Essays on Costly Charitable Fund-raising

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Economics in the Graduate School of Duke University

2013
Abstract

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Abstract

In this dissertation I present a theory of charitable fund-raising in which it is costly to solicit donors. The second chapter shows how optimizing fund-raisers will affect the equilibrium level of contributions, determine the set of givers, respond to government grants, and behave in the limit in replicator economies. The third chapter characterizes optimal fund-raising when the fund-raiser learns to become a more efficient solicitor through experience. This chapter also introduces a notion of excessive fund-raising and it shows how this is affected by learning.
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I owe special thanks to my advisor Huseyin Yildirim for his advice and for coauthoring chapter 2 with me. I also thank my committee members Timur Kuran, Leslie Marx and Tom Nechyba for helpful conversations.
In the second chapter of this dissertation Huseyin Yildirim and I provide a full characterization of the optimal solicitation strategy that maximizes donations net of fundraising costs. Optimal fund-raising dictates that the fund-raiser targets only the "net contributors" – those individuals whose equilibrium contributions exceed their solicitation costs. This chapter shows that as the income inequality increases, so does the level of the public good, despite a (potentially) non-monotonic fund-raising effort. This implies that costly fund-raising can provide a novel explanation for the non-neutrality of income redistributions and government grants often found in empirical studies. This chapter also shows that in large economies, only the "most willing" donors are solicited; and the average donation converges to the solicitation cost of these donors, which is strictly positive. The third chapter incorporates fund-raising technology into the theory of charitable giving. It specifically considers the case in which fund-raisers learn to become more efficient solicitors through fund-raising experience. The optimal solicitation strategy identifies a fund-raiser incentive to invest in learning in the form of soliciting some early donors who would give less than their solicitation costs. By defining a notion of "excessive" fund-raising, it is
shown that is higher under learning. This chapter also considers a cost structure entailing overhead costs. It is shown that the public good provision is lower the higher is the fixed cost despite an increase in fund-raising efforts as well as in gross contributions.
A Theory of Charitable Fund-raising with Costly Solicitations

2.1 Introduction

Charitable fund-raising\(^1\) is a costly endeavor. Andreoni and Payne (2003) and Andreoni and Payne (2011) indicate that an average charity spends 5 to 25 percent of its donations on fund-raising activities, including direct mailing, telemarketing, face-to-face solicitations, and staffing.\(^2\) For instance, every year more than 115,000 nonprofit organizations hire fund-raising staff and consultants, paying them 2 billion dollars Kelly (1998).\(^3\) It is thus strongly believed that both donors and charities dislike fund-raising, but view it to be a “necessary evil” for the greater good: fund-raising diverts resources away from charitable services while informing, or otherwise

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\(^1\) Charitable sector is a significant part of the U.S. economy. For instance, in 2008, total donations amounted to $307 billion. $229 billion of this total came from individuals, corresponding to 1.61% GDP (\textit{Giving USA}, 2009). See Andreoni (2006b) and List (2011) for an overview of this sector and the literature.

\(^2\) Various watchdog groups such as \textit{BBB Wise Giving Alliance} and \textit{Charity Navigator} regularly post these cost-to-donation percentages for thousands of charities in the U.S. They often recommend a benchmark of 30-35 percent for a well-run charity.

\(^3\) The estimated number of paid workers employed by nonprofits organizations in 2004 was 9.4 million, which is more than 7% of the U.S. workforce (Sherlock and Gravelle, 2009).
persuading, donors of the cause and fund-drive. Despite its significance, however, fund-raising cost has not been fully incorporated into the theory of charitable giving. This is the gap we aim to fill in this paper, and in doing so offer a new (and complementary) theory of charitable fund-raising.

Our formal setup adds an “active” fund-raiser to the “standard” model of giving in which donors care only about their private consumptions and the total supply of the public good. In particular, unlike the standard model, we assume that each donor becomes aware of the charitable fund-drive only if solicited by the fund-raiser. The solicitation is, however, costly. Our first observation is that the charity will contact an individual if he is expected to give more than his respective solicitation cost, or become a “net contributor” in equilibrium. We then show that identifying these net contributors in our model is equivalent to identifying the contributors in the standard model (without fund-raising cost) except that each donor’s wealth is reduced by his solicitation cost. This important equivalence allows us to appeal to Andreoni and McGuire’s (1993) elegant algorithm to solve for the latter. Our characterization of the optimal fund-raising strategy is simple because it does not require any equilibrium computation. More importantly, it pinpoints the exact set of donors to be targeted based on their preferences, incomes, and solicitation costs.

Using our characterization, we next address two policy-related issues, one about income redistribution and the other about government grants. When individuals differ only in their incomes, we show that the fund-raising strategy reduces to as-

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4 See, e.g., Warr (1983), Roberts (1987), Bergstrom et al. (1986), and Andreoni (1988).

5 In addition, donors and fundraisers often report that one of the most effective fundraising techniques is directly asking people. See Andreoni (2006b) for a discussion; and Yörük (2009), and Meer and Rosen (2011) for empirical evidence.

6 This is consistent with the fact that fundraising professionals often recommend a careful study of donor base for an effective campaign (Kelly 1998). For instance, several software companies such as DonorPerfect (www.donorperfect.com), DonorSearch (http://donorsearch.net), and Target Analytics (www.blackbaud.com/targetanalytics) compile donor databases and sell them to charities along with programs to identify the prospect donors.
signing each individual a cutoff cost of solicitation that depends on the incomes of others richer than him. Intuitively, with only the income heterogeneity, the charity considers contacting first the richest donor; and once this donor is in the “game”, the charity becomes more conservative about contacting the second richest donor due to the free-riding incentive, which depends on their income difference. Iteratively applied, this logic implies that unlike the well-known neutrality result predicted by the standard theory (e.g., Warr (1983), and Bergstrom et al. (1986), an income redistribution is likely to affect the fund-raising strategy and thus the provision of the public good. In particular, as the income distribution becomes more unequal in the sense of Lorenz dominance (defined below), we find that the level of the public good strictly increases in the presence of costly fund-raising despite a non-monotonic fund-raising effort. Such non-neutrality of the public good provision also manifests itself in response to a government grant to the charity. We show that a more generous grant partially crowds out fund-raising effort, leaving some donations unrealized, as well as reducing the amount of the realized donations. The importance of this additional fund-raising channel for crowding-out has been recently evidenced by Andreoni and Payne (2003) and Andreoni and Payne (2011).

Given that many charities have a large donor base, we also investigate optimal fund-raising in replica economies. We show that in a sufficiently large economy, only the donors who like the public good “the most” are contacted, whose identity jointly depends on preference, income, and solicitation cost. Thus, even in a large economy, it is not necessarily the highest income and/or the lowest solicitation cost donors who will be contacted; rather it is a combination of all the three attributes that will define the fund-raiser’s strategy. In particular, while the public good level (net of fund-raising costs) converges to a finite level, the average donation converges to the respective solicitation cost, which, unlike in the standard model, is strictly positive.

Aside from the papers mentioned above, our work relates to a relatively small
theoretical literature on strategic fund-raising as a means of: providing prestige to
donors (Glazer and Konrad (1996), Harbaugh (1998), and Romano and Yildirim
(2001), signaling the project quality Vesterlund (2003), and Andreoni (2006a)), and
organizing lotteries Morgan (2000). Our work is more closely related to the models of
strategic fund-raising to overcome zero-contribution equilibrium under a non-convex
production either by securing seed money Andreoni (1998), or by collecting donations
in piece-meals Marx and Matthews (2000). None of these papers, however, consider.endogenous, costly solicitations.

Our work is most closely related to Rose-Ackerman (1982) and Andreoni and
Payne (2003). Rose-Ackerman is the first to build a model of costly fund-raising in
which donors, as in ours, are unaware of a charity until they receive a solicitation
letter. She, however, does not construct donors’ responses from an equilibrium play.
Andreoni and Payne (2003) endogenize both the fund-raiser and donors’ responses
as in our model, but they view solicitation letters to be randomly distributed. Their
main theoretical result is that a government grant may discourage fund-raising, which
is in line with one of our results. Unlike them, we fully characterize the optimal strat-
egy that involves targeted solicitations, and provide a richer set of results, regarding
the non-neutrality and large economies.

In addition to the theoretical literature, there is a more extensive empirical and
experimental literature on charitable giving, which we will refer to as our analysis
progresses below. For recent surveys of the literature, see the reviews by Andreoni
(2006b) and List (2011).

The rest of the paper is organized as follows. In the next section, we set up the
basic model. In Section 3, we characterize the optimal fund-raising strategy as a
modified Andreoni-McGuire algorithm. In Sections 4 and 5, we consider income dis-
btributions and government grants, respectively. We examine large replica economies
in Section 6. We present the extensions in Section 7, and conclude in Section 8.
2.2 Model

Our basic setup extends the standard model for private provision of public goods (e.g., Bergstrom, Blume, and Varian 1986), which we briefly review before introducing fund-raising costs.

**Standard Model.** There is a set of individuals, \( N = \{1, \ldots, n\} \), who each allocates his wealth, \( w_i > 0 \), between a private good consumption, \( x_i \geq 0 \), and a gift to the public good or charity, \( g_i \geq 0 \). Units are normalized so that \( x_i + g_i = w_i \). Letting \( G = \sum_{i \in N} g_i \) be the supply of the public good, individual \( i \)'s preference is represented by the utility function \( u_i(x_i, G) \), which is strictly increasing, strictly quasi-concave, and twice differentiable. Individual \( i \)'s (Marshallian) demand for the public good, denoted by \( f_i(w) \), satisfies the strict normality: \( 0 < f'_i(w) \leq \theta < 1 \) for some parameter \( \theta \). The equilibrium gifts, \( \{g^*_1, \ldots, g^*_n\} \), are made simultaneously (without observing others); and under strict normality, there is a unique Nash equilibrium. We further assume that \( f_i(0) = 0 \) for all \( i \) so that \( G^* > 0 \).

**Costly Fund-raising.** Since everyone is already in the “contribution game”, there is no role for (strategic) fund-raising in the standard model. Similar to Rose-Ackerman (1982), and Andreoni and Payne (2003), we assume that person \( i \) enters the game and considers giving only if solicited by the fund-raiser. Doing so, however, costs \( c_i > 0 \) to the fund-raiser in the form of telemarketing, direct mails, or door-to-

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7 The existence of parameter \( \theta \) is not essential to our analysis but eases it by ensuring a finite \( G^*_i \) below. It is also commonly assumed in the literature (e.g., Andreoni 1988; Fries, Golding, and Romano 1991).

8 Alternatively, absent fund-raising costs, the fund-raiser would trivially ask everyone for donations since the equilibrium provision can never decrease by including a new donor (Andreoni and McGuire (1993)).

9 We envision that the charity organizes occasional fund-raising campaigns, and a solicitation, much like advertising, informs the donor of the current one. Alternatively, the donor may procrastinate in giving (O’Donoghue and Rabin (1999)), and this procrastination is minimized by the fund-raiser’s asking. See Yörük (2009), and Meer and Rosen (2011) for evidence on “the power of asking”. We, however, do not allow the fund-raiser to “pressure” people to give. In particular, a solicited person can choose not to give, though this is unlikely to occur in equilibrium.
door visits. Consider first person \( i \)'s solo decision to cover the entire fund-raising cost, \( C \). Note that person \( i \) would receive utility \( u_i(w_i, 0) \), if he contributed nothing. Otherwise, he would have to choose \( g_i \geq C \) to maximize \( u_i(w_i - g_i, g_i - C) \). Let \( V_i(w_i - C) \) be \( i \)'s indirect utility in the latter case, which is increasing in the (net) income. For \( C = 0 \), clearly \( V_i(w_i) > u_i(w_i, 0) \) because \( f_i(w_i) > 0 \), whereas for \( C = w_i \), we have \( V_i(0) \leq u_i(w_i, 0) \). Hence, there is a unique cutoff cost, \( \hat{C}_i \in (0, w_i] \) such that when alone, person \( i \) would consume some public good if and only if \( C < \hat{C}_i \).\(^{11}\) For simplicity, we assume that \( c_i \) is not too large; in particular \( c_i < \hat{C}_i \).

Let \( F \subseteq N \) be the set of donors contacted by the fund-raiser, or the fund-raiser set. In the basic model, we assume that the fund-raiser set is commonly known by the contacted donors, though we relax this assumption in Section 3.2.\(^{12}\) As in the standard setup, let \( g^*_i(F) \) be donor \( i \)'s equilibrium gift engendered by the simultaneous play in \( F \). Then, the total fund-raising cost and the gross donations are defined, respectively, by \( C(F) = \sum_{i \in F} c_i \) and \( G^*(F) = \sum_{i \in F} g^*_i(F) \), where \( C(\emptyset) = 0 \) and \( g^*_i(\emptyset) = 0 \) by convention.\(^{13}\) The charity chooses \( F \) that maximizes the supply of the public good (or net revenues):

\[
\overline{G}^*(F) = \max \{ G^*(F) - C(F), 0 \} .
\] (2.1)

Eq. (2.1) implies that if insufficient funds are received to cover the cost, then no public good is provided.\(^{14}\) We assume that when indifferent between two sets, the

---

\(^{10}\) While, to break the fund-raiser’s indifference, we do not allow for \( c_i = 0 \) (e.g., a repeat donor), our results do extend to this possibility up to a trivial non-uniqueness in the fundraiser’s strategy.

\(^{11}\) For the CES utility: \( u_i = \left( x_i^{\rho_i} + (\overline{G})^{\rho_i} \right)^{1/\rho_i} \), with \( \rho_i < 1 \), it is easily verified that \( \hat{C}_i = [1 - (1/2) \frac{1}{\rho_i}]w_i \) for \( \rho_i \in (0, 1) \), and \( \hat{C}_i = w_i \) for \( \rho_i \leq 0 \) (including the Cobb-Douglas specification at \( \rho_i = 0 \)).

\(^{12}\) The fact that prior to giving, donors may know the fund-raiser set is not completely unrealistic. For instance, universities often organize alumni re-unions and fund-raising events where contacted donors meet each other.

\(^{13}\) In the next chapter we include a fixed setup cost of fund-raising.

\(^{14}\) We assume that donations are not refunded in the case of a failed fundraising, or they are used for other causes that donors do not care about.
charity strictly prefers the one with the lower fund-raising cost. Our solution concept is subgame perfect Nash equilibrium.

2.3 Optimal Fund-raising

2.3.1 Characterization

When fund-raising entails significant costs, a carefully planned strategy of who to ask for donations seems to be of utmost importance both to control the expenses and to encourage giving. We illustrate this point with a numerical example, which also motivates our subsequent analysis.

**Example 1.** Let $N = \{1, 2, 3\}$ and $u_i = x_i^{1-\alpha}(G)^{\alpha}$, with $\alpha = 0.3$. Individuals’ wealth and solicitation costs are such that $(w_1, w_2, w_3) = (18, 18, 20)$ and $(c_1, c_2, c_3) = (0.01, 4, 6.9)$. The following table reports donor equilibrium, and highlights the optimal fund-raiser set.

Table 2.1: Donor Equilibrium for each Fund-raiser Set

<table>
<thead>
<tr>
<th>$F$</th>
<th>$g_1^*(F)$</th>
<th>$g_2^*(F)$</th>
<th>$g_3^*(F)$</th>
<th>$G^*(F)$</th>
<th>$C(F)$</th>
<th>$\bar{G}^*(F)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1}</td>
<td>5.41</td>
<td>-</td>
<td>-</td>
<td>5.41</td>
<td>0.01</td>
<td>5.40</td>
</tr>
<tr>
<td>{2}</td>
<td>-</td>
<td>8.2</td>
<td>-</td>
<td>8.2</td>
<td>4</td>
<td>4.2</td>
</tr>
<tr>
<td>{3}</td>
<td>-</td>
<td>-</td>
<td>10.83</td>
<td>10.83</td>
<td>6.9</td>
<td>3.93</td>
</tr>
<tr>
<td>{1, 2}</td>
<td>4.83</td>
<td>4.83</td>
<td>-</td>
<td>9.66</td>
<td>4.01</td>
<td>5.65</td>
</tr>
<tr>
<td>{1, 3}</td>
<td>5.20</td>
<td>-</td>
<td>7.20</td>
<td>12.40</td>
<td>6.91</td>
<td>5.49</td>
</tr>
<tr>
<td>{2, 3}</td>
<td>-</td>
<td>6.84</td>
<td>8.84</td>
<td>15.68</td>
<td>10.9</td>
<td>4.78</td>
</tr>
<tr>
<td>{1, 2, 3}</td>
<td>4.85</td>
<td>4.85</td>
<td>6.85</td>
<td>16.55</td>
<td>10.91</td>
<td>5.64</td>
</tr>
</tbody>
</table>

Table 2.1 reveals that it is optimal to contact only the donors 1 and 2. Donor 3 is not included in the set because of his high solicitation cost even though he would give the most. It also reveals that with the optimal solicitations, the $C/G$ ratio is 41%, which is not the lowest. Finally, it is clear that even with three donors, a direct approach to identifying the optimal fund-raiser set promises to be computationally
demanding. In the next section, we make some key observations about the optimal fund-raising strategy and derive a simple algorithm to find it.

Our first observation is that although donors may end up contributing nothing for an arbitrary fund-raiser set, the same cannot happen if the set is optimally chosen.\textsuperscript{15}

**Observation.** In a fund-raising equilibrium, \( F^o \neq \emptyset \) if and only if \( \mathbb{G}^o(F^o) > 0 \).

Thus, in a world of complete information, an optimizing charity would never start fund-raising if it did not expect that donations would exceed the cost. This means that in our model, the charity can fail to provide the public good despite fund-raising only because it suboptimally sets the fund-raising strategy.\textsuperscript{16}

While enlightening, the previous observation does not inform us about the composition of individual contributions. As hinted by Example 1, the optimal fund-raiser set is likely to depend on this composition. The following result offers some significant insights in this direction.

**Lemma 1.** If \( F^o \neq \emptyset \), then it is unique and exactly identified by these two conditions:

(C1) every individual \( i \) in \( F^o \) is a “net contributor” in the sense that \( g^o_i(F^o) - c_i > 0 \);

and

(C2) any individual \( i \) outside \( F^o \) would be a “net free-rider” if added to \( F^o \), in the sense that \( g^o_i(F^o \cup \{i\}) - c_i \leq 0 \).

Lemma 1 indicates that the charity will contact person \( i \) if he is expected to give more than his own cost in equilibrium. As such, the charity classifies donors

\textsuperscript{15} In our model the fundraising cost introduces a threshold to the public good provision as in Andreoni (1998). Unlike his model, however, the provision point in ours will be endogenous to fundraising strategy as opposed to being a capital requirement.

\textsuperscript{16} As noted in the Introduction, charities spend billions of dollars on professional fundraisers to presumably have a well-planned fund-drive. For instance, the Association of Fundraising Professionals (AFP) represent 30,000 such fundraisers.
as *net* contributors and *net* free-riders with respect to their solicitation costs even though there is no explicit cost-sharing agreement among them. The equilibrium gifts exhibit some (implicit) cost-sharing due simply to the charity’s decision of who to solicit. Note, however, that net contributors and net free-riders are defined in equilibrium. Thus, if the fund-drive is expected to be too costly, it is possible that donors give nothing, and in turn no fund-drive begins, $F^o = \emptyset$.

Lemma 1 essentially offers an algorithm to determine the optimal set. Consider Table 1 above. We see that $F^o \neq \{1\}$ because if included in this set, person 2 would also be a net contributor ($4.83 - 4 > 0$). $F^o \neq \{1, 2, 3\}$ either; because person 3 would be a net free-rider ($6.85 - 6.9 \leq 0$). As a result, $F^o = \{1, 2\}$.

Nevertheless, Lemma 1 is not a full characterization of the optimal strategy because it involves equilibrium choices. It does, however, point out that the optimal strategy should exactly identify the set of net contributors, or equivalently the set of net free-riders. A similar identification problem would arise in the standard model if one were to detect the (pure) free-riders. For that case, Andreoni and McGuire (1993) offer an elegant algorithm that does not require equilibrium calculation for each subset of donors. Although our point of investigation here is very different from theirs, we draw a connection owing to Lemma 1.

When finding the optimal set, it is clear from Lemma 1 that the fund-raiser can imagine each individual $i$ tentatively paying for $c_i$. Then, the optimal set problem reduces to “identifying the net free-riders” with the residual incomes, $w_i - c_i$, by using Andreoni and McGuire algorithm. Let $G^0_i > 0$ be the “drop-out” level of the public good for person $i$, which, given that $0 < f'_i \leq \theta < 1$ and $f_i(0) = 0$, uniquely

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17 This is simply the subgame perfection argument. Some fundraising may never start because, given the cost, the fundraiser believes the donors would play the zero equilibrium. In fact, given our convention that $g^*(\emptyset) = 0$, Proposition 2 is also consistent with $F^o = \emptyset$: C1 would trivially hold while C2 would imply that everyone would be a net free-rider, resulting in $F^o = \emptyset$.

18 Note that $w_i - c_i > 0$ because, by assumption, $c_i < \hat{C}_i$. In general, any donor with $w_i - c_i \leq 0$ would automatically be excluded from the fundraiser set.
solves
\[ f_i(w_i - c_i + G_i^0) = G_i^0. \] (2.2)

One interpretation of \( G_i^0 \) in our context is that person \( i \) becomes a net contributor if and only if the sum of others’ net contributions stays below \( G_i^0 \). Without loss of generality, index individuals in a descending order of their dropout levels: \( G_1^0 \geq G_2^0 \geq \ldots \geq G_n^0 \). Next, define

\[ \Phi_i(G) = \sum_{j=1}^{i} (\phi_j(G) - G) + G, \]

where \( \phi_j = f_j^{-1} \) (inverse demand), \( \phi_j > 1 \), and thus \( \Phi'_i(G) > 0 \). The following result fully characterizes the optimal fund-raising strategy.

**Proposition 1.** Define \( \Delta_i = \Phi_i(G_i^0) - \sum_{j=1}^{i} (w_j - c_j) \). Then, we have \( \Delta_1 \geq \Delta_2 \geq \ldots \geq \Delta_n \), with \( \Delta_1 > 0 \). Moreover, letting \( k \in N \) be the largest number such that \( \Delta_k > 0 \), the optimal fund-raiser set is \( F^* = \{1, \ldots, k\} \). This set generates the public good, \( \overline{G}^* = \Phi_k^{-1}(\sum_{j=1}^{k} (w_j - c_j)) \).

To understand how the optimal strategy works, note that \( \Delta_i \) can be interpreted as a measure of person \( i \)'s incentive to pay for his solicitation cost. In particular, as in Bergstrom et al. (1986), \( \Phi_i(G) \) is the minimum level of total wealth needed to sustain public good \( G \) as an equilibrium among agents, \( 1, \ldots, i \). This means that if the actual total wealth available to these agents is strictly less than \( \Phi_i(G_i^0) \), namely \( \Delta_i > 0 \), then the dropout value of person \( i \), \( G_i^0 \), cannot be reached, making him a net contributor and thus a candidate for the fund-raiser set. Given \( \Phi'_i(G) > 0 \) by the strict normality, these incentives are monotonic in that \( \Delta_i \geq \Delta_{i+1} \), and therefore, the fund-raiser considers the largest set of individuals with a positive incentive. This set will, however, be optimal if, given the total fund-raising cost, \( \sum_{j=1}^{k} c_j \), incurred, each individual decides to contribute rather than consume only the private good;
i.e., if, in equilibrium, his net cost, \( \sum_{j=1}^{k} c_j - G_{i}^{*} \), is strictly less than his cutoff, \( \hat{C}_i \). Since everyone else in the set is expected to give more than his solicitation cost, this net cost cannot exceed one’s own cost, which, by Assumption, is less than his cutoff, \( \hat{C}_i \). As a point of reference, it is worth observing that if the charity could force each contacted donor to at least pay for his solicitation cost, then there would be no incentive constraint, \( \Delta_k > 0 \), because no donor would be able to give less than his cost. This means that providing donors with the incentives to be net contributors is the reason why some in the population may not be solicited in our model.

The optimal fund-raising strategy in Proposition 1 is easy to apply given that it does not require any equilibrium computation. Moreover, it ends in at most \( n \) steps, which is often much smaller than the number of all donor subsets, \( 2^n - 1 \). Re-consider Example 1 above. From eq. (2.2), it is easily verified that \( G_1^0 = 7.71 \), \( G_2^0 = 6.0 \), and \( G_3^0 = 5.61 \). Using these, we find that \( \Delta_1 = 7.71 \), \( \Delta_2 = 2.01 \), and \( \Delta_3 = -0.17 \), which implies that \( F^o = \{1, 2\} \), as previously observed.

The optimal fund-raising strategy also has some intuitive comparative statics. Since eq. (2.2) implies that all else equal, \( G_i^0 \) is higher (1) the richer the person; (2) the greater his demand for the public good;\(^{19}\) and/or (3) the lower his solicitation cost, the fund-raiser is more likely to contact such a person. This is consistent with the anecdotal evidence that schools often exclusively solicit alumnus and parents; religious organizations first target their members; and health charities primarily ask former patients and their families for donations.

Proposition 1 also raises an interesting fund-raising question when donors’ preferences and incomes are negatively correlated. Consider, for instance, two individuals, \( a \) and \( b \), with the Cobb-Douglas utilities, and let the solicitation cost be \( c \) for each. Then, \( G_i^0 = \beta_i (w_i - c) \) where \( \beta_i = \frac{\alpha_i}{1-\alpha_i} \). Suppose that \( \beta_a > \beta_b \) but \( w_a < w_b < \frac{\beta_a}{\beta_b} w_a \).

\(^{19}\) Formally, person \( i \) has a greater demand for the public good than \( j \) if \( f_i(w) \geq f_j(w) \) for all \( w > 0 \).
It can be verified that there is some $c^* > 0$ such that $G^0_a > G^0_b$ for $c < c^*$, and $G^0_a < G^0_b$ for $c > c^*$. That is, while for small costs, the higher preference individual is more likely to be solicited than the richer one, the order switches for large costs. The reason is that the fund-raising cost has a direct income effect, which is larger for the higher preference individual.

2.3.2 Unobservability of the Fund-raiser Set

While our assumption that the fund-raiser set is observable to donors is reasonable in some settings, it may be less so in others. In particular, it may be difficult or infeasible for donors to monitor the charity’s solicitations, in which case they can only hold beliefs about them. Given the unique optimal set $F^o$, one natural belief system is as follows: if a donor in $F^o$ is contacted, he learns about the fund-drive and believes that the rest of $F^o$ will also be contacted, whereas if a donor outside $F^o$ is contacted, he attributes this to a mistake and believes that he is the only one contacted besides $F^o$.\footnote{These beliefs are similar to “passive” beliefs often used in bilateral contracting in which one party privately contracts with several others (e.g., Cremer and Riordan (1987); McAfee and Schwartz (1994)). One justification for such beliefs in our context is that the fund-raiser assigns a different staff member to contact different donors so that mistakes are perceived to be uncorrelated.} Under these beliefs, the following result shows that the unobservability of the fund-raiser set is of no consequence in equilibrium.

**Proposition 2.** Suppose that the fund-raiser set is unobservable to donors. Then, under the beliefs described above, $F^o$ is sustained as a perfect Bayesian equilibrium.

Proposition 2 mainly obtains from Lemma 1, and says that the fund-raiser does not necessarily have a commitment problem about its targeting strategy.

Armed with the optimal fund-raiser behavior, we next address two policy-related issues, the first one being the role of an income redistribution.
2.4 Income Redistribution and Non-neutrality

Suppose that individuals differ only in incomes, namely \( c_i = c \) and \( u_i = u \). Without loss of generality, rank incomes as \( w_1 \geq w_2 \geq \ldots \geq w_n \), which, from eq.(2.2), implies that \( G^0_1 \geq G^0_2 \geq \ldots \geq G^0_n \). Applying Proposition 1, the fund-raising strategy then simplifies to a cutoff solicitation cost for each donor.

**Lemma 2.** Let \( \overline{\phi}(G) \equiv \phi(G) - G \), and donor \( i \)'s cost cutoff be given by

\[
\overline{c}_i = w_i - \overline{\phi} \left( \sum_{j=1}^{i} (w_j - w_i) \right).
\] (2.3)

Then, \( \overline{c}_1 \geq \overline{c}_2 \geq \ldots \geq \overline{c}_n \), and \( F^0 = \{ i \in N | c < \overline{c}_i \} \).

In general, since \( \overline{\phi}(G) > 0 \) by the strict normality, the cutoff cost in (2.3) is strictly less than one’s income except for the richest; and the gap increases for lower income individuals.\(^{21}\) The reason is that for a given \( c \), the charity first contacts the richest person, and upon informing him of the fund-drive, the charity becomes more conservative in contacting the second richest person to alleviate the free-rider problem, which is a function of their wealth difference. Applied iteratively, this logic explains why person \( i \)'s cutoff in (2.3) is decreasing in the sum of wealth differences between him and the wealthier others. One important implication of this observation is that a redistribution of income is likely to affect the fund-raising strategy and thus the equilibrium provision of the public good.

As first observed by Warr (1983), if the set of contributors and their total wealth do not change by an income redistribution, then neither does the level of the public good in the standard model of giving.\(^{22}\) This striking theoretical prediction has,

\(^{21}\) In fact, \( \overline{c}_i < 0 \) is possible and signifies that \( i \) would contribute nothing if solicited.

\(^{22}\) Subsequent work showed the robustness of this result with varying generality. See, e.g., Bergstrom et al. (1986), Bernheim (1986), Roberts (1987), Andreoni (1988), and Sandler and Posnett (1991).
however, been at odds with empirical evidence on private charity.\textsuperscript{23} As such, several researchers have modified the standard model, but these modifications have been mostly confined to the donor side – the most prominent one being “warm-glow” giving in which people also receive a direct benefit from contributing (see Section 2.7.1).\textsuperscript{24} Here, we show that costly fund-raising can provide a complementary explanation as to the endemic breakdown of neutrality.

To develop some intuition, suppose that individuals have identical Cobb-Douglas preferences: $u_i = x_i^{1-\alpha}G^\alpha$, and consider these two income distributions: $w' = (w, w, ..., w)$ and $w'' = (\varepsilon + n(w-\varepsilon), \varepsilon, ..., \varepsilon)$, with $1/[1 + \alpha/\epsilon(n(1-\alpha))] < \varepsilon/w < 1$. It is readily verified that in the standard model, all individuals contribute under both income distributions and thus in equilibrium, $G^{\text{st}} = G^{\text{st}} > 0$. This neutrality result should extend to costly fund-raising as long as $c$ is small so that everyone is still contacted. For a sufficiently large $c$, however, the fund-raising strategy, and thus the public good provision, is likely to be affected by the income distribution. For instance, when $\varepsilon < c < w$, it is clear that whereas everyone is contacted under the egalitarian income distribution, $w'$, only the richest individual is contacted under the unequal income distribution, $w''$. This means that although there are more contributors under $w'$, there are also more fund-raising expenses. Trivial algebra shows that equilibrium public good levels are given respectively by $G^{\text{st}} = [n\alpha/(n(1-\alpha) + \alpha)](w-c)$ and $G^{\text{st}} = \alpha(\varepsilon + n(w-\varepsilon) - c)$, and comparing them reveals $G^{\text{st}} < G^{\text{st}}$. Note also that if the fund-raising were even costlier, $w \leq c < \varepsilon + n(w-\varepsilon)$, then the fund-raising effort would be reversed: no individual would be solicited under $w'$, whereas the richest person under $w''$ would still be solicited. Nevertheless, the public good provision would again imply that $0 = G^{\text{st}} < G^{\text{st}}$. Of course, if $c \geq \varepsilon + n(w-\varepsilon)$, then no


\textsuperscript{24} See, e.g., Cornes and Sandler (1984), Steinberg (1987), and Andreoni (1989).
fund-raising takes place in either case.

Overall, it seems that when fund-raising cost is significant, the neutrality result is unlikely to hold. It also seems that while the equilibrium number of solicitations responds non-monotonically to a more unequal distribution of income, the public good provision will always increase. To prove these observations generally, we employ the well-known concept of Lorenz dominance for income inequality (e.g., Atkinson 1970).

**Definition 1.** *(Lorenz Dominance)* Let \( w = (w_1, w_2, \ldots, w_n) \) be a vector of incomes whose elements are indexed in a descending order, and define \( L_i(w) = \sum_{j=1}^{i} w_j \). Consider two income vectors \( w' \neq w'' \) such that \( L_n(w') = L_n(w'') \). It is said that \( w'' \) is more unequal than \( w' \), if \( w' \) Lorenz dominates \( w'' \), i.e., \( L_i(w'') > L_i(w') \) for all \( i < n \).

Intuitively, an income distribution \( w'' \) is more unequal than \( w' \) if the total income is more concentrated in the hands of the few. In particular, the egalitarian income distribution Lorenz dominates all the others, whereas a perfectly unequal income distribution in which one person possesses all the wealth is dominated by all the others. Based on this inequality concept, we reach,

**Proposition 3.** Let \( w' \neq w'' \) be two income vectors such that \( w'' \) is more unequal than \( w' \) in the sense of Lorenz. Moreover, suppose that with the standard model, every person is a contributor under both \( w' \) and \( w'' \) so that \( G^{*I} = G^{*II} > 0 \). Then, \( G^{*I} = G^{*II} > 0 \) for \( c \in [0, \tau''_n) \), and \( G^{*I} < G^{*II} \) for \( c \in [\tau''_n, \tau''_1] \). For \( c \geq \tau''_1 \), no fund-raising takes place, yielding \( G^{*I} = G^{*II} = 0 \).

Proposition 3 generalizes our intuition from the above discussion. For a sufficiently small cost of fund-raising, every donor is solicited regardless of the income redistribution, resulting in the same level of the public good. When the cost is significant, however, the fund-raising strategy, and the level of public good, are influenced
by the income redistribution. In particular, a more unequal income distribution produces a higher level of the public good. Note from (2.3) that the interval $[\tau_n', \tau_n'']$ is likely to be wide because $\tau_1'' = w_1''$, and $\tau_n''$ can be much smaller than $w_n''$. To illustrate Proposition 3 and the non-monotonicity of the fund-raising strategy, we present the following example.

**Example 2.** Let $N = \{1, 2, \ldots, 10\}$ and $u_i = x_i^{1-\alpha}(G)\alpha$, with $\alpha = 0.3$. Table 2 below records three income distributions, $w'$, $w''$, and $w'''$. It is easy to verify that average income in each case is 19.27, and $L_i(w') > L_i(w'') > L_i(w''')$ for all $i$. Thus, $w'$ and $w'''$ exhibit the least and the most inequality, respectively. Using (3.2 cutoff solicitation cost for each donor under these income distributions.

Table 2.2: Solicitation Cost and Non-Monotone Fund-raising

<table>
<thead>
<tr>
<th>Donor $i$</th>
<th>$w'_i$</th>
<th>$w''_i$</th>
<th>$w'''_i$</th>
<th>$\tau'_i$</th>
<th>$\tau''_i$</th>
<th>$\tau'''_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.8</td>
<td>21.2</td>
<td>23.5</td>
<td>20.8</td>
<td>21.2</td>
<td>23.5</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>19.8</td>
<td>20.97</td>
<td>18.13</td>
<td>16.53</td>
<td>15.07</td>
</tr>
<tr>
<td>3</td>
<td>19.8</td>
<td>19.7</td>
<td>18.7</td>
<td>17</td>
<td>15.97</td>
<td>2.2</td>
</tr>
<tr>
<td>4</td>
<td>19.7</td>
<td>19.7</td>
<td>18.55</td>
<td>16.2</td>
<td>15.97</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>19.5</td>
<td>19.5</td>
<td>18.5</td>
<td>14.13</td>
<td>13.9</td>
<td>0.49</td>
</tr>
<tr>
<td>6</td>
<td>18.7</td>
<td>18.65</td>
<td>18.5</td>
<td>4</td>
<td>3.13</td>
<td>0.49</td>
</tr>
<tr>
<td>7</td>
<td>18.55</td>
<td>18.58</td>
<td>18.5</td>
<td>1.75</td>
<td>2.08</td>
<td>0.49</td>
</tr>
<tr>
<td>8</td>
<td>18.55</td>
<td>18.57</td>
<td>18.5</td>
<td>1.75</td>
<td>1.91</td>
<td>0.49</td>
</tr>
<tr>
<td>9</td>
<td>18.55</td>
<td>18.5</td>
<td>18.49</td>
<td>1.75</td>
<td>0.53</td>
<td>0.29</td>
</tr>
<tr>
<td>10</td>
<td>18.55</td>
<td>18.5</td>
<td>18.49</td>
<td>1.75</td>
<td>0.53</td>
<td>0.29</td>
</tr>
</tbody>
</table>

From Table 2.2, note first that for a solicitation cost, $c \leq .29$, all donors are called upon regardless of the income distribution, which implies the neutrality: $G''' = G'''$, as it should. Second, for $1.75 < c < 1.91$, the fund-raising set is non-monotonic in income inequality because clearly, $F'' = \{1, 2, \ldots, 6\}$, $F''' = \{1, 2, \ldots, 8\}$.

---

25 For instance, in the numerical example above, if solicitation costs were taken equal, then 90 and 2.3 would be the respective cutoffs for the incomes, 90 and 87.
and $F^{all} = \{1, 2, 3\}$, revealing that $F^{all} \subset F^{af} \subset F^{af}$. Nevertheless, the monotonicity in public good provision holds: $\bar{G}^{af} < \bar{G}^{all} < \bar{G}^{all}$, as predicted by Proposition 3. For instance, for $c = 1.8$, we have $\bar{G}^{af} = 7.180$, $\bar{G}^{all} = 7.185$, and $\bar{G}^{all} = 7.220$.

We should point out that strategic costly fund-raising offers a complementary explanation for the non-neutrality to those identified in the literature. In particular, as with Bergstrom, Blume, and Varian (1986), we draw attention to the endogenous nature of the contributor set to the income distribution; but unlike in their study of the standard model, the contributor set in ours is optimally chosen by the fundraiser. This means, for instance, that the non-contributors in our model are not necessarily pure free-riders; rather they are not asked for donations due to solicitation costs. We should also point out that in their Theorem 1d, Bergstrom, Blume, and Varian also observe that “Equalizing income redistributions that involve any transfers from contributors to non-contributors will decrease the equilibrium supply of the public good.” However, as is clear from Proposition 3, under strategic costly fund-raising, the non-neutrality exists even when everyone remains a contributor under both income distributions in the standard model.

2.5 Government Grants

A long-standing policy question in public economics is that if the government gives a grant to a charity, to what degree will it displace private giving? While, in light of the neutrality result, the standard model of giving predicts a complete (dollar-for-dollar) crowding out, there is overwhelming evidence that this is not the case. The empirical studies have, for the most part, attributed any crowding-out to the donors’ responses. Recently however, Andreoni and Payne (2003, 2011) have empirically showed that a significant part of the crowding out can be explained by reduced fund-

---

26 Bergstrom, Blume, and Varian use direct transfers among donors, but it is well-known that such Daltonian transfers are equivalent to Lorenz dominance (Atkinson (1970)).
raising. By simply modifying our model, we can theoretically support their findings. Let $R > 0$ be the amount of the government grant, and $F_R^o$ and $F_0^o$ denote the optimal fund-raiser sets with and without the grant, respectively.

**Proposition 4.** Suppose that, without a grant, some public good is provided, i.e., $\mathcal{G}_0^o > 0$. Then, with the grant, donor $i$ is solicited if and only if $\Delta_i > R$. Moreover,

(a) there is less fund-raising with the grant, i.e., $F_R^o \subseteq F_0^o$;

(b) each donor gives strictly less with the grant, i.e., $g_i^a(F_R^o) < g_i^a(F_0^o)$ for $i \in F_R^o$;

(c) private giving is partially crowded out, i.e., $\mathcal{G}^*(F_R^o) < R + \mathcal{G}^*(F_0^o)$, but $\mathcal{G}^*(F_R^o) > \mathcal{G}^*(F_0^o)$.

Since a government grant directly enters into public good production, part (a) implies that the charity optimally solicits fewer donors. Under a linear production, this reduced fund-raising is, however, not because the charity has diminishing returns to funds, but because it anticipates that donors will be less willing to give, as reflected by the optimal strategy. While, all else equal, cutting back fund-raising increases the public good provision by cutting costs, it also leaves some donations unrealized. Moreover, despite a smaller fund-raiser set, and thus less severe free-riding, with the grant, part (b) indicates that each contacted donor gives strictly less than he would without the grant. This is due to diminishing marginal utility from the grant that simply overwhelms the small group effect. Part (c) shows that the two effects of a government grant, namely lower fund-raising and fewer donations, never neutralize its direct production effect on the public good. That is, the crowding-out is partial because of both the fund-raiser’s and the donors’ behavioral responses.\(^{27}\)

\(^{27}\) Note that Proposition 4 ignores the financing issue of the government grant and thus may be underestimating the crowding out. In particular, having a reduced wealth $w_i - t_i$ by a tax $t_i$ toward the grant, person $i$ would have a lower $\mathcal{G}_i^o$, which would, in turn, lead to less fund-raising and a lower equilibrium gift than without taxation. Note also that any cost of receiving the grant by the fund-raiser could be absorbed by $R$. 
2.6 Large Replica Economies

Many charities have access to a large donor base. In particular, the advent of information technology has helped fund-raisers to better search and locate prospect donors. To understand fund-raising behavior in large economies, we consider a simple replica-economy in which there are \( r \) donors of each type represented by the triple \((u_i, w_i, c_i)\), resulting in the drop-out value \( G^0_i \) from (2.2). The following result is the main finding in this section.

**Proposition 5.** Suppose \( G^0_1 > G^0_2 > \ldots > G^0_n > 0 \). Then

(a) there are some replicas \( r_\alpha \leq \ldots \leq r_2 < \infty \) such that type-\( i \) donors are not solicited in any \( r \geq r_i \) replica economy.

(b) As \( r \to \infty \), only type-1 donors are solicited, in which case each donation converges to the solicitation cost, \( c_1 \), but the public good level approaches to \( G^0_1 \).

Proposition 5 says that except for the most willing type, there is a large enough replication of the economy in which no other type is solicited. The reason is that as the economy is replicated, the higher types replace the lower ones in net contributions. Proposition 5 also says that in the limit, each donation from a type-1 person converges to his respective solicitation cost. While this means that the net contribution is approximately zero, the level of the public good approaches to a finite level \( G^0_1 \). Note that even in the limit, it is not the lowest cost and/or highest income donors who are solicited; rather it is a combination of all the three attributes that determine the highest type.

Within the standard model, Andreoni (1988) finds that, all else equal, in large economies, only the richest agents contribute and others free ride. Andreoni also finds that the average contribution decreases to zero.\(^{28}\) With costly fund-raising, our

\(^{28}\) See also Fries et al. (1991) for a characterization of large economies under the standard model.
result suggests that only the richest agents contribute because they will be the only ones to be solicited in large economies. Moreover, the average donation converges to the solicitation cost, which is strictly positive.

2.7 Extensions

In this section, we briefly discuss three extensions: (A) warm-glow giving, (B) fundraising with income uncertainty, and (C) “learning-by-fundraising”. Many technical details are relegated to the online appendix.

2.7.1 Warm-Glow Giving

It is well-documented in the literature that a model of warm-glow giving in which individuals also receive a private benefit from contributing explains the data better than the purely altruistic model employed so far. Our results, however, easily generalize to such added realism. Following Andreoni (1989), let \( u_i = u_i(x_i, G, g_i) \) be person \( i \)'s utility function, which is increasing and strictly quasi-concave. In the absence of fund-raising costs, the Nash supply of person \( i \)'s gift can be written:

\[
g_i = \max \{ \hat{f}_i(w_i + G_{-i}, G_{-i}) - G_{-i}, 0 \},
\]

where partial derivatives satisfy \( 0 < \hat{f}_{i1} < 1 \) and \( \hat{f}_{i2} \geq 0 \) by normality of goods. If, in addition, \( 0 < \hat{f}_{i1} + \hat{f}_{i2} \leq \theta < 1 \), then a unique Nash equilibrium obtains. Note that for \( \hat{f}_{i2} = 0 \), the warm-glow model reduces to the standard model.

Building on Andreoni’s characterization, we define the inverse Nash supply, \( \hat{\phi}_i(G, w_i) \) such that \( \hat{f}_i(w_i + \hat{\phi}_i, \hat{\phi}_i) = G \). It is readily verified that \( \hat{\phi}_{i1} > 1 \) and \( -1 \leq \hat{\phi}_{i2} < 0 \). Analogous to \( \Phi_i \) above, we also define \( \hat{\Phi}_i(G, w) = \sum_{j=1}^{i} (\hat{\phi}_j(G, w_j) - G) + G \), which is strictly increasing in \( G \) and strictly decreasing in \( w_j \). In the presence of fund-raising costs, it can be shown that Lemma 1 continues to hold (see the online appendix).

\[\text{We assume that warm-glow is felt by the net contribution, } g_i - c_i, \text{ so that no extra utility is received by simply covering the solicitation cost. We also assume that } c_i < C_i \text{ for a cutoff similarly defined for the warm-glow model.}\]
Thus, slightly modifying eq.(2.2), let $G_0^i$ be uniquely determined by: 
$$
\hat{f}_i(w_i - c_i + G_0^i, G_i^0) = G_i^0.
$$
Next, similar to $\Delta_i$ in Proposition 1, set $\hat{\Delta}_i = \hat{\Phi}_i(G_0^i, w - c)$. Then, our results in Propositions 1 and 4 obtain by simply replacing $\Delta_i$ with $\hat{\Delta}_i$. In particular, since, without a warm-glow motive, $\hat{\phi}_i(G, w_i) = \phi_i(G) - w_i$, our previous results under pure altruism follow.

In order to perform comparative statics, we consider a general CES utility for all $i$:

$$
\begin{align*}
    u_i &= \left[(1 - \alpha) x_i^\rho + \alpha ((1 - \omega) G + \omega g_i)^\rho \right]^\frac{1}{\rho},
\end{align*}
$$
where $\rho \in (-\infty, 1)$, $\alpha \in (0, 1)$, and $\omega \in [0, 1]$. Clearly, as $\omega$ increases, person $i$ cares more about the warm-glow and less about the altruistic giving. From Proposition 4, person $i$ is contacted if and only if $\hat{\Delta}_i > R$, or the solicitation cost, $c$ is less than his cutoff:

$$
\bar{c}_i = w_i - \eta (1 - \omega) R - \frac{\eta (1 - \omega)}{1 + \eta \omega} \sum_{j=1}^{i} (w_j - w_i),
$$
where $\eta = \left(\frac{1-\omega}{\alpha}\right)^{\frac{1}{1-\rho}}$. Eq.(2.4) implies that $\bar{c}_i$ is increasing in $\omega$, and decreasing in $R$ at the rate of $\eta (1 - \omega)$. That is, as warm-glow giving becomes more pronounced, the fund-raiser solicits more people; and she is less discouraged by an outside grant.

These observations suggest that with warm-glow giving, both the fund-raiser’s and the donors’ diminished response to a government grant are responsible for the partial crowding out. It is, however, an empirical matter to quantify them. In a recent paper, Andreoni and Payne (2011) measure 73 percent crowding out and attribute all to the reduced fund-raising. We believe that the absence of the (classic) donor crowding out can be evidence of a strong warm-glow motive in their data. Given this, the high fund-raiser reaction to government grants seems inconsistent with net revenue maximization. That is, at the margin, the fund-raiser could increase net revenues by contacting more donors. This conclusion firmly supports Andreoni and
Payne’s empirical finding.\textsuperscript{30} As a policy remedy, they propose (and we agree) that “…requirements that charities match a fraction of government grants with increases in private donations could be a feasible response to crowding out.” (p.342)

In a related paper, Andreoni and Payne (2003) find that government grants crowd out fund-raising efforts in social services organizations much less than they do in the arts. In light of our analysis, this evidence points to a stronger warm-glow giving toward social services than toward the arts. This inference appears reasonable because the contributors to the arts are more likely to be the beneficiaries than the contributors to the social services. In another paper, Ribar and Wilhelm (2002) present clear evidence of warm-glow giving to international relief and development organizations. Together with our theory, we should expect substantial fund-raising by these organizations despite sizable governmental aids to international relief programs.

\subsection{2.7.2 Fund-raising with Income Uncertainty}

Up to now, we have maintained the strong assumption that the fund-raiser fully knows donors’ incomes and preferences. We partially relax this assumption here by introducing income uncertainty to our basic model. Suppose that depending on its demographics, the fund-raiser divides the population into \( m \geq 1 \) groups of donors. She believes that each member of group \( i \) independently draws his income from a discrete distribution, \( \tilde{w}_i \), with mean \( E[\tilde{w}_i] \). The fund-raiser’s strategy is to choose the number of donors to be contacted from each group. To focus the analysis on the fund-raiser, we continue to assume that donors have no uncertainty about the income profile in the population. Moreover, to simplify the analysis, we consider identical homothetic preferences so that \( f(w) = \alpha w \) for some \( \alpha \in (0, 1) \). Then, given the cost \( c \) per solicitation and the ranking of the mean group incomes: \( E[\tilde{w}_1] \geq \ldots \geq E[\tilde{w}_m] \),

\textsuperscript{30} In general, the evidence on fund-raisers’ objectives is mixed (Andreoni (2006b)). The net revenue maximization is, however, often adopted in theoretical studies.
we can write the cutoff cost for group $i$ as:

$$
\bar{c}_i = E[\hat{w}_i] - \frac{1 - \alpha}{\alpha} \sum_{j=1}^{i} n_j (E[\hat{w}_j] - E[\hat{w}_i]),
$$

(2.5)

where $n_j$ is the size of group $j$ (see the online appendix). We show that it is optimal to solicit all members of group $i$ if $c < \bar{c}_i$; and no members, otherwise. Note that if each group contains a single donor, say demographics are sufficiently informative, eq.(2.5) reduces to eq.(2.3), as it should. With income uncertainty, however, the fund-raiser optimally treats each group member as having its mean income. We show that the use of such coarse information for solicitations hurts the fund-raiser: if members of any two groups become “indistinguishable” by the fund-raiser, the equilibrium supply of the public good decreases. The reason is that with increased uncertainty, the fund-raiser is more likely to contact net free-riders and leave out net contributors. This means that information is valuable to the fund-raiser, which may explain the existence of a market for donor research.

2.7.3 Learning-by-Fundraising

As in many service and manufacturing sectors, the fund-raiser may also learn and become a more efficient solicitor over time. This raises the interesting issue that the fund-raiser may “invest” in learning by initially contacting net free-riders. To illustrate this point, we re-consider our basic model with identical individuals. Let $c(i)$ be the marginal cost of soliciting $i$th individual in sequence such that $c(1) > c(2) \geq \ldots \geq c(n)$ due to learning. Also, let $a_n = (1/n) \sum_{i=1}^{n} c(i)$ be the average cost of solicitation where $a_n < \bar{C}$. Clearly, $a_n$ is decreasing in $n$ and thus converges to some $a_\ell < c(1)$. We show that it is optimal to contact every donor in this case. More importantly, in the unique equilibrium, each (symmetric) gift, $g^*_n$, is decreasing in $n$

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31 See Benkard (2000) and the references therein.
and converges to $a_t$. This implies that $g_n^* - c(1) < 0$ for a sufficiently large $n$. That is, with learning, the fund-raiser may initially solicit some net free-riders to lower future costs – a benefit that was absent in the basic setup.

2.8 Conclusion

As part of doing business, charities often spend money to raise money. Thus, a careful planning of whom to ask for donations should be paramount for a charity aiming to control its fund-raising costs while maximizing donations. Perhaps this is why the charitable sector has grown to be highly professional and innovative. Yet, the theory of charitable fund-raising has mostly ignored its cost side. In this paper, we take a first stab at filling this void. We fully characterize the optimal fund-raising strategy that can be easily computed from the donors’ preferences, incomes, and the solicitation costs. Among other results, we show that costly fund-raising can provide a novel explanation for the non-neutrality of income redistributions and the crowding-out hypothesis often encountered in empirical studies. For future research, it may be worthwhile to consider sequential solicitations where donations are revealed in each visit. Another promising, and perhaps more challenging, direction would be to investigate the competition between charities where donors’ responses are fully accounted for.

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32 The supply of the public good, $G_n^*$ is increasing in $n$ and converges to $\phi^{-1}(w - a_t)$.

33 For instance, the Association of Fundraising Professionals (AFP) represents 30,000 professional fund-raisers.
3. Learning by Fund-raising

3.1 Introduction

It is strongly believed that fund-raising is learned on the job, raising the demand for those professionals who are more experienced. For instance, a recent survey by Cygnus Applied Research reveals that most succesful fund-raisers are on the job just three to six months before being recruited for another.\(^1\) As the president of Cygnus puts it: “Only one out of three fund-raisers experiences even a day without a job”. Professional fund-raisers also place a great value on experience as suggested in this quote from a fund-raiser’s webpage: “Fund Development Associates is the regional expert in fund-raising. No one has more direct, hands-on experience. By selecting our firm, you will have a team of professionals with more than one hundred years of combined successful fund-raising experience who have assisted hundreds of charitable organizations achieve their goals”.\(^2\)

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\(^1\) The survey includes 1,700 fund-raisers and 8,000 nonprofit chief executives. Results are available at http://www.cygresearch.com/files/AFP_Intl-Conf_Vancouver_April_2_2012-PenelopeBurk.pdf

\(^2\) See http://www.funddevelopmentassociates.com/associates.html
raising techniques is directly asking people (Andreoni and Payne 2003; Yoruk, 2009; Meer and Rosen 2011). It is believed that people often have good intentions to give, but unless they are solicited, these intentions may not turn into donations. In this paper, we contend that such direct solicitations are also the source of learning for the fund-raiser. Our main objective is to investigate how learning shapes the fund-raising strategy and if it may cause “excessive” fund-raising.

Our formal setup adds an “active” fund-raiser to the “standard” model of giving in which donors consume two goods: a private good as well as a public good.\(^3\) We consider a charity which occasionally runs fund-drives. The fund-raiser’s role then consists in informing potential donors individually about the charitable cause, as in Name-Correa and Yildirim (2013). Asking people is costly. The presence of learning economies, however, enables the charity to reduce the marginal cost of fund-raising as the charity solicits more people.

Our first observation is that the fund-raising cost introduces a provision point to the public good, but under an optimal solicitation procedure, a coordination problem among donors does not arise. The charity contacts individuals according to income, starting with the wealthiest. A sufficient condition to solicit one more individual is that she is expected to provide a gift above the marginal cost or become a “net contributor”. We show that identifying these net contributors in our model is equivalent to identifying the contributors in a model with constant marginal cost except that each donor’s wealth is reduced by the variable part of its marginal cost. This important equivalence allows us to utilize the characterization in Name-Correa and Yildirim (2013) who assume away learning.

Absent learning economies, the charity considers contacting first the richest donor; once this donor is in the “game”, the charity becomes more conservative about con-

\(^3\) See, e.g., Warr (1983); Roberts (1984); Bergstrom, Blume, and Varian (1986); and Andreoni (1988).
tacting the second richest donor due to the free-riding incentive, which depends on their income difference. Sequentially applied, this logic implies once the charity identifies a “net free-rider”, the solicitations optimally stop.

In the presence of learning, the fund-raiser may, however, solicit a net free-rider, as long as this solicitation enables the fund-raiser to substantially move down her learning curve. In this sense, negative net contributions represent the fund-raiser’s investment in learning. We provide the exact equilibrium condition determining whether investing in learning is worthy or not. While we assume that the solicitation set is observed by the contacted donors, our characterization is robust to unobservability under reasonable (off-equilibrium) beliefs.

Watchdogs groups evaluate a charity efficiency according to its cost structure. They recommend managing a low fixed cost. For instance, Charity Navigator considers that administrative costs should not represent more than 20% of total costs. My model also applies to a situation in which the presence of a fixed cost generates returns to scale in fund-raising. When a higher setup cost does not totally discourage fund-raising, it increases current donations and encourages the charity to solicit more. Despite these two positive effects, the public good provision diminishes.

I build a benchmark in which the fund-raiser establishes for each donor a minimum gift size and commits to it. We show that this commitment allows the charity to obtain extra-large gifts from the wealthiest donors. With respect to this benchmark I find that the charity conducts excessive fund-raising regardless of the solicitation technology. Moreover, we show that learning is another source of excessive fund-raising. We find, however, that a higher learning rate does not necessarily generate a greater extent of excessive fund-raising.

I extend the model to incorporate a warm-glow motive for giving (Andreoni 1989) and show that my results follow under such added realism. In another extension, we show that when the fund-raiser separates the population in groups and learning is
group specific, the charity may favor contacting groups with lower expected income but with more potential for learning. Finally, we show that under decreasing returns to scale it is never optimal to contact a net free-rider.

Given that this chapter is closely related to chapter 2, I just briefly mention here some of the theoretical works in charitable fund-raising: Rose Ackerman 1983, Andreoni and Payne 2003, Glazer and Konrad 1996, Vesterlund 2003, Andreoni 2006b, Andreoni 1998 and Marx and Matthews 2000. Some papers consider learning about the project quality by providing the charitable good within a dynamic framework. In these models learning is faster when the cumulative production of the good is larger (Bolton and Harris (2003); and Yildirim (2006)).

The closest work to this is the one presented in the previous chapter, referred here as Name-Correa and Yildirim 2013; henceforth, NY (2013). They build a model in which donors do not consider giving unless asked by the fund-raiser. They fully incorporate fund-raising costs to determine the fund-raiser’s solicitation strategy. The charity commits to that strategy and successfully launches a fund-drive. Our work is similar to theirs; instead of attaching a cost to each donor, though, we explicitly introduce a fund-raising cost structure, which is unrelated to donors’ identities. This allows us to model the learning aspect of soliciting as decreasing marginal costs in fund-raising.

Rose-Ackerman (1982) is the first to build a model of costly fund-raising in which donors, as in mine, are unaware of a charity until they receive a solicitation letter. She, however, does not construct donors’ responses from an equilibrium play. She was also the first in positing that fund-raising is likely to be conducted in excess. Her argument is that competition among charities triggers high expenses in fund-raising without bringing further benefits to donors. This happens whenever fund-raising diverts funds from one charity that donors value to another they like the same. On the contrary, in our model we build the concept of excessive fund-raising in a non-
competitive framework. The term "excessive" comes from the fact that relatively more cost is incurred when contributions are voluntary and those extra resources are wasted, valued neither value by donors nor by the charity.

In addition to the theoretical literature, more extensive empirical and experimental literature exists on charitable giving, to which we will refer below. For recent surveys of the literature, see the reviews by Andreoni (2006a) and List (2011).

The rest of the paper is organized as follows. In Section 2, we set up the model. In Section 3, we determine the optimal fund-raising strategy. In Section 4 we introduce returns to scale generated by a fixed cost. In Section 5 we consider excessive fund-raising. We present the extensions in Section 6, and conclude in Section 7.

3.2 Model

Our formal setup extends the standard model of privately provided public goods, as described in the previous chapter (e.g., Warr 1983; Roberts 1984; Bergstrom et al. 1986; and Andreoni 1988).

Costly Fund-raising. In the standard model there is no role for strategic fund-raising since all potential donors are already aware of the public good provision. Thus, as with Rose-Ackerman (1982); and Andreoni and Payne (2003), we assume that each person becomes informed of the fund-drive only if solicited by the fund-raiser. We assume for simplicity that each solicitation reaches the donor with certainty. It costs to solicit the individual in a sequence. The fixed marginal cost reflects minimum expenses in telemarketing, face to face solicitations, envelopes procurement, and mailing costs. The variable marginal cost is

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4 Alternatively, in the standard model, the fund-raiser would trivially ask everyone for donations since the equilibrium provision of the public good never decrease by adding an individual (e.g., Andreoni and McGuire 1993).

5 We envision a charity that occasionally runs fund-drives. In this scenario, it is reasonable to think that donors are unaware of the charitable good provision. However, even if a donor expects a fund-drive to be made, she may procrastinate in giving (O’Donoghue and Rabin 1999) or just wait for the solicitation to save on search costs (Andreoni and Payne 2001).
non increasing in \(i\), perhaps because of the fund-raiser learning on the job or because of scale economies purchasing inputs at a discount. We assume that this cost structure is known by contacted individuals. Absent the variable cost, our model would reduce to NY (2013) with homogeneous preferences and constant marginal cost.

Let \(F \subseteq N\) be the set of donors contacted by the fund-raiser, or the fund-raiser set. In the basic model, we assume that the contacted donors know those in the fund-raiser set, though we relax this assumption in Section 3.2. 6 As in the standard setup, let \(g^*_i(F)\) be donor \(i\)'s equilibrium gift engendered by the simultaneous play in \(F\). Then, the total fund-raising cost and the gross donations are defined, respectively, by \(C(F) = \sum_{i=1}^{\lvert F \rvert} c(i)\) and \(G^*(F) = \sum_{i \in F} g^*_i(F)\), where \(C(\emptyset) = 0\) and \(g^*_i(\emptyset) = 0\) by convention. The charity chooses \(F\) that maximizes the supply of the public good (or net donations):

\[
\overline{G}^*(F) = \max \{G^*(F) - C(F), 0\} .
\tag{3.1}
\]

Eq. (3.1) implies that if insufficient funds are received to cover the cost, then no public good is provided, which simply refers to a failed fund-raising in our model.7 We assume that the charity dislikes fundraising in that when two fund-raiser sets yield the same amount of public good, the charity prefers the one with the lower cost.8

Our fund-raising game, then, proceeds as follows. First, the charity decides whether or not to launch a fund-drive. If one is launched, then the charity reaches out to a (optimal) set \(F^o\) of potential donors, who all become aware of both the fund-drive and the others solicited. Finally, the contacted donors simultaneously contribute to the public good, leading to equilibrium gifts \(\{g^*_i(F^o)\}_{i \in F^o}\) and the public good \(\overline{G}^*(F^o)\). Our solution concept is subgame perfect Nash equilibrium in pure

6 That donors may know the fund-raiser set prior to giving is not completely unrealistic. For instance, charities organize fund-raising events where donors meet each other

7 In the case of a failed fund-raising, we assume for simplicity that either the donations are not refunded or they are used for other causes

8 One justification for this could be that the charity has some concern about its cost/donation rating by the watchdog groups. Formally, if \(F' \neq F\) are two fund-raiser sets such that \(G' - C' = G - C\) and \(C' > C\), then it follows that \(C'/G' > C/G\).
strategies.

3.3 Optimal Fund-raising

In this section we fully characterize the fund-raising equilibrium in terms of the primitives of the model. Before that, we point out that although donors may end up contributing nothing for an arbitrary fund-raiser set, the same cannot happen if the set is optimally chosen.

3.3.1 Characterization

Let \( \hat{C}_i \in (0, w_i] \) be the unique cutoff cost for person \( i \) if he were to pay for the entire fund-raising cost himself, as defined in chapter 2. The following result shows that although donors together may contribute nothing in some situations, in equilibrium a launched fund-drive is always successful.

**Proposition 1.** Fix any arbitrary fund-raiser set, \( F \neq \emptyset \), whose fund-raising cost is \( C(F) \). If \( \max_{i \in F} \hat{C}_i \leq C(F) \), then there is a zero-contribution equilibrium, generating \( \mathcal{G}^0(F) = 0 \). However, in a fund-raising equilibrium, \( F^o \neq \emptyset \) if and only if \( \mathcal{G}^0(F^o) > 0 \).

The first part of Proposition 1 says that if no person can bear the cost alone, then the zero-contribution profile becomes an equilibrium. Hence, when fund-raising entails significant costs, a carefully planned strategy of whom to ask for donations seems to be of utmost importance both to control the expenses and to encourage giving.\(^9\)

The second part of the Proposition highlights that in a setting in which the fund-raiser set is observable, an optimizing charity would never start fund-raising if it did not expect that donations would exceed the cost. Together, this proposition means that in our model, the charity can fail to provide the public good despite fund-raising.

\(^9\) Since the fund-raising cost introduces a threshold to the public good provision, Proposition 1 is a reminiscent of the equilibrium characterization in Andreoni (1998). Unlike his model, however, the provision point in ours will be endogenous to fund-raising strategy as opposed to being a capital requirement.
only because it suboptimally sets the fund-raising strategy.\textsuperscript{10} While enlightening, Proposition 1 does not inform us about the charity’s solicitation strategy.

In order to do so, note two observations for any fixed fund-raiser set; (1) the incurred cost just depends on the number of solicitations, (2) the higher the income of an individual, the more she gives, as shown in Andreoni (1988). We intuitively observe that the fund-raiser solicits the highest income individual(s). Without loss of generality, index subjects in a descending order of their wealth: \( w_1 \geq w_2 \geq \ldots \geq w_n \).

**Observation 1.** For any optimal fund-drive size \( k \), the top \( k \) individuals are the ones being solicited.

From the previous observation we may consider that soliciting individual \( i \) costs \( c(i) \) to the fund-raiser. In other words, the charity may view fund-raising costs as identity dependent, keeping in mind that soliciting individual \( i + 1 \) implies that individual \( i \) is already included in the fund-raiser set. According to NY (2013), when costs are purely identity dependent, the fund-raiser designs a strategy where individual donors are solicited at the margin whenever their gifts exceed solicitation costs; such donors are net contributors. This marginal strategy leads to an optimal fund-raiser set, \( F^o \), in which every solicited individual becomes a net contributor, even without a cost sharing agreement, since contributions are voluntary, and all of them just take into account the whole fund-raising cost \( C(F^o) \).

Once we introduce learning economies, it is possible that the fund-raiser at the margin optimally solicits an individual \( i \), who provides a gift below the marginal cost \( c(i) \); in other words, the donor is a net free rider. We illustrate this point with a numerical example, which also motivates our subsequent analysis.

**Example 1.** Let \( N = \{1, 2, 3\} \) and \( u_i = x_i^{1-\alpha} (G)^{\alpha} \), with \( \alpha = 0.3 \). Individuals’ wealth and solicitation costs are such that \( (w_1, w_2, w_3) = (20, 14, 14) \), \( c = 1 \).

Consider first no scale economies, i.e., \( s(i) = 0 \). The following table reports donor equilibrium, and highlights the optimal fund-raiser set.

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\textsuperscript{10} As noted in the Introduction, charities spend billions of dollars on professional fund-raisers. For instance, the Association of Fundraising Professionals (AFP) represents 30,000 such fund-raisers.
Table 3.1: Donor Equilibrium without Learning

<table>
<thead>
<tr>
<th>$F$</th>
<th>$g_1^* - c$</th>
<th>$g_2^* - c$</th>
<th>$g_3^* - c$</th>
<th>$G^* - C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1}</td>
<td>5.7</td>
<td></td>
<td></td>
<td>5.7</td>
</tr>
<tr>
<td>{1, 2}</td>
<td>5.82</td>
<td>-0.177</td>
<td></td>
<td>5.65</td>
</tr>
<tr>
<td>{1, 2, 3}</td>
<td>5.875</td>
<td>-0.125</td>
<td>-0.125</td>
<td>5.62</td>
</tr>
</tbody>
</table>

Table 3.1 reveals that it is optimal to contact only donor 1. Donor 2 and 3 are not included in the set because their contributions never exceed the marginal cost.

Keeping donors’ characteristics as above and $c = 1$, consider $s(i) = (7, 5, 1)$

Table 3.2: Donor Equilibrium under Learning Economies

<table>
<thead>
<tr>
<th>$F$</th>
<th>$g_1^* - c(1)$</th>
<th>$g_2^* - c(2)$</th>
<th>$g_3^* - c(3)$</th>
<th>$G^* - C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1}</td>
<td>3.9</td>
<td></td>
<td></td>
<td>3.9</td>
</tr>
<tr>
<td>{1, 2}</td>
<td>3.94</td>
<td>-0.06</td>
<td></td>
<td>3.88</td>
</tr>
<tr>
<td>{1, 2, 3}</td>
<td><strong>2.79</strong></td>
<td><strong>-1.21</strong></td>
<td><strong>2.79</strong></td>
<td><strong>4.37</strong></td>
</tr>
</tbody>
</table>

Table 3.2 shows that it is optimal to contact donors 1, 2, and 3. Without donor 3, donor 2, whose gift remains below $c(2)$, diminishes the charitable good provision. By additionally soliciting individual 3, however, the public good reaches its optimal level. Finally, it is clear that even with three donors, a direct approach to identify the extent of fund-raising is non-trivial.

To develop a simple, intuitive characterization of the fund-raiser set, we interpret sequential costs as taxes on individuals. In this sense, let $\hat{w}_i = w_i - s(i)$ be individual i’s ”disposable income”. Under this formulation, for a given set $F$, i’s gift is $g_i(F) - s(i)$. We show this in two steps. Consider person i’s maximization problem:

$$\max_{x_i, g_i} U(x_i, G - \sum_{j \in F} c(j))$$

s.t. $x_i + g_i = w_i$
As a first step, consider substituting for \(w_i \equiv w_i - s(i) - c\) and \(g_i \equiv g_i - s(i) - c\), person \(i\) can be deemed as choosing the level of the charitable good:

\[
\max_{x_i, G} U(x_i, G) \text{ s.t. } x_i + G = w_i + G_{-i} \geq G_{-i}
\]

The solution to this maximization yields \(i\)'s demand function for the charitable good given net contributions by others, \(G_{-i} : \)

\[
G = \max \{ f(w_i + G_{-i}), G_{-i} \}.
\]

As a second step, from this whole normalization, we define \(\tilde{w}_i \equiv w_i - c \equiv w_i - s(i)\) and \(\tilde{g}_i \equiv g_i - c \equiv g_i - s(i)\). This change of variables allows us to reformulate our original problem with learning economies to a constant return to scale setting with marginal cost \(c\) and nominal income distribution \{\(\tilde{w}_i\}\).

Let \(F_i\) be the set of the top \(i\) individuals. The next Lemma shows that individual \(i\)'s incentive to provide a donation above the marginal cost, \(c\), in \(F_i\) can be represented by a cost cutoff.

**Lemma 1.** Let \(\bar{\phi}(G) \equiv \phi(G) - G\), where \(\phi = f^{-1}\), and donor \(i\)'s cost cutoff be given by

\[
\bar{c}_i = \tilde{w}_i - \bar{\phi}(\sum_{j=1}^{i} (\tilde{w}_j - \tilde{w}_i)).
\]

(3.2)

Individual \(i\) is a net contributor in \(F_i\) iff \(c < \bar{c}_i\)

By strict normality \(\bar{\phi}(.) > 0\). Therefore, \(i\)'s cutoff cost decreases in others' disposable incomes and increases in \(i\)'s own.

**Observation 2.** Absent the sequential component, we obtain: \(\bar{c}_1 \geq \bar{c}_2 \geq .. \geq \bar{c}_n\) and \(F^0 = \{i \in N | c < \bar{c}_i(\tilde{w}_i)\}\) (NY, 2012)
Note first that under no sequential cost, $\hat{w}_i = w_i$. Hence, for any subeconomy $F_i$, individuals are ranked according to their net gifts $g^*_i(F_i) - c$, since $w_1 - c \geq w_2 - c \geq \cdots \geq w_n - c$. It is clear that $\tau_i$ is less than $w_i$, except for the first individual, and it diminishes in $i$. Intuitively, once the richest donor is solicited, the second individual is less likely to cover the marginal cost $c$ as a consequence of the free rider problem. In general, as the charity keeps fund-raising, free riding becomes more and more severe and it is less likely that an additional person will be solicited. Once a net free rider is identified, fund-raising must stop. Otherwise, given that individuals are ranked according to their net gifts’ sizes, additional solicitations would bring only negative net donations. This would hurt the public good provision, as shown in Lemma 1 in NY (2013). Re-consider Example 1 above, when $s(i) = 0$. From eq. (3.2), it is easily verified that $\tau_1 = 20$, $\tau_2 = 0$, and $\tau_3 = 0$, which implies that $F^o = \{1\}$.

The free-rider problem is still present when $s(i) > 0$. However, on the upside, fund-raising allows the charity to reach learning economies, thus partially counteracting free riding.

To be more precise, let $a_w(i, k)$ be the average disposable income from individuals $i$ to $k$ where $i \leq k$. By convention, $a_w(i, i) = w_i - s(i)$. By applying the next proposition iteratively we obtain a full characterization of the fund-raiser’s strategy.

**Proposition 2.** Suppose either (1) $i = 1$ or (2) $i > 1$ and individuals 1 to $i - 1$ are solicited by the fund-raiser. Then, $i$ is solicited iff there is an individual $k \geq i$ such that $c < \tau_i(a_w(i, k))$. Moreover if $k > i$ is the closest individual to $i$ satisfying the previous inequality, then donors from $i + 1$ up to $k$ must also be solicited.

Proposition 2 says that to contact an additional individual $i$, it is sufficient that she pays for her marginal cost at the margin, i.e., if the economy were $F_i$. It does not matter whether or not she becomes a net contributor in $F^o$.

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11 A donor $i$ may be a net contributor in the subeconomy $F_i$ but not in $F^o$ because the sequence of disposable incomes is not necessarily monotonically decreasing. Therefore, after soliciting individual $i$, another subject providing a higher net gift may also be contacted, thus driving $i$’s contribution below $c(i)$ as a consequence of the free rider problem.
Even though the free-rider problem is more pronounced the more a charity fundraises, it is also true that more fund-raising generates more experience for the charity. Thus, Proposition 3 also says that despite individual $i$ being a net free rider at the margin, she is solicited as long as subsequent cost decreases turn out to be substantial.

This proposition contrasts with the equilibrium characterization in NY(2013), where every individual in $F^o$ is a net contributor. In this sense, the presence of net free riders in $F^o$ can be thought of a charity’s investment in acquiring experience.

Re-consider Example 1 above, under learning economies. From eq. (3.2), it follows that $\bar{c}_1 = 13, \bar{c}_2 = -0.33 < c < \bar{c}_3 = 22.33$. Moreover, $\bar{c}_3(a_{\hat{\alpha}}(2,3)) = 6.33 > c$. Thus, according to Proposition 2, $F^o = \{1, 2, 3\}$.

Finally, the fund-raiser considers the set resulting from iteratively applying proposition 2 as a candidate equilibrium strategy. This set will be optimal if, given the total fund-raising cost, $\sum_{j=1}^{k_{\tau^o}} c(j)$, incurred, each individual decides to contribute rather than consume only the private good; i.e., if, in equilibrium, her net cost, $\sum_{j=1}^{k_{\tau^o}} c(j) - G^*_i$, is strictly less than her cutoff, $\hat{C}_i$. The next condition guarantees that this happens for every individual included in the set.

Assumption S. Let $k \in N$ be the largest index such that $c < \bar{c}_i$. Then it follows that

(i) $\sum_{i=1}^k (w_i - c(i)) > 0$, and (ii) for $i \leq k$: $f(w_i - \hat{C}_i) \leq \Phi^{-1}_k(\sum_{i=1}^k (w_i - c(i)),$

where $\Phi_k(G) \equiv \sum_{j=1}^k (\phi(G) - \overline{G}) + \overline{G}.$

We define drastic learning as a sequence of variable costs \( \{s(i)\} \) generating a monotonically increasing sequence of disposable incomes. The next corollary shows the fund-raiser’s response to drastic learning.

**Corollary 1.** Under drastic learning all potential donors are solicited.

This corollary says that in some cases the learning curve may be steep enough such that each additional solicitation would bring the greatest net gift among already requested individuals. Thus, the fund-raiser faces strong incentives to fund-raise more. Indeed, she ends up soliciting all potential donors to fully take advantage of cost savings.
3.3.2 Unobservability of the Fund-raiser Set

Our assumption regarding the observability of the the fund-raiser set is reasonable for small fund-raising campaigns. For others, it is not feasible for donors to keep track of the charity’s solicitations, but hold beliefs about them.

Given the optimal fund-raiser set $F^0$, one natural belief system is as follows: a solicited donor who is also in $F^0$ believes that the charity sticks to the solicitation strategy $F^0$, whereas a solicited donor outside $F^0$ believes that every richer individual is also solicited while lower income individuals are not.\(^{12}\) Each donor assumes that others act according to the stated beliefs. We show in the next proposition that when donors share these beliefs, the fund-raiser’s equilibrium strategy is the same whether or not it is observable.

**Proposition 3.** Suppose the fund-raiser set is unobservable to donors. Let $k$ be the highest index individual in $F^0$. Suppose gains from learning are exhausted, i.e., $c \geq c_j$ for every $j > k$. Then, under the beliefs described above, $F^0$ is sustained as a perfect Bayesian equilibrium.

It is plausible in big fund-raising campaigns that total initial donations do not cover total initial costs. Despite that, we observe that fund-drives are launched and charitable goods are provided from net donations because initial donors expect the charity to continue fund-raising up to individual $k$ to take advantage of learning economies. Thus, they know that eventually total donations exceed total costs.

Under the belief system described above, the fund-raiser does not necessarily have a commitment problem to its target strategy. Problems may arise if people hold a different belief system. For example, consider the following beliefs: if a donor in $F^0$ is contacted, he learns about the fund-drive and believes that the rest of $F^0$ will also be contacted, whereas if a donor outside $F^0$ is contacted, he attributes this to a mistake and believes that he is the only one contacted besides $F^0$.\(^{13}\) To illustrate the

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\(^{12}\) This belief system is grounded in a learning by fund-raising setting. To gain experience in the field, fund-raising may be carried out by few persons. Thus donors may not perceive deviations from $F^0$ as uncorrelated or isolated mistakes.

\(^{13}\) These beliefs are similar to “passive” beliefs often used in bilateral contracting in which one
tension between charity and donors, consider a case in which the fund-raiser solicits individuals in $F^0$ but there is still potential for learning. Notice that any solicited individual $i > k$ would take others’ contributions as: $G^*(F^0) - C(F^0) - c(|F^0| + 1)$. But, in fact, as the charity keeps fund-raising more and more, subsequent cost decreases are obtained without being noticed by additional donors. Consequently, the free rider problem is curbed to some extent, thus undermining the charity’s credibility to $F^0$. As a result, more than optimal fund-raising may be conducted at expense of the charitable good provision. This is consistent with the anecdotal evidence that schools often announce a target level of funds to be raised as well as the length of the fund-drive.  

3.4 Effects of a Fixed Cost on Optimal Fund-raising

Fixed costs, also called overhead costs—expenses such as rent, utilities, technology, accounting costs, legal costs, and marketing costs—are an important component of a charity’s cost structure. Donors and foundations are aware of the potential detrimental impact of these costs on the charitable good provision. Indeed, watchdog groups rank charities’ efficiency based on the administrative cost to total cost ratio. For instance, Charity Navigator suggests that for an acceptable charity this ratio ranges from 15% to 20%. Moreover, a study conducted by the center of philanthropy at Indiana University shows that of the 710 foundations that responded to the survey, 69% responded that their donations were intended to support charity’s overhead expenses.

To isolate the effect of a fixed cost on optimal fund-raising, we consider the following particular cost structure: a fixed cost $s$ and a constant marginal cost $c$. This is captured in my model by making $s(1) = s > 0$ and $s(i) = 0$ for every $i > 1$. Let $F^0(s)$ be the fund-raiser set when the fixed cost amounts to $s$. 

\footnote{party privately contracts with several others (e.g., Cremer and Riordan 1987; McAfee and Schwartz 1994).}

\footnote{For example, Duke University recently announced a new five-year fundraising campaign to raise $3.25$ billion for academic programs, medical education and health research, and its endowment.}
**Proposition 4.** Consider two fixed cost levels, \( s \) and \( s' \) such that \( s < s' \) and \( F^o(s) \) as well as \( F^o(s') \) are non-empty. Then,

(a) *Fund-raising increases in the setup cost*, i.e., \( F^o(s) \subseteq F^o(s') \)

(b) *Individual gross donations augments in the setup cost*, i.e., \( g_i(F^o(s)) < g_i(F^o(s')) \)

for every \( i \in F^o(s') \), but

(c) *The public good amount falls in the setup cost*, i.e., \( G^*(F^o(s)) > G^*(F^o(s')) \)

The intuition behind this Proposition is simple. From (3.2) it is clear that for individuals \( i > 1 \) cutoff costs rise in the fixed cost. Thus, given a higher fixed cost, the charity solicits more because it anticipates that individuals are more willing to give in order to *partially* recover the cost increase. Despite the rise in total gross donations generated by current and additional solicited donors, the level of the public good falls since individuals collectively do not make up for the totality of the rise in the cost. Thus, the two positive effects of the setup cost increase, more fund-raising and more gross donations, are neutralized by the negative effect of a rising cost burden on the supplied public good.

More fund-raising, even when optimally conducted, may in some cases indicate that the charity is actually less productive. This observation contrasts with our intuitive understanding of public good provision in a costless economy where, fixing individuals’ characteristics, a larger set of contributors signals a greater supply of the public good.

### 3.5 Excessive Fund-raising

Do charities spend too much in fund-raising? Does the cost structure matter in providing an answer for the previous question? To answer these questions, we first consider a benchmark setting in which the fund-raiser fixes for each donor \( i \) a minimum gift size \( t_i \). She publicly announces these and refuses donations below the
respective thresholds. In some sense the charity is exerting some individual pressure on each donor, even though giving is still voluntary. Thus, the free rider problem is still present.

For a fixed set \( F \), the fund-raiser maximization problem is:

\[
\max_{\{t_i\}_{i=1}^{\|F\|}} \sum t_i
\]

\( \text{s.t. } U(w_i - t_i, T) \geq U(w_i, \max\{T_{-i} - C(F), 0\}) \) for every \( i \in F^0 \)

where \( T = \sum_{j=1}^{\|F\|} t_j \).

The next observation shows that in the benchmark the fund-raiser also starts soliciting from the richest individual.

**Observation 3.** Individual \( i \) does not provide a gift above \( t_i^* \). Moreover \( t_{i+1}^* > 0 \) implies \( t_i^* > t_{i+1}^* \).

Observation 3 says that threshold gifts leave each individual indifferent to contributing the ”suggested” amount or not giving at all. As in the case with purely voluntary contributions and homogeneous preferences, the richer is the individual, the higher is the threshold gift imposed on her.

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15 This is actually a case of multilateral ”contracting” under positive externalities as in Segal (1999). It also resembles Andreoni 1998, in which the threshold for public good provision is determined by the production technology. In our setting, donors face individual thresholds endogenously determined by the fund-raiser.

16 The most acute form of commitment or pressure would add a target level of the charitable good such that if total donations are below that target, neither provision takes place nor refund is made. In this extreme case, the fund-raiser extracts from each individual, \( g_i^0 \), where it solves

\[
U(w_i - g_i^0, g_i^0) = U(w_i, 0).
\]

The critical public good level would be \( \sum_{i=1}^n g_i^0 \).

17 Let \( \hat{T}(w_i + T_{-i} - C, T_{-i} - C) \) represent the level of the public good that makes individual \( i \) indifferent between making up for that level given others’ contributions \( T_{-i} \), and not contributing at all. Thus, \( t_i = \hat{T}(w_i + T_{-i} - C, T_{-i} - C) - (T_{-i} - C) \). To establish some comparisons with regard to the pure altruism case, consider \( C = 0 \). There are two effects of \( T_{-i} \), on \( \hat{T} \): The first effect operates through aggregate income \( w_i + T_{-i} \) as in the voluntary case: \( \hat{T}_1 > 0 \). The second effect is negative and operates by increasing \( \hat{T}'s \) outside utility. Thus, \( \hat{T}_2 < 0 \). Moreover, if the latter effect is stronger than the former, then \( \hat{T}_1 + \hat{T}_2 < 0 \), which implies \( \frac{\partial U}{\partial T_{-i}} < -1 \). In this case, the substitution effects would be much stronger than in the pure voluntary case, as we see under Cobb-Douglas preferences with \( \alpha < \frac{1}{2} \).
Given the commitment to minimum gift sizes in the benchmark, the following observation is very intuitive:

**Observation 4.** The voluntary provision of the public good is below that in the benchmark.

Noteworthy, the fund-raiser can feasibly set a minimum gift size to individual $i$ corresponding to her voluntary contribution under $F^o, g^*_i(F^o)$. In other words, the equilibrium voluntary contribution profile $\{g^*_i(F^o)\}_i \subseteq F^o$, is a feasible solution to (3.3) when $F = F^o$. We then show that the fund-raiser may profitably deviate from that solution. To see this, suppose the charity exclusively "pressures" individual 1. By quasiconcavity of the utility function, the fund-raiser is able to extract from him a larger gift than voluntarily provided. In response, other individuals lower their contributions. Overall, the public good amount increases above the level supplied under voluntary contributions, by the strict normality assumption.

On the other hand, we observe that on the benchmark the charity aims to supply the greatest feasible level of the public good, even at the expense of donors’ aggregate welfare. In this sense, the resulting outcome is not efficient in a Samuelsian sense. However, a lack of commitment to minimum donations might lead the charity to conduct excessive fund-raising, in the sense that more solicitation expenses would have to be incurred to optimally supply a relatively low charitable good provision. To be more precise, let $F^*$ be the fund-raiser set on the benchmark.

**Definition 1.** We say that a charity conducts excessive fund-raising whenever she solicits a larger number of donors with respect to the benchmark, i.e., $F^* \subset F^o$.

The next proposition states that excessive fund-raising occurs when individuals have a low preference for the public good. For instance, when individuals have Cobb-Douglas preferences and the demand for the public good is $\alpha w$ this is the case for $\alpha < \frac{1}{2}$. Indeed, this case is the most relevant from an empirical perspective. Some works, as Zieschang (1985), have estimated the income effect to be 0.0342.

To understand the driving force causing excessive fund-raising, we first focus in
the benchmark when fund-raising is costless, and make the following assumption that is satisfied under a low preference for the public good.

Assumption: *In a costless economy* \(\frac{dt_i}{dT_i} < -1\).

A low preference for the public good generates a strong substitution effect among feasible requested donations. Thus, when the fund-raiser increases for one individual the minimum donation size by one dollar, the potential gift size for everyone else drops by more than one dollar. As a result, optimal fund-raising entails receiving an extra-larger donation from the richest individual and nothing from the rest. Now, once cost is introduced, it may be the case that the fund-raiser solicits more individuals to partially recover the initial cost, \(c(1)\). This means that each solicited individual \(i > 1\) provides a positive net donation and also that \(\sum_{i>1}(g_i(F^*) - c(i)) < c(1)\). Indeed, it is shown in the appendix that if more than two individuals are solicited, all of them are pivotal, in the sense that each individual contribution is critical to the public good provision. Since gifts are smaller when charity lacks commitment, it is then intuitive that relatively more fund-raising is conducted to recover the initial cost, as shown in Proposition 5.

**Proposition 5.** *Suppose more than two individuals are solicited under \(F^o\). Then, there is excessive fund-raising.*

Rose-Ackerman (1982) was the first to introduce the concept of excessive fund-raising in a competitive charitable market under costly fund-raising. She has in mind a benchmark in which charities act coordinately to maximize aggregate net donations. She points out that competition for donations triggers a relatively high level of fund-raising, without increasing aggregate gross donations. Rather, competition causes a switch of gifts among charities equally valued by donors. Thus, ultimately less public good is provided.

In contrast, I have a single charity in my setup, and the more fund-raising conducted, the greater the level of gross donations collected. The main source of excessive fund-raising in my model, then, is the lack of commitment to gift sizes.
Is learning an additional source of excessive fund-raising? Does excessive fund-raising worsen with a faster learning process?

To answer these questions we build on the following sequential cost function:

$$s(i) = \max\{s - \delta(i - 1), 0\}$$

where $\delta$ represents the learning rate. The next proposition shows how excessive fundraising changes when we move from constant returns to scale in fund-raising to learning by fund-raising.

**Proposition 6.** Consider two scenarios: constant returns to scale, $\delta_{nl} = 0$, and learning by fund-raising, $\delta_{l} \in (0, s)$. Excessive fund-raising is higher under learning, $\delta_{l} > 0$.

Proposition 6 says that excessive fund-raising worsens with learning. This result can be explained in terms of the effect of learning on optimal fund-raising in both cases, when charity commits to minimum gift size and when this is not feasible. On one hand, the charity fund-raises more to take advantage of cost decreases when there is no commitment. On the other hand, recall that if more than two individuals are solicited in the benchmark, it is just out of a cost recovery motive; in other words, all individuals are pivotal. Then, fund-raising shrinks with learning because for any subeconomy $F_i$, $i > 1$, total cost diminishes. Both effects push excessive fund-raising to a higher extent.

Following this logic, it seems intuitive that any increase in the rate of learning widens excessive fund-raising. Surprisingly, this statement is not necessarily correct.

**Proposition 7.** Excessive fund-raising is (potentially) non-monotonic in $\delta$.

Proposition 7 shows that excessive fund-raising is affected by the rate of learning in a complex way. The underlying force driving the previous result is that in the purely voluntary contribution case, the propensity to fund-raise an individual $i > 2$, reflected in her cutoff cost, is non-monotonic in the rate of learning, reaching an interior optimum. Now, to understand the source of this non-monotonicity, note first
from (3.4) that (i) there is some threshold rate for individual $i > 2$, $\delta_i^*$, such that $s(i)$ decreases in $\delta$ for $\delta < \delta_i^*$ and remains constant for $\delta \geq \delta_i^*$. (ii) $\delta_1^* > \delta_2^* > \delta_n^*$. From these two points we see that each disposable income difference between individual $i$ and the other lower index individuals decreases for $\delta \leq \delta_i^*$, and it increases for $\delta > \delta_i^*$. Because $i$'s cutoff cost is decreasing in the sum of these differences, (see 3.2) the marginal propensity to fund-raise individual $i$ increases for $\delta \leq \delta_i^*$ This result can be interpreted as coming from a relative cost-saving effect that makes it more likely that individual $i$ becomes a net contributor. The opposite happens when $\delta > \delta_i^*$.

Even though a slower learning process may actually bring more excessive fund-raising, it may surprisingly permit the fund-raiser to accumulate more experience as well, as formalized in the next Lemma.

**Lemma 2.** Consider two rates of learning: $\delta_h$ and $\delta_l$ such that $\delta_h > \delta_l > 0$. Then $c(\{F^o(\delta_h)\}) \leq c(\{F^o(\delta_l)\})$ is not always the case.

A slower learning process on one hand makes fund-raising a given number of individuals more costly, but on the other hand, it may encourage the charity to solicit more people, thus fostering learning. If the difference between learning rates is low enough, the latter effect may outweigh the former one. Consequently, a charity learning more slowly may end up accumulating more fund-raising experience reflected in a lower marginal cost. This may be important for a charity periodically running fund-drives because learning spillovers would also be intertemporal in this case.

In summary, a slower learning process may have negative consequences in a static sense because of excessive fund-raising. This same learning process may generate positive dynamic consequences because of deeper learning.

### 3.6 Extensions

In this section we provide three extensions. In the first one we introduce warm-glow giving. In the second one we consider the case in which population is divided among professional groups and learning is group specific. The second one addresses a setting in which there are decreasing returns to scale in fund-raising.
3.6.1 Warm-Glow Giving

In this section we consider warm-glow as an additional motive for giving and show how fund-raising incentives are affected by it. As in NY(2013), we assume that an individual gets warm-glow from her net contribution. Thus, let $u = u(x_i, \overline{G}, g_i - c(i))$ be person $i$’s utility function, which is increasing and strictly quasi-concave. Person $i$’s demand for the public good in a Nash equilibrium can be written as: $\overline{G}^* = \hat{f}(\overline{w}_i + G^*_i - C(F_{-i}), G^*_i - C(F_{-i}))$, where $\overline{w}_i = w_i - c(i)$ and $F_{-i} = F \setminus \{i\}$. Partial derivatives satisfy $0 < \hat{f}_1, < 1$ and $\hat{f}_2 \geq 0$ by normality of goods. If, in addition, $0 < \hat{f}_1 + \hat{f}_2 \leq \theta < 1$, then a unique Nash equilibrium obtains. Note that for $\hat{f}_2 = 0$, the warm-glow model reduces to the standard model.

To obtain a closed form solution that facilitates our comparative statics analysis, we consider the following utility for all $i$:

$$U_i(x_i, G, g_i) = (1 - \beta) \ln x_i + \beta \ln(\gamma G + (1 - \gamma) g_i)$$

where $\beta \in (0, 1)$, $\gamma = \frac{\alpha - \beta}{\alpha(1 - \beta)}$ and $\alpha \in (\beta, 1)$. Under this specification warm glow is a substitute for altruism. The demand for the public good in this case is $\overline{G}^* = \beta w_i + \frac{\beta}{\alpha} G_{-i}$. Ignoring the costly aspect of fund-raising, note that when $\alpha = 1$, $\overline{G}^* = \beta (w_i + G_{-i})$. Thus, individuals give out of a pure public good motive. On the other hand, when $\alpha = \beta$, then $\overline{G}^* = \beta w_i + G_{-i}$ and $g_i^* = \beta w_i$. Hence, individuals give motivated by pure warm-glow.\(^\text{18}\) Thus, the lower $\alpha$ is, the stronger is the warm-glow motive.

It can be shown that Proposition 2 holds under this utility specification, and individual $i$’s cutoff cost is given by

$$\overline{c}_i = \hat{w}_i - \frac{\alpha - \beta}{\beta} \sum_{j=1}^{i} (\hat{w}_j - \hat{w}_i).$$ (3.5)

\(^{18}\) The parameter $\alpha$ represents the altruism coefficient as introduced in Andreoni (1989). It is a measure of the relative strength of the public good motive for giving.
It is intuitive that the more warm-glow people experience, the more incentivized is the fund-raiser to solicit more, since the free rider problem is less severe. Indeed, Eq.(3.5) implies that \( c_i \) increases as \( \alpha \) decreases. Thus, fund-raisers learn more on the job when the warm-glow motive is strong. Moreover, if a net free-rider is identified, the fund-raiser is more likely to solicit her as a learning investment.

3.6.2 Group Specific Learning

Suppose the fund-raiser divides the set of potential donors into \( m \geq 1 \) groups, depending on their professional activities. She believes that each member of group \( i \) independently draws his income from a discrete distribution, \( \tilde{w}_i \), with mean \( E[\tilde{w}_i] \). We assume that the charity learns by fund-raising within a group, but this experience does not translate into cost decreases in soliciting members of other groups. Thus, let \( s_i(j) \) be the sequential cost of fund-raising the \( j \)th individual in group \( i \). The fund-raiser’s strategy is to choose the number of donors to be contacted from each group. To focus the analysis on the fund-raiser side, we continue to assume that donors have no uncertainty about the income profile in the population. Moreover, to simplify the analysis, we consider identical homothetic preferences so that \( f(w) = \alpha w \) for some \( \alpha \in (0,1) \). Without loss of generality we rank groups according to their average disposable incomes: \( E[\tilde{w}_1] - a_{s_1} \geq \ldots \geq E[\tilde{w}_m] - a_{s_m} \). The fund-raiser’s equilibrium strategy is stated in Proposition 9.

**Proposition 8.** Let group \( i \)'s cutoff be given by

\[
\bar{c}_i = E[\tilde{w}_i] - a_{s_i} - \frac{1-\alpha}{\alpha} \sum_{j=1}^{i} n_j \left[ (E[\tilde{w}_j] - E[\tilde{w}_i]) + (a_{s_i} - a_{s_j}) \right]
\]

Then \( \bar{c}_1 \geq \bar{c}_2 \geq \ldots \geq \bar{c}_n \). Moreover, every member of group \( i \) is solicited iff \( c < \bar{c}_i \).

The fund-raiser optimally treats each group member as having mean disposable income \( E[\tilde{w}_i] - a_{s_i} \). It is intuitive, then, that the fund-raiser either contacts no members of group \( i \) iff \( c \geq \bar{c}_i \) or solicits all of them iff \( c < \bar{c}_i \). Thus, group \( i \)'s cutoff cost is interpreted as the average propensity of its members to pay for \( c \). Note that
an increase in the extent of learning economies within a group $i$, either due to the presence of more members or to a higher speed of learning, augments the group’s mean disposable income. Thus, group $i$ is more likely to be solicited and any other group less so.

We say that groups $i$ and $j$ merge if the fund-raiser knows $n_i$ and $n_j$ but is not able to distinguish among members of these groups. (NY, 2012)

Consider the case in which the technology for fund-raising any given group is $s(j)$ and two groups merge. We assume full learning spillovers within the merged group. That is, the fund-raising cost function for this group is still $s(j)$. One may think that since the merger brings more potential for learning, the public good provision increases. But this is not always the case. After a merger, available information becomes coarser in the sense that the fund-raiser does not distinguish individuals in the merged group. This effect potentially hurts the public good provision as shown in the next Lemma:

**Lemma 3.** Suppose groups $i$ is solicited and group $j$ is not and they merge. If the merged group $ij$ is not solicited, i.e.,

$$E[\tilde{w}_{ij}] - a_{s_{ij}} - \frac{1 - \alpha}{\alpha} \sum_{k \neq i, j; E[\tilde{w}_k] > E[\tilde{w}_{ij}]} n_k \left[ (E[\tilde{w}_k] - E[\tilde{w}_{ik}]) + (a_{sk} - a_{s_{ij}}) \right] \leq c,$$

where $E[\tilde{w}_{ij}]$ and $a_{s_{ij}}$ are respectively the mean income and average cost of the merged group $ij$, then the public good provision after the merger diminishes.

Lemma 3 makes explicit the tradeoff generated after a merger. On one hand, learning increases, i.e., $a_{s_{ij}} < a_{s_i}$, which makes $\bar{c}_{ij}$ increase with respect to $\bar{c}_i$. On the other hand, coarser information hurts fund-raising in the sense that $E[\tilde{w}_{ij}] < E[\tilde{w}_i]$. This effect makes $\bar{c}_{ij}$ fall below $\bar{c}_i$. If the latter effect is stronger, the merged group $ij$ is not fund-raised. Thus, members of group $i$, who were optimally fund-raised before the merger, are no longer identified by the fund-raiser, learning spillovers do not justify the inclusion of members of the merger group in the solicitation set. As a result, the public good provision declines.
Even though we consider homogeneous preferences in this work, this result trivially extends to a setting in which donors may have different taste toward the public good, $\alpha_i$.

Previous works, such as Andreoni (2013), examine the effect of diversity on the public good provision from the donor’s side. The results of this section suggest that a better understanding of this matter must include as well the fund-raiser’s response given potential learning spillovers.

### 3.6.3 Decreasing Returns to Scale

In this section we consider a charity constrained by physical and human resources. We envision fund-raising as an increasingly costly process. The next proposition formalizes the fund-raiser’s solicitation strategy in this setting.

**Proposition 9.** Suppose $s(1) \leq s(2) \leq s(3) \leq \ldots \leq s(n)$. Let $\bar{\phi}(G) \equiv \phi(G) - G$, and donor $i$’s cost cutoff be given by

$$c_i(\hat{w}_i) = \hat{w}_i - \Phi(\sum_{j=1}^{i-1} (\hat{w}_j - \hat{w}_i))$$

Then, $\bar{c}_1 \geq \bar{c}_2 \geq \ldots \geq \bar{c}_n$, and $F^o = \{i \in N | c < \bar{c}_i\}$.

In general, as costs increase, the charity becomes more conservative soliciting an additional subject, because of both the free-rider problem and the increase in marginal cost. Furthermore, absent a learning motive, once a net free rider is identified, the solicitation process stops. Consequently, as in NY (2013), every solicited individual is a net contributor in $F^o$. Indeed, it is intuitive that as the charity experiences more rapid diseconomies of scale, the fund-raiser set shrinks as well as the public good provision. Moreover, the degree of excessive fund-raising tends to diminish.

### 3.7 Conclusion

In this paper we extend the literature on charitable fund-raising by bringing to the center of the analysis the role of solicitation technology in optimal fund-raising, which
is characterized in terms of donors’ preference and incomes as well as solicitation costs. We also define excessive fund-raising in a single charity environment with respect to a setting in which the fund-raiser commits to minimum gift sizes.

We specially consider a charity which becomes a more efficient solicitor through time. This fact is not innocuous in terms of optimal fund-raising and excessive fund-raising. On the contrary, on one hand, it determines an investment in learning incentive. For instance, some charities may launch a fund-drive even when initial donations are not sufficient to cover initial costs. However, it is common knowledge that the charity fund-raises more to achieve cost reductions, which ensures the charitable good provision. On the other hand, excessive fund-raising worsens when we move from a constant return to scale technology to a setting with learning by fund-raising. Moreover, excessive fund-raising is non-monotonic in the rate of learning.

From a policy perspective, the introduction of government grants, either direct or matching ones, could reinforce or counteract the advantages of learning. Our results also suggest that some sort of commitment to alleviate the free-rider problem is more valuable in environments where the fund-raiser learns on the job.

As an extension we work a setting in which the population is divided among professional groups and the fund-raiser believes that each member of a given group independently draws his income from a discrete distribution. Moreover, learning takes place exclusively within each group. We find that moving to a less diverse population by merging groups may hurt public good provision, despite full learning spillovers within merged groups. Thus, diversity may actually be beneficial for the fund-raiser.

For future work, it may be interesting to consider how experience generates wage premiums in the market for fund-raisers. On one hand, a more experienced fund-raiser is highly demanded since she rises the public good provision, but on the other hand, a higher wage increases fund-raising costs, thus dampening the charitable good.

It would also be interesting to address in a formal model the divergence of objectives between a charity and a fund-raiser. The charity’s objective is to maximize current level of the public good. However, given a high demand for experienced
fund-raisers, they may over-solicit to learn more. This sort of reasoning may justify why fund-raisers are paid a fixed wage regardless of the level of funds they raise.
Appendix A

Appendix to Chapter 2

This appendix contains the proofs of Lemmas 1, 2 and Propositions 1, 3 and 5. The remaining proofs as well as the formal details of the extensions are relegated to an online appendix. In what follows, \( \Phi_F(G) \equiv \sum_{i \in F} (\phi_i(G) - G) + G; F_C = \{i \in F | g_i^*(F) > 0\} \); and \( F_{-i} \equiv F \setminus \{i\} \). For Lemma 1, we first prove Lemma A1.

**Lemma A.1.** If \( G_p F_q \leq 0 \), then \( \Phi_F(\overline{G}^s(F)) = \sum_{i \in F_C} w_i - C(F) \) and \( \Phi_F(G^s(F)) \geq \sum_{i \in F} w_i - C(F) \).

**Proof.** Suppose \( \overline{G}^s(F) > 0 \). If \( i \in F_C \), then \( \phi_i(\overline{G}^s(F)) = w_i + G^-_{-i}(F) - C(F) \). Summing over all \( i \in F_C \), and arranging terms yield \( \Phi_{F_C}(\overline{G}^s(F)) = \sum_{i \in F_C} w_i - C(F) \). Moreover, since \( \phi_i(\overline{G}^s(F)) - G^s(F) \geq w_i \) for any \( i \in F \setminus F_C \), summing over all \( i \in F \) yields \( \Phi_F(\overline{G}^s(F)) \geq \sum_{i \in F} w_i - C(F) \), as desired. \( \square \)

**Proof of Lemma 1.** \((\Longrightarrow)\): Let \( F^o \) be the unique optimal fund-raiser set. Suppose that \( i \in F^o \) but, contrary to C1, \( g_i^*(F^o) \leq c_i \). Since \( F^o \neq \emptyset \), clearly \( \overline{G}^s(F^o) > 0 \). Next, we show that \( F^o = F_C^o \). Since \( F_C^o \subseteq F^o \) by definition, we only show that \( F^o \subseteq F_C^o \). Suppose not. Then, \( j \in F^o \) but \( j \notin F_C^o \) for some \( j \). That
is, person \( j \) is contacted even though \( g_j^*(F^o) = 0 \). Then, Lemma A1 reveals that 
\[ \Phi_{F_C}(\overline{G}^*(F^o)) < \sum_{i \in F_C^0} w_i - (C(F^o) - c_j) \leq \Phi_{F_C^0}(G^*(F^o) - C(F^o_j)) \]  
Since \( \Phi_{F_C^0} > 0 \), we have \( G^*(F_C^0) - C(F^o_j) > G^*(F^o) \). Given this, note that if \( i \notin F^o_C \) under cost \( C(F^o) \), then \( i \notin F^o_C \) under cost \( C(F^o_j) \). Thus, \( F^o_{-j,C} \subseteq F^o_C \), which implies \( \overline{C}^*(F^o_j) = G^*(F^o_C) - C(F^o_j) \), and in turn, \( \overline{G}^*(F^o_j) > \overline{G}^*(F^o) \), contradicting the optimality of \( F^o \). Hence, \( F^o = F^o_C \).

Now, recall our hypothesis that \( i \in F^o \) and \( g_i^*(F^o) \leq c_i \). We also know that \( g_i^*(F^o) > 0 \), and thus \( \phi_i(\overline{G}^*(F^o)) - \overline{G}^*(F^o) = w_i - g_i^*(F^o) \). Inserting this into the equilibrium condition in Lemma A1: 
\[ \Phi_{F^o}(\overline{G}^*(F^o)) = \sum_{j \in F^o} w_j - C(F^o) \]
we obtain
\[
\Phi_{F_{-i}^o}(G^*(F^o)) = \sum_{j \in F^o_{-i}} w_j - C(F^o) + g_i^*(F^o)
\]
\[
= \sum_{j \in F^o_{-i}} (w_j - c_j) - (c_i - g_i^*(F^o))
\]
\[
\leq \sum_{j \in F^o_{-i}} (w_j - c_j)
\]
\[
\leq \Phi_{F_{-i}^o}(\overline{G}^*(F^o_{-i}))
\]
where the last inequality is due to Lemma A1. Then, given that \( \Phi_{F_{-i}^o} > 0 \), we have \( \overline{G}^*(F^o) \leq \overline{G}^*(F^o_{-i}) \). But, this contradicts the optimality of \( F^o \) either because \( \overline{G}^*(F^o) < \overline{G}^*(F^o_{-i}) \), or because \( \overline{G}^*(F^o) = \overline{G}^*(F^o_{-i}) \) and \( C(F^o) > C(F^o_{-i}) \). As a result, \( g_i^*(F^o) > c_i \).

To prove that \( F^o \) must also satisfy C2, suppose, by way of contradiction, that individual \( i \) is not in \( F^o \), but that if added to \( F^o \), \( i \)'s contribution would satisfy \( g_i^*(F^o \cup \{i\}) - c_i > 0 \). To economize on notation, let \( F^o \cup \{i\} \equiv F^+ \) and \( F^+_{C,-i} \equiv F^+_C \setminus \{i\} \). By definition, \( F^+_{C,-i} \subseteq F^o \). Moreover, since \( c_i > 0 \), we have \( g_i^*(F^+) > 0 \), which means that \( \phi_i(\overline{G}^*(F^+)) - \overline{G}^*(F^+) = w_i - g_i^*(F^+) \). Inserting this into the equilibrium condition, 
\[ \Phi_{F^+_C}(\overline{G}^*(F^+)) = \sum_{j \in F^+_C} w_j - C(F^+) \]
we obtain
\[ \Phi_{\mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} (\mathcal{G}^\ast (F^+)) = \sum_{j \in \mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} w_j - C(F^o) + (g_i^\ast (F^+)) - c_i. \]

If \( F_{\mathcal{C}_{i} \setminus i}^+ = F^o \), then

\[ \Phi_{\mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} (\mathcal{G}^\ast (F^+)) = \sum_{j \in \mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} w_j - C(F^o) + (g_i^\ast (F^+)) - c_i > \sum_{j \in \mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} w_j - C(F^o) = \Phi_{\mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} (\mathcal{G}^\ast (F^o)), \]

where the last equality follows because \( F^o = F_{\mathcal{C}_{i} \setminus i}^o \). But, given that \( \Phi_{\mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} > 0 \), we then have \( \mathcal{G}^\ast (F^+) > \mathcal{G}^\ast (F^o) \), which contradicts the optimality of \( F^o \).

If \( F_{\mathcal{C}_{i} \setminus i}^+ \neq F^o \), or equivalently \( F_{\mathcal{C}_{i} \setminus i}^+ \subset F^o \), then, by definition of \( \Phi_{\mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} \),

\[ \Phi_{\mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} (\mathcal{G}^\ast (F^+)) = \Phi_{\mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} (\mathcal{G}^\ast (F^+)) + \sum_{j \in \mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} (\phi_j (\mathcal{G}^\ast (F^+)) - \mathcal{G}^\ast (F^+)). \]

Since \( \Phi_{\mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} (\mathcal{G}^\ast (F^+)) = \sum_{j \in \mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} w_j - C(F^o) + (g_i^\ast (F^+)) - c_i \) and \( \phi_j (\mathcal{G}^\ast (F^+)) - \mathcal{G}^\ast (F^+)) \geq w_j \) (because \( j \in \mathcal{F}_{\mathcal{C}_{i} \setminus i}^+ \) and thus a free-rider in the set \( F^+ \)), it follows that

\[ \Phi_{\mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} (\mathcal{G}^\ast (F^+)) \geq \sum_{j \in \mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} w_j - C(F^o) + (g_i^\ast (F^+)) - c_i \geq \sum_{j \in \mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} w_j - C(F^o). \]

Note again that \( \sum_{j \in \mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} w_j - C(F^o) = \Phi_{\mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} (\mathcal{G}^\ast (F^o)) \) because \( F^o = F_{\mathcal{C}_{i} \setminus i}^o \). This implies that \( \Phi_{\mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} (\mathcal{G}^\ast (F^+)) > \Phi_{\mathcal{F}_{\mathcal{C}_{i} \setminus i}^+} (\mathcal{G}^\ast (F^o)) \), which, in turn, implies that \( \mathcal{G}^\ast (F^+) > \mathcal{G}^\ast (F^o) \), contradicting the optimality of \( F^o \). As a result, \( i \) is in \( F^o \), which means \( F^o \) also satisfies C2.

(\( \iff \)): We prove the uniqueness of the equilibrium fund-raiser set. Suppose, on the contrary, that there are two distinct sets \( F \) and \( F' \) each satisfying C1 and C2. Note that \( F \subset F' \) or \( F' \subset F \) cannot be the case: otherwise, either C1 or C2 would be violated for at least one set. Next, take any \( i \) such that \( i \in F' \) but \( i \notin F \). By C2, \( i \) would be a net free-rider in \( F \cup \{i\} = F^+ \), i.e., \( g_i^\ast (F^+) - c_i \leq 0 \), which implies that \( G_{i}^0 \leq G_{-i}^\ast (F^+) - C(F) \). Therefore,

\[ G_{i}^0 = f_i (w_i - c_i + G_{i}^0) = f_i (w_i - c_i + G_{-i}^\ast (F^+) - C(F)) \leq \mathcal{G}^\ast (F^+). \]
Note also that \( \overline{G}^*(F^+) \leq \overline{G}^*(F) \) because, by the first part, removing a net free-rider increases the equilibrium public good. Together, \( G_i^0 \leq \overline{G}^*(F) \). In addition, since \( i \) is a net contributor in \( F' \) by C1, i.e., \( g_i^*(F') - c_i > 0 \), we have \( G_i^0 > G_{-i}^*(F') - C(F_{-i}') \) and thus,

\[
G_i^0 = f_i(w_i - c_i + G_i^0) > f_i(w_i - c_i + G_{-i}^*(F') - C(F_{-i}')) = \overline{G}^*(F'),
\]

implying that \( G_i^0 > \overline{G}^*(F') \). Together, the two inequalities reveal that \( \overline{G}^*(F) \geq G_i^0 > \overline{G}^*(F') \), which, in turn, reveals \( \overline{G}^*(F) > \overline{G}^*(F') \). But, a symmetric argument shows that \( \overline{G}^*(F) < \overline{G}^*(F') \), yielding a contradiction. Hence, \( F = F' \).

\[\square\]

**Proof of Proposition 1.** We first claim that if \( \overline{G}^*(F) > 0 \) for some \( F \), then \( g_i^*(F) - c_i > 0 \) if and only if \( G_i^0 > \overline{G}^*(F) \). Note that \( \phi_i(\overline{G}^*(F)) - \overline{G}^*(F) = w_i - g_i^*(F) \), or equivalently \( \phi_i(\overline{G}^*(F)) - \overline{G}^*(F) = (w_i - c_i) - (g_i^*(F) - c_i) \) if \( g_i^*(F) > 0 \); and \( \phi_i(\overline{G}^*(F)) - \overline{G}^*(F) \geq w_i \) if \( g_i^*(F) = 0 \). Since \( \phi_i(G_i^0) - G_i^0 = w_i - c_i \) by eq. (2.2), and \( \phi_i' > 1 \), the claim follows.

Next, for \( G_i^0 \geq G_{i+1}^0 \), it easily follows that \( \Delta_i \geq \Delta_{i+1} \) and \( \Delta_1 = G_1^0 > 0 \). Let \( k \in N \) be the largest index with \( \Delta_k > 0 \). Since \( \Phi_k(0) = 0 \), \( \Phi_k' > 0 \), and \( \sum_{j=1}^{k+1}(w_j - c_j) > 0 \), there is a unique solution, \( \overline{G}^* = \Phi_k^{-1}\left(\sum_{j=1}^{k+1}(w_j - c_j)\right) > 0 \) to \( \Phi_k(\overline{G}^*) = \sum_{j=1}^{k+1}(w_j - c_j) \). \( \overline{G}^* \) is an equilibrium because \( \sum_{j=1}^{k+1} c_j - G_{-i}^* \leq \sum_{j=1}^{k+1} c_j - \sum_{j \neq i} c_j = c_i \), and \( c_i < \hat{C}_i \) by assumption. Moreover, each \( i = 1, \ldots, k \) is a net contributor because \( G_i^0 > \overline{G}^* \), and thus must be solicited by Lemma 1. By the same token, each \( i = k + 1, k + 2, \ldots, n \) is a net free rider because \( G_i^0 \leq \overline{G}^* \), and thus must be left outside the fundraiser set.

\[\square\]

**Proof of Lemma 2.** From Proposition 1, define \( \Delta_i(c) \equiv \Phi_i(G_i^0(c)) - \sum_{j=1}^{i} w_j + ic \) such that \( i \in F^o \) if and only if \( \Delta_i(c) > 0 \). Substituting for \( \phi_i = \phi \), it follows that \( \Delta_i(c) = -1/\phi'(G_i^0) - 1 < 0 \) since \( \phi' > 1 \). Hence, \( i \in F^o \) if and only if \( c < c_i \), where
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. First, consider condition (1). Since,

, it follows that

, which implies that

. Next, assume condition (2), and by way of contradiction, that

. Then,

, which implies that

. Moreover,

, because individuals \{m''+1, \ldots, m'\} are net free-riders under \(w''\). Thus, \(\sum_{i=1}^{m'} (w'_i - c) \geq \sum_{i=m'+1}^{m''} (w''_i - c)\), or equivalently,

, which contradicts our hypothesis that \(w'\) Lorenz dominates \(w''\). Thus, \(G'^{\text{al}} < G'^{\text{al}}\).

Finally, consider condition (3). Let \(G'^{\text{al}}\) be the equilibrium level of the public good if agents 1, \ldots, \(m'\) constituted the whole economy under \(w''\). Since individuals
Proof of Proposition 3. Our hypothesis that \( L_i(w^n) > L_i(w') \) for every \( i < n \) implies that \( v^n_i < v_n' \) and \( v^n_i > v'_1 \). Next, \( v^n_i > 0 \) since he is assumed a contributor for \( c = 0 \) in the standard model. Hence, for \( c \in [0, v^n_1] \) all individuals are net contributors and thus \( F^{\alpha} = F^{\alpha'} \). Since \( L_n(w^n) = L_n(w') \), this means that \( \Gamma^{\alpha'} = \Gamma^{\alpha''} \). For \( c \in [v^n_1, v^n_i] \), we clearly have one of the three conditions in Lemma A2 satisfied, implying that \( \Gamma^{\alpha'} < \Gamma^{\alpha''} \). Finally, for \( c > v^n_1 \), no fund-raising takes place and so \( \Gamma^{\alpha'} = \Gamma^{\alpha''} = 0 \), completing the proof. 

Proof of Proposition 5. Suppose \( G^0_1 > G^0_2 > \ldots > G^0_n > 0 \). Note first that the order of types is preserved under replicas since \( G^0_i \) depends only on \( (u_i, w_i, c_i) \). In an \( r \)-replica economy, define \( \Delta_i = \Delta_i(r) = \sum_{j=1}^{i} r(\phi_j(G^0_i) - G^0_i) + G^0_i - \sum_{j=1}^{i} r(w_j - c_j) \), or re-arranging terms,

\[
\Delta_i(r) = G^0_i - r \sum_{j=1}^{i} [(w_j - c_j) - (\phi_j(G^0_i) - G^0_i)] . \tag{A.1}
\]

Since \( G^0_j > G^0_i \) by our indexing, and \( \phi_j > 1 \) by strict normality, it follows that \( w_j - c_j > \phi_j(G^0_i) - G^0_i \) for every \( j < i \). Moreover, since \( w_i - c_i = \phi_i(G^0_i) - G^0_i \) by definition of \( G^0_i \), eq.(A.1) implies that for \( i > 1 \), \( \Delta_i(r) \) is strictly decreasing in \( r \), whereas \( \Delta_1(r) = G^0_1 \), which is independent of \( r \).

To prove part (a), observe that since \( \Delta_1(r) = G^0_1 \) for any \( r \), type-1 donors are always solicited. Consider \( i > 1 \). Note that \( \Delta_i(1) \leq 0 \) implies that \( \Delta_i(r) \leq 0 \) for any \( r \geq 1 \). If, on the other hand, \( \Delta_i(1) > 0 \), then there exists \( r_i < \infty \) such that...
\( \bar{\Delta}_i(r_i) \leq 0 \). This means \( \bar{\Delta}_i(r) < 0 \) for \( r \geq r_i \). Moreover, given that \( \bar{\Delta}_i(r) > \bar{\Delta}_{i+1}(r) \), it follows that \( r_{i+1} \leq r_i \) for \( i > 1 \). Thus, type-\( i \) donors are not solicited in a replica economy with \( r \geq r_i \) for \( i > 1 \).

To prove part (b), note that as \( r \to \infty \), only type-1 will be asked for donations by part (a). Note also that given the symmetry within the limiting group, the equilibrium must be symmetric. Since \( G_1^0 > 0 \), it follows that \( \overline{G}^* > 0 \) and \( \overline{G}^* < G_1^0 \); otherwise \( \overline{G}^* \geq G_1^0 \) would imply no contribution, yielding a contradiction. The symmetric equilibrium means that each type-1 donor is a net contributor, which means that \( \overline{G}^* \) is monotonically converging to \( G_1^0 \). Since \( G_1^0 \) is a finite level, we must have \( g_1^* \to c_1 \); otherwise, if, in the limit, \( g_1^* - c_1 > 0 \), then \( \overline{G}^* \to \infty \), making everyone contribute nothing, a contradiction. \( \square \)
Appendix B

Appendix Chapter 3

Proof of Proposition 1. Fix an arbitrary fund-raiser set, \( F \neq \emptyset \), whose total fundraising cost is \( C(F) > 0 \). Suppose \( C(F) > \max_{\emptyset \neq F} \tilde{C}_i \). Then, individual \( i \)'s best response to \( G^*_{-j}(F) = 0 \) is \( g^*_i(F) = 0 \). Thus, the zero-contribution profile is an equilibrium, resulting in \( G^*(F) = 0 \).

Consider the optimal fund-raiser set, \( F^o \). Clearly, \( G^*(F^o) > 0 \) implies that some agents have been contacted, and thus \( F^o \neq \emptyset \). Conversely, suppose that in equilibrium, \( F^o \neq \emptyset \), but \( G^*(F^o) = 0 \). Then, since \( C(F^o) > 0 \), the charity has a strict incentive to choose \( F = \emptyset \) and incur no cost. Hence, \( G^*(F^o) > 0 \). \( \square \)

Proof of Observation 1. By closely following Andreoni (1988), equilibrium contributions in any subgame \( F \) can be characterized as

\[
    g^*_i(F) - c = (w_i - c) - w^*(F) \tag{B.1}
\]

where \( w^*(F) = \phi(G^*(F)) - G^*(F) \) and \( \phi \equiv f^{-1} \). Moreover, \( g_i(F^o) - c > 0 \) for every \( i \in F^o \). Otherwise, the fund-raiser may save on costs, which contradicts the optimality of \( F^o \). Suppose the optimal strategy is to fund-raise \( k \) individuals. Then
Note that if \( k = 1 \) the result trivially follows since \( f'(.) > 0 \) and \( w_1 - c_1 > w_j - c_j \) for any \( j > 1 \). Next, consider the case in which the first \( i \) individuals are included in the set, where \( i < k \). Denote \( G^*_{F \cup \{l\}} = G^* (F) \). Note that by including any individual \( l < i \) such that \( g_{l}^* > 0 \), by (B.1), it follows that every individual \( j \leq i \) is also a contributor. Moreover, \( G^* (F_{+i}) \) solves

\[
(i + 1)(\phi(G^* (F_{+i}) - G^* (F_{+il}))) + G^* (F_{+i}) = w_l - c_l
\]

Thus, \( G^* (F_{+(i+1)}) \geq G^* (F_{+i}) \) for any \( l \geq i + 1 \), since \( w_{i+1} - c_{i+1} \geq w_l - c_l \) and \( \phi' > 0 \).

### Definition B.1

Let \( G^0_i(c) \) be the “drop-out” level of the public good for person \( i \) under net income \( w - c \), which uniquely solves \( f(w_i - c + G^0_i) = G^0_i \). By convention \( G^0_i(c) = 0 \) whenever \( w_i - c \leq 0 \).

### Lemma B.1

If \( G^* (F_i) > 0 \) for some \( F_i \), then \( g^*_i(F_i) - c(i) > 0 \) if and only if \( G^0_i(c(i)) > G^* (F_i) \)

**Proof.** Following closely Lemma 1 in NY(2013), note that \( \phi_i(G^* (F_i)) - G^* (F_i) = w_i - g^*_i(F) \), or equivalently \( \phi_i(G^* (F_i)) - G^* (F_i) = (w_i - c(i)) - (g^*_i(F_i) - c(i)) \) if \( g^*_i(F_i) > 0 \); and \( \phi_i(G^* (F_i)) - G^* (F_i) \geq w_i \) if \( g^*_i(F_i) = 0 \). Since \( \phi_i(G^0_i(c(i))) - G^0_i(c(i)) = w_i - c(i) \), and \( \phi'_i > 1 \), the Lemma follows.

### Corollary B.1

If \( G^* (F_i) > 0 \) for some \( F_i \), then \( g^*_i(F_i) - c(i) > 0 \) if and only if \( \Delta_i(c(i)) > 0 \).

**Proof.** This follows from Lemma B.1. and Andreoni and McGuire (1993).
**Definition B.2.** Let \( \Phi_i(G) \equiv \sum_{j=1}^{i} (\phi(G) - G) + G \), where \( \phi \equiv f^{-1} \) and \( \Phi_i(G) > 0 \).

Define \( \Delta_i(c(i)) \equiv \Phi_i(G^0_i(c(i))) - \sum_{j=1}^{i} (w_j - c(j)) \)

**Proof of Lemma 1.** This proof follows closely NY(2013). Define \( \tau_i \), the value of \( c \) making \( \Delta_i(c(i)) = 0 \). Simplifying terms, \( \tau_i \) solves:

\[
i[\phi(G^0_i(c(i))) - G^0_i(c(i))] + G^0_i(c(i)) - \sum_{j=1}^{i} (w_j - s(j)) + ic = 0.
\]

Since \( \phi(G^0_i(c(i))) - G^0_i(c(i)) = w_i - s(i) - c \), from the equation above, we have

\[
G^0_i(\tau_i + s(i)) = \sum_{j=1}^{i} [(w_j - w_i) + (s(i) - s(j))].
\]

In addition, given that \( \bar{\phi}(G) \equiv \phi(G) - G \), we also have \( \bar{\phi}(G^0_i(\tau_i + s(i))) = w_i - s(i) - \tau_i = \bar{\phi} \left( \sum_{j=1}^{i} [(w_j - w_i) + (s(i) - s(j))] \right) \), which reduces to

\[
\tau_i = w_i - s(i) - \bar{\phi} \left( \sum_{j=1}^{i} [(w_j - w_i) + (s(i) - s(j))] \right).
\]

Let \( \hat{w}_i = w_i - s(i) \). Then \( \tau_i = \hat{w}_i - \bar{\phi}(\sum_{j=1}^{i} (\hat{w}_j - \hat{w}_i)) \). Finally, notice that \( \Delta_i > 0 \) iff \( c < \tau_i \), then, by the previous corollary, the proposition follows.

**Proof of Proposition 2.** By noting that if \( G^* (F_i) > 0 \) then it satisfies \( \Phi_i(G^* (F_i)) = \sum_{j=1}^{i} (w_j - c(j)) \) and by Lemma B1, it follows that \( \Delta_i(w_i - c(i)) > 0 \) iff \( g^*_i (F_i) - c(i) > 0 \). Consider first the case where \( i = 1 \). Take the lowest index individual \( k \geq i \) s.t. (i) \( \Delta_k(w_k - c(k)) > 0 \) and (ii) \( \sum_{j=1}^{k} (w_j - c(j)) \). Clearly \( G^* (F_k) > G^0_k(c(k)) > 0 = G^* (\{G\}) \). Now consider \( i > 1 \) and individuals \( 1, 2, ..., i - 1 \) are solicited. Take the lowest index individual \( k \geq i \) s.t. \( \Delta_k(w_k - c(k)) > 0 \) Notice that \( g^*_k (F_k) > 0 \). Thus, \( G^* (F_k) > 0 \). Note also that \( \Phi_{i-1}(G^* (F_k)) = \sum_{j=1}^{i-1} (w_j - c(j)) + \sum_{j=1}^{k} [g^*_j (F_k) - c(j)] \). Thus, \( G^* (F_k) > G^* (F^{i-1}) \) iff \( \sum_{j=1}^{k} [g^*_j (F_k) - c(j)] > 0 \).
Let \( w'_j - c'(j) = w_j - c(j) \) for \( j < i \) and \( w'_j - c'(j) = a_{w-c}(ik) \) for \( i \leq j \leq k \). This implies \( \Delta_i(w'_i - c'(i)) = \Delta_{i+1}(w'_{i+1} - c'(i+1)) = \ldots = \Delta_k(w'_k - c'(k)) \). Thus \( g^*_j(w'_j, F^k) - c'(j) > 0 \) for every \( j = i, i+1, \ldots, k \) iff \( \Delta_i(a_{w-c}(ik)) > 0 \). Note also that

\[
\sum_{j=i}^k g^*_j(w'_j, F^k) - c'(j) = \sum_{j=i}^k a_{w-c}(ik) + \sum_{j=1}^{i-1} (w_i - c(i)) - \sum_{j=1}^{i-1} [g^*_j(w'_j, F^k) - c'(j)] \\
= \sum_{j=1}^k (w_i - c(i)) - \sum_{j=1}^{i-1} [g^*_j(w'_j, F^k) - c(j)] \\
= \sum_{j=i}^k [g^*_j(w'_j, F^k) - c(j)]
\]

The first equality above is valid since individuals \( i, i+1, \ldots, k \) are gross contributors, under both income distributions. Thus, we obtain the result

\[
\overline{G}^*(F^k) > \overline{G}^*(F^{i-1}) \text{ iff } \sum_{j=i}^k [g^*_j(F^k) - c(j)] > 0 \text{ iff } \Delta_i(a_{w-c}(ik)) > 0.
\]

Consider the case in which \( k > i \). Since there is no \( i \leq l < k \) such that \( \Delta_i(a_{w-c}(ik)) > 0 \), then it is optimal to include \( i, i+1, \ldots, k \) in \( F^o \). Thus, by Lemma 1 the proposition follows.

Proof of Proposition 3. (i) follows by noticing that cost cutoff for individuals \( i > 1 \) are increasing in the setup cost and the fund-drive is launched for both fixed cost levels under consideration. To prove (ii) by Let \( \Phi_{F^o(s)}(\overline{G}) = \sum_{i \in F^o(s)} (\phi(\overline{G}) - \overline{G}) + \overline{G} \) and \( k \) be the number of solicitations under \( F^o(s') \). Then, by using equilibrium

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\[
\Phi_{F^o(\iota)}(G^*) = \sum_{i \in F^o(\iota)}(w_i - c) - s \\
> \sum_{F^o(\iota')} (w_i - c) - s' = \Phi_{F^o(\iota')}(G^*(F^o(\iota'))) \]

The inequality comes from \( s < s' \). By strict normality, \( \Phi'_{F^o(\iota)}(.) > 0 \). Thus, \( G^* > G^*(F^o(s')) \). Note that \( G^* \leq G^*(F^o(s)) \), by a revealed preference argument. Thus, \( G^*(F^o(s)) > G^*(F^o(s')) \). Finally, note that \( g_i^*(F^o(s')) = w_i + G^*(F^o(s')) - \phi_i(F^o(s')) \).

Since \( G^*(F^o(s)) > G^*(F^o(s')) \) and \( \phi_i > 1 \), it follows that \( g_i^*(F^o(s')) > g_i^*(F^o(s)) \) for every \( i \in F^o(s') \). Thus, (iii) follows.

**Proof of Proposition 4.** Let \( \mathcal{F}_i \) be donor \( i \)'s belief about the fund-raiser set when he is contacted. Then, as stated in the text, \( \mathcal{F}_i = F^o \) if \( i \in F^o \), and \( \mathcal{F}_i = \{1, 2, 3, \ldots, i \} \) if \( i \notin F^o \). We will show that given the beliefs \( \{\mathcal{F}_i\}_{i=1}^n \), contacting \( j \notin F^o \) is not a profitable deviation for the fund-raiser. Let \( k \) be the lowest index individual being solicited under \( F^o \). Thus, individual \( k + 1 \) provides the biggest gift among the individuals not in \( F^o \). Let \( g_{k+1}^o \) be \( k + 1 \)'s contribution under the stated belief system. We first show closely following NY(2103) that \( g_{k+1}^o \leq c(k+1) \). Suppose not. Upon being contacted, person \( k + 1 \) would expect others’ gross contributions to be \( G^*(F^o) \), resulting in

\[
\phi_{k+1}(G^*(F^o) + g_{k+1}^o - c(k+1)) - (G^*(F^o) + g_{k+1}^o - c_{k+1}) = w_{k+1} - g_{k+1}^o. \tag{B.2}
\]

On the other hand, if the individuals in \( F^o \) knew about the presence of \( k + 1 \) before contributing, then,

\[
\phi_{k+1}(G^*(F_{k+1})) - G^*(F_{k+1}) \geq w_{k+1} - g_{k+1}^o(F_{k+1}). \tag{B.3}
\]

Suppose \( g_{k+1}^o > c(k+1) \). It would directly imply \( g_{k+1}^o > g_{k+1}^o(F_{k+1}) \). Therefore, \( w_{k+1} - g_{k+1}^o(F_{k+1}) > w_{k+1} - g_{k+1}^o \). Then, since \( \phi_{k+1}^o > 1 \), the two equations above reveal that
\( G^*(F_{k+1}) > G^*(F^o) + g_{k+1}^o - c(k + 1) \). This contradicts \( G^*(F_{k+1}) \leq G^*(F^o) \). Thus, \( g_{k+1}^o \leq c(k + 1) \).

Denote \( F_k^{[k+1]} \) be the two stage game where in stage 1, individuals in \( F_k \) contributes simultaneously believing that no person outside \( F \) is contacted and in stage 2, given total contributions in stage 1, individual \( k+1 \) decides on her gift.

Consider two cases:

1. \( G^*(F_{k+1}) \geq G^*(F^o) + g_{k+1}^o(F^o) - c(k + 1) \). By strict normality, this in turn implies that
   \[
   G^*(F^o) + g_{k+1}^o(F^o) - c(k + 1) + g_{k+2}^o(F^{o+[k+1]}) \leq G^*(F_{k+1}) + g_{k+2}^o(F_{k+1})
   \] (B.4)

   Moreover, since \( g_{k+2}^o(F_{k+2}) \leq c(k + 2) \), then by exactly following the same argumentation as above, it implies \( g_{k+2}^o(F_{k+1}) \leq c(k + 2) \). Consequently
   \[
   G^*(F_{k+1}) + g_{k+2}^o(F_{k+1}) - c(k + 2) \leq G^*(F_{k+1}) \leq G^*(F^o)
   \] (B.5)

Combining (B.4) and (B.5) and since \( C(F_{k+1}) > C(F^o) \), we obtain that the fund-raiser is better off sticking to \( F^o \).

2. \( G^*(F_{k+1}) < G^*(F^o) + g_{k+1}^o(F^o) - c(k + 1) \).

   Again, \( g_{k+2}^o(F_{k+2}) \leq c(k + 2) \) implies \( g_{k+2}^o(F_{k+1}) - c(k + 2) \leq 0 \). Moreover, by strict normality \( g_{k+2}^o(F^{o+[k+1]}) < g_{k+2}^o(F_{k+1}) \). Thus, \( g_{k+2}^o(F^{o+[k+1]}) < c(k + 2) \). By recalling that \( g_{k+1}^o(F^o) - c(k + 1) \leq 0 \) we get
   \[
   G^*(F^o) + g_{k+1}^o(F^o) - c(k + 1) + g_{k+2}^o(F^{o+[k+1]}) - c(k + 2) < G^*(F^o).
   \] Thus, the fund-raiser is better off sticking to \( F^o \). By inductively applying this argument, the result follows. \( \square \)

**Definition B.3.** Let \( t_i(T_{-i}) \) be the value of \( t_i \) satisfying \( U(w_i - t_i(T_{-i}), t_i(T_{-i}) + T_{-i}) - U(w_i, T_{-i}) = 0 \)

Denote \( G^i_0(0) \) simply as \( G^i_0 \).

**Lemma B.2.** \( t_i(T_{-i}) \) may be defined as \( \hat{T}(w_i + T_{-i}, T_{-i}, w_i) - T_{-i}, \) where \( \hat{T} \) satisfies:
1. $\hat{T}(w_i + T_{-i}, T_{-i}, w_i) - T_{-i} > 0$ for every $T_{-i} \in [0, G_i^0)$

2. $\hat{T}(w_i + G_i^0, G_i^0, w_i) = G_i^0$

3. $\hat{T}_1 > 0, \hat{T}_2 < 0, \hat{T}_3 < 0$

4. $\hat{T}_1 + \hat{T}_3 > 0$

5. $\hat{T}(w_i + T_{-i}, T_{-i}, w_i) > f(w_i + T_{-i})$ for every $T_{-i} \in [0, G_i^0)$

**Proof.** By quasiconcavity of the utility function $t_i(T_{-i}) > g_i(T_{-i})$. Therefore $U_1 > U_2$. Moreover, by quasiconcavity of the utility function

$$U_{ii} < 0 \text{ and } U_{12} > 0.$$ Together, it implies

$$\frac{\partial t_i}{\partial T_{-i}} = \frac{U_2(w_i, T_{-i}) - U_2(w_i, t_i, T_{-i} + t_i)}{U_2(w_i, t_i, T_{-i} + t_i) - U_2(w_i, t_i, T_{-i} + t_i)} < 0.$$ By noting that $x_i + T = w_i + T_{-i}$, then the first term of $\hat{T}$ captures the direct positive effect of $T_{-i}$ on $T$, as in the classic public good model. The second term of $\hat{T}$ captures the negative effect of $T_{-i}$ on $T$ through a higher outside option. The third term captures the negative effect of $w_i$ on $T$ through a higher outside option. Thus, (3) follows. (4) follows by quasiconcavity of the utility function.

Let $t_i^*$ satisfies $U(w_i - t_i^*, t_i^*) = U(w_i, 0)$. By quasiconcavity of the utility function, $t_i^* > 0$. Moreover $t_i^* = \hat{T}(w_i, 0) > f(w_i) = g_i^*$. Note that by definition of $G_i^0$, $U(w_i, G_i^0) > U(w_i - g_i, G_i^0 + g_i)$ for any $g_i > 0$. Thus, (1) and (2) follows. Part (5) follows from definition of $\hat{T}$ and quasiconcavity of the utility function. 

For the following lemmas and propositions we omit the third argument of $\hat{T}$, knowing that $\hat{T}$ is increasing in $w_i$, by Lemma B2.

**Lemma B.3.** In the costless case, there exists a solution to the taxation problem unique up to total taxation $T^*$. Moreover, the solution is unique up to individual taxation if $0 < \hat{T}_1 + \hat{T}_2 < 1$
Proof. Existence follows directly from Brower’s fixed point. Uniqueness when \(0 < \tilde{T}_1 + \tilde{T}_2 < 1\) and \(\tilde{T}(w_i,0) < G_i^0\) follows from a standard contraction mapping argument, the same used in the voluntary contribution literature (Cornes and Sandler 1998). However, there is an additional point to be stressed here. Since \(\tilde{T}(w_i,0)\) may be greater than \(G_i^0\) in this framework, it may happen that the optimal solution entails taxing just one individual. Thus, if individuals are ex-ante identical, the fund-raiser is indifferent with regard to whom to tax. This generates multiplicity.

Lemma B.4. Suppose that under costless fund-raising, \(\frac{dt_i}{dT_i} < -1\), then, facing a total cost \(C\), more individual(s) are contacted if \(G_i^0 > t_1(0) - C\). If that is the case, then, they partially recover the cost, i.e., \(\sum_{i>1} t_i^* < C\). Moreover, \(\frac{dt_i^*}{dT_i} > 0\). If more than two individuals are solicited, then \(\frac{dt_i^*}{dT_i} > 0\) for every \(i > 1\) s.t. \(t_i^* > 0\)

Proof. Since in a costless economy \(t_1^* > G_1^0\) then, it also follows that \(t_i^* > G_i^0\) for \(i > 1\). Thus, once we introduce a total cost \(C\), it must be the case that \(\sum_{i>1} t_i^* < C\). Otherwise, more charitable good could be obtained by increasing individual 1’s gift size, since \(\frac{dt_i}{dT_i} < -1\) in a costless economy. Note that \(\sum_{i>1} t_i^* < C\) implies \(t_1'' > 0\).

Then, if the optimal set is \(\{1,2\}\), it must be the case that \(t_2^* < 0\). Moreover, note that if the fund-raiser receive gifts from more than two individuals, for \(2 \leq i < j\) such that \(t_i^*, t_j^* > 0\) it must be the case that \(t_i'', t_j'' > 0\). That is, all individuals are pivotal. Suppose not, let \(t_i'' < 0, i.e., T_i - C > 0\) for some \(i > 1\), then by reducing \(j\)’s gift size by one unit, the fund-raiser is enable to rise individual \(i\)’s threshold gift by more than one unit, which constitutes a profitable deviation. Contradiction.

Proof of Observation 3. Consider first the case in which \(\frac{dt_i}{dT_i} > -1\) for \(C = 0\), i.e., \(0 < \tilde{T}_1 + \tilde{T}_2 < 1\). We show that \(t_i^* > t_{i+1}^*\). By way of contradiction suppose \(t_{i+1}^* > t_i^*\). Note that there exists an inverse function \(\tilde{\phi}(T^*, w_i)\) such that \(\tilde{\phi}_1 > 0\) and \(\tilde{\phi}_2 < 0\) such that
$$t_i^* = T^* - \hat{\phi}(T^*, w_i) < t_{i+1}^* = T^* - \hat{\phi}(T^*, w_{i+1}).$$ Since $\hat{\phi}_2 < 0$, it implies $w_i < w_{i+1}$.

Now consider $\frac{d\hat{t}_i}{dt_{i-1}} < -1$ when $C = 0$, i.e., $\hat{T}_1 + \hat{T}_2 < 0$. Clearly individual 1 is solicited since she is the one providing the greatest stand-alone value. On the other hand, if the optimal solution entails 2 solicitations, then individual 2 is the other one being solicited since $\hat{T}(w_2 + t, t) > \hat{T}(w_i + t, t)$ for any $i > 2$ and $t > 0$. If 3 or more solicitations are made $t_i'' > 0$ for every $i$ s.t. $t_i^* > 0$. Thus, $\hat{T}_1 + \hat{T}_2 = \hat{T}_1 > 0$ at $T^*$ and the inverse function argument given above also applies here. Finally, if some individual provides a gift $g_i^* > t_i^*$, then by quasiconcavity of the utility function there exists $t'_i > g_i^*$ such that $U(x_i - t'_i, t'_i + T_{-i}^* - C) = U(w_i, \max\{T_{-i}^* - C, 0\})$, which contradicts the optimality of $t_i^*$.

**Proof of Observation 4.** Note that $\{g_i^*\}_{i=1}^{F^*}$ is a feasible solution. Also note that $U(w_1 - g_1^*, g_1^* + G_{-1}^* - C) > U(w_1, G_{-1}^* - C)$ Therefore, since $U()$ is quasiconcave in their arguments it follows that $g_1^* < t_1(G_{-1}^*)$. Hence, by fixing $t_1(G_{-1}^*)$ we have $G^* < t_1(G_{-1}^*) + G_{-1}(t_1)$, where the inequality comes from strict normality. Note also that the RHS of the previous inequality is still a feasible solution of the taxation problem since $G_{-1}(t_1) < G_{-1}^*$. Thus, $U(w_1 - t_1, t_1 + G_{-1}(t_1)) > U(w_1, G_{-1}(t_1))$ Hence, we found a deviation from $G^*$ where more public good is generated. Thus, $G^* < T^*$.

**Proof of Observation 5.** If $F^* \subseteq \{1, 2\}$ then trivially, there is excessive fund-raising. Consider then $|F^*| \geq 3$. We know from Lemma A4, that in this case every individual is pivotal, i.e., $\frac{dt_i}{dt_{i-1}} > 0$. Thus, $g_i^*(F^*) < t_i^*(F^*)$ for every $i \in F^*$. Thus if $G^*(F^*) > 0$, individuals are also pivotal under voluntary contributions. Therefore, $F^* \subseteq F^o$. On the other hand, if $G^*(F^*) = 0$, then $F^* \subseteq F^o$. Thus, there is excessive fund-raising.

**Lemma B.5.** *(Voluntary contributions)* For every $i > 2$ $\overline{c}_i(\delta = 0) < \overline{c}_i(\delta = s)$. Moreover, for any: $\delta'' > \delta'$ (i) $\overline{c}_i(\delta') < \overline{c}_i(\delta'')$ for $0 \leq \delta', \delta'' < \frac{1}{i-1}$ and (ii) $\overline{c}_i(\delta') \geq \overline{c}_i(\delta'')$
for \( \frac{s}{i-1} < \delta' \), \( \delta'' \leq s \)

**Proof.** Note that

\[
\bar{c}_i(s) = w_i - \Phi \left( \sum_{j=1}^{i-1} [(w_j - w_i) - s] \right) > \bar{c}_i(0)
\]

\[= w_i - s - \Phi \left( \sum_{j=1}^{i-1} [(w_j - w_i)] \right), \]

establishing the first part of the Lemma. Moreover,

\[
\bar{c}_i(\delta) = w_i - s + \delta(i - 1) - \Phi \left( \sum_{j=1}^{i-1} [(w_j - w_i) - \delta(i - 1) \frac{i}{2}] \right)
\]

for \(0 \leq \delta < \frac{s}{i-1}\). Thus, \( \frac{\partial \bar{c}_i(\delta)}{\partial \delta} = i - 1 + \Phi'(.) (i - 1) \frac{i}{2} > 0 \). On the other hand for \( \frac{s}{i-1} \leq \delta \leq s \), note that \( \bar{c}(\delta) = w_i - \Phi \left( \sum_{j=1}^{i-1} [(w_j - w_i) + \max \{ (i - 1)\delta, s \} - s] \right) \). Thus, \( \frac{\partial \bar{c}_i(\delta)}{\partial \delta} = -\Phi'(.) \left( \frac{(k-1)k}{2} \right) < 0 \) where \( k \) is the highest index individual with \( (k-1)\delta < s \).
Lemma B.6. For any $\delta_1, \delta_2$ such that $\delta_1 < \delta_2$ it must follow that $F^*(\delta_2) \subseteq F^*(\delta_1)$

Proof. Consider first the benchmark with constant marginal cost. Note that if $t_1(0) - c - s \geq G_2^0$, then, it follows that the optimal fund-raising strategy in the benchmark consists in soliciting exclusively individual 1. Moreover, this strategy is fixed for any learning rate. On the other hand, consider $|F^*| \geq 3$. By Lemma B4 we know that if an individual $i \geq 3$ is solicited, then she must be pivotal. Note that an increase in $\delta$ lowers $C(F^*)$. Therefore, if individual $i > |F^*|$ was not necessary to cover $C(F^*)$ before the $\delta-$increase, it would not be contacted once $\delta$ increases. Thus, for any $\delta_1, \delta_2$ such that $\delta_1 < \delta_2$ it must follow that $F^*(\delta_2) \subseteq F^*(\delta_1)$. 
Proof of Proposition 6. Consider the voluntary case. By Lemma B4, $c_i(0) < c_i(\delta_l)$ for any $0 < \delta_l \leq s$. Thus, $F^o(0) \subseteq F^o(\delta_l)$. By this result and Lemma A6, the proposition follows.

Proof of Proposition 7. We first show that fund-raising in the pure voluntary contribution case is potentially non-monotonic in $\delta$, consider a case in which $|N| > 2$. Let $i$ be the lowest index in the set. Fix $w_1, w_2, \ldots, w_{i-1}$ such that, (i) $w_j \geq w_{j+1} + s$ for every $j < i$, i.e., cutoff costs are monotonically decreasing, (ii) $c < c_{i-1}(0)$, i.e., every $j < i$ is a net contributor for any $0 \leq \delta \leq s$. Note that there exists $w_i > 0$ such that $c_i(w_i, s) = c$, or, equivalently $w_i - s - \phi(\sum_{j=1}^{i-1} (w_j - w_i) - s) = 0$. This follows from $c_i(w_{i-1}, s) > c$, $\frac{\partial c_i(w, s)}{\partial w} = 1 + \phi(i - 1)$. On the other hand, let $w_i > 0$ solves $c(w_i, \frac{s}{(i-1)}) = c$. That is, $w_i - s - \phi(\sum_{j=1}^{i-1} [(w_j - w_i) - \frac{s}{2}]) > 0$. Since $c_i(.)$ is increasing in $i$, it follows that $w_i > w_i$. Pick any $w_i \geq w_i > w_i$. Then:

$$c_i(0) < c_i(s) \leq s < c_i(\frac{s}{i-1}) \quad \text{(B.6)}$$

Thus, given that every $j < i$ is in $F^0$ for any $\delta$, by (ii) and from (B.6) it follows that $F^o(0) = F^o(s) \subset F^o(\frac{s}{i-1})$.

By this result and Lemma B6, it follows then that excessive fund-raising is potentially non-monotonic in $\delta$.

Proof of Lemma 2. An example works. Consider $N = \{1, 2, 3\}$. Suppose $w_1 > w_2 + c$ and $w_2 > w_3 + c$. Let $\bar{w}_3$ solves $c(\bar{w}_3, c - c_l) = c$. That is,

$$\bar{w}_3 = \bar{\phi}(\sum_{j=1}^{2} [(w_j - w_3) - (c - c_l)]).$$
Let $\overline{w}_3$ solves $\overline{c}(\overline{w}_3, \frac{c - \overline{c}}{2}) = c$. That is,

$$w_3 = \overline{\phi}(\sum_{j=1}^{2} \left((w_j - w_i) - \frac{3}{2}(c - c_l)\right))$$

Check that $w_1$ and $w_2$ are big enough such that $\overline{w}_3 > 0$. Pick any $\overline{w}_3 \geq w_3 > \overline{w}_3$. Then, $\overline{c}_3(w_3, 0) < \overline{c}_3(w_3, c - c_l) \leq c < \overline{c}_3(w_3, \frac{c - \overline{c}}{2})$. Now, let $\delta^* \text{ solves } \overline{c}_3(w_3, \delta^*) = c$. Let $\delta_h = \delta^* + \epsilon, \delta_l = \delta^*$. So, $F^o(\delta = \delta_h) = \{1, 2\}$ and $F^o(\delta = \delta_l) = \{1, 2, 3\}$. Then, $c(2, \delta_h) - c(3, \delta_l) = \delta_h - 2\epsilon > 0$ since $\delta_h > \frac{c - \overline{c}}{2}$. 

**Proof of Proposition 9.** In NY(2013) it is proven that without learning, either all members of a given group are solicited or neither of them are. Once we introduce learning, this result is reinforced in the sense that being $j$ a member of group $i$, then $E[r_i] - s(j) < E[\tilde{w}_i] - s(j + 1)$.

Thus, the cutoff cost of individual $j + 1$ is higher than the cutoff cost of individual $j$. By following the corollary of proposition 2, it, then, also follows that either all members of a given group are solicited or neither of them are. Therefore, we can redistribute income among members of group $i$ such that each of them is allocated with mean income $E[\tilde{w}_i] - a_{s_i}$. As in the proof of proposition 2, such a redistribution is neutral. Thus, the result follows by applying proposition A1 in NY (2013). 

**Proof of Lemma 3.** Notice that if group $ij$ is not solicited, then no additional learning is brought is generated by the fund-raiser strategy. On the other hand, since group $i$ was solicited before the merger, a revealed preference argument shows that a strictly lower public good provision is expected after the merger.

**Proof of Proposition 10.** Notice that under decreasing returns to scale, the cost function is non-decreasing. Therefore, $\{\tilde{w}_i\}$ is a non-increasing sequence. From (3.2) the result follows.
Bibliography


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