Essays in Macroeconomics

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in the Department of Economics
in the Graduate School of Duke University
2013
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This dissertation consists of two essays in macroeconomics. In the first essay, I explain the increase in the interbank market credit spreads during the recent financial crisis using a model with endogenous default, in which banks with different default risk borrow at different interest rates. I compare a normal times stationary equilibrium and a crisis times stationary equilibrium. In normal times there is no spread in the interbank market, because the default probability of banks is zero. In crisis times some banks default, and an interbank credit spread arises endogenously. The interbank credit spread is positively correlated with leverage and debt size, and negatively correlated with expected cash flows. Using this framework, I study the effects of equity injections, debt guarantees, and liquidity injections on the interbank credit spreads and on risky projects financed by banks. I find that debt guarantees are effective in reducing interbank credit spreads in times of crisis, but not in stimulating investment. In contrast, interbank credit spreads are relatively unresponsive to injections of equity and of liquidity; however, these policies are successful in stimulating investment.

In the second essay, I study the capacity of the Taylor principle to guarantee determinacy in the class of New Keynesian models typically used for monetary policy analysis, when firms are not able to index their prices. In a model with labor, capital accumulation, capital adjustment costs, and capital utilization, the necessary conditions for trend inflation to affect determinacy are as follows: trend inflation is
above 4%, firms are not able to index their prices, and the frequency of price changes is less than once a year. Introducing sticky wages, it is possible to find a response to inflation greater or equal than one that guarantees determinacy; however, the determinacy region is small and indeterminacy can arise even when the response to inflation is higher than one for one.
To my parents and my siblings
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Banks’ Borrowing Costs and Default Risk

1.1 Introduction

On August 9, 2007, the interbank credit spread, a leading indicator of financial instability, increased and remained at unusually high levels for almost two years. The turmoil in the money markets is summarized in Figure 1.1 that shows the USD 3M Libor-OIS spread, a proxy for the average unsecured borrowing costs in the interbank market in U.S. dollars at a 3-month horizon net of expectations of future federal funds rates.\(^1\)

The average interbank credit spread was approximately zero from the fourth quarter of 2001 until August of 2007 when BNP Paribas’s suspended the valuation of three of its hedge funds related to U.S. asset backed securities and the average interbank credit spread increased to 39 basis points. Banks’ credit spreads continued to increase following Lehman Brothers’ bankruptcy until October of 2008 when the U.S. Treasury announced the Capital Purchase Program (CPP), a plan to purchase USD 250 billion in equity from financial institutions, and the Federal Deposit Insur-

\(^1\) Recently, there has been a debate about banks misreporting Libor rates. However, the Libor broadly tracks the interest rates paid by banks in the U.S. interbank market. See Kuo et al. (2012).
ancial Corporation (FDIC) Debt Guarantee Program, a plan to guarantee the newly issued senior debt of FDIC-insured institutions.

Understanding what drives the increase in interbank credit spreads is important because of its negative effects on the real economy. Recent empirical work by Taylor and Williams (2009) and Smith (2010) suggests that default risk is positively correlated with interbank credit spreads. In addition, interbank credit spreads are positively correlated with debt size, leverage, and nonperforming loans; see Flannery and Sorescu (1996), Furfine (2001), and Afonso et al. (2011).

In this essay I build a model of borrowing costs in the interbank market that is consistent with the empirical findings mentioned above. The model belongs to the class of stationary equilibrium models with endogenous default which includes Chatterjee et al. (2007), and it is related to models of the interbank market such as Gertler and Kiyotaki (2010).

In the model there is a continuum of islands with one bank on each island. Each period a fraction of the banks have an investment opportunity to buy risky assets located on their own island but not on other islands. The risky assets produce income in the next period that is subject to an investment quality shock. Banks borrow or lend in a centralized market where a clearing house operates. Banks that borrow from the clearing house take a one period loan at a price which depends on bank characteristics and loan size; banks can default on their loans. When a bank lends to the clearing house, it takes a one period claim on it at a price that is independent of the characteristics of the bank and the size of the claim; the clearing house cannot default on its claims.

2 Motivated by the work of Gilchrist and Zakrajšek (2012), I run a structural vector autoregression using standard economic variables and I find that a one percentage point shock to banks' credit spreads decreases real GDP by approximately two percent after one year. See the appendix to this chapter.

3 Default risk is not the only explanation for the increase in the interbank credit spread, see McAndrews et al. (2008), Schwarz (2009), and Gilchrist and Zakrajšek (2011).
In equilibrium there are two sets of endogenous prices: i) the interest rate that each bank pays for its loans, and ii) the risk-free interest rate. A borrowing bank pays an interest rate that is consistent with its default probability, and a lending bank receives the risk-free interest rate which is determined so that the sum of all interbank loans equals zero. Interbank credit spreads are then given by the difference between the rate a bank pays and the risk-free rate. I examine the average interbank credit spread under a normal times stationary equilibrium, and a crisis times stationary equilibrium. In the normal times equilibrium, the investment quality shock is such that there is no spread in the interbank market because the default probability of banks is zero. In the crisis times equilibrium, I modify the distribution of the investment quality shock such that there is positive probability that the banks will receive zero income from their risky assets. In this environment, the optimal decision of some banks is to default and an interbank credit spread arises endogenously.

Furthermore, heterogeneity in the borrowing costs paid by banks allows the model to match the positive correlation of the interbank credit spreads with banks’ debt, leverage, and nonperforming loans (I use expected cash flows as a proxy for loans that are not nonperforming). This sets the model apart from models in which information asymmetry drives the interbank spread, such as Flannery and Sorescu (1996), Freixas and Jorge (2008), and Heider et al. (2009), and from models with endogenous default in which there is a common interbank interest rate, like Tsomocos (2003), Goodhart et al. (2006), and Dib (2010).

After developing a model consistent with the main features of interbank credit spreads, I use this structure to study the effects of equity injections, debt guarantees, and liquidity injections on the average interbank market credit spread. These were the main policies implemented in the U.S. and U.K. to stabilize the financial system following Lehman Brothers’ bankruptcy. In addition to the CPP and the FDIC-debt guarantee program announced by the U.S. government, the U.K. government
announced that it stood ready to inject equity into financial institutions by at least £25 billion, and offered to guarantee newly issued short-term and medium-term debt of financial institutions. At the same time, the Federal Reserve, the Bank of England, and the European Central Bank continued their injections of liquidity started in August, 2007.

To assess the effectiveness of these policies, I modify the benchmark model to allow for equity injections, debt guarantees, and liquidity injections. I analyze the effects of injecting equity into banks by adding additional income, as a fraction of risky assets, into their balance sheet equation. I introduce government guarantees into the maximization problem of the clearing house by assuming that there is a government sector that with some probability will cover the losses in case that a borrowing bank defaults. Finally, I incorporate liquidity injections by relaxing the constraint that the sum of all interbank loans equals zero. I find that debt guarantees are effective in reducing interbank credit spreads in times of crisis, but not in stimulating investment. In contrast, interbank credit spreads are relatively unresponsive to injections of equity and of liquidity; however, these policies are successful in stimulating investment.

The next subsections relate this essay to the literature and provide an empirical motivation for the link between banks’ credit spreads and default risk. Section 1.2 develops the model. Section 1.3 discusses the calibration. Section 1.4 presents the implications of the model. Section 1.5 studies the effects of equity injections, debt guarantees, and liquidity injections. Section 1.6 concludes.

1.1.1 Relationship to the Literature

This essay builds on Chatterjee et al. (2007), who develop a theory of unsecured consumer credit with risk of default. My work differs in three dimensions. First, I focus on unsecured interbank borrowing costs and the effects of equity injections,
debt guarantees, and liquidity injections on the stabilization of the interbank market and on the amount of risky projects financed by banks. Second, in my framework banks endogenously choose their exposure to risk, i.e., when a bank has an investment opportunity it is aware that by borrowing and investing in risky assets it becomes exposed to a bad realization of investment quality. In contrast, in Chatterjee et al. (2007) consumers cannot avoid risks such as shocks to their socioeconomic status, preferences, or health. Third, the motives for borrowing are different. In this essay, banks borrow to exploit investment opportunities, while in Chatterjee et al. (2007) consumers borrow when they face transitory negative shocks in order to smooth consumption.

My essay is also related to Tsomocos (2003) and Goodhart et al. (2006), who introduce a banking sector into the general equilibrium model of Arrow-Debreu with incomplete markets, developed by Dubey et al. (2000) and Dubey and Geanakoplos (2003), in order to study financial instability; however, they abstract from heterogeneity in banks’ borrowing costs, i.e., the possibility that banks with different debt sizes or risk positions borrow at different interest rates. In this essay, banks exposed to different risk borrow at different interest rates, which is consistent with the empirical evidence presented by Flannery and Sorescu (1996), Furfine (2001), and Afonso et al. (2011). Furthermore, in Tsomocos (2003) and Goodhart et al. (2006) banks default on a fraction of all their interbank loans. As a consequence, banks’ credit spreads depend on the default probability of the entire banking system. In contrast, in my framework banks’ credit spreads depend on their idiosyncratic default

4 The pricing kernel is similar to the kernel used in contemporaneous work by Bai and Zhang (2012), because the interest rate paid by borrowers is a function of assets subject to productivity shocks. However, the focus of their paper is on understanding the effects of default risk on international risk sharing.

5 Angelini et al. (2011) find that the European interbank credit spreads depend on bank characteristics during the recent financial crisis, but they also find that aggregate factors are more important.
probability.

In addition, there are papers that incorporate an interbank market structure into dynamic stochastic general equilibrium (DSGE) models, such as Gertler and Kiyotaki (2010) and Dib (2010). Gertler and Kiyotaki (2010) abstract from default risk in the interbank market by assuming that the interbank loans are fully collateralized. On the contrary, in my model loans are unsecured and default occurs in equilibrium. Dib (2010) incorporates the endogenous strategic default model of the interbank market developed by Tsomocos (2003) into the model of the financial accelerator, proposed by Bernanke et al. (1999), in order to evaluate the role of the banking sector in the transmission of real shocks, and to assess the contribution of financial shocks to business cycle fluctuations. Nevertheless, in his paper there is a negative correlation between interbank credit spreads and debt size which is not consistent with the positive correlation between debt and interbank credit spreads found by Flannery and Sorescu (1996), Furfine (2001), and Afonso et al. (2011).

Finally, there is recent theoretical work by Caballero and Krishnamurthy (2008), Allen et al. (2009), Diamond and Rajan (2011), and Acharya and Skeie (2011), in which the unwillingness of lenders to supply funds in times of crisis could cause a freeze in trade between financial institutions. In this essay the main motives for lenders to decrease their supply of funds are a decrease in the risk-free interest rate or a decrease in cash flows. While lending disruptions could be important during periods of financial turmoil, the model is consistent with the evidence presented by Afonso et al. (2011), Angelini et al. (2011), and Kuo et al. (2012), who do not find evidence of major lending disruptions in the interbank market during the recent financial crisis.
1.1.2 Empirical Motivation

In this subsection, I show that interbank credit spreads are positively correlated with default risk. First, I estimate an error components model in which each bank credit spread is a function of its default risk as measured by credit default swaps (CDSs), and a bank specific effect. Second, I run bank specific regressions of interbank credit spreads on default risk. I use quarterly data between 2002q2 and 2008q4. The explanatory variables are interacted with dummy variables for normal (2002q2 – 2007q2) and crisis (2007q3 – 2008q4) times.⁶

Table 1.1 shows the results of the error components model. There is positive correlation between banks’ credit spreads and default risk in normal times, and the correlation increases in crisis times.⁷ Table 1.2 shows the results of the bank specific regressions of the Libor-OIS spread of each bank on a constant and its CDS. Both explanatory variables are interacted with a dummy variable for normal and crisis times as defined in the error components model. Columns two and three show the results for normal times, and columns four and five show the results for crisis times. All the coefficients save for two banks, West LB and Credit Suisse, are positive and significant in both sample periods, indicating a positive correlation between interbank credit spread and default risk. Standard errors control for heteroscedasticity and serial autocorrelation.

I use the Libor-OIS spread as a proxy for the credit spread paid by banks in the interbank market. The Libor refers to the London Interbank Offered Rate, and it is a measure that broadly tracks the interest rate that banks pay for unsecured loans in the U.S. interbank market. Each day the British Bankers Association (BBA) asks a panel of major banks in the London interbank market at what rate they could

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⁶ The aggregation method for the data is the daily average of the quarter. The fourth quarter of 2008 corresponds to the daily average between 10/01/2008 and 10/14/2008.

⁷ Standard errors control for within groups and spatial correlation, see Driscoll and Kraay (1998).
obtain unsecured interbank funds in a reasonable market size. The loans could be in different currencies and maturities — I focus on loans in U.S. dollars at a 3-month horizon. The BBA computes the Libor taking a trimmed arithmetic average of the interest rates reported by the Libor panel banks. In 2007 there were 16 banks in the Libor panel: Bank of America, Bank of Tokyo Mitsubishi, Barclays, Citibank, Deutsche Bank, HSBC, J.P. Morgan Chase, Lloyds, Rabobank, RBC, RBS, UBS, West LB, HBOS, Norinchukin, and Credit Suisse. I use the CDSs of these financial institutions as a measure of their default risk. In the empirical work I exclude the RBC, the HBOS, and the Norinchukin bank because for these banks there is no available data on CDSs for the whole sample.

The OIS refers to the fixed leg in an overnight indexed swap, which is an interest rate swap between two parts. One part promises to pay a fixed rate over a notional amount at a certain maturity, the OIS rate. The other part promises to pay the geometric average of the overnight federal funds rate over the maturity of the swap. At the end of the contract, each part pays the corresponding interest rate over the notional amount. The Libor-OIS spread is the difference between the Libor and the OIS rate.

The results in Table 1.1 and Table 1.2 are unlikely to be affected by the Libor misreporting events. Consider the bank specific regressions, if banks were under-reporting their borrowing credit spreads and there is no correlation between the CDSs and the magnitude underreported, then the OLS coefficients are unbiased. If banks with higher probability of default were underreporting more, then the OLS

---

8 Market size is loosely defined. Transactions in the London interbank market are those that occur with either counterparty sitting in London, or both counterparties sitting outside of London using an intermediary based in London.

9 Currently there is a plan for a comprehensive reform of Libor, see The Wheatley Review of Libor.

10 The trimming is done by removing the highest 4 and the lowest 4 reported rates.
coefficients are biased downward, and the results on Table 1.2 are a lower bound.\textsuperscript{11}

1.2 The Model

There is a continuum of banks that choose dividends $c_{jt}$, equity $N_{jt}$, debt (claims) $b_{jt} > 0$ ($b_{jt} \leq 0$), and risky assets $k_{jt}$ in order to maximize a discounted stream of utility. Each bank is located on a different island $j$, and can buy risky assets located on its own island, but not risky assets that are located on other islands. Furthermore, in order to be able to buy risky assets a bank must have an investment opportunity, that is, there must be risky assets available in the island where the bank is located. $\chi_{jt}$ is an i.i.d random variable that can assume two values: $\chi_{jt} = 0$ or $\chi_{jt} = 1$. When $\chi_{jt}$ equals zero, $\chi_{jt} = 0$, there is no investment opportunity available in island $j$, so that a bank located there cannot buy risky assets, and when $\chi_{jt}$ equals one, $\chi_{jt} = 1$, there is an investment opportunity available in island $j$. $\chi_{jt}$ follows a Markov process with transition matrix $P_{\chi}$, where $P_{\chi}^{01}$ denotes the probability that a bank that does not have an investment opportunity at time $t$, receives an investment opportunity in period $t+1$. The stochastic process $\chi_{jt}$ is identical across islands.

Banks with investment opportunities can finance their purchases of risky assets with their cash flows or with new borrowing, and banks without an investment opportunity can lend part of their cash flows to other banks in the interbank market.\textsuperscript{12} Specifically, banks borrow or lend in a centralized market where a clearing house operates. When a bank borrows from the clearing house, it takes a one period loan. The price of the loan varies according to the characteristics of the borrowing bank, and the size of the loan; the banks can default on their loans. When a bank lends to the clearing house it buys a one period claim on it. The price of the claims is

\textsuperscript{11} A similar logic applies for the panel regressions.

\textsuperscript{12} Banks with investment opportunities can also be lenders in the interbank market when they have enough cash flows to finance their risky assets.
independent of the characteristics of the lending bank and the size of the claim; the clearing house cannot default on its claims.

Risky assets are subject to an investment quality shock $\xi_{jt}$. In normal times, the distribution of $\xi_{jt}$ is such that there is no default risk. However, in crisis times there is positive probability that banks receive a bad investment quality and find optimal to default.

1.2.1 Banks’ Operations and Preferences

Each period $t$ a bank receives income $\xi_{jt}k_{jt-1}^a$ from risky assets bought at time $t-1$, $k_{jt-1}$, where $\xi_{jt}$ is an exogenous investment quality shock. The bank also receives an exogenous endowment $\bar{e}$ every period. The bank allocates its income into dividends $c_{jt}$, equity $N_{jt}$, payments of loans taken in period $t-1$, $b_{t-1} > 0$, and an equity cost $\varphi(N_{jt})$. The relationship between income and its allocation is described by the cash flow equation,

$$\xi_{jt}k_{jt-1}^a + \bar{e} = b_{jt-1} + N_{jt} + c_{jt} + \varphi(N_{jt}).$$

Note that for banks that bought a claim in period $t-1$, $b_{jt-1} < 0$, $b_{jt-1}$ is income instead of expenses. The function $\varphi(N_{jt})$ is characterized by $\varphi(N_{jt}) = \vartheta N_{jt}^2$, and it represents the cost of holding equity. Theoretically, this cost is not necessary, and all the propositions in the essay are derived under the assumption that $\vartheta$ is equal to 0. However, setting $\vartheta > 0$ is useful to find the lower bound of the stationary distribution of debt, as described in the discussion of the computational algorithm of the model.

Each bank allocates equity and new borrowing $q_{jt}b_{jt}$ into risky assets $\chi_{jt}k_{jt}$ when $b_{jt} > 0$. Alternatively, a bank can find optimal to be a lender at time $t$, $b_{jt} < 0$, so that it allocates equity into new claims $q_{jt}b_{jt}$ and risky assets $\chi_{jt}k_{jt}$. The relationship between debt (claims), risky assets, and equity is described by the balance sheet
equation of the bank,

\[ \chi_{jt} k_{jt} = N_{jt} + q_{jt} b_{jt}, \]

where \( q_{jt} \) is a function that denotes the price of a loan if \( b_{jt} \) is positive, and the price of a claim otherwise, i.e.,

\[
q_{jt} = q(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt}) = \begin{cases} 
q^b(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt}) & \text{if } b_{jt} > 0 \\
q^i & \text{if } b_{jt} \leq 0.
\end{cases}
\]

When a bank takes a loan of size \( b_{jt} > 0 \), it obtains \( q^b(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt}) b_{jt} \) units at time \( t \) and promises to pay \( b_{jt} \) units at time \( t+1 \), so that \( q^b(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt}) \) is the price of a loan of size \( b_{jt} \) faced by a bank that arrives at time \( t \) with state variables \( k_{jt-1}, b_{jt-1}, \chi_{jt}, \) and \( \xi_{jt} \). When a bank buys a claim from the clearing house, \( b_{jt} < 0 \), it pays \( q^i (-b_{jt}) \) units at time \( t \) and receives \( (-b_{jt}) \) units at time \( t+1 \), so that \( q^i \) is the price of a claim at time \( t \). The price of a claim is independent of the state variables of the bank, and the size of the claim. A bank cannot buy a claim and take a loan simultaneously.

The banks also face a leverage constraint and a no-short in risky assets constraint. The leverage constraint,

\[ N_{jt} \geq \kappa \chi_{jt} k_{jt}, \]

implies that banks must finance a fraction of their risky assets with equity. The no-short in risky assets constraint,

\[ k_{jt} \geq 0, \]

prevents the banks from selling short risky assets.
The objective of the bank is to choose a stream of dividends $\{c_{jt}\}_{t=0}^{\infty}$, equity $\{N_{jt}\}_{t=0}^{\infty}$, debt (claims) $\{b_{jt}\}_{t=0}^{\infty}$, and risky assets $\{k_{jt}\}_{t=0}^{\infty}$ in order to maximize the expected value of a discounted stream of utility, that is,

$$\max_{\{c_{jt}, N_{jt}, b_{jt}, k_{jt}\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} (\beta \rho)^t u(c_{jt}) \right\}$$

subject to the cash flow constraint, the balance sheet constraint, the capital requirement constraint, and the no-short in risky assets constraint. The subjective discount factor is denoted by $\beta$, and $\rho$ is an exogenous probability of surviving until the next period. The utility function of the banks is characterized by $u(c_{jt}) = \log (c_{jt})$.

### 1.2.2 Market Arrangements

Unsecured loans for different types of banks are treated as different financial assets. A bank type at time $t$, $s_{jt}$, is determined by its risky assets $k_{jt-1}$, its investment quality shock $\xi_{jt}$, its borrowing or lending $b_{jt-1}$, and the availability of investment opportunities $\chi_{jt}$, i.e., $(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt})$. Given its type, a bank can take a loan of size $b_{jt}$ from the clearing house. The clearing house offers a menu of prices, $q^b(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt})$, that is a function of the bank’s type and the size of the loan. The bank can also buy a claim $b_{jt} < 0$ on the clearing house, at a price equal to $q^i$. The horizon of the loans and claims is one period.

This is a model of interbank lending, but the loans are made through the clearing house. For example, assume that there are only two banks in the economy, Bank A and Bank B. If Bank A wants to lend $X$ units to Bank B, then the following must happen:

1. At time $t$ Bank A buys a claim $q^iX$ from the clearing house.
2. At time $t$ the clearing house lends $q^b(B, X)X$ units to Bank B.
3. At time $t+1$ the clearing house receives $X$, conditional on Bank B not defaulting.

4. At time $t+1$ the clearing house pays back $X$ to Bank A.

This does not fit exactly the way in which interbank loans operate because we are forcing Bank A to buy insurance from the clearing house, that is, Bank A receives the risk-free interest rate, and always receives the principal back. Without this simplifying assumption, the model loses tractability because the price of a loan would be a function of all the possible interactions between banks.

Since there is an interest rate for each type of bank and for each loan size, the total number of financial assets available to be traded is $\mathcal{N}_{(K \times B \times X \times \Xi ^B)}$, where $\mathcal{N}_{\cdot}$ denotes the cardinality of the set $\{\cdot\}$, $K$ is the set of feasible values for risky assets, $B$ is the set of feasible values for loans/claims, $X$ is the set of feasible values for the investment opportunities, and $\Xi$ is the set of feasible values for the investment quality shocks to risky assets. The sets $K$, $X$, and $\Xi$ are closed and bounded subsets of $\mathbb{R}$, and therefore they are compact. The set $B$ is bounded above and unbounded below. However, I show in the appendix that we can redefine $B$ to be bounded below, given prices.

1.2.3 The Shocks in the Model

There are two shocks in the model, an investment opportunity shock and an investment quality shock. As I mentioned earlier, the investment opportunity shock $\chi_{jt}$ is an i.i.d random variable that can assume two values, $\chi_{jt} = 0$ or $\chi_{jt} = 1$. When $\chi_{jt}$ equals zero, $\chi_{jt} = 0$, there is no investment opportunity available in island $j$, and a bank located there cannot buy risky assets, and when $\chi_{jt}$ equals one, $\chi_{jt} = 1$, an investment opportunity is available in island $j$. $\chi_{jt}$ follows a Markov process with transition matrix $P^x$, where $P^x_{01}$ denotes the probability that a bank that does not
have an investment opportunity at time $t$, receives an investment opportunity in period $t+1$. The stochastic process $\chi_{jt}$ is identical across islands.

The investment quality shock $\xi_{jt}$ is equal to $\nu_{jt}\tilde{\xi}_{jt}$, where $\tilde{\xi}_{jt}$ is an AR(1) process distributed log normal with mean zero, standard deviation $\sigma_{\tilde{\xi}}$, and autocorrelation $\rho_{\tilde{\xi}}$, and $\nu_{jt}$ is an indicator variable that is equal to zero with probability $\rho_{\tilde{\xi}}$, and is equal to one otherwise.$^{13}$ To save on notation I do not explicitly incorporate $\nu_{jt}$ and $\tilde{\xi}_{jt}$ into the states variables of the problem, and I use $\xi_{jt}$ instead.

I compare two stationary equilibria, a normal times stationary equilibrium and a crisis times stationary equilibrium. In normal times $\rho_{\tilde{\xi}}$ is set equal to 0, and in crisis times it is set equal to a positive number that is discussed in the calibration. The idea of comparing normal and crisis times scenarios is present in several theoretical and empirical studies of the interbank market.$^{14}$

1.2.4 Timing of Events

The timing of events for survivor banks is as follows:

1. At time $t$, a bank’s type $(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt})$ is determined.

2. Given its type $(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt})$, a bank decides not to default, $d_{jt} = 0$, or to default, $d_{jt} = 1$.

3. If a bank does not default, it chooses dividends $c_{jt}$, equity $N_{jt}$, debt (claims) $b_{jt}$, and risky assets $k_{jt}$ to maximize its discounted stream of utility, subject to its constraints.

4. If a bank type $(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt})$ defaults, then equity, assets, and liabilities are set equal to zero, i.e., $N_{jt} = 0$, $b_{jt} = 0$, and $k_{jt} = 0$.

$^{13}$ The support of $\tilde{\xi}_{jt}$ is unbounded, however, it is straightforward to redefine $\xi_{jt}$ equal to $\Upsilon_{jt}\nu_{jt}\tilde{\xi}_{jt}$, where $\Upsilon_{jt} = 1$ if $\xi_{jt} \leq \bar{\xi}$, and 0 otherwise, so that the exogenous process for $\tilde{\xi}_{jt}$ is consistent with $\Xi$ being closed and bounded.

$^{14}$ See Flannery (1996), Heider et al. (2009), and Caballero and Krishnamurthy (2008).
1.2.5 Legal Environment

If a bank arrives at time $t$ and defaults, it consumes its endowment $\bar{e}$, and it is allowed to open the bank again in the next period with probability $\rho^d$. Conditional on entering, its states are $b_{jt} = 0$, and $k_{jt} = 0$. When a bank does not survive, a new bank is born in that island with state variables $b_{jt} = 0$, $k_{jt} = 0$, $\chi_{jt} = 0$, and $\xi_{jt} = 1$.

1.2.6 Clearing House

The clearing house chooses the number of contracts $a(k_{jt-1}, b_{jt-1}, \xi_{jt}, b_{jt})$ to maximize the discounted stream of profits

$$\sum_{t=0}^{\infty} \left( \frac{1}{1+i} \right)^t \pi_t,$$

where

$$\pi_t = \int_{k_{jt-2}, b_{jt-2}, \xi_{jt-1}, b_{jt-1}} \sum_{b_{jt-1}} \rho(1 - p_{jt-1}) a(k_{jt-2}, b_{jt-2}, \xi_{jt-1}, b_{jt-1}) b_{jt-1}$$

$$dk_{jt-2}db_{jt-2}d\xi_{jt-1}db_{jt-1} - \int_{k_{jt-1}, b_{jt-1}, \xi_{jt}, b_{jt}} \sum_{b_{jt}} a(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt})$$

$$b_{jt}q(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt})dk_{jt-1}db_{jt-1}d\xi_{jt}db_{jt},$$

and $i$ is the risk-free interest rate. $p_{jt} = p(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt})$ denotes the default probability in period $t + 1$ of a bank type $(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt})$ that takes a loan of size $b_{jt}$ at time $t$, i.e.,

$$p_{jt} = \begin{cases} \int_{\xi_{jt+1}}^{\chi_{jt+1}} \sum_{\chi_{jt+1}} d(k_{jt}, b_{jt}, \chi_{jt+1}, \xi_{jt+1}) P_x(\chi_{jt+1} | \chi_{jt}) dF(\xi_{jt+1} | \xi_{jt}) d\xi_{jt+1} & \text{if } b_{jt} > 0 \\
0 & \text{if } b_{jt} \leq 0 \end{cases}$$

The FOC of the clearing house is
\[ q(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt}) = \begin{cases} 
q^b(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt}) & \text{if } b_{jt} > 0 \\
q^i & \text{if } b_{jt} \leq 0. 
\end{cases} \]

where

\[ q^b(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt}) = \frac{\rho}{1 + i} (1 - p(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt})) \]

\[ q^i = \frac{\rho}{1 + i}. \]

### 1.2.7 Recursive Formulation of the Banks’ Problem

Let the set of all possible bank types be denoted by \( S \equiv \{K \times B \times \Xi \times X\} \), where \( s_{jt} \equiv \{k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}\} \subseteq S \), and \( S \) is a subset of the Euclidean space. The value of a bank type \( s_{jt} \) that faces a price function \( q(s_{jt}, b_{jt}) \) is denoted by \( v(s_{jt}) \), the dependence on the price function is implicit.\(^{15}\) This function belongs to the set of all continuous bounded functions \( V \), \( v(s_{jt}) \in V \), where \( v : S \rightarrow R \). The optimization problem of the banks is written in terms of an operator that returns the maximum lifetime utility that a bank can achieve \( T(v)(s_{jt}) : V \rightarrow V \), given prices.

The banks solve their problem in two steps. First, each bank chooses a default decision \( d_{jt} \in D = \{0, 1\} \). Next, it chooses debt \( b_{jt} \) and risky assets \( k_{jt} \), in order to maximize its objective function subject to the feasible set \( \Gamma(s_{jt}, d_{jt}) \) characterized by \( \Gamma(s_{jt}, d_{jt}) = \{(b_{jt}, k_{jt}) \in B \times K : c_{jt} > 0; k_{jt} \geq 0 \text{ if \( \chi_{jt} = 1 \), } k_{jt} = 0 \text{ if \( \chi_{jt} = 0 \), } b_{jt}(1-d_{jt}) < q_{jt}b_{jt} \leq (1-\kappa)\chi_{jt}k_{jt}(1-d_{jt}), c_{jt} = \bar{c} + (1-d_{jt}) (\xi_{jt}k_{jt-1} - b_{jt-1} - \chi_{jt}k_{jt} + q(s_{jt}, b_{jt})b_{jt} - \theta (\chi_{jt}k_{jt} - q(s_{jt}, b_{jt})b_{jt})^2) \}\), where \( b_{jt} \) is equal to the minimum \( b_{jt} \) such that \( c_{jt} = 0 \).

**Definition 1.** For \( v \in V \), let \( (Tv)(s_{jt}) \) be defined as follows

\[ (Tv)(s_{jt}) = \max_{d_{jt} \in \{0,1\}} \{v^{nd}(s_{jt}), v^{d}(s_{jt})\}, \]

\(^{15}\) The same applies to the policy functions and the stationary distribution, which I discuss later.
where \( v^{nd}(s_{jt}) \) is characterized by
\[
v^{nd}(s_{jt}) = \max_{(b_{jt}, k_{jt}) \in \Gamma(s_{jt}, d_{jt})} u(c_{jt}) + \beta \rho \mathbb{E}_t v(s_{jt+1}),
\]
and \( v^d(s_{jt}) \) is characterized by
\[
v^d(s_{jt}) = u(c^d_{jt}) + \beta \rho \{ \rho^d \mathbb{E}_t v(s^d_{jt+1}) + (1 - \rho^d) \mathbb{E}_t v^d(s^d_{jt+1}) \},
\]
with \( c_{jt} = \xi_{jt} k^0_{jt-1} - b_{jt-1} + \bar{e} - N_{jt} - \varphi(N_{jt}), N_{jt} = \chi_{jt} k_{jt} - q(s_{jt}, b_{jt}) b_{jt}, \]
c\(_{jt} = \bar{e}, \) and \( s^d_{jt} = \{0, 0, \chi_{jt}, \xi_{jt}\} \).

**Assumption 1.** \( \lim_{T \to \infty} \prod_{s=1}^T q(s_{jt+s}, b_{jt+s}) b_{jt+T} = 0 \) for all \( s_{jt+s} \) and for all \( b_{jt+s} \).

**Proposition 1.** Let \( q(s, b) \in (0, 1) \) for all \( s \) and for all \( b \), and let \( \bar{q} = \max q(s, b) \).

If Assumption 1 holds, then \( b_{jt} \geq -\bar{b}(1 - \bar{q})^{-1} \).

**Proof.** See appendix to chapter 1.

In order to prove that the solution to the problem of the banks is unique, given prices, it is useful to write the problem in terms of a vector-valued operator \( \mathbb{T} : \mathbb{V} \to \mathbb{V} \), where \( \mathbb{V} \) is the set of all continuous bounded vector-valued functions \( \mathbb{V} : \mathcal{K} \times \mathcal{B} \times \Xi \to \mathbb{R}^2 \). Each \( \mathbb{V}(k_{jt-1}, b_{jt-1}, \xi_{jt}) \) is a two by one vector, where the first component corresponds to the value of banks without an investment opportunity \( v^0(k_{jt-1}, b_{jt-1}, \xi_{jt}) \equiv v(k_{jt-1}, b_{jt-1}, 0, \xi_{jt}) \), and the second component corresponds to the value of banks with an investment opportunity \( v^1(k_{jt-1}, b_{jt-1}, \xi_{jt}) \equiv v(k_{jt-1}, b_{jt-1}, 1, \xi_{jt}) \). The control variables \( b_{jt}, k_{jt} \), and \( d_{jt} \) define an optimal policy correspondence \( h^* \), \( h^*(s) = \{ h \in \Gamma \times D : v^{x_{jt}}(k_{jt-1}, b_{jt-1}, \xi_{jt}) = \max_{h \in \Gamma \times D} v^{x_{jt}}(k_{jt-1}, b_{jt-1}, \xi_{jt}) \} \).

**Proposition 2.** Given a price function \( q \), there exists a unique \( \mathbb{V}^* \in \mathbb{V} \) such that \( \mathbb{T}(\mathbb{V}^*) = \mathbb{V}^* \), and for any \( \mathbb{V}_0 \in \mathbb{V} \), \( \delta(\mathbb{T}^n \mathbb{V}_0, \mathbb{V}^*) \leq (\beta \rho)^n \delta(\mathbb{V}_0, \mathbb{V}^*) \) for \( n = 0, 1, \ldots \).

Moreover, the optimal policy correspondence \( h^* : \mathcal{S} \to \mathcal{B} \times \mathcal{K} \times \mathcal{D} \) is compact-valued and upper hemi-continuous.
Proof. See appendix to chapter 1. □

1.2.8 Transition Function for Banks

The Markov processes for the exogenous components of $s_{jt}$, i.e., $s_{2jt} = \{s_{21jt}, s_{22jt}\} = \{\chi_{jt}, \xi_{jt}\}$, and the optimal correspondence $h(s_{jt}) = \{b(s_{jt}), k(s_{jt}), d(s_{jt})\}$ induce a law of motion for the distribution of banks. Formally, let $(\mathcal{S}, \mathcal{B}(\mathcal{S}))$ be a set of state variables and its Borel $\sigma$-algebra, and let $(\mathcal{S}, \mathcal{B}(\mathcal{S}), \Lambda)$ be a probability space. Using the optimal correspondence, we can define a mapping $\Lambda : \mathcal{S} \times \mathcal{B}(\mathcal{S}) \rightarrow [0, 1]$. The transition function can be written as

$$
\Lambda(s_{jt}, s_{jt}) = \rho \Lambda_s(s_{jt}, s_{jt}) + (1 - \rho) \Lambda_n(s_{jt}, s_{jt}),
$$

where the transition function for survivor banks $\Lambda_s$, and the transition function for new born banks $\Lambda_n$, are defined in the appendix to chapter 1. Given prices, the transition function $\Lambda$ allows us to characterize the evolution of the distribution of banks in the economy by $\mathcal{T}$

$$(\mathcal{T}\lambda)(s) = \int s \Lambda(s, s) d\lambda.$$ 

**Proposition 3.** Given a price function $q$, there is a measurable selection from $h : \mathcal{S} \rightarrow \mathcal{B} \times \mathcal{K} \times \mathcal{D}$. 

*Proof. See appendix to chapter 1.* □

**Proposition 4.** Given a price function $q$, and any measurable selection from $h : \mathcal{S} \rightarrow \mathcal{B} \times \mathcal{K} \times \mathcal{D}$, there is a unique $\lambda^* \in \Lambda(\mathcal{S}, 2^\mathcal{S})$ such that

$$
\lambda(s) = \mathcal{T}\lambda(s).
$$

*Proof. See appendix to chapter 1.* □
1.2.9 Steady-State Competitive Equilibrium

**Definition 2.** A steady-state competitive equilibrium is a price function \( q^* = q^*(s, b') \), a default probability function \( p^*(s, b') \), a policy function for the clearing house \( a^*(s, b') \), policy functions for the banks \( d^*(s), b^*(s), k^*(s) \), and a time-invariant distribution \( \lambda^*(s) \) such that

1. \( d^*(s), b^*(s), k^*(s) \) solve each bank’s optimization problem given a price function \( q^*(s, b') \).

2. Each loan market clears

\[
\int 1_{\{b^*(s,q^*)=b'\}} d\lambda^*(s) = a^*(s, b') \quad \text{for all } s, b',
\]

and loan and claim prices are consistent with default probabilities

\[
q^*(s, b') = \begin{cases} 
\frac{\rho}{1+\rho} (1 - p^*(s, b')) & \text{if } b' > 0 \\
\frac{\rho}{1+\rho} & \text{if } b' \leq 0,
\end{cases}
\]

with \( p^*(s,b') = \int_{s_{21}}^{s_{21}} \sum_{s_{22}} d^*(s') P^x(s_{22}' | s_{22}) dF(s_{21}' | s_{21}) ds_{21}' \text{ if } b' > 0 \) and \( i^* = \frac{\rho}{q^*} - 1 \).

3. The clearing house clears

\[
\int q^*(s, b^*(s)) b^*(s) d\lambda^*(s) = 0.
\]

4. \( \lambda^* \) satisfies

\[
\lambda(s) = \mathcal{T} \lambda(s).
\]
1.2.10 Computational Algorithm

1. Taking as given the risk-free interest rate and the loan prices, solve the maximization problem of each bank.

2. Taking as given the risk-free interest rate and the policy functions of the banks, solve for the loan prices. If the loan prices are consistent with default probabilities move to step 3, if not update the guess for loan prices and go bank to step 1.

3. Taking as given the risk-free interest rate and the loan prices compute the stationary distribution, and check that the clearing house clears, if not update the risk-free interest rate and go back to step 1.

The convergence criterion for step 1 is $10^{-5}$ under the sup norm. In step 2, the iteration on each loan market stops when the error is smaller than $10^{-5}$ relative to the price of a claim on the clearing house. In step 3, the convergence criterion for the stationary distribution is $10^{-5}$ under the sup norm, and I consider that the clearing house clears when the sum of all loans and claims is less than 1% of the volume of lending, i.e., the absolute sum of all loans and claims divided by 2. I discretize the continuous variables of the model as follows: I use a grid with 10 nodes for investment quality, a grid with 10 nodes for risky assets, and a grid with 50 nodes for the loan size. This together with the grid for investment opportunities yields 10000 banks.

Let us consider the equity cost of the model. Theoretically, this cost is not necessary, and we should proceed as follows. Guess an ad-hoc lower bound for the grid of loans and claims. If the lending policy is not binding, then the problem of the banks would be numerically well defined. On the contrary, if the lending policy is binding, then we need to set a lower value for the ad-hoc lower bound. However, decreasing the lower bound comes with a computational cost, see Nakajima (2007).
In order to avoid this problem, I introduce an equity cost, \( \varphi(N_{jt}) \), that facilitates finding the lower bound for \( b_{jt} \). An alternative would be to impose the cost directly on the amount of lending rather than on the amount of equity that banks hold. I choose not to do so because this would also imply a borrowing cost, which would directly affect interbank market. In order to keep the interbank market as simple as possible, I use the equity of banks to impose an endogenous lending constraint to find the lower bound of the stationary distribution of banks’ debt.

1.3 Calibration

This section discusses the calibration. I assume that the normal times equilibrium corresponds to the period 1992q1 – 2007q2, which is a time period where banks did not report losses, on average. The crisis times equilibrium corresponds to the period between BNP’s suspension of the valuation of three of its hedge funds related to U.S. asset back securities and the announcement of the U.S. financial rescue package, 08/09/2007 – 10/14/2008.

Table 1.3 shows the parameters and their targets.\(^{16}\) There are four structural parameters, four exogenous processes parameters, and two legal parameters. The structural parameters are the subjective discount factor \( \beta \), the degree of decreasing returns on risky assets \( \alpha \), the endowment \( \bar{e} \), and the parameter related to the equity cost, \( \vartheta \). The exogenous processes parameters are the transition probabilities for the investment opportunities \( P_{ij}^\chi \), the persistence of the AR(1) component of the investment quality shock \( \rho_{\xi} \), the standard deviation of the AR(1) component of the investment quality shock \( \sigma_{\xi} \), and the probability that \( \nu \) equals zero, \( p^\xi \). The legal parameters are the minimum equity requirement as a fraction of risky assets \( \kappa \), and

\(^{16}\) The survivor probability of banks \( \rho \) is not discussed in Table 1.3. I introduce \( \rho \) in the model to prove the existence of the stationary distribution given prices. Since any \( \rho \in (0,1) \) suffices, I set it to a value that is arbitrarily close to one.
the probability of opening a new bank after defaulting $\rho^d$.

The subjective discount factor $\beta$ is set equal to 0.988 in order to match the average risk-free rate of the economy in normal times (NT), 1992q1 – 2007q2, which is equal to 1.50. The degree of decreasing returns to scale on risky assets $\alpha$ is set to the value of 0.497 in order to match the average risk-free interest rate in crisis times (CT), 08/09/2007 – 10/14/2008, which is equal to 0.67. The endowment is set to 0.044 to match the standard deviation of the interbank credit spreads in crisis times, 0.06. The cost of equity $\vartheta$ is set to 0.252, which is the minimum value of $\vartheta$ that is enough to find the lower bound of the stationary distribution of banks’ debt given an ad-hoc value for $b_{jt}$.

The entries in the transition matrix for the investment opportunities $P_{ij}^X$ are set equal to 0.50. This assumption generates interbank lending. Those banks without investment opportunities or with investment opportunities and high cash flows are lenders in the interbank market, while those banks with investment opportunities and low cash flows are borrowers. In the benchmark case, I set the value of all the transition probabilities equal to 0.50 for the sake of symmetry and given that, to the best of my knowledge, there is no publicly available data set that can provide a guide for the calibration of the transition probabilities.

The persistent component and the standard deviation of the investment quality shock $\bar{\xi}$, $\rho_{\bar{\xi}}$ and $\sigma_{\bar{\xi}}$, are set equal to 0.979 and 0.0072, respectively. These values are in line with the results reported for the productivity shock process in King and Rebelo (1999). The probability of a crisis $\rho_{\bar{\xi}}$ is set equal to 0.3% in order to match the average interbank spread in times of crisis, which is equal to 1.12%. The value of $\rho_{\bar{\xi}}$ implies an average default probability equal to 0.29% that is of same order of magnitude as the marginal default probabilities of Lehman Brothers, Merrill Lynch, and Citi reported by Giglio (2012). I implicitly assume that in times of crisis the investment quality of the risky assets equals 0 with probability $\rho_{\bar{\xi}}$. This assumption
is based on the evolution of the ABX.HE BBB-price index for subprime mortgage-
backed securities, which drops from an average of 100.13 in the second quarter of
2006 to an average of 3.77 in the fourth quarter of 2008.

The core capital requirement $\kappa$ follows the requirements of the Basel I Committee
for the International Convergence of Capital Measurements and Capital Standards,
which establishes a target standard capital ratio of core capital to weighted risky
assets of at least 4%. Core capital is composed of equity capital and disclosed
reserves, and it is completely visible in the balance sheets of banks. As a result, it is
used as a reference of capital adequacy by market participants — see the International
Convergence of Capital Measurement and Capital Standards. The probability of
opening a new bank next period after a default event is set equal to 1 because of
limited liability.

The moments targeted in the calibration are computed as follows. I use the OIS
rate to approximate the nominal risk-free interest rate because it tracks well the
effective federal funds rate, and it is not related to payments of principal since it is
based on notional amounts — see Smith (2010). Unfortunately, there is no data for
the OIS rate previous to the fourth quarter of 2001. As a consequence, I use the
average daily fed funds interest rate in order to approximate the OIS rate for the
period 1992q1 – 2001q3, and I use the OIS rate for the period 2001q4 – 2007q2.\footnote{The OIS rate has been historically close to the 90 day average of the fed funds interest rates (e.g. Smith (2010)).} Then, the risk-free interbank borrowing cost is determined by the difference between
the nominal risk-free interest rate and expected inflation, which is measured by the
mean of the consumer price index forecast from the Survey of Professional Forecasters
(SPF). Finally, the average and standard deviation of the interbank credit spread
correspond to the average and standard deviation of the difference between the Libor
and the OIS rate.
Next, I study the implications of the model in terms of moments that were not specifically targeted in the calibration such as the correlation between the interbank credit spread and the balance sheet variables of banks, as well as the implications for the amount of interbank borrowing, risky assets, and output.

1.4 Model Implications

1.4.1 Interbank Credit Spreads and Banks’ Characteristics

In this subsection, I show that the model is consistent with the two empirical facts mentioned in the introduction: i) interbank credit spreads are correlated with default risk, and ii) interbank credit spreads are correlated with banks’ characteristics. The interbank spread in the model is given by

\[
    \text{Spread}(k,b,\chi,\xi) = (1 + i) \frac{p(k,b,\chi,\xi,b')}{1 - p(k,b,\chi,\xi,b')} + (1 - \rho) \frac{(1 + i)\rho}{1 - p(k,b,\chi,\xi,b')},
\]

which shows that the credit spread is a function of the idiosyncratic default probability of banks, that is a function of the banks’ characteristics implied by the banks’ type, and the risk-free rate of the economy.

Table 1.4 shows the correlation of the interbank credit spread with the default probability, loan size, leverage, and expected cash flows.\(^{18}\) These moments are computed using the time invariant distribution of the model, conditional on borrowing banks. The first row of Table 1.4 shows the correlation between banks’ credit spreads and the default probability of banks. Rows 2 to 4 in Table 1.4 show that the interbank spread is positively correlated with loan size and leverage, and negatively correlated with expected cash flows; which is consistent with Flannery and Sorescu (1996), Furfine (2001), and Afonso et al. (2011).

\(^{18}\) I omit the results for normal times because there is no default in equilibrium.
Table 1.5 shows the correlations described in Table 1.4 conditional on the lowest and highest investment quality shock in order to show that the results are not built in the investment quality shock.

1.4.2 Implications for Interbank Volume, Risky Assets, and Output

Table 1.6 shows interbank volume, risky assets, and output during crisis times relative to normal times. In crisis times, these economic variables decrease relative to normal times. Thus, the model is quantitatively consistent with the behavior of the interbank credit spread and the risk-free interest rate during the recent financial crisis, and qualitatively consistent with evidence about the correlation between banks’ credit spreads and their balance sheet variables, and with the decline in investment and output observed during the recent financial crisis. In terms of the lending volume in the interbank market, the model does not predict a freeze on interbank lending, which is consistent with the evidence found by Afonso et al. (2011), Angelini et al. (2011), and Kuo et al. (2012). Next, I used this framework to study the behavior of the crisis times economy in response to the financial rescue plan implemented by the U.S. government, which had three key components: equity injections, debt guarantees, and liquidity injections.

1.5 Policy Experiment

In this section, I study the policy response to the recent financial crisis using the framework developed in this essay. I focus on equity injections, debt guarantees, and liquidity injections. In the policy experiment, I refine the definition of equilibrium by assuming that only the prices of those loans that are taken in equilibrium are consistent with default probabilities, that is, the equilibrium condition \( \pi \) is characterized by
1.5.1 Equity Injections

On October 14, 2008, the U.S. government announced the CPP, a plan to purchase up to USD 250 billion equity from qualified financial institutions (QFIs). Each QFI could issue equity by an amount no less than 1% of its risk-weighted assets, and no more than the lesser of USD 25 billion and 3% of its risk-weighted assets. The CPP followed the Bank of England’s announcement of a financial rescue package on October 8, 2008, whereby the government stated that it stood ready to provide at least £25 billion as tier 1 capital in U.K. banks and U.K. subsidiaries of foreign banks.

We can study the effects of equity injections by modifying the balance sheet equation of banks so that it includes a fraction of risky assets as additional equity injected by the government, i.e.,

\[ q^*(s, b^*(s)) = \begin{cases} \frac{\rho}{1+i^k} (1 - p^*(s, b^*(s))) & \text{if } b^*(s) > 0 \\ \frac{\rho}{1+i^k} & \text{if } b^*(s) < 0. \end{cases} \]

where \( G = f_k k^{ct} \), and \( f_k \) is government purchased equity as a fraction of the average level of risky assets in crisis times in the absence of policy intervention, \( k^{ct} \). In addition, note that the cost of holding equity faced by each bank is given by \( \varphi(N_{jt} + G) \) so that the cash flow equation becomes,

\[ \xi_{jt} k_{jt-1}^\alpha + \bar{e} = b_{jt-1} + N_{jt} + c_{jt} + \varphi(N_{jt} + G). \]

Finally, since almost all equity injections of the CPP were in the form of warrants
and cumulative preferred stocks, they do not affect the capital requirements of the bank.

1.5.2 Debt Guarantees

The CPP was announced in conjunction with the FDIC Debt Guarantee Program. Similar to the case of equity injections, this policy followed the action of the Bank of England, which in October of 2008 announced a government guarantee for new short and medium term debt issued by eligible institutions, in any of sterling, USD, or Euro currencies.

I study this policy by adapting the problem of the clearing house to take into account the probability that the FDIC will guarantee the debt of banks in case of default. Let the bailout probability be denoted by $G_B$, then the profits of the clearing house at time $t$ are

$$
\pi_t = \int_{k_{jt-2},b_{jt-2},\xi_{jt-1},b_{jt-1}} \sum_{\chi_{jt-1}} \rho(1-p_{jt-1})(1 - G_B)a_{jt-1}(\cdot)b_{jt-1} \\
+ \int_{k_{jt-2},b_{jt-2},\xi_{jt-1},b_{jt-1}} \sum_{\chi_{jt-1}} \rho G_B a_{jt-1}(\cdot)b_{jt-1} - \int_{k_{jt-1},b_{jt-1},\xi_{jt},b_{jt}} \sum_{\chi_{jt}} a_{jt}(\cdot)b_{jt}q(\cdot),
$$

where $a_{jt}(\cdot) = a(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt})$, and $q(\cdot) = q(k_{jt-1}, b_{jt-1}, \chi_{jt}, \xi_{jt}, b_{jt})$.

1.5.3 Liquidity Injections

One common response to the financial crisis was the provision of liquidity using standard and non-standard procedures. The standard procedures consisted in the reduction of reference interest rates implemented by the Federal Reserve (FED), the Bank of England (BoE), and the European Central Bank (ECB). The non-standard procedures consisted mainly in extending the range of collateral that banks could use to borrow from central banks, and the exchange of illiquid by liquid collateral.
The FED implemented several liquidity programs such as the extension of the maturity of the primary credit discount window, the Term Auction Facility, the Primary Dealer Credit Facility, the Term Securities Lending Facility, the Commercial Paper Funding Facility, the Asset-Backed Commercial Paper Money Market Mutual Funds Liquidity Facility, and the Money Market Investor Funding Facility. The Bank of England implemented the Special Liquidity Scheme, and the European Central Bank implemented the Enhanced Credit Support Program.

The analysis of these programs in the context of the model is challenging due to the number of instruments that were used to inject liquidity into the financial system. Nevertheless, we can gauge the effectiveness of the liquidity programs, in general, by assuming that the government provides funds to the clearing house by an amount equal to a certain fraction of the volume of interbank loans in normal times. Accordingly, the interbank market clearing equilibrium condition is

\[
\int_q q^*(s, b^*(s)) b^*(s) d\lambda^*(s) = f_b b^{nt},
\]

where \(f_b \in (0, 1)\), and \(b^{nt}\) is the volume of interbank loans in normal times.

1.5.4 Evaluation

Figure 1.2 shows the effects of equity injections and the debt guarantee program on the average interbank credit spread. The lower and upper horizontal axes are independent. The lower horizontal axis indicates the probability of bailout, the upper horizontal axis indicates government purchased equity as a fraction of risky assets, and the vertical axes display the average interbank credit spread. The round (square) markers show the effect of the probability of bailout (equity injections) on the average spread. Equity injections of up to 3% of risky assets do not affect the interbank credit spread, which suggests that this policy is not effective in stabilizing
the interbank market. In contrast, as the probability of a bailout increases, the average interbank spread decreases.

Figure 1.3 shows the effects of liquidity injections in stabilizing the interbank market. The horizontal axis indicates liquidity injections as a fraction of the volume of interbank lending in normal times. The vertical axis on the left indicates the average interbank credit spread, and the vertical axis on the right indicates the risk-free rate of the economy. Liquidity injections affect the risk-free rate of the economy, but do not affect significantly the spread. For instance, a liquidity injection of 2.5% relative to the volume of interbank lending in normal times reduces the risk-free rate by 1 percentage point but has negligible effects on the average interbank credit spread. Debt guarantees reduce the interbank credit spread because the clearing house is aware that loans are going to be paid back by the government with some probability in case that a borrowing bank defaults.

In order to understand why equity injections do not reduce the interbank credit spread, note that following an equity injection, ceteris paribus the supply of funds in the interbank market increases since the lending banks have more resources available to lend. Additionally, the demand of funds in the interbank market decreases, because banks can finance a higher fraction of their risky assets with equity instead of borrowing funds. In equilibrium the risk free rate of the economy decreases which provides an incentive to borrow. As a consequence the interbank volume increases, and so thus the total amount of risky assets. Since the probability of a bad realization of the risky assets does not change, banks continue to pay a credit spread over the risky free interest rate. Liquidity injections increase the supply of funds in the interbank market which reduces the risk-free rate, but does not significantly affect the credit spread because banks’ continue to borrow to finance their investments opportunities.

The analysis so far has been focused on the effects of the government policies
on the average interbank credit spread. However, policies that are effective in terms of reducing the interbank credit spread are not necessarily effective in stimulating investment in risky projects, and hence output. Table 1.7 shows the comparison of the stabilization policies in terms of their effects on the risk-free interest rate, the interbank spread, the interbank lending volume, the amount of risky assets, and output. The first row corresponds to injecting equity by an amount equal to 3% of the average level of risky assets in crisis times without intervention. The second row corresponds to a case where the government pays the outstanding debt if a bank defaults. The third row corresponds to a liquidity injection equal to 2.5% of the volume of interbank lending in normal times.

Equity injections decrease the risk-free interest rate, do not decrease significantly the interbank credit spread, and increase interbank volume, risky assets, and output. The decrease in the risk-free rate compensates the unresponsiveness of the interbank credit spread so that the total borrowing costs of banks, i.e., the risk-free interest rate plus the spread, decreases relative to a case without intervention. This leads to higher investment on risky assets, and hence output increases. Debt guarantees reduce the interbank credit spread to zero. Therefore, other things constant, the total borrowing costs of banks decreases. Thus, for a given risk-free interest rate the demand for funds in the interbank market increases leading to a higher risk-free interest rate in equilibrium. When the increase in the risk-free is enough to compensate the reduction in the spread, banks’ total borrowing costs increase which implies a reduction in investment, and thus in output. Finally, liquidity injections decrease the risk-free rate stimulating investment and output.

1.6 Conclusion

I build a model of the interbank market with endogenous default that is consistent with two empirical findings: i) interbank credit spreads are positively correlated with
default risk, and ii) interbank credit spreads are a function of debt size, leverage, and expected cash flows. I use the model to compare a normal times stationary equilibrium and a crisis times stationary equilibrium. In normal times, there is no spread in the interbank market because the default probability of banks is zero. In crisis times, some banks default and an interbank credit spread arises endogenously. Furthermore, the credit spread is positively correlated with debt size and leverage, and negatively correlated with expected cash flows.

I use this framework to analyze the effects of equity injections, debt guarantees, and liquidity injections on the interbank credit spreads and on the amount of risky projects financed by banks. I find that debt guarantees are effective in reducing interbank credit spreads in times of crisis, but not in stimulating investment. In contrast, interbank credit spreads are relatively unresponsive to injections of equity and of liquidity; however, these policies are successful in stimulating investment.

1.7 Tables and Figures

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Normal Times</th>
<th>Crisis Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ&lt;sub&gt;nt&lt;/sub&gt;</td>
<td>0.220 (0.058)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Libor-OIS<sub>it</sub> = φ<sup>nt</sup>1<sub>it</sub> + φ<sup>ct</sup>1<sub>it</sub> + φ<sup>nt</sup>CDS<sub>it</sub> 1<sub>it</sub> + φ<sup>ct</sup>CDS<sub>it</sub> 1<sub>it</sub> + α<sub>i</sub> + ε<sub>it</sub>.
Table 1.2: Interbank Credit Spread and Default Risk: Bank Specific Data

<table>
<thead>
<tr>
<th>Bank</th>
<th>Normal Times</th>
<th></th>
<th>Crisis Times</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi_i^{nt}$</td>
<td>Std. Error</td>
<td>$\phi_i^{ct}$</td>
<td>Std. Error</td>
</tr>
<tr>
<td>Bank of America</td>
<td>0.187 (0.028)</td>
<td>1.691 (0.524)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank of Tokyo M.</td>
<td>0.160 (0.058)</td>
<td>2.463 (1.319)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barclays</td>
<td>0.416 (0.044)</td>
<td>1.764 (0.519)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Citibank</td>
<td>0.137 (0.043)</td>
<td>0.835 (0.137)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deutsche Bank</td>
<td>0.231 (0.066)</td>
<td>2.330 (0.726)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HSBC</td>
<td>0.354 (0.042)</td>
<td>2.024 (0.593)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JP Morgan C.</td>
<td>0.103 (0.043)</td>
<td>1.828 (0.757)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lloyds</td>
<td>0.429 (0.044)</td>
<td>1.747 (0.593)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rabobank</td>
<td>0.986 (0.336)</td>
<td>1.919 (0.417)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RBS</td>
<td>0.447 (0.044)</td>
<td>1.357 (0.325)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UBS</td>
<td>0.479 (0.131)</td>
<td>1.198 (0.282)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>West LB</td>
<td>0.391 (0.000)</td>
<td>1.011 (1.052)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit Suisse</td>
<td>0.081 (0.014)</td>
<td>1.855 (1.108)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Libor-OIS$_{it} = c_{it}^{nt}1_{it}^{nt} + c_{it}^{ct}1_{it}^{ct} + \phi_i^{nt}CDS_{it}1_{it}^{nt} + \phi_i^{ct}CDS_{it}1_{it}^{ct} + \epsilon_{it}$. 

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Table 1.3: Calibration Parameters and Targets

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
<th>Targets</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Structural</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>β</td>
<td>Discount factor</td>
<td>0.988</td>
<td>Risk-free interest rate in NT</td>
<td>1.63</td>
<td>1.50</td>
</tr>
<tr>
<td>α</td>
<td>Production function</td>
<td>0.497</td>
<td>Risk-free interest rate in CT</td>
<td>0.56</td>
<td>0.67</td>
</tr>
<tr>
<td>ϑ</td>
<td>Equity cost</td>
<td>0.252</td>
<td>Lending constraint</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>ε</td>
<td>Endowment</td>
<td>0.044</td>
<td>Std dev spread</td>
<td>0.11</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td><strong>Exogenous Processes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{ij}^\chi$</td>
<td>Transition $\chi$</td>
<td>0.50</td>
<td>Assumption</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$\rho_{\xi}$</td>
<td>Persistence $\xi$</td>
<td>0.979</td>
<td>Persistance of TFP</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$\sigma_{\xi}$</td>
<td>Std dev $\xi$</td>
<td>0.0072</td>
<td>Std dev of TFP</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$p_{\xi}$</td>
<td>Prob $\xi = 0$</td>
<td>(0) 0.003</td>
<td>Average spread</td>
<td>1.18</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td><strong>Legal</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>κ</td>
<td>Capital requirement</td>
<td>0.04</td>
<td>Basel I</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$\rho^d$</td>
<td>Entry post-default</td>
<td>1</td>
<td>Limited liability</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
Table 1.4: Interbank Credit Spreads and Banks’ Balance Sheets: Conditional on Borrowing Banks

<table>
<thead>
<tr>
<th>Crisis times Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
</tr>
<tr>
<td>Correlation spread and default probability</td>
</tr>
<tr>
<td>Correlation spread and loan size</td>
</tr>
<tr>
<td>Correlation spread and leverage</td>
</tr>
<tr>
<td>Correlation spread and expected cash flows</td>
</tr>
</tbody>
</table>

Table 1.5: Interbank Credit Spreads and Banks’ Balance Sheets: Conditional on Borrowing Banks and Investment Quality

<table>
<thead>
<tr>
<th>Crisis Times</th>
<th>Lowest $\xi$</th>
<th>Highest $\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation spread and default probability</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Correlation spread and loan size</td>
<td>0.42</td>
<td>0.41</td>
</tr>
<tr>
<td>Correlation spread and expected cash flows</td>
<td>-0.43</td>
<td>-0.41</td>
</tr>
<tr>
<td>Correlation spread and leverage</td>
<td>0.19</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Table 1.6: Real Variables in Crisis Times

<table>
<thead>
<tr>
<th>Interbank Volume</th>
<th>Risky Assets</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.60</td>
<td>-3.52</td>
<td>-2.04</td>
</tr>
</tbody>
</table>

Note: Annualized percentages relative to normal times.

Table 1.7: Effects of Policies on Banks’ Borrowing Costs and Real Variables

<table>
<thead>
<tr>
<th>Interbank</th>
<th>Risk-free rate</th>
<th>Spread</th>
<th>Volume</th>
<th>Risky assets</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity injections</td>
<td>-2.15</td>
<td>1.16</td>
<td>2.09</td>
<td>4.18</td>
<td>2.23</td>
</tr>
<tr>
<td>Debt guarantees</td>
<td>1.93</td>
<td>0.00</td>
<td>1.56</td>
<td>-0.32</td>
<td>-0.00</td>
</tr>
<tr>
<td>Liquidity injections</td>
<td>-0.45</td>
<td>1.17</td>
<td>6.26</td>
<td>1.29</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Note: The risk-free interest rate and the interbank credit spread are annualized and in percentages. Interbank volume, risky assets, and output are annualized and in percentages relative to the case without stabilization policies.
Table 1.8: Forecast Regressions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Specification 1</th>
<th>Specification 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Std. Error</td>
</tr>
<tr>
<td>Lag</td>
<td>0.354 (0.085)</td>
<td></td>
</tr>
<tr>
<td>Term Spread</td>
<td>-0.914 (0.214)</td>
<td></td>
</tr>
<tr>
<td>Real FFR</td>
<td>0.536 (0.154)</td>
<td></td>
</tr>
<tr>
<td>GZ Spread</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Libor-OIS</td>
<td>-2.764 (0.930)</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.219 (0.680)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1.1: Average Interbank Credit Spread

---

0 1 2 3 4
USD 3M Libor−OIS

% Annualized

2003 2005 2007 2009 2011

BNP  Lehman Brothers
Figure 1.2: CPP and FDIC Debt Guarantee

Figure 1.3: Liquidity Injections
Figure 1.4: IRF to one pp shock to Libor-OIS. $\mathbb{E}_t \left[ \log \left( \frac{GDP_{t+s}}{GDP_{t-1}} \right) \mid \varepsilon_{OIS,t} = 1 \right] - \mathbb{E}_t \left[ \log \left( \frac{GDP_{t+s}}{GDP_{t-1}} \right) \mid \varepsilon_{OIS,t} = 0 \right]$, for $s = 0, \ldots, 20$. The 95 percent confidence bands for the impulse response function are computed using a bootstrap procedure for the residuals.
Should Central Banks React More Strongly in the Presence of Trend Inflation?

2.1 Introduction

In December of 2008 the Fed hit the zero lower bound. Since then the federal funds rate has remained below 50 basis points. This has led some researchers to wonder whether the inflation target should be increased in order to relax this constraint on the capacity of monetary policy to affect output fluctuations (e.g. Blanchard et al. (2010)). Previous research suggests that greater flexibility in monetary policy comes with a cost, since raising the inflation target could lead to indeterminacy, i.e., the existence of multiple equilibria in linear rational expectations (LRE) models.

Indeterminacy is not desirable in a world populated by risk averse agents because it introduces additional volatility into the economy through agents’ beliefs.¹ In particular, expectations of higher inflation are self-fulfilling in New Keynesian models when monetary policy does not react strongly to inflation. Some authors, such as Clarida et al. (2000) and Lubik and Schorfheide (2004), argue that these shocks to

¹ A considerable amount of research has been published on the consequences of indeterminacy. For example, Woodford (2003), Benhabib et al. (2001), and Clarida et al. (2000).
beliefs, also known as sunspot fluctuations or self-fulfilling expectations, played a significant role in explaining the high volatility of aggregate economic variables during the 1970s.

Previous theoretical work by Hornstein and Wolman (2005), Kiley (2007), and Ascari and Ropele (2009), has shown that the Taylor principle, which says that central banks should respond more than one-for-one in response to inflation, is not enough to guarantee determinacy when trend inflation is positive.\(^2\)\(^3\) Based on these findings, Coibion and Gorodnichenko (2011) use a New Keynesian model without capital accumulation to revisit the importance of self-fulfilling expectations during the late 1960s and 1970s. These authors argue that it is highly probable that the US economy experienced sunspot fluctuations during the 1970s due to the high inflation levels that characterized this period. It would seem, therefore, that an increase in the inflation target could induce a return to sunspot fluctuations if the monetary authority does not react to inflation more strongly than the Taylor principle suggests.

The main contribution of this essay is to study the capacity of the Taylor principle to guarantee determinacy in the class of New Keynesian models typically used for monetary policy analysis, such as Yun (1996), Christiano et al. (2005), and Smets and Wouters (2007), when firms are not able to index their prices. In a model with labor, capital accumulation, capital adjustment costs, and capital utilization the necessary conditions for trend inflation to affect determinacy are as follows: trend inflation is above 4%, firms are not able to index their prices, and the frequency of price changes is less than once a year. Introducing sticky wages as in Erceg et al. (2000) it is possible to find a response to inflation greater or equal than one that

\(^2\) Determinacy refers to the local existence and uniqueness of equilibrium in LRE models.

guarantees determinacy; however, the determinacy region is small and indeterminacy can arise even when the response to inflation is higher than one for one.\footnote{Ascari et al. (2012) find similar results regarding the stabilization power of the Taylor principle in models with capital accumulation. However, in contrast to Ascari et al. (2012), I find that the determinacy region of New Keynesian models with capital accumulation, but without price indexation, is small once sticky wages are introduced.}

In order to understand what drives the results, I follow Sveen and Weinke (2005) and analyze whether an investment boom could be consistent with equilibrium.\footnote{Unfortunately, the dimensionally of the problem does not allow us to characterize determinacy as a function of the parameters of the model, analytically.} When economic agents consider the possibility of an investment boom they observe that there are two counteracting effects over the real interest rate. First, an investment boom increases marginal cost, which increases inflation because inflation tracks the dynamics of marginal cost. Second, marginal cost decreases in the future, ceteris paribus, due to greater availability of capital stock for production. Therefore, there will be a deflationary period in the future. When the monetary authority follows the Taylor principle, the initial increase in inflation pushes up the short term real interest rates, and the future deflationary period pulls down the long term real interest rates. The rationalization of the investment boom depends on the relative strength of these effects. When the decrease in the long term real interest rates is enough to compensate the increase in the short term real interest rates, the investment boom is consistent with equilibrium. With capital adjustment costs, the future reduction in marginal costs due to the greater supply of capital is compensated in part by future adjustment costs. As a result, it is less likely that the economy will experience a deflationary period that causes a reduction in the long term real interest rates that is able to dominate the increase in the short real interest rates. Thus, in models with capital adjustment costs, the initial beliefs of an investment boom are not self-fulfilling, under reasonable calibrations. Similarly, in models with capital utilization, future marginal costs decrease less than in models without capital utilization. As a
consequence, the decrease in long term real interest rates is not enough to compensate the increase in the short term real interest rates, which decreases the likelihood of investment booms being consistent with equilibrium. Once wage rigidities are introduced, the marginal cost are less sensitive to increases in aggregate demand so the decrease in long term real interest rate is more likely to dominate.

The second contribution of this essay is to analyze the properties of Taylor rules that include output growth, interest rate smoothing, and expected inflation, in the class of models studied in this chapter. There is a large amount of research on this topic. For instance, Sveen and Weinke (2005) find that reacting to deviations of output from the steady state, and introducing a smoothness component to the interest rate rule helps achieve determinacy in models with firm-specific capital. Coibion and Gorodnichenko (2011) find that reacting to the output gap could be destabilizing, while reacting to output growth and adding a persistent component to the Taylor rule leads to determinacy.

In New Keynesian models that include capital accumulation, capital adjustment costs, and capital utilization, even a small reaction to output growth implies that the Taylor principle yields determinacy for values of trend inflation as high as 7%. Moreover, the Taylor principle guarantees determinacy for values of trend inflation as high as 6.5% when there is enough persistence in nominal interest rates. Interestingly, these additional components become relevant only when trend inflation is above 4% and there is no price indexation, a situation which is unlikely to hold.

Recent literature has also studied the consequences of forward looking Taylor rules for the existence of a unique linear rational expectations equilibrium. Woodford (2003) and King (2000) show that in a New Keynesian model without capital accumulation, forward looking Taylor rules with moderate responses to deviations from the inflation target are ideal. More recently, Cochrane (2007) finds that the stabilization power of reacting to expected inflation disappears when certain for-
ward looking Taylor rules are used. Dupor (2001) and Carlstrom and Fuerst (2005) examine the role of investment decisions in the existence of a unique rational expectation equilibrium when the central bank reacts to expected inflation. Carlstrom and Fuerst (2005) study the determinacy properties of a New Keynesian model with rental capital. A central finding of their paper is that the determinacy region is small in models with forward looking Taylor rules. Dupor (2001) works in continuous time and finds that the Taylor principle does not induce determinacy. These studies do not consider forward looking Taylor rules with interest rate smoothing. I show that interest rate smoothing plays a crucial role in anchoring the expectations of economic agents, thereby inducing determinacy in medium-sized New Keynesian models with price stickiness and forward looking Taylor rules.

Section 2.2 presents the model. Section 2.3 shows the minimum response to inflation that is necessary to induce determinacy in models with trend inflation and no price indexation. Section 2.4 extends the results to medium-sized DSGE models that include wage rigidities. Section 2.5 shows the implications of price indexation for the long-run slope of the Phillips curve. Section 2.6 studies alternatives specifications of the Taylor rule. Section 2.7 concludes.

2.2 The Model

I use a New Keynesian model with capital accumulation similar to Yun (1996), Christiano et al. (2005), and Smets and Wouters (2007). In this section I consider four cases regarding capital adjustment costs and capital utilization: Case 1 abstracts from capital adjustment costs and capital utilization, Case 2 adds capital adjustment costs to Case 1, Case 3 adds capital utilization to Case 1, Case 4 adds capital

---

6 In continuous time the relationship between the real interest rate of the economy and the marginal productivity of capital is different than in discrete time, see Carlstrom and Fuerst (2005) for a discussion.
adjustment costs and capital utilization to Case 1. Since these models have been
developed extensively in the literature, I present a brief description of the model that
incorporates capital adjustment costs and capital utilization, in order to facilitate
the interpretation of the findings in this essay.

In the economy there is a representative household that consumes, saves, in-
vests, and supplies labor and capital services to intermediate goods producers. The
household obtains positive utility from consumption and negative utility from hours
worked. She chooses consumption $C_t$, hours worked $H_t$, investment in physical cap-
ital $X_t$, capital utilization $U_t$, next period capital $K_t$, and government bonds $B_t$ in
order to maximize the discounted value of the future streams of utility subject to
her budget constraint, and the law of motion for capital, that is,

$$\max_{\{C_t, H_t, X_t, U_t, K_t, B_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \frac{H_t^{1+\gamma}}{1+\gamma}\right)$$

subject to

$$C_t + X_t + \frac{B_t}{P_t} + \frac{T_t}{P_t} = \frac{B_{t-1}R_{t-1}}{P_t} + (r_tU_t - a(U_t))K_{t-1} + W_tH_t + \frac{F_t}{P_t}$$

and

$$K_t = (1-\delta)K_{t-1} + \left(1 - S\left(\frac{X_t}{X_{t-1}}\right)\right)X_t.$$

$P_t$ is the price level in the economy, which is the price of a unit of final goods that are
produced by final goods producers using intermediate goods as inputs. $R_t$ denotes
the gross nominal interest rate that is set by the monetary authority. The rate of

7 I set aside wage rigidities which are studied in section 2.4.
return from renting capital is denoted by \( r_t \). Capital utilization costs are described by the function \( a(U_t) \) subject to the same restrictions as Christiano et al. (2005), that is, \( a(1) = 0 \) and \( \sigma_a = \frac{a''(1)}{a'(1)} \). \( W_t \) is the cost of labor in this economy. \( F_t \) is the amount of profits that the household obtains from intermediate goods producers. \( T_t \) denote lump-sum taxes. \( \delta \) is the depreciation rate of capital. The cost of adjusting capital is given by \( S \). I assume a functional form for \( S \) such that \( S(1) = S'(1) = 0 \), and \( S''(1) = \kappa > 0 \), following Christiano et al. (2005).

There is a continuum of intermediate good producers operating in a monopolistically competitive market. Each of them produces an intermediate good that is unique, therefore, each producer has market power. The intermediate goods producers combine capital and labor using a Cobb Douglas production function with labor augmenting technology \( Z_t \), where \( \Lambda_z \) denotes the growth rate of the economy in the steady state. The production factors are hired in competitive markets. As a consequence, we can solve the maximization problem of the intermediate goods producers in two stages. In the first stage, each intermediate goods producer \( i \) chooses hours \( H_{it}^d \), and capital services \( K_{it-1}^d \) to minimize the cost of producing \( Y_{it} \) units of output, that is,

\[
\min_{H_{it}^d, K_{it-1}^d} W_t H_{it}^d + r_t K_{it-1}^d
\]

subject to the production function for good \( Y_{it} \)

\[
Y_{it} = \begin{cases} 
A_t \left(K_{it-1}^d\right)^{\alpha} \left(H_{it}^d Z_t\right)^{1-\alpha} - \phi Z_t & \text{if } A_t \left(K_{it-1}^d\right)^{\alpha} \left(H_{it}^d Z_t\right)^{1-\alpha} \geq \phi Z_t, \\
0 & \text{Otherwise.}
\end{cases}
\]

The intermediate goods producers face a fixed cost of production \( \phi Z_t \), where \( \phi \) is a parameter that is chosen to guarantee that intermediate goods producers make zero profit in the steady state. Since intermediate goods producers have market
power, they can post a price subject to the demand for their product. I use a Calvo
price setting environment where each period a firm is able to set a new price with
probability $1 - \nu$. When the firm cannot set a new price, it keeps the price posted
for the last period in which it was able to optimize. Thus, the intermediate goods
producers choose $P_{it}$ to maximize the discounted value of the future streams of profits

$$
E_t \sum_{j=0}^{\infty} (\nu \beta)^j \mathbb{E}_t \left[ Y_{it+j} \left( P_{it} \frac{P_{it}}{P_{t+j}} - MC_{t+j} \right) \right]
$$

subject to the demand for good $Y_{it}$,

$$
Y_{it+j} = \left( \frac{P_{it}}{P_{t+j}} \right)^{-\eta} Y_{t+j}^d.
$$

The discount factor of the intermediate good producers is the stochastic discount
factor of the representative household that owns the firms, $\beta^{\mathbb{E}_{t+1} \mathbb{E}_t}$, where $\mathbb{E}_t$ is the
marginal utility of consumption of the representative household. Note that the total
production cost can be written as the product of the marginal cost and the output
that is produced, $MC_t \times Y_{it}$, because we assume that the intermediate goods
producers have a constant return to scale technology. $Y_t^d$ denotes the aggregate de-
mand for goods, which is equal to the sum of aggregate consumption, investment,
and capital utilization costs. Finally, $\eta$ denotes the elasticity of substitution between
intermediate goods.

In this model there is no indexation. In the log-linearized version of the model,
this implies that intermediate goods producers set their price taking into account
not only expected inflation and current marginal cost, as in the case of zero steady
state trend inflation, but also expected marginal costs, expected deviations of output
growth from trend, and expected changes in the nominal interest rate, see Ascari and
Ropele (2009), and Coibion and Gorodnichenko (2011).

45
Next, I describe the representative final goods producer who uses the intermediate goods of the economy as inputs in the production of final goods. The market for final goods is competitive, and the final goods producer chooses inputs $Y_{it}$ to maximize profits, subject to a CES bundling technology. Formally, the final goods producer’s problem is

$$\max_{Y_{it}} P_t Y^d_{it} - \int_0^1 P_{it} Y_{it} dt$$

subject to

$$Y_{it}^d = \left( \int_0^1 \frac{Y_{it}^{-\eta} dt}{\eta} \right)^{\frac{\eta}{\eta-1}}.$$

To complete the characterization of the economy, there is a government that conducts monetary policy using the following Taylor rule

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left( \frac{\Pi_t}{\Pi} \right)^{\psi_\pi} \left( \frac{Y_{it}^d}{\Lambda_z Y_{i,t-1}^d} \right)^{\psi_y}.$$

$R$ and $\Pi$ are the steady state levels of the gross nominal interest rate and gross inflation, respectively. The response to inflation, output growth, and the smoothness component of the Taylor rule are $\psi_\pi$, $\psi_y$, and $\rho_R$ respectively. In the next section, we focus on the case in which there is neither a smoothness component nor a response to output growth, therefore, the Taylor rule is simply $\frac{R_t}{R} = \left( \frac{\Pi_t}{\Pi} \right)^{\psi_\pi}$. Finally, the government must also satisfy its budget constraint, so that the sum of debt holdings $\frac{B_t}{P_t}$ and lump-sum taxes $\frac{T_t}{P_t}$ must be equal to the debt payments $R_{t-1} \frac{B_{t-1}}{P_t}$.

2.3 Trend Inflation and Determinacy

Is there a risk of indeterminacy in the presence of trend inflation? Under plausible calibrations, I find that the Taylor principle is enough to guarantee determinacy in
New Keynesian models with capital accumulation, capital adjustments costs, and capital utilization.

2.3.1 Minimum Response to Inflation for Determinacy $\psi_{\pi}^*$

Figure 2.1 shows the minimum response to inflation that is necessary to guarantee determinacy $\psi_{\pi}^*$ in four different models when the monetary authority follows a Taylor rule specification, where the nominal interest rate reacts only to current inflation, i.e., $\frac{R_t}{R_t'} = \left(\frac{\Pi_t}{\Pi_t'}\right)^{\psi_{\pi}}$. Figure 2.1 contains four panels, in all of them I plot the minimum response to inflation that is necessary to achieve determinacy in a New Keynesian model with heterogeneous labor, Calvo price stickiness, and no capital accumulation, as in Coibion and Gorodnichenko (2011). I take the latter as a benchmark. On the left of the upper panel, I consider a model with homogeneous labor and capital accumulation for different degrees of price stickiness $\nu$. On the right of the upper panel, I consider a model with homogeneous labor, capital accumulation, and capital adjustment costs. On the left of the bottom panel, I present results for the case of a model with homogeneous labor, capital accumulation, and capital utilization costs. Finally, on the right of the bottom panel I show results for models with homogeneous labor, capital accumulation, capital adjustment costs, and capital utilization.

The different lines in each panel correspond to different values of the probability of being unable to adjust prices, controlled by the price stickiness parameter $\nu$. The range of price stickiness that I consider goes from 0.55, which implies that firms change prices once every 6 to 7 seven months on average, to 0.8230, which implies that firms change prices once every 16 to 17 months on average. The black dashed line marked with circles corresponds to the model of Coibion and Gorodnichenko (2011), the benchmark case in Figure 2.1, where the probability of firms being unable to re-optimize prices is 0.55.

I set the parameters of the models to values commonly used in the literature. I set
the Frisch elasticity of labor supply $\tau^{-1}$ to 1, the degree of temporal preference $\beta$ is equal to 0.99, the long-run growth of output $\gamma_z$ is set to 1.0037, which is equivalent to an annualized growth rate of output per capita of 1.5%, and the elasticity of substitution between intermediate goods $\eta$ is equal to 10. The share of capital on final output $\alpha$ is equal to 0.33. The capital adjustment costs parameter $\kappa$ is set to 2.48. The capital utilization parameter $\sigma_a$ is set to 0.01. The calibration for $\beta$, $\tau$, $\eta$, and $\Lambda_z$ follows Coibion and Gorodnichenko (2011). The parameters $\kappa$, and $\sigma_a$ are set following Christiano et al. (2005). The value for $\alpha$ is standard in the literature. Finally, I consider a range of values between 0.55 and 0.8230 for the Calvo parameter $\nu$ to guarantee that all models are evaluated at the same LRPC, as explained below.8

Note that different calibrations can imply different slopes for the long-run slope of the Phillips curve in models with and without capital.9 To address this issue, I set the degree of price stickiness $\nu$ to the value of 0.8230, in order that the models with capital accumulation have the same long-run slope of the Phillips curve under zero trend inflation as the benchmark case, i.e., 0.2932.

It is clear from Figure 2.1 that capital adjustment costs and capital utilization reestablish the power of the Taylor principle to guarantee determinacy, even when we set the Calvo parameter to 0.8230, which implies that firms adjust their prices once every 16 months on average.10

Although the Taylor principle is satisfied, the implications of trend inflation for the long-run slope of the Phillips curve of models that do not include price indexation are at odds with the positive long-run slope of the Phillips curve of the same models when they are log-linearized around a zero steady state inflation, or models that are

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8 In New Keynesian models the Calvo parameter refers to the probability of firms being unable to re-optimize prices. It is directly related to the frequency of price adjustment $\eta^p$, using $\eta^p = \frac{1}{1-\nu}$.

9 I compute the LRPC considering the long-term relation between output and inflation that is obtained using the supply side of the non-policy block of equations.

10 In the next subsection I explain the intuition for this result.
log-linearized around a positive value of trend inflation and assume perfect indexation of prices. Figure 2.2 shows that as trend inflation increases, the long-run slope of the long-run Phillips curve becomes negative.\(^{11}\) The upper panel shows the long-run slope of the Phillips curve when annualized trend inflation is equal to 0\%, and 0.4\%, respectively. A small increase in trend inflation is enough to induce a negative slope of the long-run Phillips curve. The bottom panel shows the slope of the long-run Phillips curve for higher values of annualized trend inflation.

The slope of the long-run Phillips curve becomes negative due to the impact that trend inflation has on price dispersion \(\nu^p\), see Yun (2005) and Ascari and Ropele (2009).\(^{12}\) Figure 2.3 shows the relation between the LRPC and trend inflation for the class of models studied in this essay. In the upper panel, we see that the steady state level of price dispersion increases with trend inflation when firms adjust prices less than once a year. The bottom panel shows the long-run response of price dispersion to trend inflation. When firms adjust their prices less than once a year, as trend inflation increases price dispersion grows exponentially.

An increase in price dispersion has a negative impact on aggregate demand because it leads to a violation of the conditions that are necessary to achieve the efficient allocation of the economy under flexible prices, see Galí (2008). Alternatively, using the CES aggregator for the composite of intermediate goods, the production of the final goods is maximized when labor and capital are allocated equally among the different intermediate goods producers.

2.3.2 Investment Booms and Determinacy

In New Keynesian models without capital accumulation, the Taylor principle can be understood by analyzing whether a consumption boom can be consistent with

\(\nu^p = \int_0^1 (\frac{\dot{P}}{\bar{P}})^{-\eta} di, \text{ and } \nu^p \approx 1 + 0.5 \ast \eta \ast \text{var}(P_t) > 1, \text{ (e.g. Galí (2008)).} \)

\(^{11}\) This has been shown by Ascari and Ropele (2009) in models that do not include capital.

\(^{12}\)
equilibrium. The logic is that if a consumption boom were to happen, then inflation would increase which triggers a reaction of the nominal interest rate under control of the monetary authority. If the increase in the nominal interest rate is higher than the increase in inflation, as the Taylor principle suggests, then the real interest rate increases, which prevents the realization of the consumption boom.

This interpretation of self-fulfilling consumption booms is appealing because it can be directly linked to the Taylor principle. However, in an insightful contribution Ascari and Ropele (2009) show that the Taylor principle is not enough to guarantee determinacy in New Keynesian models without capital accumulation. It is useful to briefly summarize their main point. For simplicity sake, I consider the case in which the monetary authority reacts to current inflation and does not react to measures of real economic activity or past nominal interest rates. In this case, a sufficient condition for determinacy when the model is log-linearized around the zero steady state level of inflation, is that the monetary authority modifies the nominal interest rate more than one-for-one in response to inflation, $\psi_\pi > 1$ (see section 3 in Ascari and Ropele (2009) or proposition 4.3 in Woodford (2003)). Nevertheless, when the model is log-linearized around a positive steady state value of inflation the conditions that guarantee determinacy in the New Keynesian model without capital accumulation are

$$\psi_\pi > 1$$

and

$$\psi_\pi > \frac{\beta \Pi^{1-\epsilon} - 1}{\varsigma}$$

where $\varsigma$ is a decreasing function of $\Pi$.\(^{13}\) The latter condition becomes binding for

\(^{13}\) See section A.4. in the appendix of Ascari and Ropele (2009) for details about the assumptions necessary for this result.
determinacy as trend inflation increases, which implies that there could be indeterminacy even when the Taylor principle is satisfied.

Once we introduce capital accumulation, it is not possible to characterize the determinacy region analytically. Therefore, following Sveen and Weinke (2005), I study the determinacy properties of the model numerically, and I rationalize the results in terms of consumption and investment booms. In particular, for a plausible calibration of the model, I analyze whether a consumption or an investment boom can be consistent with equilibrium. Consumption booms are ruled out by the Taylor principle because the real interest rate increases and prevents the rationalization of the boom. Therefore, in what follows I will focus on investment booms to provide an economic intuition for the main results of the essay.

In New Keynesian models with capital accumulation there are two counteracting effects over the real interest rate that we have to take into account to establish whether an investment boom could be consistent with equilibrium. First, an investment boom increases the marginal cost and thus inflation. Second, the marginal cost decreases in the future due to the greater supply of capital, ceteris paribus. Therefore, there will be a period of deflation in the future.14 When the monetary authority follows the Taylor principle, the initial increase in inflation pushes up the short term real interest rates, and the future deflationary period pulls down the long term real interest rates. The rationalization of the investment boom depends on the relative strength of these effects. When the decrease in future real interest rates is enough to compensate the increase in the short term real interest rates, it is more likely that the investment boom can be consistent with equilibrium.

We can observe the aforementioned effects by analyzing the impulse responses of the economy to a belief shock to investment using the method for analyzing fundamental and sunspots shocks developed by Lubik and Schorfheide (2003). A belief

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14 These counteracting effects have been pointed out by Sveen and Weinke (2005).
shock to investment $\zeta_t \tilde{X}$ is defined as

$$\zeta_t \tilde{X} = \tilde{X}_t - E_{t-1} \tilde{X}_t - \tilde{\eta}_t \tilde{X},$$

where $\tilde{X}_t$ denotes the ratio between investment and the labor-augmenting technology process, log-linearized with respect to the stationary steady state. I follow the notation introduced by Lubik and Schorfheide (2003) letting $\tilde{\eta}_t \tilde{X}$ be the revised forecast error, and $E_{t-1} \tilde{X}_t + \zeta_t \tilde{X}$ be the revised forecast. The space state representation of the log-linearized system of equations that characterize the equilibrium of the model is

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi \varepsilon_t + \Pi (\tilde{\eta}_t + \zeta_t^b),$$

where $y_t$ is the DSGE state vector of the model. $\varepsilon_t$ is a vector that contains the fundamental shocks to the economy, $\tilde{\eta}_t$ is the vector of expectational errors, and $\zeta_t^b$ is a vector of belief shocks. The solution to system 2.1 is given by

$$y_t = T_1 y_{t-1} + T_0 \begin{bmatrix} \varepsilon_t \\ \zeta_t^b \end{bmatrix} + \Theta \tilde{\zeta}_t^{b*},$$

where $\tilde{\zeta}_t^{b*}$ is a reduced form vector of sunspot shocks.

Lubik and Schorfheide (2003) characterize the full set of solutions under indeterminacy. In order to study the implications of a belief shock under indeterminacy, it is enough to select one of the infinitely many solutions available. I focus on the orthogonality solution, where the responses of the economy to fundamental shocks are not affected by indeterminacy, and I set the reduced form sunspot shocks $\tilde{\zeta}_t^{b*}$ to zero.

I calibrate the model, that includes capital accumulation without capital utilization and without capital adjustment costs, as follows: the structural parameters are set to the values of the previous section, annualized trend inflation is set to 4%,
and $\psi = 1.5$, i.e., the monetary authority reacts to inflation following the Taylor principle. Figure 2.4 shows the impulse response functions to a one percentage point positive belief shock to investment.

An investment boom is followed by an increase in the marginal cost, as can be seen on the right of the upper panel. The increase in the marginal cost pushes inflation up. This implies that the real interest rate increases since the monetary authority increases the nominal interest rate inflation more than a one for one in response to inflation. As time passes, ceteris paribus, the cost of renting capital should decrease because there is more capital in the economy. Therefore, there is an effect related to the reduction in the future marginal cost that eventually kicks in.\(^{15}\) The reduction in future marginal cost is associated with a deflationary period that exerts downward pressure on the long term real interest rates since we are assuming that the central bank reacts more than one for one to inflation. The drop in the long term real interest rate counteracts the initial increase in the short term interest rates. When this effect is strong enough, an investment boom is consistent with equilibrium. Thus, the belief shock is rationalized and investment increases initially by more than 0.60 percentage points, as shown on the left of the upper panel.

It is insightful to use the equation that relates real interest rates and the stock of capital in order to shed light on the rationalization of investment boom described above. Up to first order,

$$\hat{r}_t = \alpha \hat{mc}_t + \alpha (1 - \alpha) (\hat{h}_t - \hat{k}_{t-1}) + \alpha \hat{z}_t \quad \text{for } t = 0, 1, \ldots .$$

Hats denote that the variable is expressed in terms of deviations from a stationary steady state. In response to an investment boom, marginal cost increases on the short term to satisfy the new demand for goods. Since inflation inherits the dynamics of

\(^{15}\) In equilibrium, future marginal costs are determined endogenously. Note that in equilibrium the response of marginal cost shown in Figure 2.4 is always above zero.
the marginal cost and the monetary authority follows the Taylor principle, this is consistent with an increase in short term real interest rates. In addition, the greater supply of capital is consistent with a decrease in future interest rates. On the right of the middle panel, we can observe this pattern on the behavior of the real interest rate in response to an investment boom, that is, the short term interest rate increase and the long term interest rates decrease. This path of interest rates implies that an investment boom is consistent with equilibrium. On the contrary, an investment boom would not be consistent with equilibrium if the agents observed an increase of short term and long term real interest rates in response to their initial beliefs about the existence of an investment boom.

What is the role of trend inflation in this outcome? The effect of trend inflation in the price setting of firms is directly related to the degree of price stickiness implied by the Calvo parameter. This can be seen by writing the equations that characterize the pricing behavior of firms as

\[
\hat{p}_t^\ast = (1 - \gamma_2) \sum_{j=0}^\infty E_t \gamma_2^j \hat{m}ct_{t+j} + \sum_{j=1}^\infty \left( \gamma_2^j - \gamma_1^j \right) \left( E_t (\hat{y}_{t+j} - \hat{y}_{t+j-1}) - E_t \hat{R}_{t+j-1} \right) \\
+ \sum_{j=1}^\infty \left( \theta \gamma_2^j - \gamma_1^j (\theta - 1) \right) E_t \hat{\pi}_{t+j} + \sum_{j=1}^\infty \left( \gamma_2^j - \gamma_1^j \right) E_t \hat{z}_{t+j},
\]

where \( \gamma_1 = R^{-1}(1 + \pi)\nu \gamma_\nu \) and \( \gamma_2 = R^{-1}(1 + \pi)\nu \gamma_\nu \). Note that \( \gamma_2 \) is increasing in the degree of price stickiness \( \nu \) and trend inflation \( \pi \), and that \( \gamma_2 \) is negative related to the response of inflation to the current marginal cost \( 1 - \gamma_2 \). Therefore, as \( \nu \) or \( \pi \) increases, the impact of the current marginal cost on inflation becomes smaller, which tends to dampen the direct impact of the current marginal cost on inflation.\(^{16}\) This is consistent with Sveen and Weinke (2005), who argue that as price stickiness increases, the initial impact on inflation dampens, attenuating the

\(^{16}\) After a certain number of periods, trend inflation has a negative impact on current inflation
initial increase in the real interest rate. As a consequence, when trend inflation is high the initial increase in the real interest rate is not strong enough to avoid the rationalization of the investment boom. Thus, as trend inflation increases, it also increases the likelihood of observing negative long-term real interest rates that self-fulfill the investment boom.

Why do capital adjustment costs help restore the Taylor principle? In the presence of capital adjustment costs, the future reduction in marginal costs is compensated in part by future adjustment costs. As a result, it is less likely that the economy will experience a deflationary period that causes a reduction in future real interest rates that is able to prevent the rationalization of an investment boom.

Why does adding capital utilization restore the Taylor principle? The intuition is that in response to an investment boom, future marginal costs decrease less in a model with capital utilization than in a model without capital utilization because there is an extra margin to adjust capital that dampens the impact of changes in the capital stock on future marginal costs. As a result, the effect of a lower future real interest rate is smaller and the current increase in the real interest rate is enough to rule out an investment boom.

Why can a higher response to inflation restore determinacy in the presence of trend inflation? A more aggressive response to inflation enhances the initial increase in the real interest rate and the investment boom is not consistent with equilibrium.

2.4 Sticky Wages

In this section, I add sticky wages to the model described in section 2.2. As a result, we need to introduce a complete market structure in order to be able to through \( \gamma_2 \). To see this, note that the coefficient of the \( t + j \) marginal cost is given by \( (1 - \gamma_2)\gamma_2^j \). Then one can show that \( \frac{\gamma(1-\gamma_2)\gamma_2^j}{\gamma_2^{j+1}} > 0 \) for \( j > \frac{\gamma_2}{1-\gamma_2} \). Thus, as trend inflation \( \pi \) increases, future reductions in the marginal cost have a stronger negative effect on contemporaneous inflation.
aggregate the policy functions of the agents. In this class of models, each household supplies labor to a representative competitive firm, a labor packer, which transform the heterogeneous labor into a composite labor input that is then sold to intermediate goods producers. I present the problem of the households and the labor packer in the appendix to chapter 2 — the production side of the economy is identical to the model developed in section 2.2.

Table 2.1 presents the calibration of the model. The parameters are the same as in the model developed in section 2.2. In addition, once wage stickiness is introduced, we need to set a value for the elasticity of substitution between workers $\eta_w$, the Calvo parameter for wages $\nu_w$, and wage indexation $\iota_w$. I follow Christiano et al. (2005), setting $\eta_w$ equal to 21, $\nu_w$ equal to 0.64, and $\iota_w$ to 1. Regarding, the frequency of wage adjustment $\nu_w$, I perform sensitivity analysis to consider situations where households adjust wages once every year.

The upper panel of Figure 2.5 shows the minimum response to inflation that is necessary for determinacy. In the left panel I set the degree of wage stickiness to 0.64 and I compute the minimum response to inflation that is necessary for determinacy under different assumptions regarding the degree of price stickiness in the economy. Note that for trend inflation to affect the minimum response necessary for determinacy, the level of steady state inflation must be above 4% and the degree of price adjustment greater or equal to $\nu = 0.75$. In the right panel, I fix the degree of price stickiness $\nu$ to 0.70, such that the long-run slope of the Phillips curve of the model with sticky wages equals the one of Coibion and Gorodnichenko (2011), and I compute the minimum response to inflation that is necessary for determinacy under different assumptions regarding the degree of wage stickiness in the economy. The bottom panel of Figure 2.5 shows that these results are not robust to small changes

17 When trend inflation is above 6% annually and $\nu \geq 0.70$ we do not find determinacy even when the response to inflation is above 40.
in the response to deviations of inflation from its target. On the left panel, I consider a situation where \( \nu = 0.70, \nu_w = 0.64, \) and the inflation target is 4%, annualized. On the right panel, I consider a situation where \( \nu = 0.70, \nu_w = 0.75, \) and the inflation target is 4%. The vertical axis is equal to one when there is indeterminacy, and zero when there is determinacy. The horizontal axis is equal to the response to deviations from the inflation target \( \psi_\pi. \) Interestingly, although it is possible to find a response to inflation greater or equal than one that guarantees determinacy, the determinacy region is small and indeterminacy can arise even when the response to inflation is higher than one for one.

These findings can be rationalized using the economic intuition developed in section 2.3. In particular, with sticky wages the marginal cost is less responsive to increases in aggregate demand and as a consequence the decrease in long term interest rates are more likely to dominate, and hence induce indeterminacy.

Figure 2.6 shows the long-run slopes of the Phillips curve associated with different levels of trend inflation. The upper panel shows the long-run slope of the Phillips curve when annualized trend inflation is 0%, and 0.4% respectively. The bottom panel shows the long-run slope of the Phillips curve for higher levels of annualized trend inflation.

2.5 Price Indexation

Price indexation is a stabilizing force regarding the determinacy properties of the model, as has been pointed out by Ascari and Ropele (2009). Interestingly, in the class of models used in this essay, a small degree of indexation can reestablish the power of the Taylor principle to guarantee determinacy even for values of annualized trend inflation that are above 4%.

In this section, I use the model with capital accumulation, capital adjustment costs, and capital utilization calibrated at the parameter values used in section 2.3
including degrees of indexation in the range from 0 to 0.20 – in line with the estimates of Cogley and Sbordone (2008). The Calvo parameter $\nu$ is set equal to 0.8230. Figure 2.7 shows the minimum response to inflation that is necessary for determinacy, we can observe that the Taylor principle can be reestablished for values of annualized trend inflation as high as 6%.

2.6 Alternative Specifications of the Taylor Rule

So far, in the numerical analysis, we have considered Taylor rule specifications where the nominal interest rate responds only to inflation. In this section, I include output growth stabilization and interest rate smoothing in the Taylor rule.

2.6.1 Output Responses

Figure 2.8 shows the minimum response to inflation that is necessary to guarantee determinacy when the monetary authority reacts to inflation and to deviations of output growth, i.e.,

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\psi_{\pi}} \left(\frac{Y_{t}^{d}}{\Lambda_{t}Y_{t-1}^{d}}\right)^{\psi_{y}}.$$ 

I study different degrees of output responses $\psi_{y}$ setting the degree of price stickiness $\nu$ equal to 0.8230. On the left of the upper panel, I consider a model with capital accumulation. On the right of the upper panel, I consider a model with capital accumulation and capital adjustment costs. On the left of the bottom panel, I present the results for the case of capital accumulation and capital utilization costs. Finally, on the right of the bottom panel I show the results for the case of capital accumulation, capital adjustment costs, and capital utilization.

It is clear from Figure 2.8 that the findings of section 2.3 hold once we include output growth stabilization in the Taylor rule. For instance, on the right of the
bottom panel we can observe that the coefficient of output stabilization $\psi_y$ from 0 to 0.10, implies that the Taylor principle guarantee determinacy even when annualized trend inflation is as high as 7%.

The stabilizing properties of reacting to deviations of output growth from its trend have been emphasized by several studies, see Walsh (2003) and Coibion and Gorodnichenko (2011). I find that these results hold in the class of models studied in this essay. In addition, I show that output stabilization only becomes relevant when trend inflation is above 4%, in New Keynesian models that include capital accumulation, capital adjustment costs, and capital utilization.

2.6.2 Interest Rate Smoothing

Figure 2.9 shows the minimum response to inflation that is necessary to guarantee determinacy when the monetary authority reacts to inflation and to past values of the nominal interest rates, i.e.,

$$\frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{\bar{R}} \right)^{\rho_R} \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\psi_y(1-\rho_R)}.$$

I study different degrees of interest rate smoothing, setting the degree of price stickiness $\nu$ equal to 0.8230. On the left of the upper panel, I present results for a model with capital accumulation. On the right of the upper panel, I consider a model with capital accumulation, and capital adjustment costs. On the left of the bottom panel, I present the results for a model with capital accumulation, and capital utilization costs. Finally, on the right of the bottom panel, I show a model with capital accumulation, capital adjustment costs, and capital utilization.

Figure 2.9 shows that introducing a persistent component in the Taylor principle leads to determinacy. For instance, in the model with capital accumulation, capital adjustment costs, and capital utilization, the Taylor principle guarantees determinacy
for values of trend inflation as high as 6.5%, when \( \rho_R \) is set to 0.75.

Woodford (2003) and Coibion and Gorodnichenko (2011) have shown how reacting to past interest rates plays an important role in anchoring the expectations of economic agents, and induces determinacy. As in the case of output growth stabilization I find that interest rate smoothing contributes to determinacy in the class of models studied in this essay. Furthermore, I show that the contribution of interest rate smoothing to determinacy becomes relevant only when trend inflation is above 4%. The latter is interesting because it shows that the Taylor principle guarantees determinacy, even without interest rate smoothing, for reasonable calibration of models that include capital accumulation, capital adjustment costs, and capital accumulation.

2.6.3 Forward Looking Taylor Rules

Several authors have focused on the implications of using forward looking Taylor rules for the existence of a unique linear rational expectation equilibrium. For instance, Woodford (2003) and King (2000) show that in a simple New Keynesian model with forward looking Taylor rules, the response to deviations from the inflation target should neither be too weak nor too strong in order to guarantee the existence of a unique local equilibrium. More recently, Cochrane (2007) finds that the stabilization power of reacting to expected inflation can easily be questioned by using a Taylor rule that reacts to expected inflation, one and two periods ahead.

Surprisingly, the role of investment decisions in the existence of a unique rational expectation equilibrium when the central bank reacts to expected inflation has not received much attention. One exception is the work of Carlstrom and Fuerst (2005), which analyzes the determinacy properties of a New Keynesian model with rental capital in discrete time. One of the main findings of Carlstrom and Fuerst (2005) is that once we introduce investment decisions into the model, there is only a small
region where determinacy is possible.

Dupor (2001) also studies the determinacy properties of a New Keynesian model with rental capital. Dupor (2001) works in continuous time and reports results opposite to Carlstrom and Fuerst (2005), because in continuous time the relationship between the real interest rate of the economy and the marginal productivity of capital is different than in discrete time. However, their analysis does not consider forward looking Taylor rules with interest rate smoothing. In this essay, I study what minimum response to expected inflation is necessary for determinacy, using the following specification for the Taylor rule

\[
\frac{R_t}{\bar{R}} = \left( \frac{R_t}{\bar{R}} \right)^{\rho_R} \left( \frac{E_t \Pi_{t+1}}{\Pi} \right)^{\psi_s(1-\rho_R)}. 
\]

Coibion and Gorodnichenko (2011) have shown the stabilizing power of the smoothness parameter in this type of Taylor rule. I show that the stabilizing power of introducing persistence in the policy for nominal interest rates is also present in New Keynesian models with capital accumulation, capital adjustment costs, and capital utilization. The calibration is the same as in section 2.3, and the Calvo parameter is set to 0.8230 in order to match the long-run slope of the Phillips curve to the one in Coibion and Gorodnichenko (2011).

Figure 2.10 shows that the minimum response to expected inflation that is necessary for determinacy decreases significantly as the central bank increases the persistence of the policy for the nominal interest rate. Alas, once wage rigidities are incorporated, there is no stabilization power of interest rate smoothing when the central bank reacts to expected inflation.

\[^{18}\text{See Carlstrom and Fuerst (2005) for a discussion.}\]
2.7 Conclusion

The main contribution of this essay is to study the capacity of the Taylor principle to guarantee determinacy in the class of New Keynesian models typically used for monetary policy analysis, such as Yun (1996), Christiano et al. (2005), and Smets and Wouters (2007), when firms are not able to index their prices. In a model with labor and capital accumulation, capital adjustment costs, and capital utilization, the necessary conditions for trend inflation to affect determinacy are as follows: trend inflation is above 4%, firms are not able to index their prices, and the frequency of price changes is less than once a year. Introducing sticky wages as in Erceg et al. (2000) it is possible to find a response to inflation greater or equal than one that guarantees determinacy; however, the determinacy region is small and indeterminacy can arise even when the response to inflation is higher than one for one. An additional interesting feature of this essay is that it rationalizes the results in terms of consumption and investment booms, in line with the innovative work of Sveen and Weinke (2005). Introducing capital allows us to keep the intuition related to self-fulfilling consumption and investment beliefs that is closely tied to the Taylor principle.

The second contribution of this essay is to analyze the implications for determinacy of alternative specifications of monetary policy rules such as rules that include output growth, interest rate smoothing, and expected inflation, within the class of models studied in this essay. I find that output growth and interest rate smoothing are stabilizing forces because they contribute to achieving determinacy, extending the findings of Sveen and Weinke (2005) and Coibion and Gorodnichenko (2011) to medium-sized New Keynesian models. These additional components become relevant only when trend inflation is above 4% and there is no price indexation.
2.8 Tables and Figures

Table 2.1: Calibration under Sticky Wages

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.02</td>
</tr>
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<td>$\Lambda_z$</td>
<td>Growth rate of the labor augmenting technology</td>
<td>1.0037</td>
</tr>
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<td>$\nu_w$</td>
<td>Calvo parameter for wages</td>
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</tr>
<tr>
<td>$\nu$</td>
<td>Calvo parameter for prices</td>
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</tr>
<tr>
<td>$\iota_p$</td>
<td>Price indexation</td>
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</tr>
<tr>
<td>$\iota_w$</td>
<td>Wage indexation</td>
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</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of substitution between intermediate goods</td>
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</tr>
<tr>
<td>$\eta_w$</td>
<td>Elasticity of substitution between type of workers</td>
<td>21</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of capital on the production of final goods</td>
<td>0.33</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Inverse of the Frisch elasticity of labor supply</td>
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</tr>
<tr>
<td>$\kappa$</td>
<td>Capital adjustment cost</td>
<td>2.48</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Capital utilization cost</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Figure 2.1: Minimum Response to Inflation for Determinacy $\psi_{\pi}^*$. Case 1: NK with capital (no adjustment cost, no capital utilization). Case 2: NK with capital and adjustment cost (no capital utilization). Case 3: NK with capital and capital utilization (no adjustment costs). Case 4: NK with capital, adjustment costs and capital utilization. $\pi$ denotes the annualized trend inflation in percentages. Benchmark: New Keynesian model with heterogeneous labor and without capital accumulation.
Figure 2.2: Long-run Slope of the Phillips Curve
\[ \pi = 0.4\% \]
\[ \pi = 1.6\% \]
\[ \pi = 3.6\% \]
\[ \pi = 5.7\% \]
\[ \pi = 6.9\% \]

**Figure 2.3:** Long-run Effects of Trend Inflation on Price Dispersion
Figure 2.4: Self-fulfilling Investment Boom
Figure 2.5: Minimum Response to Inflation for Determinacy $\psi^*_{\pi}$ and Sticky Wages. The upper panel shows the minimum response to inflation for determinacy $\psi^*_{\pi}$, the bottom panel shows the robustness of determinacy to changes in $\psi^*_{\pi}$. 
Figure 2.6: Long-run Slope of the Phillips Curve
Figure 2.7: Minimum Response to Inflation for Determinacy $\psi_\pi^*$
Figure 2.8: Minimum Response to Inflation for Determinacy $\psi^*_\pi$. Case 1: NK with capital (no adjustment cost, no capital utilization). Case 2: NK with capital and adjustment cost (no capital utilization). Case 3: NK with capital and capital utilization (no adjustment costs). Case 4: NK with capital, adjustment costs and capital utilization. $\pi$ denotes annualized trend inflation in percentages.
Figure 2.9: Minimum Response to Inflation for Determinacy $\psi^*_\pi$. Case 1: NK with capital (no adjustment cost, no capital utilization). Case 2: NK with capital and adjustment cost (no capital utilization). Case 3: NK with capital and capital utilization (no adjustment costs). Case 4: NK with capital, adjustment costs and capital utilization. $\pi$ denotes annualized trend inflation in percentages.
Figure 2.10: Minimum Response to Expected Inflation for Determinacy $\psi_\pi^*$
Appendix A

Appendix to Chapter 1

A.1 Transition Function for Banks

The transition function for survivor banks $\Lambda_s$, and the transition function for newborn banks $\Lambda_n$ are characterized by

$$
\Lambda_s(s, s) = \int_s 1_{\{s_1 \leq s\}} (1 - T_d) \lambda(ds) + \int_s T_d T_k T_b \lambda(ds)
$$

and

$$
\Lambda_n(s, s) = \int_s T_k T_b T_\xi T_\chi \lambda(ds),
$$

where

$$
T_d = \begin{cases} 0 & \text{if } d(s) = 0 \\ 1 & \text{otherwise} \end{cases}, \\
T_k = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases}, \\
T_b = \begin{cases} 1 & \text{if } b = 0 \\ 0 & \text{otherwise} \end{cases}, \\
T_\xi = \begin{cases} 1 & \text{if } \xi = 1 \\ 0 & \text{otherwise} \end{cases}, \text{ and } \\
T_\chi = \begin{cases} 1 & \text{if } \chi = 0 \\ 0 & \text{otherwise} \end{cases}.
$$
s_1 denotes the endogenous components of s, i.e., s_1 = \{s_{11}, s_{12}\} = \{k, b\}, and h_1 denotes the risky asset and debt components of the optimal correspondence h^*.

### A.2 Existence and Uniqueness of the Banks’ Problem

**Assumption 1.** \[ \lim_{T \to \infty} \prod_{s=1}^{T} q(s_{jt+s}, b_{jt+s})b_{jt+T} = 0 \] for all s_{jt+s} and for all b_{jt+s}.

**Proposition 1.** Let q(s, b) ∈ (0, 1) for all s and for all b, and let \( \bar{q} = \max q(s, b) \), if assumption 1 holds, then \( b_{jt} \geq -\bar{k}(1 - \bar{q})^{-1} \).

**Proof to Proposition 1.** Using the banks’ constraints and iterating forward we can solve for the maximum resources that a bank could lend, i.e.

\[
b_{jt} = \sum_{s=0}^{T-1} (\prod_{i=1}^{s} q(s_{jt+i}, b_{jt+1+i})) (\xi_{jt+1+s}k_{jt+s}^\alpha + \bar{c} - \chi_{jt+s}k_{jt+s}) + \prod_{i=1}^{s} q(s_{jt+i}, b_{jt+1+i}) b_{jt+T}.
\]

By assumption 1, as \( T \to \infty \), \( b_t = \sum_{s=0}^{\infty} (\prod_{i=1}^{s} q(s_{jt+i}, b_{jt+1+i})) (\xi_{jt+1+s}k_{jt+s}^\alpha + \bar{c} - \chi_{jt+s}k_{jt+s}) \). Thus, using the constraints of the banks yields \( b_t \geq -\sum_{s=0}^{\infty} (\prod_{i=1}^{s} q(s_{jt+i}, b_{jt+1+i})) \bar{k} = -\frac{\bar{k}}{1-\bar{q}} \).

**Proposition 2.** For each q ∈ Q, there exists a unique \( v^* \in V \) such that \( T(v^*) = v^* \), and for any \( v_0 \in V \), \( \delta(T^n v_0, v^*) \leq \beta^n \delta(v_0, v^*) \) for \( n = 0, 1, \ldots \). Moreover, the optimal policy correspondence \( h^* : S \to B \times K \times D \) is compact-valued and upper hemi-continuous.

**Proof to Proposition 2.** For the first part of the proposition, if we can show that: (i) the set V is a complete metric space under the sup norm \( \delta = ||v|| = \sup_{k_{jt-1}, b_{jt-1}, \xi_{jt}} |v(k_{jt-1}, b_{jt-1}, \xi_{jt})| \), (ii) the vector valued operator \( T \) maps V on V , and (iii) \( T \) is a contraction mapping, then we can apply the continuous mapping theorem, Theorem 3.2 in Stokey and Lucas (1987), to show that there exist a unique \( v^* \in V \) such that \( T(v^*) = v^* \). Note that based on proposition 1 we can redefine B to be a compact set. Let’s begin by proving (i), V is a set of continuous bounded vector-valued functions.
on $\mathcal{K} \times \mathcal{B} \times \Xi$ by assumption, therefore, $\mathcal{V}$ is a complete normed vector space by adaptation of theorem 3.1 in Stokey and Lucas (1987) as in Chatterjee et al. (2007). To show that (ii) holds, note that $u(c_{jt}(k_{jt}, b_{jt}, \xi_{jt}))$ is continuous on $\mathcal{K} \times \mathcal{B} \times \Xi$. Each $u(c_{jt}(k_{jt}, b_{jt}, \xi_{jt}))$ is a two by one vector, where the first component corresponds to the value of banks without an investment opportunity $u^0(c_{jt}(k_{jt}, b_{jt}, \xi_{jt}))$, and the second component corresponds to the value of banks with an investment opportunity $u^1(c_{jt}(k_{jt}, b_{jt}, \xi_{jt}))$. In addition, the vector-valued functions $\beta \rho \mathbb{E}_t \mathbf{v}(k_{jt}, b_{jt}, \xi_{jt+1})$, and $\beta \rho (1 - \rho^d) \mathbb{E}_t \mathbf{v}^d(0, 0, \xi_{jt+1}) + \beta \rho^d \mathbb{E}_t \mathbf{v}(0, 0, \xi_{jt+1})$ are continuous. To show that the expectation operator preserves continuity, we let $\{s_{22jtn}\} \rightarrow \{s_{22jt}\}$, and we write $\mathbb{E}_t \mathbf{v}(s_{jt+1}) = \int_{s_{22jt+1}} \sum_{s_{21jtn}} \mathbf{v}(s_{jtn}) P^x(s_{21jtn}, s_{21jtn+1}) dF^x(s_{22jtn+1} | s_{22jt}) ds_{22jt+1}$, then $||\int_{s_{22jt+1}} \sum_{s_{21jtn}} \mathbf{v}(s_{jtn}) P^x(s_{21jtn}, s_{21jtn+1}) dF^x(s_{22jtn+1} | s_{22jt}) ds_{22jt+1} - \int_{s_{22jt+1}} \sum_{s_{21jtn}} \mathbf{v}(s_{jtn}) P^x(s_{21jtn}, s_{21jtn+1}) dF^x(s_{22jtn+1} | s_{22jtn}) ds_{22jt+1} ||$ converges to zero by the Helly Bray Theorem. To show (iii), let $f$ and $g \in \mathcal{V}$ with $f(k, b, \xi) \leq g(k, b, \xi)$ for all $\{k, b, \xi\} \in \mathcal{K} \times \mathcal{B} \times \Xi$, by the definition of $\mathbb{T}$ it follows that $\mathbb{T}(f)(s) \leq \mathbb{T}(g)(s)$ for all $s \in \mathcal{S}$. Discounting also follows from the definition of $\mathbb{T}$, for any $a > 0$, $\mathbb{T}(f + a)(s) \leq \mathbb{T}(f)(s) + \beta \rho a$ for all $f \in \mathcal{V}$, $s \in \mathcal{S}$. Thus, by the contraction mapping theorem there exists a unique $\mathbf{v}^* \in \mathcal{V}$ such that $\mathbb{T}(\mathbf{v}^*) = \mathbf{v}^*$. In the second part of the proposition, I follow Chatterjee et al. (2007). We need to show that the optimal policy correspondence $h^*$, $h^*(s) = \{ h \in \Gamma \times \mathcal{D} : v^{x_j}(k_j, b_j, \xi_j) = \max_{k \in \mathcal{K}, b \in \mathcal{B}, \xi \in \Xi} v^{x_j}(k_j, b_j, \xi_j) \equiv v^{x_j}(k_j, b_j, \xi_j) \}$, is compact-valued and upper hemicontinuous. First, we show that the correspondence $h^*(s)$ is compact-valued, i.e., $h^*(s)$ is bounded-valued and closed-valued. $h^*(s)$ is bounded-valued because $\mathcal{K}$, $\mathcal{B}$, and $\mathcal{D}$ are compact sets, given prices. To see that $h^*(s)$ is closed-valued, given an arbitrary $s_j$, take any sequence $(k_{jtn}, b_{jtn}, d_{jtn}) \in h^*(s_{jtn})$ converging to $(\bar{k}, \bar{b}, \bar{d})$, where $s_{jtn} \rightarrow s_{jt}$. Since $(k_{jtn}, b_{jtn}, d_{jtn}) \in h^*(s_{jtn})$, $\phi(s_{jtn}, k_{jtn}, b_{jtn}, d_{jtn}) = v^{x_j}(k_{jtn-1}, b_{jtn-1}, \xi_{jt})$, where $\phi$ is
characterized by
\[
\phi(s_{jn}, k_{jn}, b_{jn}, d_{jn}) = \max \{ v^{x_{jn}}(k_{j-1}, b_{j-1}, \xi_{j}) \}
\]
\[
v^{x_{jn}}(k_{j-1}, b_{j-1}, \xi_{j}) = u(c_{j}(s_{jn}, b_{jn}, k_{jn}, d_{jn})) + \beta \rho \mathbb{E}_t v^{x_{j+1}}(b_{jn}, k_{jn}, \xi_{j+1})
\]
\[
v^{x_{jn}}(k_{j-1}, b_{j-1}, \xi_{j}) = u(c_{j}(s_{jn}, b_{jn}, k_{jn}, d_{jn})) + \beta \rho \{ \rho d \mathbb{E}_t v^{x_{j+1}}(0, 0, \xi_{j+1}) + (1 - \rho^d) \mathbb{E}_t v^{x_{j+1}}(0, 0, \xi_{j+1}) \}.
\]

By continuity of \( \phi \), \( (s_{jn}, k_{jn}, b_{jn}, d_{jn}) \rightarrow (\tilde{k}, \tilde{b}, \tilde{d}) \) implies that \( \phi(s_{jn}, k_{jn}, b_{jn}, d_{jn}) \rightarrow \phi(s_{j}, \tilde{k}, \tilde{b}, \tilde{d}) = v^*(s_{j}) \), therefore, \( (\tilde{k}, \tilde{b}, \tilde{d}) \in h^*(s_{j}) \), i.e., \( h^*(s) \) is closed-valued. To see that \( h^*(s) \) is upper hemicontinuous, take any \( s_n \rightarrow s \) and any \( (b_n, k_n, d_n) \in \mathcal{B} \times \mathcal{K} \times \mathcal{D} \) such that \( (b_n, k_n, d_n) \in h^*(s_n) \). Since the correspondence \( h^*(s) \) is compact-valued, there exists a subsequence \( (b_{n_i}, k_{n_i}, d_{n_i}) \) that converges to some \( (\tilde{b}, \tilde{b}, \tilde{d}) \). This implies that for any \( \varepsilon > 0 \), there exists an \( \tilde{n} \) such that \( ||\phi(s_{jn}, k_{jn}, b_{jn}, d_{jn}) - \phi(s_{j\hat{n}}, k_{j\hat{n}}, b_{j\hat{n}}, d_{j\hat{n}})|| < \varepsilon/2 \) for all \( n_t \geq \tilde{n} \). By continuity of \( \phi(s, k, b, d) \) on \( s \), there exists an \( \hat{n} \) such that \( ||\phi(s_{jn}, k_{jn}, b_{jn}, d_{jn}) - \phi(s, \tilde{k}, \tilde{b}, \tilde{d})|| < \varepsilon/2 \) for all \( n_t \geq \hat{n} \). Note that we can write \( ||\phi(s_{jn}, k_{jn}, b_{jn}, d_{jn}) - \phi(s, \tilde{k}, \tilde{b}, \tilde{d})|| = ||\phi(s_{jn}, k_{jn}, b_{jn}, d_{jn}) - \phi(s_{jn}, \tilde{k}, \tilde{b}, \tilde{d}) + \phi(s_{jn}, \tilde{k}, \tilde{b}, \tilde{d}) - \phi(s, \tilde{k}, \tilde{b}, \tilde{d})|| \leq ||\phi(s_{jn}, k_{jn}, b_{jn}, d_{jn}) - \phi(s_{jn}, \tilde{k}, \tilde{b}, \tilde{d})|| + ||\phi(s_{jn}, \tilde{k}, \tilde{b}, \tilde{d}) - \phi(s, \tilde{k}, \tilde{b}, \tilde{d})|| < \varepsilon \), for all \( n_t \geq \max\{\tilde{n}, \hat{n}\} \). Hence, \( (\tilde{k}, \tilde{b}, \tilde{d}) \in h^*(s) \), i.e. the correspondence is upper-hemicontinuous. 

A.3 Existence and Uniqueness of the Stationary Distribution

Proposition 3. For each \( q \in Q \), there is a measurable selection from \( h : S \rightarrow \mathcal{B} \times \mathcal{K} \times \mathcal{D} \).


Proposition 4. For each \( q \in Q \), and any measurable selection from \( h : S \rightarrow \mathcal{B} \times \mathcal{K} \times \mathcal{D} \) there is a unique \( \lambda^* \in \Lambda(S, 2^S) \) such that
\[
\lambda(s) = T \lambda(s).
\]

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Proof to Proposition 4. Following Chatterjee et al. (2007), we need to verify that the conditions of theorem 11.10 in Stokey and Lucas (1989) are satisfied.

Condition D in Stokey and Lucas (1989) is satisfied: By exercise 11.4 in Stokey and Lucas (1989) we can focus on \( \Lambda_n \). Take an integer \( N = 1 \), and \( \varepsilon < \frac{1}{2} \). Since \( A \) is independent of \( s \), we can write \( \Lambda_n(s, A) = \phi(A) \). Thus, \( \phi(A) \leq \varepsilon \) implies \( \Lambda_n(s, A) \leq 1 - \varepsilon \).

If \( A \) is any set of positive measure \( \phi \), for each \( s \in \mathcal{S} \), \( \exists N \geq 1 \) such that \( \Lambda_n^N(s, A) > 0 \):

Take any \( A \) with positive measure \( \phi(A) > 0 \), this implies that \( \Lambda_n(s, A) > 0 \) for all \( s \in \mathcal{S} \), so that \( \Lambda(s, A) > 0 \).

\( \square \)

A.4 Interbank Credit Spreads and Real Economic Activity

In this section, I examine the link between the interbank credit spreads and real economic activity. Motivated by Gilchrist and Zakrajišek (2012), I run a structural vector autoregression (SVAR) with one lag, selected by the BIC criterium, using eight economic variables in the following order: consumption, investment, output, inflation, interbank spread, stock market excess returns, 10 year treasury bill yields, and the federal funds rate. This is the same ordering as Gilchrist and Zakrajišek (2012) except for the excess bond premium which I replace by the interbank credit spread. The model is estimated over the period 1987q1-2012q1. I choose the first quarter of 1987 as the starting point of the sample because in 1987 the Chicago Mercantile Exchange (CME) closes the Certificate of Deposits (CD) futures trading, and it becomes clear that the Libor is the interest rate of reference in the U.S. money markets, e.g. Burghardt (2003).

Consumption is the log difference of Real Personal Consumption Expenditures, PCECC96, investment is the log difference of Real Gross Private Domestic Investment, GPDIC96, real output is the log difference of the Real Gross Domestic Product, GDPC96, inflation is the log difference of the Gross Domestic Product: Implicit Price
Deflator GDPDEF. The date for these variables is from FRED at the Federal Reserve of Saint Louis. The data for the average interbank credit spread corresponds to the Libor-OIS rate and is from Bloomberg. Since the data for the OIS is only available since the fourth quarter of 2001, I used the average of the effective federal funds rate for the previous quarters given that this measure has historically been close to the OIS rate. The stock market excess returns are from Kenneth French’s website. The data for the 10 year treasury bill rate, DSG10, and the effective federal funds rate, DFF, are from FRED. All variables are expressed in percentage and annualized.

Figure 1.4 shows the impulse response function of real GDP to a one pp shock in the Libor-OIS spread. We can observe that one pp shock to the Libor-OIS spread generates a decrease in real GDP of approximately 2 percent after one year. The 95 percent confidence bands for the impulse response functions are computed using a bootstrap procedure for the residuals.

Finally, the interbank spread also contains information about future economic activity. I run forecast regressions using two specifications. In the first specification, I regress the average change in GDP between period t-1 and t+1 on the term spread at time t, the realized real federal funds rate at time t, and the Libor-OIS at time t. In the second specification, I add the Gilchrist and Zakrajšek (2012) corporate credit spread to the previous regression. The data is from 1987q1 to 2012q1. The real federal funds rate, and the Libor-OIS spread corresponds to the daily average of each quarter. Table 1.8 shows the results. In both specifications, the coefficients on the Libor-OIS are statistically significant indicating that changes in the interbank credit spread affect future real economic activity.
Appendix B
Appendix to Chapter 2

B.1 Households

In the economy there is a continuum of households indexed by $j$ that consume, save in risk-free assets (government bonds), buy or sell state contingent claims, invest in physical capital, and supply labor and capital services to intermediate goods producers. The households obtain positive utility from consumption and negative utility from hours worked. Each household chooses consumption $C_j$, hours worked $H_{jt}$, state continent claims $a_{jt}$, investment in physical capital $X_{jt}$, capital utilization $U_{jt}$, next period capital $K_{jt}$, and government bonds $B_{jt}$ in order to maximize the discounted value of the future streams of utility

$$
\max_{\{C_{jt}, H_{jt}, a_{jt}, X_{jt}, U_{jt}, K_{jt}, B_{jt}\}_{t=0}^{\infty}} E_t \sum_{t=0}^{\infty} \beta^t \left( \log(C_{jt}) - \frac{H_{jt}^{1+\tau}}{1+\tau} \right)
$$

subject to her budget constraint, and the law of motion for capital, that is,
\[ C_{jt} + X_{jt} + \sum_{s^{t+1} \in S^{t+1}} q_{t+1,t} a_{jt} (s^{t+1}) + \frac{B_{jt}}{P_t} + \frac{T_t}{P_t} \]

\[ = a_{jt-1}(s^t) + \frac{B_{jt-1} R_{t-1}}{P_t} + (r_t U_{jt} - a ((U_{jt}))) K_{jt-1} + W_{jt} H_{jt} + \frac{F_{jt}}{P_t} \]

and

\[ K_{jt} = (1 - \delta) K_{jt-1} + \left(1 - S \left[ \frac{X_{jt}}{X_{jt-1}} \right] \right) X_{jt}. \]

where \( q_{t+1,t} \) denotes the price of the state continent claims \( a_{jt} \), and the remaining notation is as described in section 2.2 with the addition of the index \( j \). The FOC condition with respect to labor takes into account the structure of the labor market. Specifically, each household supplies labor \( H_{jt} \) to a representative competitive firm, a labor packer, that transforms the heterogenous labor into a composite of labor \( L^d_t \) that is sold to the intermediate good producers. The households have market power and set their wages conditional on the demand for their labor type and conditional on a scheme à la Calvo, i.e., each period a fraction \( 1 - \nu_w \) of households are able to set wages, and a fraction \( \nu_w \) sets \( W_{jt+s} = \prod_{s=t}^{s=t} \frac{W_{jt+s}}{W_{jt+s-1}} \) \( W_{jt} \), where \( \nu_w \) denotes the indexation of wages to past inflation.

**B.2 Labor Packer**

The labor packer chooses \( H_{jt} \) in order to maximize profits,

\[ \max_{H_{jt}} W_{jt} L^d_t - \int_0^1 W_{jt} H_{jt}dj \]

subject to
\[ L_t^d = \left[ \int_0^1 H_{jt}^{\eta_w - 1} \, dj \right]^{\eta_w - 1}, \]

where \( \eta_w \) denotes the elasticity of substitution between labor types.
Bibliography


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Biography

My name is Jonas Eduardo Arias. I was born on March 9, 1984 in Villa de Merlo, Argentina. I did my undergraduate studies in Economics at Universidad Nacional de Córdoba and my graduate studies in Economics at Duke University.