Functional Metamaterials for Nonlinear and Active Applications Using Embedded Devices

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Electrical and Computer Engineering in the Graduate School of Duke University 2014
ABSTRACT

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Abstract

Metamaterials have gained extensive attention in recent years due to their ability to exhibit material properties otherwise difficult or impossible to obtain using natural materials. Nonlinear and active metamaterials in particular exhibit great promise for exploring new effects and applications, from tunability to mixing. However, nonlinear and active metamaterials have been explored significantly less than linear metamaterials to this point and much work has focused on the fundamental physics of nonlinear metamaterials. Our aim is to further extend the knowledge of practical nonlinear metamaterials and to demonstrate how they can be transformed to real-world applications through the use of embedded devices. In this dissertation, we demonstrate a variety of ways that devices can be embedded within metamaterial unit cells to provide nonlinear and active effects.

Chapter 1 introduces the basic theory of metamaterials, background of existing work, and the current limitations of nonlinear and active metamaterial design. In Chapter 2, we present the design, simulation, fabrication, and verification of an RF limiter metamaterial. We show how a metamaterial can be designed using RF engineering principles to act as an effective limiter in a new topology, relying on nonlinear devices embedded within a metamaterial. Chapter 3 shows our design and demonstration of a power harvesting metamaterial. We design a nonlinear metamaterial towards a potential application, discussing how the selection of an appropriate embedded device provides our desired functionality. In Chapter 4 we show how
nonlinear and active metamaterials can be used to realize phase conjugation, including demonstration of negative refraction and imaging through the use of these metamaterials. We also discuss design approaches to moving these metamaterials towards real-world applications. Chapter 5 discusses our work concerning metamaterials based on transistors. First we show that appropriate design of a transistor circuit allows us to tune the quality factor and resonant frequency of a metamaterial. We use this metamaterial for time-varying mixing, as well, demonstrating a mixing metamaterial that remains linear. We then illustrate how using transistors as nonlinear devices provides much greater design freedom for use with metamaterials. We show that the nonlinearity of a metamaterial can be dramatically enhanced through the use of transistors and even dynamically tuned, applying these nonlinear metamaterials to applications including phase conjugation and acoustoelectromagnetic modulation. In Chapter 6 we summarize the achievements of the presented research and directions for future work that build on the work described in this thesis.
To all the friends, family, colleagues, faculty, and everyone else who made this possible.
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List of Abbreviations and Symbols

Abbreviations

DC  Direct current
AC  Alternating current
RF  Radio frequency
TEM Transverse electromagnetic (polarization)
PIN P-i-n doping profile for diode
MOSFET Metal-oxide-semiconductor field-effect transistor
JFET Junction field-effect transistor
RFLM RF limiter metamaterial
PC  Phase conjugation
PCM Phase conjugation metamaterial
TVTM Time-varying transistor metamaterial
NTM N-type MOSFET transistor metamaterial
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Introduction

The study of electromagnetics has revolutionized human society. Advances in understanding electromagnetic phenomena, from ray optics to transformation optics, have improved the quality of life for untold numbers of human beings. This knowledge is always limited by the materials we can exploit. Better understanding of semiconductors has allowed us to create computer chips, lasers, and other microelectronics, for instance. For hundreds of years science, engineering, and technology have been driven to best use the materials we have available. We have also altered them when possible: doping semiconductors with select impurities allows us to create transistors and diodes. However, we have been fundamentally constrained in the scope of possible advances by the palette of materials available to us.

Scientists have begun to explore the possibilities of using materials with properties that are not commonly found in nature. Victor Veselago conducted a theoretical exercise [1] to predict the effects of a negative index of refraction in a material, a property not found in naturally-occurring materials. While this work presented intriguing conclusions, it was largely abandoned due to practicality: if such a material could not be found, what would we gain by further study of it?
The answer was provided decades later by the development of metamaterials. Metamaterials are similar to naturally-occurring materials - the primary difference is that the atoms of a metamaterial are manmade. A metamaterial can be broadly defined as a composite whose overall properties are determined by both the individual constituent materials and its structure. While metamaterials are most commonly designed to present effective electromagnetic properties, they can also be designed for other types of waves, such as acoustic pressure waves, elastic waves, and matter waves, among others.

The key to understanding metamaterials is that we can decouple the microscopic material properties from effective macroscopic material properties. As an example, structures made of copper and fiberglass dielectric boards can be designed such that, for a particular frequency range, they behave effectively as a plasma medium. Metamaterials give us an even greater freedom of design, however. As we design metamaterials for a particular frequency range of operation, the line between "microscopic" and "macroscopic" depends on the wavelength of operation. As long as our microscopic structures are much smaller than the wavelength of interest, we obtain a metamaterial. For microwave frequencies, with wavelengths around 30 cm, our "microscopic" structures can thus be several centimeters large. We can design structures at this size range with a number of interesting effective properties, using "meta-atoms" instead of true atoms. These effective properties may, in fact, be impossible to find in naturally-occurring materials.

1.1 (Linear) Metamaterials

Understanding of metamaterials can be gained by examining naturally occurring materials. It is not practically possible to understand the full electrodynamics of a natural material on a microscopic scale. Each atom contains numerous charged particles and enormous numbers of degrees of freedom. Natural materials are thus
modeled as a macroscopic medium. The so-called constitutive equations are used to
describe the interaction of an electromagnetic wave with matter and are given by the
following for most simple materials (isotropic materials without magnetoelectric or
electromagnetic coupling, and without chirality):

\[ D = \varepsilon_0 E + P \]  \hspace{1cm} (1.1)

\[ B = \mu_0 (H + M) \]  \hspace{1cm} (1.2)

where \( D \) and \( B \) are the electric and magnetic flux densities, \( E \) and \( H \) are the electric
and magnetic field intensities, \( P \) is the electric polarization, \( M \) is the magnetization,
and \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability of free space. For linear materials
we have \( P \propto E \) (in particular \( P = \varepsilon_0 \chi_e E \)) and similar for the magnetic equation, so we
can simplify the equations and define the material permittivity \( \varepsilon \) and permeability
\( \mu \) as

\[ \varepsilon = \varepsilon_0 (1 + \chi_e) \] \hspace{1cm} (1.3)

\[ \mu = \mu_0 (1 + \chi_m) \] \hspace{1cm} (1.4)

where \( \chi_e, \chi_m \) are the electric and magnetic susceptibilities, respectively. The suscep-
tibilities (and, thus, permittivity and permeability) are in general tensors. From a
physical intuition standpoint, \( \varepsilon \) arises due to microscopic electric dipole moments in a
material, while \( \mu \) arises due to microscopic current loops in a material. We illustrate
a sketch of the microscopic structure for a natural material. Figure 1.1(a) shows the
microscopic view of a material with \( \varepsilon \neq \varepsilon_0 \), while Fig. 1.1(b) shows the microscopic
view of a material with \( \mu \neq \mu_0 \). The continuous material properties, \( \varepsilon \) and \( \mu \), result
from homogenizing the discrete structure of the natural material.

The naturally-occurring values for \( \varepsilon \) and \( \mu \) are limited to a finite range in general.
Over a finite frequency range we can obtain materials with properties such as \( \varepsilon < 0 \),
although most materials have $\epsilon > \epsilon_0$. Similarly diamagnetic materials have $\mu < \mu_0$ and ferromagnetic materials have $\mu \gg \mu_0$, while most ("non-magnetic") materials have $\mu = \mu_0$. Veselago demonstrated that other sets of material parameters, such as $\epsilon < 0$ and $\mu < 0$ simultaneously, do not violate Maxwell’s Equations. These materials are known as negative index media (Veselago showed that the index of refraction would be negative) or left-handed media (because the $E - H - k$ triplet is left-handed rather than the normal right-handed). While the macroscopic description of materials is simple, effective, and powerful, it must be noted that it is at heart an approximation.

It was recognized as early as 1946 that metal structures could be designed such that they exhibit similar electromagnetic responses to dielectric structures by [2, 3, 4, 5]. However, these structures were only designed to mimic the response of naturally-occurring materials. It was recognized by Pendry [6] that a metal structure could be designed to mimic the properties of a plasma rather than only a simple dielectric, demonstrating that structures could be designed to provide a variety of effective responses. The field of metamaterials could be said to begin with the work by Pendry
[7] and Smith [8, 9] that designed and implemented a material with properties found in no naturally-occurring materials - namely a negative effective permeability and negative effective index of refraction. The building blocks of this metamaterial are canonical metamaterial unit cells: the thin wire medium and the split-ring resonator (SRR). The thin wire medium generates a negative effective $\epsilon$ while the SRR generates a negative effective $\mu$. If we are interested in resonant electric elements, we can use the electric inductive-capacitive (ELC) resonator. A large array of these unit cells can be homogenized and treated as a continuous material with some effective properties. We illustrate this concept with a sketch of both electric and magnetic metamaterials. Figure 1.2(a) shows an array of ELCs, electric resonant elements which result in an effective $\epsilon_{\text{effective}} \neq \epsilon_0$; Fig. 1.2(b) shows an array of SRRs, magnetic resonant elements which result in an effective $\mu_{\text{effective}} \neq \mu_0$.

We compare Figs. 1.1 and 1.2 to illustrate the basic concept of metamaterials. By designing a "meta-atom" that is much smaller than the wavelength of interest, we can describe an array of these structures as a continuous effective material.
Since this initial work, linear metamaterials have been the most studied and well-understood. A metamaterial can be described by its effective material properties such as $\epsilon$, $\mu$ or equivalently $n$, $Z$. In order for this description to be valid, the "atoms" of the metamaterial must be significantly smaller than the wavelength of interest. Linear metamaterials are fully described by these linear parameters and are useful for a wide variety of applications. Linear metamaterials have been used to create devices such as superlenses [10] and invisibility cloaks [11], among many others. However, restricting our attention to linear metamaterials dramatically restricts the useful properties and effects we can realize with metamaterials. For this reason we focus on nonlinear and active metamaterials in this work.

1.2 Nonlinear and Active Metamaterials

Nonlinear materials are ones in which the medium response depends on the amplitude of the incident field. For an illustration of how linear and nonlinear materials differ, we consider a nonlinear electric material which is nonmagnetic. A linear electric, nonmagnetic material can be described by:

$$D = \epsilon_0 (1 + \chi_e) E$$

$$B = \mu_0 H$$

as described above. A nonlinear electric material, however, is described by:

$$D = \epsilon_0 E + \epsilon_0 (\chi_e^{(1)} E + \chi_e^{(2)} E^2 + \chi_e^{(3)} E^3 + \ldots)$$

$$B = \mu_0 H$$

with the nonlinear susceptibilities $\chi^{(2)}, \chi^{(3)}, \ldots$ being used as needed depending on the material and strength of the applied field (and similarly for the magnetic nonlinear susceptibilities). Nonlinearity is required for a variety of interesting and useful
effects: efficient power amplification, power-dependent transmission, mixing, and harmonic generation are a few useful applications that generally rely on nonlinearity. In order to put our work in context, we review the existing literature on nonlinear metamaterials.

Nonlinear and tunable metamaterials have been a topic of interest since shortly after metamaterials emerged as a field. Some of the initial work, such as [12], considered an array of SRRs embedded in an intrinsically nonlinear dielectric. Resonant unit cell elements, such as SRRs, were shown to enhance the nonlinear effects due to local field enhancement: the gap in an SRR has a very strong local field, leading to enhanced nonlinear effects across the entire metamaterial. This is an example of a class of nonlinear metamaterials: we define a metamaterial comprised of \emph{nonlinear constituent materials} as a bulk nonlinear metamaterial. This class includes metamaterials composed of conductive structures embedded in (or placed on) a nonlinear dielectric substrate, as well as nonlinear conductive elements embedded in a linear dielectric substrate. This class of metamaterials has received more attention at high frequencies, such as terahertz, infrared, and optical frequencies, than at RF. This is due to a lack of alternatives at higher frequencies: any nonlinear behavior is difficult to produce with discrete nonlinear devices. While some discrete nonlinear devices, such as tunnel diodes, have been demonstrated to operate in frequencies over 1 THz [13], generally nonlinear behavior must be obtained from intrinsically nonlinear materials. This restriction is not true at RF, however. A second class of nonlinear metamaterials can be defined as nonlinear embedded-inclusion metamaterials: this consists of structures such as SRRs which have some type of \emph{nonlinear device embedded within them}. We illustrate these classes on nonlinear metamaterials in Fig. 1.3.

This class is generally restricted to lower frequencies than the first. However, there is much greater design flexibility for working with this class of nonlinear meta-
materials, and a large amount of attention has focused on these metamaterials. We are interested in applying RF design principles to nonlinear metamaterials: within this context, we restrict our attention to this class of nonlinear metamaterials.

Some of the first work involving nonlinear embedded-inclusion metamaterials was conducted in [14]. This work showed that, by including nonlinear diodes within a resonant metamaterial structure, an effective nonlinear susceptibility can be obtained for the metamaterial. A related study, published in [15], demonstrated theoretically and numerically that diode inclusions could result in metamaterials with extremely large nonlinear susceptibilities. Much of the early experimental work involving nonlinear metamaterials examined metamaterials containing varactor diodes. This element provides a simple nonlinear capacitance: the reverse-bias capacitance depends on the bias voltage as

\[ C = \frac{C_0}{(1 - \frac{V_f}{V_f})^{-M}} \]  

where \( C_0 \) is the zero-bias capacitance, \( V_f \) is the forward voltage, and \( M \) is a parameter.
that depends on the diode structure. A typical $C$ tuning range for a varactor, such as a Skyworks SMV1405, is around 4:1. The voltage across the varactor is dependent on the incident field amplitude, so the capacitance of the varactor (and thus the entire metamaterial unit cell) depends on the incident field amplitude. This nonlinearity is straightforward to model analytically and numerically, treating the metamaterial resonant element (such as the base SRR or ELC) as a resonant RLC circuit with an additional capacitance term due to the varactor diode.

Much of the focus of the literature involved obtaining tunable effective properties, whether tunable with an incident field or a DC bias voltage. For example, [16] examined varactor-embedded metamaterials with a DC bias line. This work showed that by adjusting the DC bias, the resonant frequency of a metamaterial could be actively tuned. It also demonstrated that the resonant frequency (and $Q$ factor) of the metamaterial was a function of the incident power delivered to the metamaterial, demonstrating nonlinearity. Nonlinear tuning of the resonant character of a metamaterial was also shown in [17], where the tuning was solely due to the power of the incident field (without a DC bias line).

Besides interest in a tunable/nonlinear resonant frequency, a bulk of the nonlinear metamaterial literature has involved mixing and harmonic generation processes. Work including [18, 19, 20, 21] has examined using nonlinear metamaterials to generate harmonics, in particular the second harmonic. The work of [19, 20, 21] used bulk nonlinear metamaterials, while that of [18] used varactor-embedded metamaterials. This approach was also shown to extend to resonant elements besides SRRs in [22], demonstrating power-dependent tuning behavior in an electric metamaterial (in particular an ELC). The work in [22] also used varactor-embedded unit cells to obtain nonlinear behavior.

More recent work, such as [23, 24], has also examined mixing and resonant frequency tuning with nonlinear embedded-inclusion metamaterials. These works have
also used varactors to obtain nonlinearity, embedded within resonant conductive elements (SRRs). A number of results have also been published [25, 26] demonstrating methods for considering a nonlinear embedded-inclusion metamaterial (or any nonlinear metamaterial) as a bulk nonlinear material. These include an analytical expression for and effective $\chi_m^{(2)}$ of a varactor-embedded metamaterial resulting from a perturbative method [26]. This method was derived from the nonlinear oscillator equation describing a resonant RLC approximation of a metamaterial unit cell, such as an SRR. A number of early works in the metamaterial literature, such as [27], focused on finding and formalizing an effective method to retrieve the effective material properties of a metamaterial. This approach was extended to the nonlinear effective material properties of a general nonlinear metamaterial in [25], showing how an effective nonlinear susceptibility could be retrieved to describe a metamaterial.

A large amount of the literature concerning nonlinear metamaterials at and above THz frequencies has focused on demonstrating interesting fundamental effects. A large problem with negative-index metamaterials is the fact that they are typically very lossy. The work in [28] examined a method to potentially compensate for this loss with nonlinear metamaterials. Through the use of optical parametric amplification, [28] showed that nonlinear metamaterials could be constructed with gain through the use of an auxiliary electromagnetic field. There is also interest in demonstrating bistable behavior using nonlinear metamaterials. It was shown in [29] that a nonlinear optical coupler consisting of both positive- and negative-index media would result in bistability. Although this was a theoretical study, metamaterials were shown to be promising for realizing bistable systems. Another example of a study involving fundamental physics is that conducted in [30]. In this work it was shown that a nonlinear metamaterial system could be used to obtain spatiotemporal solitons. While again this study was numerical in nature, it relied on negative in-
dex and nonlinear behavior: thus, metamaterials provided a platform for this work. These are only a few examples of the more fundamental theoretical work involving nonlinear metamaterials. However, they demonstrate some of the directions of active research involving nonlinear metamaterials.

In this survey of the literature, we have examined much of the major work involving nonlinear metamaterials. Generally the bulk of the literature involving RF nonlinear metamaterials focuses on nonlinear embedded-inclusion metamaterials, while that at THz and higher frequencies involves bulk nonlinear metamaterials. The experimental work at RF has generally involved either mixing and harmonic generation, demonstrating that nonlinear metamaterials can provide large effective nonlinear susceptibilities, or tuning of properties such as the resonant frequency due to a DC bias control or the incident wave power level. Much of this work is focused on demonstrating interesting or new fundamental physics. The nonlinear effects examined in the literature can also be used for various applications, however. The use of varactor diodes for the bulk of the RF experimental work on nonlinear metamaterials is unnecessarily restrictive: by using other types of nonlinear devices, such as PIN diodes or transistors, we may be able to obtain nonlinear metamaterials with significant advantages over those constructed using only varactors. We also wish to examine applications of some nonlinear effects. Demonstrating harmonic generation and mixing with nonlinear metamaterials provides a foundation for a number of applications, but these have been explored much less in the prior work. As an example, we examine a particular nonlinear effect and demonstrate how it can be used for a variety of applications: time reversal.

A time reversal system receives some signal, reverses it in time, and retransmits the signal. For a single frequency, time reversal is equivalent to phase conjugation, which we see from:
Time reversed signals are useful for a number of applications. In signal processing, it is well-known that for a given signal, the signal-to-noise ratio is maximized with a "matched filter" [31]. A matched filter is a time-reversed, shifted replica of the signal. Time reversal is also used for some RF applications as a retrodirective array. It can be shown that, when illuminated by a source antenna, a time reversal antenna array will retransmit a signal back to the source regardless of scatterers in the path [32, 33]. This property is also used in nonlinear optics: a thick slab of time reversal media is called a time reversal mirror. This retrodirectivity can also be considered as backward imaging: a time reversal (or phase conjugating) slab emits a signal that focuses back to an illumination source. It was shown independently by Pendry [33] and Maslovski [32] that a thin slab of time reversal (or phase conjugating) medium will not only form a backward image, but also a forward image. Thus, a time reversal (phase conjugating) medium can be used for both retrodirectivity and forward imaging, acting as a lens. This useful effect is produced using nonlinearity: in optics, typically with four-wave mixing, and with mixing (or perhaps digital conversion and direct time reversing) at RF. Time reversal is just one example of an interesting and useful property of a material or system that arises from nonlinearity. RF systems such as efficient power amplifiers (higher classes such as class C) and mixers generally rely on nonlinearity, as do some optical systems relying on frequency doubling (typically for creating visible frequency lasers).

There is also significant interest in tuning a metamaterial’s properties. This is typically, but not necessarily, accomplished through the use of varactors. It has also been accomplished through the use of embedded devices including barium strontium titanate (BST) tunable capacitors [34] and microelectromechanical (MEMS) devices.
as switches [35]. This is typically used for quasi-static tuning, such as tuning the resonant frequency of a metamaterial by using an external DC voltage bias. See, for example, [16]. For applications involving mixing, however, tuning metamaterial properties with an AC voltage bias results in the generation of sum- and difference-frequency signals. A time-varying but linear metamaterial can thus be used for mixing applications rather than a nonlinear metamaterial. One advantage of a time-varying approach to mixing is that nonlinear mixing typically requires high signal levels (voltages at a circuit level and fields for a metamaterial), since $\chi^{(2)}, \chi^{(3)}, \ldots$ are generally small in metamaterials (though still much larger than in naturally-occurring nonlinear materials). Time-varying mixing, on the other hand, can be accomplished with linear devices.

As we discussed above, the prior work at RF has focused in large part on nonlinear metamaterials using varactor diodes. While such work has generated a number of useful effects with metamaterials, there have also been a number of limitations in such an approach. First, varactor diodes offer only one type of (thus far exploited) nonlinearity: when reverse-biased, they exhibit a nonlinear voltage dependent capacitance as given by eqn. 1.9. Particular applications may require other types of nonlinearity, such as strongly current-dependent resistance (as provided by a PIN diode) or a very sharp forward conducting characteristic (as provided by a Schottky diode). Second, the nonlinearity of varactor diodes is typically fairly small and requires a large swing in voltage to fully realize it. We can obtain a very rough idea of the nonlinearity of a varactor by examining its C-V characteristic, in particular the maximum reverse bias voltage (to obtain the minimum capacitance) and the tunable range of the capacitance. This is because the smaller the applied bias voltage, the better the diode C-V characteristic is approximated by a linear fit. Typical varactor diodes have a tunable capacitance range of about 4-6:1, with a required reverse bias voltage of 10-30 V. Applying a smaller bias voltage results in a smaller tun-
able capacitance range and a smaller nonlinearity. While nonlinear metamaterials using varactors exhibit effective nonlinear susceptibilities that may be many orders of magnitude larger than natural materials, other RF devices can provide higher nonlinearity (resulting in higher conversion gain in a mixer, for instance).

In this work we detail our efforts to expand the pallette of nonlinear and effects available to the designer by incorporating other types of nonlinear elements. Rather than focusing solely on varactor diodes, we also incorporate PIN diodes, Schottky diodes, and transistor circuits. Through these new devices and circuits we can realize nonlinear and active metamaterials with previously unattainable properties, such as tunable $\chi^{(2)}_{\text{m}}$. We also demonstrate how many of these new nonlinear and active metamaterials can be used for applications, rather than solely demonstrating new effects.

1.3 Contributions

We begin with specific applications for nonlinear and/or active metamaterials. Using numerical simulations, we design metamaterials with particular desired properties. Following these simulations, we fabricate and experimentally characterize these metamaterials to validate the design. Overall this work has produced five peer-reviewed authored/coauthored journal publications (four as first author) and four conference talks.

The contributions of this work are as follows:

- A design procedure is discussed for a metamaterial that draws from the design of conventional transmission-line or circuit devices. This procedure was used to design a metamaterial which acted as an RF limiter. This RF limiter metamaterial accomplished the same function as a traditional, circuit-based limiter but in a different topology. The resulting metamaterial was fabricated and its

- We used a similar design procedure demonstrated with the RF limiter metamaterial to design a power harvesting metamaterial. We showed that treating a metamaterial platform similarly to a rectenna allows us to design a metamaterial that rectified an incident RF signal. The power harvesting metamaterial was shown with simulations to harvest significant power from an incident signal for realistic loads (between 50 and 100 Ω). The metamaterial was then fabricated and shown to harvest RF energy at up to 36.8% efficiency for a load of 68 Ω. [Hawkes, A.M., Katko, A.R., and S.A. Cummer. A microwave metamaterial with integrated power harvesting functionality. *Appl. Phys. Lett.* **103**, 163901 (2013).]

- We designed a nonlinear active metamaterial using varactor diodes to demonstrate phase conjugation. The metamaterial was shown analytically and numerically to produce a phase conjugated mixing product when properly excited. We fabricated the metamaterial and demonstrated that the desired mixing product was indeed a phase conjugated signal, then subsequently demonstrated that the phase conjugated signal is negatively refracting. This was the first experimental demonstration of a phase conjugating metamaterial and negative refraction using such a metamaterial. [Katko, A.R., Gu, S., Barrett, J.P., Popa, B.-I., and S.A. Cummer. Phase conjugation and negative refraction using nonlinear active metamaterials. *Phys. Rev. Lett.* **105**, 123905 (2010).]

- We improved on the phase conjugation metamaterial, designing it to be more
useful for practical applications. We designed, fabricated, and demonstrated experimentally an all-wireless metamaterial that also accomplished phase conjugation, obviating the need for numerous cable connections. We then further improved on the metamaterial design by including an amplifier network to compensate for free-space path loss. We also demonstrated that a phase conjugation metamaterial can be used not only for negative refraction, but in fact for negative refraction imaging, again demonstrating this for the first time with a metamaterial platform. [Katko, A.R., Shvets, G., and S.A. Cummer. Phase conjugation metamaterials: particle design and imaging experiments. J. Opt. 14, 114003 (2012).]

- We designed a transistor-loaded metamaterial to obtain tunability and mixing in a novel way for RF metamaterials. By using a transistor, we showed that a metamaterial could present a tunable Q factor and resonant frequency. We used this metamaterial to show mixing, demonstrating sum- and difference-frequency generation with a linear and time-varying, rather than nonlinear, metamaterial. We also showed experimentally that this time-varying metamaterial could be used for phase conjugation similar to the varactor metamaterial shown previously. This was the first use of a very simple, stable transistor circuit embedded in a metamaterial. [Katko, A.R., Barrett, J.P., and S.A. Cummer. Time-varying transistor-based metamaterial for tunability, mixing, and efficient phase conjugation. Accepted by J. Appl. Phys..]

- Using the time-varying transistor metamaterial as a base, we introduced nonlinearity. Doing so allowed us to both dramatically increase the conversion gain of a phase conjugation metamaterial as well as to construct a nonlinear acousto-electromagnetic metamaterial, which coupled an incident acoustic field to an incident electromagnetic field. We also demonstrated that the introduction
of nonlinearity to the transistor metamaterial allowed us to dynamically tune nonlinear parameters such as the mixing efficiency. We showed that this corresponded to providing a tunable nonlinear susceptibility in an RF metamaterial for the first time. [In preparation for submission]
As we saw in Ch. 1, nonlinear metamaterials using varactor diodes can be used to realize a number of interesting effects. A large amount of interest has focused on realizing harmonic generation (and mixing) and power-dependent resonant frequency shifting. While these clearly provide a stepping stone to applications involving resonant metamaterials, there are a large number of interesting applications requiring nonlinearity that do not require these effects. We also wish to demonstrate that metamaterials can be designed for a more practical application, rather than being used solely to demonstrate new fundamental physical phenomena. The goal of this chapter was to design a metamaterial that accomplishes the same function as a useful RF circuit-based device: an RF limiter. The RF limiter metamaterial (RFLM) has a different topology than that of a circuit-based limiter: our RF limiter metamaterial can be designed as a screen, to be placed around a device or system requiring protection from high signals. Circuit-based limiters must be plugged into the RF electronics of the system needing protection. This may be difficult or impossible to modify, showing a clear advantage of a metamaterial approach.

We begin by discussing the essential properties of an RF limiter. We then discuss
the physics of the nonlinear device we use to accomplish the limiter function, a PIN diode. After laying this groundwork, we detail the design and simulation of the RF limiter metamaterial. We then discuss our experimental demonstration of the limiter metamaterial’s efficacy. Following this we discuss the limiter metamaterial performance.

2.1 RF Limiter Properties

An RF limiter is a circuit device which is typically used to protect sensitive electronics (such as a low-noise amplifier) from high-power signals and surges. The essential characteristic of an RF limiter is that its transmitted power response depends on the input power. This is described by

\[ P_{\text{out}} = P_{\text{in}}, P_{\text{in}} < P_t \]

\[ P_{\text{out}} = P_t, P_{\text{in}} > P_t \]

where \( P_{\text{out}} \) is the transmitted power, \( P_{\text{in}} \) is the input power level, and \( P_t \) is some threshold power characteristic to the limiter. This is shown by examining a \( P_{\text{out}} \) vs. \( P_{\text{in}} \) curve. A realistic limiter exhibits some insertion loss and a non-constant \( P_{\text{out}} \) above \( P_t \). We compare \( P_{\text{out}} \) vs. \( P_{\text{in}} \) curves for a linear device, an ideal limiter, and a realistic limiter in Fig. 2.1.

We can quantify the performance of a limiter by measuring its insertion loss, 3-dB isolation bandwidth, and maximum isolation. For a limiter we define isolation by the maximum limiting at high power as

\[ I = |S_{21,P_{\text{max}}} - S_{21,P_{\text{min}}}| \]

where \( S_{21,P_{\text{max}}} \) and \( S_{21,P_{\text{min}}} \) are measured at the maximum and minimum \( P_{\text{in}} \) levels, respectively. The 3-dB isolation bandwidth is defined as the bandwidth across
Figure 2.1: RF limiter $P_{out}$ vs. $P_{in}$ characteristics

which $I \geq 3$ dB. The limiter should maximize the isolation and isolation bandwidth and minimize the insertion loss for best performance.

A typical traditional circuit-based limiter uses PIN diodes to provide the nonlinear response. The PIN diode (or diodes) is connected in shunt. For low $P_{in}$, the PIN presents a large series resistance $R_s$, so the signal bypasses the diode. For high $P_{in}$, the PIN presents a low $R_s$, so the signal is primarily shunted through the diode. PIN diodes designed for limiting applications typically can handle relatively large sustained currents, in the 1 A range or greater. For a simple single-PIN limiter, the threshold power $P_t$ is effectively set by the diode selected. We discuss the important properties of PIN diodes in the next section.

2.2 PIN Diode Properties

As we described in Ch. 1, nonlinear and active metamaterials are typically designed using various circuit elements such as varactor diodes [16, e.g.], MEMS switches in
Rather than adapt the limiter design to a different type of nonlinear element, we use a PIN diode in the design of the RF limiter metamaterial. This allows us to simply modify the design of a typical limiter to use a metamaterial topology. Our approach also exemplifies how a wide variety of circuit elements can be used in the design of a metamaterial.

PIN diodes are characterized as multi-junction semiconductor devices. There are three regions: a heavily doped p-type region, a lightly doped or intrinsic region, and a heavily doped n-type region (see [36]). PIN diodes have a nonlinear impedance which depends on the current across the diode. In particular, the diode series resistance $R_s$ depends on the current $I_D$ according to the equation

$$R_s = \frac{R_{\text{max}}A}{A + R_{\text{max}}I_D^k} + R_{\text{min}}$$

(2.4)

where $R_{\text{max}}$ is the maximum resistance, $A$ and $k$ are fitting parameters based on the particular diode used, and $R_{\text{min}}$ is the minimum resistance. We focus in this work on a typical PIN diode package containing two antiparallel diodes. In this package $I_D$ is the total current: either AC or DC current alters $R_s$. We can thus control $R_s$ by controlling the RF power level delivered to the PIN diode. We model a PIN diode in simulations by a parallel R-C circuit with varying resistance. We choose to use a lumped-element model for the PIN diode because PIN diodes, in particular, are difficult to model in nonlinear circuit simulators. Many types of diodes and transistors can be effectively modeled using SPICE-based simulators. PIN diodes, however, cannot typically be modeled with SPICE-based tools because they operate with different physical principles. The carrier lifetime of a PIN diode is a key characteristic of this type of diode, but it is not modeled in SPICE. Thus, we choose to use a simple lumped-element model to approximate the PIN diode in simulations. In the remainder of this thesis the nonlinear elements can generally be
represented with satisfactory nonlinear models, so we only use the lumped element equivalent for the PIN diode structures in this chapter. The simulated $R_s$ values are chosen based on the diode model chosen.

2.3 RF Limiter Metamaterial Design and Simulation

As stated earlier, the goal of this chapter is to design a metamaterial that accomplishes the same function as an RF limiter. In particular, for this work we focused on designing a sheet RFLM rather than a volumetric RFLM. Such an RFLM could be easily placed in front of the antenna of an existing device. With low insertion loss and wide isolation bandwidth, the RFLM can provide protection from high-power signals without altering the device itself.

For this work we selected an Avago Technologies HSMP-3822 as the PIN diode in the RFLM. This particular devices is designed for use in limiters: it has a thin intrinsic region, which is required for a low-$P_l$ limiter, and is specified for a maximum current of 1 A.

The base metamaterial particle selected to form the RFLM is a complementary electric inductive-capacitive (CELC) resonator [37]. A CELC is a planar metamaterial and thus well-suited for constructing an RF limiter sheet. As the complement of an ELC resonator, a CELC exhibits high reflection except at its resonant frequency $f_0$, while exhibiting high transmission at $f_0$. The CELC also includes a large, continuous ground plane as part of each unit cell.

The PIN diodes are used to short the interiors of the CELCs at high power, preventing them from resonating and thus decreasing transmission. At low power, the diode presents a large resistance across the gap. At high power, the diode presents a very small resistance, effectively shorting the interior of the unit cell to the ground plane. Shorting a CELC in this way destroys the resonance, decreasing the transmission of the metamaterial.
A CELC unit cell is shown in Fig. 2.2 with the PIN diode placement indicated. The orange area indicates copper, while the white interior is bare dielectric material (such as FR-4). The CELC is a polarization-dependent unit cell, so the appropriate polarization for exciting the magnetic resonant mode is shown in the figure.

![CELC unit cell with PIN diode placement](image)

**Figure 2.2**: CELC unit cell with PIN diode placement

We designed the CELCs to resonate above 2.5 GHz in simulation, because the addition of the PIN diode capacitance depresses $f_0$. This frequency range was chosen because the bands near 2.4 GHz are widely used for many applications, so test equipment in this band is readily available.

We used CST Microwave Studio to simulate a CELC array loaded with PIN diodes. As mentioned in the previous section, we modeled the PIN diodes as lumped-element impedances to approximate the diode characteristics at varying bias levels. Using Eqn. 2.4 for the PIN diodes used in this work, the diode $R_s$ varies between approximately 5000 Ω at low $P_{in}$ and 30 Ω at high $P_{in}$. The simulated results for an array of PIN diode-loaded CELCs are shown in Fig. 2.3.

The blue set of curves show that the characteristic peak in $S_{21}$ at $f_0$ is depressed as the diode $R_s$ decreases (corresponding to an increase in $P_{in}$). For this simulation we obtain $I > 10$ dB and an isolation bandwidth of over 40%.
For comparison with a traditional circuit limiter, we also designed and fabricated a limiter for a coplanar waveguide (CPW) system. A CPW system was selected over other transmission lines (such as microstrip or stripline) for the ease of constructing the limiter, since a CPW does not require vias to shunt a device.

We then fabricated the CELC RFLM. We fabricated both a single unit cell sample and a 4x2 RFLM array for testing. This size of array was chosen because it completely filled the aperture in our test setup. Photographs of the sample RFLMs are shown in Fig. 2.4.

We designed these to be tested in a closed rectangular waveguide. This was done to more easily characterize the limiting mechanism (i.e. to determine whether the limiting ”switch” was due to an increase in loss or in reflection). A TEM waveguide has some leakage of the field and typically higher loss than a closed waveguide. The
RFLM samples were mounted to metal sheets in order to ensure a good electrical connection with the waveguide.

2.4 Experiment

Both the CPW circuit limiter and the RFLM samples were tested utilizing a vector network analyzer (VNA). In order to characterize the samples under high incident power, a power amplifier was used with the signal output of the VNA, increasing the maximum available incident power to approximately +30 dBm. Attenuators were also used to avoid damage to the VNA on the return path. We found that the device was essentially linear when the incident power level was below 0 dBm, so we measured both $S_{11}$ and $S_{21}$ at power levels varying from 0 dBm to +30 dBm. We did this for the CPW-based circuit limiter in order to establish a performance baseline and for the RFLM samples in a closed rectangular waveguide. Using $S_{21}$ we can characterize the insertion loss, isolation, and isolation bandwidth. Since we wished to examine the loss within the RFLM, we also measured $S_{11}$. Since we included a power amplifier (a nonreciprocal device) in the path from the VNA to the sample, we must use a device such as a circulator or directional coupler to measure the reflected signal. This allows us to decouple the forward and reverse signals, providing a way to measure $S_{11}$. We show the measured S-parameters for the CPW limiter in Fig. 2.5.

The CPW device functions as expected, exhibiting much lower transmission for
Figure 2.5: Measured S-Parameters as functions of $P_{in}$ for CPW circuit limiter high $P_{in}$. The maximum isolation for the CPW limiter, as shown in Fig. 2.5, is approximately 12.9 dB for $P_{in}$ varying from 0 dBm to +30 dBm. The PIN diode itself is also very broadband, with approximately 37.8% 3 dB isolation bandwidth. Thus, any bandwidth limitations in the RFLM should be due to the metamaterial base structure itself. The minimum insertion loss for the CPW limiter is 1.036 dB. This is expected given the direct cable connections and use of an impedance-matched CPW transmission line.

With this baseline, we tested the RFLM samples as described above. The calibrated S-parameters are split into two figures for clarity for the two cases (single unit cell sample and 4x2 array). Figure 2.6 shows $S_{11}$ while Fig. 2.7 shows $S_{21}$. We are primarily concerned with the $S_{21}$ data since it is used to extract the three performance figures. For clarity we only show the data from +10 dBm to +30 dBm.

The maximum isolation for the single unit cell RFLM is 11.8 dB for the same $P_{in}$ variation as the CPW circuit limiter with a 3 dB isolation bandwidth of 16.4%. The maximum isolation is very close to that of the circuit limiter. While the 3 dB isolation bandwidth is smaller than that of the circuit limiter, it is still very broadband. However, the low-$P_{in}$ minimum insertion loss of 10.2 dB for the single unit cell RFLM is much higher. This is expected due to the aluminum mounting
Figure 2.6: Experimental $S_{11}$ as function of $P_{in}$ for both the single and array RFLM samples

Figure 2.7: Experimental $S_{21}$ as function of $P_{in}$ for both the single and array RFLM samples

sheet: most of the cross-section of the waveguide is taken up by aluminum rather than the resonant unit cell. Consequently the transmission should be much higher for the full RFLM array since there is no solid metal sheet filling the waveguide. The RFLM unit cell is clearly functioning as an RF limiter.

The maximum isolation for the RFLM array is 6.95 dB for the same $P_{in}$ variation as the previous tests. However, the minimum insertion loss at low $P_{in}$ is 2.7 dB, significantly better than the single unit cell RFLM and approaching that of the CPW
limiter. The performance is also very broadband, with a 3 dB isolation bandwidth of 18%. The RFLM array thus functions as an effective RF limiter. The difference in isolation and bandwidth between the single unit cell and the array cases can be expected with more unit cells, which may not be completely uniform. The individual unit cells in a metamaterial array typically resonate at slightly different frequencies, due to both process/device variation and finite array effects.

We are also interested in characterizing the nonlinearity of the RFLM. Since transmission decreases with increasing $P_{in}$, either (or both) the reflection or loss must increase due to conservation of energy. Based on the characteristics of a PIN diode, we expect the loss to increase dramatically (there should be large current flowing through the diode). We calculate the power transmitted, reflected, and absorbed for the RFLM array at both 0 dBm and +30 dBm incident power from the measured S-parameters. The results are shown in Fig. 2.8.

![Figure 2.8: Power analysis for the RFLM array at different $P_{in}$ levels](image)

At low power (Fig. 2.8(a)), most of the power is transmitted. This is consistent with the insertion loss being less than 3 dB. At high power (Fig. 2.8(b)), the transmission drops under 12% while the absorption increases to over 70%. The measurements are thus consistent with what we expect for a PIN diode-based limiter.
Finally, we calculate the $P_{out}$ vs. $P_{in}$ curves for the traditional CPW limiter, the single unit cell RFLM, and the RFLM array. The results are presented in Fig. 2.9(a) (CPW limiter) and (b) RFLM samples.

![Figure 2.9: Experimental $P_{out}$ vs $P_{in}$ curves for the CPW circuit limiter, single unit cell RFLM, and RFLM array](image)

We see from this data that the threshold power $P_t$ for the CPW limiter (Fig. 2.9(a)) and the RFLM (Fig. 2.9(b)) is approximately the same. The CPW limiter has slightly higher isolation than the RFLM array, as shown previously, but the overall performance is similar. Our RFLM array has similar performance to a traditional CPW circuit-based RF limiter. However, it is implemented as a 2D sheet of arbitrary size and can be fully extended to a volumetric limiter, as it is implemented using metamaterials. This allows the construction of a sheet or volumetric limiter to protect sensitive components without requiring circuit connections.
2.5 Conclusions

The work presented in this chapter demonstrates a design procedure for developing a functional metamaterial application. We selected a traditional circuit device and emulated its function with a metamaterial. By choosing an appropriate nonlinear circuit element and metamaterial unit cell topology, we were able to mimic the nonlinear characteristics of a traditional circuit RF limiter in a metamaterial. The performance of the RFLM was similar to that of a traditional circuit limiter but in the form of an easily deployable sheet. This work shows that a metamaterial can be designed to mimic an essential nonlinear function of a circuit by embedding a particular type of nonlinear device in a metamaterial unit cell.
In the previous chapter we provided the design of a nonlinear metamaterial based on a different type of diode than the typically-used varactor. We saw that we could obtain useful functionality by harnessing properties of other, less-used nonlinear devices embedded in metamaterial unit cells. In this chapter, we focus on another application of nonlinear metamaterials, one which requires another type of diode not commonly used in metamaterials. We wish to design a metamaterial which has an application-focused nonlinear characteristic that is distinct from previous metamaterial work.

The goal is to design a nonlinear metamaterial which can effectively harvest incident RF energy. In order to do so, we focus on providing a very nonlinear device and only harnessing the DC component. We also require that the metamaterial is extendable to higher frequencies. For the inspiration of the basic unit cell function, we examine the typical design of another type of RF power harvesting device, the rectenna. Using a rectenna as a base, we show that all the essential functions of a rectenna can be embedded within the unit cell of a metamaterial.

By using a metamaterial platform for power harvesting, we gain advantages absent from a rectenna approach. First, metamaterial unit cells are implicitly designed
to function in large arrays. Thus, we can combine many unit cells additively to provide a specific area required for a particular application. Doing the same with more traditional antennas typically results in undesirable mutual coupling, while metamaterials are designed with this in mind. Second, metamaterials are not necessarily electromagnetic. The basic design architecture of our power harvesting metamaterial could be adapted to harvest, for instance, acoustic energy simply by changing the metamaterial unit cell. Third, power harvesting can be integrated with other metamaterial functionality on a single unit cell. Many types of tunable metamaterials require a DC voltage bias. Integrating a power harvesting metamaterial with a tunable metamaterial allows us to provide a DC voltage to the metamaterial wirelessly.

We begin this chapter by discussing the essential properties of a power harvesting device. Building on this discussion, we review the architecture of a general power harvester and the types of nonlinear devices typically used. We show that Schottky diodes are well-suited for power harvesting again in a general sense. With this relevant background material, we then discuss the design of a power harvesting metamaterial, including both fullwave electromagnetic and nonlinear numerical simulations. After obtaining the design we then present experimental verification of the power harvesting metamaterial, demonstrating its effectiveness and discussing it in the context of energy harvesting as a field.

3.1 Power Harvesting Architecture

Power harvesters can be designed for harvesting various types of energy, from acoustic [38] to optical (as in a photovoltaic panel/device). Keeping with the rest of the work in this document, we focus on harvesting RF energy. A power harvester requires, at minimum, a transducer and rectifier, regardless of the type of energy harvested. An RF rectenna, for example, is composed of an antenna, which acts as a transducer; some impedance matching network; a nonlinear rectifying circuit;
and some low-pass filtering network to isolate the DC component produced by the rectifier. Acoustic power harvesters operate similarly, but with transducers such as piezoelectric membranes instead of antennas.

Typical RF power harvesters use an antenna to convert incident RF energy to an AC signal on a transmission line. Diode-based circuits are used to rectify the AC signal to a DC signal. RF power harvesters also typically have impedance matching between the antenna and rectification circuit, as well as a low-pass filter on the output of the rectification circuit.

3.2 Schottky Diode Properties

In this chapter the nonlinear characteristics of interest are different from those of either the previous chapter or prior nonlinear metamaterial work. Thus, we choose to examine a different type of nonlinear element which is especially appropriate for a power harvesting device. Here we examine the use of a Schottky diode to provide a nonlinear response. Schottky diodes are characterized by a junction between a semiconductor and metal rather than other types of diodes, which may be characterized by junctions between doped semiconductors. Because only one side is semiconductor, a Schottky diode has a lower forward voltage drop than a typical p-n junction diode. This is a significant advantage for a power harvesting application, where the efficiency of RF-to-DC conversion $\eta$, defined as

$$\eta = \frac{P_{DC}}{P_{RF}}$$  \hspace{1cm} (3.1)

where $P_{DC}$ and $P_{RF}$ are the DC and RF power, respectively, may be a major design consideration. This efficiency is impacted by a number of factors: in order to maximize $\eta$, we should ensure a good impedance match between the antenna and the rectifier; ensure all resistive losses are as low as possible, to prevent some...
energy being dissipated as heat; and use diodes with low forward voltages. Once the rectifying diode is biased to a conducting state, a (approximately) fixed voltage drops across the diode while the rest is applied to the remainder of the circuit. Thus, we should use diodes with the lowest possible forward voltage. Typical silicon p-n junction diodes have a forward voltage $V_f \approx 0.7 \text{ V}$, while a Schottky diode designed for rectifying might have $V_f \approx 0.1 - 0.2 \text{ V}$.

Also, Schottky diodes can switch significantly faster than p-n junction diodes because they do not form a depletion region at the junction. This is important because the depletion region must recover when the diode is switched from conducting to non-conducting. These properties make Schottky diodes very attractive for high-speed, low-signal rectifying applications such as RF detectors or power harvesters. In this work we are interested in the rectification properties of Schottky diodes. Thus, when we simulate Schottky diodes, we must use a nonlinear simulation algorithm such as harmonic balance or transient time-domain. This is in contrast with the PIN diodes of Ch. 2, which we modeled as lumped-element impedances.

3.3 Power Harvesting Metamaterial Design and Simulation

In this work we use a metamaterial as the platform for power harvesting. Metamaterials couple strongly to an incident field, and this is true not just for electromagnetic metamaterials but also acoustic metamaterials [39, e.g.]. In principle, acoustic metamaterials can also be designed for power harvesting, but we focus on RF electromagnetic power harvesting in this work. A recent work investigated the use of metamaterials as transducers, i.e. to convert an incident RF signal to an RF current within the metamaterial unit cells [40]. However, this work did not investigate converting the energy to a useful DC signal at all. It also involved very little experimentation, so our work is the first to apply metamaterials to a direct power harvesting structure.
As we showed in Ch. 2, nonlinear functions can be embedded within a metamaterial unit cell. This allows us to embed impedance matching, rectifying, and low-pass filtering functionality within a metamaterial unit cell. We then gain the flexibility of metamaterial design, such as designing a flexible surface that can harvest incident RF energy. This is the basic design used in this chapter. We embed all the necessary circuitry to realize an effective power harvester within an individual metamaterial unit cell, converting incident RF energy to DC energy.

Split-ring resonators (SRRs) are canonical metamaterial unit cells. They act as magnetic resonant cells, coupling primarily to an incident magnetic field and acting as resonant current loops. SRRs can also be easily modified to include some circuitry as a load, providing an ideal platform for the power harvesting metamaterial (PHM).

We design an SRR to resonate near 900 MHz. This frequency is chosen as a typical RF energy harvesting frequency range because there is usually significant energy near 900 MHz (within 100 MHz) due to CDMA and GSM bands being allocated in the 800-900 MHz band. There is also an industrial scientific, and medicine (ISM) band designated for research purposes from 902-928 MHz.

Ansys HFSS is used to simulate the fullwave characteristics of the SRR. HFSS also allows us to simulate an embedded discrete port within an SRR, which will be used for nonlinear simulations of the full PHM. We show a diagram of a typical SRR (with polarization indicated) along with an example of a fabricated SRR with embedded circuitry in Fig. 3.1.

After simulating the basic SRR (with embedded discrete port) using HFSS, we can use the results in nonlinear simulations. We use Agilent ADS to conduct the nonlinear simulations. ADS allows us to include fullwave data, from HFSS simulations as an S-parameter block, alongside nonlinear SPICE models for devices such as diodes. Thus we can cosimulate electromagnetic and circuit-level effects for nonlinear metamaterials, and this approach is taken throughout most of the rest of this thesis.
The embedded circuitry must be tailored to the application, and a number of circuits can be used for rectification. A simple single diode (also known as the Villard circuit) provides high-speed rectification without a large amount of energy dissipated in the circuit itself (this is shown by the diode forward voltage drop of a rectification circuit). However, a single diode rectifier provides a signal on a 50% duty cycle, resulting in a low average DC voltage. We can select other circuit topologies with different advantages and disadvantages, however. For this work we select a Greinacher voltage doubler as the rectification circuitry. Compared to other multiple-diode rectifiers (such as a bridge rectifier), the Greinacher circuit can operate at higher frequencies due to presenting a lower capacitance. The Greinacher circuit also provides a rectified voltage that is double the peak-to-peak AC voltage. We use an Avago HSMS-2862 Schottky diode for the Greinacher circuit in this work. The Greinacher circuit includes both a matching capacitor and smoothing capacitor, providing all the essential circuit functions of the power harvester.

In power harvesting the efficiency of the harvesting typically depends on the load impedance. This is true in general for power harvesters, and is perhaps best known by the maximum power point tracking circuitry for photovoltaics. For this work we examine a simple resistive load. Using ADS to conduct the nonlinear simulation, a simplified schematic is shown in Fig. 3.2, which includes the embedded circuitry in
The SRR S-params block is the fullwave simulated data for the PHM from HFSS. We use the harmonic balance algorithm to simulate the nonlinear characteristics of the PHM. We are primarily interested in two performance metrics of the PHM. The first is the RF-to-DC power conversion efficiency defined by eqn. 3.1. The second is the open-circuit voltage $V_{OC}$. $V_{OC}$ provides a load-independent measure of the maximum DC voltage we can harvest at a given incident power level.

When characterizing the PHM experimentally, we need to determine the effective power delivered to the PHM in order to calculate $\eta$. We specify the total power density by using a signal generator connected to an open waveguide that supports TEM waves, accounting for losses and mismatches. For a large PHM array we can safely assume that the power delivered to the array effective area is the total power delivered to the waveguide (again accounting for losses and mismatches): the effective area of the array is at least as large as the aperture of the waveguide, so all our generated power $P_{gen}$ is delivered to the array. For a single PHM element this is not the case. Given the incident power density, we need to know the effective area of a single element. Since we are using TEM waves, we can calculate the effective area $A_{e,\max}$ with knowledge of the directivity of the element $D_0$ by using [41]

\[ A_{e,\max} = \frac{P_{gen}}{D_0} \]
\[ A_{e,\text{max}} = \frac{\chi^2}{4\pi} D_0 \quad (3.2) \]

Since an SRR is effectively a small loop, \( D_0 = 1.5 \) [41]. For an SRR resonant at 900 MHz, the effective area is thus \( A_{e,\text{max}} = 5.3A_{\text{physical}} \). The full waveguide aperture is approximately \( 6.8A_{\text{physical}} \), so the effective area of the SRR is approximately 78% of the waveguide aperture. Thus the incident power delivered to the SRR is approximately \( .78P_{\text{gen}} \). This is the power we use to calculate \( \eta \) for a single PHM element.

3.4 Experiment

We place the PHM samples (single unit cell and array) in a TEM waveguide and deliver power with a signal generator, attaching leads to the resistive load to measure the output with an oscilloscope. We determine the rectified DC power using \( P = V^2/R \) and use this to calculate the efficiency, sweeping \( P_{\text{gen}} \) from +13 dBm to +24 dBm. The experiment setup is shown schematically in Fig. 3.3.

![Figure 3.3: PHM experimental test setup diagram](image)

Using this setup with the single unit cell PHM sample, we calculate an experi-
mental maximum efficiency of $\eta = 14.2\%$.

A photograph of the PHM array is shown in Fig. 3.4.

![Figure 3.4: Photograph of PHM array](image)

As described above, we calculate the efficiency for the PHM given $P_{\text{gen}}$. The experimental maximum efficiency for the PHM is 36.8%. In order to more fully characterize the PHM, we calculate both $\eta$ and $V_{OC}$ as a function of input power for the PHM array. The experimental data are shown in Fig. 3.5.

The maximum $\eta$ and $V_{OC}$ for the PHM occur at the maximum incident power, which is expected. We see that $V_{OC}$ for the PHM is 7.3 V and $V_{OC} > 2$ V for the entire power sweep, providing a usable voltage for a variety of applications. While the efficiency is lower than some antenna-based power harvesters, it is still high enough for a variety of usable applications. Anecdotally, placing a cellular phone within approximately 2-3 λ of the PHM generates a DC signal level in the 50-100 mV range. We note that this is true even given that the PHM resonates (and thus
As mentioned previously, $\eta$ generally depends on the load presented to the PHM. We sweep the load resistance and calculate $\eta$ at each load in order to find the optimal load, which corresponds to the maximum power point. We do this both to find the maximum efficiency point and to determine if our PHM can be used with realistic loads. We conducted the same sweep in simulation using ADS. Both simulated and experimental data are shown in Fig. 3.6.

Both simulated and measured data show a maximum $\eta$ in the range of approximately 70-80 $\Omega$. While the maximum experimental efficiency is only 36.8% compared to the maximum simulated efficiency of 65.3%, the measured data qualitatively...
matches the simulated data, confirming that ADS can be used for simulating PHM. Simulations of resonant conducting structures in HFSS typically underestimate resistive losses, sometimes dramatically. We expect this accounts for the quantitative difference between simulated and measured data. The efficiency could be easily increased by using lower-loss materials.

We note that the power levels used in these experiments are artificially higher than typical ambient levels. One of the primary limitations with all power harvesting technology is that, using diode- or transistor-based rectifying circuits, there is effectively a minimum power level for any operation. If the voltage delivered to the Schottky diode is well under $V_f$, the diode does not conduct and does not act as a strongly nonlinear device. This is true for all passive semiconductor device-based power harvesting systems and not solely a limitation with our approach. We discuss some possible solutions to this in the final chapter detailing future work.
3.5 Conclusions

The work presented in this chapter demonstrates an application for metamaterials containing a different type of nonlinear element compared to that of Ch. 2 or prior work. We selected a particular type of nonlinear device, a Schottky diode. Examining its characteristics we designed a metamaterial that would mimic a type of nonlinear device, a power harvester. Simulations and experiments confirmed that the PHM performs the desired power harvesting function. This works shows that we can mimic the function of other types of devices by choosing an appropriate nonlinear device to embed within a metamaterial unit cell, as well as demonstrating that metamaterials provide a potential platform for power harvesting systems.
In the first two chapters we designed metamaterials specifically for end applications. We also demonstrated how nonlinear devices not typically used in metamaterial design could provide useful functionality. In this chapter, we focus on the use of more conventional nonlinear devices: specifically, varactor diodes. We show that, even using these common nonlinear devices, we can realize useful effects and applications not previously seen in metamaterials. The primary goal of the work presented in this chapter is to demonstrate phase conjugation using metamaterials. We show that metamaterials provide an easy, ideal platform for phase conjugation of an incident signal. Phase conjugation can be used for retrodirectivity or nonlinear imaging, which we demonstrate in this chapter. We also show how, from a simple and straightforward design, we can modify a metamaterial to make it more practical for applications. Our initial design of the phase conjugation metamaterial, while effective, requires direct SMA cable connections: this makes it less applicable to practical application. We show that the unit cell can be redesigned for particular applications, demonstrating that it can be made completely wireless or, with the use of DC bias lines, wireless for RF signals with loss compensation included. Our work is the first
to apply metamaterials to phase conjugation. Here, we begin by reviewing the important properties of phase conjugation, showing why it is of interest, and discuss an approach to generate a phase conjugated signal. Following this initial discussion we analyze the important properties of the varactor diode, demonstrating why it is appropriate for realizing a phase conjugating metamaterial. We then detail the design of the metamaterial, as well as the improvements for particular applications, before validating the various metamaterial designs experimentally.

4.1 Phase Conjugation Properties

Phase conjugation or, equivalently for a monochromatic wave, time reversal is an interesting property that some systems and signals can possess. As discussed in Ch. 1, negative refraction has been a topic of particular interest to the metamaterials community beginning with Veselago’s work [1], continuing to the first experimental realization of a negative index medium [8], leading to the idea of creating a perfect lens using a negative index medium [42]. Issues with realizing a perfect lens using a negative index medium have been documented in the literature. In particular, negative index media typically rely on resonant phenomena, so they are often both narrowband and very lossy. It has been shown [33] that phase conjugating (PC) media can be used to produce negative refraction. Phase conjugating media (or individual elements) produce a time-reversed field which is reradiated. Diverging waves from a source will be reradiated and converge both back towards the source and forward to create an image. This is shown conceptually in Fig. 4.1.

Figure 4.1(a) shows the diverging waves from a source while Fig. 4.1(b) shows the reradiated, PC fields from the PC medium. The PC field produces an image which is negatively refracted. PC signals can be generated through a number of processes. If the frequency is low enough, the signal can be time-reversed directly using A/D conversion and basic digital techniques. For higher frequencies, such as
optical, four-wave mixing (FWM) is typically used. In FWM two signals are focused on a nonlinear medium, called the idler and pump signals. The nonlinear medium produces mixing products, one of which is a PC signal. Another way to generate a PC signal is to parametrically vary the resonant frequency of a resonant element, which we showed in ref [43]. We consider a basic metamaterial unit cell modeled as an RLC resonant circuit. We embed a nonlinear, voltage-dependent capacitance in the metamaterial unit cell and excite the metamaterial with two signals. For optical FWM, these signals are known as the pump and idler; for RF mixing, they are known as the local oscillator (LO) and RF signals. For this work, we refer to the signals as the pump and source, respectively.

The equation characterizing the dipole moment of such a metamaterial excited by an incident wave at a frequency $\omega_1$ is

$$\frac{d^2p_j}{dt^2} + \gamma \frac{dp_j}{dt} + \Omega_0^2 (1 + h_0 \sin (2\omega_0 t)) p_j = \sum_{k_y} \Omega_0^2 E(k_y) a^3 e^{-i(\omega_1 t - k_y y)} + c.c., \quad (4.1)$$
Here \( p_j \) is the induced dipole moment of an individual metamaterial unit cell, \( \gamma \) is the loss rate, \( \Omega_0 \) is the natural resonant frequency of the metamaterial unit cell, \( h_0 \) is the strength of the pump signal, \( \omega_0 \) is the frequency of the pump signal, and \( y_j \) is the location of an individual metamaterial unit cell. One solution of this equation for such a nonlinear metamaterial has the form of \( \exp(i\omega_2 t) \), where \( \omega_2 = 2\omega_0 - \omega_1 \), while the other solution has the form of \( \exp(-i\omega_1 t) \). In more detail, the solutions have the form:

\[
\begin{align*}
a_1^{(j)} &= \frac{\Omega^2 - \omega_2^2 + 2i\gamma\omega_2}{D(\omega_1)} e^{ik_y y_j}, \quad b_2^{(j)} = \frac{i h_0 \Omega^2}{2D(\omega_1)} e^{ik_y y_j}, \\
\end{align*}
\]

where the dispersion function \( D(\omega_1) \) is given by

\[
D = (\Omega^2 - \omega_1^2 - 2i\gamma\omega_1)(\Omega^2 - \omega_2^2 + 2i\gamma\omega_2) - h_0^2 \Omega^4/4. \tag{4.3}
\]

The solution proportional to \( \exp(i\omega_2 t) \) (the mixed signal at the difference frequency) is a phase conjugate signal. The amplitude of this signal is maximized when the following conditions are met (where \( \Omega_0 \) is the natural resonant frequency of the metamaterial without any pump signal applied):

\[
\begin{align*}
\omega_0 &= \Omega_0 \tag{4.4} \\
\omega_1 &= \omega_0 \tag{4.5}
\end{align*}
\]

This is the degenerate frequency case, so-called because the PC signal will be produced at \( \omega_2 = 2\omega_0 - \omega_0 = \omega_0 \). However, this is very difficult to measure experimentally because the generated PC signal is lower in magnitude than the illumination signal: thus, the source signal will mask any PC signal unless the mixing efficiency is extremely high. If the nondegenerate frequency case applies, the magnitude of the PC signal will be lower but more easily detected. We choose the nondegenerate case for this work to simplify our experiments.
These equations show that if we can parametrically vary the resonant frequency of a metamaterial unit cell, the signal generated at the difference frequency should be a PC signal.

4.2 Varactor Diode Properties

In order to realize a PC metamaterial, we need an element which exhibits a nonlinear capacitance or inductance. Probably the best-known nonlinear capacitance element is a varactor diode. Varactor diodes are characterized by their reverse-bias capacitance. All typical p-n junction diodes exhibit a voltage-dependent reverse-bias capacitance, but varactor diodes are specifically designed to provide a large range of capacitance tuning by using an abrupt (or hyperabrupt) junction profile. A typical varactor diode might have a tunable capacitance ratio of 4, where the tunable ratio is defined as

\[ \frac{C_{0V}}{C_{V_{\text{max}}}} \]  

where \( C_{0V} \) is the capacitance near 0 V reverse bias and \( C_{V_{\text{max}}} \) is the capacitance for the maximum rated reverse bias. A reverse-biased varactor diode thus provides the capacitance variation we require for realizing PC metamaterials. For maximum \( f_0 \) variation we must select a varactor with capacitance similar in magnitude to the capacitance of our base metamaterial element. For this work we select a Skyworks SMV1405 varactor, which provides a capacitance ratio of 4.1 with a maximum specified capacitance of 2.67 pF. Applying the pump and source signals across the varactor will produce a PC signal.
4.3 Phase Conjugation Metamaterial Design and Simulation

The base metamaterial element for our design is a split-ring resonator (SRR). As we showed in the previous chapters, an SRR can be easily loaded with a circuit element. For the basic design, we select the approximate frequency bands we wish to use for the PC process. As discussed above we choose the nondegenerate frequency case. We select the source frequency as $f_{source} = 950$ MHz and the pump frequency as $f_{pump} = 1850$ MHz, yielding a PC frequency of $f_{PC} = 900$ MHz. As mentioned in Ch. 4, 900 MHz is near an ISM band so equipment and dedicated research spectrum is readily available. We simply load an SRR with varactor diodes. The SRR couples to the source signal, providing an induced current across the varactors. Initially, we use a coaxial SMA connection to couple the pump signal to the varactor diodes, shown in Fig. 4.2(a). In order to increase the PC signal strength, we add an impedance matching network. This network is simply a set of series inductors for the basic PC metamaterial (PCM), shown in Fig. 4.2(b).

![Figure 4.2: Schematic of PC SRR design](image)

In Fig. 4.2, the red rectangles are varactor diodes, while the blue rectangles are impedance-matching inductors. The polarization and signal paths are indicated, as well.

This basic PC SRR is straightforward to fabricate and operate. The PC signal
amplitude depends upon the modulation of the resonant frequency of the meta-
material, which is determined by the capacitance of the varactor diodes. In turn,
the varactor diode capacitance is modulated by the pump signal voltage. A large
PC signal amplitude requires a large modulation of the resonant frequency of the
metamaterial and thus a large pump signal voltage. The pump signal amplitude
is maximized by direct cable connections, allowing no free space path loss for the
signal. However, for large-scale operation, many direct coaxial cable connections
are impractical and expensive, largely due to the considerable bulk required for the
cables. Ideal operation of the PC metamaterial also requires that the pump signal
at each metamaterial unit cell is in phase. Many direct cable connections introduce
small phase variations to individual unit cells, which inhibits the performance of the
PC metamaterial.

Since these characteristics limit the practical application of the PCM, we wish to
design a second PCM unit cell which does not require direct coaxial cable connections.
Using a purely wireless metamaterial unit cell we can predict the phase at each unit
cell with high accuracy as long as far field conditions hold; also, the numerous direct
cable connections and power dividers are no longer needed. For this design, we modify
the metamaterial to include two sections. The first section is the SRR; it serves as
a base to couple to the source signal, includes the varactors for generating the PC
signal, and reradiates at the PC frequency (which is generally close to both the source
frequency and resonant frequency of the SRR) as in the direct cable-connected PCM.
The SRR section does not resonate at the pump frequency, so the second section is
designed to couple to the pump signal only and provide a bias voltage across the
varactors. We choose to use an electric inductive-capacitive (ELC) resonator for the
second section as it provides a low-profile and electrically small design. The two
sections are coplanar and are simply connected, as shown in figure 4.3. As before,
the gold sections indicate conductors. This design is a double-sided layout.
Black rectangles are varactor diodes. The blue rectangle provides a (capacitive) load to harvest voltage from the ELC section. The pump signal path is shown by red arrows, with dashed arrows representing the signal on the underside of the dielectric.

The unit cell shown in Fig. 4.3 allows the use of a wireless pump signal. As we have noted in the previous chapters, metamaterials provide an excellent platform for inserting many types of devices and circuits. The particular topology chosen for this work allows insertion of other devices and functions between the ELC section and the SRR section. This could include a matching network, bandpass filters, or active circuitry, for instance. One problem with the all-wireless approach to a PCM design is that the possible pump level delivered to the diodes decreases dramatically, in comparison to the direct cable-connected PCM. This is largely due to free space path loss, which depends on the antenna used for the pump and the physical distance between the pump and PCM. There could also be an impedance mismatch between the ELC and SRR sections. In order to counteract this, we investigate the addition of a simple amplifier network embedded in each particle between the ELC section and SRR section. This topology allows the addition of active elements between the metamaterial sections which can compensate for the signal loss. A simple amplifier and bias network does add wired connections to the unit cells, losing an advantage of the all-wireless PCM. However, these connections are simply DC bias lines rather
than RF signal lines, so the issues of phase error and bulky, expensive coaxial cables are not present.

On the other hand, this approach presents a very difficult design problem, because amplifiers for wireless systems can become unstable. One of the major causes of this is the potential lack of a good, electrically large ground plane. This alters the impedances seen by any wireless elements, sometimes dramatically. Another major source of instability is coupling between the input and output ports of the amplifier, which can cause it to oscillate. In order to obtain a stable unit cell that retains good performance (in terms of the PC signal level), this design must, essentially, be accomplished iteratively. While the general approach described in Ch. 4, using HFSS to provide fullwave data and ADS to simulate the nonlinear system, can be used for this design problem, it still encounters problems. We design a partial groundplane on the back of the PCM unit cell, as well as both input and output lumped-element networks, to stabilize the amplifier. The lumped-element stabilizing networks can be designed using the HFSS-ADS approach. However, the groundplane must be designed iteratively if there are not extremely high quality (internal) models of the amplifier. We conducted this iterative design approach with fabricated PCM elements, determining appropriate groundplane layouts that yielded stable PCM cells. It is also difficult to deploy a large array of PCM unit cells while retaining stability, so careful (and again typically iterative) consideration must be used when designing the DC bias network.

Thus, we investigate three versions of the PC metamaterial. The first is the simple unit cell with direct RF cable connections. The second is an entirely passive, all-wireless unit cell. The third is an active unit cell with only DC bias connections. Photographs of the three metamaterials under investigation are shown in Fig. 4.4. Figure 4.4(a) shows the simple direct RF connected metamaterial; (b) shows the passive, all-wireless metamaterial; (c) shows the active, DC bias connected metama-
The various designs discussed and shown above allow us to navigate the tradeoffs in making a PC metamaterial. We can use the passive all-wireless metamaterial if the primary design concern is simplicity and having no wired connections, or we can use the active metamaterial if generating the maximum PC signal while not requiring direct coaxial cable connections is the primary design concern.

We use simulations to verify the performance of the PC metamaterials prior to fabricating them, beginning with the basic differential equation describing the signal dynamics (eqn. 4.1). We use Matlab with a standard Runge-Kutta method to numerically solve the equation. This allows us to examine both the full spectrum of the generated signal and properties of particular harmonics. The numerical results are shown in Fig. 4.5. The signal floor shown in Fig. 4.5 is purely numerical in nature, due to the parameters used in the Matlab solution.

Figure 4.5(a) shows the calculated amplitude of the signal at the predicted PC harmonic (the difference frequency, $f_{\text{pump}} - f_{\text{source}}$). This signal was calculated for various values of the source signal phase, including $\phi_{\text{source}} = 0, \frac{\pi}{4}, \frac{\pi}{2},$ and $\pi$. The phase of a PC signal should vary as $-\phi_{\text{source}}$. The calculated difference frequency
Figure 4.5: Numerical verification of PC signal generation

Once we know that the signal from a single element is a PC signal, the next step with simulations is to verify that a number of PC elements produce a negatively refracting forward image. This also allows us to determine the minimum requirements for an effective experimental demonstration: simulations may show a minimum number of elements required to have an effective imaging system given our measurement limitations, for instance. We use COMSOL Multiphysics to simulate an array of individual PC elements, considering a two-dimensional imaging experiment with an antenna exciting individual PC metamaterial unit cells. COMSOL is used to simulate the generated field from the PC metamaterial. This simulation takes into account nonideal factors in the imaging performance, including the non-degenerate frequency case \( f_{PC} \neq f_{source} \), the actual radiation pattern of the SRRs, and interaction between the elements. As a very thin slab of PC medium, the reradiated pattern is symmetric about the slab [33]. An array of 8 discrete metamaterial unit cells is considered, with a source excitation located 3 wavelengths normal to the array and 1 wavelength from the center of the array. A diagram of the geometry
considered is shown in Fig. 4.6. This geometry was chosen as the effective (upper) size limit of our experimental setup and to demonstrate that the PCM can generate an effective image at a reasonable distance from the PCM array itself.

![Figure 4.6: Geometry considered both for COMSOL simulations and imaging experiments](image)

The produced fields from both the source alone and the PC metamaterial using COMSOL are shown in Fig. 4.7. Figure 4.7(a) shows the field produced by the source antenna alone while Fig. 4.7(b) shows the field produced by the discrete PC metamaterial. The white circles in the figure are the image spots produced by the metamaterial, provided for clarity.

Other local extrema are sidelobes from the array. These sidelobes are produced by various nonideal factors in the setup, including the nondegenerate frequency case. The simulations show that a discrete PC metamaterial will produce a clearly distinguishable image spot, notated in the figure. There is substantial distortion very close to the metamaterial, so imaging is only feasible a few wavelengths away from the metamaterial (or further).

These same simulations also show that with a small number of elements we can obtain a negatively refracting medium, although the size of the aperture is not sufficient for effective imaging. In this case, a source produces a beam in the negatively
With validation of our design at various levels, from a numerical solution of the differential equation to COMSOL simulations, we proceed to experimentally validate the results in the following section.

4.4 Experiment

We propose a number of experiments to fully characterize the PCMs and demonstrate their effectiveness. First, we will show that mixing products are generated when the metamaterial is excited with both the source and pump, particularly the difference frequency signal. Then we will show that the signal is in fact a PC signal rather than another type of nonlinear signal. We will then demonstrate that a small number of PCM unit cells can be used to produce negative refraction, and that a large number of PCM unit cells can be used for imaging. We variously consider three metamaterial unit cells in this section: the direct RF-connected varactor-based metamaterial; the passive all-wireless varactor-based metamaterial; and the active DC-connected varactor-based metamaterial.
We begin by demonstrating that pumping a PC metamaterial generates mixing products that are easily detectable. For all the experiments, the metamaterial unit cells are placed over a large ground plane. The excitation is then provided by a monopole antenna over the ground plane. Signals are provided by signal generators while the spectra are measured with a spectrum analyzer. As we described previously, the particular frequencies we chose were also selected to avoid external interferors such as the GSM and CDMA cellular phone bands. To verify that the signals were generated by the PC metamaterials, we measured the spectrum from 800-1000 MHz with no signal generators switched on. We then measured the spectrum with each of the three PCMs present and signal generators switched on. The measured spectra are shown in Fig. 4.8.

In Fig. 4.8, the signals are as follows: solid green corresponds to background signals (no signal generators switched on); dashed red corresponds to the direct RF-connected metamaterial; solid black corresponds to the passive all-wireless metamaterial; and solid blue corresponds to the active DC-connected metamaterial. The inset of the figure is zoomed in on the PC signal, showing a clear difference frequency signal generated for each of the metamaterials. The simple RF connected metamaterial...
terial produces a signal 26 dB below the incident source signal, while the all-wireless metamaterial produces a signal 59 dB below the incident source signal. While lower than the RF connected metamaterial, this is still easily detectable above noise. The active metamaterial produces a signal 36 dB below the incident source signal, helping to compensate for free space path loss and impedance mismatch losses.

Having established that the PC metamaterials generate strong, easily detectable mixed signals at the expected frequency, we proceed to demonstrate that the difference frequency signal is in fact a PC signal. Because this is a nonlinear process, we cannot directly measure the phase using a typical vector network analyzer (although it would be possible using a large-signal network analyzer). We devise an interferometric experiment to demonstrate that the signal is PC. To verify that individual elements were PC, we measured the interference pattern generated by two isolated elements. We also conducted an experiment to examine phase conjugation with an array of elements, described later. To map interference patterns, two monopole antennas above a ground plane were connected to a signal generator and spaced 1.5\(\lambda\) apart. A microwave absorber was placed between the antennas to isolate them, eliminating crosstalk between the antennas and metamaterial elements. They were excited both in phase and \(-90^\circ\) out of phase, and the interference patterns were mapped at a constant distance \(\lambda\) from the line connecting the antennas, at the signal frequency, as illustrated in Fig. 4.9(b). For two normal, non-conjugating elements excited in phase and then \(-90^\circ\) out of phase, the maximum of the pattern moved to the right. Then we measured the same interference patterns at the mixed frequency with the SRRs in place. The absorber was used to ensure that only one antenna was illuminating each SRR. Again the antennas were excited in phase and then \(-90^\circ\) out of phase and the interference pattern was mapped along the same line as before, at the mixed frequency. The measurements were interpolated and a filter was applied in the spatial Fourier domain to remove noise. The results are shown in Fig. 3(a).
and a schematic is shown in Fig. 3(b) (to scale).

Figure 4.9: Verification of phase conjugation for metamaterial elements

The interference patterns are shown for in-phase antennas (solid blue), $-90^\circ$ out-of-phase antennas (dashed blue), in-phase SRRs (solid red), and $-90^\circ$ out-of-phase
SRRs (dashed red).

Exciting two radiating elements with 0° phase shift and the same spatial positioning yielded field patterns roughly independent of the elements used, as shown by the solid lines in Fig. 4.9(a). The small-scale features in each curve are artifacts of a low dynamic range in these measurements. However, the important feature is the clear difference in the locations of the global extrema. Introducing a phase shift to the non-conjugating elements creates a maximum to the right of the center. However, with the PC elements, the peak moves to the opposite side of the center, showing that exciting normal radiating elements with a $-90^\circ$ phase shift is equivalent to exciting the PC metamaterial elements with a $+90^\circ$ phase shift. This is a clear demonstration that the nonlinear SRRs act as phase-conjugating elements.

Having demonstrated that individual metamaterial elements are PC elements, we use them to construct a negatively refracting medium. Three direct RF-connected varactor-based elements were used to approximate a discrete slab of PC medium, with a thickness of approximately $0.12\lambda$. The SRRs were pumped in phase and illuminated with a single antenna. As before $f_{\text{pump}} = 1850\text{MHz}$, $f_{\text{source}} = 950\text{MHz}$, and $f_{\text{pc}} = 900\text{MHz}$. The SRRs were placed $\lambda/4$ apart, creating an array a total of $\lambda/2$ in width. The source antenna was placed a distance one $\lambda$ normal to the array and $\lambda/2$ from the center of the array. The field power distribution of the mixed and phase-conjugated signal was then measured spatially to create a two-dimensional field map.

The field map was generated over a measurement area of dimensions $-3\lambda/2$ to $3\lambda/2$ tangential to the array and $\lambda/4$ to $\lambda$ normal to the array again by using the spectrum analyzer. The raw data were again interpolated and a filter was applied in the spatial Fourier domain to remove noise from the measurements. Figure 4.10 shows the results overlaid on the experimental setup.

The experimental setup in Fig. 4.10 is approximately to scale. The source radi
Figure 4.10: Measured field maps demonstrating PC negative refraction

ates from the upper left in the figure and the metamaterial produces a negatively refracting beam at the difference frequency. The main beam at the phase-conjugated frequency $f_{pc}$ clearly refracts to the opposite side of the normal compared to a conventional material. Moreover, there is excellent agreement between measurement and calculation, as shown by the inset in the figure. The same calculation but assuming non-phase-conjugating elements was also conducted and yielded very different results. The field power along a line of constant distance from the array was examined to illustrate this more quantitatively, comparing conjugate calculation, non-conjugate calculation, and filtered measured data. This is presented in Fig. 4.11.

The following signals are shown in Fig. 4.11: calculated PC elements (black); calculated non-PC elements (blue); filtered measured data (red). The measured data match calculated data very well and are clearly negatively refracted.

Having demonstrated that a PC metamaterial produces a negative refraction signal, we proceed to show that it can also be used for actual imaging with an
appropriately-sized aperture. The measurement setup is similar to that used for demonstrating negative refraction, with the dimensions similar to those described above for the simulations. We used 8 PC metamaterial elements with a source located $3\lambda$ normal to the PC metamaterial and the field map covered an area over $4\lambda$ normal to the PC metamaterial and over $3\lambda$ transverse to the PC metamaterial. We use absorbers around the experiment area to minimize the effects of external scatterers and multipath propagation. In effect, the measurement is similar to a 2D anechoic chamber. We place the source to be imaged in multiple locations to demonstrate the efficacy of the PC metamaterial as an imaging system. The source is first placed $3\lambda$ normal to the metamaterial and along the center axis of the metamaterial. We compare the simulated field distributions for this arrangement with the measured field map in Fig. 4.12. We then move the source; it is kept $3\lambda$ normal to the metamaterial but moved off the center axis of the metamaterial. The simulated and measured field distributions are shown in Fig. 4.13. The simulated field distributions are shown in Figs. 4.12, 4.13(a) while the measured field distributions are shown in
Figs. 4.12, 4.13(b).

(a) (b)

\begin{figure}[H]
\centering
\includegraphics[width=\textwidth]{figure4_12.png}
\caption{Simulated and measured field patterns for centered source}
\end{figure}

\begin{figure}[H]
\centering
\includegraphics[width=\textwidth]{figure4_13.png}
\caption{Simulated and measured field patterns for off-center source}
\end{figure}

The measurements again show good agreement with simulations and they demonstrate that a PC metamaterial can be used to effectively image a source.
4.5 Conclusions

The work presented in this chapter demonstrates the design of a nonlinear, active metamaterial aimed at the specific application of phase conjugation. We showed the required properties of a metamaterial to realize phase conjugation, following by then demonstrating through simulations that a varactor-loaded metamaterial can be used to realize phase conjugation. We showed design modifications to the base phase conjugation metamaterial based on the desired end application, including conversion to an all-wireless metamaterial and to an active metamaterial with a wireless pump signal. We experimentally verified the production of a strong mixed signal for all the designs and demonstrated that the mixed signal is in fact phase conjugate through an interferometric experiment. Finally, we showed experimentally that a phase conjugation metamaterial can be used to realize negative refraction and an imaging system. This chapter shows the applicability of varactor-based nonlinear metamaterials for applications beyond harmonic generation or power-dependent resonant frequency shifting, demonstrating nonlinear imaging. It also shows how a simple metamaterial design can be modified to suit it to particular applications.
In the previous chapters we demonstrated how nonlinear metamaterials can be designed for a diverse range of applications. Chapter 2 showed that a nonlinear metamaterial could be designed to exhibit a power-dependent transmission; Chapter 3 demonstrated that a nonlinear metamaterial could be designed to produce a significant DC voltage; and Chapter 4 showed that a nonlinear metamaterial could be designed such that particular mixing products exhibit interesting properties. These previous chapters relied largely on two-terminal diodes to provide the nonlinear effects. However, restricting our attention to metamaterials loaded solely with diodes is unnecessarily limiting. By including three- and four-terminal devices, we can obtain more flexibility in the design of a metamaterial since we have more degrees of freedom. Moreover, more complex devices allow us to design metamaterials with properties unobtainable through the use of diodes alone.

In this chapter our goal is to demonstrate that, using a simple transistor circuit, we can design tunable and nonlinear metamaterials with a variety of interesting and useful properties. We begin by considering a transistor-based metamaterial biased...
in the linear regime only. Since a transistor has more than two terminals, we can actively tune its properties while it remains a linear device. This is not true of tunable metamaterials based on varactors - as discussed in the previous chapters, varactor diodes are inherently nonlinear. We show that, by applying a DC bias appropriately, we can tune the effective properties of the metamaterial, including the resonant frequency and $Q$ factor. We refer to this as quasi-static tuning, as the tuning is accomplished with a static DC bias. By applying an AC bias, we make the transistor-based metamaterial time-varying. As a time-varying system, the metamaterial can thus be used for mixing to produce the sum- or difference-frequency products. We show that this is useful for constructing a linear phase conjugation metamaterial.

Following this, we demonstrate that even more flexibility is gained by using the transistor as a nonlinear device. First, we demonstrate that by including nonlinearity, we can make highly efficient phase conjugation media. By replacing the varactor diodes of Ch. 4 with appropriate transistors, we show an improvement in mixing efficiency of over 30 dB. We also demonstrate that we can design a nonlinear acoustoelectromagnetic metamaterial: by applying an acoustic transducer as the bias to the transistor, we modulate an RF signal with an incident acoustic signal. This demonstrates that metamaterials can be designed to couple acoustic fields to electromagnetic fields. Finally, we demonstrate how an appropriately biased transistor can be used to realize a tunably nonlinear metamaterial. We show that the effective nonlinearity of a metamaterial, in particular $\chi^{(2)}$, can be dynamically tuned. Using this tunable nonlinearity we demonstrate tunable mixing efficiency in a metamaterial.
5.1 Linear Transistor Metamaterial

5.1.1 Linearly Biased Transistor Characteristics

Transistors provide extra freedom of design for realizing different functions, both linear and nonlinear. Throughout this chapter, we are primarily concerned with fundamental transistor applications rather than more complex circuits such as amplifiers or oscillators. In particular, we start by examining the effective properties presented across two terminals of a transistor. Here we focus on the effective impedance $Z_{DS}$ across the drain (D) and source (S) terminals of an enhancement-mode N-channel MOSFET. Initially we are interested in creating a simple tunable $Z_{DS}$, in particular a variable effective resistance $R_{DS}$. A resonant metamaterial can be viewed as an RLC circuit[7]. Embedding a tunable resistance within a metamaterial allows us to control the quality factor, $Q$, of the circuit: the $Q$ factor of the metamaterial depends on the total series resistance of the equivalent circuit, including the embedded transistor. We might be interested in tuning the $Q$ factor of a metamaterial for a number of reasons. For instance, reducing the $Q$ increases the operational bandwidth of the metamaterial; we could also individually tune the $Q$ of a number of unit cells to reduce the effects of variability in manufacturing. We can then dynamically tune the resonant character of a metamaterial, allowing us to damp the resonance at will (or, conversely, to have a sharp resonance). A schematic of the basic unit cell design is shown in Fig. 5.1.

We select an N-channel MOSFET initially. Such a MOSFET presents a channel where the channel resistance depends on the bias of both the gate, $V_{GS}$, and the drain $V_{DS}$. By varying the $V_{GS}$, we vary the conductance of the channel and, thus, the resistance $R_{DS}$. If we operate in the linear regime of the transistor, the channel conductance (and thus $R_{DS}$) is dependent on the gate-source bias $V_{GS}$ as [44]
Figure 5.1: Schematic of basic unit cell with tunable $Z_{DS}$ impedance provided by transistor

$$R_{DS} = \frac{L}{W\mu C_{ox}(V_{GS} - V_T - V_{DS})}$$  \hspace{1cm} (5.1)

where $L$ and $W$ are the length and width of the gate respectively, $\mu$ is the effective charge carrier mobility, $C_{ox}$ is the oxide capacitance, $V_{GS}$ is the gate-source voltage, $V_{T}$ is the threshold voltage, and $V_{DS}$ is the drain-source voltage. We can also define the overvoltage $V_{OV} = V_{GS} - V_{T}$. Assuming that $V_{DS}$ is small, we can take a Taylor expansion as an approximation of $R_{DS}$ as follows:

$$R_{DS} = \frac{L}{W\mu C_{ox}V_{OV}} \left( 1 + \frac{V_{DS}}{V_{OV}} + \left(\frac{V_{DS}}{V_{OV}}\right)^2 + \left(\frac{V_{DS}}{V_{OV}}\right)^3 \ldots \right)$$ \hspace{1cm} (5.2)

For small $V_{DS}$ we neglect higher order terms and see that $R_{DS}$ is approximately constant with $V_{DS}$, but dependent on $V_{GS}$. By operating in the linear regime, we obtain quasi-static tunability of the metamaterial without introducing nonlinear effects like harmonic generation. We ensure that we remain in the linear regime by
keeping $V_{DS}$ small, where the approximation of Eqn. 5.2 is more accurate.

A larger $V_{GS}$ increases the concentration of carriers in the channel, which increases the conductance. As seen in Eqn. 5.2, this decreases the effective resistance $R_{DS}$. A typical MOSFET for this type of application might present a tunable $R_{DS}$ range of 15 $\Omega$ to over 5 k$\Omega$, with the $R_{DS}$ range dependent on the MOSFET chosen (sometimes specified as the channel resistance on a data sheet). The equivalent resistance of a base SRR, while strongly dependent on the fabrication process, materials, and frequency range of operation, is typically on the order of 1-10 $\Omega$.

If we are only interested in tuning the $Q$ of the resonance, we have to carefully select the transistor. This is because it also presents an effective capacitance $C_{DS,eff}$ across the same terminals, which in general may be the same order of magnitude as the metamaterial capacitance $C_{SRR}$. For small $R_{DS}$, $C_{DS,eff}$ is effectively bypassed. For large $R_{DS}$, $C_{DS,eff}$ is not bypassed. If $C_{DS,eff} \sim C_{SRR}$, the combination of the two capacitances may dramatically affect the resonant frequency of the metamaterial (if $C_{DS,eff} \sim C_{SRR}$, either the SRR or MOSFET capacitance will dominate). We can use this is a design feature if desired. The resonance will effectively switch between two states ($V_{GS} < V_T$ and $V_{GS} \gg V_T$), although it will switch continuously between the two limiting cases. In this section we focus on exploiting this resonant frequency tuning. Other bias schemes can allow us to tune only the $Q$ factor without altering the resonant frequency, but these will not be examined in this work.

The effective circuit model of an SRR loaded with a transistor biased as previously described is shown in Fig. 5.2(a). We show a photograph of a fabricated N-channel MOSFET tunable metamaterial (NTM) in Fig. 5.2(b). The fabricated NTM also includes RF chokes, so the bias lines (visible as a twisted wire pair in Fig. 5.2(b)) are isolated from the NTM at RF.

It is important to note that the MOSFET is difficult to model correctly when $V_{GS} < V_T$ for the particular transistor used here. The applicable SPICE model
unfortunately does not adequately model this bias region, typically treating it as a static region of operation. That is, SPICE models in general may not model any differences between $V_{GS} = 0 \text{ V}$ and $V_{GS} \approx V_T$, and this is the case for the particular transistor used here.

5.1.2 Tunable Linear Transistor-Based Metamaterial: Design and Simulation

As discussed above, we wish to tune both the resonant frequency and $Q$ factor of the metamaterial, treating it as a metamaterial that switches (continuously) between two states. Thus, we select a MOSFET with $C_{DS,eff} \sim C_{SRR}$.

For the metamaterial base element we select an SRR designed to operate in the 500-1000 MHz range for ease of fabrication and test. This design can in principle be extended for much higher frequencies, given transistors designed for operation in the 10s of GHz range or beyond.

We design a basic SRR that will resonate in this broad frequency range. We
model the SRR as a lumped-element equivalent RLC resonant circuit for simulation purposes, allowing us to use a SPICE-based tool (in this case, LTSpice). Using Ansys Q3D Extractor we extract the effective RLC parameters for a base SRR and obtain \( C_{SRR} \approx 0.34 \text{ pF}, \ R_{SRR} \approx 1.8 \Omega, \ L_{SRR} \approx 150 \text{ nH} \), yielding a resonant frequency \( f_0 = 704 \text{ MHz} \).

We calculate the analytical effective impedance of the NTM using its lumped-element model. We select an NXP BSS83 as the transistor for this work. To form an analytical model we require knowledge of \( R_{DS} \) for this particular transistor. We measure the drain-source current \( I_{DS} \) while varying both the biases \( V_{GS} \) and \( V_{DS} \). We note that, by design, we remain in the transistor’s linear region for this measurement. Using a typical transistor \( I_{DS} - V_{DS} \) curve, we can extract the effective conductance \( g_{DS} \) and thus obtain \( R_{DS} \). The measured \( R_{DS} \) values for this particular transistor are shown in Fig. 5.3.

![Figure 5.3](image)

**Figure 5.3:** Measured \( R_{DS} \) values for the selected transistor in this work as a function of \( V_{GS} \).

As we see in Fig. 5.3, \( R_{DS} \) varies from approximately 20 \( \Omega \) to 4000 \( \Omega \). Using the circuit shown in Fig. 5.2(a), we calculate the effective impedance of the NTM as a function of \( R_{DS} \) and as a function of frequency. The NTM should have essentially
three states: low-$R_{DS}$, in which $C_{DS,eff}$ is shorted and the resonant frequency $f_0$ is determined by $C_{SRR}$; high-$R_{DS}$, in which $C_{DS,eff}$ is not shorted and $f_0$ is determined by $C_{SRR}$ and $C_{DS,eff}$; and a transition low-Q state with substantial current through both $R_{DS}$ and $C_{DS,eff}$. We show the analytical impedance in Fig. 5.4(a).

![Figure 5.4](image)

**Figure 5.4**: (a) Normalized analytical impedance and (b) SPICE simulated impedance of lumped-element NTM model

The calculated data confirm the basic features of the NTM as $R_{DS}$ varies. We note that the $Q$ factors of the low-$R_{DS}$ and high-$R_{DS}$ states are different: these are effectively determined by $R_{DS}$ and $R_{SRR}$. $R_{SRR}$ can be designed by varying the fabrication process: altering the thickness or surface roughness of the conductor, plating the copper conductor with gold, or using other conductors such as gold or silver can reduce $R_{SRR}$. On the other hand, $R_{DS}$ is determined by the choice of the transistor. Other types of transistors, such as HJ-FETs, can offer much smaller channel resistances and carrier mobilities (and thus smaller $R_{DS}$ values) than the simple MOSFET used here.

By appropriately modulating $R_{DS}$, we can vary $f_0$ between two values and modulate the $Q$ of the NTM. We face a balance in tunability of both $f_0$ and $Q$, however, since these two parameters are not entirely independent once we select a particular
transistor.

Having confirmed the analytical function of the NTM, we use LTSpice to simulate the transistor physics and verify that an NFET can be used for this NTM. Using the lumped-element equivalent model for the SRR (RLC circuit) and a level 3 model for the BSS83 transistor, we obtain the simulated impedance shown in Fig. 5.4(b).

We see that the SPICE simulations are similar to the analytical calculated impedance. The resonant frequency for the high-$R_{DS}$ state differs from that of the lumped-element analytical impedance. This is primarily because the SPICE model supplied by the MOSFET manufacturer lacks a number of parameters and is not particularly accurate. However, it does give a general validation of our design approach.

Varying $V_{GS}$ thus provides tunability of both $f_0$ and $Q$ of a metamaterial. Using these data we also plot the two tunability parameters of interest, $f_0$ and $Q$, as functions of $V_{GS}$ in Fig. 5.5(a) and (b) respectively. We compute $Q$ using $Q = \frac{f_0}{\Delta f}$, where $\Delta f$ is similar to the half-power bandwidth.

Figure 5.5 demonstrates that there are several $V_{GS}$ ranges of interest. As stated above, the SPICE model used for this simulation does not accurately simulate sub-threshold voltage effects, so for all $V_{GS} < V_T = 1.7V$ the behavior is static. For $V_{GS} > V_T$ both $f_0$ and $Q$ decrease sharply. Biasing the transistor in this range thus results in large tunability, altering $Q$ by a factor of 2 and $f_0$ by about 3%. We can gain greater tunability by selecting a different transistor with different $C_{DS,eff}$, increasing the tunable range of both $Q$ and $f_0$.

If we are interested in mixing, for instance, setting the DC voltage level of $V_{GS}$ near 1.75V will result in a mixed signal with a relatively small amplitude for the AC component $V_{GS,AC}$. For large $V_{GS}$ we maintain tunability of $Q$ while $f_0$ is nearly static.

In order to simulate mixing using the NTM, we used Agilent ADS to model the NTM with a transient simulation. We bias $V_{GS}$ with a sinusoidally varying
signal at frequency $f_{\text{pump}}$. When mixing with a source signal at $f_{\text{source}}$, we expect mixing products to be generated only at the sum and difference frequencies (because this is a time-varying linear metamaterial). In particular we expect a strong signal at the difference frequency, $f_{\text{diff}} = |f_{\text{pump}} - f_{\text{source}}|$. For the ADS simulation we select $f_{\text{source}} = 625$ MHz, $f_{\text{pump}} = 1.275$ GHz. We thus expect a strong signal from the source and the mixed signal at $f_{\text{diff}} = 650$ MHz, with no other strong mixing products present (as there would be in a nonlinear metamaterial). We show the ADS simulated data centered around $f_{\text{source}}$ in Fig. 5.6.

As expected, we see that a difference frequency signal is generated without the higher order and nonlinear mixing products at $f_{\text{mix}} = m f_{\text{pump}} + n f_{\text{source}}$ where $m, n$ are all integers. Moreover, the mixed signal level is large enough to be easily detected in laboratory experiments. We see that the NTM can thus be used as a linear, time-varying mixing metamaterial. We note from the analysis of a time-varying
resonant particle in Ch. 4 that the difference frequency produced by such a mixing metamaterial is a phase conjugated signal. Thus, we expect that the difference frequency signal generated by the NTM (acting as a mixer) will be a PC signal. With these simulation results confirming the function of the NTM, we demonstrate these effects experimentally in the following section.

5.1.3 Tunable Linear Transistor-Based Metamaterial: Experiment

We fabricated the NTM using standard RF techniques similar to those in the previous chapters. We used the TEM waveguide described in Ch. 3 and a vector network analyzer to characterize the NTM. We present only $S_{21}$ data in this work, for comparison with the calculated and simulated results in the previous section. A sample of the measured $S_{21}$ data is shown in Fig. 5.7.

Figure 5.7 clearly shows that the NTM functions as expected from the analytical calculations and SPICE simulated results presented in the previous section. As we mentioned previously, the manufacturer-supplied SPICE model was inadequate for highly accurate simulation. Despite the low quality of the model used, the NTM clearly allows us to tune both $f_0$ and $Q$ in a manner very similar to the simulated results. To better illustrate this we show the calculated $f_0$ and $Q$ of the NTM in
The experimental data validate the design and simulation data. By biasing $V_{GS}$ we can vary both $f_0$ and $Q$ for the NTM, with the $Q$ and $f_0$ qualitatively similar to the results extracted from SPICE simulations. We obtain a tunable $f_0$ range of
over 7.5% when varying $V_{GS}$ from 0 to 10 V. $f_0$ is rapidly varying near $V_{GS} \approx 2.5$ V, making this approximate bias level the most appropriate for linear mixing. The power required for this tunability is also low, requiring at most 1.8 mW. If we hold $V_{GS}$ between 0 and 2.5 V, the maximum power used is 200 $\mu$W.

If we vary $V_{GS}$ from 1 to 3 V, we obtain a $Q$-tunability of over a factor of 7.5. We thus have several bias ranges of interest depending on the desired application. If we wish to tune $Q$ but leave $f_0$ nearly static, we vary $V_{GS}$ from 1 to 1.75 V. If we wish to switch between the two high-$Q$ states, we simply switch $V_{GS}$ between 0 and 10 V. The NTM thus provides flexibility for metamaterial design in a single particle design.

As we showed in ADS simulation, we wish to use this metamaterial as a linear, time-varying mixing metamaterial. Using the frequencies specified in the previous section, we add a direct cable connection to the NTM unit cell to bias the gate (similar to the direct cable connected unit cell in Ch. 4). We expect to measure a signal at $f_{source}$ and $f_{diff}$, without other higher-order (nonlinear) mixing products present. Again similarly to the mixing demonstration in Ch. 4, we use signal generators to generate pump and source signals. We record the spectrum with a spectrum analyzer in the vicinity of $f_{diff}$. The recorded spectrum is shown in Fig. 5.9.

![Figure 5.9: Measured NTM mixing spectrum](image_url)
Figure 5.9 shows three recorded spectra: the ambient spectrum, with no signal generators turned on (solid black); the signal generator spectrum, with generators turned on but the NTM not present (dashed red); and the spectrum with generators turned on and the NTM present (solid blue).

We note that there is a strong signal at the half harmonic of the pump ($f_{\text{pump}}/2$). However, this signal is present even without the NTM, indicating that it is produced by internal mixing in the equipment. Additional filters could be used to attenuate this signal if needed. There is also a small signal at $f_{\text{diff}}$ even without the NTM present, again due to internal mixing of the equipment. When the NTM is included, the difference frequency signal is significantly stronger. We also note the lack of higher-order nonlinear mixing products, demonstrating that the NTM is an effective linear, time-varying mixing metamaterial.

We wish to verify that the difference frequency signal is in fact a PC signal. We use a similar setup to that of Ch. 4, as shown in Fig. 4.9. We can show interferometrically that a $-90^\circ$ phase shift with two NTMs is equivalent to a $+90^\circ$ phase shift in two normal elements. The results for the NTM is shown in Fig. 5.10.

![Figure 5.10: Verification of phase conjugation for NTM elements](image_url)

The solid curves are elements excited in phase, while dashed curves are elements
excited $-90^\circ$ out of phase. The blue curves are non-PC antenna elements and red curves are NTM elements. As with the varactor-based metamaterial in Ch. 4, we see that the in phase patterns are similar while the $-90^\circ$ phase shifted patterns are symmetric with each other about the center of the measurement. These data show that the NTM can be operated as a PC metamaterial.

Unlike the varactor-based metamaterial presented in Ch. 4, however, the NTM operates as a linear PC metamaterial. We have thus showed that even linear transistor-based metamaterials can accomplish useful functions. With appropriate DC biasing such a metamaterial exhibits a tunable $Q$ and resonant frequency, including switching between two resonant states. The same basic design can also be used as a linear time-varying mixing metamaterial, including for phase conjugation.

5.2 Nonlinear Transistor Metamaterials

We have demonstrated multiple interesting effects with a linear transistor-based metamaterial. However, we obtain even greater design freedom by utilizing a transistor as a nonlinear element. Nonlinear transistor circuits provide an avenue for realizing effects such as high-efficiency mixing (potentially with gain), efficient power amplification, and modulation. A full description of nonlinear transistor applications would fill a number of textbooks, so we restrict our attention in the remainder of this chapter. We primarily focus on mixing as the application of choice. We show that introducing nonlinearity (through proper biasing and signal levels) allows us to expand on the previous work in this thesis. First, we show that we can make more effective phase conjugation metamaterials by using nonlinear transistor-based unit cells. We show that simply biasing an appropriate transistor can increase the mixing efficiency (PC signal generation) by over 30 dB. We also show that we can tune the nonlinear character of a PC transistor-based metamaterial, allowing us to dynamically tune the mixing efficiency. We then demonstrate how, by using a transducer in concert with
a nonlinear transistor, we can make a metamaterial which couples acoustic waves to electromagnetic waves and effectively acts as a frequency modulating metamaterial. Finally, we demonstrate that the tunable nonlinearity of a transistor metamaterial directly extends to the effective medium description. By using a nonlinear transistor, we can construct a metamaterial with dynamically tunable $\chi^{(2)}_m$.

5.2.1 Nonlinearly Biased Transistor Properties

We assumed in Sec. 5.1 that the $V_{DS}$ bias is small such that the transconductance is linear with $V_{DS}$. If we bias a transistor with $V_{DS} \neq 0$, there are two main regions of interest. The most obvious is the saturation mode, where the transistor is approximately a current source (that is, $I_{DS} = f(V_{GS})$ for all $V_{DS} > V_{OV}$). There is also a transition region, between the linear and saturation regions. In this transition region, we can consider the transistor as a nonlinear $R_{DS}$ in parallel with $C_{DS,eff}$. Thus, the transistor is a nonlinear device for non-small $V_{DS}$. We use this analysis with the effective impedance, $Z_{DS}$. We maintain $R_{DS}(V_{GS})$, but with an extra degree of tunability: we can apply a small DC bias to $V_{DS}$ to obtain a nonlinear impedance $Z_{DS}$. We will show how this is useful for metamaterial applications in the following sections.

A transistor is more complicated than this when forward biased, however, and we can exploit other features of transistor physics. In fact, if we examine the physical structure of a MOSFET, we find that there is always a reverse-biased p-n junction seen between the drain and source [44]. Such a junction is the basis of a varactor diode, and thus presents a voltage-dependent capacitance. This dependence may not be significant, depending on the region of operation. If $Z_{DS}$ is dominated by the channel resistance, then this voltage dependence can be neglected - for instance, for large $V_{GS}$, the channel resistance dominates $Z_{DS}$. However, if we operate with $V_{GS} < V_T$, there is not a conductive channel formed between the drain and source.
Thus, when operated in the subthreshold region, we can treat the transistor similarly to a varactor diode in parallel with a large resistance (which we neglect). While applying a bias (below $V_T$) to the gate does not form a good channel, it does alter the charge distribution near where the channel will form. This affects the capacitance of the reverse-biased p-n junction: thus, a subthreshold transistor can be biased to act as a tunable varactor diode.

5.2.2 Nonlinear Phase Conjugation Metamaterial

Our first application of the nonlinearity of a transistor is to phase conjugation. We showed in section 5.1 that a linear transistor metamaterial acts as a PC metamaterial and can provide linear time-varying mixing. This application would be useful in situations that have strict spectrum requirements (as higher-order nonlinear harmonics are not generated) or with lower power levels. However, in general we may desire a particular mixing product with a very high amplitude. Nonlinearity can be used to accomplish this. In order to maximize the nonlinear signal, we select a different transistor. In particular we select an HJ-FET, a CEL NE3509M04 HJ-FET, for this work. This transistor is a depletion-mode device, so the $R_{DS}$ characteristic is different from that of the MOSFET: at low $|V_{GS}|$, with $V_{GS} < 0$, the transistor is switched on (with a conducting channel formed), while at high $|V_{GS}|$ the transistor is switched off (yielding a large $R_{DS}$). We select this transistor for a number of reasons. First, it has a smaller parasitic capacitance, $C_{DS,eff}$, than the BSS83 MOSFET. This yields a greater tunability when using quasi-static tuning. Second, it has a much larger transconductance: the BSS83 has a specified transconductance of $g_m = 22$ mS, while the HJ-FET has a specified transconductance of $g_m = 80$ mS and higher carrier mobility. The HJ-FET thus switches between the two limiting cases (large and small $V_{GS}$) much more rapidly than the BSS83. Third, the HJ-FET has a much lower threshold voltage, with $V_T \approx -0.4$ V compared to $V_T \approx 1.7$ V. The HJ-FET is thus
more suited for mixing, requiring a smaller signal on the gate bias to modulate the channel resistance. These factors make such a transistor ideal for generating a large nonlinear signal.

We use Agilent ADS to simulate a nonlinear transistor PCM. We also compare this to ADS simulations of the varactor-based PCM presented in Ch. 4. We simulate a two-tone signal (as in Ch. 4) mixing within a metamaterial. We wish to examine the mixing efficiency as a function of the pump power in this case: thus, we record the PC (difference frequency) signal as a function of pump signal power. The results are shown in Fig. 5.11.

![Figure 5.11: ADS Simulated mixing efficiency for JFET and varactor PCMs](image)

We see that the HJ-FET is significantly more nonlinear than the varactor in this simulation: the mixing efficiency, shown by the PC signal power, is over 30 dB higher in the HJ-FET PCM than the varactor PCM. This is especially important at low pump levels: the HJ-FET metamaterial will still produce a strong, detectable signal even when the varactor metamaterial signal may fall below the noise floor of our measurement system. We verify this experimentally. We use the same signal generators and spectrum analyzer as in the previous experiments. We note that the frequencies are different for the HJ-FET and varactor PCMs. The varactor
metamaterial uses the frequencies given in Ch. 4, producing a PC signal at 900 MHz. The HJ-FET metamaterial uses the frequencies given in Sec. 5.1, producing a PC signal at 650 MHz. Thus, we cannot use a static physical arrangement of the source and receiving antennas with the metamaterial. With different frequencies, the propagation loss is different over the same physical distance. Our source and receiving monopole antennas may also have different return losses at these frequencies. Our measurement setup takes these factors into account. We ensure that the source and receiving antennas are the same electrical distance from the metamaterial under test, such that the propagation loss is the same. We also use different monopole antennas, switching them such that the return loss is approximately the same between the two cases. Sweeping the pump signal power with the signal generator and recording the results with the spectrum analyzer, we obtained the data shown in Fig. 5.12.

![Figure 5.12: Measured mixing efficiency for JFET and varactor PCMs](image)

We see that the results qualitatively agree with simulations. The HJ-FET metamaterial provides a mixing efficiency of over 32 dB higher than the varactor metamaterial over the measured range of powers. We also see that at low pump levels the PC signal from the varactor metamaterial falls to the noise floor of the measurement system. The HJ-FET metamaterial, on the other hand, still produces a
very strong, detectable PC signal even at these low power levels. The ADS simulations overestimate the PC signal levels for both cases, but we see clearly that the HJ-FET metamaterial can act as an efficient phase conjugation metamaterial. The increase in mixing efficiency of over 32 dB shows that nonlinear transistor metamaterials can provide much stronger signals in mixing applications than that of varactor diode-based metamaterials.

Moreover, we gain additional tunability with the use of a nonlinear transistor. As we discussed in the previous section, we can apply a DC bias to $V_{DS}$ which will adjust the nonlinearity of our metamaterial. This allows us to dynamically tune the mixing efficiency of a nonlinear metamaterial through only the use of a DC bias. In terms of a phase conjugation application, we can dynamically adjust the PC signal level through the use of a DC bias.

For this experiment, we use a metamaterial based on the BSS83 transistor, which is rated for a higher drain current (and higher power operation) than the HJ-FET. We again characterize the PC signal power as a function of the pump signal power, this time also sweeping a DC $V_{DS,DC}$. The measured results are shown in Fig. 5.13.

We see that for low values of the pump signal, increasing $V_{DS,DC}$ decreases the PC signal level. For high pump power, however, increasing $V_{DS,DC}$ increases the PC signal level. Using ADS, we simulate the effect on the PC signal generated by a variable $V_{DS,DC}$ with low pump power ($P_{pump} = -30 \text{ dBm}$). The simulated results are shown in Fig. 5.14.

We see that the PC signal power also falls with increasing $V_{DS,DC}$. To compare the simulated and measured results more directly, we also extract similar data from that shown in Fig. 5.13, recording the PC signal level for $P_{pump} = -30 \text{ dBm}$ only. These data are shown in Fig. 5.15.

The simulated and measured data show the same qualitative behavior. However, the tunable range of PC signal level is much lower in measurement (approximately
Figure 5.13: Measured PC signal as function of both pump power and $V_{DS,DC}$ for MOSFET PCM

6 dB) than in simulation (approximately 17 dB). The difference likely comes from simplifications we made in the simulations. For this simulation we considered a lumped-element equivalent of the base metamaterial cell (an SRR). We chose to use this rather than fullwave simulation data because ADS has difficulty simulating transistor-based circuits embedded on a fullwave S-parameter block - convergence of harmonic balance simulations is often poor for using an S-parameter block, while it improves dramatically with a lumped-element equivalent. We believe that this accounts for the bulk of the difference in tunable PC signal range. This also accounts for the absolute differences between the simulated and measured results.

ADS is also unable to correctly simulate the case of high $P_{pump}$ with this transistor, likely due to these difficulties and the incomplete SPICE model for the transistor. Thus, we do not have a simulated result to compare to the data from Fig. 5.13 for
large $P_{\text{pump}}$. However, the experimental data clearly show that for $P_{\text{pump}} > 6$ dBm, the PC signal level actually increases with increasing $V_{DS,DC}$. We show the PC signal power as a function of $V_{DS,DC}$ for $P_{\text{pump}} = +10$ dBm in Fig. 5.16.

We see that adjusting $V_{DS,DC}$ provides approximately 6 dB of tunability for the mixing efficiency with high $P_{\text{pump}}$ levels. This demonstrates that the tunable PCM is actually far more useful: whether the pump for a PC application provides low or high power, we can dynamically adjust the PC signal level (to maximize it, for instance)
by simply adjusting a DC bias level. The use of a nonlinear transistor metamaterial thus provides significantly enhanced flexibility for this application.

We have shown in this section that, simply by using a transistor in the nonlinear region, we can dramatically improve the performance of a PCM. Simply selecting an appropriate transistor, such as a JFET, can increase the mixing efficiency of a PCM by over 30 dB. Moreover, we can use a DC bias $V_{DS,DC}$ to dynamically adjust the mixing efficiency of a PCM, allowing us to maximize the PC signal level whether operating with low or high $P_{pump}$.

5.2.3 Nonlinear Acoustoelectromagnetic Metamaterial

We showed that nonlinear transistor metamaterials can provide very strong and adjustable mixing in the previous section and that such metamaterials are ideal for applications requiring large nonlinearity (such as phase conjugation). In this section, we propose a new type of nonlinear metamaterial based on the concepts developed in this chapter. We demonstrate that a metamaterial can be designed to act as an acoustoelectromagnetic metamaterial (AEM): it can couple incident acoustic waves to incident RF waves. This design relies on a strongly nonlinear mixing metamaterial,
provided by the transistor design of the previous section.

For this application, we devise a method of coupling an acoustic signal to the metamaterial. We saw that an AC signal applied to the gate of a transistor can be used to generate a mixing metamaterial. Moreover, we can increase the mixing efficiency of such a metamaterial by setting the DC level of the signal near the threshold voltage of the transistor. Our design is straightforward: we use an acoustic transducer to convert an incident acoustic signal to an AC signal and apply that to the gate of a transistor. If we illuminate the AEM with an RF signal, the transistor will mix the AC and RF signals. Since \( f_{RF} \gg f_{\text{acoustic}} \), we obtain a number of mixing products in the vicinity of \( f_{RF} \). The AEM will then reradiate the mixed signal (as it is near the resonant frequency of the metamaterial). Our AEM essentially acts as a modulator: when illuminated with an RF signal near the resonant frequency of the metamaterial, it will produce a signal containing information from the acoustic signal. Thus, the AEM acts as a frequency modulating (FM) metamaterial, coupling the acoustic signal to an RF signal. We illustrate the resultant spectrum schematically in Fig. 5.17 (not to scale).

![Figure 5.17: Sketch of AEM spectrum](image)

The blue signal represents the acoustic signal; the red signal represents the
RF incident signal; the magenta signals represent the mixed acoustoelectromagnetic signals. We illustrate the rough frequency range where the AEM presents effective radiating modes, showing that the mixed products near \( f_{RF} \) (that is, at \( f = f_{RF} + m f_{acoustic} \) where \( m \) represents integers) can be effectively radiated by the metamaterial.

We use a MEMS microphone to act as the transducer. A typical MEMS microphone board will include a DC bias line and amplifier. We select a Sparkfun Breakout Board with the ADMP401 MEMS microphone as the transducer for this work. The microphone provides an AC signal out that depends on the bias voltage of the microphone, typically \( \frac{V_{bias}}{2} \pm 200 \text{ mV} \). This is an advantage for use with the BSS83 transistor: we can select the bias voltage such that the DC value of the microphone output is near \( V_T \) for the transistor, providing large sensitivity to the input acoustic signal. A sketch of the AEM cell is shown in Fig. 5.18(a), while an example fabricated unit cell is shown in Fig. 5.18(b).

![Sketch of AEM unit cell operation and photograph of fabricated AEM cell](image)

**Figure 5.18:** Sketch of AEM unit cell operation and photograph of fabricated AEM cell

In Fig. 5.18(b), we have indicated the MEMS microphone board in yellow and
the transistor in cyan. Using this unit cell as the basis for a metamaterial, we demonstrate that the AEM couples an acoustic signal to an RF signal. We begin by using a monochromatic acoustic signal for clarity. We illuminate the AEM with an RF signal at 650 MHz and a 2 kHz acoustic signal. The measured spectrum is shown in Fig. 5.19. We have indicated the carrier frequency as well as the various mixing products. In the figure, $f_{c+1}$ is shorthand for $f_{\text{carrier}} + 1f_{\text{acoustic}}$, $f_{c-2}$ is shorthand for $f_{\text{carrier}} - 2f_{\text{acoustic}}$, etc.

![Spectrum of AEM illuminated with 2 kHz acoustic signal](image)

**Figure 5.19:** Spectrum of AEM illuminated with 2 kHz acoustic signal

These data show that the AEM cell is effective in modulating an RF carrier with an acoustic signal. The AEM provides direct FM of a wireless signal with a very simple unit cell design. Moreover, the AEM only emits an RF signal when illuminated with one, potentially providing an important technology for audio surveillance. We also show that the AEM functions for typical acoustic signals, not only monochromatic ones. We record the RF spectrum for ambient sounds (those found in a typical lab environment, with background speech and equipment sounds). We then use a speaker to play sample music near the AEM and record the spectra. The recorded data are shown in Fig. 5.20.

We see that the AEM effectively modulates an RF carrier with incident acoustic signals. We note that the spectra are not precisely symmetric in the recorded
Figure 5.20: Spectrum of AEM with ambient sound and sample music modulated on RF carrier.

Data: we expect that they should be symmetric about the carrier, as there is no extra filtering to change the signal. The primary reason for this is a limitation in the test equipment used. In order for the spectrum analyzer to resolve signals in the kHz range, with a low noise floor, while being centered at RF, the spectrum analyzer requires several seconds to both generate a trace and record data. The modulated spectrum of sample music should be symmetric about the carrier at any given time, but it will change as a function of time. Thus, the recorded spectrum includes information from a number of points of time, so we expect the spectrum will be asymmetric about the carrier. Despite this difficulty with the measurement equipment, we have shown that a nonlinear transistor metamaterial effectively modulates an acoustic signal to an RF carrier signal, acting as an acoustoelectromagnetic metamaterial.

Building on the previous section, we can also apply a small DC bias $V_{DS,DC}$ to adjust the mixing efficiency of this nonlinear metamaterial. Doing so allows us to dynamically adjust the modulation level with our AEM. We again illuminate our AEM with a monochromatic acoustic signal and RF carrier. We adjust $V_{DS,DC}$ from 0 V to 2 V, recording the spectrum near the carrier at each step. We show the
spectra for $V_{DS,DC} = 0, 1, 2$ V in Fig. 5.21.

![Spectrum of AEM for monochromatic acoustic signal with varying $V_{DS,DC}$](image1)

**Figure 5.21:** Spectrum of AEM for monochromatic acoustic signal with varying $V_{DS,DC}$

These measurements clearly show that by adjusting $V_{DS,DC}$ we can dynamically adjust the mixing efficiency of the AEM. Focusing on the harmonics at $f_{RF} \pm f_{acoustic}$, we plot the modulation level in Fig. 5.22.

![Modulation Level](image2)

**Figure 5.22:** Spectrum of AEM for monochromatic acoustic signal with varying $V_{DS,DC}$

These results show that, for the first order mixing products, we can dynamically adjust the mixing (modulation) efficiency by 7.5 dB. This increases the flexibility of
adapting the AEM for applications. We can maximize the nonlinearity and, thus, the
modulation and acoustoelectromagnetic coupling by increasing a DC bias, \( V_{DS,DC} \).
If, on the other hand, we wish to decrease the higher-order mixing products, we can
decrease \( V_{DS,DC} \). We have shown experimentally that nonlinear transistor metamaterials provide a platform for coupling acoustic and nonlinear signals, with the ability
to dynamically tune the mixing efficiency using only a small DC bias.

5.2.4 Tunable \( \chi_m^{(2)} \) Metamaterial

As we discussed in section 5.2.1, a MOSFET operated in the subthreshold region is
still a nonlinear device. It presents a voltage-dependent capacitance, similar to that
of a varactor diode, across the drain and source. However, by adjusting the bias of
the gate \( V_{GS} \), we can adjust the capacitance while remaining below threshold.

It was previously shown [25] that a varactor-loaded metamaterial (in particular a
SRR) can be described by an effective \( \chi_m^{(2)} \) as well as the usual linear material prop-
eries (\( \epsilon, \mu \)). The experimental verification of this was accomplished by examining a
three-wave mixing experiment: two signal generators are used to produce incident
fields at \( \omega_1, \omega_2 \) which are incident on a nonlinear metamaterial. The fields produced
at the sum frequency \( \omega_s = \omega_1 + \omega_2 \) are proportional to the second order nonlinear
susceptibility \( \chi_m^{(2)} \) for the nonlinear magnetic metamaterial used in the experiment.
We conduct a similar experiment for a nonlinear subthreshold transistor metamate-
rial. While using different particular equipment, the setup is the same as in [25]. We
sweep the signal at \( \omega_1 \) from 500 MHz to 1 GHz while keeping the signal at \( \omega_2 = 650
MHz \), near the resonant frequency of the transistor metamaterial. The signal at the
sum frequency is measured and converted to the magnetic field value, using

\[
H = \sqrt{\frac{2P}{z_0 A}} \tag{5.3}
\]
where $P$ is the power of the wave, $z_0$ is the wave impedance in approximately free space (377Ω), and $A$ is the cross-sectional area of the waveguide, in this case 171.1 cm$^2$. Using the notation from [25], we wish to plot $|H_{3,exp}^+(\omega_{1,2})|$ as a function of both frequency and $V_{GS}$. This field is directly proportional to the effective $\chi_m^{(2)}$ for the metamaterial [25]. The metamaterial design is the same as that used for the linear transistor work described in Sec. 5.1: an SRR unit cell has a MOSFET (the BSS83 model) embedded, with DC bias lines for the gate to supply a DC $V_{GS}$.

We record the generated sum frequency signal as a function of $\omega_1$ and $V_{GS}$. The data are shown in Fig. 5.23 after Fourier processing and converting to magnetic field values using Eqn. 5.3.

![Figure 5.23: Measured $|H_{3,exp}^+(\omega_{1,2})|$ for transistor metamaterial with varying $V_{GS}$](image)

The data in Fig. 5.23 can be compared with the data in Fig. 4 of [25]. We see the same qualitative behavior as in [25] for any individual value of $V_{GS}$: the sum frequency field has a maximum value near $2f_{resonant}$, with similar field values. However, as predicted, we can adjust the sum frequency generation by adjusting $V_{GS}$. And, since $|H_{3,exp}^+|$ is directly proportional to $\chi_m^{(2)}$, adjusting $V_{GS}$ allows us to dynamically adjust the effective $\chi_m^{(2)}$ of the metamaterial. This is the first demonstration of a metamaterial with a tunable nonlinearity: metamaterials using varactor diodes, as
in [25], present an effective $\chi^{(2)}_{nm}$. However, in order to change the $\chi^{(2)}_{nm}$ of the metamaterial, it is necessary to change the varactor diode. At a particular $V_{GS}$ level we can treat the transistor as a varactor diode, since as we saw in section 5.2.1 the transistor presents a reverse-biased p-n junction. By using a nonlinear, subthreshold transistor, we are in effect changing the varactor characteristics of a nonlinear metamaterial electrically. This is a fundamental demonstration of the power of nonlinear metamaterials using transistors: we can design a metamaterial in which we can dynamically tune how nonlinear the metamaterial is.

5.3 Conclusions

The work presented in this chapter is the first work demonstrating the design power gained by using a simple transistor circuit in metamaterials. By biasing a transistor in the linear region, we showed that we can design a metamaterial with dynamically tunable $Q$ and resonant frequency with both numerical and experimental validation. We used this metamaterial, with an AC bias, to demonstrate a linear time-varying mixing metamaterial. This metamaterial provided a phase conjugated signal, which we showed experimentally.

We then examined a simple transistor-based metamaterial with the transistor nonlinearly biased. We showed that using a nonlinear transistor allowed us to dramatically improve the phase conjugation metamaterial presented in Ch. 4 as well as the linear phase conjugation metamaterial in the first section of this chapter. Simply using a nonlinear HJ-FET allowed us to increase the mixing efficiency of such a metamaterial by over 30 dB. Using the additional freedom of a third terminal for a transistor, we also showed that we could dynamically adjust the mixing efficiency of a nonlinear metamaterial. We introduced a new type of nonlinear acoustoelectromagnetic metamaterial, one which uses an incident acoustic signal to modulate
an incident RF signal. We showed that the modulation efficiency of this metamaterial could also be dynamically adjusted through the use of a nonlinear transistor bias. Finally, we showed that by using transistors, rather than diodes, we designed a nonlinear metamaterial with a dynamically tunable nonlinearity.
6

Summary and Future Work

6.1 Summary

Metamaterials provide huge design freedom for applications. The wide range of achievable material values, including those not found in natural materials, allow a designer to create metamaterials that can enhance existing applications or conceive of entirely new ones. Many useful applications rely on nonlinear effects: efficient power amplification, mixing and frequency conversion, and phase conjugation are just a few examples. The flexibility provided by metamaterials is a significant advantage for such applications: metamaterials can be designed to function in a traditional transmission line or in sheet or volumetric structures, opening new application areas. Nonlinear metamaterials have been studied primarily from a basic physics perspective, including characterizing effects such as harmonic generation or power-dependent tuning, through the use of varactor diode-based designs. The goals of this work were to expand the palette of useful nonlinear effects available to the designer, through examining the use of other nonlinear devices besides varactor diodes, and to demonstrate how nonlinear metamaterials can be applied for useful design goals.
We began by examining a particular application, an RF limiter. We presented a design strategy for a metamaterial that would realize the same nonlinear function as an RF limiter, but in the form of a sheet rather than a traditional transmission line (such as a microstrip or coplanar waveguide limiter). This RF limiter metamaterial was straightforward to design, fabricate, and experimentally validate. It showed good performance over a broad bandwidth, realizing the nonlinear function we desired. We used PIN diodes to accomplish this, demonstrating a new type of nonlinear device that could be embedded in metamaterials to realize a particular nonlinear function.

Following the RF limiter, we chose to design a nonlinear metamaterial that would act to harvest RF energy and convert it to DC energy. The basic design of this metamaterial requires a different goal than prior nonlinear metamaterial work: we wished to maximize the frequency conversion from RF to DC. We showed that traditional circuit design, such as might be applied to a rectenna, could be adapted to design such a metamaterial. Simple split-ring resonators provided a straightforward platform for the power harvesting metamaterial. We confirmed the performance of our design with cosimulation of both fullwave and nonlinear circuit data, using ANSYS HFSS and Agilent ADS, respectively. We then fabricated the metamaterial and validated its performance experimentally. We obtained good agreement between simulation and experiment, showing both that our design and simulation approach works and that a metamaterial can be designed to act as an effective power harvester.

We then chose to show how metamaterials could be used to produce a phase conjugated signal, a problem of interest for signal processing, RF, and optical engineers. A metamaterial with parametrically varying resonant frequency provided a platform for generating a phase conjugated signal: we demonstrated this numerically by solving the differential equation describing the system. We then fabricated a number of nonlinear metamaterial unit cells, showing that they produced a strong mixed signal at the frequency of interest. Using an interferometric experiment, we verified that
the nonlinear signal was a phase conjugated signal. We then demonstrated how the metamaterial unit cells could be tailored for a variety of specific applications: we first designed a unit cell that produced a phase conjugated signal with a direct coaxial cable connection, providing a large nonlinear signal output. We then modified this design to produce a phase conjugated signal completely wirelessly, with no direct connections to the unit cell. Following this we demonstrated a design that would compensate for free space path loss introduced with the all-wireless design, using an active amplifier network embedded in each cell, with only a DC bias line connected to the unit cells. With these unit cell designs available, we showed that a phase conjugation metamaterial was effective for negative refraction and imaging through both COMSOL Multiphysics simulations and measurements.

Having shown a variety of possible applications and nonlinear effects using diode-based metamaterials, we then examined the use of simple transistor circuits in metamaterials. We began by discussing how transistors biased in the linear region could be useful for tunable, linear metamaterials. Using both SPICE simulations and experiments, we demonstrated that linear metamaterials could be designed with tunable $Q$ factor and resonant frequency using a simple, low-power DC bias. We also used an AC bias with the same metamaterial to show mixing; however, this mixing metamaterial was linear rather than typical varactor-based metamaterials. The transistor-based metamaterial produced mixing products corresponding to a linear, time-varying mixer, in both Agilent ADS simulations and experiments. We applied this to produce a phase conjugated signal, showing that linear metamaterials could also be used for this application. Having demonstrated the utility of linear transistor-based metamaterials, we expanded our discussion to include transistors acting as nonlinear devices. We showed that operating a transistor in the nonlinear regime could dramatically enhance the performance of a phase conjugation metamaterial, increasing the mixing efficiency by over 30 dB. We also utilized the additional
design freedom of a three-terminal device by showing, through both ADS simu-
lations and measurements, that a phase conjugation metamaterial based on nonlinear
transistors possessed a tunable mixing efficiency. We used this to dynamically tune
the phase conjugated signal level generated by the metamaterial, demonstrating the
additional design flexibility gained through the use of transistor metamaterials. We
then illustrated the flexibility of a nonlinear transistor metamaterial by designing an
acoustoelectromagnetic metamaterial. This metamaterial modulated an incident RF
signal with an incident acoustic signal, coupling the two types of wave propagation
on an RF carrier. We again showed that utilizing nonlinear transistors allowed us
to adjust the mixing efficiency of such a metamaterial, dynamically tuning the mod-
ulation level. Finally, we examined a nonlinear transistor in the subthreshold bias
region. We showed that, in this region, the transistor behaved similarly to a varactor
diode. However, with the additional degrees of freedom inherent in a three-terminal
device, we were able to tune the effective varactor characteristics. We used this to
demonstrate, for the first time, a metamaterial with a tunable effective $\chi_n^{(2)}$ with
a very simple transistor providing the nonlinearity. Our work has illustrated some
of the design freedom we gain by using nonlinearity in metamaterials. By using a
variety of nonlinear RF devices, we showed how metamaterials could be applied to
a variety of nonlinear applications.

6.2 Suggested Future Work

6.2.1 Improvements for power harvesting metamaterials

One of the primary limitations in the power harvesting metamaterial presented here
is the use of traditional circuit elements. In order to harvest ambient and low-
level power, one of the largest obstacles is the use of semiconductor electronics to
accomplish the rectification. Using a Schottky diode as an example, the device’s
forward-bias characteristics depend strongly on its threshold voltage $V_T$. For applied
RF voltages larger than $V_T$, the diode is a very nonlinear device. Low power RF signals, however, may result in an RF bias smaller than $V_T$. If this is the case, the diode does not conduct and, more importantly, does not efficiently produce a DC signal due to nonlinearity. This is a fundamental problem with semiconductor electronics based on bandgaps: they possess some threshold voltage below which they do not act as strongly nonlinear devices. We think a fundamental shift will be required to dramatically increase the RF-to-DC conversion efficiency for low power levels, particularly near-ambient levels, using nonlinear devices that do not rely on a semiconductor bandgap. One candidate technology is a recent application of graphene: the geometric diode [45]. These produce a diode-like I-V characteristic but rely on the geometry of fabrication rather than a bandgap: carrier transport is unequal in opposite directions. These diodes rely on graphene to produce this characteristic, so they are still in the basic research stages and fabrication is difficult (requiring techniques to deposit high-quality graphene and electron beam lithography, for instance). However, power harvesting metamaterials based on geometric diodes offer great promise for increasing the RF-to-DC conversion efficiency, particularly at low power levels.

6.2.2 Use of other types of nonlinear devices

We have demonstrated in this work how PIN diodes, Schottky diodes, varactor diodes, and transistors can be used to design useful nonlinear metamaterials. However, there are a number of other classes of nonlinear devices that could be used with metamaterials. One example is the tunnel diode. Tunnel diodes are designed such that, in the forward bias region, they exhibit a region of negative differential resistance. Negative resistance is essential for applications that involve loss cancellation, amplification, and oscillation. Tunnel diodes offer a potentially straightforward method to introduce negative differential resistance to a metamaterial unit cell.
Moreover, they have been demonstrated to operate at frequencies in excess of 1 THz [13]: thus, they are promising for metamaterial designs at millimeter wave and higher frequencies.

6.2.3 Improvements of transistor metamaterials and new applications

We believe that a major area of improvement for RF metamaterials will be the application of transistor circuits at the unit cell level. The work presented in this document was the first to apply a very simple transistor circuit to a metamaterial. However, there are many avenues towards incorporating transistors, most of them available with more complex transistor circuits. We have examined using transistors for simple mixing metamaterials: improved efficiency, perhaps leading to conversion gain rather than loss, could be available by using more complicated circuits. The application of active RF circuit design techniques to metamaterials is not restricted to mixing applications, however. Using principles of amplifier design could lead to metamaterials with stable gain, rather than loss, an important goal recognized early in the metamaterial literature. Better transistor models should also lead to more accurate simulation results, yielding better quantitative predictions of performance. Single-transistor circuits are also only a platform to more complex circuits with a number of transistors, or perhaps even modules such as phase-locked loops and digital devices. Application of such circuits can lead to metamaterials with "smart" behavior, adjusting their characteristics in response to an incident signal without external control. These are just a few areas in which transistor metamaterials offer substantial improvements to existing nonlinear metamaterial designs.
Bibliography


Biography

Alexander Remley Katko was born in Columbus, Ohio on August 13, 1986. He received his B.S. in Electrical and Computer Engineering from The Ohio State University in June 2009, and his M.S. in Electrical and Computer Engineering from Duke University in May 2012. He graduated magna cum laude with Distinction in Electrical and Computer Engineering from The Ohio State University. He was a finalist for the Department of Energy Office of Science Graduate Fellowship and was awarded a student travel fellowship for the UNSC-URSI National Radio Science Meeting in 2014. Alex is a member of the IEEE and IEEE-APS and IEEE-MTTS. He is a reviewer for Applied Physics Letters, IEEE Antennas and Wireless Propagation Letters, and Journal of Applied Physics. Following is a list of his publications:


(2013).


