost and reference price are both increasing in past costs and prices, respectively. For example, a $7 unit cost will seem relatively expensive if the cost has historically been only $5 per unit. Similarly, the model assumes that the reference cost and reference price are also both increasing in the costs and prices of the other products in the decision portfolio, respectively. For example, a product that costs $5 per unit and sells for $20 per unit will seem relatively more profitable if all other products in the category cost $5 but only sell for $10.

The reference effects model predicts the following behavioral tendencies, which we refer to collectively as “reference effects.”

1. Order decisions are irrationally increasing in past costs. This is because the reference cost is increasing in past costs, and the purchase transaction utility is increasing in the reference cost.

2. Order decisions are irrationally decreasing in past prices. This is because the reference price is increasing in past prices, but the selling transaction utility is decreasing in the reference price.

3. Order decisions are decreasing in the proportion of high profit margin to low profit margin products in the decision portfolio. This is because high profit margin products have low costs and high prices, the reference cost and price are both increasing in the costs and prices of other products in the portfolio, and the transaction utility is increasing in the reference cost but decreasing in the selling price.

Three laboratory experiments in a newsvendor setting provide empirical support for the model. In the newsvendor problem a decision maker chooses an inventory order quantity to meet a one-time random demand. The goal is to choose an order quantity that optimally
balances the expected cost of over ordering (i.e., having leftover inventory) with the expected cost of under ordering (i.e., missing out on potential sales). The newsvendor problem is of particular interest because it is simple to study, is commonly used in practice (i.e., for managing products with a short selling season and limited replenishment opportunities, such as fashion apparels and high-tech products), and also serves as a building block for many types of stochastic decision problems. Furthermore, in practice, newsvendor order decisions are often made by human decision makers with an intuitive approach because, for instance, parameters are not easily estimated or there is a lack of historic data.

In the first two experiments, we set the parameters so that the expected profit maximizing solution was to order the mean demand under all conditions. In Experiment 1 we found that orders were higher than the expected profit-maximizing order if individuals were previously exposed to newsvendor rounds with high purchasing costs, keeping the price constant. Oppositely, in Experiment 2 we found that orders were higher than the expected profit maximizing order if individuals were previously exposed to low selling prices, keeping the cost constant. In Experiment 3, we presented individuals with portfolios that varied in the proportion of high profit to low profit margin products. We found that order quantities were greater for high profit products in a portfolio of mostly low profit products. Correspondingly, order quantities were smaller for low profit products in a portfolio of mostly high profit products.

These experimental findings have several immediate implications. They suggest that, first, inventory managers tend to be overly aggressive (conservative) after a sudden increase (decrease) in the profitability of a product. For example, if an economic downturn causes procurement costs to increase, newsvendors will be even more reluctant to invest in inventory than is rational. Thus, if the objective is to make profit-maximizing decisions, these are the times most critical for supervision and intervention. Second, since inventory managers over order for rare high-profit products but under order for rare low-profit products it may be beneficial to delineate categories by grouping products with similar profit margins.

Moreover, the reference effects documented in this paper work in the opposite direction
as the anchoring and insufficient adjustment heuristic that uses the previous order quantity as the anchor. It has been suggested that individuals may use such an anchor in repeated newsvendor decisions with stationary problem parameters (Schweitzer and Cachon 2000; Bostian et. al 2008). If an individual anchors on the previous order quantity and it is high, then anchoring and adjustment predicts that the subsequent order should be biased high. However, this is opposite to what we find in our Experiments 1 and 2 - after a cost or price change, individuals are biased in the opposite direction as the previous order quantity.

Reference effects also impact the entire supply chain because the newsvendor’s order decision determines its suppliers revenue and its customers service level. Using an analytical model, we demonstrate the following:

1. The supplier should set a lower wholesale price when the newsvendor is subject to reference effects compared to a rational newsvendor. In other words, there is a behavioral reason why a supplier should consider strategically lowering its wholesale price - it makes their product appear more attractive relative to other supplier’s wholesale prices. This result also suggests that a supplier may want to strategically refrain from passing a temporary cost increase on to the newsvendor’s wholesale price, since the newsvendor may react by reducing their order more than is rational due to the reference effect.

2. If the supplier sets the retail selling price to the customer, and if both the newsvendor and the final customer demand are subject to reference effects, then the optimal selling price may be higher or lower than if neither newsvendor nor customer were subject to reference effects (the direction depends on which reference effect is stronger.) That is, though the supplier may be able to inflate customer demand by lowering the selling price due to customer reference effects, the benefits may be offset because the newsvendor may be reluctant to stock aggressively enough.

3. Reference effects imply the existence of a behavioral supply chain inefficiency we call a \textit{behavioral pricing whip}. Specifically, for a high transfer price (relative to the reference
price) the newsvendor orders more than is rational while the customer demands less than is rational. Oppositely, for a low transfer price the newsvendor orders less than is rational while the customer demands more than is rational. The resulting phenomenon is similar in spirit to the bullwhip effect (Lee, Padmanabhan and Whang 1997) which states that a small variations in the demand causes large variations in upstream orders. In our case, a change in the transfer price between two supply chain nodes biases the upstream node’s order in one direction while biasing the downstream’s demand in the opposite direction, making the supply chain doubly inefficient.

The rest of the chapter is organized as follows. In Section 3.2 we review related literature. In Section 3.3 we present the reference effects model. In Section 3.4 we present our experimental methods and results. In Section 3.5 we apply the model to the supply chain setting. Finally, we conclude in Section 3.6.

3.2 Literature Review

Our paper belongs to the literature on behavioral operations management (see Bendoly et al., 2006 for a review), in which researchers study how humans make operational decisions and how these decisions often systematically differ from the expected profit-maximizing decision. Much of this research studies behavior in the newsvendor problem. There are several ways that human behavior in the newsvendor problem deviates from rationality. Human decision makers tend to place orders that are biased away from the expected profit maximizing solution in the direction towards the center of the demand distribution (Schweitzer and Cachon 2000). This is known as the “pull-to-center effect,” and may be explained by heuristics (Schweitzer and Cachon 2000; Bostian et. al 2008) or by considering additional psychological costs of stockouts and leftover inventory (Ho et al. 2011). Though this effect is generally robust, performance may improve if individuals are given the opportunity to repeat the same newsvendor problem many times, and improves at a faster rate if feedback is aggregated over many rounds (Bolton and Katok 2008; Lurie and Swaminathan 2009).

In addition to having a tendency to order too close to the mean on average, orders also
exhibit significant variation across participants and across time (Schweitzer and Cachon 2008; Bolton and Katok 2008). Su (2008) examines the effect of this “bounded rationality” on supply chains by modeling the newsvendor’s decision as a random variable. Kremer et al. (2010) further empirically examines the extent to which newsvendor behavior is captured by random errors.

Since most newsvendor experiments are conducted with students, Bolton et al. (2010) studies whether or not there are significant differences between experienced managers and students. They find that experienced managers tend to exhibit the same qualitative behavior in newsvendor decisions, making the same mistakes as inexperienced students. In a serial supply chain setting, Croson and Donohue (2006) also find managers’ and students’ inventory decisions demonstrate similar biased behavior.

Our paper differs from these newsvendor studies by examining the effect of non-stationary prices over time and non-identical prices across the product portfolio. From a theoretical standpoint, our paper contributes to the literature by examining new factors that contribute to newsvendor behavioral biases due to mental accounting (see Thaler 1999 for a review on mental accounting). Chen et al. (2011) is among the first in this line of investigation. They examine the effect of payment timing, and show that orders may be higher or lower than the expected profit maximizing solution depending on the payment scheme. They suggest that this is due to the way that newsvendors mentally account for payments associated with a transaction over time. Instead of investigating mental accounting of payments over time, our paper investigates the role of reference prices and transaction utility.

Reference prices and transaction utility have been studied extensively in the consumer behavior literature (see Kalyanaram and Winer 1995 for a review). This research has demonstrated that a consumer’s reference cost is affected by both historical costs and the costs of other products in the same category. For instance, some researchers model the consumer’s reference price over time by using the past price, a moving average, or an exponentially smoothing process (e.g., see Rinne 1981, Winer 1986). Furthermore, Rajendran and Tellis (1994) find that a customer’s reference price is sensitive to the lowest observed price at the
point-of-purchase. Reference prices and transaction utility (Thaler 1980, 1985) have been shown to affect consumers’ probability of purchase, brand choice, purchase quantity decisions, and purchase timing decisions (see Mazumdar et al., 2005). Given that consumers have reference prices and this affects their purchasing decisions, Popescu and Wu (2007) study a dynamic pricing model to determine how a firm should set a pricing policy when customer demand exhibits reference effects. Our paper differs from these literatures by examining the effect of reference prices on the inventory manager instead of the consumer.

Our work is also related to research in the supply chain setting with both behavioral and rational decision-makers. Supply chain performance is often much different with real decision-makers than it is with rational decision-makers. For instance, some researchers find that there is a behavioral reason for the bullwhip effect: human subjects do not sufficiently account for the pipeline inventory (Sterman 1989, Croson and Donohue 2005, 2006). Others researchers find that supply chain contracts do not perform the same as they should theoretically due to behavioral reasons such as social preferences (Loch and Wu 2008), trust (Ozer et al. 2011), or bounded rationality and loss aversion (Su 2008; Ho and Zhang 2008; Katok and Wu 2009). Several researchers also incorporate consumer behavior, which may deviate from rationality, in supply chain models (see Shen and Su 2007 for a review). Our paper contributes to this literature by examining how reference effects impact pricing decisions and supply chain performance. Finally, the application of our model to supply chain pricing problems in Section 5 is related to the supply chain contracting literature with rational decision-makers (see Cachon and Lariviere 2005 for a review). For instance, our analytical model in Section 5 draws from the selling to the newsvendor model of Lariviere and Porteus (2001).

3.3 Models

In this section, we briefly describe the newsvendor problem and, as a benchmark, state the expected profit maximizing solution. We then present a behavioral model of newsvendor decision-making based on the theory of reference prices and transaction utility.
3.3.1 Benchmark: Order decision without reference effects

In the newsvendor problem, a decision-maker chooses an order quantity $q$ for a product in order to meet a future random demand $D$. Let $G(\cdot)$ denote the cumulative distribution function for the random demand. We assume that backlogs are not allowed (i.e., unmet customer demand is lost) and leftover inventory has zero salvage value. The unit cost of the product is $c$ and it is sold at price $p$ (with $p > c$). The profit-maximizing problem is

$$\max_q \mathbb{E}_D\{\pi(q, D)\}$$

where

$$\pi(q, D) = -cq + p \min(q, D)$$

Denote the order quantity that solves the problem with superscript $e$ for the expected profit-maximizing solution.

$$q^e = \arg \max \mathbb{E}_D[\pi(q, D)] = G^{-1}\left(\frac{p - c}{p}\right).$$

The term $(p - c)/p$ is known as the critical fractile.

Next, consider the case when an inventory manager makes $k$ newsvendor decisions. This may represent the same product over $k$ time periods with lost sales and inventory that perishes each period, or it may represent $k$ products in a portfolio. Denote the parameters and order quantity for each decision with subscript $i$, $i \in 1, 2, ..., k$. Then the total profit function given each $q_i, D_i$ is

$$\sum_{i=1}^{k} \pi_i(q_i, D_i) = \sum_{i=1}^{k} [-c_i q_i + p_i \min(q_i, D_i)]$$

and the expected profit-maximizing ordering decisions are separable,

$$q_i^e = F_i^{-1}\left(\frac{p_i - c_i}{p_i}\right).$$
3.3.2 Order decision with reference effects

As stated in the introduction, newsvendors experience two kinds of utility: acquisition utility and transaction utility.

*Acquisition utility* can be interpreted as the “rational” utility. Given an order quantity and a realization of demand, the acquisition utility is the net profit.

\[ U^A(q, D) = -cq + p \min(q, D). \]

*Transaction utility* is purely psychological and depends on how the cost and price compare to the reference cost and reference price. It is the sum of the purchase transaction utility and the selling transaction utility. Given an order quantity and a realization of demand, the transaction utility is expressed as follows.

\[ U^T(q, D) = \tau_c(c - c^r)q + \tau_p(p - p^r) \min(q, D) \] (8)

The per unit purchase transaction utility function \( \tau_c(c - c^r) \) is increasing in \( c^r \) and equal to zero when \( c = c^r \). The per unit selling transaction utility \( \tau_p(p - p^r) \) is decreasing in \( p^r \) and equal to zero when \( p = p^r \).

The utility-maximization problem is then

\[ \max_q \mathbb{E}_D[U^A(q, D) + U^T(q, D)] \] (9)

and the utility-maximizing order quantity for the reference effects model (denoted with superscript \( r \)) is

\[ q^r = \arg \max_q \mathbb{E}_D[U^A(q, D) + U^T(q, D)] = G^{-1} \left( \frac{p + \tau_p(p - p^r) - [c + \tau_c(c - c^r)]}{p + \tau_p(p - p^r)} \right) \] (10)

Thus, \( q^r \) is increasing in \( c^r \) and decreasing in \( p^r \). Comparing Equation (10) with Equation (7) we obtain:
Proposition 15. If the decision maker is subject to reference effects, then \( q^r > q^e \) if \( p > p^r \) and \( c < c^r \), but \( q^r < q^e \) if \( p < p^r \) and \( c > c^r \).

Proof. Since \( \frac{\partial \tau^c(c-c^r)}{\partial c^r} > 0 \), \( \frac{\partial \tau^p(p-p^r)}{\partial p^r} < 0 \), and \( \tau^e(c-c^r) = \tau^p(p-p^r) = 0 \) for \( c = c^r \) and \( p = p^r \), then the critical fractile \( \frac{p^r+p^r-(c+c^r)}{p^r+p^r} > \frac{p^r}{p} \) if \( p > p^r \) and \( c < c^r \). Thus \( q^r > q^e \) if \( p > p^r \) and \( c < c^r \). The proof is similar for \( q^r < q^e \).

Thus the reference effects model predicts that orders will be higher than the expected profit maximizing solution if there is a high reference cost or low reference price.

3.3.3 Determinants of reference cost and price

As mentioned in the introduction, there are two well-established determinants of the reference cost for consumers: past prices and other current prices in the same product category. We discuss the analogs for the inventory manager in turn.

Past costs and prices Inventory managers frequently make repeated ordering decisions for the same product over time. Let \( A = \{(c^a_1, p^a_1), (c^a_2, p^a_2), \ldots, (c^a_k, p^a_k)\} \) and \( B = \{(c^b_1, p^b_1), (c^b_2, p^b_2), \ldots, (c^b_k, p^b_k)\} \) be two sequences of costs and prices in periods 1 to \( k \). It is natural to assume that past costs and past prices inform the current reference cost and the current reference price in the following way.

Assumption 2. The reference cost in period \( k+1 \) is larger under \( A \) than under \( B \) if \( c^a_i \geq c^b_i \) for all \( i \leq k \) so long as not all inequalities hold at equality. It is smaller under \( A \) than under \( B \) if \( c^a_i \leq c^b_i \) for all \( i \leq k \) so long as not all inequalities hold at equality. Similarly, the reference price in period \( k+1 \) is greater under \( A \) than under \( B \) if \( p^a_i \geq p^b_i \) for all \( i \leq k \), but smaller under \( A \) than under \( B \) if \( p^a_i \leq p^b_i \) (again, so long as not all inequalities hold at equality).

The following corollary follows directly from Assumption 1 and Proposition 1.

Corollary 7. The order in period \( k+1 \) is greater under \( A \) than under \( B \) if \( c^a_i \geq c^b_i \) and \( p^a_i \leq p^b_i \) for all \( i \leq k \) so long as not all inequalities hold at equality. It is less under \( A \) than under \( B \) if \( c^a_i \leq c^b_i \) and \( p^a_i \geq p^b_i \) for all \( i \leq k \) so long as not all inequalities hold at equality.
The proposition states that the ordering quantity is greater when past costs are high but past prices are low.

**Portfolio** Inventory managers are often responsible for managing a portfolio of products. We first define the weak majorization order.

**Definition 3.** (Muller and Stoyan 2002) $A = (a_1, \ldots, a_k)$ weakly majorizes $B = (b_1, \ldots, b_k)$ (written $A >_w B$) if and only if $\sum_{i=1}^k a_i^\downarrow \geq \sum_{i=1}^k b_i^\downarrow$ for $\kappa = 1, \ldots, k$, where $a_i^\downarrow$ and $b_i^\downarrow$ are the elements of $A$ and $B$ sorted in decreasing order.

Let $X = \{(c_{ix}^x, p_{ix}^x)\}_{i=1..k}$ and $Y = \{(c_{iy}^y, p_{iy}^y)\}_{i=1..k}$ be two sets of purchasing costs and selling prices for two portfolios of $k$ products. Denote $C^x = \{c_{ix}^x\}_{i=1..k}$, $P^x = \{p_{ix}^x\}_{i=1..k}$, $C^y = \{c_{iy}^y\}_{i=1..k}$ and $P^y = \{p_{iy}^y\}_{i=1..k}$. It is natural to assume that the reference cost and reference price are influenced by the set of costs and prices in the portfolio as follows.

**Assumption 3.** The reference cost under $X$ is greater than under $Y$ if $C^x >_w C^y$ and $C^x \neq C^y$. Similarly, the reference price under $X$ is greater than under $Y$ if $P^x >_w P^y$ and $P^x \neq P^y$.

Consider adding an identical product $i = k + 1$ to both $X$ and $Y$, to create new sets $X'$ and $Y'$. The following corollary follows directly from Assumption 2 and Proposition 1.

**Corollary 8.** If $C^x >_w C^y$, $P^x \prec_w P^y$, and $X \neq Y$, then for product $k + 1$ the newsvendor order is greater under $X'$ than under $Y'$.

The proposition states that the newsvendor orders more if the other products in the portfolio have higher costs and lower prices (lower profit margins), but less otherwise.

### 3.4 Experimental Evidence

In this section, we present results from three newsvendor experiments which test the predictions of the reference effects model. In order to isolate reference effects, our experiments do
not permit the holding of inventory or unmet demand from period to period, nor do they consider cash or space constraints. We implement demand distributions that are independent and identically distributed across time and across products.

3.4.1 Experiment 1: Effect of past purchasing costs

Experimental Design and Hypothesis The purpose of Experiment 1 is to examine the behavioral effect of past purchasing costs on inventory decisions in a newsvendor setting. We set the parameters $c = 3$ and $p = 6$. For these parameters the expected profit maximizing solution $q^e$ is the median or center of the demand distribution. Thus, they control for the pull-to-center effect. In order to examine the effect of cost changes, before asking the final inventory ordering problem, we expose decision-makers repeatedly to seven newsvendor rounds under three levels of cost parameters: low ($c = 1$), medium ($c = 3$), and high ($c = 5$). We call these conditions low, medium and high cost exposure, respectively. Thus, individuals in the low cost exposure condition experience an increase in the cost, individuals in the medium cost exposure condition experience no change in cost, and individuals in the high cost exposure condition experience a decrease in the cost. The selling price remains at $p = 6$ under all conditions for the entire experiment.

In each round, the subjects roll two six-sided dice, the sum of which determines the demand for that round. Thus, demand is independent, identically distributed, and symmetric with a minimum value of 2, maximum value of 12, and mean value of 7. In the final round, all participants make an ordering decision under the same parameters ($c = 3, p = 6$). For all conditions $q^e = 7$ in the final round. However, based on Corollary 1, we have the following hypotheses:

**Hypothesis 1a**: Orders under the high cost exposure condition will be greater than orders under the medium cost exposure condition, which in turn will be greater than orders under the low cost exposure condition.

**Hypothesis 1b**: Orders under the high cost exposure condition will be greater than $q^e$, but orders under the low cost exposure condition will be less than $q^e$. 
**Methods** We recruited 60 undergraduate and graduate students from a major American university using a lab recruiting system. The experimental conditions were assigned sequentially to the participants. In exchange for their participation, participants received a minimum of $3 plus a $1 bonus for every 20, 60, and 100 play dollars they had at the end of the game for the high, medium, and low exposure cost conditions, respectively. However, because we did not want participants to try to strategically make decisions to "reach the next threshold", we merely told each participant that "the amount of money you will collect at the end of the experiment will depend on the success of your inventory decisions. The likely range of outcomes is between $4 and $12." Similar incentive descriptions have been used to avoid drawing attention to the small stakes of each decision in a sequential investment decision making task (see Thaler et al., 1997). Each participant began with 100 play dollars.

Participants were given an instruction sheet explaining the details of the game for their given payment scheme. Instructions were also reviewed verbally by a research assistant before beginning play. Participants were told that they would be selling “widgets” (represented by poker chips) to customer demand (represented by the sum of the rolling of two standard dice). Each participant interacted one-on-one with a research assistant, who facilitated payment transfers and recorded ordering decisions and dice rolls. A participant decided an order quantity vocally, placed that many poker chips into the “store” (represented by a square drawn on an index card), and made the appropriate cost payment. Then, the participant rolled the two dice, determined how many units were sold, and received the appropriate revenue. Finally, the participant removed all chips from the store to begin the next round. Payment transfers were conducted in the form of play cash bills in denominations of 1, 5, and 10.

In the first 7 rounds, subjects purchased widgets at the cost determined by their cost condition: low ($1), medium ($3) or high ($5). In round 8, all subjects purchased widgets at the cost of $3. At the end of the experiment subjects answered the following comprehension questions: (1) "What was the minimum and maximum possible demand each round?" (2) "If you were to generate 100 more demands with these two dice, what would you expect to
be the average per-round demand?"

**Results** All 60 participants completed the study. Table 14 provides a summary of our main results, and Figure 5 shows the average order in the final round (round 8) for each exposure condition.

Orders were analyzed using a generalized linear model. We found that exposure condition significantly affects orders \( F(2, 57) = 23.59, p < .001 \). (Note that this \( p \) is the \( p \)-value which is different from the italicized \( p \) which is the selling price.) Planned contrasts show that all differences between exposure conditions were significant. Orders under high cost exposure were significantly greater than under medium cost exposure \( (F(1, 57) = 8.61, p = .005) \). Orders under medium cost exposure were significantly greater than under low cost exposure \( (F(1, 57) = 15.30, p < .001) \). Finally, orders under high cost exposure were significantly greater than orders under low cost exposure \( (F(1, 57) = 46.386, p < .001) \). We also tested whether there was significant asymmetry in these differences due to loss aversion. Loss aversion suggests that an increase in cost impacts order quantities more than an equivalent decrease in cost, because the former is a loss but the latter is a gain. We did not find any significant evidence of this type of loss aversion. That is, the difference between high and medium cost exposure orders were not significantly less than the difference between low and medium cost exposure orders \( (F(1, 57) = 0.32, p = 0.575) \).
Table 14: Mean and standard deviations of ordering quantities in Experiment 1.

<table>
<thead>
<tr>
<th>Exposure Cost</th>
<th>Average order quantity</th>
<th>round 8 (under purchasing cost 3)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>High (5)</td>
<td>6.057 (1.200)</td>
<td>9.200 (1.673)</td>
<td>20</td>
</tr>
<tr>
<td>Medium (3)</td>
<td>7.293 (1.142)</td>
<td>7.850 (1.387)</td>
<td>20</td>
</tr>
<tr>
<td>Low (1)</td>
<td>8.707 (1.572)</td>
<td>6.050 (1.276)</td>
<td>20</td>
</tr>
</tbody>
</table>

We also compared the order quantities to \( q^e = 7 \). As expected, orders under the high cost exposure condition were significantly higher than \( 7 (t(19) = 5.88, p < 0.001) \), while orders under the low cost exposure condition were significantly lower than \( 7 (t(19) = -3.33, p = 0.004) \). Though not the focus of this paper, we also found that orders under the medium cost exposure condition were also higher than \( 7 (t(19) = 2.74, p = 0.013) \). This is consistent with what Chen et al. (2011) find in their experiments, which they suggest is due to a mental accounting phenomenon called prospective accounting (Prelec and Loewenstein 1998). We conjecture that this inflated order may also be due to a desire to be risk-seeking for the last round of the game.

All participants correctly answered 2 and 12 for the minimum and maximum possible demand that could be generated by rolling 2 dice. Participant estimates of the long-run average demand with two dice revealed some variation, but were not significantly affected by condition \((F(2, 57) = 0.03, p = 0.970)\). Their average estimates by condition were 6.875, 6.875, and 6.900 for high, medium, and low cost exposure conditions, respectively. Though these averages are below 7, \( t \)-tests show the differences are not significant \((t(19) = -1.75, p = 0.096; t(19) = -1.45, p = 0.163; t(19) = -1.23, p = 0.235)\). The actual mean demands generated by rolling the two dice were 6.963, 6.994, and 7.063 under high, medium, and low cost exposure conditions, respectively.

**Transaction Utility Estimates** In order to provide further validation of the transaction utility model, and to obtain estimates for the transaction utilities, we structurally estimated the per unit purchasing transaction utility \( \tau^c \) (we assume in this experiment that
\( \tau^p = 0 \). Recall that the reference effects model reduces to the expected profit maximizing model when \( \tau^c = 0 \). Therefore, we test if \( \tau^c \) is significantly different from 0.

Our structural model assumes that the average order quantities are normally distributed with mean \( G^{-1} \left( \frac{6-3+\tau^c}{6} \right) = G^{-1} \left( \frac{3+\tau^c}{6} \right) \). Furthermore, the structural model assumes that the decision maker behaved according to the assumption that the demand distribution was triangular with minimum 1, maximum 13, and median 7. These parameters were chosen to closely match the discrete demand distribution of two dice using a continuous distribution. The resulting maximum likelihood estimates and 95% confidence intervals for \( \tau^c \) under the high and low conditions were calculated according to Casella and Berger (2002) and are shown in Table 15. The estimates show that when past costs are high, \( \tau^c > 0 \), but when past costs are low, \( \tau^c < 0 \). Recall the order cost is $3. However, individuals in the high cost exposure condition \( (c = $5) \) perceive an additional psychological gain of $1.797 per unit ordered, so they order more. Similarly, individuals in the low cost exposure condition \( (c = $1) \) perceive an additional psychological loss of $0.875 per unit ordered, so they order less.

Table 15: MLE for the per unit purchase transaction utility in Experiment 1.

<table>
<thead>
<tr>
<th>Cost Exposure Condition</th>
<th>MLE Estimate for ( \tau^c )</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>1.797</td>
<td>(1.332, 2.261)</td>
</tr>
<tr>
<td>Low</td>
<td>-0.875</td>
<td>(-1.345, -0.404)</td>
</tr>
</tbody>
</table>

3.4.2 Experiment 2: Effect of past selling prices

Experimental Design and Hypothesis The purpose of this experiment is to examine the behavioral effect of past selling prices on inventory decisions. We again set the parameters to \( c = 3 \) and \( p = 6 \) so that \( q^c \) is the median demand. This time we manipulate the exposure selling price while keeping the cost constant throughout the experiment. We expose decision-makers repeatedly to newsvendor rounds under two levels of prices: high \( (p = 12) \) and low \( (p = 4) \). We call these conditions high and low price exposure conditions,
Table 16: Mean and standard deviations of ordering quantities in Experiment 2.

<table>
<thead>
<tr>
<th>Exposure Selling Price</th>
<th>Average order quantity (under exposure selling price)</th>
<th>Average order quantity (under selling price 6)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>High (12)</td>
<td>8.795 (1.326)</td>
<td>6.000 (1.257)</td>
<td>20</td>
</tr>
<tr>
<td>Low (4)</td>
<td>6.043 (1.189)</td>
<td>9.350 (1.496)</td>
<td>20</td>
</tr>
</tbody>
</table>

respectively. Thus, individuals in the high price exposure condition experience a decrease in the selling price while individuals in the low price exposure condition experience an increase in the selling price. According to Corollary 1, we have the following hypotheses.

**Hypothesis 2a:** Orders under the high selling price exposure condition will be less than orders under the low selling price exposure condition.

**Hypothesis 2b:** Orders under the high selling price exposure condition will be less than $q^e$, but orders under the low selling price exposure condition will be greater than the $q^e$.

**Methods and Results** We recruited 40 undergraduate and graduate students from the same American university. The experimental conditions were assigned sequentially to the participants. In exchange for their participation, participants received a base payment of $3 plus a $1 bonus for every 20 and 100 play dollars they had at the end of the game for low and high price exposure conditions, respectively. Each participant began with 100 play dollars. Gameplay was the same as in Experiment 1 except for the cost and selling price parameters.

All 40 participants completed the study. Table 16 provides a summary of our main results, and Figure 5 shows the average order in the final round (round 8) for each exposure condition. Exposure condition significantly affects orders such that orders under the high price exposure condition are less than the orders under the low price exposure condition ($F(1,38) = 58.78, p < .001$). These order quantities were also significantly different from $q^e = 7$. Orders under the high selling price exposure condition were less than 7 ($t (19) = -3.56, p = 0.002$), but orders under the low selling price exposure condition were greater.
than 7 (t(19) = −7.02, p < 0.001).

All participants correctly answered 2 and 12 for the minimum and maximum possible demand that could be generated by rolling 2 dice. Participant estimates of the long-run average demand were not significantly affected by condition (F(1, 38) = 0.35, p = 0.5602) nor were they significantly different from 7 (t(19) = −1.45, p = 0.1625; t(19) = −1.00, p = 0.330). Their average estimates by condition were 6.90, 6.95 for high and low cost exposure conditions, respectively. The actual mean demands were 6.938 and 7.031 under high and low price exposure conditions, respectively.

**Transaction Utility Estimates** We followed the same procedures as in Experiment 1 to structurally estimate $\tau^p$. This time, our structural model assumes that the decision maker’s average order quantities were normally distributed with mean $G^{-1}\left(\frac{6+\tau^p-3}{6+\tau^p}\right) = G^{-1}\left(\frac{3+\tau^p}{6+\tau^p}\right)$. The resulting maximum likelihood estimates and 95% confidence intervals for $\tau^p$ under the high and low selling price exposure conditions are shown in Table 17. The estimates show that when past selling prices are low the transaction utility is significantly positive, but when past selling prices are high the transaction utility is significantly negative. Recall the actual selling price is $p = 6$. However, individuals in the high selling price exposure condition ($p = 12$) also perceive an additional psychological loss of $\$1.4$ per unit ordered due to selling transaction utility, while individuals in the low selling price exposure condition ($p = 3$) also perceive an additional psychological gain of $\$10.2$ per unit ordered due to selling transaction utility.

<table>
<thead>
<tr>
<th>Selling Price Exposure Condition</th>
<th>MLE Estimate for $\tau^p$</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>−1.404</td>
<td>(−1.556, −1.253)</td>
</tr>
<tr>
<td>Low</td>
<td>10.213</td>
<td>(8.265, 12.162)</td>
</tr>
</tbody>
</table>
3.4.3 Experiment 3: Effect of portfolio

Experimental Design and Hypothesis The purpose of Experiment 3 is to examine the behavioral effect of the product portfolio on inventory decisions. We examine three stylized portfolios that each consist of two kinds of products: high-profit products \((c = 2, p = 6)\), and low-profit products \((c = 4, p = 6)\). The three portfolios considered differ in the proportion of high and low-profit products. See Table 18 for a summary of these conditions. For each product, the subjects roll two fair six-sided dice, the sum of which determines the demand for that product. We have the following hypotheses from Corollary 2.

Hypothesis 3a: For high profit products, orders in the high-profit portfolio are greater than orders in the medium profit portfolio, which in turn are greater than orders in the low profit portfolio.

Hypothesis 3b: For low profit products, orders in the high-profit portfolio are greater than orders in the medium profit portfolio, which in turn are greater than orders in the low profit portfolio.

Table 18: Description of product portfolios in Experiment 3.

<table>
<thead>
<tr>
<th>Portfolio condition</th>
<th>Number of high-profit products ((c = 2, p = 6))</th>
<th>Number of low-profit products ((c = 4, p = 6))</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-profit</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>Medium-profit</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Low-profit</td>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

Methods We recruited 60 undergraduate and graduate students from the same American university. The experimental conditions were assigned sequentially to the participants. In exchange for their participation, participants received a minimum of $3 plus a $1 bonus for every 50 play dollars they had at the end of the game. Each participant began with 100 play dollars.

Gameplay was the same as in Experiments 1 and 2, except that there were 14 stores.
Figure 6: Order decision in Experiment 3.

(represented by 14 index cards), and all 14 ordering decisions were made before demands were generated by dice rolls. In other words, the time horizon was effectively one round, even though each participant made 14 order decisions. For each store, a participant decided an order quantity, placed that many poker chips into the appropriate store (represented by different index cards), and made the appropriate cost payment. Then, for each product, the participant rolled the two dice, determined how many units were sold, and received the appropriate revenue. Again, at the end of the experiment subjects answered the same comprehension questions as in the previous two experiments.

Results All 60 participants completed the experiment. See Table 19 for a summary of the results. Figure 6 shows the resulting average order quantities for the high and low profit products by portfolio condition.

For high-profit products, orders were significantly affected by the product portfolio ($F(2, 57) = 10.59, p < .001$). Orders were notably higher when in the low-profit portfolio - significantly greater than orders in the high profit portfolio ($F(1, 57) = 20.09, p < .001$) and the medium profit portfolio ($F(1, 57) = 9.90, p = .003$). Orders in the medium and low profit portfolio conditions were not significantly different from each other ($F(1, 57) = $
1.78, p = .187). This asymmetry is further discussed later in the section.

For low-profit products, orders were affected by the product portfolio with marginal significance ($F(2, 57) = 2.34, p = .105$). However, orders exhibited a similar pattern as for the high-profit products. Orders for low-profit products were notably lower when in the high-profit portfolio - significantly less than orders in the low-profit portfolio ($F(1, 57) = 4.00, p = 0.050$) and marginally significantly less than orders in the medium profit portfolio ($F(1, 57) = 2.48, p = 0.092$). The difference between orders in the high and medium profit portfolio conditions were not significantly different from each other ($F(1, 57) = 1.78, p = .187$).

All participants correctly answered 2 and 12 for the minimum and maximum possible demand that could be generated by rolling 2 dice. Participant estimates of the long-run average demand by condition were 6.90 , 6.85, 6.85 for high, medium, and low portfolio conditions, respectively. These were not significantly affected by condition ($F(2, 57) = 0.09, p = 0.917$) nor significantly different from 7 ($t (19) = −1.45, p = 0.1625; t (19) = −1.14, p = 0.2674; t (19) = −1.83, p = 0.0828$). The actual mean demands were 7.136, 6.968, and 7.146 under high and medium and low profit portfolio conditions, respectively.

Table 19: Mean and standard deviations of ordering quantities in Experiment 3.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Average order quantity</th>
<th></th>
<th></th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>For high profit products $\left( c = 2, p = 6 \right)$</td>
<td>For low profit products $\left( c = 4, p = 6 \right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High-profit</td>
<td>8.138 (1.184)</td>
<td>5.575 (1.150)</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Medium-profit</td>
<td>8.707 (1.491)</td>
<td>6.192 (1.188)</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Low-profit</td>
<td>10.050 (1.356)</td>
<td>6.293 (1.078)</td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

**Transaction Utility Estimates** We follow a similar procedure as in Experiment 1 to structurally estimate $\tau^c$. However, because Experiment 3 does not control for the pull-to-center effect, we estimate differences between the transaction utilities across portfolios. We assume orders are normally distributed with mean $G^{-1} \left( \frac{6−2+\tau^c−\Delta_h}{6} \right)$ for high profit products and $G^{-1} \left( \frac{6−4+\tau^c+\Delta_l}{6} \right)$ for low profit products, where $\Delta_h, \Delta_l \geq 0$ represents the pull-to-center effect for high and low profit products, respectively. We assume $\Delta_h, \Delta_l$ are equal across
portfolio conditions. Let $\tau^c_h$, $\tau^c_m$, $\tau^c_l$ denote the per unit purchase transaction utility for the high, medium, and low profit portfolio conditions, respectively. The resulting maximum likelihood estimates for the differences between these transaction utilities are shown in Table 20. The table shows that for both high and low profit products, the transaction utility decreases in the profitability of the portfolio (i.e., in the proportion of high to low profit products). This is because the reference cost is decreasing in the profitability of the portfolio (Assumption 2), and the purchase transaction utility is increasing in the reference cost.

Table 20 also provides an intuitive interpretation for the asymmetry observed in the ordering decisions. For high profit products, $\tau^c_m - \tau^c_l$ is greater in absolute value than $\tau^c_h - \tau^c_m$. On the other hand, for low profit products, $\tau^c_h - \tau^c_m$ is greater in absolute value than $\tau^c_m - \tau^c_l$. In other words, for high profit products, transaction utilities are sharply increasing as high profit products become very rare in the portfolio. For low profit products, transaction utilities are sharply decreasing as low profit products become very rare in the portfolio. This suggests that reference costs and reference prices influence order decisions the most when the product has a distinctly different cost and/or price from other products in the category, which have relatively similar costs and prices.

Table 20: MLE for transaction utility differences between portfolios in Experiment 3.

<table>
<thead>
<tr>
<th></th>
<th>High profit product</th>
<th>Low profit product</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^c_m - \tau^c_l$</td>
<td>-1.875</td>
<td>-0.088</td>
</tr>
<tr>
<td>$\tau^c_h - \tau^c_m$</td>
<td>-0.704</td>
<td>-0.502</td>
</tr>
</tbody>
</table>

3.4.4 Experiment results discussion

The three laboratory experiments support the reference effects model. Consistent with Hypotheses 1a and 1b, Experiment 1 finds that orders are biased high when past costs are higher than the current cost, but biased low when past costs are lower than the current cost. Consistent with Hypotheses 2a and 2b, Experiment 2 finds that orders are biased high when past selling prices are lower than the current price, but biased low when past selling
prices are higher than the current price. Finally, consistent with Hypotheses 3a and 3b, Experiment 3 finds that orders are sensitive to the portfolio such that orders are higher for rare high-profit products, but lower for rare low-profit products.

As discussed in the introduction, the heuristic of anchoring on previous ordering decisions and adjusting towards the optimal solution would predict the opposite behavior to what we find in Experiments 1 and 2. Thus, the reference effect in our experiments is apparently stronger than the anchoring and adjustment heuristic effect. We conjecture that this is likely because the change in the cost or price in our experiments is sufficient to reduce the salience of the previous round’s order as an anchor, or altogether eliminates the use of the previous round’s order as an anchor.

Before we continue to discuss the application of reference effects to supply chains, we note a few immediate managerial implications of our experimental results. In general, our experiments show that individuals are irrationally influenced by past costs and prices and by other costs and prices in the portfolio. Thus, if one is interested in making expected profit maximizing decisions, our results suggest that the times most important for intervention and or training are right after a sudden increase or decreases in costs or prices, and for a product that has a distinctly different cost and/or price from the other products in the category. Alternatively, it may be prudent to manipulate the environment to try and avoid reference effects altogether. For instance, to avoid the reference effect of portfolios, a firm could organize products according to their cost and prices so that all products within a manager’s category have similar costs and prices.

3.5 Application to Supply Chains

We now extend the reference effects model in Section 3 to a supply chain setting by explicitly modeling a supplier. The supplier may choose the wholesale price to the newsvendor (Section 5.2) or the retail price to the customer (Section 5.3). Finally, we discuss a supply chain inefficiency that arises due to reference effects, the behavioral price whip (Section 5.4).
3.5.1 Notation

We consider a supply chain consisting of a supplier selling to a newsvendor. Refer to Figure 7. A supplier produces at cost $c$ and sells to the newsvendor at wholesale price $w$. The newsvendor in turn decides the order quantity $q$, and sells to random customer demand $D$ at unit price $p$. Thus, the supplier’s profit is

$$\Pi^S(q) = (w - c)q$$

and the newsvendor’s profit is

$$\Pi^N(q, D) = -cq + p \min(q, D)$$

To simplify exposition, we assume the following additive customer demand form

$$D = a - bp + \varepsilon$$

where $\varepsilon \sim \text{Uniform}(0, m)$, and $a, b > 0$.

3.5.2 Wholesale price

Consider a supplier who sets $w$ while $p$ is fixed.

Rational model A rational newsvendor orders according to the expected profit-maximizing criteria, $q^e$. This is the problem studied by Larivierre and Porteus (2001). In our special
case of uniform demand, the newsvendor order is

\[ q^e(w) = a - bp + m \left( \frac{p - w}{p} \right) \]

and the supplier’s optimal wholesale price when the newsvendor is expected profit maximizing is

\[ w^{e*} = \arg \max_w \{(w - c)q^e(w)\} = \frac{(1 + \frac{a-bp}{m})p + c}{2} \]

**Reference effects model** We now show how the optimal wholesale price changes if the newsvendor is subject to reference effects. We capture the newsvendor’s purchasing transaction utility through the linear function \( \tau^e(w - w^r) = -\theta_n \left( \frac{w - w^r}{w^r} \right) \), \( 0 < \theta_n < 1 \). This model is motivated by the Weber-Fechner law in psychophysics, and Winer (1988) implements a similar form to model customer transaction utility. Assume the selling transaction utility is zero (i.e., \( p^r = p \)) and that \( w^r < p \). From Equation (8) the newsvendor’s transaction utility is

\[ U^T_n(q, D) = -\theta_n \left( \frac{w - w^r}{w^r} \right) q. \]

Denote \( q^r \) the newsvendor order quantity under reference effects. Thus, the newsvendor’s response function is

\[ q^r(w) = b \left( \frac{p - w - \theta_n \left( \frac{w - w^r}{w^r} \right)}{p} \right). \]

**Proposition 16**. The supplier’s optimal wholesale price when the newsvendor is subject to reference effects is

\[ w^{rs*} = \frac{(1 + \frac{a-bp}{m})p + (1 + \frac{\theta_n}{w^r})c + \theta_n}{2(1 + \frac{\theta_n}{w^r})} \]

which is decreasing in \( \theta_n \) and equal to \( w^{e*} \) when \( \theta_n = 0 \).

**Proof.** Taking the derivative of the supplier’s profit with respect to \( w \), we obtain

\[ \frac{d\Pi^S}{dw} = a - bp + m \left[ p - \left( 1 + \frac{\theta_n}{w^r} \right) (2w - c) + \theta_n \right] \]
The supplier’s optimal wholesale price can be solved by the first order conditions. Next, note that for $\theta_n = 0$, we have $w^{r*} = w^{e*}$. Finally,

$$
\frac{dw^{r*}}{d\theta_n} = \frac{2(1 + \frac{c}{w^r})(1 + \frac{\theta_n}{w^r}) - \frac{2}{w^r} \left[(1 + \frac{a-bp}{m})p + (1 + \frac{\theta_n}{w^r})c + \theta_n\right]}{4(1 + \frac{\theta_n}{w^r})^2}
$$

is less than zero when

$$
\frac{w^r}{p} < 1 + \frac{a-bp}{m}
$$

which holds because $w^r < p$. Thus, $w^{r*}$ is decreasing in $\theta_n$.

The proposition shows that the supplier’s optimal wholesale price is lower when the newsvendor is subject to reference effects than when the newsvendor is expected profit maximizing. This is because there is an additional behavioral value to charging a lower wholesale price - it provides the newsvendor with additional transaction utility. Furthermore, the lower the reference wholesale price is or the more sensitive transaction utilities are, the lower the optimal wholesale price should be.

This result suggests that even if there is no direct competition among suppliers for the newsvendor’s order, there may be behavioral competition. Thus, if other wholesale prices in the category are low, the supplier should also lower her wholesale prices to mitigate the transaction utility effect. Additionally, this result suggests that a supplier may not want to pass a temporary cost increase on to the newsvendor’s wholesale price. This is because the newsvendor may lower her order even more than she should rationally because the increased wholesale price will induce a negative transaction utility when compared to the regular, less expensive, wholesale price.

### 3.5.3 Selling price

Next, consider a supplier who has the power to set $p$ while $w$ is fixed.
Rational model  Again, a rational newsvendor orders according to the expected profit-maximizing criteria, $q^e$.

$$q^e(p) = a - bp + \theta_c \left( \frac{p^r - p}{p^r} \right) + m \left( \frac{p - w}{p} \right)$$ (11)

and the supplier’s optimal retail price when the newsvendor is expected profit maximizing is

$$p^{e*} = \arg \max_p \{(w - c)q^e(p)\} = \sqrt{\frac{mw}{b}}$$

Reference effects model  Under reference effects, both newsvendor and customer must be considered when choosing the retail selling price. Assume the reference price $p^r$ is the same for both the newsvendor and customer demand.

We capture reference effects in customer demand according to a linear relative difference model (see Winer 1988, Popsecu and Wu 2007).

$$D^r(p) = a - bp + \theta_c \left( \frac{p^r - p}{p^r} \right) + \varepsilon.$$ Therefore, we can interpret $\theta_c$ as a measure of customer demand’s sensitivity to reference effects. Customer demand is increasing in the reference selling price, $p^r$.

For the newsvendor, we assume $w^r = w$. From Equation (8) the newsvendor’s transaction utility is

$$U^{T}_n(q, D) = \theta_n \left( \frac{p - p^r}{p^r} \right) \min(q, D).$$ (12)

Assume the newsvendor correctly anticipates the reference effect on customer demand (see next section for further discussion). The newsvendor’s order quantity is

$$q^r(p) = a - bp + \theta_c \left( \frac{p^r - p}{p^r} \right) + m \left( \frac{p + \theta_n(p - p^r) - w}{p + \theta_n(p - p^r)} \right)$$

We have
Proposition 17. If both the newsvendor and customer are subject to reference effects, the supplier’s optimal selling price $p$ is

$$p^{**} = \left(1 + \frac{\theta_n}{p^{**}}\right) \sqrt{\frac{mw}{b + \frac{\theta_c}{p^{**}}} + \frac{\theta_n}{1 + \frac{\theta_n}{p^{**}}}}$$

which is increasing in $\theta_n$, decreasing in $\theta_c$ and equal to $p^{**}$ when $\theta_c = \theta_n = 0$.

Proof. Taking the derivative of the supplier’s profit with respect to $p$ we obtain

$$\frac{d \Pi^S}{dp} = a - bp + \frac{m}{p} \left[ p - \left(1 + \frac{\theta_n}{w^*}\right)(2w - c) + \theta_n \right]$$

The first order conditions give us $p^{**}$, and it is straightforward to show that $\frac{\partial p^{**}}{\partial \theta_n} < 0$ and $\frac{\partial p^{**}}{\partial \theta_c} > 0$. 

These results demonstrate that the optimal selling price $p$ depends on both the transaction utilities of the customer and the newsvendor. Furthermore, these transaction utilities have opposite effects on the optimal selling price. When the price is relatively low the customer enjoys additional transaction utility while at the same time the newsvendor suffers negative transaction utility. Thus, the supplier’s optimal pricing policy depends on the relative strength of these two reference effects.

This implies that suppliers must consider how a pricing strategy affects both the consumer and the retailer. For instance, setting the retail price lower than the competition may provide an additional customer demand boost due to transaction utility. However, the supplier will not enjoy an increase in profits if the low price induces a negative transaction utility for the newsvendor which causes them to be reluctant to stock enough inventory. Similarly, when a supplier runs a price promotion by lowering the retailer price to the customer, they must be careful to evaluate whether the retailer will order enough to make the promotion profitable.
3.5.4 Behavioral pricing whip

In Section 5.3, we assumed that the newsvendor could accurately predict the reference price effect on customer demand. If the newsvendor ignores consumer reference effects, we say that the newsvendor is “naive”. Next we demonstrate that if the newsvendor is naive, reference effects cause a supply chain inefficiency: the newsvendor order is biased low when demand is biased high, and vice versa.

We use a hat to denote that the newsvendor is naive to customer demand reference effects. Thus, $q^\hat{e}$ is the newsvendor’s order quantity who is expected profit maximizing and naive to the reference effect on customer demand.

$$q^\hat{e} = a - bp + m \left( \frac{p - w}{p} \right)$$

Similarly, $q^\hat{r}$ is the newsvendor’s order quantity who is subject to reference effects and naive to the reference effect on customer demand. As in Section 5.3, we focus on the reference effect of the selling price $p$ by assuming that $w^r = w$ so that the newsvendor’s transaction utility is given in Equation (12).

$$q^\hat{r} = a - bp + m \left( \frac{p + \theta_n(p - p^r) - w}{p + \theta_n(p - p^r)} \right)$$

Recall, the newsvendor’s optimal order quantity $q^e$ which accurately takes into account customer reference effects is given in Equation (11). Direct comparison of these three quantities gives the following proposition.

**Proposition 18.** $q^\hat{r} < q^\hat{e} < q^e$ if $p < p^r$ but $q^\hat{e} > q^\hat{r} > q^e$ if $p > p^r$.

The proposition shows that if the newsvendor is naive to the effect the reference price has on the customer, she is biased away from the direction of the optimal solution. This suggests that the mismatch cost due to reference effects is doubly severe: a relatively high selling price reduces the customer demand but increases the newsvendor’s order, and a relatively low selling price increases the customer demand but decreases the newsvendor’s
order. We call this this phenomenon a behavioral price whip because it is similar in spirit to the bullwhip effect.

One practical implication of the behavioral price whip is that for products that are sold over a long time horizon, high-low pricing may be inefficient for behavioral reasons. This is because each time there is a swing in the sales price, the behavioral price whip causes inefficiency. Thus, there is a behavioral advantage for everyday low pricing.

3.6 Conclusion

In this paper we argued that inventory managers experience transaction utility and are therefore subject to reference effects. Subsequently, past prices and other prices in the product portfolio may irrationally influence order decisions in predictable ways. Reference effects may deteriorate the expected profit of a firm and the performance of the supply chain. Our results serve to inform efforts to mitigate the reference effects, or from a supply chain perspective, react optimally. Finally, reference effects and transaction utility have more applications in operations management than those studied in this paper. For instance, how do reference effects influence revenue managers and service managers? We believe the application of reference effects in the modeling of operations managers can significantly enrich our understanding of real world systems.
Appendix A

Proof of Proposition 3

From (3) we have that

\[ P_j(t|o_j + \delta_j) = P_j(t - \delta_j|o_{j+1}) - I_j^f(t - \delta_j|o_{j+1}, o_j) \]
\[ P_j(t|s_j + \delta_j) = P_j(t - \delta_j|s_{j+1}) - I_j^f(t - \delta_j|s_{j+1}, s_j) \]
\[ P_j(t|r_j + \delta_j) = P_j(t - \delta_j|r_{j+1}) - I_j^f(t - \delta_j|r_{j+1}, r_j) \]

and from (1) we obtain

\[ P_j(t|o_j + \delta_j) = P_j(t|o_{j+1} + \delta_j) - B_{j+1}^p(t - \delta_j) - IT_j^p(t - \delta_j) - IT_{j-1}^p(t - \delta_j) + B_j^p(t - \delta_j) \]
\[ P_j(t|s_j + \delta_j) = P_j(t|s_{j+1} + \delta_j) - IT_j^p(t - \delta_j) - IT_j^p(t - \delta_j) \]
\[ P_j(t|r_j + \delta_j) = P_j(t|r_{j+1} + \delta_j) - IT_j^p(t - \delta_j) - IT_{j-1}^p(t - \delta_j) \]

which, with initial conditions

\[ P_j(t|o_n + \delta_j) = P_j(t - \delta_j|o_n) \]
\[ P_j(t|s_n + \delta_j) = P_j(t - \delta_j|o_n) - B_{n+1}^p(t - \delta_j) \]
\[ P_j(t|r_n + \delta_j) = P_j(t - \delta_j|o_n) - B_{n+1}^p(t - \delta_j) - IT_n^p(t - \delta_j) \]

we obtain
Proof of Lemma 4.

Proof. We will show the first equality by starting with the right hand side. First, note that the financed inventory is unchanged if one simultaneously looks forward by time \( \delta_j \) days and also delays all payment times by \( \delta_j \) days. Apply this to the first term to obtain

\[
P_j(t|o_j + \delta_j) = P_j(t - \delta_j|o_n) - \sum_{i=j+1}^{n} [IT^p_i(t - \delta_j) + P^p_i(t - \delta_j)] + B^p_{j+1}(t - \delta_j)
\]

Second, we can always divide financed inventory into two parts. We have that for any time \( \bar{\tau} \in \mathcal{T} \),

\[
I^f_j (t| (\rho_j, \rho_{j-1})) + I^f_j (t| (\rho_j + \delta_j, \rho_{j-1} + \delta_{j-1}))
\]

Third, we can always divide financed inventory into two parts. We have that for any time \( \bar{\tau} \in \mathcal{T} \),

\[
I^f_j (t| (\rho_j, \rho_{j-1})) + I^f_j (t| (\rho_j + \delta_j, \rho_{j-1} + \delta_{j-1}))
\]

The proof of the second equality follows the same logic.

Proof of Proposition 4.

Proof. From Proposition 3 we know that each payment time can be expressed as a linear combination of the physical inventory, the physical backorders and the physical in-transit

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inventory. Moreover, using Lemma 4, we can separate out the reference events from the payment delays. Thus, all that is left to show is that each coefficient is in the set \{-1, 0, 1\}. To do this, simply note that each term in the expression for \(P(t|o_j), P(t|s_j), P(t|r_j)\) has the same sign in its coefficients. Therefore, when subtracting \(P(t|\rho_j) - P(t|\rho_{j-1})\), the resulting coefficient should be either \(-1, 0\) or \(1\).

**Proof of Proposition 5.**

*Proof.* We know that the total financed inventory is

\[
\sum_j I^F_j(t|\tau_j, \tau_{j-1}) = I^F_j(\tau_n, \tau_0)
\]

\[
= P(t|\tau_n) - P(t|\tau_0)
\]

\[
= P(t|s_n) - P(s_0)
\]

\[
= \sum_{i=0}^n IT^P_i(t) + I^P_i(t)
\]

and so the result follows.

**Proof of Lemma 5.**

*Proof.* The margin backorders are also unchanged if one simultaneously looks backwards in time \(\delta_{j-1}\) days and advances all payment times by \(\delta_{j-1}\) days. Thus we have that

\[
B^f_j(t|\rho_{j-1} + \delta_{j-1}) = B^f_j(t - \delta_{j-1}|\rho_{j-1} + \delta_{j-1} - \delta_{j-1})
\]

\[
= B^f_j(t - \delta_{j-1}|\rho_{j-1})
\]

and the result follows from the fact that margin backorders do not depend on the time payments are made.

**Proof of Proposition 10.**
Proof. From Proposition 3 we know that

\[ P(t|o_j + \delta_j) = P(t - \delta_j|o_n) - \sum_{i=j+1}^{n} [IT^p_i(t - \delta_j) + P_i^p(t - \delta_j)] + B^p_{j+1}(t - \delta_j) \]

\[ P(t|s_j + \delta_j) = P(t - \delta_j|o_n) - \sum_{i=j+1}^{n} [IT^p_i(t - \delta_j) + P_i^p(t - \delta_j)] \]

\[ P(t|r_j + \delta_j) = P(t - \delta_j|o_n) - IT^p_i(t - \delta_j) - \sum_{i=j+1}^{n} \left[ I^p_i(t - \delta_j) + IT^p_{i-1}(t - \delta_j) \right] \]

and we know that under an echelon base-stock policy, we have

\[ IOP_j(t) = B_{j+1}(t) + IT^p_j(t) + P^p_j(t) - B_j(t) \]

\[ = O_j(t) - S_j(t) + S_j(t) - R_j(t) + R_j(t) - S_{j-1}(t) - O_{j-1}(t) + S_{j-1}(t) \]

\[ = O_j(t) - O_{j-1}(t) \]

\[ = y_j \]

Because \( O_n(t) = D(t) \), we have

\[ O_j(t) = D(t) - \sum_{i=j+1}^{n} y_i \]  \hspace{1cm} (13)

We can then obtain the following equalities by substitution:

\[ R_j(t) = D(t) + IN^p_j(t) - \sum_{i=j}^{n} y_i \]

\[ S_j(t) = D(t) - B^p_{j+1}(t) - \sum_{i=j+1}^{n} y_i \]
and the desired result follows

\[
P_j(t|\tau_j) = \begin{cases} 
D(t - \delta_j) - \sum_{i=j+1}^{n} y_i & \text{if } \rho_j = o_j \\
D(t - \delta_j) - \sum_{i=j+1}^{n} y_i - B^p_{j+1}(t - \delta_j) & \text{if } \rho_j = s_j \\
D(t - \delta_j) - \sum_{i=j}^{n} y_i + IN^p_j(t - \delta_j) & \text{if } \rho_j = r_j
\end{cases}
\]

\[\square\]
Appendix B: Comparison to Prelec and Loewenstein (1998)

Prospective Accounting Model

Prelec and Loewenstein (1998) model prospective accounting for a consumer through a “double-entry” model of mental accounting. In the context of the newsvendor problem, the “double-entry” feature of their model suggests that the newsvendor imputes payments twice: once from the vantage point of the order and once from the vantage point of the demand realization. Their model is quite sophisticated—including loss aversion, time-discounting, and the idea of coupling. Here, we focus on the coupling feature. In the newsvendor context, “coupling” qualifies the prospective accounting rule by allowing only partial appreciation of payments looking forward. Thus, we add a coupling term $b$, $0 \leq b \leq 1$ to denote how strongly an outgoing payment at the time of order is “buffered” by the thought of future incoming payments. Similarly, we add a coupling term $a$, $0 \leq a \leq 1$ to denote how strongly an incoming payment at the time of order is “attenuated” by the thought of future outgoing payments. The resulting rewards are:

$$R^i(q, D) = \begin{cases} 
-cq + bp \min(q, D) + p \min(q, D) & \text{if } i = O, \\
2[(p - c) \min(q, D) - c \max(q - D, 0)] & \text{if } i = S, \\
(p - c)q - ap \max(q - D, 0) - p \max(q - D, 0) & \text{if } i = C.
\end{cases}$$

The above formulation also results in the prediction $q^O > q^S > q^C$. However, for simplicity, our prospective accounting model uses a simpler formulation that merely underweights the order-time payments. Under O, the cost is “buffered,” so we use the notation $\beta < 1$. Under C, the revenue is “attenuated,” so we use the notation $\alpha < 1$. 
Appendix C: Experiment Instructions to Participants

We present excerpts from the instructions of Studies 1 and 2. Participants in each treatment group are provided with the description of the relevant payment scheme only. Study 3 instructions are the same as Study 1 except for the values of the price and cost parameters and the reduced number of rounds.

Study 1 Instructions

In this simplified game, you own a business that makes money by selling widgets for 25 simulated days. At the beginning of each simulated day you decide how many widgets to have available for sale in your store that day.

{(O) At this time you pay $1 per unit and place the units in your store. Then roll the 3 dice to determine demand for that day. Sell as many units in your store that you can (the minimum of demand and units in your store) and receive $2 per unit that you sell.}

{(S) Your supplier sends you these units and you place them in your store. Then roll the 3 dice to determine demand for that day. Sell as many units in your store that you can (the minimum of demand and units in your store). You receive $1 profit per unit that you sell, but you must pay a penalty of $1 for each leftover unit.}

{(C) At this time you actually receive $1 per unit that you place your store (advanced payment.) Then roll the 3 dice to determine demand for that day. Sell as many units in your store that you can (the minimum of demand and units in your store). You receive $0 for each unit that you sell, but you must pay a penalty of $2 for each extra unit you have leftover.}

Then discard all leftover units from your store and start empty for the beginning of the next day. Your goal is to maximize your play cash by the end of the 25 days.
Study 2 Instructions

In this simplified game, you own a business that makes money by selling widgets (represented by poker chips) for 25 simulated days. At the beginning of each simulated day you need to decide how many widgets to order to have available for sale in your store that day. Place the units ordered in your store. Then roll the 3 dice to determine demand for that day. Sell as many units in your store that you can (the minimum of demand and units in your store). Then discard all leftover units from your store and start empty for the beginning of the next day.

\{(O) At the end of each day, you settle payments for that day. You pay $1 per unit that you ordered and you receive $2 per unit that you sold.\}

\{(S) At the end of each day, you settle payments for that day. You pay $1 per unit that is leftover and you receive $1 per unit that you sold.\}

\{(C) At the end of each day, you settle payments for that day. You receive $1 per unit that you ordered, but you pay $2 per unit that is leftover.\}

Your goal is to maximize your play cash by the end of the 25 days.
References


Biography

Jordan David Tong was born in Fresno, California on July 31st, 1985. He received his bachelor’s degree in mathematics from Pomona College in Claremont, California in 2007. After completing his Ph.D. in Operations Management at Duke University’s Fuqua School of Business in 2012, he will join the Wisconsin School of Business as an Assistant Professor of Operations and Information Management.