Essays on Self-Control

by

Alexander Groves

Department of Economics
Duke University

Date: ______________________

Approved:

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Rachel Kranton, Co-Supervisor

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Philipp Sadowski, Co-Supervisor

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Curtis Taylor

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Dan Ariely

Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in the Department of Economics
in the Graduate School of Duke University
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Abstract

This dissertation concerns methods to test whether or not self-control is costly, the form of temptation, and the affects different assumptions about costly self-control and temptation have on optimal borrowing and saving mechanisms. The second chapter shows that costly self-control and temptation can be differentiated from changing impatience in a stochastic income consumption-savings environment. The third chapter describes an experiment to test whether subjects have time inconsistent preferences, whether self-control is costly, and if so, whether the cost of self-control is time dependent. The fourth chapter describes the affects on the optimal borrowing and savings mechanisms that assumptions about the myopia of temptation and the strength of costly self-control have.

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1

Introduction

The second chapter of this dissertation, Tempted by Impatience, describes a method to differentiate the Gul and Pesendorfer (GP) model and the $\beta\delta$ model in a simple stochastic income consumption-savings setting by calibrating the models using one stochastic income distribution and then comparing predictions of the two models using another stochastic income distribution. In this setting there are two choices: the level of commitment, and subsequent savings. The GP model has three parameters to calibrate: the strength of temptation, the level of impatience, and the discount factor; while the $\beta\delta$ model does not have temptation. Therefore given a particular stochastic income distribution, a bounded continuum of GP models can emulate the $\beta\delta$ model. However, once the models have been calibrated their predictions are not likely align given any other stochastic income distribution. This procedure allows for measurements of the strength of temptation, how myopic temptation is, and for the existence of costly self-control.

In the third chapter Edward Kung and I propose a simple theoretical model of time inconsistent behavior which embeds both quasi-hyperbolic discounting models and dual-self costly self-control models. We show that these two sources of time inconsistency lead to different predictions about a time inconsistent individual's willingness to pay for various forms of pre-commitment. The differences in predictions allow us to test for the presence and form of self-control costs in a laboratory setting using qualitative observations. We will be able to reject an exponential discounting model and support a time inconsistent model if subjects pay for commitment. We will be able to reject the quasi-hyperbolic discounting model and support the concept of costly self-control if the subjects pay for non-restrictive commitment. Finally we will be able to reject a time independent cost of self-control if subjects are able to complete a task for a longer period of time with committing than they can without commitment.

The main purpose of the fourth chapter is to describe how different assumptions about temptation and self-control affect the optimal borrowing and saving contracts in a simple stochastic income setting. I find that the optimal savings plan when temptation is extreme and myopic consists of a mandatory minimum level of savings, while the optimal plan for mild myopic and farsighted temptation can have several fully restrictive mandatory savings levels, with only the highest being a mandatory minimum. When temptation is mild and myopic money burning will never be part of an optimal separating contract, though it can be when temptation is farsighted. Also, when temptation is myopic and extreme an individual will strictly prefer mandatory deposits to liquidity constraints. Finally, I find that the optimal borrowing plan when self-control is costly consists of regular payments and self imposed debt limit.

2

Tempted by Impatience

Abstract

This paper describes a method to differentiate the Gul and Pesendorfer (GP) model and the $\beta\delta$ model in a simple stochastic income consumption-savings setting by calibrating the models using one stochastic income distribution and then comparing predictions of the two models using another stochastic income distribution. In this setting there are two choices: the level of commitment, and subsequent savings. The GP model has three parameters to calibrate: the strength of temptation, the level of impatience, and the discount factor; while the $\beta\delta$ model does not have temptation. Therefore given a particular stochastic income distribution, a bounded continuum of GP models can emulate the $\beta\delta$ model. However, once the models have been calibrated their predictions are not likely align given any other stochastic income distribution. This procedure allows for measurements of the strength of temptation, how myopic temptation is, and for the existence of costly self-control.

2.1 Introduction

The prevalent hypothesis for observed preference for commitment is that an individual knows that her future decision making will be influenced by temptation or additional impatience, and commitment is a method of self-control. The most common model in the literature of changing impatience is the $\beta\delta$ model of quasi-hyperbolic discounting, introduced by Phelps and Pollak (1968) and popularized by Laibson (1997). In this model an individual discounts consumption today versus consumption tomorrow at a higher rate than she does between any other two dates in the future. This results in inconsistent preferences over time and strategic behavior between the current and future selves. Commitment is valuable to this individual because it allows her current self to control her future selves.

An alternative explanation is that there exist options that an individual finds attractive at the time of temptation, but not at any other moment. In order to avoid choosing these tempting options the individual must exercise costly internal self-control. Models of this flavor have been described by Gul and Pesendorfer (2001), Fudenberg and Levine (2012), and Noor (2007), to name a few. In this setting commitment is desirable because it serves to reduce the amount of costly internal self-control exercised.

The main difference between these two explanations is that when there is costly self-control eliminating the most tempting options can be valuable at the commitment stage, even if it does not change behavior during the consumption stage, because this will reduce the amount of self-control used. This is not true when there is not costly self-control and discounting is inconsistent. This paper focuses on a simple three period stochastic income consumption-saving setting. The first observation is that the Gul and Pesendorfer (2001) (henceforth GP) model can emulate any data that the $\beta\delta$ model can describe. This can be accomplished by calibrating the discount factor, the strength of temptation, and the level of myopia of temptation in the GP model. However, once estimated for one data set (one distribution for income) the two models will no longer agree with each other when simulating choices with a different distribution for income[1].

For a given distribution, not just one, but a bounded continuum of GP models can emulate the $\beta\delta$ model[2]. This is because while the GP model has three parameters to calibrate there are only two choices to be made: the amount to save, and the level of commitment. The parameters of this continuum of models are related in a very particular way: as the level of myopia of temptation (the additional amount of discounting between consumption today and consumption tomorrow) decreases from completely myopic to the level of the $\beta\delta$ model, the strength of temptation must increase towards infinity, and the discount rate of the GP model must approach that of the $\beta\delta$ model. Using a second distribution for income pins down a particular GP model from the continuum and therefore allows for the estimation of these parameters values. The estimated parameters would reveal more information about how temptation works, specifically: are people only tempted by current consumption, or are they tempted by impatience?

Gul and Pesendorfer (2005) axiomatize the $\beta\delta$ model for finite decision problems. Given a finite decision problem they show that a GP model over a related continuous decision problem can be found that approximates the $\beta\delta$ model. Because they analyze only one decision problem at a time they cannot show that the $\beta\delta$ and approximating GP models will no longer agree given another decision problem.

Dekel and Lipman (2011) compare random preference versions of the GP and

[1] Unless temptation becomes overwhelming, in which case the GP model will always reproduce the $\beta\delta$ model. A version of the GP model with overwhelming temptation with the specific purpose of emulating the $\beta\delta$ model was introduced by Krusell et al. (2010).

[2] This is similar to what Dekel and Lipman (2011) find for a general Strotz (1955) model (of which the $\beta\delta$ is a particular example of), stochastic preferences, and a random strength of temptation.

Strotz (1955) models[3]. They show that if a preference over menus can be represented by a random GP representation then it can be represented by a random Strotz model as well. They also show that the two models can be differentiated using a combination of choices over menus and choices from menus. In this paper I present results which have a similar flavor but are quite different.

The paper proceeds as follows: section 2 introduces a general GP model and the $\beta\delta$ model; section 3 shows how the GP model can be calibrated to emulate the $\beta\delta$ model for any one distribution of income, and how the calibrated models will differ for simulations with a second income distribution; section 4 briefly discusses the relationship between the two models in settings with more time periods; section 5 concludes. All proofs are relegated to the appendix.

2.2 The GP and $\beta\delta$ Models

There are three periods: 0, 1, 2. In period 0 the individual makes a choice about a mandatory minimum to commit to save, $\hat{s}$, and consumes nothing. In period 1 income, $y > 0$, is drawn from a distribution with pdf $f(y)$ and she decides how much to save, $s \geqslant \hat{s}$, and consume, $c \leqslant y - s$. Finally in period 2 income, y_2, is deterministic, the individual receives her savings plus interest, r, and consumes $c_2 \leqslant y_2 + (1+r)s$. The budget sets for periods 1 and 2 are: $B \equiv \left\{(c,s) \in \mathbb{R}^2_+ | c + s \leqslant y, \quad s \geqslant \hat{s}\right\}$ $B_2 \equiv \{c_2 \in \mathbb{R}_+ | c_2 \leqslant y_2 + (1+r)s\}$. In the context of the models specified below assuming a deterministic y_2 is without loss of generality.

The lower bound on the distribution of income is greater than or equal to zero. Income in period 1 is stochastic and the pdf of the distribution, $f(y)$ is known by the individual.

The $\beta\delta$ model describes an individual with time inconsistent preferences who is particularly impatient between consumption now and consumption later, but less

[3] The $\beta\delta$ model is a particular formulation of the Strotz model.

impatient between any two dates in the future. This is modeled with a utility function $w : \mathbb{R}^2_+ \to \mathbb{R}$ that is increasing, continuous, and concave. The individual's expected utility in period 0 is

$$\beta\delta\left[\mathbb{E}w(y-s) + \delta w(y_2 + (1+r)s)\right].$$

Their utility in period 1 is

$$w(y-s) + \beta\delta w(y_2 + (1+r)s).$$

$\beta, \delta \in [0,1]$. It is the increased impatience, β, of the individual's future self over their current self that will lead to a preference for commitment.

The GP model describes an individual with time-consistent preferences that exhibit temptation and costly self-control, where $u : \mathbb{R}^2_+ \to \mathbb{R}$ is her *commitment* utility and is continuous, concave, increasing and time separable.

$V : \mathbb{R}^2_+ \to \mathbb{R}$ is her *temptation* utility which is continuous, and concave. $V(c, c_2) = v(c) + \delta v(c_2)$, and $v : \mathbb{R} \to \mathbb{R}$ is assumed to be continuous and increasing. A temptation utility that depends only on the immediate period's consumption, $\delta = 0$, is a model of myopic temptation. Temptation that depends on period 2 consumption as well is a model of farsighted temptation. The strength of temptation is denoted by γ.

Because the individual will always consume along her budget constraint, after period 0 there is only one choice variable, the level of savings, s; therefore c_2 can be dropped. To simplify notation I will write $\delta u(c_2) = \delta u(s)$ and $V(c, c_2) = V(s)$ with the understanding that $c_2 = y_2 + (1+r)s$, not just s.

Utility in period 1 is,

$$U_1 = u(c) + \gamma V(s) - \gamma \max_{\tilde{s} \geqslant \hat{s}} V(\tilde{s}) + \delta_{GP} u(s).$$

Period 0 expected utility is,

$$\mathbb{E}U = \mathbb{E}\left[u(c) + \gamma V(s) - \gamma \max_{\tilde{s} \geqslant \hat{s}} V(\tilde{s}) + \delta_{GP} u(s)\right].$$

$-\gamma \left[\max_{\tilde{s}} V(\tilde{s}) - V(s)\right]$ is the *net cost* of self-control and is always negative unless $s = \tilde{s}$, in which case it is zero. The *cost* of self-control is $-\max_{\tilde{s}} V(\tilde{s})$. As γ goes to zero the net cost of self-control also goes to zero and consumption is determined by the commitment utility. As γ increases towards infinity the net cost of self-control goes to infinity as well and the individual loses her ability to control herself in periods 1 and 2 and the temptation utility determines consumption decisions.

A preference for commitment is driven by the disagreement between the optimal levels of consumption for the commitment utility and the temptation utility and the resulting net cost of self-control. Commitment constrains the available options. For instance, assuming deterministic income, if the individual commits to a particular level of savings in both periods, s, then $B(y) = \{y - s, s\}$ and $B_2(s) = \{y + (1 + r)s\}$. This reduces the net cost of self-control to zero and $\mathbb{E}U = \mathbb{E}\left[u(c) + \delta u(\hat{s})\right]$.

The income level for which $\hat{s}$ is the utility maximizing savings level, y^{fb}, is defined implicitly by $u'(y^{fb} - \hat{s}) + \delta(1 + r)u'(\hat{s}) = 0$. The income for which $\hat{s}$ is the most tempting savings level, $\tilde{y}$, is defined by $V'(\tilde{y} - \hat{s}) = 0$. When $V(s)$ only depends on immediate consumption then $\tilde{y} = y_{max}$. The level of income for which $\hat{s}$ is optimal given that the individual is tempted, $\overline{y}$, is defined by $u'(\overline{y} - \hat{s}) + \gamma V'(\overline{y} - \hat{s}) + \delta(1 + r)u'(\hat{s}) = 0$.

2.3 Emulation and Differentiation

The decision problem in period 0 is to set the mandatory minimum deposit, $\hat{s} \in [0, y_{min}]$, that maximizes expected utility given the savings decisions for every feasible value of y in period 1. For the $\beta\delta$ model this optimization problem takes the following form:

$$\max_{\hat{s}\in[0,y_{min}]} \mathbb{E}\left[w(c) + \delta w(s)\right]$$

s.t.

$$s = \arg\max_{s\in B} w(c) + \beta\delta w(s)$$

For the GP model the optimization problem is,

$$\max_{\hat{s}\in[0,y_{min}]} \mathbb{E}\left[u(c) + V(s) + \delta_{GP}u(s) - \max_{\tilde{s}\in B} V(s)\right].$$

Theorem 1 illustrates how for a given distribution the GP model can be calibrated so that it replicates any $\beta\delta$ model's choices of commitment level and period 1 savings.[4]

Theorem 1. *Given a distribution of income the farsighted GP model that replicates the $\beta\delta$ model in a three period setting takes the following form:*

$$\begin{aligned} u(c) &= w(c) \\ v(c) &= \gamma\left[w(c) + \beta_{GP}\delta_{GP}u(s)\right] \end{aligned}$$

where $\frac{\delta_{GP}(1+\gamma\beta_{GP})}{1+\gamma} = \beta\delta,\ \delta_{GP} \in [\beta\delta, \delta],\ \beta_{GP} \in [0, 1],\ \gamma \in [0, \infty)$

The full proof of Theorem 1 is in the appendix. I now provide the intuition behind the result. The form of the GP model above gives us three degrees of freedom, δ_{GP}, γ, and β_{GP}, so that it can fit the $\beta\delta$ model for a given distribution. For the period 1 savings to be the same in both models the ratios of the period 1 and 2 marginal utilities must be proportional each other:

$$\frac{u'(c) + v'(c)}{\delta_{GP}u'(s)} = \frac{w'(c)}{\beta\delta w'(s)}. \tag{2.1}$$

[4] Theorem 1 is similar to Theorem 6 in Gul and Pesendorfer (2005).

When $u(c) = w(c)$ and $v(c) = \gamma\left[w(c) + \beta_{GP}\delta_{GP}w(s)\right]$ then equation (2.1) becomes the condition in Theorem 1:

$$\frac{\delta_{GP}(1+\gamma\beta_{GP})}{1+\gamma} = \beta\delta. \tag{2.2}$$

If the commitment decisions are to be the same then the period 0 first order conditions must equal zero for the same value of $\hat{s}$. The FOC for the the GP model is:

$$\begin{aligned}\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}} &= \underbrace{\int_{\overline{y}_{GP}}^{\tilde{y}} \gamma\left[w'(\hat{c}) - \beta_{GP}\delta_{GP}(1+r)w'(\hat{s})\right] f(y)dy}_{>0} \\ &+ \underbrace{\int_{y^{fb}_{GP}}^{\overline{y}_{GP}} \left[-w'(\hat{c}) + \delta_{GP}(1+r)w'(\hat{s})\right] f(y)dy}_{>0} \\ &+ \underbrace{\int_{y_{min}}^{y^{fb}_{GP}} \left[-w'(\hat{c}) + \delta_{GP}(1+r)w'(\hat{s})\right] f(y)dy}_{<0} = 0\end{aligned} \tag{2.3}$$

Where $u(c) = w(c)$ and $v(c) = \gamma\left[w(c) + \beta_{GP}\delta_{GP}u(s)\right]$. The FOC for the $\beta\delta$ model is:

$$\begin{aligned}\frac{\partial \mathbb{E}U^{\beta\delta}}{\partial \hat{s}} &= \underbrace{\int_{y^{fb}_{\beta\delta}}^{\overline{y}_{\beta\delta}} \left[-w'(\hat{c}) + \delta(1+r)w'(\hat{s})\right] f(y)dy}_{>0} \\ &+ \underbrace{\int_{y_{min}}^{y^{fb}_{\beta\delta}} \left[-w'(\hat{c}) + \delta(1+r)w'(\hat{s})\right] f(y)dy}_{<0} = 0\end{aligned} \tag{2.4}$$

When $\gamma = 0$ and $\delta_{GP} = \beta\delta$, $\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}} < \frac{\partial \mathbb{E}U^{\beta\delta}}{\partial \hat{s}}$. When $\gamma > 0$ and $\delta_{GP} = \delta$, $\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}} \geqslant \frac{\partial \mathbb{E}U^{\beta\delta}}{\partial \hat{s}}$ (these are equal given deterministic income). Because $\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}}$ varies continuously when changing the parameter values at least one set of values exist for which $\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}} = \frac{\partial \mathbb{E}U^{\beta\delta}}{\partial \hat{s}}$ and both models agree on savings. In fact, because we have three variables to adjust and two equations to satisfy there can exist a bounded but infinite set of parameter values such that the two models produce the same commitment and savings predictions. This is similar in spirit to the result in Dekel and Lipman (2011). However, they find that a random Strotz model can always be created that emulates the commitment decisions of a random GP model in a random utility setting, and that the two models will not agree on savings predictions. While here I have shown that in a random income setting a GP model can always be found that exactly emulates both the commitment and the savings decisions of any $\beta\delta$ model. Precisely why these results differ between the random income and random utility settings warrants further analysis.

Pinning down any one of these three parameters naturally reduces the set of available values for the remaining two parameters.

Theorem 2. *Given a value for the discount factor, δ_{GP}, in the stochastic income environment a GP model can be calibrated to replicate the $\beta\delta$ model either in period 0 or in period 1, but not both. When both models agree on*

1. *period 1 savings, s, then the GP models will predict a higher level of commitment than the $\beta\delta$ model: $\hat{s}_{GP} > \hat{s}_{\beta\delta}$, and $\lim_{\beta_{GP} \to \beta} \hat{s}_{GP} = \lim_{\gamma \to \infty} \hat{s}_{GP} = \hat{s}_{\beta\delta}$.*

2. *period 0 commitment, $\hat{s}$, then the GP model will predict a higher level of optional savings than the $\beta\delta$ model: $s_{GP} > s_{\beta\delta}$. s_{GP} is decreasing in β_{GP} and γ.*

The proof is in the appendix.

To work through the intuition of the theorem first consider the case where temptation is myopic, $\beta_{GP} = 0$, and then farsighted temptation, $\beta_{GP} > 0$.

1) $\beta_{GP} = 0$. If the $\beta\delta$ and myopic GP models are to agree on savings decisions in period 1 then $\gamma = (1-\beta)/\beta$. Assume for a moment that they agree on the commitment decision $\hat{s}$ as well: so $\overline{y}_{GP} = \overline{y}_{\beta\delta}$, and the second and third integrals in (2.3) are equal to (2.4). The additional positive integral in Equation (2.3) results in a higher level of commitment for the GP model. Therefore $\hat{s}_{GP} > \hat{s}_{\beta\delta}$, a contradiction. It is the first integral in Equation (2.3) that captures the difference between the GP and $\beta\delta$ models: costly self-control. Commitment benefits both models by reducing temptation (the second integral in Equation (2.3) and the first in Equation (2.4)) and is detrimental in both models because of the reduction in flexibility when income is low (the third integral in Equation (2.3) and the second in Equation (2.4)). But only the GP model benefits from the reduction in costly self-control due to commitment.

If instead the two models agree on commitment decisions then γ is reduced until $\hat{s}_{GP} = \hat{s}_{\beta\delta}$ (this occurs because $\partial\left(\partial\mathbb{E}U^{GP}/\partial\hat{s}\right)/\partial\gamma > 0$ when $\beta_{GP} = 0$, and when $\gamma = 0$: $\frac{\partial\mathbb{E}U^{GP}}{\partial\hat{s}} < \frac{\partial\mathbb{E}U^{\beta\delta}}{\partial\hat{s}}$; while when $\gamma = (1-\beta)/\beta$: $\frac{\partial\mathbb{E}U^{GP}}{\partial\hat{s}} \geqslant \frac{\partial\mathbb{E}U^{\beta\delta}}{\partial\hat{s}}$). Now $\gamma < (1-\beta)/\beta$, and consequently,

$$\frac{(1+\gamma)w'(c)}{\delta w'(s)} < \frac{w'(c)}{\beta\delta w'(s)}, \quad \forall s.$$

Therefore, period 1 savings predicted by the myopic GP model will be greater than savings predicted by the $\beta\delta$ model. At first this may seem counter-intuitive; one may think that because the GP model predicts a higher level of commitment when the two models agree on period 1 savings, forcing the models agree on the level of commitment would shift the period 1 savings predicted by the GP model down to a lower level than that of the $\beta\delta$ model. However, the cost of self-control does not play a part in the period 1 savings decision, only temptation has any relevance (when $\hat{s}$ is not binding). Because the discount factors in both models are the same, in order for the two to agree on the level of commitment γ must be decreased, which reduces the strength of temptation in the GP model. This reduction in the salience of temptation leads to larger savings decisions in period 1.

2) When $\beta_{GP} > 0$ the individual has more patience than when $\beta_{GP} = 0$, so the strength of temptation, γ, must be increased in order for the two models to agree on optimal period 1 savings. To be precise, for period 1 savings to be the same for the farsighted GP and $\beta\delta$ models, the period 1 FOCs must be equal, which reduces to:

$$\begin{aligned} \frac{1+\gamma\beta_{GP}}{1+\gamma} &= \beta \\ \Rightarrow \frac{1+\gamma\beta_{GP}}{1+\gamma} &= x\text{, a constant.} \\ \Rightarrow \gamma &= \frac{1-x}{x-\beta_{GP}} \\ \Rightarrow \frac{\partial^2\gamma}{\partial\beta_{GP}\partial x} &= \frac{-x}{x-\beta_{GP}} - \frac{2\,(1-x)\,(x-\beta_{GP})}{(x-\beta_{GP})^2} < 0 \end{aligned} \tag{2.5}$$

This equation defines the rate at which the strength of temptation, γ, must change to compensate for an increase in patience, β_{GP}, in order to keep period 1 savings constant. At $\overline{y}$, $\hat{s}$ would be the chosen level of period 1 savings if there were no commitment ($\hat{s} = s$). Therefore,

$$\frac{1+\gamma\beta_{GP}}{1+\gamma} = \frac{w'(\overline{y}-\hat{s})}{\delta w'(y_2+(1+r)\hat{s})}(1+r) \tag{2.6}$$

If $\overline{y}$ were replaced in Equation (2.6) by any $y > \overline{y}$, the rate of change for γ would have to increase, as shown by Equation (2.5). Therefore, for all $y > \overline{y}$ an increase in β_{GP} leads to a decrease in the marginal utility from $\hat{s}$. This means that if we were to start with $\beta_{GP} = 0$, and increase β_{GP} and γ in just the right way so that all the models continue to agree on period 1 savings, then this will lead to a net decrease in the marginal utility from $\hat{s}$ (Equation (2.3)). Therefore $\hat{s}(\beta_{GP} > 0) < \hat{s}(\beta_{GP} = 0)$.

Note that $\hat{s}_{GP} > \hat{s}_{\beta\delta}$ continues to hold when $\beta_{GP} > 0$ because commitment still has the added benefit of reducing the cost of self-control, but this benefit is not as great as it is in the myopic GP model.

Suppose instead all of the models agree on the optimal $\hat{s}$, and $\delta_{GP} = \delta$. Now when increasing β_{GP}, γ must be increased at a faster rate in order to keep the marginal utility of $\hat{s}$ constant than it does to make the models agree on s. Therefore $\frac{1+\gamma\beta_{GP}}{1+\gamma}$ is decreasing with increasing β_{GP}, so the optimal first period savings, s_{GP}, is decreasing, and $\lim_{\beta_{GP}\to\beta} s_{GP} = s_{\beta\delta}$. If it were otherwise then $\overline{y}_{GP} \neq \overline{y}_{\beta\delta}$ and $\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}} \neq \frac{\partial \mathbb{E}U^{\beta\delta}}{\partial \hat{s}}$. Notice that this all means that changing β_{GP} from 0 to 1 does in fact convert the GP model from the myopic GP to something that precisely emulates the $\beta\delta$ model, where temptation becomes overwhelming and so self-control is never exercised.

2.4 Empirical Test

Estimating the $\beta\delta$ model using a particular distribution for income, $f(y)$, and then calibrating the GP model defines the relationship between the three GP model parameters and the $\beta\delta$ model parameters. However, for a different distribution, $g(y)$, when the models are calibrated for $f(y)$, they generally do not continue to predict the same optimal level of commitment. At the same time, the two models would still agree on period 1 savings decisions because Equation (2.2) will still hold. A popular way to estimate an individual's discount rate, and in the context of the $\beta\delta$ model, also the level of time inconsistency, β, is to use a degenerate distribution for income (Meier and Sprenger (2010), Ashraf et al. (2006)). Then using these estimated values the individual's choices can be simulated using a non-degenerate distribution. Calibrating the GP model when the distribution is degenerate results in $\delta_{GP} = \delta$, and $\frac{1+\gamma\beta_{GP}}{1+\gamma} = \beta$. When the two models are then used for simulation with any non-degenerate distribution they differ in a systematic way.

Corollary 1. *Given an initial calibration with deterministic income the GP and* $\beta\delta$ *models will predict the same savings levels but different commitment levels for any non-degenerate distribution:* $\hat{s}_{GP} > \hat{s}_{\beta\delta}$*, where* $\hat{s}_{GP}$ *is negatively correlated with* β_{GP}*, and* $\lim_{\beta_{GP}\to\beta} \hat{s}_{GP} = \hat{s}_{\beta\delta}$.

Comparing multiple income distributions (e.g. deterministic and stochastic) not only allows for the differentiation of the $\beta\delta$ and GP models, but is also useful because all of the parameters of the GP model can be estimated. The level of β_{GP} measures how myopic temptation is, and γ measures its strength.

2.5 More Periods

As shown in Gul and Pesendorfer (2005) and Krusell et al. (2010), the functional form for the GP model above can be calibrated to emulate the $\beta\delta$ model for any number of time periods by allowing $\gamma \to \infty$, $\beta_{GP} \to \beta$, and $\delta_{GP} = \delta$, so temptation is overwhelming and self-control non-existent. However, alternative GP models in which temptation is more mild and self-control actually functions have not been examined thoroughly and is the subject of current research.

2.6 Conclusion

This paper describes a method to differentiate the GP model and the $\beta\delta$ model by calibrating the models using one stochastic income distribution and then comparing predictions using another stochastic income distribution. This approach allows for measurements of the strength of temptation, how myopic temptation is, and for the existence of costly self-control. An empirical test based on this strategy should shed light on the way temptation and self-control work, and thereby improve models of individuals' decision making. Groves and Kung (2012) present a detailed description of an experimental setup that implements this idea.

2.7 Proofs

Proof of Theorem 1

Proof.

- Step 1: show that there is a unique form of the GP model (when temptation is instantaneous) that can replicate the time one behavior of any $\beta\delta$ model.

- Step 2: show that this functional form of the GP model can be calibrated so that it also replicates the time zero behavior of a $\beta\delta$ model.

Step 1: For the period 1 savings to be the same in both models the ratios of the period 1 and 2 marginal utilities must equal each other:

$$\frac{u'(y-s)+V'(y-s,s)}{\delta_{GP}u'(y+(1+r)s)} = \frac{w'(y-s)}{\beta\delta w'(y+(1+r)s)}. \tag{2.7}$$

In order for this to hold there must exist some time separable conversion $u(c)+v(c)-v(B) = g(w(c))$, such that

$$\frac{g'(w(c_1))w'(c_1)}{\delta_{GP}g'(w(B_2))w'(B_2)} = \frac{w'(c_1)}{\beta\delta\mathbb{E}w'(B_2)}$$

$$\Rightarrow \frac{g'(w(c_1))}{g'(w(B_2))} = \frac{\delta_{GP}}{\beta\delta}. \tag{2.8}$$

In equilibrium the ratio $\frac{w'(c)}{w'(B_2)}$ is constant, which means that the ratio $\frac{w(c)}{w(B_2)}$ is not. Therefore $g'(\cdot)$ must be constant. When $u(c) = w(c)$ and $V(c,s) = \gamma\left[w(c)+\beta_{GP}\delta_{GP}w(s)\right]$ satisfies this condition. Given this form equation (2.7) becomes the condition

$$\frac{\delta_{GP}(1+\gamma\beta_{GP})}{1+\gamma} = \beta\delta. \tag{2.9}$$

$\beta_{GP} \in [0,1]$, $\gamma > 0$, and $\delta_{GP} \in [0,1]$. We can further reduce these bounds through the use of Equation (2.9). Because $\gamma > 0$ this means that $\delta_{GP} \geqslant \beta\delta$.

Step 2: If the commitment decisions are to be the same then the period 0 first order conditions must equal zero for the same value of $\hat{s}$. The FOC for the GP model is:

$$\begin{aligned}\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}} &= \underbrace{\int_{\overline{y}_{GP}}^{\tilde{y}} \gamma \left[w'(y-\hat{s}) - \beta_{GP}\delta_{GP}(1+r)w'(y_2+(1+r)\hat{s})\right] f(y)dy}_{>0} \\ &+ \underbrace{\int_{y_{GP}^{fb}}^{\overline{y}_{GP}} \left[-w'(y-\hat{s}) + \delta_{GP}(1+r)w'(y_2+(1+r)\hat{s})\right] f(y)dy}_{>0} \\ &+ \underbrace{\int_{y_{min}}^{y_{GP}^{fb}} \left[-w'(y-\hat{s}) + \delta_{GP}(1+r)w'(y_2+(1+r)\hat{s})\right] f(y)dy}_{<0} = 0 \end{aligned} \quad (2.10)$$

Where $u(c) = w(c)$ and $v(c) = \gamma\left[w(c) + \beta_{GP}\delta_{GP}u(s)\right]$ have been substituted in. The FOC for the $\beta\delta$ model is:

$$\begin{aligned}\frac{\partial \mathbb{E}U^{\beta\delta}}{\partial \hat{s}} &= \underbrace{\int_{y_{\beta\delta}^{fb}}^{\overline{y}_{\beta\delta}} \left[-w'(y-\hat{s}) + \delta(1+r)w'(y_2+(1+r)\hat{s})\right] f(y)dy}_{>0} \\ &+ \underbrace{\int_{y_{min}}^{y_{\beta\delta}^{fb}} \left[-w'(y-\hat{s}) + \delta(1+r)w'(y_2+(1+r)\hat{s})\right] f(y)dy}_{<0} = 0 \end{aligned} \quad (2.11)$$

When $\gamma = 0$ and $\delta_{GP} = \beta\delta$, $\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}} < \frac{\partial \mathbb{E}U^{\beta\delta}}{\partial \hat{s}}$. When $\gamma > 0$ and $\delta_{GP} = \delta$, $\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}} \geqslant \frac{\partial \mathbb{E}U^{\beta\delta}}{\partial \hat{s}}$ (these are equal given deterministic income, as will be shown in the proof for

Corollary 1). Because $\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}}$ varies continuously when the parameters are changed there exists at least one set of parameters for which equations (2.10) and (2.11) are equal. Notice that $\delta_{GP} \leqslant \delta$ otherwise the $\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}} > \frac{\partial \mathbb{E}U^{\beta\delta}}{\partial \hat{s}}$ for any value of γ and β_{GP}.

Rearrange Equation (2.9) so that β_{GP} is a function of the other two parameters:

$$\beta_{GP}(\gamma, \delta_{GP}) = \frac{\beta\delta + \gamma(\beta\delta - \delta_{GP})}{\gamma\delta_{GP}} \tag{2.12}$$

After this is substituted into (2.10) we find that $\frac{\partial^2 \mathbb{E}U^{GP}}{\partial \hat{s} \partial \delta_{GP}} > 0$, and that $\frac{\partial^2 \mathbb{E}U^{GP}}{\partial \hat{s} \partial \gamma} > 0$ (the trick for the latter is to remember that $\delta_{GP} - \beta\delta > 0$). This means that if we start at $\gamma = 0$ and $\delta_{GP} = \delta$, as we reduce δ_{GP} we can always increase γ by enough to equate (2.10) and (2.11). □

Proof of Theorem 2

Proof. When $\delta_{GP} = \delta$ then $\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}} \geqslant \frac{\partial \mathbb{E}U^{\beta\delta}}{\partial \hat{s}}$. First assume that $s_{GP} = s_{\beta\delta} = s$, so Equation (2.9) holds. We know from the proof of Theorem 1 that $\frac{\partial^2 \mathbb{E}U^{GP}}{\partial \hat{s} \partial \gamma} > 0$. If $\hat{s}_{GP} = \hat{s}_{\beta\delta}$ then because $\gamma > 0$ Equation (2.10) is greater than Equation (2.11), and therefore greater than zero. Hence the optimal level of commitment for any GP model with this form will be greater than $\hat{s}_{\beta\delta}$. Furthermore, because $\frac{\partial^2 \mathbb{E}U^{GP}}{\partial \hat{s} \partial \gamma} > 0$, as γ increases so does $\hat{s}_{GP}$.

Now assume that $\hat{s}_{GP} = \hat{s}_{\beta\delta}$. If $s_{GP} = s_{\beta\delta}$ then $\overline{y}_{GP} = \overline{y}_{\beta\delta}$ and $y^{fb}_{GP} = y^{fb}_{\beta\delta}$, so $\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}} > \frac{\partial \mathbb{E}U^{\beta\delta}}{\partial \hat{s}} = 0$. Increasing γ and/or decreasing β_{GP} will both decrease $\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}}$, thereby equalizing the marginal utilities of the $\beta\delta$ and GP models so that $\hat{s}_{GP} = \hat{s}_{\beta\delta}$. From equation (2.9) we can see that s_{GP} is decreasing in γ and increasing in β_{GP}. □

Proof of Corollary 1

Proof. When income is deterministic then the first order constraints for commitment for the GP and $\beta\delta$ models are as follows:

$$\frac{\partial \mathbb{E}U^{GP}}{\partial \hat{s}} = -w'(y-\hat{s}) + \delta_{GP}(1+r)w'(y_2+(1+r)\hat{s}) = 0$$

$$\frac{\partial \mathbb{E}U^{\beta\delta}}{\partial \hat{s}} = -w'(y-\hat{s}) + \delta(1+r)w'(y_2+(1+r)\hat{s}) = 0$$

It is easy to see that for these two equations to equal zero for the same $\hat{s}$ it must be true that $\delta_{GP} = \delta$. The remainder of the proof is identical to the proof for Theorem 2. $\square$

3

Testing for Self-Control Costs in a Generic Model of Time Inconsistency

Abstract

We propose a simple theoretical model of time inconsistent behavior which embeds both quasi-hyperbolic discounting models and dual-self costly self-control models. We show that these two sources of time inconsistency lead to different predictions about a time inconsistent individual's willingness to pay for various forms of pre-commitment. The differences in predictions allow us to test for the presence and form of self-control costs in a laboratory setting using qualitative observations. We will be able to reject an exponential discounting model and support a time inconsistent model if subjects pay for commitment. We will be able to reject the quasi-hyperbolic discounting model and support the concept of costly self-control if the subjects pay for non-restrictive commitment. Finally we will be able to reject a time independent cost of self-control if subjects are able to complete a task for a longer period of time with commitment than they can without commitment.

3.1 Introduction

The purpose of this experiment is to test whether self-control is costly and whether this cost depends on the time spent exerting it. While brain imaging, self-reporting, and introspection suggest that temptation, impatience, or self-control do have a neurological counterpart, currently it difficult to test for these processes directly. Choices about different types of commitment can work as an indirect test of these concepts. In particular, reducing the set of available options reveals a preference for commitment and gives evidence of temptation or impatience, a preference for non-restrictive commitment[1] supports the existence of costly self-control, and the change in an individual's behavior when using non-restrictive commitment can support or reject time dependent costly self-control.

If an individual knows that her future self has different preferences than her current self, because of temptation or impatience, then she can use commitment to control her future self's actions. When the commitment does not actually change her future self's choices then it is of no use to her. If, on the contrary, the individual has the ability to use internal self-control to control her future self's actions, and this self-control is costly to use, then even when commitment does not change her future self's choices it may still be desirable. This is because commitment can reduce the amount of internal self-control the individual has to expend. For instance, in this experiment subjects are repeatedly presented with the choice of a boring task with long-run benefits or a more tempting outside option with no long-run benefit. If a subject repeatedly chooses the task over the outside option, committing herself to the task by eliminating the outside option for some amount of time would be beneficial to her because she no longer has to control her temptation to switch from the task to the outside option during that period. Finally, if the amount of self-control is limited,

[1] If an individual is presented with a menu of options, A, and she chooses $a \in A$, a non-restrictive commitment would present her with a new menu $A' \subset A$ such that $a \in A'$.

or if its cost is increasing over time, then non-restrictive commitment will increase the amount of time before an individual succumbs to temptation. The individual will be able to complete the boring task for a longer period of time with non-restrictive commitment than she would without it.

There is a large body of evidence in cognitive neuroscience that supports the notion of an intrinsic difference between consumption now and consumption in the future (McClure (2004) for example, and see Luhmann (2009) for a survey of the neuroscience literature)[2]. Commitment can be used to control temptation by reducing future flexibility, and this is one explanation for the observed preference for commitment devices (Thaler and Benartzi (2004), Loewenstein and Prelec (1992), Beshears et al. (2011), Ashraf et al. (2006), Bucciol (2012) [3]). Additionally, there is evidence that self-control requires effort, and that exerting cognitive effort beforehand can reduce the amount of self-control available later (Shiv and Fedorikhin (1999), Baumeister et al. (1998), Dewitte et al. (2005)). However, there have not been any experiments that test whether or not exerting effort on self-control is unpleasant in itself.

Houser et al. (2010) use a similar decision problem as in this paper between a boring paid task and a tempting unpaid outside option. In their experiment subjects choose to commit to the paid task, continue the paid task, or switch to the outside option all at the same time, and the commitment lasts for the duration of the experiment. This set-up tests for a preference for commitment, but cannot test for costly self-control, or the time dependence of self-control because it is not repeated without the opportunity to commit and restrictive and non-restrictive commitment cannot be differentiated.

[2] See Loewenstein (1996) for a discussion of the consequences of visceral urges on behavior in general.

[3] For surveys of the literature see Bryan et al. (2010) and Frederick et al. (2002).

Benhabib et al. (2010) use a lab experiment to test a general model that nests exponential, hyperbolic, quasi-hyperbolic discounting, and fixed cost present bias. They find evidence for present bias in the form of a small fixed cost of $4 on average. They do not attempt to test for costly self-control or its time dependence.

The existence of costly self-control would have two major implications. First, if self-control is costly a model that ignores this fact will consistently estimate the strength of temptation incorrectly (Groves (2012b)). This would lead to inaccurate estimates and simulations. Second, costly self-control has policy implications (Groves (2012a)). Namely, if self-control is costly then people will avoid using it, even if they do not foresee themselves succumbing to temptation. This is the type of behavior that we use in this experiment to test for the existence of costly self-control.

The remainder of this unfinished paper proceeds as follows: the next section describes the experiment. Section three describes the models of time consistent individuals, and time inconsistent individuals with and without costly self-control, and presents our hypotheses. Section four discusses how these hypotheses are tested in the experiment. Section five concludes with a discussion of potential issues with the experimental design and how we will try to avoid these. The proofs are relegated to the appendix.

3.2 Experimental Design

The experiment consists of three phases:

- Phase 1: The subject begins the tedious task, being paid per second they are spend on the task. Simultaneously available is a more interesting, but unpaid activity that they can choose to do instead. If they ever choose to switch from the tedious task to the other activity they will not be able to switch back, and so will continue the interesting activity until the end of the first phase. They will keep whatever money they have earned up to the time of the switch.

- Phase 2: The subject is given two options:

  1. Commitment: the subject begins Phase 3 without the opportunity to switch from the paid task to the unpaid activity. The option to switch to the unpaid activity will instead be introduced after some set amount of time, τ_c. The length of time offered will depend on the treatment group. The first treatment group will be offered commitment times that weakly exceed the amount of time the subject spent on the tedious task before switching in Phase 1. The second treatment group will be offered commitment times that are strictly less than the amount of time spent on the tedious task in Phase 1 (non-restrictive commitment).
  2. No commitment: the subject has the option to switch present during the entirety of Phase 3.

  One treatment will be given the first option will be free while another will have to pay for it. A third treatment group will only be given the second option.

- Phase 3: The subject begins the tedious task again, with the chosen level of commitment in place. Once the commitment time has passed the option to switch to the more interesting activity will reappear.

3.3 Models, Predicted Behavior, and Hypotheses

The subject is faced with a task that begins at $t = 1$ and that the subject can do for any number of consecutive periods up to $t = T$. At any period the subject may stop performing the task, but once the subject stops then he or she can no longer perform the task in future periods. If the subject stops performing the task, then he or she engages in an outside activity for the rest of the experiment. If the subject performs the task for τ periods, then he or she is rewarded a monetary payout of τ at $t = T + 1$. The subject's utility over monetary payouts is $u(\tau)$. We assume that $u' > 0$ and $u'' \leqslant 0$, and that there is no time discounting.

Let $a_t \in \{0, 1\}$ denote the action taken at time t, with $a_t = 1$ indicating that the task was performed at time t and $a_t = 0$ indicating that the task was not performed at time t. Let $h_t = (a_1, a_2, \ldots, a_{t-1})$ denote the history of actions up to time $t - 1$. Let the flow utility to the outside activity be normalized to zero. Let the flow utility to performing the task at time t be given by $-v(\tau)$ where τ is the number of periods for which the task had been performed previously. The negative sign in front of v allows us to interpret v as the flow cost of performing the task. We allow the flow cost of performing the task to depend on the number of periods the task was performed. This allows us to capture the possibility that the task becomes more "boring" over time ($v' > 0$) or the possibility that the subject learns over time and so the mental cost of performing the task decreases with time ($v' < 0$). Regardless of whether v is increasing or decreasing, we assume that $-v(\tau) < 0$ for all τ so that myopically the task is always less attractive than the outside activity.

Hypothesis 1. *Subjects prefer the outside activity to the task if not paid for doing either.*

Subjects can reveal their preferences for the outside activity and the task if they pay for commitment or if they choose to switch from the task to the unpaid outside activity. We can also test this hypothesis by offering the choice of the outside activity and the task, both unpaid, to a group of subjects. If these subjects choose the outside activity immediately over the unpaid task this would give this hypothesis further support.

3.3.1 Time Consistency

We can easily solve for the behavior of a time consistent subject using backward recursion. Consider a subject who enters period T with a history of actions $h_T = (1, 1, \ldots, 1)$. We do not consider the case in which the subject had already stopped performing the task because such a subject has no decision to make. The subject will stop performing the task at time T if and only if:

$$u\big(T-1\big) > -v\big(T\big) + u\big(T\big)$$

So let us define the value function at time T as:

$$V^T = \max\Big\{u\big(T-1\big), -v\big(T\big) + u\big(T\big)\Big\}$$

Then at time $T-1$ the subject will stop performing the task if and only if:

$$u\big(T-2\big) > -v\big(T-1\big) + V^T$$

Continuing backwards we can define a sequence of value functions for each time period, $t = 1, \ldots, T$:

$$V^t = \max\Big\{u(t-1), -v(t) + V^{t+1}\Big\}$$

with $V^{T+1} = u(T)$.

Proposition 1. *If $v' = 0$ and $u'' = 0$, then the subject will either stop immediately at $t = 1$, or continue performing the task until the end, or will be indifferent to stopping or continuing at all times. If $v' \geqslant 0$ or $u'' \leqslant 0$ (with at least one holding strictly) then there exists a τ^*, given by $u(\tau^* - 1) = -v(\tau^*) + u(\tau^*)$, such that the subject will stop if $t > \tau^*$ and continue if $t \leqslant \tau^*$.*

In the second phase of the experiment the subject is given the choice to pay price $p \geqslant 0$ for committing to a time period $\tau_c > 0$ without the option to stop.

Hypothesis 2. *If the subject is time consistent they will never pay a positive price, p, for a positive amount of commitment,* $\tau_c > 0$.

A time consistent subject without temptation would never pay a positive price for any level of commitment because she have no use for self-control mechanisms. Such a subject would be indifferent between committing to $\tau_c \leqslant \tau^*$ for free.

3.3.2 Quasi-Hyperbolic Discounting without Self Control

Now suppose that the subject when making decisions in the current period underweights the future utilities by a factor $\beta < 1$, but weights current flow utilities by a factor of 1. This is equivalent to a $\beta - \delta$ quasi-hyperbolic discounting model with $\delta = 1$. It can also be viewed as a dual-self model in which the short-run self overweights current flows rather than behaving myopically. We assume that within phases 1 and 3 the subject is time-consistent as it simplifies notation and analysis and does not change the results in any important way. We assume for now that the "long-run" self exerts no control over the "short-run" self.

At time T, the subject will stop performing the task if and only if:

$$\beta u(T-1) > -v(T-1) + \beta u(T)$$

and the value function at time T is given by:

$$V^T = \begin{cases} \beta u(T-1) & \text{if } \beta u(T-1) > -v(T-1) + \beta u(T) \\ -v(T-1) + \beta u(T) & \text{otherwise} \end{cases}$$

At a general time t, the subject will stop performing the task if:

$$\beta u(t-1) > -v(t-1) + V^{t+1}$$

and we can write:

$$V^t = \begin{cases} \beta u(t-1) & \text{if } \beta u(t-1) > -v(t-1) + V^{t+1} \\ -v(t-1) + V^{t+1} & \text{otherwise} \end{cases}$$

Proposition 2. *If $v' = 0$ and $u'' = 0$, then the subject will either stop immediately at $t = 1$, or continue performing the task until the end, or will be indifferent to stopping or continuing at all times. If $v' \geqslant 0$ or $u'' \leqslant 0$ (with at least one holding strictly) then there exists a τ^*, given by $\beta u(\tau^* - 1) = -v(\tau^*) + \beta u(\tau^*)$, such that the subject will stop if $t > \tau^*$ and continue if $t \leqslant \tau^*$.*

In Phase 2 the subject's long-run self chooses the commitment time, τ_c. Given the short-run self's actions, the long-run self's value function will be:

$$V_{LR}^t(\tau_c) = \begin{cases} u(t-1) & \text{if } \begin{array}{l} \beta u(t-1) > -v(t-1) + V^{t+1} \\ t > \tau_c \end{array} \\ -v(t-1) + V^{t+1} & \text{otherwise} \end{cases}$$

The subject will choose to pay price p for commitment level τ_c if $V_{LR}^t(\tau_c) \geqslant V_{LR}^t(0)$. Let τ_{LR} be the time period at which the subject's long-run self would stop completing the task with no commitment, and τ_{SR} be the analogous time period for the short-run self. Given that the subject discounts all future utility at β, $\tau_{LR} \geqslant \tau_{SR}$.

Hypothesis 3. *If the subject is quasi-hyperbolic then they will be willing to pay up to price $p^* \geqslant 0$ for any level of commitment $\tau_c \in (\tau^*, \tau_{LR}]$, where $\tau^* = \tau_{SR}$.*

Because the quasi-hyperbolic subject has no self-control $\tau_{SR} = \tau^*$. A quasi-hyperbolic subject would pay for commitment only if $\tau_c \in (\tau^*, \tau_{LR}]$. If $\tau_c \leqslant \tau^*$ then it does not change her behavior. Because there is no cost of self-control this type of commitment is of no use to the subject, and she will not pay for it.

3.3.3 Dual-Self Model with Self-Control

We suppose that in each period the subject can exert self-control at a cost $c(s)$ where s is the total number of periods for which self control was previously exerted. Note that s may be different from τ, the number of periods the task was performed, if there were some periods in which the action preferred by the long-run and short-run selves did not differ, or if the subject was pre-committed to performing to task for some number of periods. We assume that the long-run self acts before the short-run self and that the short-run self plays only subgame perfect strategies (i.e. we do not consider equilibria which are supported by off-equilibrium-path threats by the short-run selves).

At time T, the short-run self will stop if:

$$\beta u(T-1) > -v(T) + \beta u(T)$$

and will unilaterally perform the task if:

$$\beta u(T-1) \leqslant -v(T) + \beta u(T)$$

The long-run self will exert self control to force the short-run self to perform the task if:

$$\begin{aligned} \beta u(T-1) &> -v(T) + \beta u(T) \\ \text{and } u(T-1) &< -v(T) + u(T) - c(s) \end{aligned}$$

i.e. if the short-run self would not unilaterally choose to continue, and if the long-run self's utility to continuing less its utility to stopping exceeds the self-control cost. We can therefore write:

$$V^T(s) = \begin{cases} -v(T-1) + \beta u(T) - c(s) & \text{if } \begin{array}{l} \beta u(T-1) > -v(T) + \beta u(T) \\ u(T-1) \leqslant -v(T) + u(T) - c(s) \end{array} \\ -v(T) + \beta u(T) & \text{if } -v(T) + \beta u(T) \geqslant \beta u(T-1) \\ \beta u(T-1) & \text{otherwise} \end{cases}$$

And more generally,

$$V^t(s) = \begin{cases} -v(t) + V^{t+1}(s+1) - c(s) & \text{if } \begin{array}{l} \beta u(t-1) > -v(t) + V^{t+1}(s) \\ u(t-1) \leqslant -v(t) + V^{t+1}(s+1) - c(s) \end{array} \\ -v(t) + V^{t+1}(s) & \text{if } -v(t) + V^{t+1}(s) \geqslant \beta u(t-1) \\ \beta u(t-1) & \text{otherwise} \end{cases}$$

Proposition 3. *If $v' = 0$, $u'' = 0$, and $c'(s) = 0$ then the subject will either stop immediately at $t = 1$, or continue performing the task until the end, or will be indifferent to stopping or continuing at all times. If $v' \geqslant 0$, $u'' \leqslant 0$, or $c'(s) > 0$ (with at least one holding strictly) then there exists a τ^*, given by $\beta u(\tau^* - 1) = -v(\tau^*) + \beta u(\tau^*)$, such that the subject will stop if $t > \tau^*$ and continue if $t \leqslant \tau^*$.*

The long-run self's value function with commitment level $\tau_c$is,

$$V^t(s, \tau_c) = \begin{cases} -v(t) + V^{t+1}(s+1) - c(s) & \text{if } \begin{array}{l} \beta u(t-1) > -v(t) + V^{t+1}(s) \\ u(t-1) \leqslant -v(t) + V^{t+1}(s+1) - c(s) \\ t > \tau_c \end{array} \\ -v(t) + V^{t+1}(s) & \text{if } \begin{array}{c} -v(t) + V^{t+1}(s) \geqslant \beta u(t-1) \\ \text{or } t \leqslant \tau_c \end{array} \\ u(t-1) & \text{otherwise} \end{cases}$$

Let $s^*(\tau_c)$ denote the level of s that maximizes $V^0_{LR}(s, \tau_c)$. The long-run self will pay p for τ_c if $V^0_{LR}(s^*(\tau_c), \tau_c) > V^0_{LR}(s^*(0), 0)$. Denote τ_1^* the switching time in Phase 1 and τ_3^* the switching time in Phase 3.

Hypothesis 4. *If the subject is DS with self-control then they will be willing to pay up to price $p^* \geqslant 0$ for any level of commitment $\tau_c \in (\tau_{SR}, \tau_{LR}]$, where $\tau_{SR} \leqslant \tau^*$. Additionally, if $\tau_c \in (\tau_{SR}, \tau_1^*]$, then $\tau_3^* > \tau_1^*$.*

Unlike a quasi-hyperbolic subject, $\tau_{SR} \leqslant \tau^*$ for a DS subject with self-control because she can use self-control to force her short-run self to continue the task for a longer time span. Such a subject would pay a positive price for any $\tau_c \in [\tau_{SR}, \tau_{LR}]$. Exerting costly self-control is what allows the subject to continue the task past time τ_{SR}, so if $\tau_c \geqslant \tau_{SR}$ and is relatively cheap then this is a better alternative to using internal self-control.

Commitment allows the subject to avoid using their internal self-control, delaying the first period in which it is used. Therefore, if the cost of self-control is increasing over time any level of commitment $\tau_c \geqslant \tau_{SR}$ will decrease the cost of self-control for all $t > \tau_c$ and thereby increase the length of time the subject will continue the task before switching.

3.4 Methods

This experiment is designed so that the hypotheses are testable through qualitative observations.

Phase 1 will be used to estimate the time at which the subject is indifferent between the net present value of the paid task and the net present value of the unpaid activity. Phases 2 and 3 are used to measure the value for different types of commitment. If a subject chooses to take the no commitment option when the commitment option is free then based on their subsequent choices in Phase 3 we will be able to say that their level of temptation is either low or nonexistent (Hypothesis 2), or that they are naive about their level of temptation. The treatment group for whom the length of the commitment time is greater or equal to the amount of time they spent working on the paid task before switching to the more unpaid activity in Phase 1 will test whether commitment is a desirable option (hypotheses (3) and (4)), i.e. whether or not they have a problem controlling themselves and know this. The treatment group who is offered commitment lengths that are strictly less than the amount of time they spent working on the paid task before switching will test for costly self-control. If self-control is costly then even when the subject is completing the paid task they are exerting self-control to avoid switching to the unpaid activity (Hypothesis 4)). Commitment, even when it does not directly affect actions, alleviates this cost, and therefore would be desirable. If the subjects in the treatment with non-restrictive commitment switch in Phase 3 after they did in Phase 1 then we know that self-control becomes more costly as it is used.

3.5 Discussion

Currently the task requires the subject to watch a computer screen where dots are displayed above and below a line for a brief period of time. When the dots are removes the subject is asked whether there were more dots above or below the line. This is repeated in steady intervals that the subject has no control over for the entirety of phases 1 and 3 or until they choose the outside option. During this task there is an option on the computer screen to quit and surf the internet instead. This option is always present in Phase 1, and will not be present for whatever the commitment length is in Phase 3.

For the experiment we are going to assume that the long run utility, $u(\cdot)$, is linear. This should be a reasonable assumption given the relatively small payouts, but can and will be tested through the use of a control group who is not given the opportunity to commit to anything in Phase 2. If it turns out that a concave long run utility is necessary to explain the subjects' actions then we may be able to use the control group for calibration, though this is not ideal.

We are assuming that subjects will learn how tedious the task is during Phase 1. We suspect that this task will become more tedious over time, and that this effect may carry over from Phase 1 to Phase 3, which would be problematic. In order to combat this we are going to let the subject rest and eat between Phases 1 and 3. If this does not work we will try holding Phases 1 and 3 on different days, though this may introduce an attrition problem.

A problem that we have run into is designing a task that is sufficiently boring that subjects will quit doing it before the end of the phase. We may try to reduce the per period payoff for performing the task, but this may lead subjects to just quit immediately in the Phase 3 because the task is not worth their while. A better solution may be to design an outside option that becomes increasingly tempting, or perhaps having them perform the task in an increasingly unpleasant situation, such as in a room that becomes hot or cold. The problem is to find an outside option that gives an immediate but ephemeral payoff.

3.6 Proofs

Proof of Proposition 1

Proof. Let us first consider the case of $v' = 0$ and $u'' = 0$. Let $v(x) = v$ and let $u(x) = x$ (slope and intercept won't matter). Suppose $v > 1$. Then it is easy to check that $V^t = t - 1$ and that the subject will stop immediately. Suppose instead that $v < 1$. In this case we can check that $V^t = -(T - t + 1)v + T$ and that the subject continues performing the task until the end. Finally, if $v = 1$ then $V^t = -(T-t+1)v+T = t-1$ and the subject is indifferent in each period between continuing or stopping.

Now let us consider cases in which $v' \geqslant 0$ and $u'' \leqslant 0$ (with one holding strictly). These assumptions imply that $u(t) - u(t-1)$ is weakly decreasing in t and that $v(t)$ is weakly increasing in t. Together they imply that $u(t-1) > -v(t) + u(t)$ for $t > \tau^*$ and $u(t-1) \leqslant -v(t) + u(t)$ for $t \leqslant \tau^*$. It is easy to check that for $\tau^* < t \leqslant T$, we have $V^t = u(t-1)$ and thus that stopping is optimal for every $t > \tau^*$. Now denote by t^* the largest integer t such that $t \leqslant \tau^*$. For this t^* we have $V^{t^*+1} = u(t^*)$ and we also know that $u(t^*-1) \leqslant -v(t^*) + u(t^*)$, so the subject will continue performing the task at t^*, and $V^{t^*} = -v(t^*) + u(t^*)$. So the proposition holds for $t = t^* \leqslant \tau^*$. To see that it holds for the remaining t, we first postulate the form of the value functions:

$$V^t = -v(t) - v(t+1) - v(t+2) - \ldots - v(t^*) + u(t^*)$$

So suppose that this holds for $t + 1$; we will show that it holds for t. We need to compare $u(t-1)$ with $-v(t)+V^{t+1}$. We know that:

$$\begin{aligned}
u(t-1) &\leqslant -v(t)+u(t) \\
&\leqslant -v(t)-v(t+1)+u(t+1) \\
&\leqslant -v(t)-v(t+1)-v(t+2)+u(t+2) \\
&\vdots \\
&\leqslant -v(t)-v(t+1)-v(t+2)-\ldots-v(t^*)+u(t^*) \\
&= -v(t)+V^{t+1}
\end{aligned}$$

which proves the rest of the proposition. □

Proof of Proposition 2

Proof. Let us first consider the case of $v'=0$ and $u''=0$. Let $v(x)=v$ and let $u(x)=x$ (slope and intercept won't matter). Suppose $v>\beta$. Then it is easy to check that $V^t=t-1$ and that the subject will stop immediately. Suppose instead that $v<\beta$. In this case we can check that $V^t=-(T-t)v+\beta T$ and that the subject continues performing the task until the end. Finally, if $v=\beta$ then $V^t=-(T-t)v+\beta T=\beta(t-1)$ and the subject is indifferent in each period between continuing or stopping.

The remainder of the proof is analogous to that for Proposition 1. □

Proof of Proposition 3

Proof. $\Rightarrow$ $v'=0$,$u''=0$, and $c'(s)=0$:

Let $v(x)=v$, $u(x)=x$, and $c(x)=c$. Note that when $c>(\frac{1}{\beta}-1)v$ the cost of self-control plus the continuation cost for the long-run self is greater or equal to the continuation cost of the short-run self. Also notice that whenever the short-run self wants to continue so does the long-run self, since $v<\frac{1}{\beta}v$.

If $c > (\frac{1}{\beta} - 1)v$ then self-control is too costly for the long-run self to exercise, so the analysis is identical to Proposition 2. Now assume that $c \leqslant (\frac{1}{\beta} - 1)v$. If $v \leqslant \beta$, $V^t = -(T-t)v + T$, the short-run self will either be indifferent or want to continue to the end and the long-run self will have a strict preference to continue to the end. We assume that when the short-run self is indifferent, the long-run self's preference determines the outcome. If $1 - c > v > \beta$, so $c \in (0, 1-\beta)$, the value function will be $V^t = -(T-t)(v-c) + T$: the short-run self will want to stop immediately, the long-run self will not and will use self-control to force the short-run self to continue until time T. If $v > 1$ then both selves will stop immediately. If $v = 1$ then the long-run self is indifferent in each period between stopping and continuing.

$\Rightarrow$ $v' \geqslant 0$, $u'' \leqslant 0$, and $c' = 0$:

Assume one inequality holds strictly. Therefore $u(t)-u(t-1)$ is weakly decreasing in t, and $v(t)$ is weakly increasing in t. Together this implies that $u(t-1) > -\frac{1}{\beta}v(t) + u(t)$ for $t > \tau_{SR}$ and $u(t-1) \leqslant -\frac{1}{\beta}v(t) + u(t)$ for $t \leqslant \tau_{SR}$. There exists a similar cutoff time period for the long-run self, τ_{LR}. Because $\beta < 1$, $\tau_{SR} < \tau_{LR}$. For any $t > \tau_{LR}$, $V^t = u(t-1)$ and therefore stopping is optimal. Denote by t_{SR} and t_{LR} the largest integers such that $t_{SR} \leqslant \tau_{SR}$ and $t_{LR} \leqslant \tau_{LR}$, respectively. At $t = t_{SR}$, $V^t(s) = u(t-1)$ and $u(t-1) \leqslant -\frac{1}{\beta}v(t) + u(t)$, so the subject will continue performing the task. The proof that this is true for $t < t_{SR}$ is analogous to the proof in propositions 1 and 2. For any $c > 0$, there exists a $\tau_c \in [0, \tau_{LR})$ such that $u(t-1) > -v(t) + u(t) - c$ for $t > \tau_c$, and $u(t-1) > -v(t) + u(t) - c$ for $t \leqslant \tau_c$. Given that $\beta < 1$ there exists a $\bar{c} > 0$ such that $-\frac{1}{\beta}v(\tau_{SR}) + u(\tau_{SR}) = -v(\tau_{SR}) + u(\tau_{SR}) - \bar{c}$. If $c \in [0, \bar{c}]$ then $\tau^* = \tau_c \in [\tau_{SR}, \tau_{LR}]$. For any $t > \tau^*$, $V^t = u(t-1)$ and stopping is optimal. Let t^* be the largest integer such that $t^* \leqslant \tau^*$. At $t = t^*$, $V^t = u(t-1)$ and $u(t-1) \leqslant -v(t) + u(t) - c$, so the subject will continue performing the task, and $V(t_c) = -v(t_c) + u(t) - c$. So the proposition holds at t^*. Above we proved that for $t \leqslant t_{SR} < t^*$ the subject will continue performing the task, therefore we need to show that for $t \in (t_{SR}, t^*)$ the subject performs the task. To do this, guess that the value function takes the following forms:

$$V^t = -v(t) - c - v(t+1) - c - ... - v(t_c) - c + u(t_c)$$

and the proof proceeds as in Proposition 1. If $c > \bar{c}$ then $-\delta v(\tau_{SR}) + u(\tau_{SR}) > -v(\tau_{SR}) + u(\tau_{SR}) - c$ and self-control is never used, so $\tau^* = \tau_{SR}$. Therefore the proposition holds for constant c.

$\Rightarrow \quad v' \geqslant 0$, $u'' \leqslant 0$, and $c' > 0$:

Assume one holds strictly. Self-control is only used when the short-run and long-run selves disagree on continuing or stopping. Therefore the analysis of this situation for $t \leqslant t_{SR}$ and after $t \geqslant t_{LR}$ is analogous to above. If $c(1) > \bar{c}$ then self-control is not used and $\tau^* = \tau_{SR}$. Assume that $c(1) \leqslant \bar{c}$. We know that $v(\tau_{SR} + s) + c(s)$ is increasing in s and $u(\tau_{SR} + s) - u(\tau_{SR} + s - 1)$ is decreasing in s, and that $u(\tau_{SR} - 1) \leqslant -v(\tau_{SR}) + u(\tau_{SR})$ by definition. This means that there exists an $s^* > 0$ and a $\tau^* \in [\tau_{SR}, \tau_{LR})$ such that $u(\tau^*) = -v(\tau^*) + u(\tau^*) - c(s^*)$. Proving that the subject continues when $t \leqslant \tau^*$ and stops when $t > \tau^*$ works the same way as previously shown.

$\Rightarrow \quad v' = 0$, $u'' = 0$, and $c' > 0$:

If $c(1) > (\frac{1}{\beta} - 1)v$ then self-control is too costly, is never used, and the subject either continues to T if $v < \beta$, stops immediately if $v > \beta$, or is indifferent between the two if $v = \beta$. Now assume that $c(1) \leqslant (\frac{1}{\beta} - 1)v$. If $v \leqslant \beta$ the short-run self weakly prefers to continue until time T and the long-run self strictly prefers to do so, therefore no self-control will be used and the subject will continue until time T. If $v < 1 - c(1)$, because $c(\tau)$ is increasing in τ there exists an $\tau^* > 0$ such that $v = 1 - c(\tau^*)$, and for $t > \tau^*$, $v < 1 - c(t)$, while for $t \leqslant \tau^*$, $v \geqslant 1 - c(t)$. If

$v \in [\beta, 1 - c(1))$ then the short-run self want to stop immediately and the long-run self wants to continue initially and so uses self-control to force the short-run self to continue. Showing that the subject will continue until τ^* follows the same reasoning as before. $\square$

Table 3.1: Summary of Results for Time Consistent Model

u	v	τ^*	Pay for commitment?
Linear	Flat	Stop immediately, continue to the end, or indifferent to all stopping times	No
Linear	Increasing	Unique stopping time	No
Linear	Decreasing	???	No
Concave	Flat	Unique stopping time	No
Concave	Increasing	Unique stopping time	No
Concave	Decreasing	???	No

Table 3.2: Summary of Results for Quasi-Hyperbolic Discounting Model

u	v	τ^*	Pay for commitment?	Stopping time with commitment?
Linear	Flat	Stop immediately, continue to the end, or indifferent to all stopping times. Stopping time is not longer than in the time-consistent case.	Only if commitment time is longer than TC stopping time	Max of commitment time vs. QH time
Linear	Increasing	Unique stopping time, shorter than under TC	Only if commitment time is longer than TC stopping time	Max of commitment time vs. QH time
Linear	Decreasing	???	???	
Concave	Flat	Unique stopping time, shorter than under TC	Only if commitment time is longer than TC stopping time	Max of commitment time vs. QH time
Concave	Increasing	Unique stopping time, shorter than under TC	Only if commitment time is longer than TC stopping time	Max of commitment time vs. QH time
Concave	Decreasing	???	???	

Table 3.3: Summary of Results for Dual-Self Model with Self Control

u	v	c	τ^*	Pay for commitment?	Stopping time with commitment?
Linear	Flat	Flat	Stop immediately, continue to the end, or indifferent to all stopping times. Stopping time is not longer than in the time-consistent case.	No	Max of commitment time vs. DS time
Linear	Flat	Increasing	Unique stopping time, between TC and QH	Only for an amount of time that falls within the SR/LR disagreement time span.	May be longer than both commitment time and DS time
Concave	Increasing	Flat	Unique stopping time, between TC and QH	Only for an amount of time that falls within the SR/LR disagreement time span.	Max of commitment time vs. DS time
Concave	Increasing	Increasing	Unique stopping time, between TC and QH	Only for an amount of time that falls within the SR/LR disagreement time span.	May be longer than both commitment time and DS time

4

Optimal Commitment and the Type of Self-Control

Abstract

The main purpose of this paper is to describe how different assumptions about temptation and self-control affect the optimal borrowing and saving contracts in a simple stochastic income setting. I find that the optimal savings plan when temptation is extreme and myopic consists of a mandatory minimum level of savings, while the optimal plan for mild myopic and farsighted temptation can have several fully restrictive mandatory savings levels, with only the highest being a mandatory minimum. When temptation is mild and myopic money burning will never be part of an optimal separating contract, though it can be when temptation is farsighted. Also, when temptation is myopic and extreme an individual will strictly prefer mandatory deposits to liquidity constraints. Finally, I find that the optimal borrowing plan when self-control is costly consists of regular payments and self imposed debt limit.

4.1 Introduction

The $\beta\delta$ model of hyperbolic discounting assumes that an individual discounts more between consumption today and tomorrow than between any two days in the future. This frame dependent impatience can be reinterpreted as an overwhelming temptation to consume more now, whenever now is. Since the individual has no internal self-control commitment is valuable because she can use it as external self-control. In the $\beta\delta$ model the individual is tempted to consume more than her previous self would have liked, though she is not tempted to consume everything now because her temptation takes into account future consumption.

Gul and Pesendorfer (2001) describe a model of temptation and costly self-control where the individual is tempted to consume everything today, without regard for the future. However, she can exert costly internal self-control. Commitment can be used to supplement or replace her internal self-control. One could also imagine combinations of these two models, such as myopic temptation that finds something less drastic than consuming everything today to be the most tempting option. Or, farsighted temptation with internal self-control.

The main purpose of this paper is to describe how different assumptions about how temptation and self-control work affect the optimal borrowing and saving contracts in a simple stochastic income setting. I use a generalized version of the Gul and Pesendorfer (2001) (henceforth GP) model that can encompass different types of myopic as well as far-sighted forms of temptation. This allows me to illustrate the affects of myopia versus farsightedness, extreme myopic temptation (desire to consume everything now) versus milder levels of myopic temptation, and limited self-control versus none.

I find that the optimal savings plan when temptation is extreme and myopic consists of a mandatory minimum level of savings, while the optimal plan for mild temptation can have several fully restrictive mandatory savings levels, with only the highest being a mandatory minimum. When an individual is tempted to consume all of her wealth immediately no matter what her level of wealth is, then there is no way to create a state contingent contract that defines different levels of saving or borrowing for different realized states. Think about this as a principal-agent model: if all of the agents have the same type of preferences then there is no way to make the IC constraints bind at different allocations for different agents. However, when the individual has a milder form of temptation, a contract can be designed that assigns state specific levels of consumption that are more tempting given one realized income level versus another, so at least some states can be separated given a subtle enough contract. In a separating contract the lower mandatory saving levels must be completely restrictive (not a mandatory minimum), otherwise agents that are assigned higher savings levels will find this lower saving level more tempting because it offers them the flexibility to reach their most tempting level of savings. Because the individual is time consistent she values the flexibility to choose to save more than a mandatory amount when it does not increase her cost of self-control, hence the highest mandatory savings level being a mandatory minimum and not fully restrictive.

In a separating contract when temptation is mild and myopic money burning will never be optimal, though it can be when temptation is farsighted. When temptation is myopic burning money is just as effective as increasing savings by the same amount because both reduce the amount of immediate consumption available. Therefore the individual is better off getting the money back and consuming more tomorrow than throwing it all away. When temptation takes into account future consumption this is no longer the case and burning money offers another tool with which temptation can be regulated.

I also analyze the difference between liquidity constraints (an IRA or CD with pre-defined date of maturity) and mandatory deposits (automatic paycheck deduction). I find that an individual would strictly prefer mandatory deposits to liquidity constraints. This is surprising given that most existing savings devices have a liquidity constraint as one of their main components. For instance, as just noted, IRAs and CDs, and additionally pensions, 401(k) plans, and life insurance to name a few. However, often times these liquidity constraints are coupled with mandatory deposits in the form of automatic paycheck deductions. Mandatory minimum deposits are preferred to liquidity constraints because they are more flexible: any liquidity constraint can be exactly reproduced by an appropriately designed series of mandatory minimum deposits, but the reverse is not true.

Finally, I find that the optimal borrowing plan when self-control is costly consists of regular payments and self imposed debt limit. Additionally, an individual would choose to borrow more with the optimal plan than without because borrowing and repayment become less costly due to the reduction in the cost of self-control.

In terms of the optimal savings mechanism literature this paper is most closely related to Amador et al. (2006) and Ambrus and Egorov (2012). Those papers analyze a stochastic preference setting using the $\beta\delta$ model. The first paper defines conditions in which a mandatory minimum savings level is optimal. The second paper extends and amends the first, showing that money burning can be optimal in separating equilibria, but only when mandatory savings is zero for some types of preference shocks. In my paper it is income that is stochastic instead of preferences, and the model of utility is more general. This results in the additional finding that for some forms of utility money burning may be optimal even when mandatory savings is greater than zero.

Fudenberg and Levine (2012) analyze general types of decision making and the affects of assumptions on the farsightedness of temptation, how the cost of self-control changes under different levels of temptation, and if and how willpower is depleted. In this paper I do not deal with willpower depletion. From the empirical data (Baumeister et al. (1998); Dewitte et al. (2005)) it seems as though depletion of willpower would come into play only when consumption decisions take place in rapid succession over a time span of minutes or maybe hours, and generally savings decisions occur on the order of days or weeks instead of minutes, though naturally a more thorough analysis should be conducted. Finally, their paper does not derive the optimal saving and borrowing mechanisms under these various assumptions on temptation, which is the main contribution of this paper.

The only other paper that I am aware of that studies borrowing and temptation is Fischer and Ghatak (2010)[1]. Their paper uses the $\beta\delta$ model shows that an individual would be willing to borrow more if she can commit to a regular repayment schedule than she would if she could not commit. They offer this as a reason why most microfinance institutions require regular predefined payments after a loan is taken out.

This paper proceeds as follows: the next section introduces the generalized GP model and the specializations that I analyze. Section 3 covers optimal commitment savings with the first subsection focusing on a two income setting, and then continuing on to the continuous income setting. Section 4 covers optimal commitment borrowing. Sections 5 and 6 extend the models to more periods. Finally Section 7 concludes. All proofs that are not present in the main paper have been relegated to the appendix.

4.2 The Model

There is a bank with which the individual can sign binding contracts. It also offers an interest rate, r, for the individual's savings that is large enough so that non-zero

[1] Basu (2008) studies a general equilibrium setting using the $\beta\delta$ model in which there is a profit maximizing bank that offers loans and a welfare maximizing microfinance bank that offers savings.

savings is optimal for some level of income. The bank has no profit or costs, and makes no decisions.

There are three periods: 0, 1, 2. In period 0 the individual makes a choice about commitment and consumes nothing. In period 1 stochastic income, y, is realized and she decides how much to save, s, and consume, $c \leqslant y - s$. The pdf of the distribution, $f(y)$, has a lower bound of $y_{min} > 0$. Finally in period 2 the individual receives income, y_2, which is deterministic, and her savings and consumes $c_2 \leqslant y_2 + s$. In the context of the model specified below, assuming that y_2 is deterministic is without loss of generality. The second period income and the pdf for period 1 income are known to both the individual and the bank. Realized period 1 income is the individual's private information. The budget sets for periods 1 and 2 are: $B \equiv \left\{(c, s) \in \mathbb{R}_+^2 | c + s \leqslant y\right\}$ $B_2 \equiv \{c_2 \in \mathbb{R}_+ | c_2 \leqslant y_2 + (1 + r)s\}$.

A generalized version of Gul and Pesendorfer (2001) model is used to describe an individual with time-consistent preferences that exhibit temptation and costly self-control. $u : \mathbb{R}_+^2 \to \mathbb{R}$ is her *commitment* utility and is continuous, concave, and increasing and time separable.

$V : \mathbb{R}_+^2 \to \mathbb{R}$ is her *temptation* utility and is continuous, but may be concave or convex. If it is convex it is increasing monotonically and depends only on consumption in the immediate period: $V(c, c_2) = v(c)$. If it is concave it is increasing initially and then decreasing, and may or may not depend on consumption in both periods. That is $V(c, c_2) = v(c)$ or $V(c, c_2) = v(c) + \beta\delta v(c_2)$. If it is dependent on both periods then it is time separable. A temptation utility that depends only on the immediate period's consumption is a model of myopic temptation. Allowing myopic temptation to peak before maximal consumption models milder forms of temptation, and will be referred to as *mild myopic* temptation. Monotonically increasing myopic temptation will be referred to as *extreme myopic* temptation. Temptation that depends on period 2 consumption is a model of *farsighted* temptation, something akin to impatience. A particularly useful type of farsighted temptation introduced by Krusell et al. (2010) is $V(c, c_2) = [u(c) + \beta\delta u(c_2)]$, and will be referred to as KKS temptation. The main purpose of this paper is to analyze how different assumptions about temptation and self-control can affect the optimal borrowing and saving contracts.

I assume that $\frac{\partial V}{\partial c} < \frac{\partial u}{\partial c}$ and $\frac{\partial V}{\partial c_2} < \frac{\partial u}{\partial c_2}$ so that temptation peaks before commitment utility.

Because $\arg\max_{\tilde{c}_2 \in B_2} V(c, \tilde{c}_2) = \arg\max_{c_2 \in B_2} u(c) + \delta u(c_2) = y_2 + (1+r)s$ the individual will always consume along her budget constraint and c_2 is only a function of s. To simplify notation I will write $\delta u(s)$ and $V(c, s)$ with the understanding that $c_2 = y_2 + s$, not just s.

Her expected utility in period 1, where γ is the strength of self-control, is

$$U_1 = u(c) + \gamma V(c, s) - \gamma \max_{\{\tilde{c}, \tilde{s}\} \in B} V(\tilde{c}, \tilde{s}) + \delta u(s).$$

The individual's period 0 expected utility is,

$$\mathbb{E}U = \mathbb{E}\left[u(c) + \gamma V(c, s) - \gamma \max_{\{\tilde{c}, \tilde{s}\} \in B} V(\tilde{c}, \tilde{s}) + \delta u(s)\right].$$

$-\gamma \left[\max_{\{\tilde{c}, \tilde{s}\}} V(\tilde{c}, \tilde{s}) - V(c, s)\right]$ is the individual's *net cost* of self-control, and is always negative unless $s = \tilde{s}$. $-\max_{\{\tilde{c}, \tilde{s}\}} V(\tilde{c}, \tilde{s})$ is the *cost* of self-control. As γ goes to zero the net cost of self-control also goes to zero and consumption is determined by the commitment utility. As γ increases towards infinity the net cost of self-control goes to infinity as well, the individual always succumbs to temptation in periods 1 and 2, and the temptation utility determines her consumption decisions. With KKS temptation it is the individual's impatience, $\beta < 1$, that results in a temptation to consume more today than she would if she could commit, and the level of impatience governs how much more she is tempted to consume. When $\gamma \to \infty$ the individual has no self-control and always succumbs to temptation, in which case the individual's actions approach those predicted by the time-inconsistent $\beta\delta$ model.

A preference for commitment is driven by the disagreement between the optimal levels of consumption for the commitment utility and the temptation utility and the resulting net cost of self-control. Commitment constrains the available options which can reduce the net cost of self-control. For instance, assuming deterministic income, if the individual commits to a particular level of savings in both periods, s, then $B(y) = \{y - s, s\}$ and $B_2(s) = \{y + (1 + r)s\}$. This reduces the net cost of self-control to zero and $\mathbb{E}U = \mathbb{E}[u(c) + \delta u(s)]$.

Given y, the first best savings is defined as $\left\{c^{fb}, s^{fb}\right\} = \arg\max_{\{c,s\}\in B} u(c) + \delta u(s)$ and could be obtained if y were verifiable instead of private information. This is the utility maximizing savings rate because the net cost of self-control is zero. The most tempting consumption and savings levels are defined as $\{\tilde{c}, \tilde{s}\} = \arg\max_{\{c,s\}\in B} V(c, s)$. The flexible savings level is defined as $\{\overline{c}, \overline{s}\} = \arg\max_{\{c,s\}\in B} u(c) + V(c, s) + \delta u(s)$. This is the level of savings chosen when either there is no commitment available or the commitment does not restrict the individual's saving decision. Non-restrictive commitment could occur if there is a mandatory minimum savings level.

4.3 Optimal Commitment

In this section I assume that $r = 1$ to simplify notation. This means that $y_2 < y_{min}$ in order for savings to be desirable for all levels of income. The following analysis is not affected by this assumption.

In period 0 the individual commits to a savings plan $\hat{S}(y) = \left\{(\hat{c}, \hat{s}) \in \mathbb{R}^2_+ | \hat{c} + \hat{s} \leqslant y\right\}$. A savings plan effectively eliminates a subset of the feasible savings levels that can be chosen in period 1. This affects the net cost of self-control by defining the set of most tempting options, $\tilde{S}$, which impacts the $V(\tilde{c}(y), \tilde{s}(y))$ term. Whereas the individual's period 1 choice of a level of savings takes the set of most tempting options as given and influences the net cost of self-control via the $V(c(y), s(y))$ term. The goal in both periods is to strike a balance between the benefits of commitment utility and the net cost of self-control.

$\hat{B}(y)$ is the feasible set of consumption and savings levels given realized income, y, and commitment choices $\hat{c}(y)$ and $\hat{s}(y)$. $\hat{B}(y) = \left\{(c, s) \in \mathbb{R}^2_+ | s + c \leqslant y, c \leqslant \hat{c}(y), s \leqslant \hat{s}(y)\right\}$ if the individual is restricted to a mandatory minimum, and $\hat{B}(y) = \{\hat{c}(y), \hat{s}(y)\}$ if she is fully restricted.

The optimal plan will maximize period 0 expected utility, taking into account the period 1 savings decision (the first constraint), and what the most tempting alternative is given realized income (the second constraint, referred to as the *temptation constraint*)

$$\max_{\{\hat{c}(y),\hat{s}(y)\}\in\hat{S}(y)} \mathbb{E}\left[u(c(y)) + V(c(y), s(y)) - V(\hat{c}(y), \hat{s}(y)) + \delta u(s(y))\right] \quad (4.1)$$

$$s.t. \qquad s(y) = \arg\max_{\{c,y\}\in\hat{B}(y)} u(c) + V(c, s) - V(\hat{c}(y), \hat{s}(y))$$

$$V(\hat{c}(y), \hat{s}(y)) \geqslant V(\hat{c}(y'), \hat{s}(y')) \quad \forall y, y' \in [y_{min}, y_{max}]$$

Because the individual is time consistent I will speak of separating and pooling maxima. A separating maximum assigns different mandatory savings levels to different subsets of possible income, while a pooling maximum assigns only one mandatory savings level for all possible levels of income.

4.3.1 Two Possible Income Levels

First we will focus on a simple two state setting to illustrate the optimization problem and the effects of assumptions about the temptation utility, $V(c(y), s(y))$. Let p be the probability that $y = y_h$ and $(1 - p)$ be the probability that $y = y_l$, where $y_h > y_l > 0$. To reduce notation, define $c_h = c(y_h)$, $s_h = s(y_h)$, and c_l, s_l similarly. Also, $\hat{B}^l_h = \hat{B}(y_h, \hat{c}_l, \hat{s}_l)$, $\hat{B}^h_h = \hat{B}(y_h, \hat{c}_h, \hat{s}_h)$, and $\hat{B}^l_l$, $\hat{B}^h_l$ similarly.

Therefore the individual's time zero optimization problem is:

$$\max_{\{\hat{c}_h,\hat{s}_h\}\in\hat{S}(y_h)} p\left[u_h(c_h) + \gamma V_h(c_h, s_h) - \gamma V_h(\hat{c}_h, \hat{s}_h) + \delta u(s_h)\right]$$

$$+\max_{\{\hat{c}_l,\hat{s}_l\}\in\hat{S}(y_l)}(1-p)\left[u_l(c_l) + \gamma V_l(c_l, s_l) - \gamma V_l(\hat{c}_l, \hat{s}_l) + \delta u(s_l)\right] \tag{4.2}$$

$$s.t. \quad \{c_i, s_i\} = \arg\max_{\{c,s\}\in\hat{B}_i^i} u_i(c) + \gamma V_i(c,s) - \gamma V_i(\hat{c}_i, \hat{s}_i) + \delta u(s), \quad i \in \{h, l\} \tag{4.3}$$

$$\max_{\{c,s\}\in\hat{B}_h^h} V_h(c,s) \geqslant \max_{\{c,s\}\in\hat{B}_h^l} V_h(c,s)$$

$$\max_{\{c,s\}\in\hat{B}_l^l} V_l(c,s) \geqslant \max_{\{c,s\}\in\hat{B}_l^h} V_l(c,s)$$

In any optimal contract $\arg\max_{\{c,s\}\in\hat{B}_h^h} V_h(c,s) = \{\hat{c}_h, \hat{s}_h\}$ and $\arg\max_{\{c,s\}\in\hat{B}_l^l} V_l(c,s) = \{\hat{c}_l, \hat{s}_l\}$, otherwise the contract would not reduce the net cost of self-control. Therefore the two temptation constraints above simplify to

$$V_h(\hat{c}_h, \hat{s}_h) \geqslant \max_{\{c,s\}\in\hat{B}_h^l} V_h(c,s)$$

$$V_l(\hat{c}_l, \hat{s}_l) \geqslant \max_{\{c,s\}\in\hat{B}_l^h} V_l(c,s)$$

There are two possible types of commitment savings devices in this setting: a fully restricted savings level, and a mandatory minimum savings level. Given a particular level of income both are equally effective at reducing the net cost of self-control when $\hat{s}_i > \tilde{s}_i$. A mandatory minimum is more flexible though, allowing the individual to choose to save above the minimum, which she will do when $\hat{s}_i < \overline{s}_i$. However, since $\hat{s}_l < \hat{s}_h$, low income commitment savings and consumption must be fully restricted, otherwise $V_h(\hat{c}_h, \hat{s}_h) < \max_{\{c,s\}\in\hat{B}_h^l} V_h(c,s)$. Also, notice that $\hat{s}_h > \tilde{s}_l$ in any optimal contract otherwise the contract would not reduce the cost of self-control for either level of income. Therefore $\max_{\{c,s\}\in\hat{B}_l^h} V_l(c,s) = V_l(\hat{c}_h, \hat{s}_h)$, and the commitment savings device for high income can be a mandatory minimum since the low income temptation constraint must hold.

Lemma 1. *The high income temptation constraint will bind unless* $\hat{s}_h = s_h^{fb}$ *and* $\hat{s}_l = s_l^{fb}$.

This lemma and the reasoning above means that we can ignore the low income temptation constraint and further simplify the high income temptation constraint to $V_h(\hat{c}_h, \hat{s}_h) = V_h(\hat{c}_l, \hat{s}_l)$. Therefore $\hat{s}_h$ can be treated as a function of $\hat{s}_l$ and vice versa, which will be useful when proving that an optimal contract exists in the next subsection.

There are two types of maxima that can exist in this situation: pooling maxima, which consist of a mandatory minimum level of savings, $\hat{s}$, that is the same for both levels of income; and separating maxima, where a mandatory minimum level of savings, $\hat{s}_h$, is set for the high level of income, and a completely restricted single level of savings, $\hat{s}_l$, is assigned for the low income realization. In a pooling maximum $\hat{s}_h = \hat{s}_l = \hat{s} \in \left[\max\left\{s_l^{fb}, \tilde{s}_h\right\}, s_h^{fb}\right]$. If $\hat{s} > s_h^{fb}$ then the individual would be strictly better off with a mandatory minimum of $\hat{s} = s_h^{fb}$ no matter what level of income is realized because s_h^{fb} is the optimal savings level when income is high, and $s_l^{fb} < s_h^{fb}$. Similarly if $\hat{s} < s_l^{fb}$. If $\hat{s} < \tilde{s}_h$ then setting $\hat{s}_l = \hat{s}$ and $\hat{s}_h = \tilde{s}_h + \epsilon$, for some small ϵ, would make the individual strictly better off.

In a separating maximum $\hat{s}_l < \tilde{s}_h < \hat{s}_h$. If instead $\tilde{s}_h < \hat{s}_l < \hat{s}_h$ then no matter which income is realized $\hat{s}_h$ would never be the most tempting option and therefore would be ineffective in reducing the net cost of self-control. Alternatively, if $\hat{s}_l < \hat{s}_h < \tilde{s}_h$ then increasing $\hat{s}_h$ would strictly increase utility when high income is realized because it would decrease the cost of self-control, and it would not affect utility when low income is realized.

The following three subsections illustrate the effects of assumptions about temptation on the possible forms the optimal savings contract can take. The next subsection and the subsequent one prove that despite the fact that the individual is time consistent money burning, $\hat{c}_i + \hat{s}_i < y_i$, can be part of an optimal savings contract. The third shows that when temptation is myopic and extreme the global maximum will always be pooling.

General Farsighted Temptation

In this subsection $V(c,s) = v(c) + \beta\delta v(s)$.

Theorem 3. *At least one maximum will exist.*

- *For any pooling maximum* $\hat{s} \in \left[\max\left\{s_l^{fb}, \tilde{s}_h\right\}, s_h^{fb}\right]$ *and no money will be burned.*

- *For any separating maximum* $\hat{s}_h \geqslant \max\left\{\tilde{s}_h, s_l^{fb}\right\}$*, and* $\hat{s}_l < \tilde{s}_h$*. For any maximum with money burned and* $\hat{s}_l > 0$ *there exists another maximum with a higher savings level and no money burned.*

Proof.

1. First it is shown when the first best allocations are obtainable.

2. Then I prove that the optimization problem is indeed continuous and an optimal contract will always exist.

3. Finally, I show when money burning will be part of a maximum.

1. If $v_h(c_h^{fb}) + \beta\delta v(s_h^{fb}) \geqslant v_h(c_l^{fb}) + \beta\delta v(s_l^{fb})$ then set $\{\hat{c}_h, \hat{s}_h\} = \left\{c_h^{fb}, s_h^{fb}\right\}$ and $\{\hat{c}_l, \hat{s}_l\} = \left\{c_l^{fb}, s_l^{fb}\right\}$. Because $v_h(c_h^{fb}) + \beta\delta v(s_h^{fb}) \geqslant v_h(c_l^{fb}) + \beta\delta v(s_l^{fb})$, the first best allocations for each income realization will be the most tempting for each income realization. Therefore the global maximum will be a separating maximum that assigns the first best allocations for each income realization.

2. It was shown above that $\hat{s} \in \left[\max\left\{s_l^{fb}, \tilde{s}_h\right\}, s_h^{fb}\right]$ in a pooling maximum and $\hat{s}_h \geqslant \max\left\{\tilde{s}_h, s_l^{fb}\right\}$, $\hat{s}_l < \tilde{s}_h$ in a separating maximum. Assuming that there is no

money burning ($\hat{c}_i + \hat{s}_i = y_i$ for $i = l, h$), the marginal expected utility for the high income realization when the optimal contract is separating are:

$$\hat{s}_h \geqslant \overline{s}_h : \quad p\left[-u_h'\left(\hat{c}_h\right) + \delta u'\left(\hat{s}_h\right)\right] \tag{4.4}$$

$$\overline{s}_h > \hat{s}_h > \tilde{s}_h : \quad p\gamma\left[v_h'(\hat{c}_h) - \beta\delta v'(\hat{s}_h)\right] \tag{4.5}$$

At $\hat{s}_h = \overline{s}_h$ the marginal utilities are equal, so the marginal utility of $\hat{s}_h$ for the high income realization is continuous with a kink at $\hat{s}_h = \overline{s}_h$. Equation (4.4) is always decreasing. It is positive when $\hat{s}_h < s_h^{fb}$ and negative when $\hat{s}_h > s_h^{fb}$. Equation (4.5) is positive and increasing.

The marginal expected utilities for the high income realization when the optimal contract is pooling are:

$$\overline{s}_h > \hat{s} \geqslant s_l^{fb} : \quad p\gamma\left[v_h'(\hat{c}_h) - \beta\delta v'(\hat{s})\right] \tag{4.6}$$

$$\hat{s} \geqslant \overline{s}_h : \quad p\left[-u_h'(\hat{c}_h) + \delta u'(\hat{s})\right] \tag{4.7}$$

Again, when $\hat{s} = \overline{s}_h$ equations (4.6) and (4.7) are equal.

By Lemma 1 the temptation constraint for the high income realization binds. Therefore we can treat $\hat{s}_h$ as an implicit function of $\hat{s}_l$, and vice versa. This allows us to create a continuous function where equations (4.4) and (4.5) are multiplied by $\frac{\partial \hat{s}_h}{\partial \hat{s}_l} < 0$, and hold when $\hat{s}_l < \tilde{s}_h$, and equations (4.6) and (4.7) hold when $\hat{s}_h = \hat{s}_l = \hat{s} \geqslant \tilde{s}_h$. This function will have kinks at $\hat{s}_l = \hat{s}_h(\overline{s}_h)$ and $\hat{s}_l = \hat{s} = \overline{s}_h$.

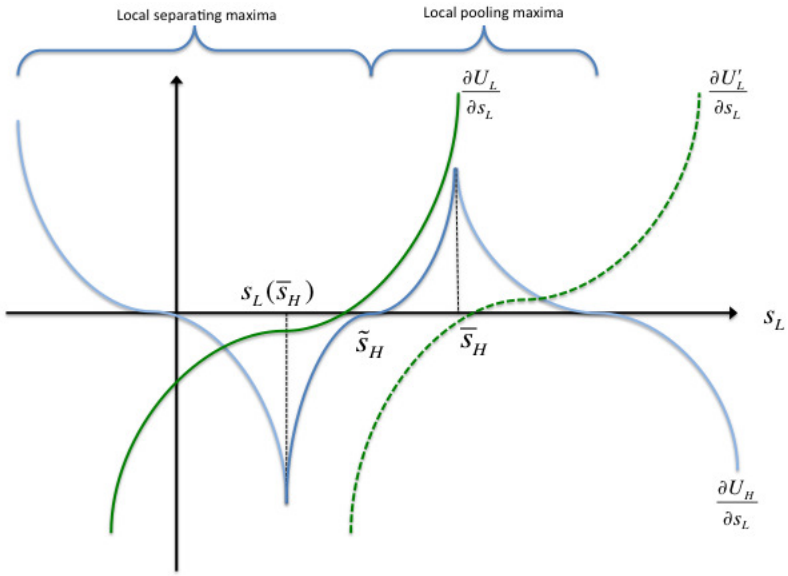

FIGURE 4.1: The blue line is the high income marginal utility. The green line is the low income marginal utility. The blue line crosses the axis at $\hat{s}_l(s_h^{fb})$, $\hat{s}_l = \hat{s} = \tilde{s}_h$, and $\hat{s} = s_h^{fb}$. The green line crosses once at $\hat{s}_l = s_l^{fb}$. When $\frac{\partial U_l}{\partial s_l}$ =the solid green line there will be a unique separating maximum without money burning, and no pooling maximum. When $\frac{\partial U_l}{\partial s_l}$ = the dashed green line the global maximum will be pooling.

Because the optimal contract is always fully restrictive for the low income realization the marginal utility for the low income realization is $(1-p)\left[-u_l'(\hat{c}_l) + \delta u'(\hat{s}_l)\right]$, which is continuous and decreasing in $\hat{s}_l$ (and $\hat{s}$). In the optimal contract the marginal expected utilities when income is high and low must equal each other. Therefore the optimization problem is continuous, and always has at least one solution.

3. Money burning will never be part of a pooling maximum because the individual will always be made strictly better off by saving this money instead. However, money burning can be part of a separating maximum. The individual's optimization problem, taking into account Lemma 1 and the fact that the contract will be fully

restricting when income is low:

$$\max_{\{\hat{c}_h,\hat{s}_h\}\in\hat{S}(y_h)} p\left[u_h(c_h)+\gamma V_h(c_h,s_h)-\gamma V_h(\hat{c}_h,\hat{s}_h)+\delta u(s_h)\right]$$
$$+\max_{\{\hat{c}_l,\hat{s}_l\}\in\hat{S}(y_l)}(1-p)\left[u_l(\hat{c}_l)+\delta u(\hat{s}_l)\right]$$
$$s.t. \quad \{c_h,s_h\}=\arg\max_{\{c,s\}\in\hat{B}_h^h} u_h(c)+\gamma V_h(c,s)-\gamma V_i(\hat{c}_h,\hat{s}_h)+\delta u(s)$$
$$V_h(\hat{c}_h,\hat{s}_h)=V_h(c,\hat{s}_l)$$

First assume that money burning is non-zero when y_l is realized but zero when y_h is realized. The first order conditions of the Lagrangian with respect to $\hat{c}_h$, $\hat{c}_l$, $\hat{s}_h$, and $\hat{s}_l$, where λ_1 and λ_2 are the multipliers for the high income temptation constraint and the budget constraint for low income, respectively, are

$\hat{s}_h\in(\tilde{s}_h,\overline{s}_h)$:

$$\frac{\partial\mathcal{L}}{\partial\hat{s}_h} = \gamma(\lambda-p)\left[-v_h'(\hat{c}_h)+\beta\delta v'(\hat{s}_h)\right]=0 \tag{4.8}$$

$\hat{s}_h\geqslant\overline{s}_h$:

$$\frac{\partial\mathcal{L}}{\partial\hat{s}_h} = p\left[-u_h'(\hat{c}_h)+\delta u'(\hat{s})\right]+\lambda_1\gamma\left[-v_h'(\hat{c}_h)+\beta\delta v'(\hat{s}_h)\right]=0 \tag{4.9}$$

The first order conditions for $\hat{s}_l$ and $\hat{c}_l$ are ($\lambda_2=0$: assuming money is burned)

$$\frac{\partial\mathcal{L}}{\partial\hat{s}_l} = (1-p)\delta u'(\hat{s}_l)-\lambda_1\gamma\beta\delta v'(\hat{s}_l)=0 \tag{4.10}$$

$$\frac{\partial\mathcal{L}}{\partial\hat{c}_l} = (1-p)u_l'(\hat{c}_l)-\lambda_1\gamma v_h'(\hat{c}_l)=0 \tag{4.11}$$

Equation (4.8) cannot hold unless $\lambda=p$. Rearranging these last two equations results in the following condition:

$$\frac{u_l'(\hat{c}_l)}{v_h'(\hat{c}_l)}=\frac{u'(\hat{s}_l)}{\beta v'(\hat{s}_l)}=\frac{p\gamma}{1-p} \tag{4.12}$$

which can be satisfied for some functional forms of $v(\cdot)$ and $u(\cdot)$ and parameter values. When $\hat{s}_h\geqslant\overline{s}_h$ Equation (4.9) holds and the first order conditions result in the slightly modified requirement:

$$\frac{u_l'(\hat{c}_l)}{v_h'(\hat{c}_l)} = \frac{u'(\hat{s}_l)}{\beta v'(\hat{s}_l)} = \frac{-p\left[-u_h'(\hat{c}_h) + \delta u'(\hat{s}_h)\right]}{(1-p)\left[-v_h'(\hat{c}_h) + \beta\delta v'(\hat{s}_h)\right]} \tag{4.13}$$

which again can be satisfied, and for a larger class of functions and set of parameters than the previous condition since there is an additional degree of freedom with the inclusion of $\hat{s}_h$. Note here that $\hat{s}_h < s_h^{fb}$ in order for the equation to hold.

The FOCs when $\lambda_2 \neq 0$ (no money burning) for $\hat{s}_h \in (\tilde{s}_h, \overline{s}_h)$ and $\hat{s}_h \geqslant \overline{s}_h$ are

$$\begin{aligned}
\frac{-u_l'(\hat{c}_l) + \delta u'(\hat{s}_l)}{-v_h'(\hat{c}_l) + \beta\delta v'(\hat{s}_l)} &= \frac{p\gamma}{1-p} \text{ and} \\
\frac{-u_l'(\hat{c}_l) + \delta u'(\hat{s}_l)}{-v_h'(\hat{c}_l) + \beta\delta v'(\hat{s}_l)} &= \frac{-p\left[-u_h'(\hat{c}_h) + \delta u'(\hat{s}_h)\right]}{(1-p)\left[-v_h'(\hat{c}_h) + \beta\delta v'(\hat{s}_h)\right]},
\end{aligned}$$

respectively. In order for either equation to hold, $\hat{s}_l \in \left(s_l^{fb}, \tilde{s}_h\right)$. The left side of this equation will vary between zero as $\hat{s}_l$ approaches s_l^{fb}, and infinity, when $\hat{s}_l \to \tilde{s}_h$. Therefore, when condition (4.12) and/or (4.13) can be satisfied, so can these equations. Therefore, for every maximum with money burning and $\hat{s}_l > 0$ there is a maximum with no money burning and a larger savings level. However, given the general forms for commitment and temptation utility we cannot conclude which results in a larger expected utility. Therefore it may be feasible to have money burned even when $\hat{s}_l > 0$.

When money burning is non-zero when y_h is realized and zero when y_l is realized results in analogous analysis. If money burning is non-zero for both income realizations then the first order conditions result in the following constraint:

$$\frac{u_l'(\hat{c}_l)}{v_h'(\hat{c}_l)} = \frac{u'(\hat{s}_l)}{\beta v'(\hat{s}_l)} = \frac{u_h'(\hat{c}_h)}{v_h'(\hat{c}_h)} = \frac{u'(\hat{s}_h)}{\beta v'(\hat{s}_h)} \tag{4.14}$$

which can never hold because $\frac{u_l'(\hat{c}_l)}{v_h'(\hat{c}_l)} \neq \frac{u_h'(\hat{c}_h)}{v_h'(\hat{c}_h)}$.

If instead $\hat{s}_l = 0$ then the $\frac{u'(\hat{s}_l)}{\beta v'(\hat{s}_l)}$ terms in equations (4.12) and (4.13) will disappear and the conditions will be satisfied by a larger set of parameters and functional forms. □

As the proof above illustrates, a pooling maximum balances the gain in marginal utility from a reduction in the cost of self-control when income is high with the loss in marginal utility due to forced over-saving when income is low. A separating maximum balances the gain from the cost reduction with the loss from either forced over or under saving. More precise analytical results about when the global maximum is separating or pooling, and when a separating maximum includes money burning requires more restrictions on the temptation utility. These additional restrictions will also help elucidate the difference between the strength of temptation and the level of impatience.

KKS Temptation

KKS temptation is defined as $\gamma\left[v(c) + \beta\delta v(s)\right] = \gamma\left[u(c) + \beta\delta u(s)\right]$. Defining the model of temptation in this way results in further restrictions on when money burning can be part of a separating maximum.

Corollary 2. *For KKS temptation money burning will be part of a maximum only if* $\hat{s}_h < s_h^{fb}$ *and* $\hat{s}_l = 0$*. When* $\gamma \to \infty$ *there exists at most one separating local maximum and one pooling local maximum.*

Proof. With KKS temptation condition (4.10) becomes

$$\delta u'(\hat{s}_l)\left[(1-p) - \lambda\gamma\beta\right] \quad = \quad 0 \tag{4.15}$$

When $\hat{s}_h \in (\tilde{s}_h, \overline{s}_h)$, $\lambda = p$ and Equation (4.15) only holds when $\gamma = \frac{1-p}{p\beta}$. Combining this with Equation (4.10) then requires

$$\frac{u_l'(\hat{c}_l)}{u_h'(\hat{c}_l)} = \frac{1}{\beta}$$

If this equation can hold for some combination of $\hat{c}_l$ and $\hat{s}_l$ such that $\hat{c}_l + \hat{s}_l < y_l$, then it will hold for any combination $\hat{c}_l$ and $\hat{s}'_l > \hat{s}_l$, since it is not a function of $\hat{s}_l$. In particular it will hold when no money is burned. Converting burned money one for one into save money strictly increases expected utility, therefore no money will be burned in a maximum when $\hat{s}_l > 0$ and $\hat{s}_h \in (\tilde{s}_h, \overline{s}_h)$.

When $\hat{s}_h \geqslant \overline{s}_h$ the Lagrange multiplier is defined by Equation (4.15): $\lambda = \frac{(1-p)}{\gamma\beta}$, and Equation (4.13) becomes

$$\frac{u'_l(\hat{c}_l)}{u'_h(\hat{c}_l)} = \frac{-p\left[-u'_h(\hat{c}_h) + \delta u'(\hat{s}_h)\right]}{(1-p)\left[-u'_h(\hat{c}_h) + \beta\delta u'(\hat{s}_h)\right]} = \frac{1}{\beta}.$$

This condition results in the same conclusion as the one above. Therefore no money will be burned in a maximum when $\hat{s}_l > 0$.

When $\gamma \to \infty$ the region $(\tilde{s}_h, \overline{s}_h)$ collapses and the marginal utility for high income approaches a discontinuous function that is negative and decreasing for $\hat{s}_l(\hat{s}_h) < \tilde{s}_h$, and positive and decreasing for $\hat{s} > \tilde{s}_h$. The marginal utility for low income is monotonically increasing, crossing zero at s_l^{fb}. Therefore there can be at most one separating local maximum and one pooling local maximum. □

As $\gamma \to \infty$ temptation becomes overwhelming because the net cost of self-control goes to infinity. Therefore in period 1 the individual succumbs fully to temptation, which means that $\overline{s}_h \to \tilde{s}_h$. So the size of the region in which equations exist (4.5) and (4.6) approaches zero, as illustrated in the Figure (4.2). This means that there can exist at most one separating and one pooling maximum.

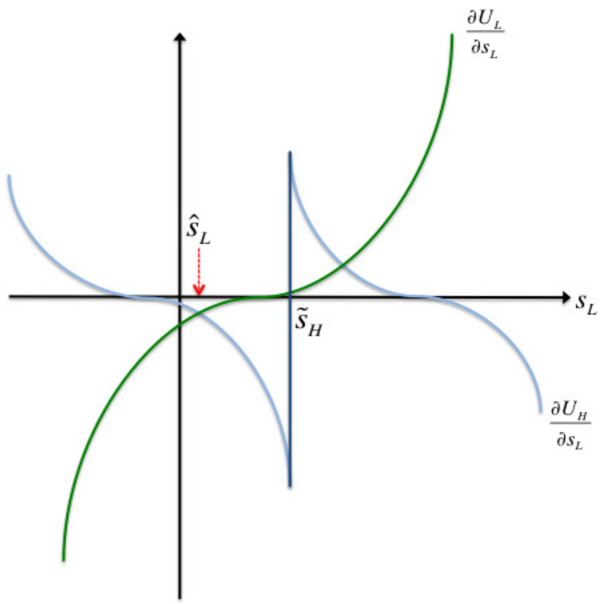

FIGURE 4.2: KKS as $\gamma \to \infty$.

Theorem 4. *When $\gamma \to \infty$, there exists a $\underline{\beta}$ such that for any $\beta > \underline{\beta}$ the unique global maximum will be separating.*

Proof. Assume that $\hat{s}$ is the pooling maximum. Therefore $\hat{s}_l = \hat{s}_h = \hat{s}$ and the high income temptation constraint naturally holds.

$$u_h(\hat{c}_h) + \beta\delta u(\hat{s}_h) = u_h(\hat{c}_l) + \beta\delta u(\hat{s}_l)$$

Suppose we decrease $\hat{s}_l$ and increase $\hat{c}_l$ so that the right side stays constant. $\hat{s}_l$ must decrease by more than $\hat{c}_l$ increases since $\hat{s}_l < \tilde{s}_h$.

$$u_h'(\hat{c}_l) - \delta\beta u'(\hat{s}_l) = 0$$

The increase in $\hat{c}_l$ will increase the marginal utility when income is low by even more: $u_l'(\hat{c}_l) > u_h'(\hat{c}_l)$. The decrease in $\hat{s}_l$ and increase in $\hat{c}_l$ will increase the objective function only if

$$\begin{aligned} u_l'(\hat{c}_l) - \delta u'(\hat{s}_l) &> 0 \\ \Rightarrow u_l'(\hat{c}_l) - \frac{1}{\beta} u_h'(\hat{c}_l) &> 0 \end{aligned}$$

Because the FOC for a pooling maximum put no constraints on β, and because $u_l'(\hat{c}_l) > u_h'(\hat{c}_l)$, there always exists a $\underline{\beta}$ such that for any $\beta > \underline{\beta}$ the separating maximum will be the global maximum.

□

Myopic Temptation

Myopic temptation is defined by $V(c, s) = v(c)$. *Mild myopic* temptation is defined by a concave increasing and then decreasing $v(c)$. When $v(c)$ is increasing monotonically it is *extreme myopic* temptation.

Corollary 3. *Separating maxima may exist if $V(c, s)$ is mild myopic. Furthermore, money burning will never be part of any maximum.*

Proof. A mild myopic $V(c, s)$ will behave similarly to a time separable $V(c, s)$ and so the proof of existence will be the same when there is no money burning. Because $V(c, s)$ depends only on consumption in period 1 burning money and saving have an identical effect on the cost of self-control. Changing burned money into savings will strictly increase the individual's utility, therefore money will never be burned in a maximum. □

Corollary 4. *Separating maxima will not exist when $V(c, s)$ is extreme myopic. If $V(c, s)$ is convex then there is a unique pooling global maximum.*

Proof. When $V(c, s)$ is increasing monotonically the only time the temptation constraint for either income level can bind is when $\hat{s}_h = \hat{s}_l$. No separating maximum can exist because as long as $\hat{s}_l < \hat{s}_h$ the individual will always be most tempted to save $\hat{s}_l$ ($\arg\max_{\{c,s\}\in B_l^l \cup B_h^l} V_l(c,s) = \arg\max_{\{c,s\}\in B_l^h \cup B_h^h} V_h(c,s) = \{\hat{c}_l, \hat{s}_l\}$), no matter what level of income is realized.

The marginal expected utility when income is high for a pooling maximum is

$$\overline{s}_h > \hat{s} \geqslant s_l^{fb} : \quad p\gamma V_h'(\hat{c}_h, \hat{s}) \quad = 0 \tag{4.16}$$

$$s_h^{fb} \geqslant \hat{s} \geqslant \overline{s}_h : \quad p\left[-u_h'(\hat{c}_h) + \delta u'(\hat{s})\right] \quad = 0 \tag{4.17}$$

This is a continuous, monotonically decreasing function with a kink at $\hat{s} = \overline{s}_h$, that equals zero when $\hat{s} = s_h^{fb}$. The marginal expected utility when income is low is continuous and increases from zero when $\hat{s} = s_l^{fb}$. Therefore a unique optimal pooling contract exists. □

When temptation is extreme the most tempting option is always to consume all wealth today, therefore $\tilde{s}_h = \tilde{s}_l = 0$. This means that the size of the separating maxima region goes to zero. Going back to KKS temptation, when β is large the optimal contract will be separating, and as Corollary (3) proves, when $\beta = 0$ the optimal contract will be pooling. As temptation becomes overwhelming, $\gamma \to \infty$, either type of contract could be optimal, and when $\gamma = 0$ the optimal contract is separating. Therefore low impatience and low temptation have a similar effect, but large impatience and overwhelming temptation do not necessarily. This is because when both are small the individual becomes a normal exponential discounter, but when impatience becomes large she becomes more myopic, while when temptation becomes overwhelming future consumption may still tempt her.

4.3.2 A Continuum of Possible Income Levels

Except for particular types of functional forms for the temptation utility, $V(c,s)$, it is difficult to say precisely when maxima will be pooling or separating in the continuous income case. Here I will focus on describing when separating maxima do not exist and when there is a unique pooling maximum. Given that there is no money burning in pooling maxima, $c = y - s$, and c is dropped to simplify notation.

For a general form for temptation the individual's expected utility given a mandatory minimum savings level, $\hat{s}$, is as follows:

$$\begin{aligned}
\mathbb{E}U \quad = \quad & \underbrace{\int_{\tilde{y}}^{y_{max}} \left(u(y-s) + \gamma V(y,s) - \gamma V(y,\tilde{s}) + \delta u(s)\right) f(y)dy}_{\text{No effect}} \\
& \underbrace{\int_{\overline{y}}^{\tilde{y}} \left(u(y-s) + \gamma V(y,s) - \gamma V(y,\hat{s}) + \delta u(s)\right) f(y)dy}_{\text{Decreased cost of self-control}} \\
& + \underbrace{\int_{y^{fb}}^{\overline{y}} \left(u(y-\hat{s}) + \delta u(\hat{s})\right) f(y)dy}_{\text{Decreased temptation}} \qquad (4.18) \\
& + \underbrace{\int_{y_{min}}^{y^{fb}} \left(u(y-\hat{s}) + \delta u(\hat{s})\right) f(y)dy}_{\text{Reduced flexibility}}
\end{aligned}$$

Where $\overline{y}$ is defined by the first order condition for savings with temptation, y^{fb} is defined by the first order condition for savings without temptation, and $\tilde{y}$ is level of income for which $\hat{s}$ is the most tempting level of savings or y_{max}, whichever is less:

$$\overline{y}: \quad -u'(\overline{y}-\hat{s}) + \gamma V'(\overline{y},\hat{s}) + \delta u'(\hat{s}) = 0 \qquad (4.19)$$

$$y^{fb}: \quad -u'(y^{fb}-\hat{s}) + \delta u'(\hat{s}) = 0 \qquad (4.20)$$

$$\tilde{y}: \quad \gamma V'(\tilde{y},\hat{s}) = 0 \qquad (4.21)$$

The optimal level of commitment, $\hat{s}$, solves the first order condition of $\mathbb{E}U$:

$$\frac{\partial \mathbb{E}U}{\partial \hat{s}} = \underbrace{\int_{\overline{y}}^{\tilde{y}} -\gamma V'(y,\hat{s}) f(y) dy}_{\substack{>0 \\ \Rightarrow \text{ Reducing the cost of self-control}}}$$

$$+ \underbrace{\int_{y^{fb}}^{\overline{y}} \left[-u'(y-\hat{s}) + \delta u'(\hat{s})\right] f(y) dy}_{\substack{>0 \\ \Rightarrow \text{ Decreasing temptation}}} \tag{4.22}$$

$$+ \underbrace{\int_{y_{min}}^{y^{fb}} \left[-u'(y-\hat{s}) + \delta u'(\hat{s})\right] f(y) dy}_{\substack{<0 \\ \Rightarrow \text{ Restricting choice in a negative way}}} = 0$$

As equations (4.19) , (4.21), and (4.21) show, the individual will choose to save less when she is tempted than when she is not, so $\tilde{y} > \overline{y} > y^{fb}$. Equation (4.22) illustrates that $\hat{s}$ only affects temptation when income is between $\overline{y}$ and y^{fb}, hence the reduction in temptation. Above $\overline{y}$ the mandatory minimum deposit reduces the cost of self-control by restricting the maximum possible consumption, but it has no effect on temptation in this range of income because $\hat{s}$ is not binding. For $y > \tilde{y}$ $\hat{s}$ does not bind nor does it reduce the most tempting option. At low levels of income, below y^{fb}, the individual would ideally save less than $\hat{s}$, but cannot, so the mandatory minimum deposit reduces utility in this range. The optimal $\hat{s}$ balances these three forces.

Because the individual is not permitted to borrow $\hat{s}$ cannot be greater than y_{min}. $y_{min} > 0$ is reasonable if the individual is entering the setting with some savings, or if she receives a nonzero income with certainty.

Theorem 5. *At least one $\hat{s}$ exists. If $V(y, \hat{s})$ is either convex or KKS with $\gamma \to \infty$, there is a unique $\hat{s}$. When $V(y, \hat{s})$ is monotonically increasing then there are only pooling maxima.*

When $V(y, s)$ is increasing monotonically the most tempting option for all levels of income is to consume everything today. Because the individual always finds the same option the most tempting no matter her realized level of income there is no way to segregate the different income levels using different savings options. Therefore a mandatory minimum is the only type of maximum possible for this type of temptation. When $V(y, s)$ is convex as well then $\frac{\partial^2 \mathbb{E}U}{\partial \hat{s}^2} < 0$, so it has a unique maximum.

When $V(y, s)$ is KKS and temptation is overwhelming self-control is never exercised, which means that $\overline{y} \to \tilde{y}$ and the first integral in Equation (4.22) disappears. The remaining integrals are strictly concave, so there is a unique maximum.

4.4 Borrowing

In this section I focus on borrowing, abstracting away from savings. The optimal borrowing plan for the GP model in the three period setting works in a very similar manner to the optimal savings plan, except a debt limit plays the part of the mandatory minimum deposit in decreasing the cost of self-control. In equation (4.23) $\overline{b}$ is the debt limit, b is the amount borrowed when the debt limit is not binding. Consumption in the second period is $c_2 = y_2 - (1 + q)b$, where q is the interest rate. The limit will not bind for higher income draws because the individual will not desire to borrow much in that situation, though it will bind for lower income draws. In this scenario borrowing has two costs, interest and the cost of self-control.

$$\begin{aligned}
\mathbb{E}U &= \underbrace{\int_{\tilde{y}}^{y_{max}} \Big(u(y+b)+V(y,b)-V(y,\tilde{b})+\delta u(b)\Big) f(y)dy}_{\text{No effect}} \\
&\quad \underbrace{\int_{\overline{y}}^{\tilde{y}} \big(u(y+b)+V(y,b)-V(y,\overline{b})+\delta u(b)\big) f(y)dy}_{\text{Decreased cost of self-control}} \\
&\quad + \underbrace{\int_{y^{fb}}^{\overline{y}} \big(u(y+\overline{b})+\delta u(\overline{b})\big) f(y)dy}_{\text{Decreased temptation}} \qquad (4.23) \\
&\quad + \underbrace{\int_{y_{min}}^{y^{fb}} \big(u(y+\overline{b})+\delta u(\overline{b})\big) f(y)dy}_{\text{Reduced flexibility}}
\end{aligned}$$

The reasoning behind the optimal borrowing mechanism is parallel to that of the optimal savings mechanism.

Theorem 6. *At least one $\overline{b}$ exists. If $V(y,\overline{b})$ is either convex or KKS there is a unique $\overline{b}$. When $V(y,\overline{b})$ is monotonically increasing then there are only pooling maxima.*

If $V(y,b)$ is concave and myopic and $f(y)$ is either uniform or monotonically decreasing then $\overline{b}$ is unique as well.

4.5 Saving, more time periods

In this section I focus on the simple GP model in an extended setting with an additional time period. This allows for the comparison of liquidity constraints (an IRA or

CD with pre-defined date of maturity) and mandatory deposits (automatic paycheck deduction). I find that an individual would strictly prefer mandatory deposits to liquidity constraints. This is surprising given that most existing savings devices have a liquidity constraint as one of their main components. For instance, as just noted, IRAs and CDs, and additionally pensions, 401(k) plans, and life insurance to name a few. However, often times these liquidity constraints are coupled with mandatory deposits in the form of automatic paycheck deductions[2]. Mandatory minimum deposits are preferred to liquidity constraints because they are more flexible: any liquidity constraint can be exactly reproduced by an appropriately designed series of mandatory minimum deposits, but the reverse is not true. This could potentially explain the anomaly presented in Noor (2007)

> Self-control problems have been put forward as an explanation for the apparent undersaving in the U.S. The earlier noted models imply that undersavers do not need added incentives to participate in saving schemes such as 401(k) and IRAs which provide a means to commit to saving for retirement. Yet such saving schemes have substantial tax benefits associated with them: all contributions are tax deductible. Furthermore, participation in these schemes is closely related to the tax benefits. For instance, IRA contributions fell by 62% when the Tax Reform Act of 1986 excluded higher-income groups from tax benefits [Venti and Wise (1987), Poterba, Venti, Wise (2001)]. The fall in participation took place although there was no change in the commitment aspect of IRAs (early withdrawal penalties). This suggests that the appeal of such saving vehicles is primarily the tax benefits, not their commitment value [Akerlof, Gale, Hall (1998)]

Perhaps the liquidity constraints imposed by the 401(k)s and IRAs are not flexible enough, and so when the incentives are reduced they become much less attractive. If instead the savings plan consisted of the more flexible mandatory minimum deposits without liquidity constraints maybe this decline would not have been so precipitous.

The liquidity constraints examined are absolute in the sense that if the person decides to save an amount s_t in the current period she will not have access to $\alpha_t \in [0, 1]$ of the savings again until the predefined date of maturity. Additionally in this

[2] Though I have not thoroughly examined them yet I suspect that other models of temptation will have a similar preference for mandatory deposits over liquidity constraints because there is nothing in these models that matures at any certain future date.

section I assume that $v(c)$ is convex for convenience. A four period model in which the savings matures in period three, with compounding interest rate, r (period 0 is the date in which the level and type of commitment is decided upon) when there is a necessary expenditure x, takes the following form:

$$\begin{aligned}\mathbb{E}U(\alpha_1) =& \mathbb{E}\left[u(y_1 - s_1) + v(y_1 - s_1) - v(y_1)\right.\\ &+ \delta\mathbb{E}\left[u(y_2 + (1+r)(1-\alpha_1)s_1 - s_2)\right.\\ &+ v(y_2 + (1+r)(1-\alpha_1)s_1 - s_2) - v(y_2 + (1+r)(1-\alpha_1)s_1)]\\ &+ \left.\left.\delta^2\mathbb{E}u(y_3 - x + (1+r)^2\alpha_1 s_1 + (1+r)s_2)\right]\right]\end{aligned}$$

The parallel four period model with mandatory deposits instead is:

$$\begin{aligned}\mathbb{E}U(\hat{s}_1, \hat{s}_2) =& \mathbb{E}\left[u(y_1 - s_1) + v(y_1 - s_1) - v(y_1 - \hat{s}_1)\right.\\ &+ \delta\mathbb{E}\left[u(y_2 + (1+r)s_1 - s_2)\right.\\ &+ v(y_2 + (1+r)s_1 - s_2) - v(y_2 + (1+r)s_1 - \hat{s}_2))\\ &+ \left.\left.\delta^2\mathbb{E}u(y_3 - x + (1+r)s_2)\right]\right]\end{aligned}$$

The mandatory deposits are assumed to be set at the optimal level for the analysis in this subsection. Because $v(c)$ is convex these exist and are unique for any continuous distribution. The individual must commit to one or the other of these two saving regimes. As they are modeled above the two savings plans are draconian versions of an IRA and an automatic paycheck deduction savings plan respectively.

Liquidity constraints do not dictate the level of savings as in the mandatory minimum deposit regime, hence some temptation and self-control costs are present: $v(y_1 - s_1) - v(y_1)$ in the first period, and $v(y_2 + (1+r)(1-\alpha_1)s_1 - s_2) - v(y_2 + (1+r)(1-\alpha_1)s_1)$ in the second. However, liquidity constraints do damp the self-control cost by tightening the available budget constraint. On the other hand, as savings becomes less liquid the individual has less access to the interest income and, naturally, to her savings. Mandatory minimum deposits allow her more flexibility in the second period. Because there are still self-control costs with liquidity constraints the person will prefer mandatory deposits to liquidity constraints.

Theorem 7. *A person with costly self-control will strictly prefer optimal mandatory deposits to optimal liquidity constraints.*

Any liquidity constraint α_t can be recreated with a mandatory deposit of $\hat{s}_{t+1} = \alpha_t s_t + r s_t$, where s_t is the amount saved in time t, and r is the interest rate. Mandatory minimum deposits are more effective in reducing the cost of self-control if the individual is expected to save part of her realized income (that is, if $\hat{s}_t > (1+r)\hat{s}_{t-1}$) because this additional reduction in the her self-control cost cannot be achieved with liquidity constraints.

In addition, during the first period a mandatory deposit can be defined and thereby reduce the cost of self-control, whereas there cannot be a liquidity constraint. Therefore even if somehow the optimal mandatory deposit level for each subsequent period is identical to the liquidity constraints this first period will make the individual strictly prefer the optimal mandatory deposits to liquidity constraints.

Liquidity constraints and mandatory payments will be identical whenever it is possible that income will be zero, but may not be otherwise. If we do not allow for borrowing then if zero income has positive probability the most a particular mandatory minimum payment can be is the amount of savings plus interest income from the previous period, $\hat{s}_t = (1 + r)s_{t-1}$. This and any amount less than this can be replicated with a liquidity constraint. Once minimum income is greater than zero then the mandatory payment can be larger than savings plus interest from the previous period and thus liquidity constraints and mandatory payments may not be equivalent. As mentioned in the proof for Theorem 7 no matter what in the first period liquidity constraints and mandatory payments would be different, because no liquidity constraint can exist in the first period.

Theorem 7 suggests an experiment that tests people's preference for mandatory payments over liquidity constraints. If the results are consistent with Theorem 7 then many current saving mechanisms (IRAs, 401(k)s and so on) should remove their focus on long term liquidity constraints. If the data reveal that people instead prefer liquidity constraints over mandatory payments then a more elaborate model will be necessary. This could include a cognitive cost from thinking about financial decisions often (see Ergin and Sarver (2010), and Conlisk (1988)).

4.6 Borrowing, extended model

This section focuses on borrowing and regular repayment with a simple GP model. With the additional period, and a simple interest rate (instead of compounding) the optimal mandatory borrowing plan has two elements: a debt limit as before, and regular mandatory payments[3].

The individual's expected utility would take the following form in a four period model with a simple interest rate and a loan that is taken out in period 1 and repaid in period 3:

$$\begin{aligned}\mathbb{E}[U] = \mathbb{E}\big[&u(y_1 + b - x) + v(y_1 + b - x) - v(y_1 + \bar{b} - x) \\ &+ \delta\mathbb{E}\left[u(y_2 - a) + v(y_2 - a) - v(y_2 - \hat{a}) + \delta\mathbb{E}\left[u(y_3 - (1+q)b + a)\right]\right]\big]\end{aligned}$$

A debt limit, $\bar{b}$, and mandatory payment, $\hat{a}$, that are identical to the desired level of borrowing and repayment would be ideal. The individual cannot rely on her own truthful report of income, and so the best that she can do is to tie both the mandatory repayment and budget constraint to expected wealth.

Theorem 8. *The optimal borrowing plan will consist of predefined payments made each period and a debt limit, both of which are functions of the income distributions.*

[3] If interest is compounding then the payments can be rolled into the debt limit since compounding interest is like taking out a new loan each period.

Loans in this setting have two types of costs: interest, and the self-control required to avoid borrowing up the the debt limit plus the self-control necessary to make oneself repay in the second period. Mandatory payments prevent the individual from delaying repayment of the loan until later periods, which increases her overall expected utility from a loan of any size. This means that instituting mandatory payments that are optimized with respect to the distribution for income actually increases the size of the loan the individual would like to take out.

Theorem 9. *The amount the person would like to borrow is greater when there are optimized mandatory payments than when payments are completely flexible.*

Theorem 9 is analogous to the main result in Fischer and Ghatak (2010), although here the effect on welfare is not ambiguous: welfare strictly increases because mandatory payments allow the individual to move closer to her first-best, full commitment borrowing level. This is one explanation of why most microfinance institutions require regular fixed payments that begin almost immediately after the loan is taken out[4].

4.7 Conclusion

Because the precise form of temptation does have an affect on the optimal borrowing and saving mechanisms further research is needed about what exactly people find tempting. Also, temptation is not the only thing that can affect borrowing and saving decisions, and may not even be the most important. Though it is likely that many of these other factors are dependent on the individual's situation (being poor versus rich, urban versus rural), it is clearly important to understand these better given that borrowing and saving decisions can have such large welfare effects.

[4] The vast majority of microfinance loans have an interest rate that is simple instead of compounding.

4.8 Proofs

Optimal Savings

Lemma. *1 The high income constraint will bind unless $\hat{s}_h = s_h^{fb}$ and $\hat{s}_l = s_l^{fb}$.*

Proof. When $V(c,s) = v(c) + \delta v(s)$, $v(\cdot)$ is concave, so

$$\begin{aligned} v_h(\hat{c}_l) - v_h(\hat{c}_h) &< v_l(\hat{c}_l) - v_l(\hat{c}_h) \\ \Rightarrow v_h(\hat{c}_l) + \delta v(\hat{s}_l) - v_h(\hat{c}_h) - \delta v(\hat{s}_h) &< v_l(\hat{c}_l) + \delta v(\hat{s}_l) - v_l(\hat{c}_h) - \delta v(\hat{s}_h) \end{aligned}$$

so the high income temptation constraint will always bind before the low income temptation constraint will. This is also true when $V(c,s)$ is myopic. The high income constraint must bind otherwise the individual's expected utility could be increased by either changing $\hat{s}_h$, or by changing $\hat{s}_l$, until the constraint does bind. If $V_h(c_h^{fb}, s_h^{fb}) \geqslant V_h(c_l^{fb}, s_l^{fb})$ then the first best allocations will be feasible and both temptation constraints may be slack. □

Liquidity constraints

Theorem. *7 A person with costly self-control will strictly prefer optimal mandatory deposits to optimal liquidity constraints.*

Any liquidity constraint α_t can be recreated with a mandatory deposit of $\hat{s}_{t+1} = \alpha_t s_t + r s_t$, where s_t is the amount saved in time t, and r is the interest rate.

The largest a liquidity constraint can be is $\alpha_t = 1$: under liquidity constraints the minimum cost of self-control is $v(y_{t+1})$. It is feasible for a mandatory minimum deposit to require the individual to save her entire income, so the minimum cost of self-control is $v(0)$. Therefore mandatory minimum deposits have the ability to reduce the cost of self-control by more than can liquidity constraints.

In this model of liquidity constraints, once a portion of savings is constrained it is not available until the date of maturity. In a model with $T > 4$ periods it can be the case that the wealth accrued through mandatory minimum deposits is greater than that available each period to consume given liquidity constraints. In this case liquidity constraints cause over consumption in period T and under consumption in some date $t < T$.

Even if neither of the two cases above occur, during the first period a mandatory deposit can be defined and thereby reduce the cost of self-control, whereas there cannot be a liquidity constraint. Therefore the individual will strictly prefer the optimal mandatory deposits to liquidity constraints.

Borrowing

Theorem. ***6*** *The optimal borrowing plan consists of a debt limit, $\bar{b}$, that is a function of that is a function of the distribution for income.*

Proof. This proof's reasoning exactly parallels that of Theorem 5. □

Theorem. ***8*** *The optimal borrowing plan will consist of predefined payments made each period and a debt limit, both of which are functions of per period expected wealth.*

Proof. Proving that neither the debt limit or the mandatory payments can be functions of reported current wealth is the same as above as well.

Step 1: Show that mandatory payments and a debt limit optimized with respect to the distributions for income exist.

Step 2: Show that both a debt limit and mandatory payments can exist simultaneously.

Step 1.

$$\begin{aligned}\mathbb{E}[U] = \mathbb{E}\big[&u(y_1 + b - x) + v(y_1 + b - x) - v(y_1 + \bar{b} - x) \\ &+ \delta\mathbb{E}\big[u(y_2 - a) + v(y_2 - a) - v(y_2 - \hat{a}) + \delta^2\mathbb{E}\left[u(y_3 - (1 + q)b + a)\right]\big]\big]\end{aligned}$$

$\hat{a}$ is the mandatory payment in time t. If a loan of size b was taken out in the first period, and there are no defaults, then the sum of the payments made by the person in the subsequent periods naturally must be equal to $(1+q)b$. In addition, as above, there will be a level $\hat{y} = y(\hat{a})$ such that when income is above $\hat{y}$ the mandatory payment is not binding, and when it is below it is binding. The result is a piece-wise expected utility function, whose derivative with respect to $\hat{a}$ takes the following form:

$$\frac{\partial \mathbb{E}U}{\partial \hat{a}} = \int_{\hat{y}}^{\bar{y}} v'(y-\hat{a})f(y)dy - \int_{\underline{y}}^{\hat{y}} \left(u'(y-\hat{a}) - \delta \mathbb{E}\left[u'(y-(1+q)b+\hat{a})\right]\right) f(y)dy \quad (4.24)$$

When $\hat{a} = 0$ then the mandatory payment is never binding, $\hat{y} = \underline{y}$, the first term above is positive and the second is zero. Given that both terms in equation (4.24) are continuous at least one value of $\hat{a} > 0$ exists that will maximize expected utility.

An optimal debt limit exists

$$\frac{\partial \mathbb{E}[U]}{\partial \bar{b}} = -\int_{\hat{y}}^{\bar{y}} v'(y+\bar{b})f(y)dy + \int_{\underline{y}}^{\hat{y}} \left(u'(y+\bar{b}) - \delta^2(1+q)\mathbb{E}\left[u'(y-(1+q)\bar{b}+a)\right]\right) f(y)dy \quad (4.25)$$

Because both integrals are continuous an optimal $\bar{b}$ will exist.

Step 2. By the implicit function theorem $\partial\bar{b}/\partial\hat{a} > 0$, and $\partial\hat{a}/\partial\bar{b} < 0$, so there does exist a stable pair $\bar{b}, \hat{a}_2, ..., \hat{a}_T$ that maximizes expected utility. □

Theorem. ***9*** *The amount the person would like to borrow is greater when there are optimized mandatory payments than when payments are completely flexible.*

Proof. Assuming four periods for simplicity, the period 1 expected utility will take the following form:

$$\begin{aligned}
\mathbb{E}U &= u(y_1 + b) + v(y_1 + b) - v'(y + \overline{b}) \\
&+\delta \int_{\hat{y}}^{\overline{y}} \left[u(y - a) + v(y - a) - v(y - \hat{a}) + \delta^2 \mathbb{E}u(y_3 - (1 + q)b + a)\right] f(y)dy \\
&+\delta \int_{\underline{y}}^{\hat{y}} \left[u(y - \hat{a}) + \delta^2 \mathbb{E}u(y_3 - (1 + q)b + \hat{a})\right] f(y)dy
\end{aligned}$$

Since we are interested in the effect of mandatory payments on borrowing the debt limit is assumed not to bind.

$\hat{a}$ is not a function of b, so neither is $\hat{y}$. Using the envelope condition the first order condition for borrowing is as follows:

$$\begin{aligned}
\frac{\partial \mathbb{E}[U]}{\partial b} &= u'(y + b) + v'(y + b) - v'(y + \bar{b}) \\
&-\delta^2(1 + q) \int_{\hat{y}}^{\overline{y}} \mathbb{E}\left[u'(y - (1 + q)\bar{b} + a)\right] f(y)dy \\
&-\delta^2(1 + q) \int_{\underline{y}}^{\hat{y}} \mathbb{E}\left[u'(y - (1 + q)\bar{b} + \hat{a})\right] f(y)dy
\end{aligned}$$

Using the implicit function theorem (note that the derivatives of the limits of the two integrals cancel out):

$$\frac{\partial b}{\partial \hat{a}} = \frac{\int_{\underline{y}}^{\hat{y}} \mathbb{E}\left[u''(y - \bar{b} + \hat{a})\right] f(y)dy}{\frac{\partial^2 \mathbb{E}[U]}{\partial b^2}} > 0$$

□

5

Conclusion

This dissertation concerns methods to test whether or not self-control is costly, the form of temptation, and the affects different assumptions about costly self-control and temptation have on optimal borrowing and saving mechanisms. The second chapter showed that costly self-control and temptation can be differentiated from changing impatience in a stochastic income consumption-savings environment. The third chapter described an experiment to test whether subjects have time inconsistent preferences, whether self-control is costly, and if so, whether the cost of self-control is time dependent. The fourth chapter described the affects on the optimal borrowing and savings mechanisms that assumptions about the myopia of temptation and the strength of costly self-control have.

This research refines our understanding of temptation and self-control and how these concepts affect behavior. In order to improve current borrowing and savings devices, future research should be conducted on the relative magnitudes of the affects that temptation and self-control, and other influences have on financial decision making.

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Biography

1. Alexander Philip Groves
2. Born 12/28/1980, Miami, FL
3. Education:
   - M.A. Economics, Duke University - 2007
   - B.S. Mechanical Engineering, University of California - Berkeley, 2004
   - B.A. Economics, University of California - Santa Cruz, 2004
4. Awards/Fellowships
   - Summer Research Fellowship, Duke University, 2011.
   - Program for Advanced Research in the Social Sciences Fellowship, Duke University, 20102011.
   - Department Summer Fellowship, Duke University, 2010.
   - Summer Research Fellowship, Duke University, 2008