Reallocation in Perishable Goods Markets

by

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Dissertation submitted in partial fulfillment of the
requirements for the degree of Doctor of Philosophy
in the Department of Economics
in the Graduate School of
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ABSTRACT

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Abstract

How does the ability to reallocate affect markets for perishable goods? I study how reallocation affects consumers and a monopolist seller, and how the effects vary depending on the method of reallocation—through resale or refund contracts—and the motive for reallocation—either demand uncertainty or arbitrage.

In Chapter 2, I evalute the performance of three common reallocation mechanisms: resale, partial refunds, and a menu of refund contracts. I show that the performance of each mechanism depends on the properties of demand uncertainty and build a model in which consumers anticipate shocks, make advance purchase decisions, and reallocate after shocks are realized. By capturing the effects of different types of shocks, the model is able to predict the relative performance of the mechanisms. Using data from the market for college football tickets, I estimate the model and find that refund contracts produce higher profit and total welfare than resale because of fees and frictions associated with resale.

In Chapter 3, I use a theory model with advance purchases and a rich set of idiosyncratic demand shocks to compare the performance of resale and refunds. For profit, the relative performance depends on the degree of aggregate uncertainty. The seller can completely insulate itself against aggregate uncertainty by owning the resale

market and selling to brokers. Aggregate uncertainty enhances the performance of resale because resale prices adjust to reflect shocks while the monopolist seller's prices do not. For welfare, both the seller and a monopolist resale market operator have an incentive to impede frictionless resale. Which strategy maximizes social welfare depends on the responsiveness of prices to the resale friction.

In Chapter 4, I consider how the ability to resell affects a monopolist's incentive to bundle. Using a model in which consumers have heterogeneous preferences over two goods and a cost of participating in a resale market, I show that the monopolist may choose to bundle even if some consumers resell. The coexistence of price discrimination and resale is novel in settings where resale harms the seller, and I show that it fundamentally changes the monopolist's pricing problem compared to models without resale.

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Chapter 1

Introduction

How to allocate goods is one of the most fundamental problems in economics. Reallocation mechanisms—such as resale markets and refund contracts—are similarly important because of their role in determining the final allocation of goods. But the overall effects of reallocation are not straightforward. Consumers who know they can reallocate later may make different initial purchases. A seller might choose different prices if its buyers have a chance to reallocate. Each change in behavior affects profit and welfare. The matter is further complicated because different methods of reallocation could have different effects. This dissertation considers the effects of reallocation through resale markets and refund contracts in perishable goods markets.

Understanding reallocation is essential because it is common for initial allocations to be flawed. Whenever a plane ticket goes unused, a purchase comes with an unwanted promotional item, or a book sits unread on the shelves, the allocation of goods is suboptimal and society can benefit from reallocation. And reallocation is common. Consumers often return goods for refunds (including partial refunds for airline tickets) or resell goods like books. A thorough study of the effects of reallocation is necessary to understand such markets.

The chapters that follow demonstrate that there is no single effect of reallocation. Instead, the effects depend on both the setting and the method of reallocation. In settings where the initial allocation is suboptimal because of preference shocks, as in Chapters 2 and 3, a capacity-constrained seller might benefit from reallocation that redirects goods to consumers with the highest values after shocks. But in settings where the seller prefers an initial allocation that is not Pareto optimal—as when the seller practices price discrimination in Chapter 4—the ability to reallocate might harm profit. The effect on consumers depends on the seller's response, which in turn depends on the motive for reallocation in the setting.

The method of reallocation also matters. In Chapters 2 and 3, I compare the performance of the two most common reallocation mechanisms, resale markets and refund contracts, in a setting where consumers make advance purchase decisions and receive preference shocks. The central difference between the two methods is that refunds require all sales to be made at the monopolist seller's prices, which are fixed and could be suboptimal after aggregate shocks, while resale transactions have flexible prices but incur fees paid to the resale market operator. Which maximizes profit and welfare hinges on the relative importance of aggregate shocks (which make flexible prices valuable) and resale fees. The choice between the mechanisms also affects social welfare. The primary market seller and resale market operator have different incentives to limit reallocation, and it is not clear which is larger. Moreover, refunds might fail to allocate all goods in states where the seller's prices are suboptimal for realized demand, but resale could incur other frictions that limit reallocation.

Each of the remaining chapters of the dissertation studies the effects of reallocation in a specific setting. In Chapter 2, I compare the performance of resale markets, partial refunds, and a menu of refund contracts when consumers make advance purchases and receive preference shocks. I provide a theoretical framework demonstrating that each strategy can be optimal for a different set of demand shocks, then evaluate the performance of each strategy empirically using a structural model of the market for college football tickets. In the model, consumers anticipate each type of demand shock and decide whether to buy tickets in advance. After shocks are realized, consumers participate in an endogenous resale market. In counterfactual experiments, I replace the resale market with each type of refund contract. Ultimately, I find that the refund strategies produce higher profit and welfare because of fees and frictions associated with resale.

In Chapter 3, I also compare the performance of resale and refunds in a setting where consumers receive preference shocks. The central difference from Chapter 2 is that the analysis is theoretical, enabling me to use a richer set of shocks that permit a deeper analysis of welfare. I show that resale can only be profitable when there are aggregate shocks and it is costly for the seller to adjust its prices. The effect on welfare is also ambiguous: resale with the profit-maximizing fee can produce higher total and consumer welfare as long as the elasticity of prices with respect to the refund is large enough. I also show that sellers profit most by owning the resale market and selling to brokers, which helps explain the close ties between sports teams and resale markets.

Finally, in Chapter 4, I study a monopolist that uses mixed bundling when consumers can resell in a market with certain demand. The central puzzle is to explain why the monopolist continues to discount its bundle when some consumers resell, a phenomenon observed in markets for products like season tickets and box sets of books. I show that bundling and resale can coexist in equilibrium when consumers have heterogeneous costs of resale. With heterogeneous costs of resale, the monopolist might profit by discriminating against some, but not all consumers; it would have to set a lower, less profitable discount to prevent all resale. Heterogeneous costs add a new dimension to the monopolist's pricing problem that is not present in models with a homogeneous cost of resale or no resale, forcing the monopolist to balance its discount against the number of consumers who resell.

Chapter 2

Resale, Refunds, and Demand Uncertainty: Evidence from College Football Ticket Sales

2.1 Introduction

It is only May, but fans of State U are already making plans for November's big football game against State A&M. Many of them have purchased tickets, but their values for using them can change. Some will have schedule conflicts that prevent them from attending. Others will be less interested if State U's star quarterback gets injured. And some will refuse to attend if a covid-19 vaccine is not widely distributed by game day. When demand is uncertain, what is the best way to allocate the tickets?

In this paper, I study the profit- and welfare-maximizing sales strategies for a seller of perishable goods facing demand uncertainty. As in the market for football tickets, the core problem is that the optimal price and allocation change after consumers receive preference shocks. I consider the performance of three sales strategies that reallocate after shocks: allowing resale, offering a partial refund, and offering a menu of refunds.

Each strategy is widely used. Sports teams and other sellers of event tickets allow resale but do not offer refunds; airlines and hotels offer refundable reservations but prohibit resale. However, we have little understanding of when each strategy is optimal. The decision to allow resale is particularly puzzling because resale platforms (e.g. StubHub) collect over 20% of the resale price as fees. Why do ticket sellers surrender so much of the gains from reallocation when they could avoid fees by using refunds?

I show that the optimal strategy depends on the properties of demand uncertainty, like whether shocks are idiosyncratic or aggregate. Despite fees, resale can be optimal when the optimal price after shocks is uncertain. A menu of refunds is valuable when the efficient allocation depends on observable states. I quantify the performance of each strategy in the market for college football tickets. Using ticket sales data covering the primary and resale markets and survey data on demand for tickets with and without a covid-19 vaccine, I develop and estimate a structural model of ticket sales in which consumers anticipate shocks and participate in resale markets. The model allows me to evaluate partial refunds and a menu of refunds in counterfactual experiments.

The analysis applies to a wide range of goods, including reservations (event tickets, airlines, hotels, rental cars, etc.), seasonal goods (fashion), and unreleased goods for which consumers pay deposits (cars). The relevant markets are large: online event ticket sales alone amounted to $56bn in 2019 (Statista (2020a)). Moreover, this paper has broad implications for our understanding of resale and aftermarkets. Why are resale markets common for some perishable goods but not others? What are the net effects of resale of perishable goods on the seller and consumers? And what is the best way to run aftermarkets? Prior research has left these questions unanswered, but this paper offers theoretical and empirical evidence on each.

Ultimately, I find that the refund strategies outperform resale, but that resale is better for all parties than not reallocating. Specifically, partial refunds raise profit by 3.6% and total welfare by 1.3% relative to resale. However, the seller and consumers benefit from resale when the alternative involves no reallocation: profit increases by 1.8%, total welfare by 4.8%, and consumer welfare by 10.1%. In an application with states of the world with and without a covid-19 vaccine, the menu of refunds (implemented as state-dependent full refund contracts) also improves significantly on not reallocating, raising profit by 10.3% and total welfare by 7.7%. In both counterfactuals, refunds perform better because of fees and frictions associated with resale.

The performance of the sales strategies depends on three sources of demand uncertainty with distinct properties, all of which are present in the market for college football tickets. The first shock is purely idiosyncratic, like schedule conflicts. It causes consumers who bought tickets to have low values for using them, motivating reallocation. The second shock is a change in a common component of consumer values, like team performance. Changes in the common value make it difficult for the seller to predict the optimal price after shocks. The third shock is a realized state of the world, modeled as whether there will be a covid-19 vaccine at the start of the season. Consumers have heterogeneous reactions to the state with no vaccine: some are willing to pay the same amount for tickets if there is no vaccine while others would not pay a penny. The shock causes different consumers to have the highest values in each state.

The analysis relies on three data sources. The main source of data is transaction-level primary and resale market sales records for one season of college football from a large U.S. university. The records contain nearly 40,000 primary market transactions (including season ticket packages) and over 6,000 resale transactions on StubHub, the largest online resale platform (Satariano (2015)). I supplement the ticket sales with 576 observations of average annual resale prices for 76 college football teams, taken from the resale market SeatGeek. To learn about the effect of a covid-19 vaccine on consumer demand for tickets, I designed and conducted a survey asking

500 consumers, 250 of whom were 50 or over, for their willingness to pay in states with and without a vaccine.

To see how the properties affect the performance of each sales strategy, consider a seller of sports tickets that offers partial refunds and suppose that it cannot change prices after shocks. Some consumers will buy in advance, have schedule conflicts, and return their tickets to the seller's inventory for a refund. Whether partial refunds are optimal turns on whether there are also aggregate shocks in the market. If not, then the seller's price is optimal after shocks and all units are profitably and efficiently reallocated, making partial refunds optimal. But if there is an aggregate shock—for example, if there is unexpectedly no covid-19 vaccine, or if the team is bad—then the seller's price will be too high after shocks, leaving returned tickets unused. In this case, resale can maximize profit and welfare because resellers adjust their prices after shocks, ensuring that more tickets are sold and used. The downside of resale is that it incurs search frictions and fees paid to the resale market operator, which reduce its value when there is no aggregate uncertainty.

The choice between resale and partial refunds thus depends on a tradeoff between the intensity of aggregate shocks and the magnitude of fees and frictions. The data suggest that both factors are important, leaving the optimal strategy unclear. Reallocation is important because many consumers choose to resell: 6% of all seats were resold by consumers on StubHub. Aggregate uncertainty makes it difficult to predict the optimal price: nearly a third of the time, annual resale prices for a school deviate from the sample average by 25% or more. And resale fees are substantial: StubHub charges over 20% of the total price paid by the buyer, amounting to over $10 on each ticket resold.

The last strategy, a menu of refunds, is valuable when the efficient allocation varies across states of the world. Suppose it is uncertain whether there will be a covid-19 vaccine by the start of the season. The consumers with the highest values in the states with and without a vaccine may be different, making the efficient allocation dependent on the state. The seller can respond by offering a menu of state-dependent refund contracts, such as a contract giving a full refund in the state with no vaccine.[1] The principle of contracting on an observed state is used for financial derivatives; the NFL has also offered Super Bowl tickets only in the state when a certain team is in the game. The key property affecting the performance of the menu of refunds is the degree of heterogeneity in consumer values across states. When consumer responses to the state without a vaccine are more varied, the efficient allocations in the two states are less similar and the menu of refunds is optimal.

The survey results show that there is significant heterogeneity in consumer reactions to the state without a vaccine and hence in the efficient allocation across states. Almost a third of consumers report that it does not matter if there is a vaccine—they would pay the same amount in either state—while a fifth would pay something with a vaccine but nothing without one.

To compare the sales strategies, I develop a structural model of the market. In the model, consumers anticipate future shocks and participate in resale markets after shocks are realized. Specifically, in the first of two periods consumers know the distribution of shocks and decide whether to buy season tickets. Shocks are realized at the start of the second period. In the second period, consumers who bought season tickets decide whether to attend or resell; all other consumers decide whether to purchase tickets in the primary or resale markets.

The model captures the effect of each source of demand uncertainty. Idiosyncratic shocks are modeled as independent Bernoulli draws for each game. Consumers re-

[1] A menu of refund contracts can still be profitable when the shock is not an observable state. For example, contracts could consist of different levels of refunds (e.g. full or no refunds), as in Courty and Li (2000), and would still have implications for profit and welfare.

ceiving idiosyncratic shocks have no value for using the tickets. The rate at which consumers receive idiosyncratic shocks is identified by the rate of resale in the ticket sales data. The common value is a shared component of consumer utility. There is one draw of the common value per season, and the variance of the distribution is identified by the variation in annual resale prices in the SeatGeek data. The effect of the state with no covid-19 vaccine is modeled as a penalty to consumer values that varies across consumers. The distribution of penalties is identified by changes in individual-level reported willingness to pay in the covid-19 survey.

Other model parameters are estimated in structural simulations that match observed values for resale prices, season ticket quantities, and single-game ticket quantities. One such parameter affecting the performance of the sales strategies is the friction associated with searching both primary and secondary markets for tickets. The friction is identified by the number of consumers who choose to purchase tickets in the primary market when equivalent tickets are cheaper in the resale market.

The estimated model allows me to evaluate two core sets of counterfactuals. In the first, I consider a baseline model without uncertainty from covid-19 and compare resale to a partial refund. I also consider benchmark cases with no reallocation (neither resale nor refunds) and flexible prices (refunds with price adjustments after shocks). In the second set of counterfactuals, I only consider uncertainty from the vaccine states[2] and compare the performance of a menu of refunds to resale.

The study has three broad implications. First, it informs our understanding of when to use resale markets. Resale is ubiquitous for some goods, like stocks, common for others, like event tickets, and rare for many more, like current fashions. What determines when resale of perishable goods is valuable? I provide a framework clarify-

[2]The menu of refunds is not mutually exclusive from other sales strategies when the full refund depends on the state. Consumers who receive tickets in the realized state might still receive idiosyncratic shocks, which leaves room for resale and partial refunds. I do not consider idiosyncratic shocks to avoid testing combinations of the sales stratgies.

ing that resale is useful in markets where the clearing price is uncertain, the primary market price does not adjust easily, and resale frictions are sufficiently low. The empirical results suggest that the harms of frictions outweigh the benefits of price flexibility in the market for football tickets, making partial refunds more valuable. In markets where primary market prices are flexible, such as airlines, refunds may perform better than resale.

Second, the study has direct implications for the design of aftermarkets. Which aftermarket strategy produces the most efficient allocation after shocks, and how large are the changes in profit in welfare? Few prior studies have compared the strategies and so the answers are, to this point, unknown.

Third, the study provides evidence on the value of resale. Despite much theoretical interest, there are few estimates of the net effects of resale of perishable goods on profit and consumer welfare because of the difficulty of assembling primary and resale market data.[3] Estimates are valuable because the net effect on profit is ambiguous in theory (Cui et al. (2014)) and the effect on consumers is informative for government policy on the right to resell. Consumers have a legal right to resell most legally purchased goods, but not event tickets or airline reservations.[4] After major concert tours prohibited ticket resale, several states passed laws extending the right to resell to ticket markets (Pender (2017)). This paper's predictions for the effects of resale on consumers provide evidence on whether such laws are beneficial.

The remainder of the introduction discusses the relevant literature. Section 2 presents several examples demonstrating how the properties of demand uncertainty affect the seller's optimal sales strategy. Section 3 discusses the data sources used,

[3]An exception is Leslie and Sorensen (2014), who provide estimates for consumer welfare. They do not consider profit because of systematic mispricing in their empirical setting.

[4]The first-sale doctrine prevents copyright holders from restricting the buyer's ability to resell in 17 U.S.C. §109. However, it does not apply to goods sold as revocable licenses, like tickets and reservations.

and Section 4 presents descriptive evidence from the data on the market for college football tickets and each source of uncertainty. Section 5 develops a structural model of the market and Section 6 details how it is estimated. Section 7 presents the counterfactual experiments and their results. Section 8 concludes.

Related Literature. This paper contributes to several literatures, notably those on resale, demand uncertainty, and price discrimination. For the resale literature, this paper provides estimates of how resale affects profit and welfare. The effects of resale on buyers and sellers are a traditional focus of the literature. Courty (2003) and Cui et al. (2014) examine whether resale is profitable in theory. Lewis et al. (2019) investigate the effect of resale on demand for season tickets in professional baseball but do not model how resale of season tickets affects sales of other tickets. This paper does so by modeling both the primary market and an endogenous resale market. Leslie and Sorensen (2014) use a similar model combining primary and resale markets to study whether resale increases welfare in the market for concert tickets, but they do not consider profit because tickets are systematically underpriced in their sample. Tickets in my setting are not underpriced, allowing me to provide estimates of the effect of resale on both profit and welfare. Other empirical studies of resale markets for event tickets include Sweeting (2012) and Waisman (2020).

A separate literature considers resale for sellers of durable goods. Unlike sellers of perishable goods, sellers of durable goods compete with past vintages of their products, and consumers may have heterogeneous preferences over the vintages. Examples of studies of durable goods include Chen et al. (2013a), who find that resale is harmful for oligopolists in the market for cars, and Ishihara and Ching (2019), who find that sellers of video games benefit from resale markets.

This paper also broadens the traditional focus on resale to consider alternative methods of reallocation. Two recent studies, Cui et al. (2014) and Cachon and Feldman (2018), have compared resale and refunds. Both use theory models and find that one strategy is always more profitable. This paper develops a model in which either can be more profitable and provides empirical evidence on their performance.

Second, this paper relates to the literature on demand uncertainty, specifically studies that consider the effect of aggregate uncertainty on firms' strategic choices. Studies of aggregate demand uncertainty in this tradition include Kalouptsidi (2014), who studies the effect of uncertainty on investment when investment is lagged and irreversible, Jeon (2020), who studies how uncertainty creates boom and bust cycles in investment, and Collard-Wexler (2013), who studies how uncertainty affects entry, exit, and market structure. This paper differs because it does not emphasize industry-level outcomes, focusing instead on the sales strategies individual firms use when there is uncertainty.

The emphasis on how individual firms cope with uncertainty ties this paper to studies of airline pricing with stochastic demand, such as Lazarev (2013), Williams (2020), and Aryal et al. (2018). In these studies, stochastic consumer arrivals make it profitable for sellers to use dynamic pricing. In contrast, this paper focuses on non-price strategies like resale and refund contracts that reallocate goods after shocks. A branch of the management literature also considers non-price strategies. For example, Chen and Yano (2010) and Su (2010) consider different responses to aggregate uncertainty (offering retailers rebates and selling to brokers), while Xie and Gerstner (2007) study whether refunds are profitable when consumers have idiosyncratic preference shocks. All of these papers, however, have no empirical component. This paper contributes by providing empirical estimates for similar sales strategies.

The emphasis on uncertainty also relates to the literature on learning, such as Erdem and Keane (1996), Ching et al. (2013), and Hitsch (2006). In the learning literature, agents are initially uncertain about unchanging model parameters and attempt to gather information about them over time. In this paper, agents are certain about the environment and uncertainty comes from the future realization of stochastic shocks.

Several features of this study have not yet been considered in the literature on demand uncertainty. Whereas earlier papers consider one non-price strategy, this study compares several and shows that their performance depends on the properties of demand uncertainty. Moreover, one property of uncertainty, heterogeneous value changes, has rarely been considered in empirical work.

Third, this paper connects to the literature on price discrimination, particularly studies in which sellers use future value shocks as a screening device. Courty and Li (2000) show that refund contracts are an optimal mechanism in a model in which consumers make purchase decisions before learning their values and different types of consumers have different distributions of values. The environment is similar to the one in this paper, where consumers can purchase season tickets before learning the state of the world. Despite significant attention from theorists, empirical evidence is rare because of the difficulty in identifying different effects of shocks for different types of consumers. I overcome the problem and estimate the returns to a menu of refund contracts by using survey data that identifies the distribution of changes in willingness to pay. One other study providing empirical evidence is Lazarev (2013), who considers two types of airline passengers with different probabilities of schedule conflicts and calculates the benefits of offering fully and non-refundable tickets. The application in this paper differs by using a full distribution of value changes rather than two types. It also considers an aggregate source of uncertainty, which allows the seller to offer state-dependent contracts. The fact that uncertainty comes from an aggregate shock also links this paper to Alexandrov and Bedre-Defolie (2014), who analyze product-state combinations as a bundling problem.

This study also relates to the literature on dynamic mechanism design. As in the dynamic mechanism design literature, consumer values evolve over time in this paper and the optimal mechanism may involve reallocation. This paper differs in that it tests the performance of several common reallocation mechanisms instead of investigating the theoretically optimal mechanisms studied in, for instance, Pavan et al. (2014) and Bergemann and Välimäki (2019). The literature on dynamic mechanism design also includes studies of dynamic pricing, such as Board and Skrzypacz (2016), Dilmé and Li (2019), and Sweeting (2012). This paper instead focuses on reallocation after preference shocks.

2.2 Uncertainty and Sales Strategies

In this section, I present three examples illustrating the connection between the types of uncertainty and sales strategies. Each example includes different types of uncertainty and implies that a different sales strategy maximizes profit and welfare.

In each example, there are two periods and the seller has one ticket to sell. The seller can set different prices for each period but, as in the data, it must commit to its menu at the start of the first period. Forward-looking consumers arrive in both periods but receive preference shocks at the start of the second period. Suppose that consumer i has value ν_i and that the value is affected by three potential shocks:

1. Independently drawn (idiosyncratic) shocks that affect consumer i with probability ψ. Consumers who receive a shock have zero value.

2. An aggregate shock to a common value V that changes all consumers' values to $\nu_i + V$.

3. A realized state ω that changes consumer i's value to $\nu_i - b_i(\omega)$. The function

$b_i(\omega)$ is specific to consumer i, so changes in values are heterogeneous. Possible states are $\omega \in \{\omega^B, \omega^G\}$. All consumers have $b_i(\omega^G) = 0$ and $b_i(\omega^B) \geq 0$.

The three shocks closely track the ones observed in the data and are modeled almost identically in the empirical model. Consequently, the idiosyncratic shocks can be thought of as schedule conflicts, the common value as the quality of the team, and the realized state as whether there is a covid-19 vaccine at the start of the season. A more detailed explanation of the equilibrium of the following examples can be found Appendix A.1.

2.2.1 Idiosyncratic Uncertainty

When there is only idiosyncratic uncertainty, refunds maximize profit and welfare. Suppose that a seller has one ticket to sell to two buyers, Alice and Bob, and that there is only idiosyncratic uncertainty, $\psi = \frac{1}{5}$. Alice arrives in the market in the first period and prefers to buy early; she has value $\nu_A = 50$ in period one, but it falls to $\nu_A = 40$ if she waits to purchase until the second period. Bob always arrives in period two with $\nu_B = 40$. The seller optimally offers a refund $r = 5$ and sets $p_1 = 41$, $p_2 = 40$.[5] Alice purchases the ticket in the first period despite the risk of a schedule conflict.

If Alice does not have a conflict, then total welfare is 50 and profit is 41. But if Alice does have a schedule conflict, she will return her ticket for a refund and the seller will sell the ticket on to Bob at $p_2 = 40$. In this case, the seller earns 76 in profit (a net of 36 from Alice and 40 from Bob) and total welfare is 40. Expected profit and total welfare equal 48.

[5]The choice of $r = 5$ is optimal but not unique. The seller could produce the same allocation and division of surplus by offering any refund r such that Alice returns her ticket if and only if she receives an idiosyncratic shock. For any such r, it can charge $p_1 = 40 + \psi r$ but will pay ψr in expected refunds.

What if the seller had not offered a refund, but had allowed resale? Suppose that resale only incurs one friction, a multiplicative fee of $\tau = \frac{1}{10}$ on each transaction that is paid to the resale market. Also suppose that Alice buys the ticket in the first period. If she does not receive a shock, then total welfare is 50 as before. If she does, then she can resell to Bob at resale price 40, generating total welfare of 40.[6] However, Alice will only receive 36 because 4 is paid to the resale market operator. The seller can thus charge Alice 50 for the state where she has no shock but only 36 for the state where she does, leading to $p_1 = 47.2$. Profit is lower than with refunds because of resale fees: the seller earned 40 when selling to Bob with refunds, but it only earns 36 when Alice resells to Bob. Total welfare is unchanged but would be lower if resale incurred other frictions, like a hassle cost of using the resale market.

2.2.2 Idiosyncratic and Common Value Uncertainty

When there is also aggregate uncertainty from a common value, resale can maximize profit and welfare. Continue to suppose that a seller has one ticket to sell to Alice and Bob and that $\psi = \frac{1}{5}$, but now there is also an aggregate shock arriving between the two periods: $V = 0$ with probability $\frac{3}{4}$, but $V = -20$ with probability $\frac{1}{4}$.

If the seller offers refunds, it will set $r = 5$, $p_1 = 37$, and $p_2 = 40$.[7] Alice is just willing to purchase the ticket. If she does not receive a shock, she uses the ticket and expected total welfare is 45. If she does receive a shock, she returns the ticket. There are two cases where the ticket is returned. Three quarters of the time, there is no aggregate shock and Bob is willing to buy the returned ticket for $p_2 = 40$, leaving

[6] In the resale market, I assume that Alice makes a take-it-or-leave-it offer to Bob. The TILI assumption is not necessary in examples with many agents, but I use it here for simplicity. Moreover, the assumption is realistic because resellers make TILI offers in online resale markets. Buyers cannot make counteroffers.

[7] As before, the choice of $r = 5$ is optimal but not unique. The division of surplus is again the same with other optimal selections of r.

total welfare of 40 and profit of 72. But one quarter of the time, Bob is only willing to pay 20 and refuses to buy the ticket. In this case, refunds do not reallocate the ticket because the seller's price is too high after shocks. When Bob refuses to purchase, total welfare is zero and profit is only 32. Refunds produce expected profit and welfare of 42.

Now suppose the seller allows resale and that Alice buys the ticket in period one. As before, Alice receives expected surplus of 45 when she does not have a shock. When she receives a shock, she can always resell to Bob, earning 36 when there is no aggregate shock and 18 when there is one. The key distinction between resale and refunds is that, because the resale price is flexible, Bob receives the ticket even when his value is low. Expected total welfare is 43 and the seller earns profit of 42.3 by charging Alice $p_1 = 42.3$, both higher than with refunds.

Resale is valuable because prices adjust to reflect aggregate shocks, but it is only more profitable when frictions are low. With a higher fee or costs of using the resale market, refunds could have remained more profitable despite reallocating the tickets less effectively. Similarly, if there had been significant resale frictions, total welfare would be lower with resale. The choice between resale and refunds reduces to a trade-off between aggregate uncertainty and the importance of fees and frictions in resale markets.

2.2.3 States of the World

For the last example, suppose that there are states of the world but no idiosyncratic or common value uncertainty. When consumers have heterogeneous value changes across the states of the world, the menu of refunds can maximize profit and welfare. Suppose that the seller only makes sales in the first period, before the state of the world is known, and that each state of the world (ω^G and ω^B) occurs with probability

$\frac{1}{2}$. Alice and Bob both enter in the first period. Alice's value is 40 both with and without the shock—her response is $b_A(\omega^B) = 0$. Bob has $\nu_B = 50$ but responds harshly to the shock, $b_B(\omega^B) = 40$.

If the seller offered a single price, it would set $p = 40$ and sell to Alice. But in state ω^G, Alice would have the ticket when Bob has a higher value. A single refund would not help because Alice would return her ticket in both states. With resale, Alice could resell to Bob in the good state, but doing so would incur frictions.

A menu of state-dependent contracts would avoid fees and maximize welfare and profit. The seller could offer a contract granting a full refund in state ω^B at price 50, which Bob would purchase, and another contract granting a full refund in state ω^G at price 40, which Alice would purchase. By implementing a state-dependent allocation, the contracts maximize welfare and profit when consumers have heterogeneous value changes.

2.3 Data

2.3.1 Primary and Resale Market Data

The main source of data is transaction-level ticket sales records from the primary and resale markets. The primary market records are provided by a large U.S. university and include all ticket sales for two years. Each record indicates the price paid, date of purchase, and seating zone. Seating zones are groups of similar seats sharing one price, which I use as a measurement of quality. The primary market records also indicate the type of sale, such as season tickets or promotions.

Similar transaction-level records for resale come from StubHub. The main difference between the resale and primary market data is that the transaction price is not included for resale transactions.

To learn about the transaction price, I supplement the transaction records with daily records of all StubHub listings for the university's football games, which I gather using a web scraper. The listings are available for two full seasons and one partial season, but the listings and official resale transactions only overlap for the one full season studied in this paper. Each listing includes a listing ID, price, number of tickets for sale, and location in the stadium (section and row).

The primary and resale market records are informative about demand for tickets, idiosyncratic shocks, and the choice between buying tickets in the primary or resale market. Resale is informative about idiosyncratic shocks because resale implies that a consumer changed her mind about whether to attend the game.

The final set of resale market data contain average annual resale prices for 76 college football teams, which I gather from SeatGeek, another online resale market. The annual prices end in 2019 and start as early as 2011, although records for some teams start later.

The SeatGeek data are informative about aggregate shocks. It shows that the average price of a resold ticket varies meaningfully from one year to the next. The changes are too large to be the result of purely idiosyncratic shocks like schedule conflicts; it is far more likely to reflect an aggregate shock to a common component of values.

I use StubHub listings to infer the distribution of resale transaction prices. Resale transaction prices are not directly observable from listings because the StubHub listings only contain tickets currently available for sale. I start by inferring transactions from changes in listings. For example, if the number of tickets offered in a listing falls by two from one day to the next, then I assume two tickets were purchased at the last observed price.

The procedure leads to false positives because some listings are removed without being sold. I take two steps to correct them. First, I remove implausibly expensive transactions.[8] Second, I compare the number of inferred and actual transactions at the game-section-time level and assume that the lowest-price inferred transactions are the true ones. The removed transactions are generally outliers, either occurring earlier or containing more seats than typical transactions.

Because the data come from only one resale market, resale is undercounted. However, StubHub is likely to account for most resale in this market for two reasons. First, the university has a partnership with StubHub and recommends that consumers resell on StubHub. Second, StubHub is one of the largest resale platforms, processing about half of all ticket resale in 2015 (Satariano (2015)).

StubHub charges fees for transactions on its platform. StubHub's exact fee structure is not public, but buyers usually pay an additional 10% of the listing price and sellers 15%, with some discounts for large sellers. I assume that the standard fees apply to all buyers and resellers.

2.3.2 Covid-19 Survey

Data on how demand with and without a covid-19 vaccine come from a survey. Respondents report the maximum they are willing and able to pay for one ticket to a college football game in several scenarios related to covid-19.[9] The scenarios are (i) the 2019 season, (ii) a covid-19 vaccine, (iii) no vaccine but the number of cases falls below the CDC's near-zero benchmark, and (iv) no vaccine and the number of

[8] These are defined as transactions sold at more than 1.5 times the 75$^{\text{th}}$ percentile of price for similar quality seats

[9] Eliciting willingness to pay by asking directly is used in other surveys, such as the one analyzed in Fuster and Zafar (2021). Eliciting assessments of probabilities in the same way is commonly used in Federal Reserve Bank of New York surveys: see Potter et al. (2017).

cases is above the CDC's near-zero benchmark.[10] (I combine responses for scenarios (iii) and (iv) because reported willingness to pay is similar.) Respondents also report their demographic information and the percent chance of each scenario in January 2021, September 2021, and January 2022. I distributed the survey to 500 users of Prolific.co, an online distribution platform, in August 2020. Half of respondents were aged 50 or over. The full survey and details can be found in Appendix A.3.

The survey directly measures changes in willingness to pay in two states of the world. Descriptive evidence presented in the next section establishes that changes in consumer values are indeed heterogeneous. I use the resulting distribution of changes in values in the empirical model.

2.4 Descriptive Evidence

2.4.1 Market Background

The university is a monopolist because it is the sole primary market seller of its tickets. In the season used in the analysis, it sells tickets to five home games.[11]

The stadium has about 50,000 seats, but only 30,000 are available to the public. Seats unavailable to the public include premium seats for athletics boosters, student seats, and seats reserved for visiting team fans.

Tickets are sold in two main phases. The first phase consists of season ticket sales, which are made months before the season—80% of season tickets are bought at least four months before the season starts. The second phase consists of single-game

[10] When the survey was distributed, public concern focused on whether a vaccine would exist rather than how quickly it could be manufactured and distributed.

[11] An additional home game was scheduled but cancelled. The cancelled game is excluded from the data provided by the university, so I exclude it from the analysis. I assume that consumers would have made the same season ticket purchases if that game had not been scheduled, and I use prorated season ticket prices in the estimation.

ticket sales and resale and occurs much later. Single-game tickets do not go on sale until the first game is about a month away. 70% of resale and full-price single-game transactions occur within a month of the game and 50% within two weeks. The gap between the two phases makes it plausible that consumers receive preference shocks. The empirical model reflects the timing of the market, with a first period in which only season tickets are sold and a second in which only single-game tickets and resale tickets are sold.

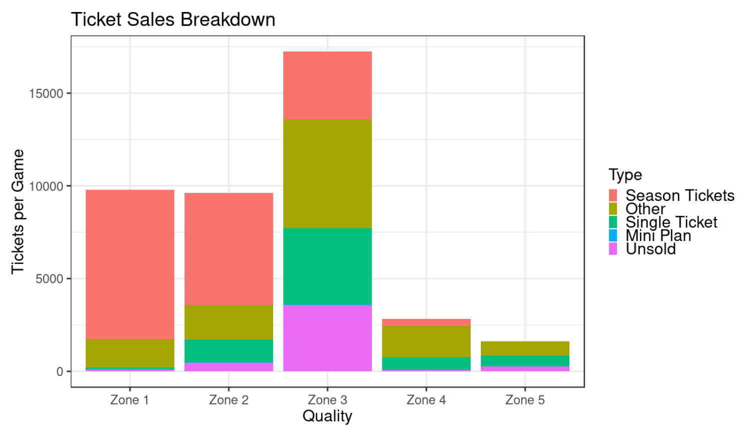


Figure 2.1: Sale types and volumes by quality group.

Figure 2.1 shows the average number of tickets sold to each game by type of sale and quality, including unsold tickets. Most tickets are sold as season tickets, and season tickets are fully 75% of tickets sold to the public (the "other" category consists of tickets that are not available to the public, like student tickets). Most of the rest are sold as single tickets or unsold. A minuscule number are sold in mini-plans, bundles of tickets to a subset of games that I exclude from the analysis. The single ticket purchases in Figure 2.1 include group sales and promotions. I only consider full-price single ticket sales in the analysis because promotions and group sales are not optimally priced and may only be available to targeted groups, like veterans.[12]

[12]Nearly 40% of promotional tickets in the season were given away for free, and 98% were sold for half-price or less. Group tickets are discounted by over 40% on average. Promotions are not used to cope with demand uncertainty because they are too steeply discounted and too targeted.

The stadium is divided into five seating zones, which I use to measure the quality of each seat. Higher zones (e.g. zone 5) contain worse seats. Zone 1 seats are close to the field and near the 50-yard line, but zone 5 seats are at the extreme edges of the upper deck.

Table 2.1: Primary market prices for each game, their sum, and season ticket prices. Table excludes the cancelled game. Season ticket prices are prorated to reflect the cancellation.

Game	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5
1	70	60	50	40	30
2	70	60	55	45	30
3	70	60	50	40	30
4	70	60	55	45	30
5	60	55	40	35	30
Season Tickets	315	270	216	179	125
Face Value Sum	340	295	250	205	150

The menu of primary market prices (excluding the cancelled game), including prorated season ticket prices, is shown in Table 2.1. Primary market prices vary mainly by seat quality. Tickets in zone 1 cost \$60–\$70 depending on the game, but zone 5 tickets always sell for \$30. Season tickets are \$25–\$35 cheaper than buying primary market tickets to each game. Prices vary slightly across games, but never by more than \$10.

2.4.2 Resale Markets

Resale is a notable feature of the market, with 5.98% of all tickets sold to consumers resold on StubHub.[13] The true resale rate is higher because some tickets are resold on other resale markets.

An important feature of resale markets is that prices are flexible. Resellers set the

[13] The figure excludes tickets sold directly to ticket brokers. I conservatively assume that all tickets sold to brokers are resold on StubHub.

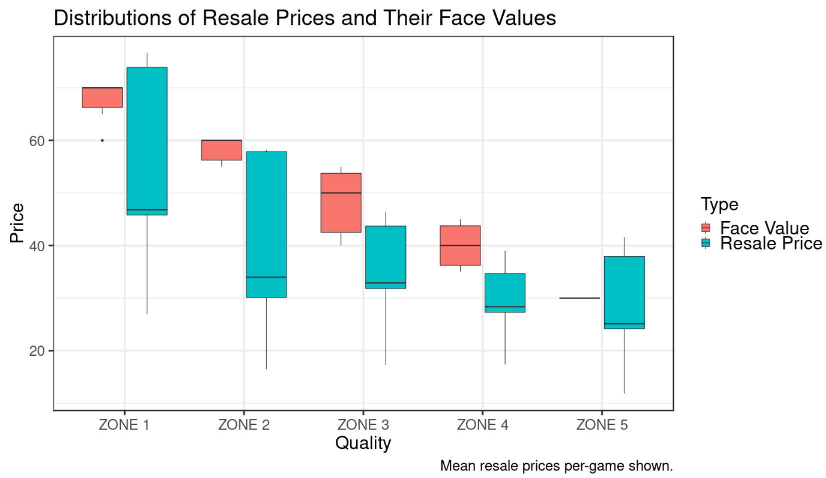


Figure 2.2: Distributions of mean per-game resale prices and face value.

price of their tickets when they list on StubHub and are free to change prices afterwards. The ability to change prices makes it plausible that the average transaction price adjusts after shocks.

Figure 2.2 establishes that resale prices adjust to differ from face value. It shows the distribution of face values and the distribution of the average resale price for each game-quality combination. The difference between the distributions suggests that resale prices adjust, presumably in response to demand. The variation in resale prices also suggests that consumers have different average values for each game.

Figure 2.3 provides further evidence of price flexibility. It shows the percent change in the quantity of single-game tickets sold for each game (in both primary and resale markets) from the season average. The changes in primary market quantities are practically always larger than the changes in resale quantities, usually by a large margin. The higher volatility in the primary market is unsurprising because its prices are fixed. In contrast, resale market prices adjust and smooth the quantity of tickets resold.

The last important feature of resale markets is that they include frictions that are not present in the primary market. StubHub charges buyers a fee of 15% of the

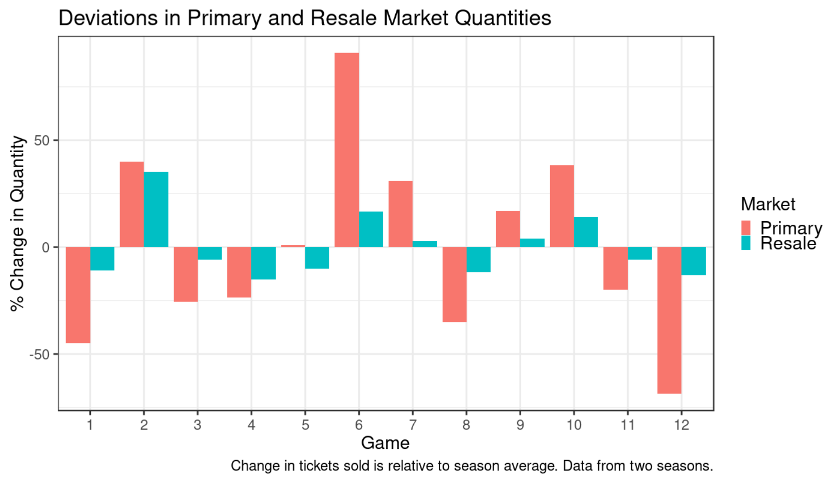


Figure 2.3: Percent deviation from season-average quantities sold for each game.

Table 2.2: Face value minus mean resale price and quantity of full-price primary market tickets sold for each game.

Game	Mean Price Diff.	PM Quantity
1	16.48	1266
2	-6.69	3373
3	16.94	991
4	-5.79	2383
5	28.21	493

advertised price and sellers a fee of 10%. The average fee is $10.71 on each ticket resold, a substantial amount when the average resale price is under $40. There is also evidence of non-monetary frictions. If there were no frictions, consumers would buy single-game tickets for a given section in whichever market is cheaper. Table 2.2 shows that this is not true in the data: hundreds of single-game tickets are sold in the primary market when cheaper resale tickets are available. For instance, the average resale ticket to game one is over $16 cheaper than the average primary market ticket, yet over 1250 single-game tickets are sold in the primary market. There are several plausible interpretations for the friction. Consumers might not like or trust the resale market, they might find searching for tickets onerous, or they might be unaware that it has cheaper tickets.

2.4.3 Annual Price Changes

Annual price changes for each team provide evidence of aggregate shocks. Using SeatGeek's records of average annual resale prices, I define the normalized price for university u in year y as

$$\text{NormalizedPrice}_{uy} = \text{AvgResalePrice}_{uy} / \left(\frac{1}{Y} \sum_{y} \text{AvgResalePrice}_{uy} \right), \tag{2.1}$$

where Y denotes the number of years in the sample. Figure 2.4 shows the distribution of normalized prices for a random sample of 20 universities. The distributions demonstrate that within-university price variation is significant and ubiquitous. All but one university has a season where prices are 25% above the sample mean, and most have a season where prices are 25% below. There are several changes of 50% or more.

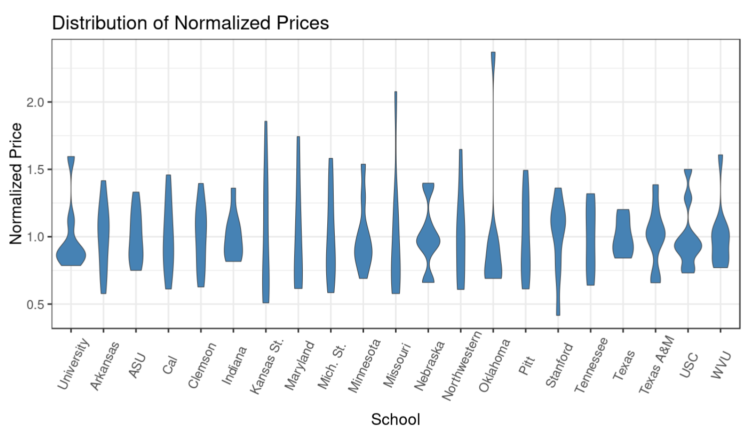


Figure 2.4: Distribution of average annual resale prices (normalized by school mean) for a random sample of 20 schools in similar conferences.

The dramatic swings in resale prices likely reflect aggregate preference shocks, like unpredictable changes in team performance. For instance, in Clemson's lowest-priced season they lost two of their first three games—as many as they lost in the entire

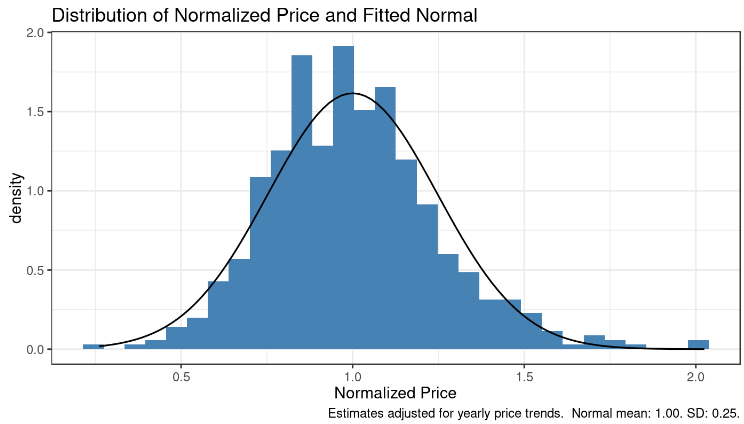


Figure 2.5: Distribution of resale prices normalized by team-mean in the sample. Adjusted for yearly trends. From SeatGeek annual average resale prices (76 teams, 576 team-seasons).

previous season—whereas in their two highest-priced seasons they either played in or won the national championship game.

Figure 2.5 shows the combined distribution for all 76 teams in the data after adjusting for time trends. The distribution is approximately normal and has an estimated standard deviation of .25, implying that there is a roughly one-third chance that prices in any given season will be more than 25% away from the mean.

2.4.4 Covid-19 Survey

Figure 2.6 shows the distribution of reported willingness to pay (WTP) in the survey in three scenarios without social distancing. There is significant variation in how much respondents will pay for college football tickets, with many reporting WTP of $100 or more in the 2019 season.

As expected, WTP is highest for the 2019 scenario. Consumers report somewhat lower WTP even if there is a covid-19 vaccine and WTP falls markedly in the case where there is no vaccine. Many consumers, however, are still willing to pay $50 or more if there is no vaccine.

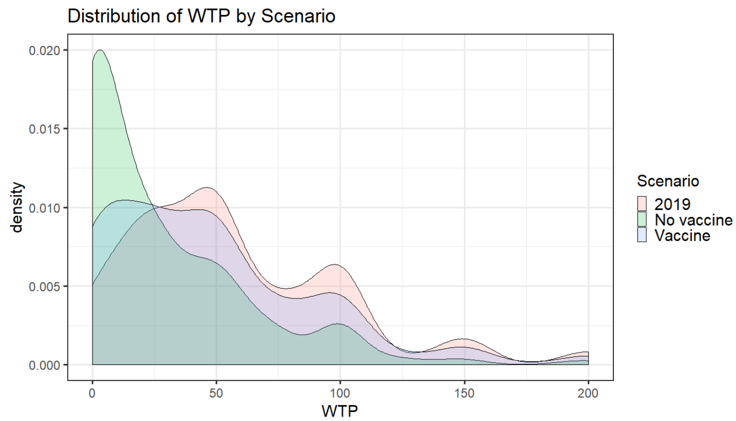


Figure 2.6: Distribution of reported willingness to pay without social distancing in 2019, with a vaccine, and with no vaccine.

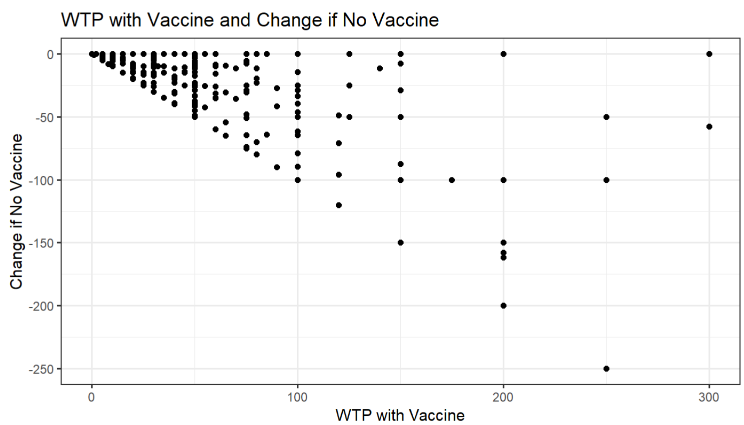


Figure 2.7: Scatterplot of reported willingness to pay with a vaccine and change in willingness to pay if there is no vaccine.

For evidence on individuals' changes in WTP across states, consider Figure 2.7. The figure shows the joint distribution of WTP with a vaccine and the change in WTP if there is no vaccine.[14] There is significant heterogeneity in the changes in WTP. Many report changes exceeding $50 while others report small or no change. Moreover, there is heterogeneity at each level of WTP with a vaccine, and the changes in WTP are not visibly correlated with initial WTP.

[14]The lower triangle is empty because the change in WTP cannot exceed reported WTP.

2.5 Model

2.5.1 Outline, Utility, and Uncertainty

Let i index consumers and j index games. A monopolist seller has capacity K_q for each of q seat qualities and sells tickets over two periods, $t = 1, 2$. In period one, it only sells a season ticket bundle including one ticket to each game, and in period two, it only sells single-game tickets. The seller sets the price p_{Bq} for a season ticket bundle containing tickets of quality q. It sets the price p_{jq} for quality q tickets to game j. It commits to its menu at the start of the first period and does not change it afterwards.

There are three sources of uncertainty, each realized at the start of the second period. The first is an idiosyncratic preference shock. Consumers receive independently drawn shocks for each game with probability ψ, and any consumer receiving a shock for game j has zero utility for that game. The idiosyncratic shock captures anything that might cause one consumer, but not others, to change her mind about attending the game. Schedule conflicts are a natural interpretation.

The second is an aggregate shock. There is a single draw $V \sim N(0, \sigma_V^2)$ for the season that affects each consumer's value for attending the game. I refer to V as the common value. It captures changes (aside from whether there will be a vaccine) that affect annual aggregate demand for tickets, like team performance. There is only one draw of V per season and it affects all games.

The third is a state of the world ω. In the application with uncertainty over a covid-19 vaccine, the state is ω^{Vax} if there is a vaccine and ω^{NoVax} if there is not. Each consumer has weakly lower utility in the state with a vaccine, but the size of the change is heterogeneous across consumers. In the baseline model without uncertainty from covid-19, a baseline state ω^{BL} is realized with certainty.

There are N consumers who each want at most one ticket. A fraction a arrive in the first period and the rest arrive in the second. In the first period, consumers decide whether to buy season tickets or wait. In the second period, consumers who bought season tickets decide whether to resell tickets or attend each game. Consumers without season tickets decide whether to purchase in the primary market, secondary market, or not at all.

The model outline is depicted in Figure 2.8, which shows the timeline of choices on the right and a flowchart of consumer decisions on the left. Consumer decisions in period two are for one game j but occur for all games.

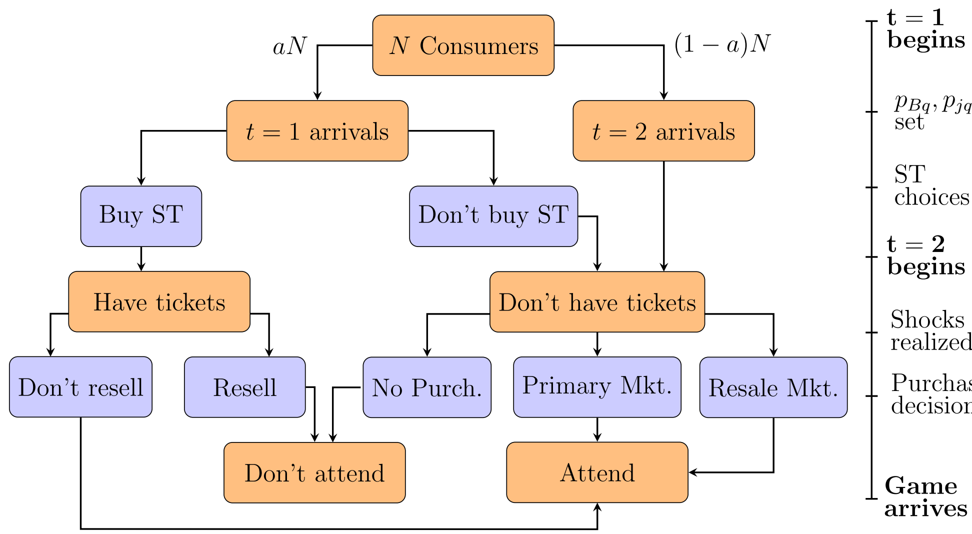


Figure 2.8: Model timeline and outline for consumer arrivals and choices. Decisions are shown in blue.

Consumer i's utility for a ticket of quality q to game j is measured in dollars (relative to an outside option with utility zero) and takes the form

$$u_{ijq}(V, \omega) = \alpha_j \left(V + \nu_i + \gamma_q - b_i(\omega) \right). \quad (2.2)$$

Consumer i's utility depends on a scalar α_j specific to game j, the common value V, a consumer-specific taste parameter ν_i, a quality-specific parameter γ_q, and consumer i's distaste for attending sporting events in state ω, $b_i(\omega)$. I assume that the taste parameters ν_i follow an exponential distribution with parameter λ_ν.

Utility can be broken into two pieces. The piece in parentheses is constant across games and can be thought of as consumer i's base utility for all games. The base utility is multiplied by the second piece, the scalar α_j that describes which games are more desirable.

Changes in V are aggregate, affecting all consumers' utilities in the same way. The penalty $b_i(\omega)$ only applies to uncertainty from covid-19. Consumers experience no change if there is a vaccine, but they are willing to pay weakly less if there is no vaccine: $b_i(\omega^{\text{Vax}}) = 0$, $b_i(\omega^{\text{NoVax}}) \geq 0$. Realizations when there is no vaccine are heterogeneous and independent of ν_i.[15] In the baseline model, all consumers have $b_i(\omega^{BL}) = 0$.

2.5.2 Period Two

At the start of period two, consumers know the realizations of idiosyncratic shocks, the common value V, and the state of the world ω. Consumers who purchased season tickets decide whether to resell or attend; all other consumers decide whether to purchase tickets in the primary or resale markets. Resale prices are noted by $p^r_{jq}(V, \omega)$. They depend on the realizations of aggregate shocks because the shocks affect consumer values.

For simplicity, consider game j. Consumers who bought season tickets resell if

$$u_{ijq}(V, \omega) \leq (1 - \tau) p^r_{jq}(V, \omega), \tag{2.3}$$

where τ is the percent commission charged by StubHub. Consumers who receive an

[15] There is no correlation between reported WTP with a vaccine and the percent change in valuation from the vaccine to the no vaccine state. The lack of correlation is evident in Figure 2.7.

idiosyncratic shock have value zero and always resell.

Consumers without season tickets decide whether and how to buy tickets to game j. They have three choices: make no purchase, purchase in the primary market, or purchase in the secondary market.

In addition to the familiar utility and price terms, surplus in the secondary market depends on the friction s_{ij}. I assume it follows an exponential distribution, $s_{ij} \sim Exp(\lambda_s)$, and is independently drawn across individuals and games. Consumers know the distribution in the first period but do not learn their realizations until the second. The friction explains why some consumers in the data purchase single-game tickets in the primary market when similar tickets are available for less in the secondary market.

Surplus from each option is

$$\textit{No Purch. Surplus}_{ij} = 0, \tag{2.4}$$

$$\textit{PM Surplus}_{ijq}(V, \omega) = u_{ijq} - p_{jq}, \tag{2.5}$$

$$\textit{SM Surplus}_{ijq}(V, \omega, s_{ij}) = u_{ijq} - p^r_{jq}(V, \omega) - s_{ij}. \tag{2.6}$$

The equilibrium resale price $p^r_{jq}(V, \omega)$ makes the number of consumers willing to resell tickets of quality q, determined in equation (2.3), equal to the number of consumers who demand a ticket of quality q on the resale market.

If all tickets were available, consumer i would select the maximizer of the set

$$\mathcal{C}_i(V, \omega, s_{ij}) = \{0, \{\textit{SM Surplus}_{ijq}(V, \omega, s_{ij})\}_{q=1}^{Q}, \{\textit{PM Surplus}_{ijq}(V, \omega)\}_{q=1}^{Q}\}. \tag{2.7}$$

But some tickets might sell out, leaving the consumer unable to acquire his preferred option. Stock-outs are possible in equilibrium because a high draw of the common value could leave single-game tickets underpriced in the primary market. I assume that tickets are rationed randomly. Let the probability of receiving a primary market ticket of quality q to game j be $\sigma_{jq}(V, \omega)$. (There is no rationing on the resale market at equilibrium resale prices.) Consumers rank all options in the choice set and request their first-choice ticket. They receive the ticket with the rationing probability and, if they do not receive it, request their next-preferred ticket.

2.5.3 Period One

In period one, aN consumers know their type $(\nu_i, b_i(\omega^{\text{NoVax}}))$ and decide whether to buy season tickets.[16] By buying season tickets, consumers receive the maximum of their value for attending game j and the after-fee resale price. Surplus depends on attendance values, resale values, the price of season tickets, and an additional parameter δ. The purpose of δ is to capture other factors that affect valuations for season tickets, such as perks for season ticket holders or diminishing returns from attending many games. Surplus from season tickets of quality q is

$$\begin{aligned} ST\ Surplus_{iq} = \sum_j \mathrm{E}_{V,\omega}\Big(\max\big\{(1-\psi)u_{ijq}(V,\omega) + \psi(1-\tau)p^r_{jq}(V,\omega), \\ (1-\tau)p^r_{jq}(V,\omega)\big\}\Big) + \delta - p_{Bq}. \end{aligned} \tag{2.8}$$

The surplus from waiting until period two requires an expectation for surplus with rationing. Without rationing, surplus is the expected maximizer of equation (2.7).

With rationing, it is possible that the consumer must choose his m^{th}-best option.

[16] In this section, I only consider the traditional season ticket package with resale markets. I modify the decision rules to reflect refunds and alternative contracts in Section 2.7.

Let $c^{(m)}(\mathcal{C})$ be the m^{th}-largest element of $\mathcal{C}$, and let $\sigma_j(V, \omega, c)$ be the probability of receiving option c. The expected utility from waiting with choice set $\mathcal{C}_i$ when the common value is V, state is ω, and resale friction is s_{ij} can be defined recursively as

$$\begin{aligned} \mathit{WaitSurplus}_i(V, \omega, s_{ij}, \mathcal{C}_i) = \; & \sigma_j(V, \omega, c^{(1)}(\mathcal{C}_i))c^{(1)}(\mathcal{C}_i) + \\ & (1 - \sigma_j(V, \omega, c^{(1)}(\mathcal{C}_i)))\, \mathit{WaitSurplus}_i(V, \omega, s_{ij}, \mathcal{C}_i \setminus c^{(1)}(\mathcal{C}_i)). \end{aligned} \tag{2.9}$$

Overall surplus from waiting is the expected value,

$$\mathit{WaitSurplus}_i = \mathrm{E}_{V,\omega,S}\left(\mathit{WaitSurplus}_i(V, \omega, S, \mathcal{C}_i(V, \omega, S))\right). \tag{2.10}$$

The consumer's choice set in period one is thus

$$\mathcal{C}_{i,ST} = \left\{ \mathit{WaitSurplus}_i, \{ST\ \mathit{Surplus}_{iq}\}_{q=1}^{Q} \right\}. \tag{2.11}$$

Without rationing, the consumer would again select the maximizer. However, it is possible that some qualities of season tickets will sell out. I again assume random rationing under the same procedure discussed for the second period.

2.5.4 Equilibrium

I search for a fulfilled-expectations equilibrium. The seller anticipates consumer demand and selects profit-maximizing prices $\{p_{Bq}\}$ and $\{p_{jq}\}$. Consumers anticipate a set of resale prices $\{p^r_{jq}(V, \omega)\}$ and primary market purchase probabilities $\{\sigma_{jq}(V, \omega)\}$. In equilibrium, consumers make optimal choices in the first period given expectations for resale prices and probabilities, and their expectations are realized in the second period when they make optimal purchase choices.

2.6 Estimation and Results

There are two stages in the estimation strategy. The first stage includes all parameters that can be estimated without structural simulations, and the second estimates the remaining parameters using the method of simulated moments. I assume that the realized state is ω^{BL} when using the sales data because the season predates the covid-19 pandemic.

2.6.1 First Stage

The fee τ is the percentage of the fee-inclusive price paid by the buyer, calculated directly from StubHub's policies. The idiosyncratic shock rate ψ is identified by the frequency of resale. In the model, observed resale is explained by idiosyncratic shocks in equilibrium, so the parameter ψ equals the ratio of tickets resold by consumers to all tickets sold.[17]

The data are not directly informative about how many consumers consider season tickets. In the absence of data on browsing, I calibrate the fraction of consumers arriving in period one based on purchse data. Specifically, I take a to be the percentage of tickets sold 30 or more days in advance.

Next, the parameters α_j and γ_q affect consumer values and hence resale prices. Recovering the parameters requires a model for the price of resale transaction k. The resale price of listing k depends on all parameters affecting the relative surplus

[17] The true number of all tickets resold is unknown because StubHub is not the only resale market. Moreover, the university sells some tickets to brokers for resale. I conservatively assume that all tickets sold to brokers are resold on StubHub and that 75% of consumers resell on StubHub. In contrast, Leslie and Sorensen (2014) assume that StubHub and eBay have a combined market share of 50%.

received in the primary and secondary markets in period two, including the realization of V, the distribution of resale market frictions, the distribution of consumer types, the menu of primary market prices, and characteristics X_k of listing k. The price can be written as a non-parametric function,

$$p^r_{jqk} = g(\alpha_j, \gamma_q, \lambda_s, V, \lambda_\nu, \mathbf{p}_j, X_k) + \varepsilon_{jqk}, \tag{2.12}$$

where X_k includes the number of tickets in the transaction and the number of days until the game. (For a full discussion of how these factors affect price, see Sweeting (2012).)

Equation (2.12) can be simplified because most of its arguments are constant in the data. For instance, the common value, primary market prices, and type distribution do not change during the season. Moreover, the resale price is approximately linear in consumers' attendance values under mild assumptions.[18] Consequently, I assume that

$$g(\alpha_j, \gamma_q, \lambda_s, V, \lambda_\nu, \mathbf{p}_j, X_k) = \alpha_j(\beta_0 + \gamma_q + X_k\beta). \tag{2.13}$$

The right-hand side of equation (2.13) is the same as consumers' values for the game plus an additional term to capture features of listing k. The approximation does not capture one source of nonlinearity, substitution to the primary market from the cost of resale s_{ij}, but estimates are very similar with a polynomial form that allows nonlinearities.

The identifying variation for α_j and γ_q comes from across-game and across-quality variation in resale prices. More precisely, α_j explains why similar tickets for different

[18]The assumptions are that the supply of tickets to the resale market does not change and that resale prices are below primary market prices. The first assumption holds in equilibrium and the second is nearly always true in the data.

games sell at different prices and γ_q explains why tickets to the same game in different quality zones sell at different prices.

The variance of the common value, σ_V^2 is estimated using the distribution of normalized resale prices shown in Figure 2.5. I multiply the distribution of normalized prices by the university's average resale price in the SeatGeek sample. Then, I adjust for the average value of α_j because the shocks enter utility as $\alpha_j V$. Finally, I take σ_V^2 as the variance of a normal fit to the distribution. The normal fit is appropriate because the distribution in Figure 2.5 is approximately normal. Details can be found in Appendix A.2.

The identifying variation for the variance is entirely within each team. The normalized prices measure year-on-year variation relative to the team average, so σ_V^2 reflects the variation an individual team can expect from year to year.

The procedure makes three assumptions. First, the year-to-year variation in the SeatGeek data is the sole source of variation in the common value. It is not clear if the assumption understates or exaggerates the variance: it could understate the variance because annual prices smooth over game-specific shocks like rain, but it could exaggerate the variance if some part of the year-to-year change is predictable. Second, shocks to common values pass through linearly to resale prices. This is the same assumption used to estimate α_j and γ_q in equation (2.13). And third, the university faces the same shocks to normalized prices as all other schools. This is plausible based on the distributions in Figure 2.4.

The last parameters estimated in the first stage define the effect of the states of the world with and without a vaccine on preferences. The survey asks consumers about WTP in 2019 and in three scenarios, one with a vaccine and two without. Consumers reported similar WTP in the two scenarios without a vaccine, so I combine them into a single no-vaccine state. The survey also asks for values with and without social distancing in each scenario. Social distancing also does not significantly affect consumer values, so I only consider reported WTP without it. See Appendix A.3 for details.

The counterfactual considers sales for the college football season beginning in September 2021. The probabilities that there will and will not be a vaccine are taken as the average percent chance of each state in the survey for September 2021, normalized to sum to one. (The normalization excludes a state in which there is no attendance at sporting events.)

There are two necessary adjustments for consumer preferences. The first is to find the function $b_i(\omega^{\text{NoVax}})$ describing the change in WTP from the vaccine to the no vaccine state. The second is to find the analogous function $b_i(\omega^{\text{Vax}})$ describing the change from the benchmark year (ω^{BL}, measured using reports for 2019) to the vaccine state. The second adjustment is necessary because the estimated distribution of values from the sales data reflects a typical year and reported values are lower with a vaccine.

I assume that each consumer's reported WTP in the survey is his utility for a representative game. I also assume that the representative game has the game-specific parameter $\bar{\alpha}$, which is an average of the estimated α_j. The change in consumer i's WTP from state ω to state ω' is

$$WTP_i(\omega) - WTP_i(\omega') = \bar{\alpha}(b_i(\omega') - b_i(\omega)). \tag{2.14}$$

I further assume that ω is a baseline state with $b_i(\omega) = 0$ and that $b_i(\omega')$ follows the parametric form

$$b_i(\omega') = \begin{cases} 0 & \text{w.p. } \rho_1 \\ \tilde{b}_i & \text{otherwise} \end{cases} \tag{2.15}$$

where $\tilde{b}_i \sim \text{Exp}(\rho_2)$. There is a mass point at zero to reflect the fact that many consumers report no change in WTP in the survey.

I estimate two sets of parameters to capture the two reported changes in WTP, $\text{WTP}_i(\omega^{\text{Vax}}) - \text{WTP}_i(\omega^{\text{NoVax}})$ and $\text{WTP}_i(\omega^{BL}) - \text{WTP}_i(\omega^{\text{Vax}})$. The parameters for the first difference identify the distribution of $b_i(\omega^{\text{Vax}})$ and are labeled ρ_1^{Vax} and ρ_2^{Vax}. The parameters for the second identify the distribution of $b_i(\omega^{\text{NoVax}})$ and are labeled ρ_1^{NoVax} and ρ_2^{NoVax}.

The reported differences in WTP almost directly identify the function b by equation (2.14). The sole complication is censoring: the change in WTP cannot be larger than WTP. I adjust for censoring and estimate by MLE using

$$\left(\text{WTP}_i(\omega) - \text{WTP}_i(\omega')\right)/\bar{\alpha} = \begin{cases} 0 & \text{w.p. } \rho_1 \\ \min\{\text{WTP}_i(\omega)/\bar{\alpha}, \tilde{b}_i\} & \text{otherwise.} \end{cases} \tag{2.16}$$

2.6.2 Second Stage

Three parameters remain for structural estimation: λ_s, which defines the distribution of resale market frictions; λ_ν, which defines the distribution of consumer values; and δ, which explains why values for season tickets differ from attendance and resale values. I estimate them using the method of simulated moments. In model simulations, I assume that there are 200,000 consumers who demand up to one ticket and weight moments by their inverse variances. Details are in Appendix A.2.

The estimation moments are the number of season tickets purchased, the average resale price for each game, and the quantity of tickets sold in the primary market for each game. With five games played, there are a total of 11 moments.

Each parameter is identified by a combination of the estimation moments. Start with the distribution of costs of purchasing on the resale market, which is parameterized by λ_s. In the model, consumers purchase in the primary market if the primary market price is less than the sum of the resale price and the cost of resale. For instance, if the resale price is $5 less than the primary market price, any consumer with $s > 5$ prefers the primary market. The distribution of s determines the number of consumers with $s > 5$ and hence the number of tickets sold in the primary market. It follows that λ_s is identified by primary market quantities and resale prices, which give an observed difference between resale and primary market prices and the number of consumers who prefer the primary market.

Next, consider the additional value of season tickets, δ. Values for season tickets equal the sum of attendance values, expected resale revenue, and the parameter δ. The role of δ is to explain why observed demand for season tickets differs from the demand predicted by attendance values and resale revenue. Consequently, it is identified by season ticket quantities, which capture demand for season tickets, and resale prices, which capture resale revenue.

The last parameter is the distribution of values for college football relative to the outside option, parameterized by λ_ν. Higher values cause purchase quantities and resale prices to rise, so λ_ν is explained by all estimation moments: season ticket quantities, primary market quantities, and resale prices.

Equilibrium requires a fixed point of the model: consumers must have correct expectations for resale prices and primary market purchase probabilities as a function of V. Finding the fixed point for each set of candidate parameters is challenging.

Moreover, each iteration of each fixed-point search requires a solution for resale prices for every realization of V.

I use several simplifications to make estimation feasible. First, shared quality preferences γ_q reduce the search for resale prices to one dimension. Second, I discretize continuous variables. Consumer types and resale prices are both assumed to be a grid, with 100 values for resale prices and 200 for consumer types. Under these assumptions, iterating to find equilibrium expectations remains difficult: expectations for resale prices and primary market purchase probabilities vary by realization of V and game, giving two $100 \times J$ matrices.

2.6.3 Results and Fit

Estimated parameters are in Tables 2.3, 2.4, 2.5, and 2.6. The resale fee is about 22% of the fee-inclusive price paid by the buyer.[19] The idiosyncratic shock rate suggests that 8% of buyers change their minds about attending the event between the first and second periods. The fraction of consumers arriving in the first period, a, is calibrated to 77%, indicating that most consumers consider whether to buy season tickets.

Consumer values vary widely across games and qualities. I normalize $\alpha_1 = 1$ and $\gamma_1 = 0$. The best game, game 2, has attendance values 67% higher than those for the baseline game; the worst game, game 5, has values nearly 50% lower. The best seats are worth roughly $23 per ticket more than the worst seats for game 1, with the difference scaled by the relevant α_j for other games.

The standard deviation of the distribution of consumer values is $7.85. The university thus faces consumer values for the baseline game that differ from the mean by more than $7.85 about a third of the time.

[19] For a listing with price p, StubHub charges the buyer $1.15p$ and gives the seller $.9p$. Ultimately, it collects $.25/1.15 \approx .22$ of the price paid by the buyer.

Parameter Description	Notation	Estimate	Std. Err.
Resale Fee (%)	τ	0.22	-
Idiosyncratic Shock Rate	ψ	0.08	-
% in First Period	a	0.77	-
Preference for Game 1	α_1	1.00	-
Preference for Game 2	α_2	1.67	0.032
Preference for Game 3	α_3	1.01	0.023
Preference for Game 4	α_4	1.60	0.029
Preference for Game 5	α_5	0.56	0.015
Preference for Quality 1	γ_1	0.00	-
Preference for Quality 2	γ_2	-12.05	0.581
Preference for Quality 3	γ_3	-17.58	0.55
Preference for Quality 4	γ_4	-22.65	0.62
Preference for Quality 5	γ_5	-21.95	0.687
SD of Common Value	σ_V	7.85	0.231

Table 2.3: Estimated parameters from the first stage.

State probabilities and parameters governing preference changes across vaccine states are contained in Tables 2.4 and 2.5. Conditional on there being attendance at sporting events, consumers report a 59% chance that there will be a vaccine in September 2021 and a 41% chance that there will not be one. 60% of consumers report no value change between the benchmark and the state with a vaccine, but other consumers report significant penalties, with a mean (uncensored) change in WTP of $43.20. For the transition from the vaccine to the no vaccine state, only 29% of consumers report no change in values. The remaining consumers again report a significant change in WTP, with a mean of $52.27. For proof of the fit of the model, see Figure A.1 in Appendix A.2.

Table 2.4: Expected state probabilities in September 2021

State	Probability
Vaccine	0.59
No Vaccine	0.41

In the second stage, the average consumer's friction associated with resale market

Table 2.5: Estimated preference change parameters.

Parameter	Value	Std. Err
ρ_1^{NoVax}	0.29	0.02
ρ_2^{NoVax}	52.27	4.50
ρ_1^{Vax}	0.60	0.02
ρ_2^{Vax}	43.20	4.58

purchases, s_{ij}, is $48.42. Although the average value is large, the consumers who purchase in the resale market have much smaller realizations. Two-thirds of frictions are $10 or less, and nearly 90% are $20 or less. The full distribution of realized costs for resale market buyers is shown as Figure A.2 in Appendix A.2.

Table 2.6: Estimated parameters from the second stage.

Parameter Description	Notation	Estimate	Standard Error
Mean Resale Friction	λ_s	48.42	1.54
Mean Consumer Type	λ_ν	16.02	0.02
Mean ST Benefits	δ	29.96	0.34

The mean of the distribution of consumer types is 16.02, suggesting that the average consumer would pay $16.02 for the worst seats to the baseline game in an average season. Finally, the benefits of season tickets are estimated to be $29.96, suggesting that the convenience and perks of season tickets outweigh diminishing marginal returns.

Table 2.7 and Figures 2.9 and 2.10 assess the model fit. Observed and model-implied resale prices are extremely close. The model captures the patterns in primary market sales across games, but does not fit them exactly. The looser fit is expected because there are no game-specific quantity parameters. Finally, the model-implied number of season tickets purchased is within 13% of the true value.

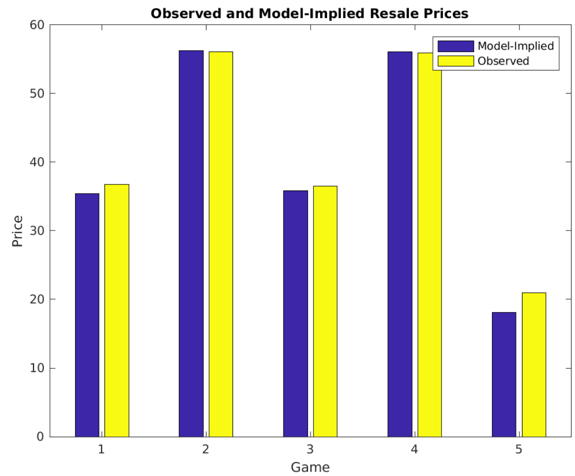


Figure 2.9: Observed and model-implied resale prices for each game.

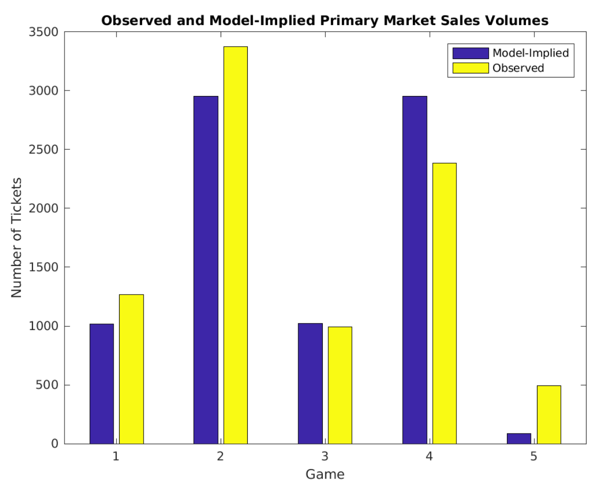


Figure 2.10: Observed and model-implied primary market quantities sold.

Table 2.7: Observed and model-implied quantities of season tickets.

Moment	Model-Implied	Observed
Season Tickets Sold	25291	22471

2.7 Counterfactuals

I use the structural estimates to evaluate several counterfactual policies. The central counterfactuals are partial refunds and a menu of refunds, but I also consider benchmark cases with no reallocation and refunds with price adjustments after shocks.

2.7.1 Benchmarks: No Reallocation and Flexible Prices

Start with counterfactuals that provide a benchmark for the focal sales strategies, which are only tested in the baseline state of the world. With no reallocation, the university prohibits resale and does not offer refunds. The comparison is useful because it measures the net effect of resale and refunds on profit and welfare. Whether sellers profit from resale is a primary concern of the theoretical literature (e.g. Courty (2003), Cui et al. (2014)), but empirical evidence is rare for sellers of perishable goods.

To evaluate the model without reallocation, I fix expected resale prices and supply at zero. Consumers who buy season tickets and have idiosyncratic shocks neither use nor reallocate them, and all other consumers can only buy tickets in the primary market. The seller maximizes profit by optimally selecting its prices p_{Bq} and p_{jq}.

A second useful benchmark is to consider refunds if the seller has flexible prices. In this counterfactual, the seller can adjust its prices after shocks, eliminating the advantage of resale. I implement the counterfactual as a partial refund (described in the next subsection) with primary market prices for game j satisfying

$$p_{jq}(V, \omega^{BL}) = p_{jq} + \alpha_j V. \tag{2.17}$$

The counterfactual is useful because it measures the reduction in profit from fixed prices.

2.7.2 Partial Refunds

To implement refunds in the model, I close down the resale market and have consumers with idiosyncratic shocks return their tickets to the seller. The exact level of the refund cannot be determined—all refunds are equally profitable as long as consumers request refunds if and only if they receive idiosyncratic shocks.[20] As before, the seller chooses prices p_{Bq} and p_{jq}. I only evaluate partial refunds for the baseline model with state ω^{BL}.

2.7.3 Menu of Refunds

The final sales strategy is only considered for the application with two states of the world, ω^{Vax} and ω^{NoVax}. The seller offers three types of state-dependent season ticket contracts: a non-refundable package sold at $\{p_{Bq}^{NR}\}$ granting consumers tickets in both realized states ω^{Vax} and ω^{NoVax}, a fully refundable package sold at $\{p_{Bq}^{FR}(\omega^{\text{Vax}})\}$ granting consumers tickets in the vaccine state ω^{Vax}, and another fully refundable package sold at $\{p_{Bq}^{FR}(\omega^{\text{NoVax}})\}$ granting consumers tickets in the no vaccine state ω^{NoVax}. The seller continues to offer single-game tickets, which are sold at prices $\{p_{jq}\}$ in both states.

In the counterfactual, I remove uncertainty from idiosyncratic shocks and the common value, $\psi = 0$ and $\sigma_V^2 = 0$. The extra sources of uncertainty are not important for the returns to screening on uncertainty and removing them simplifies the results.[21] To

[20] The reason is the same as in the example in Section 2.2. Risk-neutral consumers will pay ψr more for tickets with refund r (as long as they only return them after receiving a shock). The seller can charge them ψr more, but must pay them r with probability ψ, leaving profit unchanged.

[21] With all forms of uncertainty, the seller would also need to choose between resale and refunds

implement the counterfactual, I use the estimated changes in willingness to pay from Section 2.6 to obtain consumer values with and without a vaccine. Using preferences in the vaccine state and the changes if there is no vaccine, consumers choose between the contracts.

I compare the performance of the menu of refunds to resale markets and not allowing reallocation. The case without reallocation provides a benchmark for the benefits of price discrimination, adding new empirical evidence related to Courty and Li (2000). The comparison to resale allows the allocation of tickets to vary across states, but incurs fees and frictions as before. The performance of resale is important because it is the option ticket sellers are most likely to select for the coming season.

State-dependent contracts differ from the strategy considered in Courty and Li (2000) by allowing the seller to offer a contract for the state with no vaccine. In Courty and Li (2000), consumers choose between contracts offering different refunds. With only two states, consumers would only request a refund in the state without a vaccine, making the menu equivalent to offering a contract for tickets with a vaccine and a contract for tickets in both states. State-dependent contracts also allow a contract for the state without a vaccine.

2.7.4 Counterfactual Estimates

I test the partial refund and benchmark couterfactuals against resale in the baseline model with one state of the world. The results, shown in Table 2.8, suggest that refunds are the most profitable strategy. Profit is 3.6% higher with refunds than with resale, and 5.5% higher than with no reallocation.

The two strategies produce similar levels of welfare. Refunds also maximize total

for consumers who receive tickets and idiosyncratic shocks. Focusing solely on uncertainty over states avoids the complication.

welfare, besting resale by 1.3%. Resale, however, produces consumer welfare that is 1.6% higher than refunds. Both strategies produce significant gains relative to no reallocation. The gains in total welfare are 4.8% for resale and 6.2% for refunds. For consumer welfare, it is 10.1% for resale and 8.3% for refunds. The predicted welfare gains are larger than the 2.9% gain estimated by Leslie and Sorensen (2014), but their paper included harms of resale that are not relevant in the present market.

The patterns in season ticket volumes are unsurprising. The seller chooses to sell more season ticket packages for strategies that reallocate more profitably, in this case refunds and flexible prices. By contrast, when it cannot reallocate or when there are fees and frictions associated with reallocation, it chooses to sell fewer packages.

As expected, the counterfactual with flexible prices is more profitable than all other strategies. However, the gains relative to refunds are relatively small: 1.5% for profit, 1.3% for total welfare, and 0.8% for consumer welfare. The results suggest that aggregate uncertainty does not significantly reduce the profitability or efficiency of reallocation with partial refunds. The seller's reliance on season ticket sales explain the small gains to flexible prices. By selling tickets before shocks are realized, the seller avoids relying on single-ticket sales when its prices may be suboptimal.

Results for the screening application are in Table 2.9. The comparison between state-based refunds and no reallocation establishes that screening results in significant gains for the seller. Profit increases by 10.3% and total welfare increases by 7.7%, but consumer welfare is unchanged. Gains are the product of the seller's ability to allocate tickets to different consumers in different states: thousands of tickets are sold in each type of contract.

Again, resale is a significant improvement over not reallocating, with gains in profit of 6.8% and total welfare of 4.7%. Consumer welfare falls slightly, by 1.7%. Resale, however, is worse in all respects than state-dependent refunds. The small size of resale fees implies that resale frictions drive the difference.

	Resale	Refunds	Flex. Prices	No Reall.
Profit (mn)	7.18	7.44	7.55	7.05
Consumer Welfare (mn)	2.50	2.46	2.48	2.27
Total Welfare (mn)	9.77	9.90	10.03	9.32
Resale Fees (mn)	0.09	0.00	0.00	0.00
Season Ticket Buyers (1000)	23.75	26.91	26.91	23.75
Season Ticket Base Price	33.45	30.80	30.90	32.48
Single Game Base Price	33.29	37.81	39.30	36.58

Table 2.8: Counterfactual results for the baseline model.

	No Reall.	Menu of Refunds	Resale
Profit (mn)	6.49	7.16	6.92
Consumer Welfare (mn)	2.35	2.35	2.31
Total Welfare (mn)	8.83	9.51	9.25
Resale Fees (mn)	0.00	0.00	0.02
Non-Refund. S. Tix (1000)	20.65	12.13	26.63
Vaccine S. Tix (1000)	0.00	6.25	0.00
No Vaccine S. Tix (1000)	0.00	12.64	0.00

Table 2.9: Counterfactual results for the model with different states of the world.

There are two drawbacks to resale: non-price frictions and explicit fees. To decompose their effects, I simulate the market with the fee τ set to zero (column 2) and the non-price friction s set to zero (column 3).[22]

The results are shown in Table 2.10. They suggest that the non-price friction does not drive the difference in profit between resale and partial refunds: removing resale frictions only explains 27% of the difference in profit between resale and partial refunds. Instead, the difference is primarily due to resale fees. Eliminating fees explains 73% of the gap in profit. However, frictions drive the difference in total welfare.

[22]Frictionless resale, where τ and s both equal zero, is identical to the flexible price counterfactual.

	Resale	$\tau = 0$	$\lambda_s = 0$
Profit (mn)	7.18	7.37	7.25
Consumer Welfare (mn)	2.50	2.46	2.58
Total Welfare (mn)	9.77	9.84	10.06
Resale Fees (mn)	0.09	0.00	0.22
Season Ticket Buyers (1000)	23.75	26.91	26.91
Season Ticket Base Price	33.45	32.86	33.01
Single Game Base Price	33.29	43.85	33.23

Table 2.10: Counterfactual results for resale frictions in the baseline model.

2.8 Conclusion

Demand uncertainty causes the initial allocation of goods to be suboptimal and can make it difficult to predict the optimal price in advance. Both sellers and society can benefit from sales strategies that cope with uncertainty, but it is unclear which strategy is best. I showed that the optimal strategy depends on the properties of demand uncertainty, then estimated a structural model describing the salient sources of uncertainty in the market for college football tickets and used it to evaluate each strategy.

The results suggest that refunds, rather than the status quo of resale, maximize profit and welfare. In the counterfactual without uncertainty from covid-19, profit is 3.6% higher with partial refunds than with resale, and total welfare is 1.3% higher. With uncertainty from covid-19, profit is 3.4% higher with a menu of refunds than with resale, and total welfare is 1.7% higher. However, resale is valuable when the seller offers no way to reallocate. Resale raises profit by 1.8% and consumer welfare by 10.1% in the counterfactual without uncertainty from covid-19; in the other counterfactual it raises profit by 6.6% and lowers consumer welfare by 1.7%.

The paper has three core implications for our understanding of resale and aftermarkets. First, the theory demonstrates that resale can be valuable in markets with primary market rigidities, aggregate uncertainty, and low resale frictions. The market for college football tickets has the potential to satisfy each requirement, but the results show that fees and frictions are significant enough in the resale market for refunds to be optimal. In similar markets without primary market rigidities, like airlines and hotels, refunds are a more natural choice.

Second, the comparison between sales strategies informs how to run aftermarkets. The results imply that refund-based strategies are superior in a perishable goods market with a monopolist seller. One benefit of refunds with a monopolist seller is to reduce search costs by leaving only one seller. The effect of refunds could be different in markets with many competing sellers.

Third, the paper provides empirical evidence on the effects of resale. Whether sellers of perishable goods profit from resale is ambiguous in theory, and this paper shows that sellers benefit in practice. The effect of resale on consumer welfare informs policy on ticket resale. When sellers prohibit resale but do not offer refunds, consumer welfare falls by more than 10%. Consumers would benefit from a legal right to resell tickets provided that the seller does not offer refunds instead.

Finally, the analysis suggests several avenues for future research. The counterfactual experiments in this study suggest that refunds and menus of refunds boost profit and welfare. A next step would be to investigate the strategies' real-world performance. The results also suggest that frictions affect the performance of resale markets. Further investigation would advance our understanding of resale.

Chapter 3

How to Reallocate: Resale or Refunds?

3.1 Introduction

You bought tickets a month ago for an upcoming NBA game, but just learned you cannot attend. So you list the tickets on StubHub, where someone else buys them and pays StubHub a fee for hosting the transaction. Economists tell stories like this one to illustrate the benefits of reallocation, focusing on the happy ending where the ticket is used. But the story raises questions about how to reallocate that economists have hardly considered.

For instance, why does the sports team allow tickets to be resold when it could offer a refund? Resale lets the resale market operator seize some of the benefits of reallocation through fees, but the primary market seller could keep all of the gains if it prohibited resale[1] and offered a refund instead. The decision to allow resale is puzzling because fees are high, with StubHub collecting over 20% of the total price, and because other sellers with similar problems offer refunds, like hotels and airlines.

And if the team did ban resale and offer refunds, would it harm consumers or efficiency? Refunds would give the team market power over the extent of reallocation, but resale markets also have an incentive to impede reallocation. After all, resale markets only earn profit through fees, which drive a wedge between the valuations of buyers and resellers. Which reallocation mechanism should society prefer?

The goal of this paper is to compare the performance of resale and refunds in

[1]Ticket sellers are able to prohibit resale in most states, for example by requiring ticket holders to present the credit card used to purchase the ticket to gain entry (Pender (2017)).

a perishable goods market. Using a two-period model in which consumers purchase in advance, receive preference shocks, and have a chance to reallocate, I determine the conditions required for resale to maximize profit and welfare. Resale can be more profitable despite resale fees when there is aggregate demand uncertainty and primary market prices are rigid. Either strategy can maximize welfare, with refunds more efficient when the extent of reallocation has little effect on the market-clearing price in the second period.

Understanding the relative performance of resale and refunds matters for three reasons. First, it advances our understanding of firm behavior. Current theories, such as the one advanced in Cui et al. (2014), do not explain why it is common for sellers of perishable goods to allow resale. This paper demonstrates that resale can be profitable when primary market prices are rigid and there is aggregate demand uncertainty, as in markets for sports tickets.[2] Second, it contributes to an ongoing policy debate over rights to resell. In response to resale bans for major concert tours, some U.S. states have passed laws guaranteeing a right to resell tickets (Pender (2017)). The welfare analysis in this paper shows that consumers may prefer refunds to resale. Third, this paper informs our design of aftermarkets, suggesting which strategy is best for social welfare—and identifying why welfare differs from the optimum.

Resale can be more profitable when aggregate demand is uncertain and primary market prices are rigid. The combination of aggregate uncertainty and price rigidity allows the seller's price to be suboptimal when demand is realized. If prices were optimal, the seller could reallocate efficiently through refunds, leaving no reason to split the benefits with the resale market operator. But if prices are suboptimal, reallocation through resale may be more efficient because resale prices are flexible. Whether resale is more profitable depends on the relative importance of aggregate

[2] Ticket prices are rigid because they are often printed on the ticket and not revised. Aggregate uncertainty comes from the performance of the team, which leads to large swings in resale prices.

shocks and the fees charged in the resale market. This is the first paper studying resale and refunds in which either strategy can be optimal.

For welfare, it would be intuitive for resale to be more efficient because it lessens the market power of the primary market seller. However, the welfare-maximizing strategy is ambiguous because a monopolist resale market must introduce resale fees to profit. The fees move the market away from the efficient allocation, creating a classic pricing tradeoff between the fee earned on each unit and the quantity resold. The primary market seller introduces inefficiencies for a different reason, to prop up the price. If too many tickets are refunded, supply grows and the seller must charge a lower price to late-arriving consumers. Anticipating lower prices later on, early arrivals are willing to pay less to buy in advance. The seller will limit reallocation, and hence supply in the second period, to maintain a higher price. Which strategy is most efficient depends on the relative strength of the incentives to introduce inefficiencies. For tractability, I compare the two incentives in a setting without aggregate uncertainty.

The analysis proceeds as follows. I start by reviewing the relevant literature. I present the model in Section 3.2 and move on to analyze a market with certain aggregate demand in Section 3.3. The analysis of certain demand is illustrative for welfare, but does not explain why sellers would allow resale. To do so, I analyze a market with uncertain aggregate demand in Section 3.4. I conclude in Section 3.5.

Related Literature. The closest paper is Cui et al. (2014), who use a similar model to analyze the performance of resale and compare it to refunds (equivalently, options). This paper differs because it emphasizes the effect of uncertain demand on profit and evaluates the relative efficiency of the strategies. In contrast, Cui et al. (2014) only study profit with certain demand, concluding that refunds are always more profitable. One other paper, Cachon and Feldman (2018), compares the performance of resale and refunds. They conclude that resale is always more profitable in a setting with two buyers. The current paper is the first to illustrate the forces that could make either resale or refunds more profitable.

Some of the theory developed in this paper is also used in its empirical counterpart, presented in Chapter 2. This Chapter differs by providing a formal treatment of the effect of strategies on profit and by introducing new theory on the welfare implications of each strategy.

The analysis is closely linked to the literature on resale of perishable goods. Courty (2003) considers a model with no capacity constraints and finds that resale is not profitable for a monopolist ticket seller. Karp and Perloff (2005) finds that a seller can benefit from resale when resellers are able to discriminate among potential buyers. The basis for the analysis in this paper, by contrast, is that a capacity-constrained monopolist can benefit from resale after demand shocks are realized.

There is also a significant body of empirical work on resale markets. Leslie and Sorensen (2014) investigate the effect of ticket resale on welfare, weighing the benefits of reallocation against the harms of rent-seeking when tickets are systematically underpriced. Because of underpricing in their setting, they do not consider the effect of resale on profit. Sweeting (2012) studies dynamic pricing within online resale markets. Chen et al. (2013a) consider the effect of resale on primary market sellers in the market for cars. Their setting differs because cars are durable goods, giving consumers preferences over vintages that enrich the problem.

A smaller literature has considered the benefits of refunds. Gallego and Şahin (2010) establish that refunds can be valuable in the context of revenue management. Xie and Gerstner (2007) show that refunds can be profitable in a simpler setting.

3.2 Model

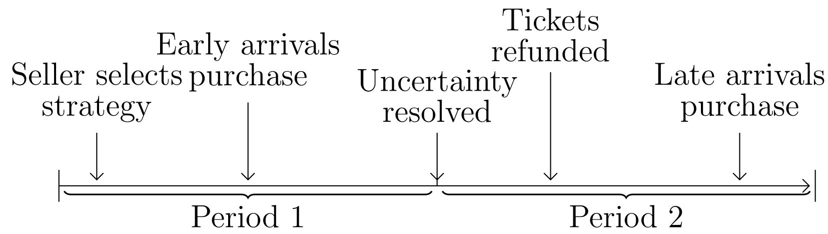


Figure 3.1: Model timeline.

A monopolist seller with fixed capacity K and marginal costs normalized to zero sells a product over two periods. A measure a_1 of buyers arrives in the first period and an additional measure a_2 arrives in the second, where $a_1 + a_2 > K$. Consumers who arrive in the first period can wait to purchase until the second period at no cost.

Consumer i's final value for the product is $V + s_i$, where S and V are random variables whose realizations are learned at the start of the second period. The consumer-specific component of values s_i satisfies $\mathrm{E}(S) = 0$, has CDF $G(s)$, and has density $g(s)$ satisfying $g(s) > 0$ at all points in its support. Because of free disposal, I assume that values $V + s_i$ always exceed zero. The random variable V is degenerate in Section 3.3 and has two possible realizations in Section 3.4.

The monopolist sets a menu of prices $\{p_1, p_2\}$ to maximize profit. The price p_1 is charged in the first period and p_2 in the second. The monopolist is unable to revise its prices after they have been set.

The assumption that primary market prices are rigid does not affect the results of Section 3.3, but it is essential to the advantages of resale in Section 3.4. The assumption holds for industries like event tickets, where prices are printed on the tickets and rarely changed, and is approximately true when sellers have too short of a sales horizon to adjust the price, as for some seasonal goods, or adjusting prices is costly.

Consumers either have the right to resell or to return their tickets for a refund.

Resale. With a resale market, consumers have the right to resell to each other in the second period. The resale market clears at a single price p_2^r, which depends on the seller's menu and the realization of V. The resale market is operated by a third party that charges a fee τ on each transaction: buyers pay p_2^r, but resellers only receive $p_2^r - \tau$. The fee τ is initially taken as exogenous, but I also consider the case where a monopolist in the resale market sets its profit-maximizing fee.

Refunds. When offering refunds, the seller chooses a refund r in addition to its menu of prices (p_1, p_2). Consumers can return the good for a payment of r at the start of the second period and the seller puts the recovered units back on sale at price p_2.

3.3 A Certain Value

Suppose that the common component of values is known to be V.

Assumption 1 Efficient Rationing. Whenever tickets are rationed, they are allocated to the consumers with the highest values $V + s_i$ who do not already have the good.

By ensuring that the highest-value consumers receive the good, Assumption 1 removes the seller's incentive to create shortages and simplifies the analysis.[3]

With efficient rationing, the seller's problem when offering refund contracts is to select a profit-maximizing refund contract and price in period two, $(p_1^{RC}, p_2^{RC}, r^{RC})$. Because the a_1 consumers arriving in the first period can wait to purchase until the second, the price p_1 equals expected surplus from buying in the second period.[4]

[3]If the seller wanted to create a shortage, then its optimal refund would be lower, making total and consumer welfare higher with resale.

[4]The seller could also choose a higher p_1 and make no advance sales. The choice to offer advance sales is optimal when consumer values depend on the ability to plan ahead, as for event tickets. I provide additional conditions under which advance sales are optimal in the appendix.

Expected surplus for early arrivals is the sum of the value of using the good (when higher than the refund), the value of the refund when the consumer's value is low, and expected surplus from waiting to purchase at p_2^{RC} in period two. The seller must pay the refund r to all consumers who request it. The contribution of refunds to willingness to pay in the first period and losses from refunds paid out in the second exactly offset, simplifying the problem to

$$\begin{aligned}\max_{p_2,r}\ & a_1\Big(\int_{r-v}^{\infty} v+s\,dG(s)-\int_{p_2-v}^{\infty} v+s- \\ & p_2\,dG(s)\Big)+p_2\Big(\min\{K-a_1(1-G(r-v)),a_2(1-G(p_2-v))\}\Big).\end{aligned} \tag{3.1}$$

With resale, the seller takes the fee τ as given and sets its profit-maximizing prices (p_1^{SM}, p_2^{SM}). The price in the second period always equals the resale price when p_s^{SM} is optimal: if the resale price were lower, the seller would not exhaust its inventory,[5] and if it were higher, the seller could raise its price and continue to sell all units. Consequently, consumers buying in the first period can resell for $p_2^{SM}-\tau$. Willingness to pay in the first period includes the ability to resell in the second and expected surplus from waiting to purchase. The seller's problem simplifies to

$$\begin{aligned}\max_{p_2}\ & a_1\Big(\int_{p_2-\tau-v}^{\infty} v+s\,dG(s)+\int_{-\infty}^{p_2-\tau-v} p_2-\tau\,dG(s)-\int_{p_2-v}^{\infty} v+s-p_2\,dG(s)\Big)+ \\ & p_2\Big(\min\{K-a_1,a_2(1-G(p_2-v))-a_1G(p_2-\tau)\}\Big).\end{aligned} \tag{3.2}$$

The key difference is that, with resale, the seller benefits from reallocation by charging consumers for their expected after-fee resale revenue in the first period,

[5] Selling all inventory is optimal by Lemma 7, which can be found in the appendix.

but with refunds, it benefits by selling the refunded goods in the second period. One drawback of resale for the seller is that some of the benefits of reallocation are captured by the resale market operator as fees. With refunds, the seller can capture the benefits of reallocation for itself.

I make a standard hazard rate assumption on the distribution function $G(s)$ to ensure that it is invertible and that both problems have montone first order conditions in the price p_2.

Assumption 2. $G(s)$ has a weakly decreasing hazard rate $\frac{1-G(s)}{g(s)}$.

Under Assumptions 2 and 3,[6] the seller's optimal price p_2 exhausts inventory for any given refund r or resale fee τ, leading to solutions $p_2(\tau)$ and $p_2(r)$. Both functions are monotone, with $p_2(\tau)$ increasing in τ and $p_2(r)$ decreasing in r. I prove in the appendix that $p_2(\tau)$ can be rewritten as $V + s_H(\tau)$, where $s_H(\tau)$ is a critical consumer type.

The difference between the price in period two and the amount received for returning the good, τ with resale or $p_2^{RC} - r^{RC}$ with refunds, defines the wedge between the highest-value arrival in period two who does not acquire the good and the lowest-value arrival in period one who keeps it. The wedge captures distortion from the optimal allocation, which facilitates the comparison of the two strategies.

Definition 1. The *distortion* δ is $\delta \equiv \tau$ for resale and $\delta \equiv p_2^{*RC} - r^{*RC}$ for refunds.

Lemma 1. Total and consumer welfare only depend on the distortion δ.

Proof. Let $\tau = \delta$ and r satisfy $p_2(r) - r = \delta$. The prices $p_2(\tau)$ and $p_2(r)$ are the same because the seller wishes to sell all of its inventory. For resale, $p_2(\tau)$ satisfies $K - a_1(1 - G(p_2(\tau) - \tau - v)) = a_2(1 - G(p_2(\tau) - v))$. For refunds, $p_2(r)$ satisfies

[6] Assumption 3 and its implications are discussed in the appendix.

$K - a_1(1 - G(r - v)) = a_2(1 - G(p_2(r) - v))$. The conditions are identical when $p_2 - r = \delta$.

Total and consumer welfare can be written to only depend on p_2 and δ and are therefore the same for a common δ,

$$\begin{aligned} TW = \; & a_1 \int_{p_2 - \delta - v}^{\infty} v + s \, dG(s) + a_2 \int_{p_2 - v}^{\infty} v + s \, dG(s) \\ CW = \; & (a_1 + a_2) \int_{p_2 - v}^{\infty} v + s \, dG(s). \end{aligned}$$

□

Proposition 1. Profit is strictly higher with refunds than with resale.

Proof. Suppose $\tau > 0$. The seller can set $p_2 = p_2^{SM}$ and $r = p_2^{SM} - \tau$. Total and consumer welfare are the same by Lemma 1, but profit is strictly higher with refunds because the $a_1 G(p_2^{SM} - \tau)\tau$ in fees paid to the resale market operator are now earned by the seller.

If $\tau = 0$, profit is strictly higher because the seller's optimal prices involve $r^{RC} < p_2^{RC}$. I show that $r = p_2$ is not optimal. The optimal prices with $r = p_2$ satisfy $(a_1 + a_2)(1 - G(v - r)) = K$. The first order condition evaluated at that refund are

$$\begin{aligned} \frac{\partial \pi^{RC}}{\partial r} &= a_1 \left(-r g(r - v) + \int_{p_2(r) - v}^{\infty} \frac{\partial p_2(r)}{\partial r} \, dG(s) + 0 \right) + \\ &\quad \frac{\partial p_2(r)}{\partial r} (K - a_1(1 - G(r - v))) + p_2(r) a_1 g(r - v) \\ &= a_1 \int_{r^0 - v}^{\infty} \frac{\partial p_2(r)}{\partial r} \, dG(s) + \frac{\partial p_2(r)}{\partial r} (K - a_1(1 - G(r - v))) < 0, \end{aligned}$$

where $\frac{\partial p_2(r)}{\partial r} < 0$ because a firm receiving more refunded tickets must lower its price to sell more units in the second period.

□

Proposition 1 establishes that refunds are strictly more profitable for the seller than refunds when the average value of the good is known. The result is not surprising: with resale, the resale market operator captures some of the benefits of reallocation, but with refunds, the seller seizes all of the gains for itself. A similar result is a core contribution of Cui et al. (2014), but it does not explain why so many sellers continue to allow resale. In Section 3.4, I show that resale can be more profitable when V is uncertain.

For welfare, the optimal strategy depends on the level of the fee τ.

Proposition 2. There exists a critical fee $\bar{\tau} = p_2^* - r^*$ such that total and consumer welfare are strictly higher with resale when $\tau < \bar{\tau}$, strictly lower when $\tau > \bar{\tau}$, and equal when $\tau = \bar{\tau}$.

Proof. Each welfare measure is maximized at $\delta = 0$ and is monotonically decreasing in the distortion in Lemma 1. □

Efficiency depends entirely on the distortion in the market, and the seller has an optimal degree of distortion with refunds. Proposition 2 formalizes that resale is more efficient when its distortion τ is lower than the optimal distortion with refunds, but that resale becomes less efficient as τ increases.

The result treats the resale fee τ as an exogenous parameter. Taking τ as exogenous is reasonable if resale fees are constant across products, like how StubHub charges the same fee structure for all events, or if it captures a cost like shipping charges that the resale market operator does not control. But if the resale market operator chooses a profit-maximizing τ, then the welfare comparison requires deeper analysis. I consider the case of a monopolist resale market operator later in this section.

3.3.1 Running the Resale Market

The discussion of Proposition 1 implies that there are two meaningful differences between resale and refunds for the seller when V is certain: with refunds, the seller chooses its optimal distortion and does not pay distortion to the resale market operator. Both differences make refunds more profitable with a certain value. However, the advantages would disappear if the seller were also the resale market operator. Proposition 3 clarifies that a resale market operated by the seller is equivalent to a refund when V is certain.

Proposition 3. Profit and welfare are the same with refunds and a resale market operated by the primary seller.

Proof. The profit function when the primary market seller operates the resale market is given by replacing the $p_2 - \tau$ integrand with p_2 in the first piece of equation (3.2). Profit for a given distortion is the same under both strategies, so the seller sets the same distortion, earns the same profit, and generates the same welfare by Lemma 1. □

3.3.2 Optimal Resale Fees

The welfare conclusions in Proposition 2 treat the fee charged in the resale market as an exogenous parameter. But what if the resale market operator chose the fee to maximize profit? In this subsection, I evaluate welfare under refunds and resale with a monopolist resale market operator.[7]

[7] There is some degree of competition in online resale markets, but the difference in incentives holds even with competition among resale platforms.

The resale market operator chooses the optimal fee τ to solve

$$\max_{\tau} \tau a_1 G(p_2(\tau) - \tau - v), \tag{3.3}$$

where $p_2(\tau)$ is the optimal price in period two given the fee. Rewrite the problem so that the resale market chooses the optimal refund $r = p_2(\tau) - \tau$, giving the problem and first-order condition

$$\max_{r} \; a_1(p_2(r) - r)G(r - v) \tag{3.4}$$

$$a_1 \left((\frac{\partial p_2(r)}{\partial r} - 1)G(r - v) + (p_2(r) - r)g(r - v) \right) = 0. \tag{3.5}$$

The reseller thus faces the classic pricing tradeoff between inframarginal losses and marginal gains as it lowers the fee (raises the refund).

If the seller offers a refund, its problem is the same as in equation (3.1). The first-order condition reduces to

$$= \; a_1(p_2(r) - r)g(r - v) + \frac{\partial p_2(r)}{\partial r}(K + a_1 G(r - v) - a_1 G(p_2(r) - v)) = 0. \tag{3.6}$$

The seller's first-order condition contains the same marginal benefit, the gain in surplus when a consumer with a higher valuation receives the good. But the inframarginal loss when r increases is different. For the seller, it comes from the lower price $p_2(r)$ paid by all consumers who hold the good at the end of the market. The seller's motive for distortion is to limit supply and increase the price in the second period, earning more from all buyers through the effect of $p_2(r)$ on the price paid in the first period. Proposition 4 shows that either distortion can be larger.

Proposition 4. Assume that the two first-order conditions are monotone in r and let r^* denote the optimal refund for the resale market operator. Total and consumer welfare are higher with resale if and only if

$$a_1 G(r^* - v) + \frac{\partial p_2(r)}{\partial r}(r^*)(K - a_1 G(p_2(r^*) - v)) > 0. \tag{3.7}$$

Proof. The difference in the first-order conditions is

$$\begin{aligned}
&\Big(a_1(p_2(r) - r)g(r - v) + \frac{\partial p_2(r)}{\partial r}(K + a_1 G(r - v) - a_1 G(p_2(r) - v))\Big) - \\
&a_1 \left((\frac{\partial p_2(r)}{\partial r} - 1)G(r - v) + (p_2(r) - r)g(r - v) \right) \\
&= \frac{\partial p_2(r)}{\partial r}(K - a_1 G(p_2(r) - v)) + a_1 G(r - v).
\end{aligned}$$

If the difference is positive at r^*, then the derivative is positive for the seller, making its optimal refund higher. □

It is thus unclear which strategy is best for consumers and society. The reason for the ambiguity is that the two agents have different incentives to depart from the socially optimal allocation. The resale market operator's distortion is due to the desire to earn a high margin on the units creating the most surplus. In contrast, the seller's distortion is due to the desire to extract surplus from consumers who arrive early by raising the price charged in the second period.

Equation (3.7) does not lead to obvious comparative statics, but a key determinant of the relative distortion is the sensitivity of the price in period two to the number of units reallocated. When reallocation significantly lowers the optimal price in period two, the seller will limit reallocation more than the resale market operator. But if the price does not respond to the volume of reallocation, the seller is willing to permit more reallocation than the resale market operator. The sensitivity is linked to the number of arrivals in period two, a_2, relative to capacity and arrivals in period one. When a_2 is large, the period two price hardly moves in response to a small increase in the number of goods to be reallocated. But when it is small, the price moves significantly.

Because refunds are equivalent to a resale market run by the seller, the result implies that the efficiency of resale markets depends on who is running them.

Corollary 1. The welfare effects of giving the seller control of the resale market are ambiguous.

Proof. If the seller ran the resale market, it would choose $\tau = p_2^{RC} - r^{RC}$. By Proposition 4, this may be larger or smaller than the resale market operator's optimal fee. □

Sellers sometimes operate their own resale market, as when sports teams display resale listings alongside available tickets in the primary market. Corollary 1 establishes that integrated primary and resale markets have different welfare implications than separate resale markets, even if the resale market operator is a monopolist.

Example. Figure 3.2 shows the optimal distortion with resale and refunds when S is uniformly distributed on $[-.5, .5]$, $V = 1$, $K = 1$, and $a_1 = .95$. When a_2 is small, the elasticity $\frac{\partial p_2(r)}{\partial r}$ is large, causing the seller to choose a low refund (high refund) in order to keep the price high and extract surplus from primary market buyers. But as a_2 increases, the sensitivity falls and the seller chooses a relatively small distortion with refunds. By contrast, the resale market operator's optimal fee does not vary significantly with a_2. It is initially lower than the distortion chosen with refunds, but as a_2 grows, it becomes significantly larger.

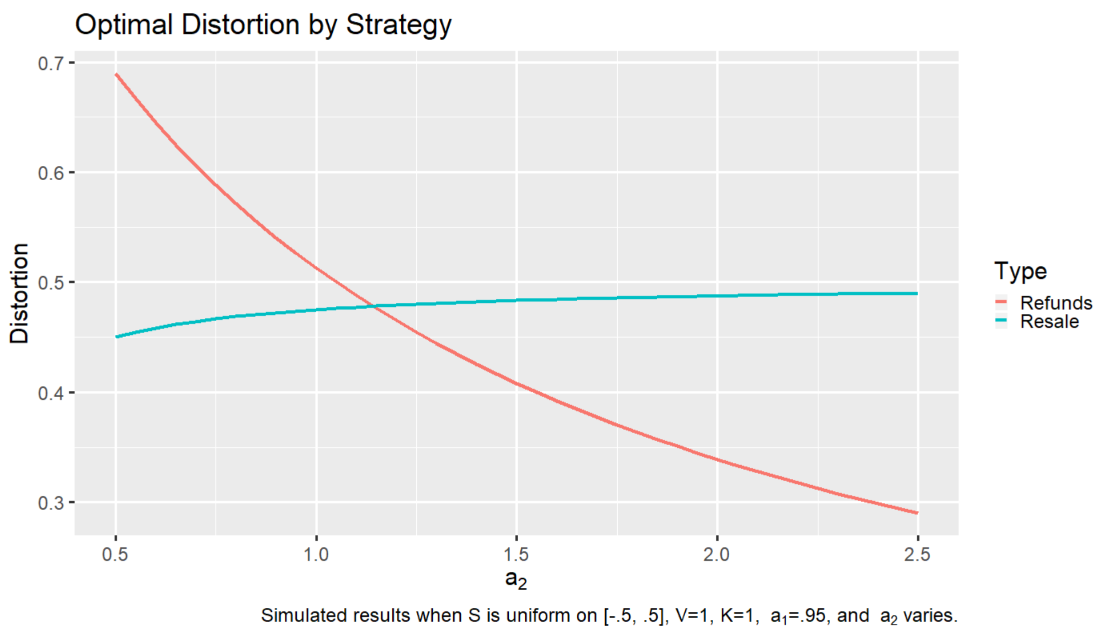


Figure 3.2: Optimal distortions for each strategy as a_2 varies.

3.4 Uncertain Values

The strong conclusions for profit in Propositions 1 and 3 rely on the seller's ability to predict the optimal price in the second period. There are cases when the seller is able to do so. For instance, the value of an established act's concert might be stable and predictable, or a luxury seller of seasonal fashion might have strong demand for all of its styles. In many other cases, however, sellers cannot predict the value of the good. Values for sports tickets vary significantly with the team's performance, and every season has surprise contenders and disappointments. Most sellers of seasonal fashions do not know which styles will be hits and which will be flops.

The difference is important to the choice between resale and refunds. When demand is uncertain and the primary market seller has rigid prices (or, more generally, that it is costly to adjust prices), resale markets have an advantage—price flexibility—that cannot be replicated with refunds. In this section, I study how demand uncertainty affects the seller's optimal strategy and makes it possible for resale to be optimal.

Modify the model so that the common value V takes the value V_L with probability α and the value $V_H > V_L$ otherwise. The seller's profit function with refunds is

$$
\begin{aligned}
\pi^{RC}(p_2, r) = a_1 \Big(& \alpha \{ \int_{r-V_L}^{\infty} V_L + s\, dG(s) - \int_{p_2 - V_L}^{\infty} V_L + s\, dG(s) \} + \\
& (1-\alpha) \{ \int_{r-V_H}^{\infty} V_H + s\, dG(s) - \int_{s^{*RC}(p_2)}^{\infty} V_H + s - p_2\, dG(s) \} \Big) + \\
& \alpha \min\{a_2(1 - G(p_2 - V_L)), K - a_1 + a_1 G(r - V_L)\} + \\
& (1-\alpha) \min\{a_2(1 - G(p_2 - V_H)), K - a_1 + a_1 G(r - V_H)\},
\end{aligned}
\tag{3.8}
$$

where the constant $s^{*RC}(p_2)$ reflects efficient rationing when all consumers with $V_H + s \geq p_2$ are not able to purchase. It satisfies

$$K - a_1 + a_1 G(r - V_H) = a_2(1 - G(s^{*RC})). \tag{3.9}$$

Similarly, profit with resale is

$$
\begin{aligned}
\pi^{SM}(p_2) = a_1\Big(\alpha\{ & \int_{p_2^r(V_L)-\tau-V_L}^{\infty} V_L + s\, dG(s) + \int_{-\infty}^{p_2^r(V_L)-\tau-V_L} p_2^r(V_L) - \tau\, dG(s) - \\
& \int_{p_2^r(V_L)-V_L}^{\infty} V_L + s - p_2^r(V_L)\, dG(s)\} + \\
& (1-\alpha)\{ \int_{p_2^r(V_H)-\tau-V_H}^{\infty} V_H + s\, dG(s) + \int_{-\infty}^{p_2^r(V_H)-\tau-V_H} p_2^r(V_H) - \tau\, dG(s) - \\
& \int_{s^{*SM}(p_2)}^{\infty} V_H + s - p_2^r(V_H)\, dG(s) - \\
& \int_{s_H}^{s^{*SM}} V_H + s - p_2^r(V_H)\, dG(s)\} + \\
& \alpha p_2 \min\{a_2(1 - G(p_2 - V_L)) - a_1 G(p_2 - V_L - \tau), K - a_1\} + \\
& (1-\alpha) p_2 \min\{a_2(1 - G(p_2 - V_H)) - a_1 G(p_2 - V_H - \tau), K - a_1\},
\end{aligned}
\tag{3.10}
$$

where $s^{*SM}(p_2)$ has an analogous role and $p_2^r(V)$ is the resale price at value V. The constant $s^{*SM}(p_2)$ satisfies

$$
K - a_1 = a_2(1 - G(s^{*SM})). \tag{3.11}
$$

The profit functions reveal a key distinction between resale and refunds that is only apparent with uncertainty over the value V. With refunds, all sales in the second period take place at the seller's price p_2, even if p_2 is not optimal for realized demand. But with resale, some sales will take place at the resale price $p_2^r(V)$, which clears the resale market for realized demand. Resale can be profitable when the seller benefits from price flexibility, but when demand is relatively certain, refunds remain more profitable.

Lemma 2. Suppose that the realization of V is known. $p_2^{RC} - r^{RC}$ does not depend on V.

Proof. Recall that the first-order condition of profit with respect to r is

$$\frac{\partial \pi^{RC}}{\partial r} = a_1\big(- rg(r-v) + \frac{\partial p_2(r)}{\partial r}(1 - G(p_2(r) - v))\big) + (K - a_1 + a_1 G(r-v)) + a_1 p_2(r) g(r-v).$$

Let p_2 and r be optimal at v, so they solve the first order condition and satisfy

$$K - a_1 + a_1 G(r - V) = a_2(1 - G(p_2(r) - V)).$$

Suppose the value changes to v' and consider candidate price $p_2 + v' - v$ and refund $r + v' - v$. From equation (3.4), the candidate menu also exhausts inventory at v' has the same derivative of p_2 with respect to r. The first order condition at the candidate prices is

$$\begin{aligned}
&= a_1\big(- (r + v' - v) g(r + v' - v - v') + \frac{\partial p_2(r)}{\partial r}(1 - G(p_2 + v' - v - v'))\big) + \\
&\quad (K - a_1 + a_1 G(r + v' - v - v')) + a_1 p_2 g(r + v' - v - v') \\
&= a_1\big(- (r + v' - v) g(r - v) + \frac{\partial p_2(r)}{\partial r}(1 - G(p_2 - v))\big) + \\
&\quad (K - a_1 + a_1 G(r - v)) + a_1 (p_2 + v' - v) g(r - v) = 0.
\end{aligned}$$

□

Corollary 2. When V is certain, the difference in refund and resale profit does not vary with V.

Proof. Because $p_2^{RC} - r^{RC}$ does not depend on V by Lemma 2, the seller's optimal menu issues refunds to consumers with $s \leq \bar{s}_1$ and sells in period two to those with

$s \geq \bar{s}_2$ for some $\bar{s}_1 < \bar{s}_2$. Hence $r = V + \bar{s}_1$ and $p_2 = V + \bar{s}_2$, giving profit

$$\begin{aligned}\pi^{RC} &= a_1\left(\int_{\bar{s}_1}^{\infty} V + s\,dG(s) - \int_{\bar{s}_2}^{\infty} s - \bar{s}_2\right) + (V + \bar{s}_2)(K - a_1 + a_1 G(\bar{s}_1)) \\ &= KV + a_1\left(\int_{\bar{s}_1}^{\bar{s}_2} s\,dG(s) + \bar{s}_2(1 - G(\bar{s}_2))\right) + \bar{s}_2(K - a_1 + a_1 G(\bar{s}_1)).\end{aligned}$$

If the seller sets $p_2 = V + s_H$ with resale, profit is

$$\begin{aligned}\pi^{SM} &= a_1\Big(\int_{s_H-\tau}^{\infty} V + s\,dG(s) + \int_{-\infty}^{s_H-\tau} V + s_H - \tau\,dG(s) - \\ &\quad \int_{s_H}^{\infty} V + s - V - s_H\,dG(s)\Big) + (K - a_1)(V + s_H) \\ &= KV + \left(s_H(1 - G(s_H)) + \int_{s_H-\tau}^{s_H} s\,dG(s) + (s_H - \tau)G(s_H - \tau)\right) + \\ &\quad (K - a_1)s_H.\end{aligned}$$

□

Proposition 5. Refunds are more profitable than resale if

1. $V_H - V_L \leq \underline{\Delta}$ for some $\underline{\Delta} > 0$, or
2. $\alpha \leq \underline{\alpha}$ for some $\underline{\alpha} > 0$, or
3. $\alpha \geq \bar{\alpha}$ for some $\bar{\alpha} < 1$.

Proof. 1. By Corollary 2, there is a constant difference $d \equiv \pi^{RC} - \pi^{SM}$ when $\alpha \in \{0, 1\}$. Suppose that the seller is able to set a different price in each state of the world with resale, earning $\pi^{SM,L}$ when V_L is realized and $\pi^{SM,H}$ otherwise, but sets its optimal menu for V_H with refunds. The difference in profit is

$$\pi^{RC} - \pi^{SM} = \alpha\Big(a_1\big[\int_{r^{RC}-V_L}^{\infty} V_L + s\,dG(s) - \int_{p_2^{RC}}^{\infty} V_L + s - p_2^{RC}\,dG(s)\big] +$$
$$p_2^{RC}(K - a_1 + a_1 G(r^{RC} - V_L)) - \pi^{SM,L}\Big) + (1-\alpha)d$$
$$\geq \Big(a_1\big[\int_{r^{RC}-V_L}^{\infty} V_L + s\,dG(s) - \int_{p_2^{RC}}^{\infty} V_L + s - p_2^{RC}\,dG(s)\big] +$$
$$p_2^{RC}(K - a_1 + a_1 G(r^{RC} - V_L)) - K(V_L + s_H)\Big).$$

When $V_L = V_H$, the difference is $d > 0$. By continuity, there exists $\underline{\Delta}$ such that for $V_L > V_H - \underline{\Delta}$ the expression is positive.

2. Consider the strategy above. For any V_L and V_H, there exists an α small enough that refunds are more profitable.

3. Suppose now that the seller still earns $\alpha\pi^{SM,L} + (1-\alpha)\pi^{SM,H}$ with resale, but suppose it sets its optimal menu for V_L with refunds. The argument proceeds as before.

□

Each of the three conditions in Proposition 5 imply that aggregate uncertainty is small relative to the idiosyncratic uncertainty from S: either the magnitude of the effect is small because V_H and V_L are close, or one realization is highly probable because α is close enough to zero or one. In line with the results in Section 3.3, refunds are more profitable whenever aggregate uncertainty is relatively unimportant.

Lemma 3. For a random variable X with density $f(x)$ and finite expectation, $\lim_{x\to\infty} x(1 - F(x+d)) = 0$ for any constant d.

Proof. Because X has a finite expectation,

$$\mathrm{E}(X) = \int_{-\infty}^{\infty} x f(x)\, dx < \infty.$$

Because

$$\lim_{u\to\infty} \mathrm{E}(X) - \int_{-\infty}^{u} x f(x)\, dx = \lim_{u\to\infty} \int_{u}^{\infty} x f(x)\, dx = 0$$

and $\int_u^\infty x f(x)\, dx \geq u \int_u^\infty f(x)\, dx = u(1 - F(u))$, we have $\lim_{x\to\infty} x(1 - F(x)) = 0$. Then

$$\lim_{x\to\infty} x(1 - F(x + d)) + d(1 - F(x + d)) = \lim_{x\to\infty} (x + d)(1 - F(x + d)) = 0$$

and the fact that $\lim_{x\to\infty} d(1 - F(x + d)) = 0$ delivers the result. □

For the statement of the next proposition, define $\pi^{RC,V}$ to be the profit from a refund contract when the value V is certain. Also define $h(v) = \alpha(V + s_L) + (1 - \alpha)(K(V + s_H) - \pi^{RC})$.

Proposition 6. Let $\alpha \in (0, 1)$ and define

$$\underline{V}_L = \inf\{V : h(v) > 0, \alpha(a_1 \int_{-V_L}^{\infty} V_L + s\, dG(s) - a_1 V_L < h(v)\}.$$

For any $V_L > \max\{\underline{V}, \underline{V}_L\}$, there exists $\bar{V}_H(V_L)$ such that resale is more profitable than refunds when $V_H \geq \bar{V}_H(V_L)$ and $\tau = 0$. For each $V_L > \underline{V}_L$ and $V_H \geq \bar{V}_H(V_L)$, there exists $\bar{\tau}(V_L, V_H) > 0$ such that resale is more profitable when $\tau < \bar{\tau}$.

Proof. The first step is to establish an upper bound for profit with refunds. Let $d > 0$ satisfy $V_H - V_L > 2d$. Define p_2^H and p_2^L to be the monopolist's optimal prices with refunds when α is zero or one, and define $\pi^{RC,H}$ and $\pi^{RC,L}$ to be the analogous profits. By Lemma 2, the prices increase linearly in values V_H and V_L. Let $p_2^{H,m} = V_H + G^{-1}(\frac{K-a_1}{a_2})$, the monopolist's optimal price at value V_H when it does not allow reallocation.

Recall that $r \leq p_2$ without loss of generality (if not, some consumers return their units for a refund and purchase again at the lower p_2). When $p_2 \leq p_2^L + d$,

$$\begin{aligned}
\pi^{RC}(p_2, r) &\leq \alpha\pi^{RC,L} + (1-\alpha)\Big(a_1\{\int_{r-V_H}^{\infty} V_H + s\,dG(s) - \int_{s^{*RC}}^{\infty} V_H + s - p_2\,dG(s)\} + \\
&\quad (K - a_1 + a_1 G(r - V_H))p_2\Big) \\
&\leq \alpha\pi^{RC,L} + (1-\alpha)\Big(a_1 \int_{-V_H}^{\infty} V_H + s\,dG(s) + \\
&\quad (K - a_1 + a_1 G(p_2^L + d - V_H))(p_2^L + d)\Big) \equiv \pi_1.
\end{aligned}$$

When $p_2 \geq p_2^H - d$ and $r > p_2^H - d$,

$$\begin{aligned}
\pi^{RC}(p_2, r) &\leq \alpha\Big(a_1\{\int_{r-V_L}^{\infty} V_L + s\,dG(s) - \int_{p_2-V_L}^{\infty} V_L + s - p_2\,dG(s)\} + \\
&\quad a_2(1 - G(p_2 - V_L))p_2\Big) + (1-\alpha)\pi^{RC,H} \\
&\leq \alpha\Big(a_1 \int_{p_2^H-d-V_L}^{\infty} V_L + s\,dG(s) + a_2(1 - G(p_2^H - d - V_L))(p_2^H - d)\Big) + \\
&\quad (1-\alpha)\pi^{RC,H} \equiv \pi_2.
\end{aligned}$$

When $p_2 \geq p_2^H - d$ and $r \leq p_2^H - d$,

$$\pi^{RC}(p_2, r) \leq \alpha\Big(a_1 \int_{-V_L}^{\infty} V_L + s\,dG(s) + a_2 \max_{p_2 \geq p_2^H - d}(1 - G(p_2 - V_L))p_2\Big) +$$

$$(1-\alpha)\Big(a_1\int_{p_2^H-d-V_H}^{\infty} V_H+s\,dG(s)+a_1G(p_2^H-d-V_H)p_2^{H,m}+$$

$$(K-a_1)p_2^{H,m}\Big)$$

$$\leq \alpha\Big(a_1\int_{-V_L}^{\infty} V_L+s\,dG(s)+a_2\max_{p_2\geq p_2^H-d}(1-G(p_2-V_L))p_2\Big)+$$

$$(1-\alpha)\Big(a_1V_H+a_1\int_{-\infty}^{p_2^H-d-V_H} p_2^{H,m}-V_H-s\,dG(s)+$$

$$(K-a_1)p_2^{H,m}\Big)\equiv\pi_3.$$

Finally, for any price in $[p_2^L+d, p_2^H-d]$ and any refund, profit is

$$\pi^{RC}(p_2,r)\leq\leq\alpha\Big(a_1\int_{-V_L}^{\infty} V_L+s\,dG(s)+\max_{p_2^L+d\leq p_2\leq p_2^H-d} a_2(1-G(p_2-V_L))p_2\Big)+$$

$$(1-\alpha)\Big(a_1V_H+a_1\int_{-\infty}^{r-V_H} p_2^H-d-V_H-s\,dG(s)+$$

$$(K-a_1)(p_2^H-d)\Big)$$

$$\leq\alpha\Big(a_1\int_{-V_L}^{\infty} V_L+s\,dG(s)+\max_{p_2^L+d\leq p_2\leq p_2^H-d} a_2(1-G(p_2-V_L))p_2\Big)+$$

$$(1-\alpha)\Big(a_1V_H+a_1\int_{-\infty}^{p_2^H-d-V_H} p_2^H-d-V_H-s\,dG(s)+$$

$$(K-a_1)(p_2^H-d)\Big)\equiv\pi_4.$$

Next I show that profit with resale when $\tau=0$ can exceed $\max\{\pi_1,\pi_2,\pi_3,\pi_4\}$. With resale, the monopolist sets $p_2=V_H+s_H$ and earns

$$\pi^{SM}\geq\pi^{SM}(V_H+s_H)=\alpha a_1(V_L+s_L)+(1-\alpha)K(V_H+s_H).$$

The difference between resale profit and each candidate upper bound is

$$
\begin{aligned}
\pi^{SM} - \pi_1 \geq\ & \alpha a_1(V_L + s_L) + (1-\alpha)K(V_H + s_H) - \alpha\pi^{RC,L} - \\
& (1-\alpha)\Big(a_1 \int_{-V_H}^{\infty} V_H + s\, dG(s) + (K - a_1 + a_1 G(p_2^L + d - V_H))(p_2^L + d)\Big) \\
=\ & \underbrace{(1-\alpha)\Big[(K - a_1)V_H + Ks_H + (K - a_1)(V_H - p_2^L - d)\Big]}_{\psi_{1.1}} - \\
& \underbrace{\alpha(\pi^{RC,L} - a_1(V_L + s_L))}_{\psi_{1.2}} - \\
& (1-\alpha)\Big(a_1 \int_{-V_H}^{\infty} V_H + s\, dG(s) - a_1 V_H + a_1 G(p_2^L + d - V_H)(p_2^L + d)\Big), \\
\pi^{SM} - \pi_2 \geq\ & \underbrace{\alpha(V_L + s_L) + (1-\alpha)(K(V_H + s_H) - \pi^{RC,H})}_{\psi_2} - \alpha\Big(a_1 \int_{p_2^L + d - V_L}^{\infty} V_L + s\, dG(s) + \\
& a_2(1 - G(p_2^H - d - V_L))(p_2^H - d)\Big), \\
\pi^{SM} - \pi_3 \geq\ & \alpha a_1(V_L + s_L) + (1-\alpha)K(V_H + s_H) - \alpha\Big(a_1 \int_{-V_L}^{\infty} V_L + s\, dG(s) + \\
& a_2 \max_{p_2 \geq p_2^H - d} (1 - G(p_2 - V_L))p_2\Big) - (1-\alpha)\Big(a_1 V_H + (K - a_1)p_2^{H,m} + \\
& a_1 \int_{-\infty}^{p_2^H - d - V_H} p_2^{H,m} - V_H - s\, dG(s)\Big) \\
\geq\ & \underbrace{(1-\alpha)\Big(a_1\big(s_H - \int_{-V_H}^{\infty} s\, dG(s)\big) - (K - a_1)\left(p_2^{H,m} - V_H - s_H\right)}_{\psi_{3.1}} + \\
& \underbrace{a_1 \int_{p_2^{H,m} - V_H}^{\infty} V_H + s - p_2^{H,M}\, dG(s)\Big)}_{\psi_{3.2}} + \\
& \alpha a_1 s_L - \alpha\big(a_1 \int_{-V_L}^{\infty} V_L + s\, dG(s) - a_1 V_L + a_2(1 - G(p_2^H - d - V_L))(p_2^H - d)\big), \\
\pi^{SM} - \pi_4 \geq\ & \underbrace{(1-\alpha)\big((K - a_1)(V_H + s_H - p_2^H + d) + a_1 s_H\big)}_{\psi_4} - \\
& \alpha\Big(a_1 \int_{-V_L}^{\infty} V_L + s\, dG(s) - a_1 V_L + \max_{p_2^L + d \leq p_2 \leq p_2^H - d} a_2(1 - G(p_2 - V_L))p_2 - a_1 s_L\Big) - \\
& (1-\alpha)\Big(\int_{-\infty}^{p_2^H - d - V_H} p_2^H - d - V_H - s\, dG(s)\Big).
\end{aligned}
$$

Let $V_L > \underline{V}_L$, so

$$\tilde{\psi}_2 \equiv \psi_2 - \alpha\big(a_1 \int_{-V_L}^{\infty} V_L + s\, dG(s) - a_1 V_L\big) > 0.$$

Select d_0 such that $\psi_4 > \psi_2$ and

$$\tilde{\psi}_1 = (1-\alpha)\Big[(K-a_1)(V_L+2d_0)+Ks_H+(K-a_1)(V_L+d_0-p_2^L)\Big]-\alpha(\pi^{RC,L}-a_1(V_L+s_L)) > 0$$

and observe that $\psi_1 \equiv \psi_{1.1} - \psi_{1.2} > \tilde{\psi}_1$ when $V_H > V_L + 2d_0$ and that ψ_1 is monotonically increasing in d. By Assumption 4, $\psi_3 \equiv \psi_{3.1} + \psi_{3.2} > 0$. Let $\epsilon = \min\{\tilde{\psi}_1, \tilde{\psi}_2, \psi_3\}$. I show that all remaining terms can be made small.

Recall that $p_2^H - V_H$ is constant by Lemma 2; $p_2^{H,m} - V_H$ is also constant by Lemma 7. Fixing V_L at the value selected above and setting $V_H = V_L + 2d$, choose $d > d_0$ so that

$$\alpha a_1 \int_{p_2^L+d-V_L}^{\infty} V_L + s\, dG(s) < \epsilon/3 \qquad (\pi_2)$$

$$(1-\alpha)a_1 \int_{-\infty}^{p_2^H-d-V_H} p_2^{H,m} - V_H - s\, dG(s) < \epsilon/3 \qquad (\pi_3)$$

$$\max_{p_2^L+d\le p_2} a_2(1-G(p_2-V_L))p_2 < \epsilon/4 \qquad (\pi_4)$$

$$(1-\alpha)a_1 \int_{-\infty}^{p_2^H-d-V_H} p_2^H - V_H - s - d\, dG(s) < \epsilon/3. \qquad (\pi_4)$$

The third selection is possible because of Lemma 3. Given the d selected above, choose $V_H > V_L + 2d$ so that

$$(1-\alpha)\Big(a_1\int_{-V_H}^{\infty} V_H + s\,dG(s) - a_1 V_H\Big) < \epsilon/2 \qquad (\pi_1)$$

$$(1-\alpha)a_1 G(p_2^L + d - V_H)(p_2^L + d) < \epsilon/2 \qquad (\pi_1)$$

$$\alpha a_2(1 - G(p_2^H - d - V_L))(p_2^H - d) < \epsilon/3 \qquad (\pi_2)$$

$$\alpha a_2 \max_{p_2 \geq p_2^H - d}(1 - G(p_2 - V_L))p_2 < \epsilon/3 \qquad (\pi_3).$$

The first selection follows from $\mathrm{E}(S) = 0$ and the third and fourth follow from Lemma 3. All differences $\pi^{SM} - \pi_i$ are thus positive. ϵ is non-decreasing in V_H and the subtracted terms are non-increasing in V_H, so resale remains more profitable at higher values of V_H. The existence of $\bar{\tau}$ follows from the continuity of resale profit in τ. □

Proposition 6 shows the opposite: that when aggregate uncertainty is large enough, resale is more profitable as long as resale fees are not too high. The intuition is simple. When the two values are far apart, the seller's primary market prices must either be too low for the high demand state, leaving money on the table, or too high for the low demand state, resulting in few sales. Resale eases the constraint by making some goods available at the resale price, which adjusts to reflect realized demand. If the losses from setting a suboptimal price are large enough to outweigh the fees associated with resale, the seller will prefer resale.

Observed choices between resale and refunds largely match the predictions of Propositions 5 and 6. Sellers with flexible primary market prices, like airlines and hotels that use dynamic pricing, derive little advantage from resale markets and so offer refunds. Sellers with inflexible prices that face aggregate uncertainty, like sellers of sports tickets, tend to allow resale. A few choices, however, do not match the predictions. Concert organizers also allow resale, but they might benefit from offering refunds as long as demand is predictable.

The required assumptions are relatively mild: resale can be more profitable for any probability α, but the low value V_L must be high enough that the seller suffers by setting a high price p_2 and making few sales when demand is low.

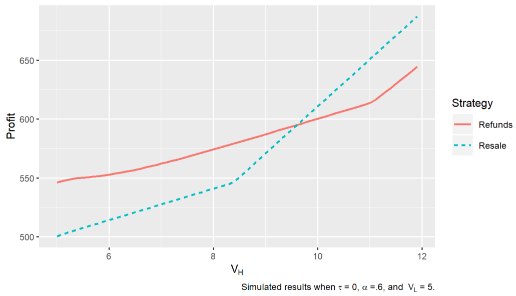
It would be intuitive for welfare to be higher with resale because it allocates goods more efficiently when there is uncertainty. However, total and consumer welfare can be higher with refunds because of the chance that the monopolist sets a high price with resale, making few sales when demand is low, but sets a low price with refunds. This can be seen in the example that follows.

Example. Suppose $K = 100$, $a_1 = 50$, $a_2 = 150$, $V_L = 5$, and $S \sim N(0, 9)$. I simulate the market for $5 \leq V_H \leq 12$ and $\tau < 5$, with results presented in Figure 3.3.

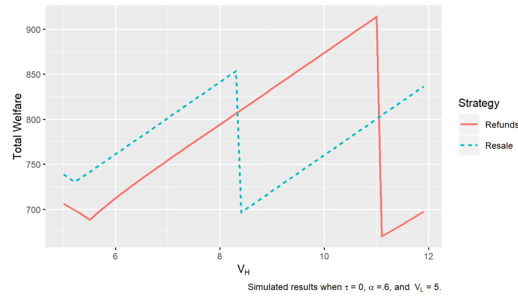
The pattern in Figure 3.3a for profit is exactly in line with Propositions 5 and 6: refunds are initially more profitable when aggregate uncertainty is low, but resale becomes more profitable as V_H grows (uncertainty increases), eventually overtaking refunds as the optimal strategy.

Figures 3.3b and 3.3c demonstrate that total and consumer welfare are not necessarily higher with resale, even when resale is frictionless. The ambiguity, and the sawtooth patterns, are the result of the monopolist changing from prices close to V_L to ones close to V_H. When the two values are close, the monopolist prefers to set its price closer to V_L because the extra margin earned at V_H from setting a high price does not justify the lost sales at V_L. As V_H grows, however, the monopolist eventually changes to set a high price, creating a discontinuity in the policy function.

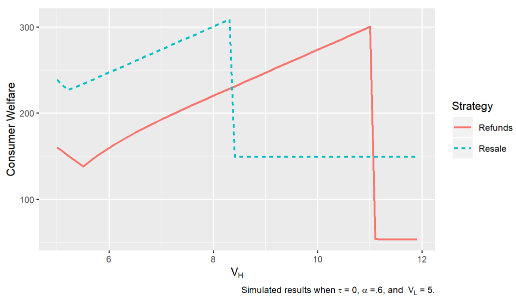
Finally, Figure 3.3d shows the regions of V_H-τ space in which resale and refunds are more profitable. As expected, the threshold value of V_H needed for resale to become more profitable is higher when the fee is higher.



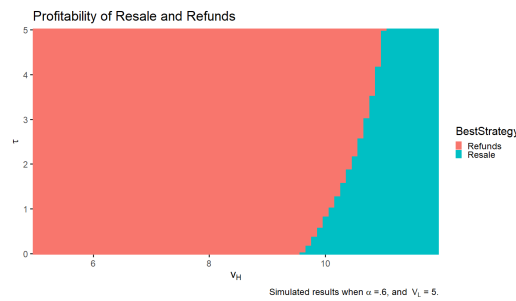
(a) Profit when $\tau = 0$



(b) Total welfare when $\tau = 0$



(c) Consumer welfare when $\tau = 0$



(d) Best strategy as V_H and τ vary

Figure 3.3: Results for the simulated example.

Resale market ownership and brokers. Propositions 5 and 6 illuminate the tradeoff facing the seller: use resale to cope with demand uncertainty and surrender fees to the resale market operator, or use refunds despite the risk that prices will be suboptimal for realized demand. The tradeoff revolves around rigidities in the primary market, and sellers do best when there are no rigidities, setting prices and refunds that vary with realized demand. But for sellers who lack flexible prices, all is not lost—they can do just as well if they own the resale market and sell to brokers.

Suppose that there are atomless brokers who purchase tickets in the first period with the sole goal of reselling in the second. Flexible resale prices ensure that the goods will be allocated efficiently in the second period. The seller contracts directly with brokers and charges them their expected resale revenue.[8]

[8]Ticket sellers often contract directly with brokers (Shepherd (2019)).

Proposition 7. Profit under resale with ownership and sales to brokers equals profit with state-contingent prices and refunds, $p_2^{RC}(V)$ and $r^{RC}(V)$.

Proof. Let $\{(p_2^{RC}(V_L), r^{RC}(V_L)), (p_2^{RC}(V_H), r^{RC}(V_H))\}$ be the firm's menu with contingent prices and refunds. By Lemma 2, $p_2^{RC}(V_L) - r^{RC}(V_L) = p_2^{RC}(V_H) - r^{RC}(V_H)$. Suppose the firm sets $\tau = p_2^{RC}(V_H) - r^{RC}(V_H)$ and contracts with brokers to sell its remaining $K - a_1$ units. It can charge them no more than $\mathrm{E}(p_2^r(V)) - \tau$ per unit, where the resale price solves

$$K - a_1 + a_1 G(p_2^r(V) - \tau - V) = a_2(1 - G(p_2^r(V) - V)),$$

which is satisfied at $p_2^r(V_H) = p_2^{RC}(V_H)$ and $p_2^r(V_L) = p_2^{RC}(V_L)$. Under both strategies, the seller earns $\mathrm{E}(p_2^{RC}(V))$ on its $K - a_1$ remaining units and can charge early arrivals their expected surplus with the ability to return the good for $p_2^{RC}(V) - \tau$ and acquire the good for $p_2^{RC}(V)$ in the second period. Profit is therefore the same.

The seller cannot earn more with resale and brokers: the seller could replicate its strategy with contingent prices and refunds by choosing $p_2^{RC}(V) = p_2^r(V)$ and setting $r(V) = p_2^{RC}(V) - \tau$. □

Corollary 3. Profit under resale with ownership and sales to brokers is strictly higher than with refunds when $\alpha \in (0, 1)$.

Proof. Because S has a positive density on its domain, the system of first order conditions in Lemma 2 cannot be satisfied at both V_L and V_H by a single menu (p_2^{RC}, r^{RC}). □

Although the seller has inflexible prices, owning the resale markets and selling to brokers let it fully capitalize on flexible resale markets and profit as if it could set perfectly adjustable contracts. Corollary 3 establishes that owning the resale market is optimal when sellers cannot adjust their refund contracts and face any uncertainty. Strikingly, the seller might as well use resale when its prices adjust perfectly.

Proposition 7 suggests that sellers can benefit from embracing resale more closely, for instance by selling to brokers and integrating primary and resale markets. The prediction resonates because many ticket sellers have chosen to partner with resale markets instead of moving towards refunds. For example, the NBA displays primary and resale market tickets side-by-side on its official ticketing platform and some teams, such as the University of Kansas men's basketball team, sell directly to brokers (Shepherd (2019)).

3.5 Conclusion

The purpose of this paper has been to investigate which of the two most commonly used reallocation mechanisms, resale and refunds, maximizes profit and welfare. The analysis demonstrates that the choice between them is not clear-cut: it is possible for either strategy to maximize profit and welfare. Resale can be more profitable when the allocative advantages of flexible resale prices outweigh the harms of resale fees, which is possible when primary market prices are rigid and aggregate demand is uncertain. Refunds can produce higher welfare when the supply of reallocated tickets has little effect on optimal, as when the number of late-arriving consumers is large. Otherwise, society may prefer resale.

The analysis helps resolve a lingering question about the firm's policy: why allow resale? The widespread use of resale for goods like event tickets makes it implausible that resale is always worse than offering a refund. This paper contributes by showing that aggregate demand uncertainty and primary market price rigidities, both of which are present in markets for event tickets, can make resale profitable. More generally, the conditions establish when resale markets offer an advantage over refunds.

The welfare results contribute to the policy dispute on rights to resell by showing that resale bans are not necessarily harmful. As long as the seller offers a refund instead, it is possible for consumers to be better off when resale is banned. Recent laws protecting the right to resell in perishable goods markets could be modified to protect resale only when the seller does not offer a refund.

The central message, however, is broader. Economists have long considered how reallocation can boost efficiency, but have paid little attention to the relative performance of commonly used reallocation mechanisms. This paper shows that both resale and refunds have unique strengths and weaknesses. Economists should not only ask whether to reallocate, but also how best to reallocate.

Chapter 4

Why Bundle When Consumers Can Resell?

4.1 Introduction

Virtually all studies of price discrimination share one assumption: that resale is impossible.[1] There are good reasons why economists do not consider resale. Second-degree price discrimination relies on the ability to design different contracts for different consumer types, but resale breaks the self-selection process by letting consumers pool the contracts. Resale and price discrimination are thus seen as incompatible.

Yet sellers often use price discrimination when there are active resale markets. Consider tickets for sports, theater, and concerts. Sellers practice mixed bundling, offering a discounted season ticket bundle alongside tickets for each event, and consumers resell tickets frequently on services like StubHub. More generally, any durable good sold in a bundle or value pack can be resold. These include trading cards, cookware sets, and box sets of books, all of which are resold individually on internet platforms. The sellers of these goods are aware that some consumers will break up the bundle and resell, but they use price discrimination anyway.

Why do sellers use price discrimination knowing that consumers will resell? The puzzle has received little formal attention even though resale is common, leaving the implications for firm behavior unknown. How does resale affect the seller's problem?

[1] Tirole (1988) and McAfee (2008) mention resale as a determinant of whether price discrimination is feasible but do not consider it in their analysis. In a review of nonlinear pricing, Armstrong (2016) does not mention resale or arbitrage.

And how do predictions for profit and welfare change?

In this paper, I examine the puzzle using a model of monopoly bundling with costly resale and show that, in equilibrium, the monopolist bundles even though some consumers will resell parts of the bundle. Resale introduces a new tradeoff to the monopolist's problem, changes its incentives to bundle, reduces profit, and has an ambiguous effect on consumer and total welfare. The combination of bundling and resale in equilibrium is important because the seller's problem, and hence the implications of resale, are different than in a model without resale in equilibrium. Bundling is an important setting because of the ubiquity of bundles that can be resold, as in the examples discussed above, and because no prior studies have considered the effect of resale on bundling.

The central tension in the model is between the profitability of bundling and consumers' gains from sharing the bundle through resale. Both usually increase with the bundle discount, the difference between the price of the bundle and the price of buying its component goods separately. The essence of bundling is that discounting the bundle can raise profit, so increasing the discount is profitable on some interval. But larger discounts tempt consumers to share the bundle through resale, which lowers profit because they only pay the discounted bundle price.

The resulting tradeoff allows bundling and resale to occur simultaneously in equilibrium when consumers have heterogeneous costs of engaging in resale. With heterogeneous costs, a higher bundle discount increases the number of consumers practicing arbitrage through resale but lets the monopolist earn more from consumers who find resale too costly. As long as the profit earned from consumers with high costs exceeds losses to resale from those with low costs, it is optimal for the monopolist to allow some resale.

The tradeoff is a significant change to the monopolist's problem, forcing the monopolist to balance the profit made by bundling against the opportunity for consumer arbitrage. Importantly, the tradeoff is not present in simpler models of price discrimination and resale. With a homogeneous cost of resale, there is no tradeoff because all consumers move to the resale market when the bundle discount exceeds the costs of resale. In equilibrium, the monopolist chooses a discount just below the threshold and no consumers resell, simplifying the pricing problem. The difference also affects comparative statics for pricing. When resale becomes more costly, the monopolist never decreases its bundle discount if there is a homogeneous cost of resale. But if there are heterogeneous costs of resale, the monopolist might lower its discount—*reducing* the extent of bundling when resale becomes more costly. The finding suggests that resale is necessary in equilibrium to understand the monopolist's problem and pricing.

The model establishes that resale limits, but does not eliminate, the returns to bundling. Resale harms the monopolist's profit by interfering with its profit-maximizing allocation, but bundling remains profitable. The effects on welfare, however, are ambiguous. It is somewhat surprising that consumers do not benefit because, for a given allocation, resale improves welfare. The resolution is that resale causes the seller to change its prices, and price discrimination has ambiguous effects on consumer and total welfare. It follows that the ability to resell can harm consumers and society.

The analysis makes two main contributions. First, it advances our understanding of how resale affects price discrimination by providing an equilibrium in which the two coexist and by investigating the implications for firm behavior. Prior studies of price discrimination and resale often feature equilibria where the seller prevents all resale, such as Alger (1999). Other studies consider settings where the seller is able to profit from resale, like McManus (2001) and Gans and King (2007). But the case where the seller discriminates, consumers resell, and resale harms the seller has not been explored until now. I show that the omission matters because the seller's problem and pricing incentives differ when there is harmful resale in equilibrium.

The second contribution is to clarify the effects of resale on bundling. No prior study has considered how resale affects bundling despite the fact that the two coexist in numerous economically significant markets. Revenue from sports tickets, sold in mixed bundles, exceeded \$10bn in 2019 (Statista (2020b)). Revenue from just one trading card game is estimated to exceed \$500m per year (Deaux (2019)). I clarify how resale affects pricing, profit, and welfare in these markets.

There are two parts to the analysis. In the first, I analyze a model in which a monopolist bundles and consumers have a homogeneous cost of participating in the resale market. The model provides a simple description of the effects of resale on bundling, but its equilibrium does not include resale. It therefore provides a benchmark for the equilibrium in which bundling and resale coexist. In the second part of the analysis, I allow consumers to have heterogeneous costs of resale. I show that bundling and resale can coexist in equilibrium and that their coexistence meaningfully changes the seller's problem.

The paper proceeds as follows. I start by reviewing the relevant literature. In Section 2, I introduce the model, and in Section 3 I study the benchmark case with homogeneous costs of resale. I consider heterogeneous costs of resale in Section 4. In Section 5, I conclude.

Related Literature. A primary focus of the bundling literature is to determine when monopoly bundling is profitable. Studies have considered how the value of bundling depends on the distribution of consumer values, as in McAfee et al. (1989) and Chen and Riordan (2013), the number of goods sold, as in Bakos and Brynjolfsson (1999), and uncertainty over future states of the world, as in Alexandrov and Bedre-Defolie

(2014). This paper contributes by demonstrating that resale is a determinant of the profitability of bundling.

Researchers have also considered bundling outside the traditional setting with a monopolist seller. Chen and Li (2018) consider the effect of bundling when a buyer must procure several products. Zhou (2017) and Zhou (2019) study the effects of bundling in a competitive environment. Pagnozzi (2009) considers bundling in auctions when consumers can resell after the auction. The implications of resale in his study are specific to auctions rather than monopoly bundling.

Bundling has also been studied empirically. Gandal et al. (2018) use data on computer software to determine the effect of correlations in consumer values on the profitability of bundling. Chu et al. (2011) use data on theater ticket sales to compare the performance of theoretically optimal bundle pricing to simpler rules. Crawford and Yurukoglu (2012) study bundling in a competitive setting, cable television, and consider its effect on upstream bargaining.

A separate literature considers the effects of resale markets. The effect of resale on sellers of durable goods has been widely studied, for instance in Chen et al. (2013b). Sellers of durable goods can benefit from resale because it allocates past vintages to the consumers who value them most, but resale forces sellers to compete against past vintages. This paper features a similar tradeoff, with resale increasing consumers' willingness to pay for the bundle but introducing competition in the market for individual goods. I find that the harms of resale always outweigh the benefits. Sellers of perishable goods can also benefit from reallocation when there is limited capacity and consumers receive preference shocks, as in Cui et al. (2014).

Chen et al. (Forthcoming) consider how to price loot boxes, random prizes that can be thought of as bundles of award probabilities. Their analysis of salvage—letting consumers return an unwanted item for a partial refund—is similar to allowing resale, but it differs in that returned goods do not compete with the seller's offerings.

Finally, my paper is related to the literature on price discrimination with resale. The most notable paper in this literature is Alger (1999), who considers pricing when consumers can make joint purchases (equivalent to frictionless resale). In equilibrium, the seller sets prices that prevent all resale. Several other papers, such as Gans and King (2007) and McManus (2001), consider settings where the seller can profit from resale.

4.2 Model

A monopolist with zero fixed and marginal costs and no capacity constraints sells two goods, called 1 and 2. The monopolist sets primary market prices $P = (P_1, P_2, P_B)$, where P_1 and P_2 are the prices of goods 1 and 2 and P_B is the price of a bundle containing both goods. The bundle price satisfies $P_B \leq P_1 + P_2$ because consumers can buy each good separately.

The market includes a mass of consumers normalized to one. Each consumer has type (v_1, v_2), where v_1 and v_2 denote the consumer's value for each good and $v_1 + v_2$ is the consumer's value for the bundle. Values are drawn from the joint distribution $F(v_1, v_2)$. I assume that $F(v_1, v_2)$ has support on $[0, 1]^2$ with a strictly positive, atomless density.

The game proceeds as follows. First, the monopolist sets primary market prices P. Next, consumers purchase goods in the primary market. Finally, consumers participate in a resale market with a vector of endogenously determined clearing prices P^s. Consumers and the monopolist know the distribution of values $F(v_1, v_2)$ throughout the game.

Participating in the resale market, either as a buyer or reseller, is costly. Consumer

k must pay a cost c_k to resell or buy from a reseller. Costs are independent of values, $c_k \perp (v_1, v_2)$, and follow the distribution $G(c)$, which has density $g(c)$ and satisfies $G(\underline{c}) = 0$ and $G(\bar{c}) = 1$ for some real numbers $0 \leq \underline{c} < \bar{c}$. The cost of resale can be interpreted as the time and effort needed to participate.[2]

I consider subgame perfect Nash equilibria of the model, which require consumers to have correct expectations for resale prices P^s when they make purchase decisions in the primary market. Equilibrium is a pair (P, P^s) of the monopolist's primary market prices P and resale market prices P^s such that (i) primary market prices P maximize profit given consumer demand, (ii) consumers make optimal primary market purchases under the expectation that the resale market will clear at prices P^s, and (iii) the prices P^s clear the resale market when consumers make optimal primary purchases anticipating P^s.

In the resale market, consumers can purchase either good or buy the bundle to resell either or both goods. For example, consumer k earns surplus $v_1 - (P_1^s + c_k)$ by buying good 1 in the resale market, but she can also earn $v_1 - (P_B - P_2^s + c_k)$ by purchasing the bundle, reselling good 2, and keeping good 1. Each consumer chooses the option maximizing surplus.

Surplus-maximizing choices define a map from types (v_1, v_2) to purchase decisions for each cost c_k. Figure 4.1 depicts the purchase regions for the case without resale. The diagonal line, $v_1 + v_2 = P_B$, separates consumers with positive and negative surplus from the bundle. The horizontal and vertical lines $v_1 = P_1$ and $v_2 = P_2$ do the same for the individual goods. Some consumers have positive surplus for several choices, so the regions denote the surplus-maximizing option.

[2]An alternative assumption would be that part of the cost is paid to the resale market operator, like the fees on sites like eBay and StubHub. Under this assumption, the cost would count towards total surplus. This would have no effect on the results in Section 4.3, but it would increase total welfare in Section 4.4.

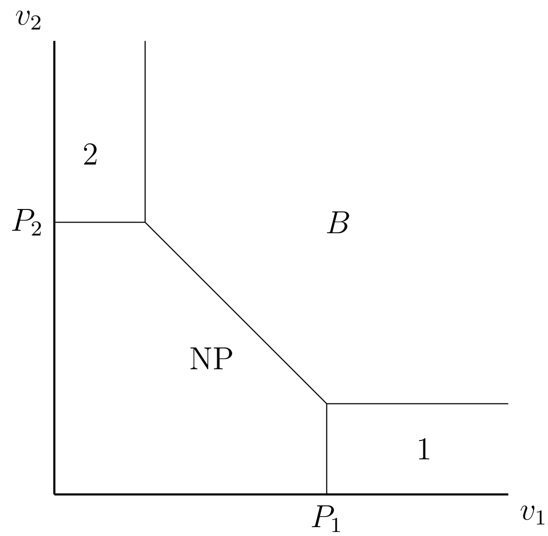


Figure 4.1: Allocations when there is no resale. Consumers in the B regions purchase the bundle, those in the 1 and 2 regions purchase only good 1 or good 2, and those in the NP region make no purchase.

4.3 Homogeneous Costs of Resale

I begin the analysis by considering the case where all consumers share the same cost of participating in the resale market: $c_k = c$ for all consumers k. With a homogeneous cost, bundling and resale do not coexist in equilibrium. Nonetheless, the analysis is valuable because it provides easily interpreted predictions for the effect of resale on bundling. Many of the predictions—such as that resale harms profit and has an ambiguous effect on consumer and total welfare—apply in both models, but can be seen more clearly with a common cost of resale. Furthermore, the predictions in this section can be compared to those of the full model in Section 4.4 where both bundling and resale occur in equilibrium.

4.3.1 Resale Equilibrium

Suppose that the monopolist has announced its price vector P and consider the subgame in which consumers make primary and then resale market purchase decisions.

An equilibrium of the subgame is a vector of secondary market prices $P^s(P)$ such that consumers make their optimal purchase decisions in the primary market believing that resale prices will be $P^s(P)$ and the vector of resale prices $P^s(P)$ clears the resale market after optimal purchases in the primary market. The goal of this subsection is to characterize equilibrium resale prices and the conditions necessary for resale in equilibrium.

The characterization of equilibrium relies on two supporting results. The first establishes a condition for resale prices.

Lemma 4. In any equilibrium with resale market transactions, $P_1^s + P_2^s = P_B$.

Proof. Assume that P^s is an equilibrium price vector and that good 2 is resold in the secondary market. Resellers of 2 buy the bundle to resell good 2, effectively paying $P_B-(P_2^s-c)$ for good 1. To be willing to resell, this must be cheaper than buying good 1 in the resale market, $P_B - P_2^s + c \leq P_1^s + c$. The condition is reversed for secondary market buyers of good 2. Satisfying both conditions requires that $P_1^s + P_2^s = P_B$. □

Lemma 4 follows from the need for a buyer and seller in each resale transaction. If the sum of resale prices is any higher, there will be no resale buyers, and if it is any lower, there will be no resellers. The only equilibria of interest satisfy $P_1^s + P_2^s = P_B$ because no other equilibrium has resale transactions.

The result is useful because it narrows the search for equilibrium resale prices. It also simplifies the consumer's choice: surplus is the same for consumers who buy the bundle to resell good 2 as for consumers who buy good 1 in the resale market.[3] For this reason, it only matters that a consumer acquires good 1 through resale, and I will hereafter discuss consumers who want to acquire good 1 or 2 through resale. Further,

[3]Observe that $v_1 - (P_1^s + c) = v_1 - (P_B - P_2^s + c)$.

no consumer buys the bundle to resell in equilibrium because doing so results in a loss of $2c$.

The second supporting result establishes that, when the monopolist only sells a bundle, there is a unique vector of equilibrium resale prices.

Lemma 5. When the monopolist only sells the bundle ($P_1 = P_2 > 1$), there exists an equilibrium vector of resale market prices $\hat{P}^s$. It is unique if $\max\{c, P_B - 1 + c\} < \min\{1 - c, P_B - c\}$.

Proof. Equilibrium requires the masses of consumers demanding each good in the resale market to be equal. When the monopolist only offers the bundle, the masses $\mu_1(P_B, P_1^s)$ and $\mu_2(P_B, P_1^s)$ of consumers who only want one good are

$$\mu_1(P_B, P_1^s) = \int_{P_1^s+c}^{1} \int_{0}^{P_B-P_1^s-c} f(v_1, v_2)\, dv_1\, dv_2$$
$$\mu_2(P_B, P_1^s) = \int_{0}^{P_1^s-c} \int_{P_B-P_1^s+c}^{1} f(v_1, v_2)\, dv_1\, dv_2.$$

Define $\delta(P_1^s) = \mu_1(P_B, P_1^s) - \mu_2(P_B, P_1^s)$ on $[0, P_B]$. There is some $\hat{P}_1^s$ satisfying $\delta(P_1^s) = 0$ because $\delta(\cdot)$ is a continuous (since F is atomless) decreasing function satisfying $\delta(0) \geq 0$ and $\delta(P_B) \leq 0$.

To be unique, $\delta(\cdot)$ can only cross zero at a single point and so $\mu_1(P_B, P_1^s)$ and $\mu_2(P_B, P_1^s)$ can never simultaneously be zero. Because F has a strictly positive atomless density, this only requires that the regions of integration not simultaneously be empty. For μ_1, this requires $P_1^s + c < 1$ and $0 < P_B - P_1^s - c$, or $P_1^s < \min\{1-c, P_B-c\}$. For μ_2, it requires $0 < P_1^s - c$ and $P_B - P_1^s + c < 1$, or $P_1^s > \max\{P_B - 1 + c, c\}$. Such a P_1^s exists when $\max\{c, P_B - 1 + c\} < \min\{1 - c, P_B - c\}$. □

The prices $\hat{P}^s$ are called the pure-bundling resale prices and would prevail in the resale market if no consumer buys an individual good in the primary market. They are useful in finding resale prices in the mixed bundling equilibrium.

To see why an equilibrium exists, consider the case of pure bundling with resale depicted in Figure 4.2. Consumers know that the bundle price is P_B (defining the diagonal line as $v_1 + v_2 = P_B$) and believe that resale prices will be P^s. Types in the region B find it optimal to buy the bundle; those in regions 1 and 2 find it optimal to acquire one good through resale. The key difference from the case with no resale is that the individual good prices (P_1, P_2) have been replaced with fee-inclusive resale prices $(P_1^s + c, P_2^s + c)$. The dashed lines from the (fee-exclusive) resale prices meet at the diagonal by Lemma 4.

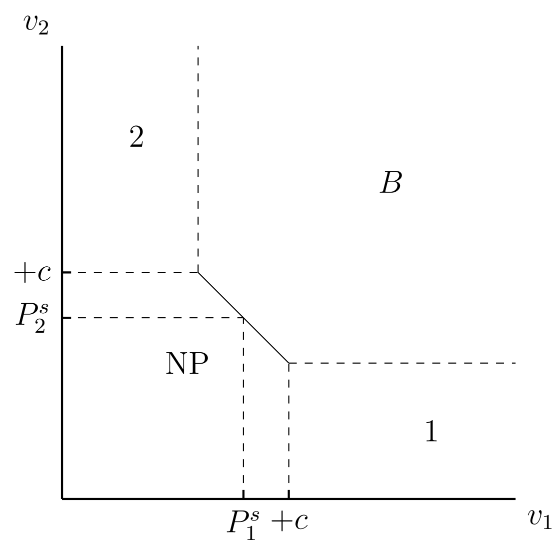


Figure 4.2: Pure bundling allocations when the resale price is P^s.

The primary market runs before the resale market. Consumers in the B region purchase the bundle and keep it. Half of consumers who want only one good purchase the bundle in order to resell.[4] Finally, consumers participate in the resale market. Consumers who bought the bundle but only want one good resell to consumers who only want one good and did not buy the bundle. The resale market only clears at the

[4]Each consumer could flip a coin, for example.

expected prices P^s if the mass of consumers in regions 1 and 2 are equal. If not, the market-clearing price differs from expectations and P^s is not an equilibrium.

Equilibrium therefore requires a vector of resale prices equating the mass of the two regions. Such a price vector exists under mild conditions. By Lemma 4, any increase in P_1^s must cause P_2^s to fall (and vice versa). The increase in P_1^s causes fewer consumers to want good 1, and more to want good 2. Graphically, the intersection of the dashed lines from P_1^s and P_2^s slides southeast along the diagonal, shrinking the 1 region and enlarging the 2 region. With a strictly positive, atomless density, there is a vector of resale prices making the mass of consumers in the two regions equal.

The condition required for uniqueness rules out the possibility of prices at which no consumer wants to participate in resale. Graphically, it requires that the regions labeled 1 and 2 cannot be empty simultaneously. For the remainder of the paper, I assume that c is small enough for the pure-bundling resale prices to be unique.

Lemmas 4 and 5 provide the tools needed to characterize the full resale equilibrium. I add one additional assumption to remove an open set problem: when all consumers are indifferent between the primary and resale markets, they choose to purchase in the primary market.

Theorem 1. Let $P = (P_1, P_2, P_B)$ be a vector of primary market prices. There is resale in equilibrium if and only if $P_1 + P_2 > P_B + 2c$. Resale prices are unique if there are resale transactions.

Proof. ($\Rightarrow$) Without loss of generality, suppose that good 1 is resold. The reseller pays $P_B + c - P_1^s$ for good 2 and the buyer pays $P_1^s + c$ for good 1. Equilibrium requires $P_B + c - P_1^s \leq P_2$ and $P_1^s + c \leq P_1$. One inequality must be strict, implying that $P_B + 2c < P_1 + P_2$.

($\Leftarrow$) Suppose $P_1 + P_2 > P_B + 2c$ and let $(\hat{P}_1^s, \hat{P}_2^s)$ be the vector of pure-bundling resale prices corresponding to P_B. There are two cases. First, $\hat{P}_1^s + c < P_1$ and $\hat{P}_2^s + c < P_2$. All consumers who want one good strictly prefer the secondary market and $\hat{P}^s$ is the unique vector of resale prices. Second, suppose $\hat{P}_1^s + c \geq P_1$ and $\hat{P}_2^s + c < P_2$. Buyers of good 2 strictly prefer the resale market, so $\hat{P}_1^s$ must fall (and $\hat{P}_2^s$ must rise) until either $P_1 = P_1^s + c$ or $P_2 = P_2^s + c$. Since $P_1 + P_2 > P_B + 2c$, it must be that $\hat{P}_1^s + c - P_1 < P_2 - \hat{P}_2^s - c$, so prices adjust until $P_1^s + c = P_1$ and $P_2^s + c < P_2$. The resulting vector $P^s = (P_1 - c, P_B - P_1 + c)$ is an equilibrium: fewer consumers only want good 2 than good 1 (since $P_1^s < \hat{P}_1^s$), and consumers who want good 1 are indifferent between the two markets, making some willing to resell and clear the resale market. The equilibrium is unique. At any higher P_1^s, there would be no supply of good 1 in the resale market. At any lower P_1^s, there would be excess demand for good 1. $\square$

Theorem 1 shows that the existence of resale in equilibrium reduces to a simple condition: whether $P_B + 2c < P_1 + P_2$. The condition has a natural interpretation. Resale involves sharing the bundle at price P_B, but incurs the extra cost $2c$. Consumers only share the bundle through resale when it is strictly cheaper than buying each individual good from the seller.

The result also connects resale to the monopolist's ability to discriminate. Bundling is profitable because it lets the monopolist discount the bundle relative to buying each good individually, $P_B < P_1 + P_2$. Doing so lets it boost sales among consumers with high average valuations, like those in the triangle bounded by $v_1 + v_2 = P_B$, $v_1 < P_1$, and $v_2 < P_2$. But discounting the bundle lets consumers share the discount through the resale market, leading to resale whenever the discount exceeds $2c$.

4.3.2 The Monopolist's Problem

The monopolist's problem is to set its profit-maximizing price vector P given the resale equilibrium described in Theorem 1. The key insight is that it is never profitable for the monopolist to allow resale. To see why, suppose that some consumers resell at prices P^s. Consumers would make the same choices if primary market prices were $(P_1^s + c, P_2^s + c, P_B)$, but they would purchase in the primary market, letting the monopolist earn an additional $2c$ on each transaction that used to involve resale.

The conclusion that it is never optimal to allow resale, coupled with the conditions for resale in Theorem 1, allow a complete description of the monopolist's problem.

Theorem 2. Let $\pi_N(P)$ be the monopolist's profit when there is no resale. There are no resale transactions in equilibrium. The monopolist's problem is

$$\max_P \pi_N(P_1, P_2, P_B) \text{ subject to } P_1 + P_2 \leq P_B + 2c.$$

Proof. Suppose there is resale in equilibrium at prices P^s. The monopolist can deviate to set $P_1 = P_1^s + c$ and $P_2 = P_2^s + c$. All consumers will receive the same goods as before, but all purchases will be made in the primary market. The monopolist earns $2c$ from a positive measure of consumers, strictly increasing profit. The monopolist thus maximizes profit subject to the condition that there are no resale transactions.

□

Theorem 2 establishes that the effect of resale is to limit the seller's bundle discount, and hence its ability to use price discrimination. Without resale, the monopolist is free to choose any bundle discount it likes, but with resale, it is limited to discounts smaller than $2c$. When resale is frictionless, the monopolist offers no discount at all.

The implication that resale is harmful for sellers rings true because sellers of bundles have attempted to prevent resale. The NFL set a price floor in the resale market before the New York Attorney General's office intervened (Belson (2016)). The Denver Broncos NFL team went so far as to revoke season tickets for consumers who resell too frequently (Thomas (2017)). Preventing the resale of durable goods is more difficult because consumers have a legal right to resell, but sellers can make resale more difficult, for example by selling sets of books in a single volume.

The condition for resale in equilibrium and the constraint on prices in the monopolist's problem are appealingly simple and demonstrate the effect of resale, but they rely on the assumption that all consumers have the same cost of resale c. With a homogeneous cost, the monopolist can prevent all resale by setting a discount less than $2c$. But the instant the discount exceeds $2c$, consumers flock to the resale market. Other studies of price discrimination and resale, such as Alger (1999), feature similar predictions but share the drawback that price discrimination and resale do not coexist in equilibrium.

4.3.3 Comparative Statics

To fully describe the effects of resale on bundling, I present comparative statics for profit, the bundle discount, and welfare as the cost of resale changes.

Corollary 4. The monopolist's profit and bundle discount are weakly increasing in the cost of resale c.

Proof. The monopolist's optimal prices for cost c are feasible at $c' > c$. □

The conclusion of Corollary 4 follows from the monopolist's constraint in Theorem 2. It confirms that resale is harmful to monopoly bundling and that the ease of resale matters for the returns to bundling. It also establishes that, when resale becomes more costly, the seller weakly increases its use of price discrimination by raising the bundle discount.

The changes in consumer and total welfare are less clear-cut. When the initial allocation of goods is fixed, consumers benefit from resale by engaging in welfare-enhancing trade. But consumers may not benefit because the monopolist revises its prices in response to resale, changing the initial allocation. As for price discrimination more generally, the effect of resale on consumer welfare is ambiguous.

To formalize the conclusion, assume that the monopolist's prices $P(c)$ are differentiable in c. Because the seller weakly increases its bundle discount as c increases, also assume that $P'_B(c) \geq 0$, $P'_1(c) \leq 0$, and $P'_2(c) \leq 0$.[5] When c increases, consumers buying the bundle benefit, but those buying the first two goods are harmed. Because marginal buyers have surplus zero, the change in consumer welfare depends only on inframarginal consumers.

Theorem 3. Let $\mu_1(c)$, $\mu_2(c)$, and $\mu_B(c)$ denote the masses of consumers buying good 1, good 2, and the bundle when the cost of resale is c. Consumer welfare weakly increases in c if and only if

$$\frac{\partial P_1(c)}{\partial c}\mu_1(c) + \frac{\partial P_2(c)}{\partial c}\mu_2(c) \leq -\frac{\partial P_B(c)}{\partial c}\mu_B(c). \tag{4.1}$$

Proof. The regions are defined as

$$\mu_1(c) = \int_{P_1(c)}^{1} \int_{0}^{P_B(c)-P_1(c)} f(v_1, v_2)\, dv_2\, dv_1$$
$$\mu_2(c) = \int_{P_2(c)}^{1} \int_{0}^{P_B(c)-P_2(c)} f(v_1, v_2)\, dv_1\, dv_2$$

[5] These assumptions hold when $F(v_1, v_2)$ is uniform.

$$\mu_B(c) = \int_{P_B(c)-P_2(c)}^{1} \int_{P_B(c)-P_1(c)}^{1} f(v_1, v_2)\, dv_2\, dv_1 - \int_{P_B(c)-P_2(c)}^{P_1(c)} \int_{P_B(c)-P_1(c)}^{P_B(c)-v_1} f(v_1, v_2)\, dv_2\, dv_1.$$

Consumer welfare is

$$\begin{aligned} CW(c) = &\Big\{ \int_{P_1(c)}^{1} \int_{0}^{P_B(c)-P_1(c)} (v_1 - P_1(c)) f(v_1, v_2)\, dv_2\, dv_1 \Big\} + \\ &\Big\{ \int_{P_2(c)}^{1} \int_{0}^{P_B(c)-P_2(c)} (v_2 - P_2(c)) f(v_1, v_2)\, dv_1\, dv_2 \Big\} + \\ &\Big\{ \int_{P_B(c)-P_2(c)}^{1} \int_{P_B(c)-P_1(c)}^{1} (v_1 + v_2 - P_B(c)) f(v_1, v_2)\, dv_2\, dv_1 - \\ &\int_{P_B(c)-P_2(c)}^{P_1(c)} \int_{P_B(c)-P_1(c)}^{P_B(c)-v_1} (v_1 + v_2 - P_B(c)) f(v_1, v_2)\, dv_2\, dv_1 \Big\}. \end{aligned} \tag{4.2}$$

Taking the derivative with respect to c and comparing it to zero yields the result.

□

The direction of the change in consumer welfare in Theorem 3 is necessarily ambiguous: bundling has an ambiguous effect, and the integral of the change in consumer welfare over c must equal the difference between component pricing and mixed bundling.

The change does not have to be monotone in the cost of resale. The sign of the change depends on how many consumers buy the bundle relative to the number buying the individual goods, and the share in each group varies with the cost of resale. The example in Section 4.3.4 illustrates the non-monotonicity.

Unlike the change in consumer welfare, the change in total welfare only depends on marginal consumers. Inframarginal consumers make the same purchases when the cost changes and thus do not contribute to the welfare change.

Theorem 4. Total welfare weakly increases in c if and only if

$$
\begin{aligned}
0 \leq\ & -\frac{\partial P_1(c)}{\partial c}\int_0^{P_B(c)-P_1(c)} P_1(c)f(P_1(c),v_2)\,dv_2 - \frac{\partial P_2(c)}{\partial c}\int_0^{P_B(c)-P_2(c)} P_2(c)f(v_1,P_2(c))\,dv_1 - \\
& \left(\frac{\partial P_B(c)}{\partial c} - \frac{\partial P_1(c)}{\partial c}\right)\int_{P_1(c)}^{1} P_B(c)-P_1(c)f(v_1,P_B(c)-P_1(c))\,dv_1 - \\
& \left(\frac{\partial P_B(c)}{\partial c} - \frac{\partial P_2(c)}{\partial c}\right)\int_{P_2(c)}^{1} P_B(c)-P_2(c)f(P_B(c)-P_2(c),v_2)\,dv_2 - \\
& \frac{\partial P_B(c)}{\partial c}\int_{P_B(c)-P_2(c)}^{P_1(c)} P_B(c)f(v_1,P_B(c)-v_1)\,dv_1.
\end{aligned}
$$

Proof. Total welfare is

$$
\begin{aligned}
TW(c) =& \Big\{\int_{P_1(c)}^{1}\int_0^{P_B(c)-P_1(c)} v_1 f(v_1,v_2)\,dv_2\,dv_1\Big\} + \\
& \Big\{\int_{P_2(c)}^{1}\int_0^{P_B(c)-P_2(c)} v_2 f(v_1,v_2)\,dv_1\,dv_2\Big\} + \\
& \Big\{\int_{P_B(c)-P_2(c)}^{1}\int_{P_B(c)-P_1(c)}^{1} v_1+v_2 f(v_1,v_2)\,dv_2\,dv_1 - \\
& \int_{P_B(c)-P_2(c)}^{P_1(c)}\int_{P_B(c)-P_1(c)}^{P_B(c)-v_1} v_1+v_2 f(v_1,v_2)\,dv_2\,dv_1\Big\}.
\end{aligned}
\tag{4.3}
$$

And so the change in total welfare with respect to c is

$$
\begin{aligned}
\frac{\partial TW(c)}{\partial c} = & -\frac{\partial P_1(c)}{\partial c}\int_0^{P_B(c)-P_1(c)} P_1(c) f(P_1(c), v_2)\, dv_2 + \\
& \left(\frac{\partial P_B(c)}{\partial c} - \frac{\partial P_1(c)}{\partial c}\right)\int_{P_1(c)}^1 v_1 f(v_1, P_B(c) - P_1(c))\, dv_1 + \\
& -\frac{\partial P_2(c)}{\partial c}\int_0^{P_B(c)-P_2(c)} P_2(c) f(v_1, P_2(c))\, dv_1 + \\
& \left(\frac{\partial P_B(c)}{\partial c} - \frac{\partial P_2(c)}{\partial c}\right)\int_{P_2(c)}^1 v_2 f(P_B(c) - P_2(c), v_2)\, dv_2 + \\
& -\left(\frac{\partial P_B(c)}{\partial c} - \frac{\partial P_1(c)}{\partial c}\right)\int_{P_1(c)}^1 v_1 + P_B(c) - P_1(c) f(v_1, P_B(c) - P_1(c))\, dv_1 + \\
& -\left(\frac{\partial P_B(c)}{\partial c} - \frac{\partial P_2(c)}{\partial c}\right)\int_{P_2(c)}^1 v_2 + P_B(c) - P_2(c) f(P_B(c) - P_2(c), v_2)\, dv_2 + \\
& -\frac{\partial P_B(c)}{\partial c}\int_{P_B(c)-P_2(c)}^{P_1(c)} P_B(c) f(v_1, P_B(c) - v_1)\, dv_1.
\end{aligned}
\tag{4.4}
$$

□

Each term in Theorem 4 captures the effect of changing c for a line segment of marginal buyers separating the purchase regions in Figure 4.1. The first two terms describe the change due to consumers who switch to buying nothing when the prices of individual goods rise. The next two are the changes from consumers who switch from buying one good to the bundle. The last term comes from consumers who switch from buying nothing to buying the bundle.

As with consumer welfare, the change in total welfare is ambiguous because bundling has an ambiguous effect of total welfare.

4.3.4 An Example

I conclude the discussion of homogeneous costs with an example in which values are distributed uniformly on the unit square, $f(v_1, v_2) = 1$. If there were no resale, the optimal prices would be $P = (\frac{2}{3}, \frac{2}{3}, \frac{4-\sqrt{2}}{3})$, constraining the monopolist whenever $c < \frac{\sqrt{2}}{6}$. I provide closed-form expressions for prices, profit, and welfare as a function of c in the appendix.

Results are shown in Figure 4.3. Profit is increasing in the cost c, in line with Theorem 4. Further, the bundle price is decreasing in c and the prices of the individual goods are increasing in c, as assumed in the welfare discussion in the last subsection.

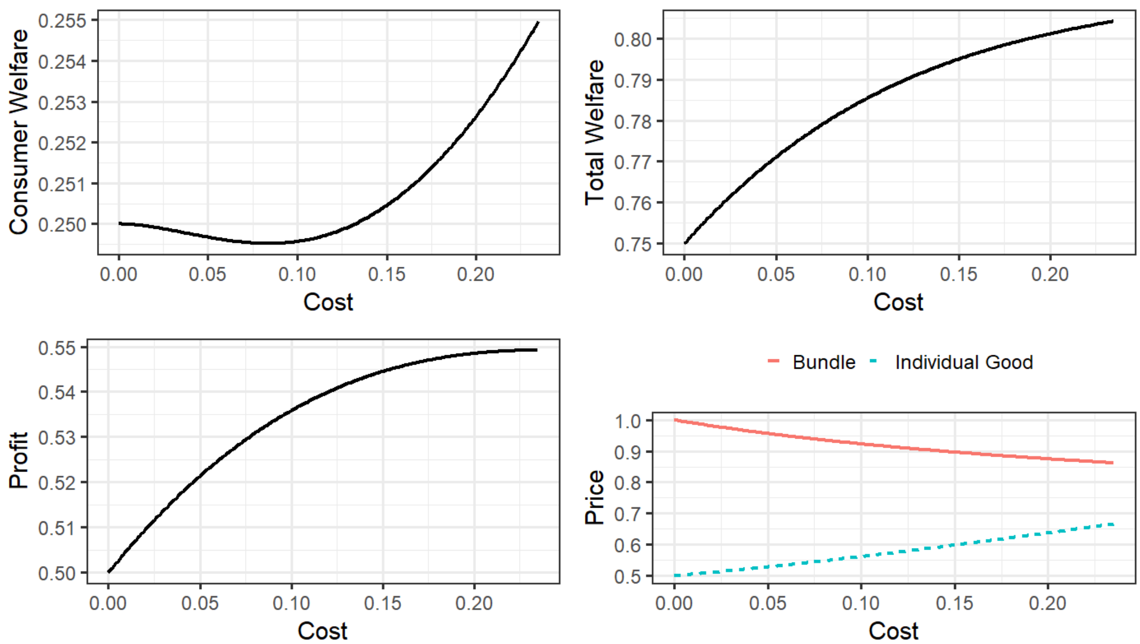
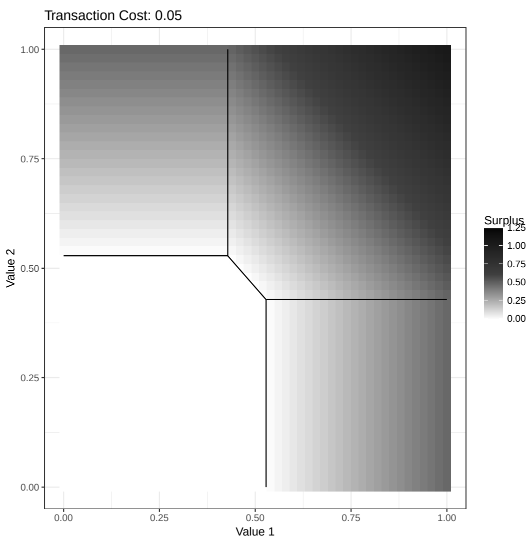


Figure 4.3: Consumer welfare, total welfare, profit, and prices when F is uniform.

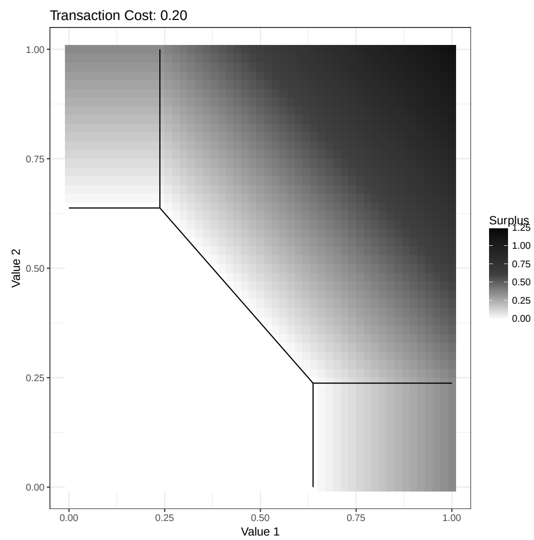
Total welfare is monotone in the cost of resale in this example, but the change in consumer welfare is not. It is decreasing at low values of c and reaches its minimum around $c = .08$. As c increases, consumer welfare starts to increase and attains its highest level at $c = \frac{\sqrt{2}}{6}$.

The non-monotonicity is consistent with Theorem 3. The change in consumer welfare is driven by inframarginal consumers, with an increase in c benefiting consumers who buy the bundle and harming consumers who buy individual goods. As c varies, the bundle price falls and the price of individual goods rises. At low values of

c, the number of consumers buying individual goods is high relative to the number buying the bundle, so the left side of equation (4.1) is large and consumer welfare falls. But as c increases and more consumers purchase the bundle, the change in consumer welfare turns positive. The change in the set of consumers purchasing each good as c increases is illustrated for $c = .05$ and $c = .2$ in Figure 4.4.



(a) $c = .05$



(b) $c = .2$

Figure 4.4: Consumer welfare for $c = .05$ (left panel) and $c = .2$ (right panel) when F is uniform. The solid lines separate the regions purchasing each good.

4.4 Heterogeneous Costs of Resale

In this section, I show that the monopolist may allow resale in equilibrium when consumers have heterogeneous costs of resale. Heterogeneous costs introduce a new tradeoff to the monopolist's problem and break the relationship between costs of resale and the bundle discount. Still, the model shares some predictions with the homogeneous cost model, such as that resale harms the seller's profit and has an ambiguous effect on total and consumer welfare.

The monopolist allows resale in equilibrium because preventing more consumers from reselling reduces the profit earned from high-cost types. Some consumers may have very low costs, forcing the monopolist to set a minuscule bundle discount if it wants to prevent resale. Instead, it may prefer a higher discount that earns more from other consumers but allows low-cost consumers to resell. The tradeoff is presented in Example 1.

Example 1. Suppose that values are distributed uniformly, half of consumers have $c = .2$, the other half have $c = 0$, and that values and costs are independent. The monopolist has two choices: it can prevent all resale by using component pricing ($P_1 = P_2 = .5$, $P_B = 1$) and earn profit .5, or it can offer a larger bundle discount to those with $c = .2$ and allow consumers with $c = 0$ to resell.

The monopolist's optimal prices are $P_B = .9231$ and $P_1 = P_2 = .6615$. The monopolist earns $\pi = .5218 > .5$ and allows the types with $c = 0$ to resell.

To prevent resale, the monopolist in Example 1 would have to abandon bundling altogether. Instead, it prefers to allow half of consumers to resell so that it can discriminate among the other half. The example also demonstrates that the monopolist's optimal prices do not maximize profit earned from consumers who do not resell. The optimal prices if all consumers have $c = .2$ involve the lower bundle price $P_B = .875$; the monopolist adjusts its prices to earn more from the consumers who resell.

Heterogeneous costs are a plausible explanation for observed resale. Some consumers are keen to pocket a few dollars by purchasing resold goods while others do not bother. Sellers are almost certainly aware that consumers can resell parts of the bundle, demonstrated by efforts to prevent resale, but they may continue to bundle as long as there is relatively little resale.

4.4.1 Equilibrium

For simplicity, assume that $F(v_1, v_2)$ is symmetric so that the monopolist's optimal prices satisfy $P_1 = P_2$. Because they are equal, I use P_i to refer to P_1 and P_2. Let $c^* \equiv \frac{1}{2}P_B - P_i$ and

$$\mu_B(P_B, c) = \int_{\frac{1}{2}P_B - c}^{1} \int_{\frac{1}{2}P_B - c}^{1} f(v_1, v_2)\, dv_1\, dv_2 - \int_{\frac{1}{2}P_B - c}^{\frac{1}{2}P_B + c} \int_{\frac{1}{2}P_B - c}^{P_B - v_1} f(v_1, v_2)\, dv_2\, dv_1, \tag{4.5}$$

$$\mu_R(P_B, c) = \int_{\frac{1}{2}P_B + c}^{1} \int_{0}^{\frac{1}{2}P_B - c} f(v_1, v_2)\, dv_2\, dv_1, \tag{4.6}$$

$$\pi_R(P_B, G, c^*) = \int_{\underline{c}}^{c^*} P_B(\mu_B(P_B, c) + \mu_R(P_B, c))\, dG(c). \tag{4.7}$$

The critical cost c^* gives the lowest-cost type that does not engage in resale. The function μ_B gives the fraction of consumers with cost $c < c^*$ who purchase the bundle and do not resell when the bundle price is P_B. Similarly, μ_R gives the fraction with $c < c^*$ who purchase the bundle to resell. The function π_R gives profit earned from consumers with $c < c^*$ when c is distributed according to $G(c)$.

Theorem 5. The monopolist's problem is

$$\max_{P}(1 - G(P_i - \frac{1}{2}P_B))\pi_N(P) + \pi_R(P_B, G, P_i - \frac{1}{2}P_B). \tag{4.8}$$

Equilibrium resale prices are $P^s = (\frac{P_B}{2}, \frac{P_B}{2})$ and consumers with $c < c^*$ resell.

Proof. Lemma 4 applies with heterogeneity in c because its proof applies for each type c. By the symmetry of $F(\cdot)$, $P_1 = P_2$ and so equilibrium resale prices are $P^s = (\frac{P_i}{2}, \frac{P_i}{2})$ in any equilibrium with resale transactions.

For prices $P = (P_i, P_B)$, all types with $c \geq P_i - \frac{1}{2}P_B$ find resale too costly. The monopolist therefore earns $\pi_N(P)$ from a fraction $1 - G(P_i - \frac{1}{2}P_B)$ consumers.

All other consumers are willing to share the bundle through resale. Surplus maximization with prices $(\frac{1}{2}P_B + c_k, P_B)$ implies that for types c_k satisfying $c_k < P_i - \frac{1}{2}P_B$, a fraction $\mu_R(P_B, c_k)$ acquire good 1 through resale and a fraction $\mu_B(P_B, c)$ purchase the bundle. For type c_k, the monopolist sells $\mu_R(P_B, c_k)$ bundles to be shared and $\mu_B(P_B, c_k)$ to consumers who do not resell. Integrating over all such types, it earns $\pi_R(P_B, G, P_i - \frac{1}{2}P_B)$ from types who resell. $\square$

The monopolist now must consider the profit earned from consumers who do not resell (the first term), consumers who resell (the second), and the number of consumers in the two groups (which depends on c^*). The result is a tradeoff: the monopolist can generally earn more from consumers who do not resell by increasing its bundle discount, but by doing so it increases the number consumers who engage in resale and contribute less to profit.

The tradeoff substantially enriches the monopolist's problem. In Section 4.3, the effect of resale was to cap its bundle discount. Now, resale forces the monopolist to strike a balance between the profitability of its prices for consumers without resale and the number of consumers who engage in arbitrage at those prices. Only the model with heterogeneous costs describes the problem facing the seller. Moreover, the tradeoff is essential for understanding the market because it explains why the monopolist might allow resale in equilibrium.

4.4.2 Resale in Equilibrium

Example 1 suggests that resale is possible in equilibrium. I provide a sufficient condition for resale in equilibrium in Theorem 6.

Theorem 6. Assume that the prices maximizing $\pi_N(P)$ subject to $P_i \leq \frac{1}{2}P_B + \underline{c}$ have $P_i = \frac{1}{2}P_B + \underline{c}$. The monopolist allows the lowest-cost type $\underline{c}$ to resell in equilibrium if $2\underline{c}\mu_R(P_B, \underline{c})g(\underline{c}) < \partial\pi_N(P)/\partial P_i$.

Proof. Assume that the monopolist prevents consumers with cost $\underline{c}$ from reselling. By assumption, the monopolist's optimal prices have $P_i = \frac{1}{2}P_B + \underline{c}$. I show that the monopolist can increase profit by increasing P_i when $g(\underline{c})$ is small relative to the change in profit. The derivative of profit with respect to P_i at $\underline{c}$ is

$$
\begin{aligned}
\frac{\partial}{\partial P_i}\pi(P') &= g(\underline{c})\left[P_B\left(\mu_B(P_B, \underline{c}) + \mu_R(P_B, \underline{c})\right) - \pi_N(P')\right] + (1 - G(\underline{c}))\frac{\partial \pi_N(P')}{\partial P_i} \\
&= g(\underline{c})\left(-2\underline{c}\mu_R(P_B, \underline{c})\right) + (1 - G(\underline{c}))\frac{\partial \pi_N(P)}{\partial P_i} \\
&= -2\underline{c}g(\underline{c})\mu_R(P_B, \underline{c}) + \frac{\partial \pi_N(P)}{\partial P_i}.
\end{aligned}
$$

The second step relies on the fact that $P_B(\mu_B(P_B, \underline{c}) + \mu_R(P_B, \underline{c})) - \pi_N(P') = -2\underline{c}\mu_R(P_B, \underline{c})$, which is true because the seller earns $2\underline{c}$ on each formerly resold transaction when type $\underline{c}$ moves to the primary market. The conclusion follows by setting the derivative greater than or equal to zero. □

Theorem 6 formalizes the intuition from Example 1. By preventing the lowest-cost type from reselling, the monopolist can gain $2\underline{c}$ from each transaction that used to involve resale. But it might not be worthwhile to do so because the monopolist must change its prices and earn less in profit from consumers who did not resell.

The result is illustrative but not necessary. It only applies to the lowest-cost type and there could be a global optimum where resale is tolerated even if there are no local improvements.[6] Nonetheless, the intuition explains why the monopolist might

[6]The complexity of the seller's problem prevents me from obtaining necessary conditions.

allow many types to resell. Table 4.1 presents examples in which more than a third of consumers have costs low enough to resell at the monopolist's optimal prices.

Theorem 6 requires a mild assumption, that the constraint $P_i \leq \frac{1}{2}P_B + \underline{c}$ binds. The assumption is needed to use the derivative at $\underline{c}$ and holds whenever the monopolist can increase profit by using a larger bundle discount.

4.4.3 Comparative Statics

Next I consider how pricing, profit, and welfare change when the distribution of costs of resale changes. Corollary 4 established an ironclad conclusion for pricing in the benchmark model: the monopolist's bundle discount is weakly increasing in the cost of resale, implying more intense price discrimination when resale becomes more costly. Surprisingly, the conclusion is no longer true with a distribution of costs.

Why might the monopolist reduce its discount and bundle less intensely when there are stronger barriers to resale? Unlike in Section 4.3, the monopolist's prices do not necessarily maximize profit from consumers who do not resell for a given bundle discount—they also affect sales to consumers who resell. It is therefore possible that, after resale becomes more costly, the seller might choose a smaller discount that earns more from consumers who do not resell.

Rows 3 and 4 of Table 4.1 provide an example. The distribution in row 4 first-order stochastically dominates the one in row 3, yet the monopolist sets a smaller discount when costs of resale are higher. The example demonstrates that simple models without resale in equilibrium do not capture the seller's incentives for pricing. However, it remains true that profit increases when resale becomes more difficult.

Lemma 6. At the monopolist's optimal prices P^*, for all $c \leq c^*$,

$$\pi_N(P^*) \geq P_B^* \left(\mu_B(P_B^*, c) + \mu_R(P_B^*, c)\right).$$

Proof. Let $\tilde{c} = \sup_{c \in [\underline{c}, c^*]} \{P_B^* \left(\mu_B(P_B^*, c) + \mu_R(P_B^*, c)\right)\}$. Assume for contradiction that $\pi_N(P^*) < P_B^* \left(\mu_B(P_B^*, \tilde{c}) + \mu_R(P_B^*, \tilde{c})\right)$. If the monopolist deviated to set $P = (\frac{1}{2}P_B^* + \tilde{c}, P_B^*)$, then it would strictly increase profit from consumers with $c \geq \tilde{c}$ and earn the same amount from consumers with $c < \tilde{c}$. □

Theorem 7. Let P be the optimal prices for the distribution $G(c)$ and let $\tilde{G}$ be a distribution such that $\tilde{g}(c) \leq g(c)$ for all $c \leq c^*$. Then the firm's profit is higher at $\tilde{G}(c)$ than $G(c)$.

Proof. I show that the monopolist earns higher profit at P under $\tilde{G}$ than under G. Let $\Delta G(c) = G(c) - \tilde{G}(c)$, which satisfies $\Delta G(c) \geq 0$ for $c \leq c^*$. Consider profit from resellers at G and use the fact that $\pi_N(P) \geq P_B \left(\mu_B(P_B, c) + \mu_R(P_B, c)\right)$ for $c \leq c^*$ by Lemma 6.

$$\begin{aligned}
&\int_{\underline{c}}^{c^*} P_B \left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) dG(c) \\
&= \int_{\underline{c}}^{c^*} P_B \left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) d\Delta G(c) + \int_{\underline{c}}^{c^*} P_B \left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) d\tilde{G}(c) \\
&\leq \int_{\underline{c}}^{c^*} \pi_N(P)\, d\Delta G(c) + \int_{\underline{c}}^{c^*} P_B \left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) d\tilde{G}(c) \\
&= (G(c^*) - \tilde{G}(c^*))\pi_N(P) + \int_{\underline{c}}^{c^*} P_B \left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) d\tilde{G}(c).
\end{aligned}$$

Let $\pi^G(P)$ give profit for prices P under the distribution of costs G. Substituting into the profit function at G gives

$$
\begin{aligned}
\pi^G(P) &= (1 - G(c^*))\pi_N(P) + \int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) dG(c) \\
&\leq (1 - G(c^*))\pi_N(P) + (G(c^*) - \tilde{G}(c^*))\pi_N(P) + \\
&\int_{\underline{c}}^{c^*} P_B\left(\mu_B(P_B, c) + \mu_R(P_B, c)\right) d\tilde{G}(c) \\
&= \pi^{\tilde{G}}(P).
\end{aligned}
$$

□

Theorem 7 is directly analogous to Corollary 4 in the original model. The assumption on the distributional shift is necessary because profit earned from resellers is not necessarily monotone in c.[7] The distributional shifts between each pair of rows in Table 4.1 satisfy the assumptions of Theorem 7. As expected, profit increases in each case.

Changes in consumer and total welfare remain ambiguous, but the expressions are far more complex than in Section 4.3. In the more stylized model, it is possible to consider the derivative of prices with respect to the cost, reducing welfare changes to the effect of price changes on marginal and inframarginal buyers. Such conclusions are not possible with a distribution of costs because a shift in the distribution does not lead to a smooth change in optimal prices.

Consider consumer welfare, which is now the integral over c of the welfare earned by consumers with each cost of resale c. Let $CW_N(P_i, P_B)$ denote consumer welfare without resale when prices are (P_i, P_B). Consumers with $c \geq c^*$ earn $CW_N(P_i, P_B)$ and all others earn $CW_N(\frac{P_B}{2} + c, P_B)$, making overall consumer welfare with distri-

[7] With non-monotonicity, it is possible to have some $\tilde{G}(c) \geq_{FOSD} G(c)$ at which P is less profitable, for instance if profit from resellers is decreasing in c on some interval and $\tilde{G}(c)$ shifts mass upwards only on that interval.

bution of costs $G(c)$

$$CW(G,P) = (1 - G(c^*))CW_N(P_i, P_B) + \int_{\underline{c}}^{P_i - \frac{1}{2}P_B} CW_N(\frac{1}{2}P_B + c, P_B)\, dG(c). \quad (4.9)$$

If the change in optimal prices were continuous, the change in consumer welfare for consumers with each cost c would depend only on inframarginal consumers, as in Theorem 3. Consumers with $c < c^*$ would only be affected by the change in P_B when the monopolist changes prices.

If prices jump when G shifts to G', however, some consumers will switch purchases and earn surplus. The overall change in consumer welfare includes both the inframarginal consumers and the switchers, adding the changes in surplus for each cost c weighted by the change in the number of consumers with that cost across the distributions. As before, the change is ambiguous: Table 4.1 contains examples in which consumer welfare moves in each direction as the distribution of costs shifts upwards.

The story is similar for total welfare. Let $TW_N(P_i, P_B)$ denote consumer welfare without resale when prices are (P_i, P_B). Total welfare $TW(G,P)$ for the distribution G is

$$TW(G,P) = (1 - G(c^*))TW_N(P_i, P_B) + \int_{\underline{c}}^{P_i - \frac{1}{2}P_B} TW_N(\frac{1}{2}P_B + c, P_B)\, dG(c). \quad (4.10)$$

Without a continuous change in prices when G shifts to G', the change in total welfare cannot be distilled to the values of marginal buyers. But the main insight of Theorem 4 applies because, for consumers with cost c, the change in total welfare is driven by consumers who change their purchase decisions at the new prices. The aggregate change is the sum of those changes, weighted by the change in the number of consumers with type c between the distributions. The change remains ambiguous, as demonstrated by the comparisons in Table 4.1.

4.4.4 Examples

To illustrate equilibrium and the comparative statics, I simulate the market for various distributions of costs of resale when values are distributed uniformly. The examples are paired, with resale becoming more costly from the first to the second distribution. The second distribution always dominates the first in the sense of first-order stochastic dominance; the changes also satisfy the stronger criterion in Theorem 7.

The examples use three types of distributions of costs: normal distributions, uniform distributions on $[0, .23]$ (ending at approximately the optimal bundle discount without resale), and a split uniform distribution. The split uniform distribution $SplitUnif(x, y)$ spreads probability y evenly on the interval $[0, x]$ and probability $1-y$ evenly on $[x, .23]$. The split uniform is used in the third, fifth, and sixth rows. For example, in the third row, it spreads .3 of the probability between 0 and .02 and the rest between .02 and .23.

In each shift, profit increases after resale becomes more difficult. In the shift from row 3 to row 4, the optimal bundle discount falls. Consumer and total welfare decline in the first three comparisons but increase in the last.

4.5 Conclusion

The goal of this paper has been to explain why sellers practice mixed bundling when consumers can resell. I showed that bundling and resale are possible in equilibrium

	P_i	P_B	c^*	π	π_N	CW	TW
$N(.1, .05)$	0.5476	0.9431	0.0761	0.5210	0.5297	0.3354	0.8564
$N(.14, .05)$	0.5615	0.9271	0.0980	0.5282	0.5354	0.3072	0.8354
$SplitUnif(.02, .3)$	0.5696	0.9481	0.0956	0.5156	0.5345	0.4129	0.9285
$Unif(0, .23)$	0.5639	0.9374	0.0952	0.5206	0.5346	0.3675	0.8881
$SplitUnif(.1, .5)$	0.5569	0.9433	0.0853	0.5187	0.5321	0.3723	0.8910
$SplitUnif(.15, .5)$	0.5785	0.9258	0.1156	0.5245	0.5390	0.3569	0.8814
$N(.08, .001)$	0.5459	0.9372	0.0773	0.5300	0.5300	0.2504	0.7804
$N(.2, .001)$	0.6347	0.8764	0.1965	0.5483	0.5483	0.2525	0.8008

Table 4.1: Examples of equilibrium with a distribution of costs of resale on $[0, .23]$ when values are uniformly distributed. The distribution SplitUnif has mass uniformly distributed on either side of a tilt point. The tilt point is the first argument and the amount of mass to the left is the second.

when consumers have heterogeneous costs of participating in the resale market. The equilibrium with resale materially changes the monopolist's problem, introducing a tradeoff between the number of consumers who resell and the profit earned from consumers who do not. The depth of the monopolist's problem is not present the equilibrium of a benchmark model with a homogeneous cost of resale and no resale in equilibrium.

Both the full model and the benchmark shed light on the effect of resale on bundling, which had not yet been explored formally despite the number of bundles that are resold. The results establish that the ability to resell is an important determinant of the returns to bundling, and that resale has an ambiguous effect on consumer and total welfare in markets where the seller bundles.

Chapter 5

Conclusions

This dissertation studies the effects of common reallocation mechanisms in markets for perishable goods. It demonstrates that there is no singular effect of reallocation—the results depend on both the motive for reallocation and the mechanism. Each chapter considers a different combination of motives and mechanisms.

The analysis has policy relevance for the ongoing debate over resale. In the market for event tickets, governments have alternately prohibited and protected resale, but for most goods, the right to resell is protected in the United States under the first-sale doctrine. A simple question lies beneath the different policies: is resale harmful or beneficial? I show that when consumer values are uncertain resale enhances profit and welfare in the market for college football tickets. In theory, resale is always welfare-enhancing and boosts profit when the seller's capacity is sufficiently small. In a separate setting where consumer values are certain, resale harms profit and has an ambiguous effect on welfare. However, it is not clear that the right to resell should be protected when consumers have uncertain values. Doing so might make it harder to offer refund contracts, which I show to be more efficient than resale in the market for college football tickets. The same conclusions are relevant when assessing sellers' attempts to limit resale.

The essays are also informative for aftermarket design. The analysis considers three criteria for the choice of reallocation mechanism: flexibility, frictions, and incentives to distort. The flexibility of resale prices makes it possible for resale to be most profitable when there is demand uncertainty, but refunds perform better empirically because of search frictions incurred when consumers browse both primary and resale markets. If there were no aggregate demand uncertainty or frictions, the most efficient reallocation mechanism remains ambiguous because the resale market operator and primary market seller both have incentives to impede reallocation. The optimal strategy depends on the responsiveness of the advance price to the ease of resale.

Lastly, the analysis sheds light on the empirical performance of resale markets. Resale could raise or reduce profit for a capacity-constrained seller of perishable goods, and I show that a seller of football tickets ultimately profits from resale. But the analysis also shows a drawback to resale: search frictions. I find that the frictions are significant and harm the performance of resale relative to refunds.

More work is required to understand the effects of reallocation in perishable goods markets. In particular, the analysis presented here cannot evaluate some frictions that may differ between resale and refunds. For example, some unused goods that are not listed for resale might be redeemed for a refund and sold again to a willing consumer. It is also possible that more goods are listed for resale than would be returned for a refund. Settling the matter will require data on usage and observed resale and refund schemes.

Appendix A

Appendix to Chapter 2

A.1 Uncertainty and Sales Strategies Details

This section provides a more detailed derivation of the equilibria of the examples in Section 2.2. Recall that there is one ticket to be sold over two periods, that the seller commits to a menu of prices at the start of the first period, and that three demand shocks have known distributions in the first period and known realizations in the second. Consumer i has valuation ν_i before shocks arrive. The shocks are

1. An independently drawn shock with probability ψ that changes consumer i's value from ν_i to zero.

2. An aggregate shock V changing consumer i's value to $\nu_i + V$.

3. A realized state ω changing consumer i's value to $\nu_i - b_i(\omega)$. States are $\omega \in \{\omega^B, \omega^G\}$ with $b_i(\omega^G) = 0$ and $b_i(\omega^B) \geq 0$.

A.1.1 Idiosyncratic Uncertainty

Alice arrives in the first period with $\nu_A = 50$ and has probability $\psi = \frac{1}{5}$ of receiving an idiosyncratic shock. If she waits to purchase until the second period, her value falls to 40. Bob arrives in the second period with $\nu_B = 40$ and does not receive a shock. When there is a resale market, the fee charged by the resale market operator is $\tau = \frac{1}{10}$.

Partial Refunds

The optimal price in the second period is $p_2 = 40$. In the first period, Alice knows that she will earn zero surplus by waiting to purchase so she can be charged up to her expected surplus from buying in the first period,

$$p_1 = (1 - \psi) \cdot 50 + \psi r. \tag{A.1}$$

Any (p_1, r) pair with $50 \geq r \geq 0$ satisfying the expression achieves the same final allocation, profit, and welfare. For simplicity, suppose that the seller offers $r = 5$, leaving $p_1 = 41$. Alice purchases the ticket.

With probability $\frac{4}{5}$, Alice does not receive an idiosyncratic shock and uses the ticket, generating total welfare of 50 and profit of 41. With probability $\frac{1}{5}$, Alice returns the ticket, yielding a net profit of $41 - 5 = 36$ on the first sale and 40 when the ticket is sold again to Bob. The total profit in this case is 76 and welfare is 40. Expected profit and welfare are 48.

Resale

The seller sets $p_2 = 40$ to ensure that Alice purchases in the first period.[1] Alice knows that if she receives a shock, she can resell to Bob at 40 and will receive $(1-\tau)40 = 36$. She is thus willing to pay

$$p_1 = (1 - \psi) \cdot 50 + \psi p_2^{\text{resale}} = 40 + \frac{1}{5}36 = 47.2. \tag{A.2}$$

The seller can again extract all of Alice's surplus and sets $p_1 = 47.2$, earning profit of 47.2. Total welfare remains 48.

[1]Doing so is optimal because the seller wants to sell to Alice in the first period: expected profit exceeds 40 when $p_2 = 40$.

A.1.2 Idiosyncratic and Common Value Uncertainty

Suppose there is also an aggregate shock: $V = 0$ with probability $\frac{3}{4}$ and $V = -20$ with probability $\frac{1}{4}$.

Partial Refunds

The seller again offers $p_2 = 40$.[2] In the first period, it can charge Alice

$$p_1 = (1 - \psi)(\frac{3}{4} \cdot 50 + \frac{1}{4} \cdot 30) + \psi r, \tag{A.3}$$

where $r \leq 30$ so that Alice only returns the ticket after an idiosyncratic shock. There are again many optimal pairs of (p_1, r). Without loss of generality, the seller offers $r = 5$ and charges $p_1 = 37$.

Alice contributes 37 to profit and 45 (in expectation) to total welfare with probability $\frac{4}{5}$. The remaining $\frac{1}{5}$ of the time, the seller earns a net of 32 from Alice and 40 from Bob with probability $\frac{3}{4}$ and 0 from Bob with probability $\frac{1}{4}$. Profit and total welfare differ from the optimal level because of the case where Alice returns the ticket and Bob does not purchase because $V = -20$ and $p_2 = 40$. Expected profit and welfare both equal 42.

Resale

With resale, the seller sets $p_2 = 40$ so that Alice buys the ticket in the first period. If Alice has an idiosyncratic shock she resells to Bob at price 40 when $V = 0$, earning 36 after fees, or 20 when $V = -20$, earning 18 after fees. The seller sets p_1 to extract Alice's full surplus,

[2]Setting $p_2 = 20$ is not optimal because, even if the seller extracted all of Alice's surplus in the first period, it would prefer to earn $\frac{3}{4} \cdot 40 + \frac{1}{4} \cdot 0 > 20$ when Alice has an idiosyncratic shock.

$$p_1 = (1-\psi)\cdot(\frac{3}{4}\cdot 50 + \frac{1}{4}\cdot 30) + \psi(\frac{3}{4}\cdot 36 + \frac{1}{4}\cdot 18) = 42.3. \quad \text{(A.4)}$$

With resale, the ticket is always allocated to the consumer with the highest value, yielding total welfare of $\frac{4}{5}\cdot 45 + \frac{1}{5}\cdot 35 = 43$. Profit is 42.3 because .7 is paid as fees to the resale market operator in expectation.

A.1.3 States of the World

The states ω^G and ω^B each occur with probability $\frac{1}{2}$. Alice has value 40 in each state, but Bob has value 50 in state ω^G and 10 in state ω^B. All sales must occur in the first period, but the state is not realized until the second period.

No Reallocation

Without reallocation, the seller prefers to sell to Alice at $p = 40$ than to Bob at $p = 30$. Profit and welfare are both 40.

Resale

With resale, Bob can resell to Alice in state ω^G at price 40, earning 36. The seller can thus charge Bob up to

$$p = \frac{1}{2}\cdot 50 + \frac{1}{2}36 = 43. \quad \text{(A.5)}$$

Profit is 43 and total welfare is maximized at 45.

State-Dependent Refund Contracts

The seller can offer a contract granting a full refund in state ω^G at price 40, which Alice is willing to purchase, and another granting a full refund in state ω^B at price

50, which Bob is willing to purchase. Total welfare is again maximized at 45. Profit is now $\frac{1}{2}40 + \frac{1}{2}50 = 45$.

A.2 Estimation Details

A.2.1 Distribution of V

The estimation procedure for the distribution of V uses the normalized prices defined in equation (2.1),

$$\text{NormalizedPrice}_{uy} = \text{AvgResalePrice}_{uy} / \left(\frac{1}{Y} \sum_{y} \text{AvgResalePrice}_{uy} \right). \tag{2.1 revisited}$$

The distribution of V is based on residuals from the regression

$$\text{NormalizedPrice}_{uy} = \beta_y Season_y + \varepsilon_{uy}. \tag{A.6}$$

The residuals form the distribution in Figure 2.5, which can be interpreted as percent deviations from mean prices. To recover the magnitude of the deviations for the university, I multiply the residuals by the university's mean price, which is adjusted to reflect time trends for the relevant year.

To recover σ_V^2, the distribution must be adjusted for the effect of α_j. The adjustment is necessary because changes in V affect utility and hence resale prices as $\alpha_j V$. Under the assumptions that changes in V linearly affect resale prices and that deviations in annual resale prices are solely due to changes in V,

$$\text{NormalizedPrice}_{uy} - 1 = V_y \sum_j w_{jy} \alpha_j \tag{A.7}$$

$$(\text{NormalizedPrice}_{uy} - 1) \left(\sum_j w_{jy} \alpha_j \right)^{-1} = V_y \tag{A.8}$$

where the vector w_{jy} sums to one and determines the relative importance of each game. SeatGeek does not describe how their averages are computed, so I assume that they are an average of all transactions on their platform and weight the α_j parameters by number of resale transactions. The resulting standard deviation is 7.85.

A.2.2 Vaccine Demand

Recall from Section 2.6 that the estimated distribution of values from structural estimation, parameterized by λ_ν, reflects demand before covid-19. The survey results suggest that demand with a vaccine is different, as illustrated in Figure 2.6.

Section 2.6 explains how the change in values $b_i(\omega^{\text{Vax}})$ is estimated. In the application with states of the world, I adjust values to reflect the change by defining

$$\nu_i' = \nu_i + b_i(\omega^{\text{Vax}}). \tag{A.9}$$

I use the distribution of ν_i' as the distribution of consumer values in the application. The value changes $b_i(\omega^{\text{NoVax}})$ are independent of ν_i'.

Figure A.1 demonstrates that the parametric form of $b_i(\omega)$ fits the data.

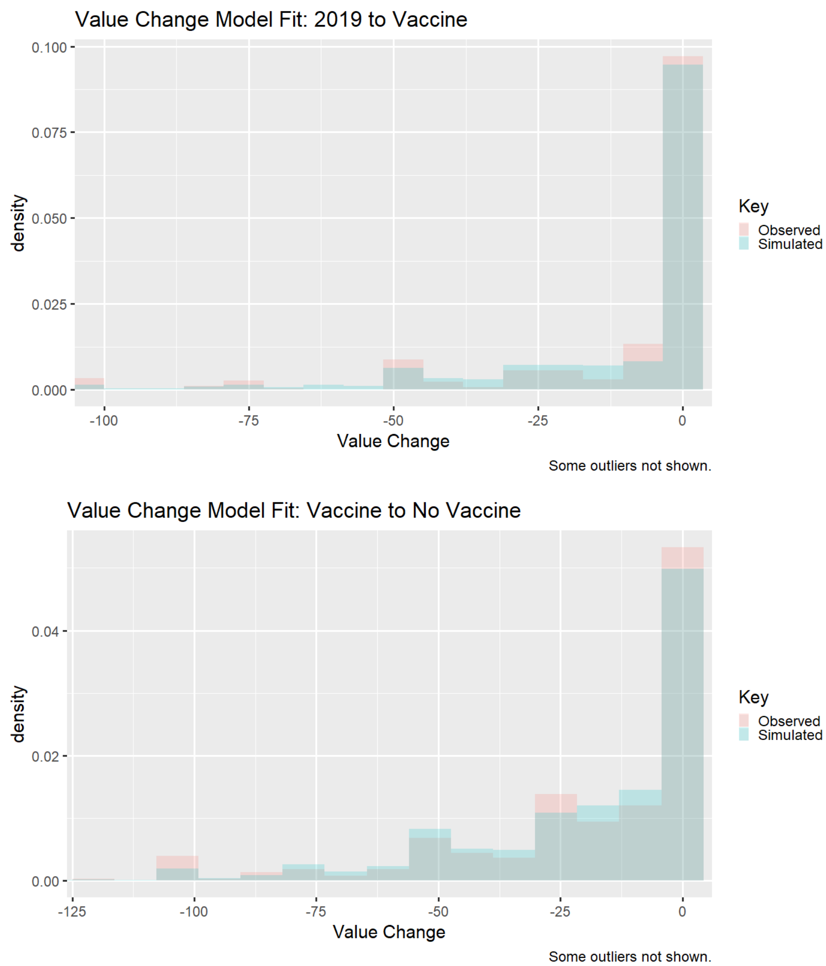


Figure A.1: Observed and simulated changes in willingness to pay. Top panel shows change from 2019 WTP to vaccine WTP. Bottom shows vaccine WTP to no vaccine WTP.

A.2.3 Weights

The weight matrix used in the second stage of estimation has moment variances on the diagonal and zeros elsewhere. Although the inverse covariance matrix is asymptotically optimal for GMM, I am unable to recover the covariances of most estimation moments because they come from separate data sources. Even for resale prices for different games, an observation only contains information about one game and so a sample is not informative about the covariance between games. The resulting weight matrix is consistent but not asymptotically optimal.

I calculate the variance of each moment using the bootstrap. Resale prices for each game are the simplest case. The data contain records of resale transactions and their prices. If there are N_j observed resale transactions for game j, I repeatedly sample N_j draws from the population of transactions and take the variance of the sample average price as the variance for game j.

Calculating the variance is less straightforward for season ticket and primary market quantities because the decision to not purchase is unobserved. In each bootstrap sample, I suppose that there are M total consumers and and take M Bernoulli draws with success probability N_j/M, where N_j is the observed number of tickets purchased. I censor each sample to ensure that no more tickets are sold than are available, then take the variance of the resulting sample means as the moment variance.

One concern with this strategy is that the variance depends on the market size M, which is assumed to be 200,000. If there were no censoring, the variance would follow from M Bernoulli draws with success probability N_j/M,

$$M\frac{N_j}{M}(1-\frac{N_j}{M}) = N_j(1-\frac{N_j}{M}). \tag{A.10}$$

The only dependence on M is mild because M is large relative to the quantity purchased. Consequently, the last term is close to one and the variance is robust to different values of M.

Moment variances are presented in Table A.1.

Table A.1: Variance of estimation moments.

Moment	Variance
Season Tickets Sold	19899.16
Avg. Resale Price: Game 1	0.30
Avg. Resale Price: Game 2	0.43
Avg. Resale Price: Game 3	0.31
Avg. Resale Price: Game 4	0.53
Avg. Resale Price: Game 5	0.16
PM Tickets Sold: Game 1	1262.01
PM Tickets Sold: Game 2	3286.64
PM Tickets Sold: Game 3	994.04
PM Tickets Sold: Game 4	2394.55
PM Tickets Sold: Game 5	495.96

I make two adjustments to model output so that it is comparable to the estimation moments. First, I only use the model's predicted resale prices and quantities for the value of V realized in the data. The model predicts resale prices and quantities for all possible realizations, but only the one for the realized V is comparable. Second, I weight resale prices by the observed average quantity of tickets resold in that quantity for the season. Weighting is necessary because the model predicts resale at the game-quality level and the mix of qualities resold affects the resale price.

A.2.4 Parameter Standard Errors

Standard errors for the first stage are calculated using the bootstrap and the properties of MLE. The errors for the α_j and γ_q parameters are calculated using the bootstrap for samples of resale prices. Similarly, the standard errors for ρ_1^{NoVax}, ρ_2^{NoVax}, ρ_1^{Vax}, and ρ_2^{Vax} are bootstrapped using repeated sampling of survey responses. The standard error of σ^V follows from maximum likelihood.

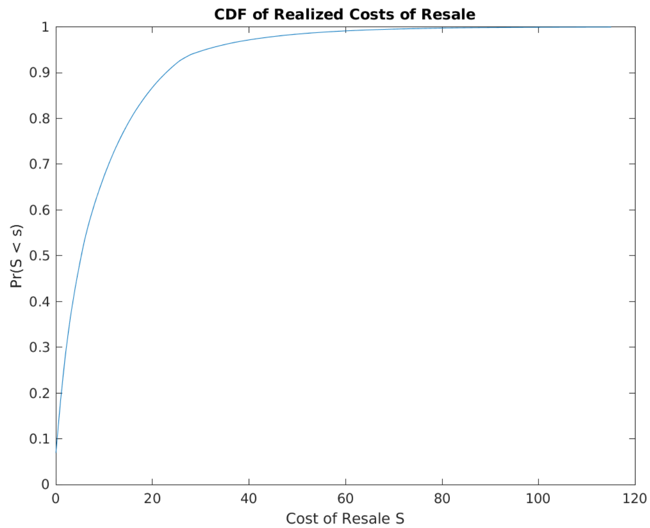


Figure A.2: CDF of realized costs of resale for resale buyers in equilibrium.

Standard errors for structural estimation are also calculated using the bootstrap. I draw a sample of 50 sets of moments from the covariance matrix used to weight moments in estimation and estimate optimal parameters for each set. The first stage parameters are fixed at their point estimates.

A.3 Survey

I surveyed 250 Americans under the age of 50 and 250 Americans aged 50 or over, ultimately receiving a total of 457 usable responses. I distributed the survey through Prolific.co, an online survey distribution platform. Respondents were paid \$9.34 per hour and live in nine states that each have one dominant college football team: Arkansas, Georgia, Louisiana, Michigan, Minnesota, Nebraska, Ohio, West Virginia, and Wisconsin. Respondents from each state were asked to consider one ticket for that team throughout the survey.

The survey refers to the CDC's benchmark for the number of new cases to be near zero, which is 0.7 new cases per 100,000 people. Respondents were given the

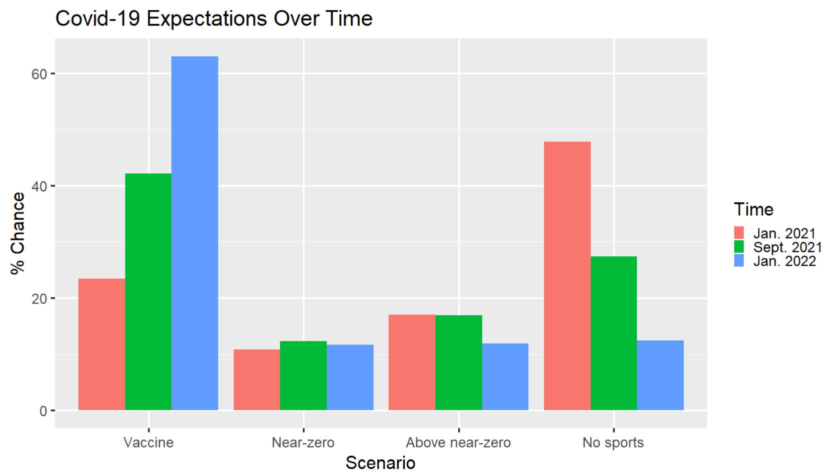


Figure A.3: Average reported percent chance of each scenario occurring in each month.

benchmark and a practical illustration, that a 25,000-seat stadium filled with randomly selected people would contain an average of 2.5 sick people if each case lasts two weeks. They were also told that the true number of infected people would be lower, on average, because some people would know that are ill and decide not to attend.

In the absence of a true measure of the probability of each scenario in the future, I ask respondents how likely they consider each one at three future dates. The average percent chances are shown in Figure A.3. Respondents do not expect a vaccine in January 2021, but think the chance exceeds 40% in September 2021 and 60% in January 2022.

Figure A.4 shows that the distribution of reported WTP is similar for the near-zero and above near-zero scenarios.[3] The distributions are not exactly the same—consumers are more reluctant to attend when there are more cases—but the differences are small enough for the two to be consolidated into a single state without a vaccine. I consolidate WTP as a weighted average, taking the relative probability of

[3]The figure shows reported WTP without social distancing. The analogous figure with social distancing is similar.

the states in September 2021 as the weights.

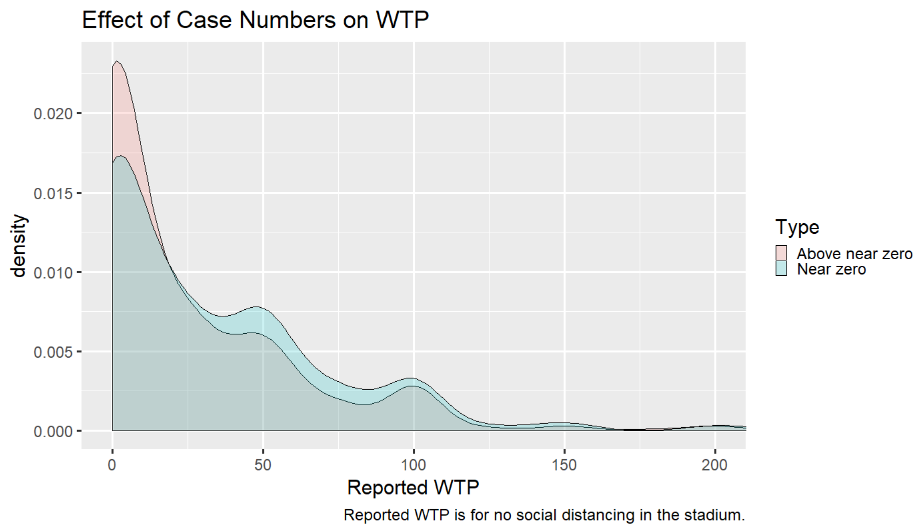


Figure A.4: WTP distributions with near-zero and above near-zero levels of cases.

Figure A.5 shows that the distribution of reported WTP is also similar with and without social distancing. As before, there are some changes, but they are not large enough to treat separately. I use reported values without social distancing because distancing would greatly reduce the number of tickets the seller can offer.

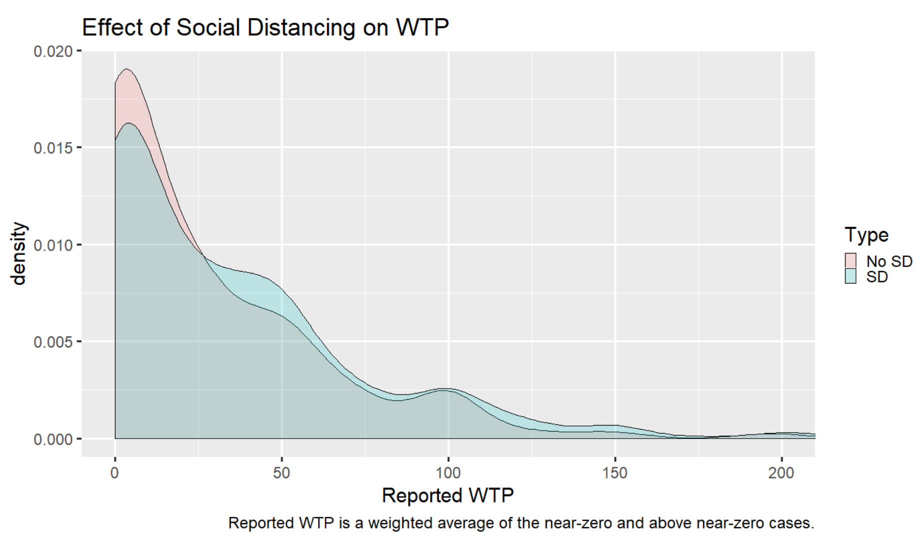


Figure A.5: WTP distributions with near-zero and above near-zero levels of cases.

The full survey is included below.

Event Expectations (General)

Start of Block: Intro

Q1 This study is conducted by Drew Vollmer, a doctoral student researcher, and his advisor, Dr. Allan Collard-Wexler, a faculty researcher at Duke University.

The purpose of the research is to design sales strategies that cope with uncertainty over the covid-19 pandemic. You will be asked about how much you would pay for tickets to an outdoor college football game under several scenarios related to covid-19. The survey should take 5-10 minutes.

We do not ask for your name or any other information that might identify you. Although collected data may be made public or used for future research purposes, your identity will always remain confidential.

Your participation in this research study is voluntary. You may withdraw at any time and you may choose not to answer any question. You will not be compensated for participating.

If you have any questions about this study, please contact Drew Vollmer. For questions about your rights as a participant contact the Duke Campus Institutional Review Board at campusirb@duke.edu.

End of Block: Intro

Start of Block: Block 4

Q16 In which state do you currently reside?

▼ Alabama (1) ... I do not reside in the United States (53)

End of Block: Block 4

Start of Block: WTP

JS

Q2
In this section of the survey, you will be asked how much you are **willing and able to pay for one ticket to a football game**. Your responses should be dollar amounts.

In some questions, you will be given a scenario related to COVID-19. You should respond with the amount you would pay if that scenario occurs. You should not consider how likely the scenario is.

---

*

Q3 What is the **maximum** you would be **willing and able to pay** for **one** ticket...

	Amount (dollars) (1)
one year ago, in Fall 2019? (1)	
if there had not been a global COVID-19 outbreak and the virus had not spread to the US? (2)	
if there is a widely available COVID-19 vaccine? (3)	

Q4
In the next two questions, suppose that there is **no COVID-19 vaccine**, but that fans are allowed to attend sporting events.

You will be asked to consider two levels of risk from the virus:

The CDC says that new cases are **near zero**. The CDC says that new cases are **more than near zero,** but **risk is low enough** to allow fans at sports games.

The CDC standard for new cases to be near zero is 0.7 new cases per 100,000 people or fewer. This means that filling a 25,000-seat stadium with randomly selected people would imply an average of **2.5 sick people** in the stadium if each case lasts two weeks. The true number of infected people at any event, however, would be lower because some people would know they are sick and would not attend.

---

*

Q5
Suppose that there is **no social distancing in the stadium**.

What is the **maximum** you would be **willing and able to pay** for **one** ticket if...

	Amount (dollars) (1)
the CDC says that the number of new cases is **near zero**? (4)	
the CDC says that the number of new cases is **higher than near-zero**, but that the risk from attending mass gatherings is **low enough** to allow fans at sports games? (5)	

---

*

Q6
Suppose that there is **social distancing in the stadium**.

What is the **maximum** you would be **willing and able to pay** for **one** ticket if...

	Amount (dollars) (1)
the CDC says that the number of new cases is **near zero**? (4)	
the CDC says that the number of new cases is **higher than near-zero**, but that the risk from attending mass gatherings is **low enough** to allow fans at sports games? (5)	

Q7
Suppose that fans can return their tickets if the number of new virus cases is higher than near-zero. Tickets are sold out, but there is a **wait list** in case fans who bought tickets return them because of the virus.

What is the maximum you would be willing to pay for a ticket on the wait list?

	Amount (dollars) (1)
No social distancing in the stadium (1)	
Social distancing in the stadium (3)	

End of Block: WTP

Start of Block: Probabilities

Q8
In this section, you will be asked about the likelihood of COVID-19 scenarios. Your answers should be *percent chances*. So, if you believe an outcome has a one-in-four chance of occurring, the percent chance is 25%.

JS *

Q34 What is the *percent chance* of each outcome in **January 2021**? Chances must sum to 100.

Current total: 0 / 100
_______ There is a widely available COVID-19 vaccine. (1)
_______ There is no COVID-19 vaccine and new cases are **near zero**, as defined by the CDC. (2)
_______ There is no COVID-19 vaccine and new cases are **higher than near-zero**, but the CDC considers the risk from mass gatherings is **low enough** to allow fans at sports games. (3)
_______ There is no COVID-19 vaccine, new cases are **higher than near-zero**, and the CDC judges that the risk from mass gatherings is **high enough** that fans cannot attend sports games. (4)

JS *

Q36 What is the *percent chance* of each outcome in **September 2021**? Chances must sum to 100.

Current total: 0 / 100
_______ There is a widely available COVID-19 vaccine. (1)
_______ There is no COVID-19 vaccine and new cases are **near zero**, as defined by the CDC. (2)
_______ There is no COVID-19 vaccine and new cases are **higher than near-zero**, but the CDC considers the risk from mass gatherings is **low enough** to allow fans at sports games. (3)
_______ There is no COVID-19 vaccine, new cases are **higher than near-zero**, and the CDC judges that the risk from mass gatherings is **high enough** that fans cannot attend sports games. (4)

Q35 What is the *percent chance* of each outcome in **January 2022**? Chances must sum to 100.

Current total: 0 / 100
_______ There is a widely available COVID-19 vaccine. (1)
_______ There is no COVID-19 vaccine and new cases are **near zero**, as defined by the CDC. (2)
_______ There is no COVID-19 vaccine and new cases are **higher than near-zero**, but the CDC considers the risk from mass gatherings is **low enough** to allow fans at sports games. (3)
_______ There is no COVID-19 vaccine, new cases are **higher than near-zero**, and the CDC judges that the risk from mass gatherings is **high enough** that fans cannot attend sports games. (4)

End of Block: Probabilities

Start of Block: Demographics

Q12 What is your year of birth?

____

Q13 What is your gender?

- ○ Male (1)
- ○ Female (2)
- ○ Prefer not to answer (3)

Q14 What is your ethnicity?

- ○ Hispanic or Latino/Latina (1)
- ○ Not Hispanic or Latino/Latina (2)

Q15 What is your race?

- ☐ White (1)
- ☐ Black or African American (2)
- ☐ American Indian or Alaska Native (3)
- ☐ Asian (4)
- ☐ Native Hawaiian or Pacific Islander (5)
- ☐ Other (6) ____

End of Block: Demographics

Appendix B

Appendix to Chapter 3

B.1 Preliminaries

Define s_H as the value solving

$$K - a_1 + a_1 G(s_H - \tau) = a_2(1 - G(s_H)) \tag{B.1}$$

and s_L as the value solving

$$a_1 G(s_L - \tau) = a_2(1 - G(s_L)). \tag{B.2}$$

If the monopolist sets price $p_2 = V + s_H$, it sells all K of its units with no excess demand. If it sets the price $p_2 = V + s_L$, then all demand in the second period is satisfied by resellers. Setting $V + s_L$ is not optimal when V is certain, but the constant s_L is useful for analyzing resale when V is uncertain.

B.1.1 Selling All Inventory

For any reallocation strategy to be worthwhile, the seller must exhaust its inventory. If it does not, then reallocation reduces profit by displacing primary market sales. Assumption 3 provides a condition under which reallocation is worthwhile.

Assumption 3. Assume that $V \geq \underline{V}$, where $\underline{V}$ solves

$$\underline{V} = \max\Big\{\frac{(a_1+a_2)(1-G(s_H)) - a_1 G(s_H-\tau)}{a_2 g(s_H) + a_1 g(s_H-\tau)} - s_H, \quad \frac{(a_1+a_2)(a_1+a_2-K)}{a_2^2 g(G^{-1}(\frac{K-a_1}{a_2}))} - G^{-1}(\frac{K-a_1}{a_2})\Big\}. \tag{B.3}$$

When there is uncertainty over V, it is sufficient that assumption 3 hold for the lowest possible value.

Lemma 7. The monopolist sells all of its inventory with resale and no reallocation. If assumption 3 does not hold, the monopolist will not sell all of its inventory for at least one strategy.

Proof. The monopolist's optimal price to sell all inventory with resale is $p_2 = V + s_H$. By Appendix Lemma 4, the first order condition is monotone in price under the assumption on $g'(s)$. Therefore the monopolist will sell all of its units if the first order condition is weakly negative at $V + s_H$. At $p_2 = V + s_H$, the condition is

$$(a_1+a_2)(1-G(s_H)) - a_1 G(s_H-\tau) - (V+s_H)(a_2 g(s_H) + a_1 g(s_H-\tau)) = 0.$$

Solving for V gives the first term in the maximum statement. Next consider profit and its first order condition with no reallocation,

$$\pi^{NR}(p_2) = a_1\left(\int_{-V}^{\infty} V + s\, dG(s) - \int_{p_2-V}^{\infty} V + s - p_2\, dG(s)\right) + p_2 \min\{a_2(1-G(p_2-V)), K-a_1\}$$

$$\frac{\partial}{\partial p_2}\pi^{NR}(p_2) = (a_1+a_2)(1-G(p_2-V)) - a_2 p_2 g(p_2-V) = 0.$$

At the price selling all inventory, $p_2 = V + G^{-1}(\frac{K-a_1}{a_2})$, the condition is weakly negative when

$$
\begin{aligned}
0 &\geq (a_1 + a_2)(1 - G(G^{-1}(\frac{K - a_1}{a_2}))) - a_2(V + G^{-1}(\frac{K - a_1}{a_2}))g(G^{-1}(\frac{K - a_1}{a_2})) \\
V &\geq \frac{(a_1 + a_2)(a_1 + a_2 - K)}{a_2^2 g(G^{-1}(\frac{K-a_1}{a_2}))} - G^{-1}(\frac{K - a_1}{a_2}).
\end{aligned}
$$

As before, the first order condition is monotone decreasing in p_2 and is monotone decreasing in V when $p_2 = V + G^{-1}(\frac{K-a_1}{a_2})$. Thus the monopolist sells all of its inventory when V exceeds the second term in the maximum statement. If V is less than the maximum, then one of the first order conditions will not be satisfied. □

Assumption 3 thus allows us to restrict attention to cases where the monopolist would like to sell all of its inventory, making reallocation worthwhile. However, it does not guarantee that the most extensive form of reallocation is worthwhile. Assumption 4 establishes conditions under which frictionless resale is more profitable than not reallocating, which is necessary for resale to be competitive with refunds.

Assumption 4.

$$
\begin{aligned}
&a_1 \left(G^{-1}(\frac{a_1 + a_2 - K}{a_1 + a_2}) - \int_{-\underline{V}}^{\infty} s \, dG(s) \right) - \\
&(K - a_1) \left(G^{-1}(\frac{a_1 + a_2 - K}{a_2}) - G^{-1}(\frac{a_1 + a_2 - K}{a_1 + a_2}) \right) + \\
&a_1 \int_{G^{-1}(\frac{a_1+a_2-K}{a_2})}^{\infty} s - G^{-1}(\frac{a_1 + a_2 - K}{a_2}) \, dG(s) > 0.
\end{aligned}
\tag{B.4}
$$

Lemma 8. The monopolist prefers resale to no reallocation when $\tau = 0$ if and only if Assumption 4 holds.

Proof. Profit with frictionless resale is $\pi^{SM} = K(V + G^{-1}(\frac{a_1+a_2-K}{a_1+a_2})$. With no reallocation, it sets $p_2 = V + G^{-1}(\frac{a_1+a_2-K}{a_2}))$ and earns

$$\pi^{NR} = a_1 \left(\int_{-V}^{\infty} V + s\, dG(s) - \int_{G^{-1}(\frac{a_1+a_2-K}{a_2})}^{\infty} s - G^{-1}(\frac{a_1 + a_2 - K}{a_2})\, dG(s) \right) + (K - a_1)(V + G^{-1}(\frac{a_1 + a_2 - K}{a_2})).$$

$\pi^{SM} - \pi^{NR}$ reduces to the expression in Assumption 4. □

Assumption 4 is satisfied whenever the total number of consumer arrivals a_1+a_2 is large relative to capacity K. The incentive to reallocate, and thus allow resale, intuitively increases with arrivals: as a_1 increases, more units are sold to consumers who will realize low values, and as a_2 increases, the value of the first unserved consumer increases.

B.1.2 Advance Sales

The analysis assumes that the seller wants to goods in the first period. Doing so could be optimal if consumers want to plan ahead, as for event tickets. However, advance sales are optimal in the model when the seller chooses its profit-maximizing sales strategy.

Lemma 9. Profit and welfare without advance sales are the same as with frictionless resale.

Proof. Without advance sales, the seller sets $p_2 = V + G^{-1}(\frac{a_1+a_2-K}{a_1+a_2})$ to sell all K units without rationing. With frictionless resale, the same price prevails in period two to clear the market; equation (3.2) shows that the seller cannot charge more than p_2 in the first period when τ is zero. Profit and welfare are thus the same. □

Corollary 5. Advance sales with refunds are more profitable than making no advance sales.

Proof. Proposition 1 establishes that the seller strictly prefers refunds to frictionless resale. The result follows from Lemma 9. □

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