Essays in Financial Econometrics

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in the Department of Economics
in the Graduate School of Duke University
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Abstract

The main goal of this work is to explore the effects of time-varying extreme jump tail dependencies in asset markets. Consequently, a lot of attention has been devoted to understand the extremal tail dependencies between of assets. As pointed by Hansen (2013), the estimation of tail risks dependence is a challenging task and their implications in several sectors of the economy are of great importance. One of the principal challenges is to provide a measure systemic risks that is, in principle, statistically tractable and has an economic meaning. Therefore, there is a need of a standardize dependence measures or at least to provide a methodology that can capture the complexity behind global distress in the economy. These measures should be able to explain not only the dynamics of the most recent financial crisis but also the prior events of distress in the world economy, which is the motivation of this paper. In order to explore the tail dependencies I exploit the information embedded in option prices and intra-daily high frequency data.

The first chapter, a co-authored work with Andrew Patton, proposes a new class of dynamic copula models for daily asset returns that exploits information from high frequency (intra-daily) data. We augment the generalized autoregressive score (GAS) model of Creal, *et al.* (2013) with high frequency measures such as realized correlation to obtain a "GRAS" model. We find that the inclusion of realized measures significantly improves the in-sample fit of dynamic copula models across a range of U.S. equity returns. Moreover, we find that out-of-sample density forecasts from

our GRAS models are superior to those from simpler models. Finally, we consider a simple portfolio choice problem to illustrate the economic gains from exploiting high frequency data for modeling dynamic dependence.

In the second chapter using information from option prices I construct two new measures of dependence between assets and industries, the Jump Tail Implied Correlation and the Tail Correlation Risk Premia. The main contribution in this chapter is the construction of a systemic risk factor from daily financial measures using a quantile-regression-based methodology. In this direction, I fill the existing gap between downturns in the financial sector and the real economy. I find that this new index performs well to forecast in-sample and out-of-sample quarterly macroeconomic shocks. In addition, I analyze whether the tail risk of the correlation may be priced. I find that for the S&P500 and its sectors there is an ex ante premium to hedge against systemic risks and changes in the aggregate market correlation. Moreover, I provide evidence that the tails of the implied correlation have remarkable predictive power for future stock market returns.

To my wife Cedel and my parents. A mi esposa Cedel y a mis padres, por su amor y soporte.

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1

Introduction

After the last financial crisis it has been an increasing interest in understanding the dynamics of potential systemic distress. Consequently, a lot of attention has been devoted to understand the extremal tail dependencies between of assets. As pointed out by Hansen (2013), the estimation of tail risks dependence is a challenging task and their implications in several sectors of the economy are of great importance. One of the principal challenges is to provide a measure systemic risks that is statistically tractable and has an economic meaning, for example Bisias et al. (2012) detects at least thirty one ways to measure systemic risk. Therefore, there is a need of a standardize systemic risk measures. These measures should be able to explain not only the dynamics of the most recent financial crisis but also the prior events of distress in the world economy, which is the motivation of this paper.

The main goal of this work is to explore the effects of time-varying extreme jump tail dependencies in asset markets. To explore the tail dependencies I exploit the information from option prices and intra-daily high frequency data. The former captures the markets' expectation of stock prices, providing us with information that can be used to predict tail dependencies. The latter contains additional information that is also useful to predict the market's behavior. Using non-parametric estimation techniques and performing inference through copula models I explore the correlation and tail dependencies in equity markets.

The first chapter, a co-authored investigation with Andrew Patton, proposes a class of models to incorporate high frequency (intra-daily) information into models of the dynamic dependence between lower frequency (e.g., daily) asset returns. We focus on copula-based models, which facilitate the specification of flexible joint distributions. Our new class of models is based on recent work on general time-varying distributions by Creal, et al. (2012), ideas on the incorporation of high frequency data into models for lower frequency conditional second moments (see Shephard and Sheppard (2010), Noureldin et al. (2012), Hansen et al. (2012, 2013)) and work on dynamic copula models for economic time series, see Patton (2012) for a review.

In the second chapter, using information from option prices I construct two new measures of dependence between assets and industries, the Jump Tail Implied Correlation and the Tail Correlation Risk Premia. The main contribution in this chapter is the construction of a systemic risk factor from daily financial measures using a quantile-regression-based methodology. In this direction, I fill the existing gap between downturns in the financial sector and the real economy. I find that this new index performs well to forecast in-sample and out-of-sample quarterly macroeconomic shocks. In addition, I analyze whether the tail risk of the correlation may be priced. I find that for the S&P500 and its sectors there is an ex ante premium to hedge against systemic risks and changes in the aggregate market correlation. Moreover, I provide evidence that the tails of the implied correlation have remarkable predictive power for future stock market returns.

Lastly, to understand the complexity behind the systemic risks in the economy I extend the possible applications of tails correlation measures, "other tales". In the first extension, which is described at the end of the second chapter, I provide a new semiparametric forward-looking beta for the diffusive and jump systematic risks. This links a one factor model with the implied correlation measures directly. I find a positive relation between the expected returns and the proposed forward-looking betas. Moreover, I extend the implications of the forward-looking betas to predict individual asset returns. The results show that the forward-looking betas have reasonable predictive power for up to twelve month horizons of individual stock returns. In the second extension, described in the Appendix, I illustrate the economic gains that arise from constructing portfolio weights with tail correlation measures. The hypothesis is that under different performance measures, the portfolio strategies under tail measures outperform the "benchmark"[1] portfolio strategy. I also provide the guidelines, under a simplistic portfolio exercise, to perform out-of-sample evaluation when the weights are related to tail measures.

The dissertation is organized as follows. Section 2 describes the GRAS model and its implications. Section 3 outlines the estimation procedures and applications of the jump tail correlation measures.

[1] Equally weighted portfolio.

2

Dynamic Copula Models and High Frequency Data

2.1 Introduction

This paper proposes a class of models to incorporate high frequency (intra-daily) information into models of the dynamic dependence between lower frequency (e.g., daily) asset returns, modeled using a copula-based approach. Our approach is based on a combination of recent work on general time-varying distributions (Creal, *et al.* (2013)), on the incorporation of high frequency data into models for lower frequency conditional second moments (Shephard and Sheppard (2010), Noureldin *et al.* (2012), Hansen *et al.* (2011, 2013)), and on dynamic copula models for economic time series (see Patton (2013) for a review).

Unlike variances and covariances, the copula of low frequency returns is not generally a known function of the copula of corresponding high frequency returns. Thus the elegant link between high frequency volatility measures (e.g., realized variance and covariance) and their lower frequency counterparts cannot generally be exploited when considering dependence via the copula function.[1] However, it is still likely that

[1] The literature on realized measures is large and still growing, see Andersen et al. (2006) and Barndorff-Nielsen and Shephard (2007) for surveys.

high frequency measures such as realized correlation contain information that is useful for modeling dynamic dependence through a copula model, to the extent that the copula model has parameters that are related to correlation. As Andersen, *et al.* (2003) note, "[t]he essence of forecasting is quantification of the mapping from the past and present into the future. Hence, quite generally, superior estimates of present conditions translate into superior forecasts of the future." It is this intuition that we seek to exploit in this paper.

Similar to the class of "GARCH-X" models, we propose augmenting a generalized autoregressive score model (GAS) of Creal *et al.* (2013) with realized measures such as realized correlation. We call such models "GRAS" models.[2] In addition to a baseline GRAS model using realized correlation, we also consider models that incorporate realized volatilities, measures of co-jumps, and market volatility. We apply these models stock returns over the period 2000 to 2010, a total of 2,773 trading days, and we consider three pairs of stocks, exhibiting low, medium and high levels of correlation.

We compare our proposed GRAS copula models with models that do not exploit high frequency data, both in-sample and out-of-sample. In sample, we find that including a measure of realized correlation significantly improves the performance of the model. This finding is robust across all three pairs of assets, across the particular choice of realized measure, and across the "shape" of the copula that we assume. (We consider the Normal, rotated Gumbel and the Student's t copulas.) We also find evidence that other high frequency measures, such as continuous and jump realized correlations and realized volatilities, contain useful information.

We compare the out-of-sample performance of the competing models in two ways. Firstly, we use the density forecast accuracy test for copulas proposed by Diks *et al.*

[2] The name "GAS-X" is already taken for a line of medications. (We thank Drew Creal and Kevin Sheppard for pointing this out to us.)

(2012). We compare the dynamic copula models both in terms of their fit across the entire support, and in particular on the joint tails of the distribution. When we look at the joint tails of the support, the GRAS model uniformly out-performs the constant parameter copula model and the GAS model, and in a majority of comparisons the difference is statistically significant. Second, we consider a simple portfolio decision problem, where a risk averse investor uses one of the competing copula-based density forecasts to compute her optimal portfolio weights, as in Patton (2004) and Jondeau and Rockinger (2012). We evaluate the out-of-sample utility of these portfolios, and find that investors are generally willing to pay a positive management fee to switch from the constant copula model and GAS model to the GRAS model.

This paper is related to work over the past decade on specifications for time-varying conditional copulas. We build on Patton (2006a), Jondeau and Rockinger (2006) and Creal, *et al.* (2013), who consider models of time-varying copulas where a parametric functional form is assumed, and the parameter is allowed to vary through time as a function of lagged information, similar to the ARCH model for volatility, see Engle (1982).[3] We attempt to bridge the gap between the existing time-varying copula models, which have almost exclusively used lower frequency data, and models from the volatility and correlation forecasting literature, which have successfully used high frequency data, see Shephard and Sheppard (2010), Noureldin *et al.* (2012), Hansen *et al.* (2011, 2013) for example. In a recent related paper, Fengler and Okhrin (2012) use a method-of-moments approach to match the covariance structure implied by a copula-based multivariate model with that estimated using high frequency data. This approach suffers from the fact that it cannot be used with copulas that have more than one free parameter (e.g., the Student's t copula), and

[3] Alternative dynamic specifications include regime switching models, see Rodriguez (2007) and Okimoto (2008), and "stochastic copula" models, see Hafner and Manner (2010).

the authors' simulation study suggests that it can lead to biased estimators when the level of dependence is high. Our proposed approach overcomes these limitations by using the GAS structure, described in the next section, to link high frequency information to low frequency dependence measures.

The remainder paper is organized as follows. Section 2.2 provides the general formulation of our GRAS models and outlines the estimation procedure. Section 2.3 presents our in-sample results, and Section 2.4 presents the out-of-sample forecasting results, for density forecasting and a portfolio choice problem. Section 2.5 concludes. An online supplemental appendix contains additional details and results.

2.2 Dynamic Copula Models and High Frequency Data

The models used in this paper are based on Sklar's (1959) theorem, extended to apply to conditional distributions in Patton (2006a). This theorem allows the researcher to decompose a (conditional) joint distribution into marginal distributions and a copula. If $\mathbf{Y}_t = [Y_{1t}, Y_{2t}]'$ has conditional joint distribution $\mathbf{F}_t$ and conditional marginal distributions F_{1t} and F_{2t}, then we can write:

$$\mathbf{Y}_t|_{t-1} \sim \mathbf{F}_t = \mathbf{C}_t\left(F_{1t}, F_{2t}\right) \tag{2.1}$$

where $\mathbf{C}_t$ is the conditional copula of $\mathbf{Y}_t$ and $_{t-1}$ is some information set, usually taken as $\sigma\left(\mathbf{Y}_{t-1}, \mathbf{Y}_{t-2}, ...\right)$. The copula contains all information about the dependence between Y_{1t} and Y_{2t}, and is sometimes called the "dependence function." From a modeling perspective, the usefulness of Sklar's theorem arises from the fact that we can construct a joint distribution $\mathbf{F}_t$ by linking together any two marginal distributions, F_{1t} and F_{2t}, with any copula; there is no need for these functions to belong to the same family. Thus it provides a great deal of flexibility in modeling joint distributions. In Section 2.2.3 we describe the models that we use for the conditional marginal distributions in our empirical work, and below we describe various models for the conditional copula. The discussion below exploits the fact that the conditional copula of $\mathbf{Y}_t$ can be interpreted as the conditional joint distribution of the probability integral transforms of these variables:

$$\begin{aligned} \text{Let } U_{it} &\equiv F_{it}\left(Y_{it}\right), \quad i = 1, 2 \\ \text{then } \mathbf{U}_t|_{t-1} &\sim \mathbf{C}_t \end{aligned} \tag{2.2}$$

2.2.1 The GAS Model

As noted above, we build on ARCH-type models for the dynamic copula, and in particular the class of GAS models proposed by Creal, *et al.* (2013). The GAS specification addresses the problem of the choice of "forcing variable" to use in the equation governing the dynamics of the time-varying parameter. For models of the conditional variance, an immediate choice for this variable is the lagged squared residual, as in the ARCH model, but for models with parameters that lack an obvious interpretation the choice is less clear. Creal, *et al.* (2013) propose using the lagged score of the density model (copula model, in our application) as the forcing variable.[4] Specifically, for a copula with time-varying parameter δ_t we have:

$$\begin{aligned} \text{Let } \mathbf{U}_t|_{t-1} &\sim \mathbf{C}(\delta_t) \\ \text{then } \delta_t &= \omega + \beta\delta_{t-1} + \alpha s_{t-1} \\ \text{where } s_{t-1} &= S_{t-1}\nabla_{t-1} \\ \nabla_{t-1} &= \frac{\partial \log \mathbf{c}(\mathbf{u}_{t-1}; \delta_{t-1})}{\partial \delta_{t-1}} \end{aligned} \tag{2.3}$$

and S_t is a scaling matrix (e.g., the inverse Hessian or its square root). While this specification for the evolution of a time-varying parameter is somewhat arbitrary,

[4] Harvey (2013) and Harvey and Sucarrat (2012) propose a similar method for modeling time-varying parameters, which they call a "dynamic conditional score," or "DCS," model.

Creal, *et al.* (2013) motivate it by showing that it nests a variety of popular and successful existing models: GARCH (Bollerslev (1986)) for conditional variance; ACD (Engle and Russell (1998)) for models of trade durations (the time between consecutive high frequency observations); Davis, *et al.*'s (2003) model for Poisson counts. Harvey (2013) further motivates this specification as an approximation to a filter for a model driven by a stochastic latent parameter, or an "unobserved components" model.

2.2.2 The "GRAS" Model

Building on the GAS specification in equation (2.3), we propose augmenting the evolution equation with information based on high frequency data, making this a "realized GAS" model, which, continuing the recent tradition of liberal acronym formation, we call the GRAS model. Consider a GRAS model based on a generic realized measure, denoted RM :

$$\delta_t = \omega + \beta\delta_{t-1} + \alpha s_{t-1} + \gamma RM_{t-1} \tag{2.4}$$

To the extent that the realized measure provides useful information about the current dependence parameter, δ_t, *beyond* that contained in the score of the copula likelihood, this model will out-perform the underlying GAS model. One can imagine that the usefulness of realized measures for copula parameters will depend on the type of dependence the copula parameter governs, and the type of dependence captured by the realized measure. For example, a copula parameter that primarily captures rank correlation might be well explained by realized (linear) correlation, while a copula parameter that primarily captures asymmetry between joint upper-tail and joint lower-tail dependence might have little to do with realized correlation.

It is worth noting here that by augmenting the GAS model with a realized measure, we are formally outside the "generalized autoregressive score" framework, and the process in equation (2.4) no longer has the interpretation as an approximate filter for an unobservable components model. Instead, it is interpreted as a parsimonious way of capturing dynamics in the conditional copula, with a forcing variable guided by the shape of the copula. Like the many extensions of the original ARCH model of Engle (1982), see Bollerslev (2009) for a glossary, the model proposed here is somewhat arbitrary, and must be judged on its empirical performance. Sections 2.3 and 2.4 are devoted to answering that question in detail.

Most parametric copulas have parameters that are constrained to lie in a particular range of values (e.g., a correlation parameter forced to take values only inside $(-1, 1)$). To ensure that this is satisfied, Creal, *et al.* (2013) suggest applying a strictly increasing transformation, h, (e.g., log, logistic, arc tan, etc.) to the parameter,

$$\varphi_t = h\left(\delta_t\right) \Leftrightarrow \delta_t = h^{-1}\left(\varphi_t\right) \tag{2.5}$$

and to model the unconstrained transformed parameter, denoted φ_t. This approach extends directly to GRAS models and we employ this in our analysis below:

$$\varphi_t = \omega + \beta\varphi_{t-1} + \alpha s_{t-1} + \gamma RM_{t-1} \tag{2.6}$$

where $s_{t-1} = S_{t-1}\nabla_{t-1}$ and $\nabla_{t-1} = \partial \log \mathbf{c}(\mathbf{u}_{t-1}; \delta_{t-1})/\partial\delta_{t-1}$ as above. In all of our GAS and GRAS specifications, we use the (Cholesky) square root of the inverse Hessian matrix as our scale matrix, that is, $S_t = I_t^{-1/2}$.

2.2.3 Estimation and Inference

We consider marginal distribution models of the following form:

$$\begin{aligned} Y_{it} &= \mu_i\left(\mathbf{Z}_{t-1};\theta_i\right) + \sigma_i\left(\mathbf{Z}_{t-1};\theta_i\right)\varepsilon_{it}, \quad i = 1, 2, \quad \mathbf{Z}_{t-1} \in_{t-1} \\ \text{where } \varepsilon_{it}|_{t-1} &\sim F_i\left(0, 1; \theta_i\right) \ \forall t \end{aligned} \tag{2.7}$$

That is, we allow for general time-varying conditional means and variances of the individual asset returns, and we assume that the standardized residuals, ε_{it}, are *iid* from some parametric distribution with zero mean and unit variance.[5,6] In our empirical work below, we use ARMA models for the conditional mean and GJR-GARCH models (see Glosten, *et al.* (1993)) for the conditional variance, with the number of lags chosen using the BIC.[7] We use the skewed t distribution of Hansen (1994) for the distribution of the standardized residuals, and we verify its goodness-of-fit using standard tests described in Section 2.3.1 below.

When combined with parametric models for the conditional marginal distributions, the GRAS model for the conditional copula defines a dynamic parametric model for the joint distribution. The joint likelihood is then

$$\begin{aligned}\mathcal{L}(\theta) &\equiv \sum_{t=1}^{T} \log \mathbf{f}_t(\mathbf{Y}_t;\theta) = \sum_{t=1}^{T} \log f_{1t}(Y_{1t};\theta_1) \\ &\quad + \sum_{t=1}^{T} \log f_{2t}(Y_{2t};\theta_2) + \sum_{t=1}^{T} \log \mathbf{c}_t\left(F_{1t}(Y_{1t};\theta_1), F_{2t}(Y_{2t};\theta_2); \theta_c\right)\end{aligned} \tag{2.8}$$

By the structure of the model above, this model can be estimated in stages, first estimating the marginal distributions and then estimating the copula model conditioning on the estimated marginal distribution parameters. This entails, in general,

[5] Inference methods for models with dynamic conditional copulas and nonparametric or semiparametric marginal distributions are not yet available in the literature, see Patton (2013), and so we are constrained to consider fully parametric marginal distributions. Also, we note here that the literature on GAS models is growing and some results on the asymptotic properties of MLE for *univariate* GAS models are available (see Blasques, et al. (2012)), but no formal results are yet available for GAS models applied to copulas, and similarly nothing that could be directly used for GRAS models. Heejoon (2013) provides some results on univariate GARCH-X processes, which is related to the extension from GAS to GRAS models, but is not directly applicable. We simply assume that ML estimation of GAS and GRAS models for copulas have the usual asymptotic properties.

[6] It is possible to allow the conditional distribution of the standardized residuals to vary through time (e.g., to have time-varying higher-order moments), but for simplicity we do not consider this here.

[7] As mentioned in the Introduction, recent work in volatility modeling has found that including high frequency variance measures in a GARCH-type equation to improve goodness of fit, see Shephard and Sheppard (2010) and Hansen, et al. (2011) and for example. In the online supplemental appendix we also present results when the "realized GARCH" model of Hansen, et al. (2011) is used in place of the GJR-GARCH model.

some loss of efficiency relative to estimating the entire joint distribution model in one step, however it greatly simplifies the computational burden, and Joe (2005) and Patton (2006b) find the loss of efficiency to generally be low.

While estimation of the entire parameter vector θ is simplified by doing it in stages, inference on the resulting copula parameter estimates is more difficult than usual, as the estimation error from the marginal distribution stages must be taken into account. White (1994) and Patton (2006b) provide methods for doing so based on a modified "Hessian" matrix. An alternative method is to use a block bootstrap, applied to the returns, see Gonçalves and White (2004) and Gonçalves, *et al.* (2013) for technical details. Unlike some applications of the bootstrap, its use here does not lead to any asymptotic refinements, rather it merely enables one to avoid having to compute large and complicated Hessian matrices. We employ the stationary bootstrap of Politis and Romano (1994), using 100 replications and an average block length of 60, for inference in this paper.

2.3 Empirical Analysis of U.S. Equity Returns

2.3.1 Data Description and Marginal Distribution Estimation

We use high frequency transaction data over the period January 2000 to December 2010, a total of 2,773 trading days, taken from the NYSE's TAQ database. We follow the cleaning rules outlined in Barndorff-Nielsen, *et al.* (2009), see Li (2013) for details. All of our "realized measures" are constructed using five-minute returns from within the trade day, with the overnight returns omitted. Our daily returns are computed as the log-difference of the close prices from the high frequency data base, and are adjusted for stock splits using information from CRSP.[8]

[8] The dates and the corresponding ratio of the splits for each stock are: (i) 2-for-1 for Microsoft on May 23, 1994; December 9, 1996; February 23, 1998; March 29, 1999; February 18, 2003. (ii) 2-for-1 for Apple on June 21, 2000; February 28, 2005. (iii) 2-for-1 for Exxon on April 14, 1997; July 19, 2001. (iv) 2-for-1 for Chevron on June 13, 1994; September 13, 2004. (v) 2-for-1 for Celgene

We consider three pairs of stocks for this analysis: Microsoft and Apple, Exxon and Chevron, Celgene and Johnson & Johnson.[9] These pairs cover a range of (average) levels of dependence, with sample linear correlations of 0.44, 0.85 and 0.16 respectively, to allow us to see whether the gains from using high frequency data are greater or lesser depending on the level of dependence. The top panel of Table 2.1 presents some summary statistics for the daily returns used in our analysis, and Figure 2.1 presents a scatter plot of the daily returns on these assets.

As noted in Section 2.2.3, we use ARMA–GJR-GARCH models to capture the marginal distribution dynamics, with lag lengths chosen using the BIC.[10] The second and third panels of Table 2.1 present the parameter estimates for the conditional mean and variance models. The estimated parameters are similar to other studies of daily equity returns: generally small AR coefficients, strongly persistent volatility, with the asymmetric ARCH coefficient being two to four times larger than the standard ARCH coefficient, indicating asymmetric volatility dynamics.

We use Hansen's (1994) skewed t for the distribution of the standardized residuals, and the estimated parameters are presented in the fourth panel of Table 2.1. The estimated degrees of freedom parameter, which controls the degree of kurtosis, ranges from 4.5 to 18.8, indicating excess kurtosis. The skewness parameter, which lies in the interval $[-1, 1]$ for this distribution, is generally small, indicating evidence of

on October 25, 2004; February 27, 2006; 3-for-1 on April 17, 2000. (vi) 2-for-1 for J&J on June 12, 1996; June 13, 2001.

[9] Moving from bivariate to higher dimension analyses significantly reduces the number of existing copula models that can be applied, and significantly increases the complexity of the problem (see Christoffersen, et al. (2012) and Oh and Patton (2013) for recent work in this area). Addressing these issues in a satisfactory way would take us beyond the scope of the current paper, and we leave this extension for future work.

[10] We consider ARMA(p, q) models up to order (5,5). For four of the six stocks the BIC-optimal model is an AR(0), i.e., just a constant, while for Exxon the optimal model is an AR(1) and for Chevron an AR(2) is selected. We consider volatility models in the set ARCH(1), ARCH(2), GARCH(1,1), GARCH(2,2), GJR-GARCH(1,1) and GJR-GARCH(2,2). For all six stocks the BIC-optimal model is a GJR-GARCH(1,1) specification.

only mild skewness. The bottom panel of Table 2.1 presents Kolmogorov-Smirnov and Cramer-von Mises tests of the specification of the marginal distribution. We use a simulation-based approach to obtain critical values that are correct in the presence of estimated parameters, see Patton (2013) for details. All of the series pass both of these tests at the 0.05 level, although there is some mild evidence of misspecification for Apple, and potentially for Exxon. We proceed with this specification and move on to the estimation of the dynamic copula models.

2.3.2 High Frequency Data and Dynamic Copula Models

In Table 2.1 we present our first empirical contribution. This table reports parameters estimates from three different GRAS copula models. The first is based on a Normal copula, and we model the arc tan of this parameter evolving according to equation (2.6). This copula rules out tail dependence and asymmetric dependence, and is presented as a benchmark model, similar to the DCC model of Engle (2002). The second is based on a rotated (or survival) Gumbel copula, which allows for lower tail dependence and asymmetry. The Gumbel copula parameter must lie in $[1,\infty)$ and we impose this by modeling $\varphi = \log(\delta - 1)$. The third model uses a Student's t copula, which allows for dependence in both tails, but imposes symmetry. The Student's t copula has two parameters, ρ and ν^{-1}, and for simplicity we only allow the correlation parameter ρ to vary through time. Like the Normal copula, we model the arc tan of this parameter. In all three models, the GRAS dynamics are obtained using equation (2.6), with the realized measure being realized correlation, computed in the usual way:

$$\begin{aligned} RCorr_t &= \frac{RV_t^{(1,2)}}{\sqrt{RV_t^{(1,1)} RV_t^{(2,2)}}} \qquad (2.9) \\ \text{where } RV_t^{(i,j)} &= \sum_{k=1}^{m} r_{t,k}^{(i)} r_{t,k}^{(j)} \end{aligned}$$

and $m = 78$ is the number of five-minute returns in a trade day. Our daily returns are whole day ("close to close"), but we use only open-to-close realized measures in the GRAS models, as this is the only period of the day in which high frequency data is available.[11]

The coefficient on lagged realized correlation in the GRAS model is denoted γ_{RC}, and is reported in the last row of parameter estimates. Bootstrap standard errors, which take into account estimation error from the marginal distribution parameters, are presented in parentheses below the estimated parameters. We see from this table that the coefficient on realized correlation is significantly different from zero for all three models across all three pairs of assets. In seven out of the nine models it is significant at the 1% level, and for the other two it is significant at the 5% level. This is strong statistical evidence of the usefulness of high frequency data for modeling dynamics in the conditional copula of lower frequency returns, confirming the general intuition of Andersen, *et al.* (2003) for this application.[12,13]

Figure 2.1 presents the estimated time series of the linear correlation[14] from the Student's t copula model using the GAS and GRAS specifications. (The Student's t GRAS model beats the Normal and rotated Gumbel specifications for all three

[11] In the online supplemental appendix we present tables corresponding to Table 2.2 but using 1-minute, 10-minute and 15-minute sampling to obtain realized correlations. The results using 10- and 15-minute sample are similar to those using 5-minute sampling, but worse when using 1-minute sampling, perhaps reflecting greater market microstructure effects at that frequency.

[12] In the online supplemental appendix we present a corresponding table of parameter estimates when γ_{RC} is imposed to be zero, leading to the GAS specification of Creal, *et al.* (2012). Comparing the values of the log-likelihoods from that model with those in Table 2.2 confirms the gains from including realized correlation in this specification.

[13] In the supplemental online appendix we consider all 15 possible pairs out of these six assets, and we present a table of the p-values from tests comparing the proposed GRAS model to the GAS model. We find that 10 out of these 15 pairs have significant p-values, indicating gains from using the GRAS model. Interestingly, all of the non-significant pairs involve Apple or Microsoft, perhaps indicating that these technology stocks have different dynamics than the other stocks under consideration.

[14] The linear correlation implied by a copula-based multivariate model is generally not a closed-form function of the parameters of the model, even when the copula has a "correlation" parameter, see Patton (2013). We use simple numerical integration to obtain the model-implied linear correlation.

pairs of assets, discussed further in Section 2.3.4 below.) The bottom panel of this figure, showing the results for Celgene–Johnson & Johnson is perhaps the easiest to interpret. We see from this figure that the GRAS model is above to capture lower and higher values of dependence than the GAS model, which is based only on daily information. As expected, a model based on higher frequency information is able to adjust to changes in dependence faster than a model based only on daily information. The same inference can be made from the other two plots in this figure, although the parameter dynamics in those models are such that it is harder to observe.

2.3.3 Extensions of the Baseline GRAS Model

We next consider extensions of the GRAS model presented in equation (2.6), where we augment that specification with additional realized measures.

Firstly, we consider an extension motivated by Engle (2002) and Andersen, *et al.* (2006), who find that conditional correlation is affected by the past level of volatility. We augment our GRAS specification from above to include not only realized correlation, but also the log realized variance of each asset. Thus the evolution equation becomes:

$$\varphi_{t+1} = \omega + \beta\varphi_t + \alpha s_t + \gamma_{RC} RCorr_t + \gamma_{RV1} \log\left(RV_t^{(1,1)}\right) + \gamma_{RV2} \log\left(RV_t^{(2,2)}\right) \quad (2.10)$$

The results for this model are presented in Table 2.3. We again see that the coefficient on lagged realized correlation is highly significant across all models. Further, the coefficients on lagged realized volatility are significant for two out of three pairs of assets. The last two rows of this table present joint tests for the significance of the coefficients on realized measures. The penultimate row tests the restriction that all realized measures have coefficients equal to zero, which simplifies this model to a GAS model, and this is strongly rejected. The bottom row tests that only realized correlation has a non-zero coefficient, and this is strongly rejected for the Microsoft-

Apple and Celgene-J&J pair, but is not rejected for the Exxon-Chevron pair. Thus this table presents evidence that other measures computed from high frequency data may be helpful in capturing the dynamics of dependence between asset returns.

Next we consider a "heterogeneous autoregressive" (HAR) version of our baseline model, to see whether longer-run dependence in realized measures is useful for modeling the dynamics of daily copulas, see Corsi (2009) and Müller, *et al.* (1997) for details on the HAR model, and Corsi and Reno (2009) and Sokolinskiy and van Dijk (2011) for the usefulness of HAR models for forecasting volatility. Consider the following specification:

$$\varphi_{t+1} = \omega + \beta\varphi_t + \alpha s_t + \gamma_{RC} RCorr_t + \gamma_W \sum_{i=1}^{5} \frac{RCorr_{t+1-i}}{5} + \gamma_M \sum_{i=1}^{22} \frac{RCorr_{t+1-i}}{22} \quad (2.11)$$

The results for this model are presented in Table 2.4. We see that the coefficient on lagged realized correlation is again statistically significant across all nine specifications. The coefficients on the past 5-day and 22-day average realized correlation are generally not significant, indicating that these additional lags of realized correlation do not much improve the fit of the model (perhaps due to the persistence that the GAS part of the model already captures). The penultimate row tests the restriction that all realized measures have coefficients equal to zero, and we are unable to reject this in any specification, indicating that high frequency information is not useful. This contradicts our findings in the previous section, where high frequency data was found to be strongly significant, and is explained by the last row of Table 2.4, where we test the restriction that the coefficients on the weekly and monthly realized correlation lags are zero. We fail to reject this null in all cases, and we conclude that by including weekly and monthly, as well as daily, realized correlation, we "dilute" its significance. Overall, it appears that just the one-day lag of realized correlation is useful for modeling the dynamic copula.

Finally, we consider a specification that decomposes realized correlation into a "continuous" and a "jump" component, as in Bollerslev, *et al.* (2013). These labels come from an assumption that the continuous time diffusion generating the observed returns is subject to occasional jumps.[15] We construct these measures using the procedure in Bollerslev, *et al.* (2013), described in detail in the supplemental online appendix. In brief, five-minute returns are categorized as "jump" or "non-jump" using standard methods, and any five-minute window where at least one of the two assets had a "jump" is excluded from the computation of a "continuous realized correlation." The difference between the usual realized correlation and the continuous component is defined as the jump realized correlation. This leads us to the specification:

$$\varphi_{t+1} = \omega + \varphi f_t + \alpha s_t + \gamma_{CTS} RCorr_t^{(CTS)} + \gamma_{Jump} RCorr_t^{(Jump)} \tag{2.12}$$

This model nests the baseline GRAS model when $\gamma_{CTS} = \gamma_{Jump}$, and a test of that restriction is presented in the bottom row of the table (denoted "$p\text{-}val_{GRAS}$"). We see that for the Microsoft-Apple pair that this restriction is rejected for all three copula specifications. For Exxon-Chevron it is rejected for just the Student's t copula, and for the Celgene-J&J pair it is not rejected in any case. Thus the additional explanatory power of jump versus continuous correlations is mixed across these assets.

As general conclusions, for all three assets pairs and for all copula specifications, we find that exploiting information from realized correlation significantly improves the fit of the model. Some additional explanatory power can be gained for certain assets by also including realized volatilities, or the continuous and jump components of realized correlations separately, however our baseline GRAS model, which includes just the one-day lag of realized correlation, generally performs very well across these

[15] In the presence of jumps, the above analyses based on realized correlation can be interpreted as the correlation implied by the quadratic covariation matrix, which combines both the continuous and jump components.

assets. In the remainder of the paper we focus on this baseline GRAS specification.

2.3.4 The Choice of Copula Functional Form

Up to this stage of the analysis, we have focused attention on the specification of the *dynamics* of the model for the conditional copula (constant, GAS, or GRAS), and paid little attention to the choice of shape. The estimation results presented above also enable us to shed light on the best-fitting copula shape across these three pairs of asset returns, and we discuss these results here.

In Table 2.6 we compare different specifications of the shape of the copula model, holding the model for the dynamics fixed. The comparison of the rotated Gumbel with the Normal or the Student's t involves testing non-nested models, and we use the test of Rivers and Vuong (2002), see Patton (2013) for a discussion of its implementation for comparing copula models. Given the models are non-nested, we use a two-sided test for this comparison. The Student's t copula nests the Normal copula, and we compare these models via a simple (one-sided) Wald test on the estimated degrees of freedom parameter.[16]

The top panel of Table 2.6 reveals that the rotated Gumbel model generally out-performs the Normal model, across all three asset pairs and all three models for dynamics, revealed through the positive t-statistics, however in no case is the difference significant. The lower two panels of this table reveal the improved fit provided by the Student's t copula: it significantly beats the Normal copula in eight out of nine comparisons, and significantly beats the rotated Gumbel in six out of nine comparisons. This finding confirms the importance of allowing for joint fat tails in multivariate models of asset returns. It is apparent evidence against the importance

[16] Specifically, we test that the estimated inverse degrees of freedom parameter is equal to zero. This is on the boundary of the parameter space, which means that the t-statistic will not be standard Normally distributed, however the right-tail critical values, which are the ones we require, remain applicable.

of allowing for asymmetric dependence (e.g., for dependence being higher during market downturns than market upturns), given that the (symmetric) Student's t copula outperforms the (asymmetric) rotated Gumbel copula, however this interpretation is muddied by the additional flexibility the Student's t copula attains from it having two free parameters compared with just one for the Gumbel copula.

2.4 Out-of-sample Forecast Comparisons

The previous section showed that high frequency data significantly improves the fit of dynamic copula models for daily asset returns. However, there remains the concern that improved in-sample fit does not guarantee an improvement in out-of-sample forecast performance. In this section we investigate whether high frequency information also leads to gains in out-of-sample forecast performance. We do so using two applications. Firstly, we consider a density forecasting application, and evaluate performance using a metric related to the Kullback–Leibler information criterion (KLIC). Secondly, we consider an illustrative portfolio decision problem, and evaluate the competing models using the realized utility from a portfolio optimized using the predictive density.

2.4.1 Multivariate Density Forecasting

We use the approach of Diks *et al.* (2012), which provides a means of comparing density forecasts over the entire support, as well as in particular regions of the support. Motivated by its connection with the Kullback–Leibler information criterion (KLIC), both of these approaches are based on the out-of-sample log-likelihood of the density forecast. Note that since all of our multivariate models use the same marginal distribution models (means, variances, and standardized residual densities), comparing the full multivariate density forecasts from two models reduces to simply comparing the copula density forecasts.

We use the period from January 2000 to December 2005 as the in-sample period, and January 2006 to December 2010 as our out-of-sample period. Given the computational complexity of the models estimated here, we use a standard rolling window estimation scheme for the marginal distribution parameters, but a fixed window estimation scheme for the copula parameters.[17] In all cases, the density forecast for day t is based only on information up until day $t-1$. Diks *et al.* (2012) propose a conditional likelihood test to compare density forecasts, where the performance of model $\mathcal{A}$ in the region $[0,q] \times [0,q]$ is based on:

$$S_t^{\mathcal{A}}(\mathbf{u}_t) = \left(\log \mathbf{c}_t^{\mathcal{A}}(\mathbf{u}_t) - \log \mathbf{C}_t^{\mathcal{A}}(\mathbf{q})\right) \mathbf{1}_{\{\mathbf{u}_t \leqslant \mathbf{q}\}} \tag{2.13}$$

where $\mathbf{q} = [q,q]'$ and $\mathbf{u}_t = [u_{1t}, u_{2t}]'$. Note that when $q = 1$ this expression simplifies to the log copula density, $\log c_t^{\mathcal{A}}(\mathbf{u}_t)$, and for $q < 1$ it can be interpreted as the log-likelihood of the model conditional on the observation lying in the region $[0,q] \times [0,q]$. The null hypothesis of equal predictive accuracy of models $\mathcal{A}$ and $\mathcal{B}$ in the region $[0,q] \times [0,q]$ is:

$$\begin{aligned} H_0 &: E\left[S_t^{\mathcal{A}}(\mathbf{u}_t) - S_t^{\mathcal{B}}(\mathbf{u}_t)\right] = 0 \\ \text{vs.} \quad H_1 &: E\left[S_t^{\mathcal{A}}(\mathbf{u}_t) - S_t^{\mathcal{B}}(\mathbf{u}_t)\right] \neq 0 \end{aligned} \tag{2.14}$$

Diks *et al.* (2012) propose testing this hypothesis based on a test on the average difference in the conditional likelihoods. Let:

$$d_t = S_t^{\mathcal{A}}(\mathbf{u}_t) - S_t^{\mathcal{B}}(\mathbf{u}_t) \tag{2.15}$$

then a test that $E[d_t] = 0$ can be used to test the null hypothesis above. This series may be serially correlated and heteroskedastic, and so robust standard errors are

[17] The asymptotic testing framework of Giacomini and White (2006), which is used by Diks et al. (2012) to implement their test, can handle both of these schemes and so this raises no theoretical issues. The out-of-sample fit of the copula models is presumably worse using a fixed window scheme than using a rolling window scheme, but should not affect our main conclusions on relative performance.

required.[18]

We consider five regions for analysis of the copula density forecasts: the joint 1%, 5% 10% and 25% lower tails, as well as the entire support. The results of these tests are presented in Table 2.7. The right-most column of this table presents the results for the entire support, and shows that across all three pairs of assets, and across three copula models (Normal, rotated Gumbel, and Student's t) our proposed GRAS model significantly beats the constant copula model in all but one case. This shows that the dynamics captured by our copula model based on high-frequency data are useful out-of-sample: the predictive copula densities from the GRAS model provide a better out-of-sample fit than those from simple, constant parameter copula models. This conclusion holds true across all joint lower tail regions but one, providing strong support for our model.

A tougher hurdle for our GRAS model is to significantly out-perform the GAS model. The GAS model provides a parsimonious way of capturing dynamics in the conditional copula, and our proposed GRAS model will only out-perform it if the high frequency information we exploit (realized correlation) has additional explanatory power out-of-sample. When looking at the entire support we find that the GRAS beats the GAS model (the t-statistics are positive in 8 out of 9 comparisons), but the out-performance is generally not statistically significant. When we use the method of Diks *et al.* (2012) to zoom in on the joint tails of the support, we find that the GRAS model uniformly out-performs the GAS model, and in a majority of comparisons the difference is statistically significant. Thus it appears that high frequency information is particularly useful for dynamic copula models when interest is focused on the tails of the distribution.

[18] Diks et al. (2012) also propose a "censored likelihood" test as an alternative to the conditional likelihood test. The results from that test are very similar to those discussed here, and are presented in the supplemental online appendix.

2.4.2 A Portfolio Decision Problem

Previous work has documented evidence of two types of asymmetries in the joint distribution of stock returns, namely skewness in the distribution of individual returns, and asymmetry in the dependence between asset returns.[19] With this motivation, we use a portfolio application to gain insights into the economic significance of using high frequency data in dynamic copula models. We approach the portfolio allocation problem following the methodology proposed in Patton (2004) and Jondeau and Rockinger (2012). The latter paper proposes approximating a CRRA utility function using a fourth-order polynomial for tractability:

$$E_t\left[\mathcal{U}\left(W_{t+1}\right)\right] = \varphi_0 + \varphi_1 m_{t+1}^{(1)} + \varphi_2 m_{t+1}^{(2)} + \varphi_3 m_{t+1}^{(3)} + \varphi_4 m_{t+1}^{(4)} \tag{2.16}$$

with $\varphi_k = (k!)^{-1}\,\partial^k \mathcal{U}\left(W\right)/dW^k|_{W=1}$, which are known functions of k and the degree of risk aversion, and $m_{t+1}^{(j)} = E_t\left[W_{t+1}^j\right]$ is the uncentered j^{th} moment, with W_{t+1} the value of the portfolio at time $t+1$.[20] We consider five different levels of relative risk aversion, $RRA = 1,\ 3,\ 7,\ 10$ and 20, similar to the range considered in Campbell and Viceira (1999) and Aït-Sahalia and Brandt (2001).

Optimal portfolio weights are found by maximizing the expected utility under the predictive density:

$$\begin{aligned} \omega_{t+1}^{*} &= \arg\max_{\omega\in\Upsilon}\ E_t\left[\mathcal{U}\left(\omega'\mathbf{Y}_{t+1}\right)\right] \qquad (2.17)\\ &= \arg\max_{\omega\in\Upsilon} \int\int \mathcal{U}\left(\omega'\mathbf{Y}_{t+1}\right)\hat{f}_{1t+1}\left(y_1\right)\hat{f}_{2t+1}\left(y_2\right)\hat{\mathbf{c}}_{t+1}\left(\hat{F}_{1t+1}\left(y_1\right),\hat{F}_{2t+1}\left(y_2\right)\right)\mathbf{dy} \end{aligned}$$

where $\omega'\mathbf{Y}_{t+1}$ is $1+\omega_1 y_1+\omega_2 y_2$, Υ is a compact subset of $\mathbb{R}^2$ for the unconstrained

[19] See Erb, *et al.* (1994), Longin and Solnik (2001), Ang and Chen (2002), and Patton (2004) for example.

[20] The optimal portfolio weights under a CRRA utility function do not depend on the level of initial wealth, and so we set the initial wealth to 1. Thus the end-of-period wealth is equal to the gross return on the portfolio.

investor, and the two-dimensional unit simplex[21] for the short-sales constrained investor. For simplicity we take the return on the risk-free asset to be zero.

We use the same in-sample and out-of-sample periods as in the previous section, but in this section we evaluate the competing models using the average utility of the portfolio returns formed using weights optimized according to the model. To make the units used in this performance study interpretable, we convert the average utility into a "management fee," which is the fixed amount, ϑ, that could be charged (or paid, if needed) each period to an investor to switch from model $\mathcal{A}$ to model $\mathcal{B}$. Alternatively, it can be interpreted as the amount that could be deducted from the daily return on portfolio $\mathcal{B}$ over the out-of-sample period and leave the investor indifferent between portfolio $\mathcal{A}$ and portfolio $\mathcal{B}$. The management fee is the solution to the following equation:

$$\frac{1}{P}\sum_{t=R+1}^{R+P} \mathcal{U}\left(1+\omega_{\mathcal{A},t+1}^{*\prime}\mathbf{Y}_{t+1}\right) = \frac{1}{P}\sum_{t=R+1}^{R+P} \mathcal{U}\left(1+\omega_{\mathcal{B},t+1}^{*\prime}\mathbf{Y}_{t+1}-\vartheta\right) \tag{2.18}$$

where R is the length of the in-sample period and P is the length of the out-of-sample period.

Table 2.8 presents the management fees estimates, and for all comparisons we consider switching from a smaller model (Constant or GAS) to the larger model (GRAS).[22,23] Consider first the comparison of the constant copula model and the GRAS model. Across all asset pairs, copula shape specifications, and levels of risk

[21] That is, for short sales constrained investors, $\Upsilon = \left\{(\omega_1,\omega_2)\in[0,1]^2 : \omega_1+\omega_2 \leqslant 1\right\}$.

[22] The supplemental online appendix presents some summary statistics on the portfolio returns, as well as results for the short-sales constrained investor, which yields conclusions that are consistent with those for the unconstrained investor. The supplemental appendix also presents plots the optimal portfolio weights for some representative models. For lower levels of risk aversion, these plots reveal substantial portfolio turnover, and in practice it may be desirable to impose turnover constraints on the optimization problem. We do not attempt this here.

[23] We use a stationary bootstrap with average block length of 60 days to determine the significance of the "management fees." Note that the quantities being bootstrapped here are asset returns, not cumulative wealth, and so issues of non-stationarity or integrated processes do not arise.

aversion, we see that the GRAS model is generally preferred: the management fees are positive in a majority of cases, and are significantly positive for two of the asset pairs (Microsoft-Apple and Celgene-J&J) when combined with the Student's t copula. For example, for the Microsoft-Apple pair, the annualized management fees that could be charged to switch from a constant t copula to a GRAS t copula range from 43.0% (RRA=1) to 1.3% (RRA=20), and is 4.0% for the intermediate level of risk aversion.[24] In no case is the management fee negative and significant. This is strong out-of-sample support for our GRAS model, at least against the simple benchmark of a constant copula.

As in the previous section, a tougher benchmark for the GRAS model is the GAS specification. In this case the results are more mixed: for Celgene-J&J we find that the GRAS copula is preferred to the GAS copula, particularly when combined with the Student's t shape specification. For Exxon-Chevron, the GRAS model is generally preferred, with the performance fees ranging from -1.9% to +17.9%, but these are not statistically significant in any case, indicating that the GAS and GRAS models are equally good out-of-sample, according to this metric. For the Microsoft-Apple pair the GRAS model is beaten by the GAS model, with the performance fees being negative and often significant. Thus the out-of-sample gains from a GRAS model relative to a GAS model from a portfolio decision perspective are somewhat sensitive to the choice of assets and the copula shape specification: in some cases positive and significant, in others negative and significant, and in yet others not different from zero. Further work on the determinants of this variability is clearly desirable, as is a more comprehensive and higher-dimension portfolio decision application.

[24] In almost all cases, we find that the absolute value of the management fee decreases with the level of risk aversion. This reflects the fact that more risk averse investors shrink their weights on risky assets towards zero, making the differences in portfolio weights from different models smaller.

2.5 Conclusion

This paper proposes a new class of dynamic copula models for daily asset returns that exploits information from high frequency (intra-daily) data. This class of models bridges the gap between the existing time-varying copula models, which have almost exclusively used lower frequency data, and models from the volatility and correlation forecasting literature, which have successfully used high frequency data. We accomplish this by augmenting the generalized autoregressive score (GAS) model of Creal, *et al.* (2013) with high frequency measures, such as realized correlation, to obtain a "realized GAS," (GRAS) model. While measures of dependence based in high frequency data cannot generally be interpreted as unbiased or consistent estimators of dependence and lower frequencies, our approach builds on the intuition that by better measuring the current degree of dependence, broadly defined, we can build better forecasts of dependence in the future.

We employ equity return data six U.S. firms over the period 2000 to 2010, and show that including high frequency information in a dynamic model of daily equity return dependence leads to improvements in both goodness of fit and out-of-sample forecasts. In sample, we find that the information in realized correlation leads to significant improvements across all assets and copula specifications considered, and we find some evidence that realized volatility, and a measure of correlation that is related to "co-jumps," improves our baseline GRAS model even more for certain assets. Out of sample, we find that density forecasts based on our GRAS specification outperform forecasts based on models that ignore high frequency information, and this outperformance is strongly significant when attention is focused on the tail of the joint distribution. We also find some gains in out-of-sample portfolio construction based on these density forecasts, though the gains vary across the assets under consideration.

While this paper provides some initial light on the value of high frequency information for lower frequency copula modeling, some important questions remain open. Firstly, it would be interesting extend the analysis in this paper to high dimension copula models, along the lines of Christoffersen, *et al.* (2012) and Oh and Patton (2013) for example. Estimates of large-dimension realized covariance matrices have recently attracted attention, see Hautsch *et al.* (2012) for example, and these could prove useful in an extension of GRAS models to high dimensions. Furthermore, extending and elaborating the portfolio decision application presented above would enable a better understanding of the economic value of high frequency information for dynamic copula modeling. We leave these important extensions for future research.

Table 2.1: The top panel of this table presents summary statistics on the daily returns for six U.S. firms, listed in the column headings. The second panel presents the parameter estimates for AR(p) models of the conditional means of these returns, and the third panel presents parameter estimates for GJR-GARCH(1,1) models of the conditional variance. The fourth panel presents parameter estimates for Hansen's (1994) skew t density for the standardized residuals. The bottom panel presents simulation-based p-values from Kolmogorov-Smirnov and Cramer-von Mises tests of the goodness-of-fit of the density specification.

Summary statistics and marginal distribution estimates

	Microsoft	*Apple*	*Exxon*	*Chevron*	*Celgene*	*J&J*
			Panel A: Summary statistics			
Mean	-0.027	0.119	0.022	0.028	0.085	0.010
Std dev	2.209	2.958	1.745	1.770	3.618	1.374
Skewness	-0.083	-0.138	0.178	0.158	-0.140	-0.788
Kurtosis	11.221	6.390	14.374	14.230	8.714	18.733
Correl (linear/rank)	0.437/0.478		0.851/0.803		0.163/0.218	
			Panel B: Conditional mean			
Constant	-0.027	0.119	0.026	0.034	0.085	0.010
AR(1)			-0.138	-0.118		
AR(2)				-0.079		
			Panel C: Conditional variance			
Constant	0.079	0.106	0.057	0.068	0.059	0.020
ARCH	0.041	0.026	0.029	0.025	0.014	0.031
Asym ARCH	0.074	0.050	0.076	0.088	0.032	0.110
GARCH	0.906	0.937	0.907	0.900	0.964	0.905
			Panel D: Skew t density			
DoF	4.498	5.977	9.464	18.768	5.027	5.881
Skew	-0.008	0.031	-0.125	-0.109	0.017	-0.020
			Panel E: GoF tests			
KS p-value	0.708	0.088	0.066	0.380	0.543	0.785
CvM p-value	0.507	0.059	0.109	0.315	0.588	0.842

Table 2.2: This table presents the estimated parameters from the GRAS model for the copula of three pairs of assets. The coefficient on realized correlation is denoted γ_{RC}. Estimates of γ_{RC} that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively. The bottom row of this table presents the p-value from a test that the realized measure is equal to zero, and thus that the GRAS model simplifies to a GAS model.

GRAS with realized correlation

	Microsoft and Apple			*Exxon and Chevron*			*Celgene and J&J*		
	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t
ω	0.099	-0.491	0.094	0.517	-0.188	0.439	0.010	-0.864	0.008
	(0.097)	(0.171)	(0.069)	(0.238)	(0.071)	(0.205)	(0.018)	(0.271)	(0.018)
α	0.054	0.083	0.084	0.141	0.153	0.166	0.025	0.030	0.028
	(0.031)	(0.050)	(0.033)	(0.034)	(0.045)	(0.040)	(0.020)	(0.073)	(0.023)
β	0.655	0.737	0.688	0.587	0.604	0.610	0.853	0.663	0.879
	(0.158)	(0.124)	(0.116)	(0.145)	(0.150)	(0.133)	(0.097)	(0.121)	(0.097)
ν^{-1}			0.111			0.129			0.035
			(0.023)			(0.021)			(0.022)
γ_{RC}	0.614***	0.659***	0.585***	0.717***	0.562***	0.779***	0.392**	1.457***	0.329**
	(0.179)	(0.188)	(0.145)	(0.201)	(0.175)	(0.212)	(0.160)	(0.421)	(0.159)
log L	326.570	331.570	353.320	1506.80	1488.70	1550.80	120.600	113.393	122.400
p-val	0.001	0.000	0.000	0.000	0.001	0.000	0.014	0.001	0.039

Table 2.3: This table presents the estimated parameters from the GRAS model for the copula of three pairs of assets. The coefficient on realized correlation is denoted γ_{RC}. Estimates of γ_{RC}, γ_{RV1}, or γ_{RV2} that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively.

GRAS with realized correlation and realized variance

	Microsoft and Apple			*Exxon and Chevron*			*Celgene and J&J*		
	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t
ω	0.085	-0.812	0.121	0.483	-0.364	0.395	-0.540	-3.254	-0.507
	(0.161)	(0.477)	(0.164)	(0.364)	(0.496)	(0.374)	(0.405)	(0.960)	(0.426)
α	0.053	0.080	0.078	0.148	0.156	0.171	-0.018	-0.046	-0.010
	(0.036)	(0.066)	(0.037)	(0.032)	(0.041)	(0.041)	(0.033)	(0.097)	(0.035)
β	0.657	0.671	0.688	0.549	0.570	0.589	0.177	0.257	0.191
	(0.200)	(0.130)	(0.114)	(0.167)	(0.157)	(0.150)	(0.185)	(0.137)	(0.175)
ν^{-1}			0.109			0.130			0.025
			(0.020)			(0.020)			(0.023)
γ_{RC}	0.528**	0.746***	0.500***	0.851***	0.641***	0.869***	0.945***	1.810***	0.924***
	(0.221)	(0.256)	(0.180)	(0.278)	(0.216)	(0.267)	(0.181)	(0.347)	(0.188)
γ_{RV1}	0.055*	0.055*	0.053**	0.122**	0.038	0.070*	-0.247***	-0.467***	-0.245***
	(0.031)	(0.033)	(0.025)	(0.055)	(0.058)	(0.039)	(0.066)	(0.101)	(0.057)
γ_{RV2}	-0.066*	-0.091**	-0.059**	-0.127*	-0.054	-0.075	0.107**	0.199**	0.110**
	(0.039)	(0.042)	(0.027)	(0.067)	(0.071)	(0.061)	(0.050)	(0.090)	(0.047)
log L	330.565	336.124	355.494	1509.90	1489.30	1551.70	131.080	126.280	132.020
p_{GAS}	0.010	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000
p_{GRAS}	0.050	0.020	0.000	0.050	0.500	0.220	0.000	0.000	0.000

Table 2.4: This table presents the estimated parameters from the GRAS model for the copula of three pairs of assets. The coefficient on realized correlation is denoted γ_{RC}. Estimates of γ_{RC}, γ_W, or γ_M that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively.

GRAS with HAR realized correlations

	Microsoft and Apple			*Exxon and Chevron*			*Celgene and J&J*		
	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t
ω	0.037	-0.748	0.054	0.535	-0.249	0.475	0.002	-0.026	0.002
	(0.089)	(0.417)	(0.079)	(0.250)	(0.153)	(0.214)	(0.048)	(0.947)	(0.046)
α	0.059	0.097	0.089	0.153	0.168	0.183	0.023	0.046	0.023
	(0.038)	(0.068)	(0.036)	(0.030)	(0.037)	(0.040)	(0.033)	(0.097)	(0.032)
β	0.570	0.628	0.643	0.553	0.538	0.540	0.968	0.990	0.970
	(0.234)	(0.211)	(0.175)	(0.182)	(0.235)	(0.179)	(0.322)	(0.366)	(0.319)
ν^{-1}			0.110			0.127			0.036
			(0.021)			(0.019)			(0.021)
γ_{RC}	0.569**	0.646*	0.582***	0.536**	0.361*	0.524***	0.463***	0.821*	0.440**
	(0.254)	(0.357)	(0.214)	(0.268)	(0.217)	(0.225)	(0.181)	(0.485)	(0.189)
γ_W	-0.313	-0.112	-0.268	-0.517	-0.227	-0.366	-0.410	-0.724	-0.386
	(0.550)	(0.589)	(0.463)	(0.571)	(0.523)	(0.500)	(0.547)	(0.829)	(0.519)
γ_M	0.734	0.533	0.507	0.805*	0.572	0.836*	0.035	-0.047	0.030
	(0.644)	(0.696)	(0.555)	(0.448)	(0.494)	(0.463)	(0.659)	(1.270)	(0.709)
log L	329.189	332.347	354.534	1510.10	1491.20	1553.20	121.710	117.400	123.470
p_{GAS}	0.270	0.480	0.360	0.260	0.370	0.230	0.840	0.670	0.860
p_{GRAS}	0.310	0.580	0.410	0.280	0.410	0.240	0.870	0.770	0.860

Table 2.5: This table presents the estimated parameters from the GRAS model for the copula of three pairs of assets. The coefficients on realized correlation is denoted γ_{CTS} and γ_{Jump}. Estimates of γ_{CTS} or γ_{Jump} that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively.

GRAS with continuous and jump realized correlations

	Microsoft and Apple			*Exxon and Chevron*			*Celgene and J&J*		
	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t
ω	0.066	-0.441	0.083	0.504	-0.192	0.441	0.010	-0.447	0.005
	(0.060)	(0.236)	(0.080)	(0.265)	(0.082)	(0.220)	(0.019)	(0.372)	(0.020)
α	0.043	0.076	0.079	0.146	0.150	0.173	0.025	0.044	0.030
	(0.027)	(0.055)	(0.033)	(0.036)	(0.046)	(0.035)	(0.024)	(0.086)	(0.023)
β	0.758	0.765	0.728	0.586	0.608	0.601	0.855	0.825	0.930
	(0.102)	(0.145)	(0.121)	(0.162)	(0.165)	(0.131)	(0.067)	(0.156)	(0.073)
ν^{-1}			0.110			0.128			0.038
			(0.019)			(0.022)			(0.022)
γ_{CTS}	0.405***	0.577*	0.495***	0.796***	0.593***	0.858***	0.390**	0.718	0.201
	(0.158)	(0.345)	(0.201)	(0.260)	(0.189)	(0.208)	(0.171)	(0.592)	(0.156)
γ_{Jump}	0.798***	0.766***	0.716***	0.418	0.375	0.499*	0.381	1.136	0.147
	(0.282)	(0.283)	(0.250)	(0.331)	(0.237)	(0.288)	(0.301)	(0.947)	(0.386)
log L	327.421	331.661	353.130	1508.10	1489.30	1552.00	120.600	114.250	122.270
p_{GAS}	0.010	0.070	0.020	0.000	0.000	0.000	0.330	0.310	0.780
p_{GAS}	0.010	0.030	0.020	0.250	0.150	0.080	0.390	0.390	0.820

Table 2.6: This table presents t-statistics for the comparison of models of copula functional form, for a given choice of model for the copula dynamics (constant, GAS or GRAS). The top and bottom comparisons involve non-nested copula models, and we employ a two-sided Rivers and Vuong (2002) test. The middle panel involves a nested comparison, and we use a standard, one-sided, Wald test, based on the bootstrap standard errors presented in Table 2. In all cases a positive t-statistic indicates that the second model listed in the panel label is preferred to the first model.

Comparison of copula functional forms

		Microsoft and Apple	*Exxon and Chevron*	*Celgene and J&J*
	Constant	0.698	0.545	0.591
Normal vs. R.Gumbel	GAS	0.557	0.166	0.248
	GRAS	0.350	-0.469	-1.478
	Constant	4.928	7.216	2.249
Normal vs. Student's t	GAS	5.981	5.565	1.682
	GRAS	4.489	5.690	1.143
	Constant	1.732	3.260	0.312
R.Gumbel vs. Student's t	GAS	2.174	3.388	0.553
	GRAS	2.311	3.767	2.023

Table 2.7: This table presents t-statistics from pair-wise comparisons of the out-of-sample likelihoods of competing density forecasts. We consider five regions of support over which to compare the competing density forecasts: the joint lower 0.01, 0.05, 0.10 and 0.25 tails, as well as the entire support. For a given copula specification (Normal, rotated Gumbel and Student's t) we compare specifications of the dynamics: Constant versus GRAS and GAS versus GRAS. Test statistics that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively.

Out-of-sample comparison of density forecasts

		Lower tail region probability				
		0.01	0.05	0.10	0.25	1.00
		Microsoft and Apple				
	Normal	1.338*	2.126**	3.232***	3.142***	1.536*
GRAS	R. Gumbel	1.291*	1.799**	2.974***	3.279***	2.000**
vs. Const	Student's t	1.724**	1.923**	3.242***	3.149***	2.014**
	Normal	1.100	1.471*	2.790***	3.164***	1.192
GRAS	R. Gumbel	0.855	0.980	2.086**	2.472***	0.997
vs. GAS	Student's t	1.703**	0.948	2.303**	2.601***	0.932
		Exxon and Chevron				
	Normal	2.649***	6.363***	7.845***	8.857***	2.770***
GRAS	R. Gumbel	0.871	3.819***	3.523***	3.678***	3.530***
vs. Const	Student's t	2.598***	6.174***	7.387***	8.934***	3.236***
	Normal	2.624***	6.120***	7.312***	7.725***	1.081
GRAS	R. Gumbel	0.132	2.546***	1.538*	1.143	1.475*
vs. GAS	Student's t	2.297**	5.562***	6.240***	7.033***	1.618*
		Celgene and J&J				
	Normal	1.328*	3.130***	4.520***	8.260***	3.100***
GRAS	R. Gumbel	1.287*	2.982***	4.191***	7.302***	1.240
vs. Const	Student's t	1.300*	3.123***	4.524***	8.308***	2.586***
	Normal	1.384*	2.894***	3.999***	7.153***	0.623
GRAS	R. Gumbel	1.397*	2.912***	4.107***	6.584***	-0.375
vs. GAS	Student's t	1.405*	2.912***	4.084***	7.203***	0.138

Table 2.8: This table presents the"management fee," in annualized percent returns, that an investor with risk aversion given in the column titles would be willing to pay to switch from the Constant or GAS model to the GRAS model. A block bootstrap is used to ascertain the significance of these fees, and estimates that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively.

Management fees and realized portfolio return performance

		Relative risk aversion				
		1	3	7	10	20
		Microsoft and Apple				
	Normal	-1.195	-0.235	-0.486	-0.305	-0.193
GRAS	R. Gumbel	-5.868	-0.010	-0.061	0.021	0.122
vs. Const	Student's t	43.003*	10.286**	3.984**	2.781**	1.327**
	Normal	-8.879	-2.402	-1.434	-1.039	-0.547
GRAS	R. Gumbel	-16.682*	-4.594	-1.700	-1.156	-0.465
vs. GAS	Student's t	-11.907	-6.609**	-3.206***	-2.295***	-1.207***
		Exxon and Chevron				
	Normal	-3.291	-0.451	-0.325	-0.178	-0.118
GRAS	R. Gumbel	6.752	0.688	0.350	0.279	0.109
vs. Const	Student's t	8.969	1.910	0.776	0.505	0.205
	Normal	-1.936	0.118	-0.368	-0.179	-0.081
GRAS	R. Gumbel	1.472	-1.176	-0.057	-0.057	0.010
vs. GAS	Student's t	17.922	4.923	1.732	1.237	0.552
		Celgene and J&J				
	Normal	0.626	1.395	0.385	0.187	0.113
GRAS	R. Gumbel	-9.207	-3.549	-1.477	-1.109	-0.534
vs. Const	Student's t	26.663***	8.015***	3.362***	2.316***	1.178***
	Normal	1.008	0.962	0.271	0.169	0.100
GRAS	R. Gumbel	0.710	-0.237	-0.049	-0.077	-0.030
vs. GAS	Student's t	12.026*	3.907*	1.496*	1.123*	0.549*

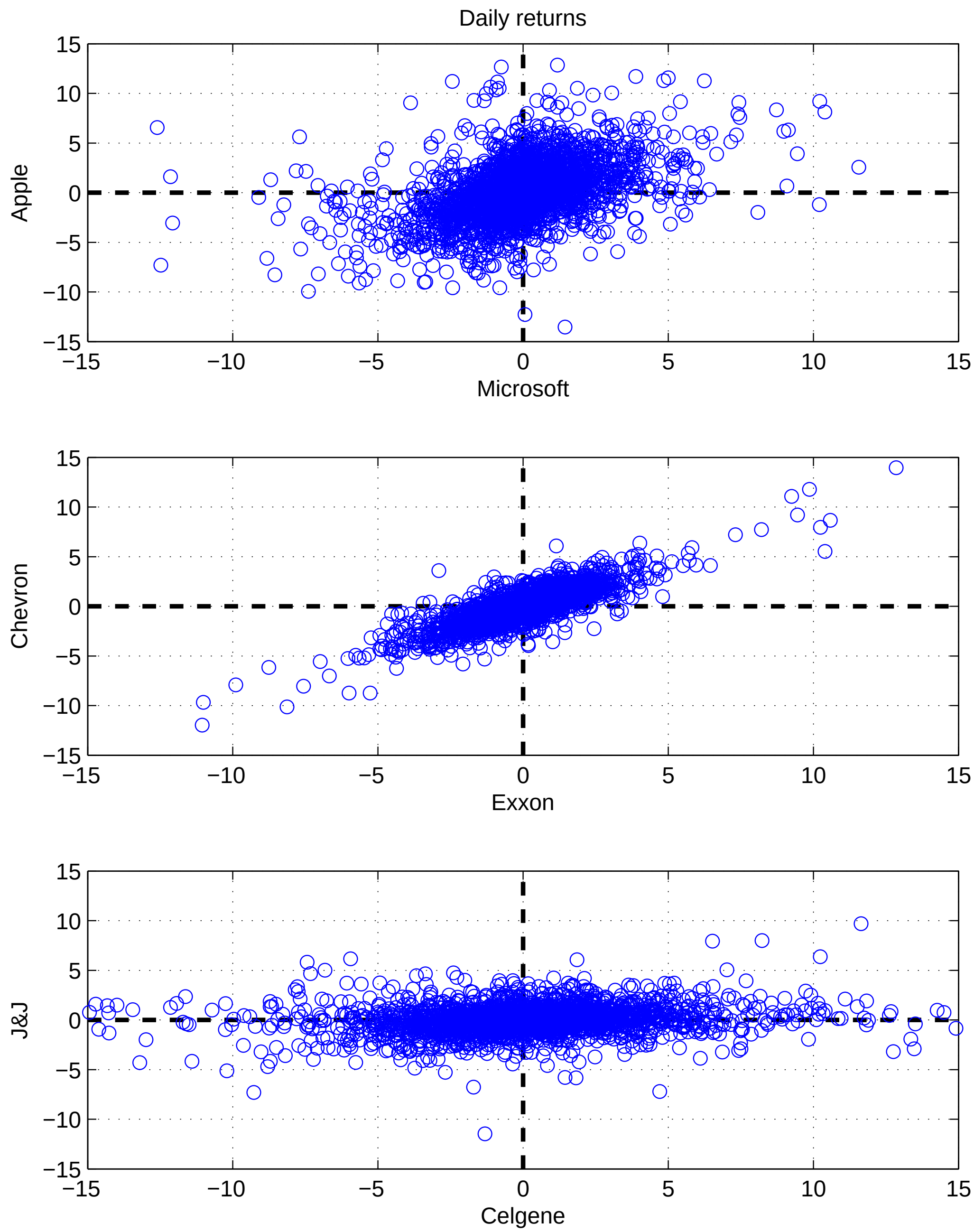


FIGURE 2.1: This figure shows a scatter plot of daily returns on three pairs of stocks. We use data over the period January 2000 to December 2010.

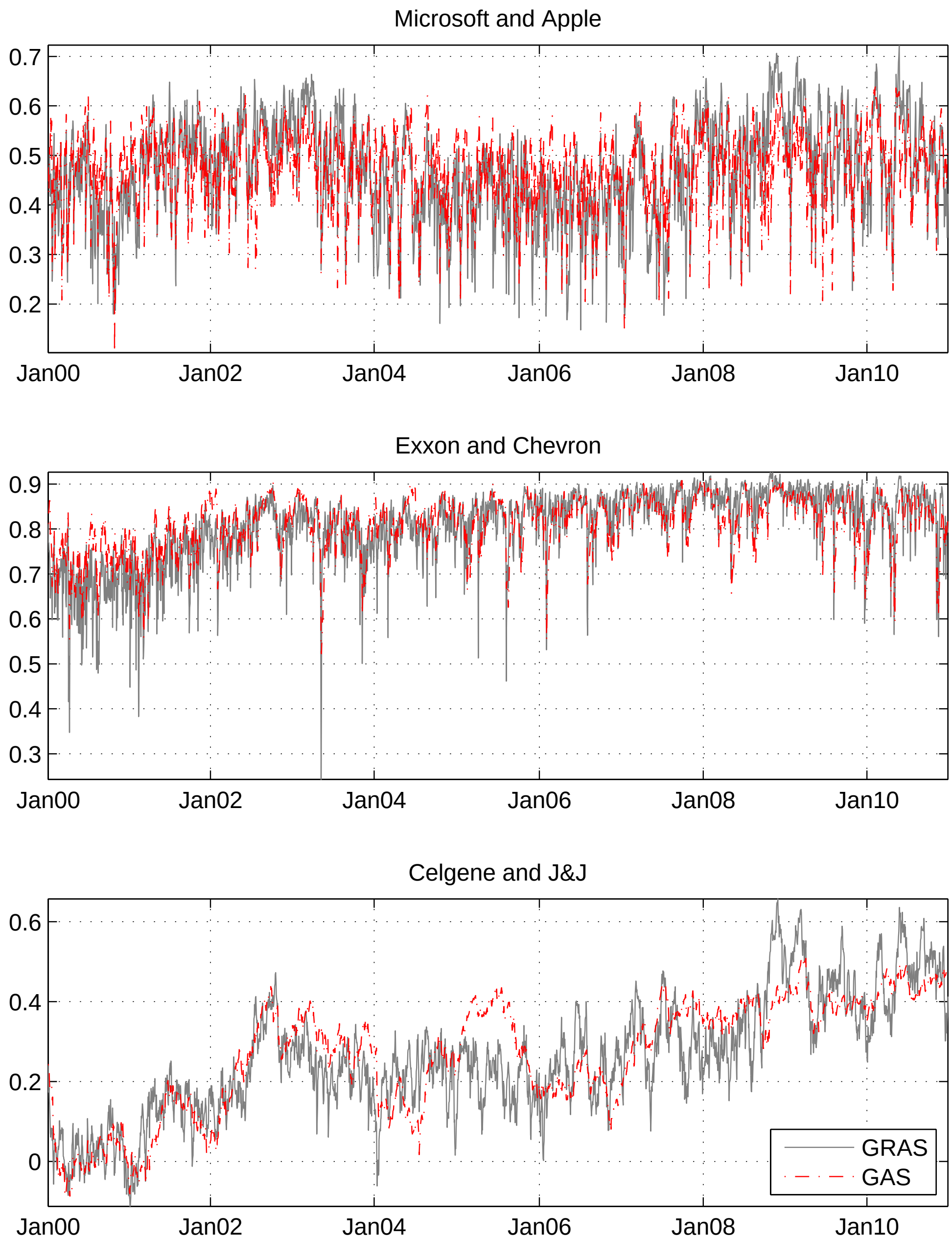


FIGURE 2.2: This figure shows the estimated correlation parameter of the Student's t GRAS model (solid line) against the estimated correlation parameter of the Student's t GAS model (dotted line) for three different pairs of stocks.

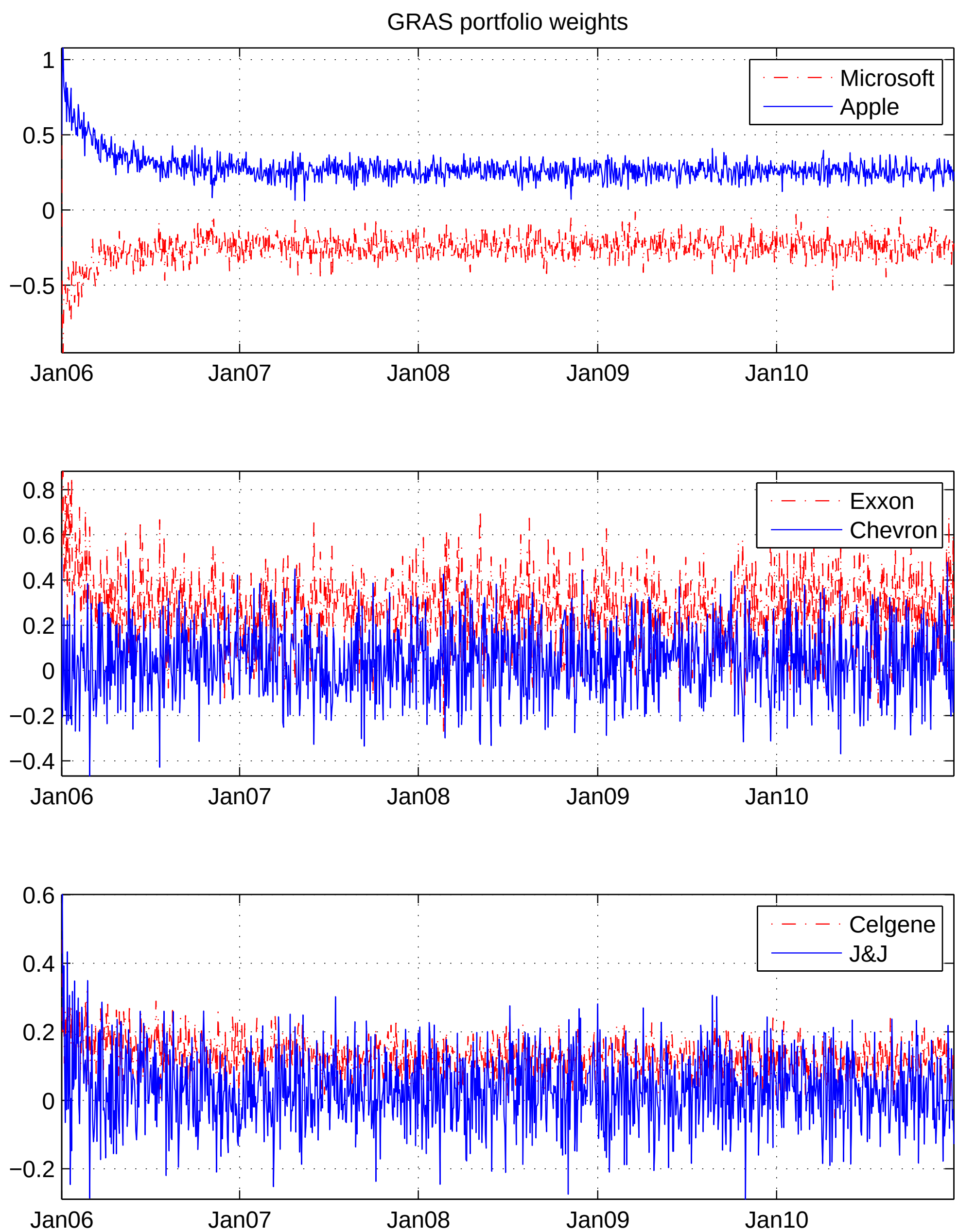


FIGURE 2.3: This figure shows the estimated optimal portfolio weights based on the Student's t GRAS model, for an investor with risk aversion of 7, for three pairs of assets.

3

Implied Correlation Tales: Systemic Risks, Fears and Return Predictability

3.1 Introduction

After the last financial crisis it has been an increasing interest in understanding the dynamics of potential systemic distress. Consequently, a lot of attention has been devoted to understand the extremal tails dependencies between of assets. As pointed out by Hansen (2013), the estimation of tail risks dependence is a challenging task and their implications in several sectors of the economy are of great importance. One of the principal challenges is to provide a measure systemic risks that is, in principle, statistically tractable and has an economic meaning. As an example, Bisias et al. (2012) detects at least thirty one ways to measure systemic risk. Therefore, there is a need of a standardize systemic risk measures or at least to provide a methodology that can capture the complexity behind global distress in the economy. These measures should be able to explain not only the dynamics of the most recent financial crisis but also the prior events of distress in the world economy, which is the motivation of this paper.

The main goal of this work is to explore the effects of time-varying extreme jump tail dependencies not only in asset markets but also in the real economy[1]. In order to achieve this goal, I construct two new measures that exploit the information from option prices and high-frequency data of the equity US market, the Jump Tail Implied Correlation and the Jump Tail Correlation Risk Premia[2]. Firstly, the paper investigates the effect of extreme dependencies in asset returns[3]. As a first step I analyze if the correlation risk can be priced. Using a simple test, I find evidence that the correlation risk carries a negative premium, for both, the tails and the jump-robust part. Consequently, the investors are willing to pay an insurance to hedge against systemic risks and changes in the aggregate market correlation. In addition, I study the forecasting power of several correlation measures proposed in this paper. I find that the implied correlation measures have high forecasting power and outperform the variance risk premium in terms of predictability power. Also, I proposed a new investor's correlation fears index that is directly related to the left tail of the implied correlation.

The main contribution of the paper is the construction of a systemic risk factor – Mixed Principal Quantile Regression (MPQR) – from daily financial measures using a quantile-regression-based methodology. In this direction, the paper fills the gap that exists between the negative shocks to the real economy and the financial sector. In a general context, the MPQR factor is directly related to a phase of any economic crises. Through history, economic downturns have had different origins, and we should be able to recognize the source of the distress, i.e. if it is coming from a war,a natural disasters or the collapse of a bubble. This paper focuses in detecting the later

[1] For example, the GDP growth, the Industrial Production, among others.

[2] The Correlation Premia is defined as the difference between the realized and the options jump tail implied correlations.

[3] In order to apply an empirical procedure to infer the behavior of the jump tails, the statistical tools in the paper rely on extreme value theory type approximations for the jump intensities.

type of rare events – bubbles –. The first two types of extremal events are in general non anticipated, ergo non predictable[4]. As pointed by Allen and Gale (2000b), asset bubbles have showed a common pattern. A first phase characterized for a boom in lending or an economic event, such a reform or liberalization. This event generates an increase in credit and an upward trend in asset prices during a certain period of time, creating a potential bubble. In a second phase, the bubble bursts and the asset prices collapse. During the final phase, agents and firms enter in default positions. The MPQR index has the advantage of characterize and anticipate the collapse of the bubble in the second phase. In this sense, I find that the MPQR index has a good out-of-sample and in-sample performance to forecast quarterly macroeconomic shocks of the US industrial production and the CFNAI economic activity index.

The results and contributions of this paper are related to a wide range of literatures. Recently, Ait-Sahalia et al. (2013), Bollerslev et al. (2013a) and Hansen (2013), study the joint dependence and systemic risks in the tails of the assets. The later discusses the challenges of future and existing approaches to measure systemic risk. He emphasizes the first two different venues of research,tail estimation (econometrics approach), dynamic macroeconomic models and network models. This paper is related to the three venues, I leave the network approach as a future work. In the econometrics literature, Ait-Sahalia et al. (2013) propose a Hawkes jump-diffusion model in which a jump in one region increases the jump intensity in other regions and in the same region. Using financial data for six countries, the authors find evidence that jumps in the US market tend to affect other markets but not the other way around. Bollerslev et al. (2013a) using high-frequency data for fifty stocks and the S&P 500 market portfolio find evidence of asymptotic tail dependence between the individual stocks and the market index. They argue that most of the tail dependence

[4] According to Barro (2006) the World War I, the Great Depression, and World War II were the principal rare events in OECD countries

is due to systematic jump tails and dependencies between the sizes of the simultaneously occurring jumps. Along with the econometrics literature, several works have focused in the contagion dynamics using macroeconomic models and baking contagion channels, see Allen and Gale (2000a), Douglas and Dybvig (1983) and Kiyotaki and Moore (2002) and Adrian and Shin (2008). Caballero and Simsek (2011) introduce a model of fire sales and market breakdowns in which a hit in one institution of the financial network has an indirect impact in other institution of the network. The main contribution of the paper to this set of works is to provide time-varying dependence measures of jump tails.

In the literature of tail measures it has been an effort to model extreme returns using time-varying intensities, for example Duffie, Pan and Singleton (2000). Kelly (2012) uses a cross-section of stock returns to estimate a time-varying conditional tail risk. His results supports the evidence that tail risk has large predictive power for aggregate stock market returns. A previous work by Bollerslev et al. (2011a) provides a framework in which high-frequency data can be used to dynamically estimate tails. Furthermore, Bollerslev et al. (2013b) use a panel of S&P 500 options to estimate the time-varying risk-neutral jump tails. They find evidence of temporal dependencies in the tails, which are stronger for the left tail during market turbulence. This paper contributes to this literature with the estimation of jump correlations under the objective and risk-neutral distribution.

In addition, this paper contributes to the literature related to bubbles detection, their economic policy implications and the asymmetry of information, see Abreu and Brunnermeier (2003), Caballero et al. (2005), Gilchrista et al. (2005), Caballero et al. (2008), Kocherlakota (2009), Caballero (2009a, 2009b) and Caballero (2010). These set of works point that the increase in the demand for safe-assets at an international level, the global imbalances and Knightian uncertainty were critical elements in the recent collapse in the world-wide economy. Additionally, Beaudry and Portier (2004) propose a Vector Error Correction Model (VECM) for quarterly US data from 1948 to 2000 to study the relation between stock prices, total factor productivity (TFP) and business cycles. They find that permanent changes in the productivity growth are preceded by stock market boom. Similar results are drawn by Beaudry and Portier (2005) for Japanese and US data. Consequently, given the evidence that changes in stock prices are followed by changes in real variables, this paper is key in providing a reliable measure that detects large movements of the market stock returns.

Lastly, I provide a new semiparametric forward-looking beta for the diffusive and jump systematic risks. This links a one factor model with the implied correlation measures directly. I find a positive relation between the expected returns and the proposed forward-looking betas. Moreover, I extend the implications of the forward-looking betas to predict individual asset returns. The results show that the forward-looking betas have reasonable predictive power for up to twelve month horizons of individual stock returns.

The paper is organized as follows. Section 3.2 provides the general setup, assumptions of the model and also shows the construction of the implied correlation jump measures. Section 3.3. outlines the estimation procedures for the jump tail parameters and the implied correlation measures. Section 3.4 studies the predictability power of the implied tail correlation measures. Section 3.5 presents the main application of the paper, the link between macroeconomic shocks and financial variable. Section 3.6 concludes.

3.2 Jump-Robust and Discontinuous Implied Correlation

This section presents the general framework to construct a jump-robust version of the implied correlation. Using this framework, I propose a new semiparametric form to estimate the tails of the implied correlation. The continuous-time – no-arbitrage

– framework used in this paper is essentially model-free. It uses semi-parametric models previously analyzed and estimated in the literature to construct the tail elements inside the implied correlation. I begin with a discussion of the basic setup and assumptions.

3.2.1 Setup and Assumptions

The underlying price of the asset P_t, $p_t \equiv \log(P_t)$, is defined on the filtered probability space $(\Omega, , \mathbb{P})$, where $(_t)_{t \geqslant 0}$ denotes the filtration. The general dynamic specification for the underlying asset is assumed to be,

$$\frac{dP_t}{P_{t-}} = \alpha_t dt + \sigma_t dW_t +_{\mathbb{R}} (e^x - 1)\, \widetilde{\mu}^{\mathbb{P}}(dt, dx) \tag{3.1}$$

where α_t and σ_t denote locally bounded drift and instantaneous volatility processes, W_t is a Brownian motion, $\mu(dt, dx)$ is a counting measure for the jumps in the asset with compensator $dt \otimes \nu_t^{\mathbb{P}}(dx)$, then $\widetilde{\mu}^{\mathbb{P}}(dt, dx) \equiv \mu(dt, dx) - dt \otimes \nu_t^{\mathbb{P}}(dx)$ denotes the corresponding martingale measure under $\mathbb{P}$[5].

Denote the discrete-time continuously compounded return from time t to $t+\tau$ as $r_{t,t+\tau} \equiv p_{t+\tau} - p_t$. Therefore, the dynamics of the underlying asset can be expressed by,

$$r_{t,t+\tau} =_t^{t+\tau} (a_s + q_s)\, ds +_t^{t+\tau} \sigma_s dW_s +_t^{t+\tau} {}_{\mathbb{R}} x \widetilde{\mu}^{\mathbb{P}}(ds, dx) \tag{3.2}$$

where q_t is the convexity adjustment associated with the transformation from arithmetic to logarithmic returns. Now, assume that there is an risk-neutral probability measure under which the dynamics of P_t follows,

$$\frac{dP_t}{P_{t-}} = (r_t - \delta_t)\, dt + \sigma_t dW_t^{\mathbb{Q}} +_{\mathbb{R}} (e^x - 1)\, \widetilde{\mu}^{\mathbb{Q}}(dt, dx) \tag{3.3}$$

where r_t and δ_t denote the risk-free rate and the dividend yield of P_t, respectively,

[5] The $\mathbb{P}$ measure is also known as the objective measure or the true probability measure.

$W_t^{\mathbb{Q}}$ is a Brownian motion under $\mathbb{Q}$[6], and $\widetilde{\mu}^{\mathbb{Q}}(dt,dx) \equiv \mu(dt,dx) - dt \otimes \nu_t^{\mathbb{Q}}(dx)$ where $dt \otimes \nu_t^{\mathbb{Q}}(dx)$ is the compensator of the jumps under $\mathbb{Q}$. Since the paper deals not only with the market index but also with individual stocks, I extend the previous setup to I financial assets, where,

$$\mathcal{I} = \left\{ \frac{dP_t^{(i)}}{P_{t-}^{(i)}} = \left(r_t - \delta_t^{(i)}\right) dt + \sigma_t^{(i)} dW_t^{\mathbb{Q},(i)} +_{\mathbb{R}} (e^x - 1)\, \widetilde{\mu}^{\mathbb{Q},(i)}(dt,dx) \right\}_{i=0}^{I+1} \tag{3.4}$$

and

$$\mathcal{T}_{[t,t+\tau]}^{(j)} = \left\{ s \in [t, t+\tau] : \Delta p_j^{(s)} \neq 0 \right\}, j = 0, ..., M \tag{3.5}$$

where $\Delta p_j^{(s)} \equiv p_j^{(s)} - p_{j-}^{(s)}$ is the set of jump prices for asset j[7]. Under this setup I can construct the co-variation measures between two stocks. The corresponding co-variation –in the interval of time $[t, t+\tau]$– between the returns of assets i and j is measured by the quadratic co-variation, defined as,

$$QV_{[t,t+\tau]}^{(i,j)} = \int_t^{t+\tau} \rho_s^{(i,j)} \sigma_s^{(i)} \sigma_s^{(j)} ds + \int_t^{t+\tau} \int_{\mathbb{R}\times\mathbb{R}} x^{(i)} x^{(j)} \mu^{(i,j)}(ds, d\mathbf{x}) \tag{3.6}$$

The quadratic co-variation in (3.6) consists in a continuous part and a discontinuous part. Denote the total continuos co-variation between the assets i and j over the $[t+\tau]$ time-interval as,

[6] The $\mathbb{Q}$ measure is also named the risk-neutral measure and it is generally related to option-prices.

[7] Denote $\mu^{(j,i)}(dt,dx)$ the jump measure for asset j for the jumps that occur at the same time as the asset i. Also, $\mu^{(j,j)}(dt,dx)$ denotes the jump measure for the asset j jumps that occur a times restricted to the set $\mathcal{T}_{[t,t+\tau]}^{(j)} \backslash \left\{ \mathcal{T}_{[t,t+\tau]}^{(j)} \cap \mathcal{T}_{[t,t+\tau]}^{(i)} \right\}$. Then, by definition:

$$\mu^{(j)}(dt,dx) = \mu^{(j,j)}(dt,dx) + \mu^{(j,i)}(dt,dx), \forall i, j \in \mathcal{I}$$

Similarly, denote $\mu^{(i,j)}(dt,dx)$ the jump measure for asset i for the jumps that occur at the same time as the asset j. In addition, the compensators $\nu_t^{\mathbb{K},(j)}(dx)$ for $\mathbb{K} = \mathbb{P}, \mathbb{Q}$, to be a valid jump must satisfy for any t,

$$_{\mathbb{R}} \left(x^2 \wedge 1\right) \nu_t^{\mathbb{K},(j,j)}(dx) +_{\mathbb{R}} \left(x^2 \wedge 1\right) \nu_t^{\mathbb{K},(i,j)} +_{\mathbb{R}} \left(x^2 \wedge 1\right) \nu_t^{\mathbb{K},(j,i)} < \infty$$

$$CV_{[t,t+\tau]}^{(i,j)} = \int_t^{t+\tau} \rho_s^{(i,j)} \sigma_s^{(i)} \sigma_s^{(j)} ds \tag{3.7}$$

Analogously, denote the corresponding total predictable jump co-variation under the objective and risk-neutral measures by,

$$\begin{aligned} JV_{[t,t+\tau]}^{(i,j),\mathbb{P}} &= \int_t^{t+\tau} \int_{\mathbb{R}\times\mathbb{R}} x^{(i)} x^{(j)} v_s^{(i,j),\mathbb{P}} (d\mathbf{x})\, ds \text{ and} \\ JV_{[t,t+\tau]}^{(i,j),\mathbb{Q}} &= \int_t^{t+\tau} \int_{\mathbb{R}\times\mathbb{R}} x^{(i)} x^{(j)} v_s^{(i,j),\mathbb{Q}} (d\mathbf{x})\, ds \end{aligned} \tag{3.8}$$

In order to make the estimation tractable, the following semiparametric structure is imposed to the jump density,

$$\begin{aligned} \nu_t^{\mathbb{K},(j,i)} (dx) &= \Big(\alpha_0^{-,(j,i)} 1_{\{x<0\}} + \alpha_0^{+,(j,i)} 1_{\{x>0\}} + \\ &\quad \Big(\alpha_1^{-,(j,i)} 1_{\{x<0\}} + \alpha_1^{+,(j,i)} 1_{\{x>0\}} \Big) \sigma_t^2 \Big) \nu^{\mathbb{K},(j,i)} (x)\, dx \end{aligned} \tag{3.9}$$

where $\nu^{\mathbb{K},(j,i)} (x)$ is a valid Lévy density for $\mathbb{K} = \mathbb{P}, \mathbb{Q}$ with $\alpha_0^{\pm,(j,i)}$ and $\alpha_1^{\pm,(j,i)}$ nonnegative free parameters. This specification allows for temporal variation in the jumps arrivals, no time-variation in the jump intensities, and it is a function of the latent volatility process σ_t^2. This form on the jump intensities will be the only restriction imposed in the construction if the implied correlation measures. Moreover, this is a very weak specification in the sense that only restricts the time variation to be the same across different jump sizes. This semiparametric form is satisfied in most of the parametric jump-diffusion models when $\alpha_0^{\pm,(j,i)} + \alpha_1^{\pm,(j,i)} \sigma_t^2$ is equal to one or equal to σ_t^2, see Bakshi, Cao, and Chen (1997), Bates (1996, 2000), Pan (2002), and Eraker (2004).

3.2.2 Jump-Robust and Jumps Implied Correlation

In this subsection I extract the implied correlation measures from an arbitrary portfolio consisting of N stocks. Any portfolio can be expressed as a weighted average of all the individual stocks members of the portfolio, $S_t = \sum_{i=1}^{N} \omega_i P_t^{(i)}$, where $\{\omega_i\}_{i=1,..,N}$ are positive weights that are fixed up front. Therefore, the quadratic variation of the portfolio can be decomposed in a continuos part and a discontinuous part, $QV_{[t,t+\tau]}^{(S)} = CV_{[t,t+\tau]}^{(S)} + JV_{[t,t+\tau]}^{(S)}$, with

$$CV_{[t,t+\tau]}^{(S)} = \sum_{i=1}^{N} \omega_i^2 CV_{[t,t+\tau]}^{(i)} + \sum_{i=1}^{N}\sum_{i\neq j} \omega_i \omega_j CV_{[t,t+\tau]}^{(i,j)} \text{ and} \tag{3.10}$$

$$JV_{[t,t+\tau]}^{(S)} = \sum_{i=1}^{N} \omega_i^2 JV_{[t,t+\tau]}^{(i)} + \sum_{i=1}^{N}\sum_{i\neq j} \omega_i \omega_j JV_{[t,t+\tau]}^{(i,j)} \tag{3.11}$$

Using the above expressions and taking the expected value under the risk-neutral measure, I can express the risk neutral expectation of the quadratic variation of the portfolio as,

$$E_t^{\mathbb{Q}}\left(QV_{[t,t+\tau]}^{(S)}\right) = E_t^{\mathbb{Q}}\left(CV_{[t,t+\tau]}^{(S)}\right) + E_t^{\mathbb{Q}}\left(JV_{[t,t+\tau]}^{(S),\mathbb{Q}}\right) \tag{3.12}$$

where the continuous part is given by,

$$E_t^{\mathbb{Q}}\left(CV_{[t,t+\tau]}^{(S)}\right) = \sum_{i=1}^{N} \omega_i^2 E_t^{\mathbb{Q}}\left[CV_{[t,t+\tau]}^{(i)}\right] + \sum_{i=1}^{N}\sum_{i\neq j} \omega_i \omega_j E_t^{\mathbb{Q}}\left[CV_{[t,t+\tau]}^{(i,j)}\right] \tag{3.13}$$

and the discontinuous part by,

$$E_t^{\mathbb{Q}}\left(JV_{[t,t+\tau]}^{(S),\mathbb{Q}}\right) = \sum_{i=1}^{N} \omega_i^2 E_t^{\mathbb{Q}}\left[JV_{[t,t+\tau]}^{(i),\mathbb{Q}}\right] + \sum_{i=1}^{N}\sum_{i\neq j} \omega_i \omega_j E_t^{\mathbb{Q}}\left[JV_{[t,t+\tau]}^{(i,j),\mathbb{Q}}\right] \tag{3.14}$$

The next two assumptions are crucial in order to construct the implied correlation of an arbitrary portfolio. Similar assumptions for continuous prices have been explored by previous literature, see Engle and Kelly (2012) and Driessen et al. (2013) among others.

Assumption 1. *Assume that a single state variable ,$\rho^{\mathbb{Q}}_{C,t}$, drives all continuous pair-wise correlations. Where $\rho^{\mathbb{Q}}_{C,t} = \rho^{(i,j),\mathbb{Q}}_{C,t}$, with*

$$\rho^{(i,j),\mathbb{Q}}_{C,t} = \frac{E^{\mathbb{Q}}_t\left[CV^{(i,j)}_{[t,t+\tau]}\right]}{\left(E^{\mathbb{Q}}_t\left[QV^{(i)}_{[t,t+\tau]}\right]\right)^{1/2}\left(E^{\mathbb{Q}}_t\left[QV^{(j)}_{[t,t+\tau]}\right]\right)^{1/2}} \tag{3.15}$$

Assumption 2. *Assume that a single state variable, $\rho^{\mathbb{Q}}_{J,t}$, drives all discontinuous pair-wise correlations. Where $\rho^{\mathbb{Q}}_{J,t} = \rho^{(i,j),\mathbb{Q}}_{J,t}$, with*

$$\rho^{(i,j),\mathbb{Q}}_{J,t} = \frac{E^{\mathbb{Q}}_t\left[JV^{(i,j),\mathbb{Q}}_{[t,t+\tau]}\right]}{\left(E^{\mathbb{Q}}_t\left[QV^{(i)}_{[t,t+\tau]}\right]\right)^{1/2}\left(E^{\mathbb{Q}}_t\left[QV^{(j)}_{[t,t+\tau]}\right]\right)^{1/2}}$$

Both assumptions, than can be called equicorrelation assumptions and are also related to strategies in derivatives trading, i.e. dispersion trading. In this sense, the risk premium of this strategies should be related to the risk derived from changes of the market-wide correlation. Under the previous assumptions, I propose the following novel correlation measures, the jump-robust implied correlation and the jump tail implied correlation[8].

Definition 3. *Under assumptions (1-2) the Jump-robust Implied Correlation and the Jump Tail Implied Correlation are given by,*

$$\rho^{\mathbb{Q}}_{C,t} = \frac{E^{\mathbb{Q}}_t\left(CV^{(S)}_{[t,t+\tau]}\right) - \sum_{i=1}^{N}\omega_i^2 E^{\mathbb{Q}}_t\left[CV^{(i)}_{[t,t+\tau]}\right]}{\sum_{i=1}^{N}\sum_{i\neq j}\omega_i\omega_j\left(E^{\mathbb{Q}}_t\left[QV^{(i)}_{[t,t+\tau]}\right]\right)^{1/2}\left(E^{\mathbb{Q}}_t\left[QV^{(j)}_{[t,t+\tau]}\right]\right)^{1/2}} \tag{3.16}$$

[8] The jump-robust implied correlation is equivalent to the risk-neutral expected average correlation by Driessen et al. (2013) in the absence of jumps (discontinuities) in the price.

and

$$\rho^{\mathbb{Q}}_{J,t} = \frac{E^{\mathbb{Q}}_t\left(JV^{(S),\mathbb{Q}}_{[t,t+\tau]}\right) - \sum_{i=1}^N \omega_i^2 E^{\mathbb{Q}}_t\left[JV^{(i),\mathbb{Q}}_{[t,t+\tau]}\right]}{\sum_{i=1}^N \sum_{i\neq j} \omega_i\omega_j \left(E^{\mathbb{Q}}_t\left[QV^{(i)}_{[t,t+\tau]}\right]\right)^{1/2} \left(E^{\mathbb{Q}}_t\left[QV^{(j)}_{[t,t+\tau]}\right]\right)^{1/2}} \tag{3.17}$$

The interpretation of the Implied Correlation comes directly from its equicorrelation structure; it can be thought as the constant correlation that will result in the same expected portfolio variance under the risk neutral measure[9]. Moreover, the implied correlation is also the market's expectation of the future correlation between the index components, for both, the continuous and discontinuous implied correlation.

Equations (3.16) and (3.17) can be extended to their "realized analogues" using the objective measures, i.e. computing the expectations under the objective measure instead of the risk neutral measure. The realized correlation measures are related to the concept of equicorrelation introduced by Engel and Kelly (2012). The main difference with the derivation by these authors is the decomposition of the equicorrelation in a continuos and a discontinuous part. The next definition provides the explicit functional form of the continuous and discontinuous realized equicorrelation.

Definition 4. *The Realized Correlation (continuous equicorrelation) and the Jump Tail Realized Correlation (discontinuous equicorrelation) are*

$$\rho^{\mathbb{P}}_{C,t} = \frac{E^{\mathbb{P}}_t\left(CV^{(S)}_{[t,t+\tau]}\right) - \sum_{i=1}^N \omega_i^2 E^{\mathbb{P}}_t\left[CV^{(i)}_{[t,t+\tau]}\right]}{\sum_{i=1}^N \sum_{i\neq j} \omega_i\omega_j \left(E^{\mathbb{Q}}_t\left[QV^{(i)}_{[t,t+\tau]}\right]\right)^{1/2} \left(E^{\mathbb{Q}}_t\left[QV^{(j)}_{[t,t+\tau]}\right]\right)^{1/2}} \tag{3.18}$$

and

$$\rho^{\mathbb{P}}_{J,t} = \frac{E^{\mathbb{P}}_t\left(JV^{(S),\mathbb{P}}_{[t,t+\tau]}\right) - \sum_{i=1}^N \omega_i^2 E^{\mathbb{P}}_t\left[JV^{(i),\mathbb{P}}_{[t,t+\tau]}\right]}{\sum_{i=1}^N \sum_{i\neq j} \omega_i\omega_j \left(E^{\mathbb{P}}_t\left[QV^{(i)}_{[t,t+\tau]}\right]\right)^{1/2} \left(E^{\mathbb{P}}_t\left[QV^{(j)}_{[t,t+\tau]}\right]\right)^{1/2}} \tag{3.19}$$

[9] The interpretation follows the idea developed by Driessen at al. (2013) for the continuous part of the correlation.

The jump tail implied correlation can be further decomposed in positive and negative jumps[10]. The left tail of the implied correlation captures the market's expectation of the future correlation between the negative jumps of the index components[11]. As I will show later, under very mild assumptions the left tail of the implied correlation measure is related to the investor's fears for simultaneous crashes of stocks within the market index. The next section discusses the economic interpretation of the correlation risk.

3.2.3 Correlation Swap Rates and Correlation Risk

As pointed out by Buraschi, Kosowski and Trojani (2013), correlation swap contracts provide a direct measure of exposure to changes in correlation. Following their reasoning, a swap buyer pays the implied correlation at the maturity $T = t + \tau$ of the contract. Therefore, the buyer pays the correlation swap, $SC_{[t,t+\tau]}$, and receives the correlation realized from the initiation to the maturity of the contract, $RC_{[t,t+\tau]}$. Notice that the initial price of the correlation price is zero. Consequently, the correlation swap rate must be equal to the arbitrage free price of the realized correlation,

$$SC_{[t,t+\tau]} = E_t^{\mathbb{Q}}\left[RC_{[t,t+\tau]}\right] \tag{3.20}$$

Taking a long position in a correlation swap entitles the buyer to a payout equal to the notional amount, L, multiplied by the difference between the subsequent realized average pairwise correlation on the basket of underlyings and the implied correlation[12], that is given by,

$$CR_{t,\tau} = \left(RC_{[t,t+\tau]} - SC_{[t,t+\tau]}\right) \tag{3.21}$$

[10] This is also true for the jump tail realized correlation.

[11] The right tail of the implied correlation captures the market's expectation of the future correlation between positive jumps of the index components. Later in the paper I show that the magnitude of the right tail of the implied correlation together with the right and left tail of the realized correlation are negligible.

[12] For simplicity it can be assumed that $L = 1$ and the interpretation won't change.

The next results provide a set of conditions under which the correlation risk premia has a negative sign

$$CRP_{t,\tau} = E_t^{\mathbb{P}}\left[RC_{[t,t+\tau]}\right] - E_t^{\mathbb{Q}}\left[RC_{[t,t+\tau]}\right] = E_t^{\mathbb{P}}\left[RC_{[t,t+\tau]}\right] - SC_{[t,t+\tau]} < 0 \quad (3.22)$$

Similarly, the result in the next proposition also provide a set of conditions under which the jump tail correlation risk premia has a negative sign,

$$\begin{aligned} TCRP_{t,\tau} &= E_t^{\mathbb{P}}\left[TRC_{[t,t+\tau]}\right] - E_t^{\mathbb{Q}}\left[TRC_{[t,t+\tau]}\right] \\ &= E_t^{\mathbb{P}}\left[TRC_{[t,t+\tau]}\right] - TSC_{[t,t+\tau]} < 0 \end{aligned} \quad (3.23)$$

where $TRC_{[t,t+\tau]}$ is the jump tail realized correlation and $TSC_{[t,t+\tau]}$ is the jump tail swap correlation. Moreover, the result can be extended to study the left and right jump tail correlation risk premia. Intuitively, a negative jump tail correlation risk premia is the ex ante premium that an investor is willing to pay to hedge against systemic risks. Whereas, a negative jump-robust correlation risk premia is the ex ante premium that the agents pay in order to hedge against changes in the average correlation.

Proposition 5. *Under assumptions (1-2) and given a set of positive fix weights,* $\{\omega_i\}_{i=1,..,N}$ *, with non-priced continuous individual variance risk,*

$$E_t^{\mathbb{Q}}\left[\int_t^{t+\tau}\left(\sigma_s^{(i)}\right)^2 ds\right] - E_t^{\mathbb{P}}\left[\int_t^{t+\tau}\left(\sigma_s^{(i)}\right)^2 ds\right] \approx 0$$

and with non-priced jump individual variance risk

$$E_t^{\mathbb{Q}}\left[\int_t^{t+\tau}\int_{\mathbb{R}} x^2 v_s^{(i),\mathbb{Q}}(dx)\, ds\right] - E_t^{\mathbb{P}}\left[\int_t^{t+\tau}\int_{\mathbb{R}} x^2 v_s^{(i),\mathbb{P}}(dx)\, ds\right] \approx 0$$

the sign of the correlation risk premium (difference between the Implied Correlation and the Realized Correlation),

$$\mathbf{sign}\left(CRP_{t,\tau}\right) = \mathbf{sign}\left(\rho_t^{\mathbb{P}} - \rho_t^{\mathbb{Q}}\right) = \mathbf{sign}\left(VRP_{t,\tau}\right)$$

where $VRP_{t,\tau} = \frac{1}{\tau}\left(E_t^{\mathbb{P}}\left(QV_{[t,t+\tau]}^{(S)}\right) - E_t^{\mathbb{Q}}\left(QV_{[t,t+\tau]}^{(S)}\right)\right)$[13].

Proposition 5 shows that testing whether correlation risk is priced or not can be done by testing whether $\rho_t^{\mathbb{P}} = \rho_t^{\mathbb{Q}}$. Moreover, it provides a one to one relationship between the variance risk premium and the correlation price. Also, Proposition 5 can be extended to provide a simple testing rule that tell us whether the tails of the correlation can be priced. The following corollary summarizes the testing rules for the jump tail correlation price,

Corollary 6. *Under the assumption of Proposition 5, the sign of the jump tail correlation risk premium (difference between the Jump Tail Implied Correlation and the Jump Tail Realized Correlation),*

$$\mathbf{sign}\left(TCRP_{t,\tau}\right) = \mathbf{sign}\left(\rho_{J,t}^{\mathbb{P}} - \rho_{J,t}^{\mathbb{Q}}\right) = \mathbf{sign}\left(VRP_{t,\tau}^{J}\right)$$

where $VRP_{t,\tau}^{J} = \frac{1}{\tau}\left(E_t^{\mathbb{Q}}\left(JV_{[t,t+\tau]}^{(S)}\right) - E_t^{\mathbb{P}}\left(JV_{[t,t+\tau]}^{(S)}\right)\right)$ *. In addition, we can decompose the price of the jump tail correlation by looking at the left and right jump tail variation as,*

$$TCRP_{t,\tau}^{+} = \rho_{J,t}^{\mathbb{P},+} - \rho_{J,t}^{\mathbb{Q},+} \text{ and } TCRP_{t,\tau}^{-} = \rho_{J,t}^{\mathbb{P},-} - \rho_{J,t}^{\mathbb{Q},-}$$

where the sign is again related one to one to the tails of the variance risk premia,

$$\begin{aligned} VRP_{t,\tau}^{J,+} &= \frac{1}{\tau}\left[E_t^{\mathbb{P}}\left(\int_t^{t+\tau}\int_{x>k_t} x^2 v_s^{\mathbb{P}}(dx)\,ds\right) - E_t^{\mathbb{Q}}\left(\int_t^{t+\tau}\int_{x>k_t} x^2 v_s^{\mathbb{Q}}(dx)\,ds\right)\right] \\ VRP_{t,\tau}^{J,-} &= \frac{1}{\tau}\left[E_t^{\mathbb{P}}\left(\int_t^{t+\tau}\int_{x<-k_t} x^2 v_s^{\mathbb{P}}(dx)\,ds\right) - E_t^{\mathbb{Q}}\left(\int_t^{t+\tau}\int_{x<-k_t} x^2 v_s^{\mathbb{Q}}(dx)\,ds\right)\right] \end{aligned}$$

with $k_t < 0$ *a time-varying cutoff pertaining to the log-jump size.*

[13] See Carr and Wu (2009), Bollerslev, Marrone, Xu and Zho (2012) and Bollerslev, Xu and Zho (2013) for Variance Risk Premium definitions.

3.3 Data and Preliminary Estimates

This section discusses the estimation procedures that are implemented throughout the paper. First, I review the estimation procedure to construct the implied measures using options data. Analogously, I briefly describe the methodology and the high-frequency data that is used to estimate the realized measures.

3.3.1 Options Data and Risk-neutral Measures

I use options data from January 1996 to December 2012 from Option Metrics. In addition, from the same database I obtain the Exchange-traded funds (ETFs) options and returns for sector(s). To construct the daily measures, I use daily data on stock and index options and returns (939 stocks and 9 sectors). As a proxy for the risk free rate I use the zero-coupon curve data. Finally, from the Compustat Index Constituents I get the Daily composition of the S&P500. To avoid obvious inconsistencies and problems with the price quotes, I apply similar filters to those used by Carr and Wu (2009) and those used by the CBOE for the VIX. In addition, I follow their methodology to estimate the implied volatilities for all the active days. The estimates for the two tail measures are based on closest-to-maturity options. Once the filters have been applied, I begin the estimation of the implied volatility of all the stocks and indices in the same way as Carr and Wu (2009). Then, using the same filtering criteria, I calculate the Black-Scholes[14] implied volatilities for all the out-of-the-money options with moneyness k using linear interpolation of options that bracket the targeted strike. From these implied volatilities I compute an out-of-the

[14] For the individual stocks, the options are American-style. As pointed out by Santa-Clara and Saretto (2007) and Driessen et al. (2009), the returns on American and European options are very similar. Therefore, there is no need to adjust for early exercise premium.

money option price with moneyness k to extract the right and left tails,

$$\begin{aligned} RT_t^{\mathbb{Q}}(k) &= \frac{1}{T-t}\int_t^T \int_{\mathbb{R}} \left(e^x - e^k\right)^+ E_t^{\mathbb{Q}}\left(v_s^{\mathbb{Q}}(dx)\right) ds \approx \frac{e^{r(t,T]}C_t(K)}{(T-t)F_{t-}} \\ LT_t^{\mathbb{Q}}(k) &= \frac{1}{T-t}\int_t^T \int_{\mathbb{R}} \left(e^k - e^x\right)^+ E_t^{\mathbb{Q}}\left(v_s^{\mathbb{Q}}(dx)\right) ds \approx \frac{e^{r(t,T]}P_t(K)}{(T-t)F_{t-}} \end{aligned} \quad (3.24)$$

where $k = \ln(K/F_{t-})$ denotes the log-moneyness. As shown in Bollerslev and Todorov (2012), the negative and positive jump quadratic variations with threshold k are proportional to $LT_t^{\mathbb{Q}}(k)$ and $RT_t^{\mathbb{Q}}(k)$, respectively. Denote the proportionality constants as $\Omega^{\mathbb{Q},\pm}\left(\alpha_{\mathbb{Q}}^{\pm}\overline{\nu}_{\psi}^{\mathbb{Q}\pm}; \xi_{\mathbb{Q}}^{\pm}; \sigma_{\mathbb{Q}}^{\pm}\right)$[15]. The authors show that using the sample analogues of the following two equations for three different levels of moneyness results in an exactly GMM estimator.

$$E\left(RT_t^{\mathbb{Q}}(k)\right) = \alpha_{\mathbb{Q}}^{+}\overline{\nu}_{\psi}^{\mathbb{Q}+}\left(tr^+\right)\frac{\sigma_{\mathbb{Q}}^{+}}{1-\xi_{\mathbb{Q}}^{+}}\left(1+\frac{\sigma_{\mathbb{Q}}^{+}}{\xi_{\mathbb{Q}}^{+}}\left(e^k - 1 + tr^+\right)\right)^{1-\frac{1}{\xi_{\mathbb{Q}}^{+}}} \quad (3.25)$$

$$\begin{aligned} E\left(LT_t^{\mathbb{Q}}(k)\right) &= \alpha_{\mathbb{Q}}^{-}\overline{\nu}_{\psi}^{\mathbb{Q}-}\left(tr^-\right)\frac{\xi_{\mathbb{Q}}^{-}}{\xi_{\mathbb{Q}}^{-}+1}\left(e^k\right)^{1+\frac{1}{\xi_{\mathbb{Q}}^{-}}}\left(\frac{\xi_{\mathbb{Q}}^{-}}{\sigma_{\mathbb{Q}}^{-}}\right)^{-\frac{1}{\xi_{\mathbb{Q}}^{-}}} \times \\ & \quad {}_2F_1\left(1+\frac{1}{\xi_{\mathbb{Q}}^{-}}; \frac{1}{\xi_{\mathbb{Q}}^{-}}; 2+\frac{1}{\xi_{\mathbb{Q}}^{-}}; \frac{tr^{-}\frac{\xi_{\mathbb{Q}}^{-}}{\sigma_{\mathbb{Q}}^{-}} - 1}{e^{-k}\frac{\xi_{\mathbb{Q}}^{-}}{\sigma_{\mathbb{Q}}^{-}}}\right) \end{aligned}$$

where $0 < tr^+ \leqslant e^k - 1$, $1 < tr^- \leqslant e^k$[16] The three levels of moneyness suggested in Bollerslev and Todorov (2012) are 0.9250, 0.9125, and 0.9000 for the left tail, and 1.0750,1.0875, and 1.1000 for the right tail. Finally, for each stock, index and sector, the estimated values of $\Omega^{\mathbb{Q},\pm}$, $RT_t^{\mathbb{Q}}(k)$ and $LT_t^{\mathbb{Q}}(k)$ are used to recover

[15] For the specific form of $\Omega^{\mathbb{Q},\pm}$ see section A of the Appendix in Bollerslev and Todorov (2012).

[16] In addition,

$$\alpha_{\mathbb{Q}}^{+} = E\left(\frac{1}{T-t}E\left(\begin{smallmatrix}\mathbb{Q}\\ t\end{smallmatrix}\int_t^T \varphi_s^{+} ds\right)\right) \text{ and } \alpha_{\mathbb{Q}}^{-} = E\left(\frac{1}{T-t}E\left(\begin{smallmatrix}\mathbb{Q}\\ t\end{smallmatrix}\int_t^T \varphi_s^{-} ds\right)\right)$$

the expected value of the discontinuous quadratic variation under the risk-neutral measure, $E_t^{\mathbb{Q}}\left(JV_{[t,t+\tau]}^{(S),\mathbb{Q}+}\right)$ and $E_t^{\mathbb{Q}}\left(JV_{[t,t+\tau]}^{(S),\mathbb{Q}-}\right)$.

The first 5 columns in Table 3.1 Panel B show the summary statistics of the implied variation measures for the S&P500 index. The table shows that the continuous part of the implied variance accounts for 93% of the total variation. In addition, the left tail of the quadratic variation is the major contributor to the jumps variation. The top panel in Figure 3.1 shows the time series of the implied volatility.

3.3.2 High Frequency Data and Statistical Measures

I use high frequency transaction data over the period January 1996 to December 2012, a total of 4,280 trading days, taken from the NYSE's TAQ database. I follow the cleaning rules outlined in Barndorff-Nielsen, et al. (2009), see Li (2013) for details. All of the "realized measures" are constructed using five-minute returns from within the trade day and taking into account the overnight returns. The daily returns are computed as the log-difference of the close prices from the high frequency data base, and are adjusted for stock splits using information from CRSP.

To estimate the jump tails from high frequency data I assume the availability of equidistant price observations for the assets in the market portfolio, over the discrete grid $0, \frac{1}{n}, \frac{2}{n}, .., T$ where $n \in \mathbb{N}$ and $T \in \mathbb{N}$. Denote the price increments over the corresponding discrete time-intervals $\left[\frac{i-1}{n}, \frac{i}{n}\right]$ by $\Delta_i^n p^{(j)} = p_{\frac{i}{n}}^{(j)} - p_{\frac{i-1}{n}}^{(j)}$. The price increments can be split into a jumps and a continuous part. The relevant realized measures that characterize the continuous and jump variations are the Realized Variance (RV), and the Bipower Variation (BV). They are defined as,

$$RV_t^{(j)} =_{i=tn+1}^{tn+n} \left|\Delta_i^n p^{(j)}\right|^2, \; BV_t^{(j)} =_{i=tn+2}^{tn+n} \left|\Delta_i^n p^{(j)}\right| \left|\Delta_{i-1}^n p^{(j)}\right| \tag{3.26}$$

under weak regularity conditions, see Andersen et al. (2003) and Barndorff-

Nielsen and Shephard (2004, 2006),

$$RV_t^{(j)} \overset{\mathbb{P}}{\to}{}_t^{t+1} \left(\sigma_s^{(j)}\right)^2 ds +_t^{t+1} {}_{\mathbb{R}} x^2 \mu^{(j)} (dt, dx) \text{ and } BV_t^{(j)} \overset{\mathbb{P}}{\to}{}_t^{t+1} \left(\sigma_s^{(j)}\right)^2 ds \tag{3.27}$$

Based on these daily realized variation measures, I subsequently estimate the total variation attributable to jumps by,

$$JV_t^{(j)} = RV_t^{(j)} - BV_t^{(j)} \overset{\mathbb{P}}{\to}{}_t^{t+1} {}_{\mathbb{R}} x^2 \mu^{(j)} (dt, dx) \tag{3.28}$$

Similarly to the estimation procedure of the implied measures, Bollerslev and Todorov (2012) show that the negative and positive jump tail quadratic variations with threshold k are proportional to the left tail and right tail under the objective measures, $LT_t^{\mathbb{P}}(k)$ and $RT_t^{\mathbb{P}}(k)$. Denote the proportionality constants as $\Omega^{\mathbb{P},\pm}\left(\alpha_{\mathbb{P}}^{\pm}\overline{\nu}_{\psi}^{\mathbb{P}\pm}; \xi_{\mathbb{P}}^{\pm}; \sigma_{\mathbb{P}}^{\pm}\right)$[17]. Once that the proportionality constants have been recovered, the next step is to estimate,

$$\begin{aligned} RT_t^{\mathbb{P}}(k) &= \left(\alpha_0^+ + \frac{\alpha_1^+}{T-t} E^{\mathbb{P}}\left(\int_t^T \sigma_s^2 ds\right)\right) \int_{\mathbb{R}} \left(e^x - e^k\right)^+ \upsilon_s^{\mathbb{P}}(dx)\, ds \\ LT_t^{\mathbb{P}}(k) &= \left(\alpha_0^- + \frac{\alpha_1^-}{T-t} E^{\mathbb{P}}\left(\int_t^T \sigma_s^2 ds\right)\right) \int_{\mathbb{R}} \left(e^k - e^x\right)^+ \upsilon_s^{\mathbb{P}}(dx)\, ds \end{aligned} \tag{3.29}$$

The only element that needs an special treatment in the above equations is $E^{\mathbb{P}}\left(\int_t^T \sigma_s^2 ds\right)$. The expected value of the integrated volatility under the objective measure can be estimated with a 22-order VAR,

$$\mathbf{X}_t = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{X}_{t-1} + \mathbf{A}_2 \sum_{i=1}^{5} \frac{\mathbf{X}_{t-i}}{5} + \mathbf{A}_3 \sum_{i=1}^{22} \frac{\mathbf{X}_{t-i}}{22} + \varepsilon_t$$

with $\mathbf{X}_t = (BV_t, JV_t, ON_t)$[18], ON_t the overnight returns. Columns 6 to 10 in

[17] For the specific form of $\Omega^{\mathbb{P},\pm}$ see section A of the Appendix in Bollerslev and Todorov (2012). In addition, section A.1 of the Appendix describes the methodology to estimate all the parameters under the physical measure.

[18] Notice that for the estimation of the realized measures I omit the overnight returns and I adjust for stock splits using information from CRSP. The overnight returns account for significant changes in the price.

Table 3.1 Panel B show the summary statistics of the realized variation measures for the S&P500 index. The table shows that the continuous part of the realized variance accounts for most of the total variation. In addition, the left tail of the quadratic variation accounts for almost all the realized jumps variation.

3.3.3 Price of Tail Risk Correlation

The previous two subsections described the methodology to estimate the variation measures that are necessary to compute the implied correlation measures for the S&P500 index. Panel A in Table 3.1 shows the summary statistics of the implied correlation and the realized correlation measures. Similarly to the variation estimates, the continuous part of the implied and realized correlation dominates the jumps implied correlation and the jumps realized correlation, respectively. Also, the right tail implied and realized correlations are close to zero, which is an indication that positive variations in the stocks of the S&P500 are uncorrelated. It is important to mention that the persistence in the correlation measures is in all the cases lower than their variation counterparts – implied volatility measures – (see the AR(1) coefficients in Table 3.1).

The bottom panel in Figure 3.1 shows the time series of the continuous implied correlation. It is clear that behavior of the variation and the correlation measures is different. First, the variation measures clearly explode during the two most recent economic crises and suddenly collapse to lower values after the recession period[19]. Whereas the correlation measure increases gradually during periods of financial turmoil and do not collapse suddenly after these episodes. This behavior can be explained by the fact that investors become more risk averse after episodes of financial turmoil. Once that the crisis has ended, the market's expectation of the future volatility of the S&P500's adjusts to lower values. However, the implied correlation

[19] The dot-com, the 2008 housing bubble and the European debt crisis.

remains high for more time since investor's fears for correlated negative jumps remains high. Notice that for the investors, the expected future correlation of negative jumps in the returns of the S&P500 stocks is around the same during the 9/11 episode and the European debt crisis. This is not the case when we turn our attention to the left tail of the implied volatility.

As I mentioned earlier, in order to test if the jump correlation risk is priced, it is enough to test wether the difference between the average of the tail jump implied correlation and the mean of the tail jump realized correlation is different from zero. The test is based on a t-test with Newey and West standard errors (1987). Panel A in Table 3.4 shows the results of the correlation risk pricing test. First notice that the correlation risk carries a positive premium not only for the market index but also in the nine sectors of the S&P500. The same is true for the jump-robust correlation risk premium. This implies that the investors pay an ex ante premium to hedge against changes in the correlation. In the same direction, the left tail of the correlation risk premium is positive and statistically different from zero. Therefore, the investors are willing to pay an insurance to hedge against systemic risks. This behavior can be observed in Figure 3.3, that shows the time series of the left tail of the correlation risk premium. It is evident that even previous to the financial crises, the left tail correlation carried a positive premium.

Analogously, Panel B in Table 3.4 presents the results of the pricing test for the variation measures. The total variance risk premium and the continuous variance risk premium are statistically different from zero and the sign of the t-value indicates a positive premium. This is true not only for the market index but also for the nine sectors. In addition, the premium in the jump variation is statistically different from zero. Investors are willing to pay a premium to hedge against changes in the market volatility and changes in the tails of the market volatility.

3.4 Return Predictability and Fears

Several studies have explored different measures that are link to future returns on the aggregate market portfolio[20]. In this section, I expand the results found in Driessen et al. (2013) by considering tail variation measures of the implied correlation. To perform the regressions, I use as a dependent variable the continuously compounds returns of the S&P500 index from the interval t to $t + h$[21], with the unit time interval corresponding to a day. The specification in the predictability regressions can be expressed as,

$$h^{-1} r_{t,t+h} = a\left(h\right) + \mathbf{b}\left(h\right) \mathbf{Z}_t + u_{t,t+h},\, t = 1, ..., T - h \tag{3.30}$$

where $\mathbf{Z}_t$ represents a vector of measures in Table 3.1, and h correspond to different monthly return horizons. I begin discussing the results for the aggregate market portfolio and later I expand the specification to allow different variation measures. Panel A in Table 3.5 shows the results of the simple multi-period regression. The values in the table represent the $R^2 s$ that arise from the multi-period equation. The first striking result is that all the implied correlation measures but the right tail implied correlation are statistically significant for a horizon of twelve months. Moreover, the R-squared values of the implied correlation measures are higher than the R-squared values of the variance risk premium beginning in the second month. This clearly shows that in a simplistic setting, the implied correlation measures outperform the predictability power of the variance risk premium; this results are similar to Driessen

[20] Some of these works are Bollerslev et al. (2009, 2014a, 2014b), Drechsler and Yaron (2011), Du and Kapadia (2012), Bekaert and Hoerova (2013), Camponovo et al. (2013), Vilkov and Xiao (2013).

[21] The compound returns are defined by,

$$h^{-1} r_{t,t+h} = h^{-1} \sum_{j=1}^{h} r_{t+(j-1)s,t+js}$$

where $s = 20$ and h represents the number of months.

et al. (2013). The top panel in Figure 3.4 shows the t-statistic time series of the coefficients of the implied correlation measures from the regressions in the Panel A. In addition, the bottom panel in Figure 3.4 shows the R^2 time series of the same set of regressions.

The results are similar when instead of using the implied correlation measures as predictors, I use different correlation risk premium measures. From the last 5 rows in Panel A of Table 3.5, it is clear that only the positive tail of the correlation risk premium is not statistically significant. However, the R-squared values of the continuous correlation risk measures are in general lower than the R-squared values of the variance risk premium. On the other hand, the discontinuous and the left tail correlation risk premium outperform the variance risk premium in terms of the R-squared values. I extend the analysis by controlling the regressions with the variance risk premium, see Panel B in Table 3.5. The first five rows in Panel B show that even if the variance risk premium is included, all the implied correlation measures continue to be significant[22].The last five rows in Panel B show that with the inclusion of the variance risk premium neither the continuous correlation risk premium nor the right tail of the correlation risk premium are statistically significant. Only the coefficients of the left tail of the correlation risk premium and the discontinuous correlation risk are statistically significant.

In order to capture which part of the implied correlation -continuous or discontinuous- is driven the predictability of the market index I run the full model regressions. The predictability results of the full model regressions are showed in Table 3.6. First, the variance risk premium is only significant in the first four months whereas the jump-robust implied correlation is significant for the twelve months ahead. Also, the left tail of the implied correlation is significant for six different predictability intervals, from the third to the eighth month. Although the continuous part clearly dominates,

[22] Notice that now the right tail implied correlation is statistically significant.

the left tail is clearly important to increase the predictability of the market returns. From the results I can conclude that in periods when the implied correlation (left tail and continuous) is high, we expect to observe higher market returns in the future. But in periods when the right tail of the implied correlation is high the expected market returns tend to be low.

To complement the results, the lower panel in Table 3.6 shows the results for the full model using the correlation risk premium. In this case, the left tail of the correlation risk premium dominates the continuous part (which is not statistically significant). Additionally, the variance risk premium becomes significant for more monthly horizons than the full model of Panel A in Table 3.6. The positive sign in the coefficient of the left tail tell us that in periods when investors are willing to pay for an insurance for total collapse in the economy, the expected market returns are in average higher. As it happened in the last financial crisis, when the correlation risk carries a premium, the expected market returns tend to be high. In the next subsections I perform two robustness checks. Firstly, I analyze if the predictability results are driven by the 2008 financial crises or not. Therefore, I perform the analysis in the pre-crises period. Secondly, I use an ad-hoc parametric connection between the realized and the implied correlation matrix.

3.4.1 Robustness Analysis

Pre crises period

This section performs the first robustness check looking at the predictability results for the pre-crises period. Since the implied correlation can be seen as a measure of systemic risks and economic turmoil in the US economy, the first concern that arises is related to effect that the 2007/2008 financial crises has in the results. Taking this into account, I analyze the return predictability of the implied correlation measures during the pre-crises period, January 1996 to December 2006. Table 3.8 Panel A shows the results of the single multi-period regressions. As we can observe, none of the results for the full sample are driven by the financial crises[23]. Moreover, in the full sample the R-squared values of the correlation risk measures were in general lower than the R-squared values of the variance risk premium. In the case of the pre-crisis period the total jump tail and the left tail of the correlation risk premium outperform the market return predictability of the variance risk premium.

Parametric Covariance connection

In this second subsection, I proposed a parametric relationship between the covariance matrix under the objective measure ($\mathbb{P}$) and the risk-neutral measure ($\mathbb{Q}$) to construct a set of parametric correlation measures[24]. This parametrization ensures that the correlation matrices are positive definite and also ensures that the implied correlation is above its realized counterpart. The next theorem summarizes the parametric connection,

Theorem 7. *The implied correlation matrix* $\Gamma_t^{\mathbb{Q}}$ *with elements* $\rho_t^{(i,j),\mathbb{Q}} = \rho_{C,t}^{(i,j),\mathbb{Q}} + \rho_{J,t}^{(i,j),\mathbb{Q}}$, *where* $\rho_{C,t}^{(i,j),\mathbb{Q}} = \rho_{C,t}^{(i,j),\mathbb{P}} - \alpha_t^C \left(1 - \rho_{C,t}^{(i,j),\mathbb{P}}\right)$ *and* $\rho_{J,t}^{(i,j),\mathbb{Q}} = \rho_{J,t}^{(i,j),\mathbb{P}} - \alpha_t^J \left(1 - \rho_{J,t}^{(i,j),\mathbb{P}}\right)$, *is positive definite if and only if the correlation matrices under true probability* $\Gamma_{t,C}^{\mathbb{P}}$ *and* $\Gamma_{t,J}^{\mathbb{P}}$ *with elements* $\rho_{C,t}^{(i,j),\mathbb{P}}$ *and* $\rho_{J,t}^{(i,j),\mathbb{P}}$, *respectively, are positive definite with* $\alpha_t^C \in (-1, 0]$ *and* $\alpha_t^J \in (-1, 0]$.

Theorem 7 can be implemented in theory for any size of a covariance matrix. However, in practice it would be computational intensive to use high frequency data to estimate all the pairwise realized correlations. For example, in a portfolio of 100 stocks, the number of parameters that need to be estimated is 5,050. The

[23] I also performed the analysis using the variance risk premium as a control variable. The results of the full sample and the pre-crises period are equivalent.

[24] This parametrization is an extension of Buss and Vilkov (2009,2012). Other parametrizations have been proposed in the literature, see Chang and Christoffersen (2009), Christoffersen and Pan (2014) among others.

S&P500 contains about 500 stocks, therefore the number of elements to be estimated is 125,250. Therefore, the next corollary help us to reduce the dimensionality using the idea of equicorrelation.

Corollary 8. *Assuming that* $\rho_{C,t}^{(i,j),\mathbb{P}} = \rho_{C,t}^{\mathbb{P}}$ *and* $\rho_{J,t}^{(i,j),\mathbb{P}} = \rho_{J,t}^{\mathbb{P}}$ *for all* i, j, $\Gamma_t^{\mathbb{Q}}$ *is positive definite if and only if* $\alpha_t^C, \alpha_t^J \in (-1, 0]$ *and,*

1. $\Gamma_{t,C}^{\mathbb{P}}$ *and* $\Gamma_{t,J}^{\mathbb{P}}$ *are positive definite matrices or;*

2. $\Gamma_{t,C}^{\mathbb{P}}$ *is negative definite with* $\rho_{C,t}^{(i,j),\mathbb{P}} - \alpha_t^C \left(1 - \rho_{C,t}^{(i,j),\mathbb{P}}\right) > 0$ *(and* $\Gamma_{t,J}^{\mathbb{P}}$ *is positive definite) or;*

3. $\Gamma_{t,J}^{\mathbb{P}}$ *is negative definite with* $\rho_{J,t}^{(i,j),\mathbb{P}} - \alpha_t^J \left(1 - \rho_{J,t}^{(i,j),\mathbb{P}}\right) > 0$ *(and* $\Gamma_{t,C}^{\mathbb{P}}$ *is positive definite) or;*

4. $\Gamma_{t,C}^{\mathbb{P}}$ *and* $\Gamma_{t,J}^{\mathbb{P}}$ *are negative definite matrices with* $\rho_{C,t}^{(i,j),\mathbb{P}} - \alpha_t^C \left(1 - \rho_{C,t}^{(i,j),\mathbb{P}}\right) > 0$ *and* $\rho_{J,t}^{(i,j),\mathbb{P}} - \alpha_t^J \left(1 - \rho_{J,t}^{(i,j),\mathbb{P}}\right) > 0$.

Using the result in Corollary 6 we get,

$$\widetilde{\beta}_{it}^{\mathbb{Q},C} = \left(\rho_{C,t}^{\mathbb{P}} - \alpha_t^C \left(1 - \rho_{C,t}^{\mathbb{P}}\right)\right) \times \frac{\left(E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(i)}\right]\right)^{1/2} \sum_{j=1}^N \omega_j \left(E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(j)}\right]\right)^{1/2}}{E_t^{\mathbb{Q}}\left(CV_{[t,t+\tau]}^{(S)}\right)} \tag{3.31}$$

and also the following equation,

$$\widetilde{\beta}_{it}^{\mathbb{Q},J} = \left(\rho_{J,t}^{\mathbb{P}} - \alpha_t^J \left(1 - \rho_{J,t}^{\mathbb{P}}\right)\right) \times \frac{\left(E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(i)}\right]\right)^{1/2} \sum_{j=1}^N \omega_j \left(E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(j)}\right]\right)^{1/2}}{E_t^{\mathbb{Q}}\left(JV_{[t,t+\tau]}^{(S),\mathbb{Q}}\right)} \tag{3.32}$$

Furthermore, substituting $\rho_{C,t}^{(i,j),\mathbb{Q}} = \rho_{C,t}^{(i,j),\mathbb{P}} - \alpha_t^C \left(1 - \rho_{C,t}^{(i,j),\mathbb{P}}\right)$ in the definition of the continuous quadratic co-variation of a portfolio, the close form solution for α_t^C is,

$$\alpha_t^C = -\frac{E_t^{\mathbb{Q}}\left(CV_{[t,t+\tau]}^{(S)}\right) - \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j \left(E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(i)}\right] E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(j)}\right]\right)^{1/2} \rho_{C,t}^{(i,j),\mathbb{P}}}{\sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j \left(E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(i)}\right] E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(j)}\right]\right)^{1/2} \left(1 - \rho_{C,t}^{(i,j),\mathbb{P}}\right)} \tag{3.33}$$

Analogously for the discontinuous part, substituting $\rho_{J,t}^{(i,j),\mathbb{Q}} = \rho_{J,t}^{\mathbb{P}} - \alpha_t^J \left(1 - \rho_{J,t}^{\mathbb{P}}\right)$ in the discontinuous quadratic co-variation of a portfolio, the close form solution for α_t^J is,

$$\alpha_t^J = -\frac{E_t^{\mathbb{Q}}\left(JV_{[t,t+\tau]}^{(S)}\right) - \rho_{J,t}^{\mathbb{P}} \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j \left(E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(i)}\right] E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(j)}\right]\right)^{1/2}}{\left(1 - \rho_{J,t}^{\mathbb{P}}\right) \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j \left(E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(i)}\right] E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(j)}\right]\right)^{1/2}} \tag{3.34}$$

Notice that under the parametric scheme, and using Corollary 6, the price of the correlation risk between two assets, i and j, can be easily determined by,

$$\frac{1}{\tau}\left(\rho_t^{(i,j),\mathbb{P}} - \rho_t^{(i,j),\mathbb{Q}}\right) = \alpha_t^C \left(1 - \rho_{C,t}^{\mathbb{P}}\right) + \alpha_t^J \left(1 - \rho_{J,t}^{\mathbb{P}}\right) \tag{3.35}$$

The summary statistics and the pricing test of the parametric implied correlation measures are presented in Table 3.7. The summary statistics in Panel A show that the dynamics of the parametric implied correlation measures are similar to the nonparametric implied correlation measures. The continuous part dominates the total correlation and the right tail correlation is barely above zero. Two main differences arise when we looked at the parametric implied correlation and related measures. First, in average the parametric implied correlation is above the nonparametric one. Therefore, the parametric specification could be seen as an upper bound. Second, the correlation risk premium is always positive when the parametric correlation is used (by construction this is true) but as shown before in Figure 3.4 the premium can be negative sometimes. Panel B presents the t-values of the pricing test using the parametric specification; from the results I reject the null that the correlation risk premium under the parametric specification is zero. This is true not only for the market index but also for all the sectors in the S&P500. Additionally, Panel C in Table 3.7 shows that the correlation between the nonparametric correlation measure and its parametric counterpart range between 0.939 to 0.992. This implies that the dynamics under the parametric connection seem to be very close to the nonparametric case.

The second robustness check for the market return predictability is shown in Panel B of Table 3.8. The results for the simple multi-period predictability regressions are not only close to the nonparametric case but in some cases stronger. In general, for all months, the parametric implied correlation measures are statistically significant. Also, the R-squared values of the parametric implied correlation measures are higher than the R-squared values of the variance risk premium beginning in the second month. The main difference is observed when the regressors are the correlation risk measures. Under the parametric connection, all the correlation risk measures outperform the variance risk premium in terms of the R^2 value. Moreover, these measures remain significant from the first to the last month in the predictability time horizon.

Both robustness checks work in favor of the predictability power of the implied correlation measures. So far, the jump-robust implied correlation and the left tail implied correlation have shown to be relevant risk factors to predict future market returns.

3.4.2 Investor´s Fears and Premia

In this subsection I propose a new "systemic risks" index based on the implied correlation measures. Similarly to Bollerslev and Todorov (2011), the investor´s risk premia due to temporal variation in the co-jump intensities can be expressed as,

$$CFI_t\left(k\right) = TCRP^{-}_{t,\tau}\left(k\right) - TCRP^{+}_{t,\tau}\left(k\right) \tag{3.36}$$

This index may be interpreted as a direct measure of investors's fears for a simultaneous crash of all the stocks members in the market. Following the results and intuition in Bollerslev, Todorov and Xu (2014), I can assume that there exists symmetry in the $\mathbb{P}$ jump tails. Moreover, we know that the magnitude of the aggregate market portfolio risk-neutral left jump tail completely dominates that of the right jump tail. Therefore, under assumptions 1 and 2 and assuming the right tail risk-neutral variation is close to zero, $\sigma_t^{\mathbb{Q},J+} \approx 0$, I may express the Investor's Correlation Fears Index (CFI) as,

$$CFI_t\left(k\right) \approx -\frac{E_t^{\mathbb{Q}}\left(\int_t^{t+\tau}\int_{x<-k_t} x^2 v_s^{(S),\mathbb{Q}}\left(dx\right)ds\right)}{\sum_{i=1}^{N}\sum_{i\neq j}\omega_i\omega_j\left(E_t^{\mathbb{Q}}\left[QV^{(i)}_{[t,t+\tau]}\right]\right)^{1/2}\left(E_t^{\mathbb{Q}}\left[QV^{(j)}_{[t,t+\tau]}\right]\right)^{1/2}} \tag{3.37}$$

The above expression provides a new measure for systemic risks in the US economy[25]. The top panel in Figure 3.5 shows the time series of the left tail variance risk premium and bottom panel the CFI. From the figure we can observe how the investors's fears for a simultaneous crash in the US economy increases after the 9/11 terrorist attack, the dot-com bubble, the subprime crises and the European debt crises. The main difference between the correlation measures and the left tail variance risk premium is the persistence. The variation measures tend to return faster to less volatile levels after a distress event. On the other hand, the correlation index

[25] Notice that the CFI reduces to the negative of the left tail of the implied correlation.

is more persistent, showing how investors confidence about a simultaneous recovery slowly adjust after the crises. Then, the investors' perception of a total collapse of the economy was about the same for the 9/11, the housing crisis and the European debt crisis.

Given the observed dynamics in the CFI, and given the predictability power of the left tail implied correlation, I explore the predictability power of the CFI controlling for the variance risk premium and the continuous correlation risk premium. The results of the predictability regressions are presented in Table 3.9. Independently of the period of time, the CFI is highly significant for the twelve months horizons. In addition, from Panel B in Table 3.9 we can observe how the significance of the variance risk premium disappears under this specification and under the pre-crises period. Since I have found that the left tail of the implied correlation is a good termometer of simultaneous crashes in several economic sectors, in the next three sections I explore the relationship between the options market and systemic risks, systematic risks and portfolio selection. I not only focus in the implications of the correlation indices but also in the variation measures of extreme negative events.

3.5 Measuring Systemic Risks

As I mentioned earlier, after the last financial crisis it has been an increasing interest in understanding the dynamics of potential systemic distress. The increasing complexity in the interaction between all the sectors of the economy makes the estimation of systemic risks a challenging task. One of the principal challenges is to provide a measure of systemic risks that is, in principle, statistically tractable and has an economic meaning. Therefore, making use of the methodology proposed by Giglio and Kelly (2013), this section proposes a new systemic risk index based only on the measures explored across this paper. This section studies the relationship between the financial market and the real economy. In addition, I assess whether the forward-looking implied measures are good predictors of future downturns in the real economy.

3.5.1 Systemic Risk and Macroeconomics

The idea follows the Principal Components Quantile Regression (PCQR) methodology proposed by Giglio and Kelly (2013). I make two variations with respect to the PCQR in Giglio and Kelly (2012). First, the individual risk measures used in the analysis have different frequency from the dependent variable. I exploit the fact that the financial markets provide daily information prior to the disclosure of the real economy indicators. Therefore, using intraday information from options and stocks, the frequency of the proposed systemic measure is in principle based on daily information. Second, the set of measures used in this section correspond only to jump tail implied variation and jump tail implied correlations measures.

Having in mind those modifications, assume that the τ^{th} quantile of a macroeconomic variable, y_{t+h} is given by,

$$Q_\tau\left(y_{t+h}|\mathcal{I}_t\right) = \alpha_0 + \alpha_1 f_t \tag{3.38}$$

where f_t is an unobservable factor, $Q_\tau\left(y_{t+h}|\mathcal{I}_t\right) = \arg\inf_q E\left[\rho_\tau\left(y_{t+h} - q\right)\right]$ and $\rho_\tau\left(x\right) = x\left(\tau - I_{x<0}\right)$. In addition, the cross section of predictors (variation measures) can defined as,

$$\mathbf{x}_t = \mathbf{\Lambda}\mathbf{F}_t + \varepsilon_t = \phi f_t + \mathbf{\Psi}\mathbf{g}_t + \varepsilon_t \tag{3.39}$$

where $\mathbf{g}_t$ is an additional factor that only drives the variation measures.

The PCQR follows a three step estimation procedure. First, perform a time series quantile regression of y_{t+h} on a constant and each individual predictor $x_{i,t}$ in order to recover the slope estimate $\widehat{\gamma}_{1,i}$. Second, calculate the cross-section covariance of $x_{i,t}$ and $\widehat{\gamma}_{1,i}$ for each t to get the factor estimate $\widehat{f}_t$. Finally, perform a time series quantile regression of y_{t+h} on a constant and $\widehat{f}_t$.

As pointed out at beginning of this section, one slight modification should be included in the first step to account for the difference in frequencies between the variation measures and the dependent variable. Therefore, the first step estimates the quantile slope coefficient of y_{t+h} on each individual predictor x_i as follows,

$$Q_\tau \left(y_{t+h} | \mathcal{I}_t \right) = \gamma_{0,i} + \gamma_{1,i} x_{i,t} \tag{3.40}$$

But the individual predictor, $x_{i,t}$, consists of daily variation measures, $z_{i-d,t}$, which are linearly filtered as follows,

$$x_{it} = \sum_{d=0}^{D} \omega_d \left(\kappa_\theta \right) z_{i-d,t} \tag{3.41}$$

where d represents the daily lag, $\omega_d \left(\kappa_\theta \right)$ represents parametrized weights with shape is captured by the κ_θ parameter. The estimation procedure is a generalization of the MIDAS type of estimation proposed by Ghysels, Plazzi and Valkanov (2011) in which the weights follow a beta parametrization. Once the modified first pass is performed, the subsequent steps remain unchanged. The estimated fitted value, $\widehat{\alpha}_1 \widehat{f}_t$, obtained in the last step is called the Mixed Principal Components Quantile Regression predictor. The next theorem shows that under very mild conditions the MPQR predictor is consistent.

Theorem 9. *Under Assumptions 1-4 in the appendix of Giglio and Kelly (2013), the MPQR predictor $Q_\tau \left(y_{t+h} | \mathcal{I}_t \right)$ equal to $\alpha_1 f_t$ is given by $\widehat{\alpha}_1 \widehat{f}_t$, where $\widehat{f}_t$ is the second stage factor estimate and $\widehat{\alpha}_1$ is the third stage quantile regression coefficient. For each t, the MPQR quantile forecast satisfies*

$$\widehat{\alpha}_1 \widehat{f}_t - \alpha_1 f_t \xrightarrow{p} 0$$

In order to measure the out-of-sample performance, I use,

$$R^2_{OOS} = 1 - \frac{\sum_{k=0}^{T} \rho_\tau \left(y_{t+h+k} - \widehat{\alpha}_1 \widehat{f}_{t+k} \right)}{\sum_{k=0}^{T} \rho_\tau \left(y_{t+h+k} - \widehat{q}_{\tau+k} \right)}$$

where $\widehat{q}_{\tau+k}$ corresponds to the historical quantile –in which case $\widehat{q}_{\tau+k} = \widehat{q}_\tau$– or corresponds to the out-of-sample fitted forecast values of the quantile regression using only one of the implied measures as explanatory variable. For the empirical investigation I consider five measures – $\{x_{i,t}\}_{i=1}^{5}$– that could potentially capture downturns in the economy, the continuous and left tail of the variance risk premium, the left tail of the implied correlation, the jump-robust implied correlation and the left tail of the correlation risk premium.

Table 3.10 shows the in-sample 20th percentile forecast results for the industrial production (IP) and the Chicago Fed National Activity Index (CFNAI)[26]. From Panel A it can be observed that all the coefficients of the implied measures and the MPQR are statistically significant for both, monthly and quarterly IP forecasts. In terms of economic policy, I are more interested in quarterly forecasts. Therefore, looking only at the quarterly in-sample R^2, we can observe that it is higher for the left tail of the variance risk premium and the MPQR index. Similarly, Panel B in Table 3.10 shows the monthly and quarterly in-sample forecast results of the CFNAI index. The results also indicate that all the individual tail implied measures are statistically significant and the quarterly in-sample R^2 is also higher for the left tail of the variance risk premium. However, the MPQR index is not statistically significant for the CFNAI.

In addition, the sign of the coefficient reveals the direction in which the implied

[26] The dependent variable used in the analysis is a quarterly or monthly shock. In order construct the macroeconomic shock, first for every month an AR process is estimated, where the optimal number of lags is selected according the Akaike information criterion. Then, I calculate the one-month ahead forecast residual, which is the monthly macroeconomic shock. The quarterly shock is just the moving sum of three month residuals. See Bai and Ng (2008), Stock and Watson (2012) and Kelly et al. (2013).

measures and the macroeconomic shocks are related. For example, an increase in the left tail of the implied correlation implies –in average– an increase in the 20th quantile of the IP. Therefore, there is positive relation between the implied correlation and the IP growth shocks. On the other side, the negative sign in the MPQR or in the left tail of the variance risk premium, indicates a negative correlation between the these individual indices and the 20th quantile of the IP. The direction of the signs is the same for the IP and the CFNAI in-sample forecast quantile regressions. The top panels in Figure 3.6 and Figure 3.7 show the observed macroeconomic shocks and the fitted in-sample values. The left tail of the variance risk premium has a better in-sample performance than the MPQR index and the latter is only significant for the in-sample forecast of the IP.

As a robustness check, I use three measures not constructed in this paper: the default return spread (long-term corporate bond return minus the long-term government bond return), the default yield spread (BAA bond yield minus AAA bond yield), and the term spread (the slope of the Treasury yield curve). All credit factors are statistically significance in the in-sample analysis but the default yield spread for the US industrial production.

I extend the analysis to an out-of-sample performance comparison. The results completely change in the out-of-sample comparison which is also the most relevant in terms of policy implications. Table 3.11 shows the out-of-sample $R^{2\prime}s$ that arise between the comparison of two different specifications. A negative value means that the row-specification outperforms the column-specification. Table 3.11 present the out-of-sample results for the IP. First, notice that the best performance is from the MPQR, for monthly and quarterly out-of-sample forecast. Second, the rest of the specifications cannot be ranked only the historical quantile. Analogously, Table 3.12 shows the out-of-sample forecast results for the CFNAI index. According to the results, the best performance is again from MPQR index, which is only significant for quarterly forecast (and monthly when compare against the credit risk indicators). The bottom panels in figures 3.6 and 3.7 show the observed macroeconomic shocks and the fitted out-of-sample values.

It is clear that a systemic risk index constructed only by tail implied measures outperforms in-sample and out-of-sample the rest of the individual implied measures. Consequently, it can be said that the proposed systemic risk index –that is also forward-looking– has a more than acceptable performance to predict macroeconomic shocks.

3.6 Conclusion

This paper proposes two new measures of dependence between the tails of the assets, the jump tail implied correlation and the jump tail correlation risk premium. First, I explore the financial implications of these measures. Using a simple test, I find evidence that the correlation risk carries a negative premium, for both, the tails and the jump-robust part. The results show evidence that the investors are willing to pay an insurance to hedge against systemic risks and changes in the aggregate market correlation. In addition, I study the forecasting power of the different correlation measures proposed in this paper. I find that the implied correlation measures have high forecasting power and outperform the benchmark factor– the variance risk premium –. Moreover, the results remain strong even in the pre-crisis period and in the case of a parametric form for the implied correlation. Also, I proposed a new investor's correlation fears index that is directly related to the left tail of the implied correlation. Consequently, the CFI shows high predictive power of the market returns. In addition, the index is more persistent that the left tail of the implied volatility and the left tail of the variance risk premium. Then, the investors' perception of a total collapse of the economy was about the same for the 9/11, the housing crisis and the European debt crisis.

The main contribution of the paper is the construction of a systemic risk factor from daily financial measures. In this direction, the paper fills the gap that exists between the negative shocks to the real economy and the financial sector. Following a quantile-regression-based methodology, I construct a systemic risk index – Mixed Principal Quantile Regression– with daily tail measures. I find that the MPQR index has a good performance to forecast quarterly macroeconomic shocks of the US industrial production and the CFNAI economic activity index. This result opens the door the continue analyzing the relation between the information embedded in the financial market and the forecast of macroeconomic shocks.

An additional "tale" investigated in this paper is the construction of implied betas. I provide a new semiparametric forward-looking beta for the diffusive and jump systematic risks. This links a one factor model with the implied correlation measures directly. I find a positive relation between the expected returns and the proposed forward-looking betas. Moreover, I extend the implications of the forward-looking betas to predict individual asset returns. The results show that the forward-looking betas have reasonable predictive power for up to twelve month horizons individual stock returns.

Finally, further work can be done in the direction of rare disasters in macro-finance models. Firstly proposed by Rietz (1988) and then brought it back by Barro (2006), these authors use heavy-tailed shocks to economic fundamentals in order to solve asset pricing puzzles. Extensions of this idea have been developed by Gabaix (2012) and Wachter (2013). The measures proposed in this paper can be used link in an elegant way in dynamic stochastic macroeconomic models.

3.7 Proofs

3.7.1 Proposition 5

Under assumptions (1-2) and given a set of positive fix weights, $\{\omega_i\}_{i=1,..,N}$, with non-priced continuous individual variance risk,

$$E_t^{\mathbb{Q}}\left[\int_t^{t+\tau}\left(\sigma_s^{(i)}\right)^2 ds\right]-E_t^{\mathbb{P}}\left[\int_t^{t+\tau}\left(\sigma_s^{(i)}\right)^2 ds\right]\approx 0$$

and with non-priced jump individual variance risk

$$E_t^{\mathbb{Q}}\left[\int_t^{t+\tau}\int_{\mathbb{R}} x^2 v_s^{(i),\mathbb{Q}}(dx)\, ds\right]-E_t^{\mathbb{P}}\left[\int_t^{t+\tau}\int_{\mathbb{R}} x^2 v_s^{(i),\mathbb{P}}(dx)\, ds\right]\approx 0$$

the sign of the correlation risk premium (difference between the Implied Correlation and the Realized Correlation),

$$\mathbf{sign}\left(CRP_{t,\tau}\right)=\mathbf{sign}\left(\rho_t^{\mathbb{P}}-\rho_t^{\mathbb{Q}}\right)=\mathbf{sign}\left(VRP_{t,\tau}\right)$$

where $VRP_{t,\tau}=\frac{1}{\tau}\left(E_t^{\mathbb{P}}\left(QV_{[t,t+\tau]}^{(S)}\right)-E_t^{\mathbb{Q}}\left(QV_{[t,t+\tau]}^{(S)}\right)\right)$.

Proof. First, to facilitate the notation, $E_t^{\mathbb{K}}\left[QV_{[t,t+\tau]}^{(i)}\right]=\left(\sigma_{\mathbb{K},t}^{(i)}\right)^2$ for $\mathbb{K}=\mathbb{P},\mathbb{Q}$ and stock i. Using the definition of correlation risk premium and the assumption of non-priced continuous and jumps individual variance risk:

$$\begin{aligned}
CRP_{t,\tau} &= \frac{\left(\sigma_{\mathbb{P},t}^{S}\right)^2-\sum_{i=1}^{N}\omega_i^2\left(\sigma_{\mathbb{P},t}^{(i)}\right)^2}{\sum_{i=1}^{N}\sum_{i\neq j}\omega_i\omega_j\sigma_{\mathbb{P},t}^{(i)}\sigma_{\mathbb{P},t}^{(j)}}-\frac{\left(\sigma_{\mathbb{Q},t}^{S}\right)^2-\sum_{i=1}^{N}\omega_i^2\left(\sigma_{\mathbb{Q},t}^{(i)}\right)^2}{\sum_{i=1}^{N}\sum_{i\neq j}\omega_i\omega_j\sigma_{\mathbb{Q},t}^{(i)}\sigma_{\mathbb{Q},t}^{(j)}}\\
&= \frac{\left[\left(\sigma_{\mathbb{P},t}^{S}\right)^2-\sum_{i=1}^{N}\omega_i^2\left(\sigma_{\mathbb{P},t}^{(i)}\right)^2\right]-\left[\left(\sigma_{\mathbb{Q},t}^{S}\right)^2-\sum_{i=1}^{N}\omega_i^2\left(\sigma_{\mathbb{Q},t}^{(i)}\right)^2\right]}{\sum_{i=1}^{N}\sum_{i\neq j}\omega_i\omega_j\sigma_{\mathbb{P},t}^{(i)}\sigma_{\mathbb{P},t}^{(j)}}\\
&= \frac{\left(\sigma_{\mathbb{P},t}^{S}\right)^2-\left(\sigma_{\mathbb{Q},t}^{S}\right)^2}{\sum_{i=1}^{N}\sum_{i\neq j}\omega_i\omega_j\sigma_{\mathbb{P},t}^{(i)}\sigma_{\mathbb{P},t}^{(j)}}
\end{aligned}$$

Since the denominator of the above expression is always positive, we have that,

$$\begin{aligned}\mathbf{sign}\left(CRP_{t,\tau}\right) &= \mathbf{sign}\left(\frac{\left(\sigma_{\mathbb{P},t}^{S}\right)^{2}-\left(\sigma_{\mathbb{Q},t}^{S}\right)^{2}}{\sum_{i=1}^{N}\sum_{i\neq j}\omega_{i}\omega_{j}\sigma_{\mathbb{P},t}^{(i)}\sigma_{\mathbb{P},t}^{(j)}}\right)\\ &= \mathbf{sign}\left(\left(\sigma_{\mathbb{P},t}^{S}\right)^{2}-\left(\sigma_{\mathbb{Q},t}^{S}\right)^{2}\right)=\mathbf{sign}\left(VRP_{t,\tau}\right)\end{aligned}$$

$\square$

3.7.2 Corollary 6

Under the assumption of Proposition 5, the sign if the tails correlation risk premium (difference between the Implied Jump Correlation and the Realized Jump Correlation),

$$\mathbf{sign}\left(TCRP_{t,\tau}\right)=\mathbf{sign}\left(\rho_{J,t}^{\mathbb{P}}-\rho_{J,t}^{\mathbb{Q}}\right)=\mathbf{sign}\left(VRP_{t,\tau}^{J}\right)$$

where $VRP_{t,\tau}^{J}=\frac{1}{\tau}\left(E_{t}^{\mathbb{Q}}\left(JV_{[t,t+\tau]}^{(S)}\right)-E_{t}^{\mathbb{P}}\left(JV_{[t,t+\tau]}^{(S)}\right)\right)$.In addition, we can decompose the price of the jump correlation by looking at the left and right jump tail variation as,

$$TCRP_{t,\tau}^{+}=\rho_{J,t}^{\mathbb{P},+}-\rho_{J,t}^{\mathbb{Q},+}\text{ and }TCRP_{t,\tau}^{-}=\rho_{J,t}^{\mathbb{P},-}-\rho_{J,t}^{\mathbb{Q},-}$$

where the sign is again related one to one to,

$$\begin{aligned}VRP_{t,\tau}^{J,+} &= \frac{1}{\tau}\left[E_{t}^{\mathbb{P}}\left(\int_{t}^{t+\tau}\int_{x>k_{t}}x^{2}v_{s}^{\mathbb{P}}\left(dx\right)ds\right)-E_{t}^{\mathbb{Q}}\left(\int_{t}^{t+\tau}\int_{x>k_{t}}x^{2}v_{s}^{\mathbb{Q}}\left(dx\right)ds\right)\right]\\ VRP_{t,\tau}^{J,-} &= \frac{1}{\tau}\left[E_{t}^{\mathbb{P}}\left(\int_{t}^{t+\tau}\int_{x<-k_{t}}x^{2}v_{s}^{\mathbb{P}}\left(dx\right)ds\right)-E_{t}^{\mathbb{Q}}\left(\int_{t}^{t+\tau}\int_{x<-k_{t}}x^{2}v_{s}^{\mathbb{Q}}\left(dx\right)ds\right)\right]\end{aligned}$$

with $k_t < 0$ a time-varying cutoff pertaining to the log-jump size.

Proof. In a similar way to *Proposition 1*, under the assumption of non-priced jumps individual variance risk, and $E_{t}^{\mathbb{K}}\left[JV_{[t,t+\tau]}^{(i)}\right]=\left(\sigma_{\mathbb{K},t}^{(i),J}\right)^{2}$ for $\mathbb{K}=\mathbb{P},\mathbb{Q}$ and stock i, we have that

$$
\begin{aligned}
TCRP_{t,\tau} &= \frac{\left(\sigma_{\mathbb{P},t}^{S,J}\right)^2 - \sum_{i=1}^N \omega_i^2 \left(\sigma_{\mathbb{P},t}^{(i),J}\right)^2}{\sum_{i=1}^N \sum_{i\neq j} \omega_i \omega_j \sigma_{\mathbb{P},t}^{(i)} \sigma_{\mathbb{P},t}^{(j)}} - \frac{\left(\sigma_{\mathbb{Q},t}^{S,J}\right)^2 - \sum_{i=1}^N \omega_i^2 \left(\sigma_{\mathbb{Q},t}^{(i),J}\right)^2}{\sum_{i=1}^N \sum_{i\neq j} \omega_i \omega_j \sigma_{\mathbb{Q},t}^{(i)} \sigma_{\mathbb{Q},t}^{(j)}} \\
&= \frac{\left[\left(\sigma_{\mathbb{P},t}^{S,J}\right)^2 - \sum_{i=1}^N \omega_i^2 \left(\sigma_{\mathbb{P},t}^{(i),J}\right)^2\right] - \left[\left(\sigma_{\mathbb{Q},t}^{S,J}\right)^2 - \sum_{i=1}^N \omega_i^2 \left(\sigma_{\mathbb{Q},t}^{(i),J}\right)^2\right]}{\sum_{i=1}^N \sum_{i\neq j} \omega_i \omega_j \sigma_{\mathbb{P},t}^{(i)} \sigma_{\mathbb{P},t}^{(j)}} \\
&= \frac{\left(\sigma_{\mathbb{P},t}^{S,J}\right)^2 - \left(\sigma_{\mathbb{Q},t}^{S,J}\right)^2}{\sum_{i=1}^N \sum_{i\neq j} \omega_i \omega_j \sigma_{\mathbb{P},t}^{(i)} \sigma_{\mathbb{P},t}^{(j)}}
\end{aligned}
$$

Since the denominator of the above expression is always positive, we have that,

$$
\begin{aligned}
\mathbf{sign}\left(TCRP_{t,\tau}\right) &= \mathbf{sign}\left(\frac{\left(\sigma_{\mathbb{P},t}^{S,J}\right)^2 - \left(\sigma_{\mathbb{Q},t}^{S,J}\right)^2}{\sum_{i=1}^N \sum_{i\neq j} \omega_i \omega_j \sigma_{\mathbb{P},t}^{(i)} \sigma_{\mathbb{P},t}^{(j)}}\right) \\
&= \mathbf{sign}\left(\left(\sigma_{\mathbb{P},t}^{S,J}\right)^2 - \left(\sigma_{\mathbb{Q},t}^{S,J}\right)^2\right) = \mathbf{sign}\left(VRP_{t,\tau}^J\right)
\end{aligned}
$$

analogously, following the same steps, the sign of the left and right jump correlation risk premia is related one-to-one with the sign of the left and right jump variation risk premia, respectively □

3.7.3 Theorem 7

The implied correlation matrix $\Gamma^{\mathbb{Q}}$ with elements $\rho_t^{(i,j),\mathbb{Q}} = \rho_{C,t}^{(i,j),\mathbb{Q}} + \rho_{J,t}^{(i,j),\mathbb{Q}}$, where $\rho_{C,t}^{(i,j),\mathbb{Q}} = \rho_{C,t}^{(i,j),\mathbb{P}} - \alpha_t^C\left(1 - \rho_{C,t}^{(i,j),\mathbb{P}}\right)$ and $\rho_{J,t}^{(i,j),\mathbb{Q}} = \rho_{J,t}^{(i,j),\mathbb{P}} - \alpha_t^J\left(1 - \rho_{J,t}^{(i,j),\mathbb{P}}\right)$, is positive definite if and only if the correlation matrices under true probability $\Gamma_C^{\mathbb{P}}$ and $\Gamma_J^{\mathbb{P}}$ with elements $\rho_{C,t}^{(i,j),\mathbb{P}}$ and $\rho_{J,t}^{(i,j),\mathbb{P}}$, respectively, are positive definite with $\alpha_t^C \in (-1, 0]$ and $\alpha_t^J \in (-1, 0]$.

Proof. The proof of this theorem is based on the proof of Theorem 1 in Buss and Vilkov (2012). Let's start with the if part of the proof. The correlation matrix under the $\mathbb{Q}$ measure can be expressed as,

$$\Gamma^{\mathbb{Q}} = \Upsilon_1 + \Upsilon_2 \text{ with}$$

$$\Upsilon_1 = \Gamma_C^{\mathbb{P}}\left(1+\alpha_t^C\right) - \alpha_t^C\left(\mathbf{1}\cdot\mathbf{1}^T\right) \text{ and } \Upsilon_2 = \Gamma_J^{\mathbb{P}}\left(1+\alpha_t^J\right) - \alpha_t^J\left(\mathbf{1}\cdot\mathbf{1}^T\right)$$

where $\mathbf{1}$ is an $N \times 1$ vector of ones, $\Gamma_C^{\mathbb{P}}$ is an $N \times N$ matrix with elements $\rho_{C,t}^{(i,j),\mathbb{P}}$ and $\Gamma_J^{\mathbb{P}}$ is an $N \times N$ matrix with elements $\rho_{J,t}^{(i,j),\mathbb{P}}$. Since $\alpha_t^C \in (-1,0]$ and $\alpha_t^J \in (-1,0]$, it follows directly that $-\alpha_t^C\left(\mathbf{1}\cdot\mathbf{1}^T\right)$ and $-\alpha_t^J\left(\mathbf{1}\cdot\mathbf{1}^T\right)$ are positive semidefinite matrices. Since $\Gamma_C^{\mathbb{P}}$ and $\Gamma_J^{\mathbb{P}}$ are positive definite matrices and $\alpha_t^C \in (-1,0]$ and $\alpha_t^J \in (-1,0]$, then $\Gamma_C^{\mathbb{P}}\left(1+\alpha_t^C\right)$ and $\Gamma_J^{\mathbb{P}}\left(1+\alpha_t^J\right)$ are positive definite. Finally, the sum of positive definite and positive semidefinite matrices results in a positive definite matrix. Therefore, $\Gamma^{\mathbb{Q}}$ is positive definite.

The only part of the proof assumes $\Gamma^{\mathbb{Q}}$ is positive definite. The only way to ensure that $\Upsilon_1 + \Upsilon_2$ is positive definite is that both matrices are positive definite or at least one is positive definite and the other is positive semidefinite. Then, Υ_1 will be positive definite only if $\Gamma_C^{\mathbb{P}}$ is positive definite and $\alpha_t^C \in (-1,0]$. Similarly, Υ_2 will be positive definite only if $\Gamma_J^{\mathbb{P}}$ is positive definite and $\alpha_t^J \in (-1,0]$. Any other case is follow by a contradiction. Therefore, if $\Gamma^{\mathbb{Q}}$ is positive definite then $\Gamma^{\mathbb{P}}$ is positive definite and $\alpha_t^C \in (-1,0]$ and $\alpha_t^J \in (-1,0]$. □

3.7.4 Corollary 8

Assuming that $\rho_{C,t}^{(i,j),\mathbb{P}} = \rho_{C,t}^{\mathbb{P}}$ and $\rho_{J,t}^{(i,j),\mathbb{P}} = \rho_{J,t}^{\mathbb{P}}$ for all i, j, $\Gamma_t^{\mathbb{Q}}$ is positive definite if and only if $\alpha_t^C, \alpha_t^J \in (-1,0]$ and,

1. $\Gamma_{t,C}^{\mathbb{P}}$ and $\Gamma_{t,J}^{\mathbb{P}}$ are positive definite matrices or;

2. $\Gamma_{t,C}^{\mathbb{P}}$ is negative definite with $\rho_{C,t}^{(i,j),\mathbb{P}} - \alpha_t^C\left(1-\rho_{C,t}^{(i,j),\mathbb{P}}\right) > 0$ (and $\Gamma_{t,J}^{\mathbb{P}}$ is positive definite) or;

3. $\Gamma^{\mathbb{P}}_{t,J}$ is negative definite with $\rho^{(i,j),\mathbb{P}}_{J,t} - \alpha^J_t \left(1 - \rho^{(i,j),\mathbb{P}}_{J,t}\right) > 0$ (and $\Gamma^{\mathbb{P}}_{t,C}$ is positive definite) or;

4. $\Gamma^{\mathbb{P}}_{t,C}$ and $\Gamma^{\mathbb{P}}_{t,J}$ are negative definite matrices with $\rho^{(i,j),\mathbb{P}}_{C,t} - \alpha^C_t \left(1 - \rho^{(i,j),\mathbb{P}}_{C,t}\right) > 0$ and $\rho^{(i,j),\mathbb{P}}_{J,t} - \alpha^J_t \left(1 - \rho^{(i,j),\mathbb{P}}_{J,t}\right) > 0$.

Proof. Theorem 7 does not impose a restriction in the form of the elements of the matrices $\Gamma^{\mathbb{Q}}$, $\Gamma^{\mathbb{P}}_C$ and $\Gamma^{\mathbb{P}}_J$. Therefore, since all the conditions in Theorem 7 are satisfied, $\Gamma^{\mathbb{Q}}$ is positive definite if and only if $\Gamma^{\mathbb{P}}_C$ and $\Gamma^{\mathbb{P}}_J$ are positive definite matrices and $\alpha^C_t \in (-1, 0]$ and $\alpha^J_t \in (-1, 0]$ with $\rho^{(i,j),\mathbb{P}}_{C,t} = \rho^{\mathbb{P}}_{C,t}$ and $\rho^{(i,j),\mathbb{P}}_{J,t} = \rho^{\mathbb{P}}_{J,t}$ for all i, j. In a similar way, if any of the conditions (2) to (4) are satisfied it follows directly that $\Gamma^{\mathbb{Q}}_t$ is positive definite (the other direction is also true). $\square$

3.7.5 Theorem 9

Under Assumptions 1-4 in the appendix of Giglio and Kelly (2013), the MPQR predictor $Q_\tau\left(y_{t+h}|\mathcal{I}_t\right) = \alpha_1 f_t$ is given by $\widehat{\alpha}_1 \widehat{f}_t$, where $\widehat{f}_t$ is the second stage factor estimate and $\widehat{\alpha}_1$ is the third stage quantile regression coefficient. For each t, the MPQR quantile forecast satisfies

$$\widehat{\alpha}_1 \widehat{f}_t - \alpha_1 f_t \xrightarrow{p} 0$$

Proof. This proof is almost by English. The set of predictors, $x_{i,t} = \sum_{d=0}^{D} \omega_d\left(\kappa_\theta\right) z_{i-d,t}$, satisfy the assumptions in White (1994) Corollary 5.12 and Kelly et al. (2013). Therefore, by Theorem 1 in Kelly et al. (2013) we have that, $\widehat{\alpha}_1 \widehat{f}_t - \alpha_1 f_t \xrightarrow{p} 0$. $\square$

3.8 A Note in Systematic Risk: Jumps and Implied Betas

In addition to the systemic risk application, in this small note I study the relation of the implied correlation with the construction of forward-looking betas. Consider the one-factor model representation,

$$r_i = \alpha_i + \beta_i r_0 + \epsilon_i \tag{3.42}$$

where r_i and r_0 represent the returns of asset i and a systemic risk factor, respectively[27]. The idiosyncratic risk is captured by ϵ_i and $\epsilon_i \perp r_0$. As proposed by Bollerslev and Todorov (2010), is natural to decompose the systemic risk factor in a continuous and discontinuous part. Therefore, equation (3.42) can be expressed as

$$r_i = \alpha_i + \beta_i^C r_0^C + \beta_i^J r_0^J + \epsilon_i \tag{3.43}$$

From (3.43) we can easily notice that $\beta_i^C = \frac{Cov\left(r_i, r_0^C\right)}{Var\left(r_0^C\right)}$ and $\beta_i^J = \frac{Cov\left(r_i, r_0^J\right)}{Var\left(r_0^J\right)}$. Using a more general setting we can extract the implied betas. Consider that the underlying asset P_t, $p_t \equiv \log\left(P_t\right)$, is defined on the filtered probability space $\left(\Omega, , \mathbb{P}\right)$, where $\left({}_t\right)_{t \geqslant 0}$ denotes the filtration. The general dynamic specification for the underlying asset is assumed to be,

$$\begin{aligned} dp_{it} \;\; = \;\; & \alpha_{it} dt + \beta_i^C \sigma_{it} dW_{it} + \sigma_{it} dW_{it} + \beta_i^J {}_{E^0} \kappa \left(\delta_0\left(t, x\right)\right) \widetilde{\mu}_0 \left(dt, d\mathbf{x}\right) \\ & + \beta_i^J {}_{E^0} \kappa' \left(\delta_0\left(t, x\right)\right) \mu_0 \left(dt, d\mathbf{x}\right) \\ & + {}_{E^i} \kappa \left(\delta_i\left(t, x\right)\right) \widetilde{\mu}_i \left(dt, d\mathbf{x}\right) + {}_{E^i} \kappa' \left(\delta_i\left(t, x\right)\right) \mu_i \left(dt, d\mathbf{x}\right) \end{aligned} \tag{3.44}$$

and the general dynamic specification for the common asset or common risk factor naturally follows,

$$dp_{0t} = \alpha_{0t} dt + \sigma_{0t} dW_{0t} + {}_{E^0} \kappa \left(\delta_0\left(t, x\right)\right) \widetilde{\mu}_0 \left(dt, d\mathbf{x}\right) + {}_{E^0} \kappa' \left(\delta_0\left(t, x\right)\right) \mu_0 \left(dt, d\mathbf{x}\right) \tag{3.45}$$

Therefore, under mild assumptions

$$\beta_{it}^C = \frac{CV_{[t, t+\tau]}^{(i,0)}}{CV_{[t, t+\tau]}^{(0)}} \text{ and } \beta_{it}^J = \frac{JV_{[t, t+\tau]}^{(i,0)}}{JV_{[t, t+\tau]}^{(0)}} \tag{3.46}$$

[27] Notice that now I denote the return of the systemic risk factor by r_0 whereas in the previous section the market factor was denoted by S, i.e. r_S.

Any portfolio consisting of N stocks can be expressed as the weighted average of the individual stocks as $P_t^{(0)} = \sum_{i=1}^N \omega_i P_t^{(i)}$. In order to be consistent with the previous section, it follows that $P_t^{(0)} = S_t$. This implies that the continuos and discontinuous betas in (3.46) can be re-expressed as,

$$\beta_i^C = \frac{\sum_{j=1}^N \omega_j CV_{[t,t+\tau]}^{(i,j)}}{CV_{[t,t+\tau]}^{(S)}} \text{ and } \beta_{it}^J = \frac{\sum_{j=1}^N \omega_j JV_{[t,t+\tau]}^{(i,j),\mathbb{Q}}}{JV_{[t,t+\tau]}^{(S),\mathbb{Q}}} \tag{3.47}$$

Therefore, the implied (or forward-looking) continuos and discontinuous betas are,

$$\beta_{it}^{\mathbb{Q},C} = \frac{\sum_{j=1}^N \omega_j E_t^{\mathbb{Q}}\left(CV_{[t,t+\tau]}^{(i,j)}\right)}{E_t^{\mathbb{Q}}\left(CV_{[t,t+\tau]}^{(S)}\right)} \text{ and } \beta_{it}^{\mathbb{Q},J} = \frac{\sum_{j=1}^N \omega_j E_t^{\mathbb{Q}}\left(JV_{[t,t+\tau]}^{(i,j),\mathbb{Q}}\right)}{E_t^{\mathbb{Q}}\left(JV_{[t,t+\tau]}^{(S),\mathbb{Q}}\right)} \tag{3.48}$$

Buss and Vilkov (2012) as well as Chang and Christoffersen (2009) where among the pioneers in proposing new forms to construct forward-looking betas using option prices. This paper is the first in construct semiparametric forward-looking betas where the only restrictions come from assumptions 1 and 2 and the semiparametric form of the jump intensities. In addition, it is the first paper in proposed a forward-looking beta that measures the systematic jump risk of the assets. The next subsection describes in detail the construction and implications of the semiparametric feasible implied betas.

We need to rely in Assumptions (1-2) in order to recover the implied betas with a semiparametric structure. The continuous implied beta can be expressed as,

$$\beta_{it}^{\mathbb{Q},C} = \rho_{C,t}^{\mathbb{Q}} \times \frac{\left(E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(i)}\right]\right)^{1/2} \sum_{j=1}^N \omega_j \left(E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(j)}\right]\right)^{1/2}}{E_t^{\mathbb{Q}}\left(CV_{[t,t+\tau]}^{(S)}\right)} \tag{3.49}$$

Similarly, the proposed implied discontinuous beta is,

$$\beta_{it}^{\mathbb{Q},J} = \rho_{J,t}^{\mathbb{Q}} \times \frac{\left(E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(i)}\right]\right)^{1/2} \sum_{j=1}^{N} \omega_j \left(E_t^{\mathbb{Q}}\left[QV_{[t,t+\tau]}^{(j)}\right]\right)^{1/2}}{E_t^{\mathbb{Q}}\left(JV_{[t,t+\tau]}^{(S)}\right)} \tag{3.50}$$

Using the semiparametric implied betas, I examine the relation that exists between them and the expected market returns. Following Black, Jensen and Scholes (1972), I perform a simple portfolio sorting exercise. At the end of each month, according to each set of the semiparametric implied betas, I sort the stocks into portfolios. Then, I compute for each portfolio the value-weighted realized return over the next month. I repeat this process for every day in the sample using a fixed rolling window of one month. The daily availability of the semiparametric implied betas allows me to perform the portfolio sorting exercise in daily horizon.

Table 3.13 shows the results of the sorting exercise using three different beta-specifications grouping the stocks in ten portfolios. The first portfolio is related to the lowest value of the implied beta. From the results in Table 3.13, it is evident a positive relation between the implied betas and the expected returns. In order to provide hard evidence of this positive relationship, I perform a nonparametric monotonicity test proposed Patton and Timmermann (2010). The results are in the bottom of Table 3.13. The penultimate row shows the p-value of the null hypothesis test that there is a negative monotone relation between the expected returns and the implied betas. For all the betas I strongly reject. The last row shows the p-value of the null hypothesis test that there is a positive monotone relationship. The null is barely rejected[28]. The results does not change when I use five and twenty portfolios.

Once that I have found a positive relationship between the implied betas and the expected returns, I perform a cross-sectional analysis in accordance to Black, Jensen,

[28] If the exercise is performed using a simple average instead of a weighted average, the null hypothesis of a positive relation cannot be rejected. In this case, a weighted average seems more pertinent to perform the analysis.

and Scholes (1972). In order to perform the cross-sectional regression; first, I sort the stocks into five, ten and twenty five portfolios according to their semiparametric implied beta. Then, I compute the time-series of the realized portfolio excess returns. The final step is to regress the weighted mean of the realized excess returns of the portfolios on a constant and the weighted mean of the semiparametric implied betas. Table 3.14 shows the cross-sectional results using different sorting conditions and different sizes for the portfolios. Notice that for all the portfolio sizes, the coefficients of the three semiparametric implied betas are statistically significant. In addition, notice that the coefficients for the full model, are statistically significant in the ten and twenty five cross-sectional regressions. Moreover, the signs of the coefficients are different for the continuous and the discontinuous implied betas. Implying that stocks with higher discontinuous forward-looking beta have in average lower returns.

The last exercise using the implied betas is to explore their predictability power of individual stock returns. Using a small subset of the 14 most liquid S&P500 stocks, I perform simple multi-period forecast regressions. Table 3.15 shows the results of the predictability regressions using a horizon of 6 months[29]. As it is shown in Table 3.15, the implied betas are reasonable factors to forecast individual asset returns. In addition, I performed the analysis for a twelve month horizon and the 100 most liquid stocks in the S&P500. I find that in average, 30% of the coefficients of the forward-looking betas in the predictability regression are statistically different from zero.

[29] The results do not change when the horizon is changed. I performed the predictability regression using one month horizon to twelve month horizon.

Table 3.1: Panel A of this table presents summary statistics on the implied correlation measures and the realized correlation measures, over the sample period from January 1996 to December 2012. Panel B shows the summary statistics on the annualized variation measures and the annualized realized variation measures.

Summary statistics, correlation and variance

Panel A: Univariate Statistics, Correlation Measures

	$\rho_t^{\mathbb{Q}}$	$\rho_{C,t}^{\mathbb{Q}}$	$\rho_{J,t}^{\mathbb{Q}}$	$\rho_{J,t}^{\mathbb{Q},+}$	$\rho_{J,t}^{\mathbb{Q},-}$	$\rho_t^{\mathbb{P}}$	$\rho_{C,t}^{\mathbb{P}}$	$\rho_{J,t}^{\mathbb{P}}$	$\rho_{J,t}^{\mathbb{P},+}$	$\rho_{J,t}^{\mathbb{P},-}$
Mean	0.312	0.293	0.019	0.0003	0.019	0.179	0.178	0.00016	0.00015	$9.7e^{-5}$
St.Dev.	0.152	0.140	0.013	0.001	0.012	0.090	0.089	0.00014	0.0001	$6.8e^{-5}$
Skewness	0.703	0.707	0.981	1.932	0.980	1.038	1.037	-3.661	-0.289	-20.013
Kurtosis	3.807	4.063	3.721	9.245	3.785	4.233	4.234	57.453	13.573	662.930
Max.	1.281	1.273	0.087	0.009	0.086	0.592	0.591	0.001	0.001	0.0001
Min.	-0.344	-0.334	-0.012	-0.006	-0.007	-0.166	-0.165	-0.002	-0.001	-0.003
AR(1)	0.886	0.876	0.919	0.773	0.916	0.975	0.975	0.725	0.879	0.393

Panel B: Univariate Statistics, Variation Measures

	$\sigma_t^{\mathbb{Q}}$	$\sigma_{C,t}^{\mathbb{Q}}$	$\sigma_{J,t}^{\mathbb{Q}}$	$\sigma_{J,t}^{\mathbb{Q},+}$	$\sigma_{J,t}^{\mathbb{Q},-}$	$\sigma_t^{\mathbb{P}}$	$\sigma_{C,t}^{\mathbb{P}}$	$\sigma_{J,t}^{\mathbb{P}}$	$\sigma_{J,t}^{\mathbb{P},+}$	$\sigma_{J,t}^{\mathbb{P},-}$
Mean	6.148	5.730	0.418	0.021	0.397	2.638	2.635	0.003	0.003	0.001
St.Dev.	5.915	5.445	0.480	0.048	0.436	3.352	3.348	0.004	0.003	0.001
Skewness	4.270	4.317	3.835	6.120	3.585	4.928	4.928	4.962	4.962	4.962
Kurtosis	31.782	32.536	24.872	53.297	22.277	37.897	37.897	38.295	38.295	38.295
Max.	73.943	68.737	5.206	0.715	4.662	37.240	37.200	0.042	0.034	0.007
Min.	0.688	0.663	0.001	0.0001	0.001	0.144	0.143	0.0003	0.0003	0.00007
AR(1)	0.948	0.945	0.959	0.917	0.959	0.997	0.997	0.998	0.998	0.998

Table 3.2: Panel A shows the summary statistics on the correlation risk premium and the variance risk premium measures. Panel B shows the structure of the options data in average.

Summary statistics, risk premia

Panel A: Univariate Statistics, Premium Measures

	CRP	CRP_C	CRP_J	CRP_J^+	CRP_J^-	VRP	VRP_C	VRP_J	VRP_J^+	VRP_J^-
Mean	0.133	0.114	0.019	0.0001	0.019	3.511	3.096	0.415	0.019	0.397
St.Dev.	0.111	0.103	0.012	0.001	0.012	3.456	3.062	0.476	0.046	0.435
Skewness	0.738	0.751	0.981	1.967	0.980	3.562	3.415	3.829	6.203	3.583
Kurtosis	6.152	7.153	3.725	9.574	3.786	24.378	23.581	24.812	55.037	22.262
Max.	1.152	1.144	0.086	0.009	0.086	40.653	35.482	5.172	0.700	4.656
Min.	-0.234	-0.275	-0.012	-0.005	-0.007	-8.414	-10.467	0.0004	-0.003	0.0004
AR(1)	0.802	0.788	0.919	0.762	0.916	0.853	0.832	0.959	0.908	0.959

Panel B: Options Data Statisitcs

	Active Days	# Strikes	# Stocks	Mcap	#Strikes within
C. Disc.	409	8.438	0.521	0.797	8.942
C. Staples	972	12.420	0.560	0.759	10.735
Energy	2,079	35.620	0.748	0.860	12.010
Financial	1,464	14.247	0.384	0.629	10.266
Health	676	9.757	0.485	0.722	9.984
Industrial	1,042	13.907	0.490	0.752	9.879
Tech	1,011	9.933	0.580	0.888	12.686
Materials	1,382	15.544	0.458	0.767	10.444
Utilities	1,229	11.087	0.128	0.243	8.102
S&P 500	4,183	79.528	0.298	0.628	10.553

Table 3.3: This table shows the contemporaneous correlation of the implied correlation measures, the variance risk premium and correlation risk premium measures, over the sample period from January 1996 to December 2012

Contemporaneous Correlations

	$\rho_t^{\mathbb{Q}}$	$\rho_C^{\mathbb{Q}}$	$\rho_J^{\mathbb{Q}}$	$\rho_J^{\mathbb{Q}+}$	$\rho_J^{\mathbb{Q}-}$	VRP_C	VRP_J	VRP_J^+	VRP_J^-	CRP	CRP_C	CRP_J	CRP_J^+	CRP_J^-
$\rho_t^{\mathbb{Q}}$	1.00	0.99	0.91	0.60	0.91	-0.24	0.30	0.29	0.30	0.89	0.76	0.91	0.57	0.91
$\rho_C^{\mathbb{Q}}$		1.00	0.89	0.59	0.89	-0.25	0.31	0.30	0.31	0.82	0.77	0.89	0.56	0.89
$\rho_{J,t}^{\mathbb{Q}}$			1.00	0.66	0.99	-0.14	0.20	0.19	0.20	0.65	0.58	0.99	0.63	0.99
$\rho_J^{\mathbb{Q}+}$				1.00	0.60	0.04	0.11	0.11	0.11	0.40	0.35	0.66	0.99	0.60
$\rho_J^{\mathbb{Q}-}$					1.00	-0.14	0.20	0.19	0.20	0.65	0.58	0.99	0.57	0.99
VRP_C						1.00	-0.98	-0.98	-0.98	-0.40	-0.42	-0.14	-0.05	-0.14
VRP_J							1.00	0.99	0.99	0.43	0.44	0.20	0.12	0.20
VRP_J^+								1.00	0.99	0.43	0.44	0.19	0.12	0.19
VRP_J^-									1.00	0.44	0.44	0.20	0.12	0.20
CRP										1.000	0.99	0.65	0.41	0.65
CRP_C											1.00	0.58	0.36	0.58
CRP_J												1.00	0.63	0.997
CRP_J^+													1.00	0.57
CRP_J^-														1.00

Table 3.4: The top panel of this table presents the t-statistics for the comparison between the time series of the implied correlation measures and the realized correlation measures. The bottom panel shows the corresponding t-statistics for the comparison between the time series of the implied variation measures and the realized variation measures. I employ a two-sided Rivers and Vuong (2002).

Correlation and Variance Risk Pricing

	Panel A: Correlation Risk Premia				
	CRP	CRP_C	CRP_J	CRP_J^+	CRP_J^-
C. Disc.	11.966	11.270	11.994	8.475	10.967
C. Staples	18.848	17.955	18.295	9.229	20.025
Energy	39.624	34.992	59.425	31.094	61.384
Financial	18.986	15.999	23.563	26.161	18.611
Health	16.839	16.228	13.200	2.710	14.078
Industrial	23.752	22.535	21.217	14.285	20.442
Tech	3.644	2.405	12.826	1.675	15.583
Materials	19.033	17.581	20.755	9.822	22.177
Utilities	15.479	15.771	5.224	4.137	4.728
S&P 500	31.287	29.118	36.261	3.253	38.098
	Panel B: Variance Risk Premia				
	VRP	VRP_C	VRP_J	VRP_J^+	VRP_J^-
C. Disc.	7.980	7.515	5.804	5.334	5.853
C. Staples	13.058	13.449	9.682	7.438	10.225
Energy	18.436	18.474	13.575	10.731	14.150
Financial	9.823	9.631	10.443	9.004	11.003
Health	12.019	12.437	7.279	5.121	7.573
Industrial	17.092	17.205	11.928	8.800	12.420
Tech	13.629	12.962	10.738	7.678	11.457
Materials	18.411	17.467	13.876	11.317	14.478
Utilities	12.605	11.831	8.375	7.033	8.570
S&P 500	23.759	24.110	18.497	8.959	19.317

Table 3.5: Panel A of this table presents the R^2s from the simple predictability regressions on the S&P500 index, over the sample period from January 1996 to December 2012. Panel B shows the corresponding R^2s from the predictability regressions controlling by the variance risk premium on the S&P500 index: Estimates that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively.

Portfolio Return Predictability Regressions

	Panel A: Single Regression						
	Months						
	1	2	3	4	6	9	12
VRP_t	2.86***	2.13***	3.79***	3.44***	0.79**	0.21	0.03
$\rho_t^{\mathbb{Q}}$	1.54***	3.79***	6.25***	6.87***	8.00***	8.59***	8.36***
$\rho_{C,t}^{\mathbb{Q}}$	1.55***	3.81***	6.22***	6.84***	7.98***	8.66***	8.43***
$\rho_{J,t}^{\mathbb{Q}}$	1.13***	2.94***	5.44***	6.07***	6.89***	6.56***	6.30***
$\rho_{J,t}^{\mathbb{Q},+}$	0.44	0.57	0.81	0.69	1.19***	1.74***	2.19***
$\rho_{J,t}^{\mathbb{Q},-}$	1.13***	3.06***	5.71***	6.43***	7.20***	6.73***	6.39***
CRP_t	0.24	0.62***	1.37***	1.07***	0.71***	0.96***	1.41***
CRP_t^C	0.15	0.41*	0.96***	0.66***	0.35***	0.55***	0.95***
CRP_t^J	1.10***	2.88***	5.34***	5.95***	6.72***	6.39***	6.15***
$CRP_t^{J,+}$	0.29	0.30	0.41	0.28	0.53*	0.89***	1.27***
$CRP_t^{J,-}$	1.12***	3.03***	5.68***	6.39***	7.14***	6.68***	6.34***
	Panel B: Control by Variance Risk Premium						
$\rho_t^{\mathbb{Q}}$	3.81**	5.13***	8.68***	8.95***	8.19***	8.59***	8.44***
$\rho_{C,t}^{\mathbb{Q}}$	3.82***	5.14***	8.66***	8.92***	8.17***	8.66***	8.51***
$\rho_{J,t}^{\mathbb{Q}}$	3.55*	4.45***	8.10***	8.37***	7.17***	6.57***	6.33***
$\rho_{J,t}^{\mathbb{Q},+}$	3.33	2.73	4.65**	4.17*	2.01***	1.96***	2.24***
$\rho_{J,t}^{\mathbb{Q},-}$	3.52*	4.51***	8.27***	8.63***	7.44***	6.74***	6.42***
CRP_t	2.87	2.35	4.36***	3.84*	1.22*	1.01***	1.42***
CRP_t^C	2.86	2.23	4.11	3.60	0.95	0.64*	0.95***
CRP_t^J	3.53*	4.38***	8.01***	8.26***	6.99***	6.40***	6.17***
$CRP_t^{J,+}$	3.18	2.45	4.24	3.74	1.34*	1.11***	1.31***
$CRP_t^{J,-}$	3.51**	4.48***	8.24***	8.59***	7.37***	6.69***	6.38***

Table 3.6: The top panel of this table presents the estimated coefficients from the full regressions –using implied correlation measures– on the S&P500 index, over the sample period from January 1996 to December 2012. The bottom panel shows the corresponding estimated coefficients from the full regressions –using correlation risk measures– on the S&P500 index: Estimates that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively. The last row in each panel corresponds to the R^2 of the predictability regressions.

Full Model Predictability Regressions

	Months						
	1	2	3	4	6	9	12
	Full Model 1						
Const	-1.02	-1.23***	-1.24***	-1.13***	-0.94***	-0.75***	-0.52***
	-(1.69)	-(2.84)	-(3.61)	-(3.77)	-(3.66)	-(3.53)	-(2.80)
VRP_t	0.21***	0.09*	0.09***	0.07*	-0.01	-0.03	-0.04
	(2.63)	(1.91)	(3.01)	(1.93)	-(0.25)	-(1.55)	-(2.19)
$\rho^{\mathbb{Q}}_{C,t}$	0.06***	0.06***	0.05***	0.04***	0.05***	0.05***	0.05***
	(2.11)	(3.17)	(3.19)	(3.53)	(4.23)	(5.52)	(5.69)
$\rho^{\mathbb{Q},+}_{J,t}$	0.32	-2.03	-2.51	-2.83**	-2.42**	-1.84*	-1.15
	(0.12)	-(0.95)	-(1.64)	-(2.14)	-(2.09)	-(1.90)	-(1.45)
$\rho^{\mathbb{Q},-}_{J,t}$	-0.10	0.10	0.31*	0.35***	0.27**	0.08	0.02
	-(0.41)	(0.51)	(1.99)	(2.61)	(2.21)	(0.80)	(0.21)
R^2	4.55	7.30	12.05	13.17	13.33	14.52	13.84
	Full Model 2						
Const	-0.45	-0.63	-0.79	-0.74	-0.53	-0.31	-0.17
	-(0.98)	-(1.84)	-(2.84)	-(2.96)	-(2.47)	-(1.78)	-(1.03)
VRP_t	0.26***	0.14***	0.14***	0.12***	0.04*	0.01	-0.01
	(3.28)	(2.82)	(4.17)	(3.54)	(1.78)	(0.46)	-(0.46)
$CRP^{C}_{t,\tau}$	-0.03	-0.03*	-0.02**	-0.03***	-0.03***	-0.02***	-0.01*
	-(1.42)	-(1.75)	-(2.02)	-(3.03)	-(3.94)	-(2.93)	-(1.76)
$CRP^{J,+}_{t,\tau}$	0.85	-1.35	-1.99	-2.39*	-1.99*	-1.30	-0.80
	(0.32)	-(0.67)	-(1.38)	-(1.93)	-(1.89)	-(1.53)	-(1.09)
$CRP^{J,-}_{t,\tau}$	0.45**	0.66***	0.73***	0.76***	0.73***	0.56***	0.43***
	(2.24)	(4.34)	(5.89)	(7.01)	(8.14)	(7.67)	(7.06)
R^2	3.82	5.04	9.26	10.49	9.76	8.01	6.89

Table 3.7: Panel A of this table presents summary statistics on the parametric implied correlation and risk premium, over the sample period from January 1996 to December 2012. Panel B shows the t-statistics for the comparison between the time series of the parametric implied and realized correlation measures. I employ a two-sided Rivers and Vuong (2002). Panel C shows the contemporaneous correlations.

Parametric Covariance Connection

Panel A: Summary Statistics

	$\rho_t^{\mathbb{Q}}$	$\rho_{C,t}^{\mathbb{Q}}$	$\rho_{J,t}^{\mathbb{Q}}$	$\rho_{J,t}^{\mathbb{Q}+}$	$\rho_{J,t}^{\mathbb{Q}-}$	C^{RP}	C_C^{RP}	C_J^{RP}	C_J^{RP+}	C_J^{RP-}
Mean	0.52	0.49	0.02	0.001	0.02	0.34	0.32	0.02	0.001	0.02
St.de.	0.22	0.21	0.01	0.001	0.01	0.14	0.13	0.01	0.001	0.01
Skew.	0.74	0.74	1.02	2.737	0.99	0.79	0.80	1.06	2.65	0.99
Kurt.	3.41	3.41	3.83	12.49	3.86	3.93	4.23	3.94	11.84	3.85
Max.	1.42	1.41	0.089	0.009	0.09	1.29	1.29	0.09	0.01	0.08
Min.	0.04	0.03	$8e^{-4}$	-0.001	$7e^{-5}$	0.05	0.05	$1e^{-4}$	$9e^{-7}$	$7e^{-5}$
AR(1)	0.94	0.94	0.92	0.73	0.92	0.88	0.87	0.92	0.74	0.91
Panel B: Pricing Test (H_0: $\rho_t^{\mathbb{Q}} = \rho_t^{\mathbb{P}}$)										
C. D.	14.84		37.95		38.71		10.61		16.63	
C. S.	24.16		38.32		39.55		16.26		28.07	
Ene.	64.49		88.27		88.25		37.63		67.60	
Fin.	38.58		65.41		65.14		29.41		38.91	
Hea.	18.38		39.47		40.63		11.36		19.32	
Ind.	40.34		62.17		63.55		22.90		43.41	
Tech	29.16		39.93		40.64		17.25		32.52	
Mat.	48.32		62.67		62.78		24.85		39.34	
Util.	21.29		36.30		36.60		12.34		25.00	
S&P	41.71		57.49		58.58		20.77		43.34	

Panel C: Contemporaneous Correlations

	$\rho_t^{\mathbb{Q}}$	$\rho_{C,t}^{\mathbb{Q}}$	$\rho_{J,t}^{\mathbb{Q}}$	$\rho_{J,t}^{\mathbb{Q}+}$	$\rho_{J,t}^{\mathbb{Q}-}$	$\rho_t^{\mathbb{Q}-\mathbb{P}}$	$\rho_{C,t}^{\mathbb{Q}-\mathbb{P}}$	$\rho_{J,t}^{\mathbb{Q}-\mathbb{P}}$	$\rho_{J,t}^{\mathbb{Q}-\mathbb{P}+}$	$\rho_{J,t}^{\mathbb{Q}-\mathbb{P}-}$
$\rho_t^{\mathbb{Q}}$	1.00	0.99	0.91	0.60	0.91	0.94	0.94	0.89	0.48	0.89
$\rho_{C,t}^{\mathbb{Q}}$		1.00	0.89	0.59	0.89	0.94	0.94	0.88	0.47	0.88
$\rho_{J,t}^{\mathbb{Q}}$			1.00	0.66	0.99	0.91	0.90	0.99	0.56	0.99
$\rho_{J,t}^{\mathbb{Q}+}$				1.00	0.60	0.59	0.59	0.63	0.93	0.58
$\rho_{J,t}^{\mathbb{Q}-}$					1.00	0.91	0.90	0.99	0.50	0.99
$\rho_t^{\mathbb{Q}-\mathbb{P}}$						1.00	0.99	0.91	0.51	0.91
$\rho_{C,t}^{\mathbb{Q}-\mathbb{P}}$							1.00	0.90	0.50	0.90
$\rho_{J,t}^{\mathbb{Q}-\mathbb{P}}$								1.00	0.57	0.99
$\rho_{J,t}^{\mathbb{Q}-\mathbb{P}+}$									1.00	0.51
$\rho_{J,t}^{\mathbb{Q}-\mathbb{P}-}$										1.00

Table 3.8: Panel A of this table presents the R^2s from the simple predictability regressions on the S&P500 index, over the sample period from January 1996 to December 2005. Panel B shows the corresponding R^2s from the simple predictability regressions – using the parametric implied correlation measures– on the S&P500 index, over the sample period from January 1996 to December 2012: Estimates that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively.

Robustness Analysis

	Panel A: Pre-crises						
	Months						
	1	2	3	4	6	9	12
VRP_t	1.19**	2.80***	1.02**	1.23**	0.75*	0.33	0.24
$\rho_t^{\mathbb{Q}}$	3.37***	6.36***	10.29***	13.12***	14.23***	16.48***	16.82***
$\rho_{C,t}^{\mathbb{Q}}$	3.35***	6.31***	10.12***	12.96***	14.23***	16.74***	17.08***
$\rho_{J,t}^{\mathbb{Q}}$	2.87***	5.47***	9.77***	11.83***	11.03***	10.11***	10.36***
$\rho_{J,t}^{\mathbb{Q},+}$	1.43***	1.19**	1.25**	0.87*	0.13	0.10	0.09
$\rho_{J,t}^{\mathbb{Q},-}$	2.85***	5.71***	10.41***	12.82***	12.33***	11.34***	11.63***
CRP_t	0.31	0.66*	1.48***	1.88***	0.94***	1.81***	3.01***
CRP_t^C	0.16	0.37	0.91**	1.17***	0.45	1.17***	2.23***
CRP_t^J	2.83***	5.39***	9.65***	11.67***	10.81***	9.89***	10.14***
$CRP_t^{J,+}$	1.06**	0.71	0.66	0.32	0.01	0.02	0.02
$CRP_t^{J,-}$	2.85***	5.68***	10.37***	12.78***	12.28***	11.29***	11.57***
	Panel B: Parametric Implied Correlation						
VRP_t	2.86***	2.13***	3.79***	3.45***	0.79**	0.21	0.03
$\rho_t^{\mathbb{Q}}$	2.24***	5.47***	8.72***	10.30***	13.17***	13.88***	12.82***
$\rho_{C,t}^{\mathbb{Q}}$	2.28***	5.56***	8.79***	10.41***	13.37***	14.14***	13.03***
$\rho_{J,t}^{\mathbb{Q}}$	1.37***	3.61***	6.62***	7.53***	8.99***	8.86***	8.62***
$\rho_{J,t}^{\mathbb{Q},+}$	0.51	0.59	1.04*	1.05*	2.03***	3.16***	4.05***
$\rho_{J,t}^{\mathbb{Q},-}$	1.27***	3.50***	6.54***	7.47***	8.52***	8.21***	7.98***
CRP_t	1.73***	4.28***	7.32***	8.21***	9.81***	10.76***	10.66***
CRP_t^C	1.75***	4.31***	7.31***	8.19***	9.82***	10.87***	10.77***
CRP_t^J	1.35***	3.56***	6.53***	7.42***	8.84***	8.69***	8.45***
$CRP_t^{J,+}$	0.72	0.99*	1.67**	1.82**	3.24***	4.71***	5.65***
$CRP_t^{J,-}$	1.27***	3.48***	6.51***	7.43***	8.47***	8.15***	7.92***

Table 3.9: The top panel of this table presents the estimated coefficients from the predictability regressions –using the Investors correlation fears' index– on the S&P500 index, over the sample period from January 1996 to December 2012. The bottom panel shows the corresponding estimated coefficients for the same specification in Panel A but over the sample period from January 1996 to December 2005: Estimates that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively. The last row in each panel corresponds to the R^2 of the predictability regressions.

Fears Index, Market Predictability

	Panel A: Full Period						
	Months						
	1	2	3	4	6	9	12
Const	-0.67	-0.72	-0.84***	-0.76***	-0.59***	-0.40***	-0.28**
	(-1.59)	(-2.34)	(-3.40)	(-3.43)	(-3.09)	(-2.49)	(-1.96)
VRP_t	0.25***	0.14***	0.15***	0.12***	0.04*	0.01	-0.01
	(3.18)	(2.75)	(3.81)	(3.24)	(1.72)	(0.50)	(-0.49)
CRP_t^C	-0.03	-0.03*	-0.03**	-0.03***	-0.03***	-0.02***	-0.01**
	(-1.47)	(-1.88)	(-2.22)	(-3.27)	(-4.31)	(-3.44)	(-2.28)
FI_t	-0.52***	-0.62***	-0.68***	-0.68***	-0.68***	-0.54***	-0.45***
	(-2.54)	(-4.35)	(-5.91)	(-7.12)	(-8.94)	(-8.71)	(-8.72)
R^2	3.87	5.26	9.51	10.61	10.39	9.21	8.46
	Panel B: Pre-crises						
Const	-0.75*	-0.75**	-0.85***	-0.82***	-0.57***	-0.38***	-0.36***
	-(1.66)	-(2.31)	-(3.49)	-(3.83)	-(3.28)	-(2.45)	-(2.35)
VRP_t	0.09	0.11	-0.06	-0.06	-0.07	-0.08***	-0.09***
	(0.74)	(1.28)	-(1.03)	-(1.21)	-(1.39)	-(2.37)	-(2.72)
CRP_t^C	-0.03	-0.02*	-0.02	-0.02*	-0.024***	-0.01*	-0.003
	-(1.14)	-(1.71)	-(1.50)	-(1.81)	-(3.11)	-(1.68)	-(0.59)
FI_t	-0.83***	-0.81***	-1.01***	-0.98***	-0.87***	-0.77***	-0.66***
	(-2.98)	(-4.12)	(-7.52)	(-8.32)	(-8.43)	(-9.16)	(-8.81)
R^2	3.55	7.51	12.94	16.08	17.71	16.73	17.29

Table 3.10: Panel A in this table reports the in-sample –monthly and quarterly– quantile forecast R^2 relative to the historical quantile model for the IP. Panel B reports the in-sample –monthly and quarterly– quantile forecast R^2 relative to the historical quantile model for the CFNAI: Estimates that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively.

In-Sample 20$^{\text{th}}$ Percentile Forecasts

	R^2	*t-val*	R^2	*t-val*
	Panel A: IP Growth			
	Monthly		*Quarterly*	
$\rho^{\mathbb{Q},-}_{J,t}$	0.017	10.799	0.009	17.023
$\rho^{\mathbb{Q}}_{C,t}$	0.013	8.428	0.002	6.429
$CRP^{J,-}_{t,\tau}$	0.012	7.516	0.002	6.039
$VRP^{C,-}_{t,\tau}$	0.048	-7.173	0.095	-26.745
$VRP^{J,-}_{t,\tau}$	0.086	-19.422	0.153	-28.543
$MPQR$	0.034	-3.311	0.098	-3.020
TMS	0.002	-5.864	0.001	-5.254
DFY	0.084	-1.401	0.145	1.487
DFR	0.002	-10.196	0.013	-10.292
	Panel B: CFNAI Shock			
	Monthly		*Quarterly*	
$\rho^{\mathbb{Q},-}_{J,t}$	0.025	24.365	0.020	12.038
$\rho^{\mathbb{Q}}_{C,t}$	0.012	14.107	0.005	11.796
$CRP^{J,-}_{t,\tau}$	0.012	14.403	0.004	8.882
$VRP^{C,-}_{t,\tau}$	0.094	-32.859	0.093	-11.175
$VRP^{J,-}_{t,\tau}$	0.154	-31.218	0.158	-22.747
$MPQR$	0.067	-0.722	0.074	-1.044
TMS	0.014	-5.004	0.010	-2.865
DFY	0.131	1.896	0.128	1.658
DFR	0.012	-6.083	0.027	-6.023

Table 3.11: In this table reports the out–of-sample quarterly quantile forecast R^2 comparisons between the different implied measures for the IP and CFNAI: Estimates that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively.

Out-of-Sample 20$^{\text{th}}$ Percentile Forecasts, quarterly

	Quarterly									
	IP Growth					*CFNAI Shock*				
	$MPQR$	Q_τ^{hist}	$\rho_{J,t}^{\mathbb{Q},-}$	$CRP_{t,\tau}^{J,-}$	$VRP_{t,\tau}^{J,-}$	$MPQR$	Q_τ^{hist}	$\rho_{J,t}^{\mathbb{Q},-}$	$CRP_{t,\tau}^{J,-}$	$VRP_{t,\tau}^{J,-}$
Q_τ^{hist}	0.145*	-	-	-	-	0.160*	-	-	-	-
$\rho_{J,t}^{\mathbb{Q},-}$	0.163*	0.021	-	-	-	0.181**	0.025*	-	-	-
$\rho_{C,t}^{\mathbb{Q}}$	0.149*	0.004	-0.017*	-	-	0.191***	0.036***	0.011	-	-
$CRP_{t,\tau}^{J,-}$	0.150*	0.005	-0.016*	-	-	0.190***	0.036***	0.010	-	-
$VRP_{t,\tau}^{C,-}$	0.375***	0.268***	0.253***	0.264***	-	0.536***	0.447***	0.433***	0.427***	-
$VRP_{t,\tau}^{J,-}$	0.327***	0.213***	0.196***	0.209***	-	0.485***	0.387***	0.371***	0.364***	-
TMS	0.815***	0.783***	0.779***	0.782***	0.725***	0.758***	0.711***	0.704***	0.701***	0.529***
DFY	0.479***	0.390***	0.377***	0.387***	0.225***	0.597***	0.520***	0.508***	0.503*	0.217***
DFR	0.655***	0.596***	0.588***	0.594***	0.487***	0.561***	0.477***	0.464***	0.458	0.147

Table 3.12: In this table reports the out–of-sample monthly quantile forecast R^2 comparisons between the different implied measures for the IP and CFNAI: Estimates that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively.

Out-of-Sample 20$^{\text{th}}$ Percentile Forecasts, monthly

	Monthly									
	IP Growth					*CFNAI Shock*				
Q_{τ}^{hist}	-0.028	-	-	-	-	0.140**	-	-	-	-
$\rho_{J,t}^{\mathbb{Q},-}$	-0.027	0.0004	-	-	-	0.143***	0.003	-	-	-
$\rho_{C,t}^{\mathbb{Q}}$	-0.026	0.001	0.001	-	-	0.136**	-0.005	-0.007	-	-
$CRP_{t,\tau}^{J,-}$	-0.027	0.001	0.001	-	-	0.135**	-0.006	-0.009	-	-
$VRP_{t,\tau}^{C,-}$	0.023	0.049	0.048	0.048	-	0.362***	0.258***	0.256***	0.263***	-
$VRP_{t,\tau}^{J,-}$	0.043	0.069*	0.068	0.068	-	0.330***	0.221***	0.219***	0.226***	-
TMS	0.784***	0.790***	0.790***	0.790***	0.774***	0.794***	0.761***	0.760***	0.762***	0.693***
DFY	0.160**	0.182***	0.182***	0.182***	0.122	0.537***	0.461***	0.460***	0.465***	0.309***
DFR	0.540***	0.553***	0.552***	0.552***	0.520***	0.590***	0.523***	0.521***	0.526***	0.388***

Table 3.13: This table shows the weighted average of forward-looking semiparametric beta and the weighted average of the realized return for the three decile portfolios sorted by their corresponding forward-looking beta, over the sample period from January 1996 to December 2012. At the end of each month, I sort, for each beta methodology, the stocks into deciles based on their forward-looking market beta. Then, I compute the weighted average beta and the next month return weighted average return for each portfolio. The last two rows in the table provide the p-values, obtained from time-series block bootstrapping, for the Patton and Timmermann (2010) monotonicity test of the hypotheses for monotonically increasing and monotonically decreasing relations between expected betas and returns.

Quintile and Decile Portfolio Betas and Returns

	β^s	*Return*	β^c	*Return*	β^d	*Return*
1	1.303	0.606	1.307	0.608	1.203	0.613
2	1.765	0.698	1.772	0.701	1.632	0.670
3	2.045	0.860	2.053	0.861	1.892	0.860
4	2.294	0.965	2.302	0.970	2.122	0.956
5	2.545	0.995	1.838	0.996	2.354	0.984
6	2.818	1.010	1.838	1.009	2.606	1.014
7	3.117	0.966	1.838	0.961	2.882	0.974
8	3.458	0.994	1.945	0.993	3.199	0.997
9	3.892	1.136	2.047	1.134	3.602	1.144
10	4.696	1.027	2.142	1.025	4.351	1.023
High-Low	3.393	0.420	3.406	0.417	3.147	0.410
PT-test		0.7510		0.7490		0.7370
		0.1170		0.1440		0.1550

Table 3.14: This table shows the estimated coefficients for the cross-sectional regressions, over the sample period from January 1996 to December 2012. At the end of each month, I sort, for each beta methodology, the stocks into five, ten and twenty five portfolios based on their forward-looking market beta. Then, I compute the weighted average beta and the next month return weighted average return for each portfolio. Then, I regress the mean realized excess return of the portfolios on a constant and the portfolios' mean forward-looking betas, separately for each methodology: Estimates that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively.

Cross-sectional regressions

	Panel A: 5 portfolios					**Panel A: 10 portfolios**				
	constant	β^s	β^c	β^d	R^2	constant	β^s	β^c	β^d	R^2
I	0.595***	0.119**	-	-	0.62	0.572***	0.127***	-	-	0.66
II	0.600***	-	0.117**	-	0.62	0.577***	-	0.124***	-	0.65
III	0.578***	-	-	0.134**	0.64	0.561***	-	-	0.140***	0.66
IV	0.446**	-	28.173	-30.386	0.72	0.425***	-	29.788*	-32.124*	0.75
	Panel C: 25 portfolios									
I	0.574***	0.119***	-	-	0.54					
III	0.579***	-	0.117***	-	0.53					
III	0.562***	-	-	0.133***	0.57					
IV	0.384***	-	32.825***	-35.377***	0.67					

Table 3.15: This table shows the estimated coefficients from the simple predictability regression of the implied betas on their sixth month expected return for 20 individual stocks, over the sample period from January 1996 to December 2012. Model I corresponds to the regression where the implied beta is the only regressor, model II considers only the continuous implied beta, model III considers only the discontinuous implied beta and model IV considers the continuous and discontinuous beta: Estimates that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively. The 14 stocks are (by ticker): IBM, QCOM, AMGN, TXN, MO, APPL, MRK, BBY, JPM, BA, DELL, HD, AMZN, INTC.

Stock Returns Predictability: Market Factor

	Model I			*Model II*			*Model III*			
	Constant	β^c	R^2	Constant	β^d	R^2	Constant	β^c	β^d	R^2
1	-9.877***	0.165***	7.89	-10.019***	0.177***	9.37	-9.567***	-0.063***	0.236***	9.48
2	-1.517	0.032	0.47	-1.874	0.041*	0.82	-1.157	-0.071	0.105	1.07
3	1.714	-0.016	0.39	1.568*	-0.014***	0.34	1.947***	-0.057	0.040	0.44
4	3.022***	-0.056***	4.60	2.639***	-0.051***	4.28	3.587***	-0.166	0.105	4.86
5	-31.775***	0.391***	14.4	-32.882***	0.447***	25.9	-20.135***	-0.559***	0.883***	30.6
6	4.955***	-0.086***	5.38	2.383***	-0.051***	2.11	-20.135***	-0.378***	0.292***	11.6
7	-3.982***	0.109***	4.22	-2.054***	0.071	2.48	6.151***	0.283***	-0.155**	5.49
8	3.691*	-0.058	0.69	2.466	-0.038***	0.38	-5.293***	-0.116	0.056	0.82
9	-16.566***	0.249***	10.2	-12.819***	0.211	8.92	-16.443***	0.197**	0.053	10.3
10	2.973***	-0.046*	0.96	1.652*	-0.017***	0.17	6.823***	-0.542***	0.438***	6.78
11	-14.349***	0.261***	33.1	-11.534***	0.227	27.6	-16.024***	0.587***	-0.318***	35.7
12	0.197	0.005	0.02	1.331*	-0.023*	0.37	-5.913***	0.911***	-0.821***	16.1
13	1.614***	-0.019	0.88	1.547***	-0.019	1.00	0.979	0.080	-0.094	1.21
14	1.009	-0.019	0.26	0.416	-0.011	0.09	1.615	-0.150*	0.131	0.96

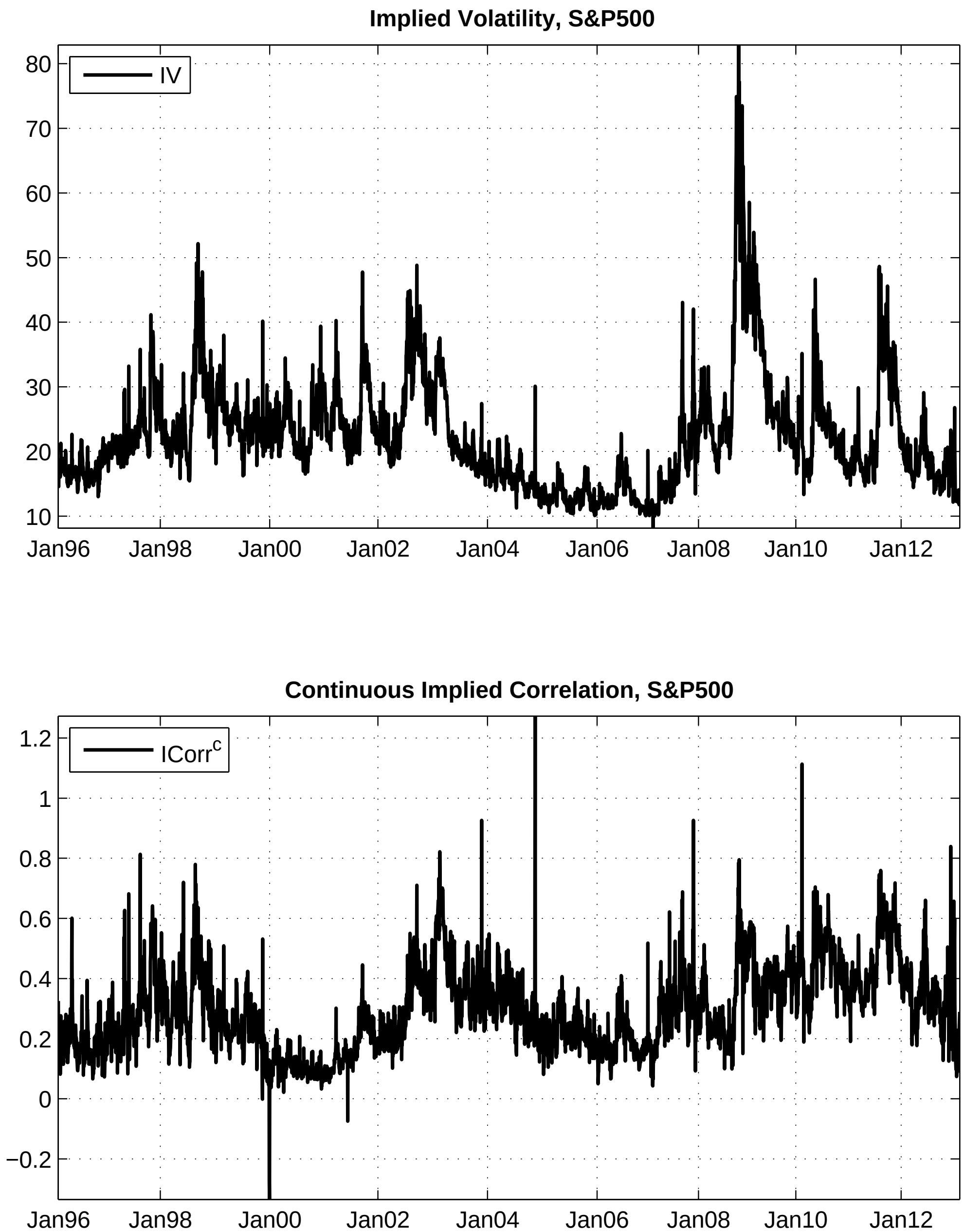


FIGURE 3.1: The top panel of this figure shows the time series of the implied volatility. The lower panel shows the times series of thejump-robust implied correlation.

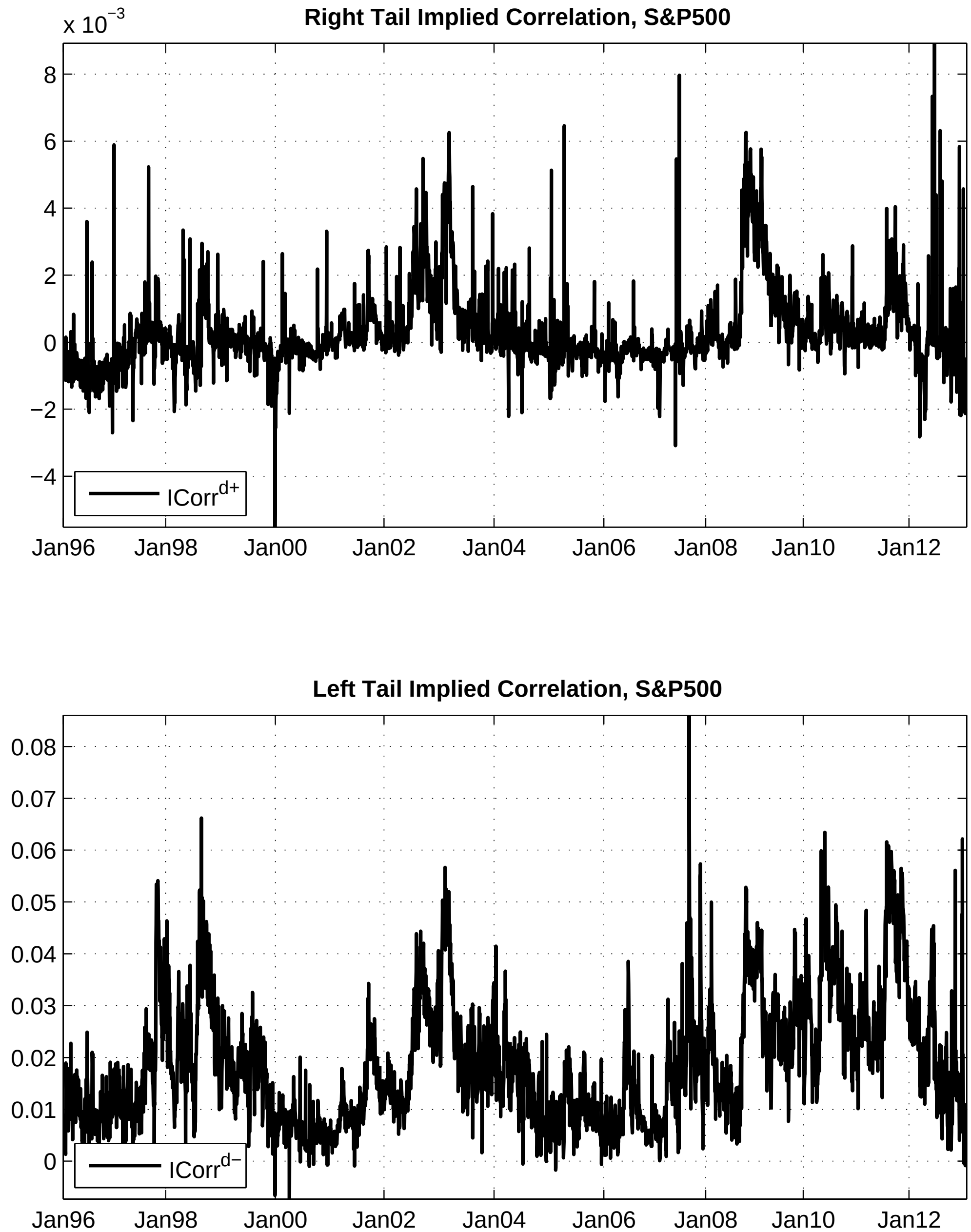


FIGURE 3.2: The top panel of this figure shows the time series of the right tail implied correlation. The lower panel shows the times series of the left tail implied correlation.

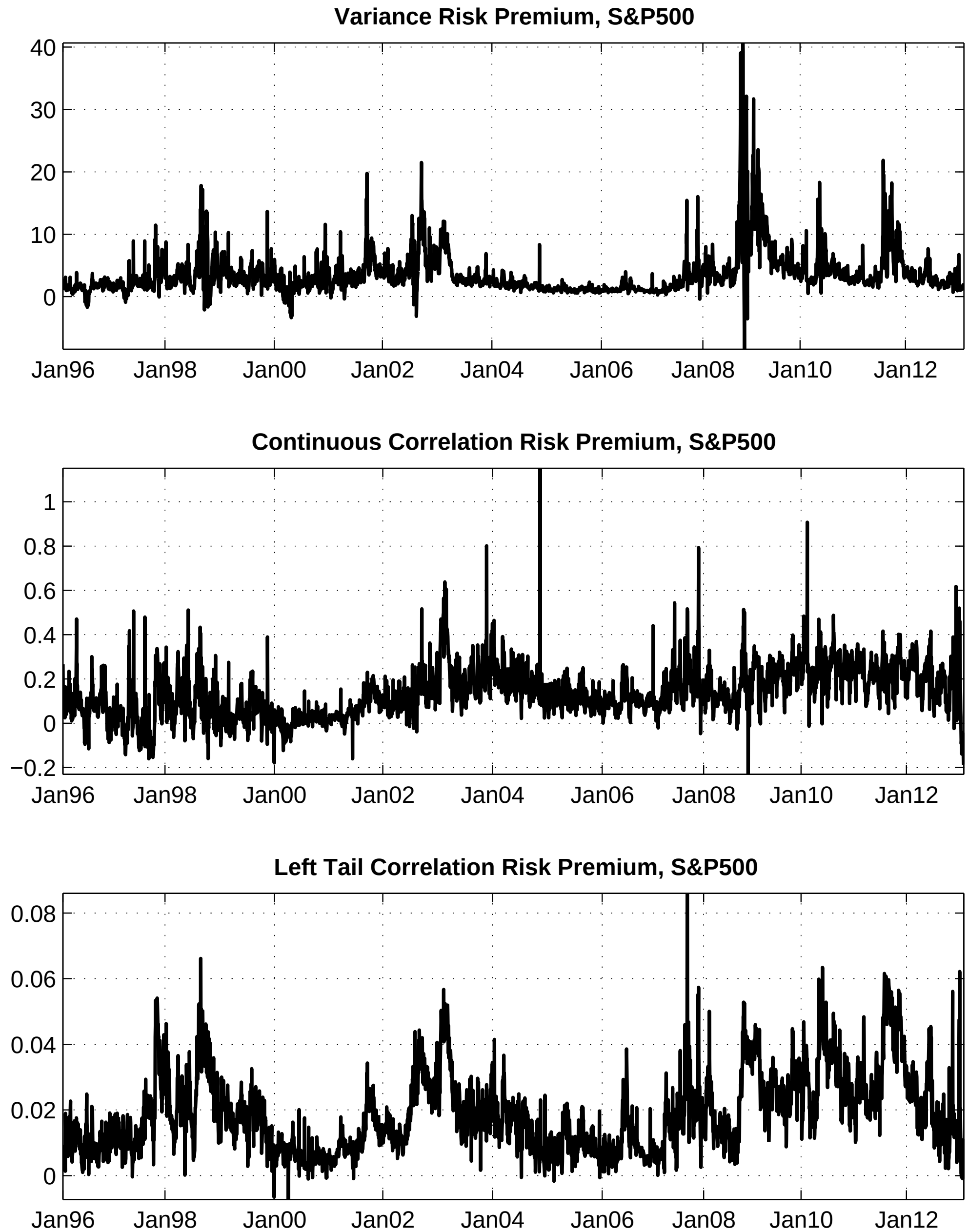


FIGURE 3.3: The top panel of this figure shows the time series of the annualized variance risk premium The middle panel shows the times series of the jump-robust correlation risk premium. The lower panel shows the times series of the left tail correlation risk premium.

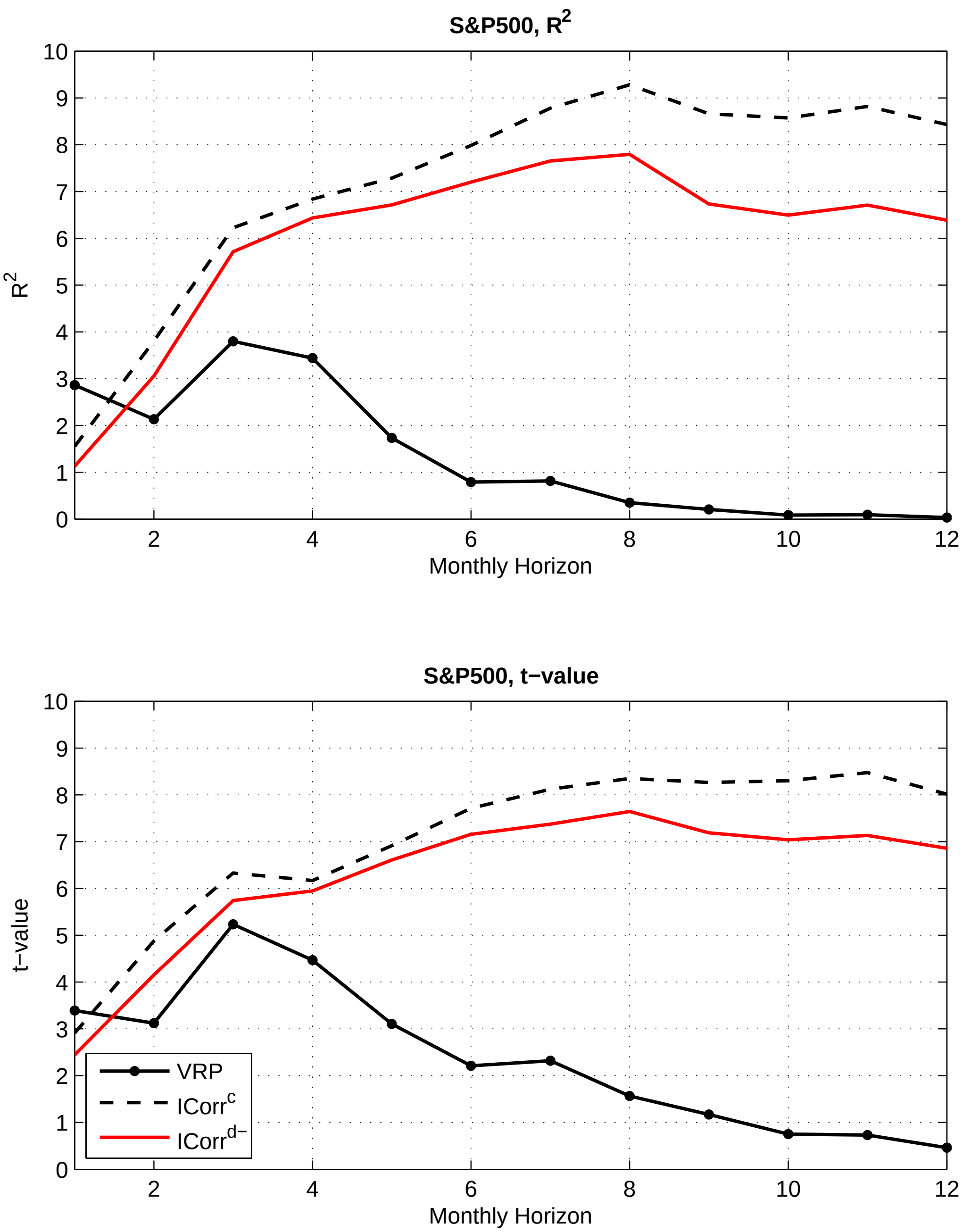


FIGURE 3.4: The top panel shows the R^2s from the simple return predictability regressions for the S&P500 based on the variance riks premium (black dot line), the jump-robust implied correlation (black line) and the left tail implied correlation (red line). The bottom panel shows the corresponding Newey-West t-values.

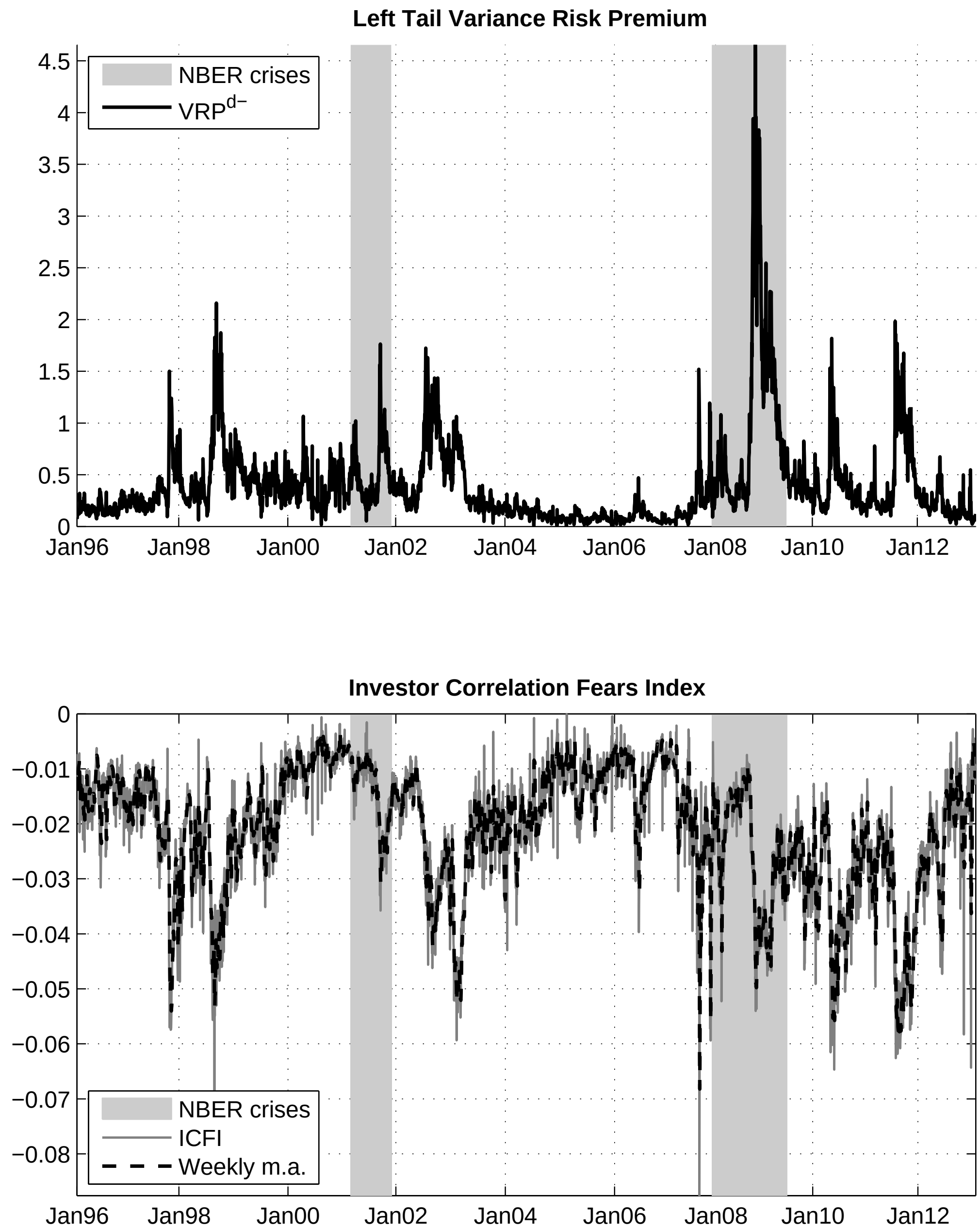


FIGURE 3.5: The top panel of this figure shows the time series for the annualized left tail of the variance risk premium and the NBER economic crises (gray shadow area). The bottom panel of this figure shows the time series for the Investors correlation fears' index (red line) and the NBER economic crises.

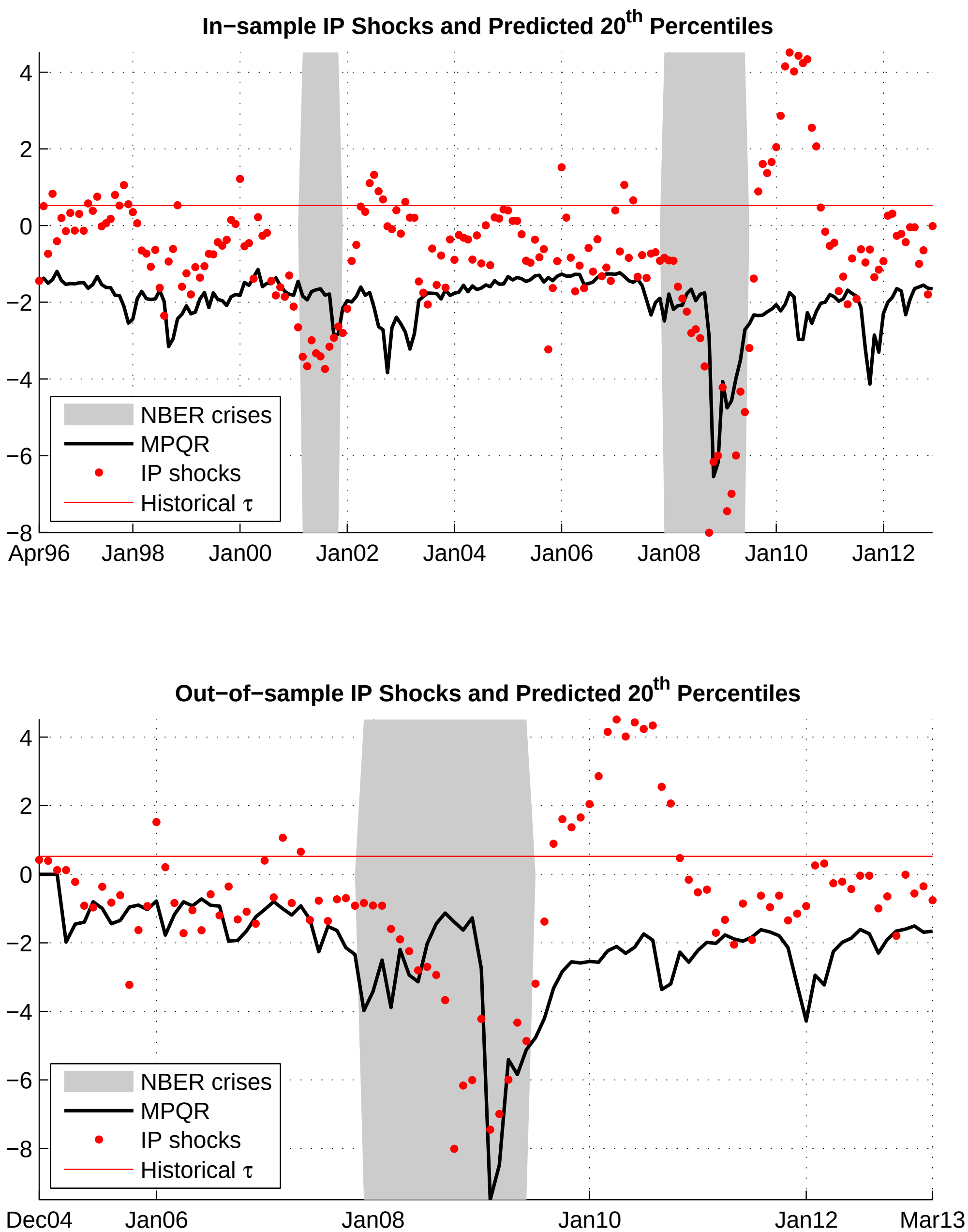
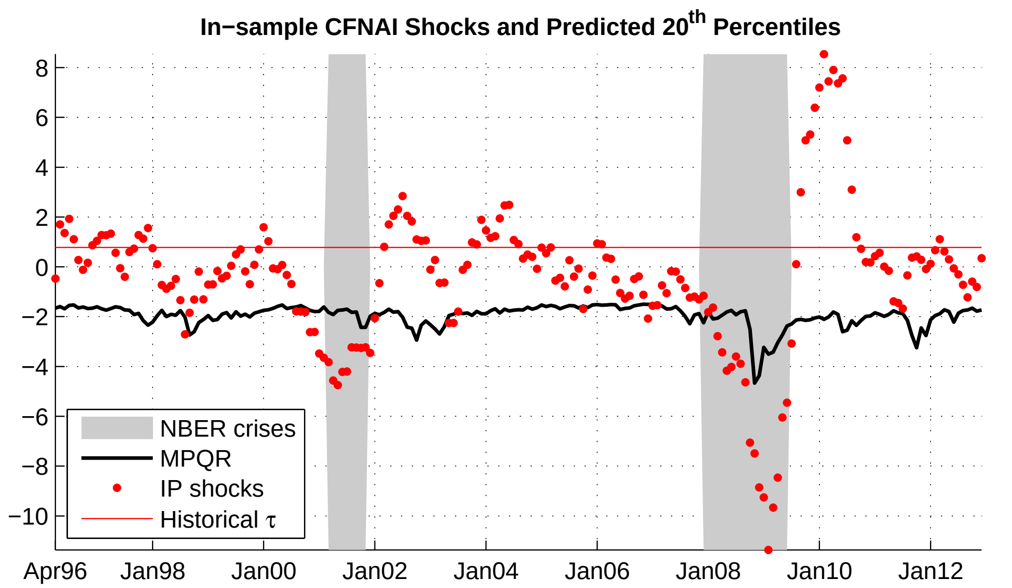


FIGURE 3.6: The top panel shows the in-sample fitted values for the 20^{th} percentile one-quarter ahead shocks to the Industrial Production (IP) growth. Historical IP (red line) , in-sample IP forecast (black line) and the in-sample IP growth shocks (red dots). The bottom panel corresponds to the out-of-sample fitted values.



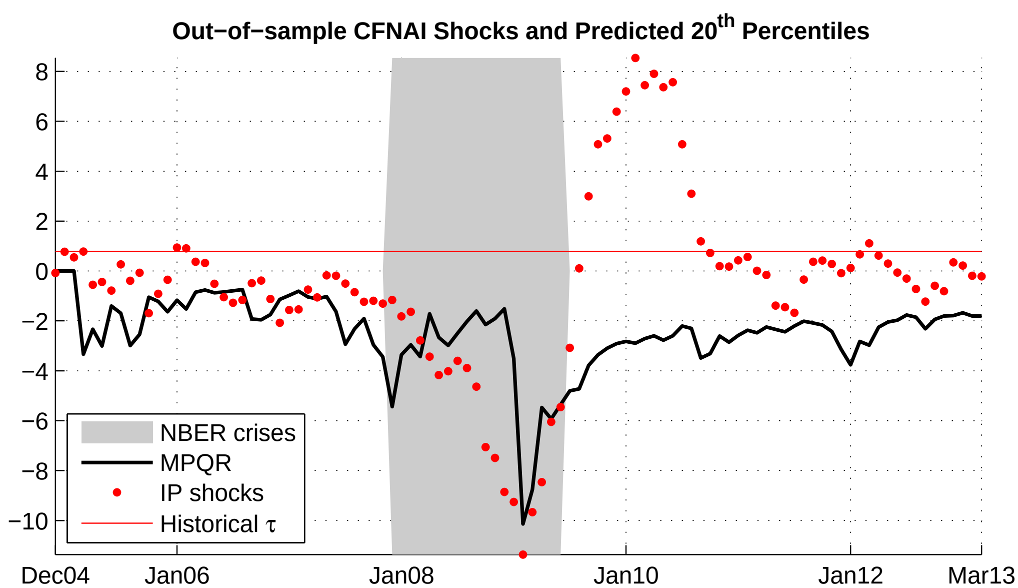


FIGURE 3.7: The top panel shows the in-sample fitted values for the 20^{th} percentile one-quarter ahead shocks to the CFNAI. Historical CFNAI (red line) , in-sample CFNAI forecast (black line) and the in-sample CFNAI growth shocks (red dots). The bottom panel corresponds to the out-of-sample fitted values.

Appendix A

Portfolio Decision with Implied Measures

In this section I describe another "tale" or application that can be studied using implied measures, a portfolio decision problem. It has been documented in previous works that there are two types of asymmetries in the joint distribution of stock returns, skewness and dependence asymmetry.[1,2] Therefore, the tails of the distribution contain information that can be translated to better performance in portfolio choice problems. Under this premise, this section explores a portfolio problems when the weights depend parametrically on tail measures, mainly the implied correlation measures constructed in this paper.

Consider the following problem face by an investor,

$$\begin{aligned} \omega_{t+1}^{*} &= \arg\max_{\omega\in\Upsilon} E_t\left[\mathcal{U}\left(r_{p,t+1}\right)\right] = \arg\max_{\omega\in\Upsilon} E_t\left[\mathcal{U}\left({}_{i=1}^{N_t}\omega_{i,t}r_{i,t+1}\right)\right] \qquad \text{(A.1)} \\ &= \arg\max_{\theta} E_t\left[\mathcal{U}\left({}_{i=1}^{N_t} f\left(x_{i,t};\theta\right) r_{i,t+1}\right)\right] \end{aligned}$$

[1] See Erb, Harvey, and Viskanta (1994), Longin and Solnik (2001), Ang and Bekaert (2002), Ang and Chen (2002), Campbell, Koedijk, and Kofman (2002), and Bae, Karolyi, and Stulz (2003).

[2] In this case, the skewness is observed in the distribution of individual stock returns. And the dependence asymmetries are present when the stock returns are more highly correlated during market downturns than during market upturns.

where $r_{i,t+1}$ is the return of stock i from date t to $t+1$, $x_{i,t}$ is a vector of stock characteristics observed at date t and the parametrization of the weights is given by,

$$\omega_{i,t} = f\left(x_{i,t};\theta\right) = \overline{\omega_{i,t}} + \frac{1}{N_t}\sum_{k=1}^{K}\theta_{k,t}x_{ik,t} \tag{A.2}$$

This problem was first implemented by Brandt et al. (2010), and it assumes that the weight of each instrument in the portfolio depends on a particular factor. This specification not only reduces the dimension of the problem but also allows the used of additional characteristics inherent to each instrument. A possible empirical implementation would consider instead of stocks, sectors of the S&P500, and the utility function will follows a standard constant relative risk aversion (CRRA) specification. Moreover, in reality equity portfolio managers face short-sale constraints. Therefore, the portfolio weights could be renormalize as follows,

$$\omega_{i,t}^{+} = \frac{\max\left\{0,\omega_{i,t}\right\}}{\sum_{j=1}^{N_t}\max\left\{0,\omega_{j,t}\right\}} \tag{A.3}$$

An empirical implementation should evaluate the out-of-sample performance of different portfolios depending on its sector-specific implied measures, the performance evaluation could be done using four criteria: (i) out-of-sample portfolio volatility (standard deviation); (ii) out-of-sample portfolio Sharpe ratio[3]; (iii) portfolio

[3] The Sharpe ratio is given by,

$$SR = \frac{\mu}{\sigma}$$

where μ is the average out-of-sample portfolio return and σ is the out-of-sample standard deviation.

turnover[4] (trading volume) and; (iv) management fee[5].

Finally, to perform an out-of-the-sample analysis, I would use a rolling window procedure to compute the portfolio weights and its performance, with an estimation-window length of 250 days ($\tau = 250$). At time t, the manager/investor maximizes his utility choosing the optimal weights. Then, he holds the portfolio for one month, yielding the corresponding out-of-sample portfolio return. The investor performs this decision problem every day.

[4] The portfolio turnover is given by,

$$TO = \frac{1}{T-\tau-1}\sum_{t=\tau}^{T-1}\sum_{j=1}^{N}|w_{j,t+1} - w_{j,t}|$$

where $T-\tau-1$ is the out-of-sample size and $\tau = 250$.

[5] The management fee is the solution to the following equation,

$$\frac{1}{P}\sum_{t=R+1}^{R+P}\mathcal{U}\left(1+\omega^{*\prime}_{\mathcal{A},t+1}\mathbf{Y}_{t+1}\right) = \frac{1}{P}\sum_{t=R+1}^{R+P}\mathcal{U}\left(1+\omega^{*\prime}_{\mathcal{B},t+1}\mathbf{Y}_{t+1}-\vartheta\right)$$

where R is the length of the in-sample period and P is the length of the out-of-sample period.

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