

Family Plans: Market Segmentation with Nonlinear Pricing

by

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Dissertation submitted in partial fulfillment of
the requirements for the degree of Doctor of Philosophy
in Business Administration
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ABSTRACT

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Abstract

In the telecommunications market, firms often give consumers the option of purchasing an individual plan or a family plan. An individual plan gives a certain allowance of usage (e.g., minutes, data) for a single consumer, whereas a family plan allows multiple consumers to share a specific level of usage. The theoretical challenge is to understand how the firm stands to benefit from allowing family plans. In this paper, we use a game-theoretic framework to explore the role of family plans. An obvious way that family plans can be profitable is if it draws in very low-valuation consumers whom the firm would choose not to serve in the absence of a family plan. Interestingly, we find that even when a family plan does not draw any new consumers into the market, a firm can still benefit from offering it. This finding occurs primarily because of the strategic impact of the family plan on the firm's entire product line. By allowing high- and low-valuation consumers to share joint allowance in the family plan, the firm is able to raise the price to extract more surplus from the individual high-valuation consumers by reducing the cannibalization problem. Furthermore, a family obtains a higher allowance compared to the purchase of several individual plans and therefore contributes more profits to the firm. We also observe different types of quantity discounts in the firm's product line. Finally, we identify conditions under which the firm offers a pay-as-you-go plan.

Dedication

To my wonderful and loving wife, Yue.

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Acknowledgements

I am deeply indebted to my advisers Preyas Desai and Debu Purohit, for their help and support during the past five years. This dissertation would not have been possible without their help. Their mentorship has been incredible, and has shaped my development in so many dimensions. I am going to miss the time spent with them in the conference room on the third floor.

I would like to thank Wilfred Amaldoss for his support, for challenging my thoughts and ideas, for generously sharing his time and feedback with me throughout the program. I would also like to thank Jim Bettman for taking the time to meet with me and offer his insights on my work. I am very grateful to Peng Sun for his invaluable comments throughout this process.

I would also like to thank Carl Mela for always being supportive and encouraging, Rick Staelin for his encouragement and feedback across various stages of the PhD program. I feel very grateful to the members of Duke Marketing Faculty for their helpful feedback on this dissertation. They have made this program as nurturing as possible. I am also grateful for the support and help from the doctoral students.

Most of all, I would like to thank my wife, Yue Qin, for her continuous love and encouragement throughout the doctoral program. Finally, I would like to thank Theodore, who has been my most loyal friend and the best listener for the past two years.

To those listed above and to those who I inevitably neglected, thank you all.

1. Introduction

In the U.S. telecommunications market, firms give consumers the option of purchasing either an individual plan or a family (or shared) plan. An individual plan gives a certain allowance of usage (e.g., minutes, data, etc.) for a single consumer, whereas a family plan allows multiple consumers to share a specific level of usage. For example, AT&T offers a 450-minute individual plan for \$39.99 and a 700-minute family plan for \$69.99. From a researcher's perspective, the theoretical challenge is to understand how a firm stands to benefit from allowing family plans. In this paper, we offer a novel and heretofore unexplored rationale for family plans and also demonstrate that when a firm allows consumers to share joint allowance by choosing a family plan, consumers and firms are both better off. Extensive research has examined bundles of products or services offered by firms, but this is the first paper to specifically consider bundling of consumers.

The problem we pose applies to situations in which consumers are uncertain about their needs for the services provided by the firm. For example, one may know that one needs a cellphone, but still have uncertainty about just how many minutes would be needed over the course of a month. Moreover, the level of need may vary from month to month, which makes estimating usage even more complicated. Knowing that consumers will face these and other types of uncertainty, firms must devise contracts that anticipate consumers' expected usage. One particular response made by firms is to offer

consumers contracts in which they pay a single fee for a block of service (e.g., 600 minutes of usage, etc.). If consumers' usage exceeds the pre-determined allotment, they pay an additional per unit fee for the additional units of service they consume (e.g., 50 cents/minute). Firms have also responded by offering "family" or shared plans that allow multiple users to share a single block of service. Although one may intuit that this approach would in some cases reduce consumers' level of uncertainty, this result is not enough to explain why a family plan makes sense from the perspective of the firm. In this paper, we show that introduction of family plans can reduce the cannibalization problem in the firm's product line design, and we identify the conditions under which the firm can maximize profits by offering both individual and family plans.

Our paper contributes to the literature on vertical differentiation and second degree price discrimination. Starting with the canonical model of Mussa and Rosen (1978), there has been a rich stream of research on vertical differentiation and consumers' self-selection. Biyalogorsky and Koenigsberg (2013) analyze the design and introduction of a product line when the firm is uncertain about consumer valuations for the products. Desai (2001) studies whether the cannibalization problem affects a firm's price and quality decisions with both horizontal and vertical differentiation, in both monopoly and duopoly settings. Dobson and Kalish (1988) suggest a heuristic algorithm to solve the monopolist's problem of positioning and pricing a product line. Johnson and Myatt (2003) analyze multiproduct monopoly and duopoly, and provide an

explanation for the common strategies of product line pruning. Kim and Chhajed (2002) consider a monopolist's product line design problem with multiple attributes. Maskin and Riley (1984) show that under a separability assumption, strong conclusions can be made about the nature of optimal incentive schemes. Moorthy (1984) studies the problem of implementing market segmentation through consumer self-selection, and shows that cannibalization comes into play so a monopolist needs to simultaneously determine the whole product line. Moorthy and Png (1992) show that sequential introduction is better than simultaneous introduction when cannibalization is a problem. Rochet and Stole (2002) introduce independent randomness into the agents' outside options and find the existence of no-distortion at the bottom. Our paper extends this line of research by incorporating consumers' uncertainty about their usage needs. Unlike the previous research, in our model even consumers within the same segment end up consuming different quantities in equilibrium. To the best of our knowledge, by modeling both individual plans and family plans, our paper is the first to simultaneously consider second degree and third degree price discrimination. We show how the firm can benefit from employing both discrimination schemes compared to only using second degree price discrimination.

Our paper is also related to the growing literature on non-linear pricing in service industries. Ascarza, Lambrecht and Vilcassim (2013) analyze the effect of tariff structure under two- and three-part tariffs and find that consumers who switch to three-

part tariffs from two-part tariffs significantly overuse minutes. Bagh and Bhargava (2013) examine the firm's choice of a pricing scheme when tariff management costs are significant enough to impose a constraint on tariff size. Gopalakrishnan, Iyengar and Meyer (2012) study the behavioral effects of multi-part tariffs in a lab setting and find consistent evidence for diminishing returns of experience with regard to reducing consumers' sub-optimal decisions. Grubb (2009) shows the optimality of the three-part tariff pricing scheme in the presence of overconfident consumers. Grubb (2012) develops a model of inattentive consumption and finds that when inattentive consumers are heterogeneous and unbiased, bill-shock regulation reduces social welfare in fairly competitive markets.

By contrast, Jiang (2012) uses billing data to estimate the welfare effects of bill-shock regulation in mobile telecommunication markets and predicts an increase in consumer surplus. Iyengar et al. (2011) use data from a field experiment to show that consumers derive lower utility from consumption under a two-part tariff than pay-per-use pricing, which results in lower retention of customers and lower usage of the service. Kolay and Shaffer (2003) show that offering a menu of price-quantity bundles is more profitable than offering a menu of two-part tariffs absent cost considerations. Lambrecht, Seim and Skiera (2007) show that demand uncertainty is a key driver of choice among three-part tariffs and that this uncertainty both decreases consumer surplus and increases provider revenue. Narayanan, Chintagunta and Miravete (2007)

develop a model for the choice and usage of local telephone service and find that the value of information to consumers is modest, and also that a major proportion of this value is in information about consumers' types. Sundararajan (2004) analyzes the non-linear pricing of information goods and finds that offering fixed-fee pricing in addition to a nonlinear usage-based pricing scheme is always profit-improving in the presence of nonzero transaction costs, and also that there may be markets in which a pure fixed fee is optimal.

In general, research in this area has demonstrated the advantages of three-part tariffs for the service providers. We build on this stream of research by incorporating shared consumption, which allows family members to join the same three-part tariff contract, and study the impact of such bundling on a service provider's pricing and profitability. Our paper is also related to the rich literature on bundling. The traditional explanation for bundling that economists have given is that it makes price discrimination strategies more powerful by reducing the role of unpredictable idiosyncratic components of valuations (see, for example, Adams and Yellen 1976, Bakos and Brynjolfsson 1999, 2002, Hitt and Chen 2005, McAfee, McMillan, and Whinston 1989, and Schmalensee 1982, 1984). Furthermore, mixed bundling (i.e., when a consumer may buy any individual component or the bundle) has been shown to be generally more profitable than pure bundling and pure component sales (Banciu, Gal-Or, and Mirchandani 2010, Basu and Vitharana 2009, Ibragimov and Walden 2010, Prasad,

Venkatesh, and Mahajan 2010, Venkatesh and Mahajan 1993, etc.). Our paper differs from the previous research by focusing on bundling consumers instead of products; in addition, we show the optimality of this approach under certain conditions.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 obtains the optimal menu of individual plans. Section 4 analyzes the optimal menu of individual and family plans, and compares the product line strategies with those in Section 3. Section 5 considers a case in which consumers are not averse to overage usage. Section 6 lays out the extensions of the base model. Section 7 presents a model that endogenizes overage aversion. Section 8 presents our conclusions.

2. Model

In this section, we lay out the assumptions related to the firm, consumers and the structure of the game between them. We start with the assumptions on the firm.

2.1 Firm

A monopoly firm provides telecommunication service to consumers. For ease of exposition, throughout this paper we denote the firm as a service provider and the product as minutes of airtime. The firm knows the distribution of different types of consumers (in terms of their valuation of a minute of airtime) but cannot directly identify each individual's type. For each type of consumer, the firm also knows the distribution of the need for minutes of communication.

The firm offers consumers a menu of individual plans from which to choose. Each plan is a three-part-tariff contract, denoted by (q, p, p_o) , where q is the included allotment of minutes for which the marginal price is zero (hereafter, q is referred to as quantity), p is the price consumers pay for q regardless of their actual consumption, and p_o is the overage price per unit for additional usage beyond the plan's quantity q . Note that the overage charge can be different across plans. We assume that the service

provider's marginal cost per unit is zero, which is a reasonable approximation for information services.¹

In addition to the individual plans, the firm may decide to offer a family plan, which allows two individuals to participate in a single contract. In this family plan, as long as the sum of both members' consumption stays within the quantity, these consumers do not incur overage charges. On the other hand, if their total consumption goes over the family plan's allotment, then they need to pay overage charges.

2.2 Consumers

There are several key characteristics of consumers in telecommunication markets. First, they differ in their valuation for telecommunication services, i.e., how much they value a minute of airtime. Second, they have different communication needs (e.g., some consumers require more minutes than others during a given consumption period). Third, they have uncertainty about their need for communication at the time of purchase (e.g., it is hard to predict the exact number of minutes one will need in the consumption stage). Fourth, they dislike being in the overage region because they incur additional costs of monitoring their usage, paying overage fees (i.e., the additional charges imposed by the service provider after consumers have exceeded their quantity permitted under their plan); importantly, these costs also ensure that consumers cut

¹ The analysis in which the marginal cost per unit is not zero is presented in the appendix. Most of our results are qualitatively the same.

back on consumption, something that they would not have done in the absence of these costs.

To model consumers' heterogeneity of valuation for telecommunication services, we assume that there are two consumer segments in the market: a high-valuation segment denoted by H and a low-valuation segment denoted by L. The sizes of the H and L segments are $(1 - \lambda)$ and λ . Consumers know their own types, and their utility function is given by

$$V(\theta_i, q) = \theta_i q - p, \text{ where } i \in \{H, L\}, \theta_H > \theta_L.$$

This utility function is increasing and weakly concave in quantities q , and decreasing in price p . Consumers' valuation parameter $\theta_H > \theta_L$ means that an H-type consumer obtains a higher utility than an L-type consumer for a given quantity q , as well as a higher marginal utility. This utility function satisfies the Spence-Mirrlees single-crossing condition: the cross partial derivative $V_{12}(\theta_i, q) > 0$, for all q .

To capture consumers' heterogeneity in terms of their communication requirements and their uncertainty about how many minutes they will need in the consumption stage, we assume consumers' consumption needs are stochastic, come from a known distribution and are independently distributed. In particular, the H segment's consumption requirement is uniformly distributed between $[\Delta, 1 + \Delta]$ ($\Delta \geq 0$), and that of the L segment is uniformly distributed between $[0, 1]$. In other words, the

exact amount of minutes consumers need is random, and the realization of the quantity needed is generated by two independent distributions, where one distribution $F_H(q)$ stochastically dominates the other $F_L(q)$. Note that H-type consumers need at least Δ minutes, below which they do not obtain sufficient utility to initiate the purchase decision. Δ thus measures the heterogeneity of consumers' usage needs. Ex ante, consumers do not expect to gain additional utility by continuing to use the service once their requirement has been met. Therefore, if the requirement realization, \hat{q} , is within a plan's quantity, consumers will stop consuming at \hat{q} . Both types of consumers gain zero utility beyond the supports of their requirement distributions; in addition, their outside option gives them zero utility.

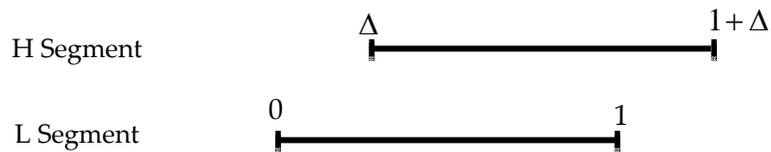


Figure 1: Consumers' Usage Distributions

Extensive behavioral research has demonstrated consumers' "pain of paying," where a difference in the transparency of payments caused by different payment mechanisms changes the pain of making a payment, and consequently influences spending and consumption behavior (Prelec and Loewenstein 1998, Soman and Gourville 2001, Soman 2003). To capture the feature that consumers dislike being in the

overage region,² we introduce a parameter k ($0 \leq k \leq 1$) to measure this degree of aversion. In particular, consumers start curtailing their consumption once they reach their plan's quantity such that they only use k proportion of the difference between the realization of their requirement and their plan's quantity. Specifically, when the draw from the distribution, \hat{q} , is beyond a plan's quantity q , consumers will use $[q + k(\hat{q} - q)]$ minutes. When $k = 0$, consumers are extremely averse to overage consumption and stop at their plan's quantity, even if their realized requirement is beyond that quantity. When $k = 1$, consumers are not at all averse to overage consumption and they keep consuming minutes up to their requirement's realization, the same way they did before reaching their plan's quantity. In reality, however, consumers may start curtailing consumption even before reaching their plan's quantity. In this case, our framework provides a first-order approximation to the intentional usage reduction near the plan's quantity and the resulting utility reduction. Note that consumers are aware of their behavior on curtailing overage, partly because they dislike paying a higher fee compared to the unit price included in the plan. There are plenty of examples in other contexts that share the same behavior pattern, such as in the auto leasing scenario where consumers actively avoid driving the car once the mileage restriction has been exceeded. To simplify our analysis and sharpen our focus on

² Consumers dislike paying overage fees more compared to the price of the plan. Overage fees are typically higher than the unit price included in the plan, thereby causing consumers' dislikes.

bundling consumers with non-linear pricing, we assume that the parameter for curtailing overage consumption, k , is the same across both consumer segments.

Now we introduce our model of a family. Families may comprise different types of consumers. To capture heterogeneity within a family in a parsimonious way, we assume that a family consists of one H-type consumer and one L-type consumer.³ We also assume that the total number of families in the market is f ($f \geq 0$). It follows that the sizes of the single H- and L-segment are $(1 - \lambda - f)$ and $(\lambda - f)$, respectively. To ensure the existence of single individual consumers (consumers who cannot buy family plans), we assume that $(1 - \lambda) \geq f$ and $\lambda \geq f$. A family's valuation for quantities, θ_F , is given by $\theta_F = \frac{\theta_H + \theta_L}{2}$ (subscript F denotes family). Because both individuals in a family curb their overage consumption by the proportion of k , a family's aversion to overage usage is also characterized by k .

Consumers are risk-neutral and make purchase decisions based on their expected utilities from different plans. Their expected utility consists of four components: (1) utility prior to reaching a plan's quantity, (2) utility after reaching that quantity, (3) the price of the plan, and (4) the overage payment. Conditional on choosing the plan (q_H, p_H, p_{OH}) , an H-type consumer's expected utility is given as:

³ The analysis in which a family comprises two H-type or two L-type consumers is presented in Section 6.

$$\begin{aligned}
EU(\theta_H, q_H, p_H, p_{OH}) &= E(V(\theta_H, q) | q \leq q_H)Pr(q \leq q_H) + E(V(\theta_H, q) | q > q_H, k)Pr(q > q_H) \\
&\quad - p_H - p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H) \tag{1} \\
&= \int_{\Delta}^{q_H} v(\theta_H, q) f_H(q) dq + \int_{q_H}^{1+\Delta} v(\theta_H, q_H + k(q - q_H)) f_H(q) dq \\
&\quad - p_H - p_{OH} \int_{q_H}^{1+\Delta} k(q - q_H) f_H(q) dq.
\end{aligned}$$

The first term in the above expression, $\int_{\Delta}^{q_H} v(\theta_H, q) f_H(q) dq$, is the H-type consumer's expected utility conditional on her requirement realization being smaller than the H plan's quantity q_H . The second term, $\int_{q_H}^{1+\Delta} v(\theta_H, q_H + k(q - q_H)) f_H(q) dq$, is her expected utility, conditional on her requirement realization exceeding the plan's quantity and her starting to curtail her overage consumption by k . The next term, p_H , is the plan's price, and the last term, $p_{OH} \int_{q_H}^{1+\Delta} k(q - q_H) f_H(q) dq$, represents the consumer's expected overage payment. Similarly, if an L-type consumer chooses the plan (q_L, p_L, p_{OL}) , her expected utility would be:

$$\begin{aligned}
EU(\theta_L, q_L, p_L, p_{OL}) &= E(V(\theta_L, q) | q \leq q_L)Pr(q \leq q_L) + E(V(\theta_L, q) | q > q_L, k)Pr(q > q_L) \\
&\quad - p_L - p_{OL}E(q - q_L | q > q_L, k)Pr(q > q_L) \tag{2} \\
&= \int_0^{q_L} v(\theta_L, q) f_L(q) dq + \int_{q_L}^1 v(\theta_L, q_L + k(q - q_L)) f_L(q) dq \\
&\quad - p_L - p_{OL} \int_{q_L}^1 k(q - q_L) f_L(q) dq.
\end{aligned}$$

2.3 Game Structure

The game between the service provider and consumers contains two stages. In the first stage, the service provider presents two individual plans, (q_H, p_H, p_{OH}) and (q_L, p_L, p_{OL}) , and consumers self-select the best plans for themselves. In addition, the service provider may decide to offer a family plan, (q_F, p_F, p_{OF}) , in which case, the consumers who comprise this family will compare this joint plan with the two individual plans that the individual members would choose. Two dimensions of uncertainty exist in this stage. First, the service provider does not know each individual consumer's type but takes this information asymmetry into account when designing the plans to maximize its expected profit. Second, neither the service provider nor the consumers know exactly how many minutes each individual will need in the next stage; they only know the H and L segments' usage distributions. Consumers choose which plan to purchase based on their expected utilities and pay the plan's price, p_H or p_L , accordingly. When the choice is a family plan, consumers with family members take the entire family's expected utility into account and pay the price p_F . In the second stage, the communication requirement is realized for each consumer. At the end of this stage, those whose requirement realizations are beyond their plans' quantities curtail their overage consumption by the proportion k and make the overage payment.

For a more interesting analysis, in the main body of this paper we consider the situation in which it is more profitable for the service provider to serve both the H and L segments than to serve only the H segment. The condition for this situation to arise in equilibrium is given by:

$$\begin{aligned} & \left(1 - k \left(1 - \Delta^2(1 - \lambda)(1 - k\lambda)\right)\right) \theta_L^2 - [(1 - k)(1 + 2\Delta)\theta_H(1 - \lambda) + (1 - k)\theta_H(1 - \lambda) \\ & (1 - \Delta^2(1 - k\lambda))]\theta_L + (1 - k)(1 + 2\Delta)\theta_H^2(1 - \lambda)^2 \geq 0. \end{aligned}$$

We assume the above condition to hold in the main text, and discuss the situation in which it is more profitable to serve only the H segment in the appendix.

3. Benchmark: Analysis of Individual Plans

In this section, we analyze the case in which the service provider attempts to maximize its profits by offering only two individual plans to consumers. The service provider chooses two sets of quantities, prices, and overage fees in these two individual plans, (q_H, p_H, p_{OH}) and (q_L, p_L, p_{OL}) , to target the high-valuation consumers (the H segment) and the low-valuation consumers (the L segment), respectively. After observing the two three-part tariff plans, consumers self-select the best plans for themselves. As is well known from the classic literature on self-selection (Mussa and Rosen 1978), the service provider must ensure that the H segment will not choose the L plan, (q_L, p_L, p_{OL}) , targeted at the L segment, and vice versa. In addition, the service provider must ensure that both segments of consumers obtain nonnegative utilities from purchasing the plans targeted at them. Thus the new elements in our framework are the three-part tariff plans and uncertainty on the requirement for quantities.

The service provider's profit, Π , is given by

$$\Pi = (1 - \lambda)[p_H + p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H)] + \lambda[p_L + p_{OL}E(q - q_L | q > q_L, k)Pr(q > q_L)].$$

The first term in this profit function, $(1 - \lambda)[p_H + p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H)]$,

gives the profit from the H segment. p_H is the price of the H plan, and the term,

$p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H)$, captures the H segment's expected overage

payment after choosing the H plan. Similarly, the second term in the profit function

gives the profit from the L segment, while the term, $p_{OL}E(q - q_L | q > q_L, k)Pr(q > q_L)$,

captures the L segment's expected overage payment after choosing the L plan. Formally, the service provider sets its menu of plans to maximize its expected profit:

$$E[\Pi] = \text{Max} (1-\lambda)[p_H + p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H)] + \lambda[p_L + p_{OL}E(q - q_L | q > q_L, k)Pr(q > q_L)] \\ = (1-\lambda)[p_H + p_{OH} \int_{q_H}^{1+\Delta} k(q - q_H)f_H(q)dq] + \lambda[p_L + p_{OL} \int_{q_L}^1 k(q - q_L)f_L(q)dq],$$

subject to

$$EU(\theta_H, q_H, p_H, p_{OH}) \geq EU(\theta_H, q_L, p_L, p_{OL}), \quad (3)$$

$$EU(\theta_L, q_L, p_L, p_{OL}) \geq EU(\theta_L, q_H, p_H, p_{OH}), \quad (4)$$

$$EU(\theta_H, q_H, p_H, p_{OH}) \geq 0, \quad (5)$$

$$EU(\theta_L, q_L, p_L, p_{OL}) \geq 0. \quad (6)$$

The left-hand side of Constraint (3), $EU(\theta_H, q_H, p_H, p_{OH})$, is an H-type consumer's expected utility of purchasing the H plan. She pays the price p_H , gets a quantity of q_H , and faces the overage charge per unit p_{OH} . The right-hand side of Constraint (3), $EU(\theta_H, q_L, p_L, p_{OL})$, is the H-type consumer's expected utility if she purchases the L plan. In this case, she pays the price p_L , gets a quantity of q_L , and faces the overage charge per unit p_{OL} . Similarly, the left and the right sides of Constraint (4) capture an L-type consumer's expected utilities of buying the L and the H plans, respectively.

Constraints (3) and (4) ensure that both consumer segments voluntarily choose the plan directed to them. Constraints (5) and (6) ensure that each segment will buy the plan directed to it rather than not buy anything at all. In the main body of this paper, we focus on the more interesting situation where the incentive compatibility (IC) constraint

for the H segment, Constraint (3), is not trivially satisfied.¹ In other words, the binding constraints are this incentive compatibility constraint for the H segment, Constraint (3), and the individual rationality (IR) constraint for the L segment, Constraint (6). In this case, H-type consumers' best outside option is the L plan, (q_L, p_L, p_{OL}) . To aid intuition, we expand the binding IC constraint for the H segment as follows:

$$\begin{aligned}
EU(\theta_H, q_H, p_H, p_{OH}) &= E(V(\theta_H, q) | q \leq q_H)Pr(q \leq q_H) + E(V(\theta_H, q) | q > q_H, k)Pr(q > q_H) \\
&\quad - p_H - p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H) \\
&\geq EU(\theta_H, q_L, p_L, p_{OL}) = E(V(\theta_H, q) | q \leq q_L)Pr(q \leq q_L) + E(V(\theta_H, q) | q > q_L, k)Pr(q > q_L) \\
&\quad - p_L - p_{OL}E(q - q_L | q > q_L, k)Pr(q > q_L).
\end{aligned} \tag{7}$$

Two points are worth highlighting in Constraint (7). First, if an H-type consumer selects the L plan, she is more likely to run into an overage region because $q_L < q_H$. Second, with the L plan, she starts curtailing the overage consumption earlier, at the quantity q_L , which leads to a reduced utility in the overage region. The advantages for an H-type consumer of selecting the L plan are a lower price p_L and a potentially lower overage charge per unit p_{OL} (confirmed ex post).

As we solve the firm's constrained optimization problem, we obtain the following optimal menu of individual plans. (We use superscript ^{*} to denote the optimal prices, quantities and overage fees in this case.)

¹ A sufficient condition for this situation to occur is that Δ is lower than a threshold. We discuss the other situation, in which the incentive compatibility constraint for the H segment is trivially satisfied and thus the only binding constraints are the two individual rationality constraints, in the appendix.

The quantities of the H and L plans are:

$$q_H^* = 1 + \Delta, \quad q_L^* = 1 - \frac{\Delta(1-\lambda)((1-k)\theta_H + k\theta_L)}{(1-k)(\theta_L - \theta_H(1-\lambda))}. \quad (8)$$

The prices for two individual plans, p_H^* and p_L^* , are given as:

$$p_L^* = \frac{\theta_L(\theta_L(1-k(1+\Delta(1-\lambda)))) - (1-k)(1+\Delta)\theta_H(1-\lambda)}{2(1-k)^2(\theta_L - \theta_H(1-\lambda))^2} \quad (9)$$

$$(\theta_L(1-k+k\Delta-k\Delta\lambda) - (1-k)(1-\Delta)\theta_H(1-\lambda)),$$

$$p_H^* = p_L^* + \frac{\Delta^2\theta_L^2(k\theta_L + (1-k)\theta_H)(1-k\lambda)^2}{2(1-k)^2(\theta_L - \theta_H(1-\lambda))^2}. \quad (10)$$

The overage fees are $p_{OL}^* = \theta_L$ and $p_{OH}^* = 0$.

First, note that the service provider can charge L-type consumers' full valuation for quantities, θ_L , as the overage price. This result occurs because as soon as L-type consumers continue to use minutes in the overage region, the firm knows their marginal utility and can fully extract it. By contrast, the firm offers free extra minutes to H-type consumers, $p_{OH}^* = 0$. This is because the quantity for the H plan, $q_H^* = 1 + \Delta$, is the upper bound of the H segment's requirement distribution. Given that no one would consume more than $1 + \Delta$, we can interpret this result, $q_H^* = 1 + \Delta$, as that H-type consumers obtain an unlimited plan. By contrast, L-type consumers never obtain their maximum requirement in their L plan, $q_L^* < 1$.

Intuitively, consumers are willing to pay a higher fixed price when their plans' included quantities are higher. In this context, the service provider faces two major

trade-offs. On the one hand, it wants to decrease the included quantity so that consumers are more likely to pay overage fees, which can be shown to be higher than the average price per unit within a plan. On the other hand, if the firm decreases a plan's quantity, it forgoes some profits with certainty because consumers are willing to pay a lower fixed price. In addition, because consumers will curtail their overage consumption if their requirement realizations are beyond a plan's quantity, the firm cannot fully reap the benefits of a large requirement realization. The curtailing behavior occurs more often with a lower quantity, which leads to another loss of revenues. Because the latter factor dominates in the offering of the H plan, given a higher valuation for quantities θ_H , the service provider does not put any restriction on the quantities of the H plan. This result is consistent with the "no distortion at the top" result from the literature on vertical differentiation.

Now we characterize the impact of the curtailing parameter, k , on the two individual plans.

Proposition 1. *The L plan's quantity, q_L^* , and its price, p_L^* , both decrease when k increases (i.e., when consumers curtail their overage consumption to a lesser extent). The H plan's quantity, q_H^* , is independent of k . The H plan's price, p_H^* , may increase or decrease as k increases.*

All the proofs are given in the appendix. Proposition 1 shows that the impact of k on the two plans is different. We discuss the intuition sequentially. The reason for the

first part of the proposition is that when consumers curb their overage consumption less (k is larger), the service provider has incentives to reduce the L plan's quantity to capture the increased benefits from the overage charges. Once the quantity, q_L^* , is reduced, the service provider must also reduce the L plan's price, p_L^* , to compensate for an L-type consumer's lowered willingness to pay.

The comparative statics, $\frac{\partial p_H^*}{\partial k}$, can be either positive or negative, depending on the heterogeneity of consumers' valuations, $(\theta_H - \theta_L)$. It is interesting to note that although the H plan's quantity, $q_H^* = 1 + \Delta$, is independent of the curtailing parameter k , its price is dependent upon k . The impact of the extent of curtailing on the H plan's price is through the binding IC constraint for the H segment. When L-type consumers' valuation, θ_L , is above a threshold, an increase in k will lead to an increase in the H plan's price p_H^* , i.e., $\frac{\partial p_H^*}{\partial k} > 0$. The intuition is that a higher k leads to a lower quantity for the L plan, which makes this best outside option for the H segment less appealing, even after accounting for its price reduction. Therefore, the service provider can raise the H plan's price to benefit from consumers' smaller aversion toward overage consumption.

Concerning the price per unit within the L plan, it is easy to show that $\frac{p_L^*}{q_L^*} < \theta_L$.

This inequality means that the price per unit within the L plan is indeed lower than its

overage price, a circumstance that is consistent with industry practice. In other words, L-type consumers are given some quantity discounts from the L plan as compared to their maximal willingness to pay per unit, θ_L . Interestingly, another form of quantity discount also exists when comparing across the individual plans.

Proposition 2. *When $\theta_L(1 - k\Delta\lambda) - \theta_H(1 - \lambda + (1 - k)\Delta\lambda) > 0$, the per unit price in the H plan*

is lower than that of the L plan, i.e., $\frac{p_H^}{q_H^*} < \frac{p_L^*}{q_L^*}$. Otherwise, the reverse is true.*

Similar to many product categories in which quantity discounts are observed, the high-valuation consumer segment purchases a product with a higher quantity (q_H^*), and enjoys a lower price per unit when $\theta_L(1 - k\Delta\lambda) - \theta_H(1 - \lambda + (1 - k)\Delta\lambda) > 0$. This condition is more likely to hold when θ_L or k increases. A higher θ_L implies an L plan with

higher quantities ($\frac{\partial q_L^*}{\partial \theta_L} > 0$), which increases the attractiveness of the H segment's best

outside option. Thus the service provider is more likely to offer quantity discounts with respect to the included quantity in the H plan, in order to prevent the H segment from selecting the less expensive L plan. However, although a higher k decreases both the L plan's price and quantity, the overall effect of k on its price per unit is positive,

$\frac{\partial \frac{p_L^*}{q_L^*}}{\partial k} > 0$. This effect increases the likelihood of quantity discounts offered to the H

segment.

To summarize, in this section we have characterized the optimal individual plans and how overage aversion affects the terms in these plans. Next we move on to examine the situation when the service provider offers family plans in addition to the individual plans.

4. Analysis of Individual and Family Plans

In the previous section, we analyzed the situation when the service provider offered two individual plans. In this section, we analyze the case when the service provider offers two individual plans, (q_H, p_H, p_{OH}) and (q_L, p_L, p_{OL}) , and a family plan, (q_F, p_F, p_{OF}) .

4.1 Model with Individual and Family Plans

The service provider uses two individual plans to target single high-valuation consumers and single low-valuation consumers, whereas it uses the family plan to target consumers who have families. Note that a family plan is jointly purchased and consumed by two family members; single, individual buyers cannot purchase a family plan. In other words, self-selection of family plans goes in one direction, and the service provider can identify family buyers' types when a family plan is sold to them. Therefore, by offering three different plans, the firm utilizes both second-degree and third-degree price discrimination. Recall that a family consists of one H-type consumer and one L-type consumer, and the size of the family segment in the market is f . It therefore follows that the size of the single H segment is $(1 - \lambda - f)$ and the size of the single L segment is $(\lambda - f)$. Furthermore, a family's valuation parameter, θ_F , is given by

$$\theta_F = \frac{\theta_H + \theta_L}{2}.$$

Before we present our analysis, we first establish a family's requirement distribution below. Note that the density function of the sum of two independent random variables, X and Y , is the convolution of the density functions of the operand distributions, $f_X(x)$ and $f_Y(y)$. The random variable, $Q = X + Y$, has density function:

$$(f_X * f_Y)(q) = \int_{-\infty}^{\infty} f_X(q-y)f_Y(y)dy = \int_{-\infty}^{\infty} f_Y(q-x)f_X(x)dx. \text{ (See Grinstead and Snell 1997.)}$$

Lemma 1. *Assume random variable Q is the sum of two independent random variables: $Q = X + Y$, where X is uniformly distributed between $(0, 1)$ and Y is uniformly distributed between $(\Delta, 1 + \Delta)$. Then Q 's probability density and cumulative distribution functions are given below:*

$$f_Q(q) = \begin{cases} 0, & q < \Delta, \\ q - \Delta, & \Delta \leq q < 1 + \Delta, \\ (2 + \Delta) - q, & 1 + \Delta \leq q < 2 + \Delta, \\ 0, & 2 + \Delta \leq q. \end{cases}$$

$$F_Q(q) = \begin{cases} 0, & q < \Delta, \\ \frac{(q - \Delta)^2}{2}, & \Delta \leq q < 1 + \Delta, \\ \frac{2 - ((2 + \Delta) - q)^2}{2}, & 1 + \Delta \leq q < 2 + \Delta, \\ 1, & 2 + \Delta \leq q. \end{cases} \quad (11)$$

Thus, the family's requirement distribution is a triangular distribution with the density peak at $1 + \Delta$. Its variance is the same as the sum of variances of the H and L segments' distributions due to independence. Note that both consumers and the firm know this requirement distribution in the first stage of the game.

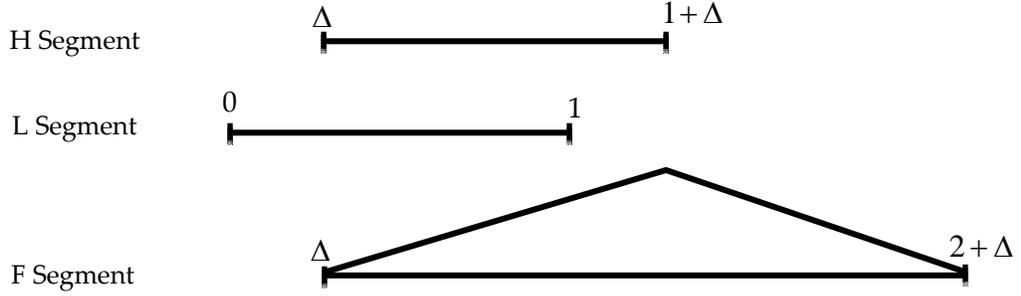


Figure 2: Individual and Family's Usage Distributions

Similar to the analysis in Section 3, the service provider must make sure that the two single consumer segments will choose individual plans targeted to them. In addition, in order to induce eligible consumers to purchase the family plan, the service provider needs to set the plan so that the expected joint utility from purchasing it is weakly greater than the sum of expected utilities obtained from two individual plans (which the two family members could have chosen).

The service provider's profit, Π , is given by

$$\Pi = (1 - \lambda - f)[p_H + p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H)] + (\lambda - f)[p_L + p_{OL}E(q - q_L | q > q_L, k)Pr(q > q_L)] + f[p_F + p_{OF}E(q - q_F | q > q_F, k)Pr(q > q_F)].$$

The first term in this profit function,

$(1 - \lambda - f)[p_H + p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H)]$, gives the expected profit from the single H segment where p_H is the price of the H plan, and the term,

$p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H)$, captures the H segment's expected overage

payment after choosing the H plan. Similarly, the second and third terms in the profit

function give the profits from the single L segment and the family segment, respectively.

The expression within the third term, $p_{OF}E(q - q_F | q > q_F, k)Pr(q > q_F)$, captures the family segment's expected overage payment after choosing the family plan. Formally, the service provider chooses three plans to maximize its expected profit:

$$E[\Pi] = \text{Max} (1 - \lambda - f)[p_H + p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H)] + (\lambda - f)[p_L + p_{OL}E(q - q_L | q > q_L, k)Pr(q > q_L)] + f[p_F + p_{OF}E(q - q_F | q > q_F, k)Pr(q > q_F)],$$

subject to

$$EU(\theta_H, q_H, p_H, p_{OH}) \geq EU(\theta_H, q_L, p_L, p_{OL}), \quad (12)$$

$$EU(\theta_L, q_L, p_L, p_{OL}) \geq EU(\theta_L, q_H, p_H, p_{OH}), \quad (13)$$

$$EU(\theta_F, q_F, p_F, p_{OF}) \geq EU(\theta_H, q_H, p_H, p_{OH}) + EU(\theta_L, q_L, p_L, p_{OL}), \quad (14)$$

$$EU(\theta_H, q_H, p_H, p_{OH}) \geq 0, \quad (15)$$

$$EU(\theta_L, q_L, p_L, p_{OL}) \geq 0, \quad (16)$$

$$EU(\theta_F, q_F, p_F, p_{OF}) \geq 0. \quad (17)$$

The left-hand side of Constraint (12), $EU(\theta_H, q_H, p_H, p_{OH})$, is an H-type consumer's expected utility of purchasing the H plan. She pays the price p_H , gets a quantity of q_H , and faces the overage charge per unit p_{OH} . The right-hand side of Constraint (12), $EU(\theta_H, q_L, p_L, p_{OL})$, is the H-type consumer's expected utility if she purchases the L plan. In this case, she pays the price p_L , gets a quantity of q_L , and faces the overage charge per unit p_{OL} . Similarly, the two sides of Constraint (13) capture an L-type consumer's expected utilities of buying the L and the H plans, respectively.

Introduction of a family plan brings two new constraints, Constraints (14) and (17), into the service provider's optimization problem. The left-hand side of Constraints (14) and (17), $EU(\theta_F, q_F, p_F, p_{OF})$, is a family's joint expected utility of purchasing the family plan. This family pays the price p_F , gets a total shared quantity of q_F , and faces the overage charge per unit p_{OF} if the sum of two family members' consumption exceeds the plan's quantity q_F . By contrast, the right-hand side of Constraint (14), $EU(\theta_H, q_H, p_H, p_{OH}) + EU(\theta_L, q_L, p_L, p_{OL})$, gives the sum of expected utilities for an H-type consumer who is buying the H plan and an L-type consumer who is buying the L plan.

Constraints (12) and (13) imply that both the H and L segments voluntarily choose the plan directed to them. By contrast, Constraint (14) implies that families will buy the family plan instead of two individual plans. Constraints (15) through (17) ensure, respectively, that the H, L, and family segments will buy the plan intended for them rather than not buy anything at all.

Next, we examine the service provider's optimal menu of plans. As in Section 3, we focus on the more interesting situation in which the incentive compatibility constraint for the H segment, Constraint (12), is not trivially satisfied.

4.2 Optimal Individual and Family Plans

When given the option of choosing a family plan, a single H-type consumer's best outside option is still the L plan, (q_L, p_L, p_{OL}) , whereas a single L-type consumer's

best outside option is no consumption. A family's best outside option is for each member to separately choose his or her best individual plan: (q_H, p_H, p_{OH}) for the H-type consumer and (q_L, p_L, p_{OL}) for the L-type consumer. In other words, the binding constraints are the incentive compatibility constraints for the H segment (Constraint (12)) and the family segment (Constraint (14)), as well as the individual rationality constraint for the L segment, Constraint (16).

By solving the firm's constrained optimization problem, we obtain the following optimal menu of plans. (We use ** to denote the optimal prices, quantities and overage fees in the presence of family plans.)

The quantities for the three plans are:

$$\begin{aligned} q_H^{**} &= 1 + \Delta, q_F^{**} = 2 + \Delta, \\ q_L^{**} &= 1 - \frac{\Delta(1-\lambda)((1-k)\theta_H + k\theta_L)}{(1-k)((1-f)\theta_L - (1-\lambda)\theta_H)}. \end{aligned} \quad (18)$$

Both the H plan and the family plan obtain their maximum requirements, $1 + \Delta$ and $2 + \Delta$. It can be interpreted that the single H segment and the family segment both obtain unlimited plans. Similar to the scenario in Section 3, the L plan's quantity, q_L^{**} , decreases in the curtailing parameter k . The prices of these three plans, which all depend on k and f , are given as:

$$\begin{aligned} p_L^{**} &= \frac{\theta_L(\theta_L((1-f)(1-k) - k\Delta(1-\lambda)) - (1-k)(1+\Delta)\theta_H(1-\lambda))}{2(1-k)^2((1-f)\theta_L - \theta_H(1-\lambda))^2} \\ &\quad (\theta_L((1-f)(1-k) + k\Delta(1-\lambda)) - (1-k)(1-\Delta)\theta_H(1-\lambda)), \end{aligned} \quad (19)$$

$$p_H^{**} = p_L^{**} + \frac{\Delta^2 \theta_L^2 (k\theta_L + (1-k)\theta_H)(1-f(1-k)-k\lambda)^2}{2(1-k)^2((1-f)\theta_L - \theta_H(1-\lambda))^2}, \quad (20)$$

$$p_F^{**} = \frac{1}{2} \left\{ (2+\Delta)\theta_L - \Delta\theta_H + \frac{\Delta^2((1-k)\theta_H + k\theta_L)\theta_L[\theta_L(1-f(2-(1-f)k) - k\lambda(\lambda-2f)) - \theta_H(1-\lambda)^2]}{2(1-k)((1-f)\theta_L - \theta_H(1-\lambda))^2} \right\}. \quad (21)$$

The overage fees are $p_{OH}^{**} = 0$, $p_{OF}^{**} = 0$, and $p_{OL}^{**} = \theta_L$, all of which are independent of k and f . Note that the overage fee structure is the same as it would be without a family plan. When a plan (the L plan) is capped in quantities, the service provider charges this segment's full valuation (θ_L) as the overage price. In the case of plans that provide unlimited quantities, the firm does not charge anything to those buyers ($p_{OH}^{**} = 0, p_{OF}^{**} = 0$) because no one would consume more than the upper supports of their requirement distributions.

Recall that f is the total number of families in the market. Next, we discuss the impact of f on the quantities and prices of the three plans.

Proposition 3. *The size of the family segment, f , affects the optimal plans in the following way:*

- (i) *The quantity of the L plan, q_L^{**} , decreases in f . The quantities of the H and the family plans, q_H^{**} and q_F^{**} , are independent of f .*
- (ii) *The price of the L plan, p_L^{**} , decreases in f . The prices of the H and the family plans, p_H^{**} and p_F^{**} , increase in f .*

Recall that when designing the product line for two individual plans, the service provider must account for the potential cannibalization problem. In particular, the firm

must ensure that the high valuation H segment purchases the more profitable H plan by obtaining at least a utility of $EU(\theta_H, q_L, p_L, p_{OL})$. In other words, the firm must balance reducing the attractiveness of the L plan, so that the H segment will not switch and thus can be further exploited, with a lowered revenue stream from the L segment due to a reduction in the L plan's quantity, q_L . With the introduction of a family plan, the size of the L plan buyers decreases, i.e., $\lambda - f < \lambda$ for $f > 0$, which means that the single L segment's importance also decreases. As the size of the family segment, f , increases, the firm can reduce the quantity of the individual L plan q_L^{**} to make this outside option less appealing to the single H segment. Once the quantity q_L^{**} is reduced, however, the price for the L plan p_L^{**} must also be decreased to account for a lower willingness to pay from the single L-type consumers. Still, when the size of the family segment, f , increases, the service provider can raise both p_H^{**} and p_F^{**} because the reduction in q_L^{**} decreases the attractiveness of the best outside option for both the single H-type consumers and the H-type consumers with families (recall the IC in Constraint (12)).

In the absence of a family plan, we have already observed quantity discounts when comparing two individual plans (Proposition 2). An interesting question is whether the quantity discounts of the H plan compared to the L plan still exists in the presence of a family plan. It is also interesting to assess the price per unit in the family

plan and compare this with those of the two individual plans. The next proposition discusses price-quantity ratios across plans and resolves these two issues.

Proposition 4. *When f is below a threshold:*

(i) *The price per unit in the H plan is lower than that of the L plan, i.e., $\frac{P_H^{**}}{q_H} < \frac{P_L^{**}}{q_L}$.*

(ii) *The price per unit in the family plan is lower than that of the H plan, i.e., $\frac{P_F^{**}}{q_F} < \frac{P_H^{**}}{q_H}$.*

The first inequality on quantity discounts of the H plan compared to the L plan holds to ensure that the single H segment would select the more expensive H plan instead of the L plan, with which they could have benefited from both a lower price and positive utility $(\theta_H - \theta_L)$ in the overage region. The second inequality results from the composition of a family. Due to the inclusion of a lower valuation consumer in the family plan, the service provider needs to lower the price per unit in the family plan compared to the H plan. Finally, it should be noted that all of these prices per unit within a plan are lower than the L plan's overage price per unit, θ_L , which displays another form of quantity discount and is also consistent with industry practice.

The condition on f is tediously complicated and is given in the appendix. As is often the case, here we discuss the more intuitive sufficient condition. When θ_L or k increases, the condition on f is more likely to be satisfied. Concerning the first inequality in Proposition 4, a higher θ_L implies an L plan with higher quantities

($\frac{\partial q_L^{**}}{\partial \theta_L} > 0$), which increases the attractiveness of the H segment's best outside option.

Thus the service provider is more likely to offer quantity discounts with respect to the included quantity in the H plan to prevent the H segment from selecting the L plan, which is less expensive. Moreover, although a higher k decreases the L plan's price as

well as its quantity, the overall effect of k on its price per unit is positive, $\frac{\partial \frac{p_L^{**}}{q_L^{**}}}{\partial k} > 0$.

This effect increases the likelihood of quantity discounts offered to the H segment.

Concerning the second inequality in Proposition 4, a higher θ_L or k imply a higher willingness to pay on the part of an L-type consumer. Therefore, the service provider no longer needs to offer as much of a quantity discount to a family in order for them to bundle an L-type consumer with an H-type consumer into the joint plan.

Next, we compare the two individual plans in the presence of the family plan with the two individual plans in the absence of the family plan (as in Section 3). This comparison provides further insights into how the introduction of the family plan changes the terms of the individual three-part tariffs.

Proposition 5. *The changes in two individual plans after the introduction of the family plan are as follows:*

(i) *Both the L plan's quantity, q_L^{**} , and its price, p_L^{**} , are lower, i.e., $q_L^{**} < q_L^*$ and $p_L^{**} < p_L^*$.*

(ii) *The H plan's quantity, q_H^{**} , stays the same, but its price, p_H^{**} , is higher, i.e., $q_H^{**} = q_H^*$ and $p_H^{**} > p_H^*$.*

In determining a plan's quantity, the service provider faces two main trade-offs. It wants to decrease the included quantity in order to induce consumers to pay overage fees with a greater probability; these fees are higher than the average price per unit within a plan (except for unlimited plans). However, it also wants to increase a plan's quantity so that it can charge buyers a higher price. Remember that consumers curtail their overage consumption if their requirement realizations are beyond a plan's quantity, so that the service provider cannot fully reap the benefits of a large requirement realization. For this reason, a higher quantity results in a smaller loss from consumers' overage consumption due to their delayed curtailing behavior. The first factor, coupled with the consideration for the IC constraint for the H segment, dominates the second factor in determining the quantity and price for the L plan. Because the detailed intuition for the resulting observations, $q_L^{**} < q_L^*$ and $p_L^{**} < p_L^*$, is presented in the discussion of Proposition 3, it is not repeated here.

By contrast, given the relatively high valuations θ_H and $\theta_F (= \frac{\theta_H + \theta_L}{2} > \theta_L)$, the firm's incentive in raising the included quantities is stronger than the opposite. Therefore, the service provider offers its maximum requirements for communication for

the H segment and the family segment, respectively. Again, this result is consistent with the “no distortion at the top” result from the literature on vertical differentiation.

By bundling some H-type consumers with some L-type consumers in the family plans, the service provider is able to better extract surplus from the single H segment by charging these consumers a higher price, $p_H^{**} > p_H^*$. In other words, the H segment with families poses some negative externalities to the single H segment. Recall that from Proposition 3, the firm utilizes the family plan to effectively reduce the number of the single L plan’s buyers and thus their importance as well. Then the firm can lower the L plan’s quantity to make it less attractive to the single H-type consumers as their best outside option. As a result, the firm can adjust its H plan to better exploit the single H-type buyers. Because the quantity in the H plan, $q_H^{**} = 1 + \Delta$ (the overage fee, $p_{OH}^{**} = 0$, does not matter in this case), has attained its maximal value, the only instrument left to change in the three-part tariff is the price, p_H^{**} . The service provider thereby raises this price to better price-discriminate the single H segment, which leads to $p_H^{**} > p_H^*$.

Now that we have analyzed how the introduction of the family plan changes the two individual plans, in the next proposition we discuss how the introduction of the family plan affects the likelihood of observing quantity discounts in the H plan as compared to the L plan.

Proposition 6. *Compared to the case without the family plan, quantity discounts between the H*

*and the L plan, $\frac{P_H^{**}}{q_H} < \frac{P_L^{**}}{q_L}$, is less likely.*

The intuition behind this proposition stems from the results in Proposition 5.

While the introduction of the family decreases the L plan's quantity and price, it also increases the H plan's price and keeps the H plan's quantity unchanged. The latter result leads to a higher price-quantity ratio for the H plan compared to that in Section 3,

$\frac{P_H^{**}}{q_H} > \frac{P_H^*}{q_H}$. Therefore, it is less likely for the service provider to offer quantity discounts

to the H plan compared to the L plan after the introduction of the family plan.

4.3 Profitability of Family Plans

Propositions 4 and 5 have shown the opposing impact of the family plan on the prices of the L plan and the H plan, as well as the quantity discounts offered to the family-plan users. Two natural questions concern whether introducing the family plan would be more profitable overall and when this strategy emerges in the equilibrium. We compare the service provider's profits with and without the family plan, $E\Pi^{**}$ and $E\Pi^*$, and provide answers to these questions in the proposition below.

Proposition 7. *When*

$\Delta\theta_L((1-k)\theta_H + k\theta_L)^2(1-\lambda)^2 - (1-k)(\theta_H - \theta_L)(\theta_L(1-f) - \theta_H(1-\lambda))^2 > 0$, *it is more profitable for the service provider to offer a family plan and two individual plans, i.e.,*

$E\Pi^{**} > E\Pi^*$. *Otherwise, offering only two individual plans is more profitable.*

This proposition validates the profitability of offering family plans as well as offering regular individual plans. When the above condition holds, offering family plans is the equilibrium strategy. Two points are worth highlighting for the parameter range of the equilibrium. First, this equilibrium condition is more likely to hold for a larger valuation θ_L , which is consistent with the firm's incentive to serve both the H and L segments assumed at the outset. Second, it is also more likely to hold for a larger number of family users f and a higher curtailing parameter k , both of which lead to a higher family plan price p_F^{**} .¹ Next, we discuss the intuition for why offering a family plan in addition to two individual plans can be more profitable.

Given that the introduction of the family plan does not expand the market (full market coverage in the absence of a family plan), the profit boost it brings can be seen from the price comparison below:

$$p_F^{**} - p_H^* - p_L^* = \frac{1}{2} \Delta \left\{ \theta_L - \theta_H + \frac{\Delta \theta_L (\theta_H - k\theta_H + k\theta_L)^2 (1-\lambda)^2 (\theta_H^2 (1-\lambda)^3 + \theta_H \theta_L (1-\lambda))}{(1-k)^2 (\theta_L - \theta_H (1-\lambda))^2 (\theta_L (1-f) - \theta_H (1-\lambda))^2} \right. \\ \left. (f(2+f-fk-2(2-k)\lambda) - 2(1-\lambda)) + \theta_L^2 (1-\lambda - f(2-fk - (2-f)(2-k)\lambda)) \right\} > 0. \quad (22)$$

This inequality states that a family of two members contributes more profits to the firm than two buyers of an individual H and L plan in the absence of the family plan.² We discuss the intuition by analyzing the binding incentive compatibility constraints. Recall

¹ The proof for these two comparative statics is given in the appendix.

² The proof for this inequality is given in the appendix. It is conditioned on f being below a threshold.

that as the single L segment always gets zero surplus, $EU(\theta_L, q_L, p_L, p_{OL}) = 0$, the IC constraint for the family plan buyers can be simplified as:

$$EU(\theta_F, q_F, p_F, p_{OF}) \geq EU(\theta_H, q_H, p_H, p_{OH}) + EU(\theta_L, q_L, p_L, p_{OL}) = EU(\theta_H, q_H, p_H, p_{OH}).$$

Note that the last term, $EU(\theta_H, q_H, p_H, p_{OH})$, is equal to $EU(\theta_H, q_L, p_L, p_{OL})$ according to the binding IC constraint of the single H segment. Based on the discussion in Proposition 5, the expected utility for the H-type consumers to choose the L plan is lower in the presence of the family plan. Therefore, it is not too costly to induce family buyers to purchase the family plan.

One important way the family plan helps the firm is through better price discriminating the L-type consumers in a family. In particular, by offering unlimited quantities, the service provider can charge a high price p_F^{**} to extract more surpluses from the L-type consumers who are family-plan buyers than from the single L-type consumers. The firm does so without worrying about the potential cannibalization problem due to the nature of the third-degree price discrimination. By contrast, with the second-degree price discrimination, the firm always faces the trade-off between a higher quantity for the L plan for a higher revenue from single L-type buyers and the resulting lower price for the H plan, in order to prevent H-type buyers from switching plans. Furthermore, although the price for the individual L plan decreases in the presence of a

family plan, $p_L^{**} < p_L^*$, the price per unit for the L plan is higher $\frac{p_L^{**}}{q_L^{**}} > \frac{p_L^*}{q_L^*}$. In addition,

the firm obtains a higher overage payment from these individual L-type buyers because of a reduced allowance, q_L^{**} . To summarize: the introduction of the family plan increases revenues from the single H segment ($p_H^{**} > p_H^*$, Proposition 5) and both types of consumers within a family ($p_F^{**} - p_H^* - p_L^* > 0$). The loss in the single L segment is not significant due to an increase in its price per unit and its overage payment. Therefore, the profitability with the family plan is higher overall than when the firm only offers two individual plans.

One interesting question in the context of a three-part tariff concerns the likelihood of running into the overage region; in particular, whether a family plan will decrease the probability of overage usage compared to that of two individual plans. We answer this question in the proposition below.

Proposition 8. *Compared to two individual plans, the probability of running into the overage region is lower in a family plan.*

Recall that only the single L-type consumers will face overage usage, because the firm offers maximum quantities for both the H segment and the family segment. In other words, the L-type consumers in family plans no longer incur overage charges such as those incurred by single L-type consumers. Therefore, no overage usage is possible in the H and the family plans, which leads to the result in Proposition 8. Note that this result is predicated upon the assumption on independent requirement distributions

between two consumer segments, in particular between two members of the same family. In situations with a negative correlation between two family members' requirements, we expect Proposition 8 to continue to hold because the need to use extra minutes within a family is further reduced.

5. Non-Aversion to Overage Usage

In the previous analysis, we focused on the situation in which consumers curtail their overage consumption by the proportion k . All the optimal quantities and prices in this case turn out to be continuous functions of the curtailing parameter when $k \in [0, 1)$. In this section, we analyze the extreme case in which consumers are not at all averse to overage usage. In other words, they do not curtail their consumption after hitting their plans' quantities, i.e., $k = 1$. We start our analysis by assuming $q_L^* \geq \Delta$ and use the resulting optimal solutions to verify this assumption ex post. Because in this case the optimal quantity, q_L^* , turns out to be zero, which violates the previous assumption, we therefore carry our following analysis with the assumption $q_L^* < \Delta$ (which is verified ex post).¹ In this situation of $q_L^* < \Delta$, we assume that an H-type consumer will not consider buying the L plan because the included quantity of the L plan cannot satisfy an H-type consumer's minimum need.² In other words, the only outside option for an H-type consumer is no consumption, which leads to zero utility. In this case, incentive compatibility constraints are trivially satisfied and are therefore ignored in the following presentation.

¹ Details of the analysis are available upon request.

² We have studied an alternative formulation in which H-type consumers still view the L plan as their best outside option and intentionally use overage minutes to satisfy their need for communication. The details are available upon request.

5.1 Analysis of Individual Plans

The firm chooses two individual plans, (q_H, p_H, p_{OH}) and (q_L, p_L, p_{OL}) , to maximize its expected profit:

$E\Pi = \text{Max} (1-\lambda)[p_H + p_{OH}E(q - q_H | q > q_H)Pr(q > q_H)] + \lambda[p_L + p_{OL}E(q - q_L | q > q_L)Pr(q > q_L)]$,
subject to

$$EU(\theta_H, q_H, p_H, p_{OH}) \geq 0, \quad (23)$$

$$EU(\theta_L, q_L, p_L, p_{OL}) \geq 0. \quad (24)$$

The two individual rationality constraints, Constraints (23) and (24), ensure that each consumer segment buys the plan intended for it rather than not buy anything at all.

Solutions to this constrained optimization lead to the following optimal plans:

$$\begin{aligned} q_L^* &= 0, & p_L^* &= 0, \\ q_H^* &= \Delta, & p_H^* &= \Delta\theta_H, \\ p_{OL}^* &= \theta_L, & p_{OH}^* &= \theta_H. \end{aligned} \quad (25)$$

The corresponding profit is $E\Pi^* = \frac{1}{2}((1 + 2\Delta)\theta_H(1 - \lambda) + \theta_L\lambda)$. The L segment obtains a free plan with zero included quantity, and pays for each unit of its consumption. The H segment gets a plan with the quantity that satisfies its minimum need, Δ . A close look at the optimal quantities and prices leads to the following proposition.

Proposition 9. *When $k = 1$, the service provider offers a pay-as-you-go plan to the L segment and the minimal required quantity to the H segment.*

When consumers do not curtail their overage consumption, it is reasonable for the service provider to offer the lowest possible quantities in order to distinguish different consumer segments and also to take advantage of their willingness to pay through overage prices. In this case, the firm no longer has to tradeoff between a lower quantity for higher overage payment with the curtailing behavior and a higher quantity for a higher fixed price with certainty. Instead, it simply pushes down the included quantities to the lower supports of two requirement distributions because consumers are going to use whatever their random draws decide. In effect, the firm is able to employ the first-degree price discrimination against both consumer segments and thereby fully extracts both segments' surpluses.

5.2 Analysis of Individual and Family Plans

Now we consider the case when the firm offers a family plan in addition to two individual plans. The firm chooses three plans to maximize its expected profit:

$$E\Pi = \text{Max} (1 - \lambda - f)[p_H + p_{OH}E(q - q_H | q > q_H)Pr(q > q_H)] + (\lambda - f)[p_L + p_{OL}E(q - q_L | q > q_L)Pr(q > q_L)] + f[p_F + p_{OF}E(q - q_F | q > q_F)Pr(q > q_F)],$$

subject to

$$EU(\theta_H, q_H, p_H, p_{OH}) \geq 0, \quad (26)$$

$$EU(\theta_L, q_L, p_L, p_{OL}) \geq 0, \quad (27)$$

$$EU(\theta_F, q_F, p_F, p_{OF}) \geq 0, \quad (28)$$

$$EU(\theta_F, q_F, p_F, p_{OF}) \geq EU(\theta_H, q_H, p_H, p_{OH}) + EU(\theta_L, q_L, p_L, p_{OL}). \quad (29)$$

Constraints (26) through (28) are individual rationality constraints which ensure that each segment purchases the plan directed to it. Constraint (29), however, ensures that a family prefers to buy the family plan instead of two individual plans.

It turns out that multiple optimal menus exist in terms of the family plan's quantity, q_F^{**} , and price, p_F^{**} , but that the service provider's profit is the same across all of these menus. One reasonable menu of the optimal plans is given as:

$$\begin{aligned} q_L^{**} &= 0, & p_L^{**} &= 0, \\ q_H^{**} &= \Delta, & p_H^{**} &= \Delta\theta_H, \\ q_F^{**} &= 1 + \Delta, & p_F^{**} &= \frac{1}{12}(5 + 6\Delta)(\theta_H + \theta_L), \\ p_{OL}^{**} &= \theta_L, & p_{OH}^{**} &= \theta_H, & p_{OF}^{**} &= (\theta_H + \theta_L) / 2. \end{aligned} \quad (30)$$

The service provider's expected profit is

$$E\Pi^{**} = \frac{1}{2}(\theta_H + \Delta((2-f)\theta_H + f\theta_L)(1-\lambda) - (\theta_H - \theta_L)\lambda).$$

The L segment still gets a pay-as-you-go plan, and the H segment still obtains the minimum quantity needed as the included quantity for the H plan. Comparison between profits with and without the family plan leads to the following proposition.

Proposition 10. *When $k = 1$, the service provider is better off by only offering two individual plans.*

This proposition states that it is no longer profitable to offer a family plan when consumers do not curtail their overage consumption. In this case, a family plan only

gives away unnecessary discounts to the quantity in the plan. The intuition can be seen from

$$\frac{p_F^{**}}{q_F^{**}} = \frac{\frac{1}{12}(5+6\Delta)(\theta_H + \theta_L)}{1+\Delta} < \frac{\frac{1}{12}(6+6\Delta)(\theta_H + \theta_L)}{1+\Delta} = \frac{(\theta_H + \theta_L)}{2} = \theta_F. \text{ Recall that}$$

when $k = 1$, the service provider can use two individual plans to perfectly distinguish two consumer segments and to fully extract all consumers' surpluses through overage prices. Because there are no other ways to beat the effectiveness of the first degree price discrimination, the service provider only offers two individual plans in the equilibrium.

6. Extensions

In this section, we present several extensions of the base model from Section 2 to Section 5. We start by discussing a model with positive marginal costs for the firm, and analyze the impact of consumers' mindless consumption on different plans.

Subsequently, we present the analysis of non-overlapping consumer segments, an alternative model for family utility, other family compositions, multiple family segments, and correlated consumer requirements within a family. Finally we analyze a deterministic model.

6.1 Full Model with Positive Marginal Costs and Mindless Consumption

In the main text, we assumed that consumers do not expect to gain additional utility by continuing to use the service once their communication needs have been met (in other words, all consumers expect to stop consuming after they have satisfied their requirement realization). In this section, we relax this assumption and allow for the existence of some myopic consumers who have inconsistent behaviors. We assume that m fraction of consumers is myopic in that they will engage in mindless consumption if their requirement realization is below their plans' quantity. These myopic consumers will use up all the remaining quantities in their plans, although ex ante they expect neither to do so nor to gain utility from this behavior. When $m = 0$, this case corresponds to the analysis in the main body of this paper. When $m = 1$, all the

consumers are myopic and may engage in mindless consumption. In addition, we assume that the marginal cost of the service provider is c , which can be positive.

The service provider's cost function, C , is comprised of two components:

$C = C_1 + C_2$. They are given as:

$$C_1 = (1-m)(1-\lambda)c \left[\int_{\Delta}^{q_H} q dq + \int_{q_H}^{1+\Delta} (q_H + k(q - q_H)) dq \right] + (1-m)\lambda c \left[\int_0^{q_L} q dq + \int_{q_L}^1 (q_L + k(q - q_L)) dq \right], \text{ and}$$

$$C_2 = m(1-\lambda)c q_H + \int_{q_H}^{1+\Delta} k(q - q_H) dq + m\lambda c \left[q_L + \int_{q_L}^1 k(q - q_L) dq \right].$$

C_1 denotes the expected cost from non-myopic consumers while C_2 denotes the expected cost from myopic consumers who engage in mindless consumption ex post.

The first terms in the above expressions are the firm's expected costs from serving the H segment, and the second terms are those of the L segment. The firm takes myopic consumers' behavior into account and chooses two individual plans to maximize its expected profit:

$$E\Pi = \text{Max} (1-\lambda)[p_H + p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H)] + \lambda[p_L + p_{OL}E(q - q_L | q > q_L, k)Pr(q > q_L)] - C,$$

subject to

$$EU(\theta_i, q_i, p_i, p_{oi}) \geq EU(\theta_j, q_j, p_j, p_{oj}), \text{ where } i, j \in \{H, L\}, i \neq j, \quad (31)$$

$$EU(\theta_i, q_i, p_i, p_{oi}) \geq 0, \text{ where } i \in \{H, L\}. \quad (32)$$

Constraint (31) ensures that each individual consumer segment prefers the plan designed for it to other plans (IC constraints), while Constraint (32) ensures that both the H and L segments will obtain nonnegative utilities from purchasing the plans designed for them (IR constraints).

By solving the firm's optimization problem, we obtain the optimal quantities and overage fees of the H and L plans:

$$q_H^* = 1 + \Delta - \frac{cm}{c(k+m-1) + (1-k)\theta_H}, p_{OH}^* = \theta_H \text{ when } q_H^* < 1 + \Delta, \text{ otherwise } p_{OH}^* = 0.$$

$$q_L^* = \frac{(1-k(1+(1-\lambda)\Delta))\theta_L - (1-k)(1+\Delta)(1-\lambda)\theta_H - (1-k)c\lambda}{(1-k)(\theta_L - \theta_H(1-\lambda)) + \lambda c(k+m-1)}, p_{OL}^* = \theta_L. \text{ The prices, } p_H^*$$

and p_L^* , and the profit, $E\Pi^*$, can be obtained by respectively plugging these quantities and overage fees into the binding IC and IR constraints and into the profit function.

If the firm decides to offer a family plan, its expected cost is given by

$C = C_1 + C_2$, where

$$C_1 = (1-m)(1-\lambda-f)c \left[\int_{\Delta}^{q_H} q f_H(q) dq + \int_{q_H}^{1+\Delta} (q_H + k(q-q_H)) f_H(q) dq \right] + (1-m)(\lambda-f)c \left[\int_0^{q_L} q f_L(q) dq + \int_{q_L}^1 (q_L + k(q-q_L)) f_L(q) dq \right] + (1-m)fc \left[\int_{\Delta}^{q_F} q f_F(q) dq + \int_{q_F}^{2+\Delta} (q_F + k(q-q_F)) f_F(q) dq \right], \text{ and}$$

$$C_2 = m(1-\lambda-f)c \left[q_H + \int_{q_H}^{1+\Delta} k(q-q_H) f_H(q) dq \right] + m(\lambda-f)c \left[q_L + \int_{q_L}^1 k(q-q_L) f_L(q) dq \right] + mfc \left[q_F + \int_{q_F}^{2+\Delta} k(q-q_F) f_F(q) dq \right].$$

Similar to the analysis on individual plans only, C_1 denotes the expected cost from non-myopic consumers while C_2 denotes the expected cost from myopic consumers. The first, second, and third terms in C_1 and C_2 are the service provider's expected costs from serving the H, the L, and the family segments, respectively. The service provider chooses three plans to maximize its expected profit:

$$E\Pi = \text{Max} (1-\lambda-f)[p_H + p_{OH} E(q-q_H | q > q_H, k) \text{Pr}(q > q_H)] + (\lambda-f)[p_L + p_{OL} E(q-q_L | q > q_L, k) \text{Pr}(q > q_L)] + f[p_F + p_{OF} E(q-q_F | q > q_F, k) \text{Pr}(q > q_F)] - C,$$

subject to

$$EU(\theta_i, q_i, p_i, p_{oi}) \geq EU(\theta_j, q_j, p_j, p_{oj}), \text{ where } i, j \in \{H, L\}, i \neq j, \quad (33)$$

$$EU(\theta_F, q_F, p_F, p_{OF}) \geq EU(\theta_H, q_H, p_H, p_{OH}) + EU(\theta_L, q_L, p_L, p_{OL}), \quad (34)$$

$$EU(\theta_i, q_i, p_i, p_{oi}) \geq 0, \text{ where } i \in \{H, L, F\}. \quad (35)$$

Constraints (33) and (34) ensure that each consumer segment prefers the plan designed for them to other plans (IC constraints), while Constraint (35) ensures that all the consumer segments will obtain nonnegative utilities from purchasing the plans designed for them (IR constraints). Solving the firm's optimization problem, we obtain the optimal quantities and the overage fees of all three plans:

$$q_H^{**} = 1 + \Delta - \frac{cm}{c(k+m-1) + (1-k)\theta_H}, \quad p_{OH}^{**} = \theta_H \text{ when } q_H^{**} < 1 + \Delta, \text{ otherwise } p_{OH}^{**} = 0;$$

$$q_F^{**} = 2 + \Delta - 2\sqrt{\frac{cm}{(1-k)(\theta_H + \theta_L) + 2c(k+m-1)}}, \quad p_{OF}^{**} = \frac{\theta_H + \theta_L}{2} \text{ when } q_F^{**} < 2 + \Delta, \text{ otherwise}$$

$$p_{OF}^{**} = 0;$$

$$q_L^{**} = \frac{((1-f)(1-k) - (1-\lambda)k\Delta)\theta_L - (1-k)(1+\Delta)(1-\lambda)\theta_H - c(1-k)(\lambda-f)}{(1-k)((1-f)\theta_L - (1-\lambda)\theta_H) - c(1-k-m)(\lambda-f)}, \quad p_{OL}^{**} = \theta_L.$$

By comparing the quantities in this case with the quantities without mindless consumption in Section 5, we obtain the following result.

Proposition 11. *If the marginal cost c is positive and a fraction of consumers engage in mindless consumption, the service provider does not offer unlimited plans.*

This proposition can be seen from: $q_H^{**} = 1 + \Delta - \frac{cm}{c(k+m-1) + (1-k)\theta_H} < 1 + \Delta$,

$$q_F^{**} = 2 + \Delta - 2\sqrt{\frac{cm}{(1-k)(\theta_H + \theta_L) + 2c(k+m-1)}} < 2 + \Delta, \forall c, m > 0. \text{ The intuition behind this}$$

result is that as long as the service provider faces a positive marginal cost (regardless of how small it is), it always has incentives to reduce quantities to discourage mindless consumption, and thus avoid excessive costs associated with mindless consumption.

Similar to the case of individual plans only, the prices for three plans, p_H^{**} , p_L^{**} , and p_F^{**} , and the firm's profit, $E\Pi^{**}$, can be obtained by respectively plugging these quantities and overage fees into the binding IC and IR constraints as well as the profit function.

6.2 Non-Overlapping Consumer Segments

In the main text, we assumed that the L and H segments' requirements were distributed between $[0, 1]$ and $[\Delta, 1 + \Delta]$, respectively. Moreover, consumers' requirement heterogeneity is moderate (i.e., Δ is below a threshold), such that the incentive compatibility constraint for the H segment is not trivially satisfied in the service provider's constrained optimization problem. For completeness, in this section we consider the situation in which consumers' requirement heterogeneity is sufficiently large so that an H-type consumer will not consider buying the L plan because the

included quantity of the L plan cannot satisfy an H-type consumer's minimum need. As a result, the incentive compatibility constraint for the H segment is trivially satisfied.

6.2.1 Individual Plans

In this subsection, we analyze the case in which the service provider only offers two individual plans. By solving the firm's constrained optimization problem, we obtain the following optimal plans:

$$\begin{aligned}
 q_L^* &= 1, p_L^* = \frac{\theta_L}{2}, \\
 q_H^* &= 1 + \Delta, p_H^* = \frac{(1 + 2\Delta)\theta_H}{2}, \\
 p_{OL}^* &= p_{OH}^* = 0.
 \end{aligned} \tag{36}$$

Both consumer segments get unlimited quantities in their individual plans. The corresponding profit is $E\Pi^* = \frac{1}{2}((1 + 2\Delta)\theta_H(1 - \lambda) + \theta_L\lambda)$.

6.2.2 Individual and Family Plans

In this subsection, we analyze the case in which the service provider offers a family plan in addition to two individual plans. Similar to the discussion in the previous subsection, the service provider only needs to ensure all consumer segments to get weakly positive utility (i.e., the only binding constraints are the H, L, and family segments' individual rationality constraints). By solving the firm's constrained optimization problem, we obtain the following optimal plans:

$$\begin{aligned}
q_L^{**} &= 1, p_L^{**} = \frac{\theta_L}{2}, \\
q_H^{**} &= 1 + \Delta, p_H^{**} = \frac{(1 + 2\Delta)\theta_H}{2}, \\
q_F^{**} &= 2 + \Delta, p_F^{**} = \frac{1}{2}(1 + \Delta)(\theta_H + \theta_L), \\
p_{OL}^{**} &= p_{OH}^{**} = p_{OF}^{**} = 0.
\end{aligned} \tag{37}$$

Again, all the plans are unlimited in terms of quantities. The corresponding profit is $E\Pi^{**} = \frac{1}{2}(((1 + 2\Delta)(1 - \lambda) - f\Delta)\theta_H + (\lambda + f\Delta)\theta_L)$. Interestingly, compared to the case without a family plan in Section 6.2.1, offering a family plan is no longer profitable.

This result can be seen from $E\Pi^{**} - E\Pi^* = -\frac{1}{2}f\Delta(\theta_H - \theta_L) < 0$.

6.3 Alternative Model for Family Utility

In the main text, we assumed that a family's valuation for one unit of consumption, θ_F , is the average of the two family members' valuations: $\theta_F = \frac{\theta_H + \theta_L}{2}$.

This simplified assumption did not take into account the potential different usage of the two members in the same family. In this extension, we use the average valuation

weighted by each family member's expected usage, $\theta_F = \frac{(1 + 2\Delta)\theta_H + \theta_L}{2 + 2\Delta}$, to assess the

robustness of our results.

By solving the firm's constrained optimization problem, we obtain the following optimal menu of plans.

The quantities for the three plans are:

$$\begin{aligned} q_H^{**} &= 1 + \Delta, q_F^{**} = 2 + \Delta, \\ q_L^{**} &= 1 - \frac{\Delta(1-\lambda)((1-k)\theta_H + k\theta_L)}{(1-k)((1-f)\theta_L - (1-\lambda)\theta_H)}. \end{aligned} \quad (38)$$

Both the H plan and the family plan obtain their maximum requirements, $1 + \Delta$ and $2 + \Delta$. It can be interpreted that the single H segment and the family segment both obtain unlimited plans. Similar to the scenario in Section 3, the L plan's quantity, q_L^{**} , decreases in the curtailing parameter k . The prices of these three plans, which all depend on k and f , are given as:

$$\begin{aligned} p_L^{**} &= \frac{\theta_L(\theta_L((1-f)(1-k) - k\Delta(1-\lambda)) - (1-k)(1+\Delta)\theta_H(1-\lambda))}{2(1-k)^2((1-f)\theta_L - \theta_H(1-\lambda))^2} \\ &\quad (\theta_L((1-f)(1-k) + k\Delta(1-\lambda)) - (1-k)(1-\Delta)\theta_H(1-\lambda)), \end{aligned} \quad (39)$$

$$p_H^{**} = p_L^{**} + \frac{\Delta^2\theta_L^2(k\theta_L + (1-k)\theta_H)(1-f(1-k) - k\lambda)^2}{2(1-k)^2((1-f)\theta_L - \theta_H(1-\lambda))^2}, \quad (40)$$

$$p_F^{**} = \frac{1}{2} \left\{ 2\theta_L + \frac{\Delta^2((1-k)\theta_H + k\theta_L)\theta_L[\theta_L(1-f(2-(1-f)k) - k\lambda(\lambda - 2f)) - \theta_H(1-\lambda)^2]}{2(1-k)((1-f)\theta_L - \theta_H(1-\lambda))^2} \right\}. \quad (41)$$

The overage fees are $p_{OH}^{**} = 0$, $p_{OF}^{**} = 0$, and $p_{OL}^{**} = \theta_L$, all of which are independent of k and f . Compared to the analysis in Section 4 where family's

valuation is given by the average of two family members, $\theta_F = \frac{\theta_H + \theta_L}{2}$, all the quantities

and prices are identical, except for the family plan price p_F^{**} , q_H^{**} , q_L^{**} , p_H^{**} , p_L^{**} , p_{OH}^{**} ,

and p_{OL}^{**} are not affected because θ_F does not influence the IC constraints for the single

H and L segments. $q_F^{**} = 2 + \Delta$ remains the same because the service provider still has incentives to offer as many quantities as the entire family requires. Not surprisingly, with the valuation weighted by each family member's expected usage,

$$\theta_F = \frac{(1+2\Delta)\theta_H + \theta_L}{2+2\Delta} > \frac{\theta_H + \theta_L}{2},$$

the service provider can charge a higher price for the family plan. These results demonstrate the robustness of the model in Section 4.

6.4 Other Family Compositions

In the main text, we focused on one family composition: a family consists of a high-valuation consumer and a low-valuation consumer. Next, we study the implications of different family compositions. We carry out the analysis under the assumption that a family consists of two high-valuation consumers (HH family), or two low-valuation consumers (LL family).

6.4.1 HH Family

In this subsection, we analyze the case in which a family comprises two H-type consumers. We assume that the total number of families in the market is f . It therefore follows that the number of single H-type consumers is $(1 - \lambda - 2f)$. A family's valuation for quantities is θ_H , and it curtails its overage consumption by k . In this case, a family's requirement distribution is:

$$f_Q(q) = \begin{cases} 0, & q < 2\Delta, \\ q - 2\Delta, & 2\Delta \leq q < 1 + 2\Delta, \\ (2 + 2\Delta) - q, & 1 + 2\Delta \leq q < 2 + 2\Delta, \\ 0, & 2 + 2\Delta \leq q. \end{cases}$$

The service provider chooses three plans to maximize its expected profit:

$$E\Pi = \text{Max} \quad (1 - \lambda - 2f)[p_H + p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H)] + \lambda[p_L + p_{OL}E(q - q_L | q > q_L, k)Pr(q > q_L)] + f[p_F + p_{OF}E(q - q_F | q > q_F, k)Pr(q > q_F)],$$

subject to

$$EU(\theta_i, q_i, p_i, p_{Oi}) \geq EU(\theta_i, q_j, p_j, p_{Oj}), \text{ where } i, j \in \{H, L\}, i \neq j, \quad (42)$$

$$EU(\theta_F, q_F, p_F, p_{OF}) \geq 2EU(\theta_H, q_H, p_H, p_{OH}), \quad (43)$$

$$EU(\theta_i, q_i, p_i, p_{Oi}) \geq 0, \text{ where } i \in \{H, L, F\}. \quad (44)$$

Constraints (42) and (43) ensure that each consumer segment prefers the plan designed for them to other plans (IC constraints), while Constraint (44) ensures that all consumer segments will obtain nonnegative utilities from purchasing the plans designed for them (IR constraints).

As we solve the firm's optimization problem, we obtain the following optimal plans:

$$q_L^{**} = \frac{(1 - k(1 + (1 - \lambda)\Delta))\theta_L - (1 - k)(1 + \Delta)\theta_H(1 - \lambda)}{(1 - k)(\theta_L - \theta_H(1 - \lambda))},$$

$$q_H^{**} = 1 + \Delta, \quad q_F^{**} = 2(1 + \Delta),$$

$$p_{OL}^{**} = \theta_L, \quad p_{OH}^{**} = \theta_H, \quad p_{OF}^{**} = \theta_H.$$

By plugging these optimal quantities and overage fees into the binding IC and IR constraints, we obtain the optimal prices: p_H^{**} , p_L^{**} , and p_F^{**} :

$$\begin{aligned}
p_L^{**} &= \frac{\theta_L(\theta_L(1-k(1+\Delta(1-\lambda))) - (1-k)(1+\Delta)\theta_H(1-\lambda))}{2(1-k)^2(\theta_L - \theta_H(1-\lambda))^2} \\
&\quad (\theta_L(1-k+k\Delta-k\Delta\lambda) - (1-k)(1-\Delta)\theta_H(1-\lambda)), \\
p_H^{**} &= \frac{1}{2(1-k)(\theta_L - \theta_H(1-\lambda))^2} \{(\theta_H - \theta_L)\theta_L(-\theta_L + (1-\Delta)(1+\Delta)((1-k)\theta_H + k\theta_L)) \\
&\quad + 2\theta_H\theta_L(\theta_L - (1-\Delta)(1+\Delta)((1-k)\theta_H + k\theta_L))\lambda + [(1-k)(1-\Delta^2)\theta_H^2\theta_L - k\Delta^2(2-k)\theta_H\theta_L^2 - k^2\Delta^2\theta_L^3]\lambda^2\}, \\
p_F^{**} &= 2p_H^{**}.
\end{aligned}$$

By comparing the profits in this case, $E\Pi^{**}$, with the profits when only two individual plans are offered, $E\Pi^*$, we obtain the following result.

Proposition 12. *When a family comprises two H-type consumers, offering a family plan is less profitable than only offering two individual plans.*

The intuition for this result is that the introduction of the HH family plan does not alter any dimension of the individual H and L plans: $q_i^{**} = q_i^*$, $p_i^{**} = p_i^*$, $p_{oi}^{**} = p_{oi}^*$, where $i \in \{H, L\}$. In order to induce the families to purchase the family plan, the service provider must give these consumers extra surpluses on top of the surplus offered through the IC constraint of the single H segment: $EU(\theta_H, q_H, p_H, p_{OH}) \geq EU(\theta_H, q_L, p_L, p_{OL})$. Therefore, offering the HH family plan is less profitable than offering two individual plans and will not be the firm's equilibrium strategy.

6.4.2 LL Family

In this subsection, we analyze the case in which a family comprises two L-type consumers. We still assume that the total number of families in the market is f . It therefore follows that the number of single L-type consumers is $(\lambda - 2f)$. A family's

valuation for quantities is θ_L and it curtails its overage consumption by k . In this case, a family's requirement distribution is:

$$f_Q(q) = \begin{cases} 0, & q < 0, \\ q, & 0 \leq q < 1, \\ 2 - q, & 1 \leq q < 2, \\ 0, & 2 \leq q. \end{cases}$$

The firm chooses three plans to maximize its expected profit:

$$E\Pi = \text{Max} (1 - \lambda)[p_H + p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H)] + (\lambda - 2f)[p_L + p_{OL}E(q - q_L | q > q_L, k)Pr(q > q_L)] + f[p_F + p_{OF}E(q - q_F | q > q_F, k)Pr(q > q_F)],$$

subject to

$$EU(\theta_i, q_i, p_i, p_{oi}) \geq EU(\theta_i, q_j, p_j, p_{oj}), \text{ where } i, j \in \{H, L\}, i \neq j, \quad (45)$$

$$EU(\theta_F, q_F, p_F, p_{OF}) \geq 2EU(\theta_L, q_L, p_L, p_{OL}), \quad (46)$$

$$EU(\theta_i, q_i, p_i, p_{oi}) \geq 0, \text{ where } i \in \{H, L, F\}. \quad (47)$$

Constraints (45) and (46) ensure that each consumer segment prefers the plan designed for it to other plans (IC constraints), while Constraint (47) ensures that all consumer segments will obtain nonnegative utilities from purchasing the plans designed for them (IR constraints).

By solving the firm's optimization problem, we obtain the following optimal plans:

$$q_L^{**} = \frac{(1 - 2f - k(1 + \Delta - 2f - \Delta\lambda))\theta_L - (1 - k)(1 + \Delta)\theta_H(1 - \lambda)}{(1 - k)((1 - 2f)\theta_L - \theta_H(1 - \lambda))},$$

$$q_H^{**} = 1 + \Delta, \quad q_F^{**} = 2,$$

$$p_{OL}^{**} = \theta_L, \quad p_{OH}^{**} = \theta_H, \quad p_{OF}^{**} = \theta_L.$$

By plugging these quantities and overage fees into the binding IC and IR constraints, we can obtain the optimal prices: p_H^{**} , p_L^{**} , and p_F^{**} . Comparison of the profits in this case $E\Pi^{**}$ with the profits when only two individual plans are offered, $E\Pi^*$, leads to the following result.

Proposition 13. *When a family comprises two L-type consumers, offering the LL family plan is more profitable than only offering two individual plans.*

To understand why the introduction of the LL family plan increases the service provider's profits, we need to assess its impact on the quantities and prices of the individual H and L plans. First, note that the single L plan's quantity and price are lower compared to the case in which only two individual plans are offered ($f = 0$):

$\frac{\partial q_L^{**}}{\partial f} < 0, \frac{\partial p_L^{**}}{\partial f} < 0$. The reduction in q_L^{**} implies that the L plan becomes less attractive to

the H-type consumers as their best outside option. Therefore, the firm can raise the price

of the H plan to better exploit the single H segment: $\frac{\partial p_H^{**}}{\partial f} > 0$. Moreover, the family plan

provides more quantities to each L-type consumer than the individual L plan does

because the firm does not need to worry about the cannibalization issue. As a result, the

firm charges a price for the family plan that is higher than the sum of two L plans in the

absence of the family plan: $p_F^{**} \geq 2p_L^*$.

6.5 Multiple Family Segments

In the main text, we assumed that there is a single family segment in the market, and each family consists of one H- and one L-type consumer. In this section, we generalize this setting and assume that there exist three different family segments: HH, HL and LL families. As the name of each family segment suggests, an HH (HL, LL) family consists of two H-type (one H- and one L-type, two L-type) consumers. To simplify the analysis, we further assume that the size of each family segment is the same, f , and the firm will only offer one family plan. In this situation, we are interested in assessing which family plan the service provider should offer to maximize its profits.

Following the logic in Section 6.4, the service provider does not have incentives to offer a family plan to cater to the HH family segment. To see whether the firm will offer an HL family plan or an LL family plan, we compare its profits in these two situations. Then we check to ensure that only the targeted family segment will purchase the family plan in the equilibrium.

Based on the analysis in Section 4, Section 6.4.1 and Section 6.4.2, the profit difference between offering an HL family plan and an LL family plan is given by:

$$\Pi_{HL}^{**} - \Pi_{LL}^{**} = -((\theta_H - \theta_L) + \frac{\Delta\theta_L((1-k)\theta_H + k\theta_L)^2(1-\lambda)^2}{(1-k)((1-2f)\theta_L - (1-\lambda)\theta_H)((1-f)\theta_L - (1-\lambda)\theta_H)}) \frac{f\Delta}{2} < 0.$$

This profit comparison implies that it is more profitable to offer an LL family plan to cater to the LL family segment than to offer an HL family plan to target the HL family segment. The next proposition formally states this result.

Proposition 14. *Offering the LL family plan is more profitable than offering the HL family plan.*

The intuition behind this result is two-fold. First, when considering a family plan, the LL family segment's best outside option is two individual L plans, which gives it zero utility. In comparison, when considering a family plan, the HL family segment's best outside option is an individual H and L plan, which gives it a positive utility. This contrast means that it is less costly to induce the LL family to purchase an LL family plan than to induce the HL family to purchase an HL family plan. Second, the single H-type consumers are paying more in the presence of an LL family plan, as can be seen from $p_H^{**}(LL) > p_H^{**}(HL)$. The reason is that after bundling two L-type consumers into a family plan, the service provider reduces the size of the individual L-plan buyers to a greater extent compared to the situation with an HL family plan. This fact allows the firm to further decrease the allowance of the individual L plan, $q_L^{**}(LL) < q_L^{**}(HL)$, to make it less attractive as the best outside option for the single H segment. Consequently, the firm can charge a higher price to better price discriminate the single H-type consumers.

Furthermore, we have checked that an HL family is better off by purchasing an individual H plan and an individual L plan than purchasing the LL family plan. Therefore, should the firm decide to offer one family plan in addition to two individual plans, it should offer an LL family plan to bundle L-type consumers into joint accounts.

6.6 Correlated Consumption Requirements

In the main text, we assumed that consumers' consumption needs are independently distributed. Now we relax this assumption and assume that consumers within a family have correlated consumption needs. To simplify the analysis, we consider two polar cases: a perfectly positive correlation within a family and a perfectly negative correlation within a family.

First, we consider the situation in which the correlation between two consumers within a family is 1. If we denote an L-type family member's need by N_L , we can write the H-type family member's need by $N_H = N_L + \Delta$. In this case, a family's total consumption requirement is given by $N_F = N_L + N_H = 2N_L + \Delta$, which is uniformly distributed between $[\Delta, 2 + \Delta]$. Note that this distribution is different from the triangular distribution discussed in the case of independent consumption requirements. In particular, the variance of a family's requirement is twice as large as that in the case of independent requirements.

The service provider's constrained optimization problem is similar to that analyzed in Section 4 and thus is not repeated here. After solving this optimization problem, we find that all three plans, $(q_H^{**}, p_H^{**}, p_{OH}^{**})$, $(q_L^{**}, p_L^{**}, p_{OL}^{**})$, and $(q_F^{**}, p_F^{**}, p_{OF}^{**})$, are all the same as those obtained in the case of independent consumption requirements. Therefore, when the correlation between two consumers within a family is 1, the profits

of offering a family plan in addition to two individual plans are the same as the profits in the independent case.

Second, we consider situation in which the correlation between two consumers within a family is -1. In this case, we can write the H-type family member's need by $N_H = 1 + \Delta - N_L$. It therefore follows that a family's total consumption requirement is given by $N_F = N_H + N_L = 1 + \Delta$, which is a constant. Note that this means that the variance of a family's requirement is reduced to zero from that in the case of independent requirements.

After solving the service provider's constrained optimization problem, we again find that all three plans, $(q_H^{**}, p_H^{**}, p_{OH}^{**})$, $(q_L^{**}, p_L^{**}, p_{OL}^{**})$, and $(q_F^{**}, p_F^{**}, p_{OF}^{**})$, are the same as those obtained in the case of independent consumption requirements. Therefore, when the correlation between two consumers within a family is -1, the profits of offering a family plan in addition to two individual plans are the same as the profits in the independent case.

The following proposition compares the current analysis with the case where consumers' consumption needs are independently distributed (Section 2 to Section 5).

Proposition 15. *When family members' consumption requirements are perfectly correlated, in equilibrium, the service provider offers the same plans as it did in the case with zero correlation (independent consumption requirements).*

Given that the equilibrium plans stay unchanged (quantities, prices and overage prices), all the results from Section 4 remain the same. In particular, it is more profitable to offer family plans.

6.7 Deterministic Consumption

In the main text, we assumed that consumers' consumption requirement is stochastic. In this section, we consider the situation without any uncertainty. In other words, we assume that consumers' needs for service are deterministic. We are interested in seeing whether the intuition on bundling consumers to reduce cannibalization and increase profits still holds.

We assume that consumers' utility function is given by $u(\theta_i, q, p) = \theta_i \sqrt{q} - p$, where $i \in \{H, L\}$. The sizes of the L and the family segments are still λ and f , respectively. A family consists of two consumers: one of them is H-type, and the other is L-type. The H-type family member consumes α fraction of the family plan's allowance. The firm's marginal cost per unit is c ($c > 0$).

6.7.1 Individual Plans

In this subsection, we analyze the case in which the service provider only offers two individual plans, (q_H, p_H) and (q_L, p_L) . By solving the firm's constrained optimization problem, we obtain the following optimal plans:

$$\begin{aligned}
q_L^* &= \frac{(\theta_L - (1-\lambda)\theta_H)^2}{4\lambda^2 c^2}, p_L^* = \frac{\theta_L(\theta_L - (1-\lambda)\theta_H)}{2c\lambda}, \\
q_H^* &= \frac{\theta_H^2}{4c^2}, p_H^* = \frac{(\theta_H - \theta_L)^2 + \theta_H\theta_L\lambda}{2c\lambda}.
\end{aligned} \tag{48}$$

Both consumer segments get unlimited quantities in their individual plans. The

$$\text{corresponding profit is } \Pi^* = \frac{(\theta_H - \theta_L)^2 - \theta_H(\theta_H - 2\theta_L)\lambda}{4c\lambda}.$$

6.7.2 Individual and Family Plans

In this subsection, we analyze the case in which the service provider offers a family plan in addition to two individual plans: (q_H, p_H) , (q_L, p_L) , and (q_F, p_F) . Similar to the discussion in the previous subsection, the service provider needs to ensure that all consumer segments prefer the plans targeted at them to other plans, and they all get weakly positive utility (i.e., the binding constraints are IC constraints for the H and family segments, and IR constraints for the L segment). By solving the firm's constrained optimization problem, we obtain the following optimal plans:

$$\begin{aligned}
q_L^{**} &= \frac{((1-f)\theta_L - \theta_H(1-\lambda))^2}{4c^2(\lambda-f)^2}, p_L^{**} = \frac{\theta_L((1-f)\theta_L - (1-\lambda)\theta_H)}{2c(\lambda-f)}, \\
q_H^{**} &= \frac{\theta_H^2}{4c^2}, p_H^{**} = \frac{(1-f)(\theta_H^2 + \theta_L^2) - \theta_H\theta_L(2-\lambda-f)}{2c(\lambda-f)}, \\
q_F^{**} &= \frac{(\sqrt{\alpha}\theta_H + \sqrt{1-\alpha}\theta_L)^2}{4c^2}, \\
p_F^{**} &= \frac{(\theta_H - \theta_L)((1-f)\theta_L - \theta_H(1-\lambda)) + (\alpha(\theta_H^2 - \theta_L^2) + \theta_L(2\sqrt{(1-\alpha)\alpha}\theta_H + \theta_L))(\lambda-f)}{2c(\lambda-f)}.
\end{aligned} \tag{49}$$

The corresponding profit is

$$\Pi^* = \frac{\theta_H^2 (1-f+f^2(1-\alpha)-\lambda+f\alpha\lambda) + 2\theta_H\theta_L \left(f \left(1-\lambda+\sqrt{(1-\alpha)\alpha(\lambda-f)} \right) - (1-\lambda) \right) + \theta_L^2 (1-f(2-f\alpha-\lambda+\alpha\lambda))}{4c(\lambda-f)}.$$

Compared to the case without a family plan in Section 6.7.1, offering a family plan can still be more profitable. This result can be seen from

$$\Pi^{**} - \Pi^* = \frac{f \left(2\theta_H\theta_L \left(\lambda \left(2-\lambda+\sqrt{(1-\alpha)\alpha(\lambda-f)} \right) - 1 \right) + \theta_L^2 (1-\lambda(2-f\alpha-(1-\alpha)\lambda)) + \theta_H^2 (1-\lambda(2-f+f\alpha-\alpha\lambda)) \right)}{4c(\lambda-f)\lambda}.$$
 If

consumers within a family optimally share the family plan's allowance, in other words, they endogenously choose α^{**} to maximize the joint family utility, then the optimal

allocation fraction is given by $\alpha^{**} = \frac{\theta_H^2}{\theta_H^2 + \theta_L^2}$. Plugging this value into the firm's profit

function, we find that offering the family plan is always more profitable:

$$\frac{\partial \Pi^{**}}{\partial f} = \frac{(\theta_H - \theta_L)^2 (1-\lambda)^2}{4c\lambda^2} > 0. \text{ The proposition below formalizes this result.}$$

Proposition 16. *When consumers' requirement is deterministic, it is more profitable for the service provider to offer family plans.*

The intuition is the same as that we discussed in the main text: Bundling some low-valuation consumers with high-valuation consumers into joint family accounts allows the firm to better price discriminate single H-type consumers and the family segment: $p_H^{**} > p_H^*$, $p_F^{**} > (p_H^* + p_L^*)$.

7. Endogenizing Overage Aversion

In the main model (Section 2 through Section 6), we introduced the parameter k ($0 \leq k \leq 1$) to capture the feature that consumers dislike being in the overage region (overage aversion) because they then need to incur costs of monitoring usage, pay overage fees, and consequently forego overage consumption. In particular, we assumed that consumers begin curtailing their consumption once they reach the limit of their plan's allowance, such that they only use k proportion of the difference between the realization of their requirement and their plan's allowance. In this section, we introduce a new model that endogenizes consumers' overage aversion.

7.1 Model

In this new model, consumers' utility function is given by

$$V(\theta_i, q) = \begin{cases} \theta_i q - p_i, & \text{if } q = \hat{q} \leq q_i, \\ \theta_i q - p_i - p_{oi}(q - q_i) - c_M(q - q_i)^2 - c_F(\hat{q} - q)^2, & \text{if } \hat{q} \geq q > q_i, \end{cases}$$

where $i \in \{H, L\}$, $\theta_H > \theta_L$. \hat{q} is the realization of consumers' requirement, and q is consumers' actual consumption level (see Figure 3 for an illustration). We assume that consumers obtain zero utility from consuming more than their requirement realization. The first line of the utility function presents consumers' utility within a plan's allowance, while the second line presents consumers' utility if they run into overages. Of note, c_M captures consumers' cost associated with monitoring their overage usage and the pain of

paying (Prelec and Loewenstein 1998). c_F denotes consumers' cost of forgone consumption because they fail to meet all their communication requirement \hat{q} .

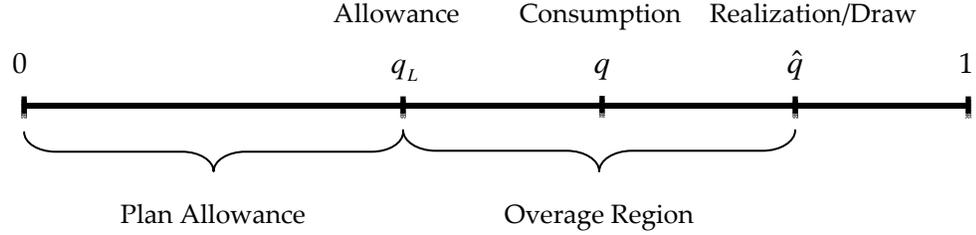


Figure 3: Illustration of a Plan

Once the realization of their requirement exceeds the plan's allowance q_i , consumers will choose a consumption level to maximize their utility:

$Max_q \theta_i q - p_i - p_{O_i}(q - q_i) - c_M (q - q_i)^2 - c_F (\hat{q} - q)^2$, if $\hat{q} \geq q > q_i$. This optimization

leads to the following consumption levels in the overage region:

$$\begin{aligned}
 q_H^* &= \frac{\theta_H + 2c_M q_H + 2c_F \hat{q} - p_{OH}}{2(c_M + c_F)}, \\
 q_L^* &= \frac{\theta_L + 2c_M q_L + 2c_F \hat{q} - p_{OL}}{2(c_M + c_F)}, \\
 q_{HL}^* &= \frac{\theta_H + 2c_M q_H + 2c_F \hat{q} - p_{OL}}{2(c_M + c_F)}.
 \end{aligned} \tag{50}$$

q_i^* , $i \in \{H, L\}$, denotes the optimal consumption level for a type i consumer if she chooses an i plan and runs into the overage region. q_{HL}^* denotes the optimal consumption level for an H-type consumer if she chooses an L plan and runs into the overage region. In other words, q_i^* and q_{HL}^* endogenously characterize consumers'

overage consumption, based on a plan's allowance, price and overage price. The two types of disutility in the overage region have opposite effects on consumers' overage usage. On the one hand, as the cost of monitoring overage usage increases, consumers' overage consumption decreases: $\frac{\partial q_i^*}{\partial c_M} = \frac{p_{O_i} - \theta_i - 2c_F(\hat{q} - q_i)}{2(c_M + c_F)^2} < 0$. On the other hand, as

the cost of forgone consumption increases, consumers' overage consumption may increase: $\frac{\partial q_i^*}{\partial c_F} = \frac{p_{O_i} - \theta_i + 2c_M(\hat{q} - q_i)}{2(c_M + c_F)^2} > 0$ (when p_{O_i} is close to θ_i). Of note, we can

rewrite consumers' optimal consumption in the overage region as follows:

$$q_i^* = \frac{(\theta_i - p_{O_i})}{2(c_M + c_F)} + \frac{c_M}{(c_M + c_F)} q_i + \frac{c_F}{(c_M + c_F)} \hat{q}. \text{ It is easy to see that this optimal}$$

consumption level decreases in the overage price p_{O_i} . Recall that in the main model

with the curtailing parameter k (Section 2), consumers would consume

$$q_i^* = q_i + k(\hat{q} - q_i) = (1 - k)q_i + k\hat{q}. \text{ Comparing the two formulations, we can see that } k \text{ is}$$

a reduced form characterization of the impact of both c_M and c_F on consumers'

decisions in the overage region. k is similar to $\frac{c_F}{(c_M + c_F)}$ in this new formulation. When

c_M increases, k decreases; while the opposite may be true for c_F .

The rest of the new model is the same as the previous setup in Section 2. We assume that there are two consumer segments in the market: a high-valuation segment denoted by H and a low-valuation segment denoted by L. The sizes of the H and L

segments are $(1 - \lambda)$ and λ , respectively. To capture consumers' heterogeneity in terms of their communication requirements as well as their uncertainty about how many minutes they will need in the consumption stage, we assume consumers' consumption needs are stochastic. They come from a known distribution and are independently distributed. In particular, the H segment's consumption requirement is uniformly distributed between $[\Delta, 1 + \Delta]$ ($\Delta \geq 0$), and that of the L segment is uniformly distributed between $[0, 1]$. In other words, the exact number of minutes consumers need is random, and the realization of the quantity needed is generated by two independent distributions, where one distribution $F_H(q)$ stochastically dominates the other $F_L(q)$. Note that H-type consumers need at least Δ minutes, below which they do not obtain sufficient utility to initiate a purchase decision. Δ thus measures the heterogeneity of consumers' usage needs. Ex ante, consumers do not expect to gain additional utility by continuing to use the service once their requirement has been met. Therefore, if the requirement realization, \hat{q} , is within a plan's quantity, consumers will stop consuming at \hat{q} . Both types of consumers gain zero utility beyond the supports of their requirement distributions; in addition, their outside option gives them zero utility.

Now we introduce our model of a family. Families may comprise different types of consumers. To capture heterogeneity within a family in a parsimonious way, we

assume that a family consists of one H-type consumer and one L-type consumer.¹ We also assume that the total number of families in the market is f ($f \geq 0$). It follows that the sizes of the single H- and L-segment are $(1 - \lambda - f)$ and $(\lambda - f)$, respectively. To ensure the existence of single individual consumers (consumers who cannot buy family plans), we assume that $(1 - \lambda) \geq f$ and $\lambda \geq f$. A family's valuation for quantities, θ_F , is

given by $\theta_F = \frac{(1 + 2\Delta)\theta_H + \theta_L}{2(1 + \Delta)}$ (weighted by each member's expected usage, subscript F denotes family). Similar to the individual optimization in the overage region, a family's

optimal consumption in the overage region is given by $q_F^* = \frac{\theta_F + 2c_M q_F + 2c_F \hat{q} - p_{OF}}{2(c_M + c_F)}$.

Consumers are risk-neutral and make purchase decisions based on their expected utilities from different plans. Their expected utility consists of four components: (1) utility prior to reaching a plan's quantity, (2) utility after reaching that quantity, (3) the price of the plan, and (4) the overage payment. Conditional on choosing the plan (q_H, p_H, p_{OH}) , an H-type consumer's expected utility is given as:

$$\begin{aligned}
 EU(\theta_H, q_H, p_H, p_{OH}) &= E(V(\theta_H, q) | q \leq q_H)Pr(q \leq q_H) + E(V(\theta_H, q) | q > q_H)Pr(q > q_H) \\
 &\quad - p_H - p_{OH} E(q - q_H | q > q_H)Pr(q > q_H) \\
 &= \int_{\Delta}^{q_H} v(\theta_H, q) f_H(q) dq + \int_{q_H}^{1+\Delta} v(\theta_H, q_H^*) f_H(q) dq \\
 &\quad - p_H - p_{OH} \int_{q_H}^{1+\Delta} (q_H^* - q_H) f_H(q) dq.
 \end{aligned} \tag{51}$$

¹ The analysis in which a family comprises two H-type or two L-type consumers is available upon request.

The first term in the above expression, $\int_{\Delta}^{q_H} v(\theta_H, q) f_H(q) dq$, is the H-type consumer's expected utility conditional on her requirement realization being smaller than the H plan's quantity q_H . The second term, $\int_{q_H}^{1+\Delta} v(\theta_H, q_H^*) f_H(q) dq$, is her expected utility, conditional on both her requirement realization exceeding the plan's quantity and her optimally adjusting her overage consumption. The next term, p_H , is the plan's price, and the last term, $p_{OH} \int_{q_H}^{1+\Delta} (q_H^* - q_H) f_H(q) dq$, represents the consumer's expected overage payment. Similarly, if an L-type consumer chooses the plan (q_L, p_L, p_{OL}) , her expected utility would be:

$$\begin{aligned}
EU(\theta_L, q_L, p_L, p_{OL}) &= E(V(\theta_L, q) | q \leq q_L) Pr(q \leq q_L) + E(V(\theta_L, q) | q > q_L) Pr(q > q_L) \\
&\quad - p_L - p_{OL} E(q - q_L | q > q_L) Pr(q > q_L) \tag{52} \\
&= \int_0^{q_L} v(\theta_L, q) f_L(q) dq + \int_{q_L}^1 v(\theta_L, q_L^*) f_L(q) dq \\
&\quad - p_L - p_{OL} \int_{q_L}^1 (q_L^* - q_L) f_L(q) dq.
\end{aligned}$$

The game between the service provider and consumers contains two stages. In the first stage, the service provider presents two individual plans, (q_H, p_H, p_{OH}) and (q_L, p_L, p_{OL}) , and consumers self-select the best plans for themselves. In addition, the service provider may decide to offer a family plan, (q_F, p_F, p_{OF}) , in which case, the consumers who comprise this family will compare this joint plan with the two individual plans that they would have chosen individually. Two dimensions of uncertainty exist in this stage. First, the service provider does not know each individual

consumer's type but takes this information asymmetry into account when designing the plans to maximize its expected profit. Second, neither the service provider nor the consumers know exactly how many minutes each individual will need in the next stage; they only know the H and L segments' usage distributions. Consumers choose which plan to purchase based on their expected utilities and pay the plan's price, p_H or p_L , accordingly. When the choice is a family plan, consumers with family members take the entire family's expected utility into account and pay the price p_F . In the second stage, the communication requirement is realized for each consumer. At the end of this stage, those whose requirement realizations are beyond their plans' quantities adjust and optimize their overage consumption and make the overage payment.

For a more interesting analysis, we focus on the situation in which it is more profitable for the service provider to serve both the H and L segments than to serve only the H segment.

7.2 Analysis of Individual Plans

In this subsection, we analyze the case in which the service provider attempts to maximize its profits by offering only two individual plans to consumers. The service provider chooses two sets of quantities, prices, and overage fees in these two individual plans, (q_H, p_H, p_{OH}) and (q_L, p_L, p_{OL}) , to target the high-valuation consumers (the H segment) and the low-valuation consumers (the L segment), respectively. After observing the two three-part tariff plans, consumers self-select the best plans for

themselves. As is well known from the classic literature on self-selection (Mussa and Rosen 1978), the service provider must ensure that the H segment will not choose the L plan, (q_L, p_L, p_{OL}) , targeted at the L segment, and vice versa. In addition, the service provider must ensure that both segments of consumers obtain nonnegative utilities from purchasing the plans targeted at them. Thus the two new elements in our framework are the three-part tariff plans and uncertainty on the requirement for quantities.

The service provider's profit, Π , is given by

$$\Pi = (1-\lambda)[p_H + p_{OH}E(q_H^* - q_H | q > q_H)Pr(q > q_H)] + \lambda[p_L + p_{OL}E(q_L^* - q_L | q > q_L)Pr(q > q_L)].$$

The first term in this profit function, $(1-\lambda)[p_H + p_{OH}E(q_H^* - q_H | q > q_H)Pr(q > q_H)]$,

gives the profit from the H segment. p_H is the price of the H plan, and the term,

$p_{OH}E(q_H^* - q_H | q > q_H)Pr(q > q_H)$, captures the H segment's expected overage payment

after choosing the H plan. Similarly, the second term in the profit function gives the

profit from the L segment, while the term, $p_{OL}E(q_L^* - q_L | q > q_L)Pr(q > q_L)$, captures the

L segment's expected overage payment after choosing the L plan. Formally, the service

provider sets its menu of plans to maximize its expected profit:

$$\begin{aligned} E[\Pi] &= \text{Max} (1-\lambda)[p_H + p_{OH}E(q_H^* - q_H | q > q_H)Pr(q > q_H)] + \lambda[p_L + p_{OL}E(q_L^* - q_L | q > q_L)Pr(q > q_L)] \\ &= (1-\lambda)[p_H + p_{OH} \int_{q_H}^{1+\Delta} (q_H^* - q_H) f_H(q) dq] + \lambda[p_L + p_{OL} \int_{q_L}^1 (q_L^* - q_L) f_L(q) dq], \end{aligned}$$

subject to

$$EU(\theta_H, q_H, p_H, p_{OH}) \geq EU(\theta_H, q_L, p_L, p_{OL}), \quad (53)$$

$$EU(\theta_L, q_L, p_L, p_{OL}) \geq EU(\theta_L, q_H, p_H, p_{OH}), \quad (54)$$

$$EU(\theta_H, q_H, p_H, p_{OH}) \geq 0, \quad (55)$$

$$EU(\theta_L, q_L, p_L, p_{OL}) \geq 0. \quad (56)$$

The procedure of solving this constrained optimization is similar to what is described in Section 4, and thus is not repeated here. In the equilibrium, the IC constraint for the H segment (Constraint (53)) and the IR constraint for the L segment (Constraint (56)) are binding. Solving the firm's constrained optimization problem yields the following optimal menu of individual plans. (We use superscript ^{**} to denote the equilibrium prices, quantities and overage fees when the service provider only offers individual plans.)

The quantities of the H and L plans are:

$$q_H^{**} = 1 + \Delta, \quad q_L^{**} = q_L^{**}(\theta_H, \theta_L, \Delta, \lambda, c_M, c_F). \quad (57)$$

Due to the complexity of this model, the analytical expression of q_L^{**} cannot be obtained.

However, we know that it is the solution to the following first order condition with respect to q_L :

$$\begin{aligned} & 4c_M q_L \theta_H - 4a(p_{OL} \Delta + c_M \Delta(2 - 2q_L + \Delta)) - 4c_M \theta_H - 2p_{OL} \theta_H - 4c_M \Delta \theta_H + \theta_H^2 + 4c_M \theta_L \\ & + 4c_F(p_{OL} \Delta + c_M(1 - q_L + \Delta)^2) \lambda + 2p_{OL} \theta_L - 4c_M q_L \theta_L - \theta_L^2 \\ & + (p_{OL}^2 + \theta_H(4c_M(1 - q_L + \Delta) - \theta_H) + 2p_{OL}(\theta_H - \theta_L)) \lambda = 0. \end{aligned}$$

The prices for the two individual plans, p_H^{**} and p_L^{**} , are functions of q_L^{**} and other parameters: Δ, λ, c_M , and c_F . The two prices can be obtained by plugging q_L^{**} back into the binding IC constraint for the H segment and IR constraint for the L segment.

The overage fees are:

$$p_{OL}^{**} = \frac{(1-\lambda)(c_F\Delta(2-2q_L^{**} + \Delta) + (1-q_L^{**} + \Delta)\theta_H - (1-q_L^{**})\theta_L)}{(1-\lambda)\Delta + (1-q_L^{**})}, p_{OH}^{**} = \theta_H. \quad (58)$$

First, note that the quantity for the H plan, $q_H^{**} = 1 + \Delta$, is the upper bound of the H segment's requirement distribution. Given that no one would consume more than $1 + \Delta$, we can interpret this result, $q_H^{**} = 1 + \Delta$, as that H-type consumers obtain an unlimited plan. By contrast, L-type consumers never obtain their maximum requirement in their L plan, $q_L^{**} < 1$.

Intuitively, consumers are willing to pay a higher fixed price when their plans' included quantities are higher. Therefore, the service provider does not put any restrictions on the quantities of the H plan. This result is consistent with the "no distortion at the top" result from the literature on vertical differentiation. In contrast, the service provider pushes down the quantities of the L plan to ensure that H-type consumers will not switch to this plan. In the equilibrium, the marginal gains from reducing q_L through a higher p_H are equal to the marginal loss through a lower p_L .

To summarize, in this section we have characterized the optimal individual plans and shown how overage aversion affects the terms in these plans. Next we move on to examine the situation when the service provider offers family plans in addition to the individual plans.

7.3 Analysis of Individual and Family Plans

In the previous subsection, we analyzed the situation when the service provider offered two individual plans. In this subsection, we analyze the case when the service provider offers two individual plans, (q_H, p_H, p_{OH}) and (q_L, p_L, p_{OL}) , and a family plan, (q_F, p_F, p_{OF}) . The two individual plans are used to target single high-valuation consumers and single low-valuation consumers, whereas the family plan is used to target consumers who have families. Note that a family plan is jointly purchased and consumed by two family members; single, individual buyers cannot purchase a family plan. In other words, self-selection of family plans goes in one direction, and the service provider can identify family buyers' types when a family plan is sold to them. Therefore, by offering three different plans, the firm utilizes both second-degree and third-degree price discrimination. Recall that a family consists of one H-type consumer and one L-type consumer, and the size of the family segment in the market is f . It therefore follows that the size of the single H segment is $(1 - \lambda - f)$ and the size of the single L segment is $(\lambda - f)$. Furthermore, a family's valuation parameter, θ_F , is given by

$$\theta_F = \frac{(1 + 2\Delta)\theta_H + \theta_L}{2(1 + \Delta)}.$$

Based on Lemma 1 in Section 4.1, a family's requirement distribution is a triangular distribution with the density peak at $1 + \Delta$. Its variance is the same as the sum of variances of the H and L segments' distributions due to independence. Note that both

consumers and the firm know this requirement distribution in the first stage of the game.

Similar to the analysis in Section 7.1, the service provider must make sure that the two single consumer segments will choose the individual plans targeted to them. In addition, in order to induce eligible consumers to purchase the family plan, the service provider needs to set the plan so that the expected joint utility from purchasing it is weakly greater than the sum of expected utilities obtained from two individual plans (which the two family members could have chosen).

The service provider's profit, Π , is given by

$$\Pi = (1 - \lambda - f)[p_H + p_{OH}E(q_H^* - q_H | q > q_H)Pr(q > q_H)] + (\lambda - f)[p_L + p_{OL}E(q_L^* - q_L | q > q_L)Pr(q > q_L)] + f[p_F + p_{OF}E(q_F^* - q_F | q > q_F)Pr(q > q_F)].$$

The first term in this profit function,

$(1 - \lambda - f)[p_H + p_{OH}E(q_H^* - q_H | q > q_H)Pr(q > q_H)]$, gives the expected profit from the single H segment where p_H is the price of the H plan, and the term,

$p_{OH}E(q_H^* - q_H | q > q_H)Pr(q > q_H)$, captures the H segment's expected overage payment after choosing the H plan and optimally adjusting overage usage. Similarly, the second and third terms in the profit function give the profits from the single L segment and the family segment, respectively. The expression within the third term,

$p_{OF}E(q_F^* - q_F | q > q_F)Pr(q > q_F)$, captures the family segment's expected overage

payment after choosing the family plan and optimally adjusting their overage usage.

Formally, the service provider chooses three plans to maximize its expected profit:

$$E[\Pi] = \text{Max} (1-\lambda-f)[p_H + p_{OH}E(q_H^* - q_H | q > q_H)Pr(q > q_H)] + (\lambda-f)[p_L + p_{OL}E(q_L^* - q_L | q > q_L)Pr(q > q_L)] + f[p_F + p_{OF}E(q_F^* - q_F | q > q_F)Pr(q > q_F)],$$

subject to

$$EU(\theta_H, q_H, p_H, p_{OH}) \geq EU(\theta_H, q_L, p_L, p_{OL}), \quad (59)$$

$$EU(\theta_L, q_L, p_L, p_{OL}) \geq EU(\theta_L, q_H, p_H, p_{OH}), \quad (60)$$

$$EU(\theta_F, q_F, p_F, p_{OF}) \geq EU(\theta_H, q_H, p_H, p_{OH}) + EU(\theta_L, q_L, p_L, p_{OL}), \quad (61)$$

$$EU(\theta_H, q_H, p_H, p_{OH}) \geq 0, \quad (62)$$

$$EU(\theta_L, q_L, p_L, p_{OL}) \geq 0, \quad (63)$$

$$EU(\theta_F, q_F, p_F, p_{OF}) \geq 0. \quad (64)$$

Introduction of a family plan brings two new constraints, Constraints (61) and (64), into the service provider's optimization problem. The left-hand side of Constraints (61) and (64), $EU(\theta_F, q_F, p_F, p_{OF})$, is a family's joint expected utility of purchasing the family plan. This family pays the price p_F , gets a total shared quantity of q_F , and faces the overage charge per unit p_{OF} if the sum of two family members' consumption exceeds the plan's quantity q_F . By contrast, the right-hand side of Constraint (61), $EU(\theta_H, q_H, p_H, p_{OH}) + EU(\theta_L, q_L, p_L, p_{OL})$, gives the sum of expected utilities for an H-type consumer who is buying the H plan and an L-type consumer who is buying the L plan.

Constraints (59) and (60) imply that both the H and L segments voluntarily choose the plan directed to them (IC constraints). By contrast, Constraint (61) implies that families will buy the family plan instead of two individual plans. Constraints (62) through (64) ensure, respectively, that the H, L, and family segments will buy the plan intended for them rather than not buy anything at all.

When given the option of choosing a family plan, a single H-type consumer's best outside option is still the L plan, (q_L, p_L, p_{OL}) , whereas a single L-type consumer's best outside option is no consumption. A family's best outside option is for each member to separately choose his or her best individual plan: (q_H, p_H, p_{OH}) for the H-type consumer and (q_L, p_L, p_{OL}) for the L-type consumer. In other words, the binding constraints are the incentive compatibility constraints for the H segment (Constraint (59)) and the family segment (Constraint (61)), as well as the individual rationality constraint for the L segment, Constraint (63).

By solving the firm's constrained optimization problem, we obtain the following optimal menu of plans. (We use ^{***} to denote the equilibrium prices, quantities and overage fees in the presence of family plans.)

The quantities for the three plans are:

$$\begin{aligned} q_H^{***} &= 1 + \Delta, q_F^{***} = 2 + \Delta, \\ q_L^{***} &= q_L^{***}(\theta_H, \theta_L, \Delta, \lambda, f, c_M, c_F). \end{aligned} \tag{65}$$

Due to the complexity of this model, the analytical expression of q_L^{***} cannot be obtained.

However, we know that it is the solution to the following first order condition with respect to q_L :

$$4c_M q_L \theta_H - 4a \left(p_{OL} \Delta + c_M \left(f(1 - q_L)^2 + \Delta(2 - 2q_L + \Delta) \right) \right) - 4c_M \theta_H - 2p_{OL} \theta_H - 4c_M \Delta \theta_H + \theta_H^2 + 4c_M \theta_L + 4c_F \left(p_{OL} \Delta + c_M (1 - q_L + \Delta)^2 \right) \lambda + 2p_{OL} \theta_L - 4c_M q_L \theta_L - \theta_L^2 + f \left(\theta_L (\theta_L - 4c_M (1 - q_L)) - p_{OL}^2 \right) + \left(p_{OL}^2 + \theta_H (4c_M (1 - q_L + \Delta) - \theta_H) + 2p_{OL} (\theta_H - \theta_L) \right) \lambda = 0.$$

The prices for all three plans, p_H^{***} , p_L^{***} , and p_F^{***} , are functions of q_L^{***} and other parameters: Δ, λ, f, c_M , and c_F . The two prices can be obtained by plugging q_L^{***} back into the binding IC constraint for the H segment and IR constraint for the L segment.

The overage fees are:

$$p_{OL}^{***} = \frac{(1 - \lambda)(c_F \Delta(2 - 2q_L^{***} + \Delta) + (1 - q_L^{***} + \Delta)\theta_H - (1 - q_L^{***})\theta_L)}{(1 - \lambda)\Delta + (1 - f)(1 - q_L^{***})}, p_{OH}^{***} = \theta_H, p_{OF}^{***} = \theta_F. \quad (66)$$

First, note that both the H plan and the family plan obtain their maximum requirements, $1 + \Delta$ and $2 + \Delta$. It can be interpreted that the single H segment and the family segment both obtain unlimited plans. Similar to the case without family plans in Section 7.1, this result is consistent with the “no distortion at the top” result from the literature on vertical differentiation. By contrast, single L-type consumers still never obtain their maximum requirement in their L plan, $q_L^{***} < 1$ (see appendix for the proof). This result means that the service provider pushes down the quantities of the L plan to ensure that H-type consumers will not switch to this plan.

Next, we compare the two individual plans in the presence of the family plan with the two individual plans in the absence of the family plan (as in Section 7.1). This comparison provides insights into how the introduction of the family plan changes the terms of the individual three-part tariffs.

Proposition 17. *The changes in two individual plans after the introduction of the family plan are as follows:*

(i) *Both the L plan's quantity, q_L^{***} , and its price, p_L^{***} , are lower, i.e., $q_L^{***} < q_L^{**}$ and $p_L^{***} < p_L^{**}$.*

(ii) *The H plan's quantity, q_H^{***} , stays the same, but its price, p_H^{***} , is higher, i.e., $q_H^{***} = q_H^{**}$ and $p_H^{***} > p_H^{**}$.*

Recall that when designing the product line for two individual plans, the service provider must account for the potential cannibalization problem. In particular, the firm must ensure that the high valuation H segment purchases the more profitable H plan by obtaining at least a utility of $EU(\theta_H, q_L, p_L, p_{OL})$. In other words, the firm must balance reducing the attractiveness of the L plan, so that the H segment will not switch and thus can be further exploited, with a lowered revenue stream from the L segment due to a reduction in the L plan's quantity, q_L . With the introduction of a family plan, the size of the L plan buyers decreases, i.e., $\lambda - f < \lambda$ for $f > 0$, which means that the single L segment's importance also decreases. As a result, the firm can reduce the quantity of the individual L plan q_L^{***} to make this outside option less appealing to the single H segment. Once the quantity q_L^{***} is reduced, however, the price for the L plan p_L^{***} must

also be decreased to account for a lower willingness to pay from the single L-type consumers.

Given that the marginal cost of providing the service is zero, the firm always offers its maximum requirements for communication for the H segment and the family segment, respectively. Again, this result is consistent with the “no distortion at the top” result from the literature on vertical differentiation. The intuition behind this result is that consumers’ marginal utility is lower in the overage region (they experience two types of disutility from their overage consumption, $-c_M(q - q_i)^2$ and $-c_F(\hat{q} - q)^2$), so that the service provider has incentives to keep increasing H plan’s quantity to capture a higher surplus from the H segment. This reason leads to $q_H^{***} = 1 + \Delta = q_H^{**}$, the same as H plan’s quantity without family plans.

By bundling some H-type consumers with some L-type consumers in the family plans, the service provider is able to better extract surplus from the single H segment by charging these consumers a higher price, $p_H^{***} > p_H^{**}$. In other words, the H segment with families poses some negative externalities to the single H segment. Recall that the firm utilizes the family plan to effectively reduce the number of the single L plan’s buyers and thus their importance as well. Then the firm can lower the L plan’s quantity to make it less attractive to the single H-type consumers as their best outside option. As a result, the firm can adjust its H plan to better exploit the single H-type buyers. Because both the quantity, $q_H^{***} = 1 + \Delta$, and the overage fee, $p_{OH}^{***} = \theta_H$, in the H plan have attained their

maximal values, the only instrument left to change in the three-part tariff is the price, p_H^{***} . The service provider thereby raises this price to better price-discriminate the single H segment, which leads to $p_H^{***} > p_H^{**}$.

Recall that f is the total number of families in the market. Next, we present the impact of f on the quantities and prices of the three plans.

Proposition 18. *The size of the family segment, f , affects the optimal plans in the following way:*

(i) *Under Condition A, the quantity of the L plan, q_L^{***} , decreases in f .² The quantities of the H and the family plans, q_H^{***} and q_F^{***} , are independent of f .*

(ii) *Under Condition A and when $\theta_H < \widetilde{\theta}_H$, the price of the L plan, p_L^{***} , decreases in f . Under Condition A, the price of the H plan, p_H^{***} , increases in f .*

The intuition is similar to the one discussed in Proposition 17, and is not repeated here.

7.4 Profitability of Family Plans

Proposition 17 has shown the opposing impact of the family plan on the prices of the L plan and the H plan. Two natural questions concern whether introducing the family plan would be more profitable overall and when this strategy emerges in the equilibrium. We compare the service provider's profits with and without the family

² Condition A is tediously complicated without providing intuition, so it is presented in the Appendix.

plan, $E\Pi^{***}$ and $E\Pi^{**}$, and provide answers to these questions in the proposition below.

Proposition 19. *When $\theta_H > \widehat{\theta}_H$, it is more profitable for the service provider to offer a family plan and two individual plans, i.e., $E\Pi^{***} > E\Pi^{**}$.³ Otherwise, only offering two individual plans is more profitable.*

This proposition validates the profitability of offering family plans as well as offering regular individual plans. When the above condition holds, offering family plans is the equilibrium strategy. Next, we discuss the intuition for why offering a family plan in addition to two individual plans can be more profitable.

Given that the introduction of the family plan does not expand the market (full market coverage in the absence of a family plan), the profit boost it brings can be partially seen from the price comparison below:

$$p_F^{***} - p_H^{***} - p_L^{***} > 0. \quad (67)$$

This inequality states that a family of two members contributes more profits to the firm than two buyers of an individual H and L plan. We discuss the intuition by analyzing the binding incentive compatibility constraints. Recall that as the single L segment always gets zero surplus, $EU(\theta_L, q_L, p_L, p_{OL}) = 0$, the IC constraint for the family plan buyers can be simplified as:

³

$$\widehat{\theta}_H = \frac{3(1-q_L^{***})\theta_L(1-\lambda) - 3c_F(2-2q_L^{***} + \Delta)\Delta(1-\lambda) + \sqrt{9\theta_L(\theta_L - 2c_M(1-q_L^{***})) - 12c_Fc_M(1-q_L^{***})^2}(\Delta + \lambda - f(1-q_L^{***}) - (q_L^{***} + \Delta)\lambda)}{3(1+\Delta-q_L^{***})(1-\lambda)}.$$

$$EU(\theta_F, q_F, p_F, p_{OF}) \geq EU(\theta_H, q_H, p_H, p_{OH}) + EU(\theta_L, q_L, p_L, p_{OL}) = EU(\theta_H, q_H, p_H, p_{OH}).$$

Note that the last term, $EU(\theta_H, q_H, p_H, p_{OH})$, is equal to $EU(\theta_H, q_L, p_L, p_{OL})$ according to the binding IC constraint of the single H segment. Based on the discussion in Proposition 17, the expected utility for the H-type consumers to choose the L plan is lower in the presence of the family plan. Therefore, it is not too costly to induce family buyers to purchase the family plan.

One important way the family plan helps the firm is through better price discrimination against the L-type consumers in a family compared to an individual L plan. In particular, by offering unlimited quantities, the service provider can charge a high price p_F^{***} to extract more surpluses from the L-type consumers who are family-plan buyers than from the single L-type consumers. The firm does so without worrying about the potential cannibalization problem due to the nature of the third-degree price discrimination. By contrast, with the second-degree price discrimination, the firm always faces the trade-off between a higher quantity for the L plan, which generates higher revenues from single L-type buyers, and the resulting lower price for the H plan, which is used to prevent H-type buyers from switching plans. Furthermore, although the price for the individual L plan decreases in the presence of a family plan, $p_L^{***} < p_L^{**}$, the firm obtains a higher overage payment from these individual L-type buyers because of a reduced allowance, q_L^{***} . To summarize: the introduction of the family plan

increases revenues from the single H segment ($p_H^{***} > p_H^{**}$, Proposition 17) and both types of consumers within a family ($p_F^{***} - p_H^{***} - p_L^{***} > 0$). The loss in the single L segment is not significant due to a reduction of its effective segment size ($\lambda - f$). Therefore, the profitability with the family plan is higher overall compared to when the firm only offers two individual plans.

One interesting question in the context of a three-part tariff concerns the likelihood of running into the overage region; in particular, whether a family plan will decrease the probability of overage compared to that of two individual plans. We answer this question in the proposition below.

Proposition 20. *In equilibrium, the probability of running into overages is zero in a family plan. The probability for single L-type consumers to run into overages is positive.*

Recall that only single L-type consumers will face overage usage, because the firm offers maximum quantities for both the H segment and the family segment. In other words, the L-type consumers in family plans no longer incur overage charges such as those incurred by single L-type consumers. Therefore, no overage usage is possible in the H and the family plans, which leads to the result in Proposition 20.

8. Discussion and Conclusion

In this paper we modeled a service provider's strategy of bundling consumers through family plans by using non-linear pricing, with a focus on the commonly observed three-part tariff contracts. Our framework captured two key features of telecommunication services: stochastic requirement for communication and general aversion to overage consumption. We showed that heterogeneity in consumers' needs and valuations, and the extent of overage-aversion all played important roles in determining the service provider's equilibrium strategy.

Our model contains two stages of the game between the service provider and the consumers. In the first stage, consumers know their valuation and their independent requirement distributions. They expect to stop using the service once their need for communication has been satisfied; i.e., when they have consumed the realizations of their requirements' distributions. They make the purchase decision based on their expected utilities during the second stage. The service provider also knows consumers' requirement distributions but cannot directly identify each consumer's type. Therefore, it must make the entire menu of plans available to all consumers for them to choose from.

In the second stage of the game, a random draw determines how many quantities each individual needs. If the requirement realization exceeds the chosen plan's quantity, consumers will start curtailing their overage consumption by a

proportion k . If the requirement realization falls below the chosen plan's quantity, consumers will stop using the service once their requirements are met. We derived the service provider's optimal product-line strategies and contrasted the three-part tariffs for individual plans both in the presence and in the absence of a family plan.

We find that using a family plan to bundle consumers is more profitable than only offering individual plans when consumers' valuation heterogeneity is moderate and their requirement heterogeneity is relatively low. By contrast, offering only individual plans is more profitable when consumers' requirement heterogeneity is high and when consumers do not curtail their overage consumption at all. In the latter case, the firm offers a pay-as-you-go plan to the low-valuation segment. Interestingly, the service provider may offer the maximum requirement quantities for the individual plan for the high-valuation segment as well as the family plan. This result can be interpreted as unlimited plans for the single high-valuation segment and for the family segment. Note that this finding echoes the fact that as operating costs continue to decrease, more wireless carriers (e.g., Verizon and T-Mobile) are starting to offer unlimited voice plans to all their customers.

We also find that the introduction of a family plan alters the terms of the two individual plans, and note that, compared to the case without a family plan, the low-valuation segment's plan quantity and price are lower. By contrast, the high-valuation segment's plan quantity stays the same but its price is higher. These findings reveal the

function of the family plan in helping the service provider better exploit the high-valuation segment's surplus.

Our model also shows that the overage probability from a family plan is lower than the sum of the overage probabilities from two individual plans. Note that this result is based on independent consumer requirement distributions. When family consumers' requirements are perfectly correlated, we have shown that this result will continue to hold.

Appendix A: Serving the H Segment Only

In the main body of this paper, we focused on the situation in which it is more profitable for the service provider to cover the entire market. In this section, we analyze the case in which the firm only serves the H segment. This case would be the equilibrium outcome when consumers' valuation heterogeneity is sufficiently large (i.e., θ_H is significantly larger than θ_L) or the size of L segment λ is sufficiently small. Then we present conditions under which each equilibrium outcome occurs. Throughout this section, we use $\tilde{\cdot}$ to denote the case in which only the H segment is served. We use * to denote the optimal plan when the firm only offers an H plan, and ** to denote the optimal plans when the firm offers an H plan and a family plan.

A.1. Individual H Plan Only

In this subsection, we analyze the case in which the service provider only offers an individual plan for the H segment. In this situation, the only outside option for an H-type consumer is no consumption, which gives her zero utility. The firm chooses

(q_H, p_H, p_{OH}) to maximize its expected profit:

$$E\Pi = \text{Max} (1 - \lambda)[p_H + p_{OH} E(q - q_H | q > q_H, k) \text{Pr}(q > q_H)],$$

subject to

$$EU(\theta_H, q_H, p_H, p_{OH}) = E(V(\theta_H, q) | q \leq q_H) \text{Pr}(q \leq q_H) + E(V(\theta_H, q) | q > q_H, k) \text{Pr}(q > q_H) - p_H - p_{OH} E(q - q_H | q > q_H, k) \text{Pr}(q > q_H) \geq 0.$$

The above constraint is the H segment's individual rationality constraint, which ensures that an H-type consumer obtains a nonnegative utility from purchasing this H plan. Solving the firm's constrained optimization problem, we obtain the following optimal plan for the high valuation H segment:

$$\begin{aligned}\widetilde{q}_H^* &= 1 + \Delta, \\ \widetilde{p}_H^* &= \frac{(1 + 2\Delta)\theta_H}{2}, \\ \widetilde{p}_{OH}^* &= \theta_H.\end{aligned}$$

Plugging the optimal prices and quantity back into the profit function, the service provider's expected profit is: $\widetilde{E\Pi}^* = \frac{1}{2}(1 - \lambda)(1 + 2\Delta)\theta_H$.

A.2. Individual H Plan and a Family Plan

In the previous subsection, we analyzed the situation in which the service provider only offers an individual plan for the H segment. In this subsection, we consider the case when the service provider offers an individual H plan and a family plan. Recall that the only outside option for a single H-type consumer is no consumption, which leads to zero utility. In order to induce eligible consumers to purchase the family plan, the firm needs to ensure that the joint utility from purchasing this family plan is weakly greater than the utility an H-type consumer can obtain from the individual H plan. Formally, the firm chooses (q_H, p_H, p_{OH}) and (q_F, p_F, p_{OF}) to maximize its expected profit:

$$E\Pi = \text{Max} (1 - \lambda - f)[p_H + p_{OH}E(q - q_H | q > q_H, k)Pr(q > q_H)] + f[p_F + p_{OF}E(q - q_F | q > q_F, k)Pr(q > q_F)],$$

subject to

$$EU(\theta_H, q_H, p_H, p_{OH}) \geq 0,$$

$$EU(\theta_F, q_F, p_F, p_{OF}) \geq EU(\theta_H, q_H, p_H, p_{OH}).$$

The first inequality (IR constraint) ensures that the single H segment purchases the H plan. The second inequality (IC constraint) ensures that a family's joint utility from a family plan is weakly greater than the utility from an individual H plan. The solutions to the firm's constrained optimization problem lead to the following optimal:

$$\begin{aligned} \widetilde{q}_H^{**} &= 1 + \Delta, & \widetilde{q}_F^{**} &= 2 + \Delta, \\ \widetilde{p}_H^{**} &= \frac{(1 + 2\Delta)\theta_H}{2}, & \widetilde{p}_F^{**} &= \frac{(1 + \Delta)(\theta_H + \theta_L)}{2}. \end{aligned}$$

In addition, the overage fees are $\widetilde{p}_{OH}^{**} = \theta_H$, $\widetilde{p}_{OF}^{**} = \frac{(\theta_H + \theta_L)}{2}$.

The first thing to notice is that the H plan is identical to the H plan in the absence of the family plan: $(\widetilde{q}_H^{**}, \widetilde{p}_H^{**}, \widetilde{p}_{OH}^{**}) = (\widetilde{q}_H^*, \widetilde{p}_H^*, \widetilde{p}_{OH}^*)$ (Refer to Section A.1.). This equation means that the addition of a family plan does not impact the contract offered to the single H-type consumers, because the single H segment still faces the same outside option. Plugging the optimal prices and quantities back into the binding IC and IR constraints, we obtain the expected profit function, $\widetilde{E\Pi}^{**}$, given by:

$$\widetilde{E\Pi}^{**} = \frac{1}{2}(f(1 + \Delta)\theta_L + (1 - \lambda + (2 - f - 2\lambda)\Delta)\theta_H).$$

One interesting question is whether the family-plan buyers obtain any quantity discounts. The next proposition answers this question.

Proposition A1. *The per unit price in the family plan is lower than the per unit price in the individual plan, i.e., $\frac{P_F^{**}}{q_F} < \frac{P_H^{**}}{q_H}$.*

Excluding the consideration of marginal costs, the service provider does not place any restriction on the included quantities on both plans. The intuition is that the benefit of increased revenues with certainty through prices overtakes the benefit of the probabilistic overage fees (after curtailing). The firm offers a quantity discount to the family-plan users because of its inclusion of the low-valuation family members.

Conversely, a family plan does extract the entire expected surplus from both family members. The next question is whether this family plan with quantity discounts can help the firm earn a higher profit. We answer this question in the following proposition.

Proposition A2. *When $(1 + \Delta)\theta_L > \Delta\theta_H$, offering both an H plan and a family plan is more profitable than offering an H plan only.*

Recall that throughout this section, we assume that the equilibrium outcome in the absence of a family plan is to offer the H plan only, because this strategy generates a higher profit than offering both the H and L plans. Intuitively, the condition for this equilibrium is a relatively small valuation θ_L and segment size λ . However, this proposition states that as long as θ_L surpasses a threshold (i.e., is not too low), the firm is better off by serving some of the L segment through a family plan. The reason is that the introduction of a family plan does not create a cannibalization problem for the firm, because the outside option for the single H-type consumers stays the same. This is the

advantage of the third-degree price discrimination, with which the firm is able to directly distinguish different consumer segments and thereby avoid cannibalization.

However, because the family plan's price $\widetilde{p}_F^{**} = \frac{(1+\Delta)(\theta_H + \theta_L)}{2}$ decreases in θ_L , when

$\theta_L < \frac{\Delta\theta_H}{(1+\Delta)}$, H-type consumers with families get "too much" quantity discount.

Therefore, when θ_L is below this threshold, the firm is better off by only offering an individual H plan.

A.3. Equilibrium Conditions

In the main body of this paper, we assumed that the service provider offering two individual plans, instead of the H plan only, is the equilibrium. We have established the expected profit in that case,

$$E\Pi^* = \frac{\theta_L(\theta_L(1-k(1-\Delta^2(1-\Delta)(1-k\lambda))) - \theta_H(1-k)(1-\lambda)(1-\Delta^2(1-k\lambda)))}{2(1-k)(\theta_L - (1-\lambda)\theta_H)}. \text{ A comparison}$$

of the expected profits in two cases shows that offering two individual plans is the

equilibrium when the following inequality holds: $E\Pi^* \geq \widetilde{E\Pi}^*$. This inequality translates

to:

$$\begin{aligned} & \left(1-k\left(1-\Delta^2(1-\lambda)(1-k\lambda)\right)\right)\theta_L^2 - [(1-k)(1+2\Delta)\theta_H(1-\lambda) + (1-k)\theta_H(1-\lambda) \cdot \\ & (1-\Delta^2(1-k\lambda))]\theta_L + (1-k)(1+2\Delta)\theta_H^2(1-\lambda)^2 \geq 0. \end{aligned}$$

Appendix B: Proof of Propositions

Proof of Proposition 1. From $\frac{\partial q_L^*}{\partial k} = -\frac{\Delta\theta_L(1-\lambda)}{(1-k)^2(\theta_L - \theta_H(1-\lambda))} < 0$ and

$$\frac{\partial p_L^*}{\partial k} = -\frac{\Delta^2\theta_L^2(k\theta_L + (1-k)\theta_H)(1-\lambda)^2}{(1-k)^3(\theta_L - \theta_H(1-\lambda))^2} < 0, \text{ we have shown the first part of Proposition 1.}$$

$$\text{From } \text{sign}\left[\frac{\partial p_H^*}{\partial k}\right] = \theta_L^2[\theta_L(1 - (2-k)k\lambda^2) - \theta_H(1 - 2\lambda + 2\lambda^2 - (2-k)k\lambda^2)], \text{ we}$$

have shown the second part of Proposition 1. Note that this comparative statics is positive when $\theta_L = \theta_H$.

Proof of Proposition 2. From $\frac{p_H^*}{q_H^*} - \frac{p_L^*}{q_L^*} = -\frac{\Delta(1-k\lambda)\theta_L^2[\theta_L(1-k\Delta\lambda) - \theta_H(1-\lambda + (1-k)\Delta\lambda)]}{2(1-k)(1+\Delta)(\theta_L - \theta_H(1-\lambda))^2}$,

we can see that the difference in average prices within quantities between the H and L plans is negative when the condition $\theta_L(1-k\Delta\lambda) - \theta_H(1-\lambda + (1-k)\Delta\lambda) > 0$ is satisfied.

Proof of Proposition 3. Comparative statics of q_L^{**} and p_L^{**} with respect to f are given as:

$$\frac{\partial q_L^{**}}{\partial f} = -\frac{\Delta(k\theta_L + (1-k)\theta_H)\theta_L(1-\lambda)}{(1-k)((1-f)\theta_L - (1-\lambda)\theta_H)^2} < 0,$$

$$\frac{\partial p_L^{**}}{\partial f} = -\frac{\Delta^2\theta_L^2(\theta_H - k\theta_H + k\theta_L)^2(1-\lambda)^2}{(1-k)^2((1-f)\theta_L - (1-\lambda)\theta_H)^3} < 0.$$

Below we show the comparative statics of p_H^{**} and p_F^{**} with respect to f :

$$\frac{\partial p_H^{**}}{\partial f} = \frac{\partial p_F^{**}}{\partial f} = \frac{\Delta^2\theta_L^2(\theta_H - k\theta_H + k\theta_L)^2(1-\lambda)(\lambda - f)}{(1-k)((1-f)\theta_L - (1-\lambda)\theta_H)^3} > 0.$$

Proof of Proposition 4. The per unit price comparison between the H and L plans is given as:

$$\frac{p_H^{**}}{q_H^{**}} - \frac{p_L^{**}}{q_L^{**}} = -\frac{\Delta\theta_L^2(1-f(1-k)-k\lambda)(\theta_L(1-f(1-k\Delta)-k\Delta\lambda)-\theta_H(1-\lambda+(1-k)\Delta(\lambda-f)))}{2(1-k)(1+\Delta)(\theta_L(1-f)-\theta_H(1-\lambda))^2} < 0,$$

when f is sufficiently small. Note that because $(1-f(1-k)-k\lambda) > 0$ always holds (recall $f < \lambda$), when $\theta_L(1-f(1-k\Delta)-k\Delta\lambda)-\theta_H(1-\lambda+(1-k)\Delta(\lambda-f)) > 0$, the above inequality holds.

At the same time, however,

$$\frac{p_F^{**}}{q_F^{**}} - \frac{p_H^{**}}{q_H^{**}} = -\frac{\Delta\left((1-k)(1+\Delta)(\theta_H-\theta_L)(\theta_L(1-f)-\theta_H(1-\lambda))^2 - (1-k)\theta_L(\theta_L(1-f)-\theta_H(1-\lambda))^2\right)}{2(1+\Delta)(1-k)(2+\Delta)(\theta_L(1-f)-\theta_H(1-\lambda))^2} \\ - \frac{\Delta^2\theta_L(\theta_H-k\theta_H+k\theta_L)\left(\theta_L\left(1+f^2(1-k)-k\lambda^2-2f(1-k\lambda)\right)-\theta_H(1-\lambda)^2\right)}{2(1+\Delta)(1-k)(2+\Delta)(\theta_L(1-f)+\theta_H(1-\lambda))^2} < 0,$$

when f is sufficiently small. Both inequalities can be seen by replacing f with 0 and utilize the continuity of these functions. Note that the conditions on f are in line with the assumption that θ_L is not too low from θ_H so it is more profitable to cover the entire market.

Proof of Proposition 5. Note that when the size of the family segment, f , is zero, the optimal individual plans with or without the family plan are the same:

$(q_H^{**}, p_H^{**}) = (q_H^*, p_H^*), (q_L^{**}, p_L^{**}) = (q_L^*, p_L^*)$. As a result, this proposition can be shown by

noting the following comparative statics: $\frac{\partial q_L^{**}}{\partial f} < 0, \frac{\partial p_L^{**}}{\partial f} < 0$, and $\frac{\partial q_H^{**}}{\partial f} = 0, \frac{\partial p_H^{**}}{\partial f} > 0$ (the

proof of Proposition 3).

Proof of Proposition 6. This proposition can be easily shown by comparing the range in

which $\frac{p_H^{**}}{q_H} < \frac{p_L^{**}}{q_L}$ with the range in which $\frac{p_H^*}{q_H} < \frac{p_L^*}{q_L}$.

Proof of Proposition 7 and the Comparative Statics After Proposition 7. From the comparative statics below, we have shown Proposition 7:

$$\frac{\partial E\Pi^{**}}{\partial f} = \frac{(1-\lambda)\Delta\left(\Delta\theta_L((1-k)\theta_H + k\theta_L)^2(1-\lambda)^2 - (1-k)(\theta_H - \theta_L)(\theta_L(1-f) - \theta_H(1-\lambda))^2\right)}{2(1-k)(\theta_L(1-f) - \theta_H(1-\lambda))^2}.$$

$$p_F^{**} - p_H^* - p_L^* = \frac{1}{2}\Delta\left\{\theta_L - \theta_H + \frac{\Delta\theta_L(\theta_H - k\theta_H + k\theta_L)^2(1-\lambda)^2(\theta_H^2(1-\lambda)^3 + \theta_H\theta_L(1-\lambda))}{(1-k)^2(\theta_L - \theta_H(1-\lambda))^2(\theta_L(1-f) - \theta_H(1-\lambda))^2}\right\} \text{ can}$$

$$\left(f(2+f-fk-2(2-k)\lambda) - 2(1-\lambda)\right) + \theta_L^2(1-\lambda-f(2-fk-(2-f)(2-k)\lambda)) \Big\} > 0$$

be shown by setting f to be zero and using the continuity of the price difference as a function of f :

$$p_F^{**} - p_H^* - p_L^* = \frac{1}{2}\Delta\left(\theta_L - \theta_H + \frac{\Delta\theta_L(\theta_H - k\theta_H + k\theta_L)^2(1-\lambda)^2}{(1-k)^2(\theta_L - \theta_H(1-\lambda))^2}\right) > 0, \text{ when } f = 0.$$

This means that when the size of the family segment is below a threshold, the price difference is positive.

$$\frac{\partial p_F^{**}}{\partial f} = \frac{\Delta^2\theta_L^2(\theta_H - k\theta_H + k\theta_L)^2(1-\lambda)(\lambda-f)}{(1-k)((1-f)\theta_L - (1-\lambda)\theta_H)^3} > 0.$$

$$\frac{\partial p_F^{**}}{\partial k} = \frac{\Delta^2 \theta_L^2 \left(\theta_L \left(1 + f^2 (1-k)^2 - (2-k)k\lambda^2 - 2f(1-(2-k)k\lambda) \right) \right)}{2(1-k)^2 (\theta_L(1-f) + \theta_H(1-\lambda))^2} \quad \text{for a sufficiently}$$

$$-\frac{\Delta^2 \theta_L^2 [\theta_H \left(1 + f^2 (1-k)^2 - 2f(1-k)^2 \lambda - \lambda(2-(2-(2-k)k)\lambda) \right)]}{2(1-k)^2 (\theta_L(1-f) + \theta_H(1-\lambda))^2} > 0$$

large θ_L . This inequality can be seen by replacing θ_L with θ_H and using the continuity of the comparative statics as a function of θ_L .

Proof of Proposition 13 and Relevant Comparative Statics. We prove this proposition by showing that the comparative statics of the profits with respect to the size of the family segment, f , is positive (recall that $f = 0$ corresponds to the case in which only two

individual plans are offered). From $\frac{\partial E\Pi^{**}}{\partial f} = \frac{\Delta^2 \theta_L (\theta_H - k\theta_H + k\theta_L)^2 (1-\lambda)^2 \lambda}{2(1-k)((1-2f)\theta_L - \theta_H(1-\lambda))^2} > 0$, we

have shown this proposition.

Regarding the comparative statics, first, we show comparative statics of q_L^{**} and p_L^{**} with respect to f .

$$\frac{\partial q_L^{**}}{\partial f} = -\frac{2\Delta\theta_L(\theta_H - k\theta_H + k\theta_L)(1-\lambda)}{(1-k)((1-2f)\theta_L - \theta_H(1-\lambda))^2} < 0,$$

$$\frac{\partial p_L^{**}}{\partial f} = -\frac{2\Delta^2\theta_L^2(\theta_H - k\theta_H + k\theta_L)^2(1-\lambda)^2}{(1-k)^2((1-2f)\theta_L - \theta_H(1-\lambda))^3} < 0.$$

Second, we show the comparative statics of p_H^{**} with respect to f .

$$\frac{\partial p_H^{**}}{\partial f} = \frac{2(\lambda - 2f)\Delta^2\theta_L^2(\theta_H - k\theta_H + k\theta_L)^2(1-\lambda)}{(1-k)((1-2f)\theta_L - \theta_H(1-\lambda))^3} > 0.$$

Third, we show that $p_F^{**} \geq 2p_L^*$.

$$p_F^{**} - 2p_L^* = \frac{\Delta^2 \theta_L (\theta_H - k\theta_H + k\theta_L)^2 (1-\lambda)^2}{(1-k)^2 (\theta_L - \theta_H (1-\lambda))^2} > 0.$$

Proof of L Plan's Quantity.

First, we show that L plan's allowance is strictly less than 1 when the firm only offers two individual plans: $q_L^{**} < 1$. Second, we show that L plan's allowance is still strictly less than 1 when the firm offers two individual plans and a family plan: $q_L^{***} < 1$.

We prove both claims by contradiction.

When the firm only offers two individual plans, the optimal p_{OL}^{**} is given by (based on the first order condition with respect to p_{OL})

$$p_{OL}^{**} = \frac{(a\Delta(2 - 2q_L^{**} + \Delta) + (1 - q_L^{**} + \Delta)\theta_H - (1 - q_L^{**})\theta_L)(1 - \lambda)}{\lambda(1 - q_L^{**}) + (1 - \lambda)\Delta}.$$

Now suppose $q_L^{**} = 1$ in the

equilibrium (note the constraint on $q_L : 0 \leq q_L \leq 1$). Plugging $q_L^{**} = 1$ into p_{OL}^{**} , we obtain

$p_{OL}^{**} = \theta_H + c_F \Delta > \theta_L$. This is a contradiction because $p_{OL} \leq \theta_L$, where θ_L is the maximal marginal utility per unit for consumers.

When the firm offers both individual and family plans, the optimal p_{OL}^{***} is given by

$$p_{OL}^{***} = \frac{(a\Delta(2 - 2q_L^{***} + \Delta) + (1 - q_L^{***} + \Delta)\theta_H - (1 - q_L^{***})\theta_L)(1 - \lambda)}{(\lambda - f)(1 - q_L^{***}) + (1 - \lambda)\Delta}.$$

Now suppose $q_L^{***} = 1$ in

the equilibrium. Plugging $q_L^{***} = 1$ into p_{OL}^{***} , we obtain $p_{OL}^{***} = \theta_H + c_F \Delta > \theta_L$. This is a contradiction because $p_{OL} \leq \theta_L$.

Therefore, we have shown that regardless of whether the firm offers family plans, L plan's allowance is strictly less than 1.

Proof of Proposition 17.

We show that L plan's allowance, q_L , increases after the firm introduces family plans. To show the above claim, we use the following two steps. In step one, we show that a change in q_L results in a smaller change in marginal utilities for L-type consumers than H-type consumers (should H-type consumers choose the L plan). In step two, we show that q_L decreases after the firm introduces family plans.

Step one requires three sub-steps. First, we show that an increase in q_L results in a higher utility for L-type consumers, holding all else constant. The marginal utility for an L-type consumer within the L plan's allowance is θ_L . In contrast, the marginal utility for an L-type consumer beyond the L plan's allowance is

$$\frac{\partial[\theta_L q_L^* - c_M (q_L^* - q_L)^2 - c_F (q - q_L^*)^2 - p_{OL} (q_L^* - q_L)]}{\partial q} = \frac{c_F (\theta_L - p_{OL} - 2c_M (q - q_L))}{(c_M + c_F)}, \text{ where}$$

q_L^* denotes the L-type consumer's optimal consumption level given her realization q , and the L plan, (q_L, p_L, p_{OL}) . Because the cost of monitoring overage usage and the cost of forgone consumption are positive, $c_M > 0, c_F > 0$, we have

$$\frac{c_F (\theta_L - p_{OL} - 2c_M (q - q_L))}{(c_M + c_F)} < (\theta_L - p_{OL} - 2c_M (q - q_L)) < \theta_L. \text{ The last inequality means that}$$

the marginal utility for an L-type consumer beyond the L plan's allowance is lower than

the marginal utility for an L-type consumer within the L plan's allowance. Therefore, an increase in q_L results in a higher utility overall for L-type consumers.

Second, we show that an increase in q_L results in a higher utility for H-type consumers if they choose an L plan, holding all else constant. The marginal utility for an H-type consumer within the L plan's allowance, if she chooses the L plan, is θ_H . In

contrast, the marginal utility for an H-type consumer beyond the L plan's allowance is

$$\frac{\partial[\theta_H q_{HL}^* - c_M (q_{HL}^* - q_L)^2 - c_F (q - q_{HL}^*)^2 - p_{OL} (q_{HL}^* - q_L)]}{\partial q} = \frac{c_F (\theta_H - p_{OL} - 2c_M (q - q_L))}{(c_M + c_F)},$$

where q_{HL}^* denotes the H-type consumer's optimal consumption level conditional on her choosing the L plan, (q_L, p_L, p_{OL}) , and her realization q . Because the cost of monitoring overage usage and the cost of forgone consumption are positive, $c_M > 0, c_F > 0$, we have

$$\frac{c_F (\theta_H - p_{OL} - 2c_M (q - q_L))}{(c_M + c_F)} < (\theta_H - p_{OL} - 2c_M (q - q_L)) < \theta_H. \text{ The last inequality means}$$

that the marginal utility for an H-type consumer beyond the L plan's allowance is lower than the marginal utility for an H-type consumer within the L plan's allowance.

Therefore, if an H-type consumer chooses the L plan, an increase in q_L results in a higher utility overall.

Third, we show that the difference between marginal utilities within the L plan's allowance and beyond L plan's allowance is greater for H-type consumers compared to L-type consumers. This claim can be shown by

$$\begin{aligned}
& \theta_H > \theta_L \\
& \Leftrightarrow \frac{c_F(p_{OL} + 2c_M(q - q_L)) + c_M\theta_H}{(c_M + c_F)} > \frac{c_F(p_{OL} + 2c_M(q - q_L)) + c_M\theta_L}{(c_M + c_F)} \\
& \Leftrightarrow \theta_H - \frac{\partial[\theta_H q_{HL}^* - c_M(q_{HL}^* - q_L)^2 - c_F(q - q_{HL}^*)^2 - p_{OL}(q_{HL}^* - q_L)]}{\partial q} \\
& > \theta_L - \frac{\partial[\theta_L q_L^* - c_M(q_L^* - q_L)^2 - c_F(q - q_L^*)^2 - p_{OL}(q_L^* - q_L)]}{\partial q}.
\end{aligned}$$

Combining the three points above, we have established that a change in q_L results in a greater change in marginal utilities for L-type consumers than H-type consumers (should H-type consumers choose the L plan): given Δq_L ,

$$\Delta u(\theta_H, L) > \Delta u(\theta_L, L).$$

Now we proceed with step two. We show that L plan's allowance, q_L , decreases after the firm introduces family plans. We prove this claim by contradiction. Suppose in the equilibrium when the firm offers both individual and family plans, the L plan's allowance weakly increases: $q_L^{***} \geq q_L^{**}$. Then suppose the firm reduces q_L^{***} by ϵ to $(q_L^{***} - \epsilon)$. Given that the individual rationality constraint for the L-segment is binding (IR_L), the firm has to decrease p_L^{***} such that $u(\theta_L, L) = 0$. Hence, the service provider loses $\Delta u(\theta_L, L)$ per user in the single L-segment. Next, consider the impact of $(q_L^{***} - \epsilon)$ on the single H-type consumers. The utility reduction of the L plan as the best outside option for H-type consumers is $\Delta u(\theta_H, L)$. Given that the incentive compatibility constraint for the H segment is binding ($u(\theta_H, H) \geq u(\theta_H, L)$), the firm can now raise the

price for the H plan by $\Delta u(\theta_H, L)$. This is the gain per user in the single H-segment.

Furthermore, given the binding incentive compatibility constraint for the family segment ($IC_F : u(\theta_F, F) \geq u(\theta_H, H) + u(\theta_L, L) = u(\theta_H, L)$), the firm can raise the family plan's price by $\Delta u(\theta_H, L)$, holding everything else constant. Then the service provider gains $\Delta u(\theta_H, L)$ per family account.

Recall that the sizes of the single L, single H, and family segments are $(\lambda - f)$, $(1 - \lambda - f)$, and f , respectively. Therefore, the total cost of reducing L plan's allowance, q_L , is $(\lambda - f)\Delta u(\theta_L, L)$. The total benefits are the sum of additional profits from the single H and family segments, $(1 - \lambda - f)\Delta u(\theta_H, L) + f\Delta u(\theta_H, L) = (1 - \lambda)\Delta u(\theta_H, L)$. Recall that from step one, $\Delta u(\theta_H, L) > \Delta u(\theta_L, L)$. Furthermore, when only individual plans were offered, $(1 - \lambda)\Delta u(\theta_H, L) = \lambda\Delta u(\theta_L, L)$ in the equilibrium. Hence, we obtain $(1 - \lambda)\Delta u(\theta_H, L) > (\lambda - f)\Delta u(\theta_L, L)$. This inequality means that the firm can increase its total profits by decreasing the allowance of the L plan, which is a contradiction that $q_L^{***} (\geq q_L^{**})$ is the equilibrium allowance. Hence we have finished step two.

To sum up, combining step one and two, we have shown that $q_L^{***} \geq q_L^{**}$ cannot hold in equilibrium. Therefore, we have established that L plan's allowance decreases after the firm introduces family plans: $q_L^{***} < q_L^{**}$. Note that the service provider can keep reducing q_L^{***} until one of the two following conditions is met: $\frac{\Delta u(\theta_H, L)}{\Delta u(\theta_L, L)} = \frac{\lambda - f}{1 - \lambda}$ or

$q_L^{***} = 0$. In comparison, when the firm only offers individual plans, q_L^{**} is set such that

$$\frac{\Delta u(\theta_H, L)}{\Delta u(\theta_L, L)} = \frac{\lambda}{1-\lambda} \text{ in the equilibrium.}$$

Proof of Proposition 18.

Condition A is given by

$$(1 + q_L^{***2} - \Delta(4 + \Delta) - q_L^{***}(2 - 4\Delta))(\lambda - \hat{\lambda}) > 0, \text{ and } \text{Max}\{\theta_{H2}, \theta_{H3}\} < \theta_H < \theta_{H4}, \text{ where}$$

$$\hat{\lambda} = \frac{f(1 - q_L^{***})(1 - q_L^{***} - \Delta) - (3 - 3q_L^{***} + \Delta)\Delta}{1 + q_L^{***2} - \Delta(4 + \Delta) - q_L^{***}(2 - 4\Delta)}.$$

Claim 1: Under Condition A, $\frac{\partial q_L^{***}}{\partial f} < 0$.

From implicit function theorem,

$$\frac{\partial q_L^{***}}{\partial f} = -\frac{\partial FOC(q_L^{***})}{\partial f} / \frac{\partial FOC(q_L^{***})}{\partial q_L} = \frac{A}{B}, \text{ where } A = a_2\theta_H^2 + a_1\theta_H + a_0,$$

$B = b_2\theta_H^2 + b_1\theta_H + b_0$, $b_2 > 0$. All the coefficients are given below:

$$a_2 = (1 - q_L^{***} + \Delta)(1 - \lambda)^2 \left(\Delta^2(1 - \lambda) + (1 - q_L^{***})^2 \lambda - f(1 - q_L^{***})(1 - q_L^{***} - \Delta) + (1 - q_L^{***})\Delta(3 - 4\lambda) \right),$$

$$a_1 = 2(1 - \lambda)^2 (c_f \Delta (f(1 - q_L^{***})) (\Delta^2 - 2(1 - q_L^{***})^2) - 6(1 - q_L^{***})^2 \Delta(1 - \lambda) - \Delta^3(1 - \lambda) - 2(1 - q_L^{***})^3 \lambda - (1 - q_L^{***})\Delta^2(6 - 7\lambda) - (1 - q_L^{***})\theta_L (f(1 - q_L^{***})^2 - 2\Delta^2(1 - \lambda) - (1 - q_L^{***})^2 \lambda + 3\Delta(1 - \lambda)(1 - q_L^{***}))) ,$$

$$\begin{aligned}
a_0 = & -a^2(2-2q_L^{***}+\Delta)\Delta^2(1-\lambda)^2(f(1-q_L^{***})(2-2q_L^{***}-\Delta)-\Delta^2(1-\lambda)-2(1-q_L^{***})^2\lambda-(1-q_L^{***})\Delta(6-7\lambda)) \\
& +\theta_L(-(f(1-f^2)(1-q_L^{***})^3-3(1-f^2)(1-q_L^{***})^2\Delta-3f(1-q_L^{***})\Delta^2+\Delta^3)\theta_L+((1-f)(1+3f)(1-q_L^{***})^3 \\
& -3(1-f)(3+f)(1-q_L^{***})^2\Delta-3(1+2f)(1-q_L^{***})\Delta^2+3\Delta^3)\theta_L\lambda+(3(2+f)(1-q_L^{***})\Delta^2-2(1-f)(1-q_L^{***})^3 \\
& -6(1-f)(1-q_L^{***})^2\Delta-3\Delta^3)\theta_L\lambda^2-\Delta^2(3-3q_L^{***}-\Delta)\theta_L\lambda^3-4c_M(1-q_L^{***})(\Delta-f(1-q_L^{***})+\lambda-(q_L^{***}+\Delta)\lambda^3) \\
& -4a(1-q_L^{***})(\Delta\theta_L(1-\lambda)^2(f(1-q_L^{***})^2+3(1-q_L^{***})\Delta(1-\lambda)+\Delta^2(1-\lambda)+(1-q_L^{***})^2\lambda) \\
& -c_M(1-q_L^{***})(\Delta-f(1-q_L^{***})+\lambda-(q_L^{***}+\Delta)\lambda^3)
\end{aligned}$$

$$b_2 = 2\Delta^2(1+f-2\lambda)^2(1-\lambda)^2 > 0,$$

$$b_1 = 4(1-\lambda)(\Delta^2(c_F\Delta(2+f-3\lambda)-\theta_L(1-\lambda))(1+f-2\lambda)(1-\lambda)+c((\lambda-f)(1-q_L^{***})+\Delta(1-\lambda))^3),$$

$$\begin{aligned}
b_0 = & 2(c_F^2\Delta^4(2+f-3\lambda)^2(1-\lambda)^2-2a\Delta^3\theta_L(2+f-3\lambda)(1-\lambda)^3+\Delta^2\theta_L^2(1-\lambda)^4-2c_M(1-f)\theta_L \\
& ((\lambda-f)(1-q_L^{***})+\Delta(1-\lambda))^3-4c_Fc_M((\lambda-f)(1-q_L^{***})-\Delta(1-\lambda))((\lambda-f)(1-q_L^{***})+\Delta(1-\lambda))^3) \cdot A
\end{aligned}$$

has two roots, denote them by θ_{H1}, θ_{H2} . Similarly, denote B's two roots θ_{H3}, θ_{H4} . It is straightforward to see that under Condition A, A is positive and B is negative. Hence, the overall comparative statics is negative.

Claim 2: Under Condition A and when $\theta_H < \tilde{\theta}_H$, $\frac{\partial p_L(q_L, f)}{\partial f} < 0$.

Proof. From the Chain rule, $\frac{\partial p_L(q_L, f)}{\partial f} = \frac{\partial p_L}{\partial q_L} \frac{\partial q_L}{\partial f} + \frac{\partial p_L}{\partial f}$. Based on the proof for

Proposition 17, $\frac{\partial p_L}{\partial q_L} > 0$. From Claim 1, $\frac{\partial q_L}{\partial f} < 0$ under Condition A. Hence the first

term in the previous equation is negative.

$$\begin{aligned}
\text{sign}\left(\frac{\partial p_L}{\partial f}\right) = & ((1-q_L^{***})(\theta_H(1-\lambda)-(1-f)\theta_L)+\Delta(\theta_H-\theta_L)(1-\lambda)+c_F((1-q_L^{***})\Delta(1-\lambda) \\
& +\Delta^2(1-\lambda)-(1-q_L^{***})^2(\lambda-f)))
\end{aligned}$$

This is a linear function of θ_H . When $\theta_H < \widetilde{\theta}_H$, the second term in the previous equation is also negative, $\frac{\partial p_L}{\partial f} < 0$.

Therefore, when Condition A holds and $\theta_H < \widetilde{\theta}_H$, $\frac{\partial p_L(q_L, f)}{\partial f} < 0$.

Claim 3: Under Condition A, $\frac{\partial p_H(q_L, f)}{\partial f} > 0$.

Proof. From the Chain rule, $\frac{\partial p_H(q_L, f)}{\partial f} = \frac{\partial p_H}{\partial q_L} \frac{\partial q_L}{\partial f} + \frac{\partial p_H}{\partial f}$. Based on the proof for

Proposition 17, $\frac{\partial p_H}{\partial q_L} < 0$. From Claim 1, $\frac{\partial q_L}{\partial f} < 0$ under Condition A. Hence the first

term in the previous equation is positive. Furthermore,

$$\frac{\partial p_H}{\partial f} = \frac{(1-q_L)^2(a\Delta(2(1-q_L)+\Delta)+(1-q_L+\Delta)\theta_H-(1-q_L)\theta_L)^2(1-\lambda)(\lambda-f)}{2(a+c)((1-q_L)(\lambda-f)+\Delta(1-\lambda))^3} > 0 \text{ always}$$

holds. Therefore, under Condition A, $\frac{\partial p_H(q_L, f)}{\partial f} > 0$.

Appendix C: Examples of Wireless Plans



Additional minutes

\$0.45 /min

Figure 4: AT&T Individual Plans



Additional minutes

\$0.45 /min

Figure 5: AT&T Family Plans

Appendix D: Notation Tables

Table 1: Notation Table for the Main Model

λ	The size of the L segment
f	The size of the Family Segment
Δ	The H segment's Minimum Requirement
k	The Curtailing Parameter
c	Marginal Cost per Unit for the Firm
m	The Fraction of Consumers with Mindless Consumption
H, L, F	The High, Low and Family Segment
$\theta_i, i \in \{H, L, F\}$	Segment- i 's Valuation
$q_i, i \in \{H, L, F\}$	i 's Plan's Quantity
$p_i, i \in \{H, L, F\}$	i 's Plan's Price
$p_{oi}, i \in \{H, L, F\}$	i 's Plan's Overage Price
Superscript *	Equilibrium Quantity/Price/Overage Price when Only Individual Plans are Offered
Superscript **	Equilibrium Quantity/Price/Overage Price when both Individual and Family Plans are Offered

Table 2: Notation Table for Endogenizing Overage Aversion (Section 7)

c_M	Cost of Monitoring Overage Usage
c_F	Cost of Forgone Consumption
\hat{q}	Realization of Usage Requirement
$\widehat{\theta}_H$	Threshold of θ_H beyond which offering family plans is more profitable
$\widetilde{\theta}_H$	Threshold of θ_H below which $\frac{\partial p_L}{\partial f} < 0$
Superscript *	Optimal Consumption Level Conditional on Overage Usage
Superscript **	Equilibrium Quantity/Price/Overage Price when Only Individual Plans are Offered
Superscript ***	Equilibrium Quantity/Price/Overage Price when both Individual and Family Plans are Offered

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