Essays on Macroeconomics

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in the Department of Economics
in the Graduate School of Duke University
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ABSTRACT

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Abstract

My dissertation consists of three self-contained essays on macroeconomics. Chapter 2 "Churning, firm inter-connectivity, and labor market fluctuations" studies the implications of firm inter-connectivity and irreversibility of inter-firm cooperation relationships on the business cycle. Chapter 3 "Inter-sector matching efficiency and sectoral comovement" examines the comovement of sectoral labor markets when there is search friction in the inter-firm matching market. Chapter 4 "Lumpy investment and endogenous investment price" (Joint work with Linxi Chen) studies the endogenous fluctuation of investment price induced by search friction in the investment goods market and partial irreversibility of capital adjustment. Each of the essays investigates the implication of market frictions, such as search friction and partial irreversibility, to the business cycle from a different perspective.

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1

Introduction

My dissertation consists of three self-contained essays on macroeconomics. Chapter 2 studies the implications of firm inter-connectivity and irreversibility of inter-firm cooperation relationships on the business cycle. Chapter 3 studies the comovement of sectoral labor markets when there is search friction in the inter-firm matching market. Chapter 4 studies the endogenous fluctuation of investment price induced by search friction in the investment goods market and partial irreversibility of capital adjustment. Each of the essays investigates the implication of market frictions, such as search friction and partial irreversibility, to the business cycle from a different perspective.

In Chapter 2 "Churning, firm inter-connectivity, and labor market fluctuations", I propose and study a business cycle model with firm inter-connectivity. The empirical motivation of this chapter is that in the US, during downturns of economic activity, firms change their profit rankings more often. Motivated by this fact, this chapter studies the effect of firms' transitions in profit distributions, or churning, on the business cycle. Specifically, I develop and estimate a modified Diamond-Mortenson-Pissarides search and matching model in which an increase in churning leads to a contraction of the labor market and a decline in output. The key feature of the model is firm inter-connectivity. Churning affects the aggregate economy by increasing firms' chances of cooperating with partners that they are reluctant to work with, which induces firms and their potential partners to reduce inter-firm cooperation. A reduction of inter-firm cooperation depresses other economic activities, such as recruiting. The main prediction of the model is that an increase in the churning of an industry causes a recession within industry and in its linked industries, which is consistent with the evidence I document in the chapter. The model's key mechanism, furthermore, is supported by microeconomic evidence.

In Chapter 3 "Inter-sector matching efficiency and sectoral comovement", I study the comovement of employment and job openings with a multi-sectoral search and matching model. The empirical motivation of this chapter is that as I document, while the sectoral job openings are highly correlated across sectors, the synchronization of sectoral labor productivities is much weaker. This fact can hardly be reconciled with conventional multi-sector models which are propagated by productivity shocks. I develop a model featured by the gain of cooperation and specialization—that is, firms are more productive when working with partners than working alone—and I show that the model can generate significant comovement of sectoral job openings even when sectoral labor productivities are independent. Moreover, the model is able to generate large labor market fluctuations matching efficiency between firms is allowed to be time varying.

In Chapter 4, "Lumpy investment and endogenous investment price", which is a joint work with Linxi Chen, we investigate the implication of investment goods market search friction on the dynamics of investment price. This chapter is related to one the most well-documented facts in macroeconomics, the negative co-movement of the quantity and the relative price of investment. This fact justifies technological shocks to investment producers (supply side story). In this chapter, we first document several new pieces of evidence that suggest a demand side story. In particular, we find that the conditional correlation between the quantity and the relative price of investment is time-varying and can occasionally become significantly positive. Moreover, we find that when fewer firms adjust capital stock along the extensive marginthat is when investment is lumpierquantity and the relative price of investment are more likely to move in the opposite direction. Motivated by these observations, we develop a heterogeneous firm model in which the investment price is endogenously determined by the composition of investment goods buyers. In the recession, only productive firms choose to invest, which decreases the elasticity of aggregate demand curve of investment. In this situation, monopolistic sellers of investment would optimally raise markup to exploit the lower elasticity of demand. While this effect is strong in periods with high investment lumpiness, it becomes much weaker when lumpiness is low.

2

Churning, Firm Inter-connectivity and Labor Market Fluctuations

2.1 Introduction

In the US, periods of rising unemployment are also periods of accelerated movement of firms' rankings across the profit distribution. That is, during downturns of economic activity, there is an increase in churning of firms' rankings in the profit distribution. Moreover, a higher churning of an industry is usually accompanied by an economic downturn within the industry and in its linked industries. Motivated by these facts, I study the effect of churning on unemployment through the lens of a Diamond-Mortenson-Pissarides (DMP hereafter) model. Based on my model, I argue that variations in churning is a significant factor explaining the cyclical labor market fluctuations, and firm inter-connectivity across industries serves as a crucial propagation mechanism.

The main idea of the paper is that firms collaborate with other firms. In choosing partners, firms care about the quality of their partners. Churning of firms' rankings increases the chance that they will cooperate with partners who they are reluctant to work with. In this situation, both the firms and their potential partners would optimally avoid inter-firm cooperation which depresses other economic activities such as recruiting of workers.

In section 4.3, I implement the idea by supplementing the canonical DMP model with firm inter-connectivity. I assume that all firms prefer to cooperate with high-ranking partners; hence, in equilibrium, firms only initiate partnership with similarly ranked partners. I show that churning increases unemployment if and only if the production function is supermodular; that is, firms have a comparative advantage in cooperating with a similarly ranked partner. The intuition is that when the production function is supermodular, cooperation between similarly ranked firms maximizes their aggregate profit. Churning generates mismatch between differently ranked firms, which decreases firms' expected profits and reduces their incentive to create jobs. The main prediction of the model is that an increase in churning within an industry causes a depression in economic activity both within the industry and in its linked industries, which is indeed consistent with the evidence I document in section 4.2.

In section 2.4, I test my model's main mechanism—particularly the supermodular production function—by conducting inference on the model's testable microeconomic implications. My model predicts that, with supermodularity, fewer firms choose long-term contracts in industries with higher churning. The intuition is that long-term contracts trap mismatched firms in inefficient relationships, while short-term contracts, in contrast, serve as an option to hedge against mismatch risk. I test my model using vertical integration and sourcing to proxy for long- and short-term cooperation. In the data I find that churning of an industry and of its linked industries has a significant negative effect on its vertical integration, which supports the main mechanism of my model.

The question of the extent to which churning contributes to the business cycle, however, remains to be seen. In section 4.4, I answer this question by embedding the simple model into a real business cycle (RBC) model disturbed by shocks to churning and several other shocks that have been commonly studied in dynamic stochastic general equilibrium (DSGE) models. After estimating the model using Bayesian method, I find that shocks to churning emerge as the major source of persistent joint movements in unemployment and other macro variables: they account for 27 percent of variation in unemployment and 32 percent of variation in aggregate output.

As a byproduct, my paper provides a theory of endogenous total factor productivity (TFP) and speaks to the Shimer puzzle (Shimer (2005)). According to my estimates, the production function is supermodular, which implies that mismatched firms are, on average, less productive. Churning of an industry raises the share of mismatched firms in the existing matches both within the industry and in its linked industries, leading to a gradual decline in aggregate TFP. This decline in aggregate TFP, however, is mild compared with the surge in unemployment.

My paper is related to the literature on uncertainty shock. Uncertainty shock, as defined by Bloom (2009) and Bloom et al. (2012), characterizes exogenous change in volatility of idiosyncratic productivity. In many settings, uncertainty shocks induce fluctuations in the rate of churning by varying firms' transition probability in the productivity distribution. While most research in this literature emphasizes the irreversibility and the non-convex adjustment cost of capital and labor input, my paper proposes a novel yet empirically plausible mechanism featured by the irreversibility of cooperation relationships and the opportunity cost of mismatch. Moreover, as shown by Schaal (2012), the standard DMP model predicts an increase in the job creation in response to a higher uncertainty, which is inconsistent with data. In contrast, my model implies that a high uncertainty generates a large decline in the job creation. Lastly, while existing models imply that uncertainty shocks at the industry level lead to recession within the industry; my paper has a new and attractive implication that industry uncertainty shocks cause recession not only within industry, but also in linked industries, which is consistent with the evidence in the data.[1]

[1] While no existing work studies the interaction between industry linkage and uncertainty shocks, Alessandria et al. (2015) show that in a two-country trade model, uncertainty shocks to one country induce negative comovement of the two countries via the non-convex adjustment cost channel.

2.2 Empirical evidence

In this section, I present the empirical evidence that motivates my research. I first construct measures of churning on both industry level and aggregate level. Then I use the measures to show that churning is associated with the condition of the labor market at both aggregate level and industry level; Moreover, I show that churning of an industry is also associated with the condition of the labor markets of its linked industries. Lastly, I discuss the difference between churning and uncertainty shock.

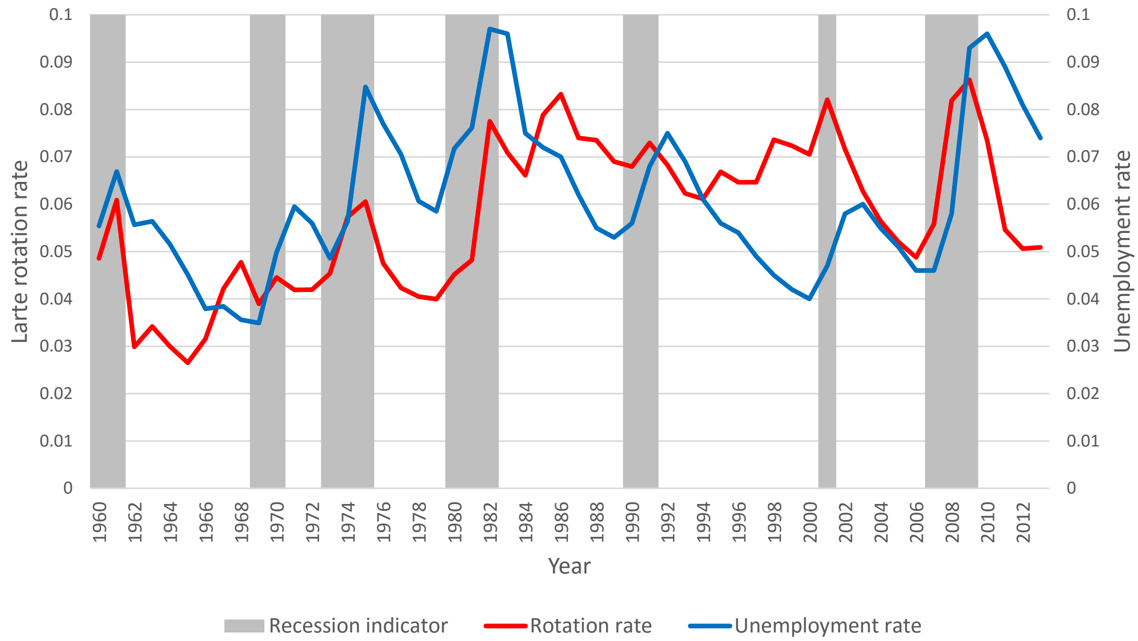
2.2.1 Measuring churning of firms' profit rankings

I construct measures of the churning of firms' profit rankings on both industry level and aggregate level. The measurement will be used to show that churning is positively correlated with unemployment rate. Moreover, I will use them as observable in the estimation of my model.

I use compustat fundamentals annual from 1960 to 2013. Compustat fundamentals annual is a data set of listed companies which contains more than 370,000 observations and covers 112 3-digit NAICS industries. For each period and within each industry I rank firms by profit,[2] which is measured by Earnings Before Interest, Taxes, Depreciation and Amortization (EBITDA). Then I categorize firms into two types, high (H) and low (L), based on ranking; a firm is H type if its profit is above median, L type if below median.

A rotation occurs when a firm changes its type in consecutive periods, either from H to L or vice versa. Industry rotation rate is the fraction of firms that have changed type.

[2] I rank firms by profit because profit is easy to measure. Ideally, firms should be ranked by their "true abilities" to promote the inter-firm match's joint pay-off, which is difficult to measure. Yet it is still reasonable to conjecture that profits are positively correlated with the "true abilities". Several studies, such as Rhodes-Kropf and Robinson (2008), found that high-earning firms are more likely to match with high-earning firms in M&A activities, which supports my conjecture and justifies profit as a legitimate proxy.



The blue curve shows the civilian unemployment rate from the BLS; the red curve displays the aggregate rotation rate constructed from Compustat. Shaded areas correspond to NBER recessions. See section 2.2.1 for detail.

FIGURE 2.1: Measure of churning is positively correlated with unemployment rate

Specifically, industry rotation rate for industry i in period t is defined as[3]

$$Rot_{i,t} = \frac{\#rotation_{i,t}}{\#firm_{i,t}}$$

Industry rotation rate $Rot_{i,t}$ measures the churning of firms' rankings in industry i in

[3] For every two years, I keep the panel balanced. The following is an example to illustrate the measurement. Industry A has four firms, A1—A4; each firm's type depends on ranking of yearly profits. Rotations occur in two of four firms in year 2; firms A1 and A3 switch their types, yielding a 0.5 rotation rate in year 2.

	Firm	*Year* 1 *profit* (*type*)	*Year* 2 *profit* (*type*)	
Industry A	*A*1	12 (*H*)	11 (*L*)	$\Longrightarrow Rot_{A,2} = 0.5$
	*A*2	10 (*H*)	15 (*H*)	
	*A*3	8 (*L*)	10 (*L*)	
	*A*4	9 (*L*)	12 (*H*)	

period t. High industry rotation rate implies that a firm is more likely to change its ranking within the industry profit distribution.

To measure churning of the aggregate economy, I construct the aggregate rotation rate by aggregating the industry rotation rates weighted by industry value-added. Specifically, aggregate rotation rate Rot_t is defined as

$$Rot_t = \sum_i Rot_{i,t} \cdot \frac{Value\ added_{i,t}}{GDP_t}$$

Aggregate rotation rate Rot_t measures churning of firms' within-industry ranking for the entire economy. Similar to the industry rotation rate, a high aggregate rotation rate indicates a higher churning in the economy. In this paper, I use the term aggregate rotation rate and rotation rate interchangeably.

It is worth noting that there exist several alternative ways to measure churning, I choose rotation rate because it is the exact empirical counterpart of churning in the model described in the next section.

Figure 4.1 plots the rotation rate and unemployment rate from 1960 to 2013. Rotation rate closely comoves with unemployment rate over the business cycles and is strongly countercyclical. For example, in the recent great recession, rotation rate hits more than 9 percent compared with the pre-recession rate of 5.5 percent.[4]

Some may think that the cyclicality of rotation rate and its comovement with unemployment rate is mainly driven by firms' small variation of profit around the median point, which would undermine the economic importance of my measurement. To address this concern, I refine the categorization of firms into four quartiles. By definition, rotation is the switching of rankings from 1st and 2nd quartiles to 3rd and 4th, or vice versa. The switchings between 2nd and 3rd quartile might contain small variation of profit around median point. Hence I ignore switchings between 2nd and 3rd quartiles and focus only

[4] This pattern is consistent with result found in Bloom et al. (2012) who found churning to be negatively correlated with consumption growth. They used manufacturing census data and ranked firms by total factor productivities (TFP).

on the ones between non-adjacent quartiles, which I denote as large rotations. By construction, large rotation is a subset of rotation and only includes very sizable changes in rankings.

I define large rotation rates in the same way as rotation rates:

$$Rot_{i,t}^{large} = \frac{\#large\ rotation_{i,t}}{\#firm_{i,t}}$$

$$Rot_{t}^{large} = \sum_{i} Rot_{i,t}^{large} \cdot \frac{Value\ added_{i,t}}{GDP_t}$$

where $Rot_{i,t}^{large}$ is large rotation rate for industry i in period t. Similarly, Rot_{t}^{large} is aggregate large rotation rate, which is the weighted sum of large industry rotation rates. Both $Rot_{i,t}^{large}$ and Rot_{t}^{large} measure the sizable changes in firms' rankings. In this paper, I use aggregate large rotation rate and large rotation rate interchangeably.

Table 4.1 reports a set of results of regressions of unemployment rate on rotation rates. In addition to the benchmark measurement of rotation when firms are ranked by profit, I also rank firms by profit margin, which is defined as the ratio of profit to sales. For each method of ranking, I use both rotation rate and large rotation rate as explanatory variables. As shown in Table 4.1, coefficient β is significant positive in all cases. According the results, measures of churning of the economy are positively correlated with unemployment rate.

It is important to clarify whether rotation rate's comovement with unemployment rate is mostly driven by a few industries, while other industries have fairly flat rotation rates. If that's the case, rotation rate reflects only some industry phenomena and might not be important for macroeconomics. To address this concern, I conduct industry panel regressions to show that measures of churning at the industry level are negatively associated with the industry employment growth. Specifically, I specify:

$$Employment\ growth_{i,t} = X_i + \gamma_t + \beta \cdot Rot_{i,t} + \varepsilon_{i,t}$$

where X_i is industry fixed effect and γ_t is year effect. $Rot_{i,t}$ is industry rotation rate or

Table 2.1: Measures of churning are positively correlated with unemployment rate

$$Unemployment_t = \alpha + \beta \cdot Rot_t + \varepsilon_t$$

Independent variable	Coefficient	$P > \|t\|$	$Adj\ R-squared$	# of years
Rot^{Profit}	0.50*** (0.18)	0.01	0.11	54
$Large\ Rot^{Profit}$	0.42*** (0.15)	0.01	0.11	54
$Rot^{Profit\ margin}$	0.54*** (0.09)	0.00	0.40	54
$Large\ Rot^{Profit\ margin}$	0.47*** (0.11)	0.00	0.24	54

$Unemployment_t$ = civilian unemployment rate t, BLS, 1960-2013
Rot_t = aggregate rotation rate at t, Compustat
$Large\ Rot_t$ =switching rate between non-adjacent quartiles
SE is in the parentheses.
1, 2, 3 asterisks denote significance at the ten, five, one percent level.

industry large rotation rate. Firms are ranked either by profit or profit margin. I use industry employment growth rate as the the dependent variable because it is well known that unemployment is not well measured at the industry level.

Table 4.2 reports the results of the industry panel regression. In all cases, an increase in measures of churning within an industry is associated with a decline in the industry employment growth.

As there is strong inter-industry linkage in the U.S. economy in terms of flow of intermediate input and output across industries,[5] it is interesting to exam if churning of an industry is also associated with the condition of its linked industries' labor market. To examine it, I conduct industry panel regressions of industry employment growth on not only the industry's own rotation rate, but also its linked industries' rotation rates. Specifically,

[5] Examples of studies of inter-industry input-output linkage in the real business cycle models can be found in Long Jr and Plosser (1983); Dupor (1999); Horvath (2000). A more recent literature on the production network, such as Atalay et al. (2011); Acemoglu et al. (2012), also documents a strong inter-industry linkage in the US economy.

Table 2.2: Measures of industry churning are negatively associated with industry employment growth

$$Employment\ growth_{i,t} = X_i + \gamma_t + \beta \cdot Rot_{i,t} + \varepsilon_{i,t}$$

Independent variable	Coefficient	$P > \|t\|$	$R-squared$	$Number\ of\ observations_{year \times industry}$
Rot^{Profit}	-0.16^{***} (0.04)	0.00	0.34	15×41
$Large\ Rot^{Profit}$	-0.21^{***} (0.05)	0.00	0.34	15×41
$Rot^{Profit\ margin}$	-0.11^{***} (0.03)	0.00	0.34	15×41
$Large\ Rot^{Profit\ margin}$	-0.17^{***} (0.05)	0.00	0.29	15×41

$Employment\ growth_{i,t}$ = industry employment growth in year t, NIPA, 1999-2013
X_i = industry and fixed effect
γ_t = year effect
$Rot_{i,t}$ = industry rotation rate in year t, Compustat
I include private non-farm industries that are in both Compustat and NIPA and have more than 8 firms from 1999 to 2013
SE is in the parentheses.
1, 2, 3 asterisks denote significance at the ten, five, one percent level.

I specify:

$$Employment\ growth_{i,t} = X_i + \gamma_t + \beta_1 \cdot Rot_{i,t} + \beta_2 \sum_j w_{i,j} Rot_{j,t} + \varepsilon_{i,t}$$

where X_i is industry fixed effect and γ_t is year effect. $Rot_{i,t}$ is industry rotation rate. $\sum_j w_{i,j} Rot_{j,t}$ is the weighted average of linked industries' rotation rates.

Table 2.3 reports the results of the above regressions. In all cases, an increase in measures of churning within an industry and an increase in its linked industry are both associated with a decline in the industry employment growth.

To summarize the empirical findings of this subsection, measures of churning are sig-

Table 2.3: Measures of industry churning and linked industries' churning are negatively correlated with industry employment growth

$$Employment\ growth_{i,t} = X_i + \gamma_t + \beta_1 \cdot Rot_{i,t} + \beta_2 \sum_j w_{i,j} Rot_{j,t} + \varepsilon_{i,t}$$

Independent variable	β_1		β_2		$Number\ of\ observations_{year \times industry}$
	Coefficient	$P > \|t\|$	Coefficient	$P > \|t\|$	
Rot^{Profit}	-0.17^{***} (0.04)	0.01	-0.24^{***} (0.09)	0.01	15×41
$Large\ Rot^{Profit}$	-0.20^{***} (0.06)	0.00	-0.27^{***} (0.10)	0.01	15×41
$Rot^{Profit\ margin}$	-0.09^{***} (0.03)	0.01	-0.17^{***} (0.07)	0.01	15×41
$Large\ Rot^{Profit\ margin}$	-0.16^{***} (0.06)	0.01	-0.15^{*} (0.09)	0.10	15×41

$Employment\ growth_{i,t}$ = industry employment growth, NIPA, 1999-2013
X_i = industry and fixed effect
γ_t = year effect
$Rot_{i,t}$ = industry rotation rate at t, Compustat
$w_{i,j}$ = share of intermediate goods from industry i to industry j, *BEA 2007 IO tables*
I include private non-farm industries that are in both Compustat and NIPA and have more than 8 firms from 1999 to 2013
SE is in the parentheses.
1, 2, 3 asterisks denote significance at the ten, five, one percent level.

nificantly associated with the condition of the labor market at both aggregate level and industry level. Moreover, measures of churning of an industry is significantly associated with the condition of labor markets of its linked industries. Given these findings, it is reasonable to conjecture that variations in churning contribute to the labor market fluctuations, and inter-firm linkage across industries serve as a propagation mechanism.

2.2.2 *Churning and uncertainty shock*

Churning is closely related to uncertainty shock. Following Bloom (2009); Bloom et al. (2012), uncertainty shock is defined as exogenous change in volatility of idiosyncratic productivity. In many settings, an increase in uncertainty leads to higher churning.[6] There are two main reasons why I distinguish churning from uncertainty in this paper.

First, churning is not equivalent to uncertainty. In fact, there are many cases in which uncertainty preserves firms' initial rankings and therefore has no effect on churning.[7] In the uncertainty shock literature, most empirical work measure uncertainty as cross-firm dispersion which characterizes the shape of the entire distribution and is unrelated to individual firms' rankings.[8]

Second, churning has straightforward interpretations that are distinct from those of uncertainty shock. Uncertainty is usually motivated as dispersion of economic outlook or as an occurrence of unexpected critical event.[9] In contrast, churning is likely to be caused by the introduction of new technologies which are then adopted by firms who are not industry leaders, or by a shift of consumer preference on different products,[10] or firms' heterogeneous exposures to aggregate shocks,[11] which are economic disturbances that are common in the economy. In the rest part of the paper, I study the effect of churning on the

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[6] This is often true if the productivity process is parameterized as an auto-regressive process. For example, if idiosyncratic productivity follows

$$z_{i,t+1} = \rho z_{i,t} + \sigma_t \varepsilon_{i,t+1},\ \varepsilon_{i,t+1} \sim \mathbf{N}(0,1)$$

and if there is positive shock to σ_t, firms' rankings in productivity distribution changes more often, which leads to a higher churning.

[7] One example is: $z_{1,t+1} = exp\left(|\sigma_{1,t}\varepsilon_{1,t+1}|\right)$ and $z_{2,t+1} = exp\left(-|\sigma_{2,t}\varepsilon_{2,t+1}|\right)$. Uncertainty shock does not change the two firms' rankings since we always have $z_{2,t+1} < z_{1,t+1}$

[8] One exception is Bloom et al. (2012), who used churning of firms' rankings in TFP as evidence of uncertainty shock.

[9] See Bloom (2009)

[10] For example, consumers previously prefer goods A to goods B, but suddenly flip the ordering. The producers of the two goods would switch ranking of output and profit.

[11] For example, a persistent increase in the oil price can possibly induce a change in the relative profitability of car markers who specialize in SUV and ones specialize in vehicles with lower fuel consumption.

economy, which also provides a new transmission and amplification mechanism for the above mentioned economic disturbances.

2.3 A simple model

Motivated by the preceding empirical findings, in this section I study the link between churning and unemployment through a simple DMP model modified with firm interconnectivity. I introduce the model in sections 3.1 to 3.3. Then I derive the key analytical results in section 3.4 to show that a higher rotation rate increases unemployment and induces an endogenous decline in measured TFP if and only if (1) there is firm interconnectivity, and (2) the production function is supermodular.

2.3.1 Baseline environment

Time is discrete. The economy has two distinct sets of firms indexed by A and B, so two labor markets. New production opportunities, corresponding to job vacancies (v_i) are created at cost χ. Each set of firms is associated with a labor market that is populated by a measure one of risk-neutral individuals who can be either employed in set i (e_i) or unemployed and searching for a job (u_i). As will be shown, in this model, the two sets always comove exactly and workers are indifferent between the two labor markets.[12] Hence, for simplicity, I assume workers cannot move across labor markets.

At the beginning of each period, unemployed workers and job vacancies are matched in the two frictional labor markets. Matching probability depends on the ratio of the number of vacancies to the number of unemployed workers. If matched, the new firms[13] draw a type from H (high productivity) or L (low productivity)[14] randomly with probability 50 percent and 50 percent. Then the firm is instantaneously assigned to a partner of the same

[12] The quantitative model in section (4) would describe the situation when the two sets do not comove and labor can move across labor markets.

[13] New firms are newly created firm–worker matches.

[14] Here firms are ranked by productivity. In the appendix 2.11, I show the case in which firms are not ranked by productivity, but by consumer's preference on different products. The two settings have similar results.

type from the other set; That is, an H type firm is assigned to an H type partner while an L type is assigned to an L type partner.[15][16]The positive assortative assignment is the unique Nash equilibrium in this economy. If not matched, the firm exits the model and the worker remains unemployed.

Production takes place directly after the assignment of partner. A firm's productivity has two components, idiosyncratic productivity and aggregate productivity. Idiosyncratic productivity depends on a firm's own type and its partner's type. I denote a firm's idiosyncratic productivity as z_i^{jk} if it is in set i with type j cooperating with a k type partner. Aggregate productivity x affects all firms in the same way. By assumption, a new firm's productivity is either $x \cdot z_i^{HH}$ or $x \cdot z_i^{LL}$.

Firm and worker split output with fixed share τ and $1-\tau$. As a firm's output is simply the productivity, its profit is $\tau \cdot x \cdot z_i^{jk}$ and the remaining part is the worker's wage. For simplicity I assume there is no unemployment insurance or disutility of working, so workers always prefer working to unemployment.[17]

The most important setting of the model is, after production of goods, type of each firm from set A rotates with a Markov switching process, which is described by a Markov switching matrix Π_t in which each row adds up to one. Here Π_t has a time subscript, meaning rotation rates are time-varying: in period t a type j firm switches to type k with probability ρ_t^{jk}. The rotation processes are i.i.d. across firms.[18]

$$\Pi_t = \begin{bmatrix} \rho_t^{HH} & \rho_t^{HL} \\ \rho_t^{LH} & \rho_t^{LL} \end{bmatrix}$$

---

[15] For simplicity, here I assume firms are assigned to their partner without searching friction. In the quantitative model, I will add searching friction to the inter-firm matching process.

[16] In appendix 2.12, I show that the main results of this section is not altered if one assumes that firms draw types *before* matching with worker.

[17] The setting of wage and unemployment benefit deviates from canonical DMP model. The quantitative model in section 4.3 considers Nash bargaining and positive unemployment benefit. The assumption of wage setting does not affect the main result.

[18] ρ_t^{jk} is also the fraction of firm with type j in set A that rotates to type k, and off-diagonal entries corresponds to the empirical rotation rate I constructed in the previous section.

As the empirical counterpart of H type and L type is the above the median and below the median in the productivity distribution, the measure of H type firms should always equal to the measure of L type firms, therefore Π_t is restricted to be symmetric with $\rho_t^{LH} = \rho_t^{HL}$.

My analysis focuses on shock to the rotation rates. For simplicity, I ignore rotation of type for firms in set B; that is, their types are permanent. However, their productivities can change due to the switching of their partner's type.

Following the canonical DMP model, worker-firm matches are destroyed exogenously at the end of each period with fixed rate δ. Upon destruction of the match, workers become unemployed in the same set since I assume that they cannot move across labor markets.

In sum, events unfold as follows: At the beginning of the period, the aggregate productivity and rotation rate of set A are observed. Firms post vacancy to match with unemployed workers. If successfully matched, firms randomly draw their type, either H or L, then are assigned to a partner with the same type. If not matched, the firm exits the model. The production takes place right after the assignment. Then the firms in set A rotate their type randomly with type rotation rate. At the end of each period, firms are destroyed and disappear from the model exogenously with a fixed rate δ, in which case the worker becomes unemployed.

2.3.2 Tightness ratios

In this subsection, I introduce some key notations of the frictional labor market. A firm can post a vacancy in either set. The cost of posting a vacancy is χ. There is free entry into vacancy posting on the part of firms.

Matching function $m(\mu_{i,t}, v_{i,t})$ determines how many matches are formed given the number of vacancies $v_{i,t}$ and the number of unemployed workers $u_{i,t}$.

Labor market tightness ratio $\theta_{i,t}$ is the ratio of the number of vacancies to the number of unemployed workers in set i:

$$\theta_{i,t} = \frac{v_{i,t}}{u_{i,t}}, \quad i \in \{A,\, B\}$$

Tightness ratios are an equilibrium object, they are taken parametrically by both firms and workers.

Job finding rate $\mu_{i,t}$ is the probability for an unemployed worker to match with a vacancy. Vacancy filling rate $q_{i,t}$ is the probability for a vacancy matching with an unemployed worker. The matching function is assumed to be homogeneous of degree one, hence job finding rate and vacancy filling rate are functions of the market tightness ratio:

$$\begin{aligned} \mu_{i,t} &= \frac{m(u_{i,t}, v_{i,t})}{u_{i,t}} = \mu(\theta_{i,t}) \quad i \in \{A,\, B\} \\ q_{i,t} &= \frac{m(u_{i,t}, v_{i,t})}{v_{i,t}} = q(\theta_{i,t}) \end{aligned}$$

Unemployment rates are determined by the joint force of job creation and job destruction. In each period, unemployed workers flow out of unemployment with the job finding rate $\mu_{i,t}$. At the same time, jobs are destroyed endogenously and workers become unemployed with the job destruction rate δ. Flow motion of unemployment is

$$u_{i,t+1} = u_{i,t} - \underbrace{\mu_i(\theta_{i,t}) \cdot u_{i,t}}_{Job\ creation} + \underbrace{\delta \cdot e_{i,t}}_{Job\ destruction} \qquad i \in \{A,\, B\} \tag{2.1}$$

$u_{i,t}$ ($e_{i,t}$) is measure of unemployment (employment) with $u_{i,t} = 1 - e_{i,t}$, because measure of workforce of each set is 1 and workers can be either working or unemployed searching for a job.

Since the job destruction rate is assumed to be fixed, fluctuation of unemployment is solely governed by the tightness ratio $\theta_{i,t}$. A lower tightness ratio halters job creation and increases unemployment. The simple model aims to find the conditions under which increases in rotation rate ρ_A^{HL} cause an increase in unemployment $u_{A,t}$ and $u_{B,t}$ by reducing tightness ratios $\theta_{A,t}$ and $\theta_{B,t}$.

2.3.3 Firm's value function

In this subsection, I demonstrate firms' value functions.

As utility function is linear in this simple model, the firm's value function is simply present value of profit. In set A, for a firm with type j and working with a partner with type k, its value function is

$$\begin{aligned} J_{A,t}^{jk} &= \tau \cdot x_t \cdot z_{A,t}^{jk} + \beta \left(1-\delta\right) E_t \left(\rho_t^{jH} J_{A,t+1}^{Hk} + \rho_t^{jL} J_{A,t+1}^{Lk}\right) \\ j,\, k &\in \{H,\, L\} \end{aligned} \tag{2.2}$$

Similar to the notation of idiosyncratic productivity, the superscript jk indicates that the firm is type j and it cooperates with another firm of type k; subscript indicates set and time period.

This value is composed of the contemporary profit, τ fraction of output $x_t \cdot z_{A,t}^{jk}$, plus the expected discounted value from the next period on. In the next period, with probability ρ_t^{jH} and complementary probabilityρ_t^{jL}, the firm becomes H type and L type and therefore has value of $J_{A,t+1}^{Hk}$ and $J_{A,t+1}^{Lk}$.

Value of firms in set B is described by a similar equation:

$$J_{B,t}^{jk} = \tau \cdot x_t \cdot z_{B,t}^{jk} + \beta \left(1-\delta\right) E_t \left(\rho_t^{kH} J_{B,t+1}^{jH} + \rho_t^{kL} J_{B,t+1}^{jL}\right) \tag{2.3}$$

The firm receives contemporary profit $\tau \cdot x_t \cdot z_{B,t}^{jk}$, and in the next period, although by assumption its own type is not subject to rotation and will always be j, the type of its partner in set A rotates from k to H and L with probability ρ_t^{kH} and ρ_t^{kL}.

In each set, there are a large number of firms that can potentially post vacancies as long as they pay the cost χ. The value of posting a vacancy in set i is

$$V_{i,t} = -\chi + f(\theta_{i,t}) \left(\underbrace{\frac{J_{i,t}^{HH}}{2}}_{draw\ H} + \underbrace{\frac{J_{i,t}^{LL}}{2}}_{draw\ L} \right) + (1 - f(\theta_{i,t})) \max_i \left[E_t(V_{A,t+1}), E_t(V_{B,t+1}), 0 \right] \quad (20)$$

$$i \in \{A, B\}$$

where $f(\theta_{i,t})$ denotes a firm's vacancy filling rate. If successfully matched with a worker, the firm draws H or L type with 50 percent and 50 percent probability and assigned to a partner with the same type. The maximization of the expectation term implies that firms who fail to match with a worker can choose to post a vacancy in either market or to be inactive in the following period.

Some useful notions

Before characterizing the equilibrium of the labor market, let me introduce some useful notions and results.

Definition 1. (Strict monotonicity)

1, Production function is strictly monotone if

$$z_i^{jH} > z_i^{jL} \quad z_i^{Hk} > z_i^{Lk} \quad i \in \{A, B\}$$

2, Value function is strictly monotone if

$$J_i^{jH} > J_i^{jL} \quad J_i^{Hk} > J_i^{Lk} \quad i \in \{A, B\}$$

Strict monotonicity holds if a firm's productivity and value are strictly increasing in both the firm's own type and its partner's type.

Definition 2. (Supermodularity)

1, Production function is supermodular if

$$z_i^{HH} - z_i^{LH} > Z_i^{HL} - z_i^{LL} \quad i \in \{A,B\}$$

2, Value function is supermodular if

$$J_i^{HH} - J_i^{LH} > J_i^{HL} - J_i^{LL} \quad i \in \{A,B\}$$

The production function is supermodular when marginal production is increasing in partner's type. Similarly, the value function is supermodular when marginal value is increasing in partner's type. Another way to interpret supermodularity is that positive assortative matching yields higher aggregate productivity or value than cross matching between different type of firms.

Lemma 1.

1. Value function is strictly monotone if production function is strictly monotone.

2. Value function is supermodular if and only if production function is supermodular.

Proof of Lemma 4 can be found in appendix 2.9. The first line of Lemma 4 shows the sufficient condition under which firms are better off as a H type firm or matching with H type partner. The second line shows that it's more efficient for firms to match with firms of same type if and only if the production function is supermodular; cross matching between different types induces efficiency loss.

Proposition 1. *Positive assortative assignment is the unique Nash equilibrium if idiosyncratic productivity is strictly monotone.*

Proposition 1 illustrates that the positive assortative assignment can be endogenized given firms are more productive matching with H type than with L type. The intuition is that, as there is no search friction between firms, H type firms only match with H type firms while L type firms are only left to match with L type firms.

2.3.4 Free entry condition and the equilibrium of the labor market

The equilibrium of the model is determined by free entry condition. In this subsection, I first derive the free entry condition. Then I illustrate the effect of rotation rate on the equilibrium of the labor market.

Free entry condition

The equilibrium level of tightness ratio $\theta_{i,t}$ is determined by the following free entry conditions

$$V_{i,t} = 0, \quad i \in \{A,\, B\}$$

By plugging the free entry condition into equation 4.3.4, we get

$$\chi = f(\theta_{i,t}) \cdot \left(\frac{J_{i,t}^{HH}}{2} + \frac{J_{i,t}^{LL}}{2} \right) \quad i \in \{A,\, B\} \tag{2.5}$$

The *LHS* of equation 2.5 is vacancy cost. The *RHS* is the expected payoff of the vacancy. Firms would post vacancies up to the point that its expected payoff exactly compensates for the cost. Because the vacancy filling rate $f(\theta_{i,t})$ is a decreasing function of $\theta_{i,t}$, a smaller expected matching value $\frac{1}{2}\left(J_{i,t}^{HH} + J_{i,t}^{LL}\right)$ would induce a lower tightness ratio $\theta_{i,t}$. The intuition is that when the expected matching value is small, fewer firms want to post or maintain vacancies which then leads to a lower tightness ratio.

The case with no firm inter-connectivity

When there is no firm inter-connectivity as in the canonical DMP model, a firm's productivity and value depends only on its own type

$$\begin{aligned} J_{i,t}^{j} &= \tau \cdot x_t \cdot z_{i,t}^{j} + \beta (1-\delta) E_t \left(\rho_t^{jH} J_{i,t+1}^{H} + \rho_t^{jL} J_{i,t+1}^{L} \right) \\ & \quad j,\, k \in \{H,\, L\} \quad i \in \{A,\, B\} \end{aligned} \tag{2.6}$$

The free entry condition becomes

$$\chi = f(\theta_{i,t}) \cdot \left(\frac{J^H_{i,t}}{2} + \frac{J^L_{i,t}}{2} \right) \quad i \in \{A,\, B\} \tag{2.7}$$

Equations 4.3.3 and 2.7 are identical to the value function and free entry condition of the canonical DMP model with two productivity states. In other words, the canonical DMP model is nested in the benchmark model with firm inter-connectivity by restricting firms' productivity to be unaffected by type of partners; that is, $z_i^{HH} = z_i^{HL}$, $z_i^{LH} = z_i^{LL}$.

In the following part, I show that when there's no firm inter-connectivity, the rotation rate has no effect on the tightness ratio $\theta_{i,t}$. To illustrate this, take vacancy filling rate to the *LHS* of free entry condition equation 2.7 and plug equation 4.3.3 into the *RHS*, we get

$$\begin{aligned} \frac{\chi}{f(\theta_{A,t})} &= \frac{1}{2} \cdot \left[\tau \cdot x_t \cdot z^H_{A,t} + \beta (1-\delta) E_t \left(\rho_t^{HH} J^H_{A,t+1} + \rho_t^{HL} J^L_{A,t+1} \right) \right] \\ &+ \frac{1}{2} \cdot \left[\tau \cdot x_t \cdot z^L_{A,t} + \beta (1-\delta) E_t \left(\rho_t^{LH} J^H_{A,t+1} + \rho_t^{LL} J^L_{A,t+1} \right) \right] \end{aligned}$$

As the Markov switching matrix $\Pi_{A,t}$ is symmetric, that is, $\rho_t^{HH} + \rho_t^{LH} = 1$, $\rho_t^{HL} + \rho_t^{LL} = 1$, which is a restriction that ensures the measure of H type and L type firms are always same. The above equation can be further derived as

$$\frac{\chi}{f(\theta_{A,t})} = \frac{1}{2} \cdot \left[\tau \cdot x_t \cdot \left(z^H_{A,t} + z^L_{A,t} \right) + \beta (1-\delta) E_t \left(J^H_{A,t+1} + J^L_{A,t+1} \right) \right] \tag{2.8}$$

As shown in equation 2.8, rotation rate becomes irrelevant in the free entry condition set A, hence does not affect the equilibrium level of set A's tightness ratio. The intuition is that with a higher ρ^{HL}, that is, H type firms are more likely to become L type; at the same time L type firms are equally likely to become H type, as ρ^{LH} also increases. Adding them together, the expected value of vacancy does not change hence rotation rate has no aggregate effect.

Because set B does not have rotation, it's free entry condition is simply

$$\frac{\chi}{f\left(\theta_{B,t}\right)} = \frac{1}{2}\cdot\left[\tau\cdot x_t\cdot\left(z_{B,t}^{H}+z_{B,t}^{L}\right)+\beta\left(1-\delta\right)E_t\left(J_{B,t+1}^{H}+J_{B,t+1}^{L}\right)\right] \tag{2.9}$$

In set B, the equilibrium level of tightness ratio $\theta_{B,t}$ is also not affected by rotation rate. I summarize the above results in the following proposition.

Proposition 2. *When there's no firm inter-connectivity, rotation rate has no effect on the equilibrium level of tightness ratio in either labor market.*

Proposition 2 shows that firm inter-connectivity is a necessary condition for rotation rate to affect the labor market. Therefore in the subsequent part, I focus on the case with firm inter-connectivity.

The case with firm inter-connectivity

When there's firm inter-connectivity, rotation rate enters into matching value and can potentially affect the equilibrium level of the tightness ratio. To illustrate this, take vacancy filling rate to the *LHS* of free entry condition equation 2.5 and plug equations 2.2 and 2.3 into *RHS*, free entry condition of set A then becomes

$$\begin{aligned}\frac{\chi}{f\left(\theta_{A,t}\right)} &= \frac{1}{2}\cdot\left[\tau\cdot x_t\cdot z_{A,t}^{HH}+\beta\left(1-\delta\right)E_t\left(\rho_t^{HH}J_{A,t+1}^{HH}+\rho_t^{HL}J_{A,t+1}^{LH}\right)\right]\\ &+ \frac{1}{2}\cdot\left[\tau\cdot x_t\cdot z_{A,t}^{LL}+\beta\left(1-\delta\right)E_t\left(\rho_t^{LH}J_{A,t+1}^{HL}+\rho_t^{LL}J_{A,t+1}^{LL}\right)\right]\end{aligned}$$

Since the Markov switching matrix is symmetric, that is $\rho_t^{HH}=1-\rho_t^{HL}$, $\rho_t^{LL}=1-\rho_t^{LH}$

and $\rho_t^{HL} = \rho_t^{LH}$, the above equation is equivalent to

$$\begin{aligned} \frac{\chi}{f\left(\theta_{A,t}\right)} &= \frac{1}{2} \cdot \left[\tau \cdot x_t \cdot z_{A,t}^{HH} + \beta\left(1-\delta\right) E_t\left(J_{A,t+1}^{HH}\right)\right] \\ &+ \frac{1}{2} \cdot \left[\tau \cdot x_t \cdot z_{A,t}^{LL} + \beta\left(1-\delta\right) E_t\left(J_{A,t+1}^{LL}\right)\right] \\ &- \frac{1}{2} \cdot \beta\left(1-\delta\right) \rho_t^{HL} E_t\left(\underbrace{J_{A,t+1}^{HH} + J_{A,t+1}^{LL} - J_{A,t+1}^{HL} - J_{A,t+1}^{LH}}_{mismatch\ loss}\right) \end{aligned} \tag{2.10}$$

As equation 4.3.9 is stochastic, I will use temporary shock to rotation rate in period t as an illustration, and defer the discussion of persistent shock to the next subsection. Suppose in period t, there is a temporary increase in the rotation rate ρ_t^{HL}. It is easy to see that the first two rows of the RHS of equation 4.3.9 are not affected as the shock is temporary and values should return to the steady state in period $t+1$. The shock to ρ_t^{HL} would affects the last row only. Specifically, with supermodularity, or equivalently $\left(J_A^{HH} + J_A^{LL} - J_A^{HL} - J_A^{LH}\right)$ is positive, a temporary increase in ρ_t^{HH} would increase expected efficiency loss, which then translates into a decline in expected matching value. As a result, fewer firms post vacancies, driving tightness ratio to a lower level.

In addition to the change in the expected matching value, a higher rotation rate increases the share of mismatched firms in the *next* period. When production function is supermodular, mismatched firms, on average, are less productive. This composition effect decreases the averaged TFP in set A in period $t+1$.

For firm in set B, its free entry condition has a similar form

$$\begin{aligned}\frac{\chi}{f\left(\theta_{B,t}\right)} &= \frac{1}{2}\left(\tau\cdot x_t\cdot z_{B,t}^{HH}+\beta\left(1-\delta\right)E_t\left(J_{B,t+1}^{HH}\right)\right) \\ &+ \frac{1}{2}\left(\tau\cdot x_t\cdot z_{B,t}^{LL}+\beta\left(1-\delta\right)E_t\left(J_{B,t+1}^{LL}\right)\right) \\ &- \frac{1}{2}\beta\left(1-\delta\right)\rho_t^{HL}E_t\left(\underbrace{J_{B,t+1}^{HH}+J_{B,t+1}^{LL}-J_{B,t+1}^{HL}-J_{B,t+1}^{LH}}_{mismatch\ loss}\right)\end{aligned} \tag{2.11}$$

Although firms in set B do not have rotation, they are faced with the risk of mismatch. When firms in set B has supermodular production and value functions, a temporary increase in ρ_t^{HH} would lower the equilibrium level of tightness ratio in period t and averaged TFP in period $t+1$.

2.3.5 *The main result: rotation rate and labor market equilibrium*

In the previous subsection, I show that an temporary increase in the rotation rate reduces labor market tightness ratio and therefore increases unemployment if two conditions hold: (1) there is firm inter-connectivity; and (2) efficiency loss caused by mismatch $E_t\left(J_{B,t+1}^{HH}+J_{B,t+1}^{LL}-J_{B,t+1}^{HL}-J_{B,t+1}^{LH}\right)$ is positive. In this subsection, I generalize the temporary shock to a persistent shock case and present the main results.

Proposition 3. *If production function is strictly monotone, and there is firm inter-connectivity, a persistent increase in the rotation rate of one set, whose persistence is bounded below* $|\psi|<1-\rho^{HL}-\rho^{LH}$*, state decreases*

1. *the equilibrium tightness ratio of both sets*
2. *the average productivity of both sets in the next period*

if and only if the production function is supermodular.

Proposition 3 is the key result of section 3. The proof of proposition 3 can be found in appendix 2.9. The first part formally shows the necessary and sufficient condition under which an increase in rotation rate reduces job creation and increases unemployment. The intuition is similar with the temporary shock case illustrated in the previous subsection: positive shock to the rotation rate of one set of firms increases the probability of mismatch for these firms and their partners. With supermodularity mismatch risk negatively affects firms' average expected value, which will reduce job creation and increase unemployment.

The second part of proposition 3 shows that the model generates endogenous change in TFP. It works in two channels. First, higher rotation rate makes more existing firms become mismatched. In the model, with the supermodular production function, mismatched firms have lower productivity on average and a larger fraction of mismatched firms leads to a lower average productivity of the economy. Second, faced with a higher mismatch risk, firms post fewer vacancies, which results in decreased creation of new firms. By construction, there's no mismatch in new firms, hence they have higher average productivity. As a result, the drop in the inflow of new firms also contributes to a decline in average productivity.

2.4 Endogenous choice of cooperation contract and testable microeconomic implication

In this section, I test the empirical relevance of my model using testable microeconomic implication that my model is not designed to match with. The main mechanism of my model is that when there's firm inter-connectivity and the production function is supermodular, churning induces mismatches which reduce output and job creation. While churning and firm inter-connectivity are observed in the economy, supermodularity cannot be directly observed. While the idea of supermodularity of inter-firm matches has been hypothesized by many studies,[19] there is no direct microeconomic evidence on it. In this section, I assess the empirical relevance of supermodularity using the model's testable microeconomic implication.

[19] For example, Becker (1973); Kremer (1993); Rhodes-Kropf and Robinson (2008); Shimer and Smith (2000)

The plan of this section is as follows: I first demonstrate the main idea of the test. Then I extend the simple model with endogenous choice of cooperation contract, which allows me to derive the model's testable microeconomic implication and implement the test. Last, I present the result of the test.

2.4.1 Intuition of the test

Supermodularity depicts the efficiency of the allocation of different types of firms, which is not directly observed. Specifically, supermodularity cannot be identified from firms' choice of partner, because firms would always initiate match with the same type partner as long as the production function is monotone, that is, firms prefer to match with a H type partner. However, it is not straightforward to tell whether the positive assortative matching is efficient or not.

To overcome this challenge, I consider a slight extension to the simple model. Specifically, I allow firms to choose the duration of the cooperation contract before matching with partners. And I show that although supermodularity does not affect firms' choice of partner, it does affect firms' choice of cooperation contract. The intuition is that, when a firm is uncertain about its own future type or its partner's future type, and then mismatch is inefficient, the firm does not want to make the commitment by choosing a long duration contract. In my empirical analysis, a long duration contract is proxied by vertical integration, a short duration contract by sourcing[20]

In particular, I show that supermodularity predicts two cross-industry patterns: first, fewer firms choose vertical integration (long-duration cooperation) in industries with higher churning; second, fewer firms choose vertical integration when their partners are in industries with higher churning. The intuition is as follows: while vertical integration helps

[20] Vertical integration means a firm internalizes the cooperation relationship, which is practically permanent cooperation; Sourcing is a temporary cooperation arrangement, the duration of which depends on maturity of the contract.

firms to avoid transaction costs,[21] it prevents firms from dissolving mismatches and reallocating to new cooperation relationships. Remember that in any mismatch between an H type and an L type, regardless of supermodularity, the H type always prefers a "divorce", while the L type always prefers to keep the match. Supermodularity, however, implies that keeping a mismatch lowers the *average* profit and value of firms, compared with dissolving the mismatch. Vertical integration traps firms in mismatches and lowers the average profit and value of the mismatched firms. In contrast, by choosing sourcing, firms have the option to dissolve from mismatches as soon as the contract expires. Hence sourcing serves as an option to hedge against mismatch risk. As churning induces mismatch risk, when a industry or its linked industries has higher churning, more firms choose sourcing and fewer choose vertical integration.

2.4.2 Extend the simple model

In this subsection, I extend the simple model by letting firms choose the type of cooperation contract either vertical integration (VI) or sourcing (SC). If a firm chooses vertical integration, it is permanently matched with its partner.[22] In contrast, if a firm chooses sourcing, it can sever mismatched partnership with probability v_i then match with a new partner. Parameter v_i is a number between zero and one depending on maturity of sourcing contract industry in industry i.[23] This rematch option comes with a price: each period, firms with sourcing contracts pay a transaction cost $\eta \sim iid\ N\left(\bar{\eta}_i, \bar{\sigma}_i^2\right)$, where η is firm fixed effect; $\bar{\eta}_i$ and $\bar{\sigma}_i$ are mean and dispersion of transaction cost within industry i.

As I do not have measurement of each industry's transaction costs or on their sourcing contracts' maturities, in the empirical analysis I treat v_i, $\bar{\eta}_i$ and $\bar{\sigma}_i$ as constant across

[21] Common transaction costs include bargaining cost, contracting cost and hold-up risk, which has been discussed extensively in Transaction Cost Economics (TCE) literature pioneered by Coase (1937); Williamson (1979, 1981)

[22] In the original simple model, firms can choose only vertical integration, hence it is nested in the extended model.

[23] For example, if a contract's maturity is 2 years, v is 1/8 in a quarterly model

industries and I d subscript i in the rest of this section.

For simplicity, as in the simple model there are two industries i and j. Only industry j has rotation, which is governed by a Markov switching matrix $\begin{bmatrix} \rho_j^{HH} & \rho_j^{HL} \\ \rho_j^{LH} & \rho_j^{LL} \end{bmatrix}$, while industry i does not have rotation.

I assume that the draw of transaction cost η and the choice of cooperation contract takes place directly after a firm matches with a worker. Hence events unfold as follows: at the beginning of the period, homogeneous firms post vacancies to match with unemployed workers. If successfully matched, a firm draws its transaction cost η from $\boldsymbol{N}\left(\bar{\eta}, \bar{\sigma}^2\right)$, then chooses between vertical integration and sourcing. Then firms randomly draw their type, either H or L, and match with a same type partner. In each period, existing firms in industry j rotate their type randomly. At the end of each period, firms are destroyed and exit the model exogenously with fixed rate δ.

Value function and free entry condition

In industry i, value functions for firms with sourcing are:

$$
\begin{aligned}
J_{i,SC}^{HH}(\eta) &= \tau \cdot \left(z_i^{HH} - \eta\right) && (2.12) \\
&+ \beta(1-\delta) E_t \left[\rho_j^{HH} J_{i,SC}^{HH}(\eta) + (1-\nu)\rho_j^{HL} J_{i,SC}^{HL}(\eta) + \nu \rho_j^{HL} J_{i,SC}^{HH}(\eta)\right] \\
J_{i,SC}^{LL}(\eta) &= \tau \cdot \left(z_i^{LL} - \eta\right) && (2.13) \\
&+ \beta(1-\delta) E_t \left[\rho_j^{LL} J_{i,SC}^{LL}(\eta) + (1-\nu)\rho_j^{LH} J_{i,SC}^{LH}(\eta) + \nu \rho_j^{LH} J_{i,SC}^{LL}(\eta)\right]
\end{aligned}
$$

The time subscripts are omitted in the above equations. The value of H type firms is composed of contemporary profit and continuation value in the next period; with rotation rate ρ_j^{HL}, the firm's partner becomes L type, and with probability ν, the firm rematches with another H type partner. The subscript SC denotes that the firm uses sourcing as cooperation contract. The value of L type firms is similar. With rotation rate ρ_j^{LH}, its partner becomes H type and they separate with probability ν, then the firm rematches with another L type partner.

The value functions for firms with vertical integration are same as those in the original simple model. A firm that chooses vertical integration is exempt from transaction cost η, but does not have a rematch option when rotation occurs.

$$J_{i,VI}^{HH} = \tau \cdot z_{i,t}^{HH} + \beta (1-\delta) E_t \left(\rho_j^{HH} J_{i,VI}^{HH} + \rho_j^{HL} J_{i,VI}^{HL} \right) \tag{2.14}$$

$$J_{i,VI}^{LL} = \tau \cdot z_{i,t}^{LL} + \beta (1-\delta) E_t \left(\rho_j^{LL} J_{i,VI}^{LL} + \rho_j^{LH} J_{i,VI}^{LH} \right) \tag{2.15}$$

The value of posting a vacancy in industry i is

$$\begin{aligned} V_i = & -\chi + f(\theta_i) \int \max \left\{ \frac{J_{i,SC}^{HH}(\eta) + J_{i,SC}^{LL}(\eta)}{2}, \frac{J_{i,VI}^{HH} + J_{i,VI}^{LL}}{2} \right\} dF(\eta) \\ & + (1 - f(\theta_i)) \max_i \{ E_t (V_i), 0 \} \end{aligned} \tag{2.16}$$

where $F(\eta)$ is cumulative distribution function (CDF) of the transaction cost distribution.

The form of vacancy value is similar to the one in the simple model. There are two differences, however. The first one is that after matching with a worker with vacancy filling rate $f(\theta_i)$, the firm chooses between vertical integration and sourcing. The second one is that there is one additional dimension of heterogeneity, the transaction cost η. In the firm chooses sourcing, firms are subject to the transaction cost η and the expected value is $\frac{1}{2}\left(J_{i,SC}^{HH}(\eta) + J_{i,SC}^{LL}(\eta)\right)$. If the firm chooses vertical integration, the expected value is $\frac{1}{2}\left(J_{i,VI}^{HH} + J_{i,VI}^{LL}\right)$. Firms decide on type of cooperation contract by comparing these two terms.

The free entry condition in industry i is

$$V_i = 0$$

From equation 2.16, we get

$$\frac{\chi}{f(\theta_i)} = \int \max\left\{\frac{J_{i,SC}^{HH}(\eta)+J_{i,SC}^{LL}(\eta)}{2}, \frac{J_{i,VI}^{HH}+J_{i,VI}^{LL}}{2}\right\} dF(\eta) \tag{2.17}$$

The choice of contract

The focus of this section is firms' choice of contract and how it relates to rotation rate. I first show that firms' choice of contract can be categorized by a threshold η_i^*: firms with transaction cost η beyond η_i^* would choose vertical integration; ones with η below η_i^* would choose sourcing.

In industry i, threshold η_j^* is the level of transaction cost that makes a firm indifferent between sourcing and vertical integration. From the free entry condition equation 2.17, we can see that η_j^* is determined by the following equation:

$$\frac{1}{2}\left(J_{i,VI}^{HH}+J_{i,VI}^{LL}\right) = \frac{1}{2}\left[J_{i,SC}^{HH}\left(\eta_j^*\right)+J_{i,SC}^{LL}\left(\eta_j^*\right)\right]$$

Proposition 4. *The threshold level of transaction cost of industry i exists and is uniquely determined by*

$$\tau\cdot\eta_i^* = \beta(1-\delta)\, g\left(\rho_j^{HL},\nu\right)\cdot\left(z_i^{HH}+z_i^{LL}-z_i^{HL}-z_i^{LH}\right) \tag{2.18}$$

with $\frac{\partial g\left(\rho_j^{HL},\nu\right)}{\partial \rho_j^{HL}} > 0$ *and* $\frac{\partial g\left(\rho_j^{HL},\nu\right)}{\partial \nu} > 0$

The LHS of equation 2.18 is the cut-off firm's transaction cost that it can save with vertical integration. The RHS is the hedgable part of mismatch risk it can hedge with sourcing, which depends on rematch probability, industry j's rotation rate and its mismatch loss. Function g is defined in appendix; it is increasing in ρ_j^{HL} and decreasing in ν. The intuition of equation 2.18 is that the cut-off firm should find transaction cost exactly equal to the hedgable part of mismatch risk.

I denote $\tilde{z}_i = z_i^{HH} + z_i^{LL} - z_i^{HL} - z_i^{LH}$ as industry i's productivity loss induced by mismatch. Hence in industry i, the threshold level of transaction cost is uniquely determined by

$$\eta_i^* = \frac{\beta (1-\delta) g\left(\rho_j^{HL}, \nu\right) \cdot \tilde{z}_i}{\tau} \tag{2.19}$$

In addition to proposition 4, in proposition 5, I show that a firm's choice of contract is categorized by the threshold.

Proposition 5. *In industry i, a firm would choose vertical integration if $\eta > \eta_i^*$, sourcing if $\eta < \eta_i^*$. The share of firms that choose vertical integration is $1 - F(\eta_i^*)$.*

The intuition of the proposition is that firms with η higher than η_j^* would choose vertical integration because the transaction cost is too high and it wants to avoid the expensive transaction cost by internalizing the inter-firm cooperation; in contrast, firms with η below η_j^* would choose sourcing, as transaction cost is not prohibitive.

Supermodularity, rotation rate, and vertical integration

In the following part, I discuss the relationship between rotation rate ρ_j^{HL} and threshold η_i^* and hence the share of firms choosing vertical integration $1 - F(\eta_i^*)$. I interrogate how the relationship is affected by supermodular production function. In particular, I show that under some exclusion assumption, rotation rate ρ_i^{HL} is positively correlated with share of firms choosing vertical integration $1 - F(\eta_i^*)$ across industries if and only if the production function is supermodular.

Specifically, I denote $\bar{z} = \frac{\sum_1^I \tilde{z}_i}{I}$ as the mean of productivity loss induced by mismatch across industries, then rewrite equation 2.19 as

$$\begin{aligned} \eta_i^* \cdot \tau &= \beta (1-\delta) g\left(\rho_i^{HL}, \nu\right) [\bar{z} + (\tilde{z}_i - \bar{z})] \\ &\Longrightarrow \\ \frac{\eta_i^* - \bar{\eta}}{\bar{\sigma}} &= -\frac{\bar{\eta}}{\bar{\sigma}} + \frac{\beta (1-\delta)}{\bar{\sigma}\tau} \cdot \bar{z} \cdot g\left(\rho_j^{HL}, \nu\right) + \frac{\beta (1-\delta)}{\bar{\sigma}\tau} g\left(\rho_j^{HL}, \nu\right) (\tilde{z}_i - \bar{z}) \end{aligned} \tag{2.20}$$

The above equations show that the threshold of industry i depends on rotation rate ρ_j^{HL}, and an interactive term between rotation rate ρ_j^{HL} and productivity loss that deviates from average level $\tilde{z}_i - \bar{z}$.

Using the same method, I show that higher rotation rate of industry j increases mismatch risk for firms in industry j. In the model, for simplicity, I assume that there is no rotation in industry i. If we allow for rotation in both industry i and j, there is a symmetric result: higher rotation rate of industry i increases mismatch risk for firms in industry i. Similar to equation 2.20, we have the following equation:

$$\frac{\eta_i^* - \bar{\eta}}{\bar{\sigma}} = -\frac{\bar{\eta}}{\bar{\sigma}} + \frac{\beta(1-\delta)}{\bar{\sigma}\tau} \cdot \bar{z} \cdot g\left(\rho_i^{HL}, \nu\right) + \frac{\beta(1-\delta)}{\bar{\sigma}\tau} g\left(\rho_i^{HL}, \nu\right)(\tilde{z}_i - \bar{z}) \tag{2.21}$$

As the last terms of equations are unobserved, I need to make the following exclusive assumption and treat the last terms as an error term in the empirical analysis.[24]

Assumption 1. *The cross industry variation in the supermodularity of production function is orthogonal to the cross industry variation in the rotation rate.*

$$E\left(\tilde{z}_i - \bar{z} \mid \rho_j, \rho_i\right) = 0, \; for\ any\ i\ and\ j$$

I present the the key result of this section in Proposition 6

Proposition 6. *With the exclusive assumption 1, the share of firms choosing vertical integration $1 - F(\eta_i^*)$ is positively correlated with rotation rates ρ_i^{HL} and ρ_j^{HL} if and only if the production function of industry j is supermodular.*

Proposition 6 is a testable result given that both rotation rate and vertical integration can be measured in the data.

My goal is to test if $\bar{J}$ is positive; that is, whether supermodularity holds on average.

[24] The instrumental variable (IV) approach can hardly exempt me from making this exclusion assumption, because any IV that is correlated with ρ_j^{HL} also correlates with $\rho_j^{HL}(z_j - \bar{z})$ unless $E(z_j - \bar{z} \mid \rho_j) = 0$. There's little prior reason to defy this assumption: $J_j - \bar{J}$ depends on production function, while ρ_j should be related to technology progress or preference shift; they seem to driven by different factors. In the following empirical analysis, I impose that mismatch loss is weakly exogenous to rotation rate and transaction cost.

2.4.3 Empirical analysis

In the empirical study, I focus on backward vertical integration and sourcing of 3-digit Naics industries. Data is described in Appendix 2.9.[25]

I use proposition 6 to test supermodularity. I combine equation 2.20 and equation 2.21, which yield the following specification[26]

$$VI_i = \alpha + \beta_1 Rot_i + \beta_2 \sum_j w_{i,j} Rot_j + \beta_3 MSRF_i + \varepsilon_i$$

The dependent variable VI_i is the share of firms choosing vertical integration in industry i. It corresponds to $1 - F(\eta_i^*)$ in the extended model I described in section 5.2. In the data I proxy it with the ratio of industry value added to industry sales.[27]

The second term on the *RHS* Rot_i is the mean of industry i's rotation rate, constructed in section 4.2.

The third term $\sum_j w_{i,j} Rot_j$ is the weighted average of supplier industries' rotation rates. Weight $w_{i,j}$ is the share of intermediate goods industry i purchases from industry j. In the model, for simplicity I have only two industries. In the data, there are many industries and each one is matched with multiple industries. In addition, each industry has its own rotation process. My model implies that the choice of contract depends on rotation rate of the industry as well as the rotation rates of its linked industries. As with Rot_i, in the data, I use both rotation rate and large rotation rate to measure Rot_j.

$MSRF_i$ is market share of the representative firm in industry i, measured by share of top 50 firms in industry sales constructed by Census Bureau. $MSRF_i$ proxies for industry i's concentration, which is not explicitly modeled in my model. I include it for two reasons:

[25] Backward vertical integration means that downstream producer buys up the upstream supplier

[26] I use a cross sectional regression instead of panel regression as I don't have a panel of measure of vertical integration.

[27] I use this approximation because I do not have firm level input-output data. See Acemoglu et al. (2010); Atalay et al. (2014) for examples of measuring vertical integration using firm level data.

First, it has been found to be important in the vertical integration literature—including it as explanatory variable avoids biased results. Second, there might be concern that rotation rate has information on industry's competition intensity and, therefore the interpretation of my result can be blurred. As a common proxy for competition intensity, $MSRF_i$ can help to address this potential concern.

The goal of the empirical analysis is to test whether β_1 and β_2 are negative as predicted by my model. The null hypothesis is $\beta_1 = \beta_2 = 0$.

As in the data, mean and standard deviation of industry rotation rate are correlated. I also consider a generalized method of moments (GMM) specification in which I use the standard deviation of industry rotation rate as an instrumental variable to increase estimation efficiency.

The empirical results are reported in Table 2.4. According to the results, industry rotation rate and supplier industries' rotation rates both have a significant negative effect on vertical integration, which suggests that production function is supermodular.

Column (1) shows the estimation result of OLS using rotation rates based on ranking of profit as explanatory variables. β_1 and β_2 are negative and significant at 10% and 1% levels. If I include standard deviation of the rotation rate as an instrumental variable and adding an additional moment condition, as shown in column (2), β_1 and β_2 are still negative and both significant at the 1% level.

When I compute the industry rotation rates by ranking firms by profit margin, I get similar results. Column (3) shows the OLS result: both β_1 and β_2 are estimated to be negative. While β_2 is still significant at the 1% level, β_1 is not significant at the 10% level due to a large standard error. By adding an additional moment condition, as shown in column (4), β_1 and β_2 become significant at 5% and 1% level.

To sum up, Table 2.4 shows that, when an industry has a higher rotation rate or its linked industries have higher rotation rates, the industry would have less vertical integration. The intuition is as follows: a higher churning in an industry or in its linked industries

Table 2.4: Measures of churning are negatively correlated with vertical integration across industries

$$VI_i = \alpha + \beta_1 Rot_i + \beta_2 \sum_j w_{i,j} Rot_j + \beta_3 MSRF_i + \varepsilon_i$$

Specification	OLS	GMM	OLS	GMM
Method of ranking	Profit		Profit Margin	
	(1)	(2)	(3)	(4)
Rot_i	-1.34^{*}	-2.22^{***}	-0.84	-1.02^{**}
	(0.75)	(0.78)	(0.52)	(0.51)
$\sum w_{i,j} Rot_j$	-3.62^{***}	-3.03^{***}	-1.81^{***}	-1.50^{***}
	(0.65)	(0.73)	(0.45)	(0.49)
$MSRF_i$	-0.16^{***}	-0.16^{***}	-0.15^{***}	-0.13^{***}
	(0.06)	(0.06)	(0.06)	(0.06)
R^2	0.44	0.42	0.38	0.37
J-stat		5.66^{**}		5.66^{**}
Number of industries	57	57	57	57

Standard deviation of rotation rate is treated as IV.
J-stat is over-identification test statistics.
SE is in the parentheses.
1, 2, 3 asterisks denote significance at the ten, five, one percent level.
VI_j = degree of vertical integration, $\frac{Value\ added_j}{Sales_j}$, *BEA 2007 IO tables*
Rot_j = mean of rotation rate, *Compustat*
$w_{j,l}$ = share of intermidiate goods from industry j, *BEA 2007 IO tables*
$MSRF_j$ = share of top 50 firms in industry sales, Census Bureau

are associated with greater mismatch risk; firms use sourcing to hedge against mismatch risk and avoid vertical integration as the inter-firm cooperation relationship. This supports the assumption of supermodularity and the main mechanism of the model.

2.5 A quantitative model

In this section, I embed the model studied in section 2.4 into a general equilibrium, real business cycle (RBC) model. The RBC model includes a number of features that are absent from the DMP framework, notably accumulation of capital and concavity of utility and production functions, which are critical in quantitative macroeconomic analysis. The primary purpose of this analysis is to evaluate the quantitative contribution of rotation shock to the business cycle, comparing with the common shocks that have been intensively studied in the literature. The second purpose is to show robustness of the cyclical properties of labor market tightness ratio and unemployment to the inclusion of these neoclassical features. The last purpose is to ensure that the model is able to replicate the stylized cyclical behavior of macro aggregates, such as output, consumption and investment. In section 4.1, I describe the model and derive the free entry condition, which determines the equilibrium of the labor market. Then in section 4.4.2, I provide a set of analytical results characterizing some key properties of the model. In section 2.5.3 I estimate the model with Bayesian method and use numerical analysis to evaluate the importance of rotation shock.

2.5.1 Description of the quantitative model

I modify Andolfatto (1996) to include firm inter-connectivity and endogenous choice of inter-firm cooperation contract. With the model studied in section 2.4, this model considers several additional features: There is capital, and both utility and production functions are concave; Firms match with partners in frictional inter-firm matching market; Labor can move across labor markets.

The economy is populated by a continuum of households. There are two distinct labor markets or sets of firms: A and B. Production of final goods requires two intermediate inputs. Each set specializes in one intermediate goods. Firms from each set cooperate with partners from the other set: they produce final goods with intermediate goods then sell final goods in a competitive market. Representative household makes two decisions. First, it optimally allocate income to consumption and investment. Second, it sends its unemployed members to search for job vacancies in either labor market.

At the beginning of each period, firms post job vacancies to match with unemployed workers in either labor market. The matching process is frictional and the search is random. If successfully matched, the firm becomes a *single firm* since it does not have a partner yet. A new single firm randomly draws a transaction cost $\eta \sim N\left(\bar{\eta}, \bar{\sigma}^2\right)$. Observing the transaction cost, the firm chooses its cooperation contract to be either vertical integration (VI) or sourcing (SC). Then the firm randomly draws a type—either H or L with probability 50 percent and 50 percent. I assume that single firms can produce final goods, but with low productivity.[28]

To have an inter-firm cooperation, single firms search for partners in frictional inter-firm matching market. Search is directed, hence single firms are then divided into submarkets depending on their own types, their target partner's type, and the choice of cooperation contract. For example, the H type firms in set A who wants to vertically integrate with an L type partner would be located into the same submarket with the L type firms in set B who want to vertically integrate with an H type partner.

In theory, there can be eight submarkets: $(H-H)-VI$, $(H-L)-VI$, $(L-H)-VI$, $(L-L)-VI$, $(H-H)-SC$, $(H-L)-SC$, $(L-H)-SC$, $(L-L)-SC$. For example, the $(H-L)-VI$ sub-market accommodates the H type firms in set A who are looking for an L type partner and the L type firms in set B who are looking for an H type partner, both sides choose vertical integration as the cooperation contract. In the model, I show that under certain conditions, firms only search for same type partner; that is, there are only four submarkets operating in the Nash equilibrium: $(H-H)-VI$, $(L-L)-VI$, $(H-H)-SC$, and $(L-L)-SC$ markets. The matching technology is similar with the one in the labor market—the probability of finding partner depends on number of single firms from each set in each sub-market. If successfully matched, the firm becomes a *cooperative firm* since it has a partner to cooperate with. If choosing SC, a firm can choose to sever its cooperation with probability ν, but needs to pay transaction cost η in every period. If choosing VI, the transaction cost is waived, yet the firm cannot separate and rematch.

Production takes place after the matching processes. In each set there are twelve cate-

[28] One interpretation is single firm needs to produce both intermediate goods, one of which is not their comparative advantage. The other interpretation is that without inter-firm cooperation, single firm can only sell intermediate goods to a commodity market which is very competitive and yields a very low price.

gories of firms: $H-VI$, $L-VI$, $H-SC$, $L-SC$, $HH-VI$, $HL-VI$, $LH-VI$, $LL-VI$, $HH-SC$, $HL-SC$, $LH-SC$, $LL-SC$; the first four are single firms and the remains are cooperative firms; each category is specified by firm's type and the cooperation contract. Production is organized by representative firms; they organize firms into production departments indexed by categories. They then rent capital from households and optimally allocate it to each production department.

After production, all firms rotate type according to a Markov switching process. Rotation rates are governed by the time varying and symmetric Markov switching matrices $\Pi_{A,t} = \begin{bmatrix} \rho_{A,t}^{HH} & \rho_{A,t}^{HL} \\ \rho_{A,t}^{LH} & \rho_{A,t}^{LL} \end{bmatrix}$ and $\Pi_{B,t} = \begin{bmatrix} \rho_{B,t}^{HH} & \rho_{B,t}^{HL} \\ \rho_{B,t}^{LH} & \rho_{B,t}^{LL} \end{bmatrix}$. For each firm-to-firm match, the two Markov switching processes are independent with each other. At the end of each period, firms are destroyed with fixed rate δ.

Notations of the labor market and the inter-firm matching market

The notations of the labor market are similar to those used in the simple model.

There are twelve types of firms, or employments.[29] The measure of firms in set i with type j choosing cooperation contract l is denoted by $n_{i,l}^{j}$.

As in the simple model, tightness ratio $\theta_{i,t}$ is the ratio of the number of vacancies posted by firms in set i to the number of workers looking for these jobs:

$$\theta_{i,t} = \frac{v_{i,t}}{u_{i,t}}$$

Job finding rate $\mu_i(\theta_{i,t})$ measures the probability of an unemployed worker matching with a vacancy. The vacancy filling rate $f_i(\theta_{i,t})$ is the probability for a vacancy matching with an unemployed worker.

Aggregate unemployment is determined by job creation and job destruction in the two

[29] As in DMP models, the production unit is firm-employment match, hence I use firm and employment interchangeably.

sets.

$$u_{t+1} = u_t - \underbrace{\left(\mu_{A,t}u_{A,t} + \mu_{B,t}u_{B,t}\right)}_{job\ creation} + \underbrace{\delta\left(1-u_t\right)}_{job\ destruction}$$

with

$$u_t = u_{A,t} + u_{B,t}$$

In the above equation, u_t is aggregate unemployment. Notice that unlike the simple model, in this model unemployed workers can freely move across labor markets. Thus $u_{A,t}$ and $u_{B,t}$ are endogenously determined by states of the economy.

Similar to the labor market tightness ratio, the tightness ratios of inter-firm matching market is the ratio of the number of single firms in two sets. In the Nash equilibrium, firms only match with firms of same type. Thus there are two endogenously segmented markets. In sum, the model has two sets and two submarkets which give rise to four tightness ratios.

$$\begin{aligned}
\tilde{\theta}^{jk}_{A,l,t} &= \frac{n^{j}_{A,l,t}}{n^{k}_{B,l,t}} \quad j,k \in \{H,L\},\ l \in \{VI,SC\} \\
\tilde{\theta}^{kj}_{B,l,t} &= \frac{n^{k}_{B,l,t}}{n^{j}_{A,l,t}}
\end{aligned}$$

where $n^{j}_{i,l,t}$ is the measure of single firms of type j in set i and plan to use inter-firm cooperation contract l.

Cooperation matching rate is the probability of a single firm matching with a partner.

$$\begin{aligned}
p^{jk}_{A,l,t} &= \frac{\tilde{M}\left(n^{j}_{A,l,t}, n^{k}_{B,l,t}\right)}{n^{j}_{A,l,t}} \quad j,k \in \{H,L\}\ l \in \{VI,SC\} \\
p^{kj}_{B,l,t} &= \frac{\tilde{M}\left(n^{k}_{A,l,t}, n^{j}_{B,l,t}\right)}{n^{k}_{B,l,t}}
\end{aligned}$$

where $\tilde{M}\left(n^{j}_{A,l,t}, n^{k}_{B,l,t}\right)$ is matching functions for inter-firm matching which is assumed to be homogeneous of degree one. As the result, set i's cooperation matching rate $p_{i,l,t}^{jk}$ is an increasing function of tightness ratio $\theta_{i,l,t}^{jk}$.

Furthermore, I impose both the worker-firm matching function and the inter-firm matching function $\tilde{M}$ to be Cobb-Douglas, that is,

$$\begin{aligned} M\left(u_{A,t}, v_{A,t}\right) &= \phi_A \left(u_{A,t}\right)^{1-\alpha_1} \left(v_{A,t}\right)^{\alpha_1} \\ M\left(u_{B,t}, v_{B,t}\right) &= \phi_B \left(u_{B,t}\right)^{1-\alpha_1} \left(v_{B,t}\right)^{\alpha_1} \end{aligned}$$

, and

$$\tilde{M}\left(n_{A,l,t}^{j}, n_{B,l,t}^{k}\right) = \psi\left(n_{A,l,t}^{j}\right)^{1-\alpha_2}\left(n_{B,l,t}^{k}\right)^{\alpha_2},\ j,K \in \{H,L\},\ l \in \{VI,SC\}$$

where I restrict the labor market matching elasticity to be same in the two labor markets. Moreover, the inter-firm matching functions are identical across submarkets.

Household

The representative household derives utility from consumption, while working incurs disutility ξ_n. A household's problem is:

$$\max E_t \left\{ \sum \beta^t \xi_{c,t} \left[log\left(C_t - \Psi_c \bar{C}_{t-1}\right) - \xi_{n,t} \sum_i \sum_j \sum_l n_{i,l,t}^{j} \right] \right\} \tag{2.22}$$

with

$$\begin{aligned} i &\in \{A,B\} \\ j &\in \{H,L,HH,HL,LH,LL\} \\ l &\in \{VI,SC\} \end{aligned}$$

where Ψ_c is an external habit parameter, C_t denotes consumption, $\bar{C}_{t-1}$ is the economy's average consumption in period $t-1$. As defined in the previous subsection, $n_{i,t}^{j}$

is the measure of employment in production department indexed by type j in set i, and has (or is willing to have) cooperation contract l. There are two exogenous preference shocks: shock to discount rate $\xi_{c,t}$ which changes household's evaluation of future utility and shock to disutility of labor $\xi_{n,t}$.

The household's problem is subject to:

(1) The transition rules of unemployment and employment

$$u_{t+1} = u_t - \left(\mu_{A,t}u_{A,t} + \mu_{B,t}u_{B,t}\right) + \delta\left(1 - u_t\right) \tag{2.23}$$

$$\underbrace{\begin{bmatrix} n^{H}_{A,VI,t+1} \\ \vdots \\ n^{LL}_{A,SC,t+1} \end{bmatrix}}_{12\times 1} = \Phi\left(\tilde{\theta}_{A,t}, \Pi_t, \Omega_t\right)\begin{bmatrix} n^{H}_{A,VI,t} \\ \vdots \\ n^{LL}_{A,SC,t} \end{bmatrix} + \Xi\left(\theta_{A,t}, \Omega_t\right)u_{A,t} \tag{2.24}$$

$$\begin{bmatrix} n^{H}_{B,VI,t+1} \\ \vdots \\ n^{LL}_{B,SC,t+1} \end{bmatrix} = \Phi\left(\tilde{\theta}_{B,t}, \Pi_t, \Omega_t\right)\begin{bmatrix} n^{H}_{B,VI,t} \\ \vdots \\ n^{LL}_{B,SC,t} \end{bmatrix} + \Xi\left(\theta_{B,t}, \Omega_t\right)u_{B,t} \tag{2.25}$$

Equation 2.23 is the flow motion of unemployment. As unemployed worker can freely move across labor markets, $u_{A,t}$ and $u_{B,t}$ are optimally chosen by the representative household given the the state of the economy.

Equations 4.3.13 and 2.25 are flow motions for each type of employment. The detail of equations 4.3.13 and 2.25 are explained in appendix 2.7. In the equation, Φ and Ξ are transition matrices whose elements are determined by vectors of cooperation tightness ratios $\tilde{\theta}_{A,t} = \left[\tilde{\theta}^{H}_{A,VI,t}, \tilde{\theta}^{L}_{A,VI,t}, \tilde{\theta}^{H}_{A,SC,t}, \tilde{\theta}^{L}_{A,SC,t}\right]$, $\tilde{\theta}_{B,t} = \left[\tilde{\theta}^{H}_{B,VI,t}, \tilde{\theta}^{L}_{B,VI,t}, \tilde{\theta}^{H}_{B,SC,t}, \tilde{\theta}^{L}_{B,SC,t}\right]$, Markov switching matrices $\Pi_t = \left[\Pi_{A,t}, \Pi_{B,t}\right]$, and labor market tightness ratios $\theta_{A,t}$ and $\theta_{B,t}$, and lastly a vector of exogenous states Ω_t

(2) The budget constraint

$$C_t + T_t + I_t = \int \sum_i \sum_j \sum_l w^{j}_{i,l,t}(\eta) \cdot n^{j}_{i,l,t}(\eta)\, d\eta + z \cdot \sum_i u_{i,t} + r_{k,t}K_t + D_t \tag{2.26}$$

The representative household's income is composed of wage income $\int \sum_i \sum_j \sum_l w^j_{i,l,t}(\eta) \cdot n^j_{i,l,t}(\eta)\, d\eta$, the unemployment insurance $z \cdot u_t$, capital rent $r_{k,t}K_t$, and dividend D_t. Household allocates income to consumption C_t, investment I_t, and receives lump-sum tax T_t.

(3) The representative household invests to accumulate capital

$$K_{t+1} = (1-d)K_t + \left[1 - S\left(\xi_{I,t}\left[\frac{I_t}{I_{t-1}}\right]\right)\right] I_t \tag{2.27}$$

I_{t-1} is investment in the previous period, and function S models investment adjustment cost which equals zero on steady state. Investment is subject to shock to marginal efficiency of investment (MEI) $\xi_{I,t}$ which affects the efficiency of the transformation of investment to productive capital.

Representative firm

The representative firm has twelve production departments indexed by type j and cooperation contract l. The firm's endogenous state variables consists of measures of current employment of each production department. Representative firms make four decisions: (1) rent capital from household then optimally allocate to each department, (2) post vacancy to match with worker,[30] (3) send single firms to search for partners in the inter-firm matching market. For the third decision, I impose that in the Nash equilibrium, single firms only search for same type partners, either SC or VI, depending on the transaction cost.

[30] Noticing that firm-worker match type is revealed only after the match is formed, firms post one type of vacancy only.

In set i, the representative firm's value function takes the following form:

$$\begin{aligned} J_i\left(\left\{n^j_{A,l}(\eta)\right\},\left\{n^j_{B,l}(\eta)\right\},\Omega\right) = & \max_{v_i,\{k^j_i\}} x\sum_j z^j_i\left\{\sum_l\int\left[\left(k^j_{i,l}(\eta)\right)^{\alpha}\left(n^j_{i,l}(\eta)\right)^{1-\alpha}\right]\right\} \\ & \underbrace{-\sum_j\left\{\int_{\Xi}\left[w^j_{i,SC}(\eta)\cdot n^j_{i,SC}(\eta)+r_k k^j_{i,SC}(\eta)+\eta\right]d\eta\right\}}_{Sourcing} \\ & \underbrace{-\sum_j\left\{\int_{\Xi^C}\left[w^j_{i,VI}(\eta)\cdot n^j_{i,VI}(\eta)+r_k k^j_{i,VI}(\eta)\right]d\eta\right\}}_{Vertical\ integration} \\ & -v_i\cdot\chi+\beta E\left[\frac{\lambda'}{\lambda}J_i\left(\left\{n^j_{A,l}(\eta)\right\},\left\{n^j_{B,l}(\eta)\right\},\Omega'\right)\right] \end{aligned} \tag{2.28}$$

with

$$\begin{aligned} i &\in \{A,B\} \\ j &\in \{H,L,HH,HL,LH,LL\} \\ l &\in \{VI,SC\} \end{aligned}$$

subject to the transition rules of employment described by equations 4.3.13 and 2.25 and also the constraint from the matching technology in the labor market

$$v_i = u_i\cdot\theta_i \tag{2.29}$$

Firm's production function is Cobb-Douglas. TFP is determined by two components: (1) the firm's idiosyncratic productivity z^j_i which depends on type j ; (2) aggregate TFP x.[31] Firms need to pay the cost on labor, capital, job vacancy. In addition, for firms who use SC as the inter-firm cooperation contract, they need to pay transaction cost η. I use Ξ to denote the set of firms who use SC. Naturally, Ξ^C contains the firms using VI, who do not

[31] Aggregate TFP contains a trend, hence vacancy posting cost χ and unemployment insurance z also grows with the trend of aggregate TFP in order to keep the tightness ratio and unemployment stationary.

pay the transaction cost. Wage $w^j_{i,l}(\eta)$ is determined by Nash bargaining, which will be described in the next subsection. Labor market tightness ratio θ_i and capital rental rate r_k are equilibrium outcomes and are taken by firms as given. It is worthnoting that in search and matching models, employment is pre-determined, which is similar to capital stock in most models; and vacancies are similar to investment. Firms adjust future employment by posting vacancies in the labor market, while they cannot adjust contemporary employment.

Nash Bargaining and wage determination

Following the DMP search and matching model, wage is determined by Nash bargaining. In each period firms and workers set wages to solve the following Nash bargaining problem:

$$w^j_{i,l}(\eta) = arg\max_w \left(\frac{\frac{\partial V}{\partial n^H_{i,l}(\eta)} - \frac{\partial V}{\partial u_i}}{\lambda} \right)^{1-\tau} \left(\frac{\partial J_i}{\partial n^H_{i,l}(\eta)} \right)^{\tau}$$

In the above equation, time subscripts are omitted. τ is a parameter of bargaining share that measures the firm's bargaining power. $\frac{\partial V}{\partial n^H_{i,l}(\eta)}$ and $\frac{\partial V}{\partial u_i}$ are the representative household's marginal value of employment and unemployment respectively. $\frac{\partial J_i}{\partial n^H_{i,l}(\eta)}$ is firm's marginal value of employment. The detail of these variables can be found in appendix 2.8. As the result of the Nash bargaining, the total surplus of each match is divided by the firm and worker with shares τ and $1-\tau$.

It can be shown that the wage has the following close form solution

$$w^j_{i,SC}(\eta) = \tau\left(z + \frac{\xi_n}{\lambda}\right) + (1-\tau)\left[xz^j_i\left(\frac{k^j_{i,SC}(\eta)}{n^j_{i,SC}(\eta)}\right)^{\alpha} - \eta + \chi\theta_i\right] \tag{2.30}$$

and

$$w^j_{i,VI} = \tau\left(z + \frac{\xi_n}{\lambda}\right) + (1-\tau)\left[xz^j_i\left(\frac{k^j_{i,VI}}{n^j_{i,VI}}\right)^{\alpha} + \chi\theta_i\right] \tag{2.31}$$

Wage is the weighted average of three components: (1) the worker's marginal product of labor (MPL) $xz_i^j\left(\frac{k_{i,l}^j(\eta)}{n_{i,l}^j(\eta)}\right)^\alpha$; (2) the flow value of unemployment, or the opportunity cost of employment, measured by unemployment insurance z plus marginal rate of substitution (MRS) of consumption for leisure, which is disutility of labor ξ_n adjusted by marginal utility; and (3) $\chi\theta_i$, the pressure from competing firms who are actively hiring workers. In the case of SC, there is a transaction cost component in the wage. It deviates from the models with a competitive labor market in which wage always equals MPL and MRS. Following the canonical DMP search and matching models, I define the total surplus of a firm-worker match as

$$TS_{i,l}^j(\eta) = \lambda\frac{\partial J_i}{\partial n_{i,l}^j(\eta)} + \frac{\partial V}{\partial n_{i,l}^j(\eta)} - \frac{\partial V}{\partial u_i}$$

And with Nash Bargaining we have

$$\begin{aligned}
\lambda\frac{\partial J_i}{\partial n_{i,l}^j(\eta)} &= \tau TS_{i,l}^j(\eta) \\
\frac{\partial V}{\partial n_{i,l}^j(\eta)} - \frac{\partial V}{\partial u_i} &= (1-\tau)TS_{i,l}^j(\eta)
\end{aligned}$$

The choice of contract and the free entry condition

In this subsection, I show the determination of cooperation contract and the free entry condition.

A firm's first order condition with regard to the measure of vacancies v_i gives the free entry condition of the model

$$\frac{\chi}{f(\theta_i)} = E\left\{\frac{\lambda'}{\lambda}\int \max\left[\frac{\frac{\partial J_i}{\partial n_{i,SC}^H(\eta)} + \frac{\partial J_i}{\partial n_{i,SC}^L(\eta)}}{2}, \frac{\frac{\partial J_i}{\partial n_{i,VI}^H} + \frac{\partial J_i}{\partial n_{i,VI}^L}}{2}\right] dF(\eta)\right\} \tag{2.32}$$

where $J_{i,n_{i,l}^{j}}$ is a firm's marginal value of employment with type j, cooperation contract l, in set i.

A single firm with transaction cost η would choose VI if and only if

$$\frac{\partial J_i}{\partial n_{i,SC}^{H}(\eta)} + \frac{\partial J_i}{\partial n_{i,SC}^{L}(\eta)} < \frac{\partial J_i}{\partial n_{i,VI}^{H}} + \frac{\partial J_i}{\partial n_{i,VI}^{L}}$$

or equivalently

$$TS_{i,SC}^{H}(\eta) + TS_{i,SC}^{L}(\eta) < TS_{i,VI}^{H} + TS_{i,VI}^{L}$$

We can categorize firms' choice of contract with the following proposition

Proposition 7. *In industry i and in period t, a single firm would choose vertical integration if $\eta \geqslant \eta_{i,t}^{*}$, sourcing if $\eta < \eta_{i,t}^{*}$. The threshold is determined by*

$$\eta_t^{*} = \beta(1-\delta)\left(\rho_{A,t}+\rho_{B,t}\right)E_t\left[\tilde{TS}_{i,VI,t+1} - (1-\nu)\tilde{TS}_{i,SC,t+1}(\eta_t^{*})\right]$$

with

$$\begin{aligned}
\tilde{TS}_{i,SC} &\triangleq TS_{i,SC}^{HH}(\eta) + TS_{i,SC}^{LL}(\eta) - TS_{i,SC}^{LH}(\eta) - TS_{i,SC}^{HL}(\eta) \\
\tilde{TS}_{i,VI} &\triangleq TS_{i,VI}^{HH} + TS_{i,VI}^{LL} - TS_{i,VI}^{LH} - TS_{i,VI}^{HL}
\end{aligned}$$

In the above proposition, as I proved in the appendix, both $\tilde{TS}_{i,SC}$ and $\tilde{TS}_{i,VI}$ are independent with η. With proposition 7, we can rewrite free entry condition equation 2.32 as

$$\chi = \tau\beta f(\theta_{i,t})E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[\begin{array}{c}\int_{-\infty}^{\eta_t^{*}}\frac{TS_{i,SC,t+1}^{H}(\eta)+TS_{i,SC,t+1}^{L}(\eta)}{2}dF(\eta)\\ +\\ (1-F(\eta_t^{*}))\frac{TS_{i,VI,t+1}^{H}+TS_{i,VI,t+1}^{L}}{2}\end{array}\right]\right\} \tag{2.33}$$

In the appendix 2.9, I prove the following result, which substantially simplifies the computation of the model.

Proposition 8. *The free entry condition of set i is*

$$\chi = \tau\beta f\left(\theta_{i,t}\right)E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[\frac{TS^H_{i,VI,t+1}+TS^L_{i,VI,t+1}}{2}+F\left(\eta^*_t\right)\Delta\hat{T}S_{i,t+1}\right]\right\}$$

with

$$\begin{aligned}\Delta\hat{T}S_{i,t} &= -\hat{\eta}_t + \\ &\quad \beta\left(1-\delta\right)E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[\Delta\hat{T}S_{i,t+1}-\left(\rho_{A,t}+\rho_{B,t}\right)\left(1-\nu\right)\tilde{T}S_{i,SC,t+1}+\left(\rho_{A,t}+\rho_{B,t}\right)\tilde{T}S_{i,VI,t+1}\right]\right\} \\ \hat{\eta}_t &\triangleq \frac{\int_{-\infty}^{\eta^*_t}\eta dF\left(\eta\right)}{F\left(\eta^*_t\right)}\end{aligned}$$

Economy resource constraint and shocks

I close the model by presenting the economic resource constraint and the processes of shocks.

The economy can allocate output—the aggregate of all departments' output net of transaction cost—to consumption, investment and vacancies. Thus the economy resource constraint is:

$$C+I+\chi\sum_i v_i = x\sum_i\sum_j\sum_l z^j_i\left(k^j_{i,l}\left(\eta\right)\right)^{\alpha}\left(n^j_{i,l}\left(\eta\right)\right)^{1-\alpha}-\sum_i\sum_j\left(\int_{\Xi}\eta d\eta\right)$$

I include exogenous shocks to six variables in the model. Besides shocks to the two rotation rates ρ^{HH}_A and ρ^{HH}_B, I consider shock to aggregate TFP, shock to investment adjustment cost ξ_I, shock to inter-temporal preference ξ_C and shock to labor disutility ξ_L.

$$log\left(x_t\right) = \mu^* + \rho_x log\left(x_{t-1}\right)+\sigma_x\varepsilon_{x,t}, \quad \varepsilon_{x,t}\sim N\left(0,1\right)$$

The shocks are governed by $AR(1)$ processes that are independent to each other:[32]

$$\begin{aligned} log\left(\xi_{w,t}\right) &= \rho log\left(\xi_{w,t-1}\right)+\sigma_w\varepsilon_{w,t}, \quad \varepsilon_{w,t}\sim N\left(0,1\right) \\ w &= x,I,C,L \end{aligned}$$

The values of all shock variables are observable to economic agents at beginning of each period. To save space, I stack them into a vector of states

$$\Omega=[x,\xi_I,\Pi_A,\Pi_B,\xi_C,\xi_L]$$

Π_A and Π_B are symmetric Markov switching matrices pinned down by rotation rates ρ_A^{HL} and ρ_B^{HL}

2.5.2 The sufficient condition for PAM

In the models studied in the previous sections, without inter-firm search friction, PAM is always the Nash equilibrium as long as the production function is monotone in partner's type. In this model with inter-firm search friction, however, monotonicity is not sufficient for PAM: there exists a possibility that a large amount of of L type single firms are willing to chase for just a few H type single firms, while the few H type firm might be happy to match with those L type if the matching probability is high enough. In proposition 9, I show the sufficient condition for PAM to be the Nash equilibrium.

Proposition 9. *The positive assortative matching is Nash equilibrium if*

$$\left(\frac{TS_{A,l}^{HH}-TS_{A,l}^{H}}{TS_{A,l}^{HL}-TS_{A,l}^{H}}\right)^{\frac{1}{\alpha_2}}\times\left(\frac{TS_{B,l}^{LH}-TS_{B,l}^{L}}{TS_{B,l}^{LL}-TS_{B,l}^{L}}\right)^{\frac{1}{1-\alpha_2}}>1 \tag{2.34}$$

[32] In another version, I specified the TFP process as a random walk process with drift as in Fernández-Villaverde and Rubio-Ramírez (2007) and gets similar result in the variance decomposition.

and

$$\left(\frac{TS^{HH}_{B,l}-TS^{H}_{B,l}}{TS^{HL}_{B,l}-TS^{H}_{B,l}}\right)^{\frac{1}{1-\alpha_2}}\times\left(\frac{TS^{LH}_{A,l}-TS^{L}_{A,l}}{TS^{LL}_{A,l}-TS^{L}_{A,l}}\right)^{\frac{1}{\alpha_2}}>1 \tag{2.35}$$

The proof of proposition 9 can be found in appendix 2.9. The intuition of the proof is to find the condition under which the H type single firms have no incentive to search for L type partner. Specifically, I first impose positive assortative matching so that inter-firm matching market is endogenously segmented to two sub-markets. Then I identify the sufficient condition under which no firm would like to deviate from the Nash equilibrium with positive assortative matching.

When solving and estimating the model, I take the same strategy as in the proof of proposition 9: I first impose positive assortative matching to be the Nash equilibrium, then verify that the inequality conditions depicted in proposition 9 hold.

2.5.3 Estimation of the quantitative model

In this subsection, I estimate the model described above and quantitatively assess the effect of rotation shock. I first describe the data and estimation strategy, then report the estimation result. With the estimation result, I conduct impulse response analysis to study the effect of rotation shock on the labor market variables and other key macro variables. I also study the role of firm inter-connectivity and supermodularity with two counterfactual experiments, one simulate a model with no firm inter-connectivity (canonical DMP model), the other one has firm inter-connectivity but with no supermodularity. Then I compare the shocks to rotation rate and TFP shocks and comment on how my model speaks to the Shimer's puzzle (Shimer (2005)). Lastly, I use variance decomposition to evaluate the importance of rotation shock relative to other shocks.

Data and methodology

Data

The sample period is 1969Q1 to 2013Q4. The estimation uses seven variables: rotation rates of sets A and B, real interest rate, aggregate unemployment rate, aggregate job opening rate, growth rates of real investment and real consumption per capita, growth rate of per hour real wage. Data sources are described in Appendix 2.10. Keeping the same notation as in the description of the model above, and writing Δ to indicate first differences, the full vector of observables is

$$\left[\rho_{A,t}^{HL}, \rho_{B,t}^{HL}, R_t, u_t, v_t, \Delta log\left(I_t\right), \Delta log\left(C_t\right), \Delta log\left(w_t\right)\right]$$

The model counterparts of the observables follow directly from the description of the model.

Methodology

I estimate the model using Bayesian method. I first solve and re-scale the model then log-linearize the non-linear model around a deterministic steady state and write the linearized equilibrium conditions in a state-space form. The resulting linear rational expectations model can then be solved by methods such as in Sims (2002). Define a vector of model variables X_t, and a data vector of observable variables Z_t. The state-space representation of the model can then be written as

$$\begin{aligned} X_t &= \Gamma X_{t-1} + \Psi \varepsilon_t \\ Z_t &= \Phi X_t + \eta_t \end{aligned}$$

where Γ and Ψ are coefficient matrices, the elements of which are typically non-linear functions of the structural parameters, and Φ is a selection matrix that maps the model variables to the observables. The innovations of the shocks are collected by ε_t. η_t is vector of observation errors.

The use of eight series of observables requires the inclusion of at least eight independent sources of variation. I consider six independent shocks: aggregate TFP shock, two set-specific rotation shocks, marginal efficiency of investment (MEI) shock, discount rate shock, and labor disutility shock. As the number of observable is larger than the number of shocks, the model cannot be identified. So I also include observation errors for three observable, $\rho^{HL}_{A,t}, \rho^{HL}_{B,t}$ and v_t, as additional disturbances.

In the implementation of the Bayesian estimation procedure, I use the Kalman-filter to evaluate the likelihood function of the observable. I then combine the likelihood function with the prior distribution of the model's parameters to obtain the posterior distribution. I then evaluate the posterior distribution using the random-walk Metropolis-Hasting algorithm. Further details on the computational procedure can be found in An and Schorfheide (2007).

Calibration and prior

Calibration

Firstly, I calibrate some parameters based on the typical values used in calibration studies. Inter-firm matching is assumed to be symmetric; that is, given the two sectors have the same measure of single firms, the inter-firm matching probability is same for the two sectors. Therefore inter-firm matching elasticity α_0 is set to 0.5. I normalize the maximum TFP z^{HH}_i as 1. As the empirical rotation rate is low on steady state (5% annually), the majority of firms fall into the HH and LL category. Hence I set z^{LL}_i to 0.44, which targets to inter-quartile ratio of productivity for manufacturing firms in U.S., as documented in Syverson (2011).[33] Vacancy posting cost χ is normalized as 1. Following Shimer (2012), I set the exogenous separation rate δ_1 as 0.05. Discount rate β is set to 0.99, as in standard models. Capital share α and depreciation rate of capital δ are set to 0.34 and 0.025. The calibrations are summarized in Table 4.3.

[33] It also implies that the inter-quartile ratio of the wage distribution is 0.62 at the posterior mode.

Table 2.5: Calibration

Description	Parameter	Calibration	Target
Firm-firm match elasticity	α_2	0.5	Inter-sector Symm.
Max TFP	z^{HH}	1	Normalization
Ratio of productivities	z^{LL}/z^{HH}	0.44	Syverson (2004)
Vacancy cost	χ	1	Normalization
Unemployment insurance	z	0.25	Department of Labor
Exog separation rate	δ_1	0.05	Shimer (2012)
Discount rate	β	0.99	2% real interest rate
Capital share	α	0.34	Capital income share in the US
Capital depreciation rate	δ	0.025	10% annual depreciation rate

Priors for the common parameters of DSGE models

The other parameters are estimated using Bayesian method. For parameters that are present in common DSGE studies, I keep priors close to previous work (e.g. Justiniano et al. (2011)). In particular, for investment adjustment cost, I choose Gamma distributions with a mean of 4 and a standard deviation of 2. Prior for habit persistence is a distribution with a mean 0.5 and standard deviation 0.1. Prior for trend productivity growth rate is a Normal distribution with mean 0.004 and standard deviation 0.001.

I follow the literature in choosing rather diffuse priors for the structural shock processes. The priors for the autocorrelation parameters are Beta distributions with mean 0.5 and standard deviation 0.1. The priors for the standard deviations of shocks are Inverse Gamma distributions: priors for standard deviations of shocks have mean 0.01 and standard deviation 0.002.

Following Ilut and Schneider (2014), the priors for the standard deviation of observables are set in the following way: for an observable W with unconditional standard deviation σ_W, the prior for the standard deviation of measurement error on W is an Inverse

Gamma distribution with mean $0.1\sigma_W$ and standard deviation $0.4\sigma_W$. Therefore, at the prior mean, measurement error would explain 1% fluctuation in W; at one standard deviation, it would explain 16% fluctuations in W.

Table 2.6: Estimation result I

Parameter	Description	Prior			Posterior	
		Type	Mean	Std	Mode	90% interval
γ_A^H	z_A^H/z_A^{HH}, z_A^L/z_A^{LL}	beta	0.5	0.1	0.46	[0.37,0.53]
γ_B^H	z_B^H/z_B^{HH}, z_B^L/z_B^{LL}	beta	0.5	0.1	0.50	[0.44,0.61]
γ_A^{HL}, γ_A^{LH}	z_A^{HL}/z_A^{HH}, z_A^{LH}/z_A^{HH}	beta	0.6	0.15	0.42	[0.47,0.78]
γ_B^{HL}, γ_B^{LH}	z_B^{HL}/z_B^{HH}, z_B^{LH}/z_B^{HH}	beta	0.6	0.15	0.43	[0.13,0.59]
ν	Separation probability for SC	beta	0.25	0.05	0.13	[0.12,0.15]
$\bar{\eta}$	Mean of transaction cost	normal	0	0.2	-0.13	[-0.14,-0.13]
$\bar{\sigma}$	Std. of transaction cost	inv gamma	2.0	0.2	1.01	[1.00,1.01]
ϕ_A	A's matching efficiency	beta	0.7	0.1	0.73	[0.67,0.78]
ϕ_B	B's matching efficiency	beta	0.7	0.1	0.75	[0.71,0.81]
ψ	Inter-firm ME	beta	0.7	0.25	0.76	[0.65,0.96]
τ	Bargaining share of firm	beta	0.5	0.1	0.76	[0.72,0.81]
$\frac{\xi_L}{\lambda}$	Mean of labor disutility	beta	0.2	0.1	0.11	[0.08,0.12]
ψ_c	Habit persistence	beta	0.5	0.1	0.04	[0.04,0.05]
S_2	Investment adj. cost	normal	4	2	4.17	[4.10,4.18]
$100 \times \mu^*$	Trend TFP growth	normal	0.4	0.1	0.14	[0.14,0.15]

Priors for the productivities

Since I have calibrated z_i^{HH} and z_i^{LL}, it remains to estimate z_i^{HL}, z_i^{LH}, z_i^H and z_i^L. I set their priors as Beta distributions. For single firms, I set the prior mean of z_i^H and z_i^L to 0.5 and

Table 2.7: Estimation result II

Para	Description	Prior			Posterior	
		Type	Mean	Std	Mode	90% interval
$100 \times \bar{rot}_A$	Mean of *Rot* in *A*	beta	1	0.1	1.24	[1.12,1.42]
$100 \times \bar{rot}_B$	Mean of *Rot* in *B*	beta	1	0.1	1.32	[1.29,1.52]
$100 \times \sigma_{rot_A}$	Std of shock to *Rot* in *A*	inv gamma	1	0.2	0.40	[0.37,0.43]
$100 \times \sigma_{rot_B}$	Std of shock to *Rot* in *B*	inv gamma	1	0.2	0.38	[0.35,0.40]
$100 \times \sigma_z$	Std of aggregate TFP	inv gamma	1	0.2	0.71	[0.68,0.75]
$100 \times \sigma_{\xi_c}$	Std of discount rate shock	inv gamma	1	0.2	1.97	[1.81,2.14]
$100 \times \sigma_{\xi_I}$	Std of investment shock	inv gamma	1	0.2	0.88	[0.78,0.98]
$100 \times \sigma_{\xi_l}$	Std of disutility shock	inv gamma	1	0.2	39.12	[31.28,46.53]
ρ_{Rot_A}	Pers of shock to *Rot* in *A*	beta	0.5	0.1	0.92	[0.90,0.94]
ρ_{Rot_B}	Pers of shock to *Rot* in *B*	beta	0.5	0.1	0.91	[0.89,0.92]
ρ_z	Pers of aggregate TFP	beta	0.5	0.1	0.95	[0.94,0.97]
ρ_{ξ_c}	Pers of discount rate shock	beta	0.5	0.1	0.75	[0.72,0.76]
ρ_{ξ_I}	Pers of investment shock	beta	0.5	0.1	0.37	[0.41,0.45]
ρ_{ξ_l}	Pers of disutility shock	beta	0.5	0.1	0.94	[0.94,0.95]

0.2, 50 percent of z_i^{HH} and z_i^{LL}. That is to say, at the prior mean, finding a cooperation can double productivity. Priors standard deviation of productivities are set to 0.1.

The prior mean of z_A^{HL} and z_B^{HL} is set to be 0.6, which means that cooperating with an L type partner can improve H type's productivity by 20%. I impose that in any inter-firm match partners split output evenly, hence $z_A^{HL} = z_B^{LH}$ and $z_A^{LH} = z_B^{HL}$.[34]

[34] As revealed in proposition **??**, it is $\left(z_i^{HH}\right)^{\frac{1}{1-\alpha}} + \left(z_i^{LL}\right)^{\frac{1}{1-\alpha}} - \left(z_i^{LH}\right)^{\frac{1}{1-\alpha}} - \left(z_i^{HL}\right)^{\frac{1}{1-\alpha}}$ that can be identified. Given that z_i^{HH} and z_i^{LL} are calibrated, I can identify $\left(z_i^{LH}\right)^{\frac{1}{1-\alpha}} + \left(z_i^{HL}\right)^{\frac{1}{1-\alpha}}$. However, I cannot identify z_i^{HL} and z_i^{LH} separately, hence I impose them to be same.

Priors for the parameters of the labor market and the cooperation contract

For most labor market parameters, I choose fairly dispersed priors and set the prior mean based on calibration studies. I set prior for labor disutility ξ_n to Beta distribution with mean 0.2 and standard deviation 0.1, combining with unemployment insurance[35] implies that unemployment benefit is 45% of average wage when the model is computed at the prior mean.

I set the prior for labor market matching efficiency to Beta distribution with a mean of 0.7 and a standard deviation of 0.1, so that steady-state unemployment rate is 5% at the prior mean. I don't have information to target the range of inter-firm matching efficiency, so I set its prior to have the same mean as labor market matching efficiency but with higher dispersion. For the bargaining share of firm τ, calibration studies use a wide range of values centering around 0.5. Therefore, I set a beta prior around 0.5. For the labor market matching elasticity α_1, instead of estimating it, I impose $\alpha_1 = \tau$ to satisfy the Hosios condition (Hosios (1990)), which ensures that the labor market matching is efficient.

For the parameters related wo the inter-firm cooperation contract, I set the prior for the separation probability of sourcing contract to a Beta distribution with mean 0.25, which implies that the average duration of sourcing contract is one year. The prior for the mean of the transaction cost distribution is Normal distributed with mean of 0 and a standard deviation of 0.2.

Parameter estimates

Tables 4.4 and 2.7 compare the posterior distributions to the priors. The posterior estimates of the parameters that are unrelated to labor market and productivities are in line with previous estimations of DSGE models. Therefore I do not comment on them.

[35] Labor disutility needs to be adjusted by marginal utility to be comparable with monetary variables such as wage and unemployment insurance.

Parameters of the labor market and the cooperation contract

Labor market matching efficiencies ϕ_A and ϕ_B have posterior mode of 0.73 and 0.75, which are close to the prior mean and imply a job finding rate of 0.56. Remember that the prior mean is set to match the steady state unemployment rate; the estimates of labor market matching efficiency well capture the empirical unemployment rate. Inter-firm matching efficiency ψ has a posterior mode of 0.9. This implies an inter-firm matching rate of 0.9; that is, on average, a firm spends 1.1 quarters to find a partner.

At its posterior mode, a firm's bargaining share is 0.76. This is similar with the result of Lubik (2009), who estimated firm's bargaining share to be 0.97. As I impose the Hosio condition to hold, the labor market matching elasticity is also 0.84. Labor disutility has a posterior mode of 0.11. Combining unemployment insurance with the adjusted labor disutility, the implied flow value of unemployment is about 37% of the average wage on steady state.

Both the bargaining weight and unemployment benefit of my estimation are in line with the conventional calibration of DMP models, such as in Shimer (2005). It is well known that in canonical DMP model which is driven only by exogenous TFP shocks, an extremely high flow value of unemployment (higher than 0.9 of mean wage) and low firm bargaining share (lower than 0.1) are often needed to generate realistic volatility of unemployment. In contrast, my model fits the data without help from high flow value of unemployment and low firm bargaining share. The main reason is that, in my model, the fluctuation of unemployment is mostly driven by rotation shocks and labor disutility shocks. As will be shown in the subsequent subsections, without the help of high flow value of unemployment and low firm bargaining share,rotation shocks and labor disutility shocks are still able to generate volatile unemployment while keeping productivity variation mild.

The separation probability of sourcing contract has a posterior mode of 0.12, which is much smaller than the prior mean and indicates that the data suggests a large friction preventing mismatched firms from reallocating to more efficient partnerships. In the posterior mode, the transaction cost distribution has a mean of -0.13 and standard deviation of 1.01, which implies the ratio of industry value added to industry sales to be 0.45, which is close to 0.53 as measured in the BEA 2007 input-output table.

Productivities

Productivities of single firms in set A z_A^H and z_A^L are 0.46 and 0.50. This means that firms can double their productivity when matching with a same type partner. For mismatched cooperative firms in set A, their productivities z_A^{HL} and z_A^{LH} are 0.43. These results imply that an H type firm would find it more productive to work alone than to cooperate with an L type partner; an L type firm would barely gain any productivity by working with an H type partner, in contrast with working with an L type partner. Productivities of firms in set B have very similar estimates.

With the above results, I can verify if the conditions in propositions 9 are satisfied at the posterior mode, so that in the Nash equilibrium, single firms match with same type partners. Specifically, I confirm that H type single firms prefers to match with H type partners in both sets, which guarantees that no firm would deviate from the Nash equilibrium in which firms only search for same type partners.

Moreover, I verify that the value function, or equivalently MPL, is supermodular in the steady state at the posterior mode. This ensures that an increase in rotation rate would lead to a rise in unemployment, which I will confirm using impulse response analysis.

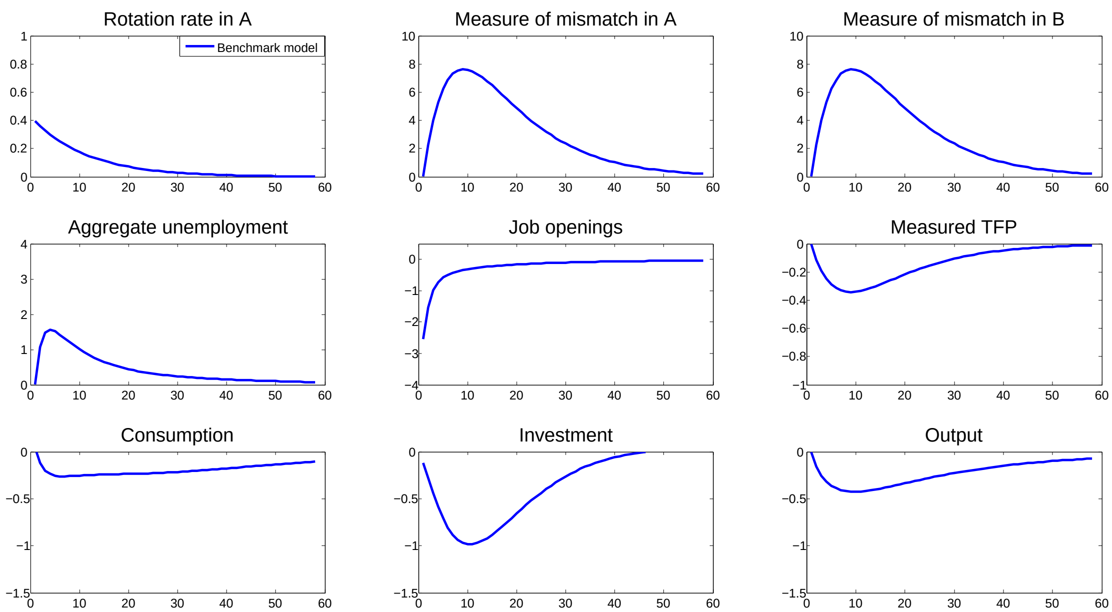
Impulse response

In order to understand the effect of rotation shock on the economy, I conduct impulse response analysis of rotation shock.

Benchmark impulse response

Consider the reaction of the economy to a positive rotation shock to set A; that is, a persistent increase in the probability of switching type for firms in set A. The lines in Figure 4.3

are the impulse responses, in percentage deviations from the steady state, to a one standard deviation increase in rotation rate.



The panels plot the percent deviation from the steady state of each induced by a one standard deviation shock to the rotation rate in set A.

FIGURE 2.2: Impulse response to 1 std. of rotation shock in set A

The shock increases rotation rates ρ_A^{HL}. As the Markov switching matrix is symmetric, it implies an increase in ρ_A^{LH}, and decline in ρ_A^{HH} and ρ_A^{LL}. A higher rotation rate generates a labor market slack in which unemployment increases and vacancies drop. This is because higher rotation rate increases the probability of mismatch between different types of firms. Notice that value function and MPL are supermodular at posterior mode, hence the risk of mismatch negatively affects firms' expected values which reduces their incentive to create jobs.

Positive rotation shock also leads to a gradual decline in measured TFP. Decline in average productivity comes from a composition effect. As demonstrated in the impulse response of the share of mismatched firms, higher churning increases the share of mismatched firms. Since they have lower production efficiencies on average, their rising share leads to a decline in average productivity.

The increase in churning also leads to a decline in macro aggregates such as consumption, investment and output. Three channels generate the aggregate recession. First, increase in churning hurts aggregate production efficiency by misallocating firms to inefficient partnerships, which causes a decline in aggregate output. Second, similar to the response of measured TFP, rotation shock leads to a decline in MPK, which makes households hold back investment. This then translates into a gradual decline in capital stock. Lastly, a weakening labor market reduces employment, which contributes to decreasing output.

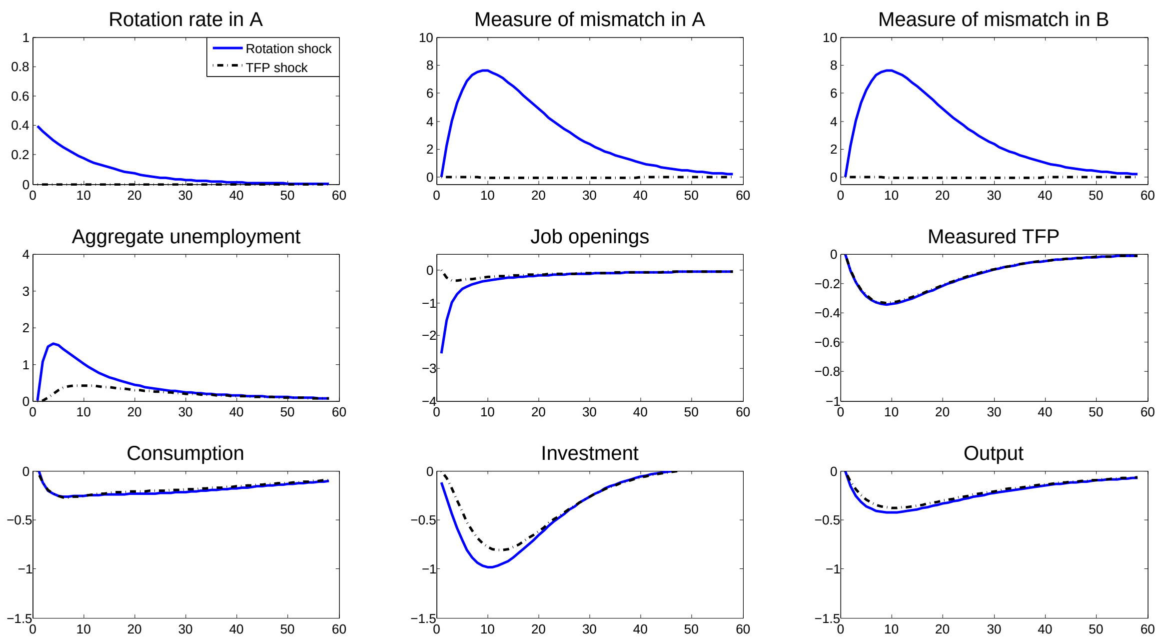
Comparing rotation shock to TFP shock

In the model, rotation shock gives rise to endogenous change in TFP. What drives the cyclical variation in the measured aggregate TFP is still one of the major open questions in macroeconomics (Rebelo (2005)). My paper provides a novel and empirically plausible mechanism: a higher rotation rate of one set of firms increases the share of mismatched firms in both sets of firms, which induces an endogenous U-shape decline in the measured aggregate TFP. The natural question is: is there any quantitative difference between rotation shock and exogenous aggregate TFP shock in terms of their impact on the labor market variables and other macro aggregates? If so, can rotation shock speak to the Shimer puzzle (Shimer (2005))? As pointed out by Shimer (2005), canonical DMP search and matching model, which is propagated by exogenous TFP shocks, has two shortcomings: (1) it under-predicts the volatility of the labor market variables, such as unemployment and job openings, relative to TFP,[36] (2) it over-predicts the correlation between the labor market variables and TFP. Exogenous TFP shock induces a perfect correlation between labor market variables and measured TFP; while they are only mildly correlated in the

[36] There is a vast literature on this, see Hall (2005); Hornstein et al. (2005); Mortensen and Nagypal (2007); Hall and Milgrom (2008); Costain and Reiter (2008); Hagedorn and Manovskii (2008); Fujita and Ramey (2009); Gertler and Trigari (2009); Pissarides (2009) for example.

data.[37]

To answer the above questions, I simulate a model with exogenous aggregate TFP shock only, then compare with the impulse response to rotation shock in Figure 4.4. Blue lines are impulse response to positive rotation shock, while the black dashed lines correspond to the TFP shock. To make the two cases comparable, I engineer the path of TFP shock so that the measured TFP is identical in the two cases.



The panels plot the percent deviation from the steady state induced by either one standard deviation shock to the rotation rate or a series of shocks to aggregate TFP whose path is set to make the measured TFP is identical as the rotation shock case.

FIGURE 2.3: Compare rotation shock and TFP shock

The responses of labor market variables are very different in the two cases. First, shock to rotation rate generates a much stronger effect in the labor market than shock to aggregate TFP. Moreover, shock to aggregate TFP generates perfect comovement between the labor market variables and the measured TFP, which is inconsistent with data (Gervais et al. (2015)). In contrast, rotation rate generates a sudden drop in job openings and tight-

[37] See Gervais et al. (2015) for an in-depth discussion

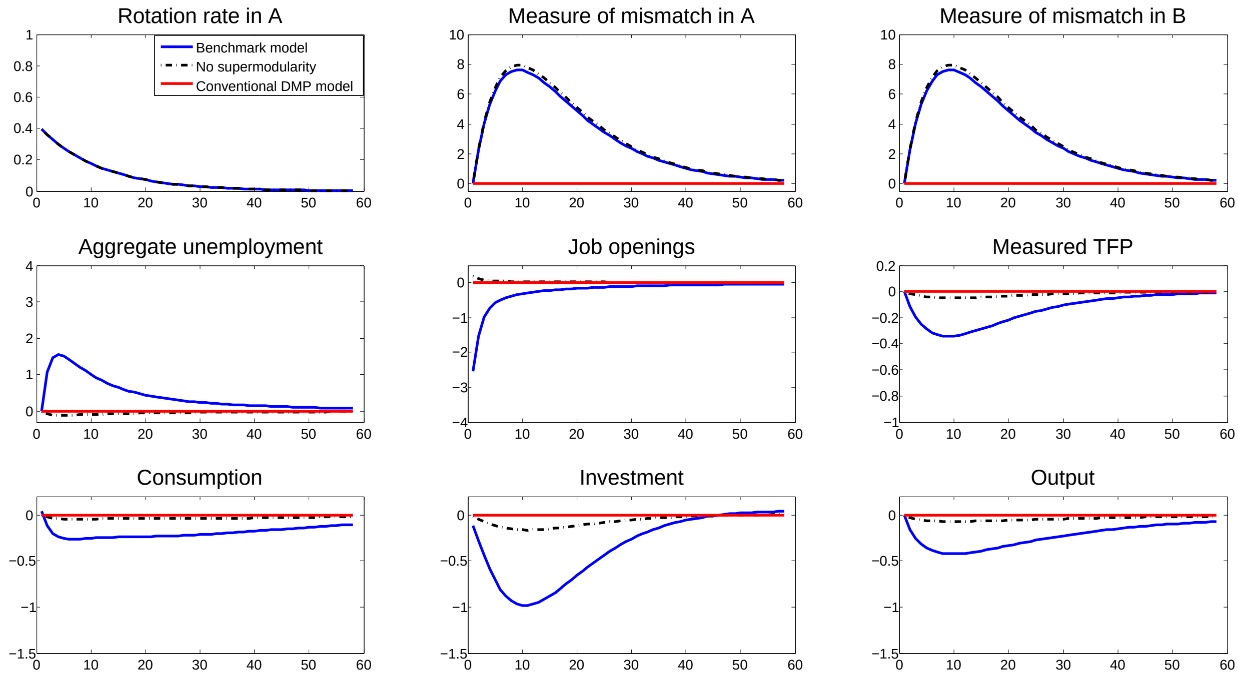
ness ratio and a gradual U-shape decline in the measured TFP, which breaks its perfect correlation with the labor market variables. Therefore, shock to rotation rate significantly improves the performance of DMP model.

Simulation without firm inter-connectivity and supermodularity

In the simple model, as shown in Proposition 3, both firm inter-connectivity and supermodularity are necessary for a positive rotation shock to increase unemployment rate. To understand the role of firm inter-connectivity and supermodularity in the quantitative model, I simulate two counterfactual models, one canonical DMP model with no firm inter-connectivity, the other one with firm inter-connectivity yet without supermodularity. Then I compare the effect of rotation shock on these two models to the benchmark case. In Figure 4.5, red lines are impulse response functions in the model with no firm inter-connectivity; black dashed lines are for model without supermodularity; blue lines correspond to the benchmark case with both firm inter-connectivity and supermodularity.

When firm inter-connectivity is removed, my model becomes identical to the canonical DMP model and rotation shock does not have any aggregate effect. In this case, churning only shifts the dispersion between the marginal values of different types. However, the mean of marginal values, which determines the aggregate job creation, is not affected.

When there is firm inter-connectivity, but MPL is not supermodular, that is $\left(z_i^{HH}\right)^{\frac{1}{1-\alpha}} - \left(z_i^{HL}\right)^{\frac{1}{1-\alpha}} = \left(z_i^{LH}\right)^{\frac{1}{1-\alpha}} - \left(z_i^{LL}\right)^{\frac{1}{1-\alpha}}$, positive rotation shock does not increase unemployment. In fact, according to the impulse response function, higher churning results in higher job creation and lower unemployment. This is a general equilibrium effect. The intuition is as follows: while MPL is not supermodular, productivity is still supermodular; that is, we still have $z_i^{HH} - z_i^{HL} > z_i^{LH} - z_i^{LL}$. Therefore a higher churning leads to diminished output and a decline in consumption growth. As a result, discount factor $\beta \frac{u_C'}{u_C}$ rises and firms are more willing to allocate resource to the activities generating future profit, including the



The panels plot the percent deviation from the steady state of each induced by a one standard deviation shock to the rotation rate in set *A*.

FIGURE 2.4: Impulse responses to rotation shock in counterfactual analysis

recruiting of worker. While this general equilibrium effect also appears in the benchmark model, it is dominated by the mismatch risk effect.

Variance decomposition

To evaluate the contribution of rotation shock to the business cycle, I conduct variance decomposition at business cycle frequency and report the results in Table 2.8. Each row reports the shares of fluctuations explained by the shocks at the business cycle frequency; that is, the share of variances explained at frequencies between 6 and 32 quarters, computed by a bandpass filter as in Stock and Watson (1999).[38]

Rotation shocks account for a significant fraction of variation of all macro variables listed in the table. In particular, rotation shocks contribute to 27 percent of labor market

[38] I thank Cosmin Ilut for generously sharing his matlab code for conducting the variance decomposition at the business cycle frequency.

Table 2.8: Variance decomposition at the business cycle frequency

Variance decomposition	TFP	DR	DL	MEI	Rot_A	Rot_B
Unemployment rate	0.03	0.02	0.67	0.01	0.14	0.13
Tightness ratio	0.02	0.02	0.68	0.01	0.14	0.13
Output	0.62	0.00	0.06	0.00	0.17	0.15
Measured TFP	0.48	0.00	0.01	0.00	0.27	0.24
Investment	0.05	0.16	0.04	0.45	0.25	0.22
Consumption	0.25	0.08	0.03	0.37	0.14	0.12

TFP is aggregate total factor productivity. DR is discount rate. DL is disutility of labor. MEI is marginal efficiency of investment. Rot_i is rotation rate for set i.

fluctuations and 32 percent of fluctuation of aggregate output. There are two main reasons for the data to assign such a big role to rotation shocks. First, as displayed in the impulse response functions, rotation shocks have strong effect on both labor market variables as well as macro aggregates without relying on extreme values of parameters. The second reason is that rotation shock is the only type of shock considered in the model that can account for the joint movement of *all* observable variables, hence it is preferred by the data.

Shocks to TFP have a very minor effect on unemployment rate and tightness ratio due to lack of amplification of productivity shock in search and matching models. However, they still account for large fractions of the variances of macro aggregates such as output and measured TFP. This is consistent with the main findings of the business cycle literature.

Shocks to discount rate, or preference shocks, contribute to a large fraction of investment and consumption fluctuations. It also explains a moderate fraction of labor market fluctuations. While it is well known that higher discount rate shifts resources from consumption to investment, its effect on labor market variables is less well known. Remember that in search and matching models, hiring is an inter-temporal action similar to investment—firms increase future employment by posting vacancies at today's cost. Thus shock to discount rate has a direct impact on firms' incentive to post vacancies by changing their evaluation of future cash flow. This effect has been emphasized by Hall (2014). In a recent paper by Albertini and Poirier (2014), they estimated a search and matching model with shocks to TFP and discount rate and found that the shock to the discount rate predominantly account for fluctuation of unemployment. In the estimation, however, they did not include real interest rate as observable, hence the discount rate shock is not disciplined. In contrast, I include real interest rate as observable and the data assigns much less role to discount rate compared to the finding by Albertini and Poirier (2014).

Shocks to marginal efficiency of investment, or shocks to investment adjustment cost, have a large effect on consumption and investment, yet has little contribution to the other variables' fluctuations.

Shocks to labor disutility have a very large effect on labor market variables. In many previous studies, fluctuation of labor disutility has been found to prominently account for variation in labor input. A positive shock to the labor disutility increases the option value of unemployment, which makes the total surplus of potential worker-firm matches shrink and hence reduces firm's incentive to create jobs.

2.6 Conclusions

This paper is motivated by two empirical facts. The first is that during downturns of economic activity, there is an increase in churning of firms' rankings in the profit distribution. Second, a higher churning of an industry is usually accompanied by an economic downturn within the industry and in its linked industries. Based on the two facts, this paper studies the effect of churning on the business cycle emphasizing the role of firm inter-connectivity.

Specifically, I construct and estimate a Diamond-Mortenson-Pissarides search and matching model featured by firm inter-connectivity and supermodular production function; that is, firms have comparative advantage in cooperating with same type partners.

The main prediction of the model is that an increase in churning of an industry causes a recession within its own industry and in its linked industries. According to the estimation, variations in churning is one of the major sources of persistent and joint movements in unemployment and other macro variables.

Lastly, I study the model's implication on the cross industry variation in the vertical integration. The model implies that when firms have comparative advantage in cooperating with same type partners; in an industry with higher churning, or whose linked industries have higher churning, firms are less likely to choose vertical integration. This implication is consistent with the evidence in the data, which supports the main mechanism of the model, particularly the supermodular production function.

Appendix

2.7 Transition rule of firms (employments)

The transition rule of firms (or equivalently, employment) of each department indexed by type in sector A is governed by the following equations.

Firms with vertical integration (VI)

- H type single firm

$$\begin{aligned} n_{A,VI}^{H'} &= (1-\delta)\left[(1-p_{A,VI}^{H})(1-\rho_A^{HL})n_{A,VI}^{H} + (1-p_{A,VI}^{L})\rho_A^{LH} n_{A,VI}^{L}\right] \\ &+\frac{1}{2}\left(1-F\left(\eta^{*}\right)\right)\mu_A u_A \end{aligned} \tag{2.7.1}$$

where P_A^H and P_A^L are cooperation matching rate with $P_{A,VI}^H = \frac{\tilde{M}\left(n_{A,VI}^H, n_{B,VI}^H\right)}{n_{A,VI}^H}$ and $P_{A,VI}^L = \frac{\tilde{M}\left(n_{A,VI}^L, n_{B,VI}^L\right)}{n_{A,VI}^L}$

- L type single firm

$$\begin{aligned} n_{A,VI}^{L'} &= (1-\delta)\left[(1-p_{A,VI}^{L})(1-\rho_A^{LH})n_{A,VI}^{L} + (1-p_{A,VI}^{H})\rho_A^{LH} n_{A,VI}^{H}\right] \\ &+\frac{1}{2}\left(1-F\left(\eta^{*}\right)\right)\mu_A u_A \end{aligned} \tag{2.7.2}$$

- H type cooperative firm matched with H type partner

$$n_{A,VI}^{HH'} = (1-\delta)\left[p_{A,VI}^{H} n_{A,VI}^{H} + \sum_{l=\{H,L\}}\sum_{q=\{H,L\}} \rho_A^{lH}\rho_B^{qH} n_{A,VI}^{lq}\right] \tag{2.7.3}$$

- H type cooperative firm matched with L type partner

$$n_{A,VI}^{HL'} = (1-\delta)\left[\sum_{l=\{H,L\}}\sum_{q=\{H,L\}} \rho_A^{lH}\rho_B^{qL} n_{A,VI}^{lq}\right] \tag{2.7.4}$$

- L type cooperative firm matched with H type partner

$$n_{A,VI}^{LH'} = (1-\delta)\left[\sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lL}\rho_B^{qH}n_{A,VI}^{lq}\right] \tag{2.7.5}$$

- L type cooperative firm matched with L type partner

$$n_{A,VI}^{LL'} = (1-\delta)\left[p_{A,VI}^{L}n_{A,VI}^{L} + \sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lL}\rho_B^{qL}n_{A,VI}^{lq}\right] \tag{2.7.6}$$

The first two equations are flow motions for single firms. Take H type as an example. Existing H type single firms may flow out in three conditions: first, with probability δ an H type single firm can be destructed exogenously; Second, with probability p_A^H it is matched with another firm; Third, with probability ρ_A^{HL} the firm rotate to a L type single firm. At the same time, there are two kinds of inflow into H type: L type single firm becomes H type with probability ρ_A^{LH}; unemployed workers becomes H type employment (firm) with job finding rate μ_A. $(1-F(\eta^*))$ fraction of the new created firms choose to do vertical integration, and half of them draw an H type.

The last four equations are flow motion for cooperative firms. Take the HH type as an example. With probability δ existing firms are exogenously destructed. And with probability $\rho_A^{lH}\rho_B^{qH}$ an lq type firm becomes HH. Lastly, H type single firms match to another H type single firm and becomes HH with probability p_A^H.

Firms with sourcing (SC)

- H type single firm

$$\begin{aligned} n_{A,SC}^{H'} &= (1-\delta)\left[(1-p_{A,SC}^{H})(1-\rho_A^{HL})n_{A,SC}^{H} + (1-p_{A,VI}^{L})\rho_A^{LH}n_{A,SC}^{L}\right] \\ &+\frac{1}{2}F(\eta^*)\,\mu_A u_A + \nu(1-\delta)\left[\sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lH}\rho_B^{qL}n_{A,SC}^{lq}\right] \end{aligned} \tag{2.7.7}$$

where P_A^H and P_A^L are cooperation matching rate with $P_{A,SC}^H = \frac{\tilde{M}\left(n_{A,SC}^H, n_{B,SC}^H\right)}{n_{A,SC}^H}$ and $P_{A,SC}^L = \frac{\tilde{M}\left(n_{A,SC}^L, n_{B,SC}^L\right)}{n_{A,SC}^L}$

- L type single firm

$$
\begin{aligned}
n_{A,SC}^{L'} &= (1-\delta)\left[(1-p_{A,SC}^L)(1-\rho_A^{LH})n_{A,SC}^L + (1-p_{A,SC}^H)\rho_A^{LH} n_{A,SC}^H\right] \\
&\quad + \frac{1}{2}F\left(\eta^*\right)\mu_A u_A + \nu\left(1-\delta\right)\left[\sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lL}\rho_B^{qH} n_{A,SC}^{lq}\right]
\end{aligned} \tag{2.7.8}
$$

- H type cooperative firm matched with H type partner

$$
n_{A,SC}^{HH'} = (1-\delta)\left[p_{A,SC}^H n_{A,SC}^H + \sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lH}\rho_B^{qH} n_{A,SC}^{lq}\right] \tag{2.7.9}
$$

- H type cooperative firm matched with L type partner

$$
n_{A,SC}^{HL'} = (1-\nu)\left(1-\delta\right)\left[\sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lH}\rho_B^{qL} n_{A,SC}^{lq}\right] \tag{2.7.10}
$$

- L type cooperative firm matched with H type partner

$$
n_{A,SC}^{LH'} = (1-\nu)\left(1-\delta\right)\left[\sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lL}\rho_B^{qH} n_{A,SC}^{lq}\right] \tag{2.7.11}
$$

- L type cooperative firm matched with L type partner

$$
n_{A,SC}^{LL'} = (1-\delta)\left[p_{A,SC}^L n_{A,SC}^L + \sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lL}\rho_B^{qL} n_{A,SC}^{lq}\right] \tag{2.7.12}
$$

The flow motion equations for SC firms are similar with the VI case, expect that mismatched firms have ν probability to sever the partnership and becomes a single firm.

Putting the equations to matrix form, I can stack the transition rule in matrix form.

For firms choosing VI

$$\begin{bmatrix} n_{A,VI}^{H'} \\ n_{A,VI}^{L'} \\ n_{A,VI}^{HH'} \\ n_{A,VI}^{HL'} \\ n_{A,VI}^{LH'} \\ n_{A,VI}^{LL'} \end{bmatrix} = \Phi_{VI} \times \begin{bmatrix} n_{A,VI}^{H} \\ n_{A,VI}^{L} \\ n_{A,VI}^{HH} \\ n_{A,VI}^{HL} \\ n_{A,VI}^{LH} \\ n_{A,VI}^{LL} \end{bmatrix} + \Xi_{VI} \times u_A \tag{2.7.13}$$

where

$$\Phi_{VI} = \begin{bmatrix} \left(1-p_{A,VI}^{H}\right)\left(1-\rho_A^{HL}\right) & \left(1-p_{A,VI}^{L}\right)\rho_A^{LH} & 0 & 0 & 0 & 0 \\ \left(1-p_{A,VI}^{H}\right)\rho_A^{LH} & \left(1-p_{A,VI}^{L}\right)\left(1-\rho_A^{LH}\right) & 0 & 0 & 0 & 0 \\ p_{A,VI}^{H} & 0 & & & & \\ 0 & 0 & \Pi_A^T & \otimes & \Pi_B^T & \\ 0 & 0 & & & & \\ 0 & p_{A,VI}^{L} & & & & \end{bmatrix} \tag{2.7.14}$$

$$\Xi_{VI} = \begin{bmatrix} \frac{1}{2}\left(1-F\left(\eta^*\right)\right)\mu\left(\theta_A\right) \\ \frac{1}{2}\left(1-F\left(\eta^*\right)\right)\mu\left(\theta_A\right) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.7.15}$$

In the Φ matrix, $\otimes$ is Kronecker product and Π_A and Π_B are Markov switching matrices that governs rotation of types.

Similarly, for firms choosing SC

$$\begin{bmatrix} n_{A,SC}^{H'} \\ n_{A,SC}^{L'} \\ n_{A,SC}^{HH'} \\ n_{A,SC}^{HL'} \\ n_{A,SC}^{LH'} \\ n_{A,SC}^{LL'} \end{bmatrix} = \Phi_{SC} \times \begin{bmatrix} n_{A,SC}^{H} \\ n_{A,SC}^{L} \\ n_{A,SC}^{HH} \\ n_{A,SC}^{HL} \\ n_{A,SC}^{LH} \\ n_{A,SC}^{LL} \end{bmatrix} + \Xi_{SC} \times u_A \tag{2.7.16}$$

where

$$\Xi_{SC} = \begin{bmatrix} \frac{1}{2}F(\eta^*)\,\mu(\theta_A) \\ \frac{1}{2}F(\eta^*)\,\mu(\theta_A) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.7.17}$$

, and

$$\Phi_{SC} = \begin{bmatrix} \left(1-p_{A,SC}^{H}\right)\left(1-\rho_A^{HL}\right) & \left(1-p_{A,SC}^{L}\right)\rho_A^{LH} & \nu\left(\Pi_A^T \otimes \Pi_B^T\right)_{(2,:)} \\ \left(1-p_{A,SC}^{H}\right)\rho_A^{LH} & \left(1-p_{A,SC}^{L}\right)\left(1-\rho_A^{LH}\right) & \nu\left(\Pi_A^T \otimes \Pi_B^T\right)_{(3,:)} \\ p_{A,SC}^{H} & 0 & \left(\Pi_A^T \otimes \Pi_B^T\right)_{(1,:)} \\ 0 & 0 & (1-\nu)\left(\Pi_A^T \otimes \Pi_B^T\right)_{(2,:)} \\ 0 & 0 & (1-\nu)\left(\Pi_A^T \otimes \Pi_B^T\right)_{(3,:)} \\ 0 & p_{A,SC}^{L} & \left(\Pi_A^T \otimes \Pi_B^T\right)_{(4,:)} \end{bmatrix} \tag{2.7.18}$$

where $\left(\Pi_A^T \otimes \Pi_B^T\right)_{(k,:)}$ denotes the kth row of $\left(\Pi_A^T \otimes \Pi_B^T\right)$ matrix.

Combine the VI and SC firms, the flow motion of employment is then described by

$$\underbrace{\begin{bmatrix} n_{A,VI}^{H'} \\ \vdots \\ n_{A,SC}^{LL'} \end{bmatrix}}_{12\times 1} = \underbrace{\Phi}_{12\times 12} \times \underbrace{\begin{bmatrix} n_{A,VI}^{H} \\ \vdots \\ n_{A,SC}^{LL} \end{bmatrix}}_{12\times 1} + \underbrace{\Xi}_{12\times 1} \times u_A \tag{2.7.19}$$

where

$$\Phi = \begin{bmatrix} \Phi_{SC} & \\ & \Phi_{VI} \end{bmatrix}$$

and

$$\Xi = \begin{bmatrix} \Xi_{SC} \\ \Xi_{VI} \end{bmatrix}$$

2.8 Non-linear system of equations

Household's F.O.Cs

- Marginal utility of consumption

$$\xi_{c,t}\left(C_t - \Psi_c \bar{C}_{t-1}\right)^{-1} = \lambda_t \tag{2.8.1}$$

where λ is the Lagrange multiplier on the household's budget constraint. In equilibrium $C = \bar{C}$

- F.O.C of investment

$$\lambda_t = q_t\left[(1 - S_t) - S_t^{'}\xi_{I,t}\frac{I_t}{I_{t-1}}\right] + \beta E\left[q_{t+1}S_{t+1}^{'}\xi_{I,t+1}\frac{I_{t+1}^2}{I_t^2}\right] \tag{2.8.2}$$

where q is Tobin's q.

- F.O.C of capital stock

$$\lambda_t r_{k,t} = -q_t(1-d) + \frac{q_{t-1}}{\beta} \tag{2.8.3}$$

Household's marginal value of employment and unemployment

- Household's marginal value of employment for each type of employee

$$\underbrace{\begin{bmatrix} V_{n_{i,VI}^H} \\ \vdots \\ V_{n_{i,SC}^{LL}} \end{bmatrix}}_{12\times 1} = -\xi_n + \lambda \underbrace{\begin{bmatrix} w_{i,VI}^H \\ \vdots \\ w_{i,SC}^{LL} \end{bmatrix}}_{12\times 1} + \beta \underbrace{\Phi^T}_{12\times 12} E \underbrace{\begin{bmatrix} V_{n_{i,VI}^H}^{'} \\ \vdots \\ V_{n_{i,SC}^{LL}}^{'} \end{bmatrix}}_{12\times 1} \tag{2.8.4}$$

where Φ is defined in the last section and T denotes transpose of matrix.

- Equilibrium condition for perfect labor mobility and household's marginal value of unemployment.

$$V_{u_A} = V_{u_B} = V_u \tag{2.8.5}$$

where

$$V_{u_A} = z \cdot \lambda + \beta\left(1-\delta-\mu\left(\theta_A\right)\right) E\left[V'_{u_A}\right] + \beta \underbrace{\Xi^T}_{1\times 12} E \underbrace{\begin{bmatrix} V'_{n^H_{A,VI}} \\ \vdots \\ V'_{n^{LL}_{A,SC}} \end{bmatrix}}_{6\times 1}$$

$$V_{u_B} = z \cdot \lambda + \beta\left(1-\delta-\mu\left(\theta_B\right)\right) E\left[V'_{u_A}\right] + \beta \Xi^T \begin{bmatrix} V'_{n^H_{B,VI}} \\ \vdots \\ V'_{n^{LL}_{B,SC}} \end{bmatrix}$$

With assumption of perfect labor mobility, the marginal value of unemployment should equalize across two sectors in any state of economy.

Firm's FOCs

- Firm's F.O.C. of capital input.

$$xz_i^H \left(\frac{k^H_{i,VI}}{xn^H_{i,VI}}\right)^{\alpha-1} = \cdots = xz_i^{LL} \left(\frac{k^{LL}_{i,SC}}{xn^{LL}_{i,SC}}\right)^{\alpha-1} = r_K \tag{2.8.6}$$

  Firm allocates capital to production departments to the point that *mpk* of each department equal to rental rate of capital.

- Firm's F.O.C. of vacancy posting.

$$\chi = \beta \int \left\{ \Xi^T_{\theta_A} \begin{bmatrix} E\left[\frac{\lambda'}{\lambda}\frac{\partial J'_i}{\partial n^H_{i,VI}(\eta)}\right] \\ \vdots \\ E\left[\frac{\lambda'}{\lambda}\frac{\partial J'_i}{\partial n^{LL}_{i,SC}(\eta)}\right] \end{bmatrix} \right\} dF\left(\eta\right) \tag{2.8.7}$$

Firm chooses the tightness ratio so that its expected marginal value equals to vacancy posting cost.

Firm's envelope conditions

- Firm, number of employee in each department

$$\lambda \begin{bmatrix} \frac{\partial J_i}{\partial n^H_{i,VI}(\eta)} \\ \vdots \\ \frac{\partial J_i}{\partial n^{LL}_{i,SC}(\eta)} \end{bmatrix} = \lambda \begin{bmatrix} mpl^H_{i,VI} \\ \vdots \\ mpl^{LL}_{i,SC} \end{bmatrix} - \lambda \begin{bmatrix} w^H_{i,VI} \\ \vdots \\ w^{LL}_{i,SC} \end{bmatrix} + \beta \Phi^T \left(\tilde{\theta}, \Pi\right) E \left(\lambda' \begin{bmatrix} \frac{\partial V'}{\partial n^H_{i,VI}(\eta)} \\ \vdots \\ \frac{\partial V'}{\partial n^{LL}_{i,SC}(\eta)} \end{bmatrix} \right) \tag{2.8.8}$$

Total surplus

Remember that the definition of total surplus TS as

$$TS^j_{i,l}(\eta) = \lambda \frac{\partial J_i}{\partial n^j_{i,l}(\eta)} + \frac{\partial V}{\partial n^j_{i,l}(\eta)} - \frac{\partial V}{\partial u_i}$$

And with Nash Bargaining we have

$$\lambda \frac{\partial J_i}{\partial n^j_{i,l}(\eta)} = \tau TS^j_{i,l}(\eta) \tag{2.8.9}$$

$$\frac{\partial V}{\partial n^j_{i,l}(\eta)} - \frac{\partial V}{\partial u_i} = (1-\tau) TS^j_{i,l}(\eta) \tag{2.8.10}$$

By combining household's F.O.C., firm's F.O.C. and envelope conditions together one can get the following total surplus representation

$$\begin{aligned} \begin{bmatrix} TS^H_{i,l}(\eta) \\ TS^L_{i,l}(\eta) \end{bmatrix} &= \lambda \left(\begin{bmatrix} mpl^H_{i,l} \\ mpl^L_{i,l} \end{bmatrix} - z - \frac{(1-\delta)(1-\tau)\chi\theta_i}{\tau} \right) - \xi_n \\ &\quad + \beta(1-\delta) E \begin{bmatrix} p^H_i TS^{HH}_{i,l} + \left(1-p^H_i\right)\left(\rho^{HH}_i TS^H_{i,l}(\eta) + \rho^{HL}_i TS^L_{i,l}(\eta)\right) \\ p^L_i TS^{LL}_{i,l} + \left(1-p^L_i\right)\left(\rho^{LH}_i TS^H_{i,l}(\eta) + \rho^{LL}_i TS^L_{i,l}(\eta)\right) \end{bmatrix} \end{aligned} \tag{2.8.11}$$

$$\begin{bmatrix} TS_{i,l}^{HH}(\eta) \\ TS_{i,l}^{HL}(\eta) \\ TS_{i,l}^{LH}(\eta) \\ TS_{i,l}^{LL}(\eta) \end{bmatrix} = \lambda \left(\begin{bmatrix} mpl_{i,l}^{HH} \\ mpl_{i,l}^{HL} \\ mpl_{i,l}^{LH} \\ mpl_{i,l}^{LL} \end{bmatrix} - z - \frac{(1-\delta)(1-\tau)\chi\theta_i}{\tau} \right) - \xi_n \tag{2.8.12}$$

$$+\beta(1-\delta)\left(\Pi_A^T \otimes \Pi_B^T\right) E \begin{bmatrix} TS_{i,l}^{HH'}(\eta) \\ TS_{i,l}^{HL'}(\eta) \\ TS_{i,l}^{LH'}(\eta) \\ TS_{i,l}^{LL'}(\eta) \end{bmatrix}$$

And the free entry condition of the labor market i becomes

$$\lambda\chi = \beta\tau \int \left\{ \Xi_{\theta_A}^T E \begin{bmatrix} TS_{i,VI}^{H}(\eta) \\ \vdots \\ TS_{i,SC}^{LL}(\eta) \end{bmatrix} \right\} dF(\eta) \tag{2.8.13}$$

2.9 Proof of lemmas and propositions

2.9.1 Proof of results of section 4.3 and 2.4

In this subsection, I will use the following notations

$\tilde{J}_i = J_i^{HH} + J_i^{LL} - J_i^{HL} - J_i^{LH}$

$\tilde{z}_i = z_i^{HH} + z_i^{LL} - z_i^{HL} - z_i^{LH}$

$\bar{J}_i = \frac{J_i^{HH} + J_i^{LL}}{2}$

$\bar{z}_i = \frac{z_i^{HH} + z_i^{LL}}{2}$

$\tilde{J}_{i,l}(\eta) = J_{i,l}^{HH}(\eta) + J_{i,l}^{LL}(\eta) - J_{i,l}^{LH}(\eta) - J_{i,l}^{HL}(\eta)$

$i \in \{A, B\},\ l \in \{VI, SC\}$

As the two sets of firms are symmetric, to save space I will drop the subscript i in some cases.

Lemma 4

(1) Value function is strictly monotone if idiosyncratic productivity is strictly monotone.

(2) Value function is supermodular if and only if production function is supermodular.

Proof. (1) Use firms' value functions, we get

$$\begin{bmatrix} J_t^{HH}-J_t^{HL} \\ J_t^{LH}-J_t^{LL} \end{bmatrix} = \tau\cdot\begin{bmatrix} z^{HH}-z^{HL} \\ z^{LH}-z^{LL} \end{bmatrix}+\beta\left(1-\delta\right)\begin{bmatrix} \rho^{HH} & \rho^{HL} \\ \rho^{LH} & \rho^{LL} \end{bmatrix}\begin{bmatrix} J_t^{HH}-J_t^{HL} \\ J_t^{LH}-J_t^{LL} \end{bmatrix}$$

$$\Rightarrow$$

$$\tau\cdot\begin{bmatrix} z^{HH}-z^{HL} \\ z^{LH}-z^{LL} \end{bmatrix} = \begin{bmatrix} 1-\beta\left(1-\delta\right)\rho^{HH} & -\beta\left(1-\delta\right)\rho^{HL} \\ -\beta\left(1-\delta\right)\rho^{LH} & 1-\beta\left(1-\delta\right)\rho^{LL} \end{bmatrix}\begin{bmatrix} J_t^{HH}-J_t^{HL} \\ J_t^{LH}-J_t^{LL} \end{bmatrix}$$

$$\Rightarrow$$

$$\begin{bmatrix} J_t^{HH}-J_t^{HL} \\ J_t^{LH}-J_t^{LL} \end{bmatrix} = \frac{\tau\cdot\begin{bmatrix} 1-\beta\left(1-\delta\right)\rho^{LL} & \beta\left(1-\delta\right)\rho^{HL} \\ \beta\left(1-\delta\right)\rho^{LH} & 1-\beta\left(1-\delta\right)\rho^{HH} \end{bmatrix}\begin{bmatrix} z^{HH}-z^{HL} \\ z^{LH}-z^{LL} \end{bmatrix}}{\left(1-\beta\left(1-\delta\right)\rho^{HH}\right)\left(1-\beta\left(1-\delta\right)\rho^{LL}\right)+\beta^{2}\left(1-\delta\right)^{2}\rho^{HL}\rho^{LH}}$$

We immediately get that $\begin{bmatrix} J_t^{HH}-J_t^{HL} \\ J_t^{LH}-J_t^{LL} \end{bmatrix} > \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ if $\begin{bmatrix} z^{HH}-z^{HL} \\ z^{LH}-z^{LL} \end{bmatrix} > \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

With the same method we can prove that $\begin{bmatrix} J_t^{HH}-J_t^{LH} \\ J_t^{HL}-J_t^{LL} \end{bmatrix} > \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ if $\begin{bmatrix} z^{HH}-z^{LH} \\ z^{HL}-z^{LL} \end{bmatrix} > \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(2) It is to show that $\tilde{J} > 0$ if and only if $\tilde{z} > 0$. Use the value functions of firms equation 2.2 and equation refE2, we get

$$\begin{aligned} \tilde{J_t} &= \tau\cdot x_t\cdot z_t^{HH}+\beta\left(1-\delta\right)E_t\left(\rho_t^{HH}J_{t+1}^{HH}+\rho_t^{HL}J_{t+1}^{LH}\right) \\ &- \left[\tau\cdot x_t\cdot z_t^{HL}+\beta\left(1-\delta\right)E_t\left(\rho_t^{HH}J_{t+1}^{HL}+\rho_t^{HL}J_{t+1}^{LL}\right)\right] \\ &+ \tau\cdot x_t\cdot z_t^{LL}+\beta\left(1-\delta\right)E_t\left(\rho_t^{LH}J_{t+1}^{HL}+\rho_t^{LL}J_{t+1}^{LL}\right) \\ &- \left[\tau\cdot x_t\cdot z_t^{LH}+\beta\left(1-\delta\right)E_t\left(\rho_t^{LH}J_{t+1}^{HH}+\rho_t^{LL}J_{t+1}^{LH}\right)\right] \end{aligned}$$

Rearranging the terms, we get

$$\begin{aligned}
\tilde{J}_t &= \tau \cdot x_t \left(z_t^{HH} + z_t^{LL} - z_t^{HL} - z_t^{LH}\right) \\
&+ \beta (1-\delta) \left(\rho_t^{HH} - \rho_t^{LH}\right) E_t \left(J_{t+1}^{HH}\right) \\
&+ \beta (1-\delta) \left(\rho_t^{LL} - \rho_t^{HL}\right) E_t \left(J_{t+1}^{LL}\right) \\
&+ \beta (1-\delta) \left(\rho_t^{LH} - \rho_t^{HH}\right) E_t \left(J_{t+1}^{HL}\right) \\
&+ \beta (1-\delta) \left(\rho_t^{HL} - \rho_t^{LL}\right) E_t \left(J_{t+1}^{LH}\right)
\end{aligned}$$

Notice that the Markov switching matrix is symmetric, we have

$$\begin{aligned}
\rho_t^{HH} - \rho_t^{LH} &= \rho_t^{LL} - \rho_t^{HL},\ 1 - 2\rho_t^{HL} = 1 - 2\rho_t^{HL} \\
\rho_t^{LH} - \rho_t^{HH} &= -\left(1 - 2\rho_t^{HL}\right),\ \rho_t^{HL} - \rho_t^{LL} = -\left(1 - 2\rho_t^{HL}\right)
\end{aligned}$$

Thus we have

$$\tilde{J}_t = \tau \cdot x_t \tilde{z}_t + \beta \left(1 - 2\rho_t^{HL}\right) (1-\delta) E_t \left(\tilde{J}_{t+1}\right) \tag{2.9.1}$$

On steady state, we have

$$\tilde{J} = \frac{\tau \cdot x \cdot \tilde{z}}{1 - \beta \left(1 - 2\rho^{HL}\right)(1-\delta)}$$

Since $1 - \beta \left(1 - 2\rho^{HL}\right)(1-\delta) > 0$, $\tilde{J} > 0$ if and only if $\tilde{z} > 0$.

□

Proposition 3

If production function is strictly monotone, and there is firm inter-connectivity, a persistent increase in the rotation rate of one set, with persistence bounded below $|\psi| < 1 - \rho^{HL} - \rho^{LH}$, state would lead to

1. a decrease in the equilibrium tightness ratio of both sets
2. a decrease in the average productivity of both sets in the next period

if and only if the production function is supermodular.

Proof. I will show when $\tilde{z} > 0$, θ is decreasing with ρ^{HL}. Notice that θ is increasing in $\bar{J}$ according to free entry condition, we only need to show $\bar{J}$ is decreasing with ρ^{HL}

Fixing aggregate productivity x to one, we have the following equations

$$\bar{J}_t = \bar{z} + \beta(1-\delta)\left[E_t(\bar{J}_{t+1}) - \frac{\rho_t^{HL} E_t(\tilde{J}_{t+1})}{2}\right] \tag{2.9.2}$$

$$\tilde{J}_t = \tilde{z} + \beta(1-\delta)\left(1-2\rho_t^{HL}\right) E_t(\tilde{J}_{t+1}) \tag{2.9.3}$$

$$\rho_t^{HL} = \psi\rho_{t-1}^{HL} + \varepsilon_t \tag{2.9.4}$$

Denote $F_{t,t+s} = \frac{1}{2}E_t\left(\rho_{t+s}^{HL}\tilde{J}_{t+s+1}\right)$, the first equation becomes

$$\bar{J}_t = \frac{\bar{z}}{1-\beta(1-\delta)} - \sum_{s=0}^{\infty} F_{t,t+s}$$

It suffices to show that $\frac{dF_{t,t+s}}{d\varepsilon_t} > 0$ for $t > 0 \quad s \geqslant 0$. Here I prove the inequality for $s = 0$, the cases for $s > 0$ are similar.

$$\begin{aligned}\frac{dF_{t,t}}{d\varepsilon_t} &= \frac{d\left[\rho_t^{HL} E_t\left(\tilde{J}_{t+1}\right)\right]}{d\rho_t^{HL}} \\ &= E_t\left(\tilde{J}_{t+1}\right) + \rho_t^{HL}\frac{E_t\left(\tilde{J}_{t+1}\right)}{d\rho_t^{HL}}\end{aligned}$$

According to equation 2.9.3, we have

$$E_t\left(\tilde{J}_{t+1}\right) = \tilde{z}\cdot\sum_{s=0}^{\infty} E_t\left[\beta^s(1-\delta)^s\prod_{k=0}^{s}\left(1-2\rho_{t+k+1}^{HL}\right)\right]$$

Therefore

$$\begin{aligned}\frac{E_t\left(\tilde{J}_{t+1}\right)}{d\rho_t^{HL}} &= \tilde{z}\cdot\sum_{s=0}^{\infty}E_t\left\{\left[\beta^s(1-\delta)^s\prod_{k=0}^{s}\left(1-2\rho_{t+k+1}^{HL}\right)\right]\cdot\sum_{k=0}^{s}\left(-\frac{2}{1-2\rho_{t+k+1}^{HL}}\cdot\frac{d\rho_{t+k+1}^{HL}}{d\rho_t^{HL}}\right)\right\} \\ &> \tilde{z}\cdot\sum_{s=0}^{\infty}E_t\left\{\left[\beta^s(1-\delta)^s\prod_{k=0}^{s}\left(1-2\rho_{t+k+1}^{HL}\right)\right]\cdot\sum_{k=0}^{s}\left(-2\cdot\frac{1-\psi^k}{1-\psi}\right)\right\} \\ &> -2\cdot\tilde{z}\cdot\sum_{s=0}^{\infty}E_t\left[\frac{\beta^s(1-\delta)^s\prod_{k=0}^{s}\left(1-2\rho_{t+k+1}^{HL}\right)}{1-\psi}\right] \\ &= -\frac{2}{1-\psi}\cdot E_t\left(\tilde{J}_{t+1}\right)\end{aligned}$$

When $\frac{1-\psi}{\rho^{HL}}>2$, we immediately get

$$\frac{dF_{t,t}}{d\varepsilon_t}>\left(1-\frac{2\rho^{HL}}{1-\psi}\right)E_t\left(\tilde{J}_{t+1}\right)>0$$

□

Proposition 4

The threshold level of transaction cost of industry i exists and is uniquely determined by

$$\tau\cdot\eta_i^*=\beta(1-\delta)\,g\left(\rho_j^{HL},\nu\right)\cdot\left(z_i^{HH}+z_i^{LL}-z_i^{HL}-z_i^{LH}\right)$$

with $\frac{\partial g\left(\rho_j^{HL},\nu\right)}{\partial\rho_j^{HL}}>0$ and $\frac{\partial g\left(\rho_j^{HL},\nu\right)}{\partial\nu}>0$

Proof. The threshold level of transaction cost is determined by

$$J_{i,VI}^{HH}+J_{i,VI}^{LL}=J_{i,SC}^{HH}\left(\eta_i^*\right)+J_{i,SC}^{LL}\left(\eta_i^*\right) \tag{2.9.5}$$

The LHS of equation 2.9.5 can be derived as

$$J_{i,VI}^{HH}+J_{i,VI}^{LL}=\tau\left(z_i^{HH}+z_i^{LL}\right)+\beta(1-\delta)E_t\begin{bmatrix}J_{i,VI}^{HH}+J_{i,VI}^{LL}\\-\rho_t\tilde{J}_{i,VI}\end{bmatrix} \tag{2.9.6}$$

The RHS of equation 2.9.5 can be derived as

$$J_{i,SC}^{HH}(\eta_i^*)+J_{i,SC}^{LL}(\eta_i^*)=\tau\left(z_i^{HH}+z_i^{LL}-2\eta_i^*\right)+\beta(1-\delta)E_t\begin{bmatrix}J_{i,SC}^{HH}(\eta_i^*)+J_{i,SC}^{LL}(\eta_i^*)\\-\rho_t(1-\nu)\tilde{J}_{i,SC}(\eta_i^*)\end{bmatrix} \tag{2.9.7}$$

Subtract equation 2.9.6 from 2.9.7, we get

$$2\tau\eta_i^*=\beta(1-\delta)\rho_t\left[\tilde{J}_{i,VI}-(1-\nu)\tilde{J}_{i,SC}(\eta_i^*)\right] \tag{2.9.8}$$

I first present a useful lemma.

Lemma. *Mismatch loss for firms choosing sourcing is constant within industry, that is*

$$\tilde{J}_{i,SC}(\eta_i^*)=\tilde{J}_{i,SC}$$

for $\eta\sim\mathcal{N}\left(\bar{\eta}_i,\sigma_i^2\right)$

Proof. By plugging value functions into the above equation and using the method of deriving equation 2.9.1, we get

$$\tilde{J}_{SC,t}(\eta)=\tau\tilde{z}_t+\beta(1-\delta)\left[1-\left(\rho_t^{HL}+\rho_t^{LH}\right)(1-\nu)\right]E_t\left(\tilde{J}_{SC,t}(\eta)\right)$$

As shown in the above equation, mismatch loss $\tilde{J_{S,t}}(\eta)$ is independent of contraction cost η, and I will drop the η term in the following proof.

□

With the above lemma, we immediately get that in the steady state,

$$\tilde{J}_{SC}=\frac{\tau\tilde{z}}{1-\beta(1-\delta)\left[1-2\rho(1-\nu)\right]} \tag{2.9.9}$$

, where $\rho\triangleq\rho^{HL}=\rho^{LH}$.

For firms who choose vertical integration, the mismatch loss is same as the simple model, and we have

$$\tilde{J}_{VI}=\frac{\tau\tilde{z}}{1-\beta(1-\delta)(1-2\rho)} \tag{2.9.10}$$

Plug equations 2.9.9 and 2.9.10 into equation 2.9.8, we get

$$\begin{aligned}\eta_i^* &= \frac{\beta(1-\delta)\tilde{z}}{2}\left[\frac{\rho}{1-\beta(1-\delta)(1-2\rho)} - \frac{(1-\nu)\rho}{1-\beta(1-\delta)[1-2\rho(1-\nu)]}\right] \\ &= \frac{\beta(1-\delta)\tilde{z}}{2}\cdot\frac{\rho\nu[1-\beta(1-\delta)]}{[1-\beta(1-2\rho)]\cdot\{1-\beta(1-\delta)[1-2\rho(1-\nu)]\}}\end{aligned}$$

Denote

$$g(\rho,\nu) = \frac{\rho\nu}{2[1-\beta(1-2\rho)]\cdot\{1-\beta(1-\delta)[1-2\rho(1-\nu)]\}}$$

Therefore,

$$\eta_i^* = \beta(1-\delta)\tilde{z}g(\rho,\nu)$$

With tedious algebra, we can show that

$$\frac{\partial g(\rho,\nu)}{\partial\rho} = \frac{\nu\left[1-\beta(1-\delta)+\beta^2(1-\delta)^2\left(1-4(1-\nu)\rho^2\right)\right]}{[1-\beta(1-2\rho)]^2\cdot\{1-\beta(1-\delta)[1-2\rho(1-\nu)]\}^2}$$

and

$$\frac{\partial g(\rho,\nu)}{\partial\nu} = \frac{\rho\left[1-\beta(1-\delta)(1-4\rho)+\beta^2(1-\delta)^2\left(1-4\rho+4\rho^2\right)\right]}{[1-\beta(1-2\rho)]^2\cdot\{1-\beta(1-\delta)[1-2\rho(1-\nu)]\}^2}$$

In the data, ρ is never larger than 0.2, hence we have

$$\frac{\partial g(\rho,\nu)}{\partial\rho} > 0$$

and

$$\frac{\partial g(\rho,\nu)}{\partial\nu} > 0$$

□

Proposition 5

In industry i, a firm would choose vertical integration if $\eta > \eta_i^*$, sourcing if $\eta < \eta_i^*$. The share of firms that choose vertical integration is $1 - F(\eta_i^*)$.

Proof. From a firm' value functions, it can be shown that

$$\begin{aligned} \eta &\lesseqgtr \eta^* = \frac{\nu\beta(1-\delta)\rho_j^{HL}\cdot\tilde{J}_i}{\tau} \\ &\Leftrightarrow \\ \frac{1}{2}\left(J_{i,VI}^{HH} + J_{i,VI}^{LL}\right) &\lesseqgtr \frac{1}{2}\left[J_{i,SC}^{HH}(\eta) + J_{i,SC}^{LL}(\eta)\right] \end{aligned}$$

□

2.9.2 *Proof for results in section 4.4*

In this subsection, I will use the following notations

$\tilde{TS}_{i,l}(\eta) = TS_{i,l}^{HH}(\eta) + TS_{i,l}^{LL}(\eta) - TS_{i,l}^{LH}(\eta) - TS_{i,l}^{HL}(\eta)$

$\tilde{z}_i = \left(z_i^{HH}\right)^{\frac{1}{1-\alpha}} - \left(z_i^{HL}\right)^{\frac{1}{1-\alpha}} - \left(z_i^{LH}\right)^{\frac{1}{1-\alpha}} + \left(z_i^{LL}\right)^{\frac{1}{1-\alpha}}$

$\bar{z}_i = \frac{\left(z_i^{HH}\right)^{\frac{1}{1-\alpha}} + \left(z_i^{LL}\right)^{\frac{1}{1-\alpha}}}{2}$

$i \in \{A, B\},\ l \in \{VI, SC\}$

As sets A and B are symmetric, to save space I will drop the subscript i in some cases.

Proposition 7

In industry i and in period t, a single firm would choose vertical integration if $\eta \geqslant \eta_{i,t}^*$, sourcing if $\eta < \eta_{i,t}^*$. The threshold is determined by

$$\eta_t^* = \beta(1-\delta)\left(\rho_{A,t} + \rho_{B,t}\right)E_t\left[\tilde{TS}_{i,VI,t+1} - (1-\nu)\tilde{TS}_{i,SC,t+1}(\eta_t^*)\right]$$

Proof. For firms choosing VI, it is easy to show that

$$\frac{TS_{VI,t}^{HH} + TS_{VI,t}^{LL}}{2} = \bar{z}_t + \frac{1}{2}\beta(1-\delta)E_t\begin{bmatrix} TS_{VI,t}^{HH} + TS_{VI,t}^{LL} \\ -\rho_t\tilde{TS}_{VI,t+1} \end{bmatrix}$$

Also, it can be shown that

$$\tilde{TS}_{VI,t} = \tilde{z}_t + \beta(1-\delta)\left[1-2\left(\rho_{A,t}+\rho_{B,t}\right)\right]E_t\left(\tilde{TS}_{VI,t+1}\right)$$

For firms choosing SC

$$\frac{TS^{HH}_{SC,t}(\eta)+TS^{LL}_{SC,t}(\eta)}{2} = (\bar{z}_t-\eta_t)+\frac{1}{2}\beta(1-\delta)E_t\begin{bmatrix} TS^{HH}_{SC,t+1}(\eta)+TS^{LL}_{SC,t+1}(\eta) \\ -\left(\rho_{A,t}+\rho_{B,t}\right)(1-\nu)\tilde{TS}_{SC,t+1} \end{bmatrix}$$

It can be shown that

$$\tilde{TS}_{SC,t} = \tilde{z}+\beta(1-\delta)\left[1-2\left(\rho_{A,t}+\rho_{B,t}\right)(1-\nu)\right]E_t\left(\tilde{TS}_{SC,t+1}\right)$$

The threshold is determined by

$$TS^{H}_{SC,t}(\eta)+TS^{L}_{SC,t}(\eta) = TS^{H}_{VI,t}+TS^{L}_{VI,t} \tag{2.9.11}$$

or equivalently

$$\begin{aligned} & p^{HH}_{SC,t}E_t\left(TS^{HH}_{SC,t+1}\right)+\left(1-p^{HH}_{SC,t}\right)E_t\left(TS^{H}_{SC,t+1}\right) \\ & +p^{LL}_{SC,t}E_t\left(TS^{LL}_{SC,t+1}\right)+\left(1-p^{LL}_{SC,t}\right)E_t\left(TS^{L}_{SC,t+1}\right) \\ & = p^{HH}_{VI,t}E_t\left(TS^{HH}_{VI,t+1}\right)+\left(1-p^{HH}_{VI,t}\right)E_t\left(TS^{H}_{VI,t+1}\right) \\ & +p^{LL}_{VI,t}E_t\left(TS^{LL}_{VI,t+1}\right)+\left(1-p^{LL}_{VI,t}\right)E_t\left(TS^{L}_{VI,t+1}\right) \end{aligned}$$

In PAM equilibrium, $p^{HH}_{SC,t} = p^{LL}_{SC,t} = p^{HH}_{VI,t} = p^{LL}_{VI,t}$

To have analytical solution, I make the assumption that

$$E_t\left(TS^{j}_{l,t+1}(\eta)\right) \approx TS^{j}_{l,t}(\eta)$$

Equation 2.9.11 can be approximated by

$$TS^{HH}_{SC,t}(\eta)+TS^{LL}_{SC,t}(\eta) = TS^{HH}_{VI,t}+TS^{LL}_{VI,t} \tag{2.9.12}$$

By solving equation 2.9.12 with the results above, the threshold of transaction cost is determined by

$$\eta_t^* = \beta(1-\delta)E_t\left[\left(\rho_{A,t}+\rho_{B,t}\right)\tilde{TS}_{VI,t+1} - (1-\nu)\left(\rho_{A,t}+\rho_{B,t}\right)\tilde{TS}_{SC,t+1}\right]$$

□

Proposition

The free entry condition of set i is

$$\lambda_t \chi = \beta f(\theta_{i,t}) E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[\frac{\frac{\partial J_i}{\partial n^H_{i,VI,t+1}} + \frac{\partial J_i}{\partial n^L_{i,VI,t+1}}}{2} + F(\eta_t^*)\Delta\hat{J}_{i,t+1}\right]\right\}$$

with

$$\Delta\hat{J}_{i,t} = -\tau\hat{\eta}_t + \beta(1-\delta)E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[\begin{array}{c}\Delta\hat{J}_{i,t+1}\\ -(1-\nu)\left(\rho_{A,t}+\rho_{B,t}\right)\left(\tilde{J}_{i,SC,t+1}+\tilde{J}_{i,VI,t+1}\right)\end{array}\right]\right\}$$

$$\hat{\eta}_t \triangleq \frac{\int_{-\infty}^{\eta_t^*}\eta\, dF(\eta)}{F(\eta_t^*)}$$

Proposition 9

The positive assortative matching is Nash equilibrium if

$$\left(\frac{\frac{\partial J_A}{\partial n^{HH}_{A,l}(\eta)} - \frac{\partial J_A}{\partial n^{H}_{A,l}(\eta)}}{\frac{\partial J_A}{\partial n^{HL}_{A,l}(\eta)} - \frac{\partial J_A}{\partial n^{H}_{A,l}(\eta)}}\right)^{\frac{1}{\alpha_2}} \times \left(\frac{\frac{\partial J_B}{\partial n^{LH}_{B,l}(\eta)} - \frac{\partial J_B}{\partial n^{L}_{B,l}(\eta)}}{\frac{\partial J_B}{\partial n^{LL}_{B,l}(\eta)} - \frac{\partial J_B}{\partial n^{L}_{B,l}(\eta)}}\right)^{\frac{1}{1-\alpha_2}} > 1$$

and

$$\left(\frac{\frac{\partial J_B}{\partial n^{HH}_{B,l}(\eta)} - \frac{\partial J_B}{\partial n^{H}_{B,l}(\eta)}}{\frac{\partial J_B}{\partial n^{HL}_{B,l}(\eta)} - \frac{\partial J_B}{\partial n^{H}_{B,l}(\eta)}}\right)^{\frac{1}{1-\alpha_2}} \times \left(\frac{\frac{\partial J_A}{\partial n^{LH}_{A,l}(\eta)} - \frac{\partial J_A}{\partial n^{L}_{A,l}(\eta)}}{\frac{\partial J_A}{\partial n^{LL}_{A,l}(\eta)} - \frac{\partial J_A}{\partial n^{L}_{A,l}(\eta)}}\right)^{\frac{1}{\alpha_2}} > 1$$

Proof. I only need to show that no H type firms want to search for L type partners. I will show this for the firms in set A. Firms in set B have the same result.

For the H type single firm in set A choosing cooperation contract l, the expected gain of searching for H type partner is

$$\frac{\tilde{M}\left(n_{A,l}^{H}, n_{B,l}^{H}\right)}{n_{A,l}^{H}}\left(\frac{\partial J_A}{\partial n_{A,l}^{HH}(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}\right)$$

where $\frac{\tilde{M}\left(n_{A,l}^{H}, n_{B,l}^{H}\right)}{n_{A,l}^{H}}$ is the probability of matching with a H type partner; $\left(\frac{\partial J_A}{\partial n_{A,l}^{HH}(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}\right)$ is the marginal benefit conditional on matching with a H type partner.

Similarly, for the L type single firm in set B, the expected gain of searching for L type partner is

$$\frac{\tilde{M}\left(n_{A,l}^{L}, n_{B,l}^{L}\right)}{n_{B,l}^{L}}\left(\frac{\partial J_B}{\partial n_{B,l}^{LL}(\eta)} - \frac{\partial J_B}{\partial n_{B,l}^{L}(\eta)}\right)$$

The Nash equilibrium is that firms search for same type partner only. Now assuming that a infinitesimal measure Δ_A^H of H firms in set A declare that they are searching for L type partner in set B. Knowing this, a certain measure of L type firms in set B will attempt to match with those H type firms; the measure is pinned down by the condition under which they are indifferent between matching with H and L. Specifically, the measure of L type firms in set B who commit deviating, Δ_B^L, is determined by

$$\frac{\tilde{M}\left(\Delta_A^H, \Delta_B^L\right)}{\Delta_B^L}\left(\frac{\partial J_B}{\partial n_{B,l}^{LH}(\eta)} - \frac{\partial J_B}{\partial n_{B,l}^{L}(\eta)}\right) = \frac{\tilde{M}\left(n_{A,l}^{L}, n_{B,l}^{L}\right)}{n_{B,l}^{L}}\left(\frac{\partial J_B}{\partial n_{B,l}^{LL}(\eta)} - \frac{\partial J_B}{\partial n_{B,l}^{L}(\eta)}\right) \tag{2.9.13}$$

Given the measure of L type firms in set B attempting to match with H type firms in set A, the H type firms in set A who search for L type partner expect to gain

$$\frac{\tilde{M}\left(\Delta_A^H, \Delta_B^L\right)}{\Delta_A^H}\left(\frac{\partial J_A}{\partial n_{A,l}^{HL}(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}\right)$$

Now I want to show that the above expected gain is less than the expected gain of searching for H type partner; that is,

$$\frac{\tilde{M}\left(\Delta_A^H,\Delta_B^L\right)}{\Delta_A^H}\left(\frac{\partial J_A}{\partial n_{A,l}^{HL}(\eta)}-\frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}\right)<\frac{\tilde{M}\left(n_{A,l}^H,n_{B,l}^H\right)}{n_{A,l}^H}\left(\frac{\partial J_A}{\partial n_{A,l}^{HH}(\eta)}-\frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}\right) \tag{2.9.14}$$

As $n_{A,l}^H=n_{A,l}^L=n_{B,l}^H=n_{B,l}^L$ in PAM equilibrium. It is easy to show that $\frac{\tilde{M}\left(n_{A,l}^L,n_{B,l}^L\right)}{n_{B,l}^L}=\frac{\tilde{M}\left(n_{A,l}^H,n_{B,l}^H\right)}{n_{A,l}^H}=\psi$, and denote $\theta_{dev}=\frac{\Delta_B^L}{\Delta_A^H}$, we can rewrite equation 2.9.13 and inequation 2.9.14 as

$$\begin{aligned}
\psi\left(\theta_{dev}\right)^{\alpha_2-1}\left(\frac{\partial J_B}{\partial n_{B,l}^{LH}(\eta)}-\frac{\partial J_B}{\partial n_{B,l}^{L}(\eta)}\right) &= \psi\left(\frac{\partial J_B}{\partial n_{B,l}^{LL}(\eta)}-\frac{\partial J_B}{\partial n_{B,l}^{L}(\eta)}\right)\\
\psi\left(\theta_{dev}\right)^{\alpha_2}\left(\frac{\partial J_A}{\partial n_{A,l}^{HL}(\eta)}-\frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}\right) &< \psi\left(\frac{\partial J_A}{\partial n_{A,l}^{HH}(\eta)}-\frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}\right)
\end{aligned}$$

which is equivalent with

$$\begin{aligned}
\theta_{dev} &= \left(\frac{\frac{\partial J_B}{\partial n_{B,l}^{LL}(\eta)}-\frac{\partial J_B}{\partial n_{B,l}^{L}(\eta)}}{\frac{\partial J_B}{\partial n_{B,l}^{LH}(\eta)}-\frac{\partial J_B}{\partial n_{B,l}^{L}(\eta)}}\right)^{\frac{1}{\alpha_2-1}}\\
\theta_{dev} &< \left(\frac{\frac{\partial J_A}{\partial n_{A,l}^{HH}(\eta)}-\frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}}{\frac{\partial J_A}{\partial n_{A,l}^{HL}(\eta)}-\frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}}\right)^{\frac{1}{\alpha_2}}
\end{aligned}$$

We immediately see that the inequation of proposition implies the above condition.

□

2.10 Data

Data for section 4.2

I obtain annual firm-level balance sheet information from Compustat for North America. My base Compustat sample covers the period 1960-2013 for 112 3-digit industries, and consists of 31069 publicly-traded firms who have NAICS code. I define profit as Earnings before Interest, Taxes and Amortization (Ebita). Profit margin is defined as the ratio of Ebida to sales. Sales growth is the growth rate of sales. For each two consecutive years, I keep the panel balanced by disregarding delist and enlist of firm in the two years.[39] In the balanced panel for each two consecutive years, each firm-year observation is categorized into H type or L type according to its position in the profit/profit margin distribution of 3-digit NAICS industry the firm belongs to: a firm is H type if its annual profit is above median; L type if below median.[40] A rotation occurs when a firm switches its type in consecutive years. Rotation rate is the ratio of number of rotation to the number of firms.

I obtain industry value added series constructed by BEA, which are used for the weights to construct the aggregate rotation rate. Yearly industry value added is available only for 1-digit industries (sector), while value added for 2-digit industries are only available every five years in Input-Output (I-O) Accounts Data. I impute value added of 2 digit industries by treating their shares of value added within each 1-digit industry as constant overtime. In particular, value added of industry i in sector j is

$$Value\ added_{i,j,t} = Value\ added_{j,t} \times S_{i,j}$$

where $S_{i,j}$ is the share of value added of industry i in sector j computed from 2007 Input-Output (I-O) table constructed by BEA.

In the regression analysis shown in tables 4.1 and 4.2, I use annual series of aggregate unemployment rate from 1970 to 2013, obtained from Current Employment Statistics (CES). I construct industry employment growth from the annual series of full-time equiv-

[39] Most of enlist and delist do not carry information on turbulence of firms' rankings. While delist due to bankruptcy and insolvency may represent a permanent rotation that is not included in my measurement, it comprise less than 3 percents of delist, which is quantitatively insignificant for my analysis.

[40] As Ebita is available only in the annual dataset, in each year, I rank firms by their profit and profit margin using annual series of Ebida and sales, then compute the annual rotation rates.

alent employees by industry from 1998 to 2013[41], obtained from National Income and Product Accounts (NIPAs) constructed by BEA. In the industry panel regression, I use the private and non-farm 3 digit NAICS industries that (1) show up in both Compustat and NIPA, and (2) have more than 8 firms in Compustat over the sample periods.

Data for section 4.4

For the estimation, I use quarterly observations on eight data series from 1969 Q1 to 2013 Q4: aggregate unemployment, aggregate job openings rate, growth rate of real consumption per capita, growth rate of real investment per capita, growth rate of real per-hour wage, real interest rate, and rotation rate for the two sectors.

The benchmark model have two sets/sectors. The real economy, however, contains much more sectors, even according to coarsest categorization. To conduct estimation, I need to construct empirical counterpart of the two sector model. I categorize the two-digit NAICS industries into two categories: industries that are more production related, including Mining and logging, Construction, Manufacturing, and industries that are more productive service related, including Trade transportation and utilities, Information, Financial activities, Professional and business services. The former category consists 19 percent of non-government employment while the latter consists 48 percent. Four industries which are inappropriate to fit into either category are dropped: Education and health services, Leisure and hospitality, Other services, and Government. I construct the two annual series of sectoral rotation rates by averaging rotation rates at the 3 digit NAICS industry level weighted by their valued added. Then I interpolate them into quarterly series.

For the other observable variables, I use Quarterly aggregate unemployment rate is from Current Employment Statistics (CES). Quarterly job openings are obtained from HWOL as described by Barnichon (2010). I do not use sectoral unemployment rate and job openings because it is nontrivial to identify the sector the unemployed workers belong to. Real interest rate series are constructed by NY Fed. The other series of macro aggregates

[41] Starting from 1998, BEA changed the classification of industries, hence I only use the series after that.

are extracted from National Income and Product Accounts (NIPAs).

2.11 Churning of rankings in consumer's preference

In the main part of the model, I assume that firms are ranked by TFP. However, the mechanism is not restricted by churning of rankings in TFP. When firms are ranked by consumer's preference on them and churning is induced by a shift in consumer's preference, one can get the similar result.

In this subsection, I briefly describe the case in which churning is caused by a shift in consumer's preference.

Same as the benchmark model, households derives utility from consumption and disutility from working. What's different here is that now households' consumption basket comprises of continuum of non-durable goods which are not perfectly substitutable to each other:

$$C_t = \left(\int \psi(j) \cdot q(j)^{\rho} \, dj \right)^{\frac{1}{\rho}} \tag{2.11.1}$$

where $\psi(j)$ is households' preference on goods j.

In each period, representative households solve the following static problem

$$\max \left(\int \psi(j) \cdot q(j)^{\rho} \, dj \right)$$

$$s.t. \int p(j) \cdot q(j) \, dj = C$$

where $p(j)$ is goods j's price and C is nominal consumption.

Define $\sigma = \frac{1}{1-\rho}$ and $P = \left(\int \psi(j)^{\sigma} \cdot q(j)^{1-\sigma} \, dj \right)^{\frac{1}{1-\sigma}}$, it's easy to derive the demand curve from the first order condition of the above problem.

$$\frac{q(j)}{c} = \left(\frac{p(j)/\psi(j)}{P} \right)^{-\sigma} \tag{2.11.2}$$

where c is real consumption with $c = C/P$. Notice that equation 2.11.2 is similar to the classic result of Dixit and Stiglitz (1977) except that the price here is adjusted by preference: given the same level of sales, goods with higher ψ charges a higher price because households gets higher preference from it. In the subsequent part, I will show how churning of ψ can play a similar role of churning of productivity.

Same as benchmark model, assume there are two sets of firms, each set has two type of firms: H and L. Firms can match with firms from the other set. Their product's preference ψ not only depends on their own type, but also on their cooperation partner's type. So within each set there are six production departments: j can be H, L, HH, HH, HH, HH, that is within in each set there are six levels of preference $\psi(H)$, $\psi(L)$, $\psi(HH)$, $\psi(HL)$, $\psi(LH)$, $\psi(LL)$

Firms' value function is similar to that of benchmark model, except here firms have same TFPs but have different preference ψ. In particular, type j firm's sales is

$$\begin{aligned} \frac{p(j)q(j)}{P} &= \left(\frac{q(j)}{c}\right)^{-\frac{1}{\sigma}} \psi(j)q(j) \qquad (2.11.3) \\ &= c^{\frac{1}{\sigma}} \psi(j)q(j)^{1-\frac{1}{\sigma}} \end{aligned}$$

Plug the production function $q(j) = zk(j)^{\alpha} n(j)^{1-\alpha}$ into the above equation and take first order conditions, it can be solved that

$$MPL(j) = \left(\psi(j)q(j)^{-\sigma} c^{\alpha+\sigma} z\alpha^{\alpha}\right)^{\frac{1}{1-\alpha}} \qquad (2.11.4)$$

It can be seen that $\psi(j)$ affects $MPL(j)$ in the same way as TFP and proposition 5 applies to this case.

2.12 Alternative assumptions of ordering of events

In the simple model, I assume that the matching process proceeds in the following steps:

1. Homogeneous firms pay fixed cost.
2. Homogeneous firms and homogeneous workers are matched in the labor market.
3. Homogeneous firms draw types from H and L hence become heterogeneous.
4. Heterogeneous firms match with same type partners from the other set.

I denote the above ordering of events as $T1$. Some might ask whether the main results of section 4.3, particularly proposition 3, depend on the assumption that firms draw types *after* matching with worker. In this subsection, I show that the main results are robust to change of ordering of events.

Consider the following ordering of events:

1. Homogeneous firms pay fixed cost.
2. Homogeneous firms draw types from H and L hence become heterogeneous.
3. Heterogeneous firms and homogeneous workers are matched in the labor market.
4. Heterogeneous firms match with same type partners from the other set.

I denote the above ordering of events as $T2$. Different with $T1$, $T2$ assumes that firms match with worker after drawing types.

As production takes place after the matching process, value function does not depend on the ordering of events in the matching process. Take set A as example, value of firms is described by equation 2.2 in section 4.3

$$\begin{aligned} J_{A,t}^{jk} &= \tau \cdot x_t \cdot z_{A,t}^{jk} + \beta (1-\delta) E_t \left(\rho_t^{jH} J_{A,t+1}^{Hk} + \rho_t^{jL} J_{A,t+1}^{Lk} \right) \\ j,\, k &\in \{H,\, L\} \end{aligned}$$

In $step$ 2 of $T2$, denote the value of drawing type H and L as $\tilde{J}^H_{A,t}$ and $\tilde{J}^L_{A,t}$. We immediately get

$$\tilde{J}^H_{A,t} = f\left(\theta_{A,t}\right) J^{HH}_{A,t} \tag{2.12.1}$$

$$\tilde{J}^L_{A,t} = f\left(\theta_{A,t}\right) J^{LL}_{A,t} \tag{2.12.2}$$

The free entry condition of set A, which takes place in $step$ 1 of $T2$, is

$$\chi = \frac{\tilde{J}^H_{A,t}}{2} + \frac{\tilde{J}^L_{A,t}}{2} \tag{2.12.3}$$

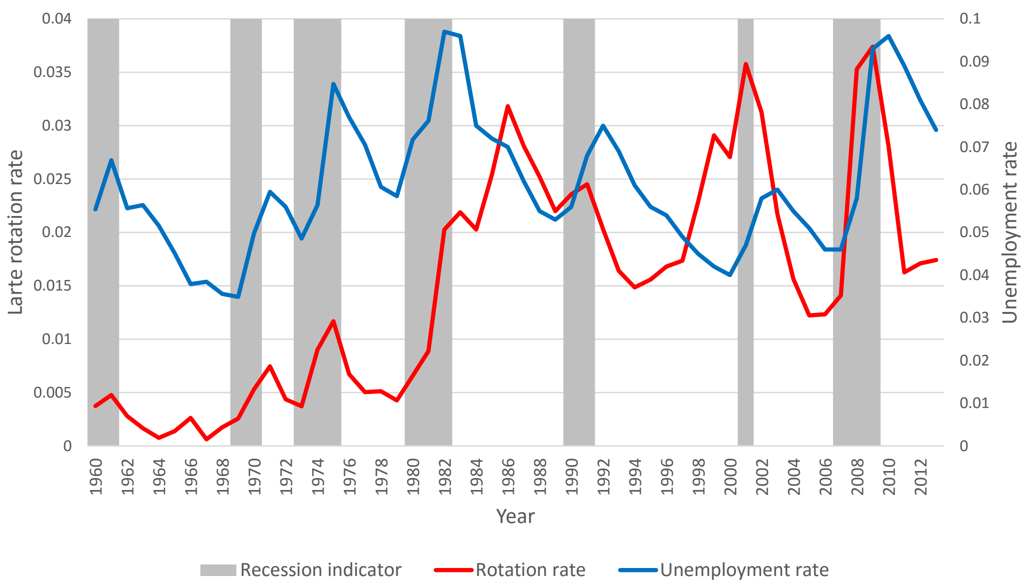
Plug equation 2.2, equation 2.12.1 and equation 2.12.2 into equation 2.12.3, the free entry condition of set A becomes

$$\begin{aligned} \chi = & \frac{f\left(\theta_{A,t}\right)\left[\tau \cdot x_t \cdot z^{HH}_{A,t} + \beta\left(1-\delta\right)E_t\left(\rho^{HH}_t J^{HH}_{A,t+1} + \rho^{HL}_t J^{LH}_{A,t+1}\right)\right]}{2} \\ + & \frac{f\left(\theta_{A,t}\right)\left[\tau \cdot x_t \cdot z^{LL}_{A,t} + \beta\left(1-\delta\right)E_t\left(\rho^{LH}_t J^{HL}_{A,t+1} + \rho^{LL}_t J^{LL}_{A,t+1}\right)\right]}{2} \end{aligned} \tag{2.12.4}$$

It is easy to see that equation 2.12.4, which is the free entry condition under $T2$, is identical with the free entry condition under $T1$. Therefore, the models under orderings of events $T1$ and $T2$ are equivalent, and should yield same results.

2.13 Additional figures

FIGURE 2.5: Large rotation rate, ranked by profit



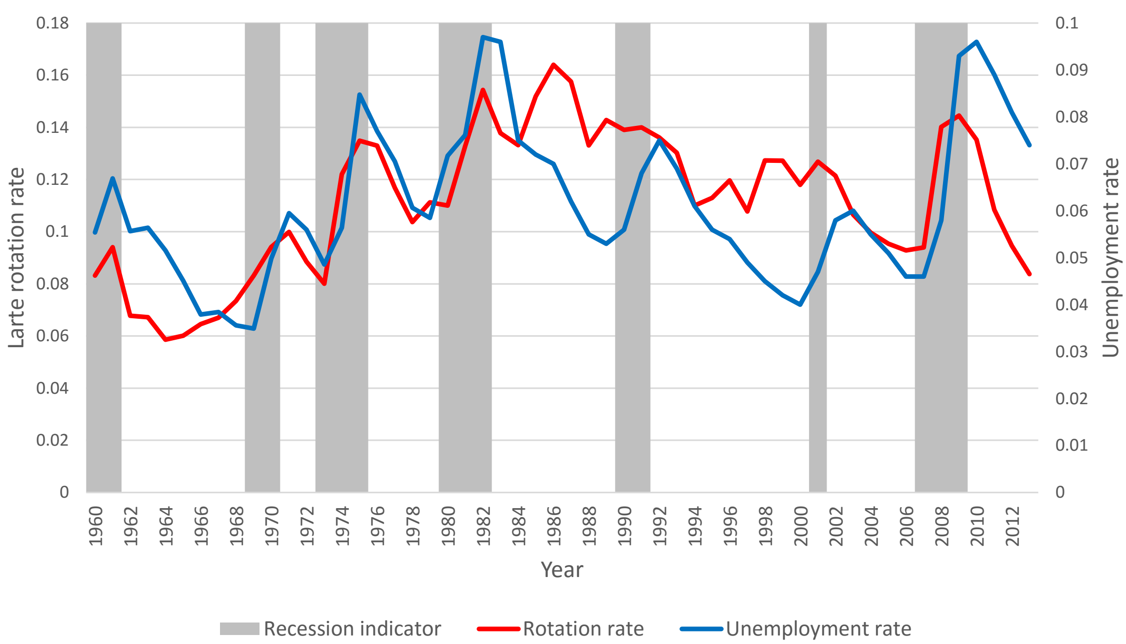


FIGURE 2.6: Rotation rate, ranked by profit margin

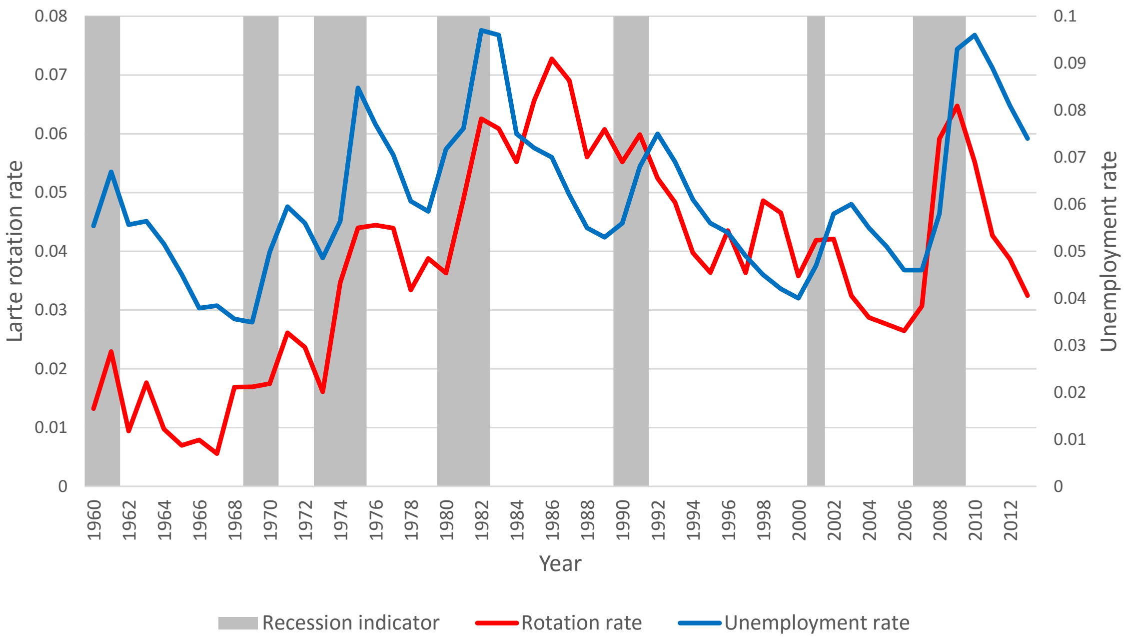


FIGURE 2.7: Large rotation rate, ranked by profit margin

3

Inter-sector Matching Efficiency and Sectoral Comovement

3.1 Introduction

Comovement is a defining characteristic of the business cycle. As Christiano and Fitzgerald (2009) stated, *"Everyone is so conscious of the business cycle because most sectors of the economy move up and down together"*. Previous work mainly focuses on the comovement of labor hours and output. In section 3, I document that the job opening and job creation rates of main sectors in US economy are highly correlated as well. In other words, firms from different sectors tend to recruit in the similar pace. This fact, however, cannot be accounted by conventional DMP models which are propagated by productivity shocks. In the data, the labor productivity appears to be very weakly correlated across sectors, the conventional DMP models predict that the job creations are at most, weakly correlated across sectors.

In this chapter, I propose a two sector DMP model to study the comovement of job creation across sectors. The model has two novel features. First, there is inter-firm cooperation. Specifically, firms are more productive when cooperating with partners from the other sectors than working alone. The intuition is that each sector possesses a comparative advantage, and inter-firm cooperation across sectors induces the gain of specialization.

Second, there is search friction in the inter-firm matching market; that is, it takes time for firms to form a partnership, and the probability of finding a partner depends on the relative measure of idled firms in each sector. As the result, a firm's expected value depends on the availability of potential partners in the other sector. When the opportunity of inter-firm cooperation is scarce, a firm has lower expected value and hence less incentive to post job openings. In section 3, I calibrate the model with reasonable parameters and show that the model generates significant comovement of job openings. Specifically, even when total factor productivity (TFP) is independent across sectors, the sectoral job openings are still positively correlated.

While successfully generating comovement of job openings across sectors, my model fails to generate a high volatility of unemployment and job creation relative to labor productivity. In other words, same as the canonical DMP model, my benchmark model suffers from the Shimer puzzle (Shimer (2005)). To tackle the problem, in section 4, I modify the benchmark model with a time-varying inter-firm matching efficiency. I show that exogenous shocks to the inter-firm matching efficiency provide strong amplification effect to the labor market. Shocks to the inter-firm matching efficiency directly shift the success rate for firms to find a partner. With a lower inter-firm matching efficiency, firms in both sectors find it harder to match with a partner and hence expects a lower payoff of posting vacancies in the labor market. As the result, the aggregate job creation decreases. On the other hand, a decrease in the inter-firm matching efficiency reduces the share of firms who have access to the inter-firm cooperation, who have higher productivity. This composition effect induces an endogenous and gradual decline in the aggregate productivity. This decline in the aggregate productivity, however, is mild compared to the drop in the job creation.

In the next section, I will do the literature review. In section 2, I document several facts of U.S. labor market at the sector level. Then I develop a model to account for the facts in section 3. In section 4, I examine some analytical properties of the model. Then in section

5, I calibrate the model and conduct simulations around the steady state to examine its quantitative implication. Lastly, I conclude the chapter in section 7.

3.2 Measuring sectoral comovement

In this section, I will illustrate some cyclical properties of labor market of U.S. economy at the sector level. Most importantly, I show that there exists strong comovement among sectoral labor markets, while correlation between sectoral labor productivities is much weaker.

In order to demonstrate sectoral comovement, I measure the correlation of sectoral job openings. In specific, I calculate the correlation between the cyclical components of job openings of eleven (one-digit industries) in U.S. economy. For n sectors, there is a n-by-n correlation matrix, and the number of correlations (the off-diagonal entries) is $n(n-1)/2$. In my analysis, $n = 11$, hence there are 55 correlations. To illustrate the sectoral comovement, I first plot the business cycle components of job opennings of five largest sectors excluding government sector, that is, manufacturing, trade-transportation-utility, information, financial activities, and professional and business services. Then I plot the histogram of the correlations for all eleven one digit sectors.

For sectoral labor productivities, I conduct the same exercise. Labor productivity is not uniquely defined at the sector level. One popular measure of labor productivity is the ratio between total output and employment of that sector, for example Carvalho and Gabaix (2012). Another common measure is the ratio between sectoral value-added and employment, such as in Sahin et. al (2013). For robustness, I use both methods of measure.

There are two points that are worthnoting. First, I do not measure sectoral comovement as the correlation of sectoral variable with aggregate variable, which is quite common in the literature. As pointed out by Christiano and Fitzgerald (2003), the 'beta-type' statistics have shortcoming: suppose a variable y_t is the sum of two independent variables $y_t =$

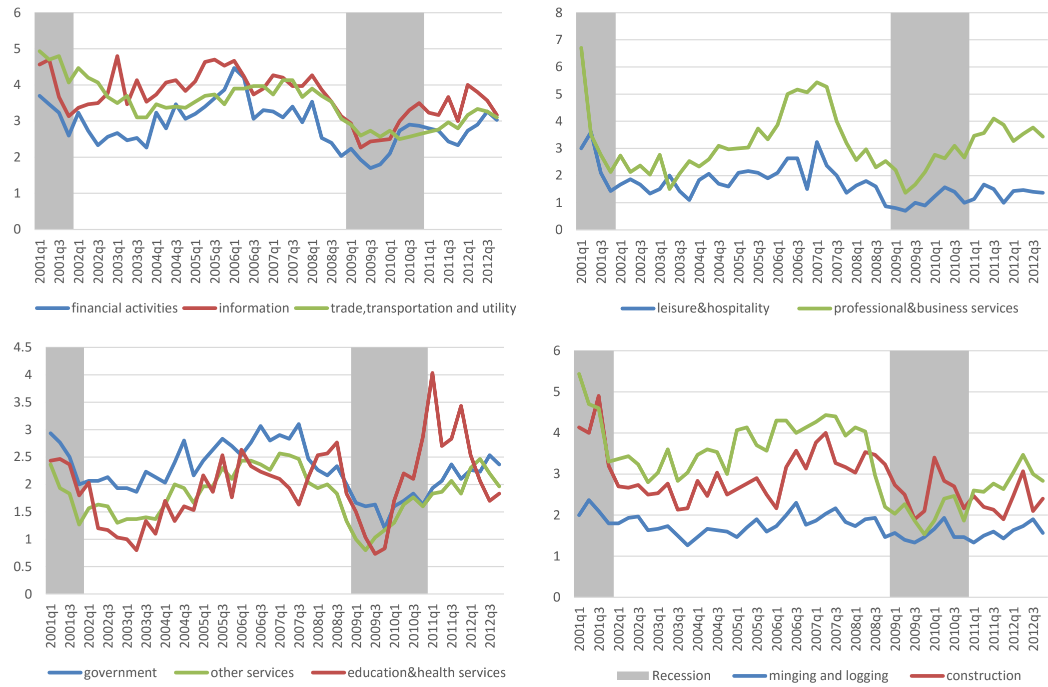


FIGURE 3.1: Job openings for major sectors

$y_{1t} + y_{2t}$. It's straightforward to show that both y_{1t} and y_{2t} are positively correlated with aggregate variable y_t. However, it does not imply there exist any correlation between y_{1t} and y_{2t}.

Second, my analysis uses job openings data instead of unemployment. While unemployment is a more crucial variable, its is not observed at the sector level, because we do not observe in which sector an unemployed worker is seeking job.

Figure 1 presents the cyclical component of job openings and labor productivity of five largest nongovernment sectors of U.S. economy from 2001 to 2012. The comovement of job openings across sectors is quite evident in the left panel. All five sectors suffer from a sharp decline in job openings in the first two years, possibly caused by the bust of the dot-com bubble. They then enter a phase of increasing hirings until the onsite of the financial crisis and economic recession. On the other hand, however, synchronization in the labor productivities is much weaker, if there exists any. Most particularly, *financial activities*

FIGURE 3.2: labor productivities for major sectors

sector and *professional and business service* sector have almost opposite cyclical labor productivity processes.

Figure 2 presents the histogram of inter-sector correlations of job openings and labor productivity for 11 major sectors of U.S. economy. The upper panel shows the distribution of correlations of job openings. Visually, most of the correlations congest between 0.5 and 0.9, and all of them are positive, which illustrates the strong comovement of job openings. The middle and bottom panel shows the distribution of correlations between labor productivity while labor productivity in the middle panel is measured using total output and in the bottom panel using value-added. As displayed in the middle panel, a much smaller proportion of the distribution stay between 0.5 and 0.9 and a some are negative. It is even more evident in the bottom panel where labor productivity is measured by value-added: almost half of the distribution is negative, and there's a significant proportion is left to -0.5, which indicates that correlations between those sectoral labor productivities are strongly negative.

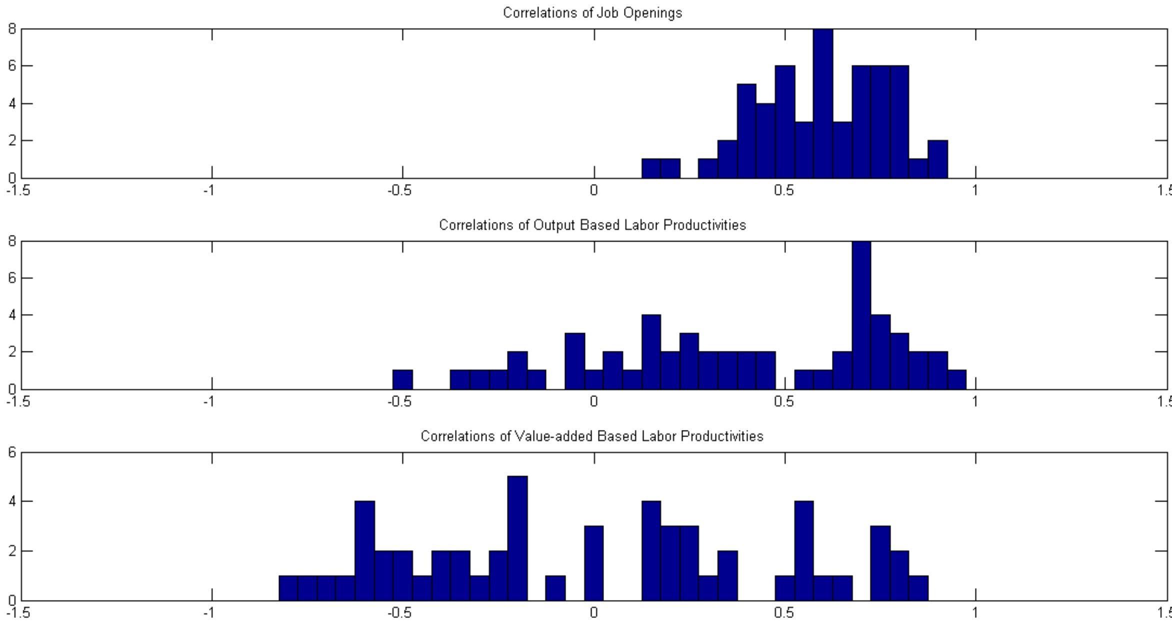


FIGURE 3.3: Histogram of correlations of job openings between sectors

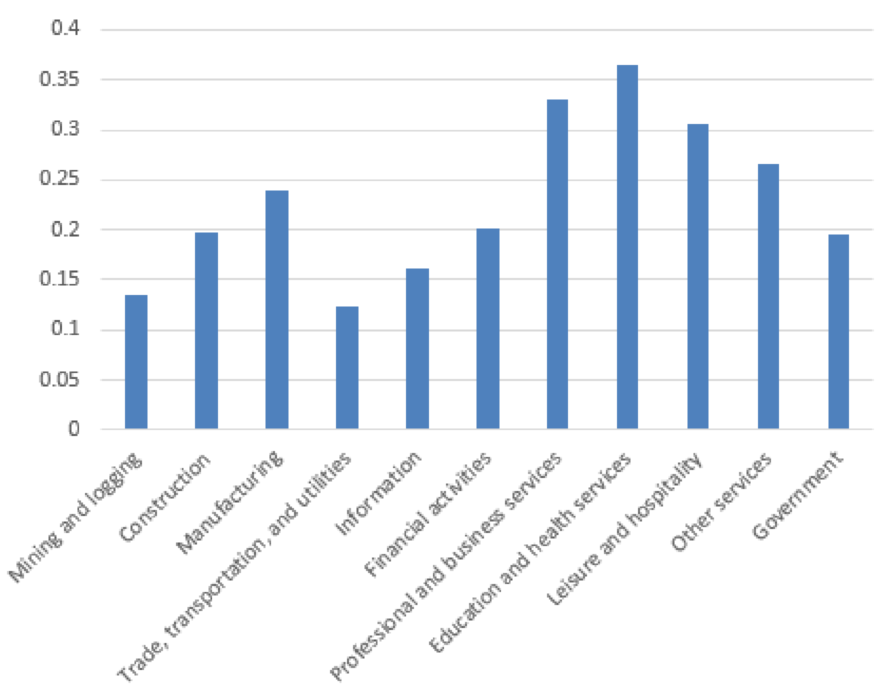


FIGURE 3.4: Standard deviations of job openings for major sectors

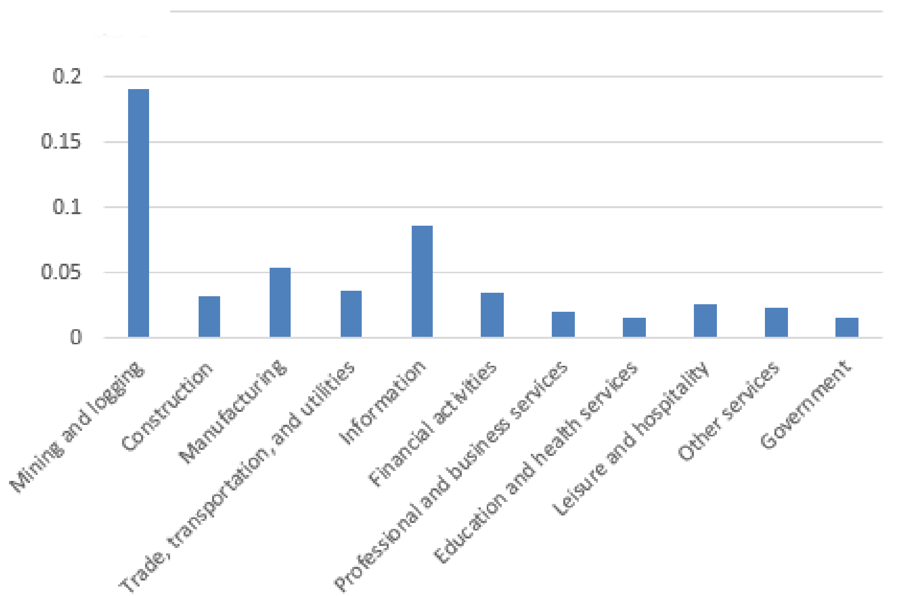


FIGURE 3.5: Standard deviations of labor productivity for major sectors

Figure 3 A and Figure 3 B show the standard deviations of (log) job openings and (log) labor productivity of 11 major sectors in U.S. economy. Both figures reflect high degree of heterogeneity across sectors. For example, as shown in Figure 3 A, *professional and business service* and *education and health service* have largest fluctuation in job openings, with volatility around 35%. In contrast, *trade transportation and utility* has very mild variation in job openings, with volatility lower than 15%. The heterogeneity of sectoral labor productivity is even more striking. For instance, as shown in Figure 3 B, *mining and logging* has the highest volatility in labor productivity, which is approaching 20%. Several other sectors such as education and health services have extremely smooth labor productivity process, with volatility smaller than 2%.

The above pattern clearly illustrates the necessity of a multisector model. One of the central questions in DMP literature is to explain the large fluctuation of job openings and unemployment propagated by mild productivity shocks. Shimer (2005) documented that in U.S. data, the aggregate volatility of job openings is about ten times as the volatility of aggregate labor productivity. He also illustrated that conventional DMP model delivers a volatility of job openings relative to the productivity of 1.3, one magnitude lower than data. A large amount of work has been devoted to studying amplification mechanisms that can help DMP model to fit data closer. Some of them successfully match to the aggregate moment, most notably Hagedorn and Manovskii (2008) and Hall (2005). However, the sectoral heterogeneity illustrated by figure 3 A and 3 B casts some doubt on those amplification mechanisms. For example, for sectors such as mining logging and information, conventional DMP model is more than enough to generate realistic labor market fluctuation. Then for what reason the amplification mechanisms become undesirable in these sectors? On the other hand, for sectors such as professional and business services, volatility of job openings is about 20 times larger than that of labor productivity, which means the amplification mechanisms are still not enough to match data.

3.3 Model

3.3.1 Baseline Environment

Time is discrete. The economy is comprised of two distinct sectors (labor markets) indexed by a and b. New production opportunities, corresponding to job vacancies (v^i) is created at cost χ. Each labor market is populated by a measure one of risk-neutral individuals who can be either employed in sector i (e^i) or unemployed and searching for a job (u^i).

In sector i, a firm's production function is $f^i\left(x^i, l, m^i\right)$: x^i is total factor productivity (TFP), l is labor input and m^i is intermediate input which is produced by firms in sector j. $f^i\left(x^i, l, m^i\right)$ is increasing in both l and m^i. For simplicity, following the canonical DMP model, firm can only employ one worker and labor hour is inelastic, thus l is either one or zero. In addition, intermediate goods m^i is also inelastic. When a firm has a partner, m^i equals one; otherwise it is zero.

Instead of purchasing labor and intermediate inputs in the competitive commodity market, firms access both labor and intermediate input through bilateral matching. The labor markets is standard. New matches (h^i) between unemployed worker (u^i) and vacancies (v^i) are determined by matching function $m\left(\phi^i, u^i, v^i\right)$, which is strictly increasing and strictly concave in u^i and v^i, and homogeneous of degree one in $\left(u^i, v^i\right)$. The ϕ^i measures matching efficiency in sector i. In both labor markets, existing firm-labor matches are destroyed exogenously at rate δ.

Inter-firm matching market is also frictional. Similar with the labor market, the inter-firm matching function is $n\left(\psi, s^i, s^j\right)$, while ψ is the inter-firm matching efficiency, and s^i and s^j are measure of single firms in each sector. Same as the matching between labor and worker, matching function n is strictly increasing and strictly concave in both arguments, and homogeneous of degree one in $\left(s^i, s^j\right)$. In the paper, I denote a firm without a partner as a single firm. A firm with a partner is denoted as cooperative firm. Single firms are subject to low productivity because they don't have access to the benefit of specialization.

Unemployed worker earn z for leisure and home production. I assume that in every state of the economy, productivity of any firm exceeds the value of leisure and home production, so there always exists bilateral gains from firm-worker matching. In all firm-worker matches, wage is determined by Nash bargaining.

In sum, events unfold as follows. At the beginning of the period, the aggregate states are observed. Then firms post vacancies in the labor markets and match with unemployed workers. Then the single firms search for partners in the inter-firm matching market. Production takes place after the matching processes. At the end of the period, firm-worker matches is destroyed randomly with probability δ.

3.3.2 Tightness Ratios

In this subsection, I define several key notations. Following the canonical DMP model, labor market tightness ratio θ^i is the ratio between number of vacancies and number of unemployed workers:

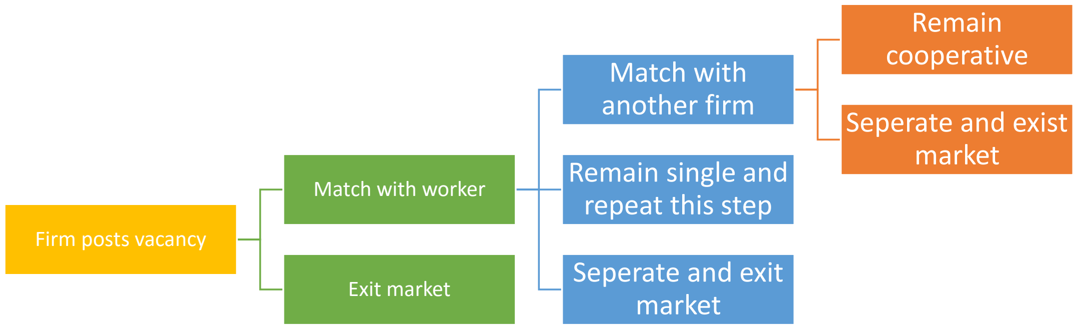


FIGURE 3.6: Timeline

$$\theta^i = \frac{v^i}{u^i},\ i = a, b$$

Job finding rate μ measures the probability for an unemployed worker to match with a vacancy. Vacancy filling rate q is the probability for a vacancy to contact an unemployed worker. Since matching function m is homogeneous of degree one, job finding rate and vacancy filling rate are functions of market tightness:

$$\mu^i = \mu^i\left(\theta^i\right) = \frac{\phi^i m\left(u^i, v^i\right)}{u^i}$$

$$q^i = q^i\left(\theta^i\right) = \frac{\phi^i m\left(u^i, v^i\right)}{v^i}$$

The notations in the inter-firm matching market is similar. Cooperation tightness is the ratio between number of single firms:

$$\tilde{\theta}^i = \frac{s^j}{s^i},\ i = a, b$$

Cooperation matching rate p measures the probability for a single firm in sector i to cooperate with a single firm in sector j is function of cooperation tightness:

$$p^i = p^i\left(\tilde{\theta}^i\right) = \frac{\psi n\left(s^i, s^j\right)}{s^i}$$

The number of unemployment and of single firm evolve according to

$$u^i_{t+1} = \delta\left(1 - u^i_t\right) + \left(1 - \mu^i_t\right) u^i_t \quad (3.3.1)$$

$$s^i_{t+1} = \mu^i_t u^i_t + \left(1 - p^i_t - \delta\right) s^i_t \quad (3.3.2)$$

Number of unemployment increases when existing match is destroyed with rate δ and decreases when unemployed worker get employed with job finding rate μ^i

Similarly, number of single firm increases when unemployed worker and vacancy match to form new single firm. It decreases either because single firm is destroyed with rate δ or single firm cooperate with single firm from another sector with rate p^i

3.3.3 First Stage: Search and Matching Between Firm and Worker

In this stage, firm posts vacancy to match with unemployed worker and unemployed worker search for matching with firm. Worker can be either unemployed or employed, the values of being unemployed and working in single firm are denoted as $\tilde{W}$ and U respectively. In sector i, values for employed and unemployed workers are

$$\tilde{W}^i_t = \tilde{w}^i_t + \beta E_t\left[p^i_t W^i_{t+1} + \delta U^i_{t+1} + \left(1 - p^i_t - \delta\right)\tilde{W}^i_{t+1}\right] \quad (3.3.3)$$

$$U^i_t = z + \beta E_t\left[\mu^i_t \tilde{W}^i_t + \left(1 - \mu^i_t\right) U^i_{t+1}\right] \quad (3.3.4)$$

The value of a worker who works in a single firm is composed of wage flow $\tilde{w}^i_t$ in this period and discounted expected value from next period on. In the next period, the single firm can become cooperative firm with probability p^i_t, hence the worker receives value W^i_{t+1}. With probability δ, the match is destroyed and worker becomes unemployed. Otherwise this worker will remain working in the single firm.

The value of an unemployed worker is flow of leisure and home production plus discounted expected value from next period on. In next period, he/she can be employed in a single firm with probability μ_t^i, otherwise this worker will remain being unemployed.

The value of single firm is denoted as $\tilde{J}$. In sector i, a single firm's value is

$$\tilde{J}_t^i = f^i\left(z_t^i, 1, 0\right) - \tilde{w}_t^i + \beta E_t\left[p_t^i J_{t+1}^i + \left(1 - p_t^i - \delta\right)\tilde{J}_{t+1}^i\right] \tag{3.3.5}$$

The value is composed of the net profit–the output minus the worker's wage–plus the expected discounted cash flow from next period on. In the next period, the single firm can become cooperative firm with probability $p^i(\tilde{\theta}_t^i)$. With probability δ, the match is destroyed and the firm receives zero value. Otherwise this firm will remain operating as a single firm.

The value of posting a vacancy in sector i is thus:

$$V_t^i = -\chi + \beta E_t\left[q_t^i\tilde{J}_{t+1}^i + \left(1 - q_t^i\right) max\left\{V_{t+1}^a, V_{t+1}^b, 0\right\}\right] \tag{3.3.6}$$

With vacancy filling rate q_t^i the firm can match a worker, otherwise it stays idle until next period, then the firm can choose to maintain the vacancy, enter the other sector, or exit the economy.

3.3.4 Second Stage: Search and Matching Between Single Firms

In the second stage, single firm search to cooperate with single firm from the other sector. When single firms cooperate with each other, they become cooperative firms and are more productive because now they enjoy intermediate goods service from each other.

The value of a worker who works in a cooperative firm is:

$$W_t^i = w_t^i + \beta E_t\left[(1-\delta)W_{t+1}^i + \delta U_{t+1}^i\right] \tag{3.3.7}$$

The worker receive wage flow w_t^i and expected discounted value from next period on. In the next period, with rate δ the match is destroyed and the worker becomes unemployed, otherwise he/she will remain working in this cooperative firm.

Similarly, the value of a cooperative firm is:

$$J_t^i = f^i\left(z_t^i, 1, 1\right) - w_t^i + \beta\left(1-\delta\right)E_t\left(J_{t+1}^i\right) \tag{3.3.8}$$

3.3.5 Equilibrium Conditions

An equilibrium is a collection of value functions, $U^i, W^i, J^i, \tilde{W}^i, \tilde{J}^i$,wage compensations, $w^i, \tilde{w}^i$, market tightness ratioθ^i, and cooperation tightness $\tilde{\theta}^i$, such that:

1. Workers are optimizing. That is, unemployed workers always search for job and wokers that are matched prefer to remain matched rather than being unemployed, $W^i > U^i, \tilde{W}^i > U^i$, and workers prefer to work in cooperative firm as opposed to single firm, $W^i > \tilde{W}^i$.

2. Firms are optimizing. That is, single firms always search for cooperation, $J^i > \tilde{J}^i$, and value of single firms equalize across sectors, $\tilde{V}^a = \tilde{V}^b$

3. Wage compensations solve the Nash bargaining problems:

$$\begin{aligned} \tilde{w}^i &= argmax\left(\tilde{W}^i - U\right)^{1-\tau}\left(\tilde{J}^i - V^i\right)^{\tau} \\ w^i &= argmax\left(W^i - U\right)^{1-\tau}\left(J^i - V^i\right)^{\tau} \end{aligned}$$

   Where τ denotes the bargaining weight of firms.

4. The free entry condition is satisfied. That is, $\tilde{V}^i = 0$

Let TS^i and $\tilde{TS}^i$ denote the total surplus from a match in single firm and cooperative firm:

$$\begin{aligned} TS^i &= W^i + J^i - U^i \\ \tilde{TS}^i &= \tilde{W}^i + \tilde{J}^i - U^i \end{aligned}$$

Under Nash bargaining, the worker and firm receive a constant, proportional share of the total surplus:

$$W^i - U^i = (1-\tau)\, TS^i \tag{3.3.9}$$

$$J^i = \tau TS^i \tag{3.3.10}$$

$$\tilde{W}^i - U^i = (1-\tau)\, \tilde{TS}^i \tag{3.3.11}$$

$$\tilde{J}^i = \tau \tilde{TS}^i \tag{3.3.12}$$

3.3.6 Analytical Results

In this subsection, I provide analytical results characterizing some key properties of the model. I begin by simplifying the model's equilibrium and characterizing the steady state equilibrium. Then I discuss the key differences between my model and the standard DMP model, and their implications for business cycle fluctuation.

3.6.1 Stationary Equilibrium

Proposition 10. (I) Equilibrium conditions 1-4 can be rewritten as the following equations

$$TS_t^i = f^i\left(z_t^i, 1, 1\right) - z + \beta E_t\left[(1-\delta)\, TS_{t+1}^i - \mu_t^i (1-\tau)\, \tilde{TS}_{t+1}^i\right] \tag{3.3.13}$$

$$\tilde{TS}_t^i = f^i\left(z_t^i, 1, 0\right) - z + \beta E_t\left\{p_t^i TS_{t+1}^i + \left[1 - p_t^i - \delta - (1-\tau)\, \mu_t^i\right] \tilde{TS}_{t+1}^i\right\} \tag{3.3.14}$$

$$\chi = \beta q_t^i \tau E_t\left(\tilde{TS}_{t+1}^i\right) \tag{3.3.15}$$

(II) $\tilde{TS}_t^i > TS_t^i$ always holds.

Proposition 1 (I) shows that with Nash bargaining, value functions of firm and worker can be summarized by total surplus; In addition, free entry condition implies equilibrium vacancy filling rate is determined by expected total surplus. Proposition 1 (II) indicates that it's always social optimal to pair single firms together as long as cooperative firm is more productive than single firm, that is $f^i\left(z_t^i, 1, 1\right) - f^i\left(z_t^i, 1, 0\right) > 0$.

Notice that the model has two sectors, we have two sets of equations demonstrated in Proposition 1. With Proposition 1, the model's equilibrium is now characterized by value functions $TS^i, \tilde{TS}^i$, market tightness θ^i and cooperation tightness $\tilde{\theta}^i$. By ignoring expectation operators and time subscripts of equation (13)-(15) and (1) (2), we immediately derive the steady state equilibrium of the model, which are summarized in Proposition 2.

Proposition 11. The steady state values of sector a. $TS^i, \tilde{TS}^i, \theta^i, \tilde{\theta}^i, u^i, s^i$, are solved by the following equations

$$
\begin{aligned}
f^i(z^i,1,1) - z &= [1-\beta(1-\delta)]TS^i + \beta\mu^i(1-\tau)\tilde{TS}^i && (3.3.16)\\
f^i(z^i,1,0) - z &= \left\{1-\beta\left[1-p^i-\delta-(1-\tau)\mu^i\right]\right\}\tilde{TS}^i - \beta p^i TS^i && (3.3.17)\\
x &= \beta q^i \tau \tilde{TS}^i && (3.3.18)\\
u^i &= \frac{\delta}{\mu^i+\delta} && (3.3.19)\\
s^i &= \frac{\mu^i u^i}{p^i+\delta} && (3.3.20)\\
\tilde{\theta}^i &= \frac{s^j}{s^i} && (3.3.21)
\end{aligned}
$$

Where the variables without time subscripts denote the steady state values.

Because there are two sectors in the model, we have two sets of steady state values and the above equations. Noticing by definition we have

$$\tilde{\theta}^a = \frac{s^b}{s^a} = \frac{1}{\tilde{\theta}^b}$$

One steady state value $\tilde{\theta}^i$ and one equation (21) are redundant. Therefore, we are left with eleven equations and eleven unknowns.

In order to showing existence and uniqueness of solution to the above equations in Proposition 3, I first present several useful results.

Lemma 2. Equations (16) (17) and (18) imply that θ^i is strictly increasing in z^i and $\tilde{\theta}^i$

The fact that the tightness ratio θ^i is increasing in the labor productivity z^i is consistent with the main prediction of DMP models: when productivity is higher, a firm finds it more profitable to hire worker. Firms would optimally post more vacancy which drives up the equilibrium level of tightness ratio θ^i. Lemma 4 shows that this prediction is robust in the modified model with inter-firm matching. The second part of lemma 4 shows that there is an increasing relationship between θ^i and $\tilde{\theta}^i$. The economic intuition is that a higher $\tilde{\theta}^i$ is due to the fact that the expected value of posting vacancy becomes higher when the productivity-upgrading opportunity (matching with a partner) is more plenty reflected by a higher cooperative tightness ratio $\tilde{\theta}^i$.

Lemma 3. Equations (19) (20) and (21) imply that s^i is strictly decreasing in θ^i and increasing in θ^j

This result simply derive from definition of cooperation tightness and accounting identity of unemployment and single firm.

Applying Lemma 3.2 to sector b, we get that s^b is strictly decreasing in θ^b and increasing in θ^a. Since $\tilde{\theta}^a$ is the ratio of s^b and s^a, we can further get:

Proposition 12. Equation (19) (20) and (21) imply that $\tilde{\theta}^a$ is strictly increasing in θ^a and strictly decreasing in θ^b

Lemma 3.3 implies that when fixing cooperation tightness $\tilde{\theta}^a$, market tightness of sector a θ^a is strictly increasing in market tightness of sector b θ^b

Now it's trivial to show the existence and uniqueness of the steady state. First, because there are 11 non-redundant equations and 11 non-redundant unknowns, solution must exist.

Second, Lemma 3.1 shows θ^a is strictly decreasing in θ^b and Lemma 3.3 shows θ^a is strictly increasing in θ^b. Therefore, the solution of (θ^a, θ^b) is unique. In addition, according to Equation (18) to (21), the other steady state variables are perfectly pinned down by (θ^a, θ^b). In sum, existence and uniqueness of steady state is guaranteed.

Proposition 13. The solution of Equation (16) to (21) exists and is unique.

This result is important for two reasons. First, steady state equilibrium exists. Second, multiple equilibrium is ruled out.

It is worth noting that Proposition 3 crucially depends on two assumptions. The first depends on homogeneous of degree one and strictly concave matching functions. It ensures that job finding rate and cooperation matching rate are strictly increasing in market tightness and cooperation tightness respectively. Secondly, strictly increasing productivity function ensures that firms are better off with higher cooperation matching rate, which guarantees that cooperation tightness is uniquely determined.[1]

When production function is not strictly increasing, single firms may not find it optimal to cooperate with other firms. Actually, in the special case where $f^i(z^i,1,1) = f^i(z^i,1,0)$, firm is indifferent between being single firm or cooperative firm. Also, Thus cooperation matching rate does not affect firm's value and the model collapses to two independent DMP model. In this situation, cooperation tightness does not affect firms' optimization decision and there would be no linkage between the two sectors.

3.6.2 Comparative Statics Analysis

In this subsection, I show some simple comparative static analysis using the results in Section 3.6.1.

[1] When matching functions are increasing return to scale, unique equilibrium is not always the case. In particular, when matching functions are increasing return to scale, increase in recruiting does not necessarily lead to lower vacancy filling rate and cooperation matching rate. With certain parameterization, there might exist a 'good equilibria' with high recruiting effort and low unemployment, and a 'bad equilibria' with low recruiting effort and high unemployment. This Thick Market Hypothesis has been discussed intensively in the literature, for example, Diamond (1982), Diamond and Fudenberg (1989) and Boldrin, Kiyotaki and Wright (1993). While multiplicity caused by increasing return to scale matching function is interesting in theory, there lacks sufficient surporting evidence.

As shown in Lemma 3.1, the free entry condition for single firm implies a negative relationship between the labor market tightness of two sectors. On the other hand, Lemma 3.3 shows that matching functions with homogeneous of degree one and strict concavity implies a positive relationship between market tightness of two sectors. For convenience I will denote the negative and positive relationship between the sectoral labor market tightness as demand and supply curve respectively and the following comparative statics analysis mainly focus on the process of adjustment in the two curves.

Experiment I, *increasing sector a's steady state productivity* z^a

When sector a has a higher productivity z^a, higher market tightness θ^a is required to push expected matching value back to zero. Thus, consistent with Lemma 3.1, θ^a would be increased for any given value of θ^b and demand curve is shifted up as a result. On the other hand, notice that supply curve, which is derived from Equation (19) to (21), does not depend on productivity z^a. Therefore, supply curve does not move. In sum, increase in steady state productivity would raise up the steady state market tightnesses of both sectors.

In the DMP model without sectoral complementarity (or equivalently when $f^i(z^i,1,1) = f^i(z^i,1,0)$), two sectors are independent. There is no demand and supply curve as defined above. Instead, sectoral market tightness is perfectly pinned down by the productivity of the sector. Thus, the diagram has a vertical and a horizontal line. Shifting one sector's productivity would not affect the other sector.

Experiment II, increasing inter-firm matching efficiency ψ

The intuition is straightforward. When inter-firm matching efficiency is increased, the probability of upgrading to cooperative firm for single firms in both sectors is raised, which implies that single firms have a higher expected value. More firms are attracted to post vacancy in both sectors and market tightness increases until expected value of posting vacancy is pushed back to zero.

Proposition 14. In the steady state, an increase in either sector's productivity z^i will raise both θ^a and θ^b

2. In the steady state, an increase in inter-firm matching efficiency ψwill raise both θ^a and θ^b

3.4 Numerical Results

In this section, I study the quantitative performance of the model. Subsection 4.1 discusses the calibration. In subsections 4.2, I present several business cycle statistics of U.S. labor market. In subsection 4.3, I first demonstrate that the standard DMP model cannot explain the sectoral labor market comovement and does poorly in matching the aggregate data. Then I show that in the benchmark model, sectoral labor market comovement shows up even when the productivity shocks are independent across sectors. However, same as the standard DMP model, the benchmark model with sectoral productivity shocks only fails to generate high volatility of the unemployment. Then I show that inter-firm matching efficiency shock can simultaneously generate 1, labor market comovement 2, high volatility of unemployment.

3.4.1 Calibration

In the benchmark model, I consider the symmetric case where sector a and sector b are identical. Many of my my model's features are standard to the DMP literature, so my calibration strategy is to maintain comparability as much as possible. The model is calibrated to a monthly frequency. As such, the discount factor is set to $\beta = 0.996$ so that annual risk free rate is 5%.

The vacancy cost χ is normalized to one. I assume that matching functions of both stages are Cobb-Douglas, so that:

$$\begin{aligned} \mu^i(\theta^i) &= \phi^i(\theta^i)^\alpha \\ p^i(\tilde{\theta}^i) &= \psi(\tilde{\theta}^i)^{\alpha_0} \end{aligned}$$

For the first matching function which describes the matching between firm and unemployed worker, I specify $\alpha = 0.4$ to be close to a wide range of estimation results in the literature. Matching efficiency of both sectors is set to 0.72 to target a monthly job finding rate of 0.45 for both sectors, as in Shimer (2005). Given the job finding rate, I set separation rate $\delta = 0.027$ to match to steady state unemployment rate of 5.5%

For the second matching function which corresponds to the matching between single firms from sector a and b, I set $\alpha_0 = 0.5$, so that the cooperation matching rates of the two sectors are same on steady state. Cooperate matching efficiency is calibrated to 0.2. It implies that on average a single firm needs five months to cooperate with a single firm from another sector. There are at least two reasons why matching between single firms is less efficient than between firm and worker. First, firms are more heterogeneous thus matching between firms are more complicated. It is natural to conjecture that it takes longer to search and bargaining with cooperation partner than recruiting a worker. Second, while the benchmark model has only two sectors for simplicity, it tries to summarize a much more complex reality where firms often need to cooperate with multiple other firms from many different sectors.

The production function is specified as

$$f^i(z^i, l, m^j) = z^i l^\rho \left[(1-\gamma^i)1^\kappa + \gamma^i (m^j)^\kappa\right]^{(1-\rho)/\kappa}$$

It is standard Cobb-Douglas production function with labor and intermediate goods service as input. I set ρ as 0.3, so that the aggregate labor share is about 30% . In the two sector model, there is only one intermediate goods service input and substitution rate across intermediate goods services κ is not identified, hence I normalize it as one. γ^i measures how much sector i relies on sector j. When γ^i is higher, the labor productivity gap between cooperative firm and single firm in sector i is larger, and the single firm has more incentive to cooperate with single firm in sector j. In the special case where $\gamma^i = 0$, single firm has the same productivity as cooperative firm and the model becomes standard

DMP model. In the benchmark model with two sectors, I set γ^i to 0.75, which implies that the cooperative firm is five times as productive as the single firm. I specify z, the flow value of unemployment to be 0.4, which lies at the upper end of the range of income replacement rates in US.

The shock processes follow independent AR(1) processes

$$
\begin{aligned}
log(z_{a,t+1}) &= (1-\rho_a)log(z_a)+\rho_a log(z_{a,t})+\varepsilon_{a,t+1} \\
log(z_{b,t+1}) &= (1-\rho_b)log(z_b)+\rho_b log(z_{a,t})+\varepsilon_{b,t+1} \\
log(\psi_{t+1}) &= (1-\rho_\psi)log(\psi)+\rho_\psi log(\psi_t)+\varepsilon_{\psi,t+1}
\end{aligned}
$$

The parameters of the shock processes are to be estimated.

Estimation of shock processes

The sample period is 2001M1 to 2014M12. The estimation uses five observation variables: unemployment rate, job opening rate in production-related sectors, job opening rate in service-related sectors, measured labor productivity in production-related sectors, measured labor productivity in service-related sectors.

I estimate the model using Bayesian method. I consider three independent shocks: TFP shock in sector a, TFP shock in sector b, inter-firm matching efficiency shock. As the number of observable is larger than the number of shocks, the model cannot be identified without other disturbances. So I also include observation errors for the observables as additional disturbances.

The estimation results are reported in 3.1.

3.4.2 Simulated Results

To investigate the quantitative predictions of the model, I simulate the model at the posterior mode to obtain 1,000,000 observations at monthly frequency. Following (**?**), I filter the logged data with smoothing parameter 10^5 to obtain second moment statistics.

Column 1 of presents selected business cycle statistics for the U.S. aggregate economy, 1953:I–2009:IV. To isolate cyclical fluctuations, I use HP filter to isolate cyclical fluctuations. Here, I highlight a number of well established observations and discuss their implication for the quantitative analysis of search and matching models.

The first is that aggregate unemployment rate is very volatile over the business cycle relative to labor productivity. The standard deviation of unemployment relative to that of labor productivity is 9.5. Hence, models that rely on shocks to productivity as the driving force require strong amplification.

Secondly, I report statistics relating to the cyclicality of job finding since this is the job finding rate, $\mu(\theta)$, is a function of the market tightness ratio θ, I present statistics for this variable as well. Column 1 of Table 1 indicates that both the job finding rate and tightness ratio are very volatile over the cycle. Relative to labor productivity, the standard deviation of these variables are 6.0 and 19.1, respectively.

Thirdly, as demonstrated in Section 2, there exists a strong comovement between sectoral job openings. The equal weighted average of correlation of job openings of six major sectors of U.S. economy is 0.7. On the other hand, comovement between sectoral labor productivities is much milder. When sectoral labor productivity is measured as ratio between sectoral output and employment, the equal weighted average of correlation of sectoral labor productivities of six major sectors of U.S. economy is 0.3.

Moreover, there exists a negative correlation between unemployment and vacancies over the business cycle–the “Beverdge Curve”. In postwar U.S. data, this correlation is -0.89. This summarizes the fact that recessions are periods when firms stop hiring and unemployment increases; booms are periods when hiring is brisk and unemployment is low.

I also document the correlation between labor productivity and unemployment. Labor productivity provides a measure of the return to work effort, while unemployment measures work effort itself. As emphasized by (Gervais et al., 2015), these two measures are only mildly (negatively) related, with a correlation of -0.41; periods when work effort rises, and unemployment falls, are only weakly associated with higher productivity.

The results for the standard DMP model are presented in column 2 of Table 2 of Table 1. It is not surprising that the standard DMP model is not able to generate or amplify sectoral labor markets' comovement. The standard DMP model also performs poorly in matching aggregate moments. In particular, the model delivers a standard deviation of unemployment relative to productivity of 1.21, which is approximately 8 times smaller than in the data. The Standard DMP model also overpredicts the correlation of labor productivity to labor market measures. Specifically, the correlation between the unemployment rate and the labor productivity is -0.86 in the model, while only -0.41 in the data.

The simulated moments for the benchmark model with sectoral productivity shocks are presented in column 3 of Table 1. This model generate sectoral comovement even when sectoral productivities are independent. When sectoral productivities are mildly correlated as in the data, the labor market comovement is amplified. However, the amplification is not enough to explain the strong comovement of sectoral labor markets.

In addition, similar with standard DMP model, the benchmark two-sector-model can hardly match to aggregate moments. In particular, standard deviation of main labor market variables are too small and their correlations with labor productivities is too large.

The results for the benchmark model with cooperate matching efficiency shocks are presented in Column 4 of Table 1. Inter-firm matching efficiency shocks alone can account for 72% of the observed volatility of unemployment, which significantly improves upon productivity shocks. The model generates a volatility of market tightness ratio relative to productivity, 13.5, which is very close to data.

In addition, inter-firm matching efficiency shocks induce a tight comovement between the two sectors– the correlation between the two sectors' unemployment 0.9. However, inter-firm matching efficiency shocks over-predict the comovement between sectors. Therefore inter-firm matching efficiency shocks alone cannot generate realistic labor market fluctuations at the sectoral level. To improve the performance of the model, other shocks such as sectoral productivity shocks are needed.

Moreover, inter-firm matching efficiency shock disentangle labor market from labor productivity shock. Specifically, inter-firm matching efficiency shocks generate a correlation between unemployment and labor productivity of -0.1, which is close to the value of -0.40 observed in the data, and far from the value near minus one generated by the standard DMP model driven by technology shocks. inter-firm matching efficiency shock disentangle labor market from labor productivity shock.

Impulse response

I use impulse response to compare standard DMP model, benchmark model with only sectoral productivity shock, and benchmark model with inter-firm matching efficiency shock.

Panel A presents standard DMP model's impulse response to a positive one standard deviation technology shock to sector a. In this case, single firm and cooperative firm have identical productivity, and technology shock has a direct impact on all matched workers' productivity in sector a. Hence, sector a's labor productivity jumps upon impact of the shock, gradually declining to steady state after that. This shock implies an immediate impact on the profit of firm in sector a. From the free entry condition, vacancies respond immediately. Because of the high empirical job finding rate that the model is calibrated to, sector a's unemployment responds quickly.

These responses clearly illustrate the unsatisfactory performance of the standard DMP model. First, the model cannot generate sectoral comovement without the help of correlated sectoral productivities. Second, because unemployment and labor productivity response to technology shock instantaneously, the two variables are strongly correlated. Third, the response of unemployment is of the same order of magnitude as that of productivity, which means the amplification mechanism is not sufficient to explain the large labor market fluctuation observed in data.

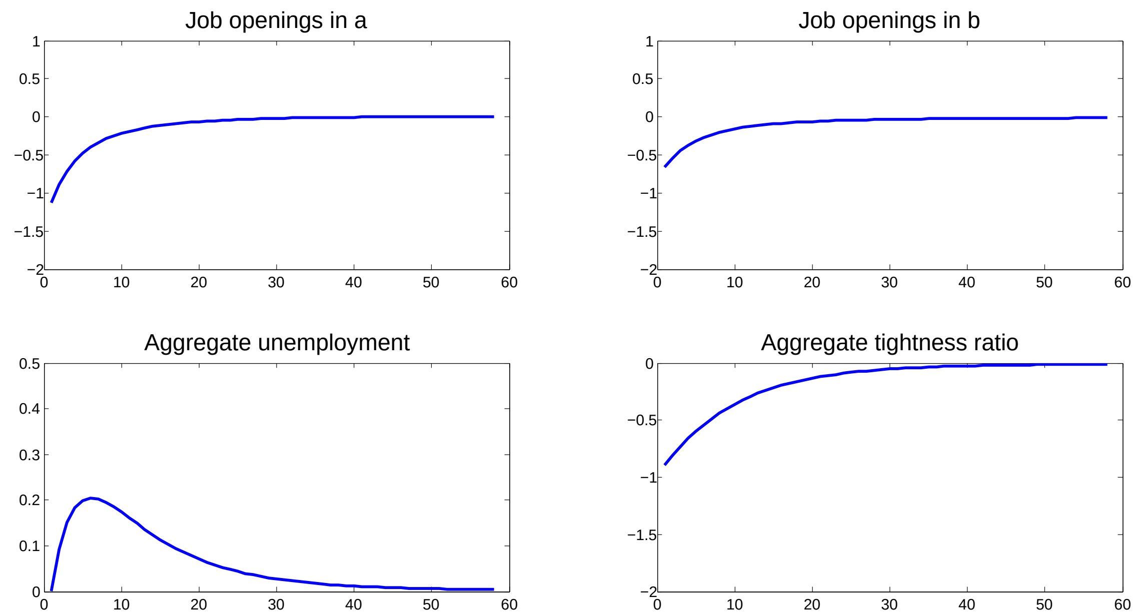


FIGURE 3.7: IRF to TFP shock to sector a

Figure presents the impulse response of benchmark model to a positive one standard deviation technology shock to sector a. For sector a, same as standard DMP model, productivities of both single firm and cooperative firm increase because of the technology shock, and then decline to steady state. Vacancy posting is enhanced immediately due to free entry condition and as a result, sector a's unemployment instantaneously react to productivity level.

As shown in the response responses, sector b also responses to sector a's technology shocks. In specific, increasing recruiting in sector a will raise the number of single firms. Sector b's single firms thus have better opportunity to match with sector a's single firm due to rising cooperation matching rate and its expected profit is enhanced as a result.

Free entry condition implies that sector b would increase vacancy posting and pull down unemployment. Since what directly affect sector b's recruiting decision is the number of sector a's single firms, which is a 'stock variable' that cannot fully adjust instantaneously, response of sector b lags of the movement in sector a.

Though successfully implying sectoral comovement in the right direction, the benchmark model is still quantitatively unsatisfactory. First, the impact of sector a's technology shock on sector b is small. Specifically, the decline in sector b's unemployment is one magnitude lower than in sector a, which means that the indirect effect transmitted sector a is mild. Second, the benchmark model still lacks sufficient amplification mechanism. The implied labor market fluctuation can hardly match to data. In sum, when there are sectoral technology shocks to both sectors, the benchmark model's performance is similar to standard DMP model.

Figure 3.8 presents the impulse responses of several key variables to a positive one standard deviation cooperate matching efficiency shock. For both sectors, the jump in the inter-firm matching efficiency creates a jump in the expected value of single firm. From the free entry condition, vacancies respond immediately, and unemployment soon after.

Variance decomposition

To evaluate the contribution of rotation shock to the business cycle, I conduct variance decomposition at business cycle frequency and report the results in Table 3.3. Each row reports the shares of fluctuations explained by the shocks at the business cycle frequency; that is, the share of variances explained at frequencies between 6 and 32 quarters, computed by a bandpass filter as in Stock and Watson (1999).

Inter-firm matching efficiency shocks account for a major fraction of variation of job openings in both sectors. It also accounts for a significant fraction of the fluctuations in the measured TFP.

Shocks to TFP have a very minor effect on the job openings. However, TFP shocks

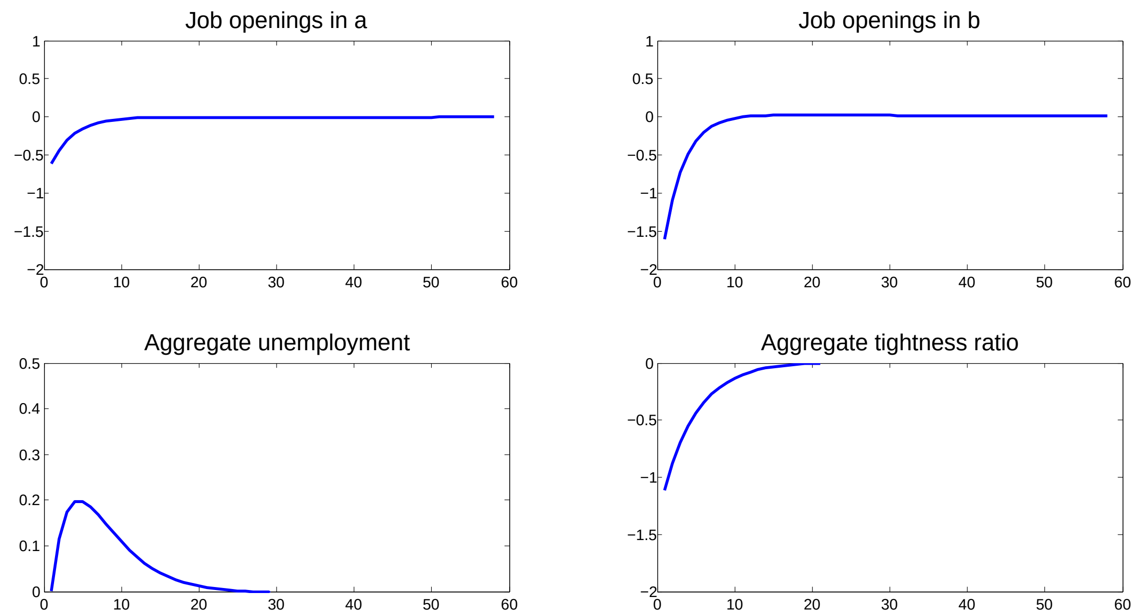


FIGURE 3.8: IRF to inter-firm matching efficiency shocks

still account for the major fraction of variation of measured TFP.

3.5 Conclusion

In this chapter, I study two stylized facts in the US labor market. The first fact is that the job openings are highly correlated across the one-digit industries. The second fact is that the labor productivity mildly correlates across the one-digit industries. Productivity shocks cannot account for this fact in a standard DMP search and matching model, as the comovement of productivity at the sector level is necessary for the comovement of job openings. To reconcile the problem, this paper proposes a two-sector search and matching model, in which: (a) firms are more productive when collaborating with trading partners than working alone, and (b) it takes time for a firm to match with a trading partner. I show that my model generates significant sectoral comovement of the labor market. Moreover, when l allow matching efficiency to be time varying, the model also generates substantial amplification in labor market variables.

3.6 Proof of results

Proof of Proposition I

(I) Equation (13) can be derived by adding equation (7) and (8) then subtracting (4)

$$\begin{aligned} J_t^i + W_t^i - U_t^i &= f^i\left(z_t^i, 1, 1\right) - z + \beta E_t[(1-\delta)\left(J_{t+1}^i + W_{t+1}^i - U_{t+1}^i\right) \\ &\quad -\mu_t^i\left(\tilde{W}_{t+1}^i - U_{t+1}^i\right)] \end{aligned}$$

Applying definition of total surplus TS and $\tilde{TS}$ and the outcome of Nash bargaining, equation (9), we immediately get equation (13).

Similarly, we derive Equation (14) by adding equation (3) and (5) then subtracting (4)

$$\begin{aligned} \tilde{J}_t^i + \tilde{W}_t^i - U_t^i &= f^i\left(z_t^i, 1, 0\right) - z + \beta E_t[p_t^i\left(J_{t+1}^i + W_{t+1}^i - U_{t+1}^i\right) \\ &\quad + \left(1 - p_t^i - \mu_t^i - \delta\right)\left(\tilde{J}_{t+1}^i + \tilde{W}_{t+1}^i - U_{t+1}^i\right) + \mu_t^i \tilde{J}_{t+1}^i] \end{aligned}$$

We get equation (14) by plugging in definition of total surplus and equation (12).

Equation (15) demonstrates the free entry condition. We can derive it simply by applying condition (2) and (4) to equation (6).

(II) Substracting Equation (14) from (13), we get the following recursive equation

$$TS_t^i - \tilde{TS}_t^i = f^i\left(z_t^i, 1, 1\right) - f^i\left(z_t^i, 1, 0\right) + \beta E_t[\left(1 - \delta - p_t^i\right)\left(TS_{t+1}^i - \tilde{TS}_{t+1}^i\right)]$$

Because both TS^i and $\tilde{TS}^i$ are bounded, by iteration on it, we have

$$TS_t^i - \tilde{TS}_t^i = \sum_{k=0}^{\infty} \beta^k E_t\left[\prod_{l=0}^{k}\left(1 - \delta - p_{t+l}^i\right)\right] * \left[f^i\left(z_{t+k}^i, 1, 1\right) - f^i\left(z_{t+k}^i, 1, 0\right)\right] + op(1)$$

Since by assumption cooperative firm is always more productive than single firm and the transition probabilities are positive in the entire state space

$$\begin{aligned} f^i\left(z_t^i,1,1\right) - f^i\left(z_t^i,1,0\right) &> 0 \\ 1-\delta-p_t^i &> 0 \end{aligned}$$

We immediately get

$$TS_t^i - \tilde{TS}_t^i > 0$$

Q.E.D

Proof of Lemma 2.1

Since p^a is strictly increasing in $\tilde{\theta}^a$, it is equivalant to show that θ^a *is strictly increasing in* z^a *and* p^a.

First, we can eliminate TS^a by manipulating Equation (16) and (17)

$$\begin{aligned} 0 = {} & \beta p^a f^a\left(z^a,1,1\right) + \left[1-\beta\left(1-\delta\right)\right] f^a(z^a,1,0) - \left[\beta p^a + \left(1-\beta(1-\delta)\right)\right] z \\ & - \left\{\beta p^a \beta\left(1-\tau\right) + \left[1-\beta\left(1-\delta\right)\right]\left[1-\beta(1-\delta)+\beta\left(p^a+(1-\tau)\mu^a\right)\right]\right\} \tilde{TS}^a \end{aligned}$$

Second, we replace $\tilde{TS}^a$ with $\frac{\chi}{\beta q^a \tau}$ using Equation (18)

$$\begin{aligned} 0 = {} & \beta p^a f^a(z^a,1,1) + (1-\beta(1-\delta)) f^a(z^a,1,0) - \left[\beta p^a + (1-\beta(1-\delta))\right] z \\ & - \left\{\beta p^a \beta(1-\tau) + (1-\beta(1-\delta))\left[1-\beta(1-\delta)+\beta(p^a+(1-\tau)\mu^a)\right]\right\} \frac{\chi}{\beta q^a \tau} \end{aligned}$$

Lastly, denote RHS as $\kappa(z^a,p^a,\theta^a)$, it suffices to show that

$$\begin{aligned} \frac{\partial \kappa}{\partial z^a} &> 0 \\ \frac{\partial \kappa}{\partial \theta^a} &< 0 \\ \frac{\partial \kappa}{\partial p^a} &> 0 \end{aligned}$$

The first two inequalities are straightforward. Noticing that $f_z^a > 0, (q^a)' < 0, (\mu^a)' > 0$

$$\begin{aligned}
\frac{\partial \kappa}{\partial z^a} &= \beta p^a f_z^a(z^a,1,1) + (1-\beta(1-\delta)) f_z^a(z^a,1,0) > 0 \\
\frac{\partial \kappa}{\partial \theta^a} &= \{\beta p^a \beta(1-\tau) + (1-\beta(1-\delta))[1-\beta(1-\delta)+\beta(p^a+(1-\tau)\mu^a)]\} \frac{\chi (q^a)'}{\beta (q^a)^2 \tau} \\
&\quad - (1-\beta(1-\delta))\beta\left[p^a + (1-\tau)(\mu^a)'\right]\frac{\chi}{\beta q^a \tau} < 0
\end{aligned}$$

The third inequality can be shown in two steps. In step one, we take derivative w.r.t p^a

$$\begin{aligned}
\frac{\partial \kappa}{\partial p^a} &= \beta [f^a(z^a,1,1) - z] - \beta[\beta(1-\tau) + (1-\beta(1-\delta))]\tilde{TS}^a \\
&= \beta\left\{[f^a(z^a,1,1) - z] - [\beta(1-\tau) + (1-\beta(1-\delta))]\tilde{TS}^a\right\}
\end{aligned}$$

In step two, we use Equation (16) and Proposition 1 (II) to show that the RHS of the above equation is positive

$$\begin{aligned}
f^a(z^a,1,1) - z &= (1-\beta(1-p^i-\delta-\mu^i(1-\tau)))\tilde{TS}^a - \beta p^a TS^a \\
&< (1-\beta(1-p^a-\delta-(1-\tau)))\tilde{TS}^a - \beta p^a TS^a \\
&= [\beta(1-\tau) + (1-\beta(1-\delta))]\tilde{TS}^a - \beta p^a (TS^a - \tilde{TS}^a) \\
&< [\beta(1-\tau) + (1-\beta(1-\delta))]\tilde{TS}^a
\end{aligned}$$

Hence we have

$$\begin{aligned}
\frac{\partial \kappa}{\partial p^a} &= \beta\left\{[f^a(z^a,1,1) - z] - [\beta(1-\tau) + (1-\beta(1-\delta))]\tilde{TS}^a\right\} \\
&< 0
\end{aligned}$$

Q.E.D

Proof of Lemma 2.2

While it is straightforward to prove by taking derivative w.r.t. s^a, θ^a, θ^b, here I will show it in a informal way. First, according to Equation (19) and (20)

$$\theta^a \uparrow \Rightarrow u^a \downarrow \Rightarrow s^a \uparrow$$

The above shows that increase in sector a's market tightness would decrease its unemployment which leads to increase in the number of its single firms.

Second, according to Equation (21) and (20)

$$s^a \uparrow \Rightarrow \tilde{\theta}^a \downarrow \Rightarrow p^a \downarrow \Rightarrow s^a \uparrow$$

The above shows increase in the number of sector a's single firms would make cooperation tightness and probability lower, which would further increase the number of single firms.

Same reasoning applies to why s^a is strictly increasing in θ^b

Q.E.D

3.7 Nash bargaining between firms

Value functions

The value for single firms are unchanged:

$$\tilde{TS}_{a,t} = \tilde{z}_{a,t} - z - \frac{(1-\tau_a)}{\tau_a}\chi\theta_{a,t} + \beta(1-\delta)E_t\left[\tilde{p}_{a,t}TS_{a,t+1} + (1-\tilde{p}_{a,t})\tilde{TS}_{a,t+1}\right] \quad (3.7.1)$$

and

$$\tilde{TS}_{b,t} = \tilde{z}_{b,t} - z - \frac{(1-\tau_b)}{\tau_b}\chi\theta_{b,t} + \beta(1-\delta)E_t\left[\tilde{p}_{b,t}TS_{b,t+1} + \left(1-\tilde{p}_{b,t}\right)\tilde{TS}_{b,t+1}\right] \quad (3.7.2)$$

In each cooperation relationship, the firms split the joint surplus $\left[\left(J_a - \tilde{J}_a\right) + \left(J_b - \tilde{J}_b\right)\right]$ with Nash bargaining. Their bargaining share is κ_a and κ_b, with $\kappa_a + \kappa_b = 1$.

The Nash bargaining problem is

$$\max \left(J_a - \tilde{J}_a\right)^{\kappa_a} \left(J_b - \tilde{J}_b\right)^{\kappa_b}$$

The solution to the above Nash bargaining problem is standard:

$$\frac{\left(J_a - \tilde{J}_a\right)}{\kappa_a} = \frac{\left(J_b - \tilde{J}_b\right)}{\kappa_b}$$

or equivalently

$$\frac{\tau_a \left(TS_a - \tilde{TS}_a\right)}{\kappa_a} = \frac{\tau_a \left(TS_b - \tilde{TS}_b\right)}{\kappa_b} \tag{3.7.3}$$

From equation 3.7.3, we get

$$\left(TS_a - \tilde{TS}_a\right) = \frac{\kappa_a}{\tau_a} X_t \tag{3.7.4}$$

and

$$\left(TS_b - \tilde{TS}_b\right) = \frac{\kappa_b}{\tau_b} X_t \tag{3.7.5}$$

where X_t measures the gain of inter-firm cooperation, with

$$X_t = \frac{\tau_a \left(TS_a - \tilde{TS}_a\right)}{\kappa_a} = \frac{\tau_a \left(TS_b - \tilde{TS}_b\right)}{\kappa_b}$$

In each period, the two partners split their joint profit $\left[(z_a - w_a) + (z_b - w_b)\right]$ in the way that satisfies equation 3.7.3. Denote that the firm in sector a receives $(z_a - w_a + \Delta)$, while the firm in sector b receives $(z_b - w_b - \Delta)$, where Δ is the inter-firm transfer to re-balance the Nash bargaining problem. Therefore the value of cooperative firm is

$$TS_{a,t} = z_{a,t} - z + \Delta + \beta\left(1 - \delta\right) E_t TS_{a,t+1} \tag{3.7.6}$$

and

$$TS_{b,t} = z_{b,t} - z - \Delta + \beta\left(1 - \delta\right) E_t TS_{b,t+1} \tag{3.7.7}$$

Combining equations 3.7.6 3.7.7 with equations 3.7.1 3.7.2, we get

$$TS_{a,t} - \tilde{TS}_{a,t} = z_{a,t} - \tilde{z}_{a,t} + \Delta_t + \beta(1-\delta)\tilde{p}_a E_t\left(TS_{a,t+1} - \tilde{TS}_{a,t+1}\right) \tag{3.7.8}$$

and

$$TS_{b,t} - \tilde{TS}_{b,t} = z_{b,t} - \tilde{z}_{b,t} - \Delta_t + \beta(1-\delta)\tilde{p}_b E_t\left(TS_{b,t+1} - \tilde{TS}_{b,t+1}\right) \tag{3.7.9}$$

Moreover, by plugging equations 3.7.4 and 3.7.5 into equations 3.7.8 and 3.7.9, we get

$$\frac{\kappa_a}{\tau_a}X_t = z_{a,t} - \tilde{z}_{a,t} + \Delta_t + \beta(1-\delta)\tilde{p}_a E_t\left(\frac{\kappa_a}{\tau_a}X_{t+1}\right)$$

and

$$\frac{\kappa_b}{\tau_b}X_t = z_{b,t} - \tilde{z}_{b,t} - \Delta_t + \beta(1-\delta)\tilde{p}_b E_t\left(\frac{\kappa_b}{\tau_b}X_{t+1}\right)$$

Therefore, X_t and Δ_t can be pinned down by

$$\left(\frac{\kappa_a}{\tau_a} + \frac{\kappa_b}{\tau_b}\right)X_t = (z_{a,t} - \tilde{z}_{a,t}) + \left(z_{b,t} - \tilde{z}_{b,t}\right) + \beta(1-\delta)\left(\tilde{p}_a\frac{\kappa_a}{\tau_a} + \tilde{p}_b\frac{\kappa_b}{\tau_b}\right)E_tX_{t+1} \tag{3.7.10}$$

and

$$\left(\frac{\kappa_a}{\tau_a} - \frac{\kappa_b}{\tau_b}\right)X_t = (z_{a,t} - \tilde{z}_{a,t}) - \left(z_{b,t} - \tilde{z}_{b,t}\right) + 2\cdot\Delta_t + \beta(1-\delta)\left(\tilde{p}_a\frac{\kappa_a}{\tau_a} - \tilde{p}_b\frac{\kappa_b}{\tau_b}\right)E_tX_{t+1} \tag{3.7.11}$$

Also, plug equations 3.7.4 and 3.7.5 into equations 3.7.1 and 3.7.2, we get

$$\tilde{TS}_{a,t} = \tilde{z}_{a,t} - z - \frac{(1-\tau_a)}{\tau_a}\chi\theta_{a,t} + \beta(1-\delta)E_t\left(\tilde{p}_{a,t}\frac{\kappa_a}{\tau_a}X_{t+1} + \tilde{TS}_{a,t+1}\right) \tag{3.7.12}$$

and

$$\tilde{TS}_{b,t} = \tilde{z}_{b,t} - z - \frac{(1-\tau_b)}{\tau_b}\chi\theta_{b,t} + \beta(1-\delta)E_t\left(\tilde{p}_{b,t}\frac{\kappa_b}{\tau_b}X_{t+1} + \tilde{TS}_{b,t+1}\right) \tag{3.7.13}$$

Finally, the four variables $\tilde{TS}_a$,$\tilde{TS}_a$,X, and Δ will be pinned down by equations 3.7.10, 3.7.11, 3.7.12, 3.7.13.

Table 3.1: Estimation result

Para	Description	Prior			Posterior	
		Type	Mean	Std	Mode	90% interval
$100 \times \sigma_{rot_A}$	Std. of TFP shock in sector *a*	inv gamma	1	0.2	0.93	[0.86,1.10]
$100 \times \sigma_{rot_B}$	Std. of TFP shock in sector *b*	inv gamma	1	0.2	0.65	[0.60,0.70]
$100 \times \sigma_z$	Std. of inter-firm matching efficiency shock	inv gamma	1	0.2	10.50	[8.64,12.53]
ρ_z	Pers. of TFP shock in sector a	beta	0.5	0.1	0.93	[0.91,0.95]
ρ_{ξ_c}	Pers. of TFP shock in sector a	beta	0.5	0.1	0.90	[0.87,0.92]
ρ_{ξ_I}	Pers. of inter-firm matching efficiency shock	beta	0.5	0.1	0.80	[0.75,0.85]
$100 \times \sigma_{obs_1}$	Std. of obs. error for unemployment rate	inv gamma	1	0.2	4.25	[4.08,4.43]
$100 \times \sigma_{obs_2}$	Std. of obs. error for job openings in sector *a*	inv gamma	1	0.2	9.59	[8.13,10.48]
$100 \times \sigma_{obs_3}$	Std. of obs. error for job openings in sector *b*	inv gamma	1	0.2	0.11	[0.06,0.17]
$100 \times \sigma_{obs_4}$	Std. of obs. error for TFP in sector *a*	inv gamma	1	0.2	0.08	[0.05,0.11]
$100 \times \sigma_{obs_5}$	Std. of obs. error for TFP in sector *b*	inv gamma	1	0.2	0.07	[0.05,0.09]

Table 3.2: Key moments of the model

Shocks	U.S. data	DMP Model TFP shocks	Benchmark Model TFP shocks	Benchmark Model Matching Efficiency Shocks
Standard Deviations				
$\sigma(u_a+u_b)/\sigma(z_{agg})$	9.5	0.40	0.99	3.40
$\sigma\left(\frac{v_a+v_b}{u_a+u_b}\right)/\sigma(z_{agg})$	19.1	1.54	2.43	13.5
Correlations				
$corr(u_a,u_b)$	0.70	0.00	0.55	0.98
$corr(u_a+u_b,v_a+v_b)$	-0.89	-0.70	-0.76	-0.93
$corr(u_a+u_b,z_{agg})$	-0.41	-0.86	-0.77	-0.07

Table 3.3: Variance decomposition at the business cycle frequency

Variance decomposition	ε_a	ε_b	ε_ψ
Job openings in a	0.069	0.004	0.927
Job openings in b	0.005	0.043	0.952
Measured TFP in a	0.909	0.000	0.091
Measured TFP in b	0.000	0.869	0.131

ε_a is shock to the TFP in sector a. ε_b is shock to the TFP in sector b. ε_ψ is shock to the inter-firm matching efficiency

4

Investment Lumpiness and Investment Goods Price

4.1 Introduction

Investment is the most volatile component of the aggregate output. Hence, the understanding of the investment fluctuations is crucial for the study of the business cycle. The relative price of investment is an essential piece of information for understanding the nature of investment fluctuations. A commonly held view in macroeconomics is that the relative price of investment increases in economic recessions (Greenwood et al. (2000); Fisher (2006)). Since the quantity of investment is strongly procyclical, this view implies that the relative price of investment increases in periods when the quantity of investment drops. In an economy with perfect competition in the investment goods market, the negative unconditional correlation between quantity and relative price justifies technological shocks to investment producers (supply side story). In this paper, we re-examine the joint dynamics of the price and quantity of investment in a framework with frictional investment goods market.

In section 4.2, we document two novel patterns of the investment market. First, we demonstrate that the correlation between the quantity and the relative price of investment exhibits substantial time-variation. Specifically, in some periods, the quantity and the relative price of investment are negatively correlated, which is consistent with the commonly held view in the literature; in some other periods, they exhibit strong positive comovement. Just to give an idea, the correlation between the quantity and the relative price of real equipment investment is about 0.6 during 2002Q3-2004Q2, but changes to about -0.8 during 2007Q3-2009Q2.

The natural question is what explains the time-varying correlation between the relative price and the quantity of investment. The second pattern we document in the paper is that the quantity and the relative price of investment are more likely to move in the opposite direction when a larger share of firms stays inactive of capital adjustment. In other words, the correlation between the two variables tends to be negative when investment is lumpy. While recent empirical literature (Khan and Thomas (2008); Gourio and Kashyap (2007); Bloom et al. (2012)) emphasizes that firm's investment decision is lumpy, that is, occurs largely along the extensive margin; we document that the relative price of investment is also tightly associated with the lumpiness of investment.

We view the two above patterns as evidence of market imperfection in the investment goods market. When there is friction prevents capital from efficiently reallocating among firms, the elasticity of demand for investment differs across firms. Specifically, given the same level of capital stock, the elasticity of demand for investment is increasing in firm's productivity. Meanwhile, low productivity firms are more likely to stay inactive in capital adjustment conditional on capital stock. The above implies that the elasticity of aggregate demand for investment gets lower in periods with higher lumpiness of investment. If sellers of investment goods possess monopolistic power, with large lumpiness of investment, they would take advantage of the low elasticity by setting a higher price of investment. In section 4.3, we illustrate this idea with a simple model with a monopolistic seller of investment.

To quantitatively assess the role of market imperfection in the observed joint dynamics of the relative price and quantity of investment, in section 4.4 we develop a dynamic stochastic general equilibrium (DSGE) model. Two main features help the model to match to the pattern of data. (1) firms are subject to non-convex adjustment cost of capital adjustment, which makes low-productive firms stay inactive in capital adjustment. (2) There is search friction in the market of investment goods. Search friction induces an endogenous monopolistic power of investment sellers because it takes time for investment buyer to find a seller.[1] In our model, movement in the relative price of investment is an endogenous product of the market imperfection of the investment market, and the pattern of the comovement of the relative price and quantity of investment is driven by the investment at the extensive margin. Specifically, we show that shocks that can shift investment lumpiness, such as uncertainty shocks (Bloom (2009); Bloom et al. (2012)), can endogenously generate realistic comovement of the relative price and quantity of investment that is consistent with the patterns we document in the data.

Our paper belongs to the literature of investment-specific technological shocks (IST shocks hereafter) (Beaudry et al. (2015); Fisher (2006); Floetotto et al. (2009); Schmitt-Grohé and Uribe (2011); Greenwood et al. (2000)) The negative unconditional correlation between the relative price and quantity of investment has been viewed as an evidence of IST shocks. The intuition is that IST shocks directly shift the supply curve of investment so that the price and quantity of investment move in the opposite directions; while the other common shocks, such as shock to the TFP, does not explain the negative comovement. Therefore the relative price of investment has been used as the empirical counterpart of IST shocks. Our paper shows that shocks which can propagate investment lumpiness can potentially account for the observed joint dynamics of the relative price and quantity of investment. Our findings suggest that the assumption of market imperfections, such as monopoly power, might markedly change the identification of shocks.

4.2 Empirical motivation

In this section, we document the empirical facts that motivate our work. We first present the historical quantities and relative prices for various types of investment. Then we show that the pattern of comovement of the quantity and the relative price of investment varies

[1] While there are other ways to model monopoly power and time-varying markup, we view our modeling approach preferable to the common approaches. One common approach is to assume a single monopoly. However, this approach can become highly intractable when there is heterogeneity on the demand side, because the seller needs take into account its effect on the composition of the demand pool when setting the price. Another common way is to assume monopolistic competition, such as in Bilbiie et al. (2012); Floetotto et al. (2009), who supplement Dixit and Stiglitz (1977) with the fluctuation of product varieties. This approach gains popularity largely due to its elegant analytical solution. When buyers are heterogeneous, however, there is no analytical solution, and again the problem becomes very intractable.

over time. Specifically, the correlation between quantity and price of aggregate investment series is time-varying. Then we examine the relationship between the investment lumpiness and the comovement of quantity-price of investment. Specifically, we regress the quantity-price correlation of investment on different measures of investment lumpiness. We find that investment lumpiness has a significant negative effect on investment's quantity-price correlation, which means that when investment is lumpier, the quantity and the relative price of investment are more likely to move in opposite direction.

Data and empirical approach

In this section, we focus on the nonresidential investment, including the investment in structures, equipment and intellectual properties product, as defined by BEA. In contrast, the residential investment, the consumption of durable goods and the change in private inventories are not considered in our empirical analysis. This is because our paper focuses on the investment made by firms, whose empirical counterpart is the nonresidential investment.

Our samples are from 1969Q1 to 2014Q3. We use investment quantity and price series from NIPA. We compute the relative investment price by dividing the investment price by the price index for nondurable and service consumption. As our paper mainly focuses on the cyclical behavior of investment, in the benchmark analysis, we use the Hodrick-Prescott (HP hereafter) with a smoothing parameter 1600 to isolate the business-cycle components of the series. As pointed out by Canova (1998), HP filter might bias the cyclical component and filter out much useful information. Moreover, as suggested by Schmitt-Grohé and Uribe (2011), investment and investment price series contain a stochastic trend with a unit root. To tackle with the above concerns, we also consider the second approach: we take log for the raw data series, then compute their first differences. Comparing with HP filter, first-order differencing keeps much high frequency and long-run components.

Figure 1 plots the series we use for our empirical analysis. The red curves are the log of per-capita investment quantities detrended by HP filter. The blue curves are the log of the relative price of investment detrended by HP filter. Shaded bands represent NBER recession dates. Unsurprisingly, investment quantities are strongly pro-cyclical.

FIGURE 4.1: Investment per capita and relative price of investment filtered by HP filter

In all recessions, the real quantities of business investment series decline substantially. Relative prices of investments are only mildly counter-cyclical. For example, in the short expansion period in 1981, relative prices of all types of business investment experienced a sharp increase.

Figure 2 plots the first order differencing of the log of per-capita investment quantity and the relative price of investment. Same as figure 1, shaded bands represent NBER recession dates. Comparing with the series detrended by HP filter as plotted in figure 1, the series in figure 2 contains many high-frequency fluctuations. Again, we see investment quantities are strongly pro-cyclical while relative prices of investments are only mildly counter-cyclical.

Table 1 displays the correlation coefficients between quantities and the relative price of various types of business investment. Column 1 and 2 report the correlation coefficients and corresponding p-values while column 3 and 4 are series filtered by HP. As shown in

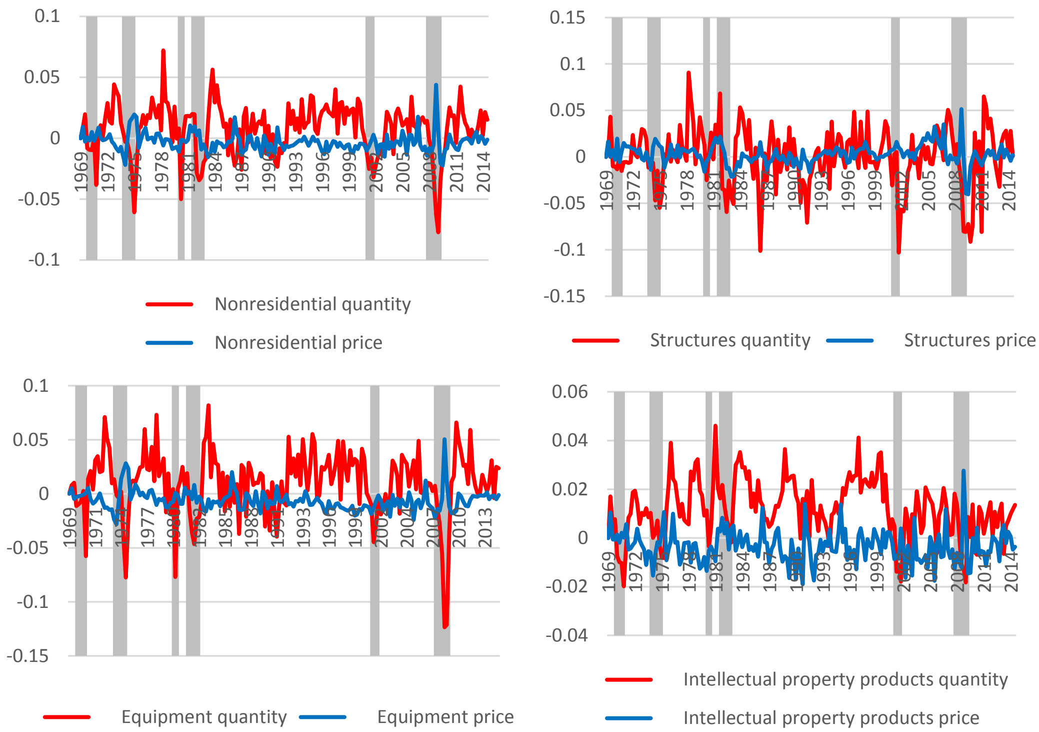


FIGURE 4.2: Investment per capita and relative price of investment filtered by log-difference

the top row, the relative price of nonresidential investment is negatively correlated with the quantity of it for both filtering methods. The correlation is -0.26 when series transformed by log-difference operator, -0.32 when filtered by HP filter. Both correlations are significant according to the p-values.

Among disaggregated investment series, the relative price of equipment and intellectual property products are significantly negatively correlated with quantities of them for both filtering methods. Otherwise, the relative price of structures is significantly positively correlated with quantity.

Rolling correlation between quantities and relative prices of business investment

As shown in Table 1, the relative prices and quantities of investment series are only mildly correlated. It most likely implies that during some periods price series positively co-move with the quantity series, while during other periods they negatively comove. To find when

Table 4.1: Unconditional correlation between investment and investment price

Type of investment	Log-difference		HP filter	
	Correlation	P-value	Correlation	P-value
	(1)	(2)	(3)	(4)
Nonresidential	-0.26	0.00	-0.32	0.00
Structures	0.15	0.04	0.40	0.00
Equipment	-0.42	0.00	-0.52	0.00
Intellectual property products	-0.13	0.07	-0.16	0.03

the price series positively/negatively co-move with the quantity series, in the following part, we compute the rolling correlations between the relative prices and quantities of investment measures. Specifically, for each year, we use quarterly data to compute the rolling correlation coefficient in an eight quarters window. For example, for investment of type i, the rolling correlation for the year 2008 is computed as

$$\rho^i_{2008} = \frac{1}{8} \sum_{t=2007Q3}^{2009Q2} \left(p^i_t - \bar{p}^i\right)\left(q^i_t - \bar{q}^i\right)$$

where p^i_t and q^i_t are the detrended relative price and quantity for type i investment in period t, and $\bar{p}^i$ and $\bar{q}^i$ are the sample means of p^i_t and q^i_t over the 2007Q3 to 2009Q2 period.

Figure 1-4 plot the rolling correlation coefficients between the relative price and quantity of business investment series from 1970 to 2013. The red curves are rolling correlation coefficients when series are detrended by the HP filter; blue curves are for the case of the first order different of the log. Instead of being flat, the rolling correlation coefficients for all business investment series demonstrate substantial time-variation. Take nonresidential investment as an example: the correlation was higher than 0.20 in 1980. It, however, becomes strongly negative in the year of 1985.

While the rolling correlations differ among different categories of business investments, there does exist some synchronization. For example, all correlations experienced a sharp decline in the early phase of the great recession. There also exists a considerable gap between the correlations series with blue and red color displayed in figure 3, which means that the correlations are also strongly sensitive to how the data series are detrended.

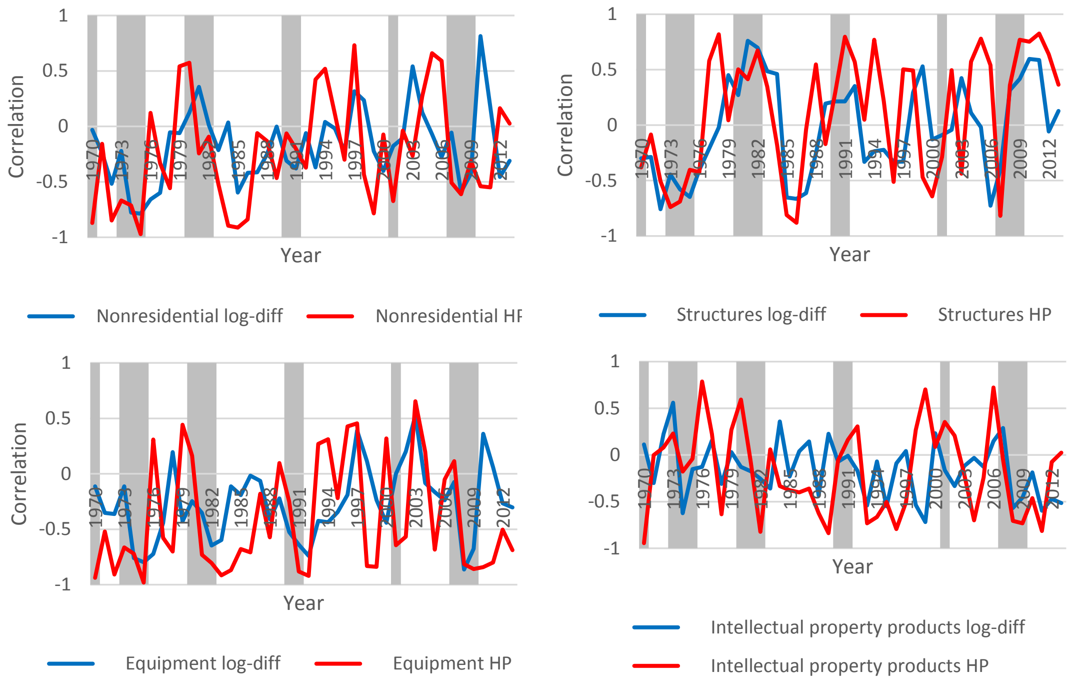


FIGURE 4.3: Rolling correlations between relative prices and quantities of investment series

Nevertheless, the two correlation series do appear to comove over our sample period.

To quantify the degree of time variation of the rolling correlation, we report the standard deviations and the share of positive correlation coefficients in our sample period in Table 2. Columns (1) and (2) report the standard deviations. When series are detrended by HP filter, the standard deviations of rolling correlation coefficients are around 0.3. When taking the first order differencing of the log to the series, the standard deviations rose to about 0.5, since lots of high-frequency components are kept in this case. Columns (3) and (4) report the share of positive correlations. According to these numbers, positive correlations occurs very often. Again take nonresidential investment as an example, the correlation between nonresidential investment's relative price and its quantity was positive in about a quarter of our sample period, while being negative in the rest of period.

Table 4.2: Statistics of rolling correlations between relative prices and quantities of investment series

Type of investment	Standard deviation		Share of positive correlations	
	Log-difference	HP filter	Log-difference	HP filter
	(1)	(2)	(3)	(4)
Nonresidential	0.34	0.48	0.25	0.27
Structures	0.43	0.55	0.43	0.59
Equipment	0.33	0.50	0.18	0.27
Intellectual property products	0.30	0.47	0.30	0.36

The role of investment lumpiness in investment's quantity-price correlation

As we argued in the introduction part, we conjecture that the time variation of investment's quantity-price correlation is caused by fluctuation of investment lumpiness. In the following part, we study the relationship between investment lumpiness and investment's quantity-price correlation.

We measure the lumpy investment using Compustat annual fundamentals. Compustat is a dataset of listed companies in the US. In the benchmark case, we define that a firm's investment is lumpy if the firm's capital expenditure is less than the amount of its depreciation and amortization within a year[2]. For each year, we count the share of inactive firms – the ones with lumpy investment – out of our sample population. We use the share to measure investment lumpiness of the economy.

Figure 4 plots the share of inactive firms according to our benchmark measurement. As shown by the blue curve, on average, around half of the firms invest less than the amount of depreciation and amortization. The share increases in recessions, which means that firms are more likely to freeze investment activities in undesirable economic condition. This is consistent with (?)

The detrended component of the share of inactive firms is also plotted in figure 4. To detrend the series of the share of inactive firms, similar to our treatment to the quantity and the relative price of investment, we consider two approaches. The first one is to detrend the log of the share of inactive firms using HP filter with parameter 6.25. The second one

[2] We do not confine the investment lumpiness as zero capital expenditure which almost never occurs in listed companies.

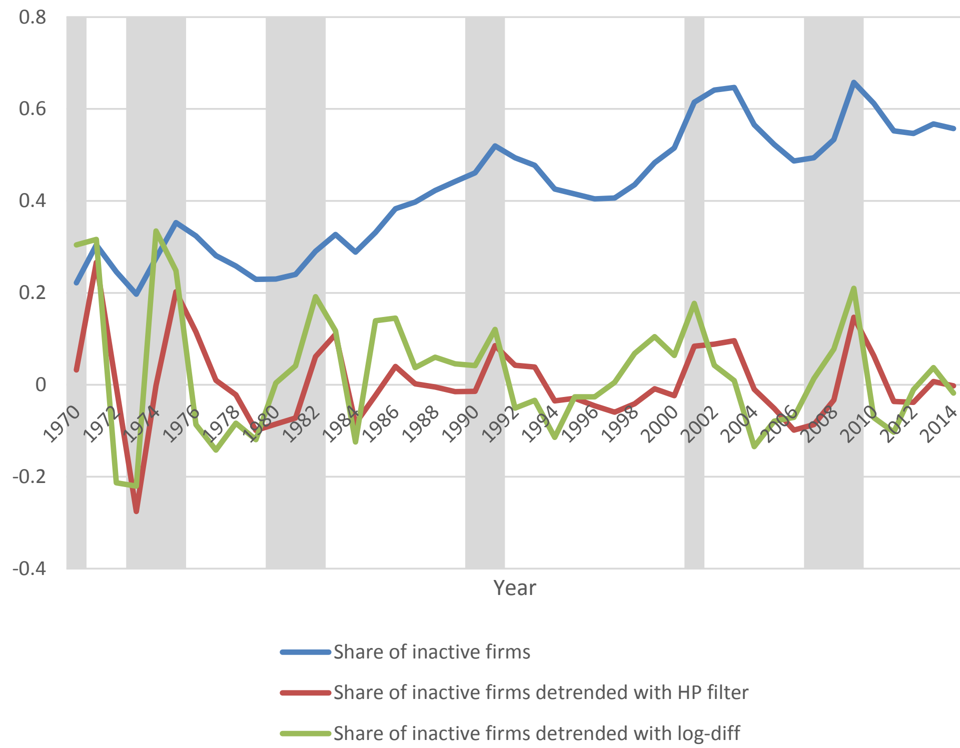


FIGURE 4.4: Measurement of investment lumpiness I

is to compute the first difference of the log of the share of inactive firms.

There might be concern that too many firms are counted as inactive firms; that is, our definition of lumpy investment might be too coarse. Therefore, we consider an alternative measurement. Specifically, we define that a firm's investment is lumpy if the firm's capital expenditure is less than the 1/4 of depreciation and amortization with a year. In other words, we count a firm as inactive firm if its annual investment is less than an averaged one-quarter's depreciation and amortization.

Figure 5 plots the raw series and detrended series of a share of inactive firms using the alternative measurement. On average, each year there is about 2 percent of firms invest less than the 1/4 of depreciation and amortization. In general, the share increases in recessions.

With the measurement of investment lumpiness, we are now able to examine if investment lumpiness is correlated with the sign of the correlation between relative price and

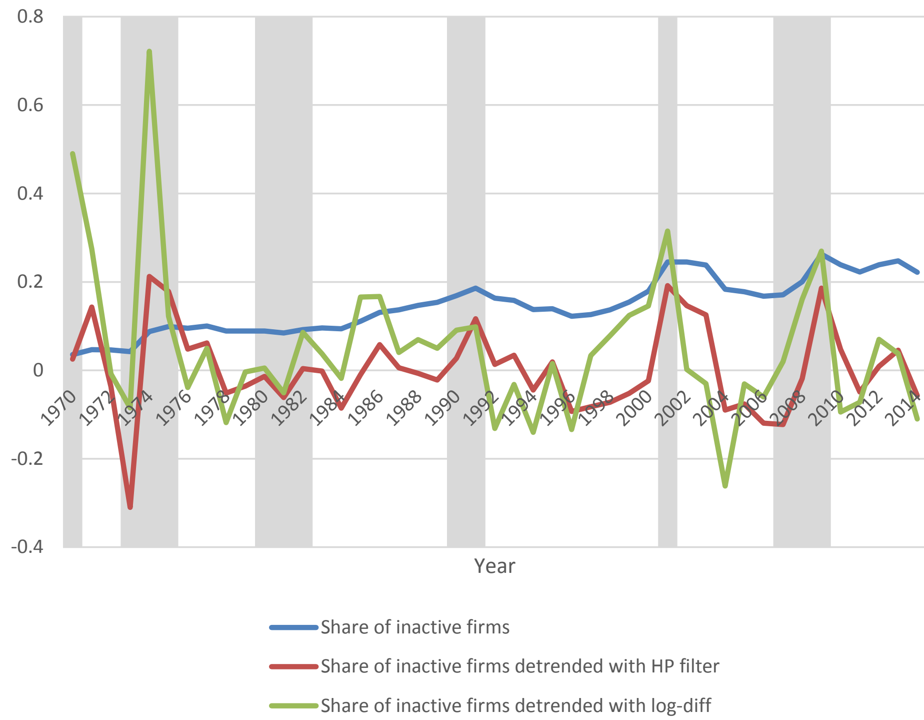


FIGURE 4.5: Measurement of investment lumpiness II

quantity of investment. Specifically, we conduct following regression

$$I\left(\rho_t^{NR} > 0\right) = \alpha + \beta \cdot lump_t$$

where ρ_t^{NR} is the rolling correlation between the quantity and relative price of nonresidential investment. We choose nonresidential investment since the capital expenditure we use to measure lumpiness fits into the nonresidential investment category. $I\left(\rho_t^{NR} > 0\right)$ is a dummy variable that equals one if ρ_t^{NR} is positive and zero otherwise. $lump_t$ is the share of inactive firms. We consider two measurements of investment lumpiness which have been described above. The original series considered in the regression, such as the measures of the relative price of investment, quantity of investment, and lumpiness, are detrended with either HP filter or the first order differencing of the log. In each regression, we keep the filtering methods same for the dependent variable and independent variable. For example, if we use HP filter for the relative price of investment, we also use HP filter for $lump_t$.

Table 4.3: When investment is lumpy, correlation is more likely to be negative

$$I\left(\rho_t^{NR} > 0\right) = \alpha + \beta_1 lump_t$$

Def. of lumpiness	Detrend method		*lump*	R-squared
$CAPX < DP$	Log-diff	(1)	-0.74 (0.50)	0.05
	HP filter	(2)	-1.29^{**} (0.49)	0.14
$CAPX < 0.25 \cdot DP$	Log-diff	(3)	-0.95^{**} (0.28)	0.12
	HP filter	(4)	-0.94^{**} (0.39)	0.09

The results are reported in Table 4.3. Coefficient β has negative estimates with all measurement methods. This means that when the share of inactive firms is higher, that is, when investment is lumpier, the relative price and quantity of nonresidential investment is more likely to be negatively correlated. The estimations have large standard errors, which might be due to the noisy measurements for both dependent and independent variables.

There might be concern that the results in table 4.3 are simply because that in recessions, $lump_t$ becomes lower and ρ_t^{NR} is more likely to be negative. So we consider the following regression

$$I\left(\rho_t > 0\right) = \alpha + \beta_1 lump_t + \beta_2 Recession_t$$

where *Recession* is a dummy variable that indicates recession years.

If the above concern is true, one should estimate β_2 to be negative and β_1 to be less significant. The results are reported in table 4.4. While *lump* does tend to become less significant, but the sign remains negative. Moreover, recession dummy does not have significant negative effect on $I\left(\rho_t > 0\right)$.

In sum, we find that when investment is lumpy, correlation is more likely to be negative.

Table 4.4: When investment is lumpy, correlation is more likely to be negative

$$I(\rho_t > 0) = \alpha + \beta_1 lump_t + \beta_2 Recession_t$$

Def. of lumpiness	Detrend method		*lump*	*Recession*	R-squared
$CAPX < DP$	Log-diff	(1)	-0.82 (0.55)	0.08 (0.16)	0.05
	HP filter	(2)	-1.03^{*} (0.54)	-0.15 (0.16)	0.16
$CAPX < 0.25 \cdot DP$	Log-diff	(3)	-1.06^{**} (0.42)	0.15 (0.15)	0.13
	HP filter	(4)	-0.66 (0.44)	-0.17 (0.16)	0.13

4.3 A simple two period model

4.3.1 Overview

In this section, we illustrate the main mechanism with a two-periods partial equilibrium model. In the model, there is a continuum of heterogeneous firms who use capital as input to produce consumption goods. To accumulate capital, the consumption goods producers purchase investment goods from a monopolistic investment goods producer. The price of investment goods is optimally set by the monopolistic investment goods producer.

We first show that an exogenous increase in lumpiness of investment, i.e. the share of firms who do not adjust capital, reduces the elasticity of demand for investment. We prove that the monopolistic seller of investment goods will raise the price as a response to the lower elasticity of demand, which means that investment lumpiness leads to an increase in the investment price. Then we show that the main result preserves when investment lumpiness is endogenous. Specifically, we employ uncertainty shock, that is, a shock to the dispersion of productivity distribution, as an example; as it is well known that shock to uncertainty induces changes in firm's capital adjustment at the extensive margin, causing an endogenous movement in the investment lumpiness.[3] We show that an increase in uncertainty reduces the elasticity of the aggregate demand for investment. Therefore, the monopolistic seller optimally raises the price in response to the steeper demand curve.

[3] see Bloom (2009); Bloom et al. (2012).

4.3.2 *Two-Period Model*

Reversible Investment

There is a continuum of consumption goods producers, indexed by $i \in [0,1]$, who are heterogeneous in capital stock and idiosyncratic productivity. For simplicity, we assume that the utility function is linear, hence the consumption goods producers simply maximize present discounted value of profit. The economy has two periods: periods 0 and 1. When the economy ends, the consumption goods producers liquidate the remaining capital $(1-\delta)k_{i,1}$ at a price P_1. Without loss of generality, we assume $P_1 = MC$, where MC denotes the marginal cost of producing investment goods.

Formally we can lay out the consumption good producer i's problem as:

$$\max_{k_{i,1}} \quad A_{i,0}k_{i,0}^{\alpha} - P_0\left[k_{i,1} - (1-\delta)k_{i,0}\right] + \beta\mathbb{E}_0\left[A_{i,1}k_{i,1}^{\alpha} + (1-\delta)k_{i,1}MC\right] \quad (4.3.1)$$

$$s.t. \quad \log(A_{i,1}) = \rho\log(A_{i,0}) + \sigma\varepsilon_{i,1}, \quad \varepsilon_{i,1} \sim N(0,1) \quad (4.3.2)$$

where $A_{i,t},\ t = 0,1$ is the idiosyncratic productivity, $k_{i,0}$ beginning of the period capital stock, and P_0 the price of investment goods set by monopolistic investment goods producer.

The consumption goods producer's investment is determined by the standard Euler equation:

$$\begin{aligned} P_0 &= \beta\mathbb{E}_0\left[\alpha A_{i,1}k_{i,1}^{\alpha-1} + (1-\delta)MC\right] \\ &= \beta\alpha\rho A_{i,0}k_{i,1}^{\alpha-1} + \beta(1-\delta)MC \end{aligned}$$

from which we can derive explicitly the demand for investment:

$$I(A_{i,0},k_{i,0};P_0) = \left(\frac{P_0 - \beta(1-\delta)MC}{\alpha\beta\rho}\right)^{\frac{1}{\alpha-1}} A_{i,0}^{\frac{1}{1-\alpha}} - (1-\delta)k_{i,0}. \quad (4.3.3)$$

From equation 4.3.3, we can derive an useful lemma.

Lemma 4. *Consumption goods producer's demand for investment is increasing in productivity $A_{i,0}$, and decreasing in investment price P_0 and capital stock in period 0 $k_{i,0}$.*

Lemma 4 shows the standard result of a firm's investment policy. With a higher productivity, the firm's expected *mpk* is higher, which increases a firm's incentive to do investment. Conditional on the productivity, when investment is more expensive or the firm has a higher initial capital stock, the firm wants less investment.

From equation 4.3.3, we can derive another useful result.

Lemma 5. *In the domain of $(A_{i,0}, k_{i,0}, P_0)$ where $I > 0$, the demand elasticity of price $\frac{\partial \log I}{\partial \log P}$ is increasing in $A_{i,0}$.*

Lemma 5 is a key component of our mechanism. As $\frac{d \log I}{d \log P}$ is negative, the size of elasticity is decreasing in $A_{i,0}$, which means that the demand becomes more inelastic when firm becomes more productive.

Equilibrium of the simple model

In this subsection, we derive the equilibrium of the model. For tractability, we assume all consumption good producers start with the same capital $\bar{k}$, that is $k_{i,0} = \bar{k}, \forall i$. In addition, the initial distribution of $A_{i,0}$ follows a Pareto distribution with with scale parameter $\underline{A}$ and shape parameter ξ which corresponds to a probability density function of $f(A_{i,0}) = \xi \underline{A}^{\xi} A_{i,0}^{-\xi-1}$. The above assumptions enable us to analytically aggregate the individual demand curves of investment goods to an aggregate demand curve.

Specifically, the aggregate demand for investment is a function of investment price P_0, and takes the following form:

$$I(P_0) = \left[\frac{P_0 - \beta(1-\delta)MC}{\alpha\beta\rho} \right]^{\frac{1}{\alpha-1}} \frac{\xi(1-\alpha)}{\xi(1-\alpha)-1} \underline{A}^{\frac{1}{1-\alpha}} - (1-\delta)\bar{k} \tag{4.3.4}$$

The monopolistic seller maximize the profit by solving

$$\max_{P_0} I(A^*, P_0) \cdot (P_0 - MC)$$

The effect of the composition of the demand pool on the equilibrium price: an exogenous case

To study the effect of composition of the individual firms on the elasticity of the aggregate demand, we impose that only buyers with productivity larger than A^* can enter the market for investment. With this restriction, the aggregate demand, denoted as $I^R(A^*, P_0)$, becomes:

$$I^R(A^*, P_0) = \left[\frac{P_0 - \beta(1-\delta)MC}{\alpha\beta\rho}\right]^{\frac{1}{\alpha-1}} \frac{\xi(1-\alpha)}{\xi(1-\alpha)-1}\underline{A}^{\xi} - (1-\delta)\bar{k}\underline{A}^{\xi}.$$

Similarly, we denote the equilibrium price set by the monopolistic seller is $P_0^R(A^*)$ with

$$P_0^R(A^*) = \arg\max_{P_0} I(A^*, P_0) \cdot (P_0 - MC)$$

It is worth noting that the case without any restriction on the entry of the buyer is nested in the restricted case with $A^* = \underline{A}$

Lemma 6. *The elasticity of aggregate demand $\frac{\partial I^R(A^*, P_0)}{\partial \log P_0}$ is increasing in A^*.*

Lemma 6 is similar to lemma 5.[4] This lemma means that the demand becomes more inelastic when the firms seeking for investment are more productive.

We can also show that the equilibrium price of investment is increasing in A^*. Hence we have the following proposition.

Proposition 15. *The equilibrium price of investment set by the monopolistic seller $P^R(A^*)$ is increasing in the productivity threshold of capital adjustment A^*.*

When the firms seeking for investment are more productive, the demand becomes more inelastic. The monopolistic seller would exploit the inelastic demand curve by setting a higher price.

[4] Notice that $\frac{\partial I^R(A^*, P_0)}{\partial \log P_0}$ is negative, the size of the elasticity of aggregate demand for investment is decreasing in A^*

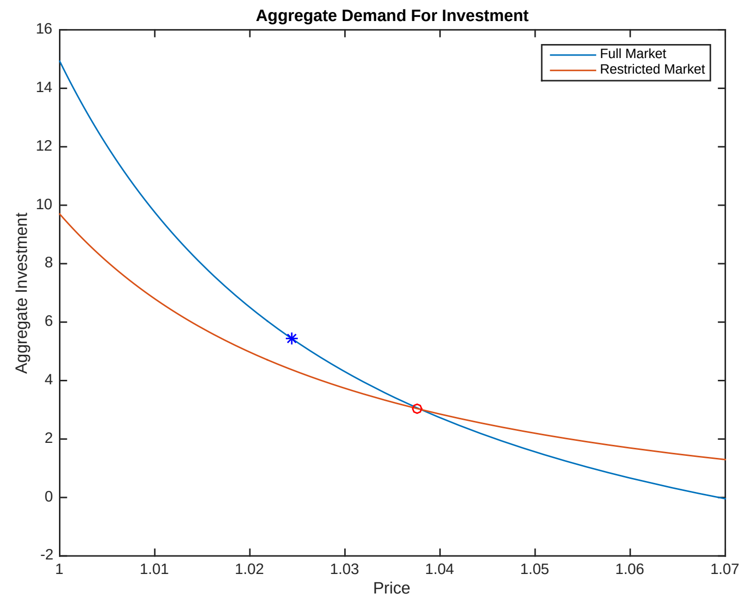


FIGURE 4.6: Effect of Disallowing Unproductive Firms To Invest

In order to show the effect of investment threshold A^* on the equilibrium of the investment market more clearly, we use Figure 4.6 to compare the demand curves of two cases, one with restriction ($A^* > \underline{A}$), the other one without ($A^* = \underline{A}$). Clearly, the elasticity of demand is lower in the first case with restriction than in the second case without restriction. The monopolist responded by charging a higher price and selling lower quantity of investment goods. Trivially, our restriction on buyer's results in a smaller mass of active buyers in the market thus a higher degree of lumpiness.

The effect of the composition of the demand pool on the equilibrium price: an endogenous case

In this subsection, we consider the case in which the decision of participating investment market is endogenous. Specifically, we modify the model with two elements. The first one is irreversible investment. The second one is a movement in the dispersion of firm's productivity.

Investment irreversibility As the result, some buyers endogenously stay inactive in capital adjustment when the desired capital stock tomorrow is lower than depreciated stock of capital today. In particular, we modify the model by adding a constraint $k_{i,1} > (1-\delta)k_{i,0}$. The consumption goods producer's problem becomes:

$$\max_{k_{i,1}} \quad A_{i,0}k_{i,0}^{\alpha} - P_0\left[k_{i,1} - (1-\delta)k_{i,0}\right] + \beta\mathbb{E}_0\left[A_{i,1}k_{i,1}^{\alpha} + (1-\delta)k_{i,1}MC\right] \tag{4.3.5}$$

$$s.t. \quad k_{i,1} \geqslant (1-\delta)k_{i,0} \tag{4.3.6}$$

$$\log(A_{i,1}) = \rho\log(A_{i,0}) + \sigma\varepsilon_{i,1}, \quad \varepsilon_{i,1} \sim N(0,1) \tag{4.3.7}$$

The irreversible constraint 4.3.6 is an extreme assumption for simplicity, which is equivalent to setting the resale price of used capital to be zero.[5] In this case, some firm might choose to not enter the investment market because it neither purchasing new investment goods nor selling used capital is preferred to simply stay inactive and let capital depreciate.

The FONC in the open set $k_{i,1} > (1-\delta)k_{i,0}$ is given by:

$$\begin{aligned} P_0 &= \beta\mathbb{E}_0\left[\alpha A_{i,1}k_{i,1}^{\alpha-1} + (1-\delta)MC\right] \\ &= \beta\alpha\rho A_{i,0}k_{i,1}^{\alpha-1} + \beta(1-\delta)MC \end{aligned} \tag{4.3.8}$$

Equation 4.3.8 depicts firm's investment decision along the intensive margin. Equation 4.3.8 is equivalent to

$$\begin{aligned} k_{i,1} &= \left[\frac{P_0 - \beta(1-\delta)MC}{\beta\alpha\rho A_{i,0}}\right]^{\frac{1}{\alpha-1}} \\ when \quad k_{i,1} &\geqslant (1-\delta)k_{i,0} \end{aligned}$$

As $\alpha - 1 < 0$, if a firm decide to invest, the level of capital stock in period 1 is increasing

[5] In the general model in the next section, the resale price of investment goods is set to be nonzero.

in productivity $A_{i,0}$ and decreasing in price of investment P_0.

It is easy to show that the cut-off price above which consumption producers would not invest is given by:

$$P^*(A_{i,0},k_{i,0}) = \beta\alpha\rho(1-\delta)^{\alpha-1}k_{i,0}^{\alpha-1}A_{i,0} + \beta(1-\delta)MC \tag{4.3.9}$$

Equation 4.3.9 depicts firm's investment decision along the extensive margin. Given the price of investment P_0, a firm would invest if and only if its idiosyncratic capital stock and productivity satisfy $P^*\left(A_{i,0},k_{i,0}\right) < P_0$. Given individual demand for investment goods we can find the aggregate demand curve faced by the investment goods monopolist. As we assume that firms have same capital stock $\bar{k}$ in period 0, we can derive the cut-off productivity as:

$$A_0^*(P_0) = \frac{P_0 - \beta(1-\delta)MC}{\alpha\beta\rho\left[(1-\delta)\bar{k}\right]^{\alpha-1}}. \tag{4.3.10}$$

A firm would adjust capital at the extensive margin if and only if its productivity is higher than $A_0^*(P_0)$.

Combine equations 4.3.8 and 4.3.9, we get the firm's investment policy is (depicted in Figure 4.9)

$$I(A_{i,0},k_{i,0};P_0) = \begin{cases} \left(\frac{P_0-\beta(1-\delta)MC}{\alpha\beta\rho}\right)^{\frac{1}{\alpha-1}} A_{i,0}^{\frac{1}{1-\alpha}} - (1-\delta)k_{i,0} & , \quad P_0 < P^*(A_{i,0},k_{i,0}) \\ 0 & , \quad otherwise \end{cases} \tag{4.3.11}$$

Given the individual firm's investment policy, we can derive analytically the aggregate demand for investment goods as a function of investment goods price P_0:[6]

$$I(P_0) = [P_0 - \beta(1-\delta)MC]^{-\xi} C(\xi,\underline{A},\bar{k})$$

,where $C(\xi,\underline{A},\bar{k})$ is a number determined by the distribution of consumption goods pro-

[6] To ensure the demand function $I\left(P_0\right)$ to be finite, we need to assume $\xi > \frac{1}{1-\alpha}+1$

ducers which is independent of P_0.

Same as the exogenous case, the monopolistic seller solves

$$P_0^M = \arg\max_{P_0} I(P_0) \cdot (P_0 - MC)$$

which yields

$$P_0^M = \frac{\xi - \beta(1-\delta)}{\xi - 1} MC. \tag{4.3.12}$$

Plugging 4.3.12 into 4.3.10 we can derive the expression for equilibrium cut-off point under monopolistic pricing

$$A_0^*(P_0^M) = \frac{MC}{\alpha\beta\rho\left[(1-\delta)\bar{k}\right]^{\alpha-1}} \frac{1-\beta(1-\delta)}{1-\xi^{-1}} \tag{4.3.13}$$

A firm would do investment if and only if its productivity is higher than $A_0^*(P_0^M)$.

An increase in the productivity dispersion and investment lumpiness In this subsection, we want to show that in our model, an endogenous increase in the lumpiness of investment would increase the investment goods price.

We first formally define lumpiness. Lumpiness is the measure of firms that doesn't investment under the price set by monopolist investment goods producer. Specifically:

Definition 3. Lumpiness is the measure of inactive firms: $L \equiv Pr\left(A_{i,0} \leqslant A_0^*\left(P_0^M\right)\right)$.

Using equation 4.3.13, we can analytically derive lumpiness as a function of the distribution of the investment buyers

$$L(\xi, \underline{A}) \quad = \quad 1 - \left[\frac{\alpha\beta\rho\left[(1-\delta)\bar{k}\right]^{\alpha-1}\mathbb{E}(A_{i,0})}{MC(1-\beta(1-\delta))}\right]^{\xi} \tag{4.3.14}$$

From equation 4.3.14, we get $\frac{\partial L}{\partial \xi} < 0$ and $\frac{\partial L}{\partial \underline{\mathrm{A}}} < 0$. It is interesting to look at how lumpiness respond to a mean-preserving spread increase, i.e. when the mean of idiosyncratic productivity remain the same but the variance increases.This implies that ξ decreases (fatter tail) but $\underline{A}$ decreases so that the mean of productivity across firms remained the same. This is illustrated in Figure 4.9. Obviously we can see this change represents a change in composition of firms: a) there are more firms with very low productivity b) there are more firms with very high productivity c) there are less firms with median productivity. That is to say, there's a mean-preserving productivity polarization taking place. From 4.3.14 we see lumpiness increases.

$$\frac{\partial L}{\partial \underline{A}} < 0, \quad \frac{\partial L}{\partial \xi} < 0$$

Moreover, from equation 4.3.12, it is easy to show that equilibrium price of investment is increasing in ξ; that is $\frac{\partial P_0^M}{\partial \xi} > 0$; the equilibrium quantity of investment is decreasing in ξ, that is $\frac{\partial I}{\partial \xi} < 0$. Hence we can summarize the above results by the following proposition.

Proposition 16. *A mean preserving increase in the dispersion of the buyers' productivity distribution (a decrease in ξ) would (1) increases the investment lumpiness and investment price and (2) reduces the investment quantity.*

When the distribution of productivity gets more fat-tailed (ξ decreases), lumpiness and investment price increases, while the quantity decreases.

4.3.3 What drives the results

The simple model is capable of generating lumpiness simultaneously with higher investment goods price, consistent with the fact we document in section 2. The simple model has two key assumptions: (1) investment irreversibility, (2) monopolistic seller. Are both assumptions necessary for matching the fact? In this subsection, we answer to the question with counterfactual analysis. Based on the counterfactual analysis, both assumptions are necessary.

No Investment Irreversibility

We first remove the investment irreversibility, hence the model becomes identical to the one we studied in section 4.3.2.

Proposition 17. *With fully reversible investment mean-preserving polarization leads to higher investment price, higher lumpiness (measure of firms not making positive investment),* but higher quantity of investment.

We give the proof in the appendix. This proposition is graphically shown in Figure 4.11. We see that mean-preserving polarization in this case imitates a "demand shock". Both price and quantity of investment go up as if everyone gets more productive and demands more capital.

When we allow individual firms to deinvest, the investment goods producer serves as a seller of used capital. The investment monopolist is allow to buy from low productivity firms and sell to high productivity firms, but at zero profit rate. Therefore the used capital supply is out-crowding the new capital goods produced by the producer, and the monopolist takes that into account. In fact there's a drastic change in the demand curve the monopolist is facing, as showed in Figure 4.10. Given the same buyers, opening the resale market induces a large downshift in demand schedule due to the out-crowding effect just described.

No Monopoly Power

Then we remove the assumption of monopolistic investment goods producer. Specifically, we replace the monopolistic investment goods producer with a continuum of producers who sell investment goods in a competitive market. When the investment goods producers do not have monopoly power, the equilibrium price of investment goods equals the marginal cost MC.

Plug $P_0 = MC$ into equation 4.3.10, we get that the cut-off productivity is constant:

$$A_0^* = \frac{[1-\beta(1-\delta)]MC}{\alpha\beta\rho\left[(1-\delta)\bar{k}\right]^{\alpha-1}}.$$

From the property of Pareto distribution, we can see that lumpiness decreases because it's strictly increases in ξ and $\underline{A}$ and a mean-preserving polarization involves lower ξ and $\underline{A}$. From Figure 4.8 we can see clearly that the quantity of investment drops. Therefore

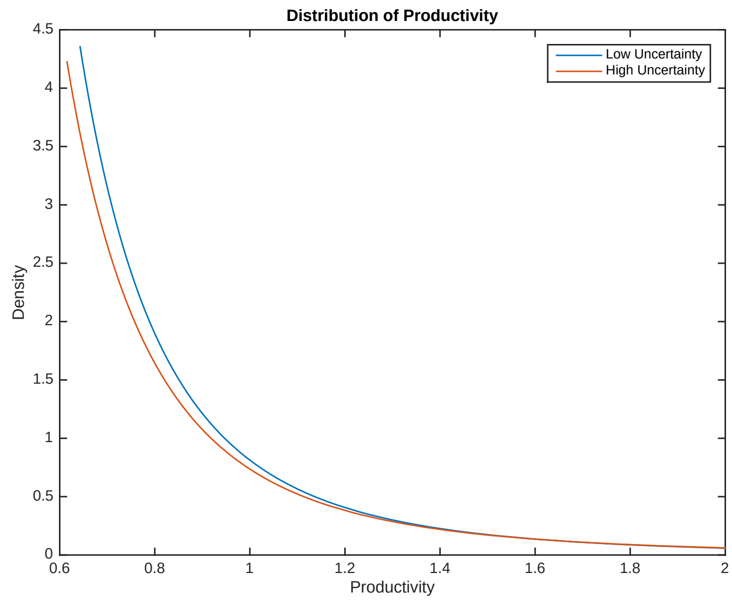


FIGURE 4.7: Change in Distribution of Productivity

we can summarize the effect of mean-preserving polarization as:

Proposition 18. *Without monopoly power mean-preserving polarization leads to the higher lumpiness, lower quantity of investment,* but the same price.

From the above analysis, we show that to generate realistic comovement of investment quantity and price and lumpiness, both investment irreversibility and monopolistic pricing are necessary.

4.4 A DSGE model with frictional investment goods market

While the simple model demonstrates the main idea, it is restricted in at least two ways. First, the simple model has only two periods hence, cannot be used for quantitative analysis. Second, the simple model assumes that investment is sold by only one monopoly producer, which is an extreme assumption, as in the economy, there are many investment goods producers. In this section, we present a DSGE model with a continuum of investment goods producers.

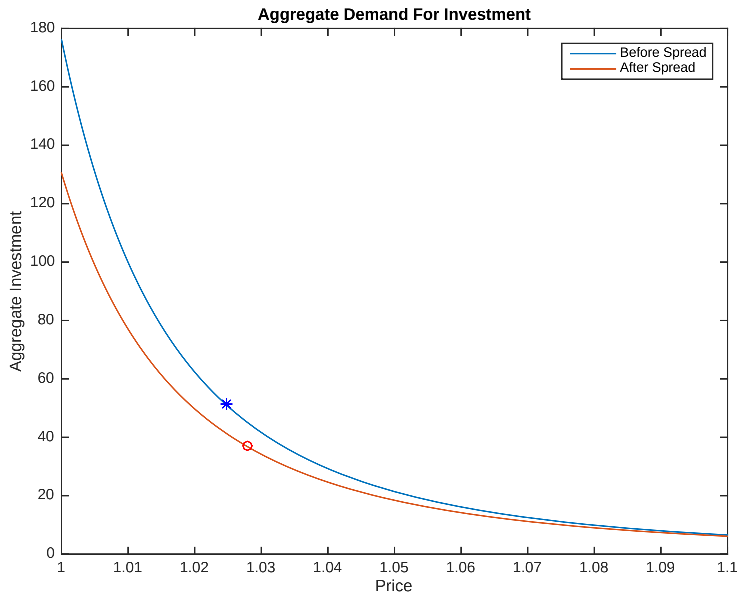


FIGURE 4.8: Change in Demand Curve

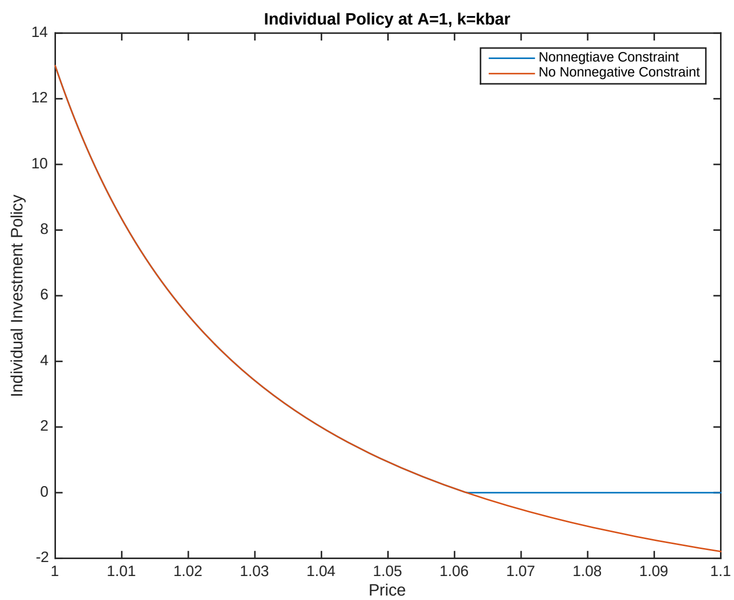


FIGURE 4.9: Policy Function

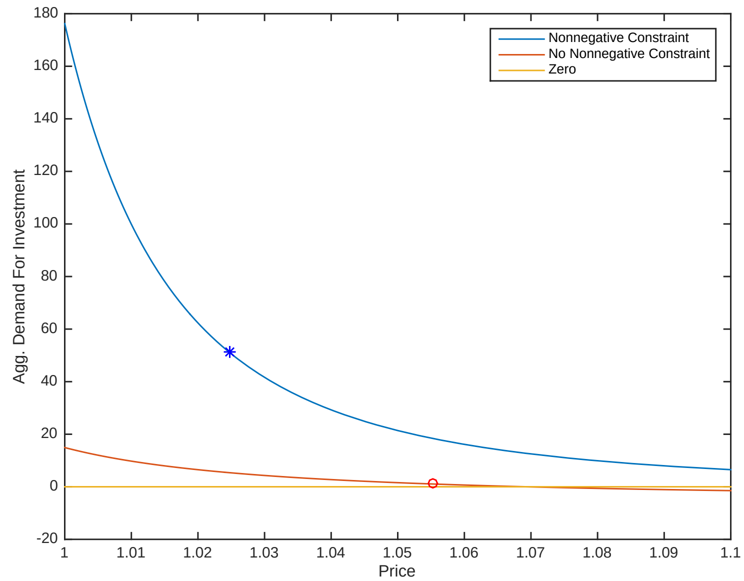


FIGURE 4.10: Opening Up Resale Market

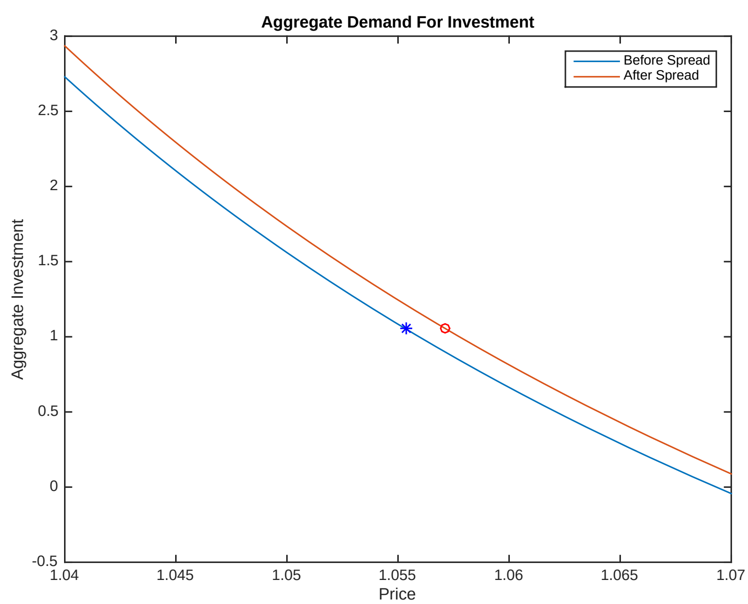


FIGURE 4.11: Effect of Polarization

Our model is an otherwise standard heterogeneous firm DSGE model except that (1) we assume that there are uncertainty shocks and irreversibility of capital adjustment, which helps the model to generate fluctuation of investment lumpiness; (2) there is a search friction in the investment goods market; that is, it takes time for the buyer and seller of investment goods to match. In the model, buyers are a continuum of final goods producers; sellers are a continuum of investment goods producers. For a buyer who is matched with a seller, there is a considerable opportunity cost of moving on to search for another seller. Therefore, with search friction in the investment goods market, sellers of investment goods have monopoly power even though they are measure zero. With the monopoly power, sellers would charge a positive markup which depends on the demand curve it is faced with. Similar to the single monopoly case, the seller of investment goods charges a higher markup when demand curve is less elastic and vise verse. It is worth noting that when the uncertainty shocks and search friction is removed, our model becomes similar to standard heterogeneous firm models, such as in Khan and Thomas (2008); Bloom et al. (2012), in which markup of investment goods is always zero.

In this section, we first demonstrate the baseline environment of the model. Then, we derive the value functions and policy functions of the buyer (final goods producers) and the seller (investment goods producers) of investment goods. The individual firm's policy function enables us to derive the aggregate demand curve and aggregate supply curve of investment goods. Given the value functions, policy functions and aggregate demand/supply, we then define the symmetric competitive equilibrium. In the end, use numerical method to solve and analyze the model.

4.4.1 Baseline environment

Technology

Final goods are produced by measure 1 of continuum of heterogeneous final goods producers. We assume that each firm operates a decreasing returns to scale production function with capital and labor as inputs. The assumption of decreasing returns to scale helps to pin down firms' size. In specific, production function of firm j is

$$y_{j,t} = A_t z_{j,t} k_{j,t}^{\alpha} L_{j,t}^{\nu}, \quad \alpha + \nu < 1$$

Each final goods producer's productivity is a product of two separate processes; aggregate productivity A_t and idiosyncratic productivity $z_{j,t}$ which are governed by the following autoregressive processes

$$\begin{aligned} log\left(A_t\right) &= \rho_A log\left(A_{t-1}\right) + \sigma_A \varepsilon_{A,t} \\ log\left(Z_{j,t}\right) &= \rho_Z log\left(Z_{j,t-1}\right) + \sigma_{Z,t-1} \varepsilon_{Z,t} \end{aligned}$$

The variance of innovation of idiosyncratic productivity $\sigma_{Z,t-1}$ follows a two-state Markov switching process which generates periods of low and high cross-firm dispersion.

There are a continuum of investment goods producers, also with measure 1. For simplicity, we assume that investment goods producers are homogeneous and use final goods as the only input to produce investment goods. Investment goods producer's production function is

$$i_{j,t}^{S} = \psi_t X_{j,t}$$

$i_{j,t}^{S}$ is unit of investment goods produced by producer j. ψ_t is the productivity of investment goods producers. $X_{j,t}$ is unit of final goods used to produce investment goods.

Productivity of investment goods $\xi_{I,t}$ follows autoregressive process

$$log\left(\xi_{I,t}\right) = \rho_I log\left(\xi_{I,t-1}\right) + \sigma_I \varepsilon_{I,t}$$

where $\varepsilon_{I,t}$ is investment-specific shock that shifts all the investment goods producers' productivity in the same way.

Frictional investment goods market

There is a standard search friction which makes it difficult for final goods producers (buyer) to meet with investment goods producers (seller). We model the search friction as random search. The sellers and buyers search and match in a decentralized investment market, and the matching probability depends on the measure of supply (seller) and demand (buyer).

Specifically, the matching function is constant returns to scale and specifies as

$$M = \frac{D \cdot S}{\left(D^{\iota} + S^{\iota}\right)^{1/\iota}}$$

This matching function, suggested by den Haan et al. (2000), ensures that the matching probability is always between zero and one.

S and D are measure of supply and demand in the matching market indexed by price q. Determination of S and D are the key part of the model, which would be discussed in the subsequent subsections.

Following standard search and matching models, we define the ratio of measure of sellers and buyers as the market tightness ratio

$$\theta = \frac{S}{D}$$

The probability for a final goods producer to find an investment goods producer is

$$\mu = \frac{M(S, D)}{D} \tag{4.4.1}$$

Since elasticity of matching function $\bar{\alpha}$ is a positive number, μ is increasing in θ. The intuition is that a higher θ indicates a larger pool of sellers relative to buyers, hence it's easier for buyers to get matched.

Similarly, the probability for an investment goods producer to match with a final goods producer is

$$f = \frac{M(S, D)}{S} \tag{4.4.2}$$

It's easy to see that f is decreasing in θ.

For simplicity, we assume that matches are temporary; that is, all matches would separate at the end of each period. In the following period, firms who want to trade investment goods need to search for match in the matching market.

4.4.2 Buyer (Final goods producer)

In this subsection, we present the value functions and policy functions of final goods producers who are the buyer of investment goods. We want to use these value functions and policy functions to characterize individual final goods producer's demand for investment goods.

We denote $V(k,z;A,\sigma_Z,\xi_I,\Pi)$ as the value function of a buyer of investment goods. The six state variables which are given by (1) a firm's capital stock k, (2) a firm's idiosyncratic productivity z, (3) aggregate productivity A, (4) volatility of idiosyncratic productivity σ_Z, (5) productivity of investment goods producers ξ_I, (6) the joint distribution of idiosyncratic productivity and firm-level capital stocks Π, which is defined for the space $R_+ \times R_+$. Distribution of final goods producers enters state space because it affects current and future prices such as tightness ratio, discount rate and investment goods price. We stack aggregate states in a vector $\Omega = \{A,\sigma_Z,\xi_I,\Pi\}$, and simplify value function as $V(k,z;\Omega)$.

Buyer's problem is to choose investment and labor input to maximize the present discounted value of future profits. Its investment problem presents at both intensive and extensive margin.

Specifically, a buyer decides whether to search for investment goods producer given the state of the economy. We denote $W(k,z;\Omega)$ and $U(k,z;\Omega)$ as values of searching for investment and not searching. By definition, we have

$$V(k,z;\Omega) = \max\left(U(k,z;\Omega), W(k,z;\Omega)\right)$$

The buyer chooses to search for investment if and only if $U(k,z;\Omega) < W(k,z;\Omega)$.

In the following part, we formally show the above mentioned value functions and the associated policy functions.

Value of being inactive

If a final goods producer decides not to search for investment goods producers, it only needs to set the labor input. Its value function is

$$U(k,z;\Omega)=\max_{l}\lambda(\Omega)[y-w(\Omega)l]+E\left[V\left((1-\delta)k,z^{'};\Omega^{'}\right)\right] \tag{4.4.3}$$

This value is composed of the contemporary profit – output y net of cost on labor input $w(\Omega)l$ – plus the expected discounted value from next period on. Since there is no investment, in the next period, future capital stock just depreciates by δ fraction, hence next period's capital stock is $(1-\delta)k$.

Value of searching for investment

If a final goods producer decides to search for investment goods producers, its needs to choose which submarket to enter. If the final goods producer successfully matches with an investment goods producer, it then chooses the optimal quantity of investment as well as the labor input. We derive the value of searching for investment backwardly in two steps. In the first step, we show the value of choosing an investment price; that is, the choice value of entering a submarket. Then we use the choice value to derive the value of searching for investment.

Following Khan and Thomas (2008), we assume that buyer pays a fixed cost ξ; Moreover, there is a resale cost to a final goods producer if the level of its investment is negative. Specifically, if the level of investment is negative, investment goods are traded at κ proportion of the price tagged in the submarket of investment goods, with $\kappa\leqslant1$. Therefore, final goods producer who invests i units of investment goods in submarket with price q pays $\phi(i)q$, where

$$\phi(i)=\begin{cases} i & when\ I_{i,t}\geqslant 0\\ \kappa i & when\ I_{i,t}<0\end{cases}$$

Denote $W(k,z;\Omega)$ as the value, or the choice value in the language of the discrete choice literature, of entering a submarket with price q, we have

$$W(k,z;\Omega) = \max_{l,i} \lambda(\Omega) \begin{bmatrix} y - w \cdot l - FC \cdot w - \mu \cdot \xi \cdot k \cdot \left(\frac{i}{k}\right)^2 \\ -\mu \cdot \phi(i) \cdot q\left(k', z'; \Omega'\right) \end{bmatrix} \quad (4.4.4)$$

$$+E\left\{\begin{bmatrix} \mu V\left(k', z'; \Omega'\right) \\ + \\ (1-\mu) V\left((1-\delta)k, z'; \Omega'\right) \end{bmatrix}\right\}$$

with

$$k' = (1-\delta)k + i \quad (4.4.5)$$

In equation 4.4.4, with probability μ, which is defined in equation 4.4.1, a final goods producer successfully matches with an investment goods producer and spends $\phi(i)q$ on investment, then receive a continuation value $V\left(k', z'; \Omega'\right)$. With probability $1-\mu$, however, it fails to find an investment goods producer in which case the next period's capital stock is $(1-\delta)k$, same as the capital stock in the case of inactivity. Equation 4.4.5 is the standard flow motion of capital stock whose last term is a quadratic adjustment cost.

Buyer's policy function and demand curve

Buyer's investment decision can be described by two policy functions. One describes the extensive margin of investment, the other one describes the intensive margin.

The first policy function $\chi(k,z;\Omega)$ controls the investment policy at the extensive margin; that is, whether to search for investment in market indexed by q

$$\chi(k,z;\Omega) = \begin{cases} 1 & if\, W(k,z;\Omega) \geqslant U(k,z;\Omega) \\ 0 & otherwise \end{cases} \quad (4.4.6)$$

The second policy function $i(k,z;\Omega)$ characterizes the policy at the intensive margin.

Specifically,

$$i(k,z;\Omega) = arg\max_{i}\left\{-\phi(i)\,q\left(k^{'},z^{'};\Omega^{'}\right) + E\left[m\left(\Omega,\Omega^{'}\right)V\left(k^{'},z^{'};\Omega^{'}\right)\right]\right\} \tag{4.4.7}$$

with

$$k^{'} = (1-\delta)\,k + i$$

Combining the two policy functions, a buyer's demand for investment goods is $\chi(k,z;\Omega)\cdot i(k,z;\Omega)$.

Aggregate demand for investment goods

Given the joint distribution of capital stock k and idiosyncratic productivity z of final goods producers and their policy function $\chi(k,z;\Omega)$ and $i(k,z;\Omega)$, we are now ready to derive the aggregate demand for investment goods. Since our model has investment lumpiness, the aggregate investment is characterized by two measures: the first one is the measure of firms who search for investment goods; the second one is the measure of investment goods bought by those final goods producers.

The measure of active firms is

$$D(\Omega) = \int \chi(k,z;\Omega)\,d\Pi \tag{4.4.8}$$

where Π is CDF of joint distribution of k and z.

$D(\Omega)$ is key variable because it affects the matching probabilities for sellers and buyers. Given the measure of sellers in a submarket, a higher $D(\Omega)$ implies that it is harder for buyers to find a match.

By integrating the buyer's demand for investment goods adjusted by the matching probability, the total measure of investment goods purchased by the buyers is

$$I(\Omega) = \mu \int i(k,z;\Omega)\,\chi(k,z;\Omega)\,d\Pi \tag{4.4.9}$$

In equation 4.4.9, the term inside the integration is the measure of investment goods that is pursued by each buyer. Only μfraction of the investment goods can be reached due to the search friction.

4.4.3 Seller (Investment goods producers)

There is a fringe of potential investment goods sellers waiting to be matched with buyers. To enter a seller needs to pay an sunk entry cost of κ. To determine the benefit of entering we study the payoff of sellers in a backward fashion. Once the buyer and seller meet and agree to transact at the price q, the seller's payoff is $\phi\left(i(k,z)\right)\cdot q(k,z) - i(k,z)\cdot\psi$. Thus the the free-entry condition is given by:

$$\kappa = \int\left[\phi\left(i(k,z;\Omega)\right)\cdot q(k,z;\Omega) - i(k,z;\Omega)\cdot\psi\right] f(\Omega)\chi(k,z;\Omega)d\Pi \tag{4.4.10}$$

which in practice pin down the tightness ratio θ (since $f(\Omega)$ is pinned down from free-entry condition) and the measure of seller entering the investment good market S. Note that $f(\Omega)$ denotes the buyer finding rate $\frac{M}{S}$.

4.4.4 Determination of investment price

For simplicity, we assume that the investment price q is determined by Nash bargaining. When matched, the seller and the buyer joint choose a price that divides the total surplus with fixed shares. The seller's and buyer's bargaining share is τ and $1-\tau$ respectively. The total surplus is $\beta\mathbb{E}V\left(k', z^{'}; \Omega^{'}\right) - \beta\mathbb{E}V\left((1-\delta)k, z^{'}; \Omega^{'}\right) - \lambda\xi\cdot k\cdot\left(\frac{i}{k}\right)^2 - \lambda(i\cdot\psi)$, where $\beta\mathbb{E}V\left(k', z^{'}; \Omega^{'}\right) - \beta\mathbb{E}V\left((1-\delta)k, z^{'}; \Omega^{'}\right)$ is the discounted expectation of the incremental value induced by capital accumulation; $\xi\cdot k\cdot\left(\frac{i}{k}\right)^2$ and $i\cdot\psi$ are the quadratic adjustment and production cost of investment; λ is the marginal utility.

By applying the Nash bargaining to the buyer's value function equation 4.4.4 we have

$$\lambda\left[\phi\left(i\right)\cdot q\left(k,z;\Omega\right)-i\left(k,z;\Omega\right)\cdot\psi\right] =$$

$$\tau\cdot\left[\beta\mathbb{E}V\left(k^{'},z^{'};\Omega^{'}\right)-\beta\mathbb{E}V\left(\left(1-\delta\right)k,z^{'};\Omega^{'}\right)-\lambda(\Omega)(i\cdot\psi)-\lambda\xi\cdot k\cdot\left(\frac{i}{k}\right)^{2}\right]$$

This implies that the investment price has the form

$$q = \frac{\frac{\tau}{\lambda}\cdot\left[\beta\mathbb{E}V\left(k^{'},z^{'};\Omega^{'}\right)-\beta\mathbb{E}V\left(\left(1-\delta\right)k,z^{'};\Omega^{'}\right)-\lambda\cdot i\cdot\psi-\lambda\xi\cdot k\cdot\left(\frac{i}{k}\right)^{2}\right]}{\phi\left(i\right)}+\frac{i\cdot\psi}{\phi\left(i\right)}$$

Plug the above equation back into the buyer's value function equation 4.4.4 , we have

$$\begin{aligned} W\left(k,z;\Omega\right) &= \max_{l,i}\lambda\left(\Omega\right)\left[y-w\cdot l-\mu(1-\tau)\cdot\xi\cdot k\cdot\left(\frac{i}{k}\right)^{2}-FC\cdot w-\mu\cdot(1-\tau)\cdot i\cdot\psi\right] \\ &\quad +\beta E\left\{\left[\begin{array}{c}\mu\left(1-\tau\right)V\left(k^{'},z^{'};\Omega^{'}\right)\\ +\\ \left(1-\mu+\mu\tau\right)V\left(\left(1-\delta\right)k,z^{'};\Omega^{'}\right)\end{array}\right]\right\}\end{aligned}$$

Since the decisions of labor and investment are independent, we can denote the maximized "operating profit" as $\pi^{*}=\max_{l}y(z,k,l;\Omega)-w\cdot l$ and re-express the active buyer's value function W as:

$$\begin{aligned} W(k,z;\Omega) &= \lambda\left[\pi^{*}-FC\cdot w\right]+\beta\mathbb{E}\left\{\mu\cdot(1-\tau)\cdot V((1-\delta)k,z';\Omega')\right\} \\ &\quad + \max_{i}\left\{\begin{array}{c}-\mu(1-\tau)\cdot\xi\cdot k\cdot\left(\frac{i}{k}\right)^{2}-\mu\cdot(1-\tau)\cdot i\cdot\psi\\ +(1-\mu+\mu\tau)\beta\mathbb{E}\left[V((1-\delta)k+i,z')\right]\end{array}\right\}\end{aligned}$$

We define the aggregate index of the price of investment goods as the ratio of the aggregate spending on investment goods to the aggregate investment.

$$\begin{aligned} q^{agg} &= \frac{\mu \int q(k,z;\Omega)\, i(k,z;\Omega)\, \chi(k,z;\Omega)\, d\Pi}{\mu \int i(k,z;\Omega)\, \chi(k,z;\Omega)\, d\Pi} \\ &= \frac{\int q(k,z;\Omega)\, i(k,z;\Omega)\, \chi(k,z;\Omega)\, d\Pi}{\int i(k,z;\Omega)\, \chi(k,z;\Omega)\, d\Pi} \end{aligned}$$

4.4.5 Households

Representative household is standard. She receives utility from consumption and disutility from labor. It chooses the consumption and labor supply to maximize its life time utility. Household's value function is:

$$J(A, \sigma_Z, \xi_I, \Pi) = \max_{C,L} u(C,L) + \beta E\left[J\left(A^{'}, \sigma_Z^{'}, \xi_I^{'}, \Pi^{'}\right)\right]$$

subject to budget constraint

$$C = L \cdot w(A, \sigma_Z, \xi_I, \Pi) + \pi$$

where C is consumption, $L \cdot w(\Omega)$ is household's wage income.π is profit received from firms – including both final goods producers and investment goods producers. The profit is transferred to household in a lump-sum way.

We assume that utility function takes the form $u(C,L) = log(C) - \xi_n l$, and stack aggregate states into the vector Ω. Household's value function becomes

$$J(\Omega) = \max_{C,L} log(C) - \xi_n l + \beta E\left[J\left(\Omega^{'}\right)\right] \tag{4.4.11}$$

subject to

$$C = L \cdot w(\Omega) + \pi \tag{4.4.12}$$

In the above value function, we stack aggregate states into the vector Ω, similar to what we did to firms' value functions.

In equilibrium, wage should be equal to marginal rate of substitution

$$w = \frac{u_L(C,L)}{u_C(C,L)} = \xi c \tag{4.4.13}$$

And stochastic discount rate is

$$m\left(\Omega,\Omega^{'}\right) = \beta E\left[\frac{u_C\left(C^{'},L^{'}\right)}{u_C(C,L)}\right] \equiv \beta E\left(\frac{\lambda'}{\lambda}\right) \tag{4.4.14}$$

4.4.6 Equilibrium condition

A competitive equilibrium is defined by final goods producer's value functions $V(k,z;\Omega)$, $W(k,z;\Omega)$, $U(k,z;\Omega)$ and household's value function $J(\Omega)$; final goods producer's investment policy functions $\chi(k,z;\Omega)$, $i(k,z;\Omega)$ and policy function of labor input $l(k,z;\Omega)$; investment goods producer's profit function $d(\Omega)$; pricing functions including tightness ratio $\theta(\Omega)$, price of investment goods $q^*(k,z;\Omega)$, wage $w(\Omega)$ and stochastic discount factor $m\left(\Omega,\Omega^{'}\right)$, that jointly solve the following problems

1. Final goods producers are optimizing: their values are maximized and satisfy

$$V(k,z;\Omega) = \max\{W(k,z;\Omega), U(k,z;\Omega)\}$$

2. Investment goods producers are optimizing; that is, their profits are maximized and satisfy

$$d(\Omega) > 0$$

3. Household is optimizing; that is, consumption choice that is consistent with stochastic discount factor $m\left(\Omega,\Omega^{'}\right)$ maximizes $J(\Omega)$.

4. There is market clearing in the investment goods market

$$I^D(\Omega) = \psi X$$

5. And market clearing in the final goods market

$$C + X + \int \{(1-\chi)[\phi(i(k,z;\Omega)) - i(k,z;\Omega)]\} d\Pi = \int Azk^{\alpha} l^{\nu} d\Pi$$

   where the integration term is the resale cost due to deinvestment.

6. And labor market

$$L = \int l(k,z;\Omega) d\Pi$$

7. And matching market of investment goods

$$\theta(\Omega) = \frac{1}{\int \tilde{\chi}(k,z;\Omega;q^*(\Omega)) d\Pi}$$

8. Finally, the evolution of the joint distribution of z, k is consistent; that is, Π is generated by policy functions $\tilde{\chi}(k,z;\Omega;q)$, $\tilde{i}^D(k,z;\Omega;q)$, and the exogenous stochastic evolution of A, z, σ_z, ξ_I.

4.4.7 Quantitative analysis

Calibration

The model is quarterly. The household's discount rate β is set to match an annual interest rate of 5%. Following Bloom et al. (2012), we set $\xi_n = 1$, which implies that that the Frisch labor supply elasticity is infinite.

We set capital depreciation rate δ to 0.025 to match a 10% annual capital depreciation rate. The capital and labor shares in the production function are set to be $\alpha = 0.25$ and $\nu = 0.5$, which implies a capital cost share of $1/3$.

The calibration of capital adjustment cost is controversial in the literature. We set adjustment cost parameters to match to moments of data. Specifically, we choose the resale loss of capital as 0.2 and fixed cost of adjusting capital to 0.002 as quarterly wage. In addition, we set the parameter of quadratic adjustment cost as 0.2.

Both the aggregate and the idiosyncratic productivity follows AR(1) processes. We follow Kahn and Thomas (2008) to set the serial auto-correlation parameters ρ^A and ρ^Z are set to 0.95.

The volatility of idiosyncratic productivity follows a two-state Markov chain:

$$\sigma_t^z \in \{\sigma_L^z, \sigma_H^z\}$$

governed by a Markov switching matrix

$$\Pi = \begin{bmatrix} \pi^{HH} & \pi^{HL} \\ \pi^{LH} & \pi^{LL} \end{bmatrix}$$

Following Bloom et. al. (2015), we set $\sigma_L^z = 0.04$ and $\sigma_H^z = 0.08$, which means that the volatility of idiosyncratic productivity doubles in high uncertainty regime compared to in low uncertainty regime. In addition, we set $\pi^{HH} = 0.92$ and $\pi^{LL} = 0.95$, which implies that the average duration of high uncertainty regime is about 12 quarters and low uncertainty regime lasts for 20 quarters.

The matching function of the investment goods market is constant returns to scale and specifies as

$$M = \frac{D \cdot S}{(D^\iota + S^\iota)^{1/\iota}}$$

with $\iota = 2$. This matching function ensures that the matching probability is always between zero and one.

Description	Parameter	Calibration	
Discount rate	β	0.99	Annual interest rate of 5%
Scale factor of labor disutility	ξ_n	1	Infinite Frisch elasticity
Capital share parameter	α	0.25	Isoelastic demand with 33% markup
Labor share parameter	ν	0.5	Labor share of 2/3, capital share of 1/3
persistence of aggregate productivity	ρ^A	0.95	Khan and Thomas (2008)
persistence of idiosyncratic productivity	ρ^z	0.95	Khan and Thomas (2008)
Capital depreciation rate	δ	0.025	Annual depreciation of capital stock of 10%
Capital re-sale loss	ϕ	0.2	Bloom et al. (2012)
Fixed cost of capital adjustment	ξ	0.002	
Scale factor of quadratic adjustment cost	ξ_i	0.2	
Volatility of innovations to aggregate productivity	σ_L^A	2%	
Volatility of innovations to idiosyncratic productivity	σ_L^z	4%	
Volatility in high uncertainty regime	σ_H^z	8%	
Transition probability from high to high uncertainty	π^{HH}	0.92	
Transition probability from low to low uncertainty	π^{LL}	0.95	

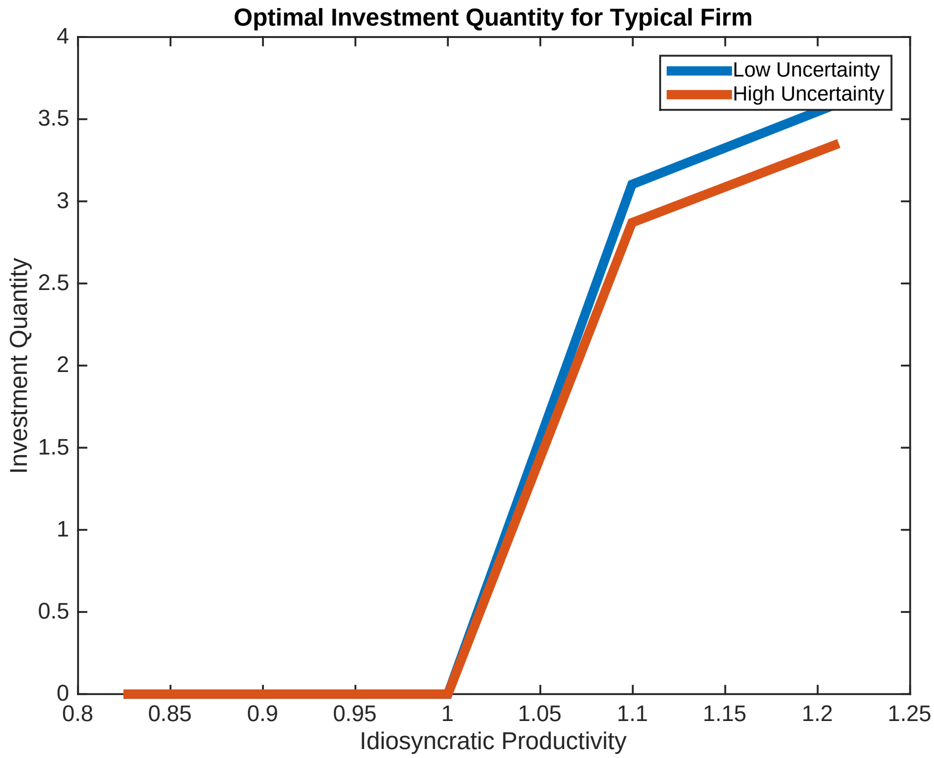


FIGURE 4.12: Investment policy at the intensive margin I

Steady state analysis

In this subsection, we examine how the investment decision of firm and the equilibrium price of investment vary with the firm's idiosyncratic states as well as the aggregate states.

Investment policy at the firm level

Figure 4.12 displays the relationship between the investment policy of firms at the intensive margin and the idiosyncratic productivity. Specifically, we plot the investment policy of a firm at the intensive margin with the median capital stock as a function of its idiosyncratic productivity. As one would expect, the optimal investment at the intensive margin is strictly increasing in the idiosyncratic productivity. As the idiosyncratic productivity is persistent, a higher idiosyncratic productivity raises the expectation of the future mpk. As the result, given the same capital level, a firm with a higher idiosyncratic productivity would want to accumulate more capital to accommodate the higher future mpk.

In Figure 4.13 , we plot the investment policy of a firm at the intensive margin with

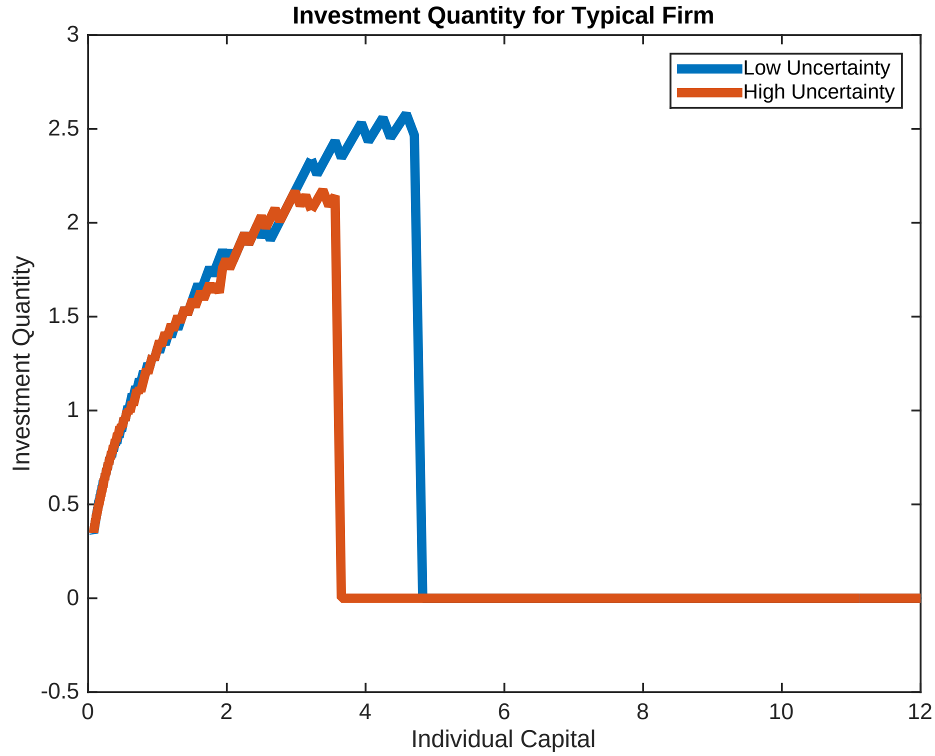


FIGURE 4.13: Investment policy at the extensive margin

the median idiosyncratic productivity as a function of its capital stock. As shown in the graph, the optimal investment at the intensive margin is strictly decreasing in the firm-level capital stock. At the intensive margin, the optimal future capital level is pinned down by the idiosyncratic productivity; therefore, the necessity of investment is decreasing in the current capital stock.

Figure 4.15 displays the price of investment as the function of the buyer firm's idiosyncratic productivity. As clearly shown in the graph, the price of investment is strictly increasing in the idiosyncratic productivity. In the model, the price is the equilibrium outcome of the Nash bargaining between the buyer and the seller. In particular, as illustrated in equation **??**, the price of investment is implicitly determined by the equilibrium profit of the seller and strictly increasing in the total surplus of the match between the buyer and the seller. Notice that the total surplus is strictly increasing in the idiosyncratic productivity when the investment is positive; strictly decreasing in the idiosyncratic productivity when the investment is negative. Given the investment is positive, a higher idiosyncratic

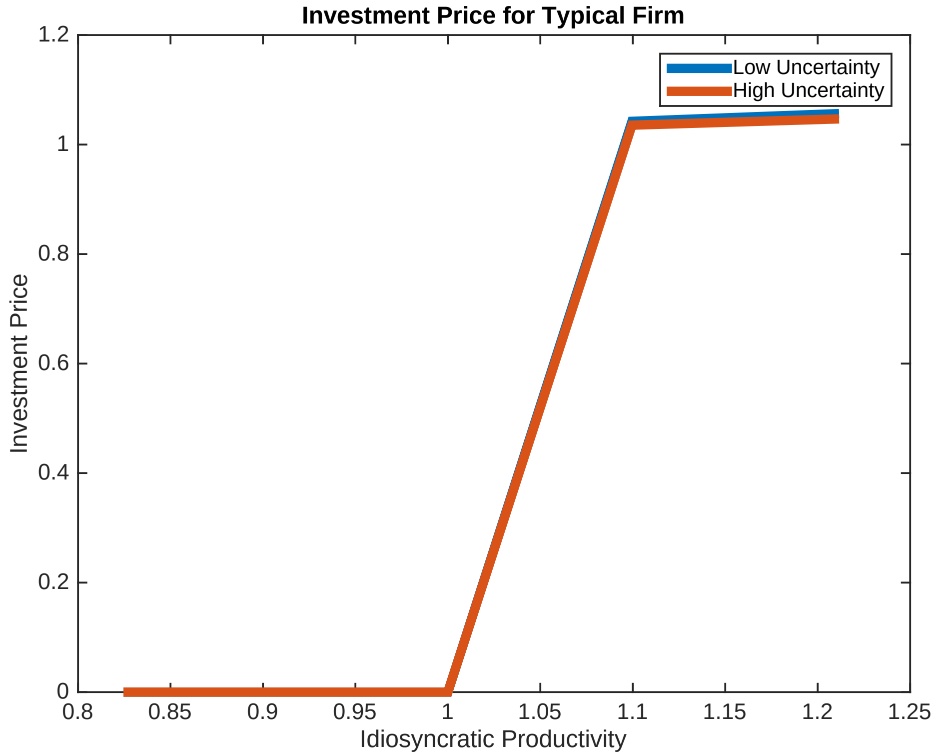


FIGURE 4.14: Investment policy at the intensive margin II

productivity would imply a higher total surplus of investment, which implies a higher price. When the investment is negative, a higher idiosyncratic productivity will imply a lower total surplus of investment, which also implies a higher price. Overall, the price of investment is strictly increasing in the idiosyncratic productivity.

In Figure 4.16 , we plot the price of investment as the function of the buyer firm's capital stock. The price of investment is strictly decreasing in capital stock. Given the same idiosyncratic productivity, a higher capital stock would suppress the incentive for investment. As the result, the seller would have to lower the price due to the lower total surplus.

Figures 4.17 and 4.18 illustrate firm's investment policy at the extensive margin. As shown in Figures **??** and 4.13, disregarding adjustment cost, positive investment occurs in firms with high productivity and low capital stock; while negative investment (deinvestment) occurs in firms with low productivity and high capital stock. With the non-convex investment adjustment cost, there exists a Ss band characterizing firm's investment policy:

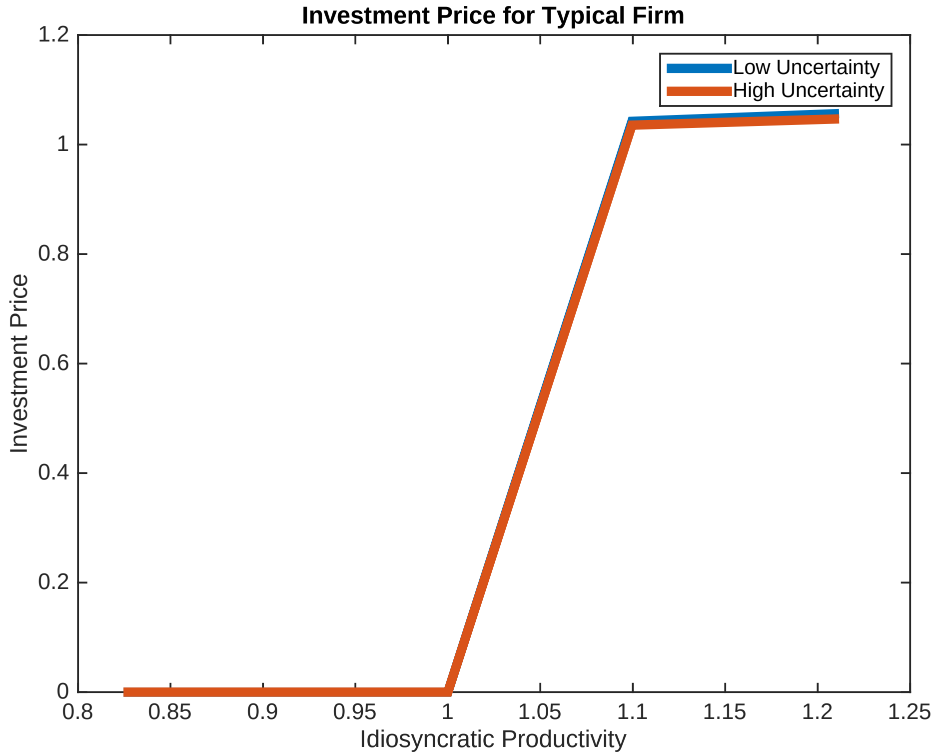


FIGURE 4.15: Equilibrium price of investment I

the buyer would actively search for a match only when the total surplus is high enough to compensate for the adjustment cost. As the result, when the capital stock and the productivity of firm is at the middle, the firm does not choose to incur the adjustment cost to search for investment seller. As shown in Figures 4.17 and 4.18, the Ss band depends on the uncertainty (the volatility of idiosyncratic productivity). Specifically, in the high uncertainty regime, the S-s band becomes wider compared to in the low uncertainty regime.

In the high uncertainty regime, condition on the capital level, fewer firms do investment. Conditional on the capital stock, the lower bound of productivity for firms choosing positive investment shift to the right and the upper bound for firms choosing negative investment shift to the left. On the one hand, the right shift of the lower bound of the productivity for firms choosing positive investment increases the aggregate index of investment price by increasing the weight of high-price firms. On the other hand, the left shift of the upper bound of the productivity for firms choosing negative investment also increases the aggregate index of investment price by reducing the weight of low-price firms (remember

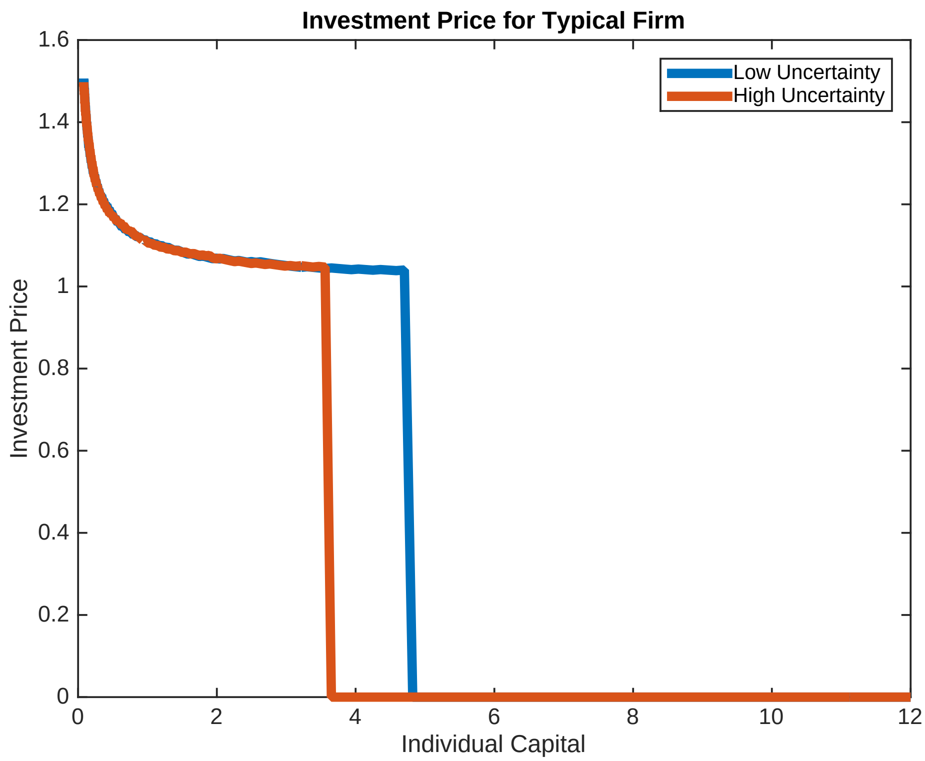


FIGURE 4.16: Equilibrium price of investment II

that the weight for deinvestment is negative). As the result, we see an endogenous increase in the aggregate index of investment price.

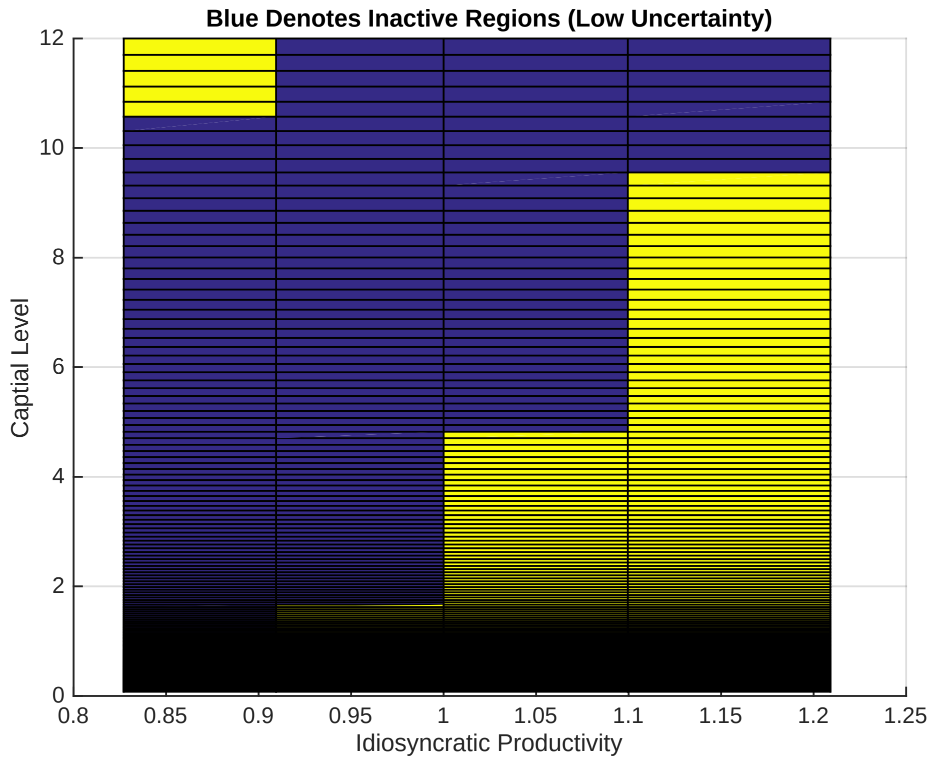


FIGURE 4.17: Investment policy at the extensive margin II

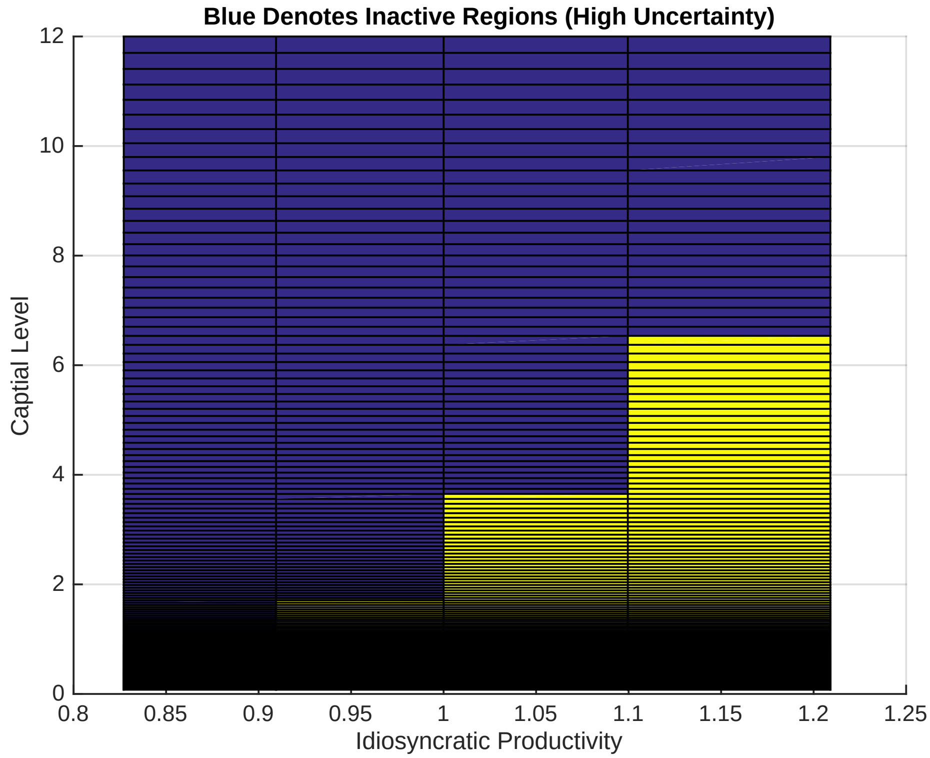


FIGURE 4.18: Investment policy at the extensive margin II

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Biography

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