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Relational contracts with and between agents

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ABSTRACT

We study a dynamic multi-agent model with a verifiable team performance measure and non-verifiable individual measures. The optimal contract can be interpreted as an explicit contract that specifies a minimum bonus pool as a function of the verifiable measure and an implicit contract that gives the principal discretion to increase the size of the pool and to allocate it among the agents. To mitigate the threat of collusion, the optimal contract often converts any exogenous productive interdependence into strategic payoff independence for the agents. Under productive complements, an unconditional bonus pool (pay without performance) can be less costly than one conditioned on the verifiable team measure.

1. Introduction

Discretion in performance-based compensation is pervasive.[1] It allows the evaluator to take into account information that is difficult to incorporate into an explicit contract (non-verifiable performance measures). A downside is an incentive for the evaluator to use the discretion in such a manner as to withhold compensation. Yet, for 42% in Murphy and Oyer's (2003) sample of 262 firms, the size of the executive bonus pool is subject to discretion; even for those firms that report following a formula to determine the size of the bonus pool, within the prior 5 years, 22% had overridden the formula to pay out a bonus when the formula called for none.[2] In this paper, we study the role of discretion in motivating team members in

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[1] See Baker et al. (1994), Murphy and Oyer (2003), Gibbs et al. (2004), and Ederhof (2010). For evidence on discretion in individual bonus plans, see Bushman et al. (1996) and Gillan et al. (2009).

[2] In recent years, "umbrella" plans have become increasingly popular as a way of allowing for discretionary increases in executive bonus pools without disqualifying the plan as incentive-based according to IRS Rule 162(m). Umbrella or "inside/outside" plans have the outside plan specify (large) maximum payouts as a function of objective metrics and allow for discretionary decreases. Actual bonuses are based on the inside plan, which typically allows for discretionary increases. According to Meridian Consultants (August 2011), an inside plan that specifies that "[t]he Compensation Committee determines that target performance goals were achieved, entitling each covered employee to a target payout … [and increases] the target payout for each employee based on individual modifiers which solely take into account the achievement of subjective performance goals" will qualify as incentive-based compensation as long as there is an outside/umbrella plan used to fund those payments. In a 2012 study of bonus plans among the top 200 S&P 500 companies by James F. Reda & Associates, 42% (50%) of the firms used such an umbrella plan in 2010 (2011). See also Eccles and Crane (1988, pp. 164) on investment banks' use of discretion in determining the size of bonus pools.

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a dynamic setting. Repeated interactions facilitate relational contracting whereby promises (implicit incentives) can be made self-enforcing by the threat of future punishments.

Teams are central to understanding firms (Alchian and Demsetz, 1972). Team members' efforts are typically non-separable: they can be (strategic) productive complements or substitutes.[3] In our model, the principal can use two types of performance measures to provide incentives: a verifiable team measure such as firm-wide or divisional earnings and non-verifiable individual measures such as direct observations of the agents' efforts. A key feature to emerge from our analysis is a link between the team members' productive interdependence and the use of discretion in providing incentives.

Strategic interdependencies enable the agents to better coordinate their actions. This aggravates the problem of collusion (agent–agent relational contracting), while at the same time enhancing principal–agent relational contracting, because agents whose effort productivities are interdependent can punish a reneging principal more severely. To simply illustrate the driving forces in our model, we first study, separately, bonus pools (to highlight collusion) and individual relational contracts (to highlight the principal's ability to commit).[4] We then derive the optimal contract and show that it can be interpreted as a modified bonus pool that gives the principal the discretion to increase the size of the pool.

We define a bonus pool as an incentive scheme under which the agents' total compensation depends only on the verifiable team measure. The non-verifiable individual measures are used only to allocate the bonus pool among the agents. The expected compensation under a collusion-proof bonus pool is particularly high for small discount rates (patient agents can collude more effectively) or strong productive interdependencies. With productive complements, it is costly to bribe an agent to unilaterally deviate from a collusive strategy that has both agents shirking in each period, because the marginal productivity of unilateral effort then is small. With productive substitutes, the most costly form of collusion to upset has the agents taking turns working. The nature of the collusive threat, therefore, is determined by the underlying production technology. Moreover, the more intertwined the agents' marginal productivities, the more severe is the collusive threat.

Conventional wisdom calls for the size of bonus pools to vary with some verifiable measure of performance, in part because fixed-prize tournaments seem especially collusion-prone (e.g., Budde, 2007). However, we show that a bonus pool with an unconditional payout—a form of “pay without performance” (Bebchuk and Fried, 2004)—deals more effectively with the collusion problem in the complements case for high discount rates. While often decried as an outgrowth of managerial power, such a payout policy can therefore be rationalized on collusion-proofness grounds.[5]

As an alternative to bonus pools, the principal can write individual relational contracts with each agent. Tying the total compensation amount to non-verifiable measures is credible only if the principal has more to gain from honoring the agreement than from reneging on it; i.e., it must be self-enforcing.[6] Strategic interdependencies facilitate relational contracts because they increase the punishment the agents can mete out on a reneging principal. Strategic effort complementarity, especially, fosters principal–agent relational contracting because of the potential for multiple equilibria in the agents' effort choice subgame under the fallback contract, which comes into play if the principal ever reneges.[7]

Moving on to general contracts, is there a way to obtain the benefit of strategic interdependence (improving principal–agent relational contracting) without exacerbating the collusion problem? It turns out the answer is yes. The severity of the collusion problem depends on the agents' overall (*strategic*) *payoff interdependence*, which commingles the exogenous productive interdependence and the endogenous wage scheme. This makes the payoff interdependence a choice variable for the principal. For the principal's own credibility in relational contracting, the key force is the *productive* interdependence, because it alone determines the efficiency of the (off-equilibrium) fallback contract, which incorporates only the team measure.

We then characterize the optimal (symmetric) contract. It combines features of bonus pools and relational contracts. For small discount rates or strong productive interdependencies, the optimal contract relies on the principal's reputation and treats the agents independently. As the discount rate increases (or the productive interdependence weakens), the agents' pay becomes intertwined, which opens the door for collusion. To mitigate the collusive threat, the optimal contract often converts any exogenous productive *inter*dependency into strategic payoff *in*dependence for the agents. For large discount rates, the optimal contract converges to a conditional bonus pool under productive substitutability and to an unconditional bonus pool under productive complementarity. The optimal contract can be interpreted as an explicit contract that specifies a minimum bonus pool as a function of the verifiable measure and allows the principal to use the non-verifiable measures to

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[3] An example of complementary efforts is a cross-functional team in which each team member's effort is necessary to pull off a success, say, because each team member provides a unique input. An example of substitutes is a team in which agent effort is interchangeable and there are decreasing returns to total effort.

[4] See Baiman and Rajan (1995) and Rajan and Reichelstein (2006, 2009) on bonus pools, and Bull (1987), MacLeod and Malcomson (1989), Baker et al. (1994), and Levin (2003) on relational contracting.

[5] Prior literature has shown that asymmetric contracts are effective in combating collusion in static settings (e.g., Demski and Sappington, 1984; Ishiguro, 2004; Rajan and Reichelstein, 2009). We show that asymmetric bonus pools can indeed achieve first-best in our setting, but only for strong productive complements. As we show in Section 3.1, in general, the problem becomes intractable because asymmetric contracts give rise to an infinite number of potential asymmetric collusion strategies.

[6] See Bull (1987), MacLeod and Malcomson (1989), Baker et al. (1994), Levin (2003), MacLeod (2003), and Malcomson (2012).

[7] Implementation in one-shot games usually means establishing a unique equilibrium (e.g., Demski and Sappington, 1984; Mookherjee, 1984), while the focus of papers on infinite-period games is typically on whether or not particular behavior can be supported by some equilibrium (e.g., Levin, 2002; Kvaloy and Olsen, 2006). In fact, multiple equilibria are a key ingredient of relational contracting models, since they rely on reneging behavior triggering players to revert to a punishment equilibrium (often the unique equilibrium of the one-shot game).

increase the size of the pool and to allocate it among the agents.[8] Our results caution against empirically interpreting wage interdependence of team members as the source of the collusive threat; what matters is the overall payoff interdependence—the combined effect of wage interdependence and productive interdependence.

Our model predicts increased discretion over compensation in settings with (i) longer collaborative horizons and (ii) stronger productive interdependencies. Our prediction (ii) provides a new perspective on earlier empirical findings. Gibbs et al. (2004) and Murphy and Oyer (2003) find evidence consistent with the idea that strong productive interdependencies create a demand for subjective performance evaluation (increased use of discretion).[9] They argue that objective, verifiable performance measures insufficiently capture such interdependencies and that subjective measures can "plug the holes."[10] Our findings imply similar empirical associations, yet the underlying logic (and causality) is different. Non-verifiable measures tend to be more targeted and hence less noisy. As a result, the principal *prefers* them to verifiable (but more aggregate) measures but is constrained by commitment problems. As our results show, these commitment problems are alleviated by productive interdependencies. Hence, productive interdependencies *facilitate* the use of implicit contracts rather than create a need for them.

Bonus pools have been studied analytically by Baiman and Rajan (1995), Rajan and Reichelstein (2006, 2009), and Ederhof et al. (2011). These authors have shown that a self-imposed budget balancing constraint in the compensation payout allows the principal to tie bonuses to non-verifiable metrics. This literature has largely focused on static models and therefore remained silent on the issue of relational contracting and collusion.[11] Arya et al. (1997) and Che and Yoo (2001) study beneficial agent–agent side contracting (mutual monitoring) but do not address principal–agent relational contracting. Kvaloy and Olsen (2006) study both kinds of relational contracting in a setting without team production but rule out bonus pools by assuming the principal cannot commit to a bonus floor.[12] Our paper is the first to study relational contracting and collusion simultaneously in a team production setting.

The remainder of the paper is organized as follows. Section 2 presents the basic model. Section 3 studies bonus pools and individual relational contracts. Section 4 derives the optimal symmetric contract, and Section 5 concludes.

2. Model

A principal contracts with two agents, $i = A, B$, over an infinite horizon. The agents simultaneously provide personally costly efforts $a_t^i \in \{L, H\}$ in period t, where $L = 0 < H$. Let $\mathbf{a}_t = (a_t^A, a_t^B)$. In a joint and stochastic fashion, these efforts result in concurrent team output, $x_t \in \{0, 1\}$. The production technology is the same in each period. In particular, let

$$p_H \equiv Pr(x_t = 1 | \mathbf{a}_t = (H, H)) > p \equiv Pr(x_t = 1 | a_t^A \neq a_t^B) > p_L \equiv Pr(x_t = 1 | \mathbf{a}_t = (L, L)).$$

We distinguish between two cases regarding the team production technology: efforts are either *(strategic) productive complements* in that

$$\Delta \equiv (p_H - p) - (p - p_L) > 0,$$

i.e., $p < (p_H + p_L)/2 \equiv p_o$, or (*strategic*) *productive substitutes* in that $\Delta < 0$ (i.e., $p > p_o$); see Footnote 3 for examples.

The team output x_t is commonly observable and *verifiable* (contractible). Aside from output, compensation contracts can also depend on signals, y_t^i, about the agents' efforts. For tractability, assume the agents' efforts can be observed by both agents and the principal without noise, i.e., $y_t^i \equiv a_t^i$. Since the individual signals y_t^i are more informative than the team measure x_t, the principal will seek ways to include them in the contract. However, y_t^i are *non-verifiable* metrics. Hence, contractual obligations based on them need to be self-enforcing.[13] As in Che and Yoo (2001), we also assume that communication from the agents to the principal is blocked, to rule out message games.

All players are risk-neutral and share a common discount rate of r, capturing the time value of money (or the probability the relationship will continue at the conclusion of the current period). The agents are protected by limited liability in that

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[8] While relational contracts between the principal and agents enhance efficiency and hence should be bolstered by cultivating long-term relationships, relational contracting among the agents (collusion) harms the principal. Job rotation might mitigate such problems as it shortens the agent–agent relationship horizon without affecting the horizon of the principal–agent relationship. A related argument is made by Tirole (1986) in a hierarchical principal–supervisor–agent model. In Arya and Mittendorf (2004), job rotation facilitates rent extraction because it precludes the ratchet effect; an employee's report about his current job will be used in the future only in dealing with other employees.

[9] Bouwens and Lent (2007) and Baiman and Baldenius (2009) find (empirically and analytically, respectively) that firms' use of non-financial performance measure, which are often non-verifiable, is positively related to interdependencies across divisions.

[10] Subjective measures allow supervisors to incorporate multiple difficult-to-measure factors. See Zimmerman (2010, p. 147) for a discussion.

[11] Ederhof et al. (2011) also address a multi-period setting but with only a single agent.

[12] Two additional papers that study multi-agent relational contracts are Levin (2002) and Rayo (2007). Levin compares bilateral with multilateral contracts and shows that the latter, by aggravating the punishment a reneging principal receives, improve her commitment power. Rayo studies a team setting without an exogenous principal (instead imposing a budget balance constraint) and shows it is sometimes optimal to have one of the agents adopt a principal-like role in serving as the budget breaker for the other agents.

[13] The earlier literature often uses the labels "non-verifiable" and "subjective" interchangeably for performance measures. We use the term "non-verifiable" throughout since in our model all parties observe all efforts without noise (hence, objectively), yet this information is not contractible because it cannot be verified by the courts.

each agent's wage $w_t^i \geq 0$ for all i, t.[14] Period-t payoffs are normalized to $w_t^i - a_t^i$ for Agent i, and to $x_t - \sum_i w_t^i$ for the principal. We assume the agents' efforts are sufficiently important that the principal always wants to elicit high effort from each agent in each period. We evaluate the efficiency of any contractual arrangement by the expected periodic cost, C_t, of eliciting the effort profile (H,H).

In our model, all actions are public. The players' strategies map the complete history into actions. At any particular time, the history consists of all past actions and outcomes. As is common in games of complete information, we focus on (pure strategy) subgame perfect equilibria.[15] Throughout the paper, we assume the agents will trust the principal's promises (principal–agent relational contracting) and play as the principal intends provided doing so constitutes a subgame perfect equilibrium in the overall game, which the agents cannot improve upon (in a Pareto sense) using self-enforcing side contracts (agent–agent relational contracting). In this sense, our contracts are *collusion-proof*.

In our complete information setting, it is without loss of generality to restrict attention to (grim) trigger strategies for the agents which, following any deviation, call for the harshest punishment that can be sustained as a stage game equilibrium. There is also no loss in restricting attention to strategies for the principal that determine current period actions as a function of only the current period's observables and a state variable that describes whether a party has reneged on some past implicit contract. We suppress the state variable in our notation, as reneging never occurs on the equilibrium path. At the beginning of each period t, the principal offers an explicit contract $w_t^i(x_t, \hat{y}_t^i, \hat{y}_t^j)$, where $\hat{y}_t^i \in \{L, H\}$ is the principal's report of the non-verifiable metric regarding Agent i's effort, a_t^i. As is common in the literature (e.g., Che and Yoo, 2001), we limit attention to short-term explicit contracts. Combining the preceding arguments with the stationary production technology, in equilibrium, the same explicit contract will be offered each period.[16] For most of the analysis, we confine attention to symmetric contracts that treat both agents the same, ex-ante. (We relax this restriction in Section 3.1.) This allows us to drop the agent and period indices in the wage contract and simply write $w(x_t, \hat{y}_t^i, \hat{y}_t^j)$ for Agent i's pay in period t.

At the beginning of each period, Agent i either accepts the contract and chooses effort or leaves the employment relationship and receives a reservation utility of zero in perpetuity. We assume that if an agent quits, the firm will shut down or, equivalently, the principal would have to incur prohibitively high search costs in order to find a replacement for the agent. Thus, the threat of dismissal cannot be used as an alternative source of incentives.[17] We also assume that the principal cannot commit to money burning or payments to a third party, which, given the principal's perfect observation of the agents' actions, would otherwise allow for the first-best to be obtained. Committing to such third-party payments is uncommon in practice, possibly due to commitment problems (e.g., the principal may not care equally about money given to employees vs. a charity and/or such gifts may not be observable). Allowing for third-party payments seems an interesting avenue for future research in a larger model with imperfect non-verifiable performance measures.

In the following, we first describe the stage (i.e., one-shot) game and then turn to relational contracting. In the first-best solution to the stage game, the principal simply reimburses the agents for their cost of effort, resulting in periodic cost of $C_t^{FB} = 2H$. As another useful benchmark, suppose the agents are subject to incentive constraints, but the relationship is a single-period one. In the stage game, assuming the principal truthfully reports the non-verifiable measures, i.e., $\hat{y}_t^i \equiv a_t^i$, for all t, i, the expected stage-game payoff to Agent i given (a_t^i, a_t^j) is

$$U_{a_t^i,a_t^j} = \mathbb{E}_{x_t}\left[w(x_t, a_t^i, a_t^j) | a_t^i, a_t^j\right] - a_t^i.$$

The principal's desired effort profile is a Nash equilibrium of the stage game if

$$U_{HH} \geq U_{LH}. \tag{StageNE}$$

Returning to the repeated interaction, we first describe principal–agent relational contracting. Given the non-verifiability of y_t^i, any promised payments tied to these measures need to be self-enforcing. Specifically, if the principal ever reneges on her promises, the agents will punish her using their grim trigger strategy, which results in a fallback payoff to the principal of F (to be specified later). The principal's reporting choice then is governed by the *reneging constraint* that requires, for any $t, x_t, \mathbf{a}_t$, and $(\hat{y}_t^A, \hat{y}_t^B)$:

$$\sum_{i=A,B}\left[w\left(x_t, a_t^i, a_t^{-i}\right) - w\left(x_t, \hat{y}_t^i, \hat{y}_t^{-i}\right)\right] \leq \frac{\mathbb{E}[x_t - 2w(x_t, H, H) | H, H] - F}{r}. \tag{R}$$

The left-hand side of (R) captures the current-period compensation the principal could save by reneging; the right-hand side, the attendant punishment as given by the difference between the principal's expected continuation payoff along the equilibrium path and under the fallback contract, in perpetuity.

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[14] The principal makes payments to the agents, not the other way around. This seems to fit practice and has become a fairly standard way to motivate contracting frictions.

[15] More generally, earlier relational contracting studies have used perfect public equilibrium as the solution concept for games of incomplete information (e.g., Levin, 2003; Rayo, 2007).

[16] Levin (2003) shows that the problem of characterizing optimal relational contracts can be restricted without loss of generality to contracts that are stationary. Kvaloy and Olsen (2006) apply Levin's arguments to settings similar to ours.

[17] Consider a larger model with the option of agent replacement. If the labor market observes only the binary outcome "dismissal or continuation," a perfect public equilibrium, in which potential future hires hold the belief that dismissal must have been caused by the principal reneging on an implicit contract, would essentially replicate the equilibrium we derive.

To study agent–agent relational contracting, note first that in our model there is no beneficial (to the principal) role for such side contracting, because the principal perfectly observes the agents' efforts. This leaves the specter of costly collusion. We study implicit rather than explicit side contracts in that any effort plans the agents can collude on must be self-enforcing.[18] Even with perfectly observable efforts, given the infinitely repeated nature of the game, the side contracting space for the two agents is unbounded. In general, the agents can agree to play any action profile $\mathbf{a} = \{a_t^A, a_t^B\}_{t=0}^{\infty}$, provided $\mathbf{a}$ is supported by some subgame perfect equilibrium.

Clearly, the agents will want to collude on an action plan, $\mathbf{a}$, only if it Pareto-dominates, from the agents' point of view, the principal's desired action plan, $(H,H)^{\infty}$. Denote by $\overline{U}_{0,i}(\sigma) \equiv \sum_{t=0}^{\infty}(1+r)^{-t}\cdot U_{\tilde{a}_i^t \tilde{a}_j^t}(\sigma)$ Agent i's Date-0 expected utility from any (collusive) action profile, $\sigma = \{\tilde{a}_t^A, \tilde{a}_t^B\}_{t=0}^{\infty}$. A sufficient condition for a contract to be collusion-proof is that:

$$\nexists\ \sigma \mid \overline{U}_{0,i}(\sigma) \geq \frac{1+r}{r}U_{\mathrm{HH}}, \quad i = A, B, \text{ with at least one inequality strict.} \tag{Pareto}$$

Collusion-proofness means that there is no action profile that both violates (Pareto) and can be sustained by the agents as a subgame perfect equilibrium. To support any implicit side contract, the agents would adopt a (grim) trigger strategy whereby each agent abides by the side contract until some Agent j defects, after which Agent i punishes him by playing the stage game equilibrium with the lowest payoff to Agent j. Despite the rich side-contracting space, as we will show in Lemma 1, attention can be confined to two specific and intuitive collusive action plans:

(i) "*Shirk*": $\mathbf{a}^{Shk} = \{\mathbf{a}_t = (L, L)\}_{t=0}^{\infty}$, i.e., each agent chooses low effort in each period.
(ii) "*Cycle*": $\mathbf{a}^{Cyc} = \left\{\{a_t^A = H, a_t^B = L\}_{t=0,2,4,\ldots} \cup \{a_t^A = L, a_t^B = H\}_{t=1,3,5,\ldots}\right\}$, i.e., the agents alternate choosing high and low effort.[19]

To break collusive agreements that call for the play of *Shirk* or *Cycle*, the principal needs to "bribe" an agent who is supposed to shirk in period t to deviate and instead choose high effort. As we show in the proof of Lemma 1, the grim trigger strategies supporting any side contract call for an agent to choose high effort forever should the other agent ever deviate. That is, reverting back to the principal's desired action profile is the harshest punishment threat that can be used by the agents to sustain collusion. The constraint to prevent *Shirk* then is

$$U_{HL} + \frac{U_{HH}}{r} \geq \frac{1+r}{r}U_{LL}. \tag{NoShirk}$$

To prevent *Cycle*, the constraint is

$$\frac{1+r}{r}U_{HH} \geq \sum_{\tau = t, t+2, t+4, \ldots} \frac{U_{LH}}{(1+r)^{\tau - t}} + \sum_{\tau = t+1, t+3, t+5, \ldots} \frac{U_{HL}}{(1+r)^{\tau - t}}. \tag{NoCycle}$$

In present value terms, the right-hand side of these collusion-proofness constraints captures an agent's payoff from sticking to the side contract; the left-hand side, his payoff from deviating in some period t, followed by a reversal to $(H,H)^{\infty}$ in the punishment phase. Our first result provides necessary and sufficient conditions for collusion-proofness. (All proofs are provided in the Appendix.)

Lemma 1. *A necessary and sufficient condition for a symmetric contract to be collusion-proof is: either both (NoCycle) and (NoShirk) are satisfied, or both (NoCycle) and*

$$U_{HH} \geq U_{LL} \tag{$\text{Pareto}_{HH \succ LL}$}$$

are satisfied.

The proof proceeds by demonstrating that (NoShirk) and (NoCycle) together are sufficient conditions for collusion-proofness. Because (NoCycle) is equivalent to the (Pareto) constraint for the *Cycle* strategy,[20] this leaves the possibility that the principal's desired strategy profile Pareto-dominates *Shirk*, which can be simplified to ($\text{Pareto}_{HH \succ LL}$). Moreover, symmetry of the wage contract does not preclude any asymmetric collusive agreements, a priori. However, as Lemma 1 shows, the relevant (binding) collusive threats are indeed symmetric in nature. By treating every shirking agent symmetrically over time, the two strategies *Shirk* and *Cycle* maximize the minimum bribe required to induce some shirking agent in some period to deviate from the side contract.

A key determinant of any collusive threat is the strategic interaction between the agents. We say a wage scheme induces *strategic complementarity in payoffs* if

$$\Delta U \equiv (U_{HH} - U_{LH}) - (U_{HL} - U_{LL}) > 0, \tag{1}$$

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[18] As Tirole (1992) writes: "[i]f, as is often the case, repeated interaction is indeed what enforces side contracts, the…approach [of modeling repeated interactions] is clearly preferable because it is the more fundamentalist."

[19] We arbitrarily label Agent A as the one to choose high effort in the first period. Alternatively, the agents could toss a coin to determine who chooses high effort first. This variant would not alter our results qualitatively.

[20] Under *Cycle*, the lower of the respective collusive payoffs, $\min_i \overline{U}_{0,i}(\sigma)$ as per the (Pareto) condition, just equals the RHS of the (NoCycle) constraint.

and *strategic substitutability in payoffs* if $\Delta U < 0$. Note that ΔU commingles the exogenous technology described by (p_L, p, p_H) and the endogenous wage scheme. With strategic complementarity in payoffs, *Shirk* is an attractive strategy for the agents. In fact, the stronger the complementarity ($\Delta U \gg 0$), the less an agent has to gain by unilaterally deviating from *Shirk*. By similar logic, the stronger any substitutability in payoffs ($\Delta U \ll 0$), the easier it is for the agents to collude on *Cycle*. The collusive threat is minimized (least costly to deter) if the contract induces *strategic independence in payoffs*, or $\Delta U = 0$. In fact, a key feature of the *optimal* symmetric contract, studied in Section 4 below, will be precisely that it often induces $\Delta U = 0$, thereby making (NoShirk) and (NoCycle) both binding constraints.

3. “Simple contracts”

In this section, we study commonly observed arrangements aimed at incorporating non-verifiable performance measures into contracts. We begin with bonus pools. Relational contracting here takes place among the agents. In Section 3.2, we consider individual relational contracts between the principal and each agent. Our goal is to develop the main determinants of the relative performance of these arrangements. In Section 4, we show that these main forces also drive the structure of the optimal contract.

3.1. Bonus pools: agent–agent relational contracting

One way to overcome the credibility issue associated with tying compensation to non-verifiable metrics is to make the total payout over all agents—the *bonus pool*—dependent only on the verifiable metrics and use the non-verifiable metrics only to allocate the pool. The principal then is indifferent as to how to split the total bonus as it is a sunk cost anyway. Earlier literature (e.g., Baiman and Rajan, 1995; Rajan and Reichelstein, 2006, 2009) has shown that bonus pools can be effective compensation tools in static settings. We now show that repeated interactions severely impede their performance as a result of collusion.

Within the class of bonus pool arrangements, various payout policies are conceivable. In general, a bonus pool prescribes a wage payment to Agent i of:

$$w^i(x, \mathbf{a}_t) = \gamma^i_{\mathbf{a}_t} B(x_t), \quad \gamma^A_{\mathbf{a}_t} + \gamma^B_{\mathbf{a}_t} = 1, \quad \text{for all } \mathbf{a}_t,$$

where $\gamma^i_{\mathbf{a}_t}$ denotes Agent i's share of the bonus pool, $B(x_t)$, which in turn only depends on verifiable team output. Unless otherwise noted, we confine attention to *symmetric* bonus pools:

$$\gamma^i_{\mathbf{a}_t} = \tfrac{1}{2}, \quad \text{if } a^A_t = a^B_t, \quad \text{and} \quad \left(\gamma^i_{\mathbf{a}_t} = 1, \gamma^j_{\mathbf{a}_t} = 0\right), \quad \text{if } a^i_t > a^j_t. \tag{2}$$

We begin with *conditional* bonus pools in that $B(0) = 0 < B(1) = B$, i.e., the bonus pool is paid out only if $x_t = 1$. The stage game incentive constraint (StageNE), applied to a conditional (symmetric) bonus pool, implies that $B \geq B^{HH} = 2H/p_H$. While repeating the stage game equilibrium in all periods is a subgame perfect equilibrium of the repeated game, other equilibria may exist for a designated bonus pool amount of B^{HH}; worse, they may Pareto-dominate (H, H) for the agents. Given the symmetric nature of the bonus pool as specified in (2), Lemma 1 applies. The principal's optimization problem is to minimize $B \geq 0$, subject to (StageNE) and collusion-proofness as per Lemma 1.

Proposition 1. *The expected per-period compensation cost of using a bonus pool with conditional payout equals* $C^C_t = p_H B^C$, *where*:

$$B^C = \begin{cases} B^{Pareto} = \dfrac{2H}{p_H - p_L}, & p < \dfrac{p_H}{2}, \\[2ex] B^{Shk} = \dfrac{2(1+r)H}{p_H + 2rp - (1+r)p_L}, & p \in \left[\dfrac{p_H}{2}, p_o\right], \\[2ex] B^{Cyc} = \dfrac{2(1+r)H}{(2+r)p_H - 2p}, & p > p_o. \end{cases}$$

For complementary efforts, the pressing collusive threat is *Shirk*. If the effort complementarity is strong ($p < p_H/2$) the principal raises B to the point where (H, H) becomes the Pareto-dominant equilibrium for the agents (i.e., $U^C_{HH} = U^C_{LL}$). For productive substitutes ($p > p_o$), the pressing collusive threat is *Cycle*. The following corollary follows immediately from Proposition 1.

Corollary 1. *The expected compensation cost of using a bonus pool with conditional payout,* C^C_t, *is*:

(i) *non-monotonic in* p: (*weakly*) *decreasing in* p *for complementary efforts* ($\Delta \geq 0$) *and strictly increasing in* p *for substitute efforts* ($\Delta < 0$);

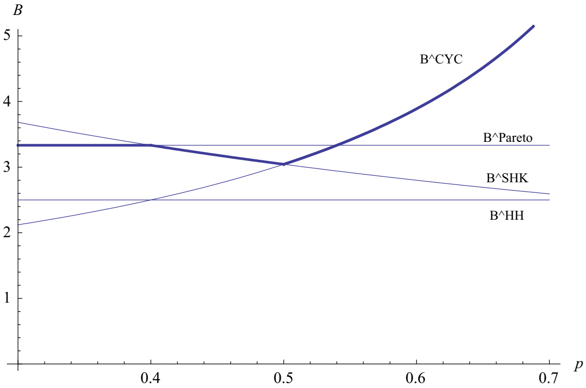

Fig. 1. Bonus pool with a conditional payout (Proposition 1) for ($p_L = 0.2$, $p_H = 0.8$, $H = 1$, $r = 0.4$).

(ii) *always greater than the first-best level*;
(iii) *weakly decreasing in r*.[21]

The threat of collusion always prevents the principal from attaining the first-best benchmark using a conditional bonus pool. Depending on the production technology, she always has to worry about the agents colluding via one of the two strategies invoked in Lemma 1. Fig. 1 illustrates the optimal conditional bonus pool, B^C, depicted in boldface.

While the effect of r is well understood (patient agents can collude more effectively), how is the threat of collusion affected by the production technology? In Section 2, we argued that the stronger the strategic payoff interdependence $|\Delta U|$, the costlier it is to preempt collusion. The conditional bonus pool simply preserves the technological strategic interaction into one of payoffs in that ΔU is increasing monotonically in $\varDelta$ (say, by varying p, all else equal), and $\Delta U = 0$ whenever $\varDelta = 0$. The threat of collusion via *Shirk* (*Cycle*) thus decreases as the productive interdependence becomes smaller, i.e., as p approaches p_o from below (from above). In line with Section 2, the principal's contracting cost under this bonus pool arrangement will reach its minimum when efforts are productively independent ($\varDelta = 0$) so that $\Delta U = 0$, as the agents' ability to collude will then be most limited. However, the nature of team production generally implies $\varDelta \neq 0$.

Unconditional payout policy: We now consider an alternative bonus pool that ensures $\Delta U = 0$. Specifically, while keeping the payout symmetric as in (2), the principal commits to paying out the pool *irrespective of the realization of* x_t, i.e., $B(1) = B(0) = B$. The requirement that (H,H) be a Nash equilibrium reduces to $B \geq 2H$, which also rules out (L,L) as a stage game equilibrium. Moreover, the relevant collusion-proofness constraint becomes (NoShirk), which requires that $B \geq \frac{1+r}{r}2H$.[22] Since this bonus pool is always paid out, the periodic contracting cost to the principal also equals $C_t^U = \frac{1+r}{r}2H$. A comparison with the contracting cost under a conditional payout, C_t^C, as given in Proposition 1, yields our next result.

Proposition 2. *Alternative payout policies for bonus pools*:

(i) *If efforts are productive complements* ($\varDelta > 0$), *the principal prefers an unconditional payout policy* ($C_t^C > C_t^U$) *for r sufficiently large*.
(ii) *If efforts are productive substitutes* ($\varDelta < 0$), *the principal prefers a conditional payout policy* ($C_t^C < C_t^U$) *for any r*.

As one might expect, Part (ii) shows dominance of the conditional payout policy for substitutes. On the other hand, Part (i) may come as a surprise. We sketch the key argument for the case of weak complements, which makes *Shirk* the pressing collusion threat. The stability of any side contract hinges on the current deviation benefit (*CDB*) and future punishment an agent anticipates for defecting from *Shirk*. The future punishment is lower under the conditional payout as the payout probability increases from p_L (under *Shirk*) to p_H (reversal to $(H,H)^\infty$), and the split of the pool remains even. The current deviation benefits equal $CDB^U = \frac{1}{2}B^U - H$ with an unconditional payout and $CDB^C = \left(p - \frac{p_L}{2}\right)B^C - H$ with a conditional payout. Scaling $B^C = \frac{B^U}{p_H}$ to adjust for the respective payout probabilities, we find that $CDB^C < CDB^U$ for all $p < p_o$. Given the lower current deviation benefit *and* the lower future punishment under a conditional payout, an unconditional payout is less costly if the agents are sufficiently impatient.

Generally speaking, a desirable feature of the unconditional pool is that, by construction, it neutralizes any productive interdependence and always delivers strategic payoff independence ($\Delta U = 0$). However, it does so at the cost of discarding

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[21] Parts (i) and (ii) of Corollary 1 are trivial. Also, note that B^{Pareto} is independent of r, whereas differentiating B^{Shk} and B^{Cyc} shows each term to be decreasing in r, if and only if $p \geq p_H/2$. Given the cutoffs for p in Proposition 1, Part (iii) of Corollary 1 follows.

[22] It is easy to see that both (NoCycle) and (StageNE) are implied by (NoShirk) under this scheme.

information that is potentially useful for contracting, namely the team output x_t. This informational disadvantage is outweighed by the strategic payoff independence for productive complements when the discount rate is high.[23]

Asymmetric payout policy: As shown above, bonus pools with a conditional, symmetric payout are plagued by two qualitatively different threats of collusion, *Shirk* and *Cycle*. The *Shirk* strategy capitalizes on the fact that with complementary efforts there may be multiple equilibria in the stage game, specifically (H,H) and (L,L). In different settings, prior literature has shown that (some) undesirable equilibria can be eliminated by treating agents asymmetrically.[24] In a dynamic setting like ours, ensuring that H is a dominant strategy for one (or both agents) in the stage game does not ensure that it is collusion-proof.

Return to a conditional payout policy $(B(0)=0<B(1)=B)$, but suppose the principal favors Agent A by allocating the entire bonus pool to him if both agents have chosen low effort. That is, the payout shares are as in (2) except $\gamma_{LL}^A=1$ and $\gamma_{LL}^B=0$. Consider the case of strong productive complements $(p<p_H/2)$. If the principal sets $B \geq B^{HH}+\varepsilon=\frac{2H}{p_H}+\varepsilon,\ \varepsilon \to 0$, then the only strategy combination that yields Agent B a positive payoff is (H,H). In this case, the asymmetric payout policy completely resolves the collusion problem, and the principal achieves her first-best payoff.

If $p>p_H/2$, two complications arise. First, the threat of side-contracting via *Cycle* remains unmitigated, so the bonus pool must be increased (*Cycle* is unaffected by the manipulation of the agents' payoffs in the (L,L) cell). Second, *the use of an asymmetric wage contract creates opportunities for asymmetric collusion*. Lemma 1, which aims to shrink the set of relevant collusion strategies to a tractable set, no longer applies. Instead, the principal needs to search for collusion-proof contracts over an unbounded side-contracting space.[25] While this renders the analysis intractable (recall that the relevant collusion-proofness constraints in our model are endogenously determined by the wage scheme), one can show that *any* asymmetric bonus pool falls short of first-best performance for $p>p_H/2$.[26] That is, strong productive complementarity $(p<p_H/2)$ is sufficient *and necessary* for asymmetric bonus pools to fully resolve the collusion problem. Given the strong performance predictions for asymmetric contracts derived from static models, this illustrates the importance of studying dynamic models to gain a more nuanced picture of the robustness of earlier findings.[27]

A bonus pool is only one way to tie compensation to non-verifiable metrics in a credible manner; another is to rely on the principal's reputation.

3.2. Individual rewards: principal–agent relational contracting

In a repeated relationship, the principal can make promises to reward agents for their effort without fixing the total payout. Any contract $w_t^i(x_t,\hat{y}_t^i,\hat{y}_t^j)$ that has the total payout $w_t^A+w_t^B$ be a nontrivial function of $\hat{y}_t^i$ inevitably takes the form of a "promise" by the principal. For the agents to trust that the principal will not renege by reporting the agents have shirked when in fact they have worked, such a contract will have to be self-enforcing in the sense described by (R) in Section 2 (and made more precise below). In this section, we restrict attention to the case in which the payment to Agent i, w_t^i, is independent of the principal's report on the other agent's non-verifiable signal, $\hat{y}_t^j,\ j \neq i$. Because the non-verifiable metrics y_t^i are perfect, the optimal (individual relational) contract can be expressed in the following additive form:

$$w_t^i=\alpha\hat{y}_t^i+\beta x_t.$$

(In our limited liability setting there is no role for a fixed salary.)

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[23] If the principal were to observe the agents' effort with noise, then the performance of the unconditional bonus pool would be reduced because of additional limited liability rents. By continuity, such an arrangement would still outperform a conditional payout policy provided the observation error is sufficiently small.

[24] See Demski and Sappington (1984), Ishiguro (2004), and Rajan and Reichelstein (2009). While Demski and Sappington (1984) consider a multi-agent adverse selection model, Ishiguro (2004), like us, models relative performance evaluation in a moral hazard setting. However, all of these models are static.

[25] For example, suppose p_L=0.2, p=0.41, p_H=0.8, and r=0.0001. Denote by U_{a_i,a_j}^i Agent i's stage game payoff under the asymmetric bonus pool described in the text $(\lambda_{LL}^A=1, \lambda_{LL}^B=0)$. One can verify that $U_{HH}^A=U_{HH}^B=0.025638, U_{HL}^A=U_{LH}^B=0.051279, U_{LH}^A=U_{HL}^B=0$, $U_{LL}^A=0.512819$, and $U_{LL}^B=0$. The agents are indifferent between colluding on *Cycle* and playing (H,H) in every period. However, there are other subgame perfect equilibria they strictly prefer, e.g., playing (L,L) for one period followed by (L,H) for ten periods, with the same 11-period play then repeated.

[26] For a bonus pool to achieve first best, it must entail an expected per-agent payment of H if both agents work. In order to satisfy the Nash constraint (StageNE), when they play (L,H) or (H,L), the shirking agent must receive a payout of 0. Hence, the only possible asymmetry in the payout is when the agents play (L,L). *Cycle* is unaffected by the payout at (L,L), so showing that any asymmetric payout cannot achieve first-best is equivalent to showing that the expected payout under B^{Cyc} is greater than $2H$, which is true if and only if $p>p_H/2$.

[27] It remains an open question how the asymmetric payout policy fares relative to a symmetric one for $p>p_H/2$. By continuity, an asymmetric payout is still preferred for p close enough to $p_H/2$. Beyond that, while a complete analysis of the asymmetric policy is intractable due to the unbounded collusion-strategy space, the performance can be bounded from above by looking at a relaxed problem that only considers collusive constraints *Shirk* and *Cycle*. If the value of this relaxed program were found to be below that of the symmetric payout policy, say for strategic substitutes, then this would be sufficient for dominance of a symmetric payout. However, this turns out not to be the case, as the following argument shows. Hold fixed the optimal bonus pool under the symmetric (conditional) payout for $\Delta<0$, B^{Cyc}, but instead of splitting it equally between the agents who have chosen efforts (H,H), increase the share paid out to one agent by ε (while lowering that of the other agent accordingly). The only binding constraint absent ε was (NoCycle). Such an asymmetric adjustment relaxes (NoCycle) for the favored agent at no cost to the principal. Hence, the cost of the relaxed program for the asymmetric payout is lower than the cost of the (unrelaxed) symmetric payout program.

Assuming, for now, the principal honors the implicit contract, the agent's periodic expected rent when playing H is $U_t^i = \alpha H + p_H \beta - H$. The stage-game incentive condition (StageNE) collapses to

$$\alpha H + (p_H - p)\beta \geq H. \tag{3}$$

We denote the resulting per-period cost to the principal under individual contracts by $C_t^{Ind}(\alpha) \equiv C_t^{Ind}(\alpha, \beta^*(\alpha))$, where $\beta^*(\alpha)$ is the explicit incentive weight that makes (3) binding for given α. In the absence of implicit incentives, $\beta^*(\alpha = 0) = \frac{H}{p_H - p} \equiv \beta^{HH}$, resulting in contracting costs of $C_t^{Ind}(0) = 2p_H \beta^{HH}$. Introducing implicit incentives reduces the need for explicit incentives, as $\beta^*(\alpha) = (1-\alpha)\beta^{HH}$. The expected periodic rents for the agents and costs to the principal, each respectively expressed as functions solely of implicit incentives, read:

$$U_t^i(\alpha) \equiv U_t^i(\alpha, \beta^*(\alpha)) = (1-\alpha)H\frac{p}{p_H - p},$$
$$C_t^{Ind}(\alpha) \equiv 2\left[\alpha H + p_H \beta^*(\alpha)\right] = C_t^{Ind}(0) - 2H\frac{p}{p_H - p}\alpha.$$

Because the non-verifiable measure y_t^i is noiseless, the greater the coefficient attached to this measure (higher α), the lower the agents' limited liability rents. In fact, the principal achieves her first-best expected payoff for $\alpha = 1$. However, the non-verifiable nature of y_t^i imposes bounds on the extent to which it can be used.

We consider relational contracts supported by the following grim trigger strategy: as long as the principal honors the implicit contract and (3) holds, the agents are willing to play the desired (H, H) strategy. If however the principal reneges on the implicit contract by claiming that $\hat{y}_t^i = L$ for some i and t, when in fact $a_t^i = H$, then both agents will not respond to implicit incentives any longer. Instead, they will punish the principal by playing (L,L) indefinitely unless (H,H) is a unique equilibrium under the fallback contract which then has to rely solely on the verifiable measure, i.e., $\alpha = 0$.[28] That is, ensuring the agents both work under the fallback contract requires that (H,H) is an equilibrium of the stage subgame *and* that (L,L) is not; more formally, $\beta \geq \max\{\beta^{HH}, \beta^{LL}\}$, where $\beta^{LL} \equiv \frac{H}{p - p_L}$ is found by solving $U_{HL} = U_{LL}$ for β.

It is well understood that strategic complementarities invite multiple equilibria. Indeed, with productive complements, $\beta^{HH} < \beta^{LL}$, and hence for any $\beta \in \left[\beta^{HH}, \beta^{LL}\right]$, there exist two symmetric pure-strategy Nash equilibria, (H,H) and (L,L). Eliminating the shirk equilibrium then requires setting $\beta > \beta^{LL}$. With productive substitutes, $\beta^{HH} \geq \beta^{LL}$, so (H,H) will be the unique equilibrium for any $\beta > \beta^{HH}$ (given $\alpha = 0$).[29] In summary, for productive substitutes, existence of an (H,H)-equilibrium ensures its uniqueness, whereas with productive complements, making (H,H) the unique, rather than just *an*, equilibrium comes at additional cost to the principal. Thus, the principal's reneging constraint is:

$$\alpha H \leq \frac{1}{r}\left[p_H \beta^{LL} - (\alpha H + p_H \beta^*(\alpha))\right] \tag{4}$$

for productive complements and

$$\alpha H \leq \frac{1}{r}\left[p_H \beta^{HH} - (\alpha H + p_H \beta^*(\alpha))\right] \tag{5}$$

for productive substitutes.[30]

The principal chooses $\alpha \in [0,1]$ so as to minimize $C_t^{Ind}(\alpha)$, subject to the reneging constraint (4) or (5), respectively.

Proposition 3. *The optimal weight on the non-verifiable measure under individual contracting is*:

$$\text{(i)}\;\; \textit{If } \Delta \geq 0,\ \alpha^* = \begin{cases} \dfrac{p_H\left(\frac{1}{p - p_L} - \frac{1}{p_H - p}\right)}{r - \frac{p}{p_H - p}} \equiv \overline{\alpha} \in (0,1), & \text{for } r > \frac{p_H - (p - p_L)}{p - p_L}, \\ 1, & \text{otherwise.} \end{cases}$$

$$\text{(ii)}\;\; \textit{If } \Delta < 0,\ \alpha^* = \begin{cases} 0, & \textit{for } r > \frac{p}{p_H - p} \\ 1, & \textit{otherwise.} \end{cases},$$

Solving the principal's reneging constraints (4) and (5) as equalities yields the conditions in Proposition 3. Regardless of the nature of the production technology, if the players are sufficiently patient, α^* attains a value of 1, and the principal achieves her first-best payoff. We now summarize how the principal's ability to engage in relational contracting turns on the time preferences and the production technology.

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[28] As in the case of bonus pools, we allow agents to play Pareto-dominated equilibria in the punishment phase.

[29] For productive substitutes, there also exist two asymmetric Nash equilibria (L, H) and (H, L) for $\beta^{LL} < \beta < \beta^{HH}$, and a unique (dominant-strategy) equilibrium (L, L) for $\beta < \beta^{LL}$.

[30] The formulation of our reneging constraints is in line with the multilateral contracting setting in Levin (2002). If the principal were to renege on one agent, both agents would insist on the fallback contract thereafter. Hence, reneging on a single agent is always dominated by reneging on both agents.

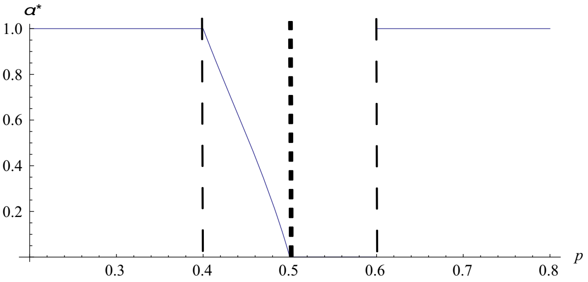

Fig. 2. Implicit incentive weight, α^*, under individual contracts for ($p_L=0.2$, $p_H=0.8$, $r=3$).

Corollary 2. *Comparative statics of the weight on the non-verifiable metric*:

(i) *If* $\Delta \geq 0$, α^* *is decreasing in* r *and increasing in the degree of strategic complementarity* (*captured by decreasing* p, *holding* p_H *and* p_L *constant*).
(ii) *If* $\Delta < 0$, α^* *is discontinuously decreasing in* r *and discontinuously increasing in the degree of substitutability* (*captured by increasing* p, *holding* p_H *and* p_L *constant*).

Generally speaking, the principal's ability to engage in relational contracting is improved by lower discount rates and greater productive interdependencies. The nature of the productive interdependence, however, matters qualitatively in that α^* varies continuously in r and p for productive complements. In contrast, for productive substitutes, the solution is "bang–bang" between first-best and no relational contracting whatsoever. The driving force is the different fallback contracts embedded in the principal's reneging constraints, (4) and (5). With complementary efforts, as argued above, to prevent the agents from playing (L,L) in all future periods, the principal has to raise the explicit bonus from β^{HH} to β^{LL}.[31] With productive substitutes, the principal can make (H,H) the agents' unique stage game equilibrium by increasing β^{HH} by any small positive amount (suppressed in the notation) because $\beta^{HH} \geq \beta^{LL}$.

Multiple equilibria in the agents' fallback stage game for productive complements allow them to punish a reneging principal more severely; this, in turn, makes it easier for the principal to commit to the relational contract. As a result, the reneging constraint (4) is more "relaxed" than (5), implying greater scope for relational contracting with productive complements, as illustrated in Fig. 2. For strong productive interdependencies (high $|\Delta|$), the principal can rely fully on implicit incentives tied to the noise-free measure, i.e., $\alpha=1$. Denote by p^{comp} (p^{sub}) the respective thresholds for p below which (above which) first-best performance obtains for productive complements (substitutes). Then, $p_o-p^{comp}=p^{sub}-p_o$, i.e., the range of parameters for which performance falls short of first-best is the same for complements and substitutes. Yet, for complements, the principal can always put *some* positive incentive weight on the non-verifiable individual measures ($\alpha^* \in [0,1]$), even if the degree of complementarity is small; not so for productive substitutes where $\alpha^* \in \{0,1\}$.[32] In that sense, the scope for implicit contracts is expanded for productive complements.

We are now in a position to compare the performance of the two forms of "simple contracts" discussed in this section. We model bonus pools based on a conditional (and symmetric) payout policy, given the prevalence in practice. As shown above, individual contracts perform well for low discount rates and for settings that exhibit strong productive interdependence. The reverse holds for bonus pools: the agents find it hard to collude for high r and small Δ. Therefore:

Corollary 3.

(i) *For any* Δ, *there exists a threshold* $r(\Delta)$ *such that a conditional bonus pool dominates individual relational contracts if and only if* $r \geq r(\Delta)$.
(ii) $r(\Delta)$ *is a continuous function that is increasing in* Δ *for* $\Delta > 0$, *and decreasing in* Δ *for* $\Delta < 0$.
(iii) *Confining attention to conditional bonus pools and individual relational contracts, for any* Δ, *the principal's contracting cost is single-peaked in* r, *with the peak located at* $r(\Delta)$. *It converges to first-best as* $r \to 0$ *and as* $r \to \infty$.

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[31] Counting on the agents punishing the principal in a way that also punishes themselves is not new, e.g., the "tit-for-tat" equilibrium in a repeated Prisoners' Dilemma. A difference between our setting and the repeated Prisoners' Dilemma is that the stage game equilibrium is unique in the repeated Prisoners' Dilemma. See also Levin (2002, 2003). In contrast, Bernheim and Whinston (1998) assume that only Pareto-unranked equilibria can be used as punishments. Under that alternative approach, the analysis for the strategic subsitutes case also applies to strategic complements. Returning to the idea of asymmetric contracts, there is a cheaper way for the principal to break the undesirable shirking equilibrium after having reneged on the implicit contract. The principal could offer β^{LL} to one of the agents and β^{HH} to the other agent. That way, the former agent will find it advantageous to deviate from shirking, and the latter agent will work hard as a best response. The revised equilibrium incentive weight on the non-verifiable metric then would read: $\overline{\alpha}=p_H\left(\frac{1}{p-p_L}-\frac{1}{p_H-p}\right)/\left(2\left[1+r-\frac{p_H}{p_H-p}\right]\right)$.

[32] Both the discontinuity under productive substitutes and the non-monotonicity of the principal's payoff in the degree of productive interdependence (when we put productive complements and substitutes together) can be traced to the fact that the off-equilibrium fallback contract uses the verifiable measure differently ($p-p_L$ comes into play) than the equilibrium contract (p_H-p comes into play). With productive substitutes, in contrast, both the on- and off-equilibrium contracts use the verifiable measure similarly (only p_H-p comes into play).

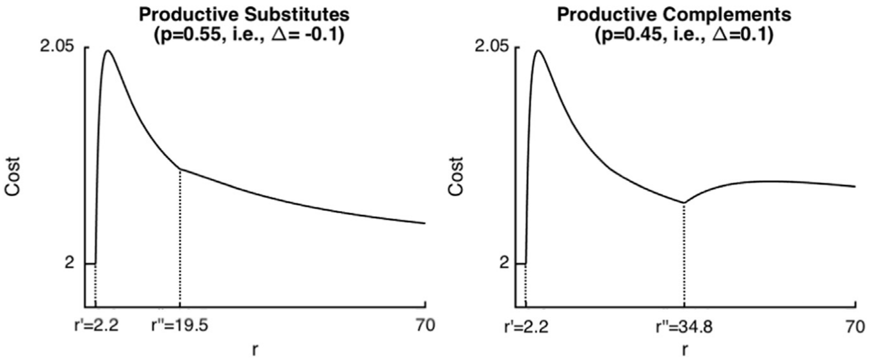

Fig. 3. Per-period compensation cost and discount rate r for ($p_L=0.2$, $p_H=0.8$, $H=1$).

Taken together, our results imply a non-monotonic relation between discount rates (or contracting horizons) and contracting costs. For small discount rates, individual relational contracts outperform bonus pools. As r increases, the principal's ability to commit to implicit incentives declines and hence the contracting cost increases, until a bonus pool becomes the lower-cost mechanism. As the discount rate increases further, the contracting cost under a bonus pool decreases as the threat of collusion subsides. The principal therefore prefers extreme discount rates (or equivalently, contracting horizons) to intermediate ones.[33]

4. Optimal symmetric contracts

So far, we have confined attention to contractual arrangements that each focus on one particular aspect of relational contracting: principal–agent relational contracting or collusion (under bonus pools). In this section, we solve for the optimal symmetric contract and show that the key forces identified in Section 3 remain central. For simplicity, we drop the period subscript t whenever possible.

Denote by $w^x_{\widehat{m}\widehat{n}}$ the wage payment to a representative agent, given a verifiable team output of $x\in\{0,1\}$ and efforts as reported by the principal of $\widehat{m}$ for the representative agent and $\widehat{n}$ for the other agent, $\widehat{m},\widehat{n}\in\{H,L\}$. The principal's reneging constraint (R) is:

$$w^x_{mn}+w^x_{nm}-w^x_{m'n'}-w^x_{n'm'}\le\frac{2\left[\frac{p_H}{p-p_L}H-\left(p_Hw^1_{HH}+(1-p_H)w^0_{HH}\right)\right]}{r}\qquad\left(\mathrm{RC}^x_{mn\succ m'n'}\right)$$

for productive complements ($\Delta>0$) and

$$w^x_{mn}+w^x_{nm}-w^x_{m'n'}-w^x_{n'm'}\le\frac{2\left[\frac{p_H}{p_H-p}H-\left(p_Hw^1_{HH}+(1-p_H)w^0_{HH}\right)\right]}{r}\qquad\left(\mathrm{RS}^x_{mn\succ m'n'}\right)$$

for productive substitutes ($\Delta<0$). The fallback contracts on the right-hand side are the same as in (4) and (5) from our analysis of “simple” individual relational contracts.

Invoking Lemma 1, the optimal symmetric, stationary contract $\{w^x_{mn}\}$ solves the following integer program, $\mathcal{P}$[34]:

$$\begin{aligned}\mathcal{P}:\quad &\min_{I\in\{0,1\},w^x_{mn}\ge0}\ p_Hw^1_{HH}+(1-p_H)w^0_{HH}\\ &\text{subject to:}\\ &(\mathrm{RC}^x_{mn\succ m'n'})\text{ if }\Delta\ge0,\\ &(\mathrm{RS}^x_{mn\succ m'n'})\text{ if }\Delta<0,\\ &(\mathrm{StageNE}),\\ &(\mathrm{NoCycle}),\\ &I\times(\mathrm{NoShirk})+(1-I)\times(\mathrm{Pareto}_{HH\succ LL}).\end{aligned}$$

---

[33] Our model is an infinite-horizon model, or, equivalently, a model in which the relationship ends at the end of each period with some probability. Consider instead a two-period version of our model. Now, backward induction comes into play, which is known to severely impede relational contracting. It is easy to show that bonus pools would achieve first-best as the agents will not be able to collude. One may expect, by similar logic, that any scope for principal–agent relational contracting also vanishes. This can be shown indeed to be the case for productive substitutes. However, for productive complements, the principal can rely on her reputation in the first period to some extent. The reason is, again, that there would be multiple equilibria in the agents' second-period subgame. Benoit and Krishna (1985) studies the finite repetition of exogenous one-shot games with multiple equilibria. The two-period game just described can be thought of as an extension of Benoit and Krishna's results to endogenous games.

[34] Note that the constraints in this program “switch,” depending on the value of the endogenous choice variable $I\in\{0,1\}$. Following Lemma 1, a necessary and sufficient condition for a symmetric contract to be collusion-proof is: either both (NoShirk) and (NoCycle) are satisfied, or both ($\mathrm{Pareto}_{HH\succ LL}$) and (NoCycle) are satisfied.

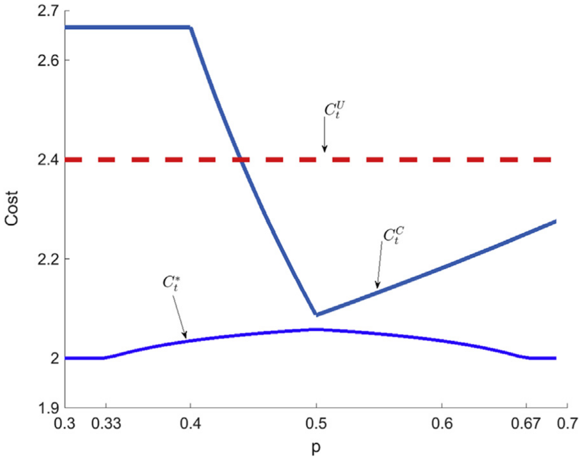

Fig. 4. Per-period compensation cost and productive interdependence (varying p) for ($p_L = 0.2$, $p_H = 0.8$, $H = 1$, $r = 5$).

As in Section 3, the principal's reneging constraint depends on the (exogenous) productive interdependence, because of the different fallback contracts. The key change from our earlier analysis of simple contracts is that, in the optimal contract, the productive interdependence is no longer the sole determinant of the overall payoff interdependence. In designing the optimal contract, the principal endogenously chooses to what extent to preserve or overturn any productive interdependence, Δ, into strategic payoff interdependence, ΔU. Our next result, which we view as the main result of the paper, bears this out.

Proposition 4. *Under the optimal symmetric contract, the principal's payoff is continuous in r, and the optimal contract depends on r as follows:*

(i) *For* $r \in (0, r']$: $w^x_{a_i,a_j} = a_i$, *for all* a_j, x; *the principal's payoff is first-best; and the agents' payoffs are trivially strategically independent* ($\Delta U = 0$).
(ii) *For* $r \in (r', r'']$: *the principal's payoff is below first-best;* $w^x_{a_i,a_j}$ *is a nontrivial function of x and* a_j; *yet* $\Delta U = 0$.
(iii) *For* $r > r''$: *if* $\Delta < 0$ *then* $\Delta U < 0$; *if* $\Delta > 0$ *then* $\Delta U \geq 0$. *As* $r \to \infty$: *the principal's payoff converges to first-best; the optimal contract converges to a conditional (unconditional) bonus pool for* $\Delta < 0$ *(for* $\Delta > 0$*).*

Here, $r' = \frac{p}{p_H - p}$ *for productive substitutes, and* $r' = \frac{p_H}{p - p_L} - 1$ *for productive complements.*

In the Appendix, we derive r'' and provide closed-form solutions for the optimal wage payments (and describe which constraints are binding) in each r-region.

Fig. 3 illustrates the proposition. The principal's payoff is first-best for a (nontrivial) range of small discount rates and again converges to first-best as all players become highly impatient. For intermediate discount rates, the principal incurs agency costs arising from two sources: her limited ability to commit to implicit incentives and the agents' ability to collude. This non-monotonicity corroborates Corollary 3(iii). It suggests that the effect of the discount rate on the principal's ability to commit is initially the dominant force, whereas for higher r, the effect on the agents' ability to collude dominates. A key insight from Proposition 4 is that, even in cases where Agent i's wage depends nontrivially on Agent j's non-verifiable performance measure, the principal still often uses the degrees of freedom embedded in the wage scheme to create payoff independence ($\Delta U = 0$). (We illustrate below how the principal achieves this.) However, it is never optimal to convert an exogenous productive complementarity ($\Delta > 0$) into a payoff substitutability ($\Delta U < 0$), or vice versa.

When the principal's credibility is at its strongest (Proposition 4(i)), she can credibly promise to reimburse each agent for his effort costs independent of the realized team output or the other agent's effort. This preempts any form of collusion. As in our earlier analysis, the principal's credibility is at its strongest when the discount rate is low (the probability of continuing the relationship is high) or the productive interdependence is strong ($|\Delta|$ is high). That is, the parameter range for which first-best can be achieved ($r \leq r'$ in Fig. 3) is expanding in the strength of productive interdependencies. The reason the cutoff r' is the same for both cases in Fig. 3 is that the fall back contract uses a bonus of $\frac{1}{p_H - p}$ for productive substitutes and of $\frac{1}{p - p_L}$ for productive complements. The symmetry in the example ($\Delta = -0.1$ or 0.1) implies $p_H - p = p - p_L$.

Corollary 4. *As the productive substitutability or complementarity becomes large* ($p \to p_H$ *or* $p \to p_L$), *the discount rate region* $r \in (0, r']$ *in which the first-best is obtained becomes arbitrarily large.*

It is important to note that, since the principal treats the agents independently in this region, the productive interdependence comes into play only in the fallback contract. Therefore, the productive interdependence works in the principal's favor as it lends credibility to her promises without aggravating collusion.

As the principal's credibility weakens ($r' < r < r''$ in Proposition 4(ii)), her reneging constraint starts to bind, so the agents' payments must be intertwined: they must be either a non-trivial function of the verifiable team measure and/or of each other's non-verifiable performance measures. As it turns out, it is optimal to design the payments in a way that it neutralizes the exogenous productive interdependence, i.e., to design $\{w_{mn}^x\}$ so as to convert $\Delta \neq 0$ into $\Delta U = 0$. For example, increasing the reward to the agents when both have exerted high effort is a way to offset productive substitutability; increasing the reward to an agent when he alone has exerted high effort is a way to offset productive complementarity.

In Proposition 4(ii), the verifiable team measure is more heavily relied on as the discount rate increases, i.e., $\frac{d[w_{HH}^1 - w_{HH}^0]}{dr} = \frac{d[w_{LL}^1 - w_{LL}^0]}{dr} > 0$.[35] An intuitive interpretation is that, as r increases in this region, there is a substitution from incentives based on non-verifiable performance measures to incentives based on verifiable measures, reflecting the principals' reduced commitment power.

Once the discount rate becomes large enough (Proposition 4(iii)), the principal's ability to commit vanishes. Nevertheless, it is precisely then that her payoff again approaches first-best as $r \to \infty$, because the agents' ability to collude also vanishes. For productive substitutes ($\Delta < 0$), the optimal contract converges to a *conditional* bonus pool and, hence, creates strategic substitutability also in the agents' payoffs ($\Delta U < 0$). The pressing collusion constraint is (NoCycle), which, as $r \to \infty$, collapses to (StageNE). For productive complements, the pressing constraint is (NoShirk), which, as $r \to \infty$, collapses to the requirement that (L,L) not be a stage game equilibrium. In this case, using a conditional bonus pool is suboptimal, because the resultant payoff complementarity would make it costlier to induce the agents to deviate from (L,L); hence $\Delta U = 0$, as long as collusion is a pressing concern. With productive complementarity, we have a substitution from incentives based on verifiable performance measures to incentives based on non-verifiable measures once r is sufficiently large. This seems counter to conventional wisdom, but it corroborates Proposition 2 on the comparison of the payout policies for bonus pools.

An interpretation of the optimal contract (once the principal's reneging constraint becomes binding) is that it specifies a minimum bonus pool as a function of verifiable output and allows the principal discretion to increase the size of the pool based on nonverifiable measures. Specifically, the contract calls for a bonus pool "floor" of $2w_{LL}^x$, $x = 0, 1$; for subjective performance measures other than (L,L), the overall pool may be increased.

Fig. 4 demonstrates how the productive interdependence affects the per-period compensation cost of the optimal contract (C_t^*), the conditional bonus pool (C_t^C), and the unconditional bonus pool (C_t^U) through an example. As shown in the "Simple Contracts" Section (and in Fig. 1), conditional bonus pools perform well in an absolute sense as productive interdependencies vanish, because of the reduced scope for collusion. By pitting them against the optimal contract, Fig. 4 shows that also the *relative* performance of conditional bonus pools is at its maximum for small $|\Delta|$. The reason is that, what bonus pools discard, namely the principal's own commitment power, is most limited for small $|\Delta|$. On the other hand, conditional bonus pools do particularly poorly in precisely those cases of high $|\Delta|$ in which the optimal contract does well. In the example shown in Fig. 4, the optimal contract achieves the first-best for strong productive interdependence (i.e., $p < 0.33$ or $p > 0.67$).

Gibbs et al. (2004) argue that productive interdependencies create a *demand* for non-verifiable metrics because cooperative activities are difficult to measure using conventional verifiable metrics. According to this logic, non-verifiable metrics are used to supplement verifiable ones whenever the latter fail to be comprehensive. Our results illustrate a complementary story that also predicts a positive association between the extent of productive interdependence and the use of non-verifiable metrics. Non-verifiable metrics are often more focused and precise than verifiable ones. Thus, the principal prefers to rely on the former but is constrained by commitment problems. Productive interdependence *facilitates* the use of non-verifiable metrics by creating harsher punishments which the agents can use against a reneging principal. Verifiable metrics emerge as supplementary so as to provide additional incentives if the principal's reneging constraint is binding (for p between 0.33 and 0.67 in our example in Fig. 4).

In general, the forces we identified in our analysis of simple incentive schemes in Section 3 remain at the heart of understanding the optimal contract. The main new insight is that the optimal symmetric contract typically converts any (exogenous) productive interdependence into strategic independence in the agents' payoffs as the least costly way to ensure collusion-proofness. Our results caution against empirically interpreting wage interdependence of team members as the source of the collusive threat; what matters is the overall payoff interdependence—the combined effect of wage interdependence and productive interdependence.

5. Conclusion

This paper studies a multi-period, multi-agent incentive model with a verifiable team performance measure and non-verifiable individual measures. The optimal contract combines features of bonus pools and individual relational contracts. It can be viewed as an explicit contract that specifies a minimum bonus pool as a function of the verifiable team output and gives the principal discretion to increase the pool and to allocate it among the agents using non-verifiable information about individual contributions to team output. As the discount rate becomes arbitrarily large, the optimal contract converges to a

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[35] The fact that the wage differential in x for the non-verifiable metrics (H,H) and (L,L) increases in r at the same rate is due to the binding reneging constraint ($\mathrm{RS}_{HH \succ LL}^x$). That is, the payments have to be raised "in lockstep."

conditional bonus pool (paid out only if team output is high) for productive substitutes, and to an unconditional bonus pool (independent of team output) for productive complements. That is, a form of pay-without-performance can be optimal in certain cases. A key feature of the optimal contract is that it often "neutralizes" the productive interdependence by converting it into strategic independence in terms of the agents' expected payoffs. (The unconditional bonus pool is one way of achieving this.) By doing so, the principal alleviates the overall threat of collusion.

Bonus pools are of particular importance in practice. A common prescription for grouping managers into bonus pools is that the tasks they perform are complementary in nature (Eccles and Crane, 1988, pp. 164–167). Our results point to an overlooked aspect of team formation, namely that strategic effort interdependencies can exacerbate collusion problems.

Our model makes a number of simplifying assumptions. The most important one is that all players observe the non-verifiable performance measure equally and without noise. Arguably, in many settings, the principal may observe the agents' effort only with noise. Other simplifying assumptions are: risk neutrality, contracts that are short-term and symmetric and designed by the shareholders (principal) directly rather than by a compensation committee (which is not the residual claimant).[36] These assumptions are fairly standard in the relational contracting literature. In future research, it seems particularly natural to incorporate intertemporal dependencies, e.g., earnings management or bonus banks.

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Appendix A

Proof of Lemma 1. We first prove two useful preliminary results. First, we derive the harshest punishment supporting the collusion. Second, we establish sufficiency of preventing the *Shirk* and *Cycle* strategies for collusion-proofness.

Claim 1. *It is without loss of generality to restrict attention to collusion constraints that have the agent punishing each other by playing* (H,H) *forever after a deviation from the collusion.*

Proof. We first argue that the off-diagonal action profiles (L,H) and (H,L) cannot be punishment strategies harsher than (H,H). We illustrate the argument for (L,H); similar logic applies to (H,L):

		Agent B	
		L	H
Agent A	L	U_{LL}, U_{LL}	U_{LH}, U_{HL}
	H	U_{HL}, U_{LH}	U_{HH}, U_{HH}

If playing (L,H) were a harsher punishment for Agent A (i.e., $U_{LH} < U_{HH}$), he would be able to profitably deviate to (H,H), implying that (L,H) is not a stage-game equilibrium and thus cannot be used as a punishment strategy. If (L,H) were a harsher punishment for Agent B (i.e., $U_{HL} < U_{HH}$), we would need $U_{HL} \geq U_{LL}$ to prevent Agent B from deviating from (L,H) to (L,L). However, $U_{HL} < U_{HH}$, $U_{HL} \geq U_{LL}$, and (StageNE) together imply $U_{HH} \geq \max\{U_{HL}, U_{LL}, U_{LH}\}$, which means there is no scope for collusion because at least one of the agents is strictly worse off under any potential collusive strategy than under the always working strategy.

To establish that (H,H) is the (weakly) harshest punishment, it remains to show that $U_{HH} \leq U_{LL}$. Suppose, by contradiction, $U_{HH} > U_{LL}$. If the wage scheme $\mathbf{w} \equiv \{w^x_{mn}\}$, $x = \{0, 1\}$, $m, n = \{L, H\}$, creates strategic payoff substitutability, i.e., $U_{HL} - U_{LL} > U_{HH} - U_{LH}$, (StageNE) again implies that (L,L) is not a stage-game equilibrium and thus cannot be used as a punishment strategy in the first place. If instead $\mathbf{w}$ creates (weak) strategic payoff complementarity, i.e., $U_{HH} - U_{LH} \geq U_{HL} - U_{LL}$, we have $U_{HL} + U_{LH} \leq U_{HH} + U_{LL} < 2U_{HH}$, where the last inequality is due to the assumption $U_{HH} > U_{LL}$. But $U_{LL} < U_{HH}$ and $U_{HL} + U_{LH} < 2U_{HH}$ together mean that at least one of the agents is strictly worse off under any potential collusive strategy than under the always working strategy, meaning there is no scope for any collusion.□

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[36] Studying the role of a compensation committee could create interesting additional collusion problems along the lines studied in a static model by Faure-Grimaud et al. (2003). Compensation committees could also strengthen the commitment to promises the principal would like to make ex ante but has difficulty carrying out ex post (Drymiotes, 2007). We thank the Editor for suggesting this idea.

Claim 2. *A sufficient condition for a symmetric contract to be collusion-proof is that neither Shirk nor Cycle can be sustained by a subgame-perfect equilibrium*:

Proof. Let

$$V_t^i(\sigma) \equiv \sum_{\tau=1}^{\infty} \frac{U_{t+\tau}^i(a_\tau^i, a_\tau^j)}{(1+r)^\tau}$$

be agent i's continuation payoff as of $t+1$, discounted to period t, from playing $\{a_{t+\tau}^A, a_{t+\tau}^B\}_{\tau=1}^{\infty}$ specified by some action profile $\sigma = \{a_t^A, a_t^B\}_{t=0}^{\infty}$, for $a_t^A, a_t^B \in \{H, L\}$. Restating (Pareto) in the main text, we allow any σ satisfying the following condition to be a potential collusive strategy:

$$\sum_{t=0}^{\infty} \frac{U_t^i(a_t^i, a_t^j)}{(1+r)^t} \geq \frac{1+r}{r} U_{HH}, \quad i = A, B, \text{ with at least one inequality strict.} \tag{6}$$

$U_t^i(a_t^i, a_t^j)$ is Agent i's stage-game payoff at t given the action pair (a_t^i, a_t^j) specified by σ, and $\frac{1+r}{r} U_{HH}$ is the agent's payoff from always working in perpetuity. The outline of the proof is as follows. Step 1: Any collusive strategy that contains only $(H,H), (H,L)$ and (L,H) (i.e., without (L,L) in any period) is cheaper for the principal to upset than *Cycle*. Step 2: Any collusive strategy that ever contains (L,L) at some period t would be cheaper for the principal to upset than either *Shirk* or *Cycle*.

Step 1: Consider any collusive strategy σ that contains only $(H,H), (H,L)$ and (L,H). The basic idea here is that, compared with *Cycle*, any reshuffling of (L,H) and (H,L) effort pairs across periods and/or introducing (H,H), can only leave some shirking agent better off in some period if it also leaves another shirking agent worse off in another period, in terms of their respective continuation payoffs. The principal would then target that latter agent. For any collusive strategy σ not including (L,L) to be attractive in the sense of (6), the joint payoff must satisfy $U_{HL} + U_{LH} > 2U_{HH}$. Therefore, we know

$$\overline{V}^{Cyc} + \underline{V}^{Cyc} \geq V_t^i(\sigma) + V_t^j(\sigma), \quad \forall t, \tag{7}$$

where $\overline{V}^{Cyc} = \sum_{t=1,3,5,\ldots} \frac{U_{HL}}{(1+r)^t} + \sum_{t=2,4,6,\ldots} \frac{U_{LH}}{(1+r)^t}$ and $\underline{V}^{Cyc} = \sum_{t=1,3,5,\ldots} \frac{U_{LH}}{(1+r)^t} + \sum_{t=2,4,6,\ldots} \frac{U_{HL}}{(1+r)^t}$ are the continuation payoffs (under *Cycle*) of the *shirking* agent and the *working* agent, respectively. We have dropped the time index in $\overline{V}^{Cyc}$ and $\underline{V}^{Cyc}$ because they are time independent. Note that (StageNE) and $U_{HL} + U_{LH} > 2U_{HH}$ together imply $U_{HL} > U_{HH} \geq U_{LH}$. Simple algebra shows

$$\overline{V}^{Cyc} > \max\{\underline{V}^{Cyc}, V^*\}, \tag{8}$$

where $V^* \equiv \frac{U_{HH}}{r}$ is each agent's continuation payoff from playing $(H,H)^\infty$.

To prove Claim 1, it is sufficient to show the following:

$$\exists t | \{(a_t^i = L, a_t^j = H) \wedge V_t^i(\sigma) \leq \overline{V}^{Cyc}\}. \tag{9}$$

That is, there will be some period in which Agent i: (i) is supposed to be the *only* "shirker" and (ii) faces a weakly lower continuation payoff (hence has stronger incentives to deviate from shirking) under the collusive strategy σ than under *Cycle*. Suppose by contradiction that (9) is violated. That is,

$$V_t^i(\sigma) > \overline{V}^{Cyc}, \quad \forall t | \{a_t^i = L, a_t^j = H\}. \tag{10}$$

We know from (7) that (10) implies the following for the other Agent j:

$$V_t^j(\sigma) < \underline{V}^{Cyc}, \quad \forall t | \{a_t^i = L, a_t^j = H\}. \tag{11}$$

Because one agent always playing L is not a subgame perfect equilibrium, there must be a switch from (L,H) to (H,L), possibly sandwiching one or more periods of (H,H). We pick any such block in the collusive strategy σ and denote by τ and $\tau+1+n$, respectively, the periods in which (L,H) and (H,L) are played (sandwiching n periods of (H,H)). We now show that for any $n \in \{0, 1, 2, 3, \ldots\}$, (10) leads to a contradiction, which then verifies (9) and proves the claim. We label the first agent in any effort pair as Agent A throughout the analysis.

- If $n = 0$, (L,H) is followed immediately by a (H,L) at $\tau+1$. Applying (11) to $t = \tau+1$, we obtain the following inequality:

  $$V_\tau^A(\sigma) = \frac{U_{HL} + V_{\tau+1}^A(\sigma)}{1+r} < \frac{U_{HL} + \underline{V}^{Cyc}}{1+r} = \overline{V}^{Cyc},$$

  which contradicts (10).
- If n is an even number ($n = 2, 4, 6, \ldots$), there are an even number of periods of (H,H) sandwiched between (L,H) and (H,L). We prove the case for $n = 2$; similar arguments apply for all even n. We can show the following for Agent A's continuation payoff at τ:

  $$V_\tau^A(\sigma) = \frac{U_{HH}}{1+r} + \frac{U_{HH}}{(1+r)^2} + \frac{U_{HL} + V_{\tau+3}^A(\sigma)}{(1+r)^3}$$

$$\begin{aligned} &< \frac{U_{HH}}{1+r} + \frac{U_{HH}}{(1+r)^2} + \frac{U_{HL} + \underline{V}^{Cyc}}{(1+r)^3} \\ &< \frac{U_{HL}}{1+r} + \frac{U_{LH}}{(1+r)^2} + \frac{U_{HL} + \underline{V}^{Cyc}}{(1+r)^3} \\ &= \overline{V}^{Cyc}, \end{aligned}$$

which again contradicts (10). The first inequality applies (11) to $t = \tau + 3$. The second inequality applies (8) and the fact that $\overline{V}^{Cyc} > V^*$ if and only if $\frac{U_{HL}}{1+r} + \frac{U_{LH}}{(1+r)^2} > \frac{U_{HH}}{1+r} + \frac{U_{HH}}{(1+r)^2}$.

- Now let n be an odd number ($n = 1, 3, 5, \ldots$). We prove the case for $n = 1$, i.e., there is one (H,H) sandwiched between (L,H) and (H,L). Similar arguments apply for all odd n. We bound Agent A's continuation payoff at τ as follows:

$$V_\tau^A(\sigma) = \frac{U_{HH}}{1+r} + \frac{U_{HL} + V_{\tau+2}^A(\sigma)}{(1+r)^2} < \frac{U_{HH}}{1+r} + \frac{U_{HL} + \underline{V}^{Cyc}}{(1+r)^2} = \frac{rV^*}{1+r} + \frac{U_{HL} + \underline{V}^{Cyc}}{(1+r)^2} = \frac{rV^*}{1+r} + \frac{\overline{V}^{Cyc}}{1+r} < \overline{V}^{Cyc},$$

which contradicts (10). The first inequality applies (11) to $t = \tau + 2$, the second equalities is from the definition of $V^* = \frac{U_{HH}}{r}$, and the last equality is by the definition of $\underline{V}^{Cyc}$ and $\overline{V}^{Cyc}$.

Step 2: We now show that any collusive strategy containing (L,L) at some period t is cheaper for the principal to upset than either *Shirk* or *Cycle*. Given any contract offered by the principal, one of the following must be true:

$$\max\{2U_{HH}, 2U_{LL}, U_{HL} + U_{LH}\} = \begin{cases} 2U_{HH} & \text{(case 1)} \\ U_{HL} + U_{LH} & \text{(case 2)} \\ 2U_{LL} & \text{(case 3)} \end{cases}$$

Case 1 is trivially collusion-proof as per (Pareto) in the main text.

Case 2 has two sub-cases: sub-case 2.1 where $\frac{U_{HL} + U_{LH}}{2} \geq U_{HH} \geq U_{LL}$ and sub-case 2.2 in which $\frac{U_{HL} + U_{LH}}{2} \geq U_{LL} > U_{HH}$. In sub-case 2.1, first note that we can ignore collusive strategy σ that contains (L,L) at some period without loss of generality. The reason is that we can construct a new strategy σ' by replacing (L,L) in σ by (H,H), and σ' is more difficult to upset than σ because both agents' continuation payoffs are at least weakly higher under σ' at every period. After ruling out collusive strategies containing (L,L), we can refer to Step 1 and show that any such collusive strategy is deterred by the (NoCycle) constraint.

In sub-case 2.2 (i.e., $\frac{U_{HL} + U_{LH}}{2} \geq U_{LL} > U_{HH}$), we argue that any collusive strategy σ that contains (L,L) at some point is easier to be upset than (thus implied by) the *Cycle* strategy. The reason is clear by comparing the action profiles between σ and *Cycle* from any $\tilde{t}$ such that $a_{\tilde{t}}(\sigma) = (L, L)$:

	$t = \tilde{t}$	$t = \tilde{t} + 1, \tilde{t} + 2, \ldots$
σ	(L,L)	$\{a^A(\sigma), a^B(\sigma)\}_{\tilde{t}+1}^{\infty}$
Cycle	(L,H)	$\{Cycle\}_{\tilde{t}+1}^{\infty}$

The maintained assumption $\frac{U_{HL} + U_{LH}}{2} \geq U_{LL} > U_{HH}$ implies $U_{HL} - U_{LL} > U_{HH} - U_{LH}$. That is, the benefit to either Agent A or B from unilaterally deviating from (L,L) at $\tilde{t}$ is higher than the benefit to Agent A from deviating from (L,H) to (H,H) at $\tilde{t}$ under the *Cycle* strategy. In addition, we know $V_{\tilde{t}}^A(Cyc) + V_{\tilde{t}}^B(Cyc) \geq V_{\tilde{t}}^A(\sigma) + V_{\tilde{t}}^B(\sigma)$ holds under Case 2, and therefore either $V_{\tilde{t}}^A(\sigma) \leq V_{\tilde{t}}^A(Cyc)$ or $V_{\tilde{t}}^B(\sigma) \leq V_{\tilde{t}}^B(Cyc)$. If $V_{\tilde{t}}^A(\sigma) \leq V_{\tilde{t}}^A(Cyc)$ holds, then A has stronger incentives to deviate at $\tilde{t}$ under σ than he would have under *Cycle*. If $V_{\tilde{t}}^B(\sigma) \leq V_{\tilde{t}}^B(Cyc)$, we make use of the observation that $V_{\tilde{t}}^B(Cyc) < V_{\tilde{t}}^A(Cyc)$ to conclude $V_{\tilde{t}}^B(\sigma) < V_{\tilde{t}}^A(Cyc)$, which means that, at $\tilde{t}$, B has stronger incentives to deviate under σ than A would have under *Cycle*. Again, once we rule out collusive strategies containing (L,L), we can refer to Step 1 and show that any remaining collusive strategies are deterred by the (NoCycle) constraint.

Case 3 implies $V_t^A(Shk) + V_t^B(Shk) = \max_\sigma\{V_t^A(\sigma) + V_t^B(\sigma)\}, \ \forall t$. If a collusive strategy σ contains $a_{t'}^A(\sigma) = a_{t'}^B(\sigma) = L$ at some period t', then either $V_{t'}^A(\sigma) \leq V_{t'}^A(Shk)$ or $V_{t'}^B(\sigma) \leq V_{t'}^B(Shk)$, which means at least one of the agents who is supposed to (jointly) shirk at t' will have a weakly stronger incentive to deviate than he would have under *Shirk* strategy. If the collusive strategy does not contain (L,L) in any period, we can refer to Step 1 and show that any such collusive strategy is deterred by the (NoCycle) constraint.□

Having proved Claims 1 and 2, it remains to prove necessity. (NoShirk) is necessary for a contract to be collusion-proof unless playing (H,H) indefinitely (i.e., $(H,H)^\infty$) Pareto-dominates *Shirk* in the sense of condition (Pareto) defined in the text (i.e., $U_{HH} \geq U_{LL}$). Likewise, (NoCycle) is necessary unless $(H,H)^\infty$ Pareto-dominates *Cycle*, which, according to (Pareto) in the text, requires $\frac{(1+r)U_{HH}}{r} \geq \sum_{t=0,2,4,\ldots} \frac{U_{LH}}{(1+r)^t} + \sum_{t=1,3,5,\ldots} \frac{U_{HL}}{(1+r)^t}$. Note that this condition is identical to (NoCycle). Collecting conditions yields Lemma 1.□

Proof of Proposition 1. As shown in the main text, for (*H*,*H*) to be an equilibrium under a conditional bonus pool, *B* has to exceed $B^{HH} = \frac{2H}{p_H}$. At the same time, (*L*,*L*) will *not* be an equilibrium, if and only if $U_{HL}^C \geq U_{LL}^C$, which is equivalent to $B \geq B^{LL} = \frac{2H}{2p - p_L}$. If $B < B^{LL}$, so that (*L*,*L*) is an equilibrium, then this equilibrium has to be Pareto-dominated by (*H*,*H*). Given the inherent symmetry among the agents, this amounts to:

$$U_{HH}^C \geq U_{LL}^C \Leftrightarrow B \geq \frac{2H}{p_H - p_L} \equiv B^{Pareto}.$$

Collusion-proofness with regard to the *Shirk* strategy in Lemma 1 requires that

$$U_{HL}^C(B) + \frac{U_{HH}^C(B)}{r} \geq \frac{1+r}{r} U_{LL}^C(B) \Leftrightarrow B \geq \frac{2(1+r)H}{p_H + 2rp - (1+r)p_L} \equiv B^{Shk}.$$

Lastly, consider agent–agent side contracting via the *Cycle* strategy in Lemma 1. Taking Agent A to be the one to choose high effort in period 0 (without loss of generality), the respective agents' expected utilities in present value terms read:

$$\overline{U}_{0,A}^{Cyc}(B) = \sum_{t=0,2,4,\ldots} \frac{pB - H}{(1+r)^t} = \frac{(1+r)^2}{r(2+r)}(pB - H),$$

$$\overline{U}_{0,B}^{Cyc}(B) = \sum_{t=1,3,5,\ldots} \frac{pB - H}{(1+r)^t} = \frac{\overline{U}_{0,A}^{Cyc}(B)}{1+r}.$$

To prevent such *Cycle* collusion, the principal has to entice the "weak link"—i.e., Agent B—to break away from the side contract:

$$\frac{1+r}{r} U_{HH}(B) \geq \overline{U}_{0,B}^{Cyc}(B) \Leftrightarrow B \geq \frac{2(1+r)H}{(2+r)p_H - 2p} \equiv B^{Cyc}.$$

In the last step, holding constant (p_H, p_L), it is a matter of straightforward algebra to derive cutoffs for p that rank the relevant *B*-values.□

Proof of Proposition 2. The precise thresholds for *r* are derived by comparing $C_t^U = \frac{1+r}{r} 2H$ with $C_t^C = p_H B^C$ as in Proposition 1. Specifically, for $p < p_H/2$ (strong complements), $C_t^U < C_t^C$ if $r > \frac{p_H - p_L}{p_L}$; whereas for $p \in [p_H/2, p_o)$ (weak complements), $C_t^U < C_t^C$ if $r > \frac{p_H - p_L}{p_H + p_L - 2p}$. For $p > p_0$, $C_t^U > C_t^C$ for all *r*.□

Proof of Proposition 3. Plugging in $\beta^*(\alpha)$ and rearranging the principal's reneging constraint (4) yields:

$$\alpha\left(r - \frac{p}{p_H - p}\right) \leq p_H\left(\frac{1}{p - p_L} - \frac{1}{p_H - p}\right). \tag{12}$$

The right-hand side of (12) is always positive for strategic complements, whereas the left-hand side is negative for $r < \frac{p}{p_H - p}$. If $r < \frac{p}{p_H - p}$, the optimal incentive weights are $\alpha^* = 1$ and $\beta^* = 0$. For $r > \frac{p}{p_H - p}$, the left-hand side is also positive and we can rewrite the reneging constraint as follows:

$$\alpha \leq \overline{\alpha} \equiv \frac{p_H\left(\frac{1}{p - p_L} - \frac{1}{p_H - p}\right)}{r - \frac{p}{p_H - p}}.$$

The optimal α^* in the case of strategic complements therefore is given by $\alpha^* = \min\{1, \overline{\alpha}\}$ and $\beta^* = (1 - \alpha^*)\beta^{HH}$. Lastly, it is easy to show that $\overline{\alpha} \geq 1$ if and only if $r \leq \frac{p_H - (p - p_L)}{p - p_L}$.□

Proof of Corollary 3. Part (i) follows from comparisons of the per-period costs given in Propositions 1 and 3. Equating $C_t^{Ind}(\alpha^*; \Delta < 0)$ with $p_H B^{Cyc}$ yields a cutoff $r_{(i)} = \frac{p}{p_H - p}$ for substitutes ($\Delta < 0$). Equating $C_t^{Ind}(\alpha^*; \Delta > 0)$ with $p_H B^{Pareto}$ yields a cutoff $r_{(ii)} = \frac{p(p_H - p)}{(p - p_L)^2}$ for strong complements (i.e., $p < \frac{p_H}{2}$). Equating $C_t^{Ind}(\alpha^*; \Delta > 0)$ with $p_H B^{Shk}$ yields a cutoff

$$r_{(iii)} \equiv \frac{2pp_L - p_L^2 + \sqrt{y_1}}{2(p - p_L)(3p - p_H - p_L)},$$

for weak complements (i.e., $p \in \left[\frac{p_H}{2}, p_o\right]$), where $y_1 = 16p^3(p_H + p_L) - 12p^4 + p_L^4 - 4p^2 p_H(p_H + 5p_L) + 4pp_L(p_H^2 + p_H p_L - p_L^2)$.

Part (ii) is established by taking derivatives of $r_{(i)}$ and $r_{(ii)}$ with respect to *p*. (Recall that an increase in *p* is equivalent to a decrease in $\Delta \equiv p_H - 2p + p_L$.) The result that $r_{(iii)}$ is decreasing in *p* follows from the fact that $C_t^{Ind}(\alpha^*; \Delta > 0)$ is increasing *p* (Corollary 2) while B^{Shk} is decreasing in *p*.□

Proof of Proposition 4. We first show that setting $w^x_{LH}=0,\ x\in\{0,1\}$, is optimal. If $w^x_{LH}>0$ in any feasible wage scheme $\mathbf{w}$, one can always lower w^x_{LH} and increase w^x_{HL} by the same amount (until $w^x_{LH}=0$) to get a new wage scheme $\mathbf{w}'$. The contract permutation does not affect the objective value and (weakly) relaxes all the constraints.

We characterize the optimal contracts (for $\Delta\geq 0$ and $\Delta<0$ separately), and then argue that they satisfy the conditions stated in Proposition 4.

Optimal contract under productive substitutes ($\Delta<0$): Making use of $w^x_{LH}=0$ for any x, we can simplify Program $\mathcal{P}$ in the text as:

$$\min_{I\in\{0,1\},\ w^x_{mn}} p_H w^1_{HH}+(1-p_H)w^0_{HH},$$

subject to, for all $m,n,m',n'\in\{H,L\},\ x=0,1$:

$$p_H w^1_{HH}+(1-p_H)w^0_{HH}-H\geq 0, \qquad (\text{StageNE}, \lambda_{SNE})$$

$$\frac{(r+1)\left(p_H w^1_{HH}+(1-p_H)w^0_{HH}-H\right)}{r}-\frac{(r+1)\left(pw^1_{HL}+(1-p)w^0_{HL}-H\right)}{r(r+2)}\geq 0, \qquad \left(\text{NoCycle}, \lambda^{Cyc}\right)$$

$$I\times\left[\frac{p_H w^1_{HH}+(1-p_H)w^0_{HH}-H}{r}+\left(pw^1_{HL}+(1-p)w^0_{HL}-H\right)-\frac{(r+1)(p_L w^1_{LL}+(1-p_L)w^0_{LL})}{r}\right]\geq 0, \qquad \left(\text{NoShirk}, \lambda^{Shk}\right)$$

$$(1-I)\times\left[p_H w^1_{HH}+(1-p_H)w^0_{HH}-p_L w^1_{LL}-(1-p_L)w^0_{LL}-H\right]\geq 0, \qquad (\text{Pareto}, \lambda^{Pareto})$$

$$\frac{2\left(\dfrac{p_H}{p_H-p}H-p_H w^1_{\mathrm{HH}}-(1-p_H)w^0_{\mathrm{HH}}\right)}{r}-\left(w^x_{\mathrm{mn}}+w^x_{\mathrm{nm}}-w^x_{m'n'}-w^x_{n'm'}\right)\geq 0, \qquad \left(\mathrm{RS}^x_{\mathrm{mn}\succ m'n'}, \lambda^x_{\mathrm{mn}\succ m'n'}\right)$$

$$w^x_{\mathrm{mn}}\geq 0. \qquad \left(\mu^x_{\mathrm{mn}}\right)$$

The program is solved in two steps. We first take the value of I as given and solve optimal contracts w^x_{mn} for $I=1$ and $I=0$ respectively, and then endogenize the value of I by comparing the optimal objective value. For a given $I\in\{0,1\}$, we form the Lagrangian $L(\mathbf{w},\lambda,\mu)$ in which the multipliers λ and μ correspond to the constraints stated above. The proof lists the optimal contract, in particular the tuple $(\mathbf{w},\lambda,\mu)$, as a function of the discount rate r. Unless otherwise noted, $I=1$ in the solution, i.e., (NoCycle) and (NoShirk) are the relevant collusion-proofness constraints. All relevant cutoffs for r are presented at the end of the proof. Zero Lagrangian multipliers are suppressed throughout to ease exhibition.

For $0<r\leq r_1$, the principal can honor the first-best contract as given by:

$$w^1_{HL}=H,\quad w^1_{HH}=H,\quad w^0_{HL}=H,\quad w^0_{HH}=H,\quad w^1_{LL}=w^0_{LL}=0,\quad \lambda^{SNE}=1.$$

The wage scheme trivially creates strategic payoff independence ($\Delta U=0$). Under the first-best contract above, the principal's reneging constraint $RS^x_{HH\succ LL},\ x\in\{0,1\}$ starts binding at $r=r_1$.

For $r_1<r\leq r_2$, the optimal contract is:

$$w^1_{LL}=\frac{H(p+pr-p_H r)}{(p-p_H)(-p_L+p_H r)},$$

$$w^1_{HL}=\frac{H\left(p(1+r)(p_L+(p_L+p_H)r)+p_H\left(p_L-(p_L+p_H)r-(p_L+p_H)r^2\right)\right)}{p(p-p_H)(1+r)(-p_L+p_H r)}-\frac{1-p}{p}w^0_{HL},$$

$$w^1_{HH}=\frac{H\left(-p(-1+p_H-r)(1+r)+p_H(p_L-r(1+r))\right)}{(p-p_H)(1+r)(-p_L+p_H r)},$$

$$w^0_{LL}=0,$$

$$w^0_{HH}=-\frac{Hp_H(p-p_L+pr)}{(p-p_H)(1+r)(-p_L+p_H r)},$$

$$\lambda^{Shk}=\frac{p_H r}{(1+r)^2(-p_L+p_H r)},\qquad \lambda^{Cyc}=\frac{p_H r^2(2+r)}{(1+r)^2(-p_L+p_H r)},$$

$$\lambda^1_{HH\succ LL}=\frac{p_L p_H}{2(1+r)(-p_L+p_H r)},\qquad \lambda^0_{HH\succ LL}=\frac{p_L-p_L p_H}{2(1+r)(-p_L+p_H r)},$$

$$\mu^0_{LL}=\frac{(-p_L+p_H)r}{(1+r)(-p_L+p_H r)}.$$

The optimal contract is not unique, but all solutions to the program create strategic payoff independence ($\Delta U=0$). The non-uniqueness arises because the principal can move payments between w^1_{HL} and w^0_{HL} at a rate of $\left(-\frac{1-p}{p}\right)$, subject to non-negative payments and the reneging constraints ($\mathrm{RS}^x_{mn \succ m'n'}$). An exhaustive characterization of the extent to which the principal can move between the two payments is lengthy and tedious. At $r = r_2$, $RS^x_{HH \succ HL}$, $x \in \{0,1\}$ starts binding and the principal's ability to freely move payments between w^1_{HL} and w^0_{HL} vanishes.

For $r > r_2$, the optimal contract is:

$$
\begin{aligned}
w^1_{LL} &= -\frac{H(p_H + (1+r)(p + pr - p_H r))}{(p - p_H)(2p(1+r) - p_H r(2+r))},\\
w^1_{HL} &= 2w^1_{LL},\\
w^1_{HH} &= \frac{H\left(p_H(-1 + p_H + r + r^2) + p\left(p_H(-1+r) - (1+r)^2\right)\right)}{(p - p_H)(2p(1+r) - p_H r(2+r))},\\
w^0_{LL} &= w^0_{HL} = 0,\\
w^0_{HH} &= \frac{H p_H (p_H + p(-1+r))}{(p - p_H)(2p(1+r) - p_H r(2+r))},\\
\lambda^{Cyc} &= \frac{p_H r(2+r)}{-2p(1+r) + p_H r(2+r)}, \quad \lambda^1_{HH \succ HL} = -\frac{p p_H}{2p(1+r) - p_H r(2+r)},\\
\lambda^0_{HH \succ HL} &= \frac{p(-1 + p_H)}{2p(1+r) - p_H r(2+r)}, \quad \mu^0_{HL} = \frac{(p - p_H) r}{2p(1+r) - p_H r(2+r)}.
\end{aligned}
$$

The contract creates strategic substitutability in agents' payoffs ($\Delta U < 0$).

Here $r_1 = \frac{p}{p_H - p}$, and $r_2 = -\frac{-4p^2 - p_H(p_L + p_H) + p(2p_L + 3p_H) - \sqrt{p_H}\sqrt{p_H(p_L + p_H)(5p_L + p_H) + p^2(8p_L + 9p_H) - 2p(2p_L^2 + 9p_L p_H + 3p_H^2)}}{2(p - p_H)(-2p + p_L + p_H)}$.□

Optimal contract under productive complements ($\Delta \geq 0$): We use the same programming approach as for $\Delta < 0$, but change the principal's reneging constraints to ($\mathrm{RC}^x_{mn \succ m'n'}$) derived in the main text for the case of productive complements.

For $0 < r \leq r'_1$, the principal can honor the first best contract as given by (all r-cutoffs are stated in closed form at the end):

$$
w^1_{HL} = H, \quad w^1_{HH} = H, \quad w^0_{HL} = H, \quad w^0_{HH} = H, \quad w^1_{LL} = w^0_{LL} = 0, \quad \lambda^{SNE} = 1.
$$

The first-best contract creates strategic payoff independence ($\Delta U = 0$). Given the contract, $RC^x_{HH \succ LL}, x \in \{0,1\}$ starts binding at $r = r'_1$.

For $r'_1 < r \leq r'_2$, the optimal contract is:

$$
\begin{aligned}
w^1_{LL} &= H\frac{p_L + p_H + p_L r - p(1+r)}{(p - p_L)(p_L - p_H r)},\\
w^1_{HL} &= H\frac{-p(1+r)(p_L + (p_L + p_H)r) + p_L\left(p_L(1+r)^2 + p_H(2 + r(2+r))\right)}{p(p - p_L)(1+r)(p_L - p_H r)} - \frac{1-p}{p} w^0_{HL},\\
w^1_{HH} &= H\frac{-(-1 + p_H)p_H(1+r) + p(-1 + p_H - r)(1+r) + p_L(1 + r(2 - p_H + r))}{(p - p_L)(1+r)(p_L - p_H r)},\\
w^0_{LL} &= 0,\\
w^0_{HH} &= -\frac{p_H(p_H + (p_L + p_H)r - p(1+r))}{(p - p_L)(1+r)(p_L - p_H r)} H,\\
\lambda^{Shk} &= \frac{p_H r}{(1+r)^2(-p_L + p_H r)}, \quad \lambda^{Cyc} = \frac{p_H r^2(2+r)}{(1+r)^2(-p_L + p_H r)},\\
\lambda^1_{HH \succ LL} &= \frac{p_L p_H}{2(1+r)(-p_L + p_H r)}, \quad \lambda^0_{HH \succ LL} = \frac{p_L - p_L p_H}{2(1+r)(-p_L + p_H r)},\\
\mu^0_{LL} &= \frac{(-p_L + p_H) r}{(1+r)(-p_L + p_H r)}.
\end{aligned}
$$

The optimal contract is again not unique, but all optimal contracts create strategic payoff independence ($\Delta U = 0$): the principal can move payments between w^1_{HL} and w^0_{HL}, subject to non-negative payments and the reneging constraints ($\mathrm{RC}^x_{mn \succ m'n'}$). For example, setting $w^0_{HL} = -\frac{2Hp_H(-p(r+1) + r(p_L + p_H) + p_H)}{(r+1)(p - p_L)(p_L - p_H r)}$ is optimal unless both $p < \frac{1}{2}$ and $r'_1 < r \leq r'$ hold for a constant $r' \in (r'_1, r'_2)$. (Note that $r' > r'_1$ if and only if $p < \frac{1}{2}$.) If both $p < \frac{1}{2}$ and $r'_1 < r \leq r'$ hold, setting $w^0_{HL} = \frac{-2p^2(r+1)^2 + p(r+1)(3p_L(r+1) + p_H(r+2)) - p_L\left(p_L(r+1)^2 + p_H(r(r+2)+2)\right)}{(p-1)(r+1)(p - p_L)(p_L - p_H r)} H$ is optimal. At $r = r'_2$, $RC^x_{HL \succ LL}$, $x \in \{0,1\}$ starts binding and the principal's ability to move payments between w^1_{HL} and w^0_{HL} vanishes.

For $r_2' < r \le r_3'$, the optimal contract is:

$$
\begin{aligned}
w_{LL}^1 &= \frac{H(1+r)(-2p_H+p(2+r)-p_L(2+r))}{(p-p_L)(2pr(1+r)-p_L(2+r(2+r)))},\\
w_{HL}^1 &= H\frac{-(2(2p^2(1+r)+p_L^2(1+r)+2p_H(1+r)+p_L(2+r)(1+p_H+r)-p(1+r)(2+3p_L+2p_H+r)))}{(p-p_L)(2pr(1+r)-p_L(2+r(2+r)))},\\
w_{HH}^1 &= H\frac{2p^2r(1+r)+p_L^2r(1+r)-p_L(2p_H+3pr(1+r))}{p_H(p-p_L)(2pr(1+r)-p_L(2+r(2+r)))}-\frac{1-p_H}{p_H}w_{HH}^0,\\
w_{LL}^0 &= 0,\\
w_{HL}^0 &= H\frac{-4p^2(1+r)+2p(3p_L+2p_H)(1+r)-2p_L(p_L+2p_H+(p_L+p_H)r)}{(p-p_L)(2pr(1+r)-p_L(2+r(2+r)))},
\end{aligned}
$$

$$
\begin{aligned}
\lambda^{Shk} &= -\frac{2pr}{(1+r)(-2pr(1+r)+p_L(2+2r+r^2))}, \quad \lambda^{Cyc} = \frac{r(2+r)(p_L-2pr+p_Lr)}{(1+r)(-2pr(1+r)+p_L(2+2r+r^2))},\\
\lambda_{HL\succ LL}^1 &= \frac{pp_L}{2pr(1+r)-p_L(2+2r+r^2)}, \quad \lambda_{HL\succ LL}^0 = \frac{(-1+p)p_L}{-2pr(1+r)+p_L(2+2r+r^2)},\\
\mu_{LL}^0 &= \frac{2(p-p_L)r}{2pr(1+r)-p_L(2+2r+r^2)}.
\end{aligned}
$$

The optimal contract is not unique, but all solutions to the program create strategic payoff independence ($\Delta U = 0$): the principal can move payments between w_{HH}^1 and w_{HH}^0, subject to non-negative payment and the reneging constraints ($\mathrm{RC}_{mn\succ m'n'}^x$). For example, we later specify a constant $r'' \in (r_2', r_3')$ and for $r'' < r \le r_3'$, setting $w_{HH}^0 = 0$ is optimal; while for $r_2' < r \le r''$, setting $w_{HH}^0 = -\frac{H(2p^2(r+1)(p_H+r)-p(r+1)(p_H(3p_L+r+2)+3p_Lr+2p_H^2)+p_H(p_L^2+2(p_L+1)p_H)+p_Lr^2(p_L+p_H)+r(p_L+p_H)((p_L+2)p_H+p_L))}{(p_H-1)(p-p_L)(2pr(r+1)-p_L(r(r+2)+2))}$ is optimal. At $r = r_3'$, $RC_{HL\succ HH}^x$ becomes binding for any $x \in \{0,1\}$, and the principal's ability to move payments between w_{HH}^1 and w_{HH}^0 vanishes.

For $r > r_3'$, the optimal contract is:

$$
\begin{aligned}
w_{LL}^1 = w_{HH}^1 &= H\frac{-2p_L+2p_H^2+(p_L+p_H)(-2+p_L+2p_H)r+p_L(p_L+p_H)r^2+2p^2r(1+r)-p(1+r)(-2+3p_Lr+p_H(2+r))}{(p-p_L)(-2p_L+2p_H-(-2p+p_L+p_H)r^2)},\\
w_{HL}^1 &= H\frac{-2(2p^2+p_L^2-2p_H^2+p_L(2+p_H)+p(-2-3p_L+p_H))+2(2p+(-2+p_H)(p_L+p_H))r+2(p-p_L)(2p-p_L-p_H)r^2}{(p-p_L)(-2p_L+2p_H-(-2p+p_L+p_H)r^2)},\\
w_{LL}^0 = w_{HH}^0 &= H\frac{p_L^2r+2p^2r(1+r)+(2p_H+p_Lr)(p_H+(p_L+p_H)r)-p(1+r)(3p_Lr+p_H(2+r))}{(p-p_L)(-2p_L+2p_H-(-2p+p_L+p_H)r^2)},\\
w_{HL}^0 &= 2H+\frac{2H(-2p^2-3p_L^2+p(5p_L-3p_H)+p_Lp_H(1+r)+p_H^2(2+r))}{(p-p_L)(-2p_L+2p_H-(-2p+p_L+p_H)r^2)},
\end{aligned}
$$

$$
\begin{aligned}
\lambda^{Shk} &= -\frac{2(p-p_H)r}{(1+r)(-2pr^2+p_H(-2+r^2)+p_L(2+r^2))}, \quad \lambda^{Cyc} = \frac{r(2+r)(p_H(-1+r)-2pr+p_L(1+r))}{(1+r)(-2pr^2+p_H(-2+r^2)+p_L(2+r^2))},\\
\lambda_{HL\succ LL}^1 &= \frac{p_L(-p+p_H)}{-2pr^2+p_H(-2+r^2)+p_L(2+r^2)}, \quad \lambda_{HL\succ LL}^0 = \frac{(-1+p_L)(p-p_H)}{-2pr^2+p_H(-2+r^2)+p_L(2+r^2)},\\
\lambda_{HL\succ HH}^1 &= \frac{(-p+p_L)p_H}{2pr^2-p_H(-2+r^2)-p_L(2+r^2)}, \quad \lambda_{HL\succ HH}^0 = \frac{(p-p_L)(-1+p_H)}{2pr^2-p_H(-2+r^2)-p_L(2+r^2)}.
\end{aligned}
$$

The optimal contract creates strategic payoff independence ($\Delta = 0$). However, if $\tilde{r}$ (as defined below) is a real number and $r \in \left(\mathrm{Max}[r_3', \tilde{r}], \tilde{\tilde{r}}\right)$, the optimal contract is as follows:

$$
\begin{aligned}
w_{LL}^1 &= \frac{H(p_L+p_H+p_Lr-p(1+r))}{(p-p_L)(p_L+p_Lr-p_Hr)},\\
w_{HL}^1 &= 2w_{LL}^1,\\
w_{HH}^1 &= \frac{H(p_L+p_H-p_H^2+p(-1+p_H-r)+p_Lr)}{(p-p_L)(p_L+p_Lr-p_Hr)},\\
w_{LL}^0 &= w_{HL}^0 = 0,\\
w_{HH}^0 &= \frac{H(p-p_H)p_H}{(p-p_L)(p_L+p_Lr-p_Hr)}
\end{aligned}
$$

$$
\lambda^{Pareto} = -\frac{p_Hr}{p_L+p_Lr-p_Hr}, \quad \lambda_{HH\succ LL}^1 = -\frac{p_Lp_H}{2(p_L+p_Lr-p_Hr)}, \quad \mu_{LL}^0 = 1-\frac{p_L}{p_L+p_Lr-p_Hr}.
$$

The last contract, which creates strategic payoff complements ($\Delta > 0$), is the *only* case where the principal chooses to use (Pareto) to break the agents' collusive strategy, i.e., $I = 0$.

The relevant r-cutoffs invoked above are:

$$r_1' = \frac{p_L + p_H - p}{p - p_L},$$

$$r_2' = \frac{4p^2 + 2(p_L + p_H)^2 - p(6p_L + 5p_H) + \sqrt{p_H}\sqrt{p^2(8p_L + 9p_H) + 4(p_L + p_H)(p_L^2 + p_L p_H + p_H^2) - 4p(3p_L^2 + 4p_L p_H + 3p_H^2)}}{2(p - p_L)(-2p + p_L + p_H)},$$

$$r_3' = \frac{2p^2 + p_L^2 + 3p_L p_H + 2p_H^2 - 3p(p_L + p_H) + \sqrt{8(p - p_L)(p - p_H)(2p - p_L - p_H)p_H + (2p^2 - 3p(p_L + p_H) + (p_L + p_H)(p_L + 2p_H))^2}}{2(p - p_L)(-2p + p_L + p_H)},$$

$$\tilde{r} = \frac{-A - \sqrt{A^2 + 4(p - p_L)p_L(p_H + p_L - 2p)(2p_H^2(p_L + p) + p_H p_L(p_L - 3p) - 2p_H^3 + p_L(-3pp_L + p_L^2 + 2p^2))}}{2(p - p_L)p_L(p_H + p_L - 2p)},$$

$$\tilde{\tilde{r}} = \frac{-A + \sqrt{A^2 + 4(p - p_L)p_L(p_H + p_L - 2p)(2p_H^2(p_L + p) + p_H p_L(p_L - 3p) - 2p_H^3 + p_L(-3pp_L + p_L^2 + 2p^2))}}{2(p - p_L)p_L(p_H + p_L - 2p)},$$

$$r' = \frac{-B + \sqrt{B^2 + 4(p - p_L)(p_H + p_L - 2p)(-p(2p_H(p_H + 2) + 3p_L) + 2p_H p_L + 2p^2(p_H + 1) + 2p_H^2 + p_L^2)}}{2(p - p_L)(p_H + p_L - 2p)},$$

$$r'' = \frac{C + \sqrt{C^2 - 4p_H(p - p_L)(-p_H - p_L + 2p)(-p(2p_H + 3p_L + 2) + 2p_H(p_L + 1) + p_L^2 + 2p^2)}}{2(p - p_L)(p_H + p_L - 2p)},$$

where

$$A = (p_H + p_L - 2p)(2(p - p_L)p_L - pp_H),$$

$$B = -2p^2(p_H + 2) + p(2p_L(p_H + 3) + p_H(2p_H + 5)) - 2(p_L + p_H)^2, \quad \text{and}$$

$$C = 2p^2(p_H + 1) - p(3p_L(p_H + 1) + p_H(2p_H + 3)) + (p_L + p_H)((p_L + 2)p_H + p_L).$$

We now turn to the properties of the optimal contract stated in Proposition 4. The continuity of the principal's payoff in r is verified by comparing the objective function values of the optimal contracts at all of the r cutoffs.

(i) r' in the proposition corresponds to r_1 and r_1' in the proof.

(ii) $(r', r'']$ in the proposition corresponds to $(r_1, r_2]$ and $(r_1', r_3']$ in the proof.

(iii) It is verified by taking the limit of the optimal contracts as r becomes arbitrarily large.□

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