

## SPACED RESPONDING AND CHOICE: A PRELIMINARY ANALYSIS<sup>1</sup>

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Pigeons were exposed to reinforcement both for short ( $2 < \text{IRT} < 3$  sec) and long ( $10 < \text{IRT} < 11$  sec) interresponse times. They developed bimodal interresponse-time distributions, which were decomposable into two independent component distributions under the control of the short and long contingencies respectively. The birds' allocation of responses between these two distributions was determined by a simple power-law relationship between reinforcement ratios, and response ratios derived from the component distributions. Comparison between this situation and concurrent choice situations raises the possibility that the power-law relation between ratios may be a more general law of choice than the matching of relative frequencies (probabilities).

Pigeons will learn to space successive key-pecks in time if such spacing is a condition of reinforcement (Ferster and Skinner, 1957; Staddon, 1965). This experiment investigated the asymptotic behavior of pigeons exposed to two concurrent spacing requirements, a procedure similar to the concurrent schedules of interresponse-time reinforcement recently reported by Malott and Cumming (1966). Malott and Cumming used probabilistic (*i.e.*, ratio-like) reinforcement contingencies and typically found the major mode of the resultant interresponse time (IRT) distribution to lie at or near the lower bound of the shorter of the two IRT classes. This experiment, while different in a number of respects from their work, employed variable-interval (VI) reinforcement for the shorter of two narrow (1-sec) and widely spaced bands and fixed-ratio one (FR 1) for the longer. A prediction by analogy with concurrent variable-interval experiments using separate operanda, is that this modification of the contingencies should yield modes in the vicinity of both reinforced IRT bands for most, if not all, VI values. This outcome would, in turn, allow a choice analysis in terms of two independent response classes. Such an analysis is the major aim of this study.

A previous investigation of DRL (differential reinforcement of low rate) schedules (Staddon, 1965) found that, in general, only points lying on the matching line relating DRL value (only responses terminating IRTs longer than the DRL value are reinforced on DRL schedules) and modal IRT were truly stable; points obtained early in the experiment being more or less displaced from the line and showing metastability (*i.e.*, non-recoverability) after interpolation of other experimental conditions. In the earlier experiment, the stability of a given empirical relation justified its adoption as, in some sense, the "true" function relating DRL value and modal IRT, only points lying close to the matching line being stable. In the present experiment, therefore, most of the experimental conditions were imposed twice to check the stability of the obtained functional relations.

Previous investigations of concurrent spaced responding (*e.g.*, Malott and Cumming, 1966) have commented on an apparent "bias" in favor of short interresponse times, *i.e.*, both rats and pigeons tend to respond too soon to maximize reinforcement rate. This failure is shown both by the location of the modal IRT on DRL schedules (at or just below the DRL value) and by the high relative frequency of short *vs.* long IRTs on concurrent DRL schedules. The present results suggest that when defined in terms of an optimal strategy, such as "momentary maximizing", the magnitude of bias in concurrent DRL is less than

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in simple DRL experiments. However, the optimality approach fails to deal with the most striking regularities in this situation, which involve obtained, rather than scheduled, variable values.

## METHOD

### *Subjects*

Four male White Carneaux pigeons, three with an experimental history involving only spaced responding (the results of these experiments are reported in Staddon, 1965) and the fourth (#244) with a varied experimental history, were maintained at approximately 80% of their free-feeding weights throughout the experiment.

### *Apparatus*

A standard Grason-Stadler two-key pigeon box, with one key occluded, was used. The response key was transilluminated with white light. A relay "feedback" click accompanied each effective key-peck. A force of about 20 g was sufficient to operate the key. Reinforcement was 3-sec access to mixed grain; during reinforcement house and key lights were out and the feeder illuminated.

During all the experimental conditions, and the later training conditions, the house and key lights were dimmed briefly after each recorded response by interposing a 500-ohm resistor in series with the 10-w key and house lights (which were connected in parallel). The duration of the dimming was equal to 10% of the lower bound of the longer class of reinforced IRTs; *i.e.*, 1 sec for the experimental conditions.

Interresponse times were recorded in 16 class intervals (cells) with a cell width of 1 sec. Cell 16 contained all IRTs longer than 15 sec.

Scheduling was accomplished by a system of relays and timers located in an adjoining room. White noise, and the noise of the ventilating fan, masked most extraneous sounds.

### *Procedure*

All pigeons were accustomed to key-pecking and to feeding from the grain magazine. Since three of the animals had experienced only DRL procedures, relatively little further training was required on the more difficult procedure used here. These three birds received a

total of 39 sessions on the following sequence of preliminary DRL and DRL with limited hold (*i.e.*, an upper as well as a lower bound on the class of reinforced IRTs) before being exposed to the experimental conditions listed in Table 1: DRL 15, DRL 15 LH 3, DRL 15 LH 1.5, DRL 10 LH 2. The other bird, #244, received an additional 43 sessions of training, comprising a sequence of DRL 5, DRL 7.5, and DRL 10, before experiencing the same preliminary sequence as the other three pigeons. All four birds were exposed to the same sequence of experimental conditions for approximately the same number of sessions. The experimental conditions, which are listed in Table 1, involved a DRL schedule in which all IRTs between 10 and 11 sec (cell 11) were reinforced, while IRTs between 2 and 3 sec (cell 3) were reinforced, on a variable-interval schedule, at the maximum rate indicated in the table. As the table indicates, the maximum frequency of reinforcement in cell 3 was gradually increased and then decreased during the course of the experiment.

Sidman (1956) and Blough (1963) have pointed out that very short, "burst" IRTs seem to differ from longer IRTs in their functional properties. For this reason it was arranged that each response produced a 1-sec dimming of house and key lights in addition to the usual relay "feedback" click. It was hoped that these brief S<sup>A</sup> stimuli would suppress the tendency to emit large numbers of "burst" IRTs, since these are presumably irrelevant to the processes of temporal discrimination being studied. Since few IRTs occurred in cell 1, these stimuli were effective in this respect.

The circuitry defined an interresponse time as the time between a key-peck and either (a) the previous key-peck, (b) the end of reinforcement, or (c) the start of the session, whichever was the shortest. Sessions lasted for 60 reinforcements. Birds performed daily throughout the experiment.

## RESULTS

### *IRT Distributions*

Interresponse time distributions for five representative experimental conditions are shown in Fig. 1 and 2. The characteristics of these distributions, and of those for the remaining conditions, may be summarized as

Table 1

Maximum (scheduled) reinforcement rate for cell-3 IRTs, ( $r_s$ ), number of sessions (range for four pigeons), obtained reinforcement rates for cells 3 and 11 ( $r_s$  and  $r_{11}$ ) and rates of production of IRTs, over the indicated intervals, per 100 min ( $n_s$  and  $n_{11}$ ), for each bird and each condition of the experiment. IRTs in cell 11 were always reinforced. Data are averaged over the last five days under each condition.

Cell 3 Max. Reinf. Rate/ 100 Min ( $r_s$ )	Ses, sions (Range)	$(IRT_s < 6 \text{ Sec}) / 100 \text{ Min}$ ( $n_s$ )				$(IRT_s > 6 \text{ Sec}) / 100 \text{ Min}$ ( $n_{11}$ )				Cell-3 Reinf./100 Min ( $r_s$ )				Cell-11 Reinf./100 Min ( $r_{11}$ )				
		420	421	422	244	420	421	422	244	420	421	422	244	420	421	422	244	
I	0.0	29-41	6.74	37.08	28.43	34.30	592.96	585.17	502.51	546.28	0.00	0.00	0.00	0.00	100.96	92.74	137.56	125.47
II	1.1	8-13	9.75	91.24	14.87	25.41	544.17	510.90	556.98	610.80	0.00	0.00	0.00	0.00	139.29	59.78	135.19	89.65
III	2.2	22	27.63	75.12	26.77	165.59	578.65	585.11	516.88	574.14	0.06	1.44	0.00	1.55	117.31	84.90	123.56	91.65
IV	3.3	26	14.96	219.13	105.31	192.13	540.19	547.05	558.71	547.61	0.00	3.79	2.03	3.06	135.73	91.06	119.95	127.95
V	5.5	71	119.27	258.30	267.54	400.17	539.88	521.63	486.56	473.28	3.70	5.06	5.61	5.17	154.63	79.35	147.55	114.05
VI	7.7	66	233.57	531.85	447.14	628.58	473.47	429.76	443.73	433.30	7.62	8.03	7.35	8.42	119.32	106.68	77.44	88.73
VII	16.7	71	563.62	703.11	770.39	614.99	368.83	399.14	389.86	398.13	17.87	16.90	14.99	18.01	89.35	61.11	84.97	94.56
VIII	33.0	53	612.83	895.58	564.07	831.02	373.22	321.23	397.10	363.53	33.59	32.04	33.18	32.88	79.63	88.12	127.37	77.95
IX	67.0	37-39	910.24	1459.62	972.94	1159.73	346.15	168.39	341.66	294.81	50.57	51.87	52.08	50.36	80.21	44.18	119.61	50.36
X	200.0	39-42	1728.78	1875.18	1796.00	1956.26	196.06	127.95	239.03	73.22	141.88	158.13	141.66	153.85	40.81	9.50	48.06	4.21
XI	67.0	29	756.38	1798.60	1186.14	1675.89	353.96	161.91	321.37	123.22	55.32	56.82	56.31	58.12	91.55	25.93	81.03	11.90
XII	33.0	27	668.36	1042.09	658.16	617.19	379.06	254.88	398.62	390.46	32.42	32.67	33.08	32.15	92.27	51.82	122.01	94.75
XIII	16.7	40	494.56	703.58	685.57	471.04	379.87	354.04	398.91	425.98	17.38	17.68	15.38	16.74	82.92	86.34	89.49	112.07
XIV	5.5	41	291.14	284.04	201.72	415.22	417.64	445.19	488.95	438.42	5.62	5.41	4.88	5.70	114.85	110.52	141.64	101.30
XV	1.1	49	117.66	157.03	18.97	108.66	470.99	500.87	500.72	508.91	1.12	1.48	0.49	1.01	111.63	110.14	149.26	100.22

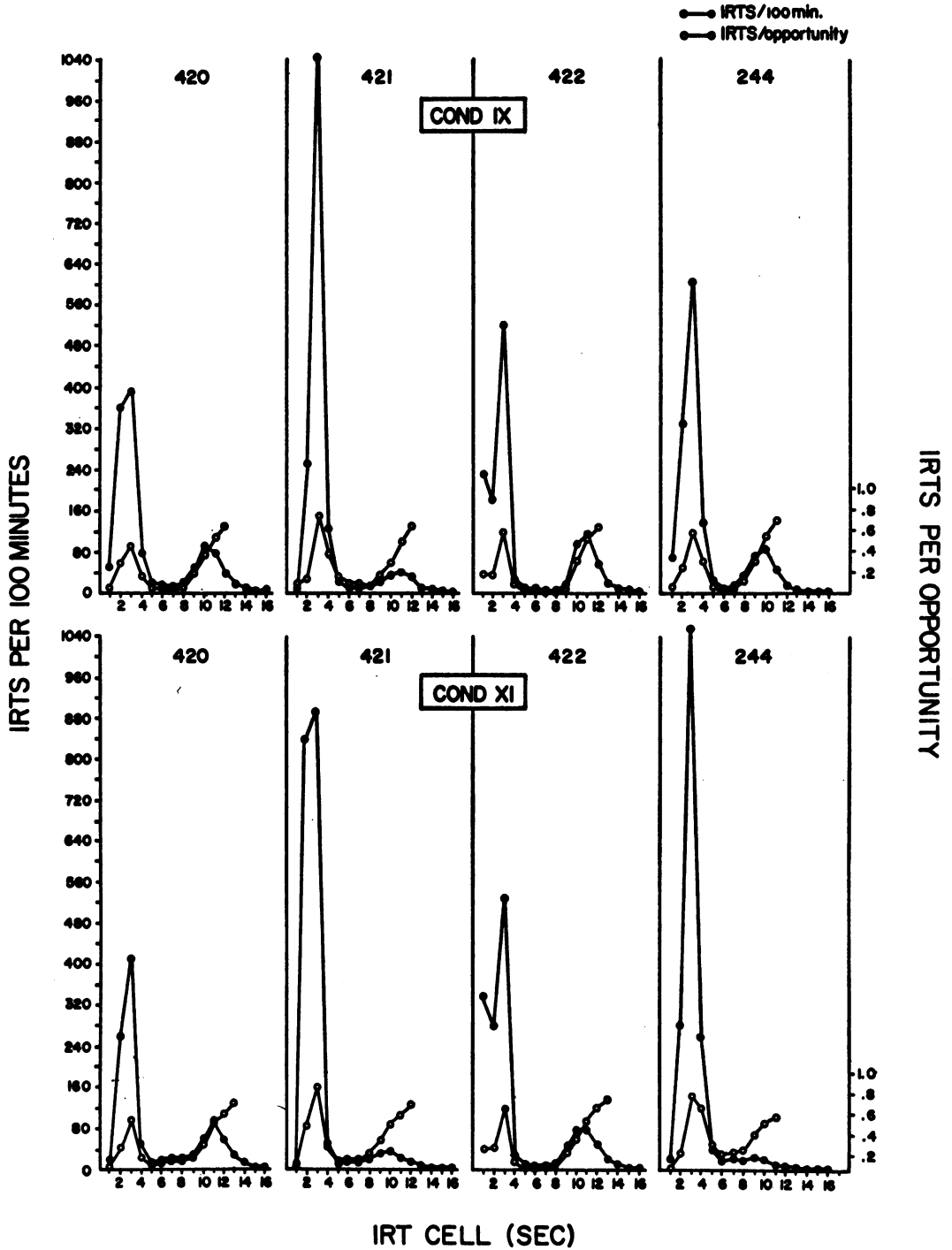


Fig. 1. Interresponse time distributions and IRTs/Op functions for Conditions IX and XI (cell-3 IRTs reinforced on VI 1.5-min, cell-11, FR 1) for the four pigeons. These distributions are the average of the last five days responding under each condition. IRTs/Op are not plotted when fewer than 50 opportunities occurred during five days.

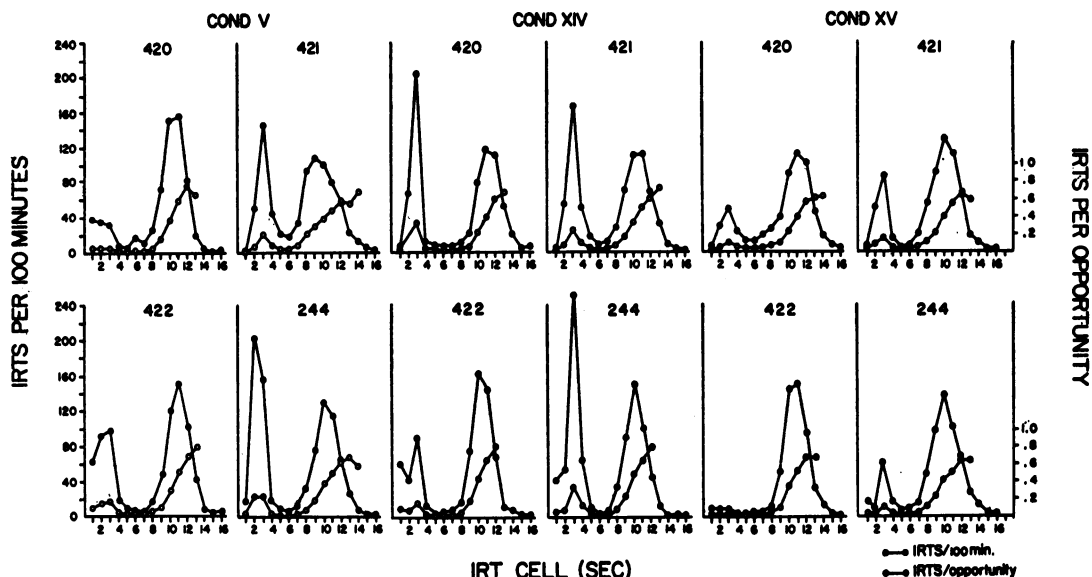


Fig. 2. Interresponse time distributions and IRTs/Op functions for Conditions V and XIV (cell-3 IRTs reinforced on VI 18-min, cell-11, FR 1), and Condition XV (cell-3 IRTs reinforced on VI 90-min, cell-11, FR 1) for the four birds. Other details as in Fig. 1.

follows: (a) Apart from Conditions I and II (all pigeons) and III (420, 421, and 422) all IRT distributions for all conditions show major modes in the neighborhood of cells 3 and 11. (b) Only 16, out of a total of 60, pigeon-conditions show any evidence of "bursting", *i.e.*, a substantial mode in the first cell. Eleven of these instances were contributed by Bird 422; nevertheless the data for this bird in Fig. 1 and 2 indicate that even here "bursting" was relatively insignificant. This absence of bursting is presumably attributable to the 1-sec S<sup>4</sup> presented after each response. (c) The two major modes of the IRT distribution were generally located in cells 3 and 11. Only Bird 244 deviated significantly from this pattern with the second mode quite consistently in cell 10. The frequency of location of modes, totaled over all four birds and all conditions, is shown in Table 2. (d) For

Table 2

Location of Modes over 60 Bird-Conditions

IRT cell	1	2	3	9	10	11	12
# modes	16	6	41	8	25	24	2

those conditions where the IRT distribution was bimodal, the minimum between the two peaks lay in the vicinity of cells 5 and 6. (e) In apparent agreement with the results of

Malott and Cumming (1966), increments in reinforcement frequency for short IRTs seem to produce disproportionately large increases in rate of production of these IRTs. (f) Conditions II and III showed metastability for most birds, since the rate of production of cell-3 IRTs was low, although the reinforcement rate for those IRTs was equal to or greater than that prevailing during Condition XV, which was accompanied by a substantial rate of 2- to 3-sec IRTs for three out of four birds (see Fig. 2). A similar lag between rate of cell-3 IRTs and reinforcement rate for those IRTs occurred between Conditions IX and XI for Bird 244. In general, however, the relation between reinforcement frequency (for cell-3 IRTs) and IRT distribution proved recoverable (*cf.* Fig. 3 and 6). (g) While the left-hand mode of the IRT distributions was disproportionately affected by the reinforcement frequency for cell-3 IRTs, the right half of the distributions, with a mode in the vicinity of cell 11, seemed little affected. When reinforcement frequency for cell 3 was high, the height of the second mode was reduced somewhat, but the range of variation of this height (when plotted as a *rate*, as in Fig. 1 and 2) was small and possibly due simply to lack of opportunities (in the sense of Anger's [1956] IRTs/Op measure) rather than any shift toward exclusive production of the shorter

IRTs. The relations among these four variables—reinforcement frequency for cells 3 and 11 and rate of production of IRTs in (and in the neighborhood of) cells 3 and 11—constitute the core of this study and are discussed below.

#### *Invariance of Component Distributions*

For those conditions which produced bimodal IRT distributions, the two modes were in the vicinity of cells 3 and 11. Both the stability of location of the two modes, and the relative invariance in the height of the longer (cell 11) mode as the height of the shorter (cell 3) mode varied over a wide range, suggest that the observed bimodal distributions result from two independent subdistributions, one with a mode between 2 and 3 sec (Distribution 3), and the other with a mode between 10 and 11 sec (Distribution 11). To test this hypothesis of independence an estimate of the best partition of the bimodal distribution is necessary. The simplest possibility is to take all IRTs shorter than the minimum between the two modes and assign them to Distribution 3, while assigning all longer IRTs to Distribution 11. Since the minimum, for most conditions, was in the vicinity of cells 5 and 6, IRTs in cells 1 through 5 were assigned to the shorter distribution (Distribution 3) and IRTs in cells 6 through 16 to the longer distribution (Distribution 11).

The hypothesis of independence asserts that while the areas of the two subdistributions may be affected by changes in the associated reinforcement frequencies, the locations of the subdistributions are not so affected. This assertion may be tested most directly by plotting the median IRTs from Distribution 3 ( $0 < \text{IRT} < 6$  sec) and Distribution 11 ( $\text{IRT} > 6$  sec) as a function of the nine different experimental conditions and their five replications. This has been done for the four birds in Fig. 3, which also shows the ratio of these two medians,  $t_3/t_{11}$ , for each condition. Three points emerge from this figure: (a) After the early conditions, the medians of both subdistributions remained relatively constant, as did the ratio, demonstrating recoverability. (b) The asymptotic location of each median seems to be close to the lower bound of the associated reinforced interval for most pigeons. (c) With the exception of small deviations under Condition X

and Conditions II and III the ratio  $t_3/t_{11}$  is between 0.2 and 0.25 for all pigeons. The assumption of invariance, both with respect to the absolute values of  $t_3/t_{11}$ , and their ratio, is well satisfied.

The agreement between data and prediction indicated by Fig. 3 suggests strongly that the appropriate dependent variables here are not simply the two classes of reinforced IRTs (*i.e.*, cells 3 and 11), but rather the two distributions centered on those classes. This assumption simplifies the search for functional relations and accommodates that search to behavioral units that are jointly determined by the situation and by the pigeons' capacities.

#### *Functional Relations*

Although Distribution 3 and Distribution 11 are invariant and independent as far as location, this independence cannot extend to their relative areas. There is necessarily an inverse relationship between the area under Distribution 3 (*i.e.*,  $n_3$ , the rate of Distribution-3 IRTs, using the coordinates of the IRT plots of Fig. 1 and 2) and the area under Distribution 11 ( $n_{11}$ , the rate of Distribution-11 IRTs) here. This is because a Distribution-3 IRT takes up an interval of time equal, on the average, to the mean of the associated distribution, as does a Distribution-11 IRT; hence an increase in the frequency of one class of IRTs must involve a decrease in the frequency of the other. The total number of Distribution-3 IRTs ( $n_3$ ) multiplied by their average duration, plus the total number of Distribution-11 IRTs ( $n_{11}$ ) multiplied by their average duration must be a constant, over any given time interval. Thus, if  $t_3$  and  $t_{11}$  are the means of the two distributions, then over 6000 sec the following relationship should hold:

$$n_3 t_3 + n_{11} t_{11} = 6000 \quad (1)$$

That the data do, in fact, conform to this constraint can be seen from Fig. 4, which is a scatter plot of  $n_3$  and  $n_{11}$  for all animals and conditions. The line in the figure is drawn by eye through the data points and represents Equation (1) with a value of 2.5 for  $t_3$  and 10.5 for  $t_{11}$ . These mean estimates are closely comparable to the medians presented in the previous figure.

The analysis thus far shows that the situation used here has both similarities to and

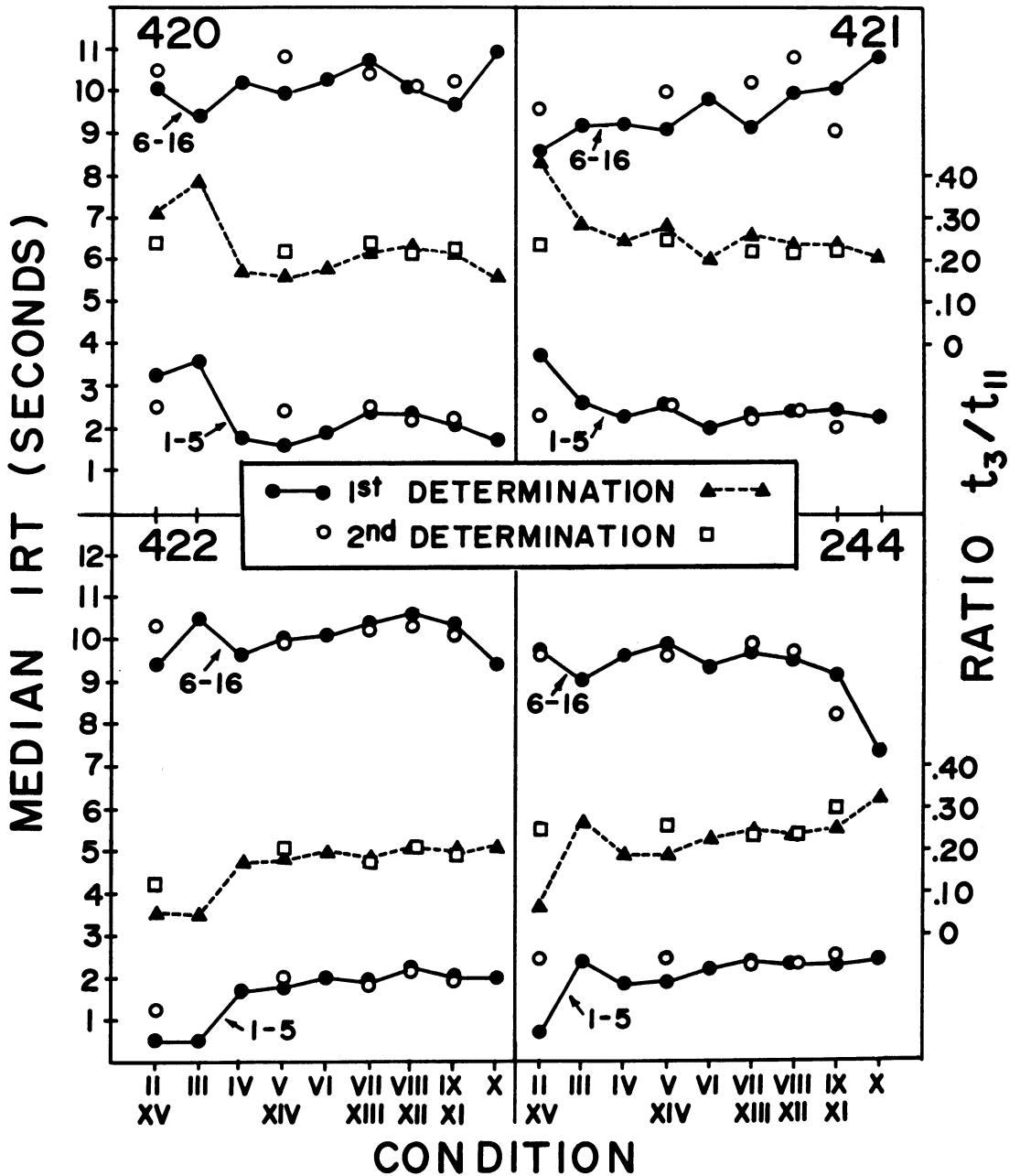


Fig. 3. Median interresponse times, estimated by linear interpolation from IRT distributions averaged over the last five days of each of the 14 concurrent reinforcement conditions, for Distribution 3 ( $0 < IRT < 6$  sec, bottom curves) and for Distribution 11 ( $6 < IRT < \infty$ , top curves) for the four pigeons. The dotted lines are the ratio of the medians,  $t_3/t_{11}$ , for each condition.

differences from the more familiar two-operandum choice experiment. The situations are similar in that two response classes appear to be involved in both cases, but different in that an increase in the rate of occurrence of one class necessarily reduces the rate of occurrence

of the other only in the spaced responding case. The differences are less striking than the similarities, however, since even when two operanda are involved, the overall response rate, taking both operanda together, tends usually to vary over a smaller range

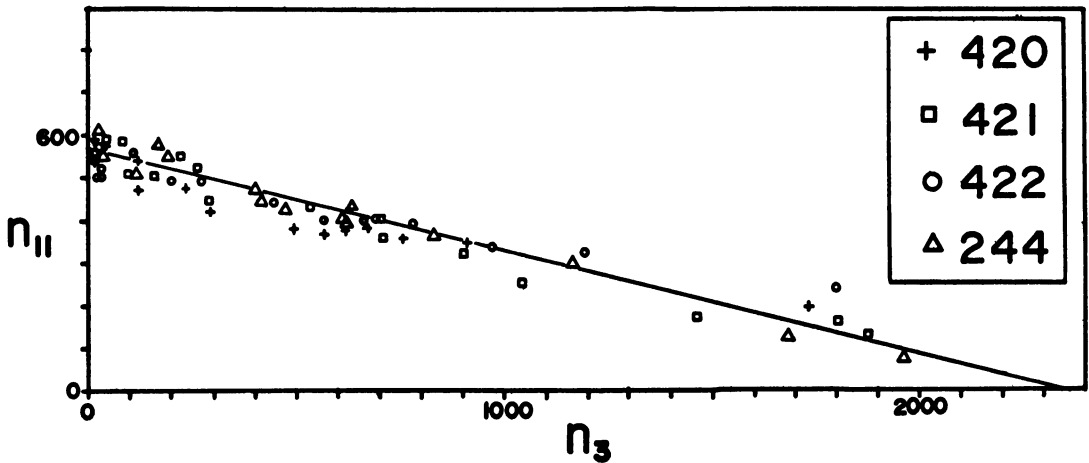


Fig. 4. Correlation plot of  $n_3$  (< 6-sec IRTs/100 min) vs.  $n_{11}$  (> 6-sec IRTs/100 min) for all birds and conditions. Each point is the mean of the last five days of a given condition. Line with negative slope is Eq. 1 in the text.

(as reinforcement rates are varied) than the rates on either operandum taken separately (cf. Catania, 1963; Herrnstein, 1964). A weak reciprocal relationship between the rate of occurrence of one response and the rate of the other can thus often be demonstrated, even in the two-operandum case. An analysis of these data in terms appropriate to a two-choice concurrent variable-interval experiment can therefore be justified both by the similarities between the two situations and, as will be indicated, by the simplicity of the functional relations that result.

The data upon which the following choice analysis is based appear in Table 1. Plots of obtained relative reinforcement rates,  $r_{11}/(r_{11} + r_3)$ , vs. relative response rates,  $n_{11}/(n_{11} + n_3)$ —a first implication of the choice paradigm—are not revealing here for two reasons. First, they are not linear, being curved towards the upper left corner of the unit probability square in all cases, a result quite different from the linear matching functions found by Herrnstein (1961) and others in concurrent variable-interval choice situations. Second, the range of reinforcement rates actually obtained was such that many points are clustered close to the ordinate. An alternative, which both spaces the data points more evenly and displays a simple relationship, is the plot of ratios of reinforcement ( $q \equiv r_{11}/r_3$ ) and response ( $s \equiv n_{11}/n_3$ ) rates shown in Fig. 5. Log-log coordinates, a natural form of representation for ratios, are used in the figure and the linear functions are least-squares lines through the

indicated points. Three comments can be made concerning these functions: (a) Variability of the points around the fitted lines is quite small and there is little difference between first and second determinations in this respect. (b) The equation of the mean line is  $s = 0.24q^{0.662}$ , suggesting that the obtained lines perhaps represent deviations around a "true" function with slope (exponent)  $2/3$ . (c) As suggested by the curvilinearity of the relative frequency plots, these functions differ from the unit-slope functions implied by Herrnstein's (1961) matching relationship.

In many free-operant experiments it has been customary to plot as independent variable scheduled maximum reinforcement rates, rather than the reinforcement rates actually obtained. Some theoretical issues raised by this distinction are discussed below, but to complement the functions using obtained reinforcement rates depicted in Fig. 5, analogous functions involving scheduled values are shown in Fig. 6. In Fig. 5 the abscissa is  $r_{11}/r_3$ , ( $q$ ); in Fig. 6 it is  $6000/r_v$ , ( $q^*$ ), where  $r_v$  is the scheduled maximum reinforcement rate for cell-3 IRTs (see Table 1). The numerator of  $q^*$  is taken as unity since the only non-arbitrary assumption that can reasonably be made about a scheduled maximum reinforcement rate for cell-11 IRTs (which are reinforced each time they occur) is that it is constant for a given animal. The linear functions in Fig. 6 are least-squares lines through the data points. Four points can be made about the indicated approximately



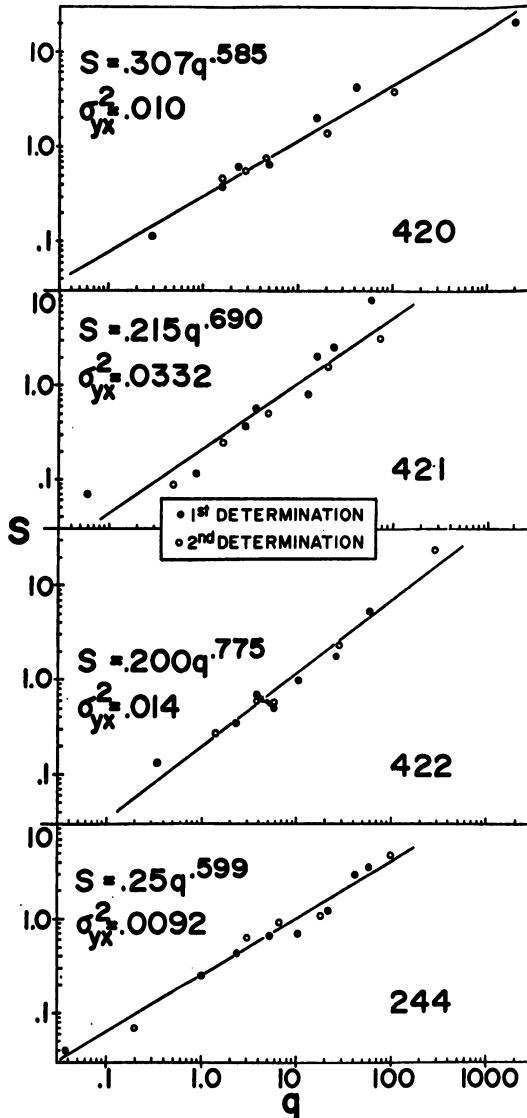


Fig. 5.  $s$  (ratio of response rates) vs.  $q$  (ratio of reinforcement rates) for the four birds (see Appendix for definitions). Points connected by an arrow represent reversals, i.e.,  $q_{n+1} > q_n$ , but  $s_{n+1} < s_n$ , or vice versa. Values of 0 and  $\infty$  are necessarily omitted. Lines are regression lines, with indicated equations and variances, fitted to the logarithms of the displayed points.

linear relationships: (a) Variability of the data points around the fitted lines is substantially greater than for the comparable plots in Fig. 5 in all but one case. Again, first and second determinations differ little in terms of fit to the linear functions. (b) The equation of the mean line is  $s = 13.27 q^{*1.00}$ , suggesting that the obtained lines represent deviations around a "true" function with unit

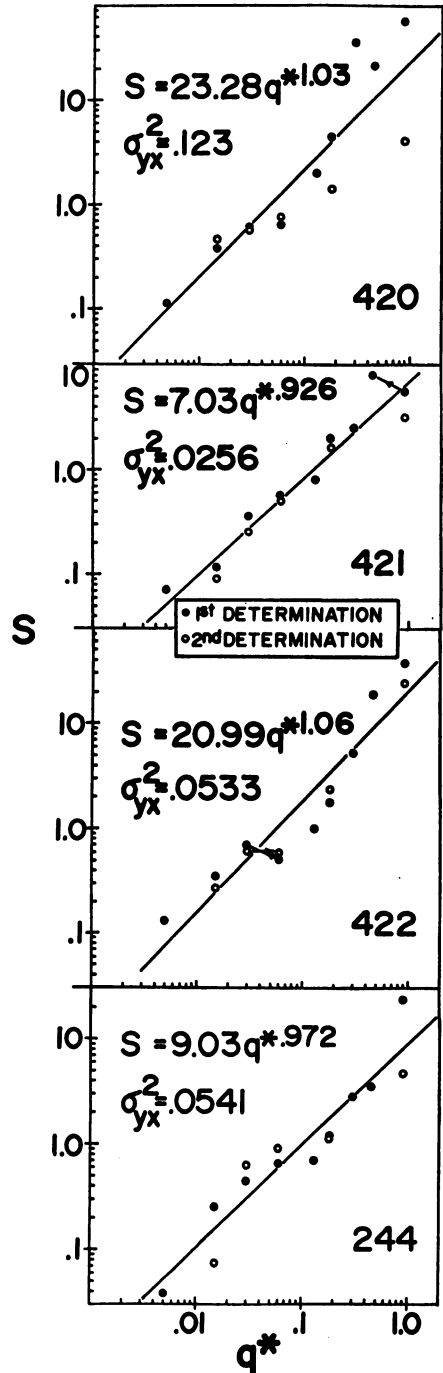


Fig. 6.  $s$  (ratio of response rates,  $n_{11}/n_3$ ) vs.  $q^*$  (VI value controlling cell-3 reinforcement/100) for the four birds (see Appendix for definitions). Other details as in Fig. 5.

slope, i.e., a linear function with zero intercept in linear coordinates. (c) Apart from the early conditions of the experiment (i.e., high values

of  $q^*$ ) the first and second determinations for a given condition are close, indicating quite good recoverability. (d) The unit slope of these functions is in agreement with the linear relationship implied by matching of relative response and reinforcement rates. The constant of proportionality here is considerably lower than would be suggested by any reasonable estimate of the maximum possible reinforcement rate for cell-11 IRTs, however. The lowest possible estimate of this rate is the actual rate of cell-11 reinforcement during Condition I. On the assumption of matching, this implies a constant in the neighborhood of 100 for most birds, since this is the order of magnitude of  $r_{11}$  during the first condition (see Table 1).

### DISCUSSION

In this experiment, only responses terminating interresponse times located in two narrow and widely separated bands were reinforced. The main finding is a power-law relation, with exponent approximately  $2/3$ , between two kinds of ratios:  $q$ , the ratio of obtained reinforcement rates for the two bands, and  $s$ , the ratio of response rates defined by the two interresponse-time subdistributions centered on those bands (Fig. 5).

This relationship holds between a pair of variables,  $s$  and  $q$ , that are both defined, *ex post facto*, by the behavior of the pigeons:  $q$  is a ratio of obtained, rather than scheduled, reinforcement rates, while  $s$  is a ratio of response rates wherein both numerator and denominator refer to IRT distributions around the reinforced IRT bands; *i.e.*,  $s$  depends both upon which bands are reinforced and upon the animals' ability accurately to locate their responses with respect to those bands. A definition of the response class in terms of just those responses which satisfy the schedule requirements (*i.e.*, IRT cells 3 and 11) yields much less orderly relationships than the definition actually used. This result, therefore, provides a measure of empirical support for arguments recently advanced in favor of a flexible, relativistic definition of the operant as a behavioral unit (Staddon, 1967).

In their experiments on concurrent spaced responding, Malott and Cumming (1966) rarely found bimodal IRT distributions. Moreover, the location of the mode in their

distributions was not fixed but varied in an undefined way with changes in the reinforcement contingencies for the two reinforced IRT bands. Both these results oppose the findings of this experiment; here most conditions yielded bimodal IRT distributions and the location of each mode varied little as a function of changes in reinforcement frequency. Two differences between the two experiments are relevant to these discrepancies. First, and probably most important, Malott and Cumming used probabilistic (*i.e.*, ratio) reinforcement contingencies for their two IRT bands. By analogy with the results of two-choice concurrent fixed-ratio experiments, one might expect probabilistic contingencies to yield fixation of choice exclusively on one or the other alternative (*cf.* Herrnstein, 1958; Catania, 1966). This result is in accordance with differences in the moment-to-moment contingencies under interval and ratio procedures: in any concurrent situation where one or more alternatives involves interval scheduling of reinforcement, the probability of reinforcement for responding to that alternative must increase monotonically to 1.0 so long as it remains unchosen. This self-correcting aspect of interval schedules made it likely that in this experiment, both alternatives would be responded to even when the interval schedule alternative (cell-3 IRTs) was reinforced only infrequently. Since there is no such self-correcting property to concurrent ratio schedules, the failure of Malott and Cumming to find bimodal IRT distributions becomes less surprising. A second difference between the two procedures concerns the discriminability of the two reinforced IRT bands. In this experiment the two bands were narrow and widely separated, hence easily discriminable; in many of the conditions of Malott and Cumming's experiment they were broad and/or adjacent, and hence hard to discriminate. For the latter conditions the choice paradigm is clearly inappropriate, since the two alternative response classes may not (and perhaps cannot) be well differentiated. The observation that the IRT distributions presented by Malott and Cumming showed the greatest lability of form and location when the reinforced intervals were adjacent accords with this view.

A second outcome of this experiment is the finding of an approximately linear rela-

tionship, with zero intercept, between  $s$ , the ratio of response rates, and  $q^*$ , the value of variable-interval schedule controlling reinforcement availability for 2- to 3-sec interresponse times. Given the assumption that one can treat the FR 1 contingency for cell 11 IRTs as equivalent to some short, but constant, interval contingency; then, but for a multiplicative constant,  $q^*$  is the equivalent for scheduled reinforcement rates of  $q$  (*i.e.*,  $r_{11}/r_3$ ) for obtained reinforcement rates. The finding of a power-law relationship, albeit with differing slopes, in both cases therefore raises questions concerning both the theoretical implications of a choice of  $q$  or  $q^*$  as independent variable, and of which is the more appropriate in terms of empirical adequacy.

Concerning the second question, the data are unambiguous: the fit of the data points to the  $s$  vs.  $q$  (*i.e.*, obtained values) curves is generally much better than to the comparable  $s$  vs.  $q^*$  functions. Moreover, in answer to the first question, acceptance of the  $s$  vs.  $q$  relationship as the more basic implies a simpler organization of behavior, although a more complex relation of that behavior to the experimental contingencies, than does acceptance of the relation between scheduled reinforcement rates and responding. This is especially true here where obtained reinforcement rate,  $r_3$ , often diverges widely from  $r_v$ , the maximum rate specified by the variable-interval schedule. To accept  $r_v$  as independent variable raises the further question of the process by which the animal "computes" this rate from the sequence of reinforcements actually obtained (which may occur at a different rate). This is a constancy problem, involving extraction of an invariance from a variable stimulus input, and, by comparison with other such problems, is by no means a difficult one. On the other hand, the existence of a simple relation between obtained values avoids the constancy issue, although it implies the existence of a rate averaging process. The choice, as far as interpretation is concerned, is between a constancy mechanism, and a process that computes average rates. Since the latter is the simpler (any conceivable constancy process here must include a rate averager in addition to performing other functions), acceptance of the  $s$  vs.  $q$  relation as the more fundamental is indicated by both empirical and theoretical considerations. On

the other hand, the  $s$  vs.  $q^*$  relation remains to be explained, although the foregoing arguments suggest that such an explanation is likely to be in terms of interactions between the reinforcement contingencies and whatever process is responsible for the  $s$  vs.  $q$  function.

Concerning the stability of the functional relationships found here, the data show good agreement, with no evidence of the kind of metastable relationship found in the previous experiment (Staddon, 1965). This stability was especially true of the fixed relationship between the medians of the IRT subdistributions and the location of the reinforced intervals, but was also true of the functional relations between response and reinforcement rates depicted in Fig. 5 and 6.

The fourth question raised at the outset concerns response bias; a possibility that has been recently discussed in connection with spaced responding procedures (*e.g.*, Malott and Cumming, 1966; Millenson, 1966; Shimp, 1967). The notion of bias implies an expectation concerning an animal's behavior in the absence of bias. The simplest such expectation is that animals will tend to maximize reinforcement frequency. In terms of this view, a bias in favor of short interresponse times exists, at least on simple spaced-responding schedules. For a given IRT distribution, the maximum reinforcement rate on spaced-responding schedules will usually occur when the mean of the IRT distribution is somewhat greater than the lower bound of the class of reinforced IRTs. Consequently, the finding (*e.g.*, Staddon, 1965) that pigeons tend to produce a mean IRT equal to the shortest reinforced IRT already indicates a tendency to respond too soon to maximize reinforcement frequency. The tendency of rats sometimes to locate their mean IRT well to the right of the shortest reinforced IRT (*cf.* Farmer and Schoenfeld, 1964), in conjunction with their apparent ability to adjust to longer spaced responding requirements than pigeons (Staddon, 1965), therefore suggests less biased responding in this species.

Evaluation of the response-bias hypothesis in the concurrent spaced-responding situation is complicated by two versions of the hypothesis of reinforcement-frequency maximization. The simpler, which can be termed *overall maximizing*, asserts that pigeons should respond exclusively to that alternative (Distri-

bution 3 or Distribution 11 in this experiment) where IRT variability and contingencies of reinforcement combine to yield the highest reinforcement rate. With the sole exception of Condition X (and possibly Condition IX) of this experiment, this view would suggest exclusive responding to the cell-11 contingency, and thus is quite false here. More generally applicable, perhaps, is the *momentary maximizing* principle discussed by Shimp (1966), which asserts that in a two-choice situation the pigeon responds to that alternative which has the highest momentary probability of reinforcement. In the concurrent variable-interval, fixed-ratio situation of the present experiment, that would mean responding to the ratio (Distribution 11) alternative until the rising probability of reinforcement for a Distribution-3 response equalled or exceeded the approximately constant reinforcement for a Distribution-11 response, and then switching to a Distribution-3 response. A cell-3 response, in turn, sets the momentary probability of reinforcement for that alternative to 0 and thus should be followed by a return to Distribution-11 responding, and so on. The derivation of predictions from this model is tedious, but it is possible, given certain rather severe simplifying assumptions, to derive a linear relationship resembling the functions of Fig. 6 on this basis. This result constitutes something less than strong support for the momentary maximizing view, however, both because the sequential data necessary for full confirmation are lacking here, and because the  $s$  vs.  $q^*$  relationship of Fig. 6, which is within the domain of the momentary maximizing view, is less convincing than the  $s$  vs.  $q$  relationship of Fig. 5, which is not. All that can be concluded, then, is that insofar as responding predicted by momentary maximizing is unbiased, the responding in this situation may be unbiased.

In sum, although intuitively reasonable, the bias notion does not seem to be helpful in understanding the birds' behavior in this experiment. Analysis in terms of an optimal strategy, such as momentary maximizing, while more promising than undefined statements about bias, is difficult here and to a large extent misses the most striking regularities.

It has already been pointed out that the power-function relationship between ratios of obtained reinforcement and response rates

here cannot be derived from simple matching of the sort found by Herrnstein (1961) in the two-choice concurrent variable-interval experiment. An approximation to the relationship of Fig. 5 can, however, be derived by analogy with the results of the concurrent-chain experiment of Herrnstein (1964). In the latter experiment, pigeons pecked either of two response keys, each transilluminated with white light. Reinforcement for responding on either key was controlled by independent but identical variable-interval schedules. Pecking the left white key changed the color of the key. In the presence of this new key-color, the other (right) key became inoperative and two food reinforcements (for pecks on the illuminated, colored key) occurred according to an independently determined schedule. After the second food reinforcement was delivered, both keys were once again illuminated with white light and the concurrent variable-interval contingencies reinstated. A similar sequence of events followed a reinforced response on the other white key. The independent variables were the obtained rates of reinforcement in the presence of the two individually-presented colored keys. The dependent variables were the rates of responding to the concurrently presented white keys. The similarity between the concurrent chain situation and the concurrent spaced-responding situation of this experiment is apparent at the following points: (a) There are two mutually exclusive response classes in both cases; pecks on the right and left white keys in Herrnstein's experiment, here the birds' emission of either a long (Distribution 11) or short (Distribution 3) IRT. An index of the relative frequency of these two choices (which are assumed to occur just after each response) is of course the relative frequency of IRTs in the two distributions ( $n_3$  and  $n_{11}$ ). (b) Following the choice which terminates in a Distribution-3 or Distribution-11 response the other choice is unavailable. The amount of time taken up by Distributions 3 and 11 is therefore analogous to the relative durations of the left-colored and right-colored stimuli, respectively, in the concurrent chain experiment. These durations may be estimated by  $n_3 t_3^-$  and  $n_{11} t_{11}^-$  here. (c) The independent variables in Herrnstein's experiment were the obtained rates of reinforcement in the presence of the two colored keys; the analogous values here, taking the

time denominators from (b), are  $r_3/n_3t_3$  and  $r_{11}/n_{11}t_{11}$ . (d) The dependent variables in Herrnstein's experiment were the rates of responding on the two white keys. The choice assumption of (a) identifies  $n_3$  and  $n_{11}$  as dependent variables here analogous to those choice frequencies.

Herrnstein found that relative rates of responding in white matched relative rates of reinforcement in the colors; thus ratios of reinforcement rates should match ratios of response rates. In this situation, with the identifications already given we have, therefore:

$$\frac{n_{11}}{n_3} = \frac{\frac{r_{11}}{n_{11}t_{11}}}{\frac{r_3}{n_3t_3}} = \frac{r_{11}}{r_3} \cdot \frac{n_3}{n_{11}} \cdot \frac{t_3}{t_{11}}$$

which yields

$$\left(\frac{n_{11}}{n_3}\right)^2 = \frac{r_{11}t_3}{r_3t_{11}}$$

or

$$s = \left(\frac{t_3}{t_{11}} \cdot q\right)^{1/2} \tag{2}$$

which is a power function similar to the relationship depicted in Fig. 5. With values of 1.5 and 9.5 for  $t_3$  and  $t_{11}$  respectively<sup>2</sup> this prediction,  $s = 0.4q^{0.5}$ , can be compared with the mean empirical function,  $s = 0.24q^{0.66}$ .

Two obvious questions are raised by this similarity between Eq. 2 and the power functions of Fig. 5. The first concerns the quite substantial difference in exponents between the theoretical (exponent = 0.5) and obtained (exponent = 0.66) functions. To this discrepancy I have no ready answer, except to draw attention to the number of assumptions necessary both to derive the theoretical prediction (it may after all be inappropriate here) and, especially, to obtain the empirical functions; such problems as the rather arbitrary partition imposed on the IRT distribution in order to obtain values for  $n_3$  and  $n_{11}$ , the qualitative difference in reinforcement contingencies for Distribution-3 and Distribution-

11 IRTs and so on, come readily to mind. A second, and perhaps more pertinent question, therefore, is raised by the similarities between found and predicted functions, *i.e.*, the fact that they are both power functions. This question concerns both the variables which are relevant to a fundamental understanding of concurrent choice situations, and the form of relation to be expected between those variables. On the basis of these results two kinds of answer to these questions can be distinguished:

- 1(a) Relative frequencies (probabilities) are the relevant variables in concurrent choice experiments and the expected form of relation is linear.
- (b) Over a group of animals the average of these linear functions will approximate the matching line.
- 2(a) Ratios of response and reinforcement rates are the relevant variables and the expected form of relation is a power function.
- (b) Although in many situations the average over a group of animals may be a power function of unit slope and unit constant of proportionality (matching), this is not true of all concurrent experiments.

The data of Fig. 5 fit the associated power functions at least as well as the relative frequency data of Herrnstein (1964) fit linear functions. Thus, the data make it hard to sustain the notion that this situation is some kind of poor approximation to the concurrent chain situation. The difference between the found exponent, and the exponent predicted by analogy with the concurrent chain situation cannot, therefore, be explained in this way. As far as the present results are concerned, empirical adequacy does not distinguish between the two alternative interpretations of choice data outlined above. Indeed, a firm decision on the relative validity of these two approaches cannot be made with the data presently available. However, since matching of relative frequencies can be regarded as a special case of a power-law relation between ratios, the present results make at least a *prima facie* case for the greater generality of the power-function relationship, and thus for the primacy of ratios over probabilities.

<sup>2</sup> $t_3$  and  $t_{11}$  are used here to compute the effective reinforcement rates associated with the Distribution-3 and Distribution-11 distributions respectively. Since a 1-sec S<sup>A</sup> followed each and every response, this time can reasonably be excluded from any reinforcement rate computation. For this reason, and because the constant thereby produced is closer to what is actually obtained, 1 sec has been deducted from the values of  $t_3$  and  $t_{11}$  estimated in Fig. 4.

APPENDIX:  
SYMBOLS AND DEFINITIONS

Distribution 3, Distribution 11: distributions of IRTs under the control of the reinforcement contingencies for IRTs between 2 and 3 sec and between 10 and 11 sec, respectively.

$t_3$ ,  $t_3$ : median and mean, respectively of Distribution 3, estimated from IRTs in the interval  $0 < \text{IRT} < 6$  sec.

$t_{11}$ ,  $t_{11}$ : median and mean respectively of Distribution 11; estimated from IRTs in the interval  $6 < \text{IRT} < \infty$ .

$n_3$ : rate (per 100 min) of production of Distribution-3 IRTs, estimated by IRTs/100 min in the interval  $0 < \text{IRT} < 6$  sec.

$n_{11}$ : rate (per 100 min) of production of Distribution-11 IRTs, estimated by IRTs/100 min in the interval  $6 < \text{IRT} < \infty$ .

$r_3$ ,  $r_{11}$ : obtained reinforcement rates (per 100 min) for IRTs in cells 3 and 11 respectively.

$s \equiv n_{11}/n_3$ ;  $q \equiv r_{11}/r_3$

$r_v$ : scheduled maximum reinforcement rate per second) for cell-3 IRTs (*i.e.*,  $1/(60 \times \text{VI value})$ ).

$q^* \equiv 6000/r_v$ , *i.e.*,  $1/(\text{scheduled maximum reinforcement rate per 100 min})$ .

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