OPTIMAL MONETARY POLICY AND OIL PRICE SHOCKS

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Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Economics in the Graduate School of Duke University

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ABSTRACT

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Abstract

This dissertation is comprised of two chapters. In the first chapter, I investigate the role of systematic U.S. monetary policy in the presence of oil price shocks. The second chapter is devoted to studying different approaches to modeling energy demand.

In influential papers, Bernanke, Gertler, and Watson (1997) and (2004) argue that systematic monetary policy exacerbated the recessions the U.S. economy experienced in the aftermath of post World War II oil price shocks. In the first chapter of this dissertation, I critically evaluate this claim in the context of an estimated medium-scale model of the U.S. business cycle. Specifically, I solve for the Ramsey optimal monetary policy in the medium-scale dynamic stochastic general equilibrium model (henceforth DSGE) of Schmitt-Grohe and Uribe (2005). To model the demand for oil, I use the approach of Finn (2000). According to this approach, the utilization of capital services requires oil usage. In the related literature on the macroeconomic effects of oil price shocks, it is common to calibrate structural parameters of the model. In contrast to this literature, I estimate the parameters of my DSGE model. The estimation strategy involves matching the impulse responses from the theoretical model to responses predicted by an empirical model. For estimation, I use the alternative to the classical Laplace type estimator proposed by Chernozhukov and Hong (2003). To obtain the empirical impulse responses, I identify an oil price shock in a structural VAR (SVAR) model of the U.S. business cycle. The SVAR model predicts that, in response to an oil price increase, GDP, investment, hours, capital utilization, and the real wage fall, while the nominal interest rate and inflation rise. These findings are economically intuitive and in line with the existing empirical evidence. Comparing the actual and the Ramsey optimal monetary policy response to an oil price shock, I find that the optimal policy allows for more inflation, a larger drop in
wages, and a rise in hours compared to those actually observed. The central finding of this Chapter is that the optimal policy is associated with a smaller drop in GDP and other macroeconomic variables. The latter results therefore confirm the claim of Bernanke, Gertler and Watson that monetary policy was to a large extent responsible for the recessions that followed the oil price shocks. However, under the optimal policy, interest rates are tightened even more than what is predicted by the empirical model. This result contrasts sharply with the claim of Bernanke, Gertler, and Watson that the Federal Reserve exacerbated recessions by the excessive tightening of interest rates in response to the oil price increases. In contrast to related studies that focus on output stabilization, I find that eliminating the negative response of GDP to an oil price shock is not desirable.

In the second chapter of this dissertation, I compare two approaches to modeling energy sector. Because the share of energy in GDP is small, models of energy have been criticized for their inability to explain sizeable effects of energy price increases on the economic activity. I find that if the price of energy is an exogenous AR(1) process, then the two modeling approaches produce the responses of GDP similar in size to responses observed in most empirical studies, but fail to produce the timing and the shape of the response. DSGE framework can solve the timing and the shape of impulse responses problem, however, fails to replicate the size of the impulse responses. Thus, in DSGE frameworks, amplifying mechanisms for the effect of the energy price shock and estimation based calibration of model parameters are needed to produce the size of the GDP response to the energy price shock.
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Chapter 1

Optimal Monetary Policy and Oil Price Shocks

1.1 Introduction.

In this paper, I characterize the optimal monetary policy response to an oil price shock in an estimated model of the U.S. business cycle. Specifically, I solve for the Ramsey optimal monetary policy in a medium-scale Dynamic Stochastic General Equilibrium model (henceforth DSGE) with the demand for oil. I estimate the key structural parameters of the model by applying the Markov Chain Monte Carlo (MCMC) approach of Chernozhukov and Hong (2003) to the Impulse Response Function Matching estimator.

I find that in response to an oil price shock the Ramsey planner tightens interest rates even more than is predicted by the model. At the same time, she allows for more inflation. The reason for higher inflation is that changes in nominal wages are associated with bigger costs compared to the costs of changes in prices. The Ramsey planner lowers real wages to achieve an equilibrium increase of labor in response to the shock. The real wage decrease is achieved at the expense of higher prices rather than lower nominal wages. The Ramsey planner tightens the nominal interest rate to prevent the real interest rate from falling.

To obtain empirical impulse responses, I identify an oil price shock in a structural VAR (SVAR) model of the U.S. business cycle. The SVAR model predicts that, in response to an oil price shock, GDP, investment, hours, capital utilization, and real wage fall, while the interest rate and inflation rise. These findings are economically intuitive and in line with the existing research. ¹

For optimal policy analysis, I turn to theoretical modeling. In particular, I combine the medium-scale DSGE model of Schmitt-Grohé and Uribe (2005) with the approach of Finn (2000) to modeling oil demand. According to this approach, utilization of capital requires oil usage.

The present paper will engage most closely with the arguments of Bernanke, Gertler, and Watson (1997) and Bernanke, Gertler, and Watson (2004), who argue that the big negative effect of the historical oil shocks on the U.S. economy does not result from the oil shocks themselves but rather from the systematic tightening of monetary policy in response to oil price increases. To evaluate the effects of the oil price shocks that are due to the systematic monetary policy response, Bernanke, Gertler, and Watson (1997) run counterfactual experiments in the VAR model identifying the oil price shock. The experiments set coefficients of a monetary policy equation in the VAR to zero and study how the response of output to the oil shock changes in this modified setup. Bernanke, Gertler, and Watson (1997) show that systematic monetary policy in these experiments is responsible for a substantial fraction of the output drop.²

The criticism of the approach used in Bernanke, Gertler, and Watson (1997) and Bernanke, Gertler, and Watson (2004) is that empirical VAR models are not well-suited to conduct policy experiments because of the Lucas critique—the estimates of coefficients would be different under alternative policies, especially if the policies considered are very far from the observed ones. Trying to minimize the distortion associated with the Lucas critique, Bernanke, Gertler, and Watson (2004) consider another counterfactual—a temporary shutdown in the response of the Federal Funds rate to an oil price shock. However, Carlstrom and Fuerst (2006) argue that the Lucas critique must be quantitatively important there, as well. In this paper, I overcome the Lucas critique by introducing a theoretical model of oil price disturbances that replicates the predictions of my empirical SVAR model.

Leduc and Sill (2004) and Carlstrom and Fuerst (2006) study a related question in the context of a theoretical model. To evaluate the contribution of systematic monetary policy to the observed output drop after the oil price rises, these papers compare the monetary policy rule to an alternative unresponsive, or “constant,” monetary policy. However, in the context of a theoretical model, it seems more appropriate to compare monetary policy to an alternative that would do the least harm to the society trying to accommodate oil price shocks. In this paper, unlike Leduc and Sill (2004) and Carlstrom and Fuerst (2006), I consider the effect of the systematic monetary policy as compared with a policy that is optimal from a welfare point of view. I show that, indeed, optimal policy is associated with a smaller output drop than the model predicts. However, it is not desirable from the welfare point of view to completely offset the negative response of the value added to the oil price shock.

Another drawback of the models presented by Leduc and Sill (2004) and Carlstrom and Fuerst (2006) is that conclusions are drawn from calibrated, rather than estimated, models. This paper differs from the extant literature in that the parameters of the model are estimated to match the empirical evidence about the response of macroeconomic variables to an oil price shock. Moreover, unlike most theoretical papers modeling oil, which assume that the price of oil is stationary, I introduce the price of oil as a nonstationary process. This is in line with many empirical studies, which assume that the growth rate of oil prices is stationary. Overall, the model that I present in this paper is more realistic than the earlier theoretical models of the oil price shock.

The plan of the paper is the following. In section 1.1, I describe the empirical strategy

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3In Leduc and Sill (2004), “constant” policy is a k-percent money growth rule. Carlstrom and Fuerst (2006) find that the estimated output effect of a given monetary policy rule depends crucially on the choice of “constant” monetary policy. Besides the money growth peg considered by Leduc and Sill (2004), Carlstrom and Fuerst (2006) suggest two other alternatives for constant policy - an interest rate peg and Wickselian monetary policy, a policy that adjusts the interest rate so that the real economy behaves as if there were no nominal rigidities. This policy, however, is not optimal from the welfare point of view because it does not take into account that nominal rigidities generate output losses.

to identify the oil price shock. Section 1.3 gives a short review of the literature on different approaches to modeling the oil sector and different views on the relative importance of oil shocks and monetary policy. Section 1.3 presents the theoretical model. Section 1.5 describes the estimation strategy and the results of estimation. In Section 1.6, I discuss the role of some assumptions of the model. Section 1.7 focuses on the evaluation of monetary policy contribution to the oil price shock effects. Section 1.8 concludes.

1.2 An Empirical Model of the Effects of Oil Price Shocks

To estimate the effects of the oil price shock, I rely on a SVAR model. In this model, I use the following quarterly data over the period 1954:III - 2006:IV\(^5\) with the total of \(T = 210\) observations:

1. \(P_{oil}^t\) - West Texas Intermediate spot oil price, monthly data from the Dow Jones & Company, from the FRED database, the series was aggregated into quarterly data using geometric average over 3 months;

2. \(GDP_t\) - seasonally adjusted quarterly GDP, per capita,\(^6\) from the NIPA tables, BEA;

3. \(Hours_t\) - seasonally adjusted quarterly index of hours of all persons in the non-farm business sector, 1992 = 100, BLS statistics, from the FRED database, per capita;

4. \(\pi_t\) - first difference of the log of the GDP deflator. The GDP deflator is calculated as the ratio of nominal to real GDP, both series are seasonally adjusted at annual rates, data from the NIPA tables, BEA;

5. \(CU_t\) - seasonally adjusted capacity utilization in the manufacturing sector, in percentages/100, quarterly data, calculated by the Federal Reserve Bank of St. Louis using monthly data from the Board of Governors of the Federal Reserve System;

---

\(^5\)The choice of the initial date is defined by the availability of data.

\(^6\)Per capita series are obtained everywhere in this paper using the monthly labor force statistics from www.bls.org. Monthly data are aggregated into quarterly by simple averaging over 3 months.
6. $W_t$ - seasonally adjusted quarterly index of the compensation per hour in the non-farm business sector, 1992 = 100, BLS statistics, from the FRED database;

7. $C_t$ - consumption of services and nondurables excluding gasoline, fuel oil, and the consumption of other energy goods, plus government consumption expenditures; all series are quarterly and seasonally adjusted at annual rates,\(^7\) data from the NIPA tables, BEA, per capita;

8. $I_t$ - nominal investment, obtained as a sum of consumption of durable goods and private investment, both series quarterly and seasonally adjusted at annual rates, data from the NIPA tables, BEA, per capita;

9. $R_t$ - Effective Federal Funds Rate, monthly data aggregated into quarterly using simple geometric averaging over three months, the Board of Governors of the Federal Reserve System, from the FRED database.

In the model, the vector of $n = 9$ random variables, $Y_t$, evolves according to the following dynamic process

$$A^0 Y_t = \alpha + A(L) Y_t + \epsilon_t$$

where $A^0$ is a square matrix of the size $9 \times 9$, $\alpha$ is a constant vector of the size $n \times 1$, $A(L) = A_1 L + A_2 L^2 + ... + A_k L^k$, the size of $Y_t - 9 \times 1$, $k$ - number of lags of VAR. The 9 variables in $Y_t$ are listed below:

\(^7\)Consumption of energy is excluded from the measure of consumption because fuel consumption is not modeled in our theoretical framework.
Wages, consumption and investment enter $Y_t$ as shares in GDP to accommodate long-term growth of these variables. The Federal Funds rate represents the monetary policy reaction to changes in macroeconomic indicators. The Federal Funds rate follows the block of macroeconomic variables that presumably do not react contemporaneously to monetary policy changes. This assumption is common in the literature estimating the monetary policy shock. The placement of the Fed Fund rate as the last element in the vector $Y_t$ is not critical for the analysis presented below, because the model does not identify the monetary policy shock. Besides, the predictions of the model are robust to the position of this variable in the VAR.

The first variable in the VAR identifies the oil price shock. I use only the increases of the nominal price of oil to disentangle the exogenous component of an oil price shock. Namely, in the oil price growth time series, I substitute negative values with zeros. Eliminating oil price drops from the model of the oil price shock helps to take into account the fact that the oil price has an asymmetric effect on the U.S. economy. This observation is documented by a number of papers. Mork (1989) shows that the effect after increases in the oil price

---

8Most VAR models use Federal Funds Rate as indicator of monetary policy (See, for example, Altig, Christiano, Eichenbaum, and Lindé (2004), Bernanke, Gertler, and Watson (1997) among others.) Bernanke and Blinder (1992) and Bernanke and Mihov (1995) show that the Federal Funds rate is a good indicator of the policy stance after 1966.

6
is bigger in magnitude than the effect observed after the drops in the oil price. Besides, Hamilton (2003) supports the hypothesis of the nonlinearity in the effect of the oil price on the U.S. economy.

Hamilton (1996) uses a different indicator to identify the oil price shock. His indicator is defined as either the difference between the current price of oil and the maximum oil price over the previous year, or zero, whichever is greater. The reason Hamilton defines the oil price variable in such a way is that the oil price increases observed after 1985 usually corrected the previous quarter decreases, and the net increase in oil prices observed over the year helps eliminate such episodes. Defined in this way, Hamilton’s oil price variable is able to capture the negative correlation between GDP and the price of oil. In the technical appendix to this paper, which is available upon request, I show that the results of my SVAR model change only marginally when I use the Hamilton’s indicator.

Different from Mork (1989), I use the nominal, rather than the real, price of oil to identify the oil price shock. The real price of oil may be affected not only by the exogenous oil shock, but also by innovations in the price level that are due to other shocks orthogonal to the shock of interest, such as technology shocks, preference shocks and others. I include the nominal as opposed to the real oil price in my model because I want to identify the macroeconomic effect of the innovation to the nominal oil price.\footnote{Rotemberg and Woodford (1996) and Blanchard and Gali (2007) pursue the same idea.}

To identify the exogenous component of the oil price growth process, which I call the oil price shock, I impose short- and long-run restrictions on the dynamics of the oil price variable. Namely, the indicator of the oil price shock may respond to endogenous variables in the VAR, however, other shocks can not affect nominal oil price contemporaneously (short-run restriction) and no shocks other than the oil price shock have a long-lasting effect on the nominal price of oil (long-run restriction). This identification strategy results in overidentification in the VAR model, and is supported by the tests of overidentifying restrictions against other plausible alternatives.
One of the alternative identification strategies for the oil price shock I consider is the assumption of fully exogenous oil price process. Theoretical research most often treats oil price process as exogenous. At the same time, there exists empirical evidence that questions the exogeneity of the oil price relative to an economy as big as the United States. I find that treatment of the oil price as an exogenous process results in impulse responses that are only marginally different from the responses in the SVAR model that I use. However, this identification scheme is not supported by the test of overidentifying restrictions.

Among other alternative identifying restrictions I consider separately a long-run and a short-run identification of the oil price shock. I find that inducing the short-run restriction is crucial to obtain statistically significant impulse response functions that are intuitive from an economic point of view.

I estimate the SVAR model with 3 lags. The choice of lags is based on Akaike, Bayesian and HC information criteria. According to both BIC and HC reveal that 2 lags is an optimal choice of lags in the VAR model. However, according to AIC, the model with 4 lags dominates the model with smaller number of lags.

The impulse responses to the oil price shock implied by the SVAR model are shown in Figure A.0.4. The dashed lines in the figures display 90% confidence bands, which are computed using a bias-adjusted double bootstrap procedure of Kilian (1998), based on 10000 draws. I show responses in percentage deviations from trend, except for inflation and the interest rate. The responses for annualized inflation and the Federal Funds rate are displayed in percentages, as deviations from their mean values. As can be seen from Figure A.0.4, GDP, hours, investment and capital utilization fall in response to a one standard deviation oil price shock, with an U-shaped responses and a trough occurring approximately 2 years after the shock. Inflation and the Federal Funds rate rise, with the peaks of the responses approximately 4 quarters after the shock. The Federal Funds rate reaches the maximum response of about 0.2% and inflation around 0.3% a year. These

\[\text{See Barsky and Kilian (2001) and Kilian (2006).}\]
results are in line with the empirical estimates of the effect of the oil price shock found in the literature.\textsuperscript{11} The real wage and consumption fall, although consumption response is not significantly different from 0 with probability 90%.

To see what is the role of oil price shocks in fluctuations of macroeconomic variables, I calculate the contribution of the shocks that are identified by my SVAR model. The numerical results are shown in Tables A.1 and A.2. Table A.1 presents the variance decomposition based on the historically observed shocks. Each number in the first column of the table estimates the variance produced by the historical sequence of the estimated shock process as a fraction of the unconditional variance of the series. The second column shows the contribution of the oil price shock to the unconditional volatility of the time series. Figure A.0.4 illustrates the results shown in Table A.1. It shows the HP-filtered time series of the variables from the SVAR model together with the series that are generated by the model where all the disturbances are shut down except for the oil price shock. Table A.2 shows conditional standard deviations of macroeconomic variables as well as the ratio of conditional to unconditional variance predicted by the SVAR model.

The results of the variance decomposition exercise suggest that the oil price shock is not the major, but still important source of business cycle fluctuations. As can be seen from Table A.1, the contribution of the shock to output volatility is around 8%. These numbers are similar to the results of Table 4 in Blanchard and Gali (2007).\textsuperscript{12} The contribution of the oil price shocks to the fluctuation of macroeconomic variables is smaller than that of the technology shocks in Altig, Christiano, Eichenbaum, and Lindé (2004)–13 and 15% for neutral and investment specific shocks correspondingly.\textsuperscript{13} Fisher (2007) comes up with a

\textsuperscript{11}Leduc and Sill (2004), Peersman (2005), and Dhawan and Jeske (2007), among others. The estimated effect of the oil price shock on the Federal Funds rate is larger (0.7\%) in Bernanke, Gertler, and Watson (1997) and Hamilton and Herrera (2004), which could result from working with monthly data series.

\textsuperscript{12}The results in the second column should be compared to the squared results in Blanchard and Gali (2007), because they show the relative standard deviations rather than relative variances.

\textsuperscript{13}The estimates of the contribution of the technology shocks to business cycle fluctuations are very different across empirical studies. The model of Altig, Christiano, Eichenbaum, and Lindé (2004) is most close to the SVAR model considered in this paper, which makes the comparisons
much larger estimate of the contribution of the investment specific shock to the business cycle. However, his results rely on a model with a smaller number of variables, thus the contribution ascribed to each of the shocks in his model is larger.

1.3 Modeling Oil in the Literature

Because oil and other sources of energy are close substitutes, I use the term oil to describe a generalized good that provides energy.

There are different ways to introduce oil sector in a theoretical macroeconomic model. The early attempts to modify the standard RBC model by adding oil as an additional factor of final good production technology, such as Kim and Loungani (1992), revealed a major flaw of the oil sector modeling - the share of oil expenditures in GDP was too small to produce significant output drops following an oil price shock.

A number of improvements were suggested by the consequent research. Rotemberg and Woodford (1996) show that modeling imperfect competition can help to deal with this anomaly. In their model, countercyclical markups allow the value added to drop by 2.5% in response to a 10% oil price shock, while competitive models generate output drops of only 0.5%.

Finn (2000) suggests that the standard RBC model can produce drops in output and wage as big as in Rotemberg and Woodford’s imperfect competition model, if utilization of capital is related to oil usage. Her result requires the assumption of a capital depreciation rate that varies with capacity utilization. In the absence of this additional propagation mechanism, the model of Finn (2000) is equivalent to a standard RBC-type oil-in-the-production model that can not explain big output drops.

Aguiar-Conraria and Wen (2007) rely on increasing returns to scale in the monopolistically competitive intermediate goods sector. The multiplier-accelerator propagation mechanism of an oil price shock in their model is very similar to the monopolistic markup...
setting of Rotemberg and Woodford. Besides, they make use of the amplification mechanism of variable depreciation. Both these features improve the ability of energy price shocks to generate sizeable recessions.

Among alternative models that are used to incorporate energy sector are Atkeson and Kehoe (1999) and Wei (2003). These models introduce a putty-clay mechanism of capital formation, which originates from Johansen (1959). The advantage of these models is that they are capable of generating enough non-linearities to produce asymmetric effects of shocks (Gilchrist and Williams (2000)). Atkeson and Kehoe (1999) use energy intensity of production as a putty-clay factor to study different substitutability of energy in the short- and long-run. Atkeson and Kehoe (1999) document, however, that compared to more standard, or putty-putty types models, the putty-clay model generates even a smaller response of output to an oil price shock.\footnote{Atkeson and Kehoe find that doubling of oil price results in only 5.3\% drop in output as opposed to their putty-putty model that delivers 33\% output drop in response to the oil price increase.}

Wei (2003) uses a putty-clay model with energy in the production function and variable capital utilization to study the relationship between oil price shocks and stock market prices. Although the response of wages in her model is large enough to account for the effect of the 1973-74 oil price shock, the model fails to generate a sizeable drop in output—the reported response of the value added, which is the real output less real oil expenditures, does not exceed 1\%, and is due mainly to the rise in the oil expenditures component of value added.\footnote{Taking into account that oil expenditure rise exclusively because of the oil price, the measure of value added composed by Rotemberg and Woodford and also used in Finn (that uses constant oil price expenditures) would drop even smaller, if not increase in response to the shock in Wei’s model.}

All in all, although putty-clay models may provide some insights into the asymmetric effect of the oil price shocks, they are not better suited to study correlation between oil price and business cycles than more standard RBC models. At the same time, these models are difficult to implement in quantitative research due to high dimensionality problem. For this reason, we proceed the next section with a more standard way to model oil sector. Namely, I take the approach of Finn (2000) to modeling oil. The next section describes the
model in more details.

1.4 A Theoretical Model of the Effect of Oil Price Shocks

While many authors have tried to model the effects of oil shocks, I know of no study that has looked at the problem in a model that can indeed explain the observed effects of oil price shocks. In this paper, I combine the DSGE model of Schmitt-Grohé and Uribe (2005) with the approach of Finn (2000) to modeling oil with the aim to present an empirically plausible model of the macroeconomic effect of oil price shocks. In this model, the features inherited from DSGE modeling provide flexibility in generating responses that better fit the responses of the data.

The model economy is inhabited by an infinite number of households, intermediate firms and competitive final good producing firms. Oil is introduced into the model using the approach proposed by Finn (2000) and also used in Leduc and Sill (2004). According to this approach, oil expenditures are tied to capital utilization. Namely, households have to use oil to supply capital services to firms. The amount of oil is proportional to the capital stock and depends on the intensity of capital utilization. I assume that oil is imported from abroad and is paid for using final goods with a zero trade balance in every period. This assumption reflects the fact that the U.S. economy is a net oil importer. Other than that, I assume that the economy is closed for capital and asset flows.

It is also assume that the capital depreciation rate depends on how intensively capital is used. To make the model better fit the data, I introduce a number of nominal and real rigidities, such as price and wage stickiness, habits in consumption, and investment adjustment costs. The monetary authority can intervene by adjusting interest rates on the risk-free bonds. The role of fiscal authorities is restricted to maintaining a balanced budget.

Although a number of empirical studies admit that the oil prices are nonstationary, the existing theoretical models most often model the oil price as a stationary process. In this paper, I assume that the nominal price of oil in the model is an $I(1)$ stochastic process.
This helps to replicate the inverse hump-shaped impulse responses observed in the empirical model.

In the model below, I use capital letters to represent the variables that grow along the equilibrium balanced growth path (except the interest rate). Lower-case letters are reserved for stationary variables. Unless mentioned specifically, all the variables are expressed in real terms.

### 1.4.1 Firms

The final good is produced by perfectly competitive firms using a continuum of differentiated goods as inputs and the technology defined by the Dixit-Stiglitz aggregation formula, where \( \eta \) is the elasticity of substitution between production factors. Differentiated goods \( Y_{i,t} \) for \( i \in [0,1] \) are produced by monopolistically competitive firms using the following production technologies

\[
Y_{i,t} \leq Y(K_{i,t}^d, Z_t h_{i,t}^d) = F(K_{i,t}^d, Z_t h_{i,t}^d) - Z_t \Psi
\]  

(1.1)

where capital services and labor, \( K_{i,t}^d, h_{i,t}^d \), are the production inputs;\(^{16}\) \( Z_t \) is a neutral labor augmenting technology, which also applies to the fixed costs, \( Z_t \Psi_t \).\(^{17}\) \( F(\cdot, \cdot) \) is a homogenous of degree one, increasing and concave in its arguments function. I assume that this function features constant elasticity of substitution in capital and labor:

\[
F(K_{i,t}^d, Z_t h_{i,t}^d) = \left[ \theta(K_{i,t}^d)^{-\varrho} + (1 - \theta)(Z_t h_{i,t}^d)^{-\varrho} \right]^{-\frac{1}{\varrho}}
\]

where \( \theta \) is a parameter determining relative factor shares, \( \varrho \) is a factor substitution parameter, such that \( \frac{1}{1+\varrho} \) is the elasticity of substitution between production factors.

The problem of firm \( i, i \in [0,1] \) is to maximize the present discounted value of its dividend payments:

\[
\max \{ \mathcal{E}_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \Phi_{i,t+s} \} 
\]  

(1.2)

\(^{16}\)In the symmetric equilibrium, \( K_{i,t}^d = u_t K_t \), where \( u_t \) determines the intensity of capital utilization, and \( K_t \) is the aggregate stock of capital.

\(^{17}\)The assumption that the neutral technology affects fixed costs is necessary for the existence of the balanced path.
where $E_t$ is the expectation conditional on time $t$, $r_{t,t+s}$ is the stochastic nominal discount factor between periods $t$ and $t+s$, $\Phi_{i,t}$ represents real dividends paid out to asset holders in period $t$. The dividends are the net profit after the trade in goods and state-contingent asset markets:

$$\Phi_{i,t} = \frac{P_{i,t}}{P_t} Y_{i,t} - r^k_t K_{i,t}^d - W_th_{i,t}^d + \frac{X_{i,t}^f}{\pi_t} - E_t r_{t,t+1} X_{i,t+1}^f$$

In the formula above, $\frac{X_{i,t}^f}{\pi_t} - E_t r_{t,t+1} X_{i,t+1}^f$ is the net gain from the trade of state-contingent assets, $X_{i,t}^f$, in real terms. I assume that in each period $t$, the net equilibrium gain of intermediate good firms from trade in the state contingent market is zero. Thus, the transversality condition in the optimal choice problem of firms will always be satisfied.

The firms are required to satisfy demand for their product, which results in an additional restriction on the monopolistically competitive producers of the intermediate good

$$Y_{i,t} \geq \left( \frac{P_{i,t}}{P_t} \right)^{-\eta} Y_t \quad (1.3)$$

where $Y_t$ is the aggregate demand for the final good in this economy.

Denoting $\beta^i mc_{i,t}$ as the Lagrange multiplier on the production function constraint of the firm $i$, (1.1), the first order necessary conditions for the choice of $K_{i,t}^d$ and $h_{i,t}^d$ of the firm $i$ respectively are

$$mc_{i,t} F_1(K_{i,t}^d, z_{i,t} h_{i,t}^d) = r^k_t$$

and

$$mc_{i,t} z_{i,t} F_2(K_{i,t}^d, z_{i,t} h_{i,t}^d) = W_t$$

To treat the steady growth of neutral technology, I rewrite the problem of the firms in terms of stationary variables as shown in the first row of Table 1.1.

With lowercase letters denoting the stationary modifications of the corresponding capital letter variables, the first order conditions with respect to $k_{i,t}^d$ and $h_{i,t}^d$ are

$$mc_{i,t} F_1(\frac{k_{i,t}^d}{\mu_{z,t}}, h_{i,t}^d) = r^k_t \quad (1.4)$$
and
\[ mc_{t,t}F_2 \left( \frac{k^d_{i,t}}{\mu_{z,t}, h^d_{i,t}} \right) = w_t \] (1.5)

I model price rigidity following Calvo (1983) and Yun (1996). The probability of not being able to change the price is \( \alpha \). Firms that can not change the price of their product today can only correct it for the previous period rate of inflation up to the degree of indexation \( \chi \); i.e. the price of firms that can not choose the price optimally is determined as

\[ P_t^i = P_{t-1}^i \pi_{t-1}^\chi \]

Firms that get the chance to change the price in period \( t \), set the price for their product to maximize (1.2). The solution to the problem of optimal price choice can be written as a solution to

\[ x_t^1 = x_t^2 \] (1.6)

where

\[ x_t^1 = \frac{\eta}{\eta - 1} y t m c_t \tilde{p}_t^{\eta - 1} + \alpha \beta \mathcal{E}_t \mu_{z,t+1} \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\pi_{t+1}}{\pi_t} \right)^{\eta} \left( \frac{\tilde{p}_{t+1}}{p_t} \right)^{\eta+1} x_{t+1}^1 \] (1.7)

\[ x_t^2 = y t \tilde{p}_t^{-\eta} + \alpha \beta \mathcal{E}_t \mu_{z,t+1} \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\pi_{t+1}}{\pi_t} \right)^{\eta-1} \left( \frac{\tilde{p}_{t+1}}{p_t} \right)^{\eta} x_{t+1}^2 \] (1.8)

and

\[ \tilde{p}_t = \frac{\tilde{P}_t}{P_t} \]

The derivation of the formulas above coincides with the technical appendix to Schmitt-Grohé and Uribe (2005).

1.4.2 Households

There are infinitely many households in the economy. Each household maximizes expected lifetime utility that is defined by the sequences of homogenous habit adjusted consumption, \( C_t - bC_{t-1} \), and leisure, \( 1 - h_t \):

\[ E_0 \sum_{t=0}^{\infty} \beta^t \bar{U}(C_t - bC_{t-1}, 1 - h_t) \] (1.9)
where $b$ is the parameter governing habit formation in consumption, and period $t$ utility function is

$$\bar{U}(C_t - bC_{t-1}, 1 - h_t) = \left[ (C_t - bC_{t-1})^{1-\sigma}(1 - h_t)^\sigma \right]^{1-\varphi} - 1$$

Every household supplies infinitely many types of labor, $h^j_t, j \in [0, 1]$, on the monopolistically competitive market. Different labor types are combined using the Dixit-Stiglitz aggregator with the elasticity of substitution $\tilde{\eta}$, and aggregate labor is supplied to the intermediate goods producers. The optimal supply of each labor type is

$$h^j_t = \left( \frac{W^j_t}{W_t} \right)^{-\tilde{\eta}} h^d_t$$

where $h^d_t$ is the aggregate labor demand.

Households are required to provide enough labor to satisfy labor demand, thus

$$h_t = \int_0^1 h^j_t dj = h^d_t \int_0^1 \left( \frac{W^j_t}{W_t} \right)^{-\tilde{\eta}} dj$$

(1.10)

Besides consuming and supplying labor services, households accumulate capital and rent it out to firms. Following Finn (2000), capital accumulation is subject to depreciation at a variable rate $\delta(u)$, which depends on the intensity of capital use. Investment changes are costly, with the costs $S\left( \frac{I_t}{I_{t-1}} \right)$ per unit of investment. The dynamics of capital is described as

$$K_{t+1} = (1 - \delta(u_t))K_t + I_t \left[ 1 - S\left( \frac{I_t}{I_{t-1}} \right) \right]$$

(1.11)

where $S(\cdot)$ satisfies the condition that $S(\mu_z) = S(\mu_z)' = 0$ and $S(\mu_z)'' > 0$. Along the balanced-growth path, investment will grow at the same rate as neutral technology, $\mu_z$. Thus, the functional form for $S(\cdot)$ implies that there are no costs of investment if the economy follows the balanced growth path. I assume that the functional form of the investment costs function is

$$S\left( \frac{I_t}{I_{t-1}} \right) = \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - \mu_z \right)^2$$

with parameter $\kappa > 0$.

---

18Investment costs here are second-order costs, which arise only if investments change relative
I assume a quadratic form for the depreciation function:

\[ \delta(u_t) \equiv \delta_0 + \omega_0(u - u^{ss}) + \omega_1(u - u^{ss})^2 \]  

(1.12)

where \( \omega_0 > 0, \ \omega_1 > 0, \) and \( u^{ss} \) is the capital utilization rate in the steady state.

Oil is used by households to provide capital services. The amount of oil is proportional to the size of the existing capital stock, \( K_t \), which reflects the idea of high complementarity between capital and oil. Although oil complements capital, the amount of oil expenditures depends on the intensity of capital utilization, \( u_t \). Thus, the ratio of oil to capital is a function of capital utilization rate, \( u_t \):

\[ \frac{E_t}{K_t} = A(u_t) \]  

(1.13)

I assume that \( A(u_t) \) is an increasing and convex function, reflecting an idea that the provision of capital services in terms of oil becomes more costly at an increasing rate if capital is utilized more intensively. Thus, one may think of \( A(u_t) \) as a technology of producing capital services, \( u_t \) in period \( t \), using oil as a production input.

Inverting this production function, one may retrieve the conditional demand for oil as a function of capital utilization rate.\(^{19}\) Because capital utilization directly enters the production technology of intermediate goods, it can be thought of as a production technology that is determined by capital, labor and oil. This means that the oil price increase will propagate into the economy by negatively affecting the marginal productivity of intermediate production and, consequently, the demands for capital and labor, similar to models where oil is explicitly assumed to be one of the production inputs.\(^{20}\) As a result, this model encompasses this alternative, oil-in-the-production, class of model. At the same time, Christiano, Eichenbaum, and Evans (2005) notice that compared to specifications in the earlier literature, where costs of changing investments are first order, the second-order investment adjustment costs help achieve a stronger and more persistent effect of monetary policy shock on output, while they do not significantly affect the estimates of the model and the response of inflation to a monetary policy shock.

---

\(^{19}\) This requires that \( A(u) \) is invertible for a range of \( u \) from 0 to 1.

\(^{20}\) See, for example, Carlstrom and Fuerst (2006).
time, the specification I use in this paper allows for an additional channel of the transmission mechanism for the oil price shock, which propagates through the capital services market. According to this mechanism, higher oil prices raise the marginal costs of providing capital services, which decreases the supply of capital services and creates an upward pressure on the rental rate $r_k^t$. Also, it creates additional downward pressure on the capital utilization rate compared to an oil-in-the-production model.

This approach to modeling oil sector was first suggested by Finn (2000). Differently from Finn (2000), however, I assume that the technology of producing capital services adjusts through time to keep up with growing oil prices. In particular, the more expensive oil is, the more efficient becomes this technology, allowing less oil expenditures to produce the same amount of capital services. This assumption also induces the long-run balanced growth as a result of growing oil prices.\footnote{This assumption is needed to guarantee the existence of the steady path. However, this is not the only way how stationarity can be induced in this model. Another way to incorporate oil price growth could be, similar to Fisher (2003) and Altig, Christiano, Eichenbaum, and Lindè (2004) to assume that the fixed costs of production grow at the rate of the oil price. The major difference between the two assumptions is that the permanent increase in the price of oil only has temporary effect on macroeconomic variables (except the quantity of oil), while in the specification of Fisher (2003) and Altig, Christiano, Eichenbaum, and Lindè (2004) the permanent increase in the oil price has a permanent effect on some macroeconomic variables.} Technically speaking, oil-to-capital requirement, which is $A(u)$, is discounted by a process $Z_t^*$ that grows in the long run at the rate of the real oil price growth, $P^E_t$. I model this process as follows:

$$Z_t^* = \alpha_z Z_{t-1}^* + (1 - \alpha_z) P^E_t$$

(1.14)

where $\alpha_z \in [0, 1)$. Then, $A(u_t)$ is

$$A(u_t) = \frac{a(u_t)}{Z_t^*}$$

(1.15)

In (1.15), $a(u_t)$ is an increasing and convex quadratic function of the capital utilization rate

$$a(u_t) = a_0 + v_0 (u - u^{es}) + v_1 (u - u^{es})^2,$$

where $v_0 > 0$ and $v_1 > 0$. 

\footnote{This assumption is needed to guarantee the existence of the steady path. However, this is not the only way how stationarity can be induced in this model. Another way to incorporate oil price growth could be, similar to Fisher (2003) and Altig, Christiano, Eichenbaum, and Lindè (2004) to assume that the fixed costs of production grow at the rate of the oil price. The major difference between the two assumptions is that the permanent increase in the price of oil only has temporary effect on macroeconomic variables (except the quantity of oil), while in the specification of Fisher (2003) and Altig, Christiano, Eichenbaum, and Lindè (2004) the permanent increase in the oil price has a permanent effect on some macroeconomic variables.}
The assumption that the oil-to-capital requirement ratio diminishes at the rate of growth of oil prices may be interpreted as follows. The growth of oil prices stimulates the development of technological progress, which helps to use energy more efficiently. Alternatively, it may represent a slow transition to alternative less expensive sources of energy. The longer it takes $A(u_t)$ to respond to the oil price shock, the larger is the negative effect of the shock on the economy. The closer $\alpha_z$ is to 1, the longer it takes the technology to improve. Thus, a permanent increase of the oil price will have a dramatic and almost permanent negative effect on the economy. On the contrary, if $\alpha_z = 0$, the technology $A(u_t)$ immediately accommodates the oil price increase. In this case, there is no effect of the oil price shock on macroeconomic variables, because the new technology level allows a decrease in the quantity of oil needed to provide capital services by exactly the same proportion as the rise in the oil price without any effect on capital and capital utilization.

Finally, I assume that markets are complete by introducing state-contingent assets for households. I denote $\mathcal{E}_t r_{t+1} X_{t+1}^h$ as the cost of the state-contingent assets acquired at time $t$ discounted at today’s price of consumption, $P_t$. $r_{t+1}$ is the stochastic nominal discount factor in the period between $t$ and $t+1$.

Based on the discussion of the household behavior above, the intertemporal budget constraint of the household in real terms is:

$$
\mathcal{E}_t r_{t+1} X_{t+1}^h + C_t + I_t + P_t E_t = \frac{X_t^h}{\Pi_t} + Tr_t + r_t u_t K_t + \int_0^1 W_j^t \left( \frac{W_j^t}{W_t^t} \right)^{-\eta} h_t^d dj + \Phi_t
$$

where $Tr_t$ is a net transfer from the government to the household, $\Phi_t$ is the dividend income the household gets from the ownership in the firms.

For the sake of convenience, the Lagrange multipliers on the budget constraint, (1.4.2), labor supply requirement, (1.10), capital accumulation (1.11) and oil-to-capital constraint (1.13) are denoted $\beta^t \Lambda_t$, $\frac{\Delta^t W_t}{\mu_t}$, $\beta^t \Lambda_t Q_t$ and $\beta^t \Lambda_t \Xi_t$, respectively. Then, it can be shown that $\Lambda_t$ is the marginal utility of wealth, $\hat{\mu}_t$ is the average wage markup of the household, $Q_t$ is the shadow price of future capital, and $\Xi_t$ is the shadow price of energy. The Lagrangian

\[\text{22This is true unless monetary policy directly responds to the oil price.}\]
of the household’s problem can now be written as:

\[ \mathcal{L} = \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \tilde{U}(C_t - bC_{t-1}, 1 - h_t) + \Lambda_t \left( \frac{\tilde{X}_t}{\tilde{\pi}_t} + T_{\tilde{r}_t} + r_t^f u_t K_t \right) + h_t^d \int_0^1 W_t \left( \frac{w_t}{W_t} \right)^{-\tilde{\eta}} d\tilde{\eta} + \Phi_t - r_{t,t+1} X_{t+1}^h - C_t - I_t - P_{tE}^E E_t \right\} + \Lambda_t \tilde{Q}_t [1 - \delta(u_t)] K_t \]

To solve the problem, it is convenient to rewrite the Lagrangian in terms of stationary transformations of the variables that grow over time in steady equilibrium due to exogenous growth of the oil price and neutral technology. Table 1.1 shows how the transformations are made.

<table>
<thead>
<tr>
<th>New variable</th>
<th>Transformed variable</th>
<th>how transformed</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_{i,t}, k_{i,t+1} )</td>
<td>( \Phi_{i,t}, K_{i,t+1} )</td>
<td>divided by ( Z_t )</td>
</tr>
<tr>
<td>( c_t, x_t^h, w_t, w_t^i )</td>
<td>( C_t, X_t^h, W_t, W_t^i )</td>
<td>divided by ( Z_t )</td>
</tr>
<tr>
<td>( \phi_t, \tau_t, k_{t+1}, i_t )</td>
<td>( \Phi_t, T_{\tau_t}, K_{t+1}, I_t )</td>
<td></td>
</tr>
<tr>
<td>( \xi_t, z_t^\ast )</td>
<td>( \Xi_t, Z_t^\ast )</td>
<td>divided by ( P_{tE}^E )</td>
</tr>
<tr>
<td>( a(u_t) )</td>
<td>( A(u_t) )</td>
<td>multiplied by ( P_{tE}^E )</td>
</tr>
<tr>
<td>( e_t )</td>
<td>( E_t )</td>
<td>multiplied by ( \frac{\tilde{z}_t}{Z_t^\ast} )</td>
</tr>
<tr>
<td>( \lambda_t )</td>
<td>( \Lambda_t )</td>
<td>multiplied by ( Z_t^{1-(1-\sigma)(1-\varphi)} )</td>
</tr>
</tbody>
</table>

The Lagrangian in terms of stationary variables is:

\[ \mathcal{L} = \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t z_t^{(1-\sigma)(1-\varphi)} [U(c_t - bC_{t-1}/\mu_{z,t}, 1 - h_t) + \lambda_t \left( \frac{x_t}{\pi_t \mu_{z,t}} + T_{\tau_t} + r_t^f u_t \frac{k_t}{\mu_{z,t}} \right) + h_t^d \int_0^1 w_t \left( \frac{w_t}{W_t} \right)^{-\tilde{\eta}} d\tilde{\eta} + \phi_t - r_{t,t+1} X_{t+1}^h - c_t - i_t - e_t] \]

\[ + \lambda_t \tilde{Q}_t [1 - \delta(u_t)] \frac{k_t}{\mu_{z,t}} \]

\[ + i_t [1 - S \left( \frac{i_t}{t-1} \mu_{z,t} \right)] - k_{t+1} + \lambda_t \xi_t [e_t - \frac{a(u_t)}{z_t} \frac{k_t}{\mu_{z,t}}] \]

where I denote \( U(c_t - bC_{t-1}/\mu_{z,t}, 1 - h_t) = \tilde{U}(C_t - bC_{t-1,1-h_t}/Z_t^{1-(1-\sigma)(1-\varphi)}) \). The first order conditions
written in terms of the stationary variables $c_t, h_t, k_{t+1}, i_t, x_{t+1}^h, u_t$ and $e_t$ for all $t \geq 0$ are respectively:

\[
U_{t,1} - \beta b E_t \frac{U_{t+1,1}}{\mu_{z,t+1}^{1-(1-\sigma)(1-\varphi)}} = \lambda_t \tag{1.16}
\]

\[
- U_{t,2} = \frac{\lambda_t w_t}{\mu_t} \tag{1.17}
\]

\[
\lambda_t q_t = \beta E_t \frac{\lambda_{t+1}}{\mu_{z,t+1}^{1-(1-\sigma)(1-\varphi)}} \left[ \bar{r}_k \lambda_{t+1} u_{t+1} + q_{t+1} (1 - \delta(u_{t+1})) - \xi_{t+1} \frac{a(u_{t+1})}{z_{t+1}^*} \right] \tag{1.18}
\]

\[
\lambda_t = \lambda_t q_t \left[ 1 - S \left( \frac{i_t}{n_t} \right) - \left( \frac{i_t}{n_t-1} \right) S' \left( \frac{i_t}{n_t} \right) \right] + \beta E_t \frac{\lambda_{t+1} q_{t+1}}{\mu_{z,t+1}^{1-(1-\sigma)(1-\varphi)}} \left( \frac{i_{t+1}}{n_t} \right) S' \left( \frac{i_{t+1}}{n_t} \right) \tag{1.19}
\]

\[
\lambda_t r_{t,t+1} = \beta E_t \frac{\lambda_{t+1}}{\mu_{z,t+1}^{1-(1-\sigma)(1-\varphi)}} \tag{1.20}
\]

\[
r_k^t = q_t \delta'(u_t) + \xi_t \frac{a'(u_t)}{z_t^*} \tag{1.21}
\]

\[
\xi_t = 1 \tag{1.22}
\]

where in equations (1.16) and (1.17), $U_{t,1}$ and $U_{t,2}$ are the derivatives of the utility function with respect to the first and the second argument correspondingly. The households supply different types of labor on a monopolistically competitive market. Wages are modeled à la Calvo (1983) and Yun (1996), with $\tilde{\alpha}$ the probability of not being able to reset a wage. If the wage of type $i$ can not be set optimally in period $t$, then it is indexed to the previous period inflation rate according to the formula:

\[
W_t^i = W_{t-1}^i (\mu_z \pi_{t-1})^\xi
\]

where $\mu_z$ is a steady growth rate of neutral technology. The optimal wage of those labor types, for which the wage can be set optimally, is defined by the following optimality conditions:

\[
f_t^1 = f_t^2 \tag{1.23}
\]

where

\[
f_t^1 = \frac{\tilde{n} - 1}{\eta} \tilde{w}_t \lambda_t \left( \frac{\tilde{w}}{\tilde{w}_t} \right)^{-\tilde{n}} h_t^d + \tilde{a} \beta E_t \mu_{z,t+1} \left( \frac{\pi_{t+1} \mu_{z,t+1}}{(\pi_t \mu_z)} \right)^\tilde{n}^{-1} \left( \frac{\tilde{w}_{t+1}}{\tilde{w}_t} \right)^{-\tilde{n} - 1} f_{t+1}^1 \tag{1.24}
\]
and

\[ f_t^2 = -U_{t,2} \left( \frac{\tilde{w}_t}{\tilde{w}'} \right)^{-\tilde{\eta}} h^d_t + \tilde{\alpha} \beta \epsilon_t \mu_{z,t+1}^{(1-\sigma)(1-\varphi)} \left( \frac{\pi_{t+1} \mu_{z,t+1}}{(\pi_t \mu_z)^{\tilde{\chi}}} \right)^{\tilde{\eta}} \left( \frac{\tilde{w}_{t+1}}{\tilde{w}_t} \right)^{\tilde{\eta}} \tilde{f}_{t+1}^2 \] (1.25)

The derivation of the formulas above coincides with the technical appendix to Schmitt-Grohé and Uribe (2005).

### 1.4.3 Aggregation and Markets Clearing

I consider a symmetric equilibrium in which monopolistically competitive firms that are able to reset the price of their product, all decide on the same price \( \tilde{P} \). The rest of the firms will update the price of their product by the previous period inflation with the degree of indexation \( \tilde{\chi} \). Taking into account the fact that whether or not a firm can set an optimal price is decided randomly, the average price before indexation of those firms that can not optimally reset their price is equal to the price level in the previous period. Using this fact and taking into account that the price level \( P_t \) is

\[
P_t = \left[ \int_0^1 P_{t,i}^{1-\eta} di \right]^{1/\eta}
\]

the equilibrium dynamics of the price level can be derived as

\[
P_{t}^{1-\eta} = \alpha (P_{t-1} \pi_{t-1}^\chi)^{1-\eta} + (1 - \alpha) \tilde{P}_{t}^{1-\eta}
\]

(1.26)

Denoting \( \tilde{\rho}_t = \frac{\tilde{P}_t}{P_t} \), equation (1.26) produces

\[
\tilde{\rho}_t = \left( \frac{1 - \alpha \pi_t^{\eta-1} \pi_{t-1}^{\chi(1-\eta)}}{1 - \alpha} \right)^{1/(1-\eta)}
\]

(1.27)

Similarly, the symmetric equilibrium induces the identical choice of \( \tilde{W} \) for all the labor types for which the wage can be reset optimally. The average wage before indexation of those labor types that can not reset their wage is equal to the previous period wage rate. The equilibrium real wage dynamics can then be derived as

\[
W_{t}^{1-\tilde{\eta}} = (1 - \tilde{\alpha}) \tilde{W}_{t}^{1-\tilde{\eta}} + \tilde{\alpha} W_{t-1}^{1-\tilde{\eta}} \left( \frac{(\mu_{z} \pi_{t-1})^{\tilde{\chi}}}{\pi_t} \right)^{1-\tilde{\eta}}
\]

(1.28)
Thus, the optimal real wage choice in stationary terms, \( \tilde{w}_t = \frac{\tilde{W}_t}{Z_t} \), is

\[
\tilde{w}_t = \left( \frac{w_t^{1-\tilde{\eta}} - \tilde{\alpha}w_{t-1}^{1-\tilde{\eta}} \left( \frac{\mu_z \pi_t}{\pi_{t-1}} \right)^{1-\tilde{\eta}}}{1 - \tilde{\alpha}} \right)^{1/(1-\tilde{\eta})} \tag{1.29}
\]

The market clearing condition in the labor market implies

\[
h_t = h \tilde{z}_t \tag{1.30}
\]

where \( \tilde{z}_t \equiv \int_0^1 \left( \frac{W_i}{W_t} \right)^{-\tilde{\eta}} di \) is the wage dispersion variable. One can show that the dynamic process for \( \tilde{z}_t \) is

\[
\tilde{z}_t = (1 - \tilde{\alpha}) \left( \frac{\tilde{w}_{t-1}}{w_t} \right)^{-\tilde{\eta}} + \tilde{\alpha} \left( \frac{\tilde{w}_{t-1}}{w_{t-1}} \right)^{-\tilde{\eta}} \left( \frac{\mu_z \pi_t}{\mu_z \pi_{t-1} \chi} \right)^{\tilde{\eta}} \tilde{z}_{t-1} \tag{1.31}
\]

Equilibrium for capital services implies

\[
u_t k_t = k^d_t \tag{1.32}
\]

The symmetric equilibrium implies that the dividends are

\[
\phi_t = y_t - r^k_t k_t - w_t h^d_t \tag{1.33}
\]

The role of the fiscal policy is minor. I assume that government expenditures are constant and financed through lump-sum taxes

\[
t r_t = -g_t \tag{1.34}
\]

The market clearing condition in the market of final goods:

\[
y = [c_t + i_t + e_t + g_t] s_t \tag{1.35}
\]

where \( y = F(u_t k_t / \mu_z, h^d_t) \) is the aggregate output and \( s_t \equiv \int_0^1 \left( \frac{P_i}{P_t} \right)^{-\eta} di \) is the measure of price dispersion in this economy. It can be shown that \( s_t \) follows a dynamic process:

\[
s_t = (1 - \alpha) \tilde{p}_t^{-\eta} + \alpha \left( \frac{\pi_t}{\pi_{t-1}} \right)^{\eta} s_{t-1} \tag{1.36}
\]

---

\(^{23}\)The derivation of the market clearing in labor and goods markets (below) coincides with the technical appendix to Schmitt-Grohé and Uribe (2005).
No arbitrage condition in the state-contingent assets market implies

\[ \lambda_t = \beta R_t \mathcal{E}_t \frac{\lambda_{t+1}}{\mu_{z,t+1}^{1-(1-\sigma)(1-\psi)} \pi_{t+1}} \]  \hspace{1cm} (1.37)

where \( R_t \) is the risk-free nominal interest rate.

The technology grows steadily at a constant rate \( \mu_z \):

\[ \frac{Z_t}{Z_{t-1}} = \mu_z \]

The oil shock enters the model through the growth of the nominal price of oil, \( \mu_{P_e,t}^N = \frac{P_{N E,t}^N}{P_{E,t-1}^N} \). The growth of the nominal oil price is an exogenous stationary stochastic process around its steady state, \( \mu_{P_e}^N \):

\[ \log \left( \frac{\mu_{P_e,t}^N}{\mu_{P_e}^N} \right) = \rho_{P_e} \log \left( \frac{\mu_{P_e,t-1}^N}{\mu_{P_e}^N} \right) + \epsilon_{P_e,t} \]  \hspace{1cm} (1.38)

with \( \epsilon_{P_e,t} \) - i.i.d stochastic process with zero mean and standard deviation \( \sigma_{P_e} \).

Monetary policy is characterized by a simple inertial rule, according to which the interest rate is inertial and contemporaneously responds to inflation, the growth of the value added, and the nominal oil price growth:

\[ \ln \left( \frac{R_t}{R_{t-1}} \right) = \alpha_R \ln \left( \frac{R_{t-1}}{R_t} \right) + \alpha_x \ln \left( \frac{\pi_t}{\pi} \right) + \alpha_{va} \ln \left( \frac{VA_t}{VA_{t-1}} \right) + \alpha_{P_e} \ln \left( \frac{\mu_{P_e,t}^N}{\mu_{P_e}^N} \right) \]  \hspace{1cm} (1.39)

1.4.4 Competitive Equilibrium

In equilibrium, at any time \( t \geq 0 \) households and firms make decisions to maximize their objectives given the monetary policy rule and exogenous oil price growth process. Besides, they possess all the information within the information set of period \( t = -1 \). Formally, a competitive equilibrium in this model as a set of stationary processes \( c_t, h_t, i_t, k_{t+1}, w_t, \bar{w}_t, \lambda_t, \bar{\mu}_t, q_t, \xi_t, f_1^t, f_2^t, \phi_t, k_t^i, h_t^d, \bar{p}_t, m c_t, x_1^t, x_2^t, r^k_t, \pi_t, w_t, s_t, \bar{s}_t, z^*_t \) for \( t = 0, 1, ..., \infty \) such that, given monetary policy rule (1.39), initial conditions \( c_{-1}, i_{-1}, k_0, \pi_{-1}, s_{-1}, \bar{s}_{-1}, z^*_{-1} \) and exogenous stochastic process for the oil price growth, \( \mu_{P_e,t}^N \), these processes satisfy the optimality conditions of firms (1.4), (1.5), (1.6), (1.7), (1.8), the optimality conditions of households (1.16) - (1.21), (1.23), (1.24), (1.25), capital evolution equation (1.11), price and
real wage dispersion dynamics (1.36) and (1.31), energy-to-capital technology evolution (1.41), market clearing conditions (1.30), (1.32), (1.35), equilibrium in state-contingent assets market (1.37), equilibrium price and wage determination equations (1.27) and (1.29).

1.5 Estimation

To get realistic predictions of the model, it is important to calibrate it properly. With this aim, I estimate the deep parameters of the model by matching the impulse responses of nine macroeconomic variables to an oil price shock generated by the empirical SVAR model from Section 1.1 and the impulse responses generated by the model. To generate the impulse responses implied by the theoretical model, I log-linearize the system of equilibrium conditions of the model around the steady state implied by the model calibration.

1.5.1 Strategy of Estimation

Some of the deep parameters are estimated, others are calibrated using the corresponding data statistics or conventional wisdom. Table 1.2 summarizes the second group of parameters.

Parameters important for the oil sector are calibrated as follows. The share of oil in GDP is fixed at 4.3%. Capital utilization is 0.81, which is the average of the utilization rate in manufacturing in the data I use in the VAR. Quarterly capital depreciation rate is fixed at a value conventional in the macroeconomic literature, 0.025. The rate of growth of the nominal oil price corresponds to the average growth rate of WTI spot oil price found in the data, 0.61%, which is 2.48% at annual frequency. The parameter determining the share of capital in production is fixed at the conventional value of 0.36. The share of government expenditures is calibrated using the data as 20% of the value added.

I calibrate the model so that inflation is equal to the average inflation rate of the GDP deflator. The real interest rate along the balanced growth path is calibrated at 4% a year.
The implied discount factor can then be derived as $\beta = \mu_z^{(1-(1-\sigma)(1-\varphi))}1.04^{-0.25}$. The quarterly growth rate of neutral technology is fixed at 0.4% (1.61% annually), which is the average of the labor productivity growth, calculated as the ratio of GDP to hours in non-farm business sector over the period of 1954:3 - 2006:4.

Finally, the parameters of the elasticity of substitution in aggregation formulas for final output and labor, $\eta$ and $\tilde{\eta}$ are fixed at 6 and 21 respectively. Similar to other studies that try to estimate these parameters, I find they are weakly identified by the model.

**Table 1.2: Calibrated parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>nominal oil price growth rate (annualized)</td>
<td>$\mu_{Pe}$</td>
<td>2.48%</td>
</tr>
<tr>
<td>standard deviation of the oil price growth process</td>
<td>$\sigma_{Pe}$</td>
<td>8.03%</td>
</tr>
<tr>
<td>share of oil in value added</td>
<td>SE</td>
<td>4.3%</td>
</tr>
<tr>
<td>capital utilization rate</td>
<td>$u$</td>
<td>82%</td>
</tr>
<tr>
<td>capital depreciation rate</td>
<td>$\delta$</td>
<td>2.5%</td>
</tr>
<tr>
<td>real interest rate (annualized)</td>
<td>$R/\pi$</td>
<td>4%</td>
</tr>
<tr>
<td>inflation (annualized)</td>
<td>$\pi$</td>
<td>3.57%</td>
</tr>
<tr>
<td>neutral technology growth rate(annualized)</td>
<td>$\mu_z$</td>
<td>1.61%</td>
</tr>
<tr>
<td>capital share in production</td>
<td>$\theta$</td>
<td>0.36</td>
</tr>
<tr>
<td>share of government expenditures in value added</td>
<td>$SG$</td>
<td>20%</td>
</tr>
<tr>
<td>shadow price of investment</td>
<td>$q$</td>
<td>1</td>
</tr>
<tr>
<td>elasticity of substitution, interm. goods</td>
<td>$\eta$</td>
<td>6</td>
</tr>
<tr>
<td>elasticity of substitution, differentiated labor types</td>
<td>$\tilde{\eta}$</td>
<td>21</td>
</tr>
</tbody>
</table>

The estimated vector of parameter $\theta$ is

$$\theta = \{ \rho_{Pe}, \alpha_z, v_0, v_1, \omega_0, \omega_1, b, \kappa, \alpha, \tilde{\alpha}, \chi, \tilde{\chi}, q, \sigma, \varphi, \alpha_R, \alpha_\pi, \alpha_{0\alpha}, \alpha_{Pe} \}$$

I use bayesian techniques to obtain the estimates. Namely, I apply the Laplace type estimator (henceforth LTE) suggested by Chernozhukov and Hong (2003). Chernozhukov and Hong show that the LTE is as efficient as a classical extremum estimator, while may be computationally more attractive. I compute the estimates as mean values of a Markov Chain sequence of draws from the quasi-posterior distribution of $\theta$, generated

---

24With the estimated parameters of the utility function $\sigma$ and $\varphi$ from table 1.3, the discount factor in the model is 0.997.
by the tailored Metropolis Hastings algorithm. The standard errors are calculated as the second moments of the sequences. The LTE of the vector $\theta$ is defined as

$$\theta = \arg \inf_{\zeta \in \Theta} [Q_n(\zeta)]$$

where $Q_n(\zeta)$ is the quasi-posterior risk function

$$Q_n(\zeta) = \int_{\theta \in \Theta} \rho_n(\theta - \zeta)p_n(\theta)d\theta$$

$\rho_n(\cdot)$ is the appropriate penalty function associated with an incorrect choice of parameter\(^{25}\) and $p_n$ is the so-called “quasi-posterior distribution”, defined using the Laplace transformation of the distance function $L_n$ and possibly the prior probability of the parameter $\theta$, $\pi(\theta)$:

$$p_n(\theta) = \frac{e^{L_n(\theta)}\pi(\theta)}{\int e^{L_n(\theta)}\pi(\theta)d\theta}$$

Specifically for this paper, the distance function $L_n$ is the weighted sum of squares of differences between the impulse responses generated by an empirical VAR model, $(\hat{IRF})$, and the theoretical model $(IRF(\theta))$:

$$L_n = -(IRF(\theta) - \hat{IRF}_n)'V^{-1}(IRF(\theta) - \hat{IRF}_n)$$

where $V$ is a diagonal weighting matrix with the sample variances of impulse responses along the diagonal\(^{26}\).

### 1.5.2 Estimation Results

Figure A.0.4 shows the impulse response functions generated by the data and the theoretical model, plotted above the empirical impulse responses together with the 90% confidence interval. Overall, the model does well in fitting the empirical impulse responses. Again,

\(^{25}\)For the mean estimator, $\rho_n(\cdot)$ is the squared loss function.

\(^{26}\)I use Impulse Responses for 20 steps in $L_n$. A more optimal number of steps to consider may be suggested by the information criterion for Impulse Response Matching estimators in Hall, Atsushi, Nason, and Rossi (2007).
Table 1.3: Estimates of the parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Estimate (s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>autocorrelation of oil price process</td>
<td>$\rho_{Pe}$</td>
<td>0.3 (0.03)</td>
</tr>
<tr>
<td>Parameter of $Z^*$ process</td>
<td>$\alpha_z$</td>
<td>0.82 (0.02)</td>
</tr>
<tr>
<td>Parameter 1 of capital utilization</td>
<td>$\upsilon_0 \times 100$</td>
<td>0.13 (0.03)</td>
</tr>
<tr>
<td>Parameter 2 of capital utilization</td>
<td>$\upsilon_1 / \upsilon_0$</td>
<td>0.33 (0.30)</td>
</tr>
<tr>
<td>Parameter of depreciation function</td>
<td>$\omega_1 / \omega_0$</td>
<td>0.013 (0.013)</td>
</tr>
<tr>
<td>habit parameter for consumption</td>
<td>$B$</td>
<td>0.83 (0.05)</td>
</tr>
<tr>
<td>investment costs parameter</td>
<td>$\kappa$</td>
<td>2.48 (0.57)</td>
</tr>
<tr>
<td>probability of not being able to reoptimize the price</td>
<td>$\alpha$</td>
<td>0.25 (0.17)</td>
</tr>
<tr>
<td>probability of not being able to reoptimize the wage</td>
<td>$\hat{\alpha}$</td>
<td>0.92 (0.05)</td>
</tr>
<tr>
<td>price indexation</td>
<td>$\chi$</td>
<td>0.51 (0.29)</td>
</tr>
<tr>
<td>wage indexation</td>
<td>$\tilde{\chi}$</td>
<td>0.75 (0.20)</td>
</tr>
<tr>
<td>production: factor elasticity of substitution parameter</td>
<td>$\vartheta$</td>
<td>-0.26 (0.04)</td>
</tr>
<tr>
<td>utility:</td>
<td>$\sigma$</td>
<td>0.53 (0.20)</td>
</tr>
<tr>
<td>utility:</td>
<td>$\varphi$</td>
<td>2.26 (0.48)</td>
</tr>
<tr>
<td>Monetary policy rule:</td>
<td>$\alpha_R$</td>
<td>0.33 (0.21)</td>
</tr>
<tr>
<td>Monetary policy rule:</td>
<td>$\alpha_\pi$</td>
<td>1.63 (0.26)</td>
</tr>
<tr>
<td>Monetary policy rule:</td>
<td>$\alpha_y$</td>
<td>0.06 (0.06)</td>
</tr>
<tr>
<td>Monetary policy rule:</td>
<td>$\alpha_{pe}$</td>
<td>0.003 (0.003)</td>
</tr>
</tbody>
</table>

All the responses except the interest rate and inflation are shown as percentages of the steady state values. The responses of the interest rate and inflation are annualized and shown as percentages in deviations from their steady state values. It can be seen from the graphs that an oil price increase as big as one standard deviation of the oil price process produces similar impulse responses for all the variables compared to the empirical model. GDP dynamics is very close to the empirically observed impulse response, with a trough of about 0.5% in 6 or 7 quarters after the shock. Capital utilization, investment, hours and consumption also decrease and the responses are U-shaped. The dynamics of these variables is within the 90% error bands of the empirical model.

The model produces the rise in inflation and the interest rate upon the oil price increase. While the response of the interest rate is close to the empirical response and shows correctly the size and timing of the peak response to the shock, the response of inflation in the model understates the empirically observed response. According to the theoretical model, inflation peaks in the second or third quarter after the shock and the inflation response does not
exceed 0.1% above the undisturbed level of inflation. On the other hand, the empirical response of inflation suggests that inflation peaks in the end of the first year after the shock with the peak around 0.3% above the steady state level.

The real wage rate falls in response to the oil price shock. Although the response lies within 90% error band of the empirical impulse response, the model response of the wage rate to the oil price shock is small, overall not exceeding 0.05% of the steady state drop. The responses of the inflation and the wage rate are the only responses that do not replicate the empirical results quite closely. However, overall the estimated model of the oil price shock still does a better job at replicating the empirical evidence than the models in the related literature, parameters of which are not estimated. For example, Leduc and Sill (2004) calibrate their model so that it approximately replicates the response of output from their empirical model. Carlstrom and Fuerst (2006) produces the peak responses of output and the interest rate of 0.3% and 1.12% respectively, in response to an oil price shock 10% size of its steady value. Both of these numbers are smaller than in the empirical study of Bernanke, Gertler, and Watson (2004), which they rely on. Neither of these studies attempts to match the response of other model variables to the shock. In the estimated model that I work with, only the dynamics of the wage rate and inflation deviate from the predictions of the empirical model. Nevertheless, they still lie almost everywhere within the error bands of the empirical impulse responses.

Table 1.3 presents the estimates of the parameters with the standard errors in parenthesis. The estimates are obtained as mean values of the MCMC sequences of the estimated parameters, while the standard errors are standard deviations of these sequences. The autocorrelation of the exogenous process for the oil price growth is 0.3. The autocorrelation parameter of the $z^*$ process, $\alpha_z$, is estimated at 0.82. The parameters that define the curvature of the oil-to-capital function imply that the slope and the curvature of this technology are smaller if compared with Finn (2000). Namely, Finn’s calibrations of $a(u)$ is such that it implies that the first and the second derivatives are 0.0088 and 0.0071 respectively. These numbers are several times larger than the values of $v_0$ and $v_1$ from Table 1.3. Besides, the
implied oil-to-capital ratio in the steady state 0.0059 is twice as large as what I estimate (0.0022). The parameters that determine the depreciation function imply that it is almost linear in capital utilization rate.

The value of parameter $\omega_0$ is pinned down by the calibration of the steady state at 0.0446, which is close to the first derivative of depreciation function in Finn (2000). The parameter for habit $b$ is 0.83, which is slightly bigger than what is usually found in the literature estimating DSGE models. The high estimate of habit formation is dictated by the weak response of consumption to the oil price shock. Investment costs parameter is 2.48, which is close to the one estimated by Altig, Christiano, Eichenbaum, and Lindé (2004). The Calvo-Yun price stickiness parameter $\alpha$ is 0.25, which is smaller than what is found in the macroeconomic literature. This parameter implies that prices change on average almost every quarter. The Calvo-Yun wage stickiness parameter is 0.92. Although the estimate is high, it is still consistent with the literature that estimates DSGE models. In much of this literature, utility is usually defined as a function of a single differentiated type of labor. In this paper, I define utility as a function of an aggregate of different labor types, similar to Schmitt-Grohé and Uribe (2005). In this setup, as is shown by Schmitt-Grohé and Uribe (2006), the higher wage rigidity coefficient is needed to obtain the same wage Phillips Curve as in other papers.

The parameters of the monetary policy rule suggest that the interest rate is moderately inertial, significantly responds to inflation with the coefficient close to 1.5, but the response to the output gap and oil price is muted and insignificant.

The next section explores the transmission mechanism of the oil price shocks and robustness of the model to the choice of parameters.

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27 In Appendix A.0.1, I show that $\omega_0$ is pinned down by the steady state.

28 Most of the literature finds this parameter between 0.6 and 0.7, including Christiano, Eichenbaum, and Evans (2005), Altig, Christiano, Eichenbaum, and Lindé (2004). This literature, however, does not focus on estimating the effect of oil price shocks.
1.6 Transmission Mechanism of the Oil Price Shock

In this section, I provide intuition for some of the results of the paper.

I introduce the shock as a nonstationary process. This assumption contrasts with the related theoretical literature, where the price of oil is modeled as a stationary process. The effect of the oil price increase in my estimated model is similar to the effect in the case where oil price is modeled as a stationary AR(1) process with autocorrelation coefficient 0.8.\textsuperscript{29} However, modeling nonstationary oil price process results in hump-shaped impulse responses of the interest rate and inflation to the shock contrary to the model with stationary AR(1) oil price process. This can be seen from Figure A.0.4.

The effect of the oil price shock is transmitted to the economy through the oil-to-capital requirement (1.15). The log-linear approximation of oil expenditures is

\[
\left(\frac{P_t^E}{A(u_t)}\right) = \frac{\nu_0}{a_0} \hat{\theta}_t - \hat{\mu}_t
\]

where I denote \(\hat{x}\) the logarithmic deviation from the steady state, \(\hat{x} = \frac{x - x_{ss}}{x_{ss}}\) and \(x_{ss}\) is the steady value of \(x\). The evolution of the stationary process \(z_t^* = \frac{Z_t^*}{P_t^E}\) can be written as

\[
z_t^* = \frac{\alpha_z z_{t-1}^*}{\mu_{pe,t}} + (1 - \alpha_z)
\]

where \(\mu_{pe,t} = \frac{P_{te}^E}{P_{te}^{E-1}}\) is the growth rate of the real oil price. The log-linear approximation of the equation above produces

\[
\hat{z}_t^* = \frac{\alpha_z}{\mu_{pe}} \hat{z}_{t-1}^* + \frac{\alpha_z}{\mu_{pe}} \hat{\pi}_t - \frac{\alpha_z}{\mu_{pe}} \hat{\mu}_{pe,t}
\]

It can easily be seen from (1.40) and (1.42) that a rise in the price of oil immediately increases the costs of capital utilization by decreasing \(z_t^*\). The parameter of the \(z^*\) process, \(\alpha_z\), determines how quickly the oil-to-capital technology adapts to the shock. In the extreme case of an immediate adjustment (\(\alpha_z = 0\)), the oil price shock does not have any effect on

\textsuperscript{29}In this case, I set \(\alpha_z = 1\).
the economy, given that monetary authorities do not directly respond to this shock. In another extreme case when the technology does not respond at all to an increase in the price of oil \( (\alpha_z = 1) \), the effect of the permanent increase in the oil price on some macroeconomic variables is permanent. In the intermediate case, when \( 0 \leq \alpha_z \leq 1 \), the technology evolves slowly to accommodate growing oil prices. The size of the effect is larger and lasts longer the higher \( \alpha_z \) is, which is demonstrated by Figure A.0.4. Eventually, however, the effect of a permanent oil price increase on the macroeconomy is temporary.

How does the oil price shock propagate in the economy? First, the shock has an immediate negative income effect through the household’s intertemporal budget constraint. The income effect reduces the demand for consumption and increases labor supply of households. Besides, the shock enters the first order conditions of the household with respect to capital utilization, (1.21). This equation determines the supply of capital services as a function of the rental price, \( r^k_t \). Log-linearized approximation of equation (1.21) is

\[
\hat{r}^k_t = \frac{q\omega_0}{r^k}(\hat{q}_t + \frac{\omega_1}{\omega_0}\hat{u}_t) + (1 - \frac{q\omega_0}{r^k})(\frac{\nu_1}{\nu_0}\hat{u}_t - \hat{z}^*_t)
\]

The above equation helps to see that the fall in \( z^* \) as a result of the rising oil price increases the marginal costs of producing the capital services and thus their supply shrinks. Also, because capital services and labor are substitutes in the production process, distortions in the capital market propagate into the labor market by affecting the labor demand. The general equilibrium determines how capital utilization and labor change after the shock. In the estimated model, both capital utilization and hours increase by a small amount on impact, but then fall in response to the shock.

Apart from the contemporaneous effect on the macroeconomic variables, the oil price shock distorts the intertemporal optimality conditions by negatively affecting the return on

\[\text{It may be useful to keep in mind that } z^*_t = \frac{Z^*_t}{P^E_t}. \text{ The case } \alpha_z = 0 \text{ is equivalent to saying that } z^*_t \text{ stays constant, because } \hat{z}^*_t = 0. \text{ This implies that in response to an increase in } P^E_t, Z^*_t \text{ increases in the same proportion, or put it differently, adjusts immediately to the shock.}\]
capital. The rate of return on capital is given by the following expression

\[
r_{t+1}^k u_{t+1} + q_{t+1} (1 - \delta(u_{t+1})) - \frac{a(u_{t+1})}{z_{t+1}^*} \frac{1}{qt}
\]

(1.43)

The increase in the price of oil today lowers \( z^* \) in future because of the autoregressive nature of this process. As a result, the next period costs of providing capital services per unit of capital rise, which creates downward pressure on the return to capital. The possibility to adjust capital utilization has a dual effect on the outcome of the shock. On the one hand, lower capital utilization helps to cut down the expenditures on energy that are necessary for the capital use and decrease the depreciation of capital. These two factors help reduce the capital costs and thus oppose to the downward pressure on the capital rate of return. On the other hand, lower utilization of capital also reduces the gains from renting out capital services, which tends to decrease the capital rate of return. Although different outcomes may emerge in equilibrium, under the baseline calibration of the oil-to-capital and capital depreciation functions, the return on capital rises after an increase in the price of oil.

The role of investment costs in the model is substantial. Namely, investment costs significantly mitigate the initial responses to the oil price shock. If the costs of investment exist, an increase in the price of oil decreases the shadow price of future capital, \( q \). This, according to (1.43), generates an downward pressure on the return on capital. Figure A.0.4 demonstrates that the negative effect of the oil price shock on all variables is much deeper in case when investment costs are absent.

Habit formation is important because it helps to get a milder short-run response of consumption to the oil price shock compared to the model without habits observed by the empirical model. By means of affecting the interest elasticity of consumption, the high habits parameter \( b \) tends to reduce the short-run impact of the real rate on consumption. The presence of habits in the model, however, does not significantly change the responses of other variables. This is demonstrated in Figure A.0.4.

Nominal rigidities take the form of the costs of price and wage changes. Without nominal rigidities, the recessions of most variables are smaller, except for the real wage. In the real economy without nominal frictions, the drop in the real wage is large enough.
to generate a big negative income effect and produce increase in hours in response to the
shock. Interestingly, wage rigidity is responsible for almost all of the differences between
the baseline and the real economy responses. Figure A.0.4 shows the dynamic responses of
the baseline model and the 3 cases where one of the nominal rigidities or both are removed
by setting $\alpha$ and/or $\bar{\alpha}$ to zero. The impulse responses of the baseline model almost coincide
with the responses of the model without price rigidities, while the impulse responses of the
real model are very close to the responses of the model without wage rigidities. The fact
that price frictions play almost no role in the model is in part due to the low estimate of
the price rigidity parameter $\alpha$.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Labor and capital market equilibrium under different calibrations of $\varrho$}
\end{figure}

In the model, I use CES of production technology in capital and labor, which is a
more general representation of production process than the Cobb-Douglass. The first order
conditions of the firm’s problem (1.5) and (1.4) help explain how this generalization affects
the equilibrium in the model. Both of these condition affect the equilibrium in capital
services and labor. Namely, (1.5) represents the demand for labor, and (1.4)–the demand
for capital services. Log-linearization of these equations produces

\[
\hat{r}_t^k = \hat{m}c_t - \frac{1 + \frac{\varrho}{1 - \varrho (\frac{uk}{\mu_h d})}}{1 + \frac{1}{1 - \varrho (\frac{uk}{\mu_h d})}} \hat{h}_t + \frac{1 + \frac{\varrho}{1 - \varrho (\frac{uk}{\mu_h d})}}{1 + \frac{1}{1 - \varrho (\frac{uk}{\mu_h d})}} \hat{u}_t \tag{1.44}
\]

and

\[
\hat{w}_t = \hat{m}c_t - \frac{1 + \frac{\varrho}{1 - \varrho (\frac{uk}{\mu_h d})}}{1 + \frac{1}{1 - \varrho (\frac{uk}{\mu_h d})}} \hat{h}_t + \frac{1 + \frac{\varrho}{1 - \varrho (\frac{uk}{\mu_h d})}}{1 + \frac{1}{1 - \varrho (\frac{uk}{\mu_h d})}} \hat{u}_t \tag{1.45}
\]

In the formulas above, \(\frac{uk}{\mu_h d}\) is calculated at the steady state and is generally greater than 1 under different plausible calibrations. The CES production function is defined for the range of parameters \(\varrho > -1\), which guarantees that isoquants are convex. Besides, \(\varrho\) determines the slopes of the demand curves for capital services and labor. The oil price shock first of all affects equilibrium in labor and capital markets by shifting the supply of capital services and labor. How big is the effect of this change depends on how steep or flat the demands for capital and services are. Thus, this parameter plays a big role in determining the effect of the oil price shock.

In the case where \(\varrho\) is close to \(-1\), capital and labor are perfect substitutes in the production of intermediate goods. The formulas above clarify that in this case, the capital utilization and hours are not related to either rental rate or hours in the first-order approximation. Thus, the demand for capital and labor are both perfectly inelastic. This case is represented by the horizontal dotted lines in Figure 1.6. On the contrary, if \(\varrho\) approaches infinity, capital and labor are close to perfect complements in the production of intermediate goods. The formulas (1.44) and (1.45) suggest that the demand for capital is absolutely elastic and represented by the vertical dotted line in the left-side graph of Figure 1.6. The demand for labor, however, is still absolutely inelastic, which is represented by a horizontal dotted line in the right-side graph of Figure 1.6. This is so because the ratio \(\frac{uk}{\mu_h d}\) is greater than one—labor is a relatively less important input in the production process. Thus, when capital and labor are close complements, the demand for labor is not driven by its wage, but rather by capital demand changes. As a result of this asymmetry, the slope of the labor demand curve rises with \(\varrho\) until \(\varrho = \varrho_0\), and for \(\varrho > \varrho_0\) the relationship between the slope of labor demand and \(\varrho\) is negative.
For the intermediate values of $\sigma$ between $-1$ and $\infty$ the demands for labor and capital services are negatively sloped, and the slopes depend on the value of $\sigma$. Introducing the parameter $\sigma$ thus provides another degree of freedom to better match the empirical evidence after the positive oil price shock.

1.7 Optimal Policy Analysis

In this section, I characterize the dynamics of the model under the optimal monetary policy. I focus on the time-invariant optimal monetary policy, or optimal from timeless perspective monetary policy, according to Woodford (2003). This definition implies that by the initial period $t = 0$ the economy has been operating for an infinite number of periods. It allows me to disregard the condition of the optimal planner’s choice at the initial period $t = 0$ and to substitute it with the first order conditions derived for an arbitrary period $t > 0$. The initial values of state variables then equal their long-run. Moreover, I assume that the optimal planner commits to decisions made in the past.

The optimal (Ramsey) monetary policy is described by the process for the interest rate $\{R_t\}_{t=0}^{\infty}$ associated with the competitive equilibrium that delivers the maximum utility for the representative agent. Formally, the Ramsey equilibrium is a set of stationary processes $c_t, h_t, i_t, k_{t+1}, w_t, \{x_{t+1,s}\}_{s\in S}, \bar{w}_t, \lambda_t, \bar{\mu}_t, q_t, \xi_t, f^1_t, f^2_t, \phi_t, k^d_t, h^d_t, \bar{\pi}_t, m_{ct}, x^1_t, x^2_t, \{x^f_{t+1,s}\}_{s\in S}; Tr_t, r_{t_t+1}, r^h_t, \pi_t, w_t, s_t, \bar{s}_t, z^*_t$ and $R_t \geq 1$, for $t$ from $0$ to $\infty$, such that they maximize the objective function of the representative household (1.9) subject to the first order conditions (1.4), (1.5), (1.6), (1.7), (1.8), (1.16)-(1.21), (1.23), (1.24), (1.25), capital evolution equation (1.11), price and real wage dispersion dynamics (1.36) and (1.31), energy-to-capital technology evolution (1.41), market clearing conditions (1.30), (1.32), (1.35), equilibrium in state-contingent assets market (1.37), equilibrium price and wage determination equations (1.27) and (1.29), and the exogenous stochastic process for the oil price growth, $\mu^{N}_{pe,t}$, (1.38), with all variables in the information set of the initial period $t = -1$ fixed at their long-run (steady) values. These variables are $c_{-1}, i_{-1}, \pi_{-1}, \pi_{-2}$,
To compute the equilibrium dynamics of the model, I log-linearize the equilibrium conditions of the Ramsey problem around the non-stochastic Ramsey steady state. This is the steady state that delivers the maximum utility to the representative household in the absence of uncertainty. The resulting impulse responses are presented in Figure A.0.4 together with the responses predicted by the baseline model. The graphs show that in response to a one standard deviation positive oil price, the optimal policy raises both the interest rate and inflation. The interest rate reaches the maximum of approximately 0.38% above the steady state value in the third quarter. Inflation stops rising after the fourth quarter with the peak of 0.2% above the steady state. GDP, investment, capital utilization and the wage rate fall with a trough observed 5 to 6 quarters after the shock. Consumption is smoothed out almost perfectly, with a slight decrease not exceeding 5 basis points. Interestingly, hours worked rise by approximately 0.2% after the shock.

The comparison of the impulse responses under the optimal policy with the responses predicted by the baseline model reveals the fact that monetary policy had a significant effect on the dynamic paths of all macroeconomic variables. Compared with the estimated model, the optimal monetary policy is associated with smaller drops of the value added, consumption, investment and capital utilization. This implies that the actual monetary policy indeed had recessionary effects on the value added. This finding confirms the results in Bernanke, Gertler, and Watson (1997) and (2004) that monetary policy exacerbated recessions. It is also in line with Leduc and Sill (2004) and Carlstrom and Fuerst (2006) who find that monetary policy could potentially help achieve a smaller drop in GDP. However, the optimal monetary policy does not justify that eliminating output drop after the oil price shock is always preferable to recession.

Another important distinction of the Ramsey model dynamics is that under the optimal policy, the interest rate rises even more in response to the oil price shock when compared
with the responses of the model with an estimated monetary policy rule. Thus, while Bernanke, Gertler, and Watson (1997) blame excessive interest rate tightening for exacerbating the recessions associated with rising oil prices, the optimal policy suggests just the opposite—the interest rate should have been raised even more to achieve smaller recessions with the least costs to the society.

Although the interest rate increases more under the optimal policy, it does not suppress inflation. In fact, under the optimal monetary policy, inflation rises more than in the estimated model. This is so because the costs of wage changes are higher than inflation costs. Thus, to lower the real wage, the Ramsey planner prefers to rely on higher price rather than lower nominal wages. Then, to prevent the real interest rate from falling, the Ramsey planner sets an increasing path for the nominal interest to keep up with higher inflation.

One of the striking differences between the Ramsey and the competitive equilibrium dynamics is the positive response of hours worked under the Ramsey policy as compared with the negative response of hours predicted by the baseline model. The reason for this is a larger fall in the real wage, which generates the negative wealth effect (compensating the substitution effect) resulting in the equilibrium increase of hours. 31

The impulse responses of all the variables other than inflation and the interest rate robust to the parameterizations of nominal rigidities in the Ramsey model. This is so because the Ramsey planner, trying to minimize the distortions created by the nominal rigidities, produces the dynamic allocations that resemble that from the model without nominal frictions. However, relative importance of the nominal rigidities may drive the optimal policy response of inflation and the nominal interest rate to the oil price shock. To check how robust are the optimal responses of these variables, I reestimate the model assuming the Calvo price and wage rigidity parameters are both set at a more conventional

31 Of course, the response of hours is defined not only by the equilibrium wage path but by general equilibrium conditions. However, the substitution effect due to the fall of the real wage in this setup is dominated by the income effect as a result of the increase of the interest rate.
value of 0.75. This is the value estimated by the Altig, Christiano, Eichenbaum, and Lindé (2004). The results are shown in Figure A.0.4. The optimal response of inflation and the interest rate still increase after the oil price shock, thus confirming the main result of this paper that the monetary policy was not raising interest rates too aggressive when it responded to oil price increases.

To provide numerical estimates of the costs of conducting the “wrong” monetary policy with respect to the oil price shock, I compare how much the value added drops in the baseline model and contrast it to the drop of the value added in the Ramsey planner problem. With this aim, I calculate the cumulative value added drop over 5 years as follows

\[ L = - \sum_{t=0}^{19} \log \left( \frac{VA_t}{VA_{ss}} \right) \]

Besides, I also evaluate the welfare costs of not following the Ramsey policy. With this aim, I estimate the welfare of the representative household in the model with the estimated monetary policy rule and compare it with the welfare estimates of a model operated by a Ramsey planner. It is well-known that the first-order approximation can not capture welfare differences of models with the same steady state. Thus, I approximate the model’s dynamics up to second-order using the apparatus developed in Schmitt-Grohé and Uribe (2004).

For the ease of presentation, I define the welfare costs as a fraction of the consumption stream in the calibrated model.\(^{32}\) The welfare costs \( \lambda \) are the minimum share of the life-time consumption pattern the representative household would demand to refuse to move to an allocation that is associated with the Ramsey equilibrium. One can show that if the welfare costs \( \lambda \) are defined in this way, then they satisfy

\[
V_{0}^{R} = (1 + \lambda)^{(1-\sigma)(1-\varphi)} V_{0}^{C} + \frac{(1 + \lambda)^{(1-\sigma)(1-\varphi)} - 1}{(1 - \beta \mu_{z}^{(1-\sigma)(1-\varphi)})} (1 - \varphi)
\]

\(^{32}\)This is similar to the definition in Schmitt-Grohé and Uribe (2005). However, they define these costs as a share of consumption in the optimal planner’s problem. I find it more intuitive to relate the costs to consumption from the calibrated model.
Here, I assume that households do not reoptimize the consumption and labor pattern after receiving the consumption transfer. From the equation above, $\lambda$ can be derived as

$$\lambda = \frac{V_t^R + \frac{1}{(1-\beta\mu z (1-\sigma)(1-\varphi))(1-\phi)}}{V_t^C + \frac{1}{(1-\beta\mu z (1-\sigma)(1-\varphi))(1-\phi)}} - 1$$

Table 1.4 presents the estimates of the cumulative value added drop for the baseline calibration of the model as well as the estimates of the welfare costs. All the numbers in the Table are shown as percentages relative to the outcome of the Ramsey economy. The first row in the Table shows the estimates for the baseline model approximated around the calibrated steady state. The second row provides the results for the baseline model approximated around the Ramsey optimal steady state.

The costs of the oil shock in terms of GDP are shown in the first column of Table 1.4. The estimates suggest that the actual suboptimal monetary policy is responsible for about 75% of the GDP loss over the period of 5 years. This finding agrees with the conclusion of Bernanke, Gertler, and Watson (1997) and (2004) who report that at least 50% of the GDP drop after the 1950s was associated with the wrong monetary policy. However, they suggest that monetary policy should not tighten interest rates in response to oil price shocks. On the contrary, I find that such a strategy is not welfare optimal, and the optimal policy would raise interest rates even more than it the Fed did responding to the oil shocks.

One can see from the second and the third columns of Table 1.4 that the response of the estimated monetary policy to a one standard deviation oil price shock leads to the welfare losses close to 5% of consumption. In terms of 2006 nominal consumption expenditures, it accounts for almost 1800 dollars a year. Thus, the welfare costs represent a significant fraction of the household’s consumption expenditures. It is important to note, however, that these costs result mostly from the actual monetary policy operating at a suboptimal steady state. This becomes clear when one looks at the second row of Table 1.4. The welfare costs of the suboptimal monetary policy response to the oil price shock are minimal if the baseline model is approximated around the Ramsey optimal steady state (less than 1 basis point of consumption). Overall, these estimates reveal the inability of the model to create non-
### Table 1.4: Welfare Costs of the Oil Price Shock

<table>
<thead>
<tr>
<th>Policy</th>
<th>L, %</th>
<th>( \lambda ), %</th>
<th>( c_{2006} \lambda ), $</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Ramsey SS</td>
<td>74.26</td>
<td>4.71</td>
<td>1767.12</td>
</tr>
<tr>
<td>Ramsey SS</td>
<td>23.27</td>
<td>0.0036</td>
<td>1.5</td>
</tr>
</tbody>
</table>

\( \lambda \) is welfare costs in terms of consumption, \( c_{2006} \) - per capita consumption expenditures in 2006, 
\( L \) - cumulative loss of value added over 5 years, expressed in percentages relative to Ramsey value added loss.

linearities that would produce significant differences in the second-order approximations.

1.8 Conclusion

In this paper, I critically evaluate the statement made by Bernanke, Gertler, and Watson (1997) that excessive tightening of monetary policy in response to the post World War II oil price shocks exacerbated recessions. I find that actual monetary policy was indeed responsible for a major part of output drops if compared with the monetary policy that maximizes the utility of the representative household. However, contrary to the claim of Bernanke, Gertler, and Watson (1997) that the Fed should decrease interest rates, I find, on the opposite, that interest rates need to be raised even more than they did to mitigate the effect of the shock.

This paper has two major contributions to the literature on monetary effects in energy models. First of all, I present an estimated DSGE model of the effect of the oil price shock on the U.S. economy. Estimation of model parameters is critical to obtain a satisfying overall picture of the economy dynamics with volatile oil prices. Although I do not get the perfect match of the inflation dynamics in the DSGE model, the model presented here, to the best of my knowledge, still better explains the macroeconomic dynamics of the empirical SVAR model than the existing models of oil sector. The second contribution of this paper is the use of the welfare approach to the analysis of the contribution of systematic monetary policy to the adverse effects generated by the oil price shocks. Using welfare based optimal monetary policy as an alternative to evaluate the contribution of the policy to the recessions from the oil shocks is more convincing, as it allows to assess how bad the policy is relative
to the best possible scenario, rather than some arbitrary, and not necessarily preferable, monetary policy alternative.
Chapter 2

Comparing Two Ways to Model Energy

2.1 Introduction

Many papers in the literature on macroeconomic effects of energy price shocks in the U.S. economy model energy as an additional factor of production in the otherwise standard Real Business Cycle (Hereafter RBC) framework. Because the share of energy in the U.S. GDP is very small, such models have been criticized for their inability to explain sizeable effects of energy price increases on the economic activity. This criticism has encouraged the development of alternative approaches to modeling the energy sector. For example, Rotemberg and Woodford (1996) suggest that to achieve a more significant effect of the energy price shock on GDP, one must reject the competitiveness assumption in the RBC model. Finn (2000), on the contrary, demonstrates that it is possible to achieve a larger drop in GDP in a model with perfectly competitive markets, if energy does not directly enter the production function.

In this paper, I compare the performance of the energy modeling strategies used by Rotemberg and Woodford (1996) and Finn (2000). I take as a basis of comparison the implied impulse responses to the energy price shock and volatilities of macroeconomic variables such as value added, investment, hours, the real wage rate and consumption, induced by the volatile energy price.

First, I abstract from the features that magnify the effect of the energy price shock and focus on modeling energy in a simple RBC framework. Thus, one of the models simply assumes that energy enters the production function directly (hereafter the EP model,) similar to Kim and Loungani (1992). The other model follows the strategy of Finn (2000). In this model, hereafter the EK model, energy does not enter the production process directly, but
it is essential for capital utilization, with the required amount of energy being proportional to the stock of capital. I show that under mild assumptions, the two different approaches to modeling energy are equivalent in terms of the log-linear dynamics and relative contribution of energy shocks to macroeconomic volatilities.

Second, I introduce the amplifying mechanisms into the two RBC frameworks for modeling energy, and evaluate the abilities of these mechanisms to magnify the effects of the energy price shocks. In the EK model, variable capital depreciation is responsible for a deeper drop in value added after the price of energy rises. In the EP model, countercyclical markups serve as an amplifying mechanism of the effect created by the energy price shock. Although I find that both amplifying mechanisms work in the right direction by worsening the recession, the amplification effect in the countercyclical markup model is smaller than in the variable depreciation model. I also confirm this result for a DSGE model with real rigidities.

Finally, I contrast the predictions of the two models with the empirical evidence on the effect of the energy price shock. I find that the size of the responses of macroeconomic variables produced by the RBC-based energy models without amplifying mechanisms are well in the range of the responses that I obtain by estimating the empirical VAR model. This empirical evidence is consistent with most of empirical studies on the effect of the energy price shocks, which questions the need for amplification of the effect of energy shocks in these models.

Rotemberg and Woodford (1996) and Finn (2000) also study the ability of their models to achieve a bigger drop in the GDP response of the energy price shock, and they also compare the performance of their models with more conventional models. The results they show, however, rely on a nonstandard definition for the energy price shock. This definition corresponds to Rotemberg and Woodford’s identification of the energy price shock in the data. Both studies demonstrate that given this definition for the energy price shock, the model without amplification mechanism fails to match the size of the GDP response, while the model with amplifying features has no problem with it. However, the shock that is used
in these papers is very different from the literature, because it produces an unusually large drop in the real GDP in the data, and it also has a complex dynamic relationship to the price of energy. This makes it hard to compare the findings of Rotemberg and Woodford (1996) and Finn (2000) with alternative studies, both empirical and theoretical. One of the goals of this paper is to use a more standard definition for the energy price shock in these models and see how well it explains the data.

There exist several other approaches to modeling energy in macroeconomic literature. Wei (2003) employs a putty-clay model of investment to model energy sector. This approach is interesting, but beyond the scope of this paper, which has the focus on explaining why the model of Finn (2000) is different from the energy-in-production models. Aguiar-Conraria and Wen (2007) use the idea of increasing returns as an amplifier of the energy price shock effect. The mechanism of increasing returns works in a similar way as the monopolistic markup setting of Rotemberg and Woodford.

The paper is organized as follows. In the next section, I present a hybrid RBC model that encompasses the two models of energy sector. In Section 2.3 I evaluate the role of amplification mechanisms in energy models. Section 2.4 switches the focus from the RBC framework to a real DSGE model. Section 2.5 is devoted to empirical evaluation of the effect of the energy price shock on the U.S. economy. Section 2.6 concludes.

## 2.2 A Hybrid RBC Model

### 2.2.1 Model

In this section, I compare two frameworks for studying the effect of the energy sector. Both of them belong to the class of the real business cycle models. One is built upon the studies of Kim and Loungani (1992), Hamilton (1983) and others. It introduces energy as an additional factor in the process of the final good production. Another frameworks was suggested by Finn (2000). On the contrary, it assumes that energy is an important factor for capital utilization. Introduced in this way, energy only indirectly influences production.
I start by describing a generalized model that covers both frameworks.

In this general setup, the production technology in period $t$ depends on capital services $u_t k_t$ derived from accumulated capital $k_t$, labor $h_t$, and, possibly, energy $e_t$ according to the following production function:

$$y_t \leq f(u_t k_t, h_t, e_t) = h^\theta \left[(1-a)(u_t k_t)^{-\theta} + ae_t^{-\theta}\right]^{-\frac{1-\theta}{\theta}}$$  \hspace{1cm} (2.1)

Firms rent capital services and hire labor from households. In the setup where firms also need energy for production, they buy it directly from the producer at an exogenous relative price $p_t$.

There are infinitely many identical consumers. Each consumer maximizes the life-time utility

$$E_0 \sum_{t=0}^{\infty} U(c_t, 1 - h_t)$$  \hspace{1cm} (2.2)

subject to the following resource constraint

$$c_t + i_t + p_t e_t \leq y_t$$  \hspace{1cm} (2.3)

where $i_t$ is investment in new capital, and $p_t$ is the price of energy in terms of final consumption. I assume that the intratemporal utility function is defined by consumption $c_t$ and labor $h_t$ in the following way

$$U(c_t, 1 - h_t) = \phi \log(c_t) + (1 - \phi) \log(1 - h_t)$$

Consumers accumulate capital $k_t$ and may use part of it in form of capital services for production. Depending on the modeling approach, energy either enters directly into the production function or is essential for capital utilization. In the latter case, the amount of energy needed to provide capital services is proportional to the existing capital stock and depends on the intensity of capital use $u_t$ according to the following formula

$$e_t \geq a(u_t) k_t$$  \hspace{1cm} (2.4)

where $a(u_t)$ is an increasing and convex function of capital utilization rate. One can think of (2.4) as a technology to produce capital services using energy, then the above assumption guarantees decreasing returns to scale for this production technology.
I also assume that capital depreciation rate varies with the intensity of capital utilization as follows
\[
\delta(u_t) = \frac{\omega_0}{\omega_1} u_t^\omega_1 
\]  
(2.5)

According to (2.5), depreciation rises at an increasing pace with higher utilization of capital, which distracts full capacity utilization. Now, the dynamics of capital can be described as
\[
k_{t+1} = (1 - \delta(u_t))k_t + i_t
\]  
(2.6)

I will assume that the depreciation rate varies with capacity utilization serves when I consider an amplifying mechanism in the EK model. Finally, I assume that producing firms may have monopolistic power. In conjunction with Rotemberg and Woodford (1996), this assumption plays the role of an amplifying mechanism of the energy price shock under countercyclical markups. I denote $\mu_t$ as a monopolistic markup charged by the firm.\(^1\)

The set of equilibrium conditions includes the resource constraint (2.3), intratemporal optimality condition for consumption demand and labor supply
\[
-\frac{U_2}{U_1} = \frac{f_2(u_tk_t, h_t, e_t)}{\mu_t}
\]  
(2.7)

and the intertemporal optimality condition for the choice of capital:
\[
U_1(c_t, 1 - h_t) = \beta U_1(c_{t+1}, 1 - h_{t+1}) \left[ \frac{f_1(u_{t+1}k_{t+1}, h_{t+1}, e_{t+1})u_{t+1}}{\mu_{t+1}} + (1 - \delta(u_{t+1})) - p_{t+1}a(u_{t+1}) \right] 
\]  
(2.8)

Besides, the optimality condition related to the the choice of energy and capital utilization depends on the approach to modeling energy. In the case of the energy-in-production model, optimal energy use is determined by its marginal productivity in production and marginal costs, which is energy price:
\[
p_t = \mu_t f_3(u_tk_t, h_t, e_t)
\]  
(2.9)

\(^1\)I postpone the discussion of markup determination till the subsection that focuses on Rotemberg and Woodford’s (1996) model.
Full capital utilization is an optimal choice in this model, because the use of capital is not associated with any costs:

\[ u_t = 1. \]

In the case of the energy-to-capital requirement model, the optimal degree of capital utilization is driven by the marginal productivity and marginal costs of capital utilization, \( f_1(u_t k_t, h_t, e_t) \) and \( p_t a'(u_t) \) correspondingly:

\[
\frac{f_1(u_t k_t, h_t, e_t)}{\mu_t} = p_t a'(u_t) + \delta'(u_t) \quad (2.10)
\]

and the amount of energy is defined by (2.4) with equality restriction.

### 2.2.2 Calibration

The model is calibrated on a quarterly basis. Several assumptions are made common in calibrating both types of the model. For example, the steady state energy share in value added across all the models is calibrated as follows:

\[ SE = \frac{p_e}{y - p_e} = 0.043 \]

The discount factor is fixed at 0.9926, which implies the annualized real rate of interest of 4%. The steady state rate of capital depreciation is 0.025. I assume that the representative consumer works 30% of his time endowment. The parameter \( \theta \) that governs the share of labor in production is fixed at 0.7. Capital utilization in the steady state is 0.82, which corresponds to the average value of the capacity utilization in manufacturing for the period from 1948:1 to 2007:4. The relative price of energy is fixed at 1, and the autocorrelation of the energy price process is set at 0.8.

The parameters described above are common across the two frameworks that I consider. The upper part of Table (B.1) summarizes the calibration strategy described above. In the subsections below, I explain how I calibrated parameters that are model-specific.
2.2.3 The EP Model

In this model, energy appears only in the production technology, so

\[ a(u_t) \equiv 0 \]

Using the first order condition (2.9),

\[ P = \mu^{-1} Y_e = \mu^{-1} (1 - \theta) ah^\theta \left[ (1 - a)(uk)^{-\theta} + ae^{-\theta} \right]^{-(1-\theta)/\theta - 1} e^{-1} \]

thus

\[ SE = \frac{pe}{y} = \frac{(1 - \theta)/\mu}{\frac{1-a}{a} \left( \frac{uk}{e} \right)^{-\theta} + 1} \]

(2.11)

Also,

\[ \frac{p}{\beta - 1 + \delta} = \frac{fe}{fuk} = \frac{a}{1 - a} \left( \frac{uk}{e} \right)^{1+\theta} \]

so,

\[ \frac{a}{1 - a} = \frac{p}{\beta - 1 + \delta} \left( \frac{uk}{e} \right)^{-1-\theta} \]

Substituting this into (2.11), I get

\[ \frac{uk}{e} = \frac{(1 - \theta)/\mu - 1}{\beta - 1 + \delta} \frac{p}{e} \]

Now, use the definition of the production function (2.1) to derive

\[ \frac{y}{e} = \frac{p}{SE} = \frac{h^\theta}{e} (1 - a) \left( \frac{uk}{e} \right)^{-\theta} + a )^{-1-\theta} \]

From the formula above, one can obtain the value of \( e \)

\[ e = \frac{h}{\left( \frac{p}{SE} \frac{(1 - a)(uk)^{-\theta} + a^{-1-\theta} \right)^{1/\theta}} \]

All the remaining variables of the model can now be inferred easily.

2.2.4 The EK Model

First of all, energy in the EK model does not directly enter into the production function, and the production function is Cobb-Douglas in capital services and labor:

\[ f(u_k, h, e) \equiv g(u_k, h) = (uk)^{1-\theta} h^\theta \]

(2.12)
Energy indirectly affects production by contributing to both the costs of accumulated capital and costs of capital utilization. The total costs of capital consist of costs that are associated with energy purchases that are proportional to capital stock, and depreciation:

$$a(u)p + \delta$$

The rise in the total costs associated with the rise in $p$ has a negative welfare effect on households, inducing a drop in consumption and stimulating labor supply. Reducing capital utilization rate in response to rising energy prices mitigates the negative effect on consumers, but hurts production through lower supply of capital services and thus higher capital rental rates.

I calibrate the EK model using the same strategy as Finn (2000). Finn (2000) makes a different assumption to model energy sector. Finn assumes that energy-to-capital requirement ratio is defined by an increasing and convex power function of capital utilization rate:

$$\frac{e}{k} = \frac{v_0}{v_1} u^{v_1}$$

(2.13)

where $v_0 > 0$ and $v_1 > 1$.

I calibrate the parameters in the Finn’s model as follows. Using the first order necessary optimality condition (2.8) in the steady state,

$$\frac{y}{k} = \frac{\frac{1}{\beta} - 1 + \delta}{1 - \theta - SE}$$

At the same time,

$$\frac{y}{k} = \left( \frac{uk}{h} \right)^{-\theta}$$

So,

$$k = \left( \frac{\frac{1}{\beta} - 1 + \delta}{u(1 - \theta - SE)} \right)^{1-\theta} \frac{h}{u}$$

All the remaining variables can now be derived easily.
2.2.5 Discussion

Section 2.2.1 presents the generalized model suitable for studying two different strategies to model the energy sector. In this section, I evaluate the performance of the two models in terms of their ability to reproduce the responses and fluctuations of macroeconomic variables that are due to the energy price shock.

Figure B.1 graphically demonstrates how the propagation of the energy price shock differ in the two models using the markets for capital services and labor. The red lines in the figures show the shifts in the curves as a result of the increase in the energy price, keeping all the endogenous variables constant. The green lines show the shift of the curves induced by the equilibrium changes of endogenous variables following the shock. As can be seen from Figure B.1(a), the energy price shock by itself shifts in the supply of capital services in the EK model, and stimulates labor supply. The supply of capital services drops immediately because the price of energy increases the marginal costs of producing capital services. Households face a negative income effect because of the higher total spending on energy after the price of energy rises, so they increase the supply of labor. In the EP model, an increase in the energy price acts like a negative technology shock, decreasing the marginal productivity of capital and labor. As a result, the shock has a direct negative impact on capital and labor demands. The demand for capital services and hours shrink in the EK models as an equilibrium response to increasing production costs. This is demonstrated by green lines in Figure B.1.

In a general equilibrium setting, both primary and secondary shifts occur simultaneously and can not be distinguished from one another. Moreover, the two models will have the same dynamics if functions $g$, $f$ and $a$ are such that

$$g(a^{-1}(\frac{e}{k})k, h) \equiv f(k, h, e)$$

This result emerges as a result of the envelope theorem. Thus, in both models the economy moves instantaneously from 0 to 1.

Thus, although the EK model explicitly assumes that energy and capital are highly
complementary, the dynamics of this model is the same as in the EP model that lacks complementarity of $e$ and $k$.$^2$ The equivalence of the models' log-linear dynamics can be extended for a DSGE framework with a number of real and nominal rigidities as long as these rigidities are not determined by energy and capital utilization.

Figure B.5 shows the impulse responses generated by the EK and the EP models. The difference in the dynamics of the two models is due to a more general CES production function with a non-unit elasticity of substitution between energy and capital in the energy-in-production model. Namely, I calibrate $g = 0.7$ as in Kim and Loungani (1992), which implies the elasticity of substitution parameter of 0.58. Interestingly, the EK model produces the drop in output as big as 0.8%, which is within the standard error bands of the estimate produced by the empirical VAR model from Section 2.5. It is also in agreement with most studies reviewed in Section 2.5. Thus, even though the simple RBC model fails to match the timing and the shape of impulse responses, the model succeeds in explaining quantitatively the effect of the energy price shock on GDP. $^3$ The reason why the EP model shows a smaller GDP effect is that the elasticity of substitution of factors in production is smaller than 1.

To summarize, this section demonstrates the equivalence of the EK and EP models. In the next section, I explain how the amplifying mechanisms work in the EK energy model.

### 2.3 Amplification of the Energy Price Shock Effect

Significant economic downturns in the U.S. economy that historically followed the energy price increases after the World War II, and in particular the two episodes in the 1970s, created concerns among the economists that a standard RBC model where energy enters the production function could not produce the observed GDP drops, given that the share

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$^2$This analysis assumes that capital depreciation is constant.

$^3$Kormiltsina (2008) shows that a richer model with a non-stationary energy price process can help to match the empirical findings across these dimensions.
of energy in value added is very small.\footnote{See, for example, Hamilton (2003), Rotemberg and Woodford (1996), Finn (2000)} This fact motivated the development of extensions to a standard RBC model that would allow to achieve a deeper macroeconomic downturns as a result of rising energy prices.

This section focuses on such an amplifying mechanism developed by Finn (2000), and tries to understand what is this mechanism and how it works. For comparison purposes, I demonstrate how the effect of this mechanism compares with the effect of Rotemberg and Woodford (1996) under an $AR(1)$ representation of the energy price shock commonly used in the literature.

Figures B.6 and B.7 show the two amplifying mechanisms at work. In both Figures, the blue lines show the impulse responses in models without amplification, and the green lines represent the responses in the models enhanced with amplifying mechanisms. One can see that on impact, both mechanisms show negative responses for all the variables about twice as large as the models that do not have them. The amplified EP model's output drop peaks at 0.8%, while the amplified EK model produces 1.5% drop in output in response to the energy price shock. However, the EK and EP models without the amplifying mechanisms produce different responses, which makes the comparisons of the effects of the new assumptions more difficult. To obtain a more direct comparison, I also present Figure B.8, where the EP model is calibrated with $\rho$ very close to 0.\footnote{I set $\rho = 0.001$.} This makes the dynamics of non-amplified models virtually indistinguishable, and the comparison of the impulse responses under the two mechanisms thus becomes more straightforward. As one can see from the graphs, the model of variable depreciation in the EK model produces output drop that is about 50% bigger than the markup model of Rotemberg and Woodford in the context of the EP model. The drop in other variables except consumption is also smaller in the countercyclical markup model. There is no much difference in the response of the real wage rate.

Below I discuss critically the amplifying mechanisms in the two models.
2.3.1 Rotemberg and Woodford (1996): Varying Markups

Rotemberg and Woodford (1996) demonstrate that a model with imperfect competition and countercyclical markups helps to amplify the effect of the energy price shock to produce a 2.5% drop in GDP following a 10% energy price shock, while in the similarly calibrated perfectly competitive model GDP drops by no more than 1%. As Rotemberg and Woodford show, an increase in the price of energy in their model raises marginal costs of energy and consequently raises firm markups. While the effect of higher marginal costs is also present in a competitive energy-in-production model, increasing markups make an additional contribution to the negative effect of the energy price shock on the economy.

The main idea behind countercyclical markups is that monopolistically competitive firms can collude in setting the markup for their products. In doing so, the firms are involved in a dynamic repeated game, and markups are determined based on a symmetric equilibrium arising in this game. Under certain assumptions, the microfoundation of markup determination does not affect the aggregate behavior in the model, it makes the markup of the firms endogenous. Rotemberg and Woodford (1992) show that the markup dynamics is described by a non-decreasing function of only the ratio of the present discounted value of future profits $X_t$ to current output. $X_t$ is defined as follows:

$$X_t = E_t \sum_{j=1}^{\infty} \alpha_j \beta_j \lambda_{t+j} \left( \frac{\mu_{t+j} - 1}{\mu_{t+j}} Y_{t+j} \right)$$

where $\alpha$ is the exogenously given probability that the game will be repeated in period $\tau + 1$ given that it is played in $\tau$. The role of this parameter is to reduce the size of possible punishments for deviations from the collusive agreements.

2.3.2 Finn (2000): Variable Capital Depreciation Rate

Finn (2000) introduces the EK model with variable capital depreciation rate to amplify the effect of the energy price increase on output. Although Finn admits that variable depreciation assumption is the feature that makes her model different from the EP model,
the intuition why the response of the economy is amplified in the presence of variable depreciation is not clear:

“when depreciation depends on utilization ... and through it on energy use ..., then energy has an additional indirect transmission channel - the indirect effect of energy on output working through the capital stock... Thus, endogenizing the capital utilization decision, with its energy and depreciation costs, leads to an important difference between the present and previous energy theories (p. 404, Finn (2000)).”

Finn also states that “total return to investment is the sum of capital’s marginal return ... and the undepreciated component $1 - \delta(u) \text{ less capital’s marginal energy costs } pa(u)...$ (p. 405)” and then explains how the direct transmission channel (working through production as in the EP model) of the energy price shock causes the decline in capital’s future marginal productivity, which “...causes a fall in capital’s future marginal return inducing a decrease in the total return to investment, and, thus, decreases in $i$ and $k$, too... (p. 406).”

However, with variable depreciation the lower intensity of capital use reduces capital depreciation and thus decreases the costs of capital, which increases future marginal return to investment. Thus, everything else equal, variable capital depreciation should work in the direction opposite to the direct channel and should thus mitigate, rather than amplify, the effect of the energy price increase.\(^6\) This reasoning, however, works only if the two models are calibrated identically. This is not the case in the model of Finn (2000), where the parameters of energy cost functions of capital are very different in constant and variable depreciation models. This fact explains why introducing variable depreciation amplifies the effect of the energy price shock.

To understand why energy costs of capital must differ in the two models, it is useful to note that introducing variable depreciation changes the marginal costs of capital from

\(^6\)It is also not clear from Finn’s analysis why lower return to investment is associated with lower $i$ and $k$. While this is true if only the supply of investment is affected by the shock, it is not usually the case in general equilibrium models.
\[ a'(u)p \] to

\[ a'(u)p + \delta'(u) \]

as well as the average costs of capital from \( a(u)p + \delta \) to

\[ a(u)p + \delta(u) \]

Another important fact is that in the steady state, both marginal and average costs of capital are the same in both types of the model. This is so because the demand for capital services is presented by the marginal productivity of capital services \( g_1(uk, h) \) and is invariant to the assumption about capital depreciation. At the same time, in equilibrium the demand for capital services equals their supply, or marginal capital costs, so in steady state, the marginal capital costs must be the same in two models. The average capital costs are invariant because in the steady state they are defined by the invariant production block of the model through the intertemporal optimality condition:

\[
1 = \beta(g_1(uk, h)u + 1 - \delta(u) - pa(u))
\]

Because the calibration strategy relies on the steady state rate of capital utilization, the parameters of \( a(u) \) must be different across the two models for the capital costs to remain the same. This calibration-induced difference in the parameters of \( a(u) \) function plays a crucial role in explaining the amplification of the effect of the energy price shock in the model where capital depreciation rate vary.

One can show that if \( a(u) \) and \( \delta(u) \) are power functions, then the average costs of capital in the model with variable depreciation is a more convex function of capital utilization than the average capital costs in the constant depreciation model. This means that the marginal capital costs in the model with constant depreciation are much steeper than the marginal capital costs in a model with variable depreciation. For the EK model described above, the absolute risk aversion coefficient in constant depreciation case is around 7, while the same parameter in the variable depreciation model is approximately 0.6, which is more than 10 times smaller.
Figure B.2 illustrates what is said above. With the initial equilibrium at point 0, the inward shift of the marginal capital costs that is due to an increase in the price of energy moves the equilibrium in the capital services market to the left to point 1 when depreciation is variable (blue line). In the case of constant depreciation the equilibrium jumps to point 2 (red line), which is between 0 and 1, so that the resulting drop in the capital utilization rate is smaller. Although this analysis is conducted assuming hours worked stay constant, this effect would not change qualitatively if hours drop in response to the shock, because the drop in hours depresses the demand for capital, but does not change the position of new equilibria 1 and 2 in the graph relative to each other.

As it becomes clear from Figure B.2, amplification of the energy shock effect can only be achieved if the slope of the marginal capital costs in case of variable depreciation is small enough comparative to the constant depreciation case. However, if parameters defining risk-aversion of capital costs could be set free, the amplification of the energy price shock may not be observed. This counterexample is presented in Figures B.3 and B.4, where I consider quadratic instead of power specifications for $a(u)$ and $\delta(u)$:

\[
\begin{align*}
  a(u) &= a_0 + a_1(u - u^{ss}) + \frac{a_2}{2}(u - u^{ss})^2 \\
  \delta(u) &= \delta_0 + \delta_1(u - u^{ss}) + \frac{\delta_2}{2}(u - u^{ss})^2
\end{align*}
\]

where $u^{ss}$ is the steady state capital utilization rate. I set parameters $a_0$, $a_1$ and $\delta_0$, $\delta_1$ to make these functions are equivalent up to the first order to the power functions (2.13) and (2.5). Namely, I define

\[
\begin{align*}
  a_0^{var} &= SE \frac{y}{pk} = 0.0067 \\
  a_1^{var} &= \upsilon_0 \upsilon_1^{-1} = 0.0124 \\
  a_2^{var} &= (\upsilon_1 - 1)\upsilon_0 \upsilon_1^{-2} = 0.0076 \\
  \delta_0 &= 0.025
\end{align*}
\]
\[ \delta_1 = \omega_0 u^{\omega_1 - 1} = 0.0354 \]
\[ \delta_2 = (\omega_1 - 1)\omega_0 u^{\omega_1 - 1} = 0.007 \]

Assuming quadratic functions helps to release the second derivatives of \( a(u) \) and \( \delta(u) \) from being constrained by the steady state equilibrium conditions. Namely, because in the steady state \( u = u^{ss} \), \( a_2 \) and \( \delta_2 \) do not enter anywhere in the steady state equilibrium of the model, so they can be assigned arbitrary positive values.\(^7\)

With the aim to construct a counterexample to the Finn’s model, I choose the values for \( a_2^{cons} \) so that the risk-aversion in the constant depreciation model is \( \frac{a_2^{cons}}{a_1^{cons}} = 0.6 \), which matches the variable depreciation rate model. At the same time, I set \( a_0^{cons} \) and \( a_1^{cons} \) to match \( a(u) \) and its first derivative in the model with power functions and constant depreciation:

\[ a_0^{cons} = SE \frac{y}{pk} = 0.0067 \]
\[ a_1^{cons} = \nu_0 u^{\nu_1 - 1} = 0.0478 \]
\[ a_2^{cons} = a_1^{cons} 0.06 = 0.0029 \]

The results of this calibration exercise are presented in Figures B.3 and B.4. The black line in Figure B.3 shows the supply of capital services before the energy shock strikes the economy, which is the same in both models. After the shock, both supply curves shift in. However, the new equilibrium denoted by point 2 moves further away to the left from the initial equilibrium at 0 than the new equilibrium under the constant depreciation assumption denoted by point 1. This is so because the slope of the new supply curve under variable depreciation is almost parallel to the new supply curve under constant depreciation, while the vertical jump of the marginal costs curve in case of constant depreciation is larger. As a result, as Figure B.4 demonstrates, the impulse responses in the EK model with variable depreciation rate are smaller than in the model with constant capital depreciation.

\(^7\)Switching away from power to quadratic function, however, has a problem that \( a(u) \) and \( \delta(u) \) functions are not invertible. However, they remain invertible in a small vicinity of the steady state, so the analysis remains valid for first-order approximation of the model.
2.4 Energy Sector in a DSGE Framework

Figures B.9 - B.12 shows the effect of the energy price shock in a DSGE framework. I assume two types of real rigidities - habits in consumption and investment adjustment costs. With consumption habits, the utility function in (2.2) becomes

\[ U(c_t - bc_{t-1}, 1 - h_t) = \phi \log(c_t - bc_{t-1}) + (1 - \phi)\log(1 - h_t) \]

The capital dynamics (2.6) under investment adjustment costs evolves as:

\[ k_{t+1} = (1 - \delta(u_t))k_t + i_t(1 - s\left(\frac{i_t}{i_{t-1}}\right)) \]

where

\[ s\left(\frac{i_t}{i_{t-1}}\right) = \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1\right)^2 \]

As shown in Tables B.3 an B.2, I assume \( b = 0.65 \) and \( \kappa = 3 \). These numbers are very close to ones estimated in DSGE literature.

The role of habit formation is to mitigate the immediate response of consumption to the shock, and investment costs serve to avoid an immediate drop in investments. As can be seen from Figure B.9, adding these features helps generate U-shaped responses in consumption, investment, and consequently, in value added. However, adding the real rigidities completely changes the response path for hours worked. In response to the energy price increase, hours worked rise.

It is important to note that adding real rigidities changes the size of the response of value added and other variables. It is also interesting to note the differences in GDP responses between the EK and EP models. The EP model produces a much smaller response of GDP than the EK model, with the deepest drop in GDP of about 0.15% in response to a 10% increase in the price of energy, while according to the EK model, GDP drops 0.3% at trough. Again, this difference between models’ impulse responses is entirely due to the fact that the elasticity of substitution between capital and labor in production in the EP model is smaller than 1, and it is exactly 1 in the EK model. Now, if one looks at Figures B.12 and B.11, one can see again the limited ability of countercyclical markups to deepen output.
drop. Even if the elasticity of factor substitution in the EP model is set at (almost) 1, the model can not produce GDP drops larger than 0.35%, as is demonstrated in Figure B.10.

2.5 The Energy Price Shock Effect Quantitatively

In this section, I summarize the findings of the previous literature on the effect of the energy price shock. The purpose of this exercise is to get a better understanding what is the size of the effect of the energy price shock on GDP for the United States.

The early studies were primarily based on the evidence of two oil shocks in the 1970s. In 1973-1974, the price of oil rose dramatically after the OPEC oil embargo as a result of the Arab-Israeli war. The resulting recession amounted to about 8% of real (detrended) GDP loss from peak to trough relative to trend.

Hamilton (2005) demonstrates that according to a standard RBC model with energy as a production factor, the share of the energy in the U.S. GDP as small as 4% implies that a 10% increase in the price of energy would produce a drop in output not bigger than 0.4%. Hamilton uses this fact together with the numbers for the 1973-1974 U.S. recession to argue for the inability of a standard RBC model to explain this episode in the U.S. history. However, it is important to take into account that during this episode, the rise in the price of oil was much larger than 10%. For example, the WTI oil price rose from 3.56 to 10.11 dollars per barrel between July 1973 and January 1974, which is the rise of 284% over just 2 quarters. In accordance with Hamilton’s calculations, a 200% rise in the price of energy implies a drop in GDP as big as 8%, which well agrees with the observed evidence.

Another influential study that undermines the ability of a standard RBC model to explain the negative effect of the energy price increase is that of Rotemberg and Woodford (1996). The authors argue that a standard model is not capable to produce a drop in the value added of the size that their empirical model produces, and propose a mechanism that amplifies the effect of the energy price shock. Rotemberg and Woodford’s analysis of impulse responses in their estimated model suggests that a 10% increase in the price of
energy results in 2.5% drop in value added. To get an idea of how this estimate fits into other research on the effects of energy price shocks, I review different papers that directly (like Blanchard and Gali (2007), Bernanke, Gertler, and Watson (1997) and others) or indirectly (Peersman (2005)) estimate output response to the oil price shock. I present the results in Table B.4. Because of the critique of RBC models, I am particularly interested in finding out what different studies have to say about the upper bound of the effect of the oil price shock on output. Because of this, I report the largest estimate of the effect of the energy price shock that I found in the papers.

As one can see from Table B.4, the estimate given by Rotemberg and Woodford (1996) is at least twice as large as other estimates. For example, the estimates in Bernanke, Gertler, and Watson (1997) suggest that a 10% oil price shock explains only 0.25% drop in GDP. Although Hamilton and Herrera (2004) suggest that Bernanke, Gertler, and Watson (1997) substantially underestimate this effect by including 7 rather than 12 lags in their VAR, the quarterly VAR model of the oil price shock that Hamilton and Herrera (2004) estimate produces 0.8% drop in GDP, which is still 3 times smaller if compared with Rotemberg and Woodford (1996). Hamilton and Herrera (2004) check the robustness of the results in Bernanke, Gertler, and Watson (1997) and present a number of impulse response estimates for different indicators of the oil price shock. The largest drop in GDP in their study (1.2%) is produced by a model where the oil price shock is identified in a structural VAR model using Hoover and Perez (1994) dummy variables. This study documents a significant effect of the oil price shock about 3 times larger than what a standard RBC model can predict according to Hamilton (2005). However, it is still twice as small as 2.5% value added drop in Rotemberg and Woodford (1996).

Hamilton (2003) estimates the nonlinear relationship between GDP and the price of oil. Hamilton presents a simple linear regression model over the period from to and finds that the U.S. GDP would drop 1.4% after a 10% oil price increase. This effect is big in

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\[8\] Lee, Ni, and Ratti (1995) (1.2%) obtain a similar result, however their measure of the shock is not comparable to the studies discussed here.
part because it is estimated on a short time interval that cover two largest in the history oil price increases. Besides, this response may be biased upward because it is based only on 1 equation. In univariate models, the endogeneity of regressors may bias the estimates upward, because these models can not identify possible feedbacks from other variables. The reason is that if there are other shocks that also negatively affect GDP but are not related to oil prices, then a univariate model will ascribe the effect of these shocks to an oil price increase.

To evaluate the effect of the oil price shock, I estimate a VAR model where the vector of endogenous variables consists of the WTI oil price increases, growth of the log of average labor productivity, inflation of the GDP deflator, logs of the growths of hours, wages, and investment, in the same ordering as presented above. I assume $A_0$ is lower triangular, and, for the results to be consistent with the theoretical model, I restrict the price of oil variable to be an exogenous AR(1) process. The time period ranges from 1948 : 1 to 2007 : 4. Figure B.13 shows the impulse responses generated by this VAR model.

As can be seen from the Figure, GDP, investment, hours, and consumption drop after the shock in a U-shaped fashion. The drop in the real wage is not significantly different from zero for most of the period. The largest drop in GDP is around 0.6%, which is similar to the findings in the related literature. The size of a drop in GDP well agrees with the finding of the simple theoretical models based on RBC theory. Thus, there seems to be no need for amplification mechanisms for simple energy models. However, these models could not replicate the timing and the shape of impulse responses. To improve upon these dimensions, models of the energy shock based on DSGE frameworks may be used. However, the latter models may need to utilize the amplification mechanisms, as the presence of rigidities mitigates the responses of variables. Moreover, the parameters of these models need to be adjusted properly to match the sizes of the variables. The DSGE

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9Hours are hours of all persons in the non-farm business sector. The wage rate is the hourly compensation in the non-farm business sector. Investment is the consumption of durable goods and private investment.
literature suggests different estimation techniques to choose the right parameter values.

In Table B.5 I show the contribution of the oil price shocks to the unconditional volatility of the macroeconomic variables that appear in the VAR model. In doing so, I evaluate the unconditional variance of these variables implied by the estimated VAR model. I also calculate the variance decomposition of then historical time series. I present the results in percentages relative to the standard deviation of the corresponding time series. As can be seen from Table B.5, the shock is responsible for the major part of the variation in the nominal oil price, however not all of it, so the identification strategy allows for a possible endogeneity in determination of the oil price. The figures showing the contribution of the oil price shock to macroeconomic fluctuations are modest, the implication of which is that the oil price shock alone does not explain all the macro volatility. Fluctuations in the price of oil are responsible for no more than 20% of output fluctuations. According to this model, the oil price shock explains a substantial part of inflation fluctuations, and a non-negligible part in fluctuations of hours. Oil price shocks also explain less than 20% of the investment growth variability, and even less variation in the wage rate growth.

Tables B.6 and B.7 evaluate the ability of energy models to capture macroeconomic volatilities induced by the stochastic nature of energy prices. In these Tables, I show the estimates of the volatility of several key macroeconomic variables that are caused by the energy price shock of the size of one standard deviation from the VAR model. The numbers are presented as percentages of the unconditional volatility of the time series. The first two columns of Table B.6 show the results of the EP and EK models without amplifying mechanisms. Several things should be noted here. First, both models explain more than 30% of the observed output fluctuations, while the energy price shock empirically identified in the VAR model attributes only 15% of output fluctuations to energy prices. It means that RBC models that are believed to underestimate the effect of energy shocks on GDP, actually overestimate this effect. The same conclusion holds for the wage rate and investment fluctuations. Both models generate fluctuations in investments and the wage rate that are about 4 times as big as is predicted by the identified VAR model of the energy
price shock. As far as hours worked are concerned, the model fails to generate fluctuations as big as in the empirical model.

The last two columns of Table B.6 show the volatilities of macroeconomic variables in the EP and EK models with amplifying mechanisms. The resulting volatilities are even bigger than the estimates from the VAR model. Using a DSGE framework helps to decrease the estimated volatilities, as can be seen from Table B.6.

Just looking at the dynamics and volatilities generated by the models with different amplifying mechanisms, it is hard to tell which of the two models is more realistic and better to use when answering energy related question. Obviously, neither of the models is perfect. The model of Rotemberg and Woodford (1996) lacks the connection between its micro- and macroeconomic foundations. Namely, the parametrization suggested by Rotemberg and Woodford (1996) is not related to the parametrization of the dynamic game, because there is no simple relationship between them. Moreover, Rotemberg and Woodford (1996) do not use any microeconomic evidence to calibrate the process for $\mu$, except a very general condition $\epsilon_\mu < \mu - 1$. However, calibrating this process consistently with the microeconomic foundations is very important, because this process determines the size of output drop after the energy shock. The shortage of the mechanism of varying depreciation rate of Finn (2000) is that it is not immediately intuitive and extremely dependent on the calibration of $a(u)$ an $\delta(u)$ functions. With a different calibration, the amplifying effect may not be observed, as is shown, for example, in Kormilitsina (2008).

The main conclusion that arises from the analysis presented above is that there is not enough evidence to state that the effect of the oil price shock is indeed so big that a simple RBC model is not able to explain it. The largest GDP drop in response to a 10 percent oil price shock that is found in this paper, does not exceed 0.6% 4 to 6 quarters after the shock. This number is almost 4 times smaller than the one obtained in Rotemberg and Woodford (1996), is of the order of magnitude similar to many other estimates presented in the literature, and it seems to be much more close to the number as suggested by Hamilton (2005). Also, I show that in terms of fluctuations, the identified oil price shock contributes
to only a moderate fraction (%) of output fluctuations. In light of this, 35% of unconditional output fluctuations that Kim and Loungani (1992) ascribe to an oil price shock in their model is definitely an overshooting of the mark. This, as a consequence, may undermine the need for amplifying mechanisms such as those used in Rotemberg and Woodford (1996) and Finn (2000).

2.6 Conclusion

In this paper, I compare two different approaches used in macroeconomics to model the energy sector: one where energy directly enters the production function, and another where energy supplements capital utilization. I compare the performance of the models with and without the amplifying mechanisms of the energy price effect and find that both amplifying mechanisms increase the GDP recession, but the amplification effect in the countercyclical markup model is smaller than in the variable depreciation model. This result is also confirmed in a DSGE model with real rigidities.

I contrast the predictions of the two models with the empirical evidence on the effect of the energy price shock. I find that the size of the responses of macroeconomic variables produced by the RBC-based energy models without amplifying mechanisms are well in the range of the responses that I obtain by estimating the empirical VAR model. This empirical evidence is consistent with most empirical studies on the effect of the oil price shocks.

Some studies focus on the effects of the energy shocks in a model where energy is just an additional factor of production in the otherwise standard RBC framework. Others use the approach of Finn (2000). I demonstrate that under very mild assumptions, the two different approaches to modeling energy are equivalent in terms of the log-linear dynamics. In this paper, I pay special attention to understanding how the shock affects the economy, and what makes its effect different from the effect of the same shock in the EP model. In particular, I explain why introducing variable depreciation in the EK model amplifies the recession after an energy price shock. Although the share of oil in the U.S. GDP is small,
the model predictions are close to those empirically observed, however, the models fail to produce the shape of impulse responses. While richer DSGE models solve this problem, more sophisticated estimation-based calibration techniques of the model parameters are crucial for a good match of the model and VAR dynamics.
Appendix A

Appendix for Chapter 1

A.0.1 Calibration of some parameters

- Given $SE$ - share of energy expenditures in value added and $Y$ - observed output, which is $Y = PF/s$,

$$SE = \frac{P^E E}{Y - P^E E} = \frac{a k/\mu_z}{z^*(y - e)} = \frac{a}{z^*} \frac{k/\mu_z}{(pf - \Psi - e)^s}$$

and that $\Psi$ is calibrated so monopolistically competitive firms earn no profits in the steady state, (thus $\Psi = (1 - smc)pf$ and $pf - \Psi = smcpf$) we can get

$$a = z^* SE \frac{mcpf}{k/\mu_z};$$

then $\frac{\mu y}{k}$ can be found from household’s F.O.C. w.r.t $k_{t+1}$ and firm’s F.O.C. w.r.t. capital factor, by solving the following equation with respect to $t$

$$(\mu^1_{\zeta}^{(1-\sigma)(1-\varphi)}) - 1 + \delta)q = \frac{t^{1+\epsilon}mc\theta}{u^\epsilon} - SEmct;$$

Now, steady state capital $k$ can be derived from

$$\frac{uk}{zh^d\mu_z} = (\frac{\mu y_{uk}}{1-\theta})^{1/\varphi}$$

$$w = zm\theta\frac{(1 - \theta)(1 - \theta + \theta(\frac{uk}{zh^d\mu_z})^{-\varphi} - 1)^{-1/\varphi}}{1 + \mu(R - 1)}$$

Fixed costs $\psi$ produce zero profits in steady states.

$$\psi = zh^d(1 - smc)(\theta(\frac{uk}{zh^d\mu_z})^{-\varphi} + 1 - \theta)^{-1/\varphi}$$

$$y = (zh^d(\theta(\frac{uk}{zh^d\mu_z})^{-\varphi} + 1 - \theta)^{-1/\varphi} - \psi)/s$$
• Capital utilization and oil-to-capital requirement

\[ a(u) = a_0 + ν_0(u - u^{ss}) + ν_1(u - u^{ss})^2 \]

\[ δ(u) = δ_0 + ω_0(u - u^{ss}) + ω_1(u - u^{ss})^2 \]

Given $δ_0$, $ω_0$ can be figured out using the FOC (1.18)

\[ ω_0 = \frac{1}{q} \left( rk - \frac{v_0}{z^*} \right) \]

Thus, $ω_0$ can not be estimated, because its value is implied by the steady state equilibrium conditions. $ω_1$, $ν_1$ and $ν_0$ are not pinned down by the steady state, however, these parameters affect the dynamics of the model and thus can be estimated by the IRF matching estimator.

### A.0.2 Production Function

The demand for capital services in log-linearized form is

\[ \dot{r}_t = \dot{mc}_t + \frac{F_{11}uk}{F_1} \dot{u}_t + \frac{F_{12}h^d_t}{F_1} \dot{h}^d_t \]

The demand for labor in log-linearized form is

\[ \dot{w}_t = \dot{mc}_t + \frac{F_{22}h^d_t}{F_2} \dot{h}^d_t + \frac{F_{21}uk}{F_2} \dot{u}_k \]

The production function is

\[ F(uk, h^d) = [θ(uk)^{-1} + (1 - θ)(h^d)^{-1}]^{-1/ε} \]

Then

\[ F_1 = θ[θ(uk)^{-1} + (1 - θ)(h^d)^{-1}]^{-1/ε-1}(uk)^{-1/ε-1} > 0 \]

and

\[ F_{12} = θ(1 - θ)(uk)^{-1}(h^d)^{-1}[θ(uk)^{-1} + (1 - θ)(h^d)^{-1}]^{-1/ε-2} \]

68
Thus

\[ \frac{F_{12}h^d}{F_1} = \frac{1 + \varrho}{1 + \frac{\varrho}{1-\theta(h^d)^{-\varrho}}} \]

Because \( F(\cdot, \cdot) \) is homogenous of degree 1, the following relation is true

\[ \frac{F_{11}uk}{F_1} = \frac{F_{12}h^d}{F_1} \]

Thus,

\[ \frac{F_{11}uk}{F_1} = -\frac{1 + \varrho}{1 + \frac{\varrho}{1-\theta(h^d)^{-\varrho}}} \]

The similar logic applies to the derivation of \( \frac{F_{21}uk}{F_2} \) and \( \frac{F_{22}h^d}{F_2} \). Here, use the fact that \( F_{12} = F_{21} \)

\[ F_{21} = (1 - \theta)\theta(h^d)^{-\varrho -1}(uk)^{-\varrho -1}|\theta(uk)^{-\varrho} + (1 - \theta)(h^d)^{-\varrho}|^{-1/\varrho -2(1 + \varrho)} \]

Thus,

\[ \frac{F_{21}uk}{F_2} = -\frac{F_{22}h^d}{F_2} = \frac{1 + \varrho}{1 + \frac{\varrho}{1-\theta(h^d)^{-\varrho}}}. \]
### A.0.3 Tables

**Table A.1: Historical contribution of the oil price shock**

<table>
<thead>
<tr>
<th>Standard Deviation</th>
<th>% of the unconditional variance explained by the oil price shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil Price</td>
<td>11.01</td>
</tr>
<tr>
<td>log(GDP)</td>
<td>0.41</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.34</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>0.28</td>
</tr>
<tr>
<td>log(CU)</td>
<td>0.83</td>
</tr>
<tr>
<td>log(H)</td>
<td>0.46</td>
</tr>
<tr>
<td>log(W)</td>
<td>0.11</td>
</tr>
<tr>
<td>log(C)</td>
<td>0.14</td>
</tr>
<tr>
<td>log(I)</td>
<td>1.47</td>
</tr>
</tbody>
</table>

The table shows the variance decomposition of the HP-filtered time series implied by the SVAR model based on the historically observed shocks. Each number in the first column of the table shows the estimate of the variance of time series produced by the SVAR model where all the shocks except the oil price shock are shut down, as a fraction of the variance of the original time series. The second column shows the ratio of the variance of the time series generated in this way to the unconditional variance of the time series, in percentages.

**Table A.2: Contribution of the oil price shock to macroeconomic volatilities**

<table>
<thead>
<tr>
<th>Standard Deviation</th>
<th>Unconditional standard deviation</th>
<th>% of the unconditional variance explained by the oil price shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil Price</td>
<td>8.03</td>
<td>8.27</td>
</tr>
<tr>
<td>GDP growth</td>
<td>0.12</td>
<td>0.22</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.034</td>
<td>0.52</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>0.028</td>
<td>0.51</td>
</tr>
<tr>
<td>log(CU)</td>
<td>0.021</td>
<td>1.65</td>
</tr>
<tr>
<td>log(H)</td>
<td>0.036</td>
<td>1.54</td>
</tr>
<tr>
<td>W growth</td>
<td>0.038</td>
<td>0.11</td>
</tr>
<tr>
<td>C growth</td>
<td>0.030</td>
<td>0.10</td>
</tr>
<tr>
<td>I growth</td>
<td>0.32</td>
<td>0.74</td>
</tr>
</tbody>
</table>

The Table shows the contribution of the oil price shock based on the SVAR model, %. The first two columns show conditional and unconditional standard deviations of macroeconomic variables implied by the SVAR model. Conditional standard deviations of macroeconomic variables are based on a 1 standard deviation oil price shock. The third column shows the squared ratio of numbers in columns 2 and 3.
A.0.4 Figures

The figures above show the impulse responses to a one standard deviation shock in oil prices, in percentage deviations from the trend along the vertical axis for all the variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, quarters of a year along the horizontal axis. The error bands are calculated using the double bootstrap method proposed by Kilian (1998), and represent $5^{th}$ and $95^{th}$ quantiles of the bootstrapped distribution.

Figure A.1: Impulse responses: SVAR model
The figures above show the impulse responses to a one standard deviation shock in oil prices as percentage deviations from the trend along the vertical axis for all the variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, quarters of a year along the horizontal axis. The error bands for the impulse responses of the SVAR model are calculated using the double bootstrap method proposed by Kilian (1998), and represent $5^{th}$ and $95^{th}$ quantiles of the bootstrapped distribution.

**Figure A.2**: Impulse responses: theoretical and empirical models
The figures above show the impulse responses to a one standard deviation shock in oil prices as the percentage deviations from the trend along the vertical axis for all the variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, quarters of the year along the horizontal axis.

**Figure A.3: Impulse responses: competitive and Ramsey economies**

The figures above show the impulse responses to a one standard deviation shock in oil prices as the percentage deviations from the trend along the vertical axis for all the variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, quarters of the year along the horizontal axis.
Figure A.4: Historical contribution
The figures show the historical contribution of the oil price shock to the dynamics of macroeconomic variables. The time series used are generated by the SVAR model based on the historically observed oil price shocks, with all the other shocks are shut down, together with the original time series.
Figure A.5: Nonstationary versus stationary oil price process
Stationary oil price process is modeled as AR(1) process with autocorrelation coefficient 0.8.

The figures above show the percentage deviations from the trend along the vertical axis for all the variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, quarters of a year along the horizontal axis.
The figures above show the percentage deviations from the trend along the vertical axis for all the variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, quarters of a year along the horizontal axis.

**Figure A.6:** Sensitivity to $\alpha_z$ parameter

The figures above show the percentage deviations from the trend along the vertical axis for all the variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, quarters of a year along the horizontal axis.
Figure A.7: The role of investment costs

The figures above show the percentage deviations from the trend along the vertical axis for all the variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, quarters of a year along the horizontal axis.
The figures above show the percentage deviations from the trend along the vertical axis for all the variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, quarters of a year along the horizontal axis.

**Figure A.8:** The role of consumption habits
Figure A.9: Sensitivity to the assumption of variable depreciation rate
The figures above show the percentage deviations from the trend along the vertical axis for all the variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, quarters of the year along the horizontal axis.
Figure A.10: The role of nominal frictions

The figures above show the percentage deviations from the trend along the vertical axis for all the variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, quarters of the year along the horizontal axis.
Figure A.11: Robustness of the optimal monetary policy

The figures present the test of the robustness of optimal policy to the calibration of Calvo price rigidity parameter. The figures above show the percentage deviations from the trend along the vertical axis for all the variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, quarters of the year along the horizontal axis.
Appendix B

Appendix for Chapter 2

B.0.1 Tables

Table B.1: Calibration of common parameters across models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.9926</td>
</tr>
<tr>
<td>$h$</td>
<td>hours</td>
<td>0.3</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation</td>
<td>0.025</td>
</tr>
<tr>
<td>$\theta$</td>
<td>labor share in output</td>
<td>0.7</td>
</tr>
<tr>
<td>$SE$</td>
<td>share of energy in VA</td>
<td>0.043</td>
</tr>
<tr>
<td>$\rho$</td>
<td>OPS: autocorrelation</td>
<td>0.8</td>
</tr>
<tr>
<td>$p$</td>
<td>relative oil price</td>
<td>1</td>
</tr>
<tr>
<td>$u$</td>
<td>capital utilization</td>
<td>0.82</td>
</tr>
<tr>
<td>$q$</td>
<td>shadow price of capital</td>
<td>1</td>
</tr>
</tbody>
</table>

Table B.2: Calibration of the model specific parameters - EP model

<table>
<thead>
<tr>
<th>Parameter/variable</th>
<th>Description</th>
<th>RBC 1</th>
<th>RBC 2</th>
<th>DSGE 1</th>
<th>DSGE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>production function</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>$b$</td>
<td>habit formation</td>
<td>0</td>
<td>0</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>investment costs</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Implied parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Scale, production</td>
<td>0.005</td>
<td>0.007</td>
<td>0.005</td>
<td>0.007</td>
</tr>
<tr>
<td>$\alpha$</td>
<td></td>
<td>-</td>
<td>0.85</td>
<td>-</td>
<td>0.85</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td></td>
<td>0</td>
<td>0.15</td>
<td>0</td>
<td>0.15</td>
</tr>
<tr>
<td>$\mu$</td>
<td></td>
<td>1</td>
<td>1.2</td>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td>$\phi$</td>
<td>utility</td>
<td>0.32</td>
<td>0.37</td>
<td>0.31</td>
<td>0.37</td>
</tr>
</tbody>
</table>
Table B.3: Calibration of the model specific parameters - EK model

<table>
<thead>
<tr>
<th>Parameter/variable</th>
<th>Description</th>
<th>RBC</th>
<th>DSGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>production function</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\beta$</td>
<td>habit formation</td>
<td>0</td>
<td>0.65</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>investment costs</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Implied parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_0$</td>
</tr>
<tr>
<td>$\nu_1$</td>
</tr>
<tr>
<td>$\omega_0$</td>
</tr>
<tr>
<td>$\omega_1$</td>
</tr>
<tr>
<td>$\phi$</td>
</tr>
</tbody>
</table>
Table B.4: The effect of the oil price shock on GDP

The numbers in the last column are either taken from the source or derived using the estimates from the source. I report the largest values of all the estimates provided.

<table>
<thead>
<tr>
<th>Study</th>
<th>Sample period</th>
<th>Oil indicator</th>
<th>GDP drop after 10% oil price increase, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>5) Bernanke, Gertler, and Watson (2004)</td>
<td>1965:Q1 - 1995:Q4</td>
<td>Hamilton</td>
<td>0.8</td>
</tr>
<tr>
<td>6) Peersman (2005)</td>
<td>1980:Q1 - 2002:Q2</td>
<td>Oil price growth</td>
<td>0.6</td>
</tr>
<tr>
<td>7) Hamilton (1983)</td>
<td>1949:Q2 - 1972:Q4</td>
<td>Oil price growth</td>
<td>0.6</td>
</tr>
<tr>
<td>9) Kormilitsina (2008)</td>
<td>1954:Q3 - 2006:Q4</td>
<td>Oil price increases</td>
<td>0.5</td>
</tr>
<tr>
<td>10) Leduc and Sill (2004)</td>
<td>1972:Q2 - 2000:Q4</td>
<td>Oil price increases</td>
<td>0.4</td>
</tr>
</tbody>
</table>
### Table B.5: Variance due to the oil price shock: VAR identification approach

<table>
<thead>
<tr>
<th>Variable</th>
<th>% of the variance explained by the oil price shock in the VAR</th>
<th>Historical variance decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP growth</td>
<td>19.1</td>
<td>16.8</td>
</tr>
<tr>
<td>Hours</td>
<td>44.8</td>
<td>15.2</td>
</tr>
<tr>
<td>Wage rate growth</td>
<td>13.3</td>
<td>13.9</td>
</tr>
<tr>
<td>Investment growth</td>
<td>20.9</td>
<td>18.6</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>2.6</td>
<td>12.3</td>
</tr>
</tbody>
</table>

### Table B.6: Second order moments: RBC framework

<table>
<thead>
<tr>
<th></th>
<th>EP</th>
<th>EK</th>
<th>EP - Countercyclical Markup</th>
<th>EK - Variable Depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>36.7</td>
<td>47.9</td>
<td>49.3</td>
<td>104.6</td>
</tr>
<tr>
<td>Hours</td>
<td>5.2</td>
<td>7.3</td>
<td>7.9</td>
<td>17.1</td>
</tr>
<tr>
<td>Wage rate</td>
<td>34.2</td>
<td>50.2</td>
<td>52.9</td>
<td>72.1</td>
</tr>
<tr>
<td>Investment</td>
<td>39.9</td>
<td>48.6</td>
<td>63.3</td>
<td>122.0</td>
</tr>
<tr>
<td>Consumption</td>
<td>30.5</td>
<td>40.9</td>
<td>44.9</td>
<td>38.3</td>
</tr>
</tbody>
</table>

### Table B.7: Second order moments: DSGE framework

<table>
<thead>
<tr>
<th></th>
<th>EP</th>
<th>EK</th>
<th>EP - Countercyclical Markup</th>
<th>EK - Variable Depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>20</td>
<td>26.5</td>
<td>24.5</td>
<td>47.8</td>
</tr>
<tr>
<td>Hours</td>
<td>3.4</td>
<td>4.1</td>
<td>4.1</td>
<td>8.02</td>
</tr>
<tr>
<td>Wage rate</td>
<td>36.4</td>
<td>54.8</td>
<td>46.5</td>
<td>77.0</td>
</tr>
<tr>
<td>Investment</td>
<td>14.4</td>
<td>17.7</td>
<td>19.5</td>
<td>41.4</td>
</tr>
<tr>
<td>Consumption</td>
<td>23.6</td>
<td>33.2</td>
<td>31.0</td>
<td>41.4</td>
</tr>
</tbody>
</table>
B.0.2 Figures

Figure B.1: Propagation mechanism: EK versus EP
**Figure B.2**: The effect of variable depreciation in the EK model
The Figure shows marginal energy costs under power cost functions. The plots are based on the calibration of deep parameters adopted in the model, and fix $k$ and $h$ at the steady state level.

The arrows denote the shift of the curves after the relative energy price $p$ increases by 100%.

**Figure B.3**: The effect of variable depreciation in the EK model - counterexample
The Figure shows marginal energy costs under quadratic cost functions. The plots are based on the calibration of deep parameters adopted in the model, and fix $k$ and $h$ at the steady state level.

The arrows denote the shift of the curves after the relative energy price $p$ increases by 100%.
Figure B.4: Impulse responses under variable depreciation - counterexample
The plots show impulse responses in a model with quadratic specification for marginal energy costs. Impulse responses to a 10% oil price shock, %.

Figure B.5: Impulse responses: EP versus EK model.
Impulse responses to a 10% oil price shock, %.
Figure B.6: Amplifying mechanism at work: EP model

Figure B.7: Amplifying mechanism at work: EK model
Figure B.8: Comparing the amplifying mechanisms
For comparison purposes, $\varrho = 0.001$ in the EP model. Impulse responses to a 10% energy price shock, %.

Figure B.9: Two energy models: DSGE framework
Figure B.10: Two amplifying mechanisms: DSGE framework
For comparison purposes, $\rho = 0.001$ in the EP model. Impulse responses to a 10% energy price shock, %.

Figure B.11: Amplifying mechanism: EP model
Figure B.12: Amplifying mechanism: EK model
Figure B.13: Impulse responses, SVAR model
The figures above show impulse responses to a 10% oil price increase in percentage deviations from the trend along the vertical axis for all the variables except inflation, percentages of the annualized values as deviations from the average for inflation, quarters of a year along the horizontal axis.

The confidence bands are calculated using the double bootstrap method proposed by Kilian (1998), and represent $5^{th}$ and $95^{th}$ quantiles of the bootstrapped distribution.
Bibliography


Biography

Anna Kormilitsina (Kozlovskaya) was born in Moscow, Russia on March 7, 1980. She attended Moscow Institute of Physics and Technology from 1997 to 2003, and New Economic School from 2001 to 2003, where she graduated with Master’s degrees in Applied Math and Economics. Before joining the graduate program at Duke University, Anna held a research position as a junior economist in the Institute for the Economy in Transition in Moscow. While at Duke, she participated in the Summer Internship program at the IMF, and she was also a Dissertation Intern at the Federal Reserve Bank of St. Louis in the Fall of 2006.